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Study of Knots in Material Culture

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ABSTRACT

In this paper we will discuss knots in material culture, giving an overview of their importance and range of usage. We will then discuss a method of characterisation for these knots and a method to study these knots through cultural evolution. We then give some preliminary results and ideas for further study.

Keywords: Material knots; anthropology; cultural evolution; Ashley Book of Knots

Mathematics Subject Classification 2000: 57M25, 57M27

1. Introduction

Knots are an important part of our everyday life. From our shoelaces to securing loads, knots are an essential tool for many of us. They are used the world over, in almost every society. The Human Relations Area Files ethnographic database [1] contains 1,900 references to knotting across 228 different cultures. It is not just humans who use knots, knots have been found elsewhere in the animal kingdom. Certain gorillas tie knots in their nests, tying granny knots out of saplings and creepers as well as the slightly more complex reef knot [2]. The Ploceidae or Weaver Bird builds its nest out of knots, weaving an intricate pattern to attract a mate [3]. Knots are part of both human and animal life, woven into important rituals and everyday practices.

There is archaeological evidence which suggests knots played a crucial role in the development of early humans, along with the use of fire and the wheel. It is difficult to say when the first knot was used, as knots are usually made of perishable material and so are subject to decay, but artefacts which probably require knots have been found dating as far back as 300,000 years ago [4]. Such material knots can be found in sites where bodies and artefacts have been preserved in conditions with sub-zero temperatures, a completely dry environment or others which prevent decomposition [5]. For example, knots were found as part of the "Ice Man's" equipment when he was discovered in 1991 south of the Italian-Austrian border. His body and equipment had been frozen solid and preserved for over 5,400 years. The knots he had in his possession were identified as single hitches, overhand and half knots, reef knots and overhand bends amongst others. These knots formed part of the "Ice Man's" net, bow and clothing. Preserved nooses [5], textiles and fishing lines have also been found in various countries in bogs, dating as far back as 3500 BC. Knots have been observed from Ancient Egypt in both archaeological remains and texts [6], the earliest dating to 1350 BC and found in Middle Egypt.

Knots play a large part in mythology, the preeminent example being the legend of the Gordian Knot [7]. The legend says that a peasant called Gordius arrived in Phrygia in an ox cart. Unbeknown to him, it had been foretold that the future king would arrive in this way and so he was crowned. In gratitude, Gordius dedicated the cart to Zeus, tying it up with a knot, named the Gordian Knot. It was then foretold that the next king would be the one who could unravel this knot. Many tried, but the knot was too complicated. In 33 BC Alexander the Great cut the knot with his sword, a solution that seemed to go against the spirit of the challenge. The phrase "Gordian Knot" is now used to refer to a complicated problem.

Knots are an important part of human's material culture and part of human history and development. But, given knots are something we use daily, have we ever stopped to wonder why we use the particular knot we do for a given purpose? For example, many of us regularly tie our shoelaces, going through the motions and not really thinking about the knot we are tying. Why do we tie our shoelaces in this way? Is it the optimal knot for the purpose or do we just blindly follow the algorithm we are taught as children?

Maybe the reason we use particular knots can be explained through cultural evolution.

Culture is an important concept in the study of human behaviour. There are many definitions of culture. Boyd and Richerson define culture as "information capable of affecting individual's behaviour that they acquire from other members of their species" [8]. This does not include information acquired genetically or learned individually. Cultural evolution can be described as a process of "descent with modification" [9] where socially learned behaviour is passed on within a population. This socially learned behaviour is referred to as a trait. Each individual may learn a trait then pass it on, adding their own modification as they go. A trait could also be misremembered or copied incorrectly, resulting in a mutation. These mutations may later on be inherited and passed to others in the population, in the same way as the original trait. Cultural traits can evolve rapidly, often much quicker than genetic evolution.

In other words, human behaviour is not only influenced by genetically inherited information, but also by socially inherited information. Evolution of culture is treated in a way analogous to genetic evolution, but the transmission of cultural traits can be passed in ways other than from parent to offspring, referred to as vertical transmission. Cultural traits can be transmitted horizontally, from those of the same generation or obliquely, from others of the parent's generation to the offspring [10]. Traits can also be transmitted via private teaching in a one-to-one way, or through mass teaching and observation in a one-to-many scenario, such as in classrooms or via the media.

In such a way the skill of knot tying can be transmitted within a population. Knots could be being used purely because that is the first knot we are taught as children by our parents, or we could be acquiring the information horizontally or obliquely using a much wider range of information to decide which knot to use.

Using knot theory and a study of knots in material culture, I am most interested in answering the following questions:

- Why are there so many knots in material culture, in particular, why not more or fewer?
- How many of the possible knots are utilised and what is the reason for this?
- Are some or many of these knots needlessly complicated for their purpose?
- What are the common features in knot design and how are these preserved?
- How does the learning environment affect the fidelity of knot transmission?

2. Knots in Literature

Many people have a keen interest in knot tying, for example sailors and climbers, but other professionals need to know and use knots regularly too. An attempt has been made by Ashley among others to collect and create an encyclopaedia of knots and their uses [11], focusing on those in modern Western cultures. From the Ashley Book of Knots we can gain insight into the range of knots used and discover some of the history behind how and why these knots are tied. Ashley's book contains over 3,800 knots and while he goes to a great length to provide as much information as he can for each knot, some knots are repeated and some knots do not have a lot of information given. However, from Ashley's work we can put together a picture of the landscape of knots used, for a range of applications, and get an idea of which knots are best suited for these applications.

Pairing Ashley's work with other studies may provide us with more information about knot usage. Studies into knot strength and suitability have been carried out in studies comparing different types of rope and different knots tied, a factor extremely important to those who use knots for purposes such as climbing. It is known that when a knot is tied in a piece of rope it weakens the rope so it is important to choose your knot and rope carefully.

Pieranski et al. [12] studied the strength of knots by finding the breaking point of knots when under strain. In order to pinpoint easily the location of the breaking point cooked spaghetti was knotted and then put under strain by being pulled gently by hand. These tests were recorded by a digital camera with high recording speed so the video could be viewed later and the knot breakage determined. The knots in this experiment were denoted by their notation in the Rolfsen Knot Table [13]. In Pieranski et al.'s study it was found that the weakest knot of all was the Overhand Knot (knot 3_1). It was also noted that knot strength increased as the crossing number of the knot increased, which is what we may expect. The exceptions to this rule were the knot 7_1, which was worse than all knots of six crossings, and the Figure-Eight Knot, (knot 4_1) which was stronger than all knots of five and six crossings, and knot 7_1. It was found that the knot breakage did not occur in the internal region of the knot, breakage was close to the entry to the knot. These studies give us an idea of the suitability of knots for certain purposes and leads us to question why the Overhand Knot (3_1) is so widely used for a range of purposes when it is shown to be the weakest knot of all.

In addition to the strength of climbing knots, the vast range of possibilities of neck tie knots has been studied. Fink and Mao [14] attempt to predict all aesthetic neck tie knots by modelling their construction through random walks. They define a neck tie knot by a sequence of moves describing the wrapping of the neck tie by the orientation and location of the tucks used to tie it. These moves can be represented as walks on a triangular lattice and so the space of possible neck tie knots can be determined. Fink and Mao demonstrate that there are 85 possible sequences and so 85 possible distinct ways to tie a neck tie. It is interesting to note that whilst there are 85 possibilities, only four of these are commonly used as ways to tie a neck tie. Whilst Fink and Mao only considered neck ties tied with the wide end of the neck tie, Hirsch et al. [15] extended the neck tie knot possibilities by including those tied with the thin end. This takes the number of possible neck ties, with up to 13 moves, up to a staggering 177,146. One thing is clear from these studies, the number of possible neck tie knots is huge, but only a fraction are observed in real life, leading us to question the reasons behind this.

The range of evidence in Ashley and that gathered through studies suggests there is a huge range of diversity in knots, but these studies do not suggest why. An answer to this may lie in the way we learn.

Transmission chain experiments are often used to explore the effect of teaching techniques on a sample of the population. The behaviour which is observed in these experiments may be indicative of the population as a whole. Linear transmission chains operate through a "grapevine" method. Information is passed through a chain of participants in which each participant learns the information, attempts to recall it, and then passes it to the next participant in the chain. The changes that occur in the chain can be measured and give an indicator of the degradation of information in the wider population [16]. Different samples can be manipulated to more accurately model the population or hypothesis which is being tested. Thin chains, in which information is passed from one person to one other, can be used to simulate one-to-one learning, for example via a teacher or a parent. In these chains the teaching method can be varied to study the effects of different forms of instruction. Fat chains, in which individuals learn in a group and are replaced by others over time, can be used to simulate group learning and are useful to see the effects of behaviours such as conformity. Multiple chains can be run at the same time with different instructions to study the effect of instruction or members of chains can be replaced to model the introduction of new members to a population. These chains are useful for studying how information is passed within a population, but they may also help understand why information is transmitted, by enabling us to analyse the effects of conformity or expert knowledge on the transmission of information.

Knot tying has been used as a tool by Muthukrishna et al. in experiments to test the effect of multiple models on learning [17]. As knot tying only requires a piece of rope it makes an accessible tool with which to experiment. In this study the group of participants were asked to tie a series of knots commonly used by rock climbers. The study ran through two chains, each with ten generations. In both chains participants would learn how to tie the knots from the generation before them. In the first chain participants were only allowed to learn from one model in the generation before them. In the second, participants could learn from five models in the generation before. The first generation in both chains were trained by the experimenter to become "experts" at tying the knots. Other generations created an instructional video for the tying of the system of knots by a camera strapped to their head. The next generation would then be given this video along with a score which measured how well the participant tied the knot series. This score was measured on a scale used when assessing sutures when training surgeons and was judged by human raters [18]. The results showed that knot tying skills declined throughout all generations but declined more slowly in those in the five-model chain than the one-model. One of the issues with the experiment was that the participants in the five-model chain did not have time to view all of the instructional videos presented to them. Another issue was the way the knots were judged. The knots were given a score based on a set of requirements observed by a rater, but the knots were not studied to determine whether they were mathematically the same. However, the way this study was set up gives us a good idea of a way in which to approach knot transmission chain experiments and that the sample size of demonstrators may affect the fidelity of transmission.

In order to explore the difference between individual and social learning, Derex et al. [19] ran a virtual experiment concerning net building and fishing. Participants were required to construct a net on a square grid using a limited amount of rope of various thicknesses and knots of various sizes. Nets were tested and given a score based on how many fish the simulated net caught. During each of the fifteen trials, participants could view their previous net and score. The participants were placed into different groups under three different treatments, participants were unaware of who was in their group and which treatment they were in. In the individual learning treatment, participants could see the last trial and cumulative score of the rest of their group members. In the product copying treatment, participants could see the different scores of each of their group members and the corresponding nets. In the process copying treatment, participants could see the different scores of each of their group members, the corresponding nets and the step-by-step information for building that net. Participants had 30 seconds in the individual learning treatment to view the information and 90 seconds for the other two treatments. The nets were scanned pixel by pixel for similarity and scored. The process similarity was judged by viewing the net building actions as characters in a string and so the similarity of the string was measured. Scores for net building improved throughout all treatments and younger participants generally performed better than other participants. The difference between performance in the individual and product learning treatments was not significant but the process copying treatment demonstrated a significant advantage. The importance of social learning of the knotting process is indicated by this virtual net building task, however the results could have been skewed by the fact that the task was virtual and the observation that the age of participants made a difference on performance. We may expect social learning mechanisms to be also important for knot learning as nets are made up of a system of knots.

Pairing knot studies with an assessment of the knot learning environment, we attempt to answer our research questions using the methods described in the next section.

3. Mathematical analysis of ABOK

The Ashley Book of Knots (ABOK) is regarded as the authority on knotting. As it contains over 3,800 knots we may wonder exactly how many different knots appear. This book seems the natural place to start to search for answers to our questions; why are there so many knots in material culture, what are the common features in knot design and how many of the possible knots are utilised?

I have created a database of the knots found in the Ashley Book of Knots [11]. The layout of the ABOK database and explanation of the fields is given below.

Table 1. Layout of the Ashley Book of Knots (ABOK) database

ABOK No.	The knot number it first appears as in Ashley
Also appears as	Any other numbers the knot appears as in Ashley
Knot Name	Name as given in Ashley
Crossing number ABOK picture	Crossing number as given by picture in Ashley
Handedness	Whether the knot diagram has positive or negative writhe for chiral knots, otherwise noted as amphichiral
Knot	Common name used by mathematicians (if there is one)
Prime	Whether the knot is prime or composite
Knotplot input	Knotplot input if known (based on Conway notation)
Knot Atlas notation	Knot Atlas notation if known (for larger knots Knotscape notation is used)
Crossing number	Reduced diagram crossing number
Link	Whether or not knot is a link of two or more components
Number of components	If link how many components it is made of
Linking no	Linking number if knot is a link of two or more components
Notes	Any notes relating to considering knot as joined ends
Related knots	Any related knots mentioned by Ashley
Uses	Uses given by Ashley
Use comments	Any comments on usage
ABOK classification	Ashley's classification, Important, Strong, Practical etc.
Alternative names	Any alternative names given by Ashley
ABOK Image	Original image from Ashley
KnotPlot Video	Video showing deformation from Ashley's knot to a known mathematical knot

To identify the knots in ABOK, I take Ashley's image of a knot and join the free ends in such a way as to create no new crossings. If there is no such way to join the ends without creating new crossings, I join the ends by creating the minimal amount of new crossings. This may result in choices of whether to create an over or under crossing and so, I consider all cases.
For knots tied in more than one piece of string, or around an object, the knot (and object) is considered as a link, with the free ends joined in the way best suited to the function of the knot. If no way is immediately obvious, all ways to join the ends are considered.

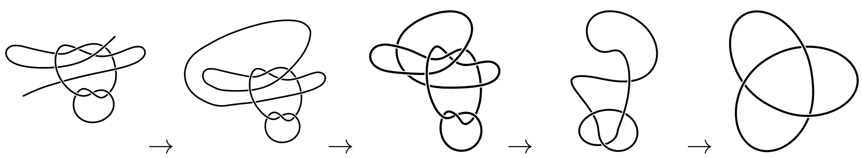

Fig. 1. This figure shows the Bowknot (ABOK number 1212) with its ends closed, then reduced to the Trefoil (3_1).

If the knot is prime with 16 crossings or less or a link with 12 or less crossings, I identify it with its knot name as found on KnotAtlas [20] or Knotscape [21] if the knot is larger. The knots are distinguished using their Dowker-Thistlethwaite notation [22] and their HOMFLY-PT polynomials [23][24], using the identification tools on KnotInfo [25], LinkInfo [26] and Knotscape. I also create a video using the program KnotPlot [27] showing this reduction, as shown in figure 1.

For some knots, joining the ends in any one way is not appropriate or too many cases need to be considered. Knots like these have been excluded from analysis for now.

4. Results

4.1. *Overall Trends*

In the ABOK database I have listed the material knots found in ABOK, then given the mathematical name of the knot they relate too. First, let us look at the number of times each mathematical knot appears as a distinct knot in ABOK.

4.1.1. *Prime Knots*

Let us look first to the number of prime knots found in the database. Table 2 shows a table showing the amount of times each prime knot appears in ABOK of up to 16 crossings.

Unsurprisingly, we see the Trefoil (knot 3_1) is the most common knot occurring in ABOK. This may be expected, given it is the simplest non-trivial prime knot. Overall we see the trend that as crossings increase, occurrence decreases. If we equate complexity with higher crossing numbers, this gives the conclusion that as complexity increases, popularity of usage decreases.

Perhaps the most interesting thing to note is the crossing numbers that do not fit this decreasing pattern. We see there are relatively high occurrences of knots of five and eight crossings. Comparative to the one knot of four crossings (the Figure-Eight Knot), both knots of five crossings occur frequently. As the Figure-Eight Knot is of relatively low crossing number it does not occur as often as we may expect.

Another thing to note is that all knots of under eight crossings appear at least once, we start to see gaps when we look to knots of eight crossings or more. Is this purely because there are so many knots of eight or more crossings that it would be unnecessary to tie them all, or something more? We also look to common families of knots, for example the -foil series (Trefoil (3_1), Cinquefoil (5_1), Septafoil (7_1), etc) we have all knots in this sequence up to knot $11a367$. Why do further knots in this sequence not appear, given they can be formed from the former by adding only one twist?

The unknot has been excluded from this chart although it occurs in ABOK often in the guise of a slipknot. These knots do not function as the unknot so an alternative way to analyse these knots may be needed.

Table 2. Prime knots found in ABOK

Crossing number	Knot	Occurrences	Crossing number	Knot	Occurrences
3	3_1	45	10	10_{121}	1
4	4_1	14		10_{124}	1
5	5_1	12		10_{139}	2
	5_2	14		10_{140}	1
6	6_1	6		10_{157}	1
	6_2	5	11	$11a351$	2
	6_3	6		$11a362$	1
7	7_1	3		$11a367$	1
	7_2	4		$11n19$	1
	7_3	1		$11n38$	1
	7_4	3		$11n98$	1
	7_5	1		$11n138$	1
	7_6	1		$11n141$	1
	7_7	3		$11n145$	1
8	8_1	1	12	$12n488$	1
	8_3	1		$12n647$	1
	8_5	1		$12n764$	1
	8_{12}	1	13	$13a3097$	1
	8_{13}	1		$13a3861$	1
	8_{16}	1		$13n4694$	1
	8_{18}	2		$13n4003$	1
	8_{19}	4		$13n4639$	1
	8_{20}	3	14	$14n21324$	1
	8_{21}	4		$14n27326$	1
9	9_1	1	15	$15a69858$	1
	9_{30}	1		$15a84903$	1
	9_{35}	1		$15n103184$	1
	9_{40}	2		$15n125031$	1
	9_{41}	2		$15n133979$	1
	9_{44}	3		$15n135983$	1
	9_{47}	1		$15n41185$	1
	9_{48}	1		$15n52069$	1
10	10_1	2	16	$16a357530$	1
	10_{109}	1		$16n259418$	1
	10_{120}	1		$Total$	189

4.1.2. *Prime Links*

Table 3. Prime links found in ABOK

Crossing number	Link	Occurrences	Crossing number	Link	Occurrences
2	$L2a1$	48	8	$L8n2$	1
4	$L4a1$	14	9	$L9a37$	1
5	$L5a1$	8		$L9a40$	1
6	$L6a1$	3		$L9n10$	2
	$L6a2$	2		$L9n11$	4
	$L6a3$	6		$L9n13$	1
	$L6a4$	1		$L9n17$	1
	$L6n1$	1		$L9n9$	2
7	$L7a1$	5	10	$L10a48$	1
	$L7a2$	1		$L10a89$	1
	$L7a3$	1		$L10a98$	1
	$L7a5$	2		$L10n30$	1
	$L7a6$	1		$L10n32$	1
	$L7a7$	1		$L10n44$	1
	$L7n1$	1		$L10n50$	1
8	$L8a11$	1		$L10n53$	1
	$L8a13$	2		$L10n63$	1
	$L8a6$	1	11	$L11n195$	2
	$L8a8$	4		$L11n252$	1
	$L8a9$	2		$Total$	130

In a similar way to prime knots, we give a table of prime links in table 3. Again this table shows the occurrence of each individual link of up to 11 crossings. The unlink is omitted for the same reasons as the unknot previously.
It is unsurprising to see the prevalence of the Hopf Link, being the simplest non-trivial link. We start to see more links with lower crossing numbers missing than for the prime knots, with two links of under eight crossings not appearing, $L6a5$ and $L7n2$. We see a high amount of links with seven and nine crossings appearing, relative to those of four and five crossings. Is this solely because there is a much wider range of higher crossing links to choose from?

4.1.3. *Composite Knots (and Links)*

Table 4. Composite knots found in ABOK

No. factors	Knot	Occurrences
2	$3_1\#3_1$	12
	$3_1\#4_1$	1
	$3_1\#5_1$	8
	$3_1\#5_2$	3
	$3_1\#7_3$	1
	$3_1\#7_7$	1
	$3_1\#8_{20}$	1
	$3_1\#9_{14}$	1
	$3_1\#L2a1$	4
	$3_1\#L4a1$	1
	$4_1\#4_1$	1
	$4_1\#L2a1$	4
	$5_1\#L2a1$	1
	$5_2\#L2a1$	2
	$6_1\#6_1$	2
	$6_1\#L2a1$	1
	$6_3\#L8n6$	1
	$6_3\#L9n12$	1
	$L2a1\#L2a1$	1
	$L2a1\#L4a1$	1
	$L2a1\#12n437$	1
	$L4a1\#L6a1$	1
2	$L5a1\#8_{20}$	1
	$L5a1\#L7n2$	1
3	$3_1\#3_1\#3_1$	6
	$3_1\#3_1\#5_2$	1
	$3_1\#3_1\#7_3$	1
	$3_1\#3_1\#L4a1$	1
	$3_1\#3_1\#L6a3$	2
	$3_1\#3_1\#L4a1$	1
	$3_1\#3_1\#L5a1$	2
	$3_1\#3_1\#L6a4$	1
	$3_1\#4_1\#L2a1$	1
	$3_1\#5_1\#L2a1$	1
	$3_1\#5_2\#L2a1$	1
	$3_1\#L2a1\#3_1$	1
	$3_1\#L9n19\#3_1$	1
	$4_1\#4_1\#4_1$	1
	$6_3\#L2a1\#6_3$	1
4	$3_1\#3_1\#3_1\#3_1$	1
	$3_1\#3_1\#L2a1\#6_3$	1
	$3_1\#L2a1\#3_1\#L5a1$	1
6	$3_1\#3_1\#3_1\#3_1\#3_1\#3_1$	1
	Total	78

Table 4 shows the number of composite knots formed of individual prime knots and links. Unsurprising the most common prime knot appearing in a composition is the Trefoil (3_1). Composite knots range from those formed of only two prime knots up to those formed of six prime knots. The prime knots and links appearing in most compositions are of relatively low crossing number, giving a composition with higher crossing number.

4.1.4. *Handedness and Chirality*

Chirality of all prime knots seen of eight crossings or fewer has been included in the ABOK database. For knots which are chiral, the handedness of the knot has been included. Of all 137 prime knots of eight crossings or fewer, 24 were amphichiral and 113 chiral knots. Of the 113 chiral knots, the diagram given by Ashley was found to have positive writhe for 47 knots and negative for 66 knots.

This does not necessarily mean that whenever a specific knot is tied, the right or left handed version is always used, it could just mean that when Ashley has drawn that specific knot he happens to have favoured the right or left handed version. Looking specifically at the Trefoil (3_1), the Trefoil appears as the right handed version 22 times and the left handed 23 times, suggesting no real preference for either.

5. Conclusion

In this paper we have discussed knots in material culture and looked to the Ashley Book of Knots to study their range. We have discussed a method to categorise and create a database of these knots in order to answer our research questions. So far, the overall trends of prime and composite knots and links have been discussed. In further work I will look closer at the distribution of these knots through usage, and explore the levels of redundant features in these knots, aiming to answer the question of are some knots needlessly complicated for their purpose.

Once we have these findings, the aim will be to explore the reasons for knot tying through transmission chain experiments. In these experiments, the idea would be to explore how knot tying information passes within a population. We can use thin chains, where information is passed from one person to another, to look at how knots are taught individually, and fat chains to look at how this information passes in a group setting. We will be able to manipulate the learning environment and gain answers to our question; how does the learning environment affect the fidelity of knot transmission?

We have seen that knot tying is a wide and varied skill, utilised by many. We have made some steps into determining how many different knots are used, but much more work needs to be done to explore the reasons behind this.

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References

[1] Yale University, "eHRAF World Cultures." [Online]. Available: http://ehrafworldcultures.yale.edu/ehrafe/ [Accessed: 2015-03-18]

[2] C. Herzfeld and D. Lestel, "Knot tying in great apes: etho-ethnology of an unusual tool behavior," *Soc. Sci. Inf.*, vol. 44, no. 4, pp. 621–653, 2005.

[3] P. T. Walsh, M. Hansell, W. D. Borello, and S. D. Healy, "Individuality in nest building: Do Southern Masked weaver (Ploceus velatus) males vary in their nest-building behaviour?" *Behav. Processes*, vol. 88, no. 1, pp. 1–6, 2011.

[4] C. Warner and R. G. Bednarik, "Pleistocene Knotting," in *Hist. Sci. Knots*, P. C. Turner, J.C. Van de Griend, Ed. World Scientific, 1998, pp. 3–18.

[5] G. van der Kleij, "On Knots and Swamps: Knots in European Prehistory," in *Hist. Sci. Knots*, P. C. Turner, J.C. Van de Griend, Ed. World Scientific, 1998, pp. 31–42.

[6] W. Wendrich, "Ancient Egyptian Rope and Knots," in *Hist. Sci. Knots*, P. C. Turner, J. C. van de Griend, Ed. World Scientific, pp. 43–68.

[7] Encyclopedia Britannica, "Gordian knot." [Online]. Available: http://www.britannica.com/EBchecked/topic/239059/Gordian-knot [Accessed: 2015-04-10]

[8] P. J. Richerson and R. Boyd, *Not by Genes Alone: How Culture Transformed Human Evolution.* Chicago, IL: University of Chicago Press, 2005.

[9] A. Mesoudi, *Cultural evolution: How Darwinian theory can explain human culture and synthesize the social sciences.* Chicago, IL: University of Chicago Press, 2011.

[10] L. L. Cavalli-Sforza and M. W. Feldman, *Cultural Transmission and Evolution: A Quantitative Approach.* Princeton University Press, 1981.

[11] C. W. Ashley, *Ashley Book of Knots.* Faber and Faber Limited, 1993.

[12] P. Pieranski, S. Kasas, G. Dietler, J. Dubochet, and A. Stasiak, "Localization of breakage points in knotted strings," *New J. Phys.*, vol. 3, pp. 0–13, 2001.

[13] D. Rolfsen, *Knots and Links.* American Mathematical Soc., 1976.

[14] T. M. A. Fink and Y. Mao, "Tie knots, random walks and topology," *Phys. A Stat. Mech. its Appl.*, vol. 276, pp. 109–121, 2000.

[15] D. Hirsch, I. Markström, M. L. Patterson, A. Sandberg, and M. Vejdemo-Johansson, "More ties than we thought," *PeerJ Comput. Sci.*, vol. 1, no. e2, pp. 1–15, 2015. [Online]. Available: http://arxiv.org/abs/1401.8242

[16] A. Mesoudi and A. Whiten, "The multiple roles of cultural transmission experiments in understanding human cultural evolution." *Philos. Trans. R. Soc. Lond. B. Biol. Sci.*, vol. 363, no. 1509, pp. 3489–3501, 2008.

[17] M. Muthukrishna, B. W. Shulman, V. Vasilescu, and J. Henrich, "Sociality influences cultural complexity." *Proc. Biol. Sci.*, vol. 281, p. 20132511, 2014.

[18] M. G. Tytherleigh, T. S. Bhatti, R. M. Watkins, and D. C. Wilkins, "The assessment of surgical skills and a simple knot-tying exercise," *Ann. R. Coll. Surg. Engl.*, vol. 83, pp. 69–73, 2001.

[19] M. Derex, B. Godelle, and M. Raymond, "Social learners require process information to outperform individual learners," *Evolution (N. Y).*, vol. 67, pp. 688–697, 2013.

[20] "Knot Atlas." [Online]. Available: http://katlas.org [Accessed: 2015-03-19]

[21] M. Thistlethwaite, "Knotscape." [Online]. Available: http://www.math.utk.edu/{~}morwen/knotscape.html [Accessed: 2015-03-19]

[22] C. H. Dowker and M. Thistlethwaite, "Classification of knot projections," vol. 16, pp. 19–31, 1983.

[23] P. Freyd, D. Yetter, J. Hoste, W. B. R. Lickorish, K. Millett, and A. Ocneanu, "A new polynomial invariant of knots and links," *Bull. Am. Math. Soc.*, vol. 12, no. 2, pp. 239–247, 1985.

[24] P. Traczyk and J. H. Przytycki, “Conway Algebras And Skein Equivalence,” vol. 100, no. 4, pp. 744–748, 1987.
[25] J. C. Cha and C. Livingston, “KnotInfo: Table of Knot Invariants.” [Online]. Available: http://www.indiana.edu/{~}knotinfo/ [Accessed: 2015-03-19]
[26] J. C. Cha and C. Livingston, “LinkInfo: Table of Knot Invariants.” [Online]. Available: http://www.indiana.edu/{~}linkinfo/ [Accessed: 2015-03-19]
[27] R. G. Scharein, “The KnotPlot Site.” [Online]. Available: http://www.knotplot.com/ [Accessed: 2015-03-19]