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Oliver Heaviside's Electromagnetic Theory

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Summary

The year 2018 marks the 125th anniversary of the first of three published volumes on *Electromagnetic Theory* by the eminent Victorian electrical engineer, physicist and mathematician, Oliver Heaviside FRS. This commemorative issue of Philosophical Transactions of the Royal Society A celebrates the publication of this work by collecting papers on a broad spectrum across the field of electromagnetic theory, including innovative research papers interspersed between historical perspectives and relevant reviews. Heaviside was a remarkable man, an original thinker with brilliant mathematical powers and physical insight who made many significant contributions in his fields of interest, though he is remembered primarily for his 'step function', commonly used today in many branches of physics, mathematics and engineering. Here, we celebrate the man and his work by illustrating his major contributions and highlighting his great success in solving some of the great telegraphic engineering problems of the Victorian era, in part due to his development and detailed understanding of the governing electromagnetic theory. We celebrate his *Electromagnetic Theory*; three volumes of insights, techniques and understanding from mathematical, physical and engineering perspectives - as dictated by J.C. Maxwell FRS, but interpreted, reformulated and expanded by Heaviside to advance the art and science of electrical engineering beyond all expectations.

125 Years of *Electromagnetic Theory*

One hundred and twenty-five years ago, Oliver Heaviside FRS published the first of his seminal masterpiece series on electromagnetic theory. The works contained within this Special Commemorative Issue are scattered across a broad range of electromagnetic topics, including original research articles in the physical and engineering sciences as well as in the history of science and technology, including new perspectives on the work of Heaviside and his subsequent influences. The issue is not intended to provide a comprehensive overview of his work or life, nor is it intended to provide the reader with a showcase of the latest research articles relating to the work of Heaviside. The intention is to act as a mark to commemorate his work *Electromagnetic Theory* on the anniversary of the publication of his first dedicated book on the subject in 1893. It is the view of the editors of this commemorative issue, and many learned scholars, that Heaviside was a remarkable man, an original thinker with brilliant mathematical powers and physical insight – his most important work can be found in his three volumes entitled *Electromagnetic Theory* and his connected works *Electrical Papers*. Here, a brief overview of the man Oliver Heaviside and his main contributions are presented

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in the broadest of terms. Secondly we illuminate the main points of interest contained within the three published volumes of *Electromagnetic Theory* and discuss the 'missing' fourth volume. These volumes of contain his significant works in the subject across a broad spectrum of ideas and it is hoped that the readers of those original works and this special issue are as captivated as the editors by the charm, intrigue and sheer brilliance of Oliver Heaviside FRS – an eminent *electromagnetician*.

The Man

Oliver Heaviside FRS was born on 18th May 1850 and died in Torquay on 3rd February 1925, at the age of 74. From humble beginnings, he was able to make very many significant scientific achievements to rival the most revered of his university-educated peers in the great Victorian era of scientific discovery. Heaviside was an English self-taught electrical engineer, physicist, and mathematician, who changed the face of telecommunications, mathematics and science for years to come, up to and well beyond his death. He adapted complex numbers to the study of electrical circuits, coining such terms as *inductance, impedance,* and *reluctance*; among many others, all in common use today. Heaviside invented many mathematical techniques, one is his 'operational calculus', a method of solution of systems of differential equations (today's equivalent is the Laplace transform) representing the great telegraphic and electromagnetic problems of his time. He reformulated James Clerk Maxwell's field equations in terms of the electric and magnetic forces ('murdering' Maxwell's potentials along the way, due to their non-observability) while independently co-formulating vector analysis and Prof. Henry Poynting's energy-flux theorem. This impressive catalog of achievements in the realms of the electromagnetic theory was complemented by his many technical achievements in electrical engineering practice. He also invented the coaxial cable, for which he was granted a patent. He was responsible for the discovery of the necessary conditions required for 'distortionless' transmission of telegraph signals and invented the practical means by which this could be achieved - perhaps amongst the most important of his achievements. This list is complemented by the many improvements in practical telegraphic and telephonic systems he contributed over his lifetime.

His achievements (with the correct attribution) are not well known, yet they are profound and wide-reaching. The so-called 'Maxwell Equations' or 'General Equations of the Electromagnetic Field' [1], as presented in every undergraduate physics textbook in the world are in fact in the form presented first by Heaviside, in the language that he independently developed for the purpose (alongside W. Gibbs, known for 'Gibbs phenomena' in Fourier theory). This 'recasting' alone is a major achievement, for which Heaviside was given credit by Heinrich Hertz [2]. Maxwell had published his two-volume work *A Treatise on Electricity and Magnetism* in 1873 [3]. Heaviside first came across this seminal work as part of his 'self-study' regime while he was living in Newcastle upon Tyne in late 1873, while working for the Great Northern Telegraph Company (his only paid work, which he left in 1873 at the age of 23 to concentrate on his studies). It was his time in Newcastle that would influence his research interests for the rest of his life. It is the aim of this Special Issue to raise awareness of his very many contributions that have been documented so well in the biographies of Heaviside.

There have been two widely published biographies, the first (and mathematical) biography was written by P. J. Nahin and published in 1988 [4], a second one was written in 2009 by B. Mahon [5]. Both biographies have a broad scope regarding their content covering his ideas, life, and works - they are recommended reading for those keen to know the details of Heaviside's fascinating life. The two other biographies are very much less known. The first is a personal account of Heaviside as written by his lifelong friend G. F. C. Searle FRS. The Searle biography was never officially published as a complete manuscript, Searle having written the account in 1949/50 for the Institution of Electrical Engineers *Heaviside Centenary Volume* (celebrating 100 years since Heaviside's birth), in which a short version is published [6]. It is understood that the full manuscript was later uncovered in the 1970s by I. Catt, who then self-published *Oliver Heaviside, The Man* in 1987 [7], attributing the authorship to Searle with Catt as editor. This 'personal sketch' by Searle, presents a

unique insight into the personality of Oliver Heaviside, written, perhaps, by the most fitting person to portray Heaviside an authentic light. H. J. Josephs wrote the second unpublished biography of Heaviside based on his extensive research [8]. The manuscript was unfortunately never published and only two copies are known to exist – one in the Institution of Engineering & Technology Archives [9] and one in the Science Museum Archives in London. Arnold Lynch (1914-2004, known for the optical tape reader which was used in the construction of the Colossus computer during WW2), compiled a list of sources for a biography of Oliver Heaviside [10]. It is left to these references for the reader to indulge in learning the intriguing details of the life, work and times of Heaviside.

He was elected a Fellow of the Royal Society in 1891 for his contributions to the mathematical description of electromagnetic phenomena; this was his first accolade of many. In 1905 he was conferred an honorary doctorate from the University of Göttingen (where K. F. Gauss spent much of his career), in 1908 he received Honorary Fellowship of the Institution of Electrical Engineers, from which he then received the first Faraday Medal in 1921.

The First Three Volumes of *Electromagnetic Theory*

Heaviside's first installment of his three volumes of *Electromagnetic Theory* was published in 1893 (aged 43) by *The Electrician Printing and Publishing Co, London*, based in Salisbury Court, Fleet Street, London. The publishing company had its own weekly trade journal which was at the forefront of publishing papers relating to all manner of electrical matters, including telegraphy and signaling through the transatlantic cable. The *Electrician* described itself as 'A weekly journal of telegraphy, electricity, and applied chemistry' and in May 1895 claimed that it was 'The Oldest and Best English Electrical Journal', a bold claim.

Heaviside was a regular contributor among many eminent electrical men of the time – it is in this journal that Heaviside published many of the articles (in fact he had a long and fruitful relationship with the editors of *The Electrician*) that would ultimately end up published in book form. His *Electrical Papers* and *Electromagnetic Theory*, which were essentially a collection of previously published papers, are peppered with new insights and humorous (with some serious) commentary. Volume I was published in 1893 [11] and focuses on topics such as a review of current electromagnetic theory from the point of view of Heaviside, the development of his 'Vectorial Algebra and Analysis' and moving these principles to his 'Theory of Plane Electromagnetic Waves'. Here he considers the mathematical description of the guidance of waves by transmission lines, including his distortionless circuit and the practical means by which it is to be achieved. This work sets out Heaviside's major ideas about his electromagnetic theory as interprets Maxwell's prior work and it paves the way for his own and others future studies of the subject.

Electromagnetic Theory Volume II came six years later in 1899 [12] (aged 49), the first chapter being a departure from electromagnetic theory, considering not wave propagation, but the *Age of the Earth* – here he uses his mathematics and knowledge of electromagnetic theory to discuss methods of answering such a physical, if not philosophical, question. Heaviside's venture outside the world of electromagnetic theory is perhaps linked to an appendix in Volume I where he discusses *A Gravitational & Electromagnetic Analogy*, utilizing his vector language and the 'potential' function to describe the gravitational field and its propagation. The bulk of Volume II comprises work relating to the generation, propagation and behavior of transverse electric and magnetic waves in various media and circuital configurations. He also discusses the existence of compressional electromagnetic waves and devotes pages to his mathematics, particularly his use of divergent series and differential operators arising from the study of natural electromagnetic systems. His non-rigorous use of divergent series was an abhorrence to the pure mathematicians, as discussed in [4]; despite this difficulty regarding his peers' acceptance of his methods, he managed to utilize them rather

successfully in his work and remarked about his general love for mathematical series in his *Electrical Papers Vol. II* [13];

> *The subject of the decomposition of an arbitrary function into the sum of functions of special types has many fascinations. No student of mathematical physics, if he possesses any soul at all, can fail to recognise the poetry that pervades this branch of mathematics.*

The final published installment, *Volume III*, followed in 1912 [14] when Heaviside was 62 years old. Perhaps his most mathematical work to date, this volume comprises two main sections *Waves from Moving Sources* and *Waves in the Ether*. The former deals at great length with the generation of electromagnetic waves by moving sources, including that of the 'electron', as discovered by the British physicist J. J. Thomson PRS in 1896. Here he discusses the acceleration of charged particles, including 'faster-than-light in the medium' particles, leading to predictions of what we now know as Cherenkov radiation and various relativistic effects, including length contraction that he first worked on with G. F. C Searle in 1888 with *Motion of Electrification through a Dielectric* [15]. The latter section presents mathematical descriptions of the movement of energy through the ether, radiation pressure and connected subjects while interspersing the electromagnetic theory with a discussion of *deep-water waves* and another on *the solution of definite integrals by differential transformation*, amongst other interesting excursions.

These three volumes of 'Electromagnetic Theory' offer a plethora of insights, techniques and understanding from mathematical, physical and engineering perspectives. Despite being published as separate volumes, their combined count exceeding some 1500 pages, the three volumes were later combined and published as a single volume in 1950 as *Electromagnetic theory: The complete & unabridged edition* [16], complete with a *Critical and Historical Introduction* by Ernst Weber (first president of the Institution of Electrical and Electronic Engineers, USA), in which he describes Heaviside;

> *Oliver Heaviside, one of the most unusual characters among great modern scientists, could probably be classified best as an outstanding applied mathematician. He was truly a pioneer in this new branch of science.*

Later in 1971, another edition of the unabridged three volumes was published again [17], this time with a foreword by Edmund Whittaker FRS, who ranked Heaviside with that of both Poincare and Ricci, stating that Heaviside's operational calculus is one of the three most important discoveries of the late 19th Century.

The Mysterious *Volume IV*

The three volumes of *Electromagnetic Theory* contain his most significant works in the subject across a broad spectrum of ideas[1]. That said, at the time of his death in 1925, and for some time before (it was published in 1912, in memory of his friend George Francis Fitzgerald FRS), a fourth volume was in preparation, to which he makes some reference in his preface in *Volume III*;

[1] His other significant work on pure electromagnetic theory is in his two volumes of *Electrical Papers* with the multiple part articles on *Electromagnetic Induction and its Propagation*.

> *Long ago, I had the intention, if circumstances were favorable, of finishing a third volume of this work in about 1904 and a fourth in about 1910. But circumstances have not been favorable.*

These unfavorable circumstances are somewhat to do with his health and his disagreements with the new editors of his publisher, *The Electrician Printing and Publishing Co* [4]. It is understood that Heaviside intended unpublished notes, amongst some of the articles intended initially for *Volume III*, to be published in this fourth instalment; some of which are discussed in the Institution of Electrical Engineers *Heaviside Centenary Volume* in an insightful article by the General Post Office (GPO) Research Engineer H. J. Josephs [6]. It is claimed that Heaviside had finished the manuscript for *Volume IV* in 1916/17 and that perhaps a different publisher may have taken on the task of disseminating his work. For whatever reason, this was not to be, and the works intended for *Volume IV* were never published. A reasonable question to ask is, 'Where is the manuscript?', if it was completed and ready for publication. The answer to this question has been sought by many ever since Heaviside's death. Following the purchase of Heaviside's books and papers by the Institution of Electrical Engineers in 1927 [18], (the extensive collection now held at the Institution of Engineering & Technology Archives and known as the *Heaviside Collection* [18]); extensive searches by some followers of Oliver Heaviside (e.g. H.J. Josephs and Dr. W. G. Radley, both of the GPO), a complete manuscript has never been found. However, it is now understood that scattered papers in the *Heaviside Collection* were intended to form part of the lost IVth volume [6], with some of these papers being found under the floorboards in Heaviside's old Newton Abbot house in the late 1950s [19]. The fate of the manuscript has been much debated; it has been claimed that it was stolen soon after Heaviside's death, that there is a copy held somewhere in MIT and that there was never a completed manuscript. Perhaps we will never know the full truth, but the loose and scattered unpublished papers contained within the *Heaviside Collection* may hold some of the answers and contain even more mathematical and electromagnetic gems – undiscovered since Heaviside first uncovered them. For more information, the reader is directed to the work of H. J. Josephs [19, 20] and B.R. Gossick [21] on this intriguing aspect of his *Electromagnetic Theory*.

Editors' Remarks

Each of the papers contained within this special issue has been prepared for this volume. Some of the papers explicitly explore either the work or life of Oliver Heaviside, whereas others have their technical roots founded in Heaviside's Electromagnetic Theory, the remainder are linked only by their relevance to the topic of Electromagnetic Theory as a scientific discipline. It is hoped that this diverse mix of articles appeals to the reader, giving historical insight and perspectives interspersed between original research articles and reviews – reminiscent of reading any of Heaviside's five volumes of *Electrical Papers* and *Electromagnetic Theory*.

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