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Mapping Dirac gaugino masses

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Abstract: We investigate the mapping of Dirac gaugino masses through regions of strong coupling, focussing on SQCD with an adjoint. These models have a well-known Kutasov duality, under which a weakly coupled electric UV description can flow to a different weakly coupled magnetic IR description. We provide evidence to show that Dirac gaugino mass terms map as

$$\lim_{\mu\to\infty} \frac{m_D}{g\kappa^{\frac{1}{k+1}}} = \lim_{\mu\to 0} \frac{\tilde{m}_D}{\tilde{g}\tilde{\kappa}^{\frac{1}{k+1}}}$$

under such a flow, where the coupling κ appears in the superpotential of the canonically normalised theory as $W \supset \kappa X^{k+1}$. This combination is an RG-invariant to all orders in perturbation theory, but establishing the mapping in its entirety is not straightforward because Dirac masses are not the spurions of holomorphic couplings in the $\mathcal{N}=1$ theory. To circumvent this, we first demonstrate that deforming the Kutasov theory can make it flow to an $\mathcal{N}=2$ theory with parametrically small $\mathcal{N}=1$ deformations. Using harmonic superspace techniques we then show that the $\mathcal{N}=1$ deformations can be recovered from electric and magnetic FI-terms that break $\mathcal{N}=2\to\mathcal{N}=1$, and also show that pure Dirac mass terms can be induced by the same mechanism. We then find that the proposed RG-invariant is indeed preserved under $\mathcal{N}=2$ duality, and thence along the flow to the dual $\mathcal{N}=1$ Kutasov theories. Possible phenomenological applications are discussed.

Keywords: Dirac gauginos, Seiberg duality, Kutasov duality, S-duality, harmonic superspace, mapping, SUSY breaking, supersymmetry

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Contents

1 Introduction and Summary

There is continuing interest in the role that Dirac gauginos may play in supersymmetry due to their possible physical advantages over the Majorana variety [1–33]. Most of this work however considers Dirac gaugino masses in a perturbative setting: supersymmetry is broken at some high scale, and this leads to mass terms that can be calculated within the perturbation theory of the low energy effective theory. This is true even if, as in [23], the adjoint fermions that partner the gauginos in the Dirac mass term (part of the so-called ESP supermultiplet) are themselves the mesinos of some strongly coupled $\mathcal{N} = 1$ gauge theory. Likewise, within supersymmetric Randall-Sundrum set-ups, Dirac mass terms can appear when the gaugino zero-modes of bulk gauge fields marry with the lowest lying KK modes [34–36]. In either case the gauge symmetry of interest is just a flavour symmetry of the strongly coupled physics. Moreover if Dirac mass terms do originate from operators of an ultra-violet (UV) theory that becomes strongly coupled, they are trivial to map to the infra-red (IR) due to holomorphy, being given simply by a combination of SUSY breaking, transmutation and fundamental scales. For example in SQCD the adjoint field ψ_X can be a mesino mapped (upto an unknowable normalisation factor) as $\psi_X \sim \Lambda^{-1}\tilde{Q}.\psi_Q$, where Q indicate (s)quarks of the confining UV theory. Then the effective Dirac mass term coupling this state to a flavour gaugino would arise from the non-renormalizable operator

$$W \supset \frac{\tilde{Q}.Q\, W^\alpha W'_\alpha}{M^2}, \tag{1.1}$$

where $\langle W'_\alpha \rangle \sim \theta_\alpha D$ is a supersymmetry breaking spurion D-term, and M is some fundamental scale. Obviously $m_D \sim \Lambda D/M^2$ is about all one can say in this case.

A more interesting question is what happens to Dirac mass terms involving the gauginos of the colour gauge symmetry that becomes strongly coupled. Can such terms be mapped from UV to IR and if so how do they appear in the IR physics? Conversely, can Dirac mass terms in the IR be mapped from operators in the UV? To make the question precise, we will focus on the $\mathcal{N} = 1$ adjoint+QCD duality of ref. [37] which we refer to as SQCD+X (occasionally as Kutasov) duality. These models and many variants were analysed in refs.[38–40], and phenomenological applications have been suggested in many works. For our current purposes SQCD+X is precisely the context in which the mapping of Dirac gaugino masses becomes important. In particular in the so-called free-magnetic phase, an asymptotically free electric $SU(N_c)$ theory with N_f flavours of quarks and a chiral adjoint X with superpotential $W \supset \kappa X^{k+1}$, flows to an IR-free $SU(n)$ theory with N_f flavours of magnetic quark and a chiral adjoint, x with superpotential $W \supset \kappa x^{k+1}$. The question is how would a Dirac mass in the electric theory manifest itself the IR magnetic theory?

Various techniques have been developed to map soft-terms in $\mathcal{N} = 1$ SUSY [41–52]. It is well known for example, that one can recover the RG flow of a Majorana gaugino mass by expressing it as a spurion contribution to the holomorphic gauge coupling [45, 46, 48, 51]

$$\mathcal{L} \supset \int d^2\theta\, S\,\mathcal{W}^2 + \text{h.c.}, \qquad S = \frac{1}{2g_S^2} - \frac{i\Theta}{16\pi^2} + \theta^2 \frac{m_{\lambda S}}{g_S^2}, \tag{1.2}$$

where the physical gauge coupling and masses are functions of $S + S^\dagger$ (and real normalisation superfields $\mathcal{Z}$). The fact that one can construct a holomorphic RG invariant

$$\Lambda_S = \mu \exp\left(-\frac{16\pi^2 S(\mu)}{b}\right) \tag{1.3}$$

where $b = 3t_G - \sum_r t_r$ is the usual beta function coefficient, shows that the quantity

$$\frac{m_{\lambda S}}{g_S^2} = -\frac{b}{16\pi^2} \left[\ln \Lambda_S\right]_\theta^2 \tag{1.4}$$

is preserved. In other words the gaugino mass can be understood perturbatively as arising from F-terms in the threshold contributions to the one-loop beta function, but because it is related to a holomorphic invariant of the RG-flow, one can argue that this ratio is also mapped through any non-perturbative regions of strong coupling. In particular, when an asymptotically free theory flows to an IR-free magnetic description we can deduce,

$$\lim_{\mu\to\infty} \frac{m_{\lambda S}}{g_S^2} = \lim_{\mu\to 0} \frac{\tilde{m}_{\tilde{\lambda} S}}{\tilde{g}_S^2}$$

where tilde's represent the quantities in the dual description. Similar treatments are possible for the squark masses by constructing invariants involving the field renormalisation superfields $\mathcal{Z}(\mu)$. An alternative method is that in [52] where the Majorana mass is related through the ABJ anomaly to the anomalous trace current, which is in turn related to the R-current. If the latter is broken only by the gaugino masses themselves, one obtains a mapping up to corrections suppressed by factors of m_λ^2/Λ^2.

Unfortunately similar techniques are not instantly available for Dirac masses. The operator that would generate the Dirac mass is

$$W \supset \frac{X W^\alpha W'_\alpha}{M}, \tag{1.5}$$

where the effective Dirac mass is $m_D = D/M$. Unlike the gauge coupling, the non-spurion part of this non-renormalizable term is not one that was in the theory before we required it for the Dirac mass. Likewise there is no equivalent to the conserved R-current technique of [52].

However, because the SUSY breaking is supersoft, one can establish the lack of anything other than field and one-loop gauge coupling renormalisation to all orders in perturbation theory, which implies that

$$\frac{\beta_{m_D}}{m_D} = \frac{\gamma_X}{2} + \frac{\beta_g}{g}\,, \tag{1.6}$$

where γ_X is the anomalous dimension of the adjoint (ESP) field X [4, 31, 53, 54][1]. If the theory contains a superpotential term $W \supset \kappa x^{k+1}$, then we can always trade γ_X for β_κ. By definition (and non-renormalization) we have $\kappa^{-1}\beta_\kappa = \frac{k+1}{2}\gamma_X$. eq. 1.6 can then be solved to give an RG invariant, $m_D/g\kappa^{\frac{1}{k+1}}$. Therefore it seems reasonable to suppose that Dirac masses in an asymptotically-free UV SQCD+X theory are mapped directly to Dirac masses in an IR-free SQCD+x theory as

$$\lim_{\mu\to\infty} \frac{m_D}{g\kappa^{\frac{1}{k+1}}} = \lim_{\mu\to 0} \frac{\tilde{m}_D}{\tilde{g}\tilde{\kappa}^{\frac{1}{k+1}}}. \tag{1.7}$$

The purpose of the present paper is to establish this map. As we mentioned above, in SQCD+X there is no RG-invariant that can be built from the couplings of the $\mathcal{N}=1$ theory which yields the Dirac mass as a spurion. Therefore the mapping cannot be done directly. However within $\mathcal{N}=2$ theories it *is* possible to map Dirac masses, as discussed in [51]. There X becomes part of the $\mathcal{N}=2$ gauge supermultiplet, $\mathcal{A}$, with the Yang Mills lagrangian arising from the canonical prepotential $\mathcal{L} \supset \int d^2\theta_1 d^2\theta_2 \Sigma \mathcal{A}^2$ where the indices label the two thetas of $\mathcal{N}=2$ in some basis. Both the Dirac and Majorana gaugino masses can be generated from spurions in the chiral $\mathcal{N}=2$ superfield Σ, out of which an RG-invariant *can* be constructed.

Our task therefore is to extend this mapping to the $\mathcal{N}=1$ SQCD+X theory. This is a much more difficult proposition than it might at first seem, because of the so-called *2 into 1 won't go* theorem of [55, 56]. Ideally one would like to first deform the $\mathcal{N}=2$ theory to $\mathcal{N}=1$ SQCD+X, and then add a second deformation for the Dirac (and Majorana) masses. But the theorem of [55, 56] greatly restricts the form that breaking of $\mathcal{N}=2$ to $\mathcal{N}=1$ can take: essentially it has to be driven through a combination of electric and magnetic Fayet-Iliopoulos (FI) terms as shown by Antoniadis, Taylor and Partouche (ATP) [57–59] in a mechanism inspired by [60, 61]. In particular the gauge coupling between the quarks and the adjoint field, which in the $\mathcal{N}=1$ language is $\mathcal{L} \supset \tilde{Q}XQ$ (and which is not present in SQCD+X), cannot be removed by FI-terms. It can at best be made inconsequential by generating a holomorphic mass for X and reducing the theory to simple $\mathcal{N}=1$ SQCD – without the X.

We are therefore forced to proceed by the following circuitous route. We consider the $N_f = 2N_c$ version of the $\mathcal{N}=1$ $SU(N_c)$ SQCD+X theory. As well as the necessary $W \supset \kappa X^{k+1}$ operator, the theory is deformed with the operator $h\tilde{Q}XQ$ – where $h \ll g$ is parametrically small. It turns out that this theory flows to the $\mathcal{N}=2$ fixed line in the IR, where $h \to g$ and $\kappa \to 0$. Therefore we arrive at an $\mathcal{N}=2$ theory deformed by an operator $W \supset \kappa X^{k+1}$ where now κ is parametrically small. It also turns out that the magnetic description of the original deformed SQCD+X theory flows to the dual $\mathcal{N}=2$ theory: in fact we will find that it is the h coupling which induces the necessary higgsing in the dual description. Once we have seen how to flow to $\mathcal{N}=2$ duality with parametrically small $\mathcal{N}=1$ deformations, we next establish that those deformations can be

[1]We define $\gamma_X = -\partial \ln Z_X/\partial t$ so that $\dim(X) = 1 + \gamma_X/2$ – hence the factor of 1/2 compared to these references.

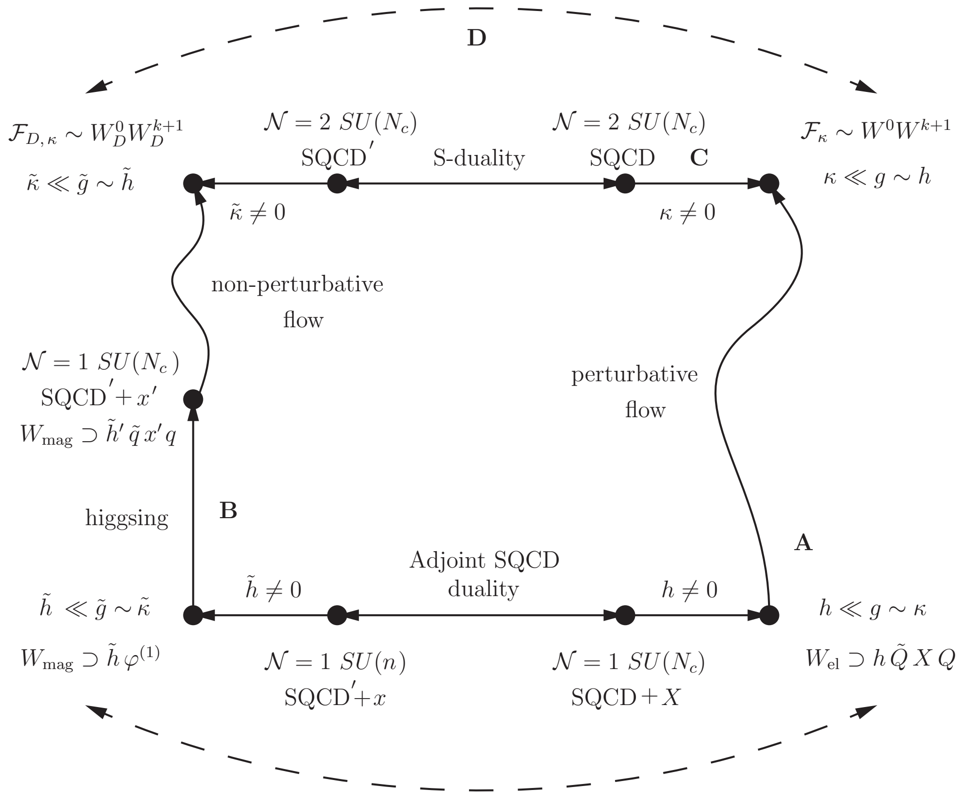


Figure 1. *The flow between $\mathcal{N}=2$ S-duality and $\mathcal{N}=1$ SQCD+X duality.* ***A****: The duality of [37] is deformed with a parametrically small $\mathcal{N}=2$ quark gauge interaction. The resulting perturbative flow to $\mathcal{N}=2$ SQCD is analysed in section 2.1.* ***B****: The the magnetic dual of the $\mathcal{N}=2$ quark gauge interaction is observed to Higgs the magnetic theory down to a gauge group of the same rank as the electric theory. This theory then flows to $\mathcal{N}=2$ SQCD$'$, as discussed in section 2.2.* ***C****: The electric theory of [37] is now written as an $\mathcal{N}=2$ theory broken to $\mathcal{N}=1$ at low energies by electric and magnetic FI terms, as discussed in section 3.3.2.* ***D****: By considering the Eguchi-Yang equations [62], the existence of a small dual x^{k+1} deformation is shown to be required in the presence of a small electric X^{k+1} deformation.*

generated by electric and magnetic FI-terms in an $\mathcal{N}=2$ theory with an appropriate prepotential (unfortunately necessitating the paraphernalia of harmonic superspace). We thus complete a route that allows us to go from an electric $\mathcal{N}=1$ $SU(N_c)$ SQCD+X theory to its magnetic dual via an intermediate pair of $\mathcal{N}=2$ duals. Dirac masses can now be added into the theory by further FI-deformations but now they *can* be mapped directly across the $\mathcal{N}=2$ duality, and then tracked down the dual RG-trajectories to the dual SQCD+X theories using eq. 1.6. A schematic of the overall picture (before adding soft terms) is shown in figure 1. The conclusion is that the proposed mapping in eq. 1.7 seems to be correct.

2 From $\mathcal{N}=1$ SQCD+X to $\mathcal{N}=2$ duality

The programme outlined above naturally splits into two parts. The first – the subject of this Section – is to understand the RG flow from dual $\mathcal{N}=1$ SQCD+X theories to dual $\mathcal{N}=2$ theories. The second part is to investigate the induction in the latter of $\mathcal{N}=1$ deformations and Dirac masses through FI terms, and also to determine explicitly how they map. As mentioned this requires some harmonic superspace technology, so it is postponed to the following Section and Section 4.

2.1 Perturbative flow to $\mathcal{N}=2$ SQCD

The electric theory is $\mathcal{N}=1$ SQCD+X duality of ref.[37] deformed by an additonal $\tilde{Q}XQ$ coupling:

$$W_{\rm el} = h\,\tilde{Q}XQ + \frac{\kappa}{k+1}\,{\rm tr}_G\, X^{k+1}, \tag{2.1}$$

where X is the chiral adjoint field of the $SU(N_c)$ gauge group. The content and global symmetries of the $\mathcal{N}=1$ model with no superpotential are given in table 1.

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	$\frac{1}{N_c}$	$1-R_X\frac{N_c}{N_f}$
$\tilde{Q}$	$\bar{\square}$	$\mathbf{1}$	$\bar{\square}$	$-\frac{1}{N_c}$	$1-R_X\frac{N_c}{N_f}$
X	**Ad**	$\mathbf{1}$	$\mathbf{1}$	0	R_X

Table 1. *The matter content of the electric SQCD+X model. All the flavour charges are anomaly-free with respect to the gauge symmetry. In the* $W\sim X^{k+1}$ *SQCD+X* $R_X=\frac{2}{k+1}$.

The $h=g$ and $\kappa=0$ model corresponds to $\mathcal{N}=2$ SQCD, while $h=0$ corresponds to the pure SQCD+X model of [37]. SQCD+X has a conformal window for

$$\frac{1}{k-\frac{1}{2}}N_c < N_f < 2N_c, \tag{2.2}$$

and is in the free magnetic phase for

$$\frac{1}{k}(N_c+1) < N_f \le \frac{1}{k-\frac{1}{2}}N_c. \tag{2.3}$$

In the present context we are envisaging flowing from this theory to the $\mathcal{N}=2$ theory with small κ induced by an FI term. Therefore we are interested in the influence of the operator h, and anticipate that the RG flow will be dominated by either h or κ in different regions of the flow. Defining the dimensionless coupling $\eta_\kappa = \kappa\mu^{k-2}$, the supersymmetric RG equations are given by

$$
\begin{aligned}
\frac{dg^2}{dt} &= 2g\beta_g\,, \ \frac{dh^2}{dt} = h^2(\gamma_X + 2\gamma_Q)\,, \ \frac{d\eta_\kappa^2}{dt} = \eta_\kappa^2\left[(k+1)(\gamma_X+2)-6\right], \\
\gamma_Q = \gamma_{\tilde{Q}} &= \frac{1}{4\pi^2}C_2(\Box)\left(h^2 - g^2\right), \\
\gamma_X &= \frac{1}{4\pi^2}\left(N_f T(\Box)\, h^2 + \delta_{k,2}(4C_2(\Box) - \frac{3}{2}T(\mathbf{Ad})\,\eta_\kappa^2 - C_2(\mathbf{Ad})\,g^2\right), \\
\beta_g &= -\frac{g^3}{16\pi^2}\frac{(3C_2(\mathbf{Ad}) - 2N_f T(\Box)(1-\gamma_Q) - T(\mathbf{Ad})(1-\gamma_X))}{(1-N_c\alpha/2\pi)}, \\
C_2(\Box) &= \frac{(N_c^2-1)}{2N_c}, \qquad C_2(\mathbf{Ad}) \,=\, T(\mathbf{Ad}) = N_c, \qquad T(\Box) = \frac{1}{2},
\end{aligned}
\tag{2.4}
$$

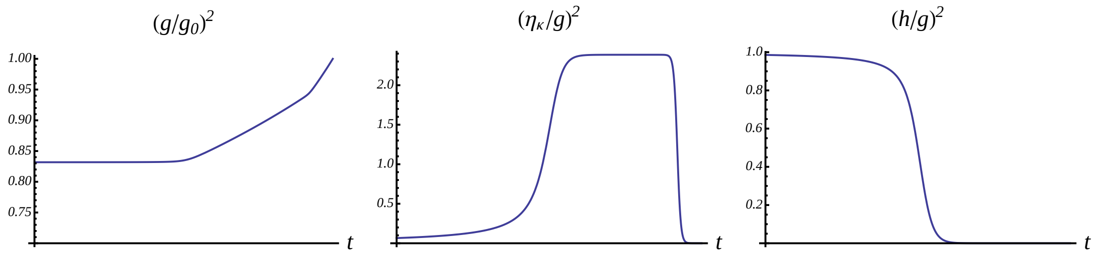
where the first line and β_g (the NSVZ beta function) are all orders, and the anomalous dimensions are leading order. Note that there is a η_κ^2 contribution to γ_X at one-loop order for $k=2$, but at two loop order for $k=3$ and successively higher order for higher values of k.

At the BZ fixed point where h is marginal, $2R_Q + R_X = 2$ so that $R(\tilde{Q}XQ) = 2$ as required. More generally, the vanishing of the NSVZ β-function agrees with the R_Q-charges shown in table 1 which are determined from absence of mixed $SU(N_c)^2 \times U(1)_R$ anomalies. Unless $N_f = 2N_c$ precisely, the values of R_X consistent with the h or κ coupling are $R_X = 0$ and $R_X = 2/(k+1)$ respectively, so that h breaks the R symmetry of SQCD+X: therefore if $N_f \neq 2N_c$ there can be no fixed point behaviour unless either h or k are zero. If and only if $N_f = 2N_c$, one can find fixed point solutions of the RGEs for any k with non-zero h and η_κ. They are at $\gamma_Q = (-2+k)/(1+k)$ and $\gamma_X = (4-2k)/(1+k)$ which (using $\gamma = 3R-2$) correspond to the required values for the superconformal R-symmetry at $N_f = 2N_c$, namely

$$
R_Q = 1 - \frac{1}{k+1}, \qquad R_X = \frac{2}{k+1}. \tag{2.5}
$$

But because h, κ preserve precisely the same R-symmetry with R-charges completely constrained, the a-theorem [63–65] now tells us that SQCD+X at a fixed-point cannot flow to the $\mathcal{N}=2$ fixed line (otherwise the flow would occur without any decrease in a). We conclude that it is impossible for any N_f and N_c. However it *is* possible to flow from SQCD+X with no superpotential via a slowly running Kutasov theory with non-zero κ to the $\mathcal{N}=2$ fixed line. We can see this explicitly in the $k=2$ case since that theory is weakly coupled and can be solved numerically.

Specifically, starting in the UV with a weakly coupled theory with η_κ and h arbitrarily small but $h \ll \eta_\kappa$, one finds that the theory first flows to $\gamma_X = 0$ and $\eta_\kappa^2/g^2 = \frac{2C_2(\mathbf{Ad})}{8C_2(\Box)-3T(\mathbf{Ad})}$, with g experiencing two-loop running. Eventually, h turns on and the theory flows to the $\mathcal{N}=2$ theory with η_κ flowing to zero. A numerically solved example is shown below, with the horizontal axis being proportional to t:



2.2 Higgsing in the dual theory and flow to $\mathcal{N}=2$ SQCD′

Now consider the same flow from the point of view of the magnetic description which must of course yield identical results. In particular while the dual of the $\mathcal{N}=2$ theory is also an $SU(N_c)$ gauge theory, the dual of SQCD+X is an $SU(n)=SU(kN_f-N_c)$ gauge theory: i.e. for $k=2$ it is an $SU(3N_c)$ theory. One finds that it is the growing h coupling which induces the necessary breaking $SU(n)\to SU(N_c)$ in the IR of the magnetic dual description.

Let us see this in detail. The spectrum of the magnetic SQCD+X theory has mesons denoted m_j identified as;

$$m^{(j)}=\tilde{Q}X^{j-1}Q, \qquad j=1\dots k, \tag{2.6}$$

with canonically normalized fields $\varphi^{(j)}\sim\Lambda^{-j}m^{(j)}$. The field content of the magnetic theory is q, $\tilde{q}$, $\varphi^{(j)}$ and x, where x is an adjoint of the $SU(n)=SU(kN_f-N_c)$ magnetic gauge group, as summarised in Table 2.

	$SU(n)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
q	$\square$	$\bar{\square}$	$\mathbf{1}$	$\frac{1}{n}$	$1-R_x\frac{n}{N_f}$
$\tilde{q}$	$\bar{\square}$	$\mathbf{1}$	$\square$	$-\frac{1}{n}$	$1-R_x\frac{n}{N_f}$
x	$\mathbf{Ad}$	$\mathbf{1}$	$\mathbf{1}$	0	R_x
$\varphi^{(j)}$	$\mathbf{1}$	$\square$	$\bar{\square}$	0	$2-2\,R_x\,\frac{N_c}{N_f}-R_x(j-1)$

Table 2. *The matter content of the magnetic theory in the dual SQCD+X model;* $n=kN_f-N_c$.

In general the magnetic superpotential is

$$W_{\rm mag}=h\,\varphi^{(2)}_{nm}\delta_{nm}+\frac{\tilde{\kappa}}{k+1}\,{\rm tr}_G(x^{k+1})+\sum_{j=1}^{k}\tilde{c}_j\,\varphi^{(j)}_{nm}\,{\rm tr}(\tilde{q}_m x^{k-j}q_n) \tag{2.7}$$

where n,m are flavour indices, and for the special case $k=2$ we have (dropping the indices)

$$W_{\rm mag}=h\varphi^{(2)}+\frac{\tilde{\kappa}}{3}x^3+\left(\tilde{c}_1\varphi^{(1)}\tilde{q}xq+\tilde{c}_2\varphi^{(2)}\tilde{q}q\right). \tag{2.8}$$

Not surprisingly the SU($3N_c$) dual theory is at the strongly coupled boundary of the conformal window,

$$\frac{1}{k-\frac{1}{2}}\,n\,=2N_c=N_f. \tag{2.9}$$

Because the anomalies must match, the a-parameters of the electric and magnetic theories also match at the endpoints of the flow, so thanks to the a-theorem, even though we cannot numerically solve the RG equations in the dual description, we know that it too flows to the $\mathcal{N}=2$ theory in the IR. Therefore in the UV we expect a strongly coupled theory with $\tilde{h} \propto \tilde{c}_1 = \tilde{\kappa} = 0$, which we expect to flow to an intermediate strongly coupled $SU(3N_c)$ SQCD+X theory with adjoint coupling $\tilde{h}=0$, and thence to an $SU(N_c)$ theory with adjoint coupling $\tilde{h}=\tilde{g}$.

Indeed the $\varphi^{(2)}$ eq. of motion sets

$$\tilde{c}^{(2)}\tilde{q}q = -h. \tag{2.10}$$

These equations have rank $N_f = 2N_c$ and thus, once it turns on, the coupling h induces the required higgsing $\mathrm{SU}(3N_c) \hookrightarrow \mathrm{SU}(N_c)$. By using colour and flavour rotations, we can arrange the VEVs for the magnetic quarks in a form that makes explicit the $N_c \times N_c$ blocks:

$$q = \tilde{q} = -\sqrt{\frac{h\Lambda^2}{\tilde{c}^{(2)}}} \begin{pmatrix} \mathbb{I}_{N_c\times N_c} & \cdot \\ \cdot & \mathbb{I}_{N_c\times N_c} \\ \cdot & \cdot \end{pmatrix}. \tag{2.11}$$

Writing the $\mathrm{SU}(3N_c)$ adjoints as

$$x = \begin{pmatrix} z & y \\ \tilde{y} & \hat{x} \end{pmatrix} \tag{2.12}$$

where z is $2N_c \times 2N_c$ and $\hat{x}$ is $N_c \times N_c$, the $\tilde{c}_1$ coupling then becomes an effective mass term for the adjoint z and the traceless mesons $\bar{\varphi}^{(1)} = \varphi^{(1)} - \frac{1}{2N_c}\mathrm{tr}(\varphi^{(1)})$, of the form

$$-\frac{h\tilde{c}_1}{\tilde{c}_2}\bar{\varphi}^{(1)}z. \tag{2.13}$$

Note that colour-flavour is broken to the diagonal, $\mathrm{SU}(3N_c) \times SU(N_f) \hookrightarrow \mathrm{SU}(N_c) \times \mathrm{SU}(N_f)_D$, and this term represents a Dirac mass for two adjoints of the remaining diagonal flavour group. In addition $\varphi^{(2)}$ gets a mass together with the higgsing $2N_c$ block of q. In detail writing $q = \begin{pmatrix} v+\eta \\ \rho \end{pmatrix}$ and $\tilde{q} = \begin{pmatrix} v+\tilde{\eta} \\ \tilde{\rho} \end{pmatrix}$, we find a mass term term $W \supset \tilde{c}_2\,(\eta+\tilde{\eta})\,\varphi^{(2)}v$, with the $8N_c^2$ massless $\eta-\tilde{\eta}$ Goldstone modes being eaten by the $8N_c^2$ heavy gauge bosons of the broken $\mathrm{SU}(3N_c)$. Meanwhile $\rho, \tilde{\rho}$ are the light quarks of the remaining unbroken $SU(N_c)$. The superpotential for the remaining effective $SU(N_c)$ theory is

$$W_{\mathrm{mag}} = \frac{\tilde{\kappa}}{3}x^3 + \tilde{h}\tilde{\rho}x\rho \tag{2.14}$$

where $\tilde{h} = \tilde{c}_1\mathrm{tr}(\varphi^{(1)})$ is dynamical in the dual theory. As stated above, the a-theorem tells us that this SQCD+X theory flows to the dual $\mathcal{N}=2$ fixed point.

It is straightforward to extend the above discussion to arbitrary k, to check that the h coupling induces the required breaking $SU((2k-1)N_c) \times SU(N_f) \hookrightarrow SU(N_c) \times SU(N_f)$. From eq. 2.8 we find that the X and φ equations of motion are

$$\varphi^{(j)}: \quad 0 = h\,\delta_{nm}\,\delta_{2j} + \tilde{c}_j\,\mathrm{tr}(\tilde{q}_m x^{k-j} q_n) \tag{2.15}$$

$$x: \quad 0 = \tilde{\kappa}\,x^k + \sum_{j=1}^{k} \tilde{c}_j\,\varphi^{(j)}_{nm} \sum_{r=0}^{k-j-1} x^{k-j-1-r}\,q_n\,\tilde{q}^T_m\,(x^r)^T\,. \tag{2.16}$$

From the first condition we see for $k \geq 3$ and non-zero $\tilde{c}_j$

$$\mathrm{tr}\langle\tilde{q}_m x^{k-1} q_n\rangle = \mathrm{tr}\langle\tilde{q}_m x^{k-3} q_n\rangle = \ldots = \mathrm{tr}_G\langle\tilde{q}_m x q_n\rangle = \mathrm{tr}\langle\tilde{q}_m q_n\rangle = 0 \tag{2.17}$$

$$\mathrm{tr}\langle\tilde{q}_m x^{k-2} q_n\rangle \neq 0. \tag{2.18}$$

Let us write x, q and $\tilde{q}$ as

$$x = \begin{pmatrix} z & y \\ \tilde{y} & \hat{x} \end{pmatrix}, \quad q = \begin{pmatrix} v+\eta \\ \rho_1 \\ \rho_2 \end{pmatrix}, \qquad \tilde{q}^T = \begin{pmatrix} \tilde{\rho}_1 \\ v+\tilde{\eta} \\ \tilde{\rho}_2 \end{pmatrix}, \tag{2.19}$$

where z is an $(k-1)N_f \times (k-1)N_f$ matrix, v, η, and $\tilde{\eta}$ are $N_f \times N_f$ matrices, ρ_1 and $\tilde{\rho}_1$ are $(k-2)N_f \times N_f$ matrices, and ρ_2 and $\tilde{\rho}_2$ are $N_c \times N_f$ matrices. We can solve equations 2.17 and 2.18 by taking z as

$$\langle z\rangle \sim \begin{pmatrix} 0_{N_f\times N_f} & \mathbb{I}_{N_f\times N_f} & \cdot & \cdot \\ \cdot & \cdot & \ddots & \cdot \\ \cdot & \cdot & \cdot & \mathbb{I}_{N_f\times N_f} \\ \cdot & \cdot & \cdot & 0_{N_f\times N_f} \end{pmatrix} \tag{2.20}$$

such that

$$\langle z^{k-2}\rangle \sim \begin{pmatrix} \cdot & \mathbb{I}_{N_f\times N_f} \\ \cdot & \cdot \end{pmatrix} \tag{2.21}$$

and then seperating the VEVs of q and $\tilde{q}$ by $k-2$ permutations,

$$\langle\tilde{q}\rangle \sim \begin{pmatrix} \mathbb{I}_{N_f\times N_f} \\ \cdot \end{pmatrix}, \qquad \langle q\rangle \sim \begin{pmatrix} 0_{(k-2)N_f\times N_f} \\ \mathbb{I}_{N_f\times N_f} \\ \cdot \end{pmatrix}, \tag{2.22}$$

so that clearly

$$\langle x^{k-2} q\rangle \sim \begin{pmatrix} \mathbb{I}_{N_f\times N_f} \\ \cdot \end{pmatrix} \sim \langle\tilde{q}\rangle, \tag{2.23}$$

as required. Then $\langle z\rangle$ which is rank $(k-2)N_f$, together with $\langle q\rangle$, leave the bottom $\rho_2, \tilde{\rho}_2$ block and hence $SU(N_c) \times SU(N_f)$ unbroken.

2.3 Flow away from $N_f = 2N_c$

Having established this connection it is simple to reach the more general SQCD+X configurations that have arbitrary N_f and N_c. From our $N_f = 2N_c$ electric theory we can add Δ additional heavy quarks $Q', \tilde{Q}'$ with mass terms $W \supset m_{Q'} Q'\tilde{Q}'$ with $m_{Q'}$ being chosen to be in the SQCD+X period of running. Instead of running to a free field theory, the original electric theory now heads towards a Landau pole in the UV. Meanwhile in the magnetic dual description, the mass term becomes a linear term for the new meson $\varphi' = \Lambda^{-1} Q'.\tilde{Q}'$, which induces a higgsing for the new magnetic quarks $\tilde{q}'.q' \sim m'_Q \Lambda$. This strongly coupled theory is asymptotically free in the UV. Conversely, as well as Q' and $\tilde{Q}'$, one can also add a meson Φ into the electric theory together with a linear coupling $W \supset \mu^2 \Phi$: this implies a symmetry restoration $\mathrm{SU}(N_c) \hookleftarrow \mathrm{SU}(N_c + \Delta)$ at the scale μ, and the theory can flow to the SQCD+X conformal fixed point of that theory. (Note that the R-symmetry associated with this fixed point can be compatible with the previous $\mathcal{N} = 2$ R-symmetry because we have integrated in more degrees of freedom.) Note that the mass deformations may be already introduced at the $\mathcal{N} = 2$ level [66, 67].

3 Duality of the deformed $\mathcal{N} = 2$ theory

3.1 Overview

Having seen that dual SQCD theories deformed by $\mathcal{N} = 2$ operators naturally flow to dual $\mathcal{N} = 2$ theories deformed by $\mathcal{N} = 1$ operators, we are now ready to study $\mathcal{N} = 2$ duality itself. In particular we would like to confirm that the residual $\mathcal{N} = 1$ deformations can be understood as being induced by the ATP mechanism, and that they map consistently into each other under the $\mathcal{N} = 2$ duality.

The origin of the ATP mechanism of spontaneous partial SUSY breaking [57–59] as electric and magnetic FI terms [68] was clarified in [69]. The effect of coupling this mechanism to $\mathcal{N} = 2$ $SU(N_c)$ Yang-Mills theories was studied in [70–74], and a formulation in harmonic superspace (HSS)[2] of $\mathcal{N} = 2$ SQCD coupled to the ATP mechanism was given in [77]. A further generalisation to quiver theories can be found in [78].

The necessary HSS language for constructing a model of $\mathcal{N} = 2$ SQCD broken by $\mathcal{N} = 1$ $SU(N_c)$ SQCD+X deformations and soft terms will be developed in the following 3 subsections. In Subsection 3.2 we first describe how $\mathcal{N} = 2$ SQCD is written in the HSS formalism, and recast the theory in $\mathcal{N} = 1$ superspace. In Subsection 3.3, we identify the X-dependent deformations consistent with $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking and the particular form of prepotential required to produce them. Noting the restriction from the *2 into 1 won't go* theorem [55, 56], we review the FI-terms that are required in order to preserve $\mathcal{N} = 1$ supersymmetry in the stable vacuum. In this context, the $SU(N_c)$ gauge theory of SQCD+X is extended to $SU(N_c) \times U(1)$, and the effective

[2]See [75, 76] for a review.

theory considered in the decoupling infinite FI term limit. In Subsection 3.4, we then discuss the full $\mathcal{N}=2 \to \mathcal{N}=0$ breaking, which induces the Dirac gaugino masses that are our subject of interest. This requires a further extension of the gauge symmetry to $SU(N_c)\times U(1)^3$; we can then assign a combination of FI terms to pick out an $\mathcal{N}=1$ preserving direction, and as a perturbation, assign a different combination of FI terms to fully break SUSY. This provides us with a description of an $SU(N_c)$ $\mathcal{N}=2$ theory augmented by both $\mathcal{N}=1$ deformations and soft-terms that can, as we shall see in the next section, all be mapped under electric-magnetic duality.

3.2 $\mathcal{N}=2$ $SU(N_c)$ SQCD

3.2.1 in $\mathcal{N}=2$ harmonic superspace

First the HSS construction of $\mathcal{N}=2$ SQCD. The low energy effective action (LEEA) for the $SU(N_c)$ theory is [79][3]

$$S^{\mathcal{N}=2}_{\rm QCD} = S^{\mathcal{N}=2}_{\rm SYM} + S^{\mathcal{N}=2}_{Q} \tag{3.4}$$

$$S^{\mathcal{N}=2}_{\rm SYM} = \frac{1}{16\pi i}\int d^4x\,(D)^4\mathcal{F}(W) + \text{h.c.}, \qquad S^{\mathcal{N}=2}_{Q} = -\int du\, d\zeta^{-4}\tilde{Q}^+\,\mathcal{D}^{++}Q^+, \tag{3.5}$$

where Q^+ is a Fayet-Sohnius (FS) hypermultiplet [66, 80], V^{++} is a $\mathcal{N}=2$ vector multiplet, and (in contrast to the upcoming $\mathcal{N}=1$ formalism), W is the full $\mathcal{N}=2$ gauge field strength. Note that $\tilde{Q}^+$ in this eq. refers to the antipodal$\times$hermitian conjugation of the Q^+ hypermultiplet and should not be confused with the $\mathcal{N}=1$ superfield $\tilde{Q}$ (See Appendix B for details). In addition we canonically normalise the hypermultiplets in contrast with the usual convention. $\mathcal{F}(W)$ is the prepotential, and is a gauge invariant function of only $W=W^a t_a$, $a=1,\ldots,N_c^2-1$, whose general form is

$$\mathcal{F}(W) = \sum_M \frac{1}{M!}\sum_{m_1\,\ldots\,m_M}\frac{c_{m_1\ldots m_M}}{m_1!\ldots m_M!}\,\text{tr}_G\left(W^{m_1}\right)\ldots\text{tr}_G\left(W^{m_M}\right), \tag{3.6}$$

where tr_G is a trace over the $SU(N_c)$ gauge indices, the m_i represent powers and not gauge indices, and the coefficients $c_{m_1\ldots m_M}$ arise from integrating out microscopic degrees of freedom, and have been exactly determined in specific cases, for example in [60, 61]. We define derivatives of the prepotential and the metrics[4]

$$\mathcal{F}_{a_1\,\ldots\,a_N}(W) \equiv \frac{\partial^N\mathcal{F}(W)}{\partial W^{a_1}\,\ldots\,\partial W^{a_N}}, \qquad h_{ab}\equiv \text{Re}\,\mathcal{F}_{ab}|, \qquad g_{ab}\equiv \text{Im}\,\mathcal{F}_{ab}|, \tag{3.7}$$

[3] HSS expansions for Q^+, V^{++}, and W are

$$Q^+(\zeta,u)\supset Q^i(x_A)u_i^+ + \theta^+\psi_Q(x_A) + \bar{\theta}^+\psi_{\tilde{Q}}(x_A), \tag{3.1}$$

$$V^{++}(\zeta,u)\supset i\sqrt{2}(\bar{\theta}^+)^2X(x_A) + 4(\bar{\theta}^+)^2\theta^+\lambda^i(x_A)u_i^- - 2i\theta^+\sigma^\mu\theta^+v_\mu(x_A) + 3(\theta^+)^2(\bar{\theta}^+)^2D^{ij}(x_A)u_i^-u_j^-, \tag{3.2}$$

$$W(\zeta,u)\supset i\sqrt{2}X(x_A) - 2\theta^+\lambda^i(x_A)u_i^- + \theta^i\sigma^{\mu\nu}\theta_i v_{\mu\nu}(x_A) + \theta^i\theta^j D_{ij}(x_A), \tag{3.3}$$

and the gauge covariant derivative $\mathcal{D}^{++}$ and further HSS definitions are provided in Appendix B.

[4] Note that by subscript-a we will always mean $\partial/\partial W^a \equiv (i\sqrt{2})^{-1}\partial/\partial X^a$.

where $\mathcal{O}| \equiv \mathcal{O}(\theta = \bar{\theta} = 0)$.

	$SU(N_c)$	$SU(N_f)$
Q^+	$\square$	$\square$

Table 3. *$\mathcal{N} = 2$ superfield representations in $\mathcal{N} = 2$ SQCD*

Considering the renormalizable part of this theory, allowing up to two derivatives in $\mathcal{F}$ we find gauge kinetic terms, yukawa interactions and a scalar potential

$$-\mathcal{L}_{\text{kin}} = \frac{g_{ab}}{4\pi}\left(\mathcal{D}^\mu X^a\,\mathcal{D}_\mu \bar{X}^b + i\,\lambda^{i,\,a}\,\sigma^\mu\,\mathcal{D}_\mu\,\bar{\lambda}^b_i - \frac{1}{4}\,F^a_{\mu\nu}\,F^{b,\,\mu\nu}\right) + \frac{h_{ab}}{16\pi}\,F^a_{\mu\nu}\,\tilde{F}^{b,\,\mu\nu}$$
$$+\,\bar{Q}^i\,\mathcal{D}^\mu\mathcal{D}_\mu\,Q_i + \frac{i}{2}\left(\bar{\psi}_Q\bar{\sigma}^\mu\mathcal{D}_\mu\psi_Q + \psi_{\tilde{Q}}\sigma^\mu\mathcal{D}_\mu\bar{\psi}_{\tilde{Q}}\right), \tag{3.8}$$

$$-\mathcal{L}_{\text{yuk}} = \frac{i g_{ab}}{4\pi\sqrt{2}}\,f^b_{cd}\,\lambda^{a,i}\,\bar{X}^c\,\lambda^d_i + i\left(\bar{Q}^i\,\lambda_i\,\psi_Q - \psi_{\tilde{Q}}\,\lambda^i\,Q_i\right) - \frac{1}{\sqrt{2}}\,\psi_{\tilde{Q}}\,X\,\psi_Q + \text{h.c.}, \tag{3.9}$$

$$V = \bar{Q}^i\,\{\bar{X}, X\}\,Q_i - \frac{g_{ab}}{4\pi}\left(\frac{1}{2}\,f^a_{cd}\,f^b_{ef}\bar{X}^c X^d \bar{X}^e X^f + \frac{1}{2}\,D^{a,\,A}|\,D^{b,\,A}|\right), \tag{3.10}$$

where we have rewritten traced $SU(2)_R$ tensor products as three vector dot products

$$a^i{}_j \equiv i\,a^A\left(\sigma^A\right)^i{}_j, \qquad a^{ij}\,b_{ij} = -a^i{}_j\,b^j{}_i = a^A\,b^B\,\text{tr}_R\left(\sigma^A\cdot\sigma^B\right) = 2\,a^A\,b^A, \tag{3.11}$$

where tr_R is a trace over the $SU(2)_R$ indices and we use the conventions of Appendix A. Integrating out $D^{a,A}$ and taking the canonical prepotential

$$\mathcal{F}(W) = \tau\frac{(W^a)^2}{2}, \qquad \tau \equiv \frac{\theta_{\text{YM}}}{2\pi} + \frac{4\pi i}{g^2} \tag{3.12}$$

gives standard kinetic terms in the holomorphic basis

$$-\mathcal{L}_{\text{kin}} = \frac{1}{g^2}\left(\mathcal{D}^\mu X^a\,\mathcal{D}_\mu\bar{X}_a + i\,\lambda^{i,\,a}\,\sigma^\mu\,\mathcal{D}_\mu\,\bar{\lambda}_{i,\,a} + \frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a\right) + \frac{\theta_{\text{YM}}}{32\pi^2}\,F^a_{\mu\nu}\,\tilde{F}^{\mu\nu}_a, \tag{3.13}$$

as well as the familiar yukawa interactions and scalar potential. Allowing for $\mathcal{F}(W)$ to describe an effective theory introduces terms involving higher derivatives of the prepotential, yielding extra terms involving $D^{a,\,A}$,

$$4\pi\mathcal{L}_{\text{fermion}} = \frac{i}{2}\,\mathcal{F}_{abc}|(\lambda^a\lambda^b)^A D^{c,\,A}| + \text{h.c.}\,, \tag{3.14}$$

as well as four-fermion interactions and higher derivative terms that we do not write. Once FI-terms are introduced, eq. 3.14 can be a source of chiral $\mathcal{N} = 1$ masses and yukawa interactions, as well as $\mathcal{N} = 0$ gaugino masses.

In this section we will be mainly concerned with the classical extension to the theory that generates the correct $\mathcal{N} = 1$ deformations, and therefore we will often work by deforming around a canonical prepotential:

$$\mathcal{F}(W) = \frac{\tau_{ab}}{2} W^a W^b + \text{deformation.} \tag{3.15}$$

This is sufficient to consider $\mathcal{N}=1$ deformations of a weakly coupled $\mathcal{N}=2$ theory where there is negligible RG-flow. The discussion of duality in section 4 will require more careful treatment of the leading part.

3.2.2 in $\mathcal{N}=1$ superspace

Because we are ultimately interested in $\mathcal{N}=1$ SQCD+X, we briefly recall how $\mathcal{N}=2$ SQCD can be recast in $\mathcal{N}=1$ superspace [81]. The appropriate $\mathcal{N}=1$ superfield content [60, 82] is given in table

	$SU(N_c)$	$SU(N_f)$	$U(1)_R$
Q	$\square$	$\square$	$1-R_X\frac{N_c}{N_f}$
$\tilde{Q}$	$\bar{\square}$	$\bar{\square}$	$1-R_X\frac{N_c}{N_f}$
X	**Ad**	**1**	R_X
Λ^b	**1**	**1**	$2\,b$

Table 4. *$\mathcal{N}=1$ superfield representations in $\mathcal{N}=2$ SQCD. The operator $W_{el}\supset\sqrt{2}\tilde{Q}XQ$ fixes $2R_Q+R_X=2$, consistent with $N_f=2N_c$ in the presence of the operator $W_{el}\supset trX^{k+1}$ which fixes $R_X=2/(k+1)$.*

4, and the $\mathcal{N}=2$ SQCD action composed of two parts as in 3.4. From the full $\mathcal{N}=2$ superspace point of view, after fixing an $SU(2)_R$ direction so that $\mathcal{Q}_1$ is the canonical $\mathcal{N}=1$ SUSY, the field content of Q^+ and W is most easily seen diagramatically in component superfield 'diamonds' [83],

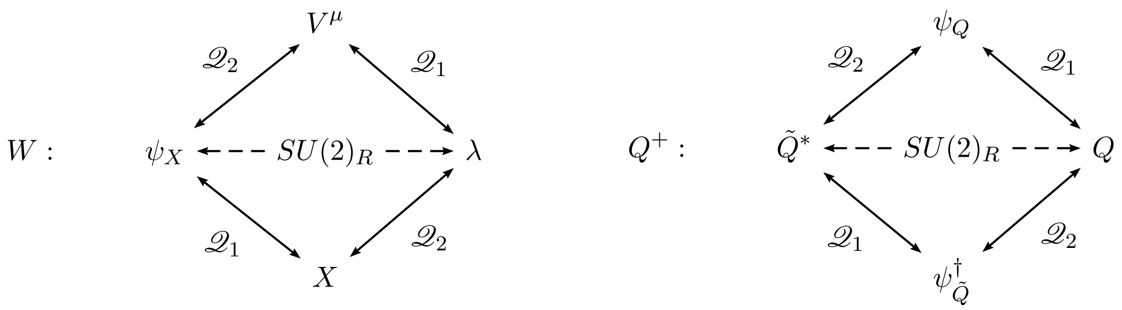


Figure 2. *$\mathcal{N}=2$ SQCD particle content*

The SYM part is written in terms of an analytic prepotential $\mathcal{F}(i\sqrt{2}X)=\mathcal{F}(\mathcal{A})$ [84],

$$S^{\mathcal{N}=2}_{\text{SYM}} = \frac{1}{16\pi i}\int d^4x\, d^2\theta \left(\mathcal{F}_{ab}\mathcal{W}^a\mathcal{W}^b - \int d^2\bar{\theta}\,\frac{i}{\sqrt{2}}\mathcal{F}_a(e^V)^a{}_b\bar{X}^b\right) + \text{h.c.} \tag{3.16}$$

whereas the QCD part is

$$S_Q^{\mathcal{N}=2} = \int d^4x\, d^2\theta \left(\sqrt{2}\,\tilde{Q}\,X\,Q + \frac{1}{2}\int d^2\bar{\theta}\left[K_Q + K_{\tilde{Q}}\right]\right) + \text{h.c.} \tag{3.17}$$

and K_ϕ is the Kähler potentials for the superfield ϕ. The Kähler potential for X and effective gauge coupling for the standard renormalisable $\mathcal{N}=2$ theory can be recovered by taking 3.12,

$$\mathcal{F}(\mathcal{A}) = \tau\frac{(\mathcal{A}^a)^2}{2} \implies S_{\text{SYM}}^{\mathcal{N}=2} = \frac{\tau}{4\pi i}\int d^4x\, d^2\theta \left(\frac{1}{4}\mathcal{W}^2 + \frac{1}{2}\int d^2\bar{\theta}\, K_X\right) + \text{h.c.} \tag{3.18}$$

3.3 Breaking $\mathcal{N}=2 \to \mathcal{N}=1$

3.3.1 in $\mathcal{N}=1$ superspace

Having established how the $\mathcal{N}=2$ SQCD theory appears in $\mathcal{N}=1$ superspace, we now wish to introduce $\mathcal{N}=1$ preserving but $\mathcal{N}=2$ breaking X deformations $W_{\text{def}}(X)$. For later comparison it is convenient to write them in the familiar $\mathcal{N}=1$ superspace language first of all. They are

$$S_{\text{SQCD}}^{\mathcal{N}=2} \to S_{\text{SQCD}}^{\mathcal{N}=2} + S_{\text{def}}^{\mathcal{N}=1}, \qquad S_{\text{def}}^{\mathcal{N}=1} = \int d^4x\, d^2\theta\, W_{\text{def}}(X) + \text{h.c.} \tag{3.19}$$

yielding the additional terms in the lagrangian

$$V_{\text{def}} = \frac{4\pi}{\tau_2}K_X^{ab}\left[\frac{\partial W_{\text{def}}}{\partial X^a} + \sqrt{2}\,\tilde{Q}\,t_a\,Q\right]\left[\frac{\partial W_{\text{def}}}{\partial X^b} + \sqrt{2}\,\tilde{Q}\,t_b\,Q\right]^\dagger \tag{3.20}$$

$$\mathcal{L}_{\text{def}}^{\text{fermion}} = -\frac{1}{2}\frac{\partial^2 W_{\text{def}}}{\partial X^a \partial X^b}\,\psi_X^a\,\psi_X^b + \text{h.c.}, \tag{3.21}$$

where K_X^{ab} is the inverse of the Kähler metric for the physically normalised X

$$(K_X)_{ab} \equiv \frac{\partial^2 K_X}{\partial X^a \partial \bar{X}^b}. \tag{3.22}$$

Note that the term in $V \sim (\tilde{Q}Q)^2$ is present in the limit $W_{\text{def}} \to 0$, and is the familiar squark scalar potential of $\mathcal{N}=2$ SQCD.

3.3.2 in $\mathcal{N}=2$ harmonic superspace

Now for the HSS formalism of these deformations, which requires the ATP mechanism generalised to couple to hypermultiplets. To do this we first extend the gauge theory as $SU(N_c) \to SU(N_c) \times U(1)_\circ$, where Q^+ is charged under the $U(1)_\circ$ factor as per table 5. The resulting action is the

	$SU(N_c)$	$U(1)_\circ$	$SU(N_f)$
Q^+	$\square$	1	$\square$

Table 5. *$\mathcal{N}=2$ superfield representations in $\mathcal{N}=2$ SQCD coupled to $U(1)_\circ$*

same as in 3.4 and 3.5 with prepotential $\mathcal{F}(W, W^\circ)$ written as a general expansion in W's, and the covariant derivative

$$\mathcal{D}^{++} = D^{++} + i\,(V^{++} + V_\circ^{++}). \tag{3.23}$$

The $\circ$-index on $V_\circ^{++}$ or W° is equivalent to the trace $U(1)$ element of the $U(N_c)$ gauge group in [77], in the sense that we can define a Kähler metric for the whole gauge theory from $\mathcal{F}_{a_1\,\dots\,a_N}(W, W^\circ)$. From now on we use the following notation to distinguish $SU(N)$ or $U(1)$ indices; $a = \circ, 1, \dots, N_c^2 - 1$, $\tilde{a} = 1, \dots, N_c^2 - 1$, and we take $t_\circ = \frac{1}{\sqrt{2N_c}}\mathbb{I}_{N_c\times N_c}$.

$\mathcal{N} = 2$ SUSY can be broken spontaneously by giving the D terms of the $U(1)_\circ$ gauge field a VEV. Dynamically this is done with FI terms [68], the simplest of which includes a linear piece in the lagrangian

$$4\pi\, S^{\mathcal{N}=2}_{\text{El FI},\,\circ} = \int du\, d\zeta^{(-4)}\, \xi^{++}(V^\circ)^{++} + \text{h.c.} = 2\int d^4x\, \xi^A D^{\circ,\,A} \; + \text{h.c.} \tag{3.24}$$

where $\xi^{++} \equiv \xi^{ij}u_i^+u_j^+$. Writing the whole action as an integral over the analytic subspace [85] and varying $V_\circ^{++}$ yields the eq. of motion [69]

$$(D^+)^2\,\mathcal{F}_\circ - \text{h.c.} = 4\,i\,\xi^{++}. \tag{3.25}$$

Because $\mathcal{F}_\circ = W_{D,\,\circ} \supset 2(\theta\theta)^A D_\circ^A$, the eq. of motion 3.25 shifts the magnetic dual D term $D^A_{D,\,\circ}$ by an imaginary part on-shell[77]:

$$\mathbf{D}^A_{D,\,\circ} = D^A_{D,\,\circ} + 4\,i\,\xi^A, \qquad \bar{\mathbf{D}}^A_{D,\,\circ} = \bar{D}^A_{D,\,\circ} - 4\,i\,\bar{\xi}^A. \tag{3.26}$$

The above analysis is analogous to including $\int d^4x\,\xi D \sim \int d^4x\,(D)^2(\bar{D})^2\xi\, V$ in pure $\mathcal{N} = 1$ SYM. This would lead to the eq. of motion

$$(D)^2(\bar{D})^2 V \sim \xi \tag{3.27}$$

after varying the full action with respect to V. The difference between the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ case is that clearly 3.27 shifts the electric D term by a constant, rather than the magnetic D term. To accomplish a shift in the electric D term in the $\mathcal{N} = 2$ theory, we therefore include FI terms for the magnetic dual gauge field which turns out to be of the form

$$4\pi\, S^{\mathcal{N}=2}_{\text{Mag FI},\,\circ} = 2\int d^4x\, \xi_D^A\left[(D)^4(\theta\theta)^A\left(\mathcal{F}_\circ + \mathcal{F}_{\circ\circ}\,4\,i\xi_D^B(\theta\theta)^B\right) - 2\,\mathcal{Q}_\circ^A\right] + \text{h.c.} \tag{3.28}$$

where

$$\mathcal{Q}_a^{ij} \equiv 4\pi\bar{Q}^{(i}t_aQ^{j)} = -\bar{\mathcal{Q}}_a^{ij}, \tag{3.29}$$

and our symmetrization conventions are $a^{(i_1\dots i_n)} \equiv \frac{1}{n!}\left(a^{i_1\dots i_n} + \text{permutations}\right)$. For later reference the explicit form of the $\mathcal{Q}^A$'s is

$$\frac{\mathcal{Q}_a^1}{2\pi i} = -(\overline{Q^2}t_a Q^1 + \overline{Q^1}t_a Q^2)\,; \quad \frac{\mathcal{Q}_a^2}{2\pi} = \overline{Q^2}t_a Q^1 - \overline{Q^1}t_a Q^2\,; \quad \frac{\mathcal{Q}_a^3}{2\pi i} = \overline{Q^2}t_a Q^2 - \overline{Q^1}t_a Q^1\,. \tag{3.30}$$

The effect of $S^{\mathcal{N}=2}_{\text{Mag FI},\circ}$ is that the electric D term $D^A_\circ$ is shifted by an imaginary constant off-shell, allowing us to write

$$S^{\mathcal{N}=2}_{\text{SQCD}} + S^{\mathcal{N}=2}_{\text{Mag FI},\circ} = \left(\frac{1}{16\pi i}\int d^4x\,(D)^4\mathcal{F}(W,W^\circ) - \frac{1}{2}\int du\,d\zeta^{-4}\tilde{Q}^+\,\mathcal{D}^{++}\,Q^+\right)\Big|_{D^A_\circ\to\mathbf{D}^A_\circ} + \text{h.c.}, \tag{3.31}$$

where

$$\mathbf{D}^A_\circ = D^A_\circ + 4\,i\,\xi^A_D, \qquad \bar{\mathbf{D}}^A_\circ = D^A_\circ - 4\,i\,\bar{\xi}^A_D. \tag{3.32}$$

Taking the full off-shell action as

$$S_{\text{Off-shell}} = S^{\mathcal{N}=2}_{\text{SQCD}} + S^{\mathcal{N}=2}_{\text{El FI},\circ} + S^{\mathcal{N}=2}_{\text{Mag FI},\circ} \tag{3.33}$$

and solving the D term equations of motion up to third derivatives in the prepotential, we finally arrive at the desired on-shell action for $\mathcal{N}=2$ SQCD coupled to the ATP mechanism:

$$S_{\text{On-shell}} = \int d^4x\,\left(\mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{4-\text{fermion}} + \mathcal{L}_{\text{H.D.}} - V\right), \tag{3.34}$$

where $\mathcal{L}_{\text{kin}}$ is 3.8, $\mathcal{L}_{\text{yuk}}$ is 3.9, $\mathcal{L}_{\text{H.D.}}$ are the higher derivative operators, and

$$4\pi\mathcal{L}_{\text{mass}} = \frac{i}{2}\,\mathbf{D}^{a,\,A}|\,\mathcal{F}_{abc}|\,(\lambda^b\lambda^c)^A + \text{h.c.} \tag{3.35}$$

$$\begin{aligned} 4\pi V &= \frac{1}{2}\,g_{ab}\,\mathbf{D}^{a,\,A}_\phi|\,\bar{\mathbf{D}}^{b,\,A}_\phi| + 4\pi\overline{Q^i}\{\bar{X},X\}\,Q^i - \frac{1}{2}\,g_{ab}f^a_{cd}f^b_{ef}\bar{X}^cX^d\bar{X}^eX^f \\ &\quad + 4\,i\,(\xi^A + \bar{\xi}^A)(\xi^A_D - \bar{\xi}^A_D), \end{aligned} \tag{3.36}$$

where the solutions of the D term equations of motion have the convenient decomposition

$$\mathbf{D}^{a,\,A} = \mathbf{D}^{a,\,A}_X + D^{a,\,A}_Q + D^{a,\,A}_\lambda, \quad \mathbf{D}^{a,\,A}_\phi = \mathbf{D}^{a,\,A}_X + D^{a,\,A}_Q, \quad \boldsymbol{\xi}^A_a \equiv (\xi^A + \bar{\xi}^A)\delta^\circ_a + (\xi^A_D + \bar{\xi}^A_D)\bar{\mathcal{F}}_{\circ a}| \tag{3.37}$$

$$\mathbf{D}^{a,\,A}_X = -2\,g^{ab}\,\boldsymbol{\xi}^A_b, \quad D^{a,\,A}_Q = -2\,i\,g^{ab}\,\mathcal{Q}^A_b, \quad D^{a,\,A}_\lambda = -\frac{i}{2}\,g^{ab}\,\mathcal{F}_{bcd}|(\lambda^c\lambda^d)^A + \text{h.c.}. \tag{3.38}$$

We shall refer back to these equations frequently below.

3.3.3 Scalar potential

Having put the on-shell action into the present context, let us first ensure that the properties of $\mathcal{N}=2$ SQCD in the presence of the SUSY breaking FI terms are indeed those of the desired $\mathcal{N}=1$ theory of subsection 3.3.1. There are three conditions that one could consider for the vacuum to respect $\mathcal{N}=1$:

- *Vacuum stability*
- *Zero vacuum energy*
- *A scalar potential corresponding to the $\mathcal{N}=1$ preserving superpotential in 3.20.*

As we shall see the first two of these provide a constraint on the FI terms while the third is observed to be generally true, and relates the prepotential to the desired $\mathcal{N}=1$ deformations. In addition, although it is possible to set $\langle V\rangle = 0$ it is not obligatory for preserving $\mathcal{N}=1$ SUSY [59], but it is natural to apply it. Results for the first two are available in the literature but somewhat scattered, so it is worth collating all three elements here.

Vacuum stability : Stable SUSY breaking vacua exist on the coulomb branch (i.e. with $\langle Q\rangle = 0$) which can be achieved by assuming $X^\circ \neq 0$ [70, 72–74, 77, 78] or on the higgs branch when $X^\circ = 0$. In order to study the latter without breaking $SU(N_c)$ one could introduce hypermultiplets charged only under $U(1)_\circ$, but this case is more complicated to analyse as the goldstino comes from a linear combination of the new quarks and the λ's, so we will restrict the discussion to the former case.

Noting that the scalar potential contains

$$-4\pi V \supset \frac{1}{2}\, g_{ab} f^a_{cd} f^b_{ef} \bar{X}^c X^d \bar{X}^e X^f,$$

it follows that $\langle X^{\hat{a}}\rangle = 0$ where $t_{\hat{a}}$ are non-Cartan generators. Therefore only $\langle X^{\underline{a}}\rangle \neq 0$ is possible, where $t_{\underline{a}}$ are Cartan generators. The vacuum condition is [73]

$$4\pi \left\langle \frac{\partial V}{\partial(W^a|)} \right\rangle = \frac{i}{4} \langle \mathcal{F}_{abc} \mathbf{D}^{b,\,A} \mathbf{D}^{c,\,A} \rangle = 0. \tag{3.39}$$

The only non-vanishing $\langle \mathcal{F}_{ab}\rangle$ are the diagonal elements $\langle \mathcal{F}_{\hat{a}\,\hat{a}}\rangle$ and $\langle \mathcal{F}_{\underline{a}\,\underline{a}}\rangle$, whilst the only non-vanishing $\langle \mathcal{F}_{abc}\rangle$ are $\langle \mathcal{F}_{\underline{a}\,\underline{a}\,\underline{a}}\rangle$ and $\langle \mathcal{F}_{\underline{a}\,\hat{b}\,\hat{b}}\rangle$. It follows that $\langle \mathbf{D}^{\hat{a}}\rangle = 0$ and so condition 3.39 becomes

$$\langle \mathcal{F}_{\underline{a}\,\underline{a}\,\underline{a}} \mathbf{D}^{\underline{a},\,A} \mathbf{D}^{\underline{a},\,A} \rangle = 0. \tag{3.40}$$

The choice $\langle \mathcal{F}_{\underline{a}\,\underline{a}\,\underline{a}}\rangle = 0$ corresponds to unstable saddle points, and so a stable vacuum must satisfy

$$\langle \mathbf{D}^{\underline{a},\,A} \mathbf{D}^{\underline{a},\,A} \rangle = 0 \tag{3.41}$$

for every $\underline{a}$. By fixing the $SU(2)_R$ direction appropriately, this condition is solved by

$$\langle \mathcal{F}_{\circ\circ}\rangle = -\frac{1}{m}\,(e + i\xi)\,, \qquad \xi^A + \bar{\xi}^A = (0, e, \xi)^A, \qquad \xi^A_D + \bar{\xi}^A_D = (0, m, 0)^A, \tag{3.42}$$

where e, m and ξ are real constants. Without loss of generality, taking $\frac{\xi}{m} < 0$ fixes the sign of the solution as we demand a positive metric, $\langle g_{\circ\,\circ}\rangle = -\frac{\xi}{m} \geq 0$.

<u>*Zero vacuum energy*</u> : The vacuum energy is given by

$$\langle 4\pi V\rangle = -4\,\xi\,m - 4\,i\,(\xi^A + \bar{\xi}^A)(\xi_D^A - \bar{\xi}_D^A), \tag{3.43}$$

so that the choice

$$\xi_D^A - \bar{\xi}_D^A = (0, 0, i\,m)^A \tag{3.44}$$

makes it vanish [57, 78]. The form of ξ_D^A is then completely fixed, whereas the imaginary part of ξ^A is still undetermined,

$$\mathrm{Re}\,\xi^A = \frac{1}{2}\,(0, e, \xi)^A, \qquad \xi_D^A = \frac{m}{2}\,(0, 1, i)^A. \tag{3.45}$$

<u>*$\mathcal{N} = 1$ superpotential*</u> : Our third requirement is that we can describe $W_{\rm def}$ correctly in this setup. The first term in 3.36 is

$$4\pi V \supset 2\,g^{ab}\,\left[\boldsymbol{\xi}_a - i\,\mathcal{Q}_a\right]^A \left[\boldsymbol{\xi}_b - i\,\mathcal{Q}_b\right]^{A\,\dagger}. \tag{3.46}$$

From the above, 3.30 and 3.36, the $U(1)_\circ$ part of the potential takes the form

$$V = |X^\circ|^2|Q^i|^2 + \frac{g^2}{2}|\overline{Q^2}Q^1 - \overline{Q^1}Q^2|^2 + \frac{g^2}{2}\xi^2 + \frac{g^2}{2}|\xi - |Q^1|^2 + |Q^2|^2|^2\,, \tag{3.47}$$

confirming that it is stable if $X^\circ > g\,\xi$. Note for later reference that along the coulomb branch the quarks all gain masses and decouple.

Now consider the $SU(N)$ part. The kinetic terms already identify $g_{ab} = \tau_2 K_{ab}$, so in order to reproduce the scalar potential 3.20, the above together with eq. 3.29 suggest the identification

$$|\boldsymbol{\xi}_a^{(2)}| \leftrightarrow \frac{4\pi}{\sqrt{2}}\left|\frac{\partial W_{\rm eff}}{\partial X^a}\right|. \tag{3.48}$$

Defining a rescaled superpotential $\hat{W}_{\rm eff} = 4\pi W_{\rm eff}$ (noting that $W^a| = i\sqrt{2}X^a$), this implies

$$\hat{W}_{\rm eff} \supset (e\,W^\circ + m\,\mathcal{F}_\circ)| + \ldots. \tag{3.49}$$

Hence a reasonable guess is that in order to preserve an $\mathcal{N} = 1$ SUSY gauge theory with an effective rescaled superpotential $\hat{W}_{\rm def}$ for the traceless $SU(N)$ adjoint matter (which we will henceforth denote $\tilde{X}$), one should take

$$\mathcal{F}(W) = \frac{\tau}{2}\,W^a W^a + \frac{W^\circ}{\Lambda^2}\hat{W}_{\rm def}, \tag{3.50}$$

where $\Lambda^2 = m$ (which has dimension 2) is the scale of new physics integrated out to form the effective prepotential, and the conditions above give $\mathrm{Im}(\tau) = -\frac{\xi}{m}$. For example deformations of the Kutasov type would be encoded by simply choosing,

$$\hat{W}_{\rm def} \supset 4\pi \frac{\kappa}{k+1} {\rm tr}\, \tilde{X}^{k+1}. \tag{3.51}$$

Note that in order to reduce clutter, until further notice the κ we refer to will be the holomorphic coupling, not the running coupling of the canonically normalised theory.

Let us check that the $\mathcal{N}=1$ scalar lagrangian is recovered in the decoupling limit with this prepotential. Sending e, m, ξ to infinity and keeping τ finite, from eq. 3.50 we have

$$g^{ab} = \frac{1}{\tau_2^2} \begin{pmatrix} \tau_2 + \frac{1}{m^2 \tau_2} {\rm Im}(\partial_{\tilde{a}} \hat{W}_{\rm def})^2 & -\frac{1}{m} {\rm Im}(\partial_{\tilde{a}} \hat{W}_{\rm def}) \\ -\frac{1}{m} {\rm Im}(\partial_{\tilde{b}} \hat{W}_{\rm def}) & \tau_2 + \frac{1}{m^2 \tau_2} {\rm Im}(\partial_{\tilde{a}} \hat{W}_{\rm def})^2 \end{pmatrix} + \dots . \tag{3.52}$$

After inserting this into eq. 3.46, multiple cancellations eventually yield

$$4\pi V \supset \frac{2}{\tau_2} \left| \frac{1}{i\sqrt{2}} \frac{\partial \hat{W}_{\rm def}}{\partial X^a} + \mathcal{Q}_a^{(3)} - i \mathcal{Q}_a^{(2)} \right|^2 . \tag{3.53}$$

Consulting eq. 3.30 we see that $\mathcal{Q}_a^{(3)} - i\mathcal{Q}_a^{(2)} = 2\pi i (Q^1 - Q^2)(\overline{Q^1} + \overline{Q^2})$. Therefore the $\mathcal{N}=1$ superfields can be identified as

$$Q \equiv \frac{1}{\sqrt{2}} (Q^1 - Q^2)\, ; \tilde{Q} \equiv \frac{1}{\sqrt{2}} (\overline{Q^1} + \overline{Q^2})\, , \tag{3.54}$$

and we find

$$V \supset \frac{4\pi}{\tau_2} \left| \partial_a W_{\rm def} + \sqrt{2} Q t_a \tilde{Q} \right|^2 , \tag{3.55}$$

in accord with the $\mathcal{N}=1$ expression in eq. 3.20. The $U(1)_R$ symmetry of the $\mathcal{N}=1$ theory is then identified with the σ^1 generator of $SU(2)_R$, under which Q and $\tilde{Q}$ have the same charge. As discussed above, on the coulomb branch we have $X^\circ > g\,\xi$ for stability, so the quarks will decouple as well, although one can arrange to keep them in the spectrum by choosing $g_\circ \ll g_{SU(N)}$.

3.3.4 Gauginos

The terms providing the fermion contributions coming from the partial SUSY breaking 3.21 are

$$4\pi \mathcal{L}_{\rm fermion} = \frac{i}{2} \mathbf{D}^{a,\,A} |\, \mathcal{F}_{abc} |\, (\lambda^b \lambda^c)^A + {\rm h.c.}. \tag{3.56}$$

Together with the yukawa interaction

$$\frac{i}{\sqrt{2}}\, g_{ab}\, f^b_{cd}\, \lambda^{a,i}\, \bar{X}^c\, \lambda^d_i + {\rm h.c.}$$

these give rise to the adjoint fermion masses. Since we are only interested in the phase where $\langle X^{\hat{a}} \rangle = 0$, we can ignore the yukawa term for a spectrum analysis for the $SU(N_c)$ part. For the

$U(1)_\circ$ theory this coupling does not exist because there are no abelian self interactions. Noting that $\langle \mathcal{F}_{\hat{a}\circ\circ}\rangle = 0$, we can decompose 3.56 into the $U(1)_\circ$ and $SU(N_c)$ parts as

$$- \mathcal{L}_{\text{fermion}} = \frac{1}{2} M_\circ^{ij} \lambda_i^\circ \lambda_j^\circ + \frac{1}{2} M^{ij} \lambda_i^{\tilde{a}} \lambda_j^{\tilde{a}} + \text{h.c.} \tag{3.57}$$

where the fermion mass matrices are

$$M_\circ^{ij} = \frac{i\, g^{\circ\circ}}{4\pi} \begin{pmatrix} e + m\,\bar{\mathcal{F}}_{\circ\circ}| & -i\,\xi \\ -i\,\xi & e + m\,\bar{\mathcal{F}}_{\circ\circ} \end{pmatrix}^{ij} \mathcal{F}_{\circ\circ\circ}, \quad M^{ij} = \frac{i\, g^{\circ\circ}}{4\pi} \begin{pmatrix} e + m\,\bar{\mathcal{F}}_{\circ\circ} & -i\,\xi \\ -i\,\xi & e + m\,\bar{\mathcal{F}}_{\circ\circ} \end{pmatrix}^{ij} \mathcal{F}_{\circ\tilde{a}\tilde{a}}|. \tag{3.58}$$

In the vacuum determined above 3.3.3 these become

$$M_\circ^{ij} = -\frac{m}{4\pi} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{ij} \langle \mathcal{F}_{\circ\circ\circ}\rangle, \qquad M^{ij} = -\frac{m}{4\pi} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{ij} \langle \mathcal{F}_{\circ\tilde{a}\tilde{a}}\rangle. \tag{3.59}$$

Note that the latter term can be rewritten as

$$M^{ij} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{ij} \frac{\partial^2 W_{\text{def}}}{\partial X^{\tilde{a}} \partial X^{\tilde{a}}} \tag{3.60}$$

as expected.

Since for m, $\langle \mathcal{F}_{\circ\circ\circ}\rangle$, and $\langle \mathcal{F}_{\circ\tilde{a}\tilde{a}}\rangle$ all non-zero we have

$$\det\, M_\circ \;=\; \det\, M \;=\; 0, \qquad \operatorname{tr} M_\circ \neq 0, \qquad \operatorname{tr} M \neq 0, \tag{3.61}$$

the $U(1)_\circ$ fermions and the $SU(N_c)$ fermions each have one linear combination that corresponds to a massless eigenstate, and one linear combination that corresponds to an eigenstate of mass $\frac{m\langle \mathcal{F}_{\circ\circ\circ}\rangle}{2\pi}$ and $\frac{m\langle \mathcal{F}_{\circ\tilde{a}\tilde{a}}\rangle}{2\pi} = \partial_{X^{\tilde{a}}} \partial_{X^{\tilde{b}}} W_{\text{def}}$ respectively. The massless $U(1)_\circ$ combination is the Nambu-Goldstone fermion of partial SUSY breaking, and the massless $SU(N_c)$ combination is the gaugino of the unbroken gauge symmetry[5]. In the $\mathcal{N} = 1$ preserving vacuum, note that the massless $SU(N_c)$ gaugino does not enter the superpotential, only the (potentially) massive $SU(N_c)$ combination will.

3.4 $\mathcal{N} = 2 \to \mathcal{N} = 0$: gaugino masses from additional $U(1)$'s

Having learned how to write $\mathcal{N} = 1$ SQCD+X theories as the low energy limit of spontaneously broken $\mathcal{N} = 2$ theories, we are in a position to deform the theory further with soft perturbations that arise from the complete spontaneous breaking of SUSY from $\mathcal{N} = 2 \to \mathcal{N} = 0$ by the same

[5] This can be seen by calculating the SUSY transformations where one finds [74]

$$\langle \delta_Q \lambda^\circ_{\text{massless}}\rangle \sim \langle \mathbf{D}^\circ_{\text{massless}}\rangle \neq 0, \qquad \langle \delta_Q \lambda^{\tilde{a}}_{\text{massless}}\rangle \sim \langle \mathbf{D}^{\tilde{a}}_{\text{massless}}\rangle = 0. \tag{3.62}$$

mechanism. In the present context we are particularly focussed on Dirac gaugino masses so it is useful to begin with some general observations.

We will be thinking of the additional $U(1)$'s as a perturbation on the $\mathcal{N}=1$ theory (in the sense that $m_D \ll \Lambda$) and will take the FI-terms for $U(1)_\circ$ to be as described above. Although Dirac mass-terms can famously preserve an R-symmetry, in the context of Kutasov duality they will break it (since the $\mathcal{N}=1$ gauginos have R-charge 1 and therefore the Dirac mass requires $\tilde{X}$ to have R-charge zero, in conflict with $W_{\text{def}} \supset \kappa X^{k+1}$). Therefore the FI-terms for the new $U(1)$'s must have some component along the σ^1 direction of $SU(2)_R$ which as we saw in section 3.3.3 is the $U(1)_R$ direction of the $\mathcal{N}=1$ theory. Furthermore the contribution from FI-terms to the fermion mass matrix M^{ij} are $M^{ij} \sim \boldsymbol{\xi}^A(\sigma^A\varepsilon)^{ij}$ where ε is the $SU(2)_R$ metric. But the stability condition essentially fixes $\boldsymbol{\xi}$ to be null. We can parameterise this generally by taking $\boldsymbol{\xi}^A = (\alpha, i\sqrt{\alpha^2+\beta^2}, \beta)$ regardless of the origin of α and β. The stability conditions for $\boldsymbol{\xi}$ then simply fix the VEVs of the F_{abc} to satisfy this condition (the specific case above has $\alpha=0$, $\beta=\xi$). Shifting to the basis in which the $\mathcal{N}=1$ created by $U(1)_\circ$ is diagonal, we find that additional terms from a single extra $U(1)$ are of the form

$$\delta M^{ij} \sim \begin{pmatrix} -\beta+\sqrt{\alpha^2+\beta^2} & -\alpha \\ -\alpha & \beta+\sqrt{\alpha^2+\beta^2} \end{pmatrix}. \tag{3.63}$$

Clearly for any choice of α and β one can never set the δM^{11} and δM^{22} components to zero unless α is zero as well, and it is therefore impossible to introduce a pure Dirac mass with a single extra $U(1)$. On the other hand it is always possible (by tuning parameters) to do this with two extra $U(1)$'s.

Consider therefore an $SU(N_c)\times U(1)_\circ \times U(1)_{\text{𝅗𝅥}} \times U(1)_{\text{♩}}$ theory, where the Q^+ is charged under only the $U(1)_\circ$ as displayed in table 6. This theory is in the same form as in 3.4 and 3.5 with the

	$SU(N_c)$	$U(1)_\circ$	$U(1)_{\text{𝅗𝅥}}$	$U(1)_{\text{♩}}$	$SU(N_f)$
Q^+	$\square$	1	0	0	$\square$

Table 6. *$\mathcal{N}=2$ superfield representations in $\mathcal{N}=2$ SQCD coupled to $U(1)_\circ \times U(1)_{\text{𝅗𝅥}} \times U(1)_{\text{♩}}$.*

prepotential $\mathcal{F}(W, W^\circ, W^{\text{𝅗𝅥}}, W^{\text{♩}})$ again being a generic function of $\mathcal{N}=2$ gauge superfields, and the gauge covariant derivative acting on the hypermultiplets remaining unchanged. The corresponding additional FI-pieces in the action take the same form as in equations 3.24 and 3.28 with the obvious replacement of gauge group. As we mentioned in the preamble to this section, the vacuum stability conditions in the $\mathcal{N}=0$ theory still set

$$\langle \mathbf{D}^{\underline{a},A}\mathbf{D}^{\underline{a},A}\rangle = 0 \tag{3.64}$$

for $\underline{a}$'s corresponding to each of the $U(1)$ factors, where as before there is summation over A but not over $\underline{a}$.

There are many combinations that one could consider for the prepotential and the new FI-terms. A simple solution is to allow only $\mathcal{F}_{\circ\text{𝅗𝅥}}$ and $\mathcal{F}_{\circ\text{♩}}$ mixing, and just electric FI-terms for the $U(1)_{\text{𝅗𝅥}}$ and $U(1)_{\text{♩}}$ factors in the σ^1 and σ^2 directions (i.e. we are going to add two $\beta = 0$ type solutions and make the Majorana masses cancel). The three vacuum stability equations then translate into the following conditions;

$$g_{\circ\circ}\text{Re}(\xi^{(2)}_{D,\circ}) = \text{Re}(\xi^{(3)}_{\circ}) \ ; \ g_{\circ\text{𝅗𝅥}}\text{Re}(\xi^{(2)}_{D,\circ}) = \text{Re}(\xi^{(1)}_{\text{𝅗𝅥}}) \ ; \ g_{\circ\text{♩}}\text{Re}(\xi^{(2)}_{D,\circ}) = -\text{Re}(\xi^{(1)}_{\text{♩}})\,. \tag{3.65}$$

The first of these is essentially the same condition as in eq. 3.42. The imaginary parts can be set to satisfy the zero vacuum energy conditions if desired. In order to get non-zero gaugino masses the prepotential is of the form

$$\mathcal{F}(W) = \frac{\tau_{ab}}{2}\,W^aW^b + \frac{W^\circ}{\Lambda^2}\hat{W}_{\text{def}} + \frac{1}{2\Lambda}(W^{\text{𝅗𝅥}} - W^{\text{♩}})W^{\tilde{a}}W^{\tilde{a}}\,, \tag{3.66}$$

where $\tau_{ab} = \mathcal{F}_{ab}|$, and we neglect higher order terms in the leading part. Note that the mass-inducing third term only involves the two additional $U(1)$'s. The contribution to the gaugino masses is of the form

$$\delta M^{ij} = -\frac{(\sigma^A\varepsilon)^{ij}}{4\pi\Lambda}\left\{\boldsymbol{\xi}^A_\circ(g^{\circ\text{𝅗𝅥}} - g^{\circ\text{♩}}) + (g^{\text{𝅗𝅥}\text{𝅗𝅥}}\boldsymbol{\xi}^A_{\text{𝅗𝅥}} - g^{\text{♩}\text{♩}}\boldsymbol{\xi}^A_{\text{♩}})\right\}\,. \tag{3.67}$$

In order to forbid additional $\mathcal{N} = 1$ mass terms for the adjoints $X^{\tilde{a}}$, we must choose $g^{\circ\text{𝅗𝅥}} = g^{\circ\text{♩}}$ to make the first term vanish. By eq. 3.65 we then have $\boldsymbol{\xi}^{(1)}_{\text{𝅗𝅥}} = -\boldsymbol{\xi}^{(1)}_{\text{♩}}$. Choosing for simplicity $g_{\circ\text{𝅗𝅥}} = g_{\circ\text{♩}} \ll g_{\circ\circ}, g_{\text{𝅗𝅥}\text{𝅗𝅥}} = g_{\text{♩}\text{♩}}$ together with $g_{\text{𝅗𝅥}\text{♩}} = 0$, we then have $g_{\circ\text{𝅗𝅥}} = g_{\circ\text{♩}} \equiv -\alpha/m$. Hence $\boldsymbol{\xi}_{\text{𝅗𝅥}} = (\alpha, i\alpha, 0)$ and $\boldsymbol{\xi}_{\text{♩}} = (-\alpha, i\alpha, 0)$, giving a gaugino mass matrix of the form

$$\delta M^{ij} = -\frac{\alpha}{2\pi\Lambda}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{3.68}$$

as required. Along with these terms we expect the super-soft operators of [4] to be induced in the scalar potential. Consulting eq. 3.46 it is clear that these arise from the cross terms $g^{\circ\text{𝅗𝅥}}\mathcal{Q}^\dagger_{\text{𝅗𝅥}}\boldsymbol{\xi}_\circ + g^{\circ\text{♩}}\mathcal{Q}^\dagger_{\text{♩}}\boldsymbol{\xi}_\circ + \text{h.c.}$.

It is much easier of course to generate pure Majorana masses: it requires only a single additional $U(1)_{\text{𝅗𝅥}}$, and a prepotential of the form

$$\mathcal{F}(W) = \frac{\tau_{ab}}{2}\,W^aW^b + \frac{W^\circ}{\Lambda^2}\hat{W}_{\text{def}} + \frac{1}{2\Lambda}W^{\text{𝅗𝅥}}W^{\tilde{a}}W^{\tilde{a}}\,, \tag{3.69}$$

choosing FI-terms such that $\alpha = 0$ in eq. 3.63. Furthermore, to avoid this becoming just another $\mathcal{N} = 1$ mass-term for the adjoint fields, the sign of β is chosen so that the non-zero eigenvalue falls in the block that has just been identified by the $U(1)_\circ$ FI-terms as belonging to the $\mathcal{N} = 1$

gauginos. That is with $\boldsymbol{\xi}_{\circ}^{A} = (0, i\xi, \xi)$ we choose $\boldsymbol{\xi}_{\lrcorner}^{A} = (0, i\beta, -\beta)$, with both ξ and $\beta > 0$.

4 Duality relations for the $\mathcal{N}=2 \to \mathcal{N}=0$ theory

4.1 $\mathcal{N}=1$ couplings and gaugino masses

Let finally return to our objective, which (recall) is to determine how couplings as well as Dirac gaugino masses map under $\mathcal{N}=2$ duality, and that the prepotential maps consistently under $\mathcal{N}=2$ duality. We should at this point make clear that we are not about to solve the $\mathcal{N}=2$ system for arbitrary numbers of colours and flavours. Nevertheless it is possible to make general statements about the constraints such a duality should give on the prepotential. This is enough to establish that it contains all the same operators as the weakly coupled electric superpotential. After this we can use the spurion technique of [51] to determine the precise coefficients.

The theory can be written in either electric variables

$$W(\phi, \lambda, D, v), \quad \mathcal{F} \tag{4.1}$$

or dual magnetic ones,

$$W_D(\phi_D, \lambda_D, D_D, v_D), \quad \mathcal{F}_D\,, \tag{4.2}$$

with the relations[69]

$$W_D^a = \frac{\partial \mathcal{F}}{\partial W_a}, \qquad W^a = -\frac{\partial \mathcal{F}_D}{\partial W_{D,a}}. \tag{4.3}$$

Differentiating this eq. gives the functional relation $\tau_D = -\tau^{-1}$, for any prepotential. The mapping of the FI-terms is given by

$$\xi \to \xi_D \ , \ \ \xi_D \to -\xi\,. \tag{4.4}$$

Now, it is known that generally the prepotential obeys (in $\mathcal{N}=1$ language) [62]

$$\mathcal{A}_{ii}\frac{\partial \mathcal{F}}{\partial \mathcal{A}_{ii}} - 2\mathcal{F} = 8\pi i \beta u \tag{4.5}$$

where the adjoint modulus u is related to the fields at weak coupling (large $|u|$) as $\mathrm{tr}(\mathcal{A}^2) \approx 2u$. If we set $\beta = 0$, we find that $\mathcal{F} = \frac{\tau}{2}\mathcal{A}^2$, so that the RHS of this eq. is encoding the one-loop running of the $\mathcal{N}=2$ theory. From this eq. we infer

$$W = W_D/(\tau - 8\pi i \beta). \tag{4.6}$$

(The dual version of this eq. has of course a much more complicated u because the theory is strongly coupled.)

Satisfying eq. 4.5 for both electric and magnetic theories gives

$$\mathcal{F}_D = \mathcal{F} - \mathcal{A}_{D,ii}\mathcal{A}_{ii}\,, \tag{4.7}$$

so we can also infer that

$$\mathcal{F}_D(W_D) = \mathcal{F}(W(W_D)) - W_D W(W_D)\,. \tag{4.8}$$

In other words the magnetic prepotential is given by taking the electric one and replacing W with $W(W_D)$ determined as a function of W_D. In general this is extremely complicated, but eq. 4.6 tells us that $W = W_D/(\tau - 8\pi i\beta)$. This is the result we need, because it tells us that, while $\tau(W_D)$ will in general be a complicated function of W_D, it is clear that every operator of the electric theory has a direct equivalent in the magnetic theory.

Indeed, suppose one knows the dual prepotential $\mathcal{F}_D^{(0)}(W_D)$ of an undeformed $\mathcal{N}=2$ theory, with prepotential $\mathcal{F}^{(0)}(W)$. If the theory is then deformed to $\mathcal{F}(W) = \mathcal{F}^{(0)} + \kappa\mathcal{F}_\kappa$, where κ is parametrically small, then in a κ expansion, a dual prepotential of the form

$$\mathcal{F}_D(W_D) = \mathcal{F}^{(0)}(W^{(0)}(W_D)) + \kappa\mathcal{F}_\kappa(W^{(0)}(W_D)), \tag{4.9}$$

where $W^{(0)}(W_D)$ is the function determined from $W_D = \partial\mathcal{F}^{(0)}/\partial W$, is seen to correctly solve equations 4.3 and 4.8 to $\mathcal{O}(\kappa^2)$.

Having established this fact, we can utilise the spurion technique of [51] to fix the coefficients of the terms in the κ-deformation of the magnetic prepotential. The technique used there extends trivially to give two sets of invariants in the $\mathcal{N}=2$ theory, namely gaugino mass invariants of the form m_{gaugino}/g^2 and κ invariants of the form κ/g^{k+1} where we now switch back to κ being the physical coupling in the canonically normalised theory. Hence the combination $m_D/g\kappa^{\frac{1}{k+1}}$ is obviously also invariant. Focusing on the Dirac mass, we see that m_D/g^2 is an RG invariant but of course only in the $\mathcal{N}=2$ theory (as in [51]); away from $\mathcal{N}=2$, the h and g couplings go their separate ways and m_D/g^2 will begin to pick up corrections of order κ^2, but as we know the combination $m_D/g\kappa^{\frac{1}{k+1}}$ remains an RG invariant even as we flow back to $\mathcal{N}=1$.

The dual prepotential required for this mapping to be correct is of the form

$$\mathcal{F}(W_D) = \mathcal{F}^{(0)}(W_D) + \frac{W_D^{\circ}}{\Lambda_D^2}\hat{W}_{\text{def}}(W_D) + \frac{1}{2\Lambda_D}(W_D^{\text{𝅗𝅥}} - W_D^{\text{♩}})W_D^{\tilde{a}}W_D^{\tilde{a}} + \mathcal{O}(\kappa^2)\,, \tag{4.10}$$

where $\partial^2_{W_D}\mathcal{F}_0(W_D)| = -\tau|^{-1}$, and the magnetic scale is $\Lambda_D = -(e+i\xi)$. To check that the latter is correct, we can study the dual version of the pre-factor in $\mathcal{L}_{\text{fermion}}$ of eq. 3.56 which is

$$g^{\circ\circ}\boldsymbol{\xi}_\circ^A F_{\circ\tilde{a}\tilde{a}} = -m(0,i,1)\mathcal{F}_{\circ\tilde{a}\tilde{a}}. \tag{4.11}$$

In the dual variables we first note that the stability conditions for $\boldsymbol{\xi}_\circ^A = (0,-m,0) + (0,e,\xi)\bar{\mathcal{F}}_{D,\circ\circ}$, consistently give $\mathcal{F}_{D,\circ\circ} = m/(e+i\xi) = -1/\mathcal{F}_{\circ\circ}$. Then straightforward manipulation leads to

$$\tilde{g}^{\circ\circ}\tilde{\boldsymbol{\xi}}_\circ^A\mathcal{F}_{D,\circ\tilde{a}\tilde{a}} = (e+i\xi)(0,i,1)\mathcal{F}_{D,\circ\tilde{a}\tilde{a}}. \tag{4.12}$$

Note that the magnetic FI-term is usurped by electric ones. Comparison with eq. 4.11 shows that one factor of τ correctly cancels from the dual coupling, $\Lambda_D^2 = \tau\Lambda^2$. It is also straightforward to check, although we do not show it explicitly (it is quite a bit more tedious as we need to solve the dual stability conditions with all three FI-terms), that the tuning of FI-terms that gave Dirac masses in the electric theory is the correct tuning for Dirac masses in the magnetic theory – i.e. we consistently map Dirac to Dirac gaugino, and Majorana to Majorana.

4.2 Quarks under Electric-Magnetic duality

Let us briefly comment on the mapping of the quark hypermultiplet Q^+ under the $\mathcal{N} = 2$ S-duality. By considering finiteness, the mapping of gauge invariants, and requiring that known non-self dual points are not mapped onto each other, refs.[86, 87] argue that a natural map for $SU(N_c)$ $\mathcal{N} = 2$ SQCD deformed by a mass for the chiral adjoint in the unbroken phase is into a similar theory $SU(N_c)$ $\mathcal{N} = 2$ SQCD$'$ with the charge conjugation acting on the flavour structure. The new hypermultiplets q^+ are interepreted as the general N_c case of the semi-classical monopoles of [60, 61], and the mass for the chiral adjoint is mapped to itself. For our purposes, we have already shown that a mass for the chiral adjoint is mapped to itself in section 4.1, and so we expect the conclusions of [86, 87] to apply here as well.

5 Conclusions

We have presented evidence for the invariance of

$$\frac{m_D}{g\kappa^{\frac{1}{k+1}}} \tag{5.1}$$

under Kutasov duality, where m_D is the Dirac gaugino mass. This was achieved by flowing to the $\mathcal{N} = 2$ dual theories in which the $\mathcal{N} = 1$ deformations and gaugino masses were generated by FI-terms of additional $U(1)$ factors coupling in deformations in the prepotential. Along the way, we discussed the generalities of embedding $\mathcal{N} = 1$ terms within manifestly $\mathcal{N} = 2$ supersymmetric theories using the techniques of harmonic superspace. Although the formalism is somewhat cumbersome, it has the advantage that quarks can be treated appropriately off-shell, $SU(2)_R$ symmetry breaking is made manifest and dynamical, and the interplay between $\mathcal{N} = 0$ terms (i.e. gaugino masses) and $\mathcal{N} = 1$ terms is evident. Aside from its obvious direct application to Dirac gaugino phenomenology, our results could therefore be useful for constructing an entirely dynamical realisation of $\mathcal{N} = 2$ sectors within an $\mathcal{N} = 1$ theory, as has often been proposed for the higgs and gauge sectors (see [4] and more recently [27]).

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A Index and $SU(2)_R$ conventions

Our index conentions are

Label	Type	Range
μ, ν, ρ, σ	space-time	0 to 3
$\alpha, \dot\alpha, \beta, \dot\beta$	spinor	$1, 2$
i, j, k, l	$SU(2)$	$1, 2$
$\tilde a, \tilde b, \tilde c, \tilde d$	$SU(N_c)$ adjoint	1 to $(N_c^2 - 1)$
a, b, c, d	all adjoints	𝅝, 𝅗𝅥, ♩, 1 to $(N_c^2 - 1)$

Our $SU(2)_R$ conventions are $\varepsilon^{12} = +1$, and that if $a^i{}_j \equiv i\, a^A (\sigma^A)^i{}_j$ then clearly $a^A = \frac{1}{2i}\mathrm{tr}(\sigma^A\, a)$, and in components

$$a^i{}_j = \begin{pmatrix} i\,a^3 & i\,a^1 + a^2 \\ i\,a^1 - a^2 & -i\,a^3 \end{pmatrix}, \quad a^{ij} = \begin{pmatrix} i\,a^1 + a^2 & -i\,a^3 \\ -i\,a^3 & -i\,a^1 + a^2 \end{pmatrix}, \quad a_{ij} = \begin{pmatrix} -i\,a^1 + a^2 & i\,a^3 \\ i\,a^3 & i\,a^1 + a^2 \end{pmatrix}. \tag{A.1}$$

B HSS notation

B.1 Conjugation rules

Complex conjugation $\bar{\mathcal{O}}$ is defined as

$$\overline{\theta_{\alpha i}} = \bar\theta^i_{\dot\alpha}, \qquad \overline{\theta^i_\alpha} = -\bar\theta_{\dot\alpha i}; \tag{B.1}$$

$$\overline{u^{+i}} = u^-_i, \qquad \overline{u^+_i} = -u^{-i}; \tag{B.2}$$

$$\overline{f^{i_1\ldots i_n}} \equiv \bar f_{i_1\ldots i_n}, \qquad \overline{f_{i_1\ldots i_n}} = (-1)^n \bar f^{i_1\ldots i_n}. \tag{B.3}$$

Antipodal conjugation $\mathcal{O}^\star$

$$(u^{+i})^\star = u^{-i}, \qquad (u^+_i)^\star = u^-_i, \tag{B.4}$$

$$(u^{-i})^\star = -u^{+i}, \qquad (u^-_i)^\star = -u^+_i. \tag{B.5}$$

Combined complex and antipodal conjugation $(\bar{\mathcal{O}})^\star = \overline{(\mathcal{O})^\star} \equiv \widetilde{\mathcal{O}}$

$$\widetilde{(u^\pm_i)} = u^{\pm i}, \qquad \widetilde{(u^{\pm i})} = -u^\pm_i. \tag{B.6}$$

It is convenient to note that

$$\overline{Q^1} = \bar Q_1 = \varepsilon_{12}\,\bar Q^2 = -\bar Q^2 = \overline{Q_2}, \qquad \overline{Q^2} = \bar Q_2 = \varepsilon_{21}\,\bar Q^1 = \bar Q^1 = -\overline{Q_1}. \tag{B.7}$$

B.2 Basis and measures

The harmonic analytic basis is defined as

$$x_A^\mu \equiv x^\mu - 2i\theta^{(i}\sigma^\mu\bar{\theta}^{j)}u_i^+u_j^-\,, \qquad (x_A, \theta^+\bar{\theta}^+, u_i^\pm) \equiv (\zeta, u)\,. \tag{B.8}$$

The covariant derivative and scalar projection are

$$\mathcal{D}^{++} = D^{++} + iV^{++}, \qquad \mathcal{O}| \equiv \mathcal{O}|_{\theta^\pm=\bar{\theta}^\pm=0}. \tag{B.9}$$

The measures are defined as

$$\int du\, d^{12}X \equiv \int du\, d^4x\, d^8\theta = \int du\, d^4x_A\, d^4\theta^+ d^4\theta^- = \frac{1}{256}\int du\, d^4x_A\, (D^-)^2(\bar{D}^-)^2(D^+)^2(\bar{D}^+)^2, \tag{B.10}$$

$$\int du\, d\zeta^{(-4)} \equiv \int du\, d^4x_A\, d^4\theta^+ = \frac{1}{16}\int du\, d^4x_A\, (D^-)^2(\bar{D}^-)^2, \tag{B.11}$$

with normalisations

$$\int d^8\theta\, \theta^8 = \int d^4\theta^+\, (\theta^+)^4 = \int d^4\theta\, (\theta)^4 = \int d^4\bar{\theta}\, (\bar{\theta})^4 = 1. \tag{B.12}$$

where

$$\theta^8 = (\theta^+)^4(\theta^-)^4 = (\theta)^4(\bar{\theta})^4, \qquad (\theta^\pm)^4 = (\theta^\pm)^2(\bar{\theta}^\pm)^2, \tag{B.13}$$

$$(\theta)^4 = (\theta^+)^2(\theta^-)^2, \qquad (\bar{\theta})^4 = (\bar{\theta}^+)^2(\bar{\theta}^-)^2. \tag{B.14}$$

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