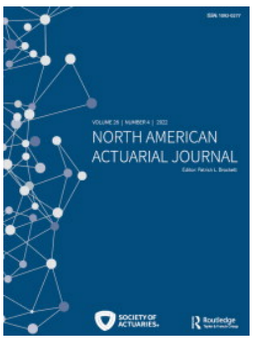

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Discounting the Discounted Projection Approach

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U.K. equity release actuaries are using a flawed approach to value the no-negative equity guarantees in their equity release mortgages. The approach they use, the discounted projection approach, incorrectly uses projected future house prices as the underlying prices in their put option pricing equations. The correct approach uses forward house prices. The discounted projection approach entails significant undervaluations of no-negative equity guarantees and overvaluations of equity release mortgages and can produce valuations that violate rational pricing principles. The discounted projection approach is also inconsistent with both actuarial and accounting standards. Our results have significant ramifications for equity release industry practice and prudential regulation.

Over the last few years there has been considerable controversy in the United Kingdom over the valuation of the no-negative equity guarantees (NNEGs) in equity release mortgages (ERMs). The issues involved will be familiar to those who remember the Equitable Life debacle of a couple of decades ago.

The world's oldest mutual insurer, the Equitable Life Assurance Society (ELAS), was founded in 1762. From 1957 to 1988, it sold with-profit pension annuities with "guaranteed annuity rates" (GARs). It also offered with-profit policies that included "guaranteed interest rates" (GIRs). However, ELAS failed to value these options properly, and in some cases, it didn't value them at all. Its risk management was also highly inadequate. After 1988, "the ELAS with-profit scheme began to take on the characteristics of a Ponzi scheme" (Blake 2001, 8) and became unsustainable. These GARs were eventually valued at £1.5 billion in July 2000, and it was clear by that point that its with-profit fund was technically insolvent or close to becoming so. To quote David Blake, writing in December 2000:

> Once again the ELAS actuaries have been exposed for their poor understanding of financial market risk and the consequences of offering guarantees without investing in appropriate matching assets. Just as they failed to understand the risks underlying guaranteed annuity rate options, so they have failed to understand the risks underlying guaranteed minimum annual returns or GIRs. (Blake 2001, 5)

The company was then closed to new business and policy holders experienced significant losses. After the resulting public outcry, the insurance regulatory system was overhauled and the new European Union (EU)-wide "solvency" regulatory regime was designed to ensure that an Equitable-style fiasco never happened again.

Both cases—Equitable then and equity release now—involve intellectual errors relating to the valuation of opaque and apparently innocuous long-term guarantees. In the Equitable case, actuaries convinced themselves that their treatment of GAR and GIR options was appropriate when it was clearly not.[1] In the equity release case, the intellectual error involves a profound misunderstanding of option pricing theory by professional actuaries, combined with a mind set on the part of industry leaders that puts short-term profitability and "competitiveness" ahead of notions of long-term sustainability. When it comes to NNEG valuation, this mind set prioritizes *low* NNEG valuations over *sound* NNEG valuations, and the rest is obvious.

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[1]The management had sought to resolve the solvency problem with its with-profit fund by proposing a "differential final bonus policy," the details of which are explained by Blake (2001, para. A6). This policy reflected how the actuaries at Equitable convinced themselves that they could "get around" the GAR guarantee in such a way that those policyholders with guaranteed rates each got their "fair share" of the assets. However, this "solution" was unsound in principle and was subsequently rejected in a ruling by the U.K. House of Lords.

This intellectual error centers around the underlying price that enters into the option pricing formula. An NNEG involves a portfolio of put options and we are dealing with puts on forward contracts. For example, if a customer takes out an ERM at the age of 70 years, there is a put for the possibility that the ERM loan might end when the customer is 71, another put for the possibility that the ERM loan might end when the customer is 72, and so forth. Each of these put options is issued now but has a horizon of 1, 2, and so on years in the future. The price that enters into each put option pricing equation is the forward price and the deliverable is a house. So for the put option that ends in future year t, the underlying price is the forward house price for year t, the price agreed now for the house to be delivered and paid for in year t. This approach is based on standard option-pricing theory and is exemplified by Black (1976). In actuarial circles, this approach is often referred to as a "market consistent" (MC) approach to NNEG valuation.

The problem is that a number of practicing actuaries in the U.K. equity release sector have convinced themselves that the underlying price that is relevant for put option pricing is not the *forward* house price for year t but the *future* house price for year t. However, forward and future prices are *very different* and to confuse the two is to commit a major error. This error is a *big deal* because inputting a projected future house price into the option-pricing equation can give very low NNEG valuations, whereas inputting forward house prices into it gives much larger NNEG valuations. This second, incorrect, approach is commonly referred to in actuarial circles as the "discounted projection" (DP) or "real world" approach.

A difference, however, between the Equitable Life and equity release cases is that when Equitable started issuing GARs in the 1950s, the valuation of options was not well understood. The option pricing breakthrough only occurred in 1973 with the publication of the famous articles by Black, Scholes, and Merton (Black and Scholes 1973; Merton 1973), followed shortly afterward by Black (1976). Both the principles and the nuances of option valuation have been well known for decades and are taught in universities all over the world.

Despite copious protestations to the contrary, the U.K. actuarial profession appears to have learned nothing from the lessons of Equitable Life.

This article sets out the case against the DP approach.

It is organized as follows: Section 1 discusses the NNEG and ERM valuation practices used by U.K. equity release lenders. Section 2 sets out the DP approach. Section 3 explains its central error. Section 4 discusses the origin of and rationale for the DP approach. Section 5 provides some illustrative NNEG and ERM valuations. Section 6 examines whether the Prudential Regulation Authority (PRA) current minimum required deferment rate of 1% is high enough to rule out indefensible valuations.[2] Sections 7 and 8 examine whether the DP approach is consistent with actuarial professional standards and accounting standards, respectively, and Section 9 concludes. The article is followed by two appendices dealing with technical issues.

1. NNEG AND ERM VALUATION PRACTICES AMONG U.K. EQUITY RELEASE LENDERS

Consider the following 2017 quotes from U.K. equity release firms discussing the methodologies they use to value their no-negative equity guarantees (NNEGs):

> When calculating the value of the no-negative equity guarantee on the lifetime mortgages, certain economic assumptions are required within the variant of the Black-Scholes formula. … In the absence of a reliable long-term forward curve for UK residential property price inflation, the [firm] has made an assumption about future residential property price inflation. … This results in a single rate of *future house price growth* of 4.25%.[3]

> [The value of the NNEG] is calculated using a variant of the Black Scholes option pricing model. The key assumptions used to derive the value of the no-negative equity guarantee include current property price, *property growth* and property volatility.[4]

> Stochastic modelling is used to capture the expected cost of [the NNEG], which will depend on the expected rate and volatility of *future house price growth*.[5]

> Equity release and securitised mortgage loans … are valued using an internal model. Inputs to the model include primarily *property growth rates*, mortality and morbidity assumptions.[6]

[2]The PRA is the regulator responsible for the prudential regulation of banks and insurance companies in the United Kingdom.
[3]JRP Group plc, *2016 Annual Report*, p. 163.
[4]Hodge Life Assurance Company Limited, *2016 Solvency and Financial Condition Report*, p. 31.
[5]Retirement Advantage Group, *2016 Solvency and Financial Condition Report*, p. 49.
[6]Aviva plc, *2016 Annual Report*, p. 187.

> The fair value of the guarantee is determined using a stochastic model. The fair value of the loans is determined using assumptions for interest rates, *future house price inflation* and its volatility.[7]

What these responses have in common is that they all use property growth assumptions, but no correct option pricing models include property growth variables. Their use of an irrelevant variable therefore indicates that they are not valuing their NNEGs properly.

To its credit, the PRA has been aware of this problem for some time. Referring to the results of an earlier survey, CP 48/16 states (p. 25):

> Many respondents mentioned a version of the Black–Scholes formula known as 'Black 76', where the underlying price is the 'forward price' of the property. This version uses the current price of a forward contract. Some respondents appeared to *conflate* this with the forecast future price of the property, but provided *no justification* for why house price inflation was relevant to the current price of a forward contract. (our italics)

The key word is "conflate." The reason why these correspondents provided no justification for using projections of future house price inflation to value these guarantees is that no such justification exists.

To spell it out: Some firms say that they are using *assumptions about future house price growth*, but the PRA correctly says that this is *obviously wrong*. From this, it follows (1) that some firms are using a method wholly at odds with the one endorsed by the PRA and (2) that the PRA would not be bothering to state this point at all, particularly through a protracted consultation period, if it had not experienced substantial pushback from firms. We can then infer (3) that firms with equity release exposure have been undervaluing their no-negative equity guarantees. We can make this inference because the PRA would not be publishing on the subject or seeking industry consultation if it thought that these guarantees were correctly valued. Consequently, some firms are presumably undervaluing them. Also, (4) by a similar logic, if firms are dedicating substantial resources to pushing back, they must think that the valuation of guarantees is a material issue.

In fact, we are not aware of a *single firm* that has *demonstrated* that it is valuing its NNEGs using a defensible methodology.

2. THE DISCOUNTED PROJECTION APPROACH

2.1. Standard Option Pricing Theory

In *standard* option pricing theory, the price of any option is the sum of its intrinsic value and its time value. The intrinsic value is the value of the underlying instrument or asset minus the strike price. The intrinsic value is therefore driven by the value of the underlying, which for a Black '76 option is the (current) price of a forward contract.

A forward contract is for deferred possession of the underlying asset with payment due at the deferment date. The price of the forward contract (denoted by F) will reflect the cost to the buyer of not receiving the income (e.g., rental income or use value, also known as the benefits of having a roof over one's head) from the asset until the deferment date, and the cost to the seller of deferring payment until the same date, namely, imputed interest.

We can then express the price of the forward contract in terms of a discount relative to the price S of the spot contract, where the discount is expressed in terms of a discount rate q, and a premium relative to S, where the premium is expressed in terms of the risk-free interest rate r. Hence the price of the forward contract is

$$F = Se^{ft} = Se^{(r-q)t} \tag{1}$$

with t as the time to maturity of the forward.

Note that there is no element of forecasting involved in determining F : We infer F from Equation (1) using our best calibrations of r and q, bearing in mind that S and t will be known. To reinforce this point, we emphasize that the formulas for the Black–Scholes (1973) and Black '76 option prices involve no element of forecasting: all inputs are either known or calibrated at the time when the option contract is agreed.

We call q the deferment rate, defined as the discount rate that when applied to the freehold price of vacant possession (i.e., S) results in the price of deferred possession. We have shown elsewhere that q is equal to the net rental yield, which we define as d/S, where d is the net nominal annual rental amount.[8]

[7] LV=, *2016 Annual Report*, p. 126.

[8] The equality of the deferment rate and the net rental yields is proven in Buckner and Dowd (2020a, 34–35).

Equation (1) implies

$$f = r - q. \tag{2}$$

We should note that in a low-interest-rate environment, we would usually expect $r < q$, in which case Equation (1) implies that $F < S$. Intuitively, in such an environment $F < S$ because the loss of rental income exceeds the benefit of not paying interest on the deferred settlement.

2.2. Discounted Projection

In contrast, firms use a different approach known as the discounted projection (DP) approach. The DP approach was proposed as an alternative to the MC approach, which we can consider for present purposes as using the Black '76 put option pricing equation with the underlying prices being forward house prices. The DP approach replaces the current price of the forward contract with some projected future house price based on some arbitrary *hpi* assumption:

$$projected\ future\ house\ price = Se^{hpi \times t} \tag{3}$$

where *hpi* is the projected rate of house price inflation. Equivalently, the DP approach replaces the forward rate f in Equation (1) with *hpi*.

Since $f = r - q$, replacing f by *hpi* gives

$$hpi = r - q. \tag{4}$$

Given also that *hpi* has been specified and r can be easily calibrated from the spot rate curve, Equation (4) implies that we can back out the following implied q :

$$q = r - hpi. \tag{5}$$

To give an example, if we set $r = 0.25\%$ and use the 4.25% *hpi* assumed recently by Just Group,[9] then we can back out q as

$$q = 0.25\% - 4.25\% = -4\%. \tag{6}$$

The negative sign in front of the 4% on the right-hand side of Equation (6) is not a typo. For this calibration, and indeed for any calibration in which *hpi* exceeds the risk-free rate, the DP approach produces a *negative* q rate.[10]

One might note that a negative q rate is peculiar, because the q rate is equal to the net rental rate, which in turn equals d/S, and we would usually expect the net nominal annual rental d to be positive. We have here a red flag regarding the soundness of the DP approach.

3. CENTRAL ERROR OF THE DISCOUNTED PROJECTION APPROACH

The central error of this DP approach is that it confuses future and forward prices. The original Black (1976) article tells us that the underlying is a forward contract.[11] Presuming, as seems reasonable, that Black's derivation is correct (it is) and that

[9]The firm reported using this number in both its 2016 and 2017 *Annual Reports* (see pp. 163 and 110, respectively). The same number also appears in its 2018H1 results (p. 18).

[10]It has been put to us (in conversation and by e-mail with actuaries and consultants) that with the deferment rate currently higher than the risk-free interest rate, the forward price is lower than spot, with the implausible implication that house prices will fall over a long period. This objection is mistaken, however. Our pricing formula in Equation (1) tells us that if q is greater than r, then the current price F of the forward contract will be lower than the spot price. The formula also tells us that as time to the deferment date decreases and approaches zero, the spot and forward price will converge—a contract to possess the house in the future eventually turns into possession itself. But the formula is *silent about the price in the future at which* they will converge. The (future) forward price may remain unchanged, in which case the house price will fall in order to meet it. Alternatively, the house price may remain unchanged, in which case the (future) forward price will have to rise to meet that. Or both may rise or both may fall. The current value of the forward contract has no implications whatever for future (spot) house prices or for future prices of forward house contracts. The future house price (and hence the future price of the forward contract) is free to do whatever it wants and can't possibly be tied down by any present conditions.

[11]Strictly speaking, Black '76 focuses on futures contracts, but futures are a standardized form of forward contract.

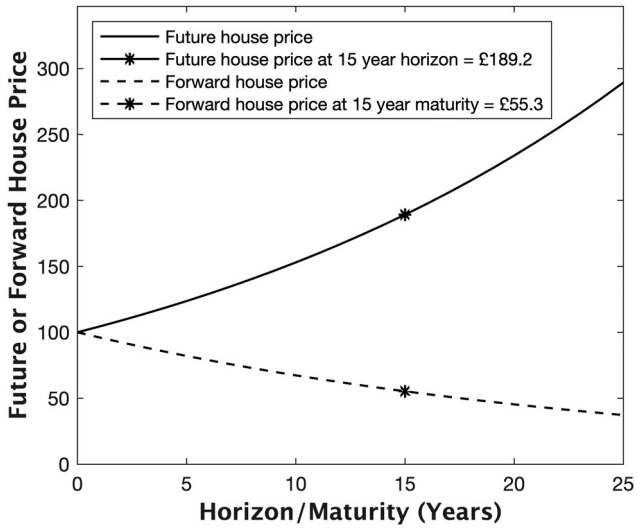

FIGURE 1. Future vs Forward House Prices. Based on current house price = £100, $hpi = 4.25\%$, $r = 0.25\%$, and q=4.2%.

his model is applicable (it is[12]), then there are no grounds to replace the price of the forward contract in the option pricing equation with some other price, such as the projected future house price or the price of something else. To insert any price other than that of the forward into the option pricing equation will give an incorrect valuation unless the two prices are the same.

To illustrate the magnitude of this price difference, consider the plots in Figure 1. The lower plot gives the correct price to use in the option pricing formula, that is, the forward house price. For a maturity of, say, 15 years, the forward house price in this case is £55.3. The upper plot gives the price that would be used in the DP approach, that is, the future house price, in this case one based on an assumed 4.25% *hpi* rate. The projected future price in 15 years is £189.2.

Based on these calibrations, the DP approach implies that we can use an underlying value of £189.2 in the put pricing equation when the correct value is £55.3. If you believe that the DP approach is sound, then you must believe that an asset, a forward worth £55.3, is actually worth £189.2.

In short, the correct approach is to start with the forward price $Se^{(r-q)t}$ (see Eq. [1]) as the underlying price. However, the DP approach incorrectly replaces the forward price with the projected future price $Se^{hpi \times t}$. The DP approach then gives the wrong answers in general except in the special case where $hpi = r - q$.

4. ORIGIN AND RATIONALE OF THE DISCOUNTED PROJECTION APPROACH

So where does the DP approach come from?

4.1. 2005 Equity Release Working Party Report on NNEG Valuation

In 2005, an equity release working party published a report on NNEG valuation (Equity Release Working Party 2005). This report confirmed that it was reasonable to use Black–Scholes (BS) methodology when seeking to obtain a "market consistent" NNEG valuation, while noting that it "is not without its difficulties and shortcomings." We have no argument with that assessment. However, it also noted that "Others may however, prefer to approach the assessment of the NNEG using more of a 'real world' stochastic modelling approach," whatever that might be, and they did not explain. So we have the juxtaposition of BS as a reasonable approach to "market consistent" NNEG valuation, versus an alternative unspecified "real world" or DP approach that gives a different valuation.

4.2. The Hosty et al. Report on NNEG Valuation

Two years later, the Institute and Faculty of Actuaries (IFoA) issued another report on NNEG valuation, Hosty et al. (2007). This report started with some concerns about the decline of profitability and its impact on the development of the ERM market:

[12]For a full treatment, see Buckner and Dowd (2020a; 2002b).

> The competitive environment that has driven product innovation has … resulted in lower product margins. This is all good for the consumer, but it is increasingly difficult for providers to reach target returns on capital, and this is deterring some prospective new entrants. One of the purposes of this paper is to investigate the profitability of typical schemes in the market at present, and so to address the question of whether competition has forced the market to function at non-profitable levels. … We will aim to provide a rational pricing methodology which can be adopted by any organisation active in the market, and we hope that this can support the market as it expands over the coming years.
>
> There is now concern that providers *may not be able to offer a product profitably at current margins*. Some competitive pressure is clearly a good thing, as it will force providers to find more efficient ways of providing their product to consumers. In the equity release market, too much competitive pressure may be a bad thing. (pp. 1–2, our emphasis)

To cut to the chase: Their main concern is that overly high NNEG valuations might undermine the ability of firms to meet their profit targets. We are sure they are right, but the question is how to reconcile these concerns with their obligations under actuarial (see Section 7) and accounting standards (see Section 8) to provide unbiased and fair value valuations.

If there weren't a conflict between these two objectives (i.e., profits and unbiased/fair value), then there would be no issue to discuss. The fact that ERM industry leaders *emphasize* the conflict between the two objectives and their *preference* for commercial considerations over unbiased/fair valuation shows that these objectives are in conflict. Otherwise, they would argue for unbiased/fair valuations and wouldn't need to argue for the primacy of commercial considerations over unbiased or fair values.

They then examine what they understand the "market consistent" approach to be. They do not define the term "market consistent," however, and the nearest we get to an explanation is that this approach is based on an

> approximate market consistent basis similar to the pricing of options on stocks. … The main challenge with a market consistent basis is the fact that there is no underlying market to speak of. Accordingly we have tried to create a proxy market consistent basis using techniques that are standard in similar markets, specifically Black Scholes style modelling. (p. 26)

The counterargument is that there is always an underlying market! Almost all property transactions are forwards—admittedly, short-maturity forwards, but there are no legal barriers to longer maturity forwards and ERM firms could always approach investment banks for quotes. Whether ERM firms would wish to trade at those rates is another matter.

Hosty et al. then explain what they mean by proxy valuation:

> Using a risk neutral basis, *house price inflation* should be linked to the return on long term risk free instruments (i.e. government stocks) less an assumption for rental income (net of expenses). (p. 26, our emphasis)

That is, they draw on Equation (4), which states that $hpi = r - q$. The Hosty et al. use of the incorrect term "house price inflation" instead of the correct term "forward house price" suggests that they consider that the future house price (or *hpi* rate, depending on the formulation one wishes to use) should go into the Black '76 pricing equation, but we have already explained in the previous section that neither the assumed future house price nor the assumed future *hpi* rate belongs in those equations. You can input them if you insist, but you shouldn't, because the model gives you no leave to do so. The use of the term "house price inflation" in this context indicates a serious misunderstanding of how BS option pricing works and is in fact the same confusion between future and forward house prices that we discussed earlier.

But Hosty et al. go on to make plain that they do not like the MC approach:

> In reality the absence of an underlying market means that this proxy market consistent approach is only of limited academic value. (p. 27, our italics)

By "absence of an underlying market" they mean the absence of a liquid market in which the option can be hedged using, for example, a zero-arbitrage trading strategy. The counterargument is that it is perfectly feasible to apply the MC approach in the U.K. property market context (see Buckner and Dowd 2020b, chap. 10), and Hosty et al. have no license *whatever* to replace the forward house price in the option pricing equation with some guess-estimate of the future house price.

The "only of limited academic value" jibe is presumably meant to suggest that the MC approach—or "proxy market consistent approach" as they put it—is of no practical "real world" use, and perhaps to hint that practitioners should be looking for a more "real world"-friendly alternative. Again, we disagree. The MC approach is not only feasible but has no feasible alternative.

Then they make a further criticism of the MC approach:

> For providers attempting to price the NNEG on a market consistent basis there is insufficient product margin in order to provide a competitive product unless they have strong competitive advantages in one or more of the other cost areas. (p. 30)

Whether or not this claim is true, this statement begs the central issue, namely, whether the MC-based valuations are reliable, and Hosty et al. provide no convincing scientific grounds to question them.

Thus, the main objection of Hosty et al. to MC valuation boils down to it giving valuations that they don't like. But remember the problems that Equitable Life got into 20 years ago when it was discovered to have been undervaluing its long-term guarantees!

Section 7.3.2 of their report examines their preferred alternative, an "insurance pricing basis using 'real world' assumptions." What these assumptions might be they do not explain; nor do they explain how this "real world" approach might be consistent with a very un-real-world negative net rental rate. In fact, they don't explain what their "real world" approach even is.

Section 7.3.2 consists of only 143 words and is here reproduced almost in full:

> 7.3.2 *"Real world: assumptions"*
>
> The alternative method we have used is to calculate the option cost using "real world" basis. The methodology we have used is as follows:
>
> - Use the log normal model as before (with same volatility).
> - A best estimate of 4.5% p.a. for HPI in the future (see Section 4.4). This is then the mean return under the model.
> - We have assumed that a real world discount rate of 4.75% per annum.
> - We have not assumed a "mean reversion" so that the random walk in each future period is applied independently of the position is [sic] preceding periods. The authors acknowledge that use of a "mean reversion" approach is equally valid. …
>
> As can be seen [from table shown], the resulting costs are significantly below those assessed using our proxy market consistent basis.

There is not a word of explanation as to why we should regard this "real world" approach as reliable, but the phrase that jumps out is "A best estimate of 4.5% for HPI in the future"—that is, the RW approach[13] is based on a guess about future HPI!

We now see the seed germinate. The 2005 IFoA report introduced the Trojan Horse of house price inflation, but at least did the calculations correctly. This error could be forgiven as an innocuous terminological one, except that the passage just quoted opens the door to full-scale misuse and seems to confirm that the Hosty et al. (2007) "real world" valuation approach is based on exactly that error. The inclusion of HPI is no longer a mere mislabeling, but a bedrock principle of the RW/DP approach.

To spell it out, HPI is now a key input *in its own right*.

These points confirm that this approach is inconsistent with established option pricing theory and therefore wrong.

Section 7.3.3 clarifies the authors' views on which approach is to be preferred. We reproduce part of it here:

> *7.3.3 Market consistent or real world?*
>
> On our proxy market consistent approach we have derived a cost for the NNEG which would render the product non-profitable, whilst real world modelling has produced a significantly lower cost.

This statement has major repercussions. If NNEG valuations on an MC basis would make ERMs unprofitable and if there is no justifiable alternative to NNEG valuations on an MC basis, then doesn't that make the ERM sector unprofitable? The answer appears to be "yes" and our own results support that conclusion (Buckner and Dowd 2020c). But if the sector only appears to be profitable because "real world" NNEG valuations make it seem so, then doesn't that mean that the profits that the firms claim to have been making might have been more apparent than real?

However, the issue is not whether the firms' NNEG valuations would go up if they used the MC approach. The issue is whether firms are using the right approach *in the first place*. If firms are using a valuation approach that undervalues their NNEGs, then they have underestimated their costs, and those costs are already being borne by firms and their investors, regardless of whether firms acknowledge that fact or not. Firms should be facing up to this problem instead of denying it. For their part, analysts should be wondering how big this problem might be and asking themselves about the potential impact on firms' financial conditions. Undervalued costs mean hidden losses and overestimated equity capital, potentially on a large scale.

[13]The RW [real world] approach is another name for the DP approach.

TABLE 1
Baseline ERM and NNEG Valuations: Market Consistent vs. Discounted Projection Approaches

Approach	L	NNEG	ERM
Market consistent	£66.7	£29.4	£37.3
Discounted projection	£66.7	£4.6	£62.1

Note: Exit probabilities are based on M5-CBD model (Cairns et al. 2006; 2009) projections using England and Wales male deaths rate data spanning years 1971–2017 and ages 55–89 years.

Imagine if Bosch were undervaluing the guarantees it issues with its washing machines. Its management then discover that the costs of replacing or repairing their washing machines are higher than they had expected, but they don't yet know how much higher. The problem might only be a small problem but then again it might not. So what is the most appropriate response from the management when they are informed of it? Should they deny it on the grounds that they wouldn't like it if their guarantees turned out to be more costly than they had previously thought, or should they look into the issue with a view to fixing the problem before it gets any worse?

We would have thought that the answer to that question was obvious, but then why would the answer be any different if it was ERMs rather than washing machines whose guarantees were being undervalued? And if the losses involved might potentially be on a large scale, then doesn't that reinforce the need to address the problem as a matter of some urgency, lest a potentially large problem grow into an even larger problem down the road if nothing is done about it?

5. ILLUSTRATIVE VALUATIONS

We now provide some illustrative valuations based on the following plausible baseline parameter values:

- Current age of customer = 70 years, a typical age for ERMs.[14]
- Loan to value ratio = 35%.[15]
- Risk-free rate $r = 0.25\%$.
- ERM loan rate $l = 4.11\%$.[16]
- Deferment rate $q = 4.2\%$ for the MC approach.[17]
- Volatility $\sigma = 20\%$ for males aged 70.[18]

We also assume an implied $q = -4\%$ for the DP approach.[19]

All rates are in percent per annum (p.a.).

We assume an illustrative house price of £100, which, combined with the assumed loan to value ratio of 35%, implies a loan amount (L) of £35.

Table 1 shows the resulting valuations from the MC and DP approaches.

In this case, the DP approach gives NNEG valuations that are 16% of those produced by the MC approach. The result is a considerable overvaluation of the ERM, in this case by 66%.

But ask yourself: Do the DP valuations even *look* right? If you believe them, then you have to believe that the "true" NNEG is 7% of L. This NNEG/*L* ratio looks awfully low when you consider the spread between the loan rate and the risk-free rate, which is 4.11% – 0.25% = 3.86%. If the loan has little risk because the NNEG is so low, then why is the risk spread so high?

[14]Implicitly, we are assuming a single male just turned 70. In the case of a single female, we would expect death/exit to occur somewhat later, which would increase the value of the NNEG. In the case of a couple, we would expect even later exit, when the longest surviving member of the couple exits the house.

[15]A 35% LTV ratio for a 70-year-old appears to be approximately in line with current industry practice for new ERM loans (see Equity Release Council 2020a, p. 14).

[16]4.11% is the latest average loan rate for reported by the Equity Release Council (2020b).

[17]For the justification see Buckner and Dowd (2020b, chap. 7).

[18]For the justification see Buckner and Dowd (2020b, chap. 9).

[19]See Equation (5).

TABLE 2
Baseline ERM and NNEG Valuations: Discounted Projection Valuations vs. PRA Principle II and Principle III Rational Valuation Bounds

Approach	NNEG	ERM
Discounted projection	£4.6	£62.1
	NNEG lower bound	ERM upper bound
PRA Principle II bounds	£22.1	£44.6
PRA Principle III bounds	£14.8	£51.9

Note: As per Table 1.

It is also interesting to compare the DP valuations with the PRA's two sets of "rational valuation bounds" calibrated to the baseline parameter inputs. These bounds are the Principle II and Principle III minimum NNEG valuations and the Principle II and Principle III maximum ERM valuations (see PRA SS 3/17). We can think of these bounds as being derived from a set of rational pricing inequalities. The bounds are model-free in that they do not depend on any assumed option-pricing model. We can then use the bounds to test any proposed NNEG valuation model. If such a model produces any NNEG and ERM valuations that violate these bounds, then the model is indefensible. A more detailed discussion of these principles is provided in Appendix A.

These results are shown in Table 2.

The DP valuations violate all these bounds. The DP NNEG valuation (£4.6) falls below the NNEG lower bounds (£22.1 and £14.8) and the DP ERM valuation (£62.1) exceeds the upper bounds (£44.6 and £51.9).

Thus, in each case considered, the DP approach produces results that violate the PRA's rational valuation bounds.

6. IS THE PRA'S MINIMUM REQUIRED q RATE HIGH ENOUGH?

The PRA has sought to counter the NNEG undervaluations and corresponding ERM overvaluations produced by the DP approach by imposing bounds on the minimum q rates that firms are allowed to use in their valuation models, and the current minimum required q rate is 1%.[20]

The question is then whether this minimum required q rate is high enough to rule out valuations that violate the PRA's rational valuation bounds.

Consider the results based on a minimum required q rate of 1% reported in Table 3.

The DP valuation for the NNEG is now £18.2 but the Principle II lower bound is £22.1, so the NNEG valuation is is still below the Principle II lower bound. Correspondingly, the DP ERM valuation (£48.5) is above the Principle II upper bound (£44.6).

As for the Principle III bounds, it is trivial, since $q = 1\% > q = 0\%$, that the Principle III bounds will be satisfied.

We believe we have made our point, however: The current minimum q rate required by the PRA is not high enough to rule out indefensible valuations.

7. IS THE DISCOUNTED PROJECTION APPROACH CONSISTENT WITH ACTUARIAL PROFESSIONAL STANDARDS?

Then there is the question of whether the DP approach is consistent with compliance to actuarial professional standards.

There is, first, the Actuarial Code,[21] the ethical Code of Conduct that all members of the IFoA must adhere to, and which came into force on 18 May 2019. To quote:

> 2. Competence and care—Members must carry out work competently and with care.
>
> 3. Impartiality—Members must ensure that their professional judgement is not compromised, and cannot reasonably be seen to be compromised, by bias, conflict of interest, or the undue influence of others.

[20]See PRA *Policy Statement 31/18*, Prudential Regulation Authority 2018, p. 11.
[21]https://www.actuaries.org.uk/upholding-standards/standards-and-guidance/actuaries-code

TABLE 3
Discounted Projection Valuations vs. PRA Bounds

Approach	NNEG	ERM
Discounted projection ($q = 1\%$)	£18.2	£48.5
	NNEG lower bound	ERM upper bound
PRA Principle II bounds	£22.1	£44.6
PRA Principle III bounds	£14.8	£51.9

Note: As per Table 1 but with first line based on $q = 1\%$.

On point 2, given the problems entailed by the DP approach (including, e.g., that it is based on a confusion between future and forward prices and produces not inconsiderable NNEG undervaluations) the question is whether/how it meets the requirement of "competently and with care."

On point 3, there is the issue of whether it is appropriate to prefer one approach over another because of commercial considerations.

Then there are the following from TAS 100:

> Technical Actuarial Standard 100: Principles for Technical Actuarial Work (TAS 100) promotes high quality technical actuarial work. It supports the Reliability Objective that users for whom actuarial information is created *should be able to place a high degree of reliance on that information's relevance,* transparency of assumptions, completeness and comprehensibility, including the communication of any uncertainty inherent in the information. (our emphasis)

The question is how users can place "a high degree of reliance" on valuations produced by an approach that can produce indefensible valuations.

> 1. *Judgement shall be exercised in a reasoned and justifiable manner*; material judgements shall be communicated to users so that they are able to *make informed decisions* understanding the matters relevant to the actuarial information.

The issue here is to explain what is "reasoned and justifiable" about the DP approach and to explain the sense in which decisions based on an approach that can produce indefensible valuations are to be considered "informed."

> 2.1 Data shall be *relevant* for the purpose of the technical actuarial work.

Actuaries are using assumptions about *hpi* to price the forward in their NNEG valuation models, but we have explained earlier that *hpi* is irrelevant.[22] The question then is to explain the sense in which an irrelevant variable is relevant.

> 3. Assumptions used, or proposed for use, in technical actuarial work shall be *appropriate* for the purpose of that work so that *users can rely* on the resulting actuarial information.

How can it be "appropriate" to use an unreliable approach that depends on an irrelevant variable? Also, how are results based on an inappropriate assumption about an irrelevant variable to be considered reliable for users?

8. IS THE DISCOUNTED PROJECTION APPROACH CONSISTENT WITH ACCOUNTING STANDARDS?

A final question is whether the DP approach is consistent with compliance to accounting standards. Under modern accounting standards such as International Financial Reporting Standards (IFRS), which is used in the EU and the United Kingdom, valuations must be based on the principle of "fair value." IFRS defines a "fair value" price as

> The price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date.[23]

[22]See also, e.g., PRA SS 3/17 (Prudential Regulatory Authority 2017, p. 13, para 3.17), which states: "It is important to note that views on future property growth play no role in preferring one contract over the other. Investors in both contracts will receive the benefit of future property growth (or suffer any property depreciation) because they will own the property at the end of the deferment period. Hence expectations of future property growth are *irrelevant*" (our emphasis).

[23]See, e.g., https://www.iasplus.com/en/standards/ifrs/ifrs13.

IFRS does not define "fair," but the assumption is that a market participant, that is, someone who is independent, knowledgeable, and able and willing to enter into the transaction, would not be duped into an *unfair* transaction. Thus, current market prices must be deemed to be fair values, because a market participant would not be duped into buying at greater than the market price or be duped into selling at less than the market price. Consequently, fair value equals market price, where the market price exists.

But what is fair value if the market price does not exist?

The answer comes from the Level 1/Level 2/Level 3 fair value hierarchy set out in IFRS 13, the "Fair Value Measurement Reporting Standard" published by the International Accounting Standards Board.[24]

Level 1 fair value[25] is the market price, where the market price exists.

Where no Level 1 fair values exist, that is, where there are no market prices, IFRS uses Level 2 fair values; these are the prices of related instruments that can be used as proxies for unobservable values.[26] An example in the equity release context would be the use of leasehold and freehold market prices as proxies for the values of the notional "leasehold" granted to the equity release borrower when an ERM is taken out.

Where no Level 2 prices are available, IFRS uses Level 3 or mark-to-model fair values; that is, Level 3 involves the use of a model to obtain fair values.[27] However, the model and its calibrations should still reflect "the assumptions that market participants would use when pricing the asset or liability, including assumptions about risk." In the equity release context, the natural example is an NNEG model. Such a model, which is by definition mark to model, would be required under Level 3 to be calibrated using assumptions that market participants would make. One such assumption would be Principle II (see Appendix A), that no value of the ERM can exceed the value of forward contract; another is Principle III, that in any equity release context the deferment house value must be less than the current spot house value, reflecting the point that a market participant would want compensation for the income or use that was lost through deferment. In general, one can say that fair value translates into market-consistent valuations.

Whatever level is used, the underlying principle is always the same. To quote Financial Reporting Standard 102:

> 2.2 The objective of financial statements is to provide information about the financial position, performance and cash flows of an entity that is *useful for economic decision-making by a broad range of users* who are not in a position to demand reports tailored to meet their particular information needs.[28]

This information should enable users to take a neutral and objective view of the company and ensure that they are not being cheated.

Qualities of this information include understandability, substance over form, completeness, comparability, and timeliness, as well as the following (quoting FRS 102, emphasis ours):

- *Relevance*: 2.5 The information provided in financial statements must be relevant to the decision-making needs of users. Information has the quality of relevance when it is capable of influencing the economic decisions of users by helping them evaluate past, present, or future events or confirming, or correcting, their past evaluations.
- Materiality: 2.6 Information is material—and therefore has relevance—*if its omission or misstatement, individually or collectively, could influence the economic decisions of users taken on the basis of the financial statements*.
- *Reliability*: 2.7 The information provided in financial statements must be reliable. Information is reliable when it is *free from material error and bias and represents faithfully that which it either purports to represent or could reasonably be expected to represen*t. Financial statements are not free from bias (ie not neutral) if, by the selection or presentation of information, they are intended to influence the making of a decision or judgement in order to achieve a predetermined result or outcome.
- Prudence: 2.9 The uncertainties that inevitably surround many events and circumstances are acknowledged by the disclosure of their nature and extent and by the exercise of prudence in the preparation of the financial statements. Prudence is the inclusion of a degree of caution in the exercise of the judgements needed in making the estimates required under conditions of uncertainty, *such that assets or income are not overstated and liabilities or expenses are*

[24]For more details, see https://www.iasplus.com/en/standards/ifrs/ifrs13.
[25]IFRS 13:76.
[26]IFRS 13:81.
[27]IFRS 13:86.
[28]Financial Reporting Standard 102, The Financial Reporting Standard applicable in the UK and Republic of Ireland. Financial Reporting Council, September 2015.
https://www.frc.org.uk/getattachment/e1d6b167-6cdb-4550-bde3-f94484226fbd/FRS-102-WEB-Ready-2015.pdf

> *not understated*. However, the exercise of prudence does not allow the deliberate understatement of assets or income, or the deliberate overstatement of liabilities or expenses. *In short, prudence does not permit bias*.

Relevant, reliable, free from material error and bias, assets not overstated, liabilities not understated, and so on.

The accountant is then hired by the management of the company to draw up accounts on the basis of these principles and in accordance with IFRS rules and existing law (e.g., the Companies Act). These accounts would be approved by the directors, who are deemed to have prepared the accounts, and then presented to the auditors for sign-off. "An auditor is an independently qualified person who is appointed to give shareholders an independent, professional and informed opinion on the financial statements prepared by the directors,"[29] and the

> auditor's objectives are to obtain reasonable assurance about whether the financial statements as a whole are free from material misstatement, whether due to fraud or error, and to issue an auditor's report that includes the auditor's opinion. Reasonable assurance is a high level of assurance … *Misstatements* can arise from fraud or error and *are considered material if, individually or in the aggregate, they could reasonably be expected to influence the economic decisions of users taken on the basis of these financial statements*.[30] (our emphasis)

Suffice it to note that it is doubtful that the DP approach is consistent with these standards either.

9. CONCLUSIONS

The Discounted Projection approach (i) has never been explained, let alone convincingly justified, by those who advocate it; (ii) is based on an elementary error about the underlying price, namely, the confusion between a future price (i.e., a price in the future) and a forward price (i.e., the price of a forward contract now); (iii) violates the principle that options be calibrated using inputs that are currently known or calibratable, as opposed to using inputs that are unknown future variables or projections of unknown future variables; (iv) has not been endorsed by a single recognized independent expert; (v) does not appear in the corpus of recognized scientific research journals that are subject to rigorous peer review[31]; and (vi) is contradicted by alternative approaches such as Black '76 that are used and taught all over the world and have been published in top-tier refereed academic journals. Thus, to use the DP approach is to defy modern finance theory. In addition, the DP approach (vii) entails significant undervaluations of no-negative equity guarantees and overvaluations of equity release mortgages; (viii) can produce valuations that are known to be indefensible; (ix) is open to abuse in that it allows practitioners to obtain low NNEG valuations by inputting arbitrarily high *hpi* assumptions; and (x) is being promoted by practitioners with a vested commercial interest who are dismissive of the only approach that is scientifically respectable because they do not like the valuations it produces. Last but not least, there is a prima facie case that the DP approach is inconsistent with both (xi) actuarial and (xii) accounting standards.

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[29]See, e.g., https://blog.corplaw.ie/bid/337442/The-Responsibility-Of-Auditors.

[30]Financial Reporting Council, "Description of the Auditor's Responsibilities for the Audit of the Financial Statements Applicable for Audits of Financial Statements for Periods Commencing on or after 17 June 2016,." https://www.frc.org.uk/auditors/audit-assurance/auditor-s-responsibilities-for-the-audit-of-the-fi/description-of-the-auditor%E2%80%99s-responsibilities-for

[31]Admittedly, Hosty et al. (2007) was later published in the *British Actuarial Journal* (Hosty et al. 2018), but it is not clear whether *British Actuarial Journal* articles (or, for that matter, any articles and reports published by the IFoA, such as Tunaru and Quaye [2019]) are subject to "rigorous peer review," and the only thing that is clear about the review process, whatever that might be, is that it is unclear. To quote from the *British Actuarial Journal* website, "*British Actuarial Journal* contains the sessional research programme of the Institute and Faculty of Actuaries along with transcripts of the discussions and debates. It also contains Presidential addresses; memoirs and papers of interest to practitioners." There is nothing about rigorous scientific peer review.

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Discussions on this article can be submitted until July 1, 2023. The authors reserve the right to reply to any discussion. Please see the Instructions for Authors found online at http://www.tandfonline.com/uaaj for submission instructions.

APPENDIX A. THE PRA'S RATIONAL VALUATION PRINCIPLES

In its Supervisory Statement SS 3/17 published in July 2017, the U.K. Prudential Regulation Authority set out certain good practice principles relating to the rational valuation of ERM portfolios. These principles include two that impose upper bounds on ERM valuations and, by implication, lower bounds on NNEG valuations. Valuations that violate these bounds are not defensible. By implication, models that can generate valuations that violate these bounds are also indefensible.

A.1. Principle II

Principle II states:

> The economic value of ERM cash flows cannot be greater than either the value of an equivalent loan without an NNEG or the present value of deferred possession of the property providing collateral.

That is:

$$ERM \leq L \text{ and } ERM \leq PV(F) \tag{A1.1}$$

where PV(.) is the present value of the term in parentheses.

We first prove that $ERM \leq L$.

Start with

$$ERM = L - NNEG. \tag{A1.2}$$

We know that

$$NNEG \geq 0. \tag{A1.3}$$

If $NNEG > 0$ then

$$ERM = L - NNEG < L. \tag{A1.4}$$

If $NNEG = 0$ then

$$ERM = L - NNEG = L. \tag{A1.5}$$

Hence

$$ERM \leq L \tag{A1.6}$$

which was to be proved.

We now prove that $ERM \leq PV(F)$.

The present value (PV) period t payoff to ERM_t is $\min[L_t, PV(F_t)]$, where L_t is the present value of the loan assuming it matures in t years and assuming that there is no NNEG involved, and $PV(F_t)$ is the present value of the period t forward contract, for all t. But

$$\min[L_t, PV(F_t)] \leq PV(F_t) \text{ for all } t. \tag{A1.7}$$

Therefore

$$ERM_t \leq PV(F_t) \text{ for all } t. \tag{A1.8}$$

Hence

$$ERM \leq PV(F) \tag{A1.9}$$

which was to be proved.

A.2. Principle III

Principle III states:

> The present value of deferred possession of a property should be less than the value of immediate possession,

that is,

$$\text{Deferment house value} < \text{spot house value.} \tag{A1.10}$$

A heuristic demonstration of the validity of Principle III goes as follows. Recall

$$R_t = \textit{current house price} \times e^{-qt} \tag{A1.11}$$

where q is the *deferment rate* and R_t is the deferment price, and note that the spot house value and the current house price will be equal.

Buckner and Dowd (2020b, 35–36) prove that

$$q = net\ \ rental\ \ yield = d/S \tag{A1.12}$$

where d is the *current* net nominal annual rental amount, the current time being the *beginning* of the year, where "net" means the gross or headline rental paid by tenants, less the costs incurred by the lessor such as management, maintenance, and the expected costs of void or empty periods while the property is being re-let. Given that it is reasonable to presume that $d > 0$, then $e^{-qt} < 1$ so Equation (A1.11) implies

$$R_t < spot\ \ house\ \ value. \tag{A1.13}$$

It is then reasonable to suppose that R_t will be equal to the deferment house value, and Principle III follows.[32]

APPENDIX B. THE DEFERMENT RATE AND THE ILLIQUIDITY PREMIUM

The deferment rate is usually taken to be equal to the net rental yield, that is,

$$q \ = \ \text{rent rental yield.} \tag{A2.1}$$

However, a recent (February 2020) "discussion note" issued by the IFoA ERM working party proposes that Equation (A2.1) should be replaced by

$$q \ = \ \text{net rental yield – illiquidity premium.} \tag{A2.2}$$

The discussion note explains (pp. 8–9):

> There is a technical argument, presented in recent actuarial ERM valuation research,[33] that the presence of an illiquidity premium in the underlying house price should reduce the cost of the NNEG (note that the illiquidity premium of the residential property is distinct from the illiquidity premium of the mortgage). Specifically, the present value of the house price illiquidity premium that will be earned over the life of the option should be added to the house price that is used in the NNEG valuation. This is equivalent to deducting the house price illiquidity premium from the deferment rate used in the NNEG valuation.

If Equation (A2.2) is correct, then q would be less than the net rental yield, and if the illiquidity premium (ILP) were large, then q would be considerably less than the net rental yield. In that case, there *might* be a justification for low deferment rates.

However, Equation (A2.2) is demonstrably incorrect.

For the sake of argument, suppose that the ILP does exist. The ILP should then appear in the formula for the discount factor that applies to future cash flows. According to bedrock finance theory in the form of the mathematics of the discount dividend model (DDM), *it can appear nowhere else*.

Let d be the current net nominal annual rental amount, the current time being the beginning of the year. "Net" means the gross or headline rental amount paid by tenants, less the costs incurred by the lessor such as management, maintenance, and the expected costs of void or empty periods while the property is being re-let.

Assume that the value S of a perpetual income-producing asset is the sum of the present values of its individual cash flows. This assumption is the bedrock of practically all financial theory.[34] Assume that the discount factor is given by $1/(1 + r + \pi + \lambda)$, where r is the risk-free rate, π the risk premium demanded by investors for taking on risky cash flows, and λ the ILP demanded by investors for cash flows that cannot easily be exchanged at market. Assume also that cash flows grow at a constant rate g, so that future cashflows are $d(1 + g)$, $d(1 + g)^2$, and so on. Define y as follows:

[32]We note, however, that it is conceivable that under certain extreme circumstances that have no relevance to equity release, the condition $d > 0$ might not hold. An example would be a property in Chernobyl, but then Chernobyl is not an equity release asset class. A fuller discussion of the validity of Principle III is provided in Buckner and Dowd (2020b, 94–110).

[33]The discussion note adds a foot note here: "See Section 5.3.5 and 5.3.6, Jeffery & Smith (2019)."

[34]It underlies, for example, the discount dividend model (e.g., Gordon 1959), with property prices and rentals taking the place of stock prices and dividends. See, e.g., https://en.wikipedia.org/wiki/Dividend_discount_model.

$$1/(1+y) = (1+g)/(1+r+\pi+\lambda). \tag{A2.3}$$

Then from the preceding definitions and assumptions,

$$S = d[1/(1+y) + 1/(1+y)^2 + 1/(1+y)^3 ...] \tag{A2.4}$$

and from the definition of the deferment price R_n as the present value of the future cash flows *minus* the present value of the first n cash flows, that is, minus the "foregone income,"

$$R_n = d[1/(1+y)^{n+1} + 1/(1+y)^{n+2} + 1/(1+y)^{n+3} ...]. \tag{A2.5}$$

Divide every term in Equation (A2.5) by $(1+y)^n$, express in terms of S, and note that y meets the definition of q as the discount rate that we apply to the "spot" price S to give the deferment price R_n :

$$\begin{aligned} R_n &= d[1/(1+y) + 1/(1+y)^2 + 1/(1+y)^3 ...]/(1+y)^n \\ &= S/(1+y)^n \\ &= S/(1+q)^n. \end{aligned} \tag{A2.6}$$

Define X as S/d. Then, substituting X into Equation (A2.4), and substituting q for y,

$$X = \frac{1}{1+q} + \frac{1}{(1+q)^2} + \frac{1}{(1+q)^3} + \ \ ... \tag{A2.7}$$

Multiply both sides by $(1+q)$

$$\begin{aligned} X(1+q) &= 1 + \frac{1}{1+q} + \frac{1}{(1+q)^2} + \ \ ... \\ &= 1 + X. \end{aligned} \tag{A2.8}$$

Divide both sides by X and subtract 1, substitute S/d for X, and rearrange:

$$q = 1/X = d/S \tag{A2.9}$$

which is equivalent to Equation (A2.1). Thus, Equation (A2.1) is correct and Equation (A2.2) is not.

To avoid any misunderstanding, we are *not* saying that the ILP does not exist or does exist but has a zero value. Nor are we arguing about its size. We are saying that if the ILP does exist, whatever its size, its impact on the deferment rate gets canceled out (along with the impacts of the growth rate g, the risk-free rate r and any risk premium π), because Equation (A2.9) is always true.