

# The Marginal Cost of Public Funds in Growing Economies

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## Abstract

The marginal cost of public funds (*MCF*) measures the cost to the economy of raising government revenue. The *MCF* can be used to guide reform of the tax system and to determine an efficient level of government expenditure. It can also be used as an input into cost-benefit analysis. Previous applications of the concept have developed a methodology in a context of a static economy. We develop the methodology of the *MCF* to extend the concept to growing economies. The extended concept is then applied to variants of the Barro endogenous growth model with a productive public input. The *MCF* is used to address the choice between labour and capital taxes and to explore the implications of infrastructural spill-overs across regions.

*JEL Classification:* E6, H4, H7

*Key Words:* Growth, externalities, infrastructure, public funds

## 1 Introduction

All implementable government tax instruments are distortionary and, as a consequence, impose deadweight losses upon the economy. These distortions are the inevitable cost of collecting the finance required to support public spending.

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The correct level of spending achieves a compromise between these distortions and the benefits of spending. There are large literatures on cost-benefit analysis (Mishan and Quah, 2007) and optimal taxation (Myles, 1995) that describe how this should be done. The Marginal Cost of Public Funds (*MCF*) provides the link between these literatures since it measures the cost of raising revenue which is determined by the degree of efficiency of the tax system. The existing literature on the *MCF* is almost entirely restricted to static economies.<sup>1</sup> In contrast, there is now a considerable body of work on optimal tax structures in growing economies. The purpose of this paper is to show how the concept of the *MCF* can be extended and applied to economies with growth. The results from using the extended *MCF* are then linked to the Chamley-Judd (Chamley 1986, Judd 1985) results on optimal taxation.

What matters for the spending decision is whether the marginal benefit of spending exceeds the marginal cost of taxation. If it does, then additional public spending is justified. This principle of contrasting marginal benefit and marginal cost underlies the methodology of cost-benefit analysis. The survey of Drèze and Stern (1987) provides a summary of the standard approach to cost-benefit analysis that emphasizes the role of the *MCF*. That approach is based on the Arrow-Debreu representation of the competitive economy and its extensions. The generality of the model permits it to encompass many situations but for specific problems alternative models can be advantageous. The questions we wish to focus upon involve economic growth. Although growth can be handled by dating commodities in the Arrow-Debreu framework it does seem preferable to employ a more specific model of the growth process. The models we analyze in this paper are extensions of the Barro model of endogenous growth with productive public expenditure. We also combine the endogenous growth model with fiscal federalism and the public input interpreted as a form of infrastructure that has spill-overs between jurisdictions.

A government has access to a wide range of different tax instruments. Taxes can be levied on consumption or on income. Different forms of consumption, and different sources of income, can be taxed at different rates. Taxes can also be levied on firms, using profit, turnover, or input use as a base. The cost of collecting revenue will depend upon the tax base that is chosen and the structure of rates that are levied. Each tax instrument has an associated *MCF* which is a measure of the cost of raising tax revenue using that tax. The values of the *MCF* for individual tax instruments can be used to identify changes to the tax structure that raise welfare keeping expenditure constant. They can also identify the best tax instruments to use for raising additional revenue. When the tax system is efficient the value of the *MCF* is equalized across tax instruments and can be used to determine the optimal level of expenditure. The *MCF* is a practical tool that permits consistent analysis of taxation choices. Dahlby (2008) provides a detailed summary of the existing methods for calculating the *MCF* in a wide variety of circumstances and for a range of tax instruments. However, there is very little literature on the derivation of the *MCF* for a growing economy.

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<sup>1</sup>The two exceptions are Dahlby (2006) and Liu (2003).

Section 2 of the paper provides a brief review of the *MCF* from a cost-benefit perspective and a summary of several typical applications. Some of the issues involved in applying the *MCF* to growing economies are discussed in Section 3. A general discussion of the *MCF* for a growing economy is given in Section 4. Section 5 analyses a basic model of endogenous growth and Section 6 extends the model to incorporate infrastructural spill-overs and tax externalities. Section 7 provides concluding comments.

## 2 Application of the *MCF*

The *MCF* provides a numerical summary of the welfare cost of raising additional revenue from each tax instrument. Consider the effect of a marginal increase in a single rate of tax. This has two effects: it will raise additional revenue for the government (provided the economy is on the “correct” side of the Laffer curve) but will reduce social welfare. The *MCF* is defined as (minus) the ratio of the welfare effect to the revenue effect so it gives the welfare reduction per unit of additional revenue.

A formal construction of the *MCF* can be provided as follows. Consider an economy that can employ  $m$  tax instruments to finance  $g$  public goods. Denote the level of tax instrument  $i$  by  $\tau_i$  and the quantity of public good  $j$  by  $G_j$ . Welfare is given by the social welfare function  $W(\tau, G)$  where  $\tau = (\tau_1, \dots, \tau_m)$  and  $G = (G_1, \dots, G_g)$ , the level of revenue by  $R(\tau, G)$ , and the cost of the public good supply by  $C(G)$ . Note that there is an interaction in the revenue function between taxes and public good supply but the cost of public goods is determined by technology alone.

The optimization problem for the government is<sup>2</sup>

$$\max_{\{\tau, G\}} W(\tau, G) \quad \text{s.t.} \quad R(\tau, G) \geq C(G). \quad (1)$$

To analyze the solution to this optimization define  $\lambda_i$  by

$$\lambda_i = -\frac{\partial W / \partial \tau_i}{\partial R / \partial \tau_i}, \quad i = 1, \dots, m. \quad (2)$$

The term  $\lambda_i$  is the Marginal Cost of Funds from tax instrument  $i$ . It measures the cost, in units of welfare, of raising an additional unit of revenue.<sup>3</sup> If the tax system is efficient then  $\lambda_i$  is equalized across the different tax instruments:  $\lambda_i = \lambda$ , for all  $i$ , and equals the Lagrange multiplier associated with the government’s budget constraint.<sup>4</sup> If the value of  $\lambda_i$  differs across tax instruments then welfare can be raised by collecting more revenue from taxes with low (but positive)  $\lambda_i$  and less revenue from those with high  $\lambda_i$ .

<sup>2</sup>The solution to this optimization is studied in detail in Atkinson and Stern (1974).

<sup>3</sup>In applications revenue is typically denominated in monetary units but in this formal (non-monetary) model it is denominated in units of the numeraire commodity.

<sup>4</sup>The converse does not necessarily hold: the common value can be negative, implying that the level of revenue is falling as taxes increase, or even zero. Neither situation is efficient.

Now assume that the tax instruments have been optimized so that  $\lambda_i = \lambda > 0$ . The use of the *MCF* in project choice can be illustrated by writing the optimality condition for the level of public good  $j$  as

$$\frac{\partial W}{\partial G_j} = \lambda \left[ \frac{\partial C}{\partial G_j} - \frac{\partial R}{\partial G_j} \right], \quad j = 1, \dots, g. \quad (3)$$

This conditions says that the quantity of provision of each public good is optimal when the marginal benefit of that extra provision ( $\partial W/\partial G_j$ ) is equal to the net cost of the public good ( $\partial C/\partial G_j - \partial R/\partial G_j$ ) converted into welfare units using the *MCF*. This analysis can be extended to include asymmetric information by adding appropriate incentive compatibility constraints to programme (1). In such a case, however, the optimum need not involve a common value of  $\lambda_i$  for all tax instruments since the multipliers on the incentive compatibility constraints will also enter the optimality conditions.

The properties of  $\lambda_i$  and  $\lambda$  are investigated in Hashimzade and Myles (2013). Assume that  $\partial W/\partial \tau_i$  is negative for low  $\tau_i$  but may become zero for high  $\tau_i$  (if the tax is sufficiently high to discourage the respective activity), and that  $\partial R/\partial \tau_i$  is positive for low  $\tau_i$  (the “upward” slope of the Laffer curve), possibly negative for higher  $\tau_i$ , and, eventually, may become zero. Under these assumptions it is shown that  $\lambda_i$  can, in principle, take any value, including  $\pm\infty$ , or can be undefined (when both  $\partial W/\partial \tau_i$  and  $\partial R/\partial \tau_i$  are zero). It also has a discontinuity at the value of  $\tau_i$  that maximizes revenue (given the value of other tax instruments).

The definition in (2) shows that  $\lambda_i$  and  $\lambda$  are denominated in units of welfare/revenue and that can be used to convert monetary costs into welfare equivalents. Furthermore,  $\lambda_i$  and  $\lambda$  are unique only up to multiplicative transformation, both in the cases when  $W(\tau, G)$  is ordinal or cardinal. That is, given  $n$  tax instruments, for every permissible transformation of the social welfare function there is a choice of  $n$  different numeraire for revenue and each results in a different numerical value for  $\lambda_i$  and  $\lambda$ . Another way to look at this is that  $\lambda_i$  is denominated in units of welfare/revenue, and its value changes when we change the units of measurement for either welfare or revenue. One response to this fact is to accept that  $\lambda_i$  provides only a ranking of tax instruments (and  $\lambda$ , respectively, provides a ranking of the efficient tax systems) for a given transformation of the welfare function and choice of numeraire. In other words, the ranking is ordinal, and so the numerical value is not in itself meaningful. An alternative response is to develop a unit-free form of the *MCF* that is not affected by transformation of the social welfare function or choice of numeraire.

To produce a unit-free version of the  $\lambda_i$  let  $\nu$  be any quantity measured in welfare/revenue units. Then a normalized form of the  $\lambda_i$  is given by

$$\lambda_i^N = \frac{\lambda_i}{\nu}. \quad (4)$$

For given  $\nu$  the value of  $\lambda_i^N$  is uniquely defined since any transformation of the welfare function or choice of numeraire must be applied to  $\nu$  as well as  $\lambda_i$ . However, this does not make the measure unique since the tax rates and public good

levels at which the normalizing factor,  $\nu$ , is evaluated, need to be chosen. Different choice will provide different numerical values of  $\lambda_i^N$ . This point can be seen by considering a model with a representative consumer. In that case one can use the private marginal utility of income<sup>5</sup> as the normalizing factor. The value of the marginal utility will depend on  $\tau$  and  $G$  at which it is evaluated. The choice of specific values, denoted  $\{\hat{\tau}, \hat{G}\}$ , is constrained by the requirement at  $R(\hat{\tau}, \hat{G}) \geq C(\hat{G})$ ; each choice satisfying this condition is equally acceptable but will result in a different numerical value for  $\lambda_i^N$ . Hashimzade and Myles (2013) also observe that a unit-free *MCF* can be obtained by using the compensating variation or equivalent variation to measure welfare change (see also Diamond and McFadden, 1974).

We would like to emphasize that using the comparison of the numerical values for the *MCF* obtained under different normalization and different welfare measures is inappropriate. Furthermore, using the extent to which the *MCF* for a particular tax instrument differs from 1 as the measure of the degree of distortion of that instrument can be misleading. This is an important point since many applications implicitly assume that an *MCF* of 1 represents a non-distortionary tax instrument and that any distortionary tax must have an *MCF* in excess of 1. In fact, as shown in Atkinson and Stern (1974), the *MCF* calculated using normalization by the private marginal utility of income can be less than 1 for a distortionary tax system (labour income tax) if the income effect on labor supply is sufficiently negative. It should also be stressed that the use of “money-metric” welfare measures avoids the issues connected with normalization but raises another difficulty: the aggregation of the money-metric measure across heterogeneous consumers. This is an issue for which there is no compelling resolution (Hammond, 1994).

This non-uniqueness in the definition of the *MCF* is linked to the discussion of the use of the differential approach versus the balanced-budget approach to the analysis of the welfare cost of taxation (Ballard, 1990). Differential analysis compares alternative means of raising the same amount of revenue, whereas the balanced-budget analysis is concerned with a change in the level of revenue and a simultaneous change of the tax system to finance this additional revenue. The numerical value can be very sensitive to the choice of the welfare measure and the reference point, as shown in Triest (1990). While a chosen normalization can be used to compare the extent of welfare loss under two different distortionary tax instruments, it cannot be meaningfully used to measure the extent of welfare loss under one particular distortionary tax system.

To summarize, a set of techniques has been developed in the formal literature that permit the *MCF* to be calculated and to be integrated into cost-benefit analysis. Numerous further examples of such applications are described in the comprehensive text of Dahlby (2008). However, despite this extensive literature there are still a number of significant issues that need to be addressed if the

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<sup>5</sup>The private marginal utility of income is defined as the derivative of the indirect utility function with respect to income.

*MCF* is to be used effectively in policy design.

### 3 The *MCF*, Economic Growth, and Externalities

The analysis of the *MCF* given above is very general and, conceptually, can be applied to any economy. The cost of this generality is an absence of detail concerning the economic model behind the welfare function and the cost function. This observation becomes important when the *MCF* is applied to practical policy questions. For example, applying the *MCF* to the case of an economy with multiple jurisdictions or an economic union, such as the EU, raises several questions. First, the use of a single objective function is an issue in an integrated economic area but with subsidiarity permitting independence in policy. Secondly, the economic integration amplifies externalities between member states of the EU or between jurisdictions in a federal state, that are not captured by the general formulation. Finally, the relationship between fiscal policy and economic growth has emerged in recent years as a focal point for the attention of EU policy makers. In contrast, almost all analysis of the *MCF* has been undertaken for static economies.

The implication of these comments is that there are benefits to be obtained by refining the analysis of the *MCF* to apply in a growth model with multiple jurisdictions. The formal developments of the *MCF* have been based upon the Arrow-Debreu model which can incorporate time in the form of dated commodities but must have all contracts agreed prior to the commencement of economic activity. Consequently, it is not a compelling representation of the growth process. Similarly, the applications of the *MCF* reviewed are typically set within a single-country model. Such a refinement would need to include the role of individual member states and the externalities that link the member states. Dahlby and Wilson (2003) make some progress in this respect by analyzing a model that includes fiscal externalities. Their model has both a central government and a local government. Both governments levy taxes on labour incomes and profit, which gives rise to vertical tax externalities linking the two levels of government. The main result is the demonstration that a local government that does not take into account the effect of its choices on the central government may have an *MCF* that is biased up or down.

Endogenous growth occurs when capital and labour are augmented by additional inputs in a production function that otherwise has non-increasing returns to scale. One interesting case for understanding the link between government policy and growth is when the additional input is a public good or public infrastructure financed by taxation. The need for public infrastructure to support private capital in production provides a positive role for public expenditure and a direct mechanism through which policy can affect growth. Introducing infrastructure permits an analysis of the optimal level of public expenditure in an endogenous growth model.

The importance of infrastructure is widely recognized, not least by the EU which pursues an active programme to support the investment activities of member states. The policy problem facing the EU is to ensure that member states undertake an efficient level of infrastructural expenditure that ensures the maximum rate of growth. The determination of the level has to take into account the full consequences of an infrastructure project for the EU, not just the direct benefits for the member state undertaking the investment. The *MCF* can be used to evaluate public infrastructure provision but its use has to recognize three significant issues. First, infrastructural investment has significant spill-overs across member states. Second, mobility of the tax base results in tax externalities between the member states, and between the member states and the EU. Third, the EU is faced with a decision on how to allocate support for infrastructural expenditure across the different member states. This interacts with the process of revenue-raising, and with the extent to which the projects are financed jointly by the EU and member states. The same is true of the relationship between the local and central government financing in a federal state.

The economic modelling of the impact of infrastructure on economic growth has focussed on the Barro (1990) model of public expenditure as a public input and its extensions (Chen *et al.* 2005, Turnovsky, 1999). This literature has identified the concept of an optimal level of expenditure, and has highlighted the deleterious effects of both inadequate and excessive expenditure. These are important insights, but they do not address the spill-over issues that confront a federation. Infrastructural spill-overs between the members of a federation can be positive, which occurs when improvements in infrastructure in one region raise productivity in another, or they can be negative if they induce relocation of capital between regions. In either case, it is important to develop an appropriate concept of the *MCF* that would correctly capture the benefits and costs of public investment in the presence of spill-overs.

## 4 Dynamic Setting

There has been little investigation of the *MCF* in growing economies. Two exceptions are Liu (2003) who computes the *MCF* as a component of a cost-benefit analysis (but taking the intertemporal path of wage rates and interest rates as exogenous) and Dahlby (2006) who uses the *MCF* to analyze public debt in an *AK* growth model. Our approach is similar to Dahlby but we employ a more general model of endogenous growth. In principle, it is possible to treat the economy lying behind (1) as intertemporal but the analysis needs to be more specific to generate worthwhile conclusions.

Consider an intertemporal economy set in discrete time. The time path for tax instrument  $i$  is a sequence  $\{\tau_i^1, \tau_i^2, \dots\}$ . The *MCF* is computed for a variation in this sequence. A *pulse variation* takes the form of a change in the tax instrument in a single time period,  $t$ . The new sequence would then be  $\{\tau_i^1, \dots, \tau_i^{t-1}, \hat{\tau}_i^t, \tau_i^{t+1}, \dots\}$ . Alternatively, a *sustained variation* in the tax in-

strument from period  $t$  onwards changes the sequence to  $\{\tau_i^1, \dots, \tau_i^{t-1}, \widehat{\tau}_i^t, \widehat{\tau}_i^{t+1}, \dots\}$ . We choose to focus on sustained variations. Correspondingly, we extend the definition of the *MCF* for a static economy to an intertemporal economy by using

$$\lambda_i^t \equiv -\frac{\partial W / \partial \tau_i}{\partial R / \partial \tau_i},$$

to denote the *MCF* of a sustained variation in tax instrument  $i$  from period  $t$  onwards. In this setting  $W$  is the intertemporal social welfare function, and  $R$  is discounted value of tax revenue.

In a dynamic, infinite horizon economy

$$R = \sum_{t=0}^{\infty} d(t) R_t,$$

where  $R_t$  is tax revenue in period  $t$ , and  $d(t)$  is the discount factor applied to revenues in period  $t$ . There are many well-known issues involved in the choice of the sequence of discount factors  $\{d(t)\}$ . We choose to remain with the standard convention (see, for example, Nordhaus, 2008) of appealing to market equilibrium to determine the social rate of time preference endogenously. In this case

$$d(t) = \frac{\partial W / \partial C_t}{\partial W / \partial C_0},$$

where  $C_t$  is consumption at time  $t$ . If the welfare function is time-separable with exponential discounting, that is,  $W = \sum_{t=0}^{\infty} \beta^t U_t(C_t, \cdot)$ , then

$$d(t) = \beta^t \frac{\partial U_t / \partial C_t}{\partial U_0 / \partial C_0}. \quad (5)$$

The purpose of the analysis is to apply the *MCF* in settings where economic growth is occurring. This requires us to embed the *MCF* within a model of endogenous growth. The major difficulty here is that, generally, the entire intertemporal path for the economy must be computed from the present into the indefinite future. To overcome this difficulty we focus upon balanced growth paths. Along a balanced growth path all real variables grow at the same rate, so such a path can be interpreted as describing the pattern of long-run growth. All the commonly used growth models have the property that the economy will converge to a balanced growth path from an arbitrary initial position.

#### 4.1 Public Infrastructure in Barro Model

This section develops the *MCF* for a growing economy by building on the Barro (1990) model of productive public expenditure. In particular, the model is used to illustrate the benefits of focussing on the balanced growth path. This analysis provides the developments that we need to combine endogenous growth with fiscal federalism and infrastructural spill-overs in Section 6.



Public infrastructure is introduced by assuming that the production function for the representative firm at time  $t$  has the form

$$Y_t = AL_t^{1-\alpha} K_t^\alpha G_t^{1-\alpha},$$

where  $A$  is a positive constant and  $G_t$  is the quantity of public infrastructure. The form of this production function ensures that there are constant returns to scale in labour,  $L_t$ , and private capital,  $K_t$ , for the firm given a fixed level of public infrastructure. Although returns are decreasing to private capital as the level of capital is increased for fixed levels of labour and public input, there are constant returns to scale in public input and private capital together.

With  $\tau$  denoting the tax upon output, the profit level of the firm is

$$\pi_t = (1 - \tau) Y_t - r_t K_t - w_t L_t,$$

where  $r_t$  is the interest rate and  $w_t$  the wage rate. Profit maximization requires that the use of capital satisfies the necessary condition

$$\frac{\partial \pi_t}{\partial K_t} = (1 - \tau) \alpha A \left( \frac{G_t}{K_t} L_t \right)^{1-\alpha} - r_t = 0. \quad (6)$$

This can be solved to give

$$r_t = (1 - \tau) \alpha A (g_t L_t)^{1-\alpha}, \quad (7)$$

where  $g_t \equiv G_t/K_t$ .

The firm belongs to a representative infinitely-lived household whose preferences are described by an instantaneous utility function,  $U(C_t, L_t)$ . The household maximizes the infinite discounted stream of utility,  $W = \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$ , subject to the sequence of intertemporal budget constraints

$$C_t + K_{t+1} = (1 - \delta_K + r_t) K_t + w_t L_t + \pi_t, \quad (8)$$

and with the sequence of taxes and government infrastructure taken as given. Here  $\delta_K$  is the rate of depreciation of private capital. We can write the first-order conditions for the optimal consumption path as

$$\frac{\partial U / \partial C_t}{\partial U / \partial C_{t+1}} = \beta (1 - \delta_K + r_{t+1}), \quad (9)$$

The public capital input is financed by the tax on output, and there is no government debt, so that the government budget constraint is

$$G_t = \tau Y_t.$$

On a balanced growth path the real variables  $(Y_t, C_t, K_t, G_t)$  grow at the same constant rate,  $\gamma$ . Markets also clear in every period and  $r_t = r$  for all  $t$ . We assume for this analysis that labour supply is constant, and normalize it to one,  $L_t = 1$ . On a balanced growth path  $C_{t+1} = (1 + \gamma) C_t$  where  $\gamma$  is the rate of

growth. Assuming the logarithmic form for the instantaneous utility function,  $U(C_t) = \ln(C_t)$ , the first-order conditions (9) become

$$\frac{C_{t+1}}{C_t} = \beta(1 - \delta_K + r_{t+1}). \quad (10)$$

Using (7), (8), and (10), the following set of equations obtains for the balanced growth path:

$$c = (1 - \tau)Ag^{1-\alpha} - \delta_K - \gamma, \quad (11)$$

$$\gamma = \beta(1 - \delta_K + (1 - \tau)\alpha Ag^{1-\alpha}) - 1, \quad (12)$$

$$g = (A\tau)^{1/\alpha}, \quad (13)$$

where  $c \equiv C_t/K_t$  is the consumption to capital ratio. The model permits a closed-form solution for all endogenous variables. In particular, for the growth rate we have

$$\gamma = \beta \left( 1 - \delta_K + \alpha A^{1/\alpha} (1 - \tau) \tau^{(1-\alpha)/\alpha} \right) - 1. \quad (14)$$

The growth rate is maximized at  $\tau = 1 - \alpha$ .

## 4.2 MCF with Output Tax

Along the balanced growth path when utility is logarithmic, the discount factor defined in (5) is

$$d = \left[ \frac{\beta}{1 + \gamma} \right]^t, \quad (15)$$

which implies

$$R = \sum_{t=0}^{\infty} \left( \frac{\beta}{1 + \gamma} \right)^t \tau Y_t = \frac{\tau K_0}{1 - \beta} Ag^{1-\alpha}. \quad (16)$$

The *MCF* is calculated with  $g$  constant so assuming the sustained variation in the tax occurs from period 0 onwards:

$$\frac{\partial R}{\partial \tau} = \frac{K_0}{1 - \beta} Ag^{1-\alpha}. \quad (17)$$

For the welfare function with logarithmic utility we have

$$W = \sum_{t=0}^{\infty} \beta^t \ln(C_t) = \frac{\ln(cK_0)}{1 - \beta} + \frac{\beta}{(1 - \beta)^2} \ln(1 + \gamma), \quad (18)$$

where  $c \equiv C_t/K_t$ . This implies

$$\frac{\partial W}{\partial \tau} = -\frac{Ag^{1-\alpha}}{1 - \beta} \left[ \frac{1 - \alpha\beta}{c} + \frac{\alpha\beta^2}{1 - \beta} \frac{1}{1 + \gamma} \right]. \quad (19)$$

Combining (17) and (19) allows the *MCF* to be computed as

$$\lambda = -\frac{\partial W/\partial \tau}{\partial R/\partial \tau} = \frac{1}{K_0} \left[ \frac{1 - \alpha\beta}{c} + \frac{\alpha\beta^2}{1 - \beta} \frac{1}{1 + \gamma} \right]. \quad (20)$$

Following Dahlby (2006) we define the normalized *MCF* by dividing  $\lambda$  in (20) by the marginal utility of income at time 0 (denoted by  $I_0$ ),

$$\lambda^N = -\frac{1}{\partial W/\partial I_0} \frac{\partial W/\partial \tau}{\partial R/\partial \tau}$$

This provides a unit-free measure of the cost of public funds. For the specification of utility in this model the marginal utility of income at time 0 is given by  $1/C_0 = 1/cK_0$ . Hence, using (20),

$$\lambda^N = 1 - \alpha\beta + \frac{\alpha\beta^2}{1 - \beta} \frac{c}{1 + \gamma}. \quad (21)$$

From (11)-(13) it is easy to obtain

$$c = \frac{1 - \alpha\beta}{\alpha\beta} (1 + \gamma) - \frac{1 - \alpha}{\alpha} (1 - \delta_K),$$

and, therefore,

$$MCF^N = \frac{1 - \alpha\beta}{1 - \beta} - \frac{(1 - \delta_K)\beta^2}{1 - \beta} \frac{1}{1 + \gamma}, \quad (22)$$

where  $\gamma$  is given by (14).

Figure 1 plots the values of the key endogenous variables against the level of the tax. Here, and in other model specifications used later in the paper, we employ values for the model's parameters that are broadly consistent with the calibration of business cycle and growth models; see, for example, Cooley and Prescott (1995). In this specification of the economy the level of welfare is an increasing function of the growth rate, and so is at a maximum when the growth rate is highest, that is, at  $\tau = 1 - \alpha$ . The same is true of the consumption to capital ratio and the normalized marginal cost of funds. For low levels of the tax rate growth is negative because of insufficient provision of public infrastructure. It can be seen that  $\lambda^N$  in this calibration is everywhere above 1.

We can now use the expression for  $\lambda^N$  in (21) to verify that the optimality condition (3) holds in this model, which is the implication of the Diamond and Mirrlees (1971) result. To derive the Diamond-Mirrlees result, observe that in the balanced growth path equilibrium  $Y_t = AK_t g^{1-\alpha}$ , and in the dynamic setting the marginal benefit calculated as the infinite discounted sum of the marginal productivities in every time period, with the discount factor (15),

$$\frac{\partial C}{\partial g} = \sum_{t=0}^{\infty} \left( \frac{\beta}{1 + \gamma} \right)^t \frac{dY_t}{dg} = \sum_{t=0}^{\infty} \left( \frac{\beta}{1 + \gamma} \right)^t (1 - \alpha) AK_t g^{-\alpha} = \frac{1 - \alpha}{1 - \beta} AK_0 g^{-\alpha}. \quad (23)$$

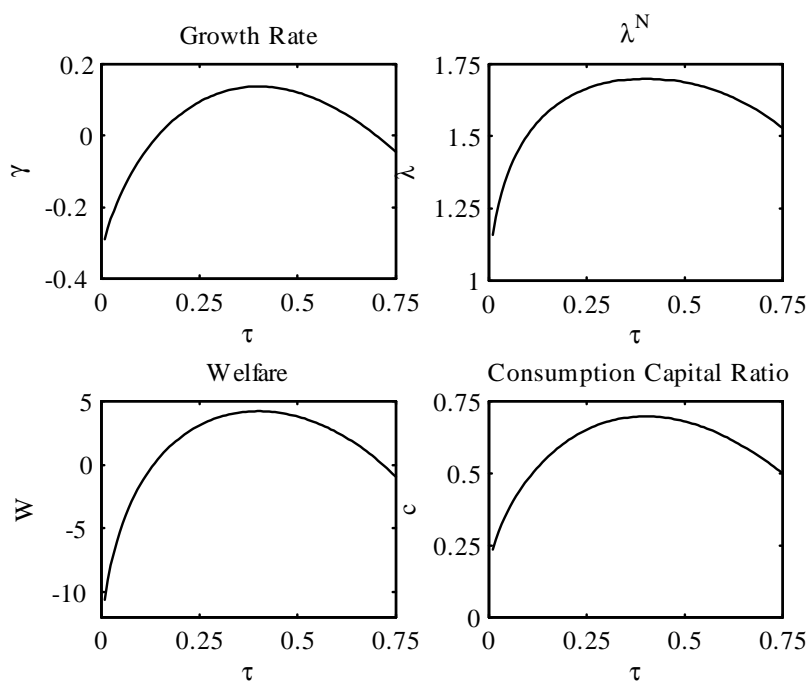


Figure 1: Tax on output ( $\alpha = 0.6$ ,  $\beta = 0.8$ ,  $A = 2$ ,  $\delta_K = 0.2$ ,  $K_0 = 2$ )

With the tax on output the marginal benefit is an increase in welfare (normalized by the marginal utility of income at time 0):

$$MB_g = \frac{\partial W / \partial g}{\partial W / \partial I_0}.$$

From (18),

$$\frac{\partial W}{\partial g} = \frac{1}{1-\beta} \frac{1}{c} \frac{\partial c}{\partial g} + \frac{\beta}{(1-\beta)^2} \frac{1}{1+\gamma} \frac{\partial \gamma}{\partial g},$$

and, using (11)-(12),

$$\frac{\partial W}{\partial g} = \frac{(1-\tau)(1-\alpha)}{1-\beta} Ag^{-\alpha} \left[ \frac{1-\alpha\beta}{c} + \frac{\alpha\beta^2}{1-\beta} \frac{1}{1+\gamma} \right],$$

so that

$$MB_g = \frac{(1-\tau)(1-\alpha)}{1-\beta} AK_0 g^{-\alpha} \left[ 1 - \alpha\beta + \frac{\alpha\beta^2}{1-\beta} \frac{c}{1+\gamma} \right]. \quad (24)$$

The optimality condition requires

$$MB_g = \lambda^N \left( \frac{\partial C}{\partial g} - \frac{\partial R}{\partial g} \right),$$

or, using (16), (21), and (24),

$$\begin{aligned} \frac{\partial C}{\partial g} &= \frac{\partial R}{\partial g} + \frac{MB_g}{\lambda^N} \\ &= \frac{\tau K_0}{1-\beta} (1-\alpha) AK_0 g^{-\alpha} + \frac{(1-\tau)(1-\alpha)}{1-\beta} AK_0 g^{-\alpha} \\ &= \frac{1-\alpha}{1-\beta} AK_0 g^{-\alpha}, \end{aligned}$$

which is exactly the same as (23) for the lump-sum tax financing of the public input, *i.e.* the Diamond and Mirrlees (1971) result that optimal tax system does not distort production decision holds in this situation.

## 5 *MCF* with Input Taxes

We now analyze a dynamic economy described in the previous section under alternative assumptions about financing of the public input in production. In particular, it is interesting to compare the *MCF* in equilibrium under the output tax with the *MCF* in equilibrium under taxes on capital and labour. First, we analyze the case with inelastic labour supply and a tax on private capital. Next, we add labour supply as a choice variable to the model and consider government spending financed by taxes on capital input and labour income. These taxes distort the choice of inputs so the *MCF* will reflect this. We also assume that public infrastructure can be accumulated as a stock.

## 5.1 Capital Tax

In this section we assume that government spending is funded from a tax levied on the private capital input. The consumers' optimization problem remains the same as described in section 4.1. The tax on the private capital input is denoted by  $\tau_K$ <sup>6</sup>. We now assume that the public capital stock accumulates over time by assuming that the depreciation rate is  $\delta_G \in [0, 1]$ . Again, there is no government debt so the government budget constraint at time  $t$  is

$$G_{t+1} = (1 - \delta_G) G_t + \tau_K K_{t+1}.$$

On the balanced growth path the real variables  $(Y_t, C_t, K_t, G_t, w_t)$  grow at the same constant rate,  $\gamma$ . Markets also clear in every period, the interest rate is constant, and the available labour is normalized to one. The normalized version of the *MCF* in this economy is given by

$$\lambda^N = \frac{1 - \alpha\beta}{1 - \beta} + \frac{(1 - \alpha)\beta^2}{1 - \beta} \frac{1 - \delta_K - \tau_K}{1 + \gamma}. \quad (25)$$

The details of the derivation can be found in the Appendix. This model does not permit a closed-form solution; however, we show that  $\lambda^N$  is strictly increasing in the tax rate, whereas the growth rate is maximized when the tax solves

$$\tau_K^{\alpha/(1-\alpha)} = [\alpha(1 - \alpha)A]^{1/(1-\alpha)} \left[ 1 + \frac{1 - \delta_G}{\beta \left( 1 - \delta_K + \frac{\alpha}{1-\alpha} \tau_K \right) - 1 + \delta_G} \right]. \quad (26)$$

Using parameterization illustrated in Figure 2 ( $\beta = 0.8$ ,  $\delta_K = \delta_G = 0.15$ ,  $A = 1$ ,  $K_0 = 2$ ) this gives for the growth-maximizing tax and the corresponding growth rate,  $(\tau_K, \gamma) = (0.294, 0.0328)$  for  $\alpha = 0.6$ ,  $(0.249, 0.0511)$  for  $\alpha = 0.65$ , and  $(0.211, 0.0734)$  for  $\alpha = 0.7$ . We also show that there is no longer a direct link between the growth rate and the level of welfare. In fact, the maximum rate of growth is achieved before welfare is maximized.

The relationships between the endogenous variables and the tax are plotted in Figure 2 for three different values of  $\alpha$ . The  $\lambda^N$  and the consumption-capital ratio are decreasing in  $\alpha$ , whereas the growth rate is increasing. These competing effects produce a single-crossing property in welfare: it increases with  $\alpha$  for low  $\tau_K$  but decreases for high  $\tau_K$ . The  $\lambda^N$  reaches high values for modest levels of the tax – so that the capital tax rapidly becomes increasingly distortionary. This is not surprising given the important role that capital plays in sustaining growth in this economy.

<sup>6</sup>Note that the tax,  $\tau_K$ , on capital input is equivalent to a tax on income from capital at rate  $\tilde{\tau}_K$ , with  $\tilde{\tau}_K = \frac{\tau_K}{r_t + \tau_K}$ . Provided  $r_t$  satisfies  $0 < r_t \leq K < \infty$ , then  $\tilde{\tau}_K \rightarrow 0$  as  $\tau_K \rightarrow 0$ , and  $\tilde{\tau}_K \rightarrow 1$  as  $\tau_K \rightarrow \infty$ . This should be borne in mind when contrasting with the labour income tax.

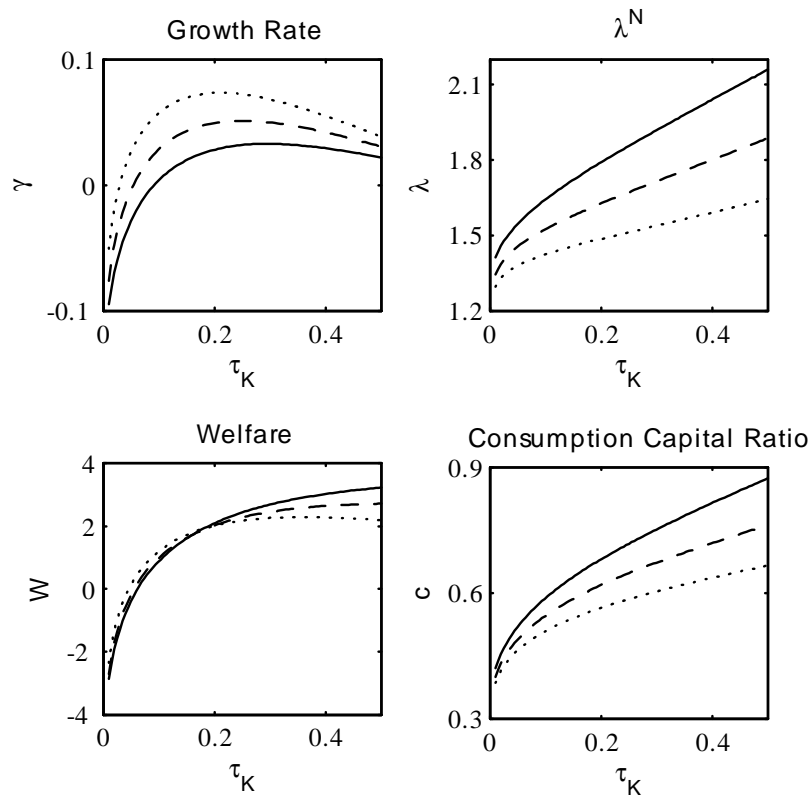


Figure 2: Tax on capital ( $\alpha = 0.6$  (solid), 0.65 (dash), 0.7 (dot),  $\beta = 0.8$ ,  $\delta_K = \delta_G = 0.15$ ,  $A = 1$ ,  $K_0 = 2$ )

## 5.2 Tax on capital and labour

The model is now extended to make labour supply elastic. This makes it interesting to analyze a capital tax and a labour tax since both instruments are distortionary. We assume that the instantaneous utility has the Cobb-Douglas form

$$U(C_t, L_t) = \theta \ln(C_t) + (1 - \theta) \ln(1 - L_t). \quad (27)$$

Labour income is taxed at rate  $\tau_L$ , and a tax of  $\tau_K$  is levied on the private capital input. The public capital input is financed by the tax on capital input and on the labour income. We assume, as before, that the government does not issue debt. The government budget constraint in period  $t$  is therefore

$$G_t = (1 - \delta_G) G_{t-1} + \tau_K K_t + \tau_L w_t L_t.$$

The achievement of the balanced growth path when public capital is modelled as a stock variable has been analyzed in Gómez (2004) and Turnovsky (1997). Turnovsky assumes that investments in public capital and private capital are reversible. This allows immediate adjustment to the balanced growth path via a downward jump in one of the capital stock variables. Without reversibility it is shown by Gómez that the optimal transition path requires investment in one of the two capital variables to be zero until the balanced growth path is reached.

The normalized *MCFs* for the two instruments are<sup>7</sup>

$$\begin{aligned} \lambda_K^N &= \frac{c}{\tau_K} \left[ (1 - \tau_L) \frac{\omega}{c} \varepsilon_L^K - \frac{\beta}{1 - \beta} \frac{\gamma}{1 + \gamma} \varepsilon_\gamma^K - \varepsilon_c^K \right], \\ \lambda_L^N &= \frac{c}{\omega \tau_L (1 + \varepsilon_\omega^L)} \left[ (1 - \tau_L) \frac{\omega}{c} \varepsilon_L^L - \frac{\beta}{1 - \beta} \frac{\gamma}{1 + \gamma} \varepsilon_\gamma^L - \varepsilon_c^L \right], \end{aligned}$$

where

$$\omega \equiv w_t L_t / K_t, \quad \varepsilon_c^i \equiv \frac{\tau_i}{c} \frac{\partial c}{\partial \tau_i}, \quad \varepsilon_L^i \equiv \frac{\tau_i}{L} \frac{\partial L}{\partial \tau_i}, \quad \varepsilon_\gamma^i \equiv \frac{\tau_i}{\gamma} \frac{\partial \gamma}{\partial \tau_i}, \quad i = K, L.$$

are elasticities with respect to the tax rates. These results are first interpreted, and then numerically evaluated.

The terms in the expression evaluate the three effects that the tax changes have upon the economy. The first elasticity,  $\varepsilon_L^K$ , reflects the distortion of labour supply. The second,  $\varepsilon_\gamma^K$ , concerns the effect of the tax on the growth rate. The final elasticity,  $\varepsilon_c^K$ , captures the impact on intertemporal allocation since it relates to the change in consumption relative to saving. It should be noted that these are not behavioral elasticities but are instead the *elasticities of equilibrium values* and that they depend on the tax rates at which they are evaluated. To help understand the contribution of each elasticity to the overall values of  $\lambda_K^N$  and  $\lambda_L^N$ . Table 1 reports the values of the elasticities for our baseline equilibrium. The table shows that the elasticity of the equilibrium consumption-capital ratio,  $c$ , and the elasticity of the equilibrium quantity of labour are negative but

<sup>7</sup>See Appendix for details.



$\varepsilon_c^K$	$\varepsilon_L^K$	$\varepsilon_\gamma^K$	$\varepsilon_c^L$	$\varepsilon_L^L$	$\varepsilon_\gamma^L$
-0.0225	0.0347	-15.91	-0.3878	-0.0629	-1.648

Table 1: Elasticities ( $t_K = t_L = 0.25$ )

are small in value. The elasticity of the growth rate (which reflects changes in capital accumulation) is negative and large for both taxes. However, the latter is weighted by a term involving the growth rate which for practical values (captured by the range 0 to 0.06 in our simulations) will be small. An increase in the (absolute) value of any of the elasticities will increase the two normalized *MCFs*.

The expressions we have derived for two tax instruments are now numerically analyzed for a calibrated version of the model. Figure 3 shows how the growth rate, level of welfare, consumption-capital ratio, and  $\lambda_K^N$  for the capital tax change with the rate of tax on private capital input, for three different levels of labour income tax. In Figure 4 the roles of the capital tax and the labour tax are reversed and the  $\lambda_L^N$  is plotted for three values of the tax on labour income. Over the range plotted in Figure 3 one can see that the decrease in consumption with a higher rate of labour income tax in this economy is more than offset by the increase in the growth rate, so that for a given level of the capital tax the welfare level is higher with a higher labour tax rate. A similar pattern is observed for the capital tax.  $\lambda_K^N$  is reduced by an increase in labour tax but, conversely,  $\lambda_L^N$  is raised by an increase in the capital tax. Both  $\lambda_K^N$  and  $\lambda_L^N$  increase rapidly as the taxes are raised and  $\lambda_L^N$  is clearly convex in the tax rate. Both are less than one for low taxes since the growth-enhancing effect is dominant and exceed two for moderate values of the tax rates. The fact that  $\lambda_K^N$  is greater than  $\lambda_L^N$  is a reflection of the Chamley-Judd result that the optimal capital tax in a growth model should be zero (Chamley 1986, Judd 1985).

## 6 *MCF* with Infrastructural Spill-Overs

This section analyzes *MCF* in a two-country model that incorporates infrastructural spill-overs between countries. The discussion that follows refers to the EU context; the same analysis applies to any federal state with multiple jurisdictions. The model builds upon that in Hashimzade and Myles (2010) with the intention of capturing the important feature of the EU that productive investments by one member state have benefits for other neighboring states. In general, independent policy-setting by countries will lead to under-investment in infrastructure in such a setting because of the positive externality generated by the spill-over. This provides a role for a supra-national body to coordinate the decisions of individual countries so as to secure an increase in the growth rate. It also has implications for the value of the *MCF*. We are interested in discovering how these externalities affect the value of the *MCF*.

The first step is to introduce the externality between countries caused by

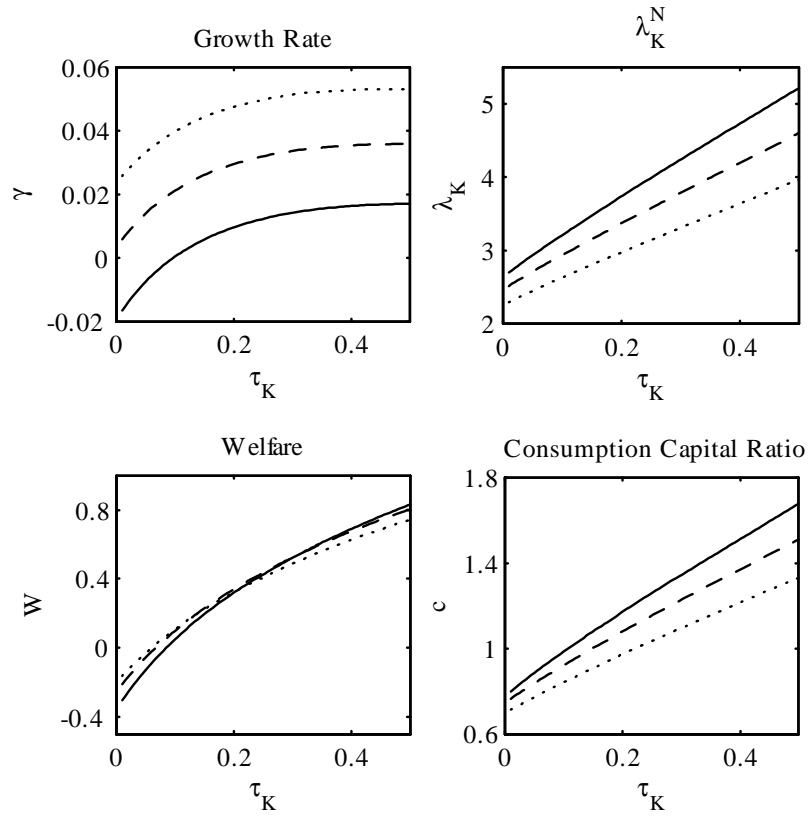


Figure 3: Tax on capital and labour  
( $\tau_L = 0.3$  (solid),  $0.4$  (dash),  $0.5$  (dot),  $\alpha = 0.3$ ,  $\beta = 0.8$ ,  $\delta_K = \delta_G = 0.1$ ,  
 $A = 1.5$ ,  $K_0 = 2$ ,  $\theta = 0.3$ )

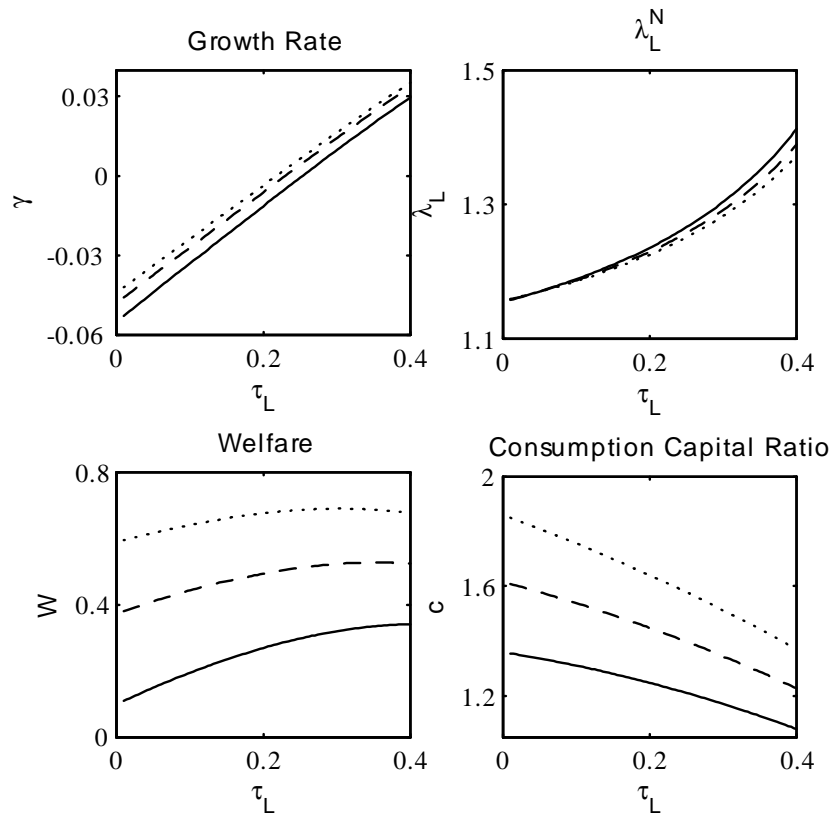


Figure 4: Tax on capital and labour  
 $(\tau_K = 0.3$  (solid),  $0.4$  (dash),  $0.5$  (dot),  $\alpha = 0.3$ ,  $\beta = 0.8$ ,  $\delta_K = \delta_G = 0.1$ ,  $A = 1.5$ ,  $K_0 = 2$ ,  $\theta = 0.3$ )

productive public input. Let  $G_t$  and  $G_t^*$  denote productive public input in the home and in the foreign country, respectively, and let  $\Gamma_t = G_t + G_t^*$  be the total public input at home and abroad. The level of output in the home country is given by

$$Y_t = AL_t^{1-\alpha} K_t^\alpha \left( G_t^{1-\rho} \Gamma_t^\rho \right)^{1-\alpha}. \quad (28)$$

There is no externality when  $\rho = 0$ . To simplify the analysis we assume labour is inelastic and normalize the quantity to one. The optimization problem of the home consumer is to maximize intertemporal utility taking as given the levels of capital and public good as well as the rate of growth in the foreign country.

We assume that there is no redistribution of the tax revenues across countries, and that the home and the foreign countries set their tax rates independently. The home (foreign) country finances their public spending by a tax  $\tau_K$  ( $\tau_K^*$ ) on private capital input at rate  $\gamma$ , and there is no government debt. Thus, for the home country

$$G_t = (1 - \delta_G) G_{t-1} + \tau_K K_t. \quad (29)$$

We focus on balanced growth paths along which all real variables in all countries grow at the same rate and the tax rates are constant over time. The equality of the growth rates across countries here is imposed, since the law of motion of the public capital in one country only ensures that the growth rates of the stock of public and private capital are equal in that country, but there is no reason of why the growth rates should be equal across countries. If we did not impose this assumption then the output of one country would eventually become arbitrarily small relative to the output of the other.

When the externality from the infrastructural spill-over is present,  $W$  depends on the growth rate in both home and foreign country. Thus, in the presence of externalities, the welfare of the home consumer depends on the home tax rate through its own growth rate as well as the growth rate in the foreign country,

$$W = \frac{\ln K_0}{1 - \beta} + \sum_{t=0}^{\infty} t \beta^t \ln(1 + \gamma) + \sum_{t=0}^{\infty} \beta^t \ln c(\gamma, \gamma^*) \quad (30)$$

where

$$c(\gamma, \gamma^*) = Ag^{1-\alpha} \left[ 1 + \left( \frac{1 + \gamma^*}{1 + \gamma} \right)^{t+1} \frac{G_0^*}{G_0} \right]^{\rho(1-\alpha)} - \gamma - \delta_K - \tau_K.$$

and

$$\frac{1 + \gamma}{\beta} = \alpha A \left[ 1 + \left( \frac{1 + \gamma^*}{1 + \gamma} \right)^t \frac{G_0^*}{G_0} \right]^{\rho(1-\alpha)} + 1 - \delta_K - \tau_K, \quad (31)$$

$$\frac{1 + \gamma^*}{\beta} = \alpha A^* \left[ 1 + \left( \frac{1 + \gamma}{1 + \gamma^*} \right)^t \frac{G_0}{G_0^*} \right]^{\rho(1-\alpha)} + 1 - \delta_K - \tau_K^*. \quad (32)$$

The details of the calculations are provided in Hashimzade and Myles (2010). For the normalized *MCF* we have

$$\lambda^N = (1 - \beta)c \left[ \frac{\partial W}{\partial \tau_K} \Big|_{\gamma, \gamma^*} + \frac{\partial W}{\partial \gamma} \frac{\partial \gamma}{\partial \tau_K} + \frac{\partial W}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \tau_K} \right]_{\gamma = \gamma^*},$$

where the partial derivatives of  $\gamma$  and  $\gamma^*$  by taking the total differential of (31)-(32). The resulting expressions are cumbersome and do not provide much insight. To illustrate the pattern we present a numerical solution for a symmetric equilibrium calculated for a range of parameters.

Figure 5 depicts the solution for symmetric equilibrium in the model with two identical countries. It can be seen that an increase in the extent of the spill-over (measured by  $\rho$ ) reduces  $\lambda^N$ , so that greater spill-overs decrease the cost of funding projects. One explanation for an increased spill-over could be economic integration, which suggests that the single-market programme may have consequences for the cost of financing public projects. The  $\lambda^N$  increases with the tax rate but over the range displayed so do the growth rate and welfare.

## 7 Conclusions

The marginal cost of public funds has a central role in the assessment of tax policy and in cost-benefit analysis. The *MCF* provides a measure of the cost of raising revenue through distortionary taxation that can be set against the benefits of a public sector project. Despite the importance of the concept the current literature has focussed upon the *MCF* in static settings. Only a very small literature has so far considered it within a growth setting.

In this paper we have computed the *MCF* in a variety of endogenous growth models with public infrastructure. To do this we have built upon the definition of the *MCF* in an intertemporal setting provided by Dahlby (2006). The models that have been analyzed are extensions of the Barro model of productive public expenditure, but with the public input represented as a stock rather than a flow. In addition, we have also introduced externalities between countries which are a consequence of spill-overs from public infrastructure. To evaluate the *MCF* we assume that the economy is on a balanced growth path which permits the evaluation of welfare in terms of a balanced growth rate. This technique provides a basis for determining the *MCF* for a variety of tax instruments in a form that can be empirically evaluated.

We have employed the standard parameter values for calibration used in real business cycle and growth models to simulate the models. Our results demonstrate that there is a link between the *MCF* and the growth rate, and that the *MCF* is sensitive to the tax rate. In the calibrated simulation the normalized *MCF* can take high values for quite reasonable values of tax rates. This indicates that the effect upon the growth rate can exacerbate the static distortions caused by taxation. In every case the *MCF* is increasing and monotonic over a range of capital input and labour income tax rates similar to those seen in practice.

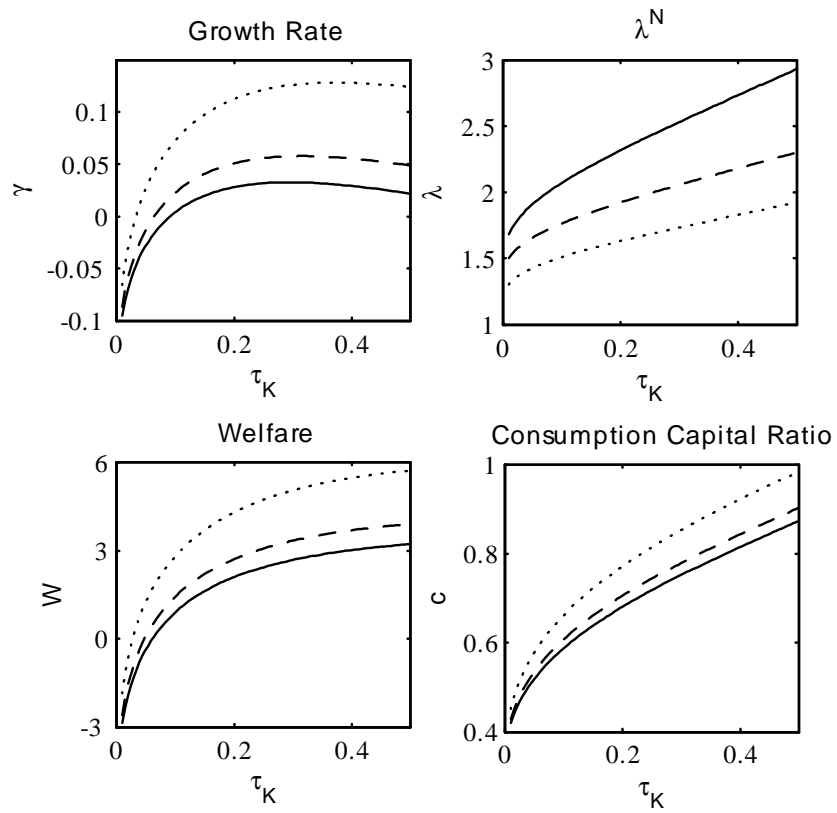


Figure 5:  $\alpha = 0.6$   $\beta = 0.8$ ,  $\delta_K = \delta_G = 0.15$ ,  $A = 1$ ,  $K_0 = 2$ ,  $\rho = 0$  (solid), 0.3 (dash), 1 (dot).

In the model with an infrastructural spill-over it is interesting to observe that an increase in the spill-over effect reduced the *MCF*.

The analysis has been restricted here by the focus on balanced growth paths. It might be thought necessary to consider the transition path but there is limited evidence on the length of such transition. We have considered only sustained variations in tax rates and have implicitly assumed credibility of government announcements and commitment to announced policies. This removed any need to consider the formation of expectations or games played between the public and private sectors. If there is any strategic interaction this would change the value of the *MCF*.<sup>8</sup> The benefit of these restrictions is the simplification they provide to the analysis and the fact that they can be applied in a similar manner to more complex models.

The *MCF* is an important concept in tax policy and cost-benefit analysis. Although it generally appears in a static setting it can be extended to growth models. Our approach to the *MCF* is suitable for numerical evaluation in more complex economic environments. An avenue for exploring the practical value of this methodology could be to consider the effect of capital mobility upon the *MCF*, and to embed it in a more general model that can incorporate several countries and a broader range of tax instruments. This latter model will form the basis of empirical implementation using a combination of calibration and estimation.

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<sup>8</sup> An example can illustrate the potential effects of game-playing. A tax on the profit from developing land has been introduced twice in the past 100 years in the UK. On both occasions developers held back from undertaking development in the belief that the next government would repeal the tax. This belief was proved correct. Hence, a significant short-term welfare cost was incurred for the generation of very little revenue.

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## A Appendix

### A.1 Balanced growth path and the *MCF* with capital tax

The consumers’ optimization problem remains the same as in the model with output tax. The tax on the private capital input is denoted by  $\tau_K$ . Net of tax profit is

$$\pi_t = Y_t - (r_t + \tau_K) K_t - w_t L_t.$$

The profit maximization condition implies that

$$r_t = \alpha A (g_t L_t)^{1-\alpha} - \tau_K.$$

The government budget constraint at time  $t$  is

$$G_{t+1} = (1 - \delta_G) G_t + \tau_K K_{t+1}.$$

On the balanced growth path the real variables  $(Y_t, C_t, K_t, G_t, w_t)$  grow at the same constant rate,  $\gamma$ . Markets also clear in every period and  $r_t = r$  for all  $t$ . Normalizing available labour to one, we obtain the following set of equations describing the balanced growth path

$$r = \alpha A g^{1-\alpha} - \tau_K, \quad (33)$$

$$c = A g^{1-\alpha} - \delta_K - \gamma - \tau_K, \quad (34)$$

$$\gamma = \beta (1 - \delta_K - \tau_K + \alpha A g^{1-\alpha}) - 1, \quad (35)$$

$$g = \frac{1 + \gamma}{\gamma + \delta_G} \tau_K. \quad (36)$$

In this case there is no closed form solution for  $\gamma$  in terms of  $\tau_K$  when the dependence of  $g$  upon  $\tau_K$  is taken into account.

For the present value of tax revenues we have

$$R = \sum_{t=0}^{\infty} \left( \frac{\beta}{1 + \gamma} \right)^t \tau_K K_t = \frac{\tau_K K_0}{1 - \beta},$$

from which it follows that

$$\frac{\partial R}{\partial \tau_K} = \frac{K_0}{1 - \beta}.$$

The welfare function remains as described by (18), and so

$$\lambda = \frac{1}{K_0} \left[ \frac{1 - \alpha\beta}{c} + \frac{\alpha\beta^2}{1 - \beta} \frac{1}{1 + \gamma} \right].$$

This *MCF* can be used directly or, dividing by the marginal utility of income at time 0, converted into the normalized version

$$\lambda^N = 1 - \alpha\beta + \frac{\alpha\beta^2}{1-\beta} \frac{c}{1+\gamma}. \quad (37)$$

Using (35) in (34) we obtain

$$c = \frac{1-\alpha\beta}{\alpha\beta} (1+\gamma) - \frac{1-\alpha}{\alpha} (1-\delta_K - \tau_K), \quad (38)$$

which implies

$$\lambda^N = \frac{1-\alpha\beta}{1-\beta} - \frac{(1-\alpha)\beta^2}{1-\beta} \frac{1-\delta_K - \tau_K}{1+\gamma}.$$

To investigate how the growth rate and the  $\lambda^N$  depends on the tax rate we use implicit differentiation in (35)-(36):

$$\frac{\partial\gamma}{\partial\tau_K} = \beta \frac{\alpha(1-\alpha)Ag^{1-\alpha} - 1/\tau_K}{1+\beta\alpha(1-\alpha)Ag^{-\alpha}\tau_K(\gamma+\delta_G)/(\gamma+\delta_G)^2}. \quad (39)$$

The expression in the numerator equals zero at the value of the tax rate given implicitly by

$$\tilde{\tau}_K = \alpha(1-\alpha)A\tilde{g}^{1-\alpha}, \quad (40)$$

where  $\tilde{g}$  is evaluated at  $\tau = \tilde{\tau}_K$ . Using this in (35) gives the corresponding maximum rate of growth,

$$\tilde{\gamma} = \beta \left( 1 - \delta_K + \frac{\alpha}{1-\alpha} \tilde{\tau}_K \right) - 1.$$

From this equation

$$\tilde{\tau}_K = \frac{1-\alpha}{\alpha} \left( \frac{1+\tilde{\gamma}}{\beta} - 1 + \delta_K \right),$$

and substitution into (36) gives

$$\tilde{g} = \left( 1 + \frac{1-\delta_G}{\beta \left( 1 - \delta_K + \frac{\alpha}{1-\alpha} \tilde{\tau}_K \right) - 1 + \delta_G} \right) \tilde{\tau}_K.$$

Finally, using this in (40) gives, after obvious rearrangement, an implicit expression for the growth-maximizing tax rate:

$$\tilde{\tau}_K^{\alpha/(1-\alpha)} = [\alpha(1-\alpha)A]^{1/(1-\alpha)} \left[ 1 + \frac{1-\delta_G}{\beta \left( 1 - \delta_K + \frac{\alpha}{1-\alpha} \tilde{\tau}_K \right) - 1 + \delta_G} \right].$$

The right-hand side is positive and increasing function of  $\tilde{\tau}_K$  on  $(0, \infty)$ , and the left-hand side is positive and decreasing function of  $\tilde{\tau}_K$  on  $(\tau_K^*, \infty)$ , where  $\tau_K^* = \frac{1-\alpha}{\alpha} \left( \frac{1-\delta_G}{\beta} - 1 + \delta_K \right)$ . Therefore, a unique solution exists on  $(\max\{0, \tau_K^*\}, \infty)$ .

Next, for  $\lambda^N$  differentiation with respect to the tax rate gives

$$\frac{\partial \lambda^N}{\partial \tau_K} = \frac{(1-\alpha)\beta^2}{(1-\beta)(1+\gamma)} \left[ 1 + \frac{\tau_K}{1+\gamma} \frac{\partial \gamma}{\partial \tau_K} \right].$$

Using (39) and (36), after straightforward manipulations, we obtain

$$\frac{\partial \lambda^N}{\partial \tau_K} = \frac{(1-\alpha)\beta^2}{(1-\beta)(1+\gamma)} \frac{\alpha(1-\alpha)Ag^{1-\alpha} + (\gamma + \delta_K) \left( \frac{1}{\beta} - \frac{\tau_K}{1+\gamma} \right)}{\alpha(1-\alpha)Ag^{-\alpha}(g - \tau_K) + (\gamma + \delta_G)/\beta}.$$

This is strictly positive, since  $g - \tau_K = \tau_K \frac{1-\delta_G}{\gamma + \delta_G} \geq 0$  and  $\frac{1}{\beta} > 1 > \frac{\tau_K}{1+\gamma}$ .

Therefore,  $\lambda^N$  is monotonically increasing in the tax rate.

For the welfare function we have

$$W = \frac{\ln(cK_0)}{1-\beta} + \frac{\beta}{(1-\beta)^2} \ln(1+\gamma),$$

where  $c$  is given by (38). Differentiation with respect to the tax rate gives

$$\begin{aligned} \frac{\partial c}{\partial \tau_K} &= \frac{1-\alpha}{\alpha} + \frac{1-\alpha\beta}{\alpha\beta} \frac{\partial \gamma}{\partial \tau_K}, \\ \frac{\partial W}{\partial \tau_K} &= \frac{1}{1-\beta} \frac{1}{c} \left[ \frac{1-\alpha}{\alpha} + \frac{1-\alpha\beta}{\alpha\beta} \frac{\partial \gamma}{\partial \tau_K} \right] + \frac{\beta}{(1-\beta)^2} \frac{1}{1+\gamma} \frac{\partial \gamma}{\partial \tau_K}. \end{aligned}$$

In particular, this implies that at the tax rate where growth is maximized ( $\frac{\partial \gamma}{\partial \tau_K} = 0$ ) the consumption to capital ratio and the welfare are increasing ( $\frac{\partial c}{\partial \tau_K} > 0$ ,  $\frac{\partial W}{\partial \tau_K} > 0$ ). That is, the maximum growth rate is achieved before welfare is maximized.

To verify the optimality condition in this model observe that, from (34)-(35),

$$MB_g = cK_0 \frac{\partial W}{\partial g} = \frac{1-\alpha}{1-\beta} AK_0 g^{-\alpha} \left[ 1 - \alpha\beta + \frac{\alpha\beta^2}{1-\beta} \frac{c}{1+\gamma} \right].$$

and  $\frac{\partial R}{\partial g} = 0$ . Therefore, using (37),

$$\frac{\partial C}{\partial g} = \frac{MB_g}{\lambda^N} = \frac{1-\alpha}{1-\beta} AK_0 g^{-\alpha},$$

which, again, is the marginal benefit of the public input financed by lump-sum tax.

## A.2 Balanced growth path and the *MCF* with tax on capital and labour

The private capital input is taxed at rate  $\tau_K$ . Net of tax profit is

$$\pi_t = Y_t - (r_t + \tau_K)K_t - w_tL_t.$$

From the necessary conditions for the choice of capital and labour inputs we obtain

$$r_t = \alpha A (g_t L_t)^{1-\alpha} - \tau_K,$$

and

$$w_t = (1 - \alpha) A (g_t L_t)^{1-\alpha} \frac{K_t}{L_t},$$

where, as before,  $g_t \equiv G_t/K_t$ .

The representative consumer has intertemporal preferences

$$W = \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \quad (41)$$

where the instantaneous utility has the Cobb-Douglas form

$$U(C_t, L_t) = \theta \ln(C_t) + (1 - \theta) \ln(1 - L_t). \quad (42)$$

Labour income is taxed at rate  $\tau_L$ , and so the consumer's budget constraint is

$$C_t + K_{t+1} = (1 - \delta_K + r_t)K_t + (1 - \tau_L)w_tL_t + \pi_t. \quad (43)$$

Upon substitution of (42) and (43) into (41) we can write the first-order conditions for the intertemporal paths of consumption and labour supply as

$$\begin{aligned} \frac{C_{t+1}}{C_t} &= \beta(1 - \delta_K + r_{t+1}), \\ \frac{C_t}{1 - L_t} &= \frac{\theta}{1 - \theta}(1 - \tau_L)w_t. \end{aligned}$$

The public capital input is financed by the tax on capital input and on the labour income. We assume, as before, that the government does not issue debt. The government budget constraint in period  $t$  is therefore

$$G_t = (1 - \delta_G)G_{t-1} + \tau_K K_t + \tau_L w_t L_t.$$

Employing the conditions developed above the balanced growth path is described by the following set of equations

$$r = \alpha A (gL)^{1-\alpha} - \tau_K, \quad (44)$$

$$\omega = (1 - \alpha) A (gL)^{1-\alpha}, \quad (45)$$

$$c = (r - \delta_K - \gamma) + (1 - \tau_L)\omega, \quad (46)$$

$$\gamma = \beta(1 - \delta_K + r) - 1, \quad (47)$$

$$\frac{1}{L} = \frac{1 - \theta}{\theta} \frac{c}{(1 - \tau_L)\omega} + 1, \quad (48)$$

$$g = \frac{1 + \gamma}{\gamma + \delta_G} (\tau_K + \omega\tau_L). \quad (49)$$

where  $c \equiv C_t/K_t$  and  $\omega \equiv w_t L_t/K_t$ .

In the balanced growth path equilibrium the present value of tax revenues are given by

$$R = \sum_{t=0}^{\infty} \left( \frac{\beta}{1+\gamma} \right)^t (\tau_K K_t + \tau_L w_t L_t) = \frac{K_0}{1-\beta} (\tau_K + \omega \tau_L),$$

so that

$$\frac{\partial R}{\partial \tau_K} = \frac{K_0}{1-\beta}, \quad \frac{\partial R}{\partial \tau_L} = \frac{K_0 \omega}{1-\beta} (1 + \varepsilon_{\omega}^L), \quad (50)$$

where  $\varepsilon_{\omega}^L \equiv \frac{\tau_L}{\omega} \frac{\partial \omega}{\partial \tau_L}$ . The welfare function can be written as

$$\begin{aligned} W &= \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) = \sum_{t=0}^{\infty} \beta^t [\theta \ln C_t + (1-\theta) \ln(1-L_t)] \\ &= \sum_{t=0}^{\infty} \beta^t \left[ \theta \ln [c K_0 (1+\gamma)^t] + (1-\theta) \ln(1-L) \right] \\ &= \frac{1}{1-\beta} [\theta \ln(c K_0) + (1-\theta) \ln(1-L)] + \frac{\beta}{(1-\beta)^2} \theta \ln(1+\gamma). \end{aligned} \quad (51)$$

Differentiation with respect to the tax rates gives

$$\begin{aligned} \frac{\partial W}{\partial \tau_i} &= \frac{1}{1-\beta} \left( \frac{\theta}{c} \frac{\partial c}{\partial \tau_i} - \frac{1-\theta}{1-L} \frac{\partial L}{\partial \tau_i} \right) + \frac{\beta}{(1-\beta)^2} \frac{\theta}{1+\gamma} \frac{\partial \gamma}{\partial \tau_i} \\ &= \frac{1}{1-\beta} \frac{\theta}{\tau_i} \left[ \varepsilon_c^i - (1-\tau_L) \frac{\omega}{c} \varepsilon_L^i \right] + \frac{\beta}{(1-\beta)^2} \frac{\theta}{1+\gamma} \frac{\gamma}{\tau_i} \varepsilon_{\gamma}^i, \end{aligned} \quad (52)$$

where

$$\varepsilon_c^i \equiv \frac{\tau_i}{c} \frac{\partial c}{\partial \tau_i}, \quad \varepsilon_L^i \equiv \frac{\tau_i}{L} \frac{\partial L}{\partial \tau_i}, \quad \varepsilon_{\gamma}^i \equiv \frac{\tau_i}{\gamma} \frac{\partial \gamma}{\partial \tau_i}, \quad i = K, L.$$

Using (50) and (52) the *MCF* for the two tax instruments can be expressed in terms of elasticities by

$$\begin{aligned} \lambda_K &= \frac{\theta}{\tau_K K_0} \left[ \varepsilon_c^K - (1-\tau_L) \frac{\omega}{c} \varepsilon_L^K + \frac{\beta}{1-\beta} \frac{\gamma}{1+\gamma} \varepsilon_{\gamma}^K \right], \\ \lambda_L &= \frac{\theta}{\omega \tau_L (1 + \varepsilon_{\omega}^L) K_0} \left[ \varepsilon_c^L - (1-\tau_L) \frac{\omega}{c} \varepsilon_L^L + \frac{\beta}{1-\beta} \frac{\gamma}{1+\gamma} \varepsilon_{\gamma}^L \right]. \end{aligned}$$

The marginal utility of income is now  $\theta/(cK_0)$ . Thus, the normalized *MCF*s for the two instruments are

$$\begin{aligned} \lambda_K^N &= \frac{c}{\tau_K} \left[ \varepsilon_c^K - (1-\tau_L) \frac{\omega}{c} \varepsilon_L^K + \frac{\beta}{1-\beta} \frac{\gamma}{1+\gamma} \varepsilon_{\gamma}^K \right], \\ \lambda_L^N &= \frac{c}{\omega \tau_L (1 + \varepsilon_{\omega}^L)} \left[ \varepsilon_c^L - (1-\tau_L) \frac{\omega}{c} \varepsilon_L^L + \frac{\beta}{1-\beta} \frac{\gamma}{1+\gamma} \varepsilon_{\gamma}^L \right], \end{aligned}$$

To calculate the elasticities we use (44) to eliminate  $r$  from (45) to (48) and take the total differential of each resulting equation holding  $g$  constant. This process produces the matrix equation

$$\mathbf{A} [ dc \quad d\gamma \quad d\omega \quad dL ]^T = \mathbf{B} [ d\tau_K \quad d\tau_L ]^T,$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{1}{\beta} & -\frac{\alpha}{1-\alpha} & 0 \\ -1 & \frac{1-\beta}{\beta} & 1-\tau_L & 0 \\ \frac{1}{c} & 0 & -\frac{1}{\omega} & \frac{1}{L(1-L)} \\ 0 & 0 & -1 & (1-\alpha)\frac{\omega}{L} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -1 & 0 \\ 0 & \omega \\ 0 & -\frac{1}{1-\tau_L} \\ 0 & 0 \end{bmatrix}.$$

This equation can be solved to yield the derivatives and, hence, the elasticities with respect to the tax rates.