

Electroweak oblique parameters as a probe of the trilinear Higgs boson self-interaction

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(Received 16 March 2017; published 30 May 2017)

We calculate the two-loop contributions from a modified trilinear Higgs self-interaction, $\kappa_\lambda \lambda_{\text{SM}} v h^3$, to the electroweak oblique parameters S and T . Using the current bounds on S and T from electroweak measurements, we find the 95% C.L. constraint on the modified trilinear coupling to be $-14.0 \leq \kappa_\lambda \leq 17.4$. The largest effects on S and T arise from two insertions of the modified trilinear coupling that result in $T/S \approx -3/2$; remarkably, this is nearly parallel to the axis of the *tightest* experimental constraint in the S - T plane. No contributions to S and T arise from a modified Higgs quartic coupling at two-loop order. These calculations utilized a gauge-invariant parametrization of the trilinear Higgs coupling in terms of higher-dimensional operators $(H^\dagger H)^n$ with $n \geq 3$. Interestingly, the bounds on κ_λ that we obtain are comparable to constraints from di-Higgs production at the LHC as well as recent bounds from single Higgs production at the LHC.

DOI: 10.1103/PhysRevD.95.093004

I. INTRODUCTION

With the Higgs boson discovered [1,2], a major goal for current and future high-energy experiments is to provide precision measurements of Higgs couplings in order to thoroughly test the Standard Model and uncover any deviations. A key ingredient to the Higgs mechanism [3,4] is the shape and structure of the scalar potential, which, after spontaneous symmetry breaking, gives rise to trilinear and quartic Higgs self-interactions. The self-couplings of the Higgs boson are, at present, the least-constrained Higgs interactions of the Standard Model. This motivates exploring a variety of techniques using a wide array of experimental data to constrain them. In this paper, we evaluate how well electroweak precision data, expressed using the electroweak oblique parameters S and T [5,6], can constrain modifications of the trilinear Higgs self-interaction.

In the Standard Model (SM), the coefficients governing the shape of the scalar potential are determined by well-measured parameters in the broken phase—the vacuum expectation value and the Higgs boson mass. In order to study deviations from the SM, we consider a modified Higgs potential,

$$V_{\text{mod}}(h) \supset \frac{m_h^2}{2} h^2 + \kappa_\lambda \lambda_{\text{SM}} v h^3 + \kappa_4 \frac{\lambda_{\text{SM}}}{4} h^4, \quad (1.1)$$

where only $m_h \approx 125$ GeV has been directly experimentally measured. In general, new physics that would result in modifications to the Higgs potential would also cause modifications to other couplings of the Standard Model. In this paper, we consider *only* the effects of modifying of the trilinear and quartic couplings *in isolation* from the other Standard Model couplings. This is reasonable if we can formulate these modifications in a gauge-invariant way, and we can understand the impact of possible operator mixing through the renormalization group.

The formulation we use to implement the modified Higgs potential in Eq. (1.1) is to add gauge-invariant higher-dimensional operators $\mathcal{O}_{2n} = -(H^\dagger H)^n / \Lambda^{2n-4}$ with cutoff scale Λ . It will be convenient to write the coefficient as $1/\Lambda^{2n-4} \equiv \bar{c}_{2n} \lambda_{\text{SM}} / v^{2n-4}$ (see e.g., Refs. [7,8]). In this formulation, the operators \mathcal{O}_{2n} only affect the scalar potential, and moreover \mathcal{O}_6 is known to not induce other dimension-6 operators under one-loop renormalization [9,10]. Hence, this modification satisfies the requirements. It is, however, also a potentially dangerous expansion since the Higgs potential receives corrections from higher-dimensional operators with, as we will see, $\Lambda \sim v$. There are several possible ultraviolet completions of these higher-dimensional operators. The simplest completion would involve new gauge singlets that interact with $H^\dagger H$ (but do not lead to singlet-Higgs boson mixing), such that integrating them out generates the tower of higher-dimensional operators. Other completions could lead to auxiliary modifications of other Higgs couplings; in this case, our analysis would be valid only if the other effects accidentally cancelled out leaving just the modified trilinear Higgs coupling. In any case, our interest in this paper is to

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determine a model-independent bound on the trilinear Higgs coupling and leave the model-dependent interpretations to future work.

While the precise measurement of the trilinear Higgs self-coupling is highly challenging [11–25], first constraints have been obtained from direct searches for multi-Higgs boson final states at the LHC. Only recently have corrections from the modified trilinear couplings been considered in precision observables [26,27]. Furthermore, loop corrections to single Higgs production and associated Higgs production were used in Refs. [28–30].

In principle, we include the complete tower of operators \mathcal{O}_{2n} given that the cutoff scale that we are considering is comparable to the vacuum expectation value of the Higgs boson. In practice, we actually include just the effects of \mathcal{O}_6 on the explicit calculation of the electroweak oblique parameters S and T . This is not because the dimension-8 and higher order terms are unimportant, but instead one can show that the modified trilinear coupling captures the full effects of the tower of operators on S and T up to two-loop order. Specifically, as we will see, no corrections to the quartic coupling enter our calculation of S and T . Consequently, to two-loop order, we can simply calculate corrections with \mathcal{O}_6 and reinterpret the correction, without loss of generality, in terms of a modified trilinear coupling. A very clear discussion of this was also very recently presented in Ref. [27].

There is another critical consequence of the observation that S and T do not depend on the modified quartic coupling to two loops. Ordinarily, global questions of vacuum stability of the Higgs potential, such as whether the minimum is local or global, bounded from below, etc., may place severe constraints on the coefficients of a truncated theory, i.e., stopping at dimension 6 [31–33]. Once at least dimension-8 terms are added, these concerns become parameter-dependent on the coefficients of the truncated tower. This does not mean there are *no* concerns with the stability of the potential—only that these concerns require knowledge of the new physics beyond just the modified trilinear coupling. If we stick to the “high ground” of model independence, we can tacitly ignore Higgs potential stability issues.

In Sec. II and III, we outline our calculation of the effect that \mathcal{O}_6 has on the electroweak oblique parameters S and T . We discuss the obtained limits, including a projection to future colliders, in Sec. IV, and present our conclusions in Sec. V. We give the analytic expressions for S and T with the inclusion of the dimension-6 operator in the Appendix.

II. HIGGS EFFECTIVE FIELD THEORY AND THE MODIFIED HIGGS POTENTIAL

We begin by briefly reviewing the scalar potential in the SM in order to define the SM couplings and the associated modifications. The Higgs potential in terms of the Higgs doublet field H is

$$V_{\text{SM}}(H) = \mu_{\text{SM}}^2 H^\dagger H + \lambda_{\text{SM}} (H^\dagger H)^2. \quad (2.1)$$

After electroweak symmetry breaking, the potential can be expanded around the vacuum expectation value v of the neutral component of the Higgs doublet, $\text{Re}[H^0] \equiv (h + v)/\sqrt{2}$. The potential in terms of the physical Higgs field h at the electroweak symmetry breaking minimum becomes

$$V_{\text{SM}}(h) \supset \frac{m_h^2}{2} h^2 + \lambda_{\text{SM}} v h^3 + \frac{\lambda_{\text{SM}}}{4} h^4, \quad (2.2)$$

where $m_h^2 = -2\mu_{\text{SM}}^2 = 2\lambda_{\text{SM}}v^2$ and $v \simeq 246$ GeV.

The modified Higgs potential, Eq. (1.1), contains the multiplicative factors κ_λ and κ_4 that parametrizes the (potentially sizeable) corrections to the trilinear and quartic couplings. We implement the modified trilinear and quartic couplings using higher-dimensional operators that only affect the Higgs potential,

$$\mathcal{L}_{\text{EFT}} = - \sum_{n \geq 3} \frac{\bar{c}_{2n} \lambda_{\text{SM}}}{v^{2n-4}} (H^\dagger H)^n, \quad (2.3)$$

where we have normalized the couplings with a factor of $\lambda_{\text{SM}} \equiv m_h^2/(2v^2)$. The modified Higgs scalar potential becomes

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \sum_{n \geq 3} \frac{\bar{c}_{2n} \lambda_{\text{SM}}}{v^{2n-4}} (H^\dagger H)^n, \quad (2.4)$$

where now μ^2 and λ are in general different from the SM values.

For now, consider extending the SM with just the additional dimension-6 operator \mathcal{O}_6 . The minimization conditions are shifted, and so μ^2 and λ develop different relations in terms of the physical Higgs boson mass m_h and vacuum expectation value v , which remain fixed to their experimental values. These relations are

$$\mu^2 = -\lambda_{\text{SM}} v^2 \left(1 - \frac{3}{4} \bar{c}_6\right), \quad \lambda = \lambda_{\text{SM}} \left(1 - \frac{3}{2} \bar{c}_6\right). \quad (2.5)$$

Expanding the potential around the vacuum expectation value once again, the Higgs potential becomes Eq. (1.1) with the identifications,

$$\kappa_\lambda - 1 = \bar{c}_6, \quad \kappa_4 - 1 = 6\bar{c}_6. \quad (2.6)$$

At this stage, we have a gauge-invariant *correlated* modification of the trilinear and quartic Higgs self-couplings. This can be generalized to two separate uncorrelated modifications by including also the dimension-8 operator

from Eq. (2.3) with coefficient \bar{c}_8 . The modified trilinear and quartic Higgs self-couplings become

$$\begin{aligned}\kappa_\lambda - 1 &= \bar{c}_6 + 2\bar{c}_8, \\ \kappa_4 - 1 &= 6\bar{c}_6 + 16\bar{c}_8.\end{aligned}\quad (2.7)$$

If we include even higher-dimensional operators $(H^\dagger H)^n$ with $n \leq n_{\max}$, we again find two different linear combinations,

$$\begin{aligned}\kappa_\lambda - 1 &= \sum_{n=3}^{n_{\max}} a_{2n} \bar{c}_{2n}, \\ \kappa_4 - 1 &= \sum_{n=3}^{n_{\max}} b_{2n} \bar{c}_{2n}.\end{aligned}\quad (2.8)$$

The coefficients a_{2n} and b_{2n} , where in general $a_{2n} \neq b_{2n}$, have to be evaluated for the chosen n_{\max} . We will see that it is not necessary to include operators beyond the additional dimension-6 operator \mathcal{O}_6 since the quartic coupling, and hence κ_4 , will be shown to not contribute to S and T at two loops. The result will therefore be expressed in terms of \bar{c}_6 , which will allow a direct translation in terms of the κ_λ trilinear self-coupling modification. Also, the higher-dimensional operators in Eq. (2.3) generate even higher order Higgs boson interactions $\mathcal{O}(h^n)$ with $n \geq 5$, but since they do not contribute to the observables at the order to which we calculate, we do not need to consider them further.

III. ELECTROWEAK OBLIQUE PARAMETERS

In the electroweak sector, the effect of new physics, if heavy, is expected to have its dominant contribution through the modification of gauge boson propagators via vacuum polarisation functions, or self-energies. These so-called oblique corrections can be parametrized in terms of the three Peskin-Takeuchi parameters, S , T and U [5,6]. Since U is only constrained by the W boson mass and width, it is relatively insensitive to new physics, and so it is usually set to zero. S and T can therefore be used as a probe of the effects of new physics in the electroweak sector. They are defined by [34]

$$\begin{aligned}S &= \frac{4c^2 s^2}{\alpha_e m_Z^2} \text{Re} \left(\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0) - \frac{c^2 - s^2}{cs} \right. \\ &\quad \left. \times [\Pi_{Z\gamma}(m_Z^2) - \Pi_{Z\gamma}(0)] - \Pi_{\gamma\gamma}(m_Z^2) \right),\end{aligned}\quad (3.1)$$

$$T = \frac{1}{\alpha_e} \left(\frac{\Pi_{WW}(0)}{m_W^2} - \frac{c^2}{m_W^2} \left[\Pi_{ZZ}(0) + \frac{2s}{c} \Pi_{Z\gamma}(0) \right] \right).\quad (3.2)$$

In these equations, $\Pi_{AB}(p^2)$ represents the part of the self-energy proportional to the metric tensor $g^{\mu\nu}$ of the gauge boson A propagating into the gauge boson B with an external momentum p . α_e is the electromagnetic coupling constant, and we use the notation $s \equiv \sin \theta_W$ and $c \equiv \cos \theta_W$ where θ_W is the Weinberg angle. S and T are defined to arise solely due to the effects of new physics, and so when calculating these quantities, the SM contribution must be subtracted. The experimentally allowed values of the electroweak oblique parameters can be obtained by performing global fits to the electroweak precision observables and comparing the results to the SM prediction [35].

Contributions to S and T involving the dimension-6 operator \mathcal{O}_6 first appear at the two-loop level. At this order in perturbation theory, self-energy diagrams containing both trilinear and quartic Higgs self-interactions appear, which, due to their modifications from \bar{c}_6 outlined above, are manifest as nonzero corrections to S and T . However, as we will see later, contributions from the quartic Higgs self-interaction exactly cancel in these observables. It is also important to note that at this order in perturbation theory, there are no vertex or box diagrams that depend on \bar{c}_6 involving light external fermions (i.e., light enough that their Yukawa couplings can be neglected). Since two-loop corrections to vertex or box diagrams involving both \bar{c}_6 and heavy external fermions do not enter the electroweak observables, the relevant two-loop \bar{c}_6 contributions to the self-energies must be separately gauge invariant.

A. Self-energy diagrams

To evaluate the electroweak oblique parameters S and T , all two-loop self-energy diagrams involving corrections from \bar{c}_6 need to be calculated. From the definitions of S and T , all SM contributions are subtracted, and so only terms proportional to \bar{c}_6 and \bar{c}_6^2 can remain. Working in the Feynman gauge, and discarding all two-loop diagrams that do not contain a contribution from \bar{c}_6 , there are 26 diagrams for ZZ , 26 for WW , 5 for $Z\gamma$ and 5 for $\gamma\gamma$. An example Feynman diagram for each of the self-energies is shown in Fig. 1. From Eqs. (3.1) and (3.2), it is apparent that the ZZ , $Z\gamma$ and $\gamma\gamma$ self-energies need to be evaluated at both zero and nonzero external momenta, whereas the WW self-energies are only required with zero external momenta.

The two-loop self-energies can be reduced to linear combinations of a set of basis integrals using the reduction algorithm from O. V. Tarasov [36], based on integration by parts relations [37]. This reduction procedure is implemented in the *Mathematica* package TARCER [38], which is part of the program FeynCalc [39,40]. The amplitudes for the self-energy diagrams were generated using a model file in FeynArts [41], before using TARCER for the integral reduction. The reduction algorithm allows for the calculation of self-energies with nonzero external momenta and requires a

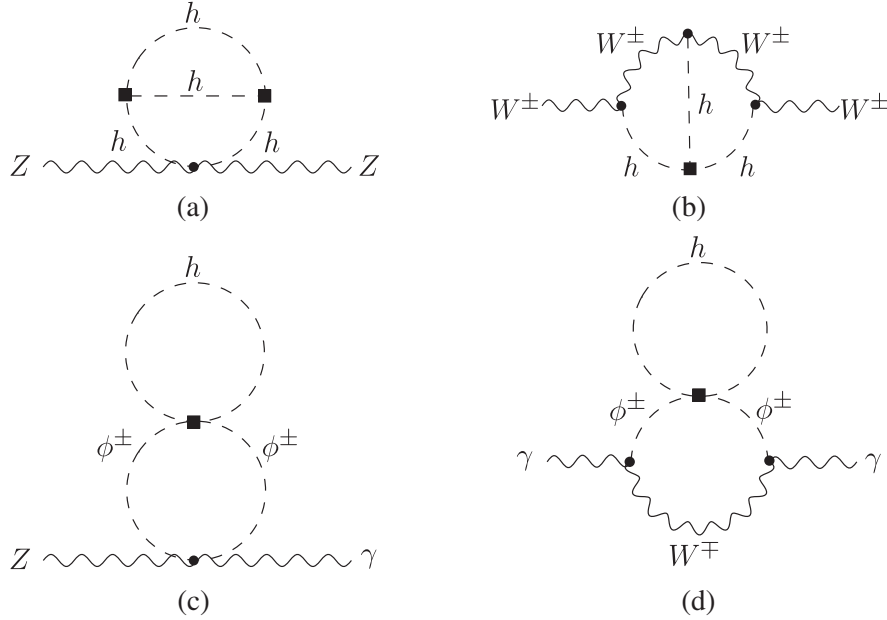


FIG. 1. Example Feynman diagrams for the (a) ZZ , (b) WW , (c) $Z\gamma$ and (d) $\gamma\gamma$ two-loop self-energies. The square represents a vertex where there is a contribution from the dimension-6 operator.

total of eight basis integrals, but this reduces to a simplified set of two basis integrals when the external momenta are zero. A numerical implementation for the evaluation of all the basis integrals is given by the TSIL package [42]. The correspondence between the notations for the basis integrals in both TARCER and TSIL is given in the Appendix of Ref. [42].

As a cross-check of our results, we have performed a second calculation of S and T based on an almost completely independent setup. After deriving the Feynman rules with the help of FeynRules [43], the self-energy diagrams were generated with QGRAF [44] and reduced to basis integrals using Laporta's algorithm [45] as implemented in FIRE [46] and Crusher [47]. Intermediate algebraic manipulations were performed with FORM [48]. Finally, the basis integrals were again evaluated numerically with TSIL. As a further check of our results, we used the *Mathematica* program TwoCalc [49] to verify the analytic expressions for the self-energy diagrams resulting from the FeynArts model file.

B. Renormalization

The leading order contribution to the electroweak oblique parameters from the Standard Model (and modifications to the renormalizable couplings) begins at one loop.¹ This means, for the calculation of these parameters at next-to-leading (two-loop) order, no actual two-loop counterterms are needed. However, all the tree-level parts

entering into the one-loop leading order result, such as vertices and propagators, obtain a one-loop counterterm contribution in the next-to-leading order calculation of the oblique parameters. Since contributions of the \mathcal{O}_6 operator and the corresponding \bar{c}_6 parameter only enter at the two-loop level, no renormalization condition is needed for this parameter. All the other parameters are SM parameters, and we perform the renormalization procedure analogously to Ref. [50], which uses the on-shell scheme.

As already stated, we only take \bar{c}_6 -dependent corrections into account. Since the one-loop results for S and T are independent of \bar{c}_6 , in order to obtain a \bar{c}_6 -dependent contribution at the two-loop level, the one-loop counterterm insertions must depend on \bar{c}_6 . In the counterterm vertices, the only \bar{c}_6 -dependent contributions originate from the field renormalization constant of the Higgs boson, but these field renormalization constants cancel together with the field renormalization constants from the counterterm insertions in the Higgs boson propagator. The only contributing counterterms are the Higgs mass and tadpole counterterms inserted into the Higgs boson and the Goldstone boson propagators.

It should be noted that the counterterm insertion into the Higgs boson propagator contains a part that is proportional to the quartic Higgs self-coupling. It originates from the on-shell Higgs mass counterterm, $\delta m_h^2 = \Sigma_{hh}(m_h^2)$, and the corresponding contribution to the Higgs self-energy Σ_{hh} shown in Fig. 2(a). The correction due to Feynman diagrams with a counterterm insertion into the Higgs propagator, an example of which is shown in Fig. 2(b), cancels the corresponding quartic Higgs self-couplings

¹We have assumed throughout the paper that the only higher-dimensional operators present are \mathcal{O}_{2n} , and in particular, the dimension-6 operators that give tree-level contributions to S and T are absent.

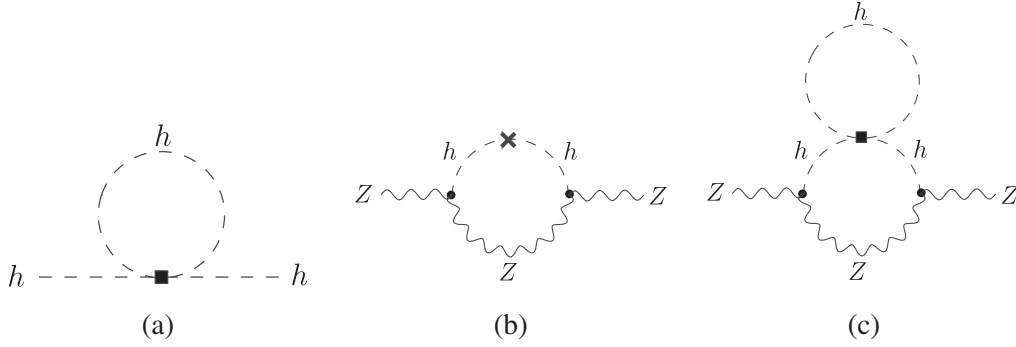


FIG. 2. Feynman diagrams demonstrating the cancellation of the quartic Higgs self-coupling. (a) shows the quartic contribution to the Higgs self-energy, and (b) shows a counterterm insertion containing the quartic Higgs self-coupling which cancels with the contribution arising in (c). The square represents a vertex where there is a contribution from the dimension-6 operator, and the cross represents a counterterm insertion.

arising in the two-loop self-energy diagrams, such as in Fig. 2(c).

IV. CURRENT AND FUTURE LIMITS FROM ELECTROWEAK OBLIQUE PARAMETERS

We have performed the calculation of the contribution from the dimension-6 operator \mathcal{O}_6 to the electroweak oblique parameters S and T , and we find that after renormalization all ultraviolet divergences from the loop integrals cancel out, leaving nonzero and finite contributions to S and T . Analytic expressions for the two-loop contributions to S and T from the \bar{c}_6 modification are given in the Appendix. For the numerical analysis, we take as input parameters [51]

$$\begin{aligned} m_W &= 80.385 \text{ GeV}, \\ m_Z &= 91.1876 \text{ GeV}, \\ m_h &= 125 \text{ GeV}, \\ G_F &= (1.16637870 \times 10^{-5}) \text{ GeV}^{-2}. \end{aligned} \quad (4.1)$$

The W and Z boson masses are the pole masses, and the electroweak scheme is specified by the tree-level relations between the parameters [52]. We find that the contribution of \bar{c}_6 to S and T is

$$\begin{aligned} S &= -0.000138\bar{c}_6^2 + 0.000180\bar{c}_6, \\ T &= 0.000206\bar{c}_6^2 - 0.000324\bar{c}_6. \end{aligned} \quad (4.2)$$

As there are no contributions from the quartic Higgs self-coupling, we can use the relation between \bar{c}_6 and κ_λ in Eq. (2.6) to write this result as

$$\begin{aligned} S &= -0.000138(\kappa_\lambda^2 - 1) + 0.000456(\kappa_\lambda - 1), \\ T &= 0.000206(\kappa_\lambda^2 - 1) - 0.000736(\kappa_\lambda - 1). \end{aligned} \quad (4.3)$$

The distinction between the contribution from two insertions of a modified Higgs self-coupling and a single insertion is made explicit here, since a term proportional to $(\kappa_\lambda^2 - 1)$ is exactly the contribution we get from two insertions.

The path of the κ_λ contribution in the S - T plane is shown in Fig. 3. The light blue ellipse shows the current 95% C.L. bound on the S and T parameters, as obtained by The Gfitter Group [35]. Also shown in the plot are possible future bounds on these parameters. The ellipses are constructed for $U = 0$ and are centred on $(0,0)$. From the intersection points of the path of κ_λ in the S - T plane

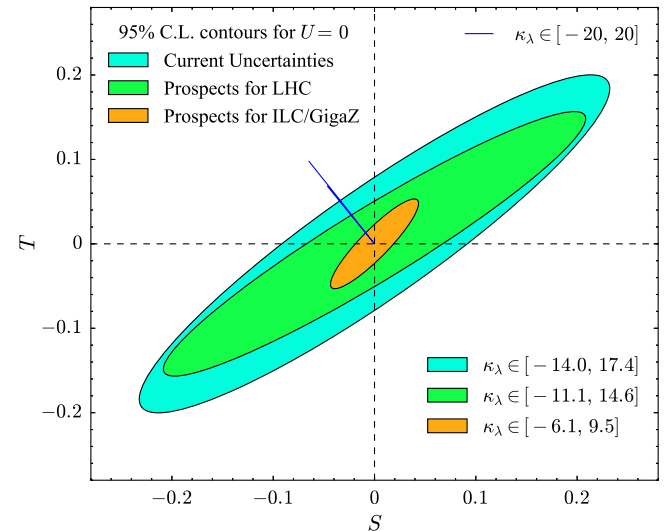


FIG. 3. Current limits and projected sensitivities of κ_λ from the electroweak oblique parameters S and T . The light blue area in the S - T plane corresponds to the 95% C.L. region based on measurements at LEP and the LHC. The green and orange areas correspond to projected LHC and ILC/GigaZ sensitivities respectively. The longer (shorter) thin blue lines show the shift in S and T as κ_λ extends up to -20 ($+20$). The intersection of these lines with the current limits and projected sensitivities gives the ranges of κ_λ as shown in the figure.

with the current ellipse, we estimate for the 95% C.L. a bound of

$$-14.0 \leq \kappa_\lambda \leq 17.4. \quad (4.4)$$

Similar bounds have been derived using the observables m_W and $\sin\theta_W$ instead of S and T [27]. The limits of Eq. (4.4) can be compared to existing bounds from searches for di-Higgs final states and Higgs coupling measurements. Direct searches constrain κ_λ to $-14.5 \leq \kappa_\lambda \leq 19.1$ [29,53] and $-8.4 \leq \kappa_\lambda \leq 13.4$ [30,54] using Run I and Run II data respectively. In addition, Higgs coupling measurements performed in single Higgs production result in the combined bound of $-9.4 \leq \kappa_\lambda \leq 17.0$ [28]. While current limits from single Higgs production are stronger than bounds derived from electroweak precision measurements, they provide complementary information and can be used to extract a combined limit.

V. CONCLUSIONS

Detailed knowledge of the self-interactions of the Higgs boson is of crucial importance to improve our understanding of the underlying mechanism of electroweak symmetry breaking and the nature of the Higgs boson itself. Only very recently have investigations of constraints on the trilinear self-interaction from (di-)Higgs production at the LHC begun to appear. However, in the absence of a signal in di-Higgs production (and thus a determination of the Higgs self-coupling), alternative ways of studying Higgs self-interactions can help to shed light on the dynamics of the scalar interactions of the Higgs boson. For example, loop-induced single Higgs production has recently been investigated and found to provide comparable limits to those from di-Higgs cross section measurements.

In this study, we have focused on the effect of Higgs self-interactions on the electroweak oblique parameters S and T in order to set limits on a modified trilinear self-coupling. Since the self-energies needed for S and T do not involve external Higgs bosons, the effects of a modified trilinear self-coupling appear only at the two-loop level and above. We found that at this order the quartic Higgs self-coupling has no effect, enabling us to set model-independent limits on κ_λ from its effects on S and T using a gauge-invariant effective field theory approach.

Our estimate for the current 95% C.L. bound on κ_λ is comparable to bounds derived from single Higgs processes. As the two approaches are orthogonal in nature, with independent uncertainties, they can be used to check the self-consistency of the bounds and, in combination (see e.g., Ref. [27]), set better limits on the trilinear Higgs self-coupling.

Reinterpreting limits on κ_λ as bounds on the scale of the higher-dimensional operators $(H^\dagger H)^n/\Lambda^{2n-4}$ implies a lower bound on the cutoff scale of order

$\Lambda \gtrsim (v/\sqrt{2}) \times \sqrt{15.5/\bar{c}_6}$. Should evidence for such a large deviation in the Higgs trilinear self-coupling appear, this clearly implies the scale of the new physics must be very close to the scale of electroweak breaking. The simplest models of new physics would involve singlets that couple only to $(H^\dagger H)$ but without mixing with the Higgs boson. We leave for future work the investigation of such models and whether they could permit large deviations in the trilinear self-coupling without having appeared in any other collider search. It is tempting to also consider the implications on the electroweak phase transition. The presence of the dimension-6 operator with $\bar{c}_6 \lesssim 2$ has been known for some time to suggest the transition becomes first order [33,55]. Larger values of \bar{c}_6 run into trouble with the global properties of the Higgs potential (global vs local minimum), but obviously once \bar{c}_6 is large enough to suggest Λ is near the electroweak scale, it no longer makes sense to truncate to dimension 6. Should evidence for large deviations in the trilinear self-coupling be observed, the electroweak phase transition is undoubtedly drastically modified. If new physics causing deviations in the trilinear self-coupling at the level that could be probed from future electroweak precision tests existed so close to the electroweak scale, it seems unavoidable that the full theory realizing the effects of the effective operators is needed to fully understand and characterize the electroweak phase transition.

ACKNOWLEDGMENTS

The authors thank P. Marquard for providing the program Crusher for integral reduction. G. D. K. is supported in part by the U.S. Department of Energy under Grant No. DE-SC0011640. A. M. is supported by a European Union COFUND/Durham Junior Research Fellowship under EU Grant No. 267209. H. R.'s work is partially funded by the Danish National Research Foundation, Grant No. DNRF90. M. S. is supported in part by the European Commission through the ‘‘HiggsTools’’ Initial Training Network PITN-GA-2012-316704.

Note added.—As this paper was being completed, Ref. [27], which also considered electroweak precision bounds on the trilinear Higgs self-coupling, appeared. Their approach was to calculate the two-loop contributions to m_W and $\sin^2\theta_{\text{eff}}^{\text{lep}}$, and the bounds they obtained (at 95% C.L.) can be read off from their Fig. 4, roughly $-14 \leq \kappa_\lambda \leq 17$, fully consistent with our results.

APPENDIX: ANALYTIC RESULTS

In the following, we present the analytic results for the \bar{c}_6 contributions to S and T . The notation for the basis integrals closely follows Ref. [42]. For the self-energy diagrams B , S , T , U , and M , the first argument is the square of the external momentum,

$$\begin{aligned}
S = & \frac{\alpha_e \bar{c}_6}{1024\pi^2 s^2 m_W^2 m_Z^4 (m_h^2 - 4m_Z^2)(m_h^2 - m_Z^2)^2} \\
& \times \{36(2 + \bar{c}_6)m_h^2(m_h^2 - m_Z^2)B(m_h^2, m_h^2, m_h^2)(-m_Z^2(m_h^6 - 3m_h^4 m_Z^2 + 4m_h^2 m_Z^4 + 16m_Z^6) \\
& + 2(m_h^2 - 2m_Z^2)^3(m_h^2 - m_Z^2)B(m_Z^2, m_h^2, m_Z^2)) \\
& + 8m_h^2 A(m_Z^2)(-4(m_h^2 - 4m_Z^2)(m_h^2 - 2m_Z^2)(m_h^2 - m_Z^2)^2 B(m_Z^2, m_h^2, m_Z^2) \\
& - (m_h^2 - m_Z^2)[(10 + 3\bar{c}_6)m_h^6 - 3(18 + 5\bar{c}_6)m_h^4 m_Z^2 \\
& + 48(3 + \bar{c}_6)m_h^2 m_Z^4 - 4(34 + 9\bar{c}_6)m_Z^6] \\
& - 9(2 + \bar{c}_6)(m_h^8 - 6m_h^6 m_Z^2 + 14m_h^4 m_Z^4 - 8m_h^2 m_Z^6 + 8m_Z^8)B(m_h^2, m_h^2, m_h^2)) \\
& + 8A(m_h^2)(-2m_h^2(2m_h^6 - 9m_h^4 m_Z^2 + 16m_Z^6)A(m_Z^2) \\
& - m_h^2(m_h^2 - m_Z^2)[(14 + 3\bar{c}_6)m_h^6 - 6(10 + \bar{c}_6)m_h^4 m_Z^2 \\
& + 12(7 + \bar{c}_6)m_h^2 m_Z^4 + 8(20 + 9\bar{c}_6)m_Z^6] \\
& + 2(m_h^2 - 4m_Z^2)(m_h^2 - m_Z^2)^2[(4 + \bar{c}_6)m_h^4 - 4(3 + \bar{c}_6)m_h^2 m_Z^2 \\
& + 12(2 + \bar{c}_6)m_Z^4]B(m_Z^2, m_h^2, m_Z^2) \\
& + 9(2 + \bar{c}_6)m_h^2(m_h^8 - 7m_h^6 m_Z^2 + 19m_h^4 m_Z^4 - 24m_h^2 m_Z^6 + 20m_Z^8)B(m_h^2, m_h^2, m_h^2)) \\
& + m_h^2(-8[(12 + 7\bar{c}_6)m_h^8 - 9(9 + 5\bar{c}_6)m_h^6 m_Z^2 + 99(2 + \bar{c}_6)m_h^4 m_Z^4 \\
& - 8(15 + 8\bar{c}_6)m_h^2 m_Z^6 + 12(6 + 7\bar{c}_6)m_Z^8]I(m_h^2, m_h^2, m_h^2) \\
& + (m_h^2 - 4m_Z^2)[8(4m_h^4 - 5m_h^2 m_Z^2 - 2m_Z^4)A(m_Z^2)^2 \\
& + 24m_Z^2((m_h^2 - 2m_Z^2)A(m_h^2)^2 + m_Z^2[20m_Z^2 - (20 + 9\bar{c}_6)m_h^2]I(m_h^2, m_h^2, m_Z^2)) \\
& - 8(m_h^2 + 2m_Z^2)(2m_h^4 - 9m_h^2 m_Z^2 + 16m_Z^4)I(m_h^2, m_h^2, m_Z^2)] \\
& + (m_h^2 - m_Z^2)\{128m_h^8 + 32\bar{c}_6 m_h^8 - 554m_h^6 m_Z^2 - 99\bar{c}_6 m_h^6 m_Z^2 + 986m_h^4 m_Z^4 \\
& + 279\bar{c}_6 m_h^4 m_Z^4 + 432m_h^2 m_Z^6 + 268\bar{c}_6 m_h^2 m_Z^6 + 304m_Z^8 + 168\bar{c}_6 m_Z^8 + 8(m_h^2 - m_Z^2) \\
& \times [(22 + 9\bar{c}_6)m_h^4 - 12(8 + 3\bar{c}_6)m_h^2 m_Z^2 + 8(22 + 9\bar{c}_6)m_Z^4]S(m_h^2, m_h^2, m_h^2, m_Z^2) \\
& + 8(m_h^2 - m_Z^2)[(4 + \bar{c}_6)(m_h^4 - 4m_h^2 m_Z^2) + 12(2 + \bar{c}_6)m_Z^4]T(m_h^2, m_h^2, m_h^2, m_Z^2) \\
& + (m_h^2 - 4m_Z^2)(2(m_h^6 - 12m_h^4 m_Z^2 + 24m_Z^6)M(m_h^2, m_h^2, m_h^2, m_Z^2, m_Z^2, m_h^2) \\
& + [(2 + \bar{c}_6)m_h^4 - 4(2 + \bar{c}_6)m_h^2 m_Z^2 + 4(10 + 3\bar{c}_6)m_Z^4]B(m_h^2, m_h^2, m_h^2) \\
& - 4m_Z^2(m_h^2 - 2m_Z^2)[B(m_h^2, m_h^2, m_h^2)^2 + 2U(m_h^2, m_h^2, m_h^2, m_h^2, m_h^2)] \\
& - [(10 + 7\bar{c}_6)m_h^6 - 2(32 + 19\bar{c}_6)m_h^4 m_Z^2 + 4(36 + 13\bar{c}_6)m_h^2 m_Z^4 \\
& + 24(\bar{c}_6 - 2)m_Z^6]U(m_h^2, m_h^2, m_h^2, m_h^2, m_h^2)]\} \}, \tag{A1}
\end{aligned}$$

$$\begin{aligned}
T = & \frac{3\alpha_e \bar{c}_6 m_h^2}{512\pi^2 s^4 m_W^4 (m_h^2 - m_W^2)^2 (m_h^2 - m_Z^2)^2} \\
& \times \{A(m_h^2)((22 + 9\bar{c}_6)m_h^2(m_h^2 - m_W^2)m_Z^2(m_h^2 - m_Z^2)s^2 \\
& + 2(m_h^2 - 2m_W^2)(m_h^2 - m_Z^2)^2 A(m_W^2) - 2(m_h^2 - m_W^2)^2(m_h^2 - 2m_Z^2)A(m_Z^2) \\
& - 9(2 + \bar{c}_6)m_h^2 m_Z^2[m_h^2(m_W^2 + m_Z^2) - 2m_W^2 m_Z^2]s^2 B(m_h^2, m_h^2, m_h^2)) \\
& - m_Z^2 s^2([m_h^2(m_W^2 + m_Z^2) - 2m_W^2 m_Z^2]A(m_h^2)^2 \\
& - 3m_h^2[2(2 + \bar{c}_6)m_h^4 - 2(1 + 2\bar{c}_6)m_W^2 m_Z^2 + (\bar{c}_6 - 1)m_h^2(m_W^2 + m_Z^2)]I(m_h^2, m_h^2, m_h^2) \\
& + (m_h^2 - m_W^2)(m_h^2 - m_Z^2)[(20 + 9\bar{c}_6)m_h^4 + 2m_W^2 m_Z^2 \\
& - 2m_h^2(m_W^2 + m_Z^2) - 9(2 + \bar{c}_6)m_h^4 B(m_h^2, m_h^2, m_h^2)]\}
\end{aligned}$$

$$\begin{aligned}
& + (m_h^2 - m_Z^2)^2 (-m_h^2 - 2m_W^2) A(m_W^2)^2 \\
& + m_W^2 [A(m_W^2) [4(m_h^2 - m_W^2) - 9(2 + \bar{c}_6) m_h^2 B(m_h^2, m_h^2, m_h^2)] \\
& + [(20 + 9\bar{c}_6) m_h^2 - 20m_W^2] I(m_h^2, m_h^2, m_W^2)] \\
& + (m_h^4 - 4m_h^2 m_W^2 + 12m_W^4) I(m_h^2, m_W^2, m_W^2) \\
& + (m_h^2 - m_W^2)^2 ((m_h^2 - 2m_Z^2) A(m_Z^2)^2 - (m_h^4 - 4m_h^2 m_Z^2 + 12m_Z^4) I(m_h^2, m_Z^2, m_Z^2) \\
& + m_Z^2 [A(m_Z^2) [-4m_h^2 + 4m_Z^2 + 9(2 + \bar{c}_6) m_h^2 B(m_h^2, m_h^2, m_h^2)] \\
& - [(20 + 9\bar{c}_6) m_h^2 - 20m_Z^2] I(m_h^2, m_h^2, m_Z^2)]) \}. \tag{A2}
\end{aligned}$$

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