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Making Sense of 'Mastery': Understandings of a Policy Term Among a Sample of Teachers in England

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Abstract
This paper considers the term 'mastery' as used in mathematics education across different times and locations. A case study from England is offered to show how these pedagogical approaches morph as they move from one territory to another, in the context of each territory's history. The paper first examines English policy documents, research and published curricula for their use of the term. This suggests that 'mastery' in England has become associated with mathematics teaching practices used in high-performing territories such as Singapore and Shanghai (China). But, the efforts to transport approaches predominately from East Asian sources, against the background of an existing Western set of meanings for the term, have led to considerable inconsistency in interpretations and definitions of the 'mastery approach'. A subsequent case study of eight teachers involved in developing mastery approaches in England explores how they make sense of 'mastery' in the context of these inconsistent messages. We suggest that this generates challenges for teachers tasked with implementing mastery approaches, with the danger that anything can be done in the name of mastery.

Keywords Mathematics mastery · Pedagogy · Teaching for mastery · Variation theory

Introduction

It seems reasonable that governments and other policymakers should look to countries they perceive as more successful as sources of innovation in education policy. However, transporting policy or practice from one context to another is fraught with difficulties. Cartwright et al. (2015) discuss the complex combination of factors that need to come together to make a new intervention work in a given context. In the area

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specifically of pedagogy, Elliott (2014) argues that efforts to import teaching practices without considering the broader cultural context tend not to work, perhaps because factors supporting the original success might be absent in the new context. It is important, therefore, to examine situations in which jurisdictions have come to adopt and adapt approaches from a source context—often another territory—to their own target context. Exploring such cases reveals the range of influences impacting the translation. This paper offers the current example of 'mastery', in the context of England's mathematics pedagogy, as a case study of such translation.

In England, 'mastery' has come to stand for a key policy intervention associated with mathematics teaching practices from territories perceived as high performing, such as Singapore and Shanghai. The issue of how some primary teachers responded to the push for 'mastery' in England has been explored in Clapham and Vickers (2018), who focused on teachers' views of 'borrowing', using mastery as an example. The two core messages identified were concerns about 'mastery as policy', focusing on teachers' worries about a policy 'parachuted in' to their classrooms, and 'mastery and culture', focusing on teachers interpreting this borrowed policy as a challenge to respect for learning, for teachers and for human rights. However, Clapham and Vickers' (2018) starting point was policy which advocated 'dropping mastery "lock, stock and barrel" into England' (p. 799) viewing it as a well-defined approach to teaching mathematics that can be lifted from source context and replicated exactly in a target context. The present paper examines the extent to which policymakers, curriculum designers and researchers were indeed coherent in their definitions of mastery, what aspects of the source context they captured and how a group of secondary teachers made sense of 'mastery'. Moreover, while the focus of Clapham and Vickers was borrowing policy (which could apply to source-to-target translation involving any discipline), our focus is specifically on mathematics teaching, albeit that we cannot remove the wider educational context. That is, while Clapham and Vickers focused on the issue of borrowing mastery as a policy, we focus on the term 'mastery' itself.

The paper is in two parts. The first examines the routes through which England has arrived at the current meanings of 'mastery', the meanings associated with it at its sources and the ways in which policymakers, curriculum designers and researchers in the target context have made sense of it. The second looks at how a specific set of teachers involved in implementing mastery approaches in their schools make sense of the term and the ways in which those front-line interpretations fit (or fail to fit) the policy, curricular and research interpretations.

The Route to Mastery in England

Many large-scale international assessments in recent years (Organisation for Economic Co-operation and Development [OECD], 2010, 2014, 2016) place the UK nations[1] close to global average achievement levels in Mathematics, Science and Reading, consistently below some Asian territories such as Shanghai and Singapore. Despite

[1] The UK consists of four constituent nations (England, Wales, Scotland and Northern Ireland). Some education policy is undertaken at the UK level, but most is devolved to individual nations and many elements can be different. The discussion in this paper generally refers to the situation in England.

substantial concerns about international comparisons (Stronach, 2009), many governments use them to justify often radical policy shifts (Wiseman, 2013). After a 2014 tour of Shanghai, UK government officials observing mathematics teaching practices concluded this apparent performance gap was linked to differences in teaching methods (Department for Education, 2014). Shortly before this, in around 2012, the influential Ark group was developing a new mathematics curriculum called Mathematics Mastery, which adopted ideas from the Singapore curriculum. The adoption of the term 'mastery', referring to the general idea of pedagogical approaches from apparently successful East Asian contexts, appears to have grown from this.

The government in England set aside £41 million in 2016 to fund approaches under the 'mastery' heading (Department for Education, 2016b). Ofsted (The Office for Standards in Education, Children's Services and Skills: the government's schools inspectorate for England) endorsed the mastery approach in 2016, stating that 'what their school is doing as it delivers/moves towards a mastery approach' was an inspection focus (Maths Hubs, n.d.). That is, implementing mastery approaches effectively became mandatory. Schools thus faced at least three interrelated challenges to implementing mastery: understanding what 'mastery' means, deciding how it might improve their students' performance and choosing how to measure its impact.

The promotion of this mastery approach has come through the two main layers of policymaking for education in England. The Department for Education (DfE) has set 'mastery' approaches as a goal since 2014; their mathematics policy arm, the National Centre for Excellence in the Teaching of Mathematics (NCETM), has added detailed explanations of their interpretation of the approach and promoted it through their Maths Hub[2] networks.

Of course, the development of the mastery approach and the translation of features from East Asian source to English target contexts did not take place in a vacuum. Indeed, the choice of the word 'mastery' will doubtless have been influenced by existing meanings, both general uses in everyday language and its previous use as a specialised term in education.

We suggest that the current meaning of 'mastery' in English schools may be influenced by how the term's historical use in education literature interacted with its employment by Ark, the Government and the NCETM to characterise (or mischaracterise) teaching observed in some Asian contexts.

Figure 1 traces publications contained in the Education Resources Information Centre database (ERIC) that refer to both 'mastery' and 'mathematics' as a proportion of all those mentioning 'mathematics' since 1985. The use of the term virtually disappears across the 1990s before it re-emerges in the latter half of the 2000s. We argue that the two parts of the graph represent two phases in the use of 'mastery' and even two rather different meanings which have become conflated.

The initial use of 'mastery' followed the particular technical sense deployed by Bloom (1968): the demonstration of high levels of competence on an assessment task by a high proportion of the class, with those not achieving this 'mastery' level revisiting the material and being retested until they do. This notion of mastery appears to have dominated the use of the word in Western educational discourse for decades and continues to be part of the discourse. Guskey (2010) summarises the procedures of the mastery learning approach as (1) diagnostic pre-assessment, (2) high-quality group-

[2] Funded by the NCETM to reach schools in each region of England.

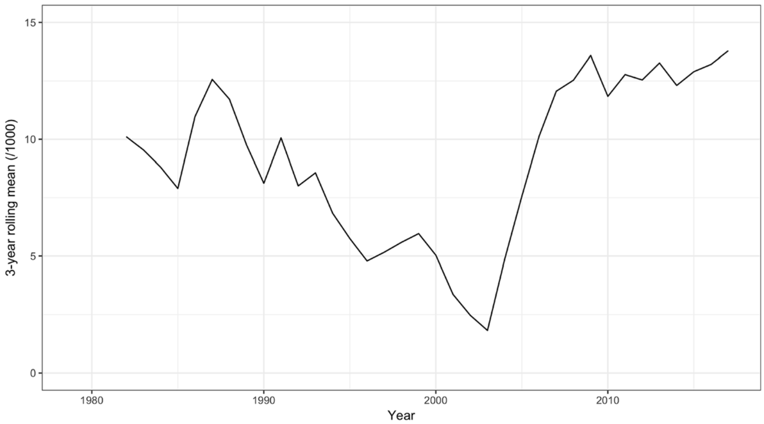

Fig. 1 Proportion of uses of mastery within mathematics education literature from the ERIC database

based initial instruction, (3) regular formative assessment, (4) high-quality corrective instruction, (5) second, parallel formative assessments, (6) enrichment or extension activities, (7) sustaining and extending success. It was into this historical but diminishing context that the association of the word 'mastery' with some East Asian methods arrived in England.

East Asian Mastery in the English Context

In this context, we look at three perspectives on how the East Asian methods have been drawn under the umbrella of 'mastery'. First, we examine the widely used commercial programme in England (from the curriculum provider Ark), giving a curriculum designer's perspective. Second, we examine the policy-level messages from four key mathematics teacher exchange research reports, which were highly influential in shaping the meaning of mastery in the English context, as well as from the NCETM: this gives a perspective from key policymakers. Third, we examine some relevant research literature on mastery in England: this gives a perspective from researchers.

Curriculum Design Perspectives from Ark

Ark is one of the largest academy chains in England, with a network of 38 academies. They are also the originators of the commercial teaching framework called the Mathematics Mastery Programme (MMP), which is now used in more than 500 secondary schools across the UK (Mathematics Mastery, n.d.). This programme has run since 2012, after a group of teachers visited Singapore. Indeed, prior to its full development under the 'mastery' title, the approach used Singapore mathematics textbooks (Department for Education, 2016a). The underlying belief is that all children from all backgrounds can succeed in mathematics. Drury (2014) explains the mastery approach from Singapore as being about fostering deep understanding using multiple representations, language and communication, and thinking mathematically. In particular, the MMP introduced the widespread integration of the 'bar model' representation into their curriculum from Singapore (Kaur, 2019).

From 2016, influenced by the increased government interest in other East Asian territories' apparent success in international comparisons, the MMP identified Shanghai as a second source of influence. The MMP was influential at both classroom and institutional levels. For the classroom, the MMP's interpretation involves six interconnected aspects: success for all, problem-solving, mathematical language, deeper understanding, mathematical thinking and multiple representations. For the institution, the curriculum package includes a 'mastery curriculum map' across primary and secondary levels, detailed lesson designs for each topic, formative assessment systems and supplementary materials tackling differentiation and the structure of weekly department meetings. Key to the Ark interpretation is the idea of assessment and, particularly, the frequent measurement of student progress. While this does not fully fit with Bloom's use of the term (there is no predetermined criterion level), a similarity is that unsatisfactory progress is met with further intervention.

Two large-scale cluster randomised controlled trials of Ark's mastery against a 'business as usual' comparison have been conducted: one in primary schools and one in secondary. Neither found statistically significant difference in progress on standardised mathematics tests (Jerrim et al., 2015; Vignoles et al., 2015), but the evaluators nonetheless gave the programme a cautious welcome.

Policy-Level Interpretations

Around the time of the development of the Ark curriculum, the government was sponsoring visits to China to learn the methods of apparently successful provinces. The first visit was to Shanghai and Zhejiang Province by 23 national leaders of education, principals of teaching schools[3] and subject specialists from primary and secondary schools in England.

The subsequent Mathematics Teacher Exchange project (MTE) between England and China involved in-service teachers visiting Shanghai to observe teaching and to participate in Continuing Professional Development (CPD) activities, along with Shanghai teachers teaching in England. The aim appeared to involve learning to adopt the Shanghai approach in English primary and secondary schools (Yuan & Huang, 2019), but naturally what gets adopted is what gets noticed in the visits and observations.

The resulting report indicated that observed secondary mathematics teaching in Shanghai generally 'focused on the understanding of concepts first and then mastery of technique' (National College for Teaching and Leadership, 2013, p. 16). The report highlights some aspects of teaching in Shanghai (often related to classroom organisation—how to organise the students), but it downplays or omits others which are perhaps core to a reified notion of a 'Shanghai style of teaching' (such as how to organise subject content). It also omits what might be a key element of the approach: *bianshi*.

The *bianshi* approach was developed by an educational reform group commissioned by the Qingpu District Education Bureau, Shanghai, and headed by Prof. Lingyuan Gu in the early 1980s (Gu et al., 2017). The aim is to deepen students' understanding of

[3] Teaching schools play an important role in their local area, supporting other schools and providing professional and leadership development across their network.

mathematics. *Bianshi* has been summarised as teaching using a combination of *conceptual variation* and *procedural variation* (Gu et al., 2017).

Conceptual variation encourages students to identify essential aspects of a mathematical concept. These aspects are normally taught by varying them individually to help students gain a deeper understanding of the concept from multiple perspectives. The method's success hinges on teachers having sufficiently varied illustrative materials that present the critical aspects in different ways, while also bringing in a variety of non-essential aspects associated with other concepts, to enable students to understand not only the essential aspects, but also how the concept is embedded in different mathematical contexts. Figure 2 illustrates conceptual variation for the concept of altitudes of a triangle, highlighting positive and negative instances, which might avert misconceptions (Gu et al., 2017, p. 17).

A procedural variation uses different kinds of tasks or activities to highlight the range of strategies that might be useful in solving a problem. It has two critical features: 'solving problems through transferring figures' and 'building connections among different types of knowledge through categorization and building a hierarchical system of categories' (Gu et al., 2017, p. 20). Figure 3 offers an example of how to use properties of midpoints to address similar problems in triangles and quadrilaterals. Teachers are encouraged to consider ways of linking these connected tasks to other topics.

Of course, the *bianshi* approach does not encapsulate all of Shanghai's mathematics teaching. It is one part of a specific learning and teaching context, different aspects of which contribute in varying degrees to the apparent success of mathematics teaching in Shanghai. When considering the current notion of mastery as a case study of the translation of policy and practice from a source to a target context, it is important to consider what aspects are identified as central to the translation.

Four evaluation reports, Boylan et al. (2016), Boylan et al. (2017), Boylan et al. (2019) and Demack et al. (2017) examined the impact of the teacher exchange programme and are important sources for understanding the policy level meaning of mastery. These highlighted what the authors saw as the essential elements of mastery, expressed in terms of promoting conceptual understanding (Boylan et al., 2016). This has been reinforced by the NCETM, noting the need for students 'acquiring a deep, long-term, secure and adaptable understanding of the subject' (NCETM, 2016b, para. 1). The reports and the NCETM also highlight key support factors for successful

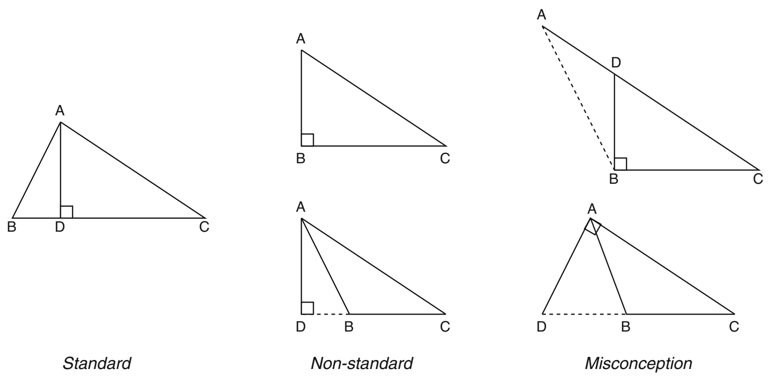

Fig. 2 An example of conceptual variation for altitudes of a triangle (Gu et al., 2017, p. 17)

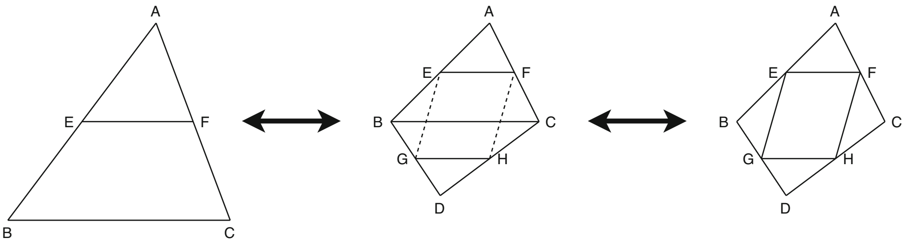

Fig. 3 An example of procedure variation (Gu et al., 2017, p. 20)

implementation: use of textbooks, resources being mathematically coherent, frequent classroom interaction, the whole class working on the same content, professional development and timetable changes.

Despite this, neither the DfE nor the NCETM offer a clear and unambiguous definition of the core elements of their interpretation of 'mastery'. This may be a source of inconsistency. Different documents highlight different aspects as core or supporting factors.

The NCETM (2014) proposed six key features: (1) curriculum design, (2) teaching resources, (3) lesson design, (4) teaching methods, (5) pupil support and differentiation and (6) productivity and practice. Two years later, the NCETM (2016a) highlighted another set of core factors: (1) a 'growth mindset' approach with an underlying belief that everyone is capable of learning the material; (2) linking procedural fluency and conceptual understanding with procedures and algorithms; (3) a high level of classroom interaction; (4) whole-class teaching with levels of support and challenge needed; (5) lessons designed with variation theory, (6) developing deep understanding, highlighting a sense of structure and connections within mathematics and (7) the use of the intelligent practice to achieve the three aims of the English National Curriculum's Mathematics Programme of Study: fluency, reasoning and mathematical thinking.

A further document from the NCETM (2017) provides yet another interpretation of mastery in the form of a pictorial representation of a model of 'Teaching for Mastery' (Fig. 4). This has five components at its core: coherence; representation and structure; mathematical thinking; variation and fluency. This diagram includes one element—variation—with apparent connections to *bianshi*.

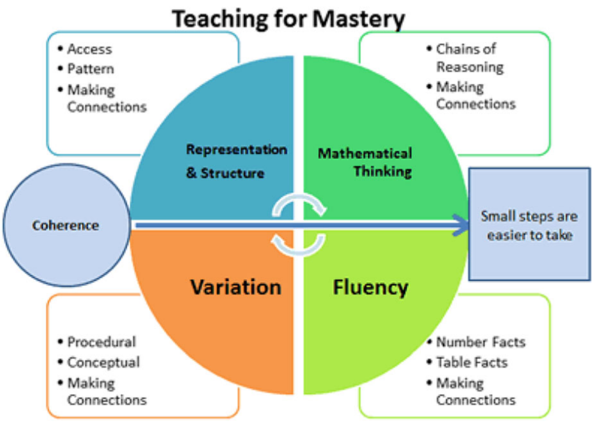

Fig. 4 Teaching for Mastery by the NCETM (NCETM, 2017)

Across these policy messages, then, there are inconsistencies and complexities—but no simple message about what mastery means. Sometimes, it refers to a curriculum, sometimes to understanding and sometimes (perhaps in a nod to Bloom's meaning) to proficiency. Aspects considered core to the meaning vary: for example, whole-class teaching, which is prominent in the reports and in one of the NCETM documents, is missing from other models.

Research Perspectives

In academic research too, there is evidence of different and non-overlapping definitions. Again, one issue is that different researchers have studied different approaches and phenomena under the heading 'mastery'.

Brown (2017) argued that the absence of a widely accepted definition of mastery in mathematics has led to confusion in data collection and thus potentially problematic findings. For example, Al-Murani et al. (2019) consider mastery to be that all pupils should learn core mathematical concepts together, with appropriate extra support. In contrast, Knowles (2017) summarises three essential elements of mastery: (1) deep understanding through stages: concrete, pictorial and abstract (CPA); (2) problem-solving and (3) success for all with whole-class teaching. From here, the core elements have been interpreted as the same pace being kept in the classroom, deep understanding, problem-solving ability and belief in all students, while the subsidiary elements include the use of representations, extra support and whole-class teaching.

Some research has also investigated the Singapore approach—again under the umbrella 'mastery' title—mainly through considering textbooks. For example, Hall et al. (2016) identified mastery with reliance on multiple representations of mathematics concepts, the CPA approach. Boyd and Ash (2017) explained mastery at the level of lesson structure: (1) beginning with an anchor task, such as exploring, (2) moving on to structuring and reflecting and then (3) refining and practicing.

There is also research related to mastery distinguishing different kinds of academic performance. For example, Mouratidis et al. (2018) highlight students viewing as improvement in understanding, rather than as outperforming peers, as in a standard performance approach.

Mastery as an overarching title also appears in larger-scale summaries of research. For example, the EEF teaching and learning toolkit rank 'mastery learning' as one of the most effective approaches. This is based on aggregating data from meta-analyses and single studies. However, under this one heading, they combine pre-2000 studies in which 'mastery' relates to Bloom's work and evaluations of the MMP which relates to Singapore-derived notions. Yet, as the argument above suggests, these two meanings bear little resemblance to each other. Care needs to be taken in aggregating research involving the title 'mastery' when it admits many potentially disjoint interpretations.

In summary, there is a wide variety of interpretations in documents from policymakers, the curriculum designer and academic researchers. This naturally leads to the question of how teachers construct their meaning of 'mastery' and how they contextualise it in normal practice. The second part of this paper explores this for the case of a small number of teachers and examines how their interpretations fit, or fail to fit, with those discussed above.

A Case Study of Teachers Making Sense of 'Mastery'

The Archimedes Maths Hub and the Hermitage Academy (one of the Teaching Schools in North-East England) commissioned the authors to lead a CPD project involving the mastery approach, with input from local educational agencies such as Education Durham, Durham County Council, the Further Mathematics Support Programme and local secondary schools in North-East England. The project focused on problems arising with how teachers in this professional learning community utilise the notion of mastery, with reference to particular mathematical concepts. It ran for two years: the first year focused on Key Stage 4 and the second on Key Stage 5.[4] In this project, teachers were exposed to how several topics (such as vectors and quadratics) are taught in Shanghai and England, and they were asked to collaboratively develop a sequence of lessons for their own classrooms taking account of Shanghai approaches. After one year of implementation, the teachers' perspectives on mastery were sought.

The teachers involved in the study had all encountered ideas about mastery from a variety of sources, had incentives from their schools and the wider national context to develop mastery approaches and thus were in the process of making their own meaning of mastery.

Given the research focus on meaning-making that likely differs between people, we recognised the need to collect data in depth. Analysis of data gathered from semi-structured interviews was seen as suitable here because this interview approach allows participants to expand on a core subject—in this case, what they understood by the term mastery—and allows the interviewer to probe and clarify (Wengraf, 2001).

Methodology

Sample

The eight interviewees work at four schools participating in the CPD programme. There were two key considerations when selecting participants: the role of the teachers and the role of mastery in their mathematics departments.

Each interviewee had extensive experience of teaching Mathematics in secondary schools and, at the time of the interviews, was Head of Mathematics (HoM) or Assistant Head of Mathematics (AHoM) in their school (see Table 1). These roles bring responsibility for the effective teaching and learning of mathematics, through working with members of their department. They can also influence change in mathematics teaching and learning at the school level and so the meaning they make of key curriculum concepts can be critical to the teaching of mathematics across the school. The individual interviewees participating in the CPD programme who were responsible for delivering the learning outcomes to the department are indicated with an asterisk in the table. It is important to note that schools A and C followed the MMP for their Year 7 and Year 8 cohort during the project period, and this may influence their interpretations.

[4] Key stage 4 is for students aged 14–16 years; Key stage 5 is for those aged 16–18 years. Key stage 5 is also referred to as 'College' or 'Sixth form', with students usually studying fewer subjects in more depth.

Table 1 Demographic information of participants (names are pseudonyms)

	School A	School B	School C	School D
Visited Shanghai schools?	Yes	Yes	Observed the Shanghai teaching style through local Maths Hub	Observed the Shanghai teaching style through local Maths Hub
Using MMP?	Yes	No	Yes	No
HoM	*Abbie (F)	*James (M)	Vic (F)	Peter (M)
AHoM	Dan (M)	*Lee (M)	*Helen (F)	*Sarah (F)

Data Collection and Analysis

All interviews were conducted by the second author at each interviewee's workplace. Interviews began with an open-ended question: 'What does teaching Mathematics for mastery mean to you?' The conversation was steered towards the key issues of adopting/adapting the mastery approach for their classroom and their use of materials or understandings from the project. Interviewees' ideas were allowed to emerge freely, with the interviewer encouraging them to expand and clarify. Each interview, lasting approximately 30 min, was audio recorded.

Data analysis followed a grounded theory approach (Cohen et al., 2013), with interviews transcribed and natural units of meaning generated. These were open coded to develop categories and find themes within the data until the themes appeared saturated. Three interviews were chosen for the first author to check transcription quality and repeat the processes of generating units of meaning and coding. These were compared to identify commonalities and differences, and the codes were clarified. Finally, using these agreed codes, the remaining transcripts were coded.

Results

Summary

Table 2 summarises the analysis of individual interviewees' understandings of mastery. The analysis identified features of the interpretation of mastery which were then classified according to the extent to which they were coherent with the features identified in the policy, curriculum and research (PCR) above. These broader classifications were:

1. Features of mastery presented in many interviews which were consistent with the PCR analysis ('understanding', 'professional development' and, to a lesser extent, 'monitoring and assessment')
2. Features in the interviews which were absent from the PCR analysis ('didactic contract' and 'teachers' confidence in stretching students')
3. Features of the PCR analysis interpreted substantially differently by the teachers ('fluency' and 'whole-class approach')
4. Features of the PCR analysis absent from the interviews ('everyone is capable', 'representation and structure' and 'mathematical thinking')

Table 2 Subject leads' overview of mastery

	School A		School B		School C		School D	
	Abbie	Dan	James	Lee	Vic	Helen	Pete	Sarah
Aspects interpreted consistently with the PCR analysis								
Understanding	C	C			C	C		C
Continuing professional development (CPD)	C		C	S	S	C		
Monitoring and assessment			C		S			
Problem-solving	C			C	C	C	C	
Curriculum and resources	S	S		S		S		
Bianshi/variation						C		
Mathematics language								S
Aspects absent from the PCR analysis								
Didactic contract	S	S			S		C	
Teachers' confidence in stretching students		S				S		

Aspects interpreted distinctly from the PCR analysis: Fluency and whole-class approach

Aspects absent from the interviews: Everyone is capable, representation and structure, and mathematical thinking

For each teacher, we identified the properties of mastery discussed in their interview and distinguished those we felt they saw as *necessary* factors for mastery (core factors, labelled 'C') and those that might be associated with it or might *support* the delivery of mastery (subsidiary factors, 'S'). The absence of a symbol in a cell is an indication that the theme was not apparent in that interview. That may not mean that the teacher did not have a view on that theme or that it played no role in their thinking, just that it was not sufficiently at the forefront of their thinking for them to express it.

We look first at those aspects which are consistent between the teachers and the PCR analysis, then at aspects which are distinct (that is, seen differently or absent from one or other perspectives). In what follows, we do not cover each element separately: many overlap in the responses (for example, 'understanding' and 'problem-solving' were often discussed together). We focus on what stood out from the data as interesting or distinctive in relation to the interpretation of mastery.

Understandings of Mastery Consistent with the PCR Analysis

Some elements of the implicit definitions of mastery given by different interviewees are also present in the range of interpretations in the PCR documents discussed in the first part of the paper. While only one participant mentioned 'variation' as a core element, aspects such as problem-solving, curriculum and resources, and language were more common. Therefore, we focus here on 'understanding', 'professional development' and 'monitoring' as common core facets to teachers' meanings for mastery, given the particular role for them, which emerged from the data.

Understanding. Four of the eight participants placed significant emphasis on students' *depth of understanding* when asked to explain the meaning of mastery. Their views revolved around changing the priorities guiding the learning process. Abbie explained:

> I think on one hand, it's trying to develop more that students actually understand why they are doing something… rather than just rote learning something… But then, alongside that, once they have understood something, I think it's developing the ability to apply that in lots of different situations.

Abbie's sense mirrors the distinction between relational and instrumental understanding of mathematical concepts (Skemp, 1976), and she sees relational understanding as the basis of developing problem-solving. She implies that the mastery approach helped her discover the relation between the two. These perceptions regarding the nature of understanding emerged as a crucial point in interviewees' appreciation of mastery. Many noted the role of understanding in facilitating problem-solving.

While Abbie and Helen recognised *understanding* as the foundation for mastery teaching, Dan illustrated this further by discussing the different ways of approaching a topic to deepen understanding:

> It's just developing a deeper understanding of the topics so that [students] can deal in a number of different ways with the questions, and if they've embedded the total understanding by going backwards and forwards between different methods, then they'll have a better chance of problem-solving, questioning, reasoning; they will be able to improve. It's a deeper understanding of how a particular concept works [that] is our goal.

While one may argue that Dan recognises aspects of variation ('going backwards and forwards between different methods'), it does not come across strongly as a definitional element. Nonetheless, his responses resonate with key ideas in variation theory, especially procedural variation. It suggests some resonance between the policy-level discussion about mastery teaching styles and pedagogy among at least some of these teachers.

These interviewees' responses show their reflections on mastery are intimately linked to the balance between procedural and conceptual understanding and problem-solving abilities. As the extracts above illustrate, this view of mastery plays an important role in guiding or shaping their teaching plans towards the depth of understanding.

Continuing Professional Development. In the first part of the paper, we saw curriculum designers (but not policymakers) pointing to the need for CPD in implementing mastery in the classroom. Here, interviewees had mixed views about CPD in relation to subject knowledge and pedagogy. On the one hand, teachers reflected thoughtfully on the relationship between their current pedagogical approaches and the collaborative work to develop pedagogy undertaken in the MMP:

> I think it was really interesting… having the time to sit down and talk with other teachers, just about how they teach something, because we don't tend to do that very often… It's having time outside of when you're actually teaching, one to

> plan the lesson effectively, but also to have time to sit with others, and bounce ideas off each other. (Abbie)

The usual practice in teacher meetings at the interviewees' schools does not include time for teachers to talk about issues in their subject area, such as activities for improving teaching:

> we don't as a department very often sit and talk about Maths. We do for Year 7 and Year 8 because we have the Mathematics Mastery [the MMP commercial programme package] meetings to talk about maths, but it is very seldom that as a whole department for Key Stage 4, we sit down and talk about actual maths, we talk about logistical things like marking... (Helen)

On the other hand, Vic and Lee were worried about not having more time for planning. Vic was concerned that mastery 'takes more planning time', something not available with his existing workload. They viewed CPD as a subsidiary element, while planning with colleagues or individually was seen as more important. This matches both the message of policymakers and the view of the commercial designer on how to facilitate mastery.

Increasingly, schools and mathematics departments are moving towards the notion of mastery-CPD, leading to the elaboration of mastery approaches. For example, school B, which does not use the MMP, had staffing resource issues at the time of the interview, but 'the CPD model, that is where next year our development will be looking'.

Of course, these participants were undertaking a mastery-focused CPD programme at the time so the emphasis on CPD may be an artefact of this. Nonetheless, that many of them tied CPD so strongly to their sense of mastery suggests that it plays either a core or support role for them. Moreover, while CPD is not an essential element of either the policy or research perspectives discussed above, it is at least a support element from the curriculum perspective.

Monitoring and Assessment. Again, this element was mentioned in curriculum design documents. The idea is that teachers and students are naturally keen to know the effectiveness or the outcome of what they are doing. How can the schools include mastery in a holistic process, rather than seeing it as an isolated element of a classroom activity?

To monitor learning outcomes, two schools modified their normal practices. All students in school B were entered together for the 'higher' exam paper[5] aiming to better monitor their progress. The HoM in this school, who had previously visited Shanghai, was confident this change would lead to positive outcomes, reflecting the mastery principle of an underlying belief that all students can achieve in mathematics:

> We band our sets. I know in Shanghai things are far more – completely – mixed together. We band slightly, so we have our most able students, half of the year group, in two parallel sets…But ultimately [it is] with the aim [that] – after two or

[5] In England, students' academic performance process is split into higher or foundation tiers, with schools deciding who to enter into each tier according to ability. Prior to their transition to a mastery approach, standard practice in school B was to enter students for both tiers.

> three weeks – they've covered around about the same thing... And we can see where students are doing well, and where some students aren't doing so well, and we can have a little bit more of a discussion with class teachers…So being able to compare as many different sets as possible is an advantage for us. (James)

Vic emphasised frequency of assessment as central to embedding mastery in Year 11: 'they [students] can see the improvement here… [T]hey can see the grades going up, the total going up. So, they can see the difference, and I can see how many people are moving slowly ….'

These interviewees both spoke to the importance of assessment in their understanding of mastery, the results of assessment being used to motivate students' learning and teachers' planning. However, this academic performance-focused view is clearly a core element from Bloom's sense of mastery. Although it is not consistent with what more recent policymakers have promoted for mastery, the commercial curriculum prioritises assessment. The implication is there is a gap to fill regarding the quantitative evidence teachers have access to about mastery's effectiveness in their school, and how to measure it compared with other approaches used in their schools.

Understanding of Mastery Absent from the PCR Analysis

Didactic Contract. Interviewees talked about trying to build on mastery's successes in East Asia but feeling hampered by implicit assumptions about what is and is not acceptable in the classroom and the roles played by the various actors therein. Some aspects of mastery teaching observed by the interviewees might be seen as breaches of implicit, local and culturally specific didactic contracts (Brousseau, 1997). Abbie commented:

> with the mastery in Shanghai, say, if students aren't on track, they'll come back after school or you'll catch up with them during school, or they'll get a tutor or whatever to help with it, so then you can… be reassured that, OK they might not have got it that lesson, but they'll have gone away and tried their best to get back on track with everyone else. But that's not going to happen in England so how can you actually…?

Peter was also highly aware of different wider educational landscapes playing a part: 'in this culture, it seems to be more our responsibility to make sure they can access it'. Teachers expressed the idea that mastery involves the support of a wider classroom culture, but it is worth noting that addressing local, culturally contextualised didactic contracts when introducing new approaches was not in the PCR documents analysed in the first part of the paper.

Teachers' Confidence in Stretching Students. Participants described their apprehensions about stretching students through mastery; they were afraid of students losing interest and of testing their own faith in the can-do attitude the NCETM advocated. Lee spoke of a positive experience in a challenging lesson about vectors with a Year 11 top set, where the class struggled when the material was introduced, but quickly improved: 'I think that was a really important moment for me, just throw it at them: then [allow students to experience] confusion'.

This example highlights the critical point that fostering deep understanding requires teachers with the confidence to embrace initial difficulties and deal with their own anxieties. Helen recalled similar feelings when encouraging students to tackle challenging questions: 'whereas before, I was a little bit frightened to let them get stuck, because I thought you are a bad teacher'.

One element the NCETM proposed is the belief that 'all can do mathematics'. This seemed to be focused on encouraging a different view of lower achieving students. However, teachers reported another consequence: struggling to stretch higher achieving students.

Understandings of Mastery Distinct from the PCR Analysis

Several elements raised in the first part of the paper were absent in the interviews. For example, no teacher highlighted the importance of representations, mathematical thinking or the viewpoint that everyone is capable of learning as important facets of their understanding of mastery. Other elements from the interview study, while recurring across responses, were either seen differently by the teachers in comparison with the PCR analysis or were absent from that analysis.

Fluency. The NCETM consider procedural fluency to be a core element of mastery. However, Vic and Helen (both at School C) said 'mastery is depth of understanding, but mastery is not fluency'. School C had been implementing the MMP for 2 years, and these teachers' understandings of mastery also came from using this programme. By reflecting on how the lesson was structured by the MMP, teachers found it difficult to make sufficient progress in practice.

In response, the teachers adapted lessons to include periods of practice (e.g. 10 to 15 min at the end of sessions). To overcome this perceived drawback to mastery (as they understand it) in the longer term, they proposed to the school's senior leadership team (SLT) a policy of adjusting homework schedules. The existing policy was to set 1 h per week, and the HoM, Vic, decided to break this into 15 min per school day, to better consolidate students' understanding.

> I told the kids 'why not… print this out, put in your planner, do ten questions or fifteen...?' So they all agreed, but I need to put this in front of the staff… in front of the SLT… because the… parents will complain about the homework.

Fluency is one of three aims of the English National Curriculum in mathematics. Procedural fluency and conceptual understanding are often mentioned in the same sentence, but the teachers' concerns reveal that classroom teaching involves balancing depth of understanding and practice in a way not consistent with the PCR analysis in the first part of this paper.

Whole-Class Teaching Approach. The whole-class teaching approach is regarded as an important subsidiary element by the NCETM and the DfE. However, this idea does not seem fully incorporated in reality, and it certainly confused some interviewees. Sarah raised concerns relating to her experience of observing a teacher from Shanghai:

> I don't know whether when I observed the Shanghai teacher, in her lesson I wouldn't be comfortable if somebody had finished and was just sat there waiting for everybody else to finish and get to that point.

Dan highlighted similar concerns regarding the challenges of keeping students together at the same pace, comparing this aspect of mastery unfavourably with non-mastery principles of managing different students' working pace:

> I suppose, at the other end of the scale, if you've got three different topics going on and everyone can cope with what they're doing and it's working them at the right ability, then maybe they're going to be more focused.

The teachers clearly noted whole-class teaching as a way to keep all students working on the same content, and this is a core element of the Shanghai approach. But, it is not a core element from the NCETM, for example. Also, teachers did not feel whole-class teaching is possible to implement.

Discussion and Conclusions

In England, 'mastery' has been promoted as a promising method for transforming mathematics teaching and learning. In the first part of this paper, evidence of how mastery came to be translated from East Asian source contexts, through the routes of policymakers, curriculum designers and researchers (in the context of previous technical meanings of the term), suggests the mathematics education community has no consensus about the meaning of 'mastery'. The sheer number of aspects of mastery (even, at times, promoted by the same institution) makes it difficult for teachers to discern a consistent meaning. The use of 'mastery' could point to the borrowing of a policy from Shanghai following the teacher exchange project: the implementation of a particular programme/textbook such as the MMP with roots in Singapore or a pedagogical shift towards 'Teaching for Mastery'. These different activities, among others, may have affected the way secondary schools understand mastery approaches, and where their focus has fallen in their own context. Indeed, the idea that 'mastery' has been 'borrowed' whole—transported from a single source context to a target—is contradicted immediately when one sees that the same term is used for ideas taken from at least two sources (Singapore and Shanghai) which do not obviously share a common pedagogical approach. For example, the bar model has come from Singapore (and is not present in Shanghai) and variation appears to come, to some extent, from Shanghai's notion of *bianshi*. In fact, it is not obvious that the use of 'variation' in some

PCR documents does overlap precisely with *bianshi*, and there may again be only a partial transfer between contexts.

The second part of the paper explored how teachers make sense of mastery in this confused and confusing situation. This case study focused on a small number of teachers in a particular context, albeit ones with considerable experience and knowledge of trying to develop mastery in their schools. Before the interview, they had attended four CPD sessions which may have focused the participants' responses disproportionately on the MMP that most participating schools were using. It is also likely to have made them disproportionately consistent in their views.

However, this sample demonstrates something of the range of interpretations consistent or inconsistent with the PCR analysis outlined in the first part of the paper. Interviewees showed some consistency with core elements such as understanding, CPD and monitoring, suggesting these elements may have been well received and understood. However, on other matters, such as fluency and whole-class approaches, the picture is less clear. Imposing an approach on top of existing didactic contracts and teachers' confidence to stretch students have been seen as barriers to implementing.

While focused on the notion of mastery in the English context, the paper can be read as a case study of the difficulty of transporting approaches between source and target contexts. Effective transportation depends on clarifying aspects of the source context that should be developed in the target context; recognising the availability (or substitutability) of support factors in the target context and evaluating the influence of the history of the approach (or even approaches with similar names) in the target context. Without clarity in these areas, the translation will likely result in the kinds of inconsistencies and confusions outlined here in the case of mastery.

Satisfactory solutions look beyond simply implementing a few features of mastery that fit in most comfortably with the current circumstances, then claiming this as successful 'mastery'. Our analysis suggests ways of pursuing more ambitious targets by clarifying shared meanings.

First, it is necessary to clearly state the core elements of mastery and to recognise that the subsidiary elements might vary from school to school. The core elements should come with an explicit structure of expectations against which to judge schools' efforts. Second, as teachers consider how to deal with these core elements in their school context, CPD can help them address the issues they face, especially if geared towards helping teachers embrace external analyses that perhaps challenge their assumptions about the approach.

There may be a need for policymakers to re-evaluate what they are trying to achieve with the mastery approach now there have been several years since the first implementation. This might involve reconsidering theories of change in managing implementation—in relation to assessment, training arrangements and how teachers come to adopt and adapt new approaches in the context of their reflection upon and evaluation of those new approaches in their own context. For the mathematics education community in general, it is important to strive to understand the processes of internalisation and of how teachers' understandings develop. Otherwise, we may be in the position that virtually anything can be done in the name of mastery.

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