Asymptotically safe standard model extensions?

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We consider theories with a large number N_F of charged fermions and compute the renormalization group equations for the gauge, Yukawa and quartic couplings resummed at leading order in $1/N_F$. We construct extensions of the standard model where SU(2) and/or SU(3) are asymptotically safe. When the same procedure is applied to the Abelian U(1) factor, we find that the Higgs quartic can not be made asymptotically safe and stay perturbative at the same time.

I. INTRODUCTION

The Large Hadron Collider (LHC) data collected at $\sqrt{s} = 13$ TeV are in line with the standard model (SM) predictions and provide strong bounds on SM extensions, in particular on those that were introduced to tame the quadratic sensitivity of the Higgs mass operator on the scale of new physics, known as natural extensions. The time is therefore ripe to explore alternative approaches to naturalness and even better, new guiding principles, that can help selecting a more fundamental theory of nature.

Weak-scale extensions of the SM valid up to infinite energy bypass the issue of quadratically divergent corrections to the Higgs mass [1] (we will ignore gravity, as an extension of Einstein gravity can have this property [2,3]). Extensions of the SM realizing total asymptotic freedom can be built by embedding the Abelian $U(1)_Y$ into non-Abelian gauge groups that explain the observed values of the hypercharges. Natural possibilities, where the extended gauge group is broken around the electroweak scale [1], have been proposed based on the groups $SU(2)_L \otimes SU(2)_R \otimes SU(4)_c$ and especially $SU(3)_L \otimes SU(3)_R \otimes SU(3)_c$ [4]. However, a fine-tuning at the % level is needed in order to make the extra vectors above present bounds, such as $M_{W_R} > 2.5$ TeV [5].

This is one of the motivations behind the search of alternative fundamental SM extensions that are asymptotic safe, rather than asymptotically free. In addition, asymptotically safe theories are an intriguing and yet much unexplored possibility. Only recently the first controllable perturbative example of a gauge-Yukawa theory able to display asymptotic safety in all couplings [6] was discovered, where the Veneziano-Witten limit $N, N_F \gg 1$ has been employed to unquestionably establish the existence of such a scenario. Quantum stability of the theory and the determination of the vacuum and potential of the theory were established in [7]. The original model did not feature gauged scalars nor radiative symmetry breaking and is a vectorlike theory. Gauged scalars and related asymptotically safe conditions were introduced for the first time in [8] while chiral gauge theories were investigated in [8,9], and radiative breaking in [10]. Extensions to semisimple groups with SM-like chiral matter appeared first in [11] while semisimple gauge theories with vectorlike fermions appeared in [12]. The Veneziano limit leads to interesting phenomenological applications once spontaneous breaking occurs [13]. Another virtue of a controllable perturbative limit is that in these theories the Higgs mass can be naturally lighter than the transition scale Λ, given that nonperturbative corrections are exponentially suppressed, $\delta M_h^2 \sim \Lambda^2 e^{-\mathcal{O}(1)/\alpha}$ [8].

Supersymmetric asymptotically safe quantum field theories, where exact nonperturbative results have been established in [14,15], are also an intriguing possibility albeit presumably not of natural type because of the tension with the Large Electron-Positron Collider (LEP) and LHC bounds [16].

In this work, we depart from the Veneziano limit and supersymmetric extensions by taking another interesting theoretical limit that can help taming the ultimate fate of one or all of the SM $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ gauge factors. This makes use of the large number of flavor expansions discussed in [17–19]. In the presence of $N_F \gg 1$ extra fermions one can resum corrections at leading order in $1/N_F$ [20,21]. As we will review in Sec. II, these fixed points occur at predicted nonperturbative values of the product of their gauge couplings times the associated large number of extra flavors. Unsuccessful attempts of constructing perturbative asymptotically safe extensions of the SM appeared in [22]. Here both the hypercharge and scalar quartics were still under the spell of Landau poles, and in any event the constructions depart from the rigorous limit of [6].

The renormalization group equations (RGEs) for the Yukawa couplings get modified by the resummation [23]. We here compute how the RGE for the quartic Higgs coupling gets modified and apply our results to the case of the standard model.

The paper is structured as follows. In Sec. II, we review how the introduction of many extra fermions allows for an ultraviolet interacting fixed point for the gauge couplings, and compute how the RGE for Yukawa and scalar quartic couplings get modified. In Sec. III, we show that this allows us to make the $SU(2)_L$ and/or $SU(3)_c$ factors of the SM gauge group asymptotically safe. However we find that $U(1)$ cannot be made asymptotically safe in a controlled regime. We offer our conclusions in Sec. IV.

II. FIXED POINTS IN THE LARGE NUMBER OF FLAVORS

We now discuss the fate of gauge theories at short distance in the presence of a large number of vectorlike fermions ($N_F \gg 1$). We do this by first summarizing the associated β functions resummed at leading order in $1/N_F$. We denote with $\alpha_i \equiv g_i^2/4\pi$ for $i = Y, 2, 3$ the SM gauge couplings (we define g_Y as the hypercharge gauge coupling in the normalization where the Higgs H has $|Y| = 1/2$).

A. Resummed gauge β functions

We conveniently write the gauge β functions as

$$\frac{\partial \alpha_i}{\partial \ln \mu} = \beta_{\alpha_i} = \beta_{\alpha_i}^{\text{SM}} + \beta_{\alpha_i}^{\text{extra}}, \qquad (1)$$
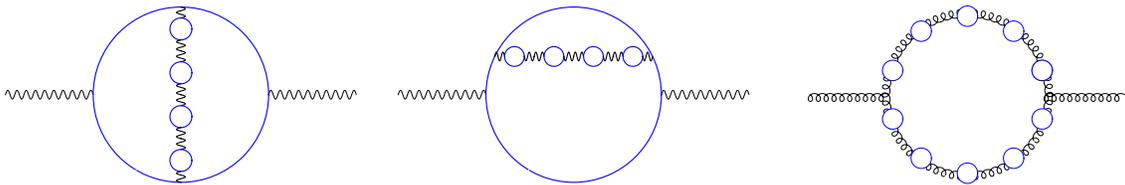
where $\beta_{\alpha_i}^{\text{SM}}$ are the perturbative SM contribution: at one-loop $\beta_{\alpha_i}^{\text{SM}} = b_i^{\text{SM}} \alpha_i^2/2\pi$, with $b_Y^{\text{SM}} = 41/6$, $b_2^{\text{SM}} = -19/6$ and $b_3^{\text{SM}} = -7$. The contribution of the $N_F \gg 1$ extra fermions can be written as their one-loop contribution plus their resummation at leading order in $1/N_F$ (see [17,24] and reference therein):

$$\beta_{\alpha_i}^{\text{extra}} = \frac{\alpha_i^2}{2\pi} \Delta b_i + \frac{\alpha_i^2}{3\pi} F_i\left(\Delta b_i \frac{\alpha_i}{4\pi}\right). \qquad (2)$$

The one-loop coefficients are well known: for Dirac fermions in the representation R_i with dimension D_{R_i} and Dynkin index S_{R_i} they are given by

$$\Delta b_Y = \frac{4}{3} Y^2 N_F D_{R_2} D_{R_3}, \qquad \Delta b_2 = \frac{4}{3} N_F S_{R_2} D_{R_3},$$

$$\Delta b_3 = \frac{4}{3} N_F S_{R_3} D_{R_2}. \qquad (3)$$

We write the resumed contributions in the simpler limit where only one of the Δb_i is nonvanishing, in order to the neglect the mixed contributions. At leading order in $N_F \gg 1$, the result is dominated by the Feynman diagrams in Fig. 1 (extra diagrams are present for non-Abelian groups). Their resummation gives the functions $F_i(A)$

$$F_1(A) \equiv 2 \int_0^A I_1(x) dx,$$

$$F_n(A) \equiv \int_0^A I_1(x) I_n(x) \quad \text{for } n = 2, 3 \qquad (4)$$

$$I_1(x) \equiv \frac{(1+x)(2x-1)^2(2x-3)^2 \sin^3(\pi x)\Gamma(x-1)^2\Gamma(-2x)}{\pi^3(x-2)}, \qquad (5)$$

$$I_n(x) \equiv \frac{n^2-1}{2n} + \frac{(20-43x+32x^2-14x^3+4x^4)n}{2(2x-1)(2x-3)(1-x^2)}. \qquad (6)$$

The F_i functions are plotted in Fig. 2. Note that F_1 ($F_{2,3}$) has a logarithmic singularity at $A = 5/2$ ($A = 1$)
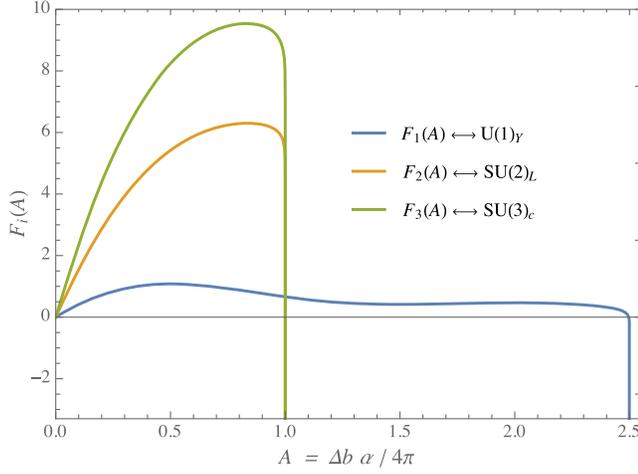

FIG. 1.   Feynman diagram topologies that dominate at leading order in $1/N_F$ (we do not show tadpoles).

FIG. 2. The functions giving the resummed gauge β functions in Eqs. (4), (5) and (6).

$$F_1(A) \overset{A \to 5/2}{\simeq} \frac{14}{15\pi^2} \ln\left(1 - \frac{2A}{5}\right) + 0.611 + \cdots,$$

$$F_n(A) \overset{A \to 1}{\simeq} \frac{n}{8} \ln(1 - A) + \cdots \quad (7)$$

where $\cdots$ in the latter expression represents contributions which remain finite at the singularity. As clear from Fig. 2, the locations of these singularities coincide to a very good approximation with the points where the β functions vanish, leading to the fixed points

$$\alpha^*_{2,3} = \frac{4\pi}{\Delta b_{2,3}}, \qquad \alpha^*_Y = \frac{10\pi}{\Delta b_Y}. \quad (8)$$

Such fixed-point values reproduce the observed values of the SM gauge couplings renormalized at a few TeV for $\Delta b_Y \approx 1800$, $\Delta b_2 \approx 400$, $\Delta b_3 \approx 150$: higher values of Δb_i are thereby not allowed.

A word of caution is in order here: the fixed point resulting from the large-N_F resummation is not on the same rigorous footing as the perturbative fixed points arising in the limit of large N, N_F [6], or the supersymmetric fixed points [14,15]. The physical meaning of the logarithmic singularity needs to be investigated further, especially because extra singular behaviors emerge at subleading orders in N_F [17]. Of course, the important issue is whether the first singularity is unaffected by the emergence of new singularities that might very well imply the existence of different physical branches not linked to the original one as argued in [17]. In fact, we can start to understand how the first UV fixed point starts to emerge within perturbation theory [18]. Singularities in beta functions are not a pathology of the theory as the well known exact supersymmetric beta functions show [25]. In fact, using alternative large number of colors limits one can even map these supersymmetric beta functions in the one of one flavor

QCD [26,27]. Furthermore beta functions are scheme dependent while fixed points are physical. This means that one can, in principle, find another scheme in which the beta function has a different behavior while the theory retains the UV fixed point. This again happens in supersymmetric field theories when going from the all-order exact beta function to the exact Wilsonian (holomorphic scheme) one-loop exact beta function, see [28] for a discussion of the scheme transformations and their impact on their derivation of the beta functions via string theory. Clearly the transformation among the schemes is also singular.

Another possible issue is that the resummation of a perturbative series can produce meaningless results when the series is asymptotic, not necessarily convergent (this happens in the SM for the series in the quartic SM Higgs coupling). In our case, the fermionic path integral gives a functional determinant which is an analytic function of the gauge coupling, such that the final path integral over vectors can be expanded in a convergent series. The predictions from resummation can be confirmed via lattice simulations for which neither technical nor theoretical impediment exist. Furthermore, lattice simulations can test whether fixed points exist also away from the $N_F \gg 1$ limit. No fixed point was found in QED with one electron, and this computation can be repeated with larger N_F.

B. Resummed Yukawa β function

As pointed out in [23], one needs to resum corrections to the RGE of Yukawa couplings. We consider a Yukawa coupling $y\psi_1\psi_2 H + \text{H.c.}$, where $\psi_{1,2}$ are Weyl fermions and H is a scalar field. All particles are in generic representations of the gauge group $G = \prod_i G_i$. The one-loop β function improved by resumming the gauge propagators can be written as

$$\frac{\partial y}{\partial \ln \mu} = \beta_y$$
$$= -3\frac{y}{4\pi}\sum_i (C_{\psi_1 i} + C_{\psi_2 i})\alpha_i \times R_y(A_i) + \mathcal{O}(y^3) \quad (9)$$

where $C_{\psi i}$ is the quadratic Casimir of ψ under G_i. The well known $\mathcal{O}(y^3)$ terms are not shown because not affected by the resummation, encoded in the function $R_y(A_i)$, equal to $R_y(0) = 1$ in the limit of vanishing $A_i = \Delta b_i \alpha_i/4\pi$. This is computed in Appendix A 2, with the result

$$R_y(A) = \frac{(3 - 2A)^2 (2 - A)\sin(\pi A)\Gamma(2 - 2A)}{9\pi A \Gamma(3 - A)^2}$$
$$\times \left(2 + A\frac{C_H}{C_{\psi_1} + C_{\psi_2}}\right) \quad (10)$$

which agrees with the computation in [23]. For a U(1) one has $C_p = q_p^2$, where q_p is the charge of particle p, that

satisfies $q_{\psi_1} + q_{\psi_2} + q_H = 0$. If multiple U(1) are present, the expressions above hold in a basis where they do not mix.

Particularly relevant are the values of R_y close to the fixed point of gauge couplings: $A = 1$ for a non-Abelian coupling, and $A = 5/2$ for an Abelian coupling:

$$R_y(1) = \frac{1}{18}\left(2 + \frac{C_H}{C_{\psi_1} + C_{\psi_2}}\right),$$

$$R_y(A) \overset{A \to 5/2}{\simeq} \frac{9 + 10 q_{\psi_1} q_{\psi_2}/(q_{\psi_1}^2 + q_{\psi_2}^2)}{270\pi^2(5/2 - A)}. \tag{11}$$

The pole at $A \to 5/2$ implies that Yukawa couplings of fermions charged under an asymptotically safe U(1) are driven to negligibly small values at large energies.

C. Resummed quartic β function

At leading order in $1/N_F$, β_λ is given by

$$\frac{\partial \lambda}{\partial \ln \mu} = \beta_\lambda$$

$$= -\frac{\lambda}{4\pi}\sum_i C_i \alpha_i R_\lambda(A_i) + \sum_{ij} C_{ij} \alpha_i \alpha_j R_g(A_i, A_j)$$

$$+ \mathcal{O}(\lambda^2, \lambda y^2, y^4), \tag{12}$$

where C_i and C_{ij} are the well known one-loop β-function coefficients, and the terms of order λ^2, λy^2, y^4 are not affected by the resummation (unless there is a large number N_F of Yukawa couplings; in this case one needs to resum them too). The values of C_i and C_{ij} in a generic QFT can be found in [29]. We compute the functions $R_\lambda(A)$ and $R_g(A) \equiv R_g(A, A)$ in Appendix A 1, finding

$$R_g(A, A) = \frac{[(2A - 3)A(H_A - 3H_{1-A} + 2H_{3-2A}) - 4A + 3]\Gamma(4 - 2A)}{18\Gamma(2 - A)^3\Gamma(A + 1)}, \tag{13}$$

$$R_g(A, 0) = \frac{(3 - 2A)\Gamma(4 - 2A)}{18\Gamma(2 - A)^3\Gamma(A + 1)}, \tag{14}$$

$$R_\lambda(A) = \frac{2(3 - 2A)\Gamma(4 - 2A)}{9A(4 - 2A)\Gamma(2 - A)^3\Gamma(A)}, \tag{15}$$

with H_n the nth harmonic number. These functions are plotted in Fig. 3 and are regular at $A = 1$, close to the fixed point of non-Abelian gauge couplings: $R_\lambda(1) = 1/9$, $R_g(1, 1) = -2/9$, $R_g(1, 0) = 1/18$. On the other hand, they diverge for $A \to 5/2$, close to the fixed point of Abelian gauge couplings. The leading behavior as $A \to 5/2$ is

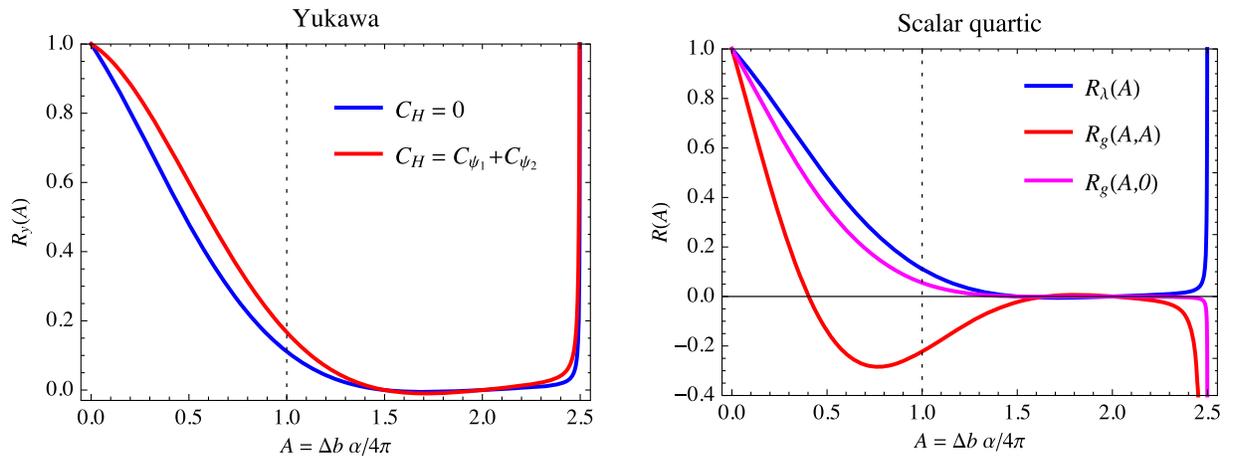$$R_\lambda(A) \simeq -\frac{2}{135\pi^2(A - 5/2)} + \cdots \tag{16}$$

FIG. 3. The functions R_y (left) and R_λ, R_g (right) encoding the correction due to the extra fermions in the large-N_F limit to the β function of the Yukawa and quartic couplings.
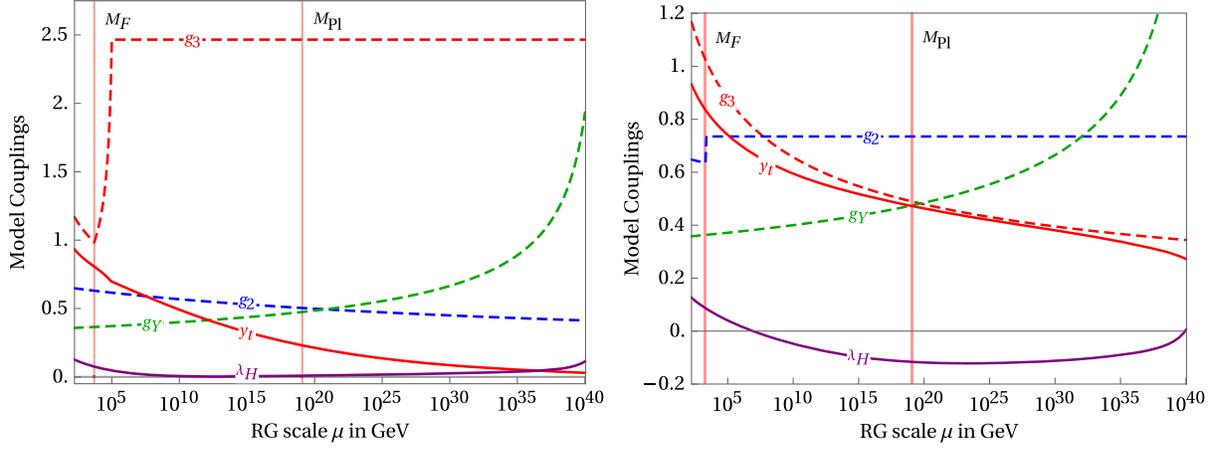
FIG. 4. Left panel: running couplings in the SM plus $N_F = 13$ ($\Delta b_3 = 26$) fermion color octets with mass $M_F \approx 5$ TeV: $SU(3)_c$ becomes asymptotically safe; the electroweak vacuum becomes stable. Right panel: running couplings in the SM plus $N_F = 220$ ($\Delta b_2 = 880/3$) $SU(2)_L$ fermionic triplets with mass $M_F \approx 2$ TeV: $SU(2)_L$ becomes asymptotically safe; the electroweak vacuum becomes unstable.

and

$$R_g(A, 0) \simeq \frac{1}{270\pi^2(A - 5/2)} + \cdots,$$

$$R_g(A, A) \simeq -\frac{1}{108\pi^2(A - 5/2)^2} + \cdots, \quad (17)$$

where the dots represent terms that are regular as $A \to 5/2$.

III. ASYMPTOTICALLY SAFE SM EXTENSIONS

A. Asymptotically safe $SU(3)_c$

We add to the SM N_F Majorana fermion color octets with no weak interactions, such that $\Delta b_3 = 2N_F$, while $\Delta b_2 = \Delta b_Y = 0$. Then, the resummed one-loop beta functions for the top Yukawa coupling is

$$(4\pi)^2 \frac{dy_t}{d\ln\mu} = \frac{9}{2}y_t^3 - y_t\left(8g_3^2 R_y(A_3) + \frac{9}{4}g_2^2 + \frac{17}{12}g_Y^2\right) \quad (18)$$

while the RGE for the Higgs quartic remains as in the SM. Close to the fixed point $R_y(1) = 1/9$, such that the running of y_t is mildly modified. In our numerical example[1] in Fig. 4 (left) this has a minor indirect effect on the running of the Higgs quartic. Nevertheless, by choosing $N_F = 13$ (or smaller values) it is possible to make the electroweak vacuum stable. To solve the RG equations we take the central value for the top quark mass from recent measurements performed by ATLAS and CMS, $M_t = 172.5$ GeV [31].

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[1] For fermions in the adjoint representation the lower boundary of the safe conformal region has been estimated to be around seven flavors [30]. It is consistently and considerably lower than for fermions in the fundamental representation.

B. Asymptotically safe $SU(2)_L$

We add to the SM N_F Majorana fermion triplets with no hypercharge and no color, such that $\Delta b_2 = 4N_F/3$, while $\Delta b_3 = \Delta b_Y = 0$. Then, the resummed one-loop beta functions for the top Yukawa coupling and for the Higgs quartic λ_H (defined writing the tree-level SM potential as $V = -\frac{1}{2}M_h^2|H|^2 + \lambda_H|H|^4$) are

$$(4\pi)^2 \frac{dy_t}{d\ln\mu} = \frac{9}{2}y_t^3 - y_t\left(8g_3^2 + \frac{9}{4}g_2^2 R_y(A_2) + \frac{17}{12}g_Y^2\right) \quad (19)$$

$$(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} = 24\lambda_H^2 + \lambda_H(12y_t^2 - 9g_2^2 R_\lambda(A_2) - 3g_Y^2)$$
$$+ \frac{9g_2^4}{8}R_g(A_2) + \frac{3g_Y^4}{8} + \frac{3g_2^2 g_Y^2}{4}R_g(A_2, 0)$$
$$- 6y_t^4. \quad (20)$$

We provide a numerical example in Fig. 4 (right). Once g_2 approaches its fixed point, y_t runs in a way slightly different way than in the SM: it can become larger or smaller depending on the fixed-point value of g_2. More importantly, the negative value of $R_g(1) \simeq -2/9$ together with the enhanced g_2 makes the Higgs quartic more negative at large energies, conflicting with bounds from vacuum metastability, $\lambda_H \gtrsim -0.05$. The conflict is reduced by keeping g_2 as small as in the SM. However, in order to avoid this problem, one needs to extend the SM in a way that avoids vacuum instability. The simplest option is adding one extra scalar that gives a tree-level positive correction to λ_H [32].

C. Asymptotically safe $U(1)_Y$?

Since $SU(2)_L$ and $SU(3)_c$ in the SM are anyhow asymptotically free, while hypercharge has a possible Landau pole around 10^{40} GeV, it would be especially

interesting to bypass it by making hypercharge asymptotically safe. We add to the SM N_F fermions with hypercharge $\pm Y$ and singlet under $SU(2)_L$ and $SU(3)_c$ such that $\Delta b_Y = 4N_F Y^2/3$, while $\Delta_3 = \Delta_2 = 0$. Then, the resummed one-loop beta functions for the top Yukawa coupling and for the Higgs quartic λ_H are

$$(4\pi)^2 \frac{dy_t}{d\ln\mu} = \frac{9}{2}y_t^3 - y_t\left(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}g_Y^2 R_y(A_Y)\right) \quad (21)$$

$$(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} = 24\lambda_H^2 + \lambda_H(12y_t^2 - 9g_2^2 - 3g_Y^2 R_\lambda(A_Y))$$
$$+ \frac{9g_2^4}{8} + \frac{3g_Y^4}{8}R_g(A_Y) + \frac{3g_2^2 g_Y^2}{4}R_g(A_Y, 0)$$
$$- 6y_t^4 \quad (22)$$

The fixed point for g_Y corresponds to

$$1 - \frac{2}{5}A_Y \simeq \exp\left[-\frac{45}{28\pi^2}(b_Y^{\text{SM}} + \Delta b_Y + 0.4)\right] \quad (23)$$

showing that it is exponentially close to the pole at $A_Y \simeq 5/2$. The functions R_y, R_g, R_λ too have poles at $A_Y = 5/2$. As a result, when g_Y approaches its fixed point, y_t is driven to small values [23] and λ_H is driven to nonperturbatively large values. Thereby it is not possible to make the Abelian factor asymptotically safe.

Even adding N_F fermions charged under all the SM group factors, the problems related to the Abelian factor prevents us from building a full asymptotically safe extension of the SM. This can be build embedding the SM in non-Abelian groups, even at the weak scale, similarly to what already done for asymptotically free extension [1,4].

IV. CONCLUSIONS

Theories where all couplings can be extrapolated up to infinite energy are interesting *per se*, and offer alternative solutions to the Higgs mass hierarchy problem. However, in the SM, the hypercharge gauge coupling grows with energy. Naively, adding a large number N_F of extra charged fermions goes in the wrong direction, as the hypercharge coupling grows even faster than in the SM. Fortunately, the very large number of fermion limit helps to tame the high energy growth of the coupling. The leading contribution of the large number of fermions can be resummed.

By computing all the resummed RGE at leading order in $1/N_F$, we found that the non-Abelian factors of the SM gauge group can be made asymptotically safe. However, when hypercharge is made asymptotically safe, the Higgs quartic flows out of perturbative control.

It should be noted that the large N_F limit is merely a mathematical tool that allows us to determine the location of the asymptotically safe fixed points. Lattice simulations

may very well find that fixed points exist for moderate values of N_F—after all, large-N_c approximations are used in QCD where $N_c = 3$. In the meantime, it is interesting to discuss the unusual physics resulting from having many extra degrees of freedom. Electroweak corrections to the W precision parameter get enhanced [33,34], tails of $d\sigma(pp \to \ell^+\ell^-)/dm_{\ell^+\ell^-}$ at large invariant mass would exhibit the pattern typical of fast running g_2 coupling [35,36]. For extra colored vectorlike fermions, the modified high energy behavior would affect the three to two jet ratio [37]. More interestingly, freeze-out of a keV-scale relativistic sterile neutrino from a plasma with $N_F \gg 1$ extra degrees of freedom provides an acceptable cold Dark Matter candidate (rather than the usual too warm DM). Furthermore, one can gauge the $SU(N_F)$ symmetry that rotates the N_F fermions, that can be identified as "dark baryons" in models of composite dark matter. A new feature of $N_F \gg 1$ is that the model is phenomenologically acceptable even when stable dark baryons are charged: their charge grows with N_F, but their relic abundance gets suppressed by 2^{-N_F}. This discussion exemplifies the new spectrum of possibilities with atypical phenomenology in (astro)particle physics and cosmology that these constructions open up.

APPENDIX: RESUMMED GAUGE CORRECTIONS

We consider a simple gauge group G with gauge coupling g and $N_F \gg 1$ fermions ψ_j in a generic representation of G. We here compute the β functions of the scalar quartics and Yukawa couplings with the gauge field propagator obtained by resumming the effects of the fermions ψ_j at the leading order in $1/N_F$.

We define $A \equiv \Delta b\alpha/4\pi$, where $\alpha \equiv g^2/4\pi$ and Δb is the contribution of the N_F fermions to the one-loop coefficient of the gauge β function, $\beta_\alpha^{\text{one-loop}} = \Delta b\alpha^2/2\pi$. The limit $N_F \to \infty$ is taken by keeping A fixed.

In order to perform the resummation of the leading terms in the expansion in $1/N_F$ we use the resummed gauge field propagator $\mathcal{D}_{\mu\nu}(k)$, where k is the momentum. We choose the Landau gauge, where the tree-level propagator $D_{\mu\nu}(k)$ is transverse.

$$D_{\mu\nu}(k) = -i\frac{P_{\mu\nu}(k)}{k^2}, P_{\mu\nu}(k) = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}. \quad (A1)$$

The Feynman $i\epsilon$ is left implicit. In this gauge the resummed propagator $\mathcal{D}_{\mu\nu}(k)$ is

$$\mathcal{D}_{\mu\nu}(k) = -i\frac{P_{\mu\nu}(k)}{k^2}\sum_{n=0}^{\infty}\Pi(k^2)^n$$

$$= -i\frac{P_{\mu\nu}(k)}{k^2}\frac{1}{1-\Pi(k^2)} \qquad (A2)$$

where $\Pi(k^2)$ is defined in terms of the correction to the vector self-energy $\Pi_{\mu\nu}(k)$ due to a loop of the N_F fermions as follows

$$\Pi_{\mu\nu}(k) = k^2 P_{\mu\nu}(k)\Pi(k^2). \qquad (A3)$$

In dimensional regularization ($d = 4 - \epsilon$) one finds [23]

$$\Pi(k^2) = \left(\frac{\mu^2}{k^2}\right)^{\epsilon/2}\Pi_0,$$

with $\quad \Pi_0 \equiv -6(-4\pi)^{\epsilon/2}A_0\frac{\Gamma(\epsilon/2)\Gamma(2-\epsilon/2)^2}{\Gamma(4-\epsilon)}, \quad$ (A4)

where A_0 is the bare value of A.

1. QUARTIC β FUNCTION

We here compute the β functions of the scalar quartic couplings. We consider a set of real scalars ϕ_a in a generic representation S of G. The covariant kinetic terms of the ϕ_a appear in Lagrangian as $\frac{1}{2}D_\mu\phi_a D^\mu\phi_a$, where the scalar covariant derivative is given by $D_\mu\phi_a = (\partial_\mu\phi_a + ig\theta^B_{ab}A^B_\mu\phi_b)$ and the θ^A are the generators in the representation S. We write their quartic interactions in the Lagrangian as $-\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d/4!$. Here we show that the λ_{abcd} obey the following RGEs at leading order in $1/N_F$:

$$(4\pi)^2\frac{d\lambda_{abcd}}{d\ln\mu} = 6g^4 R_g(A)\theta_{abcd} - 3g^2 R_\lambda(A)\lambda_{abcd}\sum_{k=a,b,c,d}C^k_H$$

$$+ \mathcal{O}(\lambda^2, \lambda y^2, y^4), \qquad (A5)$$

where

$$\theta_{abcd} \equiv \frac{1}{16}\sum_{\text{perms}}\{\theta^A,\theta^B\}_{ab}\{\theta^A,\theta^B\}_{cd}. \qquad (A6)$$

The sum runs over all permutations of $abcd$, the C^a_H are defined by $\theta^A_{ac}\theta^A_{cb} = C^a_H\delta_{ab}$ and the functions $R_g(A)$ and $R_\lambda(A)$ are given in Eqs. (13) and (15). For $A \approx 0$ one has $R_g = 1-8A/3 + \mathcal{O}(A^2)$ and $R_\lambda = 1-5A/6 + \mathcal{O}(A^2)$ in agreement with known 2-loop results [29].

a. Scalar field renormalization and R_λ

In Landau gauge R_λ is given only by the scalar field renormalizations (diagram in Fig. 5, left). We define Z_k through $\phi_{0k} = Z_k^{1/2}\phi_k$, where the ϕ_{0k} are the bare scalar
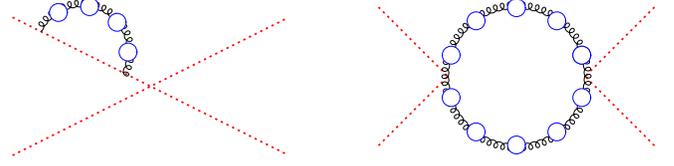

FIG. 5. Contributions at leading order in $1/N_F$ to the β functions of the quartic couplings from the scalar field renormalizations (left) and the vertex contribution (right).

fields. The (amputated) Feynman amplitude for the scalar field renormalization of the kth external line with the insertion of n bubbles is denoted as $-iS_k^{(n)}(p^2)$, where p is the momentum flowing through the diagram. Their resummation is $S_k \equiv \sum_n S_k^{(n)}$. We have

$$Z_k = 1 + \left.\frac{dS_k(p^2)}{dp^2}\right|_{\text{poles}}. \qquad (A7)$$

A calculation of Z_k in the Landau gauge was presented in [23]; however, we find a different result and, therefore, we provide here the details of the calculation. By using Eqs. (A2) and (A4), we find

$$\frac{dS_k(p^2)}{dp^2} = -\frac{2C^k_H}{\Delta b}\sum_{n=1}^{\infty}(-2A_0)^n\frac{A_\phi(n,\epsilon)}{n\epsilon^n}. \qquad (A8)$$

We do not provide the explicit expression for $A_\phi(n,\epsilon)$ for $n > 0$ because it is complicated and, as we will see, the only quantity we need is $A_\phi(0,\epsilon)$. Following [20] we expand $A_\phi(n,\epsilon)$ as

$$A_\phi(n,\epsilon) = \sum_{j=0}^{\infty}A_\phi^{(j)}(\epsilon)(n\epsilon)^j. \qquad (A9)$$

One can explicitly check that the coefficients $A_\phi^{(j)}(\epsilon)$ do not have poles at $\epsilon = 0$. By inserting the expansion Eq. (A9) in (A8), one obtains

$$\frac{dS_k(p^2)}{dp^2} = -\frac{2C^k_H}{\Delta b}\sum_{n=1}^{\infty}(-2A_0)^n\sum_{j=0}^{\infty}A_\phi^{(j)}(\epsilon)\frac{n^{j-1}}{\epsilon^{n-j}}. \qquad (A10)$$

In order to obtain the β function, we express the poles in terms of the renormalized couplings. In the case of the gauge coupling this consists in using the relation

$$A_0 = \frac{A}{Z_3} \quad \text{where} \quad Z_3 = 1 - \frac{2A}{\epsilon} + \mathcal{O}\left(\frac{1}{N_F}\right). \qquad (A11)$$

Then, by using the binomial series we obtain

$$A_0^n = A^n \sum_{i=0}^{\infty} \binom{-n}{i} \left(-\frac{2A}{\epsilon}\right)^i + \mathcal{O}\left(\frac{1}{N_F}\right) \tag{A12}$$

which, inserted in Eq. (A10), gives (dropping subleading powers in $1/N_F$)

$$
\begin{aligned}
\frac{dS_k(p^2)}{dp^2} &= -\frac{2C_H^k}{\Delta b} \sum_{n=1}^{\infty} (-2A)^n \sum_{j=0}^{\infty} A_\phi^{(j)}(\epsilon) \frac{n^{j-1}}{\epsilon^{n-j}} \sum_{i=0}^{\infty} \binom{-n}{i}\left(-\frac{2A}{\epsilon}\right)^i \\
&= -\frac{2C_H^k}{\Delta b} \sum_{n=1}^{\infty} (-2A)^n \sum_{j=0}^{\infty} A_\phi^{(j)}(\epsilon) \frac{n^{j-1}}{\epsilon^{n-j}} \sum_{i=0}^{\infty} (-1)^i \binom{n+i-1}{i}\left(-\frac{2A}{\epsilon}\right)^i \\
&= -\frac{2C_H^k}{\Delta b} \sum_{n=1}^{\infty} (-2A)^n \sum_{j=0}^{\infty} \frac{A_\phi^{(j)}(\epsilon)}{\epsilon^{n-j}} \sum_{i=0}^{n-1} (n-i)^{j-1}(-1)^i \binom{n-1}{i}.
\end{aligned}
\tag{A13}
$$

In the last step, we substituted $n \to n - i$, which requires the sum over i to be truncated at $i = n - 1$. Keeping only the poles at $\epsilon = 0$, which are the only thing we need to compute Z_k [see Eq. (A7)], we obtain

$$
\begin{aligned}
\frac{dS_k(p^2)}{dp^2}\bigg|_{\text{poles}} &= -\frac{2C_H^k}{\Delta b} \sum_{n=1}^{\infty} (-2A)^n \sum_{j=0}^{n-1} \frac{A_\phi^{(j)}(\epsilon)}{\epsilon^{n-j}} \\
&\quad \times \sum_{i=0}^{n-1} (n-i)^{j-1}(-1)^i \binom{n-1}{i}.
\end{aligned}
\tag{A14}
$$

The sum over i in the last expression is [20]

$$
\begin{aligned}
&\sum_{i=0}^{n-1} (n-i)^{j-1}(-1)^i \binom{n-1}{i} \\
&= -\delta_{j0} \frac{(-1)^n}{n} \quad \text{(for } 0 \le j \le n-1\text{)}
\end{aligned}
\tag{A15}
$$

so only the term with $j = 0$ matters and we find

$$\frac{dS_k(p^2)}{dp^2}\bigg|_{\text{poles}} = \frac{2C_H^k}{\Delta b} \sum_{n=1}^{\infty} (2A)^n \frac{A_\phi^{(0)}(\epsilon)}{n\epsilon^n}. \tag{A16}$$

As anticipated before, only $A_\phi^{(0)}(\epsilon) \equiv A_\phi(0, \epsilon)$ is relevant. We have

$$A_\phi(0, \epsilon) = \frac{2^{2-\epsilon}(1 - \frac{\epsilon}{3})\Gamma(\frac{5}{2} - \frac{\epsilon}{2})}{\sqrt{\pi}(1 - \frac{\epsilon}{2})\Gamma(1 - \frac{\epsilon}{2})\Gamma(3 - \frac{\epsilon}{2})\Gamma(\frac{\epsilon}{2} + 1)}. \tag{A17}$$

This expression agrees with the known 2-loop expressions, and does not agree with the corresponding expression in [23] that computed the resummed Yukawa β function (on which we will agree).

In order to compute the correction R_λ to the quartic β function, we only need the simple pole at $\epsilon = 0$. Using that $A_\phi(0, \epsilon)$ is regular in $\epsilon = 0$ we expand

$$A_\phi^{(0)}(\epsilon) = \sum_{i=0}^{\infty} \tilde{A}_\phi^{(i)} \epsilon^i,$$

$$\text{where} \quad \tilde{A}_\phi^{(i)} = \frac{1}{i!} \frac{d^i A_\phi^{(0)}}{d\epsilon^i}(\epsilon = 0) \tag{A18}$$

which, once inserted in (A16), gives the simple pole

$$\frac{dS_k(p^2)}{dp^2}\bigg|_{\text{simple pole}} = \frac{2C_H^k}{\Delta b} \frac{1}{\epsilon} \sum_{n=1}^{\infty} (2A)^n \frac{\tilde{A}_\phi^{(n-1)}}{n}. \tag{A19}$$

This result allows us to compute the term proportional to $g^2 \lambda_{abcd}$ in the β function of λ_{abcd}. Using the general formula provided in [38] one sees that the β function in this case is obtained by taking a logarithmic derivative with respect to A of (A19). Then, the n in the denominator of (A19) disappears and one finds a closed form for the β function. The result is given in Eqs. (A5) and (15).

b. Correction to the quartic vertex and R_g

Next, we compute R_g in Eq. (A5) by considering the four-point loop vertex at the leading order in $1/N_F$ at vanishing external momenta. In the Landau gauge only the diagram in the right panel of Fig. 5 contributes. The relative Feynman amplitude iV_{abcd} is given by

$$
\begin{aligned}
\frac{V_{abcd}}{\mu^\epsilon} &= -4i\mu^\epsilon \theta_{abcd} g_0^4 \\
&\times \sum_{n,m=0}^{\infty} (\Pi_0 \mu^\epsilon)^{n+m} \int \frac{d^d q}{(2\pi)^d} \frac{d-1}{(q^2 - m_\gamma^2)^2 (q^2)^{(n+m)\epsilon/2}},
\end{aligned}
\tag{A20}
$$

where m_γ is a tiny vector mass added to regulate IR divergences. The β function of λ_{abcd} is insensitive to this divergence and so we set $m_\gamma \to 0$ at the end. Equation (A20) has been obtained by substituting in the

relevant one-loop diagram the tree-level gauge field propagator $D_{\mu\nu}$ with the resummed propagator $\mathcal{D}_{\mu\nu}$ of Eq. (A2). The double sum over n and m is due to the presence of two gauge field propagators in the diagram. Computing the loop integrals in Eq. (A20) gives

$$\frac{V_{abcd}}{\mu^\epsilon} = \frac{16\theta_{abcd}}{9\Delta b^2} \sum_{n=2}^{\infty} (-2A_0)^n \frac{A_4(n,\epsilon)}{n\epsilon^{n-1}} \qquad (A21)$$

for some coefficients $A_4(n,\epsilon)$. We do not display $A_4(n,\epsilon)$ because it is complicated for generic n, and because we only need $A_4(0,\epsilon)$ and $A_4(1,\epsilon)$. The double sum in Eq. (A20) has been reduced to a single sum through the identity (valid for any sequence c_n)

$$\sum_{n,m=0}^{\infty} c_{n+m} = \sum_{n=0}^{\infty} (n+1)c_n. \qquad (A22)$$

We then follow an approach similar to the one used for the scalar field renormalization. We expand

$$A_4(n,\epsilon) = \sum_{j=0}^{\infty} A_4^{(j)}(\epsilon)(n\epsilon)^j, \qquad (A23)$$

which, once inserted in the expression above for V_4, gives

$$\frac{V_{abcd}}{\mu^\epsilon} = \frac{16\theta_{abcd}}{9\Delta b^2} \sum_{n=2}^{\infty} (-2A_0)^n \sum_{j=0}^{\infty} A_4^{(j)}(\epsilon) \frac{n^{j-1}}{\epsilon^{n-1-j}}. \qquad (A24)$$

By using again the relation between the bare A_0 and the renormalized A, Eq. (A11), and performing steps similar to those done for the scalar field renormalization one finds

$$\frac{V_{abcd}}{\mu^\epsilon} = \frac{16\theta_{abcd}}{9\Delta b^2} \sum_{n=2}^{\infty} (-2A_0)^n \sum_{j=0}^{\infty} A_4^{(j)}(\epsilon) \frac{n^{j-1}}{\epsilon^{n-1-j}}$$
$$\times \sum_{i=0}^{\infty} (-1)^i \binom{n+i-1}{i} \left(-\frac{2A}{\epsilon}\right)^i. \qquad (A25)$$

We now replace $n \to n-i$, which requires here to stop the sum over i at $i = n-2$, so

$$\frac{V_{abcd}}{\mu^\epsilon} = \frac{16\theta_{abcd}}{9\Delta b^2} \sum_{n=2}^{\infty} (-2A)^n \sum_{j=0}^{\infty} \frac{A_4^{(j)}(\epsilon)}{\epsilon^{n-1-j}}$$
$$\times \sum_{i=0}^{n-2} (n-i)^{j-1}(-1)^i \binom{n-1}{i}. \qquad (A26)$$

Next, we rewrite the sum over i as

$$\sum_{i=0}^{n-2} (n-i)^{j-1}(-1)^i \binom{n-1}{i}$$
$$= (-1)^n + \sum_{i=0}^{n-1} (n-i)^{j-1}(-1)^i \binom{n-1}{i}, \qquad (A27)$$

which leads to

$$\frac{V_{abcd}}{\mu^\epsilon} = \frac{16\theta_{abcd}}{9\Delta b^2} \sum_{n=2}^{\infty} (2A)^n \frac{1}{\epsilon^{n-1}} \left[\sum_{j=0}^{\infty} \epsilon^j A_4^{(j)}(\epsilon) - \frac{A_4^{(0)}(\epsilon)}{n} \right], \qquad (A28)$$

where we used that $\epsilon A_4^{(j)}(\epsilon)$ are regular at $\epsilon = 0$ and, therefore, we have used Eq. (A15) to compute the second term proportional to $A_4^{(0)}(\epsilon)$. An explicit calculation shows $A_4^{(0)}(\epsilon) = 0$ and we are therefore left with

$$\frac{V_{abcd}}{\mu^\epsilon} = \frac{16\theta_{abcd}}{9\Delta b^2} \sum_{n=2}^{\infty} (2A)^n \frac{A_4(1,\epsilon)}{\epsilon^{n-1}}, \qquad (A29)$$

having used Eq. (A23). The explicit expression of $A_4(1,\epsilon)$ is

$$A_4(1,\epsilon) = -\frac{3\pi^2(\epsilon-3)\Gamma(4-\epsilon)}{\Gamma(2-\epsilon/2)^3 \Gamma(\epsilon/2+1)}. \qquad (A30)$$

In order to compute the β function, we only need the simple pole, so we expand

$$A_4(1,\epsilon) = \sum_{i=0}^{\infty} \tilde{A}_4^{(i)} \epsilon^i \qquad (A31)$$

and insert this expansion in the last expression of V_{abcd}, to obtain

$$\left. \frac{V_{abcd}}{\mu^\epsilon} \right|_{\text{simple pole}} = \frac{16\theta_{abcd}}{9\Delta b^2 \epsilon} \sum_{n=2}^{\infty} (2A)^n \tilde{A}_4^{(n-2)}$$
$$= \frac{64\theta_{abcd}A^2}{9\Delta b^2 \epsilon} A_4(1,2A). \qquad (A32)$$

Using again the general formula provided in [38] (which allows us to extract the β function from the simple pole) leads to the result in Eqs. (A5) and (13).

It is worth noting that one can perform these calculations by using directly the resummed gauge field propagator in the last equality in Eq. (A2) before doing the loop integral. Let us illustrate this method in the calculation of V_{abcd}. In this case

$$\frac{V_{abcd}}{\mu^\epsilon} = -4i\mu^\epsilon \theta_{abcd} g_0^4 \int \frac{d^d q}{(2\pi)^d} \frac{d-1}{(q^2 - q^2\Pi(q^2))^2}. \qquad (A33)$$

Using the expression of $\Pi(k^2)$ given in Eq. (A4), the loop integral can be done analytically after performing the Wick rotation. After expressing the bare A_0 in term of the renormalized A by means of Eq. (A11), one obtains

$$\frac{V_{abcd}}{\mu^\epsilon} = \frac{32\theta_{abcd}}{9\Delta b^2} \frac{A A_4(1,\epsilon)}{1 - 2A/\epsilon}, \qquad (A34)$$

where $A_4(1,\epsilon)$ is given in (A30). To compute the β function, we only need the simple pole in this expression. Therefore, we use the expansion in (A31) and

$$\frac{1}{1 - 2A/\epsilon} = \sum_{j=0}^{\infty} \left(\frac{2A}{\epsilon}\right)^j \qquad (A35)$$

to obtain

$$\begin{aligned}
\frac{V_{abcd}}{\mu^\epsilon}\bigg|_{\text{simple pole}} &= \frac{32\theta_{abcd}A}{9\Delta b^2 \epsilon} \sum_{j=1}^{\infty} (2A)^j \tilde{A}_4^{(j-1)} \\
&= \frac{64\theta_{abcd}A^2}{9\Delta b^2 \epsilon} A_4(1, 2A), \qquad (A36)
\end{aligned}$$

which coincides with Eq. (A32), obtained instead by first performing the loop integral and then resumming. This provides another check for the β function of λ_{abcd}.

The derivation of $R_g(A, 0)$ is not explicitly presented because it is very similar to the derivation of $R_\lambda(A)$.

2. YUKAWA β FUNCTION

We now present the derivation of the β function of the Yukawa coupling discussed in Sec. II B. A calculation of β_y was presented in [23] and we do agree with their final expression. However, their derivation mixes the Feynman and Landau gauge in an *apparently* inconsistent way, so we preferred to provide another independent calculation performed in the Landau gauge.

To compute β_y three ingredients are required: the scalar field renormalization (already computed in Sec. A 1 a), the fermion renormalization and the loop correction to the Yukawa vertex. We provide the derivation of the remaining pieces in the following part of the Appendix. Once the simple poles of these three ingredients are obtained one can determine β_y with the general formula provided in [38] and the result is the one given in Sec. II B.

a. Fermion field renormalization

One ingredient to obtain β_y is the fermion field renormalization. The (amputated) Feynman amplitude of the self-energy of the fermions ψ_1 and ψ_2 is

$$-i\Sigma_{1,2}(p) = (ig_0)^2 \mu^\epsilon C_{\psi_{1,2}} \int \frac{d^d k}{(2\pi)^d} \gamma_\mu \frac{i}{\slashed{p} - \slashed{k}} \gamma_\nu \frac{-iP_{\mu\nu}(k)}{k^2(1 - \Pi(k^2))}, \qquad (A37)$$

where $C_{\psi_{1,2}}$ is the quadratic Casimir of $\psi_{1,2}$ under G.[2] After expanding $1/(1 - \Pi)$ in a power series of Π and dealing with γ-matrices we extract $d\Sigma_{1,2}/d\slashed{p}$ at zero external momentum:

$$\frac{d\Sigma_{1,2}}{d\slashed{p}}\bigg|_{p=0} = -i\mu^\epsilon g_0^2 C_{\psi_{1,2}} \frac{4 + d^2 - 5d}{d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^4} \sum_{n=0}^{\infty} \Pi(k^2)^n \qquad (A38)$$

as this is the quantity needed to obtain the fermion renormalizations $Z_{1,2}$:

$$Z_{1,2} - 1 = \frac{d\Sigma_{1,2}}{d\slashed{p}}\bigg|_{p=0,\text{poles}}. \qquad (A39)$$

Once again, we rewrite the relevant expression as

$$\frac{d\Sigma_{1,2}}{d\slashed{p}}\bigg|_{p=0} = \frac{C_{\psi_{1,2}}}{\Delta b} \sum_{n=1}^{\infty} (-2A_0)^n \frac{A_\psi(n,\epsilon)}{n\epsilon^n} \qquad (A40)$$

for some $A_\psi(n,\epsilon)$. Using a method similar to Sec. A 1 a, we find the following simple pole:

$$\frac{d\Sigma_{1,2}}{d\slashed{p}}\bigg|_{p=0,\text{simple pole}} = -\frac{C_{\psi_{1,2}}}{\Delta b} \frac{1}{\epsilon} \sum_{n=1}^{\infty} (2A)^n \frac{\tilde{A}_\psi^{(n-1)}}{n}, \qquad (A41)$$

where the coefficients $\tilde{A}_\psi^{(n-1)}$ are defined by

$$A_\psi(0, \epsilon) = \sum_{i=0}^{\infty} \tilde{A}_\psi^{(i)} \epsilon^i,$$

$$\text{where} \quad \tilde{A}_\psi^{(i)} = \frac{1}{i!} \frac{d^i A_\psi^{(0)}}{d\epsilon^i}(\epsilon = 0) \qquad (A42)$$

and

$$A_\psi(0, \epsilon) = \frac{(3 - \epsilon)\epsilon\Gamma(4 - \epsilon)}{6(\epsilon - 4)\Gamma(2 - \epsilon/2)^3 \Gamma(1 + \epsilon/2)}. \qquad (A43)$$

b. Correction to the Yukawa vertex

The remaining ingredient to calculate β_y is the loop correction to the Yukawa vertex, whose (amputated) Feynman amplitude is denoted here with $-i\Lambda_y$. We can

---

[2]We displayed explicitly this form of $\Sigma_{1,2}$ to make the comparison with the corresponding expression in [23] easier. While we use the Landau gauge consistently it seems that [23] simply replaced $P_{\mu\nu}$ with $\eta_{\mu\nu}$, which appears to be an inconsistent mixing of the Landau and Feynman gauge (as the resummation of the gauge field propagator, Eq. (A2), is performed in the Landau gauge both here and in [23]).

set the external momenta to zero as the Yukawa coupling is a nonderivative interaction. We obtain

$$\frac{\Lambda_y}{\mu^\epsilon} = -\frac{6y}{\Delta b}(C_{\psi_1} + C_{\psi_2} - C_H)\sum_{n=1}^{\infty}(-2A_0)^n\frac{A_y(n,\epsilon)}{n\epsilon^n} \quad \text{(A44)}$$

for some $A_y(n,\epsilon)$ and, by using a technique similar Sec. A 1 a, we find the simple pole:

$$\left.\frac{\Lambda_y}{\mu^\epsilon}\right|_{\text{simple pole}} = \frac{6y}{\Delta b}(C_{\psi_1} + C_{\psi_2} - C_H)\frac{1}{\epsilon}\sum_{n=1}^{\infty}(2A)^n\frac{\tilde{A}_y^{(n-1)}}{n}$$

$$\text{(A45)}$$

where the coefficients $\tilde{A}_y^{(n-1)}$ are defined by

$$A_y(0,\epsilon) = \sum_{i=0}^{\infty}\tilde{A}_y^{(i)}\epsilon^i,$$

$$\text{where} \quad \tilde{A}_y^{(i)} = \frac{1}{i!}\frac{d^iA_y^{(0)}}{d\epsilon^i}(\epsilon = 0) \quad \text{(A46)}$$

and

$$A_y(0,\epsilon) = \frac{(3-\epsilon)\Gamma(4-\epsilon)}{24\Gamma(2-\epsilon/2)^3\Gamma(1+\epsilon/2)}. \quad \text{(A47)}$$

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