Nurturing mathematical literacy at lower primary level: Impacts on student understanding of formal mathematical concepts

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Abstract (100 words)

This study investigates to what extent mathematical literacy in lower primary students (aged 6 to 8) in England was developed by a sequence of three lessons. The lessons focused on understanding of two mathematical concepts, tessellation and self-similarity, and their links with lattices and crystals. 117 students completed a pencil-and-paper based open-ended questionnaire six months after the lessons were implemented. The results show that students could articulate the meanings of these concepts. Follow-up interviews with 15 of the students illustrate that visual representations have an important role to play in making students aware of these mathematical concepts, and in developing their capabilities to reason and communicate through use of these concepts.

Key Words

Mathematical literacy, tessellation, self-similarity, visual representations

Introduction

PISA (the Programme of International Student Assessment) has proposed mathematical literacy (ML) as the main focus of its assessment. It is well acknowledged that mathematical literacy is a capacity that students should display if they are to be prepared for their future lives (OECD, 2018), as they are expected to analyse, reason, and communicate mathematical ideas effectively as they pose, formulate, solve and interpret mathematical problems. Thus, a mathematically literate student recognises the role that mathematics plays in the world (OECD, 2003). Kilpatrick has synthesised research findings about *mathematical proficiency* to provide practical recommendations for early years schooling (up to the eighth grade) on ML (Kilpatrick, 2001). Mathematical proficiency is made up of five strands: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence; (4) adaptive reasoning; and (5) productive disposition (National Research Council, 2001, p. 5). In teaching practice, ML is sometimes mixed up with the well-known term ‘numeracy’. Numeracy mainly refers to a student’s ability to perform fundamental maths skills, such as basic calculations, to use and interpret statistical information, and to think critically about mathematical information. These two concepts are indeed closely related. However, ML requires of students a high level of mathematical understanding, both of text and symbol, while numeracy is not necessarily linked to specific and technical understanding (Gal & Tout, 2014). Numeracy and literacy skills are bound to blend into many mathematics lessons, but they are rarely the focus. This study has provided a sequence of lessons to demonstrate a possible way, as how to link these elements together, such as numeracy and literacy, and

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mathematical proficiency. The study reported here works with in-service teachers to explore what ML looks like in the lower primary setting classroom, and more importantly, it aims to explore students' learning outcomes and their feedback about lessons focused on ML.

Nonetheless, research findings on numeracy in lower primary settings will be useful in informing the directions of our research on ML. Research on numeracy at lower primary level mainly focuses on language development, with it being argued that there exists a significant mutual relationship between language skills and learning early mathematics (Toll & Van Luit, 2014; Purpura, et al., 2017). Researchers suggest that parents or main caregivers practicing numeracy at home enhance their children's language development, while language development has a causal relationship with numeracy outcomes (LeFevre, et al., 2008; Napoli & Purpura, 2018). Therefore, we need nuanced discussions with students to discern the impact of ML teaching on their understanding of mathematics concepts. From the curriculum design perspective, Sarama and her colleges (2012) have investigated the impact of a particular early mathematics curriculum in linking literacy. They suggest that students' confidence to verbally express their thinking on mathematical concepts is crucial. Therefore, this study explores if students in lower primary settings are confident enough to explain their understandings of school-based mathematics concepts to researchers.

Literature Review

In this literature review, we focus on two relevant research areas: from the teaching perspective, research on teaching strategies for shaping ML, and from the learner's perspective, research on the roles that learning-related emotions play.

Several studies show that mathematics teachers relate ML with the ability to solve problems in daily life, communicating by using mathematics concepts and showing possession of basic mathematics knowledge (Afifah, Khoiri & Qomaria, 2018; Genc & Erbas, 2019). Problem-solving here is, in particular, about raising students' awareness of how they can use their skills to solve problems in real life, rather than about improving their problem-solving skills per se. This brings with it several challenges for educators and researchers, such as on how to develop the instructional process accordingly. Sumirattana, Makanong and Thipkong (2017) have proposed five steps in this instructional process: (1) posing real life problems; (2) solving problems individually or in a group; (3) presenting and discussing; (4) developing formal mathematics, and (5) applying knowledge. However, implementing this instructional process is not straightforward, as there is an underlying question to address: what sort of real-life problems should be part of the curriculum and teaching practice for facilitating ML? Gatabi, Stacey and Gooya (2012) have proposed four aspects mathematics problems should have or involve: (1) an extra-mathematical context, (2) multiple steps and the making of connections, (3) formulation, and (4) interpretation and/or checking. In this model, the purpose of teaching mathematics goes beyond calculating right answers, extending towards developing higher-order thinking skills and creating opportunities to see the world through mathematical lenses. Furthermore, this teaching even transcends problem-solving, as it involves recognising the features of mathematics in natural phenomena (Steen, Turner & Burkhardt, 2007). Drawing on these research findings, we first reviewed all the key mathematical concepts at lower primary level in the area of shapes. We found that concepts such as tessellation, similarity and self-similarity could be linked to natural phenomena, such as snowflakes, lattices and crystals. The Science National Curriculum (DfE, 2015) holds up having pupils "experience and observe phenomena" as the "principal focus of science teaching Key Stage 1". These natural phenomena are visible to all students, regardless of their background or academic ability.

ML also places an emphasis on improving students' ability to make use of mathematics in different contexts. However, the foundation of this improvement rests on the assumption that students have sufficient mathematics knowledge, and it depends on students' openness to mathematics (Amit & Fried, 2002). The focus in current education on memorisation of mathematics skills has been blamed for causing lack of confidence in the subject, and even for the shortage of people with the desire to become mathematics teachers (Breen, Cleary & O'Shea, 2009). Researchers and educators have thoroughly recognised the effects of emotions in the learning process (Valiente, Swanson & Eisenberg, 2011). A large amount of research investigates the influence of these emotions on ML, in phenomena such as mathematics anxiety, emotional intelligence, self-confidence, self-efficacy, etc. Hwang (2019) observes the strong relationship between students' feelings of helplessness and their ML. He points out that helplessness is learnt, and students tend to see it as stemming from their lack of ability – a connection that is often based on emotional states more than their actual capacities.

Although this research has highlighted these various intersections in the classroom at secondary level, findings are also instructive in the primary setting. The aim of the present study is to extend the understanding of ML into lower primary settings, testing the idea of nurturing abilities, and measuring the impacts. We employ a holistic approach, involving a ML-focused three-lesson sequence and professional development programme for teachers working with local educational agencies and academia – all with the aim that teachers will be better placed to help students develop their ML. We present student feedback on the intervention, helping us to evaluate what can be done in primary settings.

The Study

The Snowflake Bentley project was funded by Education Durham, Durham County Council. It was designed to empower teachers to think differently about how they structure tasks in order to deepen the learning experience for all pupils – in particular their ML, and in the link between numeracy and literacy. This study aimed to provide evidence of the mathematical concepts students understand and recall from lessons focused on ML, and to show their feelings towards these lessons.

The project involved a package of curriculum design (a sequence of three lessons designed and delivered by the third author, with teachers from each school asked to observe the delivery) with strong ML elements. It ran in the 2017/2018 academic year. The design of the package was in line with four aspects proposed by Bansilal, Webb and James (2015): (1) using contextual language, such as the example of the snowflake, to highlight 2D shapes, e.g. regular hexagons, (2) employing contextual signifiers, such as crystal from minerals, in describing 3D shapes, e.g. cubic, octahedral; (3) introducing contextual rules, such as self-similarity; and (4) drawing on contextual graphs to engage students with these contextual resources, thus going beyond just numeracy and literacy in the teaching.

The third author, who designed the lesson materials, led a Continuing Professional Development (CPD) session with participating schools after the full delivery of all lessons. Teachers were asked to reflect on the underlying lesson planning structure, and on how the mathematical concepts and the natural phenomena linked together. Teachers were also asked to reflect on what they observed in terms of their students interactions with and responses to the activities in the lesson. They were asked to collaboratively develop a sequence of lessons for their own classroom taking account of this curriculum design.

Given the sample in this research focuses on students aged 6 to 8, we recognise the need to collect interview data to follow up on the paper-based questionnaire as paper-and-pencil questionnaire might not capture all their understandings. Analysis of data gathered from semi-structured interviews is suitable here because it allows participants to express their views and feelings (Cohen, Manion & Morrison, 2007), as well as to explain further their understanding of certain concepts in ML.

We put forward the following two research questions in the context of lower primary settings:

1. In which ways and to what extent do students understand the mathematics concepts presented?
2. What feelings do students express in relation to ML?

Methods

The collection and processing of the data set

The research design used involved mixed methods, in the form of a descriptive study. Three primary schools located in North East England participated in the project. Students from the three schools first completed a questionnaire (see Appendix A). Four questions (Question 1, 2, 4 and 5) were related to the mathematics concepts they had been taught six months previously, without revealing the name of the concept. Question 5 was related to their understanding of crystals and how they view crystals through mathematical eyes. Two questions (Question 3 and 6) were related to their feelings. Part of the rationale for this step was that it would increase the likelihood of students being willing talk about their feelings in the subsequent interviews (See Appendix B), to help answer Research Question 2. The implementation of the questionnaire and interviews took place between 21st May and 8th June 2018, following on from the implementation in the classroom during the Autumn Term of 2017. The purpose of this schedule was to gather information on its impacts six months after the implementation.

Two members of the research team visited each primary school setting to conduct the student questionnaires and to interview students. During the visits to the schools, two members of the research team and the classroom teachers gathered the students together to complete the questionnaire. The questionnaire was designed to take no longer than 10 minutes to complete. After completing the questionnaire, one-to-one interviews were set up for 15 pupils in total in the three schools, subject to consent. Each interview took no longer than 10 minutes. The School of Education at Durham University granted outline ethics approval on 4th May 2018.

Participants

The classroom teachers from either Year 2 or Year 3 in each school first attended a briefing meeting about the curriculum package. The package was then implemented in their classes during the Autumn Term of the 2017 academic year. 88 Year 2 students and 29 Year 3 students completed the questionnaire (as one school considered the Year 3 class was the best fit for this project). 15 of them consented in interviews. During the interview day, The interview was conducted either with one student with two members from the research team, or a group of students with two members from the research team depending if participants were more comfortable to speak to the research team with their peers present.

Limitations

The limitations in this study are mainly as follows:

(1) As a small-scale pilot study, there is no comparative data, so the outcomes in the three schools could not be equated;
(2) The intervention has three lessons rather than a longer-term structured type, this short intervention, to form the ML is an experience for teachers as well as students.

More topics or exemplary lesson designs are needed in practice to form an intervention package.

Results

Findings from the questionnaire

Results for Research Question 1

The activities in the intervention for the Year 2 and 3 students were mainly related to concrete representation. The majority of the students (79 out of 117, 68 percent) reported that the project was related to ‘shapes’ and ‘book’ (referring to the book *Snowflake Bentley* that was introduced in the lessons). Nearly half of them (51 out of 117, 44 percent) were able to recall the concrete representations or manipulatives used in the project, such as dice, magnet balls, rocks and crystals. A few recalled the formal mathematics terms from the project (notwithstanding some spelling errors): 18 out of the 117 students (15 percent) mentioned the words ‘triangle’ and ‘square’, while 10 out of 117 (8.5 percent) mentioned ‘hexagon’.

The notions of being tessellate or symmetrical were reported as the key ideas by 17 of the students (14.5 percent). These words were generally not spelled correctly; ‘semetrical’ and ‘tesselate’ were common errors. The majority of these students (12 out of the 17, 70.6 percent) mentioned the word ‘atom’. These students also recognised formations, such as hexagonal, square and triangular formations, as the key ideas. When asked about number related activities in the lesson, there were various answers. 12 students reported that it was about making patterns or big shapes.

52 out of the 117 total students (44 percent) described a crystal as a ‘shiny’ shape. 17 of them (14.5 percent) made connections with the atomic level. 13 of them connected crystals with snowflakes.

Results for Research Question 2

70 out of the 117 students (60 percent) reported that their feeling about the lessons was ‘happy’, and 45 of them (38.5 percent) felt ‘excited’. 40 of them (34 percent) expressed mixed feelings, such as ‘happy and confused’, ‘happy and scared’, ‘happy, excited, worried’, or ‘a little bit sad and angry and happy at the end’. 83 out of the 117 students (71 percent) would recommend the lessons to other students as they found them ‘interesting’, ‘amazing’ and ‘fun’. 12 of them (10 percent) reported that they would not recommend them as they were boring, or not that fun. In summary, this indicated that feelings towards the mathematical-literacy focused lessons were positive, in relation to its mathematics contents, literacy and numeracy.

Findings from the interviews

Positive response towards visual representations linked to literacy

The students were presented with the concrete representations, such as triangles, hexagons and squares, pyramids using ball bearings, in a table. When presented with these visual prompts in the form of the original resources used during the project, and prompted with questions such as “How does this link to tessellation/self-similarity etc” the children could recall and replicate many of the activities encountered:

> “I remember that day…I loved it!!”
>
> “I remember now…”
>
> “Oh…and we had to put them in the grid…”

"My favourite part was the atom…all bound together to make this…we stacked them up to make a 3D from the 2D one" [child builds a pyramid using the ball bearings]

"I remember we tried to make a big hexagon from all the hexagons, and a square from all the squares and a triangle from all the triangles…"

"My favourite activity was the balls…(I) didn't think they would balance properly"

"This was my favourite activity [points to pile of 2D shapes]. You could go about and design your own snowflake pattern…I like making stuff"

Capabilities to reason and communicate ideas

Some groups of students recalled more than the sum of their individual children; the recollection of one child often sparked a memory in another:

"Thank you; you are recharging my brain!" [one child shows the other how they had built a 2 by 2 by 2 cube]

In one school, pairs of students were invited to explore together the full learning sequence, which had been laid out in chronological order in terms of the resources used, without adult intervention. They were then quizzed on what they could remember about the project. These children were able to provide a more complete account of the activities and specialist vocabulary used, successfully explaining many of the key concepts covered with minimal prompting. For example, one pair of children demonstrated self-similarity using dice, and when asked "What is a crystal?" one of the children replied: "(It) is an array of atoms…a perfect array." When probed for what was different about glass they responded: "Atoms are in a jumbled order not an array."

Another pair were able to build a 'triangular array' and explain self-similarity in 2D, and when prompted to explain why they had been asked to 'play around with cubes', they recognised the link to self-similarity.

Although several children were not able to recall the specialist vocabulary (i.e. array, self-similarity, tessellation, and lattice) without prompting, many recalled the ideas behind them. Once re-introduced to these specialist vocabulary items, they were frequently able to use them appropriately in context. For example, having revisited the meaning of self-similarity and tessellation in a 2D context, when prompted: "Cubes build bigger cubes, so cubes are…?" the child was able to describe this correctly as "self-similarity." Likewise, having revisited the meaning of tessellation, the same child correctly noted that neither circles nor pentagons tessellate and why: "they leave gaps."

When one of the Year 2 children was asked what makes a crystal a crystal, he pointed to a hexagon and said:

"There are atoms in one of these that look like that [points to a bag of balls] but in a crystal they look like that [points to the model of a square lattice]"

In the schools which used the book *Snowflake Bentley* to support a literacy task, the children were able to recall the full story in depth and spoke with empathy and sadness about the book's contents, the life and death of William Bentley, who pioneered snowflake photography:

"The book is sad because he dies"

"He tried to draw snowflakes, but his drawing was rubbish, before he got his camera"

> “He wanted to take the first pictures of a snowflake…became famous but died of pneumonia”

> “It’s a really good book…can’t keep snowflakes forever. He died as he did so much”

> “He kept trying to take photos but (they) kept melting…he went to study snowflakes in a storm and he died”

Many of the children’s understandings of what a crystal was remained linked to it being “shiny” or “precious” with “different colours” and “different shapes,” and to the idea that it “comes from the ground.” They were unable to recall how it was built up of atoms arranged in a lattice. Some recalled that snowflakes were crystals but were unable to explain why.

In general, Year 3 children were able to recall substantially more of the learning sequence without prompting. Some of the students recognised the cross-curricular nature of the learning: “We’ve mixed maths with art…!”

> “Your hexagons look like honeycomb…”

> “They’re like bathroom tiles”

> “We sometimes use arrays in number”

They went on to ask the interviewer a number of questions above and beyond the scope of the original lessons. This included exploring the importance of electron bonds and divergence angles between these bonds as a possible explanation as to why snowflake crystals were ‘flat’ and not spherical.

In another instance, when invited to ask questions at the end of the data collection interview, one child in Year 2 took the opportunity to quiz the interviewer about his understanding of whether atoms are arranged differently in solids, liquids and gases. The conversation extended to exploring what plasma is, a topic well beyond the scope of the initial project.

Thus, the interviews also provided strong evidence of the importance of providing opportunities which extend what is covered in the normal curriculum, to allow students to be stretched and challenged.

Conclusion

This study extends previous research towards lower primary settings, and investigates students’ learning outcomes regarding a certain curriculum design package that targets ML. The study reveals that students held positive views of ML lessons six months after their implementation. As expected, students in lower primary settings have difficulty in spelling the mathematical terms introduced, but they can often articulate the meanings. This suggests that nurturing ML skills is broadly achievable on this timescale. A promising finding is that students in this study consider cross-curriculum learning among their interests. The students assert that in order to understand why a mathematical concept is important, there is a need to connect the ideas to their existing experiences of daily life. More importantly, they show confidence in identifying where their mathematical knowledge can be applied in wider life. The image of snowflakes links closely to the English winter and Christmas time. Therefore, students’ ability to organise their mathematics knowledge into a coherent whole with deep conceptual understanding of the nature of snowflake is potentially boosted whenever it snows. A next step could be to design several packages on ML in lower primary settings. Conducting follow-up interviews, as evidenced in this study, can provide insight into the integrated parts of the learning process.

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Appendix A Student questionnaire

SNOWFLAKE QUESTIONNAIRE

1. During the 3 lessons we did lots of different hands on activities, using different things and objects. Write down any of the activities, things or objects that you can remember:

………
………

2. The activities that you did were designed to help you understand some key ideas and vocabulary (words). Write down any of the key ideas or words that you can remember. If you can explain any of them or give examples please do so:

………
………

3. Can you remember how you felt during any of the lessons? You may have had different feelings at different times, so multiple answers are okay. Please write down what feelings you can remember:

………
………

4. Can you remember any number work or mathematics that you did during the lessons? Please write down anything that you can remember:

………
………

5. Can you describe what a crystal is?

………
………

6. Would you recommend the lessons to other students? If possible, please explain why, or why not:

………
………

Appendix B Student Interview questions

- What do you remember about the project? *Drill down into the meaning of key words- tessellation, self-similarity and lattice and their understanding of crystals*
- How did you feel about the experience?
- Have you talked about this experience with others (i.e. parents, friends, siblings etc)? If so, what did you tell them?