

# The problem of underpowered rivers

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**ABSTRACT:** This study has hypothesized that for many rivers the trade-off between flow accumulation and the decrease in slope along channel length means that stream power increases downstream and, moreover, that given the low slope angles in headwater and low-order streams, they would have insufficient stream power to erode let alone transport sediment. The study considered the stream power profile, the particle travel distances and the application of the Hjulström curve based on the velocity profile of nine, large UK catchments. The study showed that:

- i Some rivers never showed a maximum in their longitudinal stream power profile, implying that some rivers never develop a deposition zone before they discharge at the tidal limit.
- ii Particle travel distances during a bankfull discharge event showed that for some rivers 91% of the upper main channel would not be cleared of sediment. Furthermore, while some rivers could transport a 2 mm particle their entire length in one bankfull event, for another river it would take 89 such events.
- iii The Hjulström curve shows that for three of the study rivers the upper 20 km of the river was not capable of eroding a 2  $\mu\text{m}$  particle.
- iv The study has shown that for all rivers studied, erosion is focused downstream and deposition upstream. Many UK rivers have a dead zone where, on time scales of the order of centuries, no erosion or transport occurs and erosion only occurs in the lower courses of the channel where discharge rather than slope dominates – we propose these as underpowered rivers.

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**KEYWORDS:** stream power; Hjulström curve; Schumm's fluvial system model

## Introduction

We propose that sediment processes in rivers can be dominated, not by changes in slope, but by accumulation of discharge, which means that headwaters are inactive compared with lower courses of the river. The description and analysis of river channel long profiles has been a recurrent theme in fluvial geomorphology. Following Gilbert (1877), there was a general agreement that the long profile should be a diagnostic of a graded or equilibrium condition between slope and bed material transport (Clifford, 2008). Gilbert (1877) defined *grade* as a balance between erosion and deposition maintained by a process later to become known as negative feedback. For Gilbert, the assumption of a tendency for a river channel long profile to achieve grade was ubiquitous in time and space (Chorley, 2008). In the mid-twentieth century, influential papers by Horton (1945) and Mackin (1948) extended the concept from river channels to valley-side slopes, with the whole fluvial landscape viewed as a complex process-response system capable of operating in a steady state. Subsequent research

focused on the concept of grade, but whether a curve-fitting exercise or more analytical approach using sediment transport equations linking particle size, slope and discharge, Clifford (2008) notes that the graded condition remained hard to identify. In due course, the hydraulic geometry approach of Luna Leopold and co-workers (Leopold et al., 1964) provided a multivariate approach to adjustments of alluvial channel form, but less emphasis was therefore paid to the role of channel gradient per se since continuity of sediment transport would be achieved by mutual adjustment of channel planform and cross-sectional dimensions as well as by the long profile. Setting to one side studies of hillslope erosion, the in-channel focus has more recently tended towards studies of process and form at the reach scale (where reach here is defined as the river between two tributaries), and whilst a basin-scale conceptual model has emerged that low-order channels erode, middle-order channels transport and high-order channels deposit (Schumm, 1977), there has been a lack of studies of sediment transport throughout the channel system to link to the changing downstream form of the fluvial channel system.

Our purpose here is to revisit the set of conceptual models that underpin the way in which graded rivers are thought to behave in terms of erosion, deposition, storage and transport of sediment. Unlike traditional fluvial geomorphology, our purpose is not to focus on channel landforms but simply to identify zones of erosion, transport and deposition within the drainage basin system. We apply our analysis to UK catchments, recognizing that conclusions derived there may not be applicable to other climatic regions or to much larger drainage basins. Our analytical methods have been used in fluvial geomorphology for decades: the stream power profile; the travel distance of a particle; and, the velocity profile of the river used in conjunction with the Hjulström curve to predict areas of erosion, transport or deposition. As in the approach of Mackin (1948), we examine the entire channel network, not just individual reaches.

The ability of any river to erode, transport and deposit sediment is limited by its stream power. Along a river's course, slopes tend to decrease but discharge generally increases. Low-order tributaries have received much attention in the past: slopes are high, but discharge is relatively low. There has been particular attention on sediment delivery from adjacent hillslopes and on the behaviour of coarse sediments and the transition from gravel to sand channel-forming material. Less attention is paid to sediment transport in lowland, high-order channels where channel slopes are low and the channel is formed from fine-grained alluvium. Our null hypothesis is that a graded river system should reflect Schumm's three zones, with a transition from net erosion to net deposition downstream. Our alternative hypothesis is that flows in the headwaters may be too low to move sediment, even where channel slopes are relatively high; in contrast, despite the apparent dominance of deposition in lowland floodplains, it may be that here, despite the very low slopes, the high discharge and adjustments in channel width, depth, roughness and planform mean that this zone becomes the main locus of erosion and sediment transport, not of deposition. Given that stream power, perhaps alongside sediment supply, is a main control on erosion, transport and deposition, then it is the increase in stream discharge that is more important than the decrease in slope. Thus, in essence, we are testing whether Schumm's (1977) idealized tripartite model is correct in its identification of the major zones of erosion, transport and deposition within a catchment.

To test this concept, we have studied the nine catchments across the UK where there was sufficient river gauging information to estimate the stream power profile. We consider three lines of evidence:

- 1 The stream power profile – the stream power profile for chosen rivers was derived according to the method of Worrall et al. (2014);
- 2 The travel distance of a particle based upon a travel distance approach from Wainwright et al. (2008);
- 3 The velocity profile of the river used in conjunction with the Hjulström curve to predict areas of erosion, transport or deposition.

## Methods

### Longitudinal stream power and velocity profiles

The power of a river is generated from the amount of water moving down a slope, and the stream power at a point along a river channel ( $\Omega$  in W/m length of stream channel) can be defined as

$$\Omega_x = \rho g Q_x S_x \quad (1)$$

where  $\rho$  is the density of the water ( $\text{kg/m}^3$ ),  $g$  is the acceleration due to gravity ( $\text{m}^2/\text{s}$ ),  $Q_x$  is the river discharge at point  $x$  ( $\text{m}^3/\text{s}$ ) and  $S_x$  is the slope at point  $x$  ( $\text{m/m}$ ). Stream power as defined in Equation 1 has been referred to as cross-sectional stream power (Fonstad, 2003) or total stream power (Reinfelds et al., 2004). For a rectangular channel cross-section, this is

$$\Omega_x = \rho g v_x d_x w_x S_x \quad (2)$$

where  $v_x$  is the average stream velocity at point  $x$  ( $\text{m/s}$ ),  $d_x$  is the flow depth at point ( $\text{m}$ ), and  $w_x$  is the stream width at point  $x$  ( $\text{m}$ ). Note that the unit stream power is not used as we wish to examine how power increases downstream.

The velocity profile of any river can be calculated using the approach of Worrall, Howden and Burt (2014). The mean velocity of a river at any point in sub-critical conditions (Froude number,  $Fr < 1$ ) can be estimated from the Manning equation (Manning, 1891):

$$v = \left(\frac{1}{n}\right) \left(\frac{a_{\text{cross}}}{p}\right)^{\frac{2}{3}} s^{\frac{1}{2}} \quad (3)$$

where  $a_{\text{cross}}$  is the cross-sectional area of the river at point  $x$ ,  $p$  is the the wetted perimeter,  $s$  is the water surface slope and  $n$  is the Manning coefficient. It is common for the longitudinal slope profile of a river to be expressed as an exponential function of river length (Putzinger, 1919):

$$S_x = S_0 e^{-\lambda x} \quad (4)$$

where  $S_x$  is the bed slope at point  $x$ ,  $S_0$  is the bed slope at source and  $\lambda$  is a constant. At the scale of the entire river length and at steady state, it can be assumed that the bed slope is a good approximation of the water surface slope in Equation 3 (Wilson, 1994). Leopold and Langbein (1962) attempted to justify the general applicability of the exponential form. That approach has received support from Morris and Williams (1999), who have shown theoretically that streams with small sediment loads and limited lateral inflows develop exponential profiles if bed material undergoes either abrasion or hydraulic sorting during downstream transport. Given also that downstream change in grain size is frequently modelled by an exponential function (Knighton, 1980), and given that slope and grain size are closely related, an exponential profile equation is reasonable. Equation 4 can be readily calibrated for any catchment; here this was done by reference to altitudes of gauging stations.

If it is assumed that the river has a rectangular cross-sectional area, then

$$\frac{a_{\text{cross}}}{p} = \frac{dw}{(2d + w)} \quad (5)$$

where  $d$  is the river channel depth and  $w$  is the river channel width. For a rectangular cross-section, the width of the river does not vary with discharges up to bankfull, so it is only necessary to find an expression for river width change with river length.

To calibrate Equation 3 with respect to width, we used the equation of Worrall et al. (2014) which augmented the bankfull width data of Dangerfield (1999) to create an empirical equation for river width variation with catchment area.

$$w = 0.061C + 9.0 \quad r^2 = 0.73, n = 129 \quad (6)$$

where  $C$  is the catchment area ( $\text{km}^2$ ). Based on data from the UK Flood Studies Report (NERC, 1975), it is possible to predict that:

$$l = 1.75C^{0.54} \quad r^2 = 0.77, n = 129 \quad (7)$$

River channel depth, the other component of Equation 5 and therefore Equation 3, will vary with flow, and we propose the following form of equation:

$${}^f d_x = {}^f d_m - \beta e^{\left(\frac{x}{\gamma}\right)^\delta} \quad (8)$$

where:  ${}^f d_x$  is the depth at exceedance flow  $f$  (e.g. 10% exceedance) at river length  $x$  (m),  ${}^f d_m$  is the depth of the river at the monitoring point  $m$  for exceedance flow  $f$  and  $\beta$ ,  $\gamma$ ,  $\delta$  are constants, where  $\beta$  approximates to  ${}^f d_m - {}^f d_0$ .

This approach assumed that the flow, even at bankfull, is sub-critical, i.e. that the Froude number ( $Fr$ )  $< 1$ , and that wave velocity has not enhanced stream velocity. Worrall et al. (2014) have already shown that this is a reasonable assumption for many UK rivers across many flow states, but given the data and approaches used above, it was possible to check this assumption by calculating the  $Fr$  profile for each river. For a channel of rectangular cross-section, the velocity condition to exceed critical  $Fr$  is

$$v_x > \sqrt{gd_x} \quad (9)$$

where the terms are as described above. When  $Fr > 1$ , the one-dimensional (1-D) Saint-Venant equation was used:

$$\frac{\partial Q}{\partial t} + v_c \frac{\partial Q}{\partial x} = 0 \quad (10)$$

where  $v_c$  is the wave speed or celerity (m/s). Note that this approximates to the Manning equation at sub-critical velocities. The Saint-Venant equation was solved using a kinematic wave assumption and an explicit finite difference approach (Novak et al., 2010) with the relevant section of each river discretized into 1 km lengths.

Where observations of discharge at high flows are available for a catchment, the discharge change with channel length can also be taken as a Weibull function (Weibull, 1951; Worrall et al., 2014):

$${}^f Q_x = {}^f Q_m - \epsilon e^{\left(\frac{x}{\theta}\right)^u} \quad (11)$$

Equation 11 can be calibrated against observations of river depths at gauging stations for a given exceedance flow.

The above approach means that the longitudinal profiles of velocity and stream power profile can be estimated and calibrated. Only the bankfull discharge flow regime was considered, as this would be the most powerful state in the river for which erosion and transport might occur. Some previous studies have used the median annual flood (Jain et al., 2006; Barker et al., 2009), which has a return period of 2 years. Nixon (1959) found that 29 English rivers were at or exceeded their bankfull discharge between 0.1% and 2.9% of the time; i.e. overbank flow would be occurring between 1 day every 3 years and 11 days per year. The latter then becomes the return period of bankfull discharges on a river. Bankfull discharge has been equated with the effective, or channel-forming, discharge (Hey and Thorne, 1986; Biedenharn et al., 2000).

## Particle travel distance

To predict the distance of particle travel ( $L$ ) for a given stream power ( $\Omega$ ), the approach of Wainwright et al. (2008) was used, so for a particle of known size

$$L = 0.728e^{(7.33 \times 10^{-3}\Omega)} e^{(-6.127D)} \quad (12)$$

where  $D$  is the particle diameter (m). The total travel distance was then considered with respect to the tidal limit for each study catchment, based upon the stream power profile derived for that catchment at bankfull discharge and for  $D$  between 2  $\mu\text{m}$  and 2 mm. The distance travelled relative to the tidal limit means that it is possible to calculate the point along the river channel ( $L_{dz}$ ) from which, on the bankfull discharge, an entrained particle of set grain size would be transported to the tidal limit and so out of the river system:

$$L_{dz} = L_{tl} - L \quad (13)$$

where  $L_{tl}$  is the river length to the tidal limit (km). The river length  $L_{dz}$  can be considered to be the river reach from which a particle would not be removed from the river under conditions of bankfull discharge and, therefore, the zone of the river where a particle would take at least two episodes of bankfull discharge to leave the catchment.

## Application of the Hjulström curve

Finally, given the information estimated for each catchment, the velocity profile could be compared with the predicted behaviour of particles based upon the (Hjulström, 1939) curve. The estimated velocity profile at bankfull discharge was compared with the multinomial approximation to the Hjulström curves calculated by Worrall et al. (2018) dividing, firstly, deposition from transport and, secondly, transport from erosion. The equations were applied for grain sizes between 2  $\mu\text{m}$  and 2 mm, and then, given the bankfull stream velocity predicted for each reach for each of the study rivers, each reach of the river could be classified as depositing, transporting or eroding for a given particle size. Note that for 2  $\mu\text{m}$  there is predicted to be no deposition for any flowing water.

## Study catchments

The choice of study catchments is dictated by the availability of gauging stations within the catchments for which data were available to fit the above equations. As this study used the method of Worrall et al. (2014), study catchments were chosen if they had five or more river gauging stations on their channel. Unlike Worrall et al. (2014), this study also chooses to include tributaries for which this criterion was met. The details of the nine catchments selected are given in Table 1, and locations are shown in Figure 1. Some of these rivers sourced in peat-covered headwaters typical of western and northern Britain with moorland and rough pasture (i.e. Tees, Severn, Trent-Dove, Tweed, Clyde, Spey and Wye). Conversely, the other catchments (i.e. Thames, Trent-Soar and Bedford Ouse) are solely covered by mineral soils and more intensive land uses (e.g. arable farming). However, the basins concerned are sufficiently large that they transition across different soils, land-use and are underlain by both permeable and impermeable geology. To check Equation 4 was a reasonable for these sites, the longitudinal profile of the chosen rivers was measured

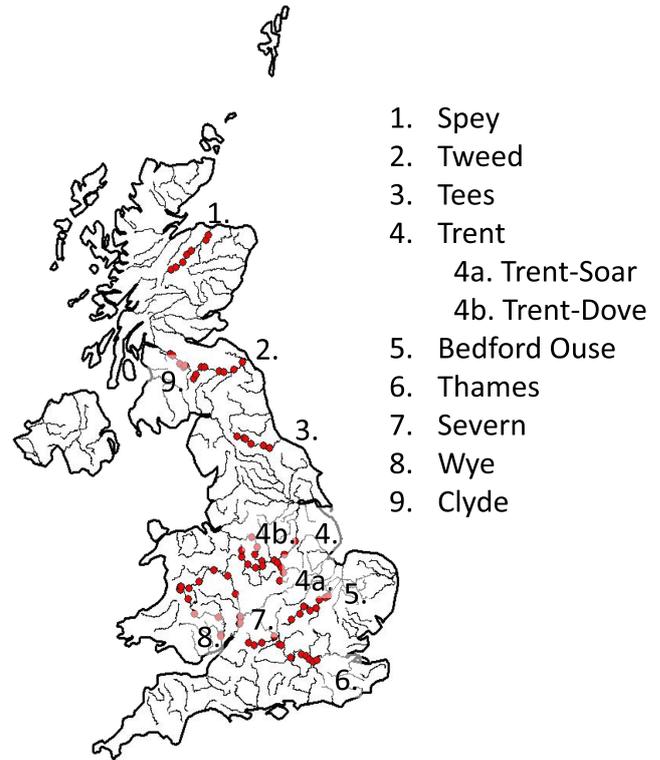
based upon measuring the position of each 10m contour along the length of the study rivers. Based upon Radoane et al. (2003), four simple empirical approaches to describe river longitudinal profiles were used (linear, exponential (Equation 4), power and logarithmic, Supporting Information). Results for example rivers are detailed in the supporting information and show that the exponential approach was the best fit for all but one river, where the linear function was a better fit.

## Results

The critical Froude number ( $Fr$ , Equation 9) was exceeded at bankfull discharge in only three out of the nine study rivers. For the Tweed, the critical value was only exceeded in the last 1 km before the tidal limit for a river 141 km long, and so this super-criticality was ignored. For the Severn,  $Fr > 1$  was only achieved for bankfull discharge for a 4 km stretch between 3 and 7 km from its source out of a length of 273 km. However, for the Tees,  $Fr > 1$  was exceeded at bankfull discharge for the upper 14 km of the river channel, and so Equation 10 was applied on the River Tees. At present we do not seek to generalize the observation that even at bankfull discharge there is no contribution from wave celerity to the flow of most UK rivers.

## Stream power profiles

The graphs of the stream power profiles at bankfull discharge show a wide variation in the stream power (Figure 2). The stream power profile shows two types of behaviour: one set of catchments where  $\Omega$  increases all the way along the main channel length, and a second set where  $\Omega$  reaches a maximum and then declines. Graf (1983) and Knighton (1984) predicted that there would be a maximum in stream power rather than a continuous increase to the tidal limit. Maxima in longitudinal stream power profiles have been observed by Abernethy and Rutherford (1998), Knighton (1999) and Reinfelds et al. (2004). Barker et al. (2009) studied the longitudinal stream power profile of five UK rivers (all different from those studied here) and found that two showed stream power increases to the tidal limit, two showed a maximum in stream power, and one showed a more complex pattern with multiple maxima. Bizzi and Lerner (2015) found maxima in the longitudinal stream power profile of two further UK rivers. This study only used simple descriptions of the longitudinal profile of rivers, and a more realistic representation of the stream power profile



**FIGURE 1.** Location of the study rivers and each river gauging station used on those rivers. [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

through more accurate topography may show that the stream power is not a smooth function but could include local maxima and local erosion may occur.

The conditions for whether or not a catchment reaches a maximum  $\Omega$  within its catchment can be considered by formulating Equation 1, combining with Equation 4 and assuming that  $Q$  can be expressed as a function of  $x$ ,  $Q(x)$  (just as in Equation 10), then

$$\Omega_x = \rho g Q(x) S_0 e^{-\lambda x} \quad (14)$$

Differentiating with respect to  $x$  shows that a maximum in  $\Omega$  will occur if the following condition is met, if there is some distance  $x$  along the channel where

**Table 1.** The flow and catchment characteristics of the rivers chosen for this study. The details are for the lowermost gauging station on that river. Names in *italics* are for tributaries, and in these cases the main channel length is given for the length of the tributary

River	Exceedance flow			Catchment characteristics			
	10	50	95	Area (km <sup>2</sup> )	Main channel river length (km)	Max. altitude (m asl)	Rainfall (mm)
Tees	5.5	13.8	34.9	818	79	885	1,141
Thames	25.4	151	444	9,948	239	329	706
Severn	47.5	96	208	9,895	272	827	792
Trent	22.1	44.4	73.5	8,231	155	634	747
<i>Trent-Dove</i>	28.6	9.86	3.61	883	72	546	935
<i>Trent-Soar</i>	25.38	7.17	3.44	1,386	95	151	641
Bedford Ouse	63.8	181	584	1,460	127	237	636
Tweed	13.1	27.2	61.6	4,390	140	838	955
Clyde	7.2	17.5	34.2	1904	112	745	1,129
Spey	11.6	18.5	34.6	2,861	151	1,303	1,120
Wye	18.9	43.2	105.8	4,010	224	749	1,011

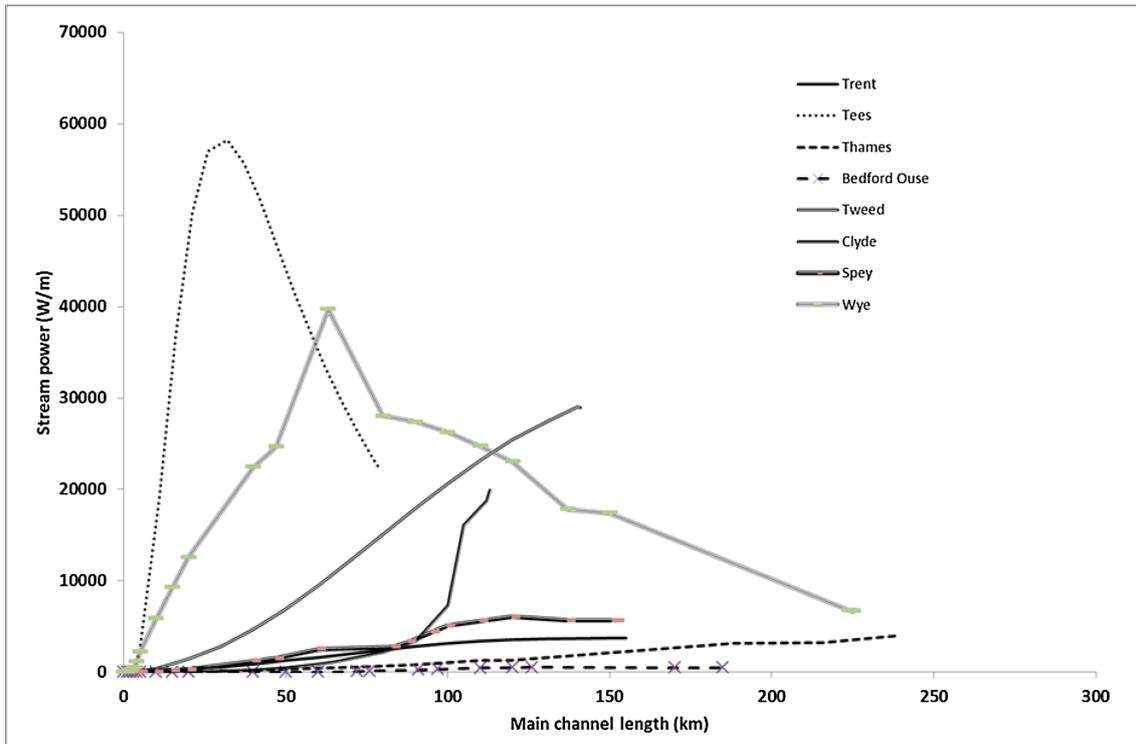


FIGURE 2. Stream power profiles for the study rivers. [Colour figure can be viewed at wileyonlinelibrary.com]

$$\frac{Q'(x)}{Q(x)} = \lambda \quad \text{with } x < L_{tl} \quad (15)$$

where  $L_{tl}$  is the main channel length from source to tidal limit (km). Given the simplest case where discharge accumulation is linear with distance (i.e.  $Q = kx$ ), then a dimensionless criterion for a maximum in stream power can be derived as:

$$L_{tl}\lambda > 1 \quad (16)$$

This predicts that a maximum in stream power will occur if the river is long enough or if the slope decline is rapid with respect to the length of the river. Equation 16 can be simply tested against the study catchments, and indeed in all cases it correctly classifies those that show a stream power maximum from those that did not – confirming the approach taken. If a maximum in stream power exists, then it is reasonable to predict that the greatest potential for erosion is at the point of maximum stream power, and in all the cases considered this was not in the upper portion of the river but in its lower portion. Furthermore, a stream power maximum means that there is a point in the river’s lower course where stream power is declining, and this may also mean stream velocity is declining, and therefore sites of deposition in the lower course could exist, otherwise stream power rises to tidal limit implying that deposition occurs in estuaries beyond the tidal limit.

### Particle travel distance

This study has hypothesized that the current conceptualization of rivers means that it should be possible for a river to transport a reasonably sized particle along its entire length during bankfull discharge. Therefore, we would expect the travel distances predicted by Equation 12 to be longer than the main channel length. For the range of grain sizes considered by this

study ( $D = 2 \mu\text{m}$  to  $2 \text{mm}$ ), Equation 12 is not sensitive to variation in  $D$  and so a single value of  $L_{dz}$  (Equation 13) can be reported for each of the study catchments. The distance  $L_{dz}$  varied from  $<1 \text{km}$  (Severn) to  $170 \text{km}$  (Bedford Ouse) river channel length, which is between 0.3% and 91% of the river channel length (Table 2). Therefore, for the Severn, approximately the entire river length is cleared out every bankfull discharge, whereas for the Bedford Ouse it is not. However, the next question would be how long would it take a particle to leave the catchment from within the  $L_{dz}$  region? Assuming a  $2 \text{mm}$  diameter particle enters the river at its source ( $x = 0$ ) and can be entrained whenever a bankfull discharge event occurs, it is possible to count how many bankfull discharge events it would require for this particle to leave at the tidal limit. For the Severn it is 1 event, but for the Bedford Ouse it would be 89 events.

As an alternative approach to understanding whether rivers have sufficient power to transport sediment, it is possible to consider what critical values of stream power would be required for a particle to be transported along the entire main channel of a major UK river. For UK rivers, main channel lengths are typically greater than  $100 \text{km}$  (Table 1); setting

Table 2. Estimated  $L_{dz}$  for the study rivers estimated from particle travel distance and the values of  $S_0k$  for each study river

River	$L_{dz}$ (km)	$1/S_0k$
Tees	4	2.7
Thames	89	549.5
Severn	1	0.5
Trent	35	10.4
Trent -Dove	68	1,783.5
Trent-Soar	51	186.9
Bedford Ouse	179	1,008.5
Tweed	20	14.9
Clyde	60	2.3
Spey	40	126.8
Wye	4	9.8

$L = 100\text{km}$  in Equation 12, then  $\Omega > 650\text{W/m}$ . Equation 11 shows that discharge accumulation is linear for low values of main channel length ( $x < 50\text{km}$ ), therefore Equation 14 could be adapted, assuming that  $Q(x)$  approximates to a straight line for values of  $x < 50\text{km}$ :

$$650 < \rho g k x S_0 e^{-\lambda x} \quad (17)$$

where  $k$  is the discharge accumulation rate ( $\text{m}^3/\text{s}/\text{km}$ ). It is possible to estimate values of  $k$ ,  $S_0$  and  $\lambda$  for each of the study catchments to assess what controls the channel length at which the river has sufficient power to erode, such that a bankfull event would clear particles. To assess critical values of  $S_0$ ,  $k$  and  $\lambda$  at which stream power would exceed  $650\text{W/m}$ , values of  $S_0$ ,  $k$  and  $\lambda$  were chosen, at random, from within the ranges defined by those values for the study catchments. The distribution between the ranges was assumed to be uniform, then values of  $W$  were calculated for values of  $x$  between 2 and 25 km. Combinations of  $S_0$ ,  $k$  and  $\lambda$  that resulted in  $\Omega > 650\text{W/m}$  were distinguished from those combinations which gave  $\Omega < 650\text{W/m}$  using logistic regression. This approach shows that the following is true whereby a dead zone of length  $L_{dz}$  will occur if:

$$\frac{0.0738}{S_0 k} > L_{dz} \quad (18)$$

The value of  $e^{-\lambda x}$  is approximately zero for low  $x$  within the range of  $\lambda$  found for the study catchments.

Alternatively, rather than considering the bankfull discharge, we could find the flow at which  $L_{dz} < 1\text{km}$  for a study catchment and compare this with the highest recorded flows for the gauging sites on that river. For the River Trent, the flow required for  $L_{dz} < 1\text{km}$  to do this was 16 times the bankfull discharge. However, the annual maximum flow series for the most upstream gauging station on the Trent included in this study shows that the largest flow ever recorded was 5.8 times the bankfull discharge (the highest recorded flow on the River Trent had a return period of 56 years by the Gringorten formula) and even at this flow,  $L_{dz} = 20\text{km}$  for the River Trent. For the River Thames the same could be achieved at 8.7 times bankfull discharge, but the largest ever flow recorded in the 132-year flow record for the Thames was 8.6 times the highest annual maximum flow (return period of 236 years), and so even assuming that the lowest annual maximum flow represents bankfull discharge means that the  $L_{dz}$  on the Thames exists for periods of centuries. For the Bedford Ouse, the flow required to reduce  $L_{dz}$  to 0 was 31 times the bankfull discharge whereas the largest flow recorded for the river was 13.5 times the lowest annual maximum flow and this had a return period of 75 years, i.e. the  $L_{dz}$  would be stable for centuries.

## Hjulström curve

The above approach based on particle travel distance assumes that the particle is entrained or can be readily entrained. By plotting our data on a Hjulström curve, the analysis could be extended to consider whether the velocities predicted would entrain particles or just transport them. It is important to consider the zone within the Hjulström plot marked out by transport alone, as we have to consider the possibility of a powerful tributary joining the main channel, which brings particles entrained in the flow of a size that could not be entrained by the flow conditions in the main channel but could be transported by that flow. However, Barker et al. (2009) did observe increases in the stream power of a main channel when

it was joined by a tributary. Equally, it may not be a tributary but a waste discharge that introduces particles into the flow. Considering the velocity profile of each study river and particles from  $2\ \mu\text{m}$  to  $2\text{mm}$ , we find that all rivers would be capable of transporting a  $2\ \mu\text{m}$  particle anywhere along their length, but for three rivers (Trent, Bedford Ouse and Clyde) there was insufficient stream velocity to cause erosion of such a particle for at least  $20\text{km}$  from their sources. A zone not capable of eroding a  $2\ \mu\text{m}$  particle within  $20\text{km}$  of its source means that, for these three rivers, there is only erosion of a  $2\ \mu\text{m}$  for as little as 73% of the river channel length. For the Spey, Thames and Tweed, for a  $2\ \mu\text{m}$  particle, there was river channel erosion within  $3\text{km}$  of the source and, given the lack of gauging information in the upper course, we can assume that these rivers are capable of eroding a  $2\ \mu\text{m}$  particle along their entire main channel length.

When a  $2\text{mm}$  particle was considered, only the River Severn was capable of transporting the particle all the way along its length and eroding it anywhere along its length at bankfull discharge (Table 3; Figure 3). For the River Wye this was also true for 96% of its main channel length. However, in contrast, the Bedford Ouse cannot erode a  $2\text{mm}$  particle anywhere along its entire length and furthermore cannot even transport it for the upper 30% of its length. The Hjulström plot and the use of Equation 16 give a similar picture, i.e. that for the rivers in this study, erosion is focused in the lower course of the river, with some rivers having an upper course where there is insufficient power or stream velocity to transport or erode sediment.

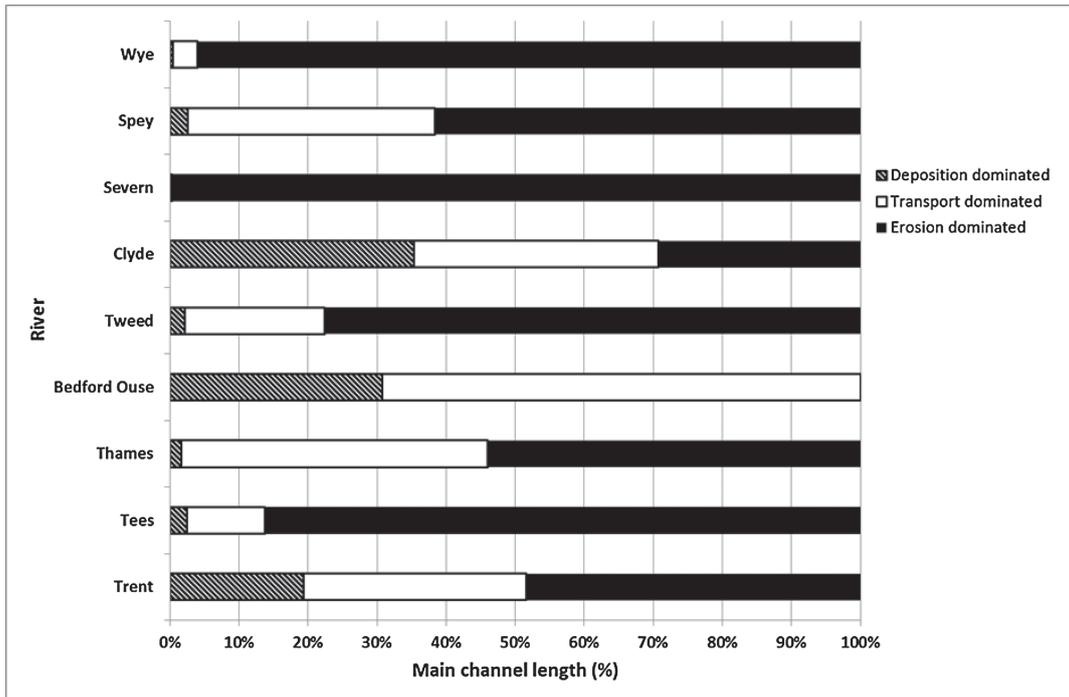
## Discussion

### Towards a new model of rivers

It is possible to propose a conceptual model of the behaviour of the underpowered rivers in the UK where stream power is dominated by flow accumulation rather than slope (Figure 4). The river is underpowered as it cannot erode or transport sediment for much of its length over sustained periods of decades. This definition is distinct from that of underfit streams, as that term refers to the planform and cross-sectional profile of river valleys with relation to their valleys, whereas underpowered refers to longitudinal profile and development. The upper course is now the stagnant or dead zone where there is insufficient slope on the channel and insufficient flow accumulation for either erosion or transport of typical particles. This zone must be dominated by in-channel storage, by virtue of its low power. And, if

**Table 3.** The estimated main channel length dominated by depositional, transport or erosion processes for a particle of  $2\text{mm}$  based upon use of the Hjulstrom curve

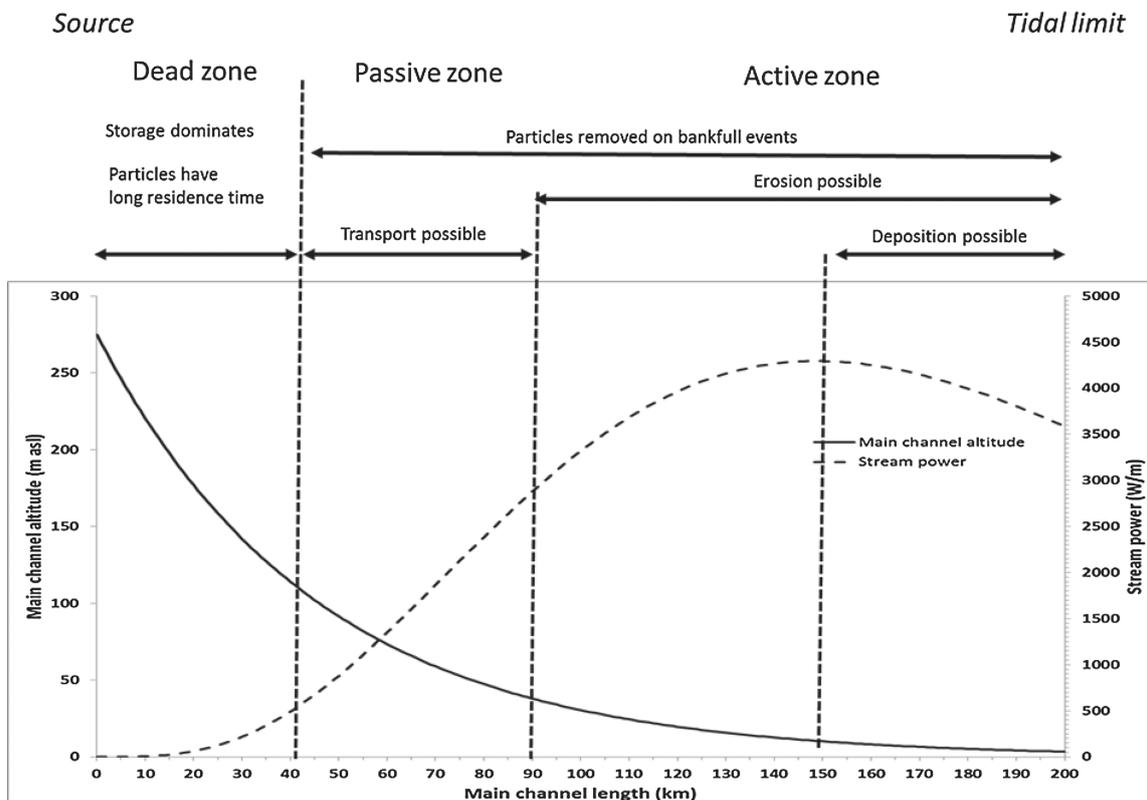
River	Main channel length (km)	Deposition	Transport	Erosion
Tees	79	2	9	69
Thames	239	4	106	129
Severn	272	1	0	272
Trent	155	30	50	75
Trent-Dove	169	11	47	101
Trent-Soar	122	20	47	56
Bedford Ouse	127	60	135	0
Tweed	140	3	28	107
Clyde	112	40	40	33
Spey	151	4	56	96
Wye	224	1	8	216



**FIGURE 3.** The proportion of the main channel length that is dominated by deposition, transport or erosion for a 2 mm diameter particle based upon Hjulstrom curve.

it is not to be clogged by sediment, then it may also be poorly connected to its catchment. The dead zone in such rivers can be stable for centuries. There is a transition to a zone where particles may be transported though entrainment may be limited; this can be viewed as a passive zone; i.e. entrained particles may be transported across it on timescales of bankfull events. Only in the lower courses of the river is there sufficient stream power for erosion and transport to occur – we term this

an active zone. In this active zone, particles are transported through it on bankfull events and erosion is also taking place, but we cannot comment upon whether that erosion is horizontal or vertical. It is possible that stream power reaches a maximum and therefore that there is deposition in the lowest portion of the river prior to flowing through the tidal limit. Conversely, the rivers that have attracted focus are those where there is no dead zone (e.g. Severn) and that events on the time



**FIGURE 4.** Conceptual model of an underpowered river with an exponential decline in long profile of the river showing where the dead, passive and active zones would be relative to the river’s ability to erode, transport or deposit particles.

scales of bankfull events can clear the entire length of the channel, i.e. there is no persistent dead zone.

Nanson and Huang (2017, 2018) describe some river regimes as being overpowered where a river has excess energy that will be expended on erosion that lessens the stream gradient as meanders develop. Nanson and Huang (2017, 2018) describe the converse situation where a river has insufficient energy, and Nanson and Huang (2018) describe such rivers as 'underpowered systems'. The underpowered case is characterized by a river having insufficient energy to transport its sediment and so deposition will occur leading to aggradation and progradation. Indeed, Nanson and Huang (2018) give possible examples of this occurring in deltas or the formation of braided rivers, however, they also suggest that 'If such low-energy rivers are sinuous then they can straighten and increase their energy gradient' – a curious breach of the second law of thermodynamics. The underpowered case proposed by the studies of Nanson and Huang would correspond to the case above where a maximum in stream power exists and deposition would occur downstream of this point. However, such a definition implies there is sediment supply upstream to be transported into a reach where energy gradient is lower and deposition can occur. But this study also shows that for many rivers and for large stretches of rivers there simply is not enough energy to erode or transport sediment and so there would be little or no supply to the lower reaches of the river. Further, this study shows that if anything the longitudinal erosion pattern is reversed and lower reaches are more likely to be erosive.

Our research highlights contrasts in the rate of transport of particles along river channels of major rivers. Initially, this study looked to consider events based upon timescales of years and not decades or centuries; in fact, the study has considered timescales of the recurrence of the bankfull discharge, which is one to six years in this case. The study has not considered flooding conditions as that may slightly increase velocities and would certainly increase discharge, but we do not have sufficient information or constraints on flooding conditions in any of the rivers to consider this facet in any detail. It would not be reasonable in the catchments considered in this study to predict that flooding would occur along the entire length of a river in any one flood event, and indeed this is also true for the bankfull conditions. The assumption of this study that bankfull conditions occur along the entire length of any of the study rivers is a conservative one. The recurrence interval of the flow conditions we have considered is in fact longer than just bankfull conditions at any one of the sites, and so our prediction of movement is an overestimation.

The results presented here could be dismissed on the basis that floods, by definition events with a greater discharge than a bankfull discharge, would do the work of the river, i.e. flood events would generate sufficient stream power even in the headwaters for the stream to be erosive and transport those particles the length of the river. However, examination of the annual maximum flow series for the rivers studied here showed that the flow needed to create erosion within the headwaters of the catchments had never occurred, even within a 132-year flow record as observed for the River Thames. Secondly, time between flood events on any part of the river would be expected to occur on decadal timescales, and decadal timescales may be a long time for the reactive components of the suspended particulates such as particulate organic matter (PM), part of which is particulate organic carbon (POC) – Moody et al. (2013) found that between 38% and 83% of peat-derived POC was lost over a 10-day period. Furthermore, reliance on flood events to do the work can lead to a 'jerky conveyor belt' approach (Ferguson, 1981) with sediment being a long time in temporary, in-channel stores, which could mean

the switch of allochthonous to autochthonous carbon dominating the POC flux. Worrall et al. (2018) have shown that POC is not fractionated into the highest flows, i.e. the proportion of the suspended sediment concentration that is POC decreases with increasing flow because POC is concentrated in the finer fractions and not the coarse fraction that would be mobilized on the higher flows. Furthermore, concentrated sources of POC such as sewage outfalls are less flow dependent than erosive sources or in-channel stores.

Considering bankfull events predicts that some rivers will have a zone where, on the timescale of the recurrence of bankfull events, there is no erosion and no transport; this zone is essentially dead. In a dead zone the lack of transport or erosion power means that in-channel storage is dominant when sediment is supplied to the stream, but it may also be that these streams, or zones, are poorly connected to sediment sources and so are in effect receiving no sediment inputs. Residence times of organic particles entering this zone will be long, possibly many decades, and as suggested above there will be a transfer from the importance of allochthonous to autochthonous sources with processing in the stream bed. Furthermore, the source of sediment within these zones (including particulate organic matter, POM) cannot be erosion from the channel or stream banks as this study has shown there to be insufficient stream power for erosion to occur; rather any sediment must be supplied by the surrounding slopes or is autochthonous.

There will be a point downstream where on bankfull events the flow accumulation and slope are sufficient that an entrained particle will be transported, yet there is insufficient power to erode particles within the channel. In this zone particles could be supplied by tributaries or by point source discharges in this zone. However, there could be substantial in-channel storage. In sluggish rivers there is an active zone where there is sufficient power and stream velocity during bankfull events to cause erosion, but it will always be focused on the lower course of the river. Nevertheless, there will be some rivers where this pattern is not the case and there is sufficient slope and/or flow accumulation for erosion to be possible along the entire length of the main channel during bankfull events. For all rivers studied, it was possible that stream power reached a maximum and therefore, as stream power declines, the possibility of deposition arises. This latter prediction would be comparatively easy to test as depocentres could be mapped down the profile of a river where a stream power maximum was predicted to see whether depositional features existed.

The results of this modelling approach do provide at least two testable outcomes: Firstly, that for some of the study rivers there is a point along their length where deposition will occur (e.g. Tees) while in others there is essentially none. Furthermore, the position of this deposition centre is predicted and therefore, the presence and nature of the deposition centre could be tested. Secondly, the approach also predicts where erosion will begin and so surveys of erosion features should coincide with the river lengths predicted in this study. Bizzi and Lerner (2015) did find an association between geomorphic features and a stream power maximum in two UK rivers.

Is the UK peculiar? We have been constrained by the rivers with sufficient gauging station information, but we might consider that UK rivers are rather short and that for longer rivers the condition  $L_r \lambda > 1$  would always be met and so a lower course where deposition was occurring could exist. However, it would be difficult to argue that the lowland rivers of England are exceptional and could be readily compared to many of the lowland rivers of Western Europe and especially those without headwaters in the mountain belts, e.g. the River Seine. Alternatively, the longest river in the UK is 280km and so the

results here would be true for rivers sourced in many wide coastal plains, e.g. the Atlantic coastal plain of the eastern United States.

The study has chosen to consider data only from main channels from rivers and not tributaries. It is perfectly possible that tributaries are steeper than the main channel – but this is not necessarily true, and indeed the River Trent is a good example where the main channel is sourced at 324 m asl, the River Dove is sourced at 546 m asl, whereas for the River Soar it is only 151 m asl. When the particle travel distance is considered, then for the River Dove the  $L_{dz}$  is 60 km and for the River Soar  $L_{dz}$  is 20 km compared with 35 km for the main channel of the River Trent (Figure 1). Considering the Hjulström curve, then for the River Dove a 2  $\mu\text{m}$  particle transport dominates for the upper 3 km of the river and erosion for the rest. When a 2 mm particle was considered for the River Dove, then storage dominates for the upper 11 km of the Dove, transport for the middle 47 km and erosion was possible along the final 101 km. This means that within the main channel of the Trent, on a bankfull event, the passive zone of the main channel could be transporting particles supplied by at least one northern tributary. In contrast, for a 2 mm particle in the River Soar, the upper 20 km is storage dominated with the middle 47 km dominated by transport and the lower 56 km dominated by erosion. Critically, this means that the River Soar is still transport dominated when it meets the main channel of the Trent, and so this southern tributary contrasts with the Dove flowing in from the north. Barker et al. (2009) have noted step increases in stream power at confluences.

A limitation of this study is that in the approaches taken it was only suspended sediment transport that was assumed to be occurring, and therefore, that bedload transport was negligible and that sediment supply was not limiting. In the relatively low-gradient rivers of the UK, bedload flux is likely to be a small proportion of total sediment flux. Foster and Walling (1994) measured bedload flux to be 21% of the total sediment flux for a lowland grass-dominated catchment, while Labadz et al. (1991) found a proportion up to 14% of the total sediment load for eroding peatlands in Northern England. The presence of bedload transport would use a proportion of the available stream power, limiting that which was available for suspended sediment transport. However, the method used here estimated the total stream power available and then we considered how that stream power could erode, transport or deposit, and if there was insufficient power to erode or transport suspended sediment of 2  $\mu\text{m}$  in size, then the amount of energy for bedload transport would have been negligible. This study has assumed that bedload transport in these rivers is not the mechanism which is controlling the river morphology of the majority of the river course. Equally, if there was no sediment supply then there would be transport, but again this study has shown that for much of the course of these rivers there is simply not enough power to do either erosion or transport and so no sediment supply would make no difference.

## Conclusions

We would contend that:

- For many UK rivers, the upper course of the river is marked by inactivity with insufficient discharge and slope to generate sufficient stream power to transport sediment, let alone erode. With low-channel slopes the processes within the channel are dominated by flow accumulation and, therefore, erosion and transport will be focused into the lower course.

- If there are actively eroding and transporting headwaters, then they are orders of magnitude below presently gauged catchment areas and even in the steepest rivers this would amount to only a few kilometres of channel length. Nevertheless, further work in such headwater catchments is needed, to acquire relevant field data to allow the approach presented here to be applied in low-order basins.
- We can define a dead zone of a river where there is insufficient stream power to erode any particles and insufficient power to transport these particles out of this zone on a bankfull event, and this zone must be poorly connected to the surrounding area.
- The dead zone is defined by stream powers less than 650 W/m of channel and for a 5 km long dead zone, defined by  $S_oK < 0.013 \text{ m}^3/\text{s}/\text{km}$ .
- There is a passive zone in which stream power is sufficient to transport particles but not to erode them.
- We can define the active zone as being the length of the river for which a 2 mm particle could be eroded and would be removed from the basin at bankfull discharge.
- In some rivers, stream power peaked before reaching the tidal limit, which means that deposition would be possible. For a stream power maximum to be reached, then  $L_{\mu\lambda} > 1$

We conclude that many UK rivers can be classed as underpowered and have had inactive headwaters even for many centuries. For suspended sediment, the channel network divides between sections where in-stream storage dominates to the lower course where sediment residence times will last from one bankfull event to the next. This means that for sediment there will be long sections of the river which act to change allochthonous particulates (e.g. particulate organic matter, POM) into autochthonous POM.

## Conflict of Interest Statement

The authors declare no conflicts of interest.

## Data Availability Statement

The data sets used in this study are publicly available from the sources quoted in the manuscript. The data sets used and/or analysed during the current study are available from the corresponding author on reasonable request.

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## Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

**Table S1.** The fit of functions to the slope of three example rivers.

**Figure S1.** The longitudinal profile of the River Trent with four functions fitted to the altitudinal profile.

**Figure S2.** The longitudinal profile of the River Tees with four functions fitted to the altitudinal profile.

**Figure S3.** The longitudinal profile of the River Spey with four functions fitted to the altitudinal profile. Although the longitudinal profile is best described by a single linear function, it does appear that two exponential functions would be better.