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## Simulation of High Precision Process Control for Set-Up Dominant Processes

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### Abstract

The main focus of this paper is to use discrete-event simulation models, to test the robustness of two process control methods against processes with different statistical distributions. The two methods under scrutiny are the Small-Batch  $\bar{X}$  & R chart and the Set-Up Process Algorithm (SUPA). These have been developed for ‘setup dominant processes’, where the major source of product variation is detected between batches. Minimizing this type of variation is critical to ensure spare parts produced at a later date will fit in operating assemblies, maintaining a Through-life Engineering Service. This paper shows their suitability to industry.

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### 1. Introduction

In order to maintain the availability of high value capital equipment, it is necessary for manufacturing firms to produce spare parts at short notice. High value capital equipment is usually made in low volumes and can often be in service for in excess of twenty years. This means production of original and spare parts takes place in small batches with large time periods between production runs. These types of processes are known as ‘set-up dominant’ [1]. The largest cause of variation in the parts is due to set-up changes between batches.

There is an expectation that parts will be interchangeable. This requires high precision, critical-to-quality features (CtQ) that form functional-fits with other components in a final assembly. To find an economically sound solution, design engineers often use statistical tolerances on those features [2].

The cost to quality of defining statistical tolerances can be quantified by the Taguchi quality-loss function [3], Fig. 1.(a). This requires CtQ features to be produced as close as possible to the nominal specification; the further away the CtQ is from its nominal specification, the greater the cost to quality, as fewer parts will function optimally and, ultimately, will not work altogether. Fig. 1.(a) shows that the Taguchi

model’s relationship between the CtQ feature and the financial cost of quality, is a quadratic function, with the cost to quality shaded in grey. Therefore, the bygone attitude of ‘within-tolerance is good enough’, Fig. 1.(b) has resulted in spare parts being dispatched for service, but not fitting in an assembly, due to tolerance stack-up. Unfortunately, in low-volume production, this bygone attitude is still common place and being supported by process control procedures which lack statistical rigour.

Companies involved with low-volume production have been reported to be increasingly using subjective approaches to quality management [4]. It has been observed that control of machining processes, such as precision turned components,

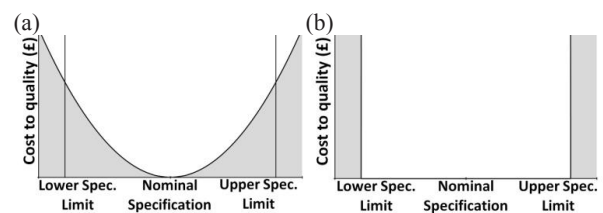


Fig. 1. (a) Taguchi quality loss function; (b) Worst-case model.

had a ‘heavy reliance on operator’s experience’ [5]. ‘Rule of thumb’ methods controlling low-volume production, have been identified as: first-offs and 100% inspection [6].

Statistical process control procedures aimed at safeguarding the consistency of CtQs in low-volume production were reviewed and presented by Cox *et. al.* [7]. This work reviewed Acceptance Control Charting [8,9], Small-Batch  $\bar{X}$  & R [6] and PRE-Control stage 1 [10], whilst introducing a new method called the Set-Up Process Algorithm (SUPA). Discrete-event simulation models were used to assess the robustness of the methods when controlling manufacturing processes that follow a Gaussian distribution. The results in [7] were improved upon in [11], where a refined SUPA method was presented and updated simulation models were used to analyse the respective process control procedures. These updated simulation models improved the accuracy of results, when compared to analytical predictions of performance. This gave greater confidence in results produced by these models, which could not be derived analytically.

It was found that having any statistically derived process control method, for monitoring ‘set-up dominant’ processes, would be an improvement on industry practices, such as first-offs [7,11]. PRE-Control offered a simple to use ‘traffic light’ system of control, based on specification limits. The main problem with this method is that its rigid control zones make it poor at centring highly capable processes.

Small-Batch  $\bar{X}$  & R and SUPA, overall, were the best performing methods of statistically derived process control. The former is a capability based approach; whereas, the latter is a ‘traffic light’ approach based on specification.

This paper will further the results in [11], by testing the two most effective statistically derived process control procedures against processes of different statistical distributions. This assessment is an important property, since a single type of part may be made on multiple machines, with inherently different statistical distributions characterising their performance. Since this takes place over the service life of the final assembly, having a consistent process control procedure prescribed during the design tolerance stage is critical to future in-service operation.

The following sections of this paper will: describe the process control procedures to be tested; present the generic discrete-event simulation model used to test these process control procedures; then the results of these simulations will be presented; finally, the key findings of this paper will be concluded.

## 2. Set-up dominant process control procedures

### 2.1. Capability metrics

The process capability metrics  $C_{pk}$  and  $C_p$  are used to quantify process variation.  $C_p$  uses process standard deviation ( $\sigma$ ) to estimate process performance against the upper ( $U$ ) and lower ( $L$ ) specification, by:

$$C_p = \frac{U - L}{6\sigma}. \quad (1)$$

$C_{pk}$  uses  $\sigma$  and process mean ( $\mu$ ) to estimate process performance against  $U$  and  $L$ , by:

$$C_{pk} = \min \left[ \frac{U - \mu}{3\sigma}, \frac{\mu - L}{3\sigma} \right]. \quad (2)$$

These metrics will be used throughout this paper to describe process performance.

### 2.2. Small-Batch $\bar{X}$ & R

A small batch  $\bar{X}$  and  $R$  chart adapts the classic  $\bar{X}$  and  $R$  chart method, to recalculate control limits as the sample size increases from one to five parts [6]. A mean ( $\bar{X}$ ) chart is used to detect if the process is off-target by a statistically significant amount. A subgroup mean is calculated by:

$$\bar{X} = \frac{\sum X_n}{n}. \quad (3)$$

If a sample subgroup’s mean falls outside the charts control limits it indicates the process is off-target. By using the nominal specification or design target ( $T$ ), historical standard deviation ( $s$ ) and subgroup size ( $n$ ), the upper ( $UCL_{\bar{X}}$ ) and lower control limits ( $LCL_{\bar{X}}$ ) can be calculated as follows:

$$UCL_{\bar{X}} = T + 3 \frac{s}{\sqrt{n}}. \quad (4)$$

$$LCL_{\bar{X}} = T - 3 \frac{s}{\sqrt{n}}. \quad (5)$$

A range, or  $R$ , chart is used to detect if there is too much variation in a process. The range,  $R$ , of a subgroup is the difference between the maximum and minimum values of  $X$ :

$$R = X_{\max} - X_{\min}. \quad (6)$$

Using Hartley’s constant ( $d_2$ ) and  $s$ , the mean range ( $\bar{R}$ ) can be estimated by [6;12]:

$$\bar{R} = d_2 s. \quad (7)$$

Finally, the upper control limit ( $UCL_R$ ) can be calculated using the statistical coefficient  $D_4$ , as follows [6;12]:

$$UCL_R = D_4 \bar{R}. \quad (8)$$

This method overcomes the issue of waiting for a complete subgroup to indicate an issue. A decision can be made on whether a process needs to be re-centred after one unit and whether there is too much process variation after two units. A significant disadvantage with this method is that it is based on the assumption that the monitored process follows a Gaussian distribution. This leads to the potential to calculate capability based control limits on subjective assumptions, engineering knowledge, or possibly scarce surrogate/historical process data.

### 2.3. Set-Up Process Algorithm (SUPA)

The SUPA method proposed by [11] is a development, or hybrid, of PRE-Control stage 1, see [10]. PRE-Control stage 1 is a method of validating a set-up, using a traffic light control chart based on tolerance and simple decision rule, see Table 1.

The decision rules outlined in Table 1, show that the measured CtQ of the sampled units are categorized as Green, Yellow or Red. To validate a new process set-up or an adjustment made to an existing process, stage 1 rules, shown in Table 1, are applied. Consecutive units are sampled from the process. If a sampled unit is Red it signals that the process

Table 1. Outline of stage 1 decision rules for PRE-Control.

Sampled Units	Observation	Action
1	Red Unit	Stop and Adjust
1 2	Two Consecutive Yellow Units Same Side of Target	Stop and Adjust
1 2	Two Consecutive Yellow Units Opposite Sides of Target	Stop and Investigate
1 2 3 4 5	Five Consecutive Green Units	Continue Process

is off-target. Two consecutive units in the same Yellow Zone signals that the process is off-target. Two consecutive units in opposite Yellow Zones signals the process variation is too great. Five consecutive Green units demonstrate the process is capable and it is allowed to continue without further checks. These decision rules are maintained for SUPA.

SUPA also maintains the use of a ‘traffic light’ control chart, based on specification, but will introduce a sliding scale Green Zone. This addition improves the centring of highly capable processes within a CtQ specification maximising the limited data. It also provides a link between statistical tolerances and process capability.

There are two types of  $\alpha$ -risk: the chance of adjusting an on-target process (“hunting”) and the chance of signalling a capable process as incapable. SUPA achieves 98% confidence for the probability of qualifying ( $P(q)$ ) a valid process. Meaning a 2%  $\alpha$ -risk (probability of not qualifying a valid process), by the fact that sampling five consecutive Green units will validate a set-up and two consecutive Yellow units initiates action [13]. To obtain a value of  $P(q)=0.98$  for different values of  $C_{pk}$ , the probability of sampling a unit in the Green zone  $P(g)$  and the probability of sampling a unit in the Yellow zone  $P(y)$  need to be used according to:

$$P(q) = P(g)^5 \frac{1 + P(y)}{1 - P(y) \sum_{i=1}^4 P(g)^i} \tag{9}$$

Based on Equation (9), the values of the look-up table shown in Table 2 are calculated.

Table 2: Look-up table of percentage green zone and minimum  $C_{pk}$  at 98% confidence.

Green Zone	$C_{pk}$
0.471353	1.333333
0.418294	1.500000
0.376396	1.666666
0.313658	2.000000
0.268850	2.333333
0.250926	2.500000
0.235244	2.666666
0.209106	3.000000

Based on the values and data shown in Tables 1 and 2, the SUPA follows the sequence:

1. Select the Green Zone limits for the required minimum  $C_{pk}$ , using the look-up Table 2.
2. Sample, Measure and classify the CtQ of consecutive units as: Red, Yellow or Green.
3. Follow the PC rules to validate a process, i.e. a Red unit signals an adjustment is needed, five consecutive Green units signals the process is valid.

The Red zones are always set at the specification limits. The final SUPA chart will have zones and limits as in Fig. 2.,

showing a SUPA chart monitoring a process requiring a minimum  $C_{pk} = 2.0$ . Given the application, where the cost of adjusting a process is not significant compared to the cost of producing out of specification units, allowing a 2%  $\alpha$ -risk is acceptable [14].

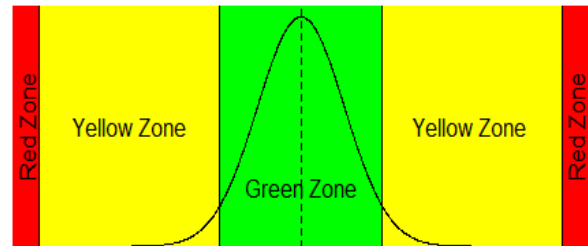


Fig. 2: A SUPA Chart monitoring a process with minimum  $C_{pk} = 2.0$  using a Green Zone of 31%.

If the process is significantly off-target, SUPA will allow quick adjustments to be made after only one or two units. Whereas, classic SPC would require a subgroup of four or five units before a change is made. Furthermore, SUPA is a nonparametric method, deriving its limits based on the specification of the product’s CtQ being monitored. The Green zone is selected based on the minimum  $C_{pk}$  required from the process, to protect the CtQ’s statistical tolerances. Selecting the size of the Green zone before the manufacturing process starts will aid the process operator to make parts as close to the nominal specification is reasonably possible. This in turn will help avoid tolerance stack-up in final assemblies. Once five consecutive units are produced in the Green zone, SUPA signals the process is valid.

### 3. Discrete-event simulation model

#### 3.1. Generic model

In order to assess the effectiveness of the different methods of validating the set-up of a process, a discrete-event simulation model was built using WITNESS 12. The model simulated a generic process applying a CtQ to a unit, which could represent a lathe machining the outer diameter of a gear. The process has a  $U$  and  $L$  of  $\pm 100$  and a process target ( $\mu_T$ ) of 200. The current process mean,  $\mu$ , can be offset at the start of the simulation. The model adjusts  $\mu$  based on the decision rules of the Control method analysed. For example, a model using the SUPA method would use the decision rules in Table 1. Capability is set prior to the simulation and remains constant throughout.

The simulation model applies adjustments by finding the mean of the units signalling an adjustment ( $\mu_A$ ). Then it subtracts the difference between  $\mu_A$  and the process target,  $\mu_T$ , from the current mean ( $\mu^i$ ) to find the new process mean ( $\mu^{i+1}$ ), i.e.

$$\mu^{t+1} = \mu^t - (\mu_A - \mu_T) \tag{10}$$

The general model can be seen in Fig. 3. At the start the experimenter sets the initial parameters of capability and process mean (boxes 1 and 2). The model is allowed to run. Units enter the model (box 3) with a generic process applying a CtQ to each unit (box 4), based on capability and process mean. The model then samples consecutive units (box 5). Based on the decision rules of the Control method utilised, a decision is made on whether or not the process is valid (box 6). In the case of SUPA, if there were five consecutive units in the green zone the model is considered validated. If a model is validated sampling immediately stops (box 7). If the model is not validated, it decides whether an adjustment is needed to the current process mean (box 8). If adjustment is not needed, sampling continues (box 5). If an adjustment is needed, the mean is recalculated (box 9) by Equation (10). The adjustment is applied to the process mean (box 2).

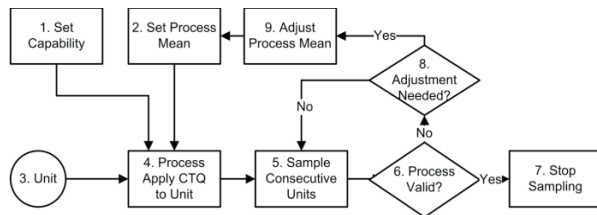


Fig. 3: Process flow of general simulation model.

3.2. Model validation

To test the validity of the simulation model, the function of adjusting the process was switched off. The simulation was stopped if it was validated, incapable or in-need of adjustment. From this the simulations estimate of confidence was compared with the statistically derived results.

Each of Small-Batch  $\bar{X}$  and  $R$  and SUPA had models built around the general framework in Fig. 3. Fig. 4. demonstrates the operating curves for the analytical and simulation results, by plotting the probability of qualifying ( $P(q)$ ) the SUPA method against a range of  $C_p$ . The analytical results for SUPA, shown in Fig. 4., are derived from the Equation (9).

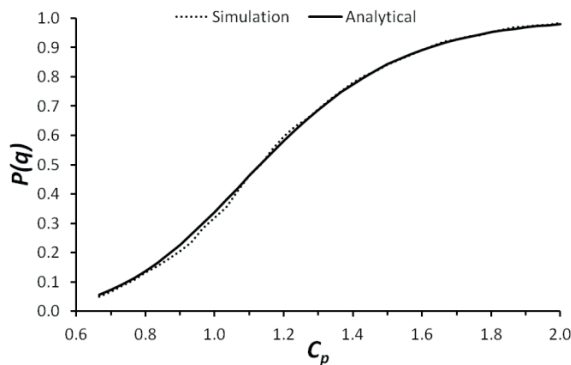


Fig. 4. Comparison of simulation and analytical operating curves for the SUPA method.

The SUPA results (Fig. 4.) are typical for the methodologies simulated. This close alignment allows credible results to be derived from the simulation.

4. Simulation results

4.1. General Result

Two control methods were tested that monitored processes with three different statistical distributions. The statistical distributions modeled were the Gaussian, Uniform and skewed Triangle distributions. In order to make the analyses comparable, the variances of the three distributions used were made equal to one another.

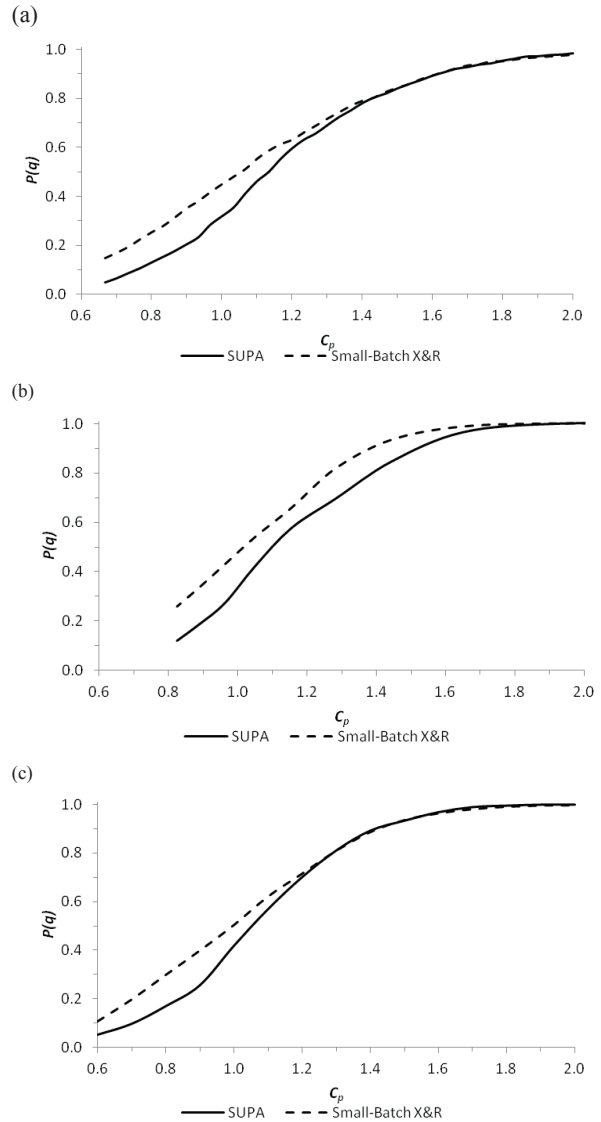


Fig. 5. Probability of qualifying ( $P(q)$ ) against the  $C_p$  of the simulated process under a centred: (a) Gaussian; (b) Uniform; (c) skewed Triangle distributions.

As Small-Batch  $\bar{X}$  and  $R$  and SUPA have different philosophies, setting small-batch  $\bar{X}$  and  $R$ 's control limits based on a historical process  $C_p=2.0$  and setting SUPA's control limits based on a required  $C_{pk}=2.0$ , allowed equivalent analyses.

In order to prevent deterministic effects, of the simulations

pseudo-random number generator disrupting the outcome, each setting was replicated 1,000 times. In these, unlike the simulation validation run, if a false adjustment signal was made, the process was adjusted and continued until the control procedure determined the process as either valid or incapable.

Two events lead to a control chart signal: a) when a process is on-target but not capable; b) when a process is capable but off-target. These two ‘out of control’ tests were applied to the three process distribution cases: (a) Gaussian; (b) Uniform; (c) skewed Triangle.

4.2. On-target, not capable

In these cases, the process had (a) Gaussian, (b) Uniform and (c) skewed Triangle distributions, whose means were on-target with respect to the nominal specification. However, the distributions’ variances were increased in each simulation, to test how effective the respective control methods were at detecting that the process was not capable. As  $C_p$  is inversely proportional to the standard deviation, when the process variation is increased,  $C_p$  is reduced.

Results for the probability of qualifying, despite adjustment, against the process  $C_p$ , are given in Fig. 5.(a;b;c) These results demonstrate that if  $C_p=2.0$ , both methods will validate the process on approximately 98% of occasions. As  $C_p$  decreases it is desirable that  $P(q)$  also decreases, i.e. as a process becomes incapable it is not validated. This demonstrates that SUPA is the most sensitive control method. If in case (a), see Fig. 5.(a), the  $C_p=1.4$ , there is still little difference between SUPA and Small-Batch  $\bar{X}$  and  $R$  validating approximately 78% and 79% of processes respectively. However, if  $C_p$  drops to 1.0, the control methods will validate 32% and 45% of process respectively. This type of separation in performance is also seen in cases (b;c) and is an important point. This indicates that SUPA is rejecting more invalid processes than the small batch method.

An important result from this test is the number of units sampled, required to make a decision. Fig. 6. plots these results under a Gaussian distribution.

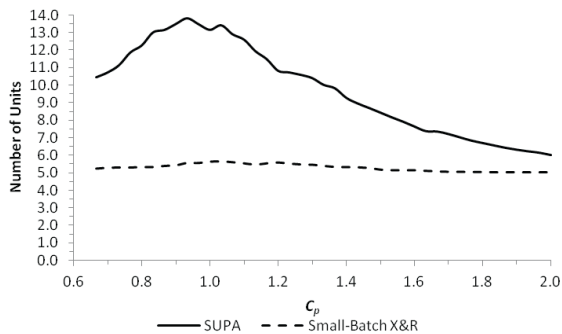


Fig. 6. Mean number of units needed to reach a valid/not-valid decision against the  $C_p$  of the simulated process under a centred a Gaussian distributions.

It can be seen that if  $C_p=2.0$ , SUPA requires a sample of 6 units on average to make a decision, whereas, Small-Batch  $\bar{X}$  and  $R$  needs 5 units. When  $C_p$  reduces, Small-Batch  $\bar{X}$  and  $R$  requires a sample of between 5-6 units; however, SUPA requires more units until it peaks at an average sample of 14

units. Similar performance profiles are also seen in cases with Uniform and skewed Triangle distributions.

4.3. Off-target, capable

The second ‘out of control’ test, was for a capable process, i.e. had a  $C_p=2.0$ , but with a process mean not on target. Fig. 7. highlights the final capability with respect to process variation and mean ( $C_{pk}$ ) against the starting position of the process mean as a coefficient of the process standard deviation. This model measures if the adjustments made to the process by the control methods were accurate.

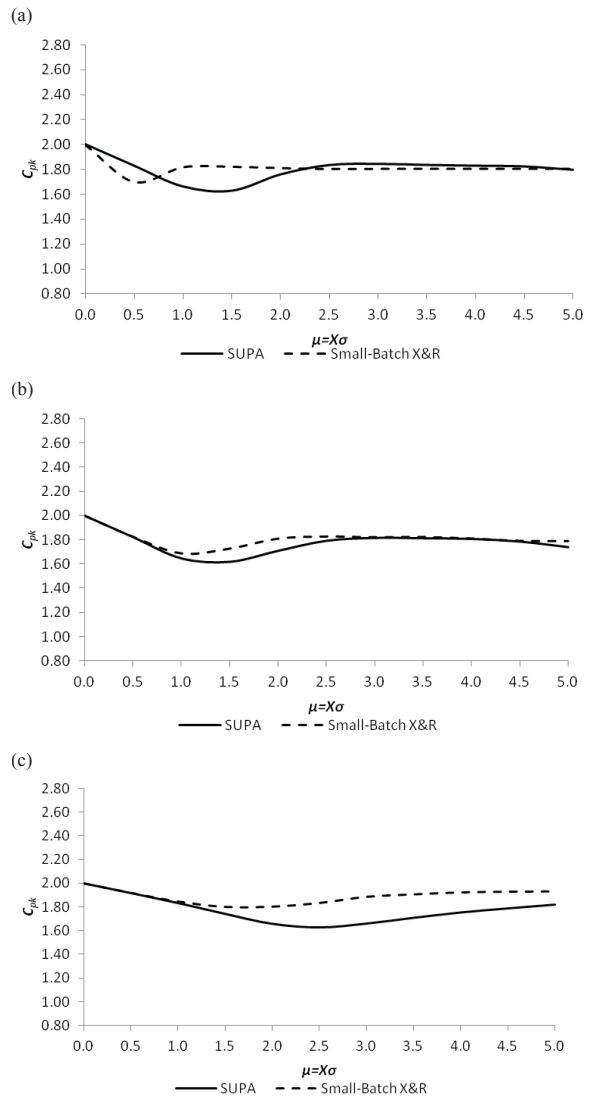


Fig. 7. Final  $C_{pk}$  of process, after adjustments, against the process mean ( $\mu$ ) as a coefficient ( $X$ ) of standard deviation ( $\sigma$ ) of a process under: (a) Gaussian; (b) Uniform; (c) skewed Triangle distributions.

It can be seen, that in case (a), see Fig. 7.(a), both control methods kept the  $C_{pk}$  above 1.6 on average, with a maximum potential  $C_{pk}=2.0$ . Both control methods have ‘dips’ in performance of similar size, with that of Small-Batch  $\bar{X}$  and  $R$  occurring at  $0.5\sigma$  and SUPA’s occurring at  $1.5\sigma$ . In case (b),

profiles are closer in performance and in case (c) the dip in performance is at  $2.5\sigma$ , see Fig. 7.(b;c).

Fig. 8. illustrates the number of units required to make a final decision for a process with a Gaussian distribution. The profiles for other distributions follow a similar trend. It was noted that the typical number of units sampled was in the range 5-8 for Small-Batch  $\bar{X}$  and  $R$  and 6-14 for SUPA.

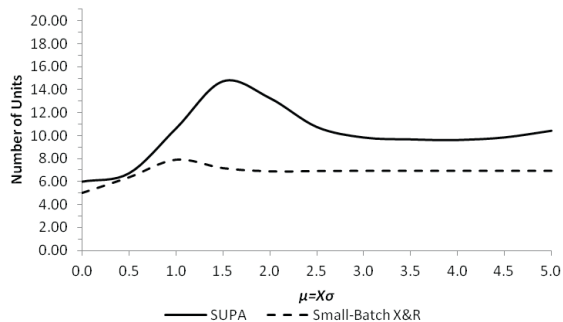


Fig. 8. Mean number of units needed to reach a valid/not-valid decision against the process mean ( $\mu$ ) as a coefficient ( $X$ ) of standard deviation ( $\sigma$ ) of a process under a Gaussian distribution.

## 5. Conclusions

This paper introduced two methods of set-up dominant process control: Small-Batch  $\bar{X}$  and  $R$  and SUPA. The former provides a statistical process control procedure, which maintains the statistical stability of a process using capability limits. Whereas, the latter is a non-parametric approach to defect prevention, with control limits and tests based on specification and derived using probability theory.

The method's effectiveness with respect to: (a) detecting an incapable process; (b) adjusting an off-target process, were studied. To investigate this, the methods were used to control a simulated process with three different statistical distributions.

Small-Batch  $\bar{X}$  and  $R$  was not as powerful as SUPA at detecting an incapable process. It was found that for the Gaussian and skewed Triangle cases, the two methods performed similarly as the  $C_p$  dropped from 2.0 to 1.4. Then, as the process  $C_p$  dropped below 1.4, the SUPA method was more powerful at detecting deterioration in capability. For the uniform case, the SUPA method was more sensitive after the process  $C_p$  dropped below 1.7. Using the SUPA control method would alert an operator earlier, than the Small-Batch method that the process was no longer capable, avoiding the risk of producing parts at the extremes of the specification or even worse that are not in specification.

The Small-Batch  $\bar{X}$  and  $R$  performs slightly better than the SUPA method at adjusting a process, whose mean is off target, closer to the nominal specification or design target. It was shown, that both methods have 'dips' in performance, that are associated with how far off target the process mean was initially. However, the Small-Batch  $\bar{X}$  and  $R$  dips in performance are typically shallower, i.e. it held or adjusted the process closer to its design target than the SUPA method.

A final consideration from the simulation results is the number of units required to make a final valid or not valid decision. It was shown in all cases that the SUPA method used more samples than Small-Batch  $\bar{X}$  and  $R$ . When the

process was capable and on-target it was shown that the difference between the two methods is marginal. However, as a process becomes less capable, or starts further from the target, the SUPA method requires many more samples to make a final decision. This can be seen as a useful feature, for the situation where the process is not capable. The SUPA method is more powerful in this situation, as a result of additional sampling when processes become incapable. Also, if the process is not capable it is more likely to produce defects; therefore, by collecting more samples it is more likely that any defective units will be captured at this stage. The high sample numbers collected by SUPA in the second situation, where a process is off target, is less desirable. It is preferable in this situation that an adjustment is made and then validated as quickly as possible.

It is also noted, that Small-Batch  $\bar{X}$  and  $R$  control method performs well against different statistical distributions. This is despite it being designed to operate with Gaussian processes.

A practical consideration, which cannot be explored by simulation, is how easy it is for the end user to implement the methods. These methods will typically be used by machine operators, who are unlikely to have a background in statistics. They are, therefore, likely to find a method which uses control limits linked to design specification easier to utilise than ones which use statistical limits.

To extend this work further, simulations of parallel processes will be built. These processes will produce parts that have CtQs that will form functional fits with one another in an assembly.

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