2019 | Hau: *Journal of Ethnographic Theory* 9 (1): 75–85

SPECIAL ISSUE
SCIENCE IN THE FOREST, SCIENCE IN THE PAST

Mathematical traditions in Ancient Greece and Rome

Serafina Cuomo, *Durham University*

There were different ways of doing mathematics in the ancient Greek and Roman world. This essay will explore historiographical approaches to this diversity, from the claim that there were different traditions, to explorations of the social status of mathematicians, to attempts to go beyond written traditions in order to reconstruct practices. I will draw on Jean Lave's studies on situation-specific mathematics to try and tease out the power relationships and underlying assumptions behind different histories of the evidence available to us.

Keywords: Hero of Alexandria, two cultures, pure mathematics, applied mathematics, code-switching, Lave

In his dialogue *Republic*, the fourth-century BCE Athenian philosopher Plato laid the foundations for the idea of two cultures in ancient Greek mathematics. While discussing how best to educate the future leaders of the ideal state, Socrates says:

> It would be appropriate . . . to legislate this subject for those who are going to share in the highest offices in the city and to persuade them to turn to calculation and take it up, *not as laymen do*, but staying with it until they reach the study of the natures of the numbers *by means of understanding itself, not like tradesmen and retailers*, for the sake of buying and selling, but for the sake of war and for ease in turning the soul around, away from becoming and towards truth and being. (Plato, *Republic* 525b–527a, Loeb tr.; italics mine)

Possibly drawing on Pythagorean ideas, Plato set up a contrast on more than one level. Different ways of doing mathematics corresponded to different expertise, purpose, and people. Indeed, in another dialogue, the *Philebus*, Plato has Socrates ask: "Are there not two kinds of arithmetic, that of the many (*oi polloi*) and that of philosophers" (Plato, *Philebus* 56d, modified Loeb tr.)?

The term *hoi polloi* used here implies that one of the essential features of the philosophers' arithmetic is its segregated, elitist character.

Fast forward a few centuries. Around 45 CE, the temporarily exiled Roman senator and translator of Plato's *Republic* Marcus Tullius Cicero wrote:

> With the Greeks, geometry was regarded with the utmost respect, and consequently none were held in greater honour than mathematicians, but we Romans have restricted this art to the practical purposes of measuring and reckoning. (Cicero, *Tusculanae Disputationes* I.2, Loeb tr.)[1]

Mathematics is only one of the ways in which Greeks and Romans differ, according to Cicero, but his characterization has remained especially influential, shifting Plato's dichotomy toward a distinction on the basis of "national" or "cultural" identity. To simplify a long and complicated story, Plato and Cicero are significant milestones in the genealogy of the idea that there were two mathematical cultures, or traditions, in classical antiquity: one theoretical, the other practical; one aimed

1. Similar sentiments appear in Horace, *Epistulae* II 3.323–332.

HAU: Journal of Ethnographic Theory. Volume 9, number 1. DOI: http://dx.doi.org/10.1086/703797

at general truths, the other at solutions to specific problems; one achieving persuasion through rigorous logical proof, the other didactic and "algorithmic"; one interested only in knowledge, the other open to applications.

Prima facie, the idea of two mathematical traditions appears to be supported by the textual evidence. There is a relatively well-defined group of texts, explicitly and intertextually linked with each other, which has often been identified as "mainstream" Greek mathematics: Euclid's *Elements*, most of Archimedes's treatises, Apollonius's *Conics*, and so on. This tradition operates for the most part within an axiomatico-deductive demonstrative framework, which means that both its theorems and its problems are formulated in general and abstract terms. On the other hand, there is a sprawling tradition of texts in Greek, arguably sometimes intertextually linked with texts in cuneiform languages, in ancient Egyptian languages, in Latin, and possibly in Arabic, which has been identified as "folk" or "practical" mathematics, and consists of procedures for solutions carried out on specific instances of a problem. It bears no authenticated authorial identification, although some of it goes under the umbrella of pseudo-Heronian tradition (Høyrup 1997).

Let us look at one example: the equivalence between the square on the hypotenuse of a right-angled triangle, and the sum of the squares on its cathetes (see Figure 1). Today this equivalence (let's call it P) is known as the theorem of Pythagoras, even though the attribution to Pythagoras, alleged to have lived in the sixth century BCE, is not found in our sources until much later. Euclid's *Elements*, originally compiled around the early third century BCE, contains P in the following form:

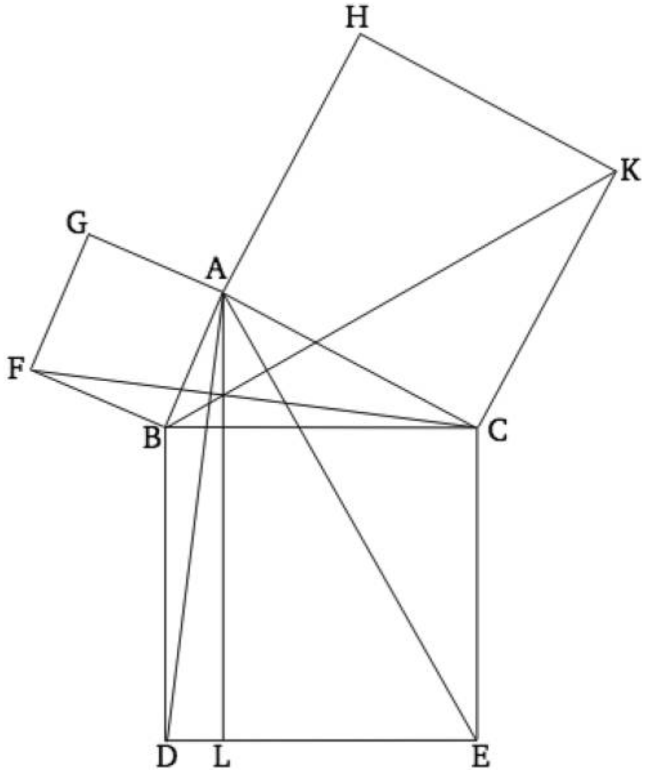

Figure 1: Diagram for Euclid, *Elements* I 47, the so-called theorem of Pythagoras (free source: https://archive.org/details/JL_Heiberg___EUCLIDS_ELEMENTS_OF_GEOMETRY/page/n45).

> In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides surrounding the right angle.
>
> Let ABC be a right-angled triangle having the angle at BAC right. I say that the square on BC is equal to the squares on BA and AC.
>
> For let a square, the BDEC, be described on BC; on BA and AC the squares GB and HC, and the AL have been drawn through A parallel to either BD or CE, and let AD and FC have been joined. And because each of the angles at BAC and BAG are right, two straight lines AC and AG, not lying on the same side, make the adjacent angles with a random straight line BA and a point A on it, equal to two right angles. Therefore CA is on a straight line with AG. Because of these things then also the BA is on a straight line with AH. And because the angle at DBC is equal to the angle at FBA, for both are right angles, let the angle at ABC be added in common. Therefore the whole angle at DBA is equal to the whole angle at FBC. And because DB is equal to BC, ZB to BA, and the two DB, BA to the two FB, BC, respectively, and the angle at DBA is equal to the angle at FBC, therefore the basis AD is equal to the basis FC, and the triangle ABD is equal to the triangle FBC. And the parallelogram BL is double the triangle ABD, for they have the same basis BD and are between the same parallels BD, AL. The square GB is double the triangle FBC, for again they have the same basis FB and are between the same parallels FB, GC. Therefore the parallelogram BL is also equal to the square GB. Similarly the AE, BK being joined, it will be proved that the parallelogram CL is also equal to the square HC. Therefore the whole square BDEC is equal to the two squares GB, HC. And the square BDEC is described on BC, while the squares GB, HC on BA, AC.
>
> Therefore the square on the side BC is equal to the squares on the sides BA, AC. Therefore in right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides surrounding the right angle. As it was necessary to prove. (Euclid, *Elements* I.47; my translation)

Euclid's formulation can be taken as emblematic of the "theoretical" tradition: it is axiomatico-deductive in that it starts with a general statement, and then proceeds logically from undemonstrated premises and from statements that have been proved earlier in the *Elements*, to conclusions that, provided the reader has agreed with the initial premises and with the "rules of the game," are logically necessary. Characteristically, it deploys a lettered diagram.[2]

Now consider the following text, dating to the second century CE:

> Let there be a right-angled triangle with the vertical side 3 feet long, the hypotenuse 5, find the basis.
>
> We will find it like this.
>
> The 5 multiplied by itself makes 25. And the 3 multiplied by itself makes 9. And from the 25 take away the 9, the remainder is 16. Its root is 4. It will be the basis: 4. Similarly too we will find with other numbers. (*P Geneva* III 124 *verso*; my translation)

This text also sets off from a(n implicit) statement of/that P, but is organized very differently from the passage in the *Elements*: P becomes a means to solving a problem rather than the focus of a proof; the text deals with a particular case and specific measurements; it appeals directly to the reader, taking them through a sequence of steps and calculations (a sequence sometimes referred to in modern scholarship as an "algorithm"), and it uses, as visible in the figure above, a numbered diagram, where the key geometrical objects of the problem are marked by a number expressing their length or area.

These differences are what gives substance to the idea of two cultures of Greek mathematics. Most recently, Markus Asper has described them as follows:

1. General mathematical knowledge emerged from practical mathematical knowledge.[3] In Asper's words, "To think of [practical mathematics, *ndr*] as "sub-scientific" makes sense, as long as one remembers that *our* understanding of what science is has been heavily influenced by Greek *theoretical* mathematics. The "sub" here should be taken literally: ancient practical mathematical traditions were certainly all-pervasive in ancient Greece, on top of which theoretical mathematics suddenly emerged, like a float on a river's surface—brightly colored and highly visible, but tiny in size"(Asper 2009: 114; italics in original). This claim has some corollaries:

   a) There is a sense in which, even though "it is doubtful whether the notion of an abstract rule was present behind all the actual procedures" of practical mathematics (Asper 2009: 113), the reiteration of procedures and the accumulation of cases led toward understanding at a general and abstract level;[4]
   b) The emergence of the theoretical tradition from the substratum of the practical tradition is a case of Bourdieu-type social distinction (Asper 2009: 123–25). The theoretical tradition was a sort of closed club, almost a "game" played by a small group, who set their way of doing mathematics, and consequently themselves, in contrast to the larger groups engaged in practical mathematics. The crucial context for this Bourdieu-type social-distinction operation is, in Asper's view, classical Athens (fifth and fourth centuries BCE). On this view, the Plato we have cited at the beginning is, as it were, channelling the *Zeitgeist*, rather than creating *ex novo* the notion of a mathematics of the *kaloi kagathoi* ("the fine and the good") and one of *hoi polloi*.

2. The *locus classicus* on lettered diagrams is Netz (1999). For the deductive structure of the *Elements*, see Mueller (1981).

3. Asper: "Manipulating pebbles on an abacus can lead to the discovery of general arithmetical knowledge" (2009: 108); "These two cases show how specialized, practical knowledge could become abstract and move beyond the circle of specialists" (109).

4. Asper: "The numbered diagram is meant to 'ensure that the reader understands the actual procedure and, thereby, the abstract method'" (2009: 119, 122; in particular 110); "Strangely, the method itself is never explained in general terms, nor is its effectiveness proved. . . . Obviously, the reader is meant to understand the abstract method by repeatedly dealing with actual, varied cases. The leap, however, from the actual case to the abstract method is never mentioned in these texts. Learning a general method is achieved in these texts by repeatedly performing a procedure, understanding its effectiveness and memorizing the steps by repetition" (111).

2. By contrast with the theoretical tradition, which thus has an inception point, the practical tradition is presented as essentially ahistorical: its roots go deep within Egyptian and Mesopotamian mathematics and, looking forward, it continues within Arabic mathematics. Despite some qualifications, the terminology used by Asper (and others) to refer to the persistence and stability of the practical tradition implies that it remained to a large extent unchanged.[5]
3. The people active within either culture were socially distinct (this as a result but also a precondition of 1b, above). In particular, mathematicians within the practical tradition, and possibly including a greater portion of "foreigners" or "migrants," "must have been of a rather low social level" (Asper 2009: 114), whereas the theoretical mathematicians were "at home in the upper circles of Athenian society" (123).
4. The language of both traditions was highly standardized and remote from oral discourse (Asper 2009: 119–20). Nonetheless, the texts of the practical tradition were accompanied by oral, personal explanation—they presupposed a teacher and a "live" situation. By contrast, the texts of the theoretical tradition are seen by Asper as autonomous—that is, constructed in such a way that they could, and still can, be understood on their own (Asper 2009: 126).

Asper's idea of distinction is an interesting twist on G. E. R. Lloyd's examination of competitive social practices in classical Athens and antiquity more generally (e.g., Lloyd 1987), but not everything in his picture, albeit sophisticated and nuanced, is equally convincing. He probably overemphasizes the extent to which the language of practical mathematics was standardized,[6] and there are other issues that I shall raise below. Let's assume for the moment, however, that Asper's two-cultures model is correct, and apply it to our example.

It would go something like this: At some point in the very remote past, somewhere or in more than one place, somehow—possibly through repeated experience on concrete cases—people became aware of P. Since then, they have been both applying P in everyday measurement situations, and teaching P through specific examples to the next generations, presumably so that they are equipped in their turn to solve measurement problems. At a certain point, possibly in classical Athens, having honed their demonstrative skills through the exercise of competitive rhetoric, a small group formulates P in a general form, and constructs a proof of it. A discourse is thus created for the purposes of social distinction, according to which the general formulation of P is the only true knowledge of P, and people who can participate in the language in which the proof is formulated are the only true mathematicians. The majority of mathematicians outside this small elite continue to do their thing, same as it ever was, but find themselves operating within a practice that is now distinct from, and construed as incompatible with, the first one.

This could be described as a clash of ontologies, in line with the theme of this volume, but, crucially, Asper highlights the fact that despite social differences there was a shared cultural substratum, and that the clashing ontologies have been constructed rather than simply being there or "emerging." Even so, once the distinction has been made and become successful, thanks to favorable historical circumstances, such as the need of Hellenistic monarchs for cultural legitimation (which led, for instance, to the compilation of Euclid's *Elements*, and to the patronage of Archimedes by the kings of Syracuse), the two cultures appear, both to ancient observers and to future generations, hypostasized, clearly distinct in status and cultural capital. In other words, what were in origin epistemic constructs can become ontologies at a later stage, in a successful example of what Bruno Latour and Steve Woolgar described as an "inscription": a process whereby an epistemic construction becomes the scientific truth, and the scaffolding of its initial construction is dismantled and erased (Latour and Woolgar 1979).

5. Asper: "Long and remarkably stable tradition . . . (but, admittedly, may have changed along the way)," Babylonian scribes "must have used essentially the same accounting board," "the tradition resurfaces" (2009: 109; see also 112). Asper cites Høyrup, probably the most influential "continuist" regarding the practical tradition's expanse across time and space.

6. Linguistic analyses of this type are not unproblematic: for instance, they define "standardization" by contrast with what they refer to as "oral discourse," and yet in a context like that of the classical or Hellenistic Greek world, our knowledge of oral discourse ultimately derives from written texts.

Asper's picture is, as I said, sophisticated and, for several aspects, persuasive. Nonetheless, there are some threads left hanging.

I. Despite the recurrence and persistence of certain features, the practical tradition is arguably as subject to change and as context-dependent as any other cultural and mathematical practice. To adapt Angela Carter's (1990) words about fairy tales, "Who first invented meatballs? In what country? Is there a definitive recipe for potato soup? Think in terms of the domestic arts. 'This is how *I* make potato soup'"; there is no mathematical Potato Soup of the Folk, nor any solution to the problem of measuring a field that has simply been passed down the generations. Even if we encounter the same problem about right-angled triangles, with the same set of numbers, in different cultural contexts, that may be how a particular person or group "made potato soup"—we should question whether it is legitimate to erase its specificity and just label it as "yet another instantiation of Potato Soup." The latter of which is also a very Platonic thing to do.

II. Several of the authors in the theoretical tradition also contributed to the practical tradition by engaging—for instance, in the problem of measuring the circle and producing numerical values for the ratio between circumference and radius.[7] Indeed,

III. Some mathematical texts or authors from antiquity are hard to classify—for instance, Diophantus or Ptolemy, or Archimedes's *Sand-reckoner*. In fact, many treatises on astronomy, optics, or harmonics may be difficult to unambiguously ascribe to one tradition to the exclusion of the other. This would seem to imply that some mathematicians—*most* mathematicians?—were active in both traditions, and that Asper's social differentiation as described at point 3 (above) needs revising. Arguably, this went both ways—not only were theoretical mathematicians occasionally "slumming it" in the practical tradition but also practical mathematicians may have been aware of the texts and practices of the theoretical tradition.

IV. While it is true that the practical tradition is mostly transmitted through papyrus and the theoretical tradition mostly through manuscript (Asper 2009: 109–10), there is theoretical mathematics on papyrus or ostrakon (specifically, material that has been identified as Euclidean),[8] and, equally, there is practical mathematics transmitted through manuscripts—for instance, the *Corpus Agrimensorum Romanorum* or, as mentioned, the so-called pseudo-Heronian material. In the rare cases where such information is available, the archaeological context for "practical" mathematics (including arithmetical tables) does not seem significantly different from the archaeological context of material that one might expect to be associated with the culture or status of theoretical mathematics, such as classical Greek literature, or official documents denoting an elevated place in society.[9] Also, we find Greek and "Egyptian" material (meaning both material in demotic, and material pertaining to "Egyptian culture," such as temple texts) in the same archaeological context. In other words, the differentiation of theoretical mathematics and practical mathematics along lines of social status or cultural identity is not borne out by evidence external to our interpretation of the text.

V. There is the small matter of Hero of Alexandria's *Metrica*, to which we now turn.

Written around the second half of the first century CE, the *Metrica*'s potential to revolutionize our picture of ancient Greek and Roman mathematics has yet to be fully realized. Here is a representative passage (see Figure 2):

> Let there be a right-angled triangle ABC, having the right angle in correspondence of B and let the AB be of 3 units, while BC is of 4 units. To find the area of the triangle and the hypotenuse.
>
> Let the ABCD be completed. Because the area of the rectangular parallelogram ABCD, as was proved above, is 12, the triangle ABC is half of the parallelogram ABCD, therefore the area of the triangle ABC will be six. And because the angle at ABC is right, and the squares on AB, BC are equal to the square on AC, and the squares on AB, BC are of 25 units,

7. For example, Archimedes, *Measurement of the circle* 3; see also other authors in Eutocius's commentary on Archimedes's treatise.

8. The most complete list is still in Fowler (1999).

9. See Cuomo (forthcoming) with further references.

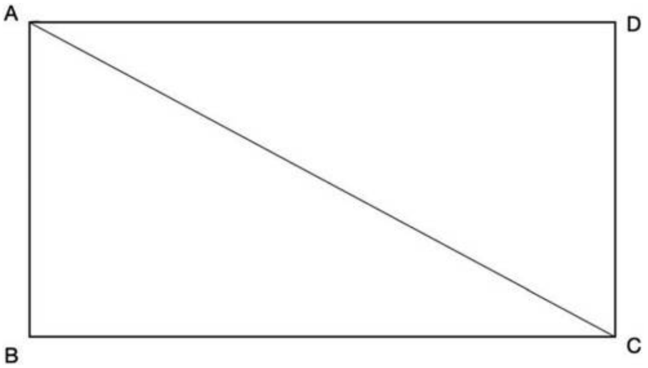

Figure 2: Diagram for Hero, *Metrica* I 2, author's drawing.

> and the square on AC therefore will be of 25 units. Therefore that side, the AC, is of 5 units.
>
> The method is this. Having multiplied the 3 by the 4, take their half. It makes 6. Of these the area of the triangle. And the hypotenuse: having multiplied the 3 by themselves and similarly having multiplied the 4 by themselves, put them together. And they make 25. And having taken the root of these, have the hypotenuse of the triangle. (Hero, *Metrica* I.2; my translation)

The *Metrica*'s special position in the history of Greek mathematics lies in its approach: measurement is tackled *both* as a general problem, solved via a proof applicable to all geometrical objects of a certain type, *and* as a specific problem, solved by measuring a particular geometrical object of that type. Historians of mathematics used to be dismissive. Van der Waerden thought that it was:

> A very childish little book. . . . Nothing but numerical examples, without proofs. Just like a cuneiform text. . . . There is no doubt in my mind that similar cookbooks have always existed. . . . Occasionally something is added, sometimes found by a real mathematician . . . but usually the source is anonymous. Some of the numerical examples in Heron are already found in cuneiform texts. . . . It is next to impossible to prove their dependence or to trace the road along which they were transmitted. And, after all, it is not very important. It is mankind's really great thoughts that are of importance, not their dilution in popularizations and in collections of problems with solutions. Let us rejoice in the masterworks of Archimedes and of Apollonius and not mourn the loss of numberless little arithmetic books after the manner of Heron. (van der Waerden 1954: 277–78)

The idea that the *Metrica* has something non-Greek ("cuneiform") about it was echoed by Otto Neugebauer in *The exact sciences in antiquity*:

> As a particularly drastic example might be mentioned the elementary geometry represented in the Hellenistic period in writings which go under the name of Heron of Alexandria (second half of the first century AD). These treatises on geometry were sometimes considered to be signs of the decline of Greek mathematics, and this would indeed be the case if one had to consider them as the descendants of the works of Archimedes or Apollonius. But such a comparison is unjust. In view of our recently gained knowledge of Babylonian texts, Heron's geometry must be considered merely a Hellenistic form of a general oriental tradition. (Neugebauer 1957: 146)

More recently, the characterization has shifted to what we could call "hybridity": *Metrica* has been called a blend (Fowler 1999: 9), a combination of elements from several traditions (Tybjerg 2004: 31, 35), a mélange of subgenres, linked to an "algorithmic" approach.[10] Toward the conclusion of a nuanced analysis of Hero's metrological work, which eschews strong commitment to the "hybridity" thesis, Vitrac nonetheless suggests:

> In any case, the *Metrica* does not raise from either of the modalities that we have distinguished, because its explicit aim is precisely to articulate the metrological steps of a sort of algorithm on the one hand and the outcomes of demonstrative geometry on the other, so as to validate the former by means of the latter. (Vitrac 2011: 14; my translation)

The implication here appears to be that demonstrative geometry is viewed as epistemologically superior to metrology, and thus able, in Hero's project, to validate it.[11] I find Karin Tybjerg's analysis to be better balanced: "In general, the techniques employed by Hero show that it is not possible to maintain the notion that Euclidean-Archimedean geometry was sealed off from

10. Acerbi and Vitrac (2014: 41, 58), are also careful to point out that the *Metrica* is not unique, but that it has obvious similarities with other algorithmic—and specifically metrological—texts, both from the Graeco-Roman tradition and from other mathematical traditions. Their metrological tradition is basically congruent with what Asper terms the "practical tradition."

11. Similar criticism appears in Tybjerg (2004: 39).

the traditions of professional problems and calculation techniques" (Tybjerg 2004: 34–35).

The debate around the the role of Hero's *Metrica* vis-à-vis the "two cultures" of Greek mathematics is meaningful because it reveals underlying assumptions, not only in the use of labels like "Greek" or "Oriental" and the respective values they are made to carry but also in the attempt at a resolution of what is perceived as its singular cultural identity. The existence itself of Hero's *Metrica* is a potential threat to the idea of two cultures, because rather than bridging them (Asper 2009: 127), it may be taken to collapse them. Conversely, Hero's *Metrica* may demand a more complex vocabulary of identity, a model other than a binary one.

With that in mind, I would like to explore the potential fruitfulness of a couple of ideas borrowed from anthropology and linguistics: the notion of situation-specificity, or situated learning, as advanced by Jean Lave, both as sole author and in joint authorship with Etienne Wenger (Lave 1986, 1988; Lave and Wenger 1991); and the notion of code-switching, which is primarily a linguistic notion, but has fruitfully been applied to issues of cultural identity and imperialism, both metaphorically and more literally, given that language was crucial for the articulation of identity in ancient Greece and Rome.[12] I will also try to apply some insights about identity and free spaces articulated by Kostas Vlassopoulos (Vlassopoulos 2007, 2009, with references to earlier bibliography).

Let's start with situation-specificity. This is not an entirely new concept for historians of science,[13] but in Lave's work it stems from observations about mathematical practice, which makes it particularly helpful, and arguably relevant, for our historical case.

In observations of the mathematical behaviour of late twentieth-century Californians, Lave found that people who appeared to be mathematically incompetent (or not very proficient) in a school context, proved to be mathematically proficient when asked to deploy the same mathematical knowledge (same in the sense that P is the same across our two previous examples) in a different, nonschool context, such as the supermarket or the home. Lave's supermarket "experiments" were in the vein of similar research conducted in different countries and situations, from tailors in Liberia to street kids selling goods at the market in Brazil.[14] In Lave's own words: "The same people differ in their arithmetic activities in different settings in ways that challenge theoretical boundaries between activity and its settings, between cognitive, bodily, and social forms of activity, between information and value, between problems and solutions" (Lave 1988: 3).

Given that school proficiency, or the lack thereof, can often be mapped onto class, gender, and race, and given that the observations about the situatedness of mathematical knowledge have almost always involved participants who are in some way disadvantaged in comparison to the stereotypically mathematically proficient white middle-class, college-educated man, situation-specificity can be deployed as a powerful political statement, even if Lave's account itself is not overtly political. Western-style school mathematics, or the "theoretical tradition," or "mental arithmetic" as opposed to, say, finger calculation, are only some among many possible mathematical "situations." They just happen, for historical reasons that are often as well known as they are ultimately ignored, to have become institutionalized, to the point where they stand in for "numeracy" or "mathematical knowledge" or "calculating skill" tout court, respectively (Harouni 2015).

The advantage, in my view, of using the notion of "situation" instead of "tradition," "culture," and even "ontology," is that "situations" are finer grained and more flexible, and also better suited to exploring use and practice, rather than "systems" (Johnstone 2011); action rather than theory. Moreover, "situations" make more room for unauthorized agency and interaction, can be similar across time and space, but are also historically localized, and they can be characterized in terms of issues of access and power (which suits both Lloyd's competitive context, and Asper's context of social distinction).

Next, consider the idea of code-switching, or a speaker's ability to alternate between two or more languages, depending on situation and context. It has long been recognized that language was key to articulating cultural identity in antiquity. At the same time, bilingualism and code-switching have become useful metaphors to talk about cultural identities in antiquity. A passage in the *Dissoi Logoi*, in the context of debating

12. I have drawn extensively on the following: Heller (1995); Webster (2001); Cooley (2002); Adams (2003); Gardner-Chloros (2005); Wallace-Hadrill (2008).

13. For example, Chemla (2012) is very much in tune with it (without explicitly using the concept).

14. Examples include Ginsburg, Posner, and Russell (1981); Carraher, Carraher, and Schliemann (1985); Lave (1986).

the question whether one can teach and learn wisdom and virtue, states:

> And if someone is not convinced that we learn our words, but rather that we are born knowing them, let him gain knowledge from this: if someone sends a child to Persia as soon as the child is born and has it brought up there without ever hearing Greek sounds, the child will speak Persian. If someone brings the child from Persia to Greece, the child will speak Greek. That is how we learn words, and we do not know who it was who taught us. (*Dissoi Logoi* 6.12, Loeb tr.)

The facts that language crops up in the discussion, and that a discussion about virtue implicates the difference between Greek and barbarian, are significant here. Examples could multiply: the notion of *paideia*, literary education, often seen as the dominant cultural paradigm of the elite in the Roman period, rested on strong competence in the Greek language, and is a very good example of the fact that, in Lave and Wenger's words, "learning involves the construction of identities" (Lave and Wenger 1991: 53). It is also well known that code-switching in antiquity could be about "the expression of different types of identity" (Adams 2003: 302; cf. also 356–82, 413–15).

All this suits *Metrica* rather well. The text has been seen as an instantiation of hybridity, but the problem with that term is its passivity. To talk in terms of code-switching, which is a more agent-centered concept, does more justice to Hero's very deliberate ("marked," in linguistic terms) combination of the reference frame of axiomatico-deductive mathematics,[15] and of calculations. The ability to speak more than one language may still leave space for a distinction between "mother tongue" and others, including pidgin languages, which are recognizably "acquired," and creolization (which could be another way to describe *Metrica*), but the main point is not competence—rather, it is the fact that agents switch linguistic codes and indeed cultural identities, according to the context of performance and communication (the situation).[16]

Situation-specificity thus creates a plausible framework for the switching of codes, and supports the possibility that *Metrica* may not have been such a rare beast—perhaps multilingualism, mathematically speaking, was not as exotic as we might think. Together, these two notions approximate a better model than the two cultures, for understanding mathematicians who "crossed boundaries" in either direction. Rather, the new model dissolves the idea of crossing boundaries, thus making sense of the fact that, as I mentioned above, whenever we are able to reconstruct a more localized context for mathematical knowledge, we are faced with "multilingualism."

Similarly to situation-specificity, there are underlying political connotations to code-switching. For a long time, this way of "mixing things up" was associated with incompetence, displacement, and subordination—a linguistic phenomenon associated with the immigrant, the insufficiently educated, and the geographically marginal. And yet, the prime example of code-switching in classical antiquity is the member of the elite but also *homo novus* Cicero, whose usage of Greek represents very complex code-switching (Adams 2003: passim). Even today, according to Penelope Gardner-Chloros, the perception of code-switching, even on the part of many of the code-switchers themselves, can be one of laziness, surprise, and embarrassment, although she notes that "approval of CS tends to coincide with a laid-back attitude towards authority" (Gardner-Chloros 2005: 14–15).

Moreover, and briefly, the idea of cultural identity has been problematized by Vlassopoulos in a way that is relevant to our purposes. While acknowledging the existence of discourses advocating strong cultural identity differences (Greek v. barbarian, Athenian v. non-Athenian, slave v. free), Vlassopoulos draws on abundant ancient evidence to make the point, specific to classical Athens in his work but in my view easily extendable to other ancient contexts, that identities were confused, confusing, and subject to continuous negotiation and renegotiation. He points out that, despite a rhetoric of separation and distinction, there were many

15. See Tybjerg (2004) for a nuanced analysis.

16. I do not find the notion of linguistic incompetence as a reason for code-switching (see Adams 2003: 305–8) useful in this context, for two reasons. First, code-switching is here a way of talking about cultural identity, so that the equivalent of incompetence would be the lack of authority in self-definition, which I reject. Second, I take Lave and Wenger's (1991) notion of legitimate peripheral participation as a useful model of learning. On that model, and particularly if code-switching is a way of talking about knowledge practices, participation is *always* legitimate, and competence or lack of competence are therefore not useful concepts. As well put by Gardner-Chloros (2005: 18): "Code-switchers upset the notion of performance errors by contravening and rewriting the expected rules."

communal "free spaces" (the *agora*, the *ergasterion* [workshop, *ndr*], the household, the harbor, the ship) where people from across alleged cultural boundaries met, interacted, and communicated. As Vlassopoulos points out, at least in some cases those free spaces must have involved literal as well as metaphorical multilingualism and code-switching (Vlassopoulos 2007, 2009).

Thus, the notion of "tradition" or "culture" seems compatible with a scenario where cultural identity is relatively unproblematic, and as such susceptible to relatively easy identification (e.g., a certain piece of mathematics looks unmistakably Greek or, conversely, non-Greek), subject to, at most, "mixing." However, more recent and self-reflective discourses, such as Vlassopoulos's, take it that cultural identity is *always* a construct, and therefore always problematic; that identification could be, and was, contested, thus raising the question of whom should be qualified to assign or deny a certain cultural identity attribution or label, particularly if we accept that it is possible for an individual to activate or switch different identities at will, without asking for external authorization.[17]

In conclusion, what are the consequences of applying situation-specificity, code-switching, and "free-space" cultural identity—in preference to "tradition" or "culture," in the sense in which they have been used in the relevant literature—to the study of mathematical practices in ancient Greek and Roman worlds? First of all, "theoretical" and "practical" turn out to be not different cultures or different traditions but different situations. It is no longer enough to populate mathematical practices with just minds at work, seen through the lens of texts. Situated learning requires that we get a better sense of the "nexus of relations between the mind at work and the world in which it works."[18] That is compatible with Asper's idea of a shared original substratum, but it also allows for people unproblematically to participate in more than one situation, and it does not map social status or institutional status onto a certain way of doing mathematics, while leaving open the possibility of mapping social or institutional status in terms of the specific situation to which that person would have had access. It leaves us open to the possibility of situations that mix things up a little, or a lot.

Situation-specificity is conducive to greater symmetry. The situation-specificity of theoretical mathematics is not essentially different from the situation-specificity of practical mathematics. Consequently, they both have a history, and the abstract, general quality of theoretical mathematics does not rest on ontological grounds. Ontologies are a feature of situations, but not the only feature. Indeed,

> a theory of situated activity challenges the very meaning of abstraction and/or generalization. . . . An important point about such sequestering when it is institutionalized is that it encourages a folk epistemology of dichotomies, for instance, between "abstract" and "concrete" knowledge. These categories do not reside in the world as distinct forms of *knowledge*, nor do they reflect some putative hierarchy of forms of knowledge among practitioners. Rather, they derive from the nature of the new practice generated by sequestration. *Abstraction* in this sense stems from the disconnectedness of a particular cultural practice. Participation in that practice is neither more nor less abstract or concrete, experiential or cerebral, than in any other. (Lave and Wenger 1991: 37; see also 33–34, 104)

Second, even though the practices we are discussing are largely textual, shifting the focus to situation-specificity and to code-switching

> emphasizes the inherently socially negotiated character of meaning and the interested, concerned character of the thought and action of persons-in-activity. This view also claims that learning, thinking, and knowing are relations among people in activity in, with, and arising from the socially and culturally structured world. (Lave and Wenger 1991: 50–51)

I think we should recognize the inevitability of personal, tacit knowledge even when all we have are texts, and cast doubt over the possibility of truly autonomous texts—even Archimedes first learned mathematics from some other person. Basic numeracy skills, which are situation-specific and include a component of tacit, interpersonal knowledge, are the sine qua non of mathematical knowledge. In this sense, again, Asper is right.

17. For a modern but relevant parallel, witness the recent discussions around LGBT, trans- and cis-gender, and gender identity.

18. Lave: "These studies converge towards a view that math 'activity' (to propose a term for a distributed form of cognition) takes form differently in different situations. The specificity of arithmetic practice wthin a situation, and discontinuities between situations, constitute a provisional basis for pursuing explanations of cognition as a nexus of relations between the mind at work and the world in which it works" (1988: 1).

Third, especially when marked, code-switching emphasizes self-determination and situation-specific agency. The authority to ascribe identity thus shifts from an external classification of people and mathematical activities (including at the hands of modern historians), to self-definition or, in the absence of explicit statements, the presumption of self-definition. The mathematicians active both within the theoretical tradition and the practical tradition, the big authors and the anonymous ones, and others involved in mathematical practices who may have not produced any texts, under this model ought to be all recognized as agents. The idea that we can separate out "real" mathematicians from those who never wrote a text, and specifically a text proving a theorem, is, in my view, unacceptably arbitrary (*pace* Netz 2002). You could see this as an extreme version of preferring actors' to observers' categories, which is again very much one of Lloyd's seminal contributions to the history of ancient science.

A perspective that frames things in terms of situation-specificity rather than culture or tradition recognizes that there are power relationships to do with mathematical practices and mathematical knowledge, but does not attribute power exclusively to the group or tradition that happen to be more similar to modern mathematics, or to modern scholars. Monica Heller has drawn on (again) Pierre Bourdieu to argue that code-switching can be used to gain entry to groups with cultural capital. Using the metaphor of a game with rules,

> specific groups set the rules of the game by which resources can be distributed. . . . It is necessary to display appropriate linguistic and cultural knowledge in order to gain access to the game, and playing it well requires in turn mastery of the kinds of linguistic and cultural knowledge which constitute its rules. Buying into the game means buying into the rules, it means accepting them as routine, as normal, indeed as universal, rather than as conventions set up by dominant groups in order to place themselves in the privileged position of regulating access to the resources they control. (Heller 1995: 160)

And yet, Heller does not allow for participants to change the game, deliberately in an act of subversion, or less deliberately by not playing the game well. She surrenders control of the game to the already-established participants, not simply in terms of setting or abiding by the rules but also in terms of who should access the game, and how well they are playing. Transferring this to cultural identity, if both situated learning and code-switching are ways to reclaim and construct—to own—cultural identities, then Lave and Wenger, compared to Heller, reaffirm the significance of self-definition over authorization by other parties. Transferred to Hero, this means that we, historians, ought to take seriously his claim to belong to the same tradition as Eudoxus and Archimedes, while recognizing it as an operation of code-switching. Transferred to wider discourses about cultural capital and learning in the ancient Greek and Roman worlds, this creates an alternative, and possibly a subversion, to the concept of *paideia*, which can be easily recognized as a Bourdieu-type social-distinction linguistic and cultural game. This would seem to work particularly well for ancient Greece and Rome because, while there were political, social, and economic hierarchies and inequalities, culture was not deeply institutionalized, and for many forms of knowledge there was, as Lloyd has repeatedly demonstrated, a marketplace.

References

Acerbi, Fabio, and Bernard Vitrac, eds. 2014. *Héron d'Alexandrie: Metrica*. Pisa: Fabrizio Serra.

Adams, James Noel. 2003. *Bilingualism and the Latin language*. Cambridge: Cambridge University Press.

Asper, Markus. 2009. "The two cultures of mathematics in ancient Greece." In *The Oxford handbook of the history of mathematics*, edited by Eleanor Robson and Jackie Stedall, 107–32. Oxford: Oxford University Press.

Carraher, Terezinha Nunes, David William Carraher, and Analúcia Dias Schliemann. 1985. "Mathematics in the streets and in schools." *British Journal of Developmental Psychology* 3 (1): 21–29.

Carter, Angela. 1990. *The Virago book of fairy tales*. London: Virago Press.

Chemla, Karine. 2012. "Historiography and history of mathematical proof: A research programme." In *The history of mathematical proof in ancient traditions*, edited by Karine Chemla, 1–68. Cambridge: Cambridge University Press.

Cooley, Alison, ed. 2002. *Becoming Roman, writing Latin? Literacy and epigraphy in the Roman West*. Portsmouth, RI: Journal of Roman Archaeology Supplementary Series.

Cuomo, Serafina. Forthcoming. *Ancient numeracy*. Cambridge, MA: Harvard University Press.

Fowler, David H. 1999. *The mathematics of Plato's Academy: A new reconstruction*. Rev. ed. Oxford: Clarendon Press.

Gardner-Chloros, Penelope. 2005. *Code-switching*. Cambridge: Cambridge University Press.

Ginsburg, Herbert P., Jill K. Posner, and Robert L. Russell. 1981. "The development of mental addition as a function of schooling and culture." *Journal of Cross-Cultural Psychology* 12 (2): 163–78.

Harouni, Houman. 2015. "Toward a political economy of mathematics education." *Harvard Educational Review* 85 (1): 50–74.

Heller, Monica. 1995. "Code-switching and the politics of language." In *One speaker, two languages: Cross-disciplinary perspectives on code-switching*, edited by Lesley Milroy and Pieter Muysken, 158–74. Cambridge: Cambridge University Press.

Høyrup, Jens. 1997. "Hero, Ps.-Hero, and Near Eastern practical geometry: An investigation of *Metrica*, *Geometrica*, and other treatises." In *Third International Conference on Ancient Mathematics: Proceedings of the Third International Meeting, Delphi, Greece, July 30–August 3, 1996*, edited by John Lennart Berggren, 93–116. Burnaby, BC: Simon Fraser University Press.

Johnstone, Steven. 2011. *A history of trust in ancient Greece*. Chicago: University of Chicago Press.

Latour, Bruno, and Steve Woolgar. 1979. *Laboratory life: The construction of scientific facts*. Beverly Hills, CA: Sage Publications.

Lave, Jean. 1986. "The values of quantification." In *Power, action and belief: A new sociology of knowledge*, edited by John Law, 88–111. London: Routledge.

———. 1988. *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.

Lave, Jean, and Etienne Wenger. 1991. *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Lloyd, G. E. R. 1987. *The revolutions of wisdom*. Berkeley: University of California Press.

Mueller, Ian. 1981. *Philosophy of mathematics and deductive structure in Euclid's* Elements. Cambridge, MA: MIT Press.

Netz, Reviel. 1999. *The shaping of deduction in Greek mathematics*. Cambridge: Cambridge University Press.

———. 2002. "Greek mathematicians: A group picture." In *Science and mathematics in ancient Greek culture*, edited by Christopher J. Tuplin and Tracey E. Rihll, 196–216. Oxford: Oxford University Press.

Neugebauer, Otto. 1957. *The exact sciences in antiquity*. 2nd ed. Providence, RI: Brown University Press.

Tybjerg, Karin. 2004. "Hero of Alexandria's mechanical geometry." *Apeiron* 37 (4): 29–56.

Vitrac, Bernard. 2011. "Faut-il réhabiliter Héron d'Alexandrie?" In *Faut-il réhabiliter Héron d'Alexandrie?*, 281–96. Montpellier: Les Belles Lettres. https://hal.archives-ouvertes.fr/hal-00454027.

Vlassopoulos, Kostas. 2007. "Free spaces: Identity, experience and democracy in classical Athens." *Classical Quarterly* 57 (1): 33–52.

———. 2009. "Beyond and below the *polis*: Networks, associations, and the writing of Greek history." In *Greek and Roman networks in the Mediterranean*, edited by Irad Malkin, Christy Constantakopoulou, and Katerina Panagopoulou, 12–23. London: Routledge.

van der Waerden, Bartel. 1954. *Science awakening*. Translated by Arnold Dresden. Groningen: Noordhoff.

Wallace-Hadrill, Andrew. 2008. *Rome's cultural revolution*. Cambridge: Cambridge University Press.

Webster, Jane. 2001. "Creolizing the Roman provinces." *American Journal of Archaeology* 105 (2): 209–25.

Serafina Cuomo is a Professor of Greek and Roman History at Durham University. She has held positions at Imperial College London, and Birkbeck, University of London. Cuomo's main area of expertise is the history of ancient Greek and Roman science and technology.

Serafina Cuomo
Department of Classics and Ancient History
Durham University
38 North Bailey
Durham DH1 3EU
England
serafina.cuomo@durham.ac.uk