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An examination of beta dose attenuation effects in coarse grains located in sliced samples

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**Abstract**

Previous work has demonstrated the feasibility of performing spatially resolved luminescence measurements with sliced solid materials (e.g., rock and ceramic) to determine the cumulative absorbed dose to individual luminescent grains *in situ*. In the present study, aspects of the dosimetry of individual grains that are truncated during slicing were examined using radiation transport simulations employing simplified geometries. The results of the simulations were applied to model the effect of grain truncation on the dosimetry of grains, in particular the beta attenuation factor and the laboratory beta source dose rate for whole and partial volumes of individual spherical grains. Where a material contains a wide range of coarse grain sizes there is the potential for misinterpreting the size of the parent grain on the basis of an examination of the shape and size of the truncated grain section exposed in the slice surface. If the original grain size is underestimated, which is likely, the simulations predict an overestimation of both the beta attenuation factor and the laboratory source beta dose rate, the maximum extent of which depends on the range of grain sizes present in the material. The simulations also indicate that by limiting the depth from the surface within which the average absorbed dose is determined, approximating the effects of opacity of the mineral, the magnitude of these deviations is reduced. However, a consequence of particular interest deriving from these results is that, when evaluating the age equation, the changes in the two quantities compensate, acting to moderate the overall effect on the calculated age for an individual grain, depending on the geometry of the grain and the composition of the sample material.

## 1 **1. Introduction**

2 Developments in the instrumentation for luminescence dating measurements, based on  
3 modified luminescence readers (Duller, 2004), spatially resolved detectors (Greilich et al.,  
4 2002; 2005; Clark-Balzan and Schwenniger, 2012; Chauhan et al., 2014; Kook et al, 2015)  
5 and laser scanning systems (Bailiff and Mikhailik, 2003), have provided the capability to  
6 measure OSL from individual grains of quartz and feldspar, enabling the determination of the  
7 equivalent dose,  $D_e$ , with sufficiently bright individual grains. Previous work with solid materials  
8 such as rock and ceramic (Greilich et al., 2005; Bailiff, 2006) has demonstrated the feasibility  
9 of performing spatially resolved measurements with sliced samples to determine  $D_e$  that, in  
10 principle, allow estimates of age to be obtained for individual grains *in situ*. In this paper  
11 dosimetry issues that arise when performing  $D_e$  determinations with individual grains within cut  
12 slices are identified and investigated by performing radiation transport simulations using  
13 simplified geometries, with a particular interest in examining their potential effect on the  
14 calculation of the luminescence age

15

### 16 *1.1. Dosimetry issues*

17 For spherical grains that are optically transparent, free of radioactive sources, of 'coarse' grain  
18 size and set in a medium containing a uniform distribution of lithogenic radionuclides, the value  
19 of  $D_e$  determined by applying OSL techniques to individual grains extracted from a  
20 disaggregated sample medium that were fully reset is expected to decrease systematically  
21 with increasing grain size. The form of reduction is governed largely by the effects of beta  
22 attenuation. Adopting the convention commonly applied in dating (Aitken, 1985), the  $\beta$   
23 component of the dose rate to a coarse grain (averaged over its volume) is calculated from the  
24 product of the point absorber infinite matrix dose rate (IMDR) and the beta attenuation  
25 coefficient,  $(1-\phi)$ . The latter is the complement of the absorbed dose fraction,  $\phi$ , because of a  
26 superposition property which follows from Mayneord's reciprocal dose rate theorem (Fitzgerald  
27 et al, 1967), which can be further exploited to advantage in radiation transport modelling  
28 calculations. Within an infinite medium of homogeneous activity, the removal of activity within  
29 a sphere of lesser radius (e.g., a grain) contained in the medium leads to a reduction of the  
30 average dose rate within the sphere by an amount corresponding to the integral dose within a  
31 sphere of the same physical composition and activity as the medium external to the sphere.  
32 This relationship is well approximated providing charged particle equilibrium is obtained within  
33 both the sphere and the surrounding medium, a condition that is usually met for quartz and  
34 feldspar grains in a silicate matrix (Brennan, 2003).

35

36 In previous work Brennan (2003), updating the earlier work of Mejdahl (1979), obtained values  
37 of the attenuation coefficient for different sizes of spherical grains by calculating the absorbed

1 dose fraction,  $\phi$ , using a scaled dose point kernel function (DPK),  $F(s)$ . The concept of the DPK  
 2 had been introduced (Loevinger, 1956) to calculate the distribution of absorbed dose (in water)  
 3 within spherical volumes about a point source as part of radioimmunological studies of  
 4 biological cells and applied to uniform media. The DPKs calculated by Berger (1973) advanced  
 5 earlier work on point beta source dose distributions (Spencer, 1959; Cross, 1968) by  
 6 employing Monte Carlo methods to simulate electron transport that accounts for multiple  
 7 scattering and energy loss fluctuations. The beta dose distribution about a unit point source of  
 8 a particular isotope is referred to as the beta dose point kernel (DPK; Prestwich et al. 1983),  
 9 which is usually calculated as a scaled DPK where the radial distance is expressed as the  
 10 fraction of the CSDA range (continuous-slowing-down approximation; ICRU, 1984). Using the  
 11 DPK, the variation in absorbed dose with distance,  $s$ , from a point source (energy  $E_0$ ) in a  
 12 homogeneous medium (density,  $\rho$ ),  $D(s)$ , is given by

$$13 \quad D(s) = \frac{E_0 F(s)}{4\tau\rho s^2}, \quad (1)$$

14 where the scaled DPK is required to meet the condition

$$15 \quad \int_0^\infty F(s) ds = 1 \quad (2)$$

16  
 17 Brennan calculated the DPKs for point sources in quartz for the three lithogenic source beta  
 18 spectra using a Monte Carlo transport code (ITS ACCEPTP) and evaluated the (average)  
 19 fractional absorbed dose,  $\phi(r)$ , within a sphere of radius  $r$  (representing a grain) by integrating  
 20 Eqn 1 over the spherical volume, obtaining the expression

$$21 \quad \phi(r) = \int_0^{2r} \left[ 1 - \frac{3s}{4r} + \frac{1}{16} \left(\frac{s}{r}\right)^2 \right] F(s) ds, \quad (3)$$

22  
 23 which is equivalent to that derived by Berger (1973), and where  $\phi(r)$  is expressed as a fraction  
 24 of the total energy released by the source (i.e., the IMDR). Integration of Eqn 1 over a spherical  
 25 segment, which does not appear to have been reported previously, results in an expression  
 26 for the integrand that is considerably more complex than Eqn 3, requiring three stages of  
 27 integration and containing quartic terms in  $s$ . Its full evaluation is underway and will be reported  
 28 elsewhere as it is beyond the scope of this paper (Bailliff and Slim, forthcoming), although some  
 29 preliminary results are referred to in the discussion below.

30  
 31 A significant advantage obtained by using radiation transport simulation codes is the  
 32 calculation of absorbed dose for any shape and size of volumes, providing their geometry can  
 33  
 34  
 35

1 be specified (Nathan, 2010; Guérin et al, 2012). In the simulations discussed below, the  
2 sources are uniformly distributed within a volume that is also defined as the detector volume,  
3 within which an average absorbed dose is calculated. For the purposes of the calculations  
4 explored in this paper, a simple spherical geometry has been used to represent a quartz grain,  
5 which is located within a solid sample medium (e.g., a ceramic or encapsulated sediment).

6  
7 Where an experimental determination of  $D_e$  is performed with slices of a sample containing  
8 transparent coarse grains truncated by the cutting process, the form of reduction in  $D_e$  with  
9 grain size is (theoretically) expected to be the same as for the parent grains providing the grains  
10 are bisected (Bailiff, 2006). However, for a material containing spherical grains of equal radius  
11 (monodispersed), a distribution of grain radii will be observed in the surface the slice, reflecting  
12 differences in the degree of truncation of the grains. The distribution will be further extended  
13 where a range of grain sizes (polydispersed) is present in the material. If the sample matrix is  
14 opaque, an examination of the grain radius in the slice surface,  $R_s$ , may give rise to a potential  
15 ambiguity in the estimation of the original (parent) grain radius, and hence increased  
16 uncertainty in the value of the attenuation factor appropriate to the parent grain when it was  
17 located in the intact sample matrix. As the distribution of absorbed dose within the interior of  
18 a single luminescent grain due to  $\alpha$  and  $\beta$  radiation from sources external to the grain is  
19 spatially not uniform (and the distribution of recombination centres within a grain may also not  
20 be uniform), the relative contributions to  $D_e$  are affected by grain truncation. In these  
21 circumstances uncertainty in the assessment of the parent grain size has implications for the  
22 calculation of the beta attenuation factor  $(1-\phi)$ .

23  
24 A second issue related to truncation that requires examination is the dose rate delivered by a  
25 laboratory beta source to a truncated grain exposed in the surface of a slice, where the source  
26 dose rate has been calibrated using whole grains. In addition to the (known) variation in the  
27 dose rate with radial distance from the central axis of the source, the dose rate to the grain can  
28 be expected to vary with the spatial extent of the grain below the cut surface.

29  
30 Both issues are further influenced by the opacity of the mineral from which the grain is formed,  
31 affecting the volume within which the absorbed dose is determined using OSL techniques.

## 32 33 **2. Dosimetry Models**

34 Two dosimetry models, A and B, were constructed for use with the general-purpose  
35 radiation transport code MCNP (Goorley et al., 2013). The physical geometry of a model coded  
36 for use with MCNP forms a central role in defining the simulation environment, as outlined in  
37 the following sections for models A and B (further details in the Supplementary Material Sec.

1 SM1). The simulations provided a calculated precision of 1-2% in most cases (after  $10^6$ - $10^{10}$   
2 particle histories), and this level of uncertainty does not reflect the larger uncertainties that  
3 would be encountered with experimentally determined quantities. Although diameter is  
4 commonly used in physical descriptions of granular materials, the radius is specified in the  
5 MCNP code, and is retained in the following discussion.

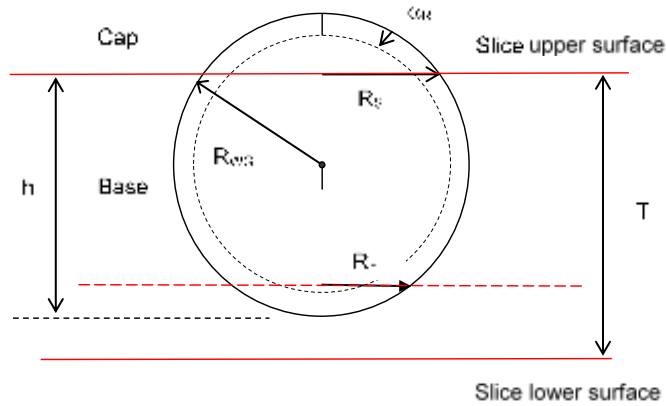
### 6 7 *2.1 Model A geometry: $\beta$ attenuation factor*

8 The model was configured to examine the calculation of  $\phi$  for an individual spherical quartz  
9 grain, located at the centre of an inert ceramic matrix containing a uniform distribution of  
10 sources of one of the lithogenic radionuclides (U, Th or K). Truncation of a grain arising from  
11 the cutting process was represented by the intersection of a sphere and the cutting plane,  
12 creating two volumetric sections defined as the detector volumes, where the smaller and larger  
13 volumes are referred to as the cap and base respectively (Fig. 1). For a sphere of radius  $R_{WG}$ ,  
14 the volume is divided into two parts at height,  $h$ , and for the purposes of modelling, the division  
15 is without loss of volume, where the surface radius,  $R_s$ , of both cap and base are the same  
16 (which is not a necessary physical requirement in applying the results of the simulations). If a  
17 second cut is made to produce a slice of thickness  $T$ , and the size of the sphere is sufficient to  
18 cause  $h$  to exceed  $T$ , a large sphere will be truncated twice and, in this case, the base volume  
19 is further divided, having a radius in the lower surface of  $R_b$ . Additional variations were also  
20 included in the model geometry, comprising layers of 100  $\mu\text{m}$  and 300  $\mu\text{m}$  thickness below and  
21 parallel to the upper cut surface (L1, 100  $\mu\text{m}$  and L3, 300  $\mu\text{m}$ ; not shown in Fig. 1 but included  
22 in Fig. SM2, Supplementary Material) within the base section to approximate the effect of grain  
23 opacity. The model A geometry was constructed with a sample slice thickness of 1 mm and  
24 with the parent grain radius ( $R_{WG}$ ), ranging from 60 to 2000  $\mu\text{m}$ . The simulations calculate the  
25 energy deposited in each of the two detector volumes. To relate the simplified geometry of the  
26 model to the physical application, the spherical detector volume is referred to in the following  
27 discussion as a (spherical) grain.

### 28 29 *2.2 Model B geometry: Laboratory $\beta$ source dose rate*

30 The calculation of the beta dose rate delivered by a  $^{90}\text{Sr}/^{90}\text{Y}$  laboratory source to quartz  
31 detector volumes located within and below the cut surface of a 1 mm-thick ceramic slice was  
32 simulated using model B. The simulations employed several geometric forms of quartz  
33 detector volumes, including a) discs and b) whole and truncated spheres presenting a specified  
34 radius at the cut surface, but having differing values of parent grain radius before cutting,  
35 ranging from 60 to 2000  $\mu\text{m}$  (examples of several grain cross-sections are given in the  
36 Supplementary Material, Fig. SM2). The simulations take into account a contribution to dose

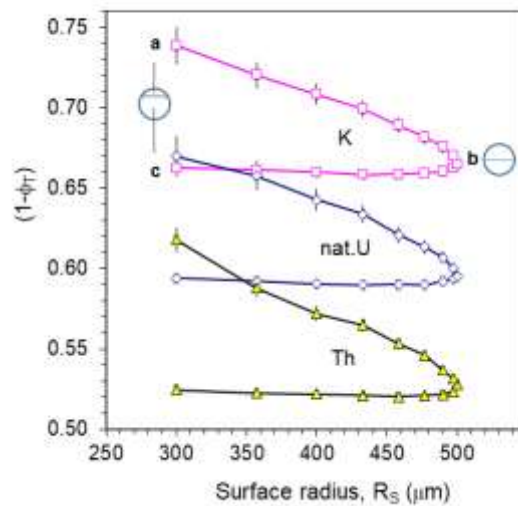
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**Figure 1**

Details of spherical geometry of parent grain, radius  $R_{WG}$ , used in the MCNP simulations showing the cross-section of a spherical grain and the positions of the upper and lower surfaces of a slice of thickness  $T$  (in this case  $T < 2 \cdot R_{WG}$ ) caused by cutting. The surface radii of the doubly truncated (hypothetical) grain in the upper and lower surfaces are given by  $R_S$  and  $R_B$  respectively. For the purposes of the simulation, the two grain volumes produced by the cutting process are referred to as the cap and base sections, as indicated. The inner circle (dotted line) indicates the outer volume of the grain penetrated by alpha particles to an average depth  $\alpha_R$  and is relevant to the assessment of the total dose rate to a whole grain.



13

**Figure 2**

Variation in the attenuation factor,  $(1-\phi_T)$ , for  $\beta$  radiation with extent of truncation for a quartz grain ( $\rho=2.65 \text{ g cm}^{-3}$ ) of  $500 \text{ }\mu\text{m}$  radius set in a ceramic medium (average density  $1.8 \text{ g cm}^{-3}$ ) containing a uniform distribution of sources ( $^{40}\text{K}$ ,  $^{238}\text{U}/^{235}\text{U}$  and  $^{232}\text{Th}$ ). The points a, b and c indicate, in the case of the  $^{40}\text{K}$  data, key stages in the change in grain volume with truncation height,  $h$ , translated to  $R_S$  (see Fig. SM1, Supplementary Material), where point b represents a bisected grain; in moving from **a** to **b** the cap volume increases and in moving from **c** to **b** the base volume increases. The lines are drawn to indicate trends in data values and are not represent functions fitted to the data.

21

1 within the slice from backscattered radiation generated at the interface with a stainless steel  
2 sample mounting plate.

3  
4 The detector volumes in the simulations comprised:

5 i) A continuous stack of quartz discs (1 mm rad. and thicknesses of 25  $\mu\text{m}$  to a depth of 200  
6  $\mu\text{m}$ , and 50  $\mu\text{m}$  thickness thereafter) extending from the surface to the base of a 1 mm-thick  
7 slice;

8 ii) Truncated spherical quartz grains of various parent grain sizes ( $R_{\text{WG}}$ , 500 to 1700  $\mu\text{m}$ )  
9 presenting a surface radius ( $R_{\text{s}}$ ) of 500  $\mu\text{m}$ . The model was also configured with layers, parallel  
10 to the cut surface, of increasing depth (100-600  $\mu\text{m}$ ) within the truncated grain to represent the  
11 effects of grain opacity (examples in Fig SM2, Supplementary Material).

12  
13 The simulations were performed for detector volumes located on the central vertical axis and  
14 along a parallel axis offset by 5 mm to examine for radial dependence. The source dose rate  
15 to each detector volume was calculated using an expression similar to Eqn 4 (below), where  
16 the value of the source activity used was  $7.5 \cdot 10^8$  Bq.

17  
18 A simplified irradiation geometry was employed comprising the ceramic slice mounted on a  
19 stainless steel base plate (thickness 0.5 mm) that rests on a 0.5 mm-thick nickel-chromium  
20 heater plate. The geometry was constructed with two source-to-sample separation distances,  
21 representing the configurations of a typical Risø reader irradiator (6.5 mm; also including a 125  
22  $\mu\text{m}$ -thick beryllium screen interposed between the sample and source) and a manually  
23 operated irradiator (14 mm) constructed in this laboratory (Bailiff, 1980).

### 24 25 *2.3 Source terms*

26 *Model A.* The simulations were performed separately for each of the three lithogenic sources  
27 distributed in the ceramic medium surrounding the grain. These included  $^{40}\text{K}$  and the  $^{238}\text{U}/^{235}\text{U}$   
28 and  $^{232}\text{Th}$  decay series (referred to as K, nat. U and Th sources, where the abundance of  $^{235}\text{U}$   
29 in natural uranium was taken to be 0.72%). The uranium and thorium progeny were assumed  
30 to be in secular equilibrium, although the simulations can be performed with any selected chain  
31 members. Details of the sources of the energy spectra for the lithogenic radionuclides are  
32 given in the Supplementary Material (Sec. SM1.2)

33  
34 *Model B.* The source in model B was based on a reported (Aitken, 1985; p118) construction of  
35 the Amersham type SIPQ source installed in our irradiators, where the  $^{90}\text{Sr}/^{90}\text{Y}$  sources were  
36 uniformly deposited on a silver disc (6 mm rad. x 20  $\mu\text{m}$ ) and covered by an additional 100  $\mu\text{m}$ -

1 thick layer of electrodeposited silver. The primary  $\beta$  particle energy spectrum was obtained  
2 from the MIRD database.

3

#### 4 *2.4 Calculation of dose rate*

5 The statistical parameter in MCNP that forms the basis of absorbed dose calculations is the  
6 \*F8 tally (Forster et al., 2004), the average energy deposited (MeV) in the detector volume(s)  
7 specified in the model. The average dose rate within the detector volume was calculated using  
8 the dose rate coefficient  $\dot{D}_c$  (mGy a<sup>-1</sup>),

$$9 \quad \dot{D}_c = \frac{E(*F8)}{m_D} \times C \times P_D \times A_P \times m_S, \quad (4)$$

10 where  $m_D$  is the mass of the detector (g),  $C$  is a lumped scaling factor (5.0558) for energy and  
11 time units,  $P_D$  is the probability of decay per unit parent disintegration,  $A_P$  is the specific activity  
12 of the parent (Bq g<sup>-1</sup>) and  $m_S$  is the mass of the source (g).

13

14 The beta absorbed dose fraction,  $\phi$ , was calculated using the values of  $\dot{D}_c$  and the beta IMDR  
15 calculated on the basis of full energy release using the same energy spectra data as employed  
16 in the simulations (for K, nat. U and Th, assuming uniformly distributed sources. The subscript  
17 T is applied to denote where the detector volume is a spherical segment.

18

### 19 **3. Simulation results**

20

#### 21 *3.1 Model A: beta attenuation factor*

22 The dose rate coefficients produced by each run of simulations of model A were used to  
23 calculate the values of  $\phi_T$ , for both base and cap sections of a truncated grain where,

$$24 \quad \phi_T = \dot{D}_c^T / \dot{D}_c^{WG} \quad (5)$$

25 and where  $\dot{D}_c^T$  and  $\dot{D}_c^{WG}$  are the dose rates for the cap or base sections and the parent grain,  
26 respectively, calculated using Eqn 4.

27

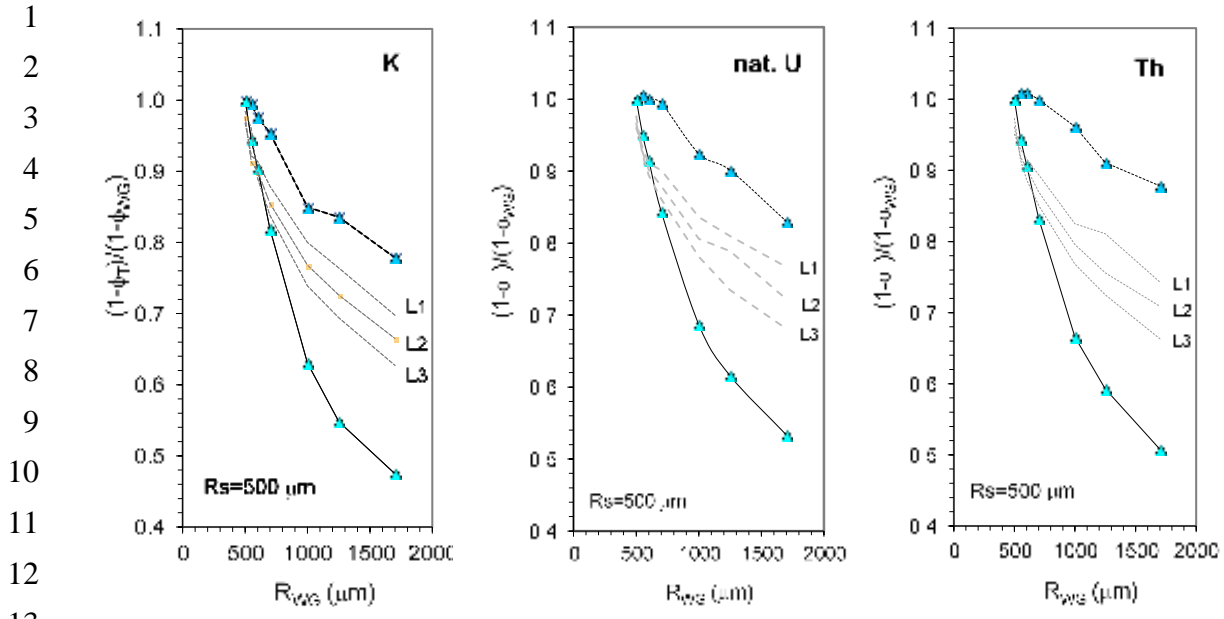
28 The change in the value of the attenuation factor,  $(1-\phi_T)$ , with extent of truncation ( $h$ ), translated  
29 to surface radius,  $R_S$ , is illustrated in Fig. 2 for a grain of radius 500  $\mu\text{m}$  ( $R_{WG}$ ) and for the three  
30 sources (K, nat. U and Th). At point **a** (Fig. 2, K) the corresponding value of  $h$  is small (100  
31  $\mu\text{m}$ ; see Supplementary Material Fig. SM1 for conversion of  $h$  to  $R_S$ ), associated with a cap  
32 section, and the value of  $(1-\phi_T)$  is higher than that for a whole grain due to the predominance  
33 of the outer volume of the grain within the cap section. At point **b**, where the grain is bisected,  
34 the value of  $(1-\phi_T)$  is equivalent to that for a whole grain and thereafter, as the volume of the  
35 base section progressively increases (to point **c**), the change in the calculated value of the



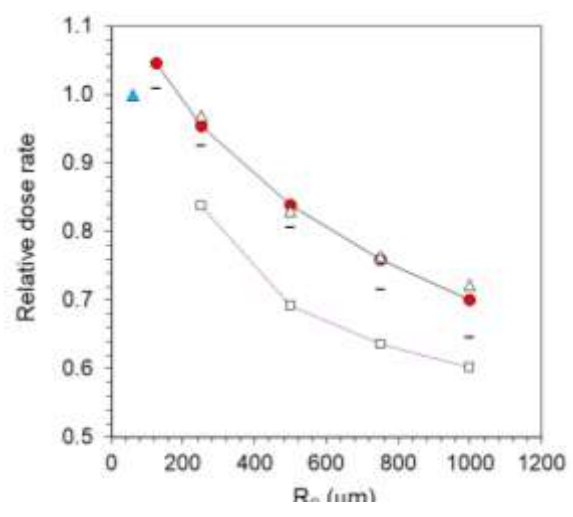
1 attenuation factor is only slight. Preliminary evaluation of Eqn 1 for a spherical segment using  
2 the transport equation developed by Loevinger (1956) in the place of Berger's DPK, produced  
3 the form of variation in the relative attenuation coefficient similar to that shown in Fig. 2. In  
4 general terms, the variation observed reflects the effect of a volumetric weighting factor on the  
5 non-uniform distribution of dose within a sphere when integrating Eqn 1 over a spherical  
6 segment (further discussion in Supplementary Material, Section SM3).

7  
8 For the range of values of truncation depth examined, the maximum relative difference in the  
9 values of  $(1-\phi_T)$  for  $R_S=300 \mu\text{m}$  (**a** vs **c**) is 11%, 13% and 18% for K and nat. U and Th sources  
10 respectively. Simulations performed with larger grains of 1 and 2 mm radius ( $R_{WG}$ ) show similar  
11 behaviour, but with larger relative differences in the attenuation factor for an equivalently  
12 scaled size of  $R_S$  (60%  $R_{WG}$ ). The relative increases in the value of  $(1-\phi_T)$  are 30%, 27% and  
13 33% for a 1 mm radius grain and 75%, 62% and 61% for a 2 mm radius grain (given in the  
14 order of K, nat. U and Th sources). The increases are broadly proportional to grain size for the  
15 nat. U and Th sources, but there is a greater divergence for the K source within this size range,  
16 reflecting its lower energy  $\beta$  spectrum. Hence, providing at least half the grain remains within  
17 the slice (between points **b** and **c** in Fig. 2), these simulations indicate that the resulting error  
18 would be small for monodispersed (single radius) truncated detector grains if the attenuation  
19 factor were calculated assuming a bisected grain.

20  
21 In a sample medium containing a population of polydispersed detector grains (i.e., with  
22 different values of radius), however, truncation gives rise to an observed value of  $R_S$  that may  
23 correspond to parent grains of differing sizes truncated at various heights,  $h$ . Using the data  
24 from the model A simulations, the beta attenuation factors were calculated for truncated grains  
25 that presented three sizes of surface radius ( $R_S = 250, 500$  and  $1000 \mu\text{m}$ ), and expressed as  
26 a fraction of the beta attenuation factor for the parent grain of the same radius (i.e.,  $R_{WG}=R_S$ ).  
27 The values obtained are plotted in Figs. 3a-c for K, nat. U and Th sources for one of the above  
28 values of surface radius ( $R_S=500 \mu\text{m}$ ). The form of the changes in the relative attenuation  
29 factor with  $R_{WG}$ , as plotted, is broadly similar for the three source types, although the values of  
30 attenuation factor differ, reflecting differences in their  $\beta$  particle energy spectra. As in Fig. 2,  
31 the point of inflexion ( $(1-\phi_T)/(1-\phi_{WG})=1$ ) corresponds to a bisected grain ( $R_S=R_{WG}$ ) and the data  
32 obtained for the cap and base sections are indicated by shaded symbols linked by broken and  
33 solid lines respectively. The bars linked by a broken line represent the values of relative  
34 attenuation factors calculated for layers L1-L3 in the base section; these have progressively  
35 lower values with increasing depth, as shown for L1 ( $100 \mu\text{m}$ ) and L3 ( $300 \mu\text{m}$ ). The values of  
36



14 **Figure 3**  
 15 Values of the beta attenuation factor  $(1-\phi_T)$  relative to that for a whole grain  $(1-\phi_{WG})$  vs whole grain  
 16 radius,  $R_{WG}$ , for spherical grains presenting a truncated surface radius of  $R_S = 500 \mu\text{m}$ , calculated  
 17 using model A. These characteristics are shown for a) K sources, b) nat. U sources and c) Th  
 18 sources. The values obtained for the cap and base sections are indicated by broken and solid lines  
 19 respectively that link the data points; the point of inflexion corresponds to a bisected grain, where  $(1-$   
 20  $\phi_T)/(1-\phi_{WG})=1$ . The bar symbols linked by a broken line indicate the values calculated for three layers  
 21 lying below the cut surface of the grain, L1-L3, each of increasing thickness from 100 to 300  $\mu\text{m}$ ,  
 22 defined as a proxy for grain opacity, as discussed in the main text. The lines are drawn to indicate  
 23 trends in data values and do not represent functions fitted to the data.  
 24



25  
 26 **Figure 4**  
 27 Dose-rate ( $\beta$ ) to quartz grains of various values of surface radius  $R_S$  in a ceramic slice of 1 mm  
 28 thickness calculated using Eqn 2, as discussed in the main text. The radionuclide composition of the  
 29 matrix is typical (Th, 40 Bq  $\text{kg}^{-1}$ ; nat. U, 40 Bq  $\text{kg}^{-1}$  U; K, 300 Bq  $\text{kg}^{-1}$ ) and the quartz grains are free of  
 30 sources; the alpha efficiency is 6%. The symbols indicate differing geometries and treatment of quartz  
 31 grains: filled circles (bisected truncated grains in slice); filled triangle, HF etched; open squares and  
 32 open triangles represent, respectively, base and cap sections of truncated grains in the slice where  
 33 the parent grain radius is  $2 \times R_S$ ; bar (300  $\mu\text{m}$  deep layer from cut surface of truncated grain). The lines  
 34 are drawn to indicate trends in data values and do not represent functions fitted to the data.  
 35

1 the relative attenuation factor for other values of  $R_s$  (250 and 1000  $\mu\text{m}$ ) exhibit broadly similar  
2 behaviour (not shown due to space limitations).

3 The influence of these potential variations in the beta attenuation factor on the calculation of  
4 the dose rate to an individual grain in a slice is illustrated in Fig. 4 for quartz grains, free of  
5 radioactivity, in a slice cut from a homogeneous (dry) sample matrix of typical radionuclide  
6 composition (Th, 40 Bq  $\text{kg}^{-1}$ ; nat. U, 40 Bq  $\text{kg}^{-1}$ ; K, 300 Bq  $\text{kg}^{-1}$ ). The total dose rate,  $\dot{D}_{\text{SGn}}^{\text{tot}}$ , was  
7 calculated using a formulation based on that described previously (Bailliff, 2006;  
8 Supplementary Material, Sec. SM2), taking into account external  $\alpha$ ,  $\beta$ ,  $\gamma$  and cosmic radiation,  
9 for the following detector volumes of individual quartz grains a) bisected grain b) cap and base  
10 sections of grains presenting a surface radius,  $R_s$ , where  $R_{\text{WG}}$  was set to  $2R_s$ , c) layer L3 of  
11 300  $\mu\text{m}$  depth within the base section of the grains described in b). All the values of dose rate  
12 were normalised to the dose rate to a 60  $\mu\text{m}$  radius HF etched quartz grain (filled triangle). The  
13 linked data points for bisected grains (filled circles;  $R_{\text{WG}} = 125, 250, 500, 750$  and  $1000 \mu\text{m}$ )  
14 trace the progressive reduction in dose rate with increasing parent grain size, reducing by  
15 some 30% between 250 and 1000  $\mu\text{m}$  radius. The dose rate to the cap (open triangles) and  
16 base (open squares) sections of truncated grains (b) were calculated for four sizes of surface  
17 radius ( $R_s = 250, 500, 750$  and  $1000 \mu\text{m}$ ) for each size of the parent grain ( $R_{\text{WG}} = 2R_s$ ).

18

19 For coarse grains with sizes extending to the millimetre range, free of internal sources and  
20 dispersed in a uniform matrix, the values of cumulative dose are expected to lie on or below  
21 the profile defined by the bisected grains. With an imposed restriction on the relative size of  
22 the parent grain ( $R_{\text{WG}} = 2R_s$ ), the dose rate to the truncated grains would be underestimated  
23 by up to 15% (average) if the grains were assumed to have been bisected, but it would be  
24 effectively unchanged for cap sections (+1% average). Although the dose rate to the cap  
25 sections might be expected to be lower than that to the bisected grain ( $R_{\text{WG}}=R_s$ ), its volume  
26 contains the outer layers in the grain which receives a proportionately higher dose contribution  
27 from external  $\alpha$  and  $\beta$  radiation, giving rise to a dose rate similar to that for a bisected grain.  
28 For layer L3 (300  $\mu\text{m}$ ) in the base section the alpha dose contribution to the dose-rate is  
29 proportionately much less than for the cap section and, following a similar trend, the dose rate  
30 values fall between those for the bisected and base sections of grains.

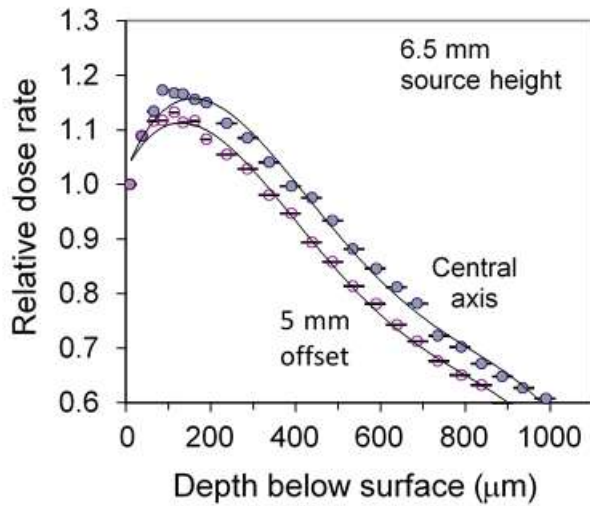
31

32

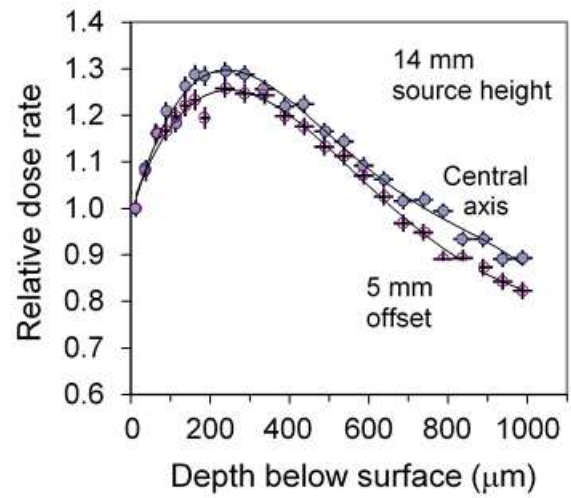
33

34

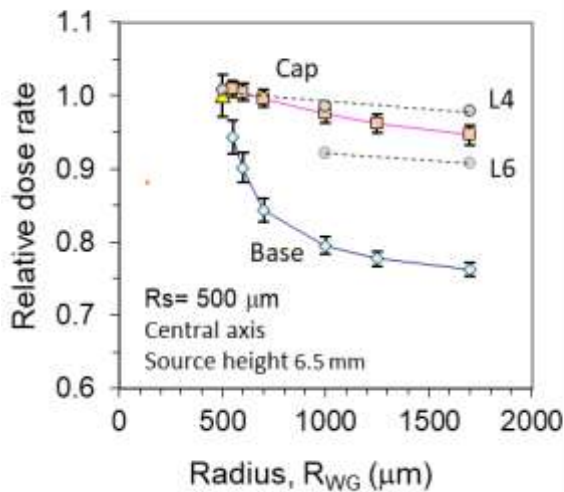
35



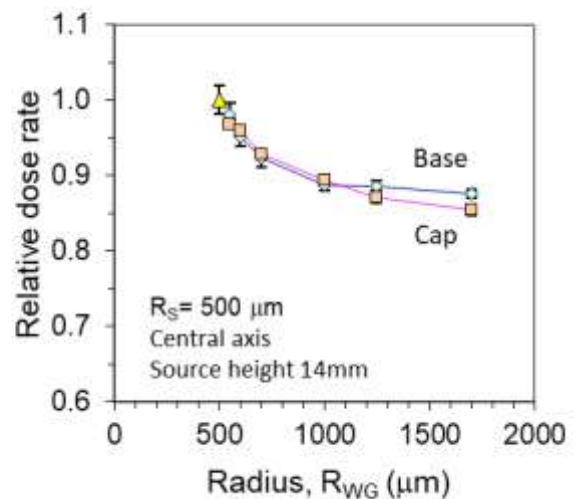
a)



b)



c)



d)

**Figure 5**

a,b) Depth-dose profiles of the  $\beta$  dose rate from the  $^{90}\text{Sr}/^{90}\text{Y}$  beta source located at heights of 6.5 mm (a) and 14 mm (b) to quartz disc detector volumes in the 1 mm-thick ceramic slice, as discussed in the main text. The profiles have been calculated along the central axis of and at an offset radius of 5 mm; the dose rate values are normalised to the first sub-surface detector disc.

c,d) Variation of relative dose rate to the base and cap sections of truncated polydispersed spherical quartz grains of various parent grain sizes,  $R_{\text{WG}}$ , that present the same surface radius,  $R_s$ , of 500  $\mu\text{m}$ , calculated using model B, where the source heights of c) 6.5 mm and d) 14 mm are indicated. The lines are drawn to indicate trends in data values and do not represent functions fitted to the data.

### 3.2 Model B: Laboratory beta source dose rate

The calculated depth-dose rate profiles for the disc detectors of configuration (i) in a 1 mm-thick ceramic slice with the two source height geometries used in these simulations are shown in Fig. 5a (6.5 mm) and Fig 5b (14 mm) as relative values along the central axis and along the offset axis. The form of the profiles is governed by the depth of the build-up layer in the sub-surface of the slice, the progressive effects of attenuation and by backscattered radiation in the region of the interface with the steel substrate. The main differences in the profiles for the two source heights are: a) the extent of the build-up layer (ca 100 and 200  $\mu\text{m}$  for 6.5 and 14 mm source heights respectively), b) the rate of reduction in dose rate is significantly greater for the lower source height (e.g., 60% vs 90%, respectively, at a depth of 1000  $\mu\text{m}$ , relative to the sub-surface dose rate) and c) the radial dependence of dose rate, where a reduction of ca 35% vs 16% is predicted between the central axis and the 5 mm offset for the lower and higher source heights respectively. The profile along the central axis shown in Fig. 5b is qualitatively similar in form to the experimental profile obtained by Wintle and Murray (1977) using a dosimetry phosphor and aluminium absorbers, and also to a calculated profile reported by Greilich et al. (2008).

The results of the simulations performed using configuration (ii) of model B to examine the variation in dose rate to truncated spherical polydispersed spheres within the slice reflect the behaviour of the depth-dose profiles. Using the above example ( $R_s=500 \mu\text{m}$ ; source height 6.5 mm), the simulations for a truncated grain (Fig. 5c) predict a modest reduction in the dose rate (relative to a bisected grain) with parent grain size for the cap sections, but a stronger reduction to the base sections, leading to a ~25% lower value for larger parent grains. For a source height of 14 mm, the changes in the dose rate to the cap and base sections with parent grain size are similar (Fig. 5d). However, the simulations indicate a complex change in the relationship between the cap and base section characteristics for the other two surface grain sizes ( $R_s=250$  and  $1000 \mu\text{m}$ ; not shown) that switch within an overall span of  $\pm 10\%$  about the value for a bisected grain, with the characteristics shown in Fig. 5d being at a cross-over point. The calculated dose rates to two layers (L4, 400  $\mu\text{m}$ ; L6, 600  $\mu\text{m}$ , shown as bars in Fig. 5c), indicate a much-reduced effect for relatively large truncated grains.

## 4. Discussion

### 4.1 Beta attenuation factor

The use of the model A simulations to obtain values of  $\phi$  indicate that for the simplest sample matrix with a monodisperse population of coarse grains, the effect of changing the surface

1 radius,  $R_s$ , of truncated grains is relatively benign for base sections providing more than half  
2 the grain remains within the slice. As  $R_s$  progressively reduces, the value of the attenuation  
3 factor for the cap increases, leading to an underestimation of the beta dose rate by 12-18%  
4 (depending on the lithogenic source) if the cap sections were assumed to be base sections. In  
5 the case of a polydisperse population of coarse grains, there is the potential for a greater  
6 overestimation of the beta dose rate to either cap or base sections of a truncated grain,  
7 depending on the full range of sizes of parent grains present in the sample matrix. The degree  
8 of variation in the beta attenuation factor (Fig. 3) is significantly larger for the base section  
9 compared with that for the cap section in the example shown ( $R_s=500 \mu\text{m}$ ).

10  
11 By applying OSL techniques to an individual grain to determine  $D_e$ , the volume from which the  
12 luminescence is detected effectively corresponds to the volume within which the absorbed  
13 dose is determined. Where a grain is transparent to UV wavelengths (i.e., the detection window  
14 used for quartz OSL), the calculation of  $(1-\phi_T)$  for the full base and cap section volumes is  
15 appropriate. In the case of opaque grains, the values of  $(1-\phi_T)$  calculated for layers in the base  
16 section indicate the potential for reducing a potential underestimation of the external grain beta  
17 component of the total dose rate, if the grains are (erroneously) assumed to be bisected (where  
18  $R_{WG}>R_s$ ). In the absence of published data on the opacity of single grains, the measurements  
19 by Aitken and Wintle (1977), although measured at wavelengths ( $\sim 400 \text{ nm}$ ) higher than the  
20 quartz OSL emission band, are of relevance: the flint with the highest opacity they examined  
21 produced a value of  $150 \text{ cm}^{-1}$  for the optical attenuation coefficient,  $\mu$ , which corresponds to a  
22 reduction of 80% in optical transmission at a depth of  $300 \mu\text{m}$ .

#### 23 24 *4.2 Laboratory source dose rate*

25 For a slice containing a population of polydisperse grains, the dose rate (Fig. 5c,d) to a  
26 truncated grain,  $\dot{D}_{SGn}^{Sr}$ , varies, for a given source height, in a complex manner according to grain  
27 size and geometry. If a wide range of grain sizes is present in a sample the dose rate to the  
28 base section may be substantially lower than that for bisected grains, although typical materials  
29 are likely to have a smaller range than that explored in these simulations. With the more  
30 commonly encountered source height of 6.5 mm, the dose rate to a truncated grain with a  $500 \mu\text{m}$   
31 surface radius where the parent grain was double this radius ( $R_{WG}=2R_s$ ) is predicted to be  
32 20% lower than for a bisected grain (located on the central axis). Again, if the 'optically active'  
33 volume within the grain is limited (L4 and L6 in Fig. 5c), the differences in the dose rate relative  
34 to a bisected (transparent) grain are predicted to be substantially reduced. In these  
35 circumstances the source dose rate may also be similar to that calculated for a cap section,  
36 the volume of which is likely to be comparable. The characteristics predicted by the simulations

1 for a 14 mm source height indicate (consistent with expectation) smaller changes in dose rate  
2 (Fig. 5d) with parent grain size. Using the same example ( $R_s=500\ \mu\text{m}$ ;  $R_{WG}=2R_s$ ), the dose  
3 rate to the base section reduces by  $\sim 12\%$  relative to a bisected grain, but the cap section also  
4 shows a similar reduction, which differs to that calculated for the lower source height.

5  
6 The simulations highlight further issues related to the calibration of the laboratory source.  
7 Following the procedures applied with granular samples (e.g., Hansen et al., 2015), the  
8 delivery of a known dose to a slice from a reference gamma source provides the opportunity  
9 to calibrate the dose rate delivered by the laboratory source to individual grains exposed in a  
10 slice surface using a spatially-resolved OSL measurement procedure. Where the minerals are  
11 transparent, the dose rate to an individual grain is expected to reflect the size of the extant  
12 grain. In practice, the dose rate is unlikely to be proportional to volume because of variations  
13 in opacity between and within grains. Although, for opaque grains, the procedure would  
14 account for the effects of an optically active depth that is less than the size of the grain, the  
15 laboratory source dose rate evaluated is expected to approach that for a bisected grain (Fig.  
16 5c). In these circumstances an assessment of the size of the parent grain made on the basis  
17 of the grain-specific calibrated dose rate for the purposes of calculating the attenuation factor  
18 is likely to be incorrect (e.g., in the case of L4 shown in Fig. 5c, an outer shell of the parent  
19 grain is the dosimetry volume of interest).

#### 21 *4.3 Effect on luminescence age calculation*

22 The above simulations indicate that if a truncated grain is incorrectly assumed to have been  
23 bisected, the omission of the adjustment to the (total) dose rate due to lithogenic sources of  
24 radiation,  $\dot{D}_{SGn}^{tot}$ , and the laboratory beta source dose rate,  $\dot{D}_{SGn}^{Sr}$ , fortunately act in a  
25 compensating manner if both quantities require adjustment in the same direction (i.e., reduced  
26 or increased) because of their reciprocal positions in the age equation. Continuing to use the  
27 same example ( $R_s=500\ \mu\text{m}$ ;  $R_{WG}=2R_s$ ), the beta attenuation coefficient, and in turn the total  
28 dose rate,  $\dot{D}_{SGn}^{tot}$  is predicted to be overestimated by  $\sim 20\%$  (Fig. 4) if the truncated grain is  
29 assumed to be bisected. Where the laboratory source height is ca 6.5 mm, and the same  
30 assumption concerning parent grain size is made, the dose rate used in evaluating  $D_e$  would  
31 also be overestimated, by  $\sim 20\%$ . Consequently, when evaluating the quotient ( $\text{Age} = D_e / \dot{D}_{SGn}^{tot}$ ),  
32 the changes in values of the two quantities compensate and, for this example, the effect on  
33 the luminescence age is predicted to be close to neutral. If the grains were opaque and the  
34 optically active volume is defined by L3 (upper  $300\ \mu\text{m}$ -thick layer in the base section), for  
35 example, similar compensation would also occur, but with much smaller overestimates of  $D_e$   
36 and  $\dot{D}_{SGn}^{tot}$ . At a greater source height (14 mm), the calculated overestimate of the dose rate

1 ( $\dot{D}_{SGn}^{Sr}$ ) is significantly lower (~10%), leading to a small overestimate in the age. The  
2 compensating effect will apply to other grain sizes for the reason discussed above, but as the  
3 value of  $\dot{D}_{SGn}^{Sr}$  varies according to both grain size and radial offset, a spatially-resolved (i.e.,  
4 grain specific) approach to the analysis needs to be taken when examining for the effects of  
5 truncation on the evaluation of the age equation.

6

## 7 **5. Conclusions**

8 This study has used radiation transport simulations with simple spherical geometries to  
9 examine the effect of the truncation of grains in sliced samples on the calculation of the beta  
10 attenuation factor and the laboratory beta source dose rate. The results have highlighted the  
11 potential effect on the values of these two variables where a material contains a wide range of  
12 coarse grain sizes and where the size of the parent grain is misinterpreted on the basis of an  
13 examination of the grain radius in the cut surface. In general, an underestimation of the parent  
14 grain size is likely to arise, resulting in an overestimation of both the beta attenuation coefficient  
15 and the laboratory beta source dose rate, the maximum extent of which depends on the range  
16 of grain sizes in the material. The simulations also indicate that by limiting the depth from the  
17 surface of the truncated grain within which the absorbed dose is determined, approximating  
18 the effects of opacity of the mineral, the magnitude of these deviations is reduced. However,  
19 a consequence of particular interest deriving from these results is that, when evaluating the  
20 age equation, the changes in the two quantities compensate, acting to moderate the overall  
21 effect on the calculated age for a particular grain, depending on the geometry of the grain and  
22 the composition of the surrounding sample material. While radiation transport modelling is  
23 computationally direct and powerful in terms of extension of this approach to modelling  
24 heterogeneous materials, establishing a link to the physical models underpinning the analytical  
25 approach employing dose point kernels is nonetheless important, as it may enable analytical  
26 functions to be derived that can be applied in dosimetry calculations without the need to resort  
27 to simulations. However, in the case of complex materials containing heterogeneous  
28 distributions of lithogenic sources (e.g., Guerin et al., 2015; Martin et al., 2015), more elaborate  
29 computational models are required to calculate the dose to individual coarse grains in cut slices  
30 of solid (or encapsulated) material, and these can provide valuable predictive tools to augment  
31 experimental measurements that ultimately aim to derive age estimates for individual grains.

32

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## 6 7 **References**

- 8 Aitken, M.J., 1985. Thermoluminescence Dating. Academic Press, London.
- 9 Aitken, M.J., Wintle, A.G., 1977. Thermoluminescence dating of calcite and burnt flint: the age relation  
10 for slices. *Archaeometry* 19, 100-105.
- 11 Bailiff, I.K., 1980. A beta irradiator for use at the Durham TL dating laboratory. *Nucl. Instrum. & Meth.*  
12 175, 224-226.
- 13 Bailiff, I.K., 2006. Development of single grain OSL dating of ceramic materials: spatially resolved  
14 measurement of absorbed dose. *Radiat. Meas.* 41, 744-749.
- 15 Bailiff, I.K., Mikhailik, V.B., 2003. Spatially resolved measurement of optically stimulated luminescence  
16 and time-resolved luminescence. *Radiat. Meas.* 37, 151-159.
- 17 Berger, M.J., 1973. Improved point kernels for electron and beta-ray dosimetry. NBSIR 73-107; National  
18 Bureau of Standards, Washington D.C.
- 19 Brennan, B.J., 2003. Beta doses to spherical grains. *Radiat. Meas.* 37, 299-303.
- 20 Chauhan, N., Adhyaru, P. Vaghela H., Singhvi, A.K., 2014. EMCCD based luminescence imaging  
21 system for spatially resolved geo-chronometric and radiation dosimetric applications. *J Instrum.*  
22 P11016.
- 23 Clark-Balzan, L., Schwenninger, J-L, 2012. First steps toward spatially resolved OSL dating with  
24 electron multiplying charge-coupled devices (EMCCDs): System design and image analysis.  
25 *Radiat. Meas.* 47, 797-802.
- 26 Cross, W.G., 1968. Variation of beta dose attenuation in different media. *Phys. Med. Biol.* 13, 611–618.
- 27 Duller, G.A.T., 2004. Luminescence dating of Quaternary sediments: recent developments, *J. Quat.*  
28 *Science* 19, 183–192.
- 29 Fitzgerald, J. J., Brownell, G. L., Mahoney, F. J., 1967. Mathematical theory of radiation dosimetry.  
30 Gordon and Breach, New York.
- 31 Forster, R.A., Cox, L.J., Barrett, R.F., Booth, T.E., Breimeister, J.F., Brown, F.B., Bull, J.S., Geisler,  
32 G.C., Goorley, J.T., Mosteller, R.D., Post, S.E., Prael, R.E., Selcow, E.C., Sood, A., 2004. MCNP™  
33 version 5. *Nucl. Instrum. and Methods B* 213, 82-86.
- 34 Goorley, T., James, M., Booth, T., Brown, F., Bull, J., Cox, L.J., Durkee, J., Elson, J., Fensin, M., Forster,  
35 R.A., Hendricks, J., Hughes, H.G., Johns, R.B., Kiedrowski, R., Martz, Mashnik, S., McKinney, G.,  
36 Pelowitz, D., Prael, R., Sweezy, J., Waters, L., Wilcox, T., Zukaitis, T., 2013. Initial MCNP6 release  
37 overview, MCNP6 version 1.0. LA-UR-11-05198.  
38 ([http://laws.lanl.gov/vhosts/mcnp.lanl.gov/pdf\\_files/la-ur-13-22934.pdf](http://laws.lanl.gov/vhosts/mcnp.lanl.gov/pdf_files/la-ur-13-22934.pdf))
- 39 Greulich, S., Glasmacher, U.A., Wagner, G.A., 2002. Spatially resolved detection of luminescence – a  
40 unique tool for archaeochronometry. *Naturwissenschaften* 89, 371-375.

1 Greilich, S., Glasmacher, U.A., Wagner, G.A., 2005. Optical dating of granitic stone surfaces.  
2 *Archaeometry* 47, 645-665.

3 Greilich, S., Bøtter-Jensen, L., Murray, A.S., 2008. Simulation of electron transport during beta  
4 irradiation. *Radiat. Meas.* 43, 748-751.

5 Guérin, G., Mercier, N., Nathan, R., Adamiec, G., Lefrais, Y., 2012. On the use of the infinite matrix  
6 assumption and associated concepts: a critical review. *Radiat. Meas.* 47, 778-785.

7 Guérin, G., Jain, M., Thomsen, K.J., Murray, A.S., 2015. Modelling dose rate to single grains of quartz  
8 in well-sorted sand samples: The dispersion arising from the presence of potassium feldspars and  
9 implications for single grain OSL dating. *Quaternary Geochronology* 27, 52-65.

10 ICRU, 1984. International Commission on Radiation Units and Measurements. ICRU Report 37,  
11 Stopping Powers for Electrons and Positrons. International Commission on Radiation Units and  
12 Measurements, Bethesda, USA.

13 Kook, M.H., Lapp, T., Murray, A., Thomsen, K.J., Jain, M 2015, A luminescence imaging system for the  
14 routine measurement of single-grain OSL dose distributions *Radiation Measurements*, 81, 171–177.

15 Loevinger, R. 1956. The Dosimetry of Beta Sources in Tissue. The Point-Source Function, *Radiology*  
16 66, 55-62.

17 Martin, L., Mercier, N., Incerti, S., Lefrais, Y., Pecheyran, C., Guérin, G., Jarry, M., Bruxelles, L., Bon,  
18 F., Pallier, C., 2015. Dosimetric study of sediments at the beta dose rate scale: Characterization and  
19 modelization with the DosiVox software. *Radiat. Meas.* 81, 134-141.

20 Mejdahl, V., 1979. Thermoluminescence dating: dose attenuation in quartz grains. *Archaeometry* 21,  
21 61–72.

22 Nathan, R.P., 2010. Numerical modelling of environmental dose rate and its application to trapped-  
23 charge dating. Unpublished DPhil thesis. University of Oxford.

24 Prestwich, W.Y., Nunes, J., Kwok, C.S., 1989. Beta dose point kernels for radionuclides of potential  
25 use in radioimmunotherapy. *J. Nuclear Med.* 30, 1036-1046.

26 Spencer, L. V., 1959. Energy Dissipation by Fast Electrons, National Bureau of Standards Monograph  
27 1, National Bureau of Standards, Washington D.C.

28 Wintle, A.G., Murray, A.S., 1977. Absorbed dose from a beta source as shown by  
29 thermoluminescence dosimetry. *Int. J. Appl. Radiat. & Isotopes* 28, 625-627.

30