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Abstract. We study gauge theories arising on D-branes on quotients of the conifold. They exhibit meta-stable SUSY breaking along the lines of the model by Intriligator, Seiberg and Shih. We propose a candidate for the extrapolation to large 't Hooft coupling of the meta-stable state. It involves anti D3-branes in a smooth gravity dual of a cascading gauge theory.

Keywords: Meta-stable Supersymmetry Breaking, Gauge/gravity Correspondence.
PACS: 11.25.Uv, 11.25.Tq

INTRODUCTION

Supersymmetry (SUSY) is a leading candidate for physics beyond the Standard Model. Low energy SUSY stabilizes the hierarchy between the Weak and Planck scales. Furthermore, a large value of the hierarchy can be naturally explained in theories in which SUSY is broken dynamically.

Recent years have seen a lot of activity in the study of models with SUSY breaking in meta-stable vacua. There are two broad classes of constructions that have been explored. On one hand, there are string constructions based on anti D-branes in smooth warped geometries. The original work on this class is the KPV model [1]. In short, a small number of anti D-branes in the warped deformed conifold [2] can be trapped in a meta-stable vacuum, which decays by tunneling to a SUSY vacuum via a brane/flux annihilation transition. The second class of models is purely field theoretic. The original ISS model is based on supersymmetric QCD with massive flavors [3]. The full power of Seiberg duality is exploited to prove the existence of a meta-stable vacuum using an IR-free magnetic description. By now, there is a vast number of models in both classes, following the same logic of the original examples.

Interestingly, the KPV construction admits a dual field theory interpretation via the dictionary of the gauge/gravity correspondence. The warped throat is dual to a large N cascading gauge theory. The smooth bottom is also interpreted in field theoretic terms as representing confinement and chiral symmetry breaking. Anti D-branes in this background are then reasonably identified with some meta-stable state in the corresponding field theory.

There are various similarities between KPV and ISS states. It is natural to conjecture that some connection between the two classes of meta-stable states might hold. We can

envision that an ISS-like vacuum remains meta-stable as we continuously increase the 't Hooft coupling until it finally reaches a regime in which it admits a simpler description in terms of anti D-branes in some warped throat, a la KPV. Strictly speaking, since meta-stable states are not protected by SUSY, they need not survive extrapolation from weak to strong 't Hooft coupling or viceversa. We investigate this question in orbifolds of the conifold, which are particularly well suited for this purpose, since both the gauge theories and dual geometries are relatively simple.

This note is based on the work in [4] and [5].

THE MODEL

As explained in the introduction, our goal is to use (fractional) D-branes on a Calabi-Yau singularity to engineer a gauge theory which breaks SUSY in a meta-stable vacuum, along the lines of ISS. Furthermore, we want the geometry to be relatively simple, in order to be able to discuss meta-stability from a gravity dual perspective.

Simple non-chiral orbifolds of the conifold are sufficient for this purpose. The resulting gauge theories are derived from that of the conifold [6] via standard orbifold techniques. The $\mathbb{Z}_N$ orbifold of the conifold contains $2N$ gauge groups and $4N$ chiral multiplets in bifundamental representations. The superpotential follows from projecting that of the parent theory and reads

$$W = h \sum_{i=1}^{2N} (-1)^{i+1} X_{i,i+1} X_{i+1,i+2} X_{i+2,i+1} X_{i+1,i} \,, \tag{1}$$

where the index i is understood mod$(2N)$.

It will be sufficient for us to consider the $N = 3$ case. The corresponding quiver is shown in Figure 1.
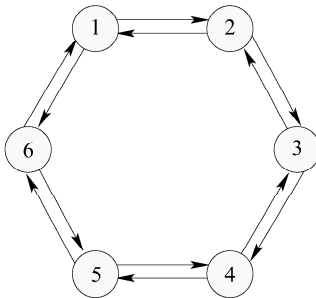

FIGURE 1.   Quiver diagram for a $\mathbb{Z}_3$ non-chiral orbifold of the conifold.

Fractional (*i.e.* D5-branes wrapped over collapsed 2-cycles) and regular D3-brane choices are mapped to anomaly free rank assignments in the quiver. Since this gauge

theory is fully non-chiral, we have complete freedom to choose the ranks of all nodes in the quiver. We are interested in the following set of ranks

$$(N_c \, , \, N_c \, , \, N_c \, , \, 1, \, 0 \, , \, 0) \, . \tag{2}$$

Following the classification in [7], the corresponding set of fractional branes can be viewed as N_c $\mathcal{N} = 2$ branes at nodes 1 and 2, N_c deformation branes at node 3 and a single deformation brane at node 4. This characterization of the fractional branes is, of course, basis dependent. Figure 2 shows the corresponding quiver diagram.
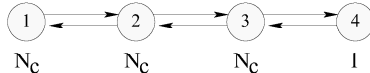
FIGURE 2. The quiver diagram for the rank assignments in (2).

The tree-level superpotential takes the form

$$W = h \left(M_{22} X_{23} X_{32} - X_{23} X_{34} X_{43} X_{32} \right) + m X_{43} X_{34} \, , \tag{3}$$

where we have assumed that node 2 is confining and defined $M_{22} = X_{21} X_{12}$. The first two terms follow from (1), while the third one is generated by a D-brane instanton as we later discuss[1].

Let us explain what motivates the choice in (2). Our goal is to construct a gauge theory that in the infrared (IR) reduces to (some extension of) the ISS model, *i.e.* SQCD with massive flavors in the free-magnetic range (probably perturbed by some additional superpotential interactions). This is the case for our example, if the dynamical scales satisfy $\Lambda_1 \gg \Lambda_3 \gg \Lambda_2$. Node 3 plays the role of the $SU(N_c)$ gauge group in ISS. Nodes 2 and 4 provide it with $N_c + 1$ flavors. The dynamics of node 1 is responsible for giving masses to N_c of them. The extra flavor contributed by node 4 is necessary to put the theory in the free-magnetic range. Its mass is generated via a D-brane instanton.

Towards the IR, node 1 is the first one to become strongly coupled. The low energy physics is described in terms of its gauge invariant mesons $M_{22} = X_{21} X_{12}$ and baryons $B \sim X_{12}^{N_c}$ and $\tilde{B} \sim X_{21}^{N_c}$. Node 1 has an equal number of colors and flavors and hence a quantum modified moduli space. On the mesonic branch[2], we have

$$\det M_{22} = \Lambda_1^{2N_c} \, . \tag{4}$$

As it is clear from (3), the expectation value of M_{22} acts as a mass matrix for the X_{23} and X_{32} fields. From the point of view of node 3, we obtain $SU(N_c)$ SQCD with $N_c + 1$ massive flavors, deformed by the quatic interaction in (3). The quartic term has some interesting consequences. It breaks R-symmetry explicitly, allowing for gaugino masses.

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[1] In fact the theory with this additional term comes from an orientifold of a larger $\mathbb{Z}_N$ orbifold of the conifold, with $N \geq 5$ [5]. Our discussion applies without changes to this geometry and we expect the reader to keep it in mind.

[2] Later we discuss the stability of our model against moving into the baryonic branch.

In addition, it causes one of the pseudo-moduli to be stabilized away from the origin. By switching to the IR-free Seiberg dual, we can show that the theory has a meta-stable SUSY breaking vacuum. The dual theory contains mesons that combine bifundamentals of the electric theory $X_{i3}X_{3j} = \Lambda_3\phi_{ij}$, as well as dual "quarks" Y_{i3} and Y_{3i}. The vacuum energy is dictated by the masses of the N_c lightest flavors [3]

$$V_{tree} = |h\Lambda_3|^2 \sum_{i=1}^{N_c} |M_i|^2 = N_c \left| h\Lambda_3\Lambda_1^2 \right|^2 , \tag{5}$$

where we obtain the final result by extremizing on the eigenvalues of the matrix M_{22} given the constraint (4) on its determinant.

GENERATING A MASS TERM BY A STRINGY INSTANTON

So far we have analyzed the IR dynamics of a gauge theory with superpotential given by (3) and concluded that it has a meta-stable vacuum. The last term in (3) is generated by a stringy instanton. It corresponds to an Euclidean D1-brane (ED1) wrapped over the 2-cycle associated with the unoccupied node 5.

There are four fermionic zero modes in the ED1-ED1 sector. Two of them are goldstinos associated with breaking 1/2 of the $\mathcal{N} = 1$ SUSY. The two extra zero modes arise because, in fact, the ED1-ED1 sector sees an accidental $\mathcal{N} = 2$ SUSY. In order to saturate the superspace measure and produce a non-zero contribution to the effective superpotential, we must have only two fermionic zero modes on the ED1. A straightforward way of achieving this is by placing the ED1 on top of an O-plane with an $O(1)$ projection.
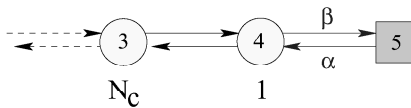
FIGURE 3. The extended quiver describing the interaction between the fractional brane system and a Euclidean D1 brane on node 5, represented by a square. The relevant coupling is a quartic one, involving α, β, X_{34} and X_{43}, as in eq.(6).

In addition, there are fermionic zero modes α and β connecting the ED1 and the spacefilling D-branes. They transform in the $(N_4, -1)$ and $(\overline{N_4}, 1)$ representations (in our case, $N_4 = 1$). These zero modes are the same that would appear if we replaced the ED1 by a spacefilling D5, while the bosonic ones are missing. It can be shown that the same reasoning applies to D-brane instantons wrapping unoccupied nodes in general quivers. Their couplings to the chiral multiplets can be determined in the same way. The situation can be encoded in an extended quiver as shown in Figure 3. The instanton action contains a coupling of the form

$$L = \alpha X_{43} X_{34} \beta . \tag{6}$$

97

Integrating over α and β, we obtain the instanton contribution to the superpotential in the 4d effective action

$$c\,X_{43}\,X_{34}\,e^{-\text{Area}},\tag{7}$$

where c is a dimensionful constant and the area is that of the curve corresponding to node 5. This indeed has the form of the mass term in (3) and we can identify $m = c\,e^{-\text{Area}}$.

A similar instanton can give rise to a mass term for the baryons of node 1

$$c'\,B\,\tilde{B}\,e^{-\text{Area}'}\,.\tag{8}$$

STABILITY OF DYNAMICAL MASSES

Like in the original ISS model, the meta-stable vacuum is identified and studied in the IR-free magnetic dual. For fixed non-zero values of the dynamical masses, it is straightforward to compute the 1-loop lifting of pseudomoduli directions, which proceeds as in similar models. An interesting feature is that the R-symmetry breaking quartic term in (3) results in a non-zero vev for the ϕ_{22} Seiberg meson, $|\phi_{22}| \sim |h\Lambda_3^2|^2$. This fact will become important in the discussion that follows. Since all the masses in our problem are dynamically generated, a central question is whether they are stable. The energy at the meta-stable vacuum is given by (5). The masses of N_c of the flavors appear when sitting on the mesonic branch for node 1, which has $N_f = N_c$. But, in fact, the full quantum constraint for the moduli space reads

$$\det M_{22} - B\tilde{B} = \Lambda_1^{2N_c}\,.\tag{9}$$

We can thus wonder whether $\det M_{22}$ can relax to zero, with the consequent disappearance of the meta-stable vacuum, by condensing B and $\tilde{B}$.

Combining (5) and (9), we are lead to the following off-diagonal terms in the matrix of second derivatives of the potential

$$V_{,B\tilde{B}} = V_{,\tilde{B}B} = -h^2\Lambda_3^2/\Lambda_1^{2N_c-4}\,.\tag{10}$$

This contribution appears at tree-level and favors the condensation of baryons as discussed above.

Two effects contribute to the stabilization of the baryonic directions. One of them is the quadratic coupling for the baryons generated by a D-brane instanton (8). The other one is the coupling to ϕ_{22}. The final result for the stabilizing diagonal terms is [5]

$$V_{,BB} = V_{,\tilde{B}\tilde{B}} = \frac{2}{\Lambda_1^{2N_c+2}}\,(c'\Lambda_1^{2N_c} - h\Lambda_1^2\Lambda_3\,\phi_{22})^2\,.\tag{11}$$

Both h and the instanton coefficient c_1 are suppressed by M_s^*, as $h \sim M_s^{*-1}$ and $c' \sim M_s^{*3-2N_c}$. Here, M_s^* indicates the string mass scale effectively warped down to a lower value due to the renormalization group (RG) flow, which manifests itself as a duality cascade. For the field theory interpretation to be valid, we need M_s^* to be bigger than

any of the dynamical scales of the gauge groups involved in the quiver. The baryonic directions are stable provided that $V_{BB} \geq |V_{B\bar{B}}|$. As we have explained, ϕ_{22} has a non-zero expectation value in the vacuum. Furthermore, its magnitude is sufficient for stabilizing the baryons. Then we can consider tuning the instanton contribution in (11) to zero. Doing so, we obtain the simplified expression

$$V_{BB} = V_{\bar{B}\bar{B}} = h^4 \Lambda_3^6 / \Lambda_1^{2N_c - 2} . \tag{12}$$

Comparing (10) and (12), we conclude that the baryonic directions (and hence the dynamically generated masses) are stable provided that

$$(\Lambda_1/\Lambda_3) < (\Lambda_3/M_s^*) . \tag{13}$$

We can always satisfy this inequality, as well as the ones coming from the hierarchy of mass scales in the low-energy model (as in *e.g.* [8]), by imposing the following hierarchy

$$\Lambda_1 \ll \Lambda_3 < M_s^* \qquad \text{and} \qquad m < \Lambda_3 . \tag{14}$$

In addition, m also has to satisfy $(\Lambda_3/M_s^*)^2 < (m/\Lambda_3)$ for the vacuum to be stable [8].

GRAVITY DUAL

Let us now consider whether it is possible to provide a gravity description of the meta-stable vacuum. Our D-brane system can be embedded in a weakly curved gravity background by adding a large number of regular D3-branes. The resulting gauge theory exhibits a duality cascade that progressively reduces the effective number of D3-branes until reaching the theory in Figure 2 at the IR bottom.

The Z_3 orbifold of the conifold is given by the following equation in $\mathbb{C}^4$

$$x^3 y^3 = uv . \tag{15}$$

It admits three independent complex deformations, resulting in the smooth geometry

$$\prod_{i=1}^{3} (xy - \varepsilon_i) = uv . \tag{16}$$

Then, there are three independent 3-cycles A_i, with sizes given by

$$\int_{A_i} \Omega = \varepsilon_i . \tag{17}$$

We now focus on the case in which only two of the 3-cycles are blown up and have the same size

$$(xy - \varepsilon)^2 xy = uv . \tag{18}$$

This deformation is triggered by the N_c deformation branes at node 3. This geometry has a $\mathbb{C}^2/\mathbb{Z}_2$ line of singularities (also called A_1-singularities, that should not be confused with the label of the 3-cycles above) at the locus $xy = \varepsilon$, $u = v = 0$.

The gravity dual arises via a geometric transition that turns the fractional branes on node 3 into flux

$$\int_A G_3 = N_c \,, \qquad \int_B G_3 = \frac{i}{g_s} k \,. \tag{19}$$

Here, A is the compact 3-cycle on which the RR 3-form flux is turned on. Simultaneously, there are k units of NS flux (dual to the number of steps in the duality cascade) in the dual non-compact 3-cycle B.

The meta-stable vacuum lives on the mesonic branch of node 1. This means that the gravity dual contains N_c explicit D5-branes whose positions are parametrized by the expectation values of the meson M_{22}. They are the $\mathcal{N} = 2$ fractional branes on nodes 1 and 2 and can move along the curve of A_1 singularities. In the meta-stable vacuum, all of them sit at a distance $(\det M_{22})^{1/N_c} \propto \Lambda_1^2$ from the origin.

The ideas in the previous discussion can be intuitively captured by a toric web diagram as shown in Figure 4.
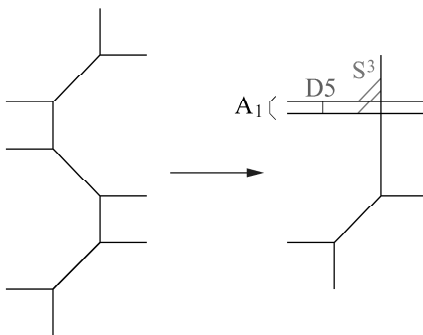
FIGURE 4. Web diagram describing the basic features of the geometry under consideration. After a geometric transition that grows two equal size 3-spheres, the geometry contains a curve of A_1 singularities over which N_c D5-branes are wrapped.

Following the ideas in [1], we propose that a meta-stable state can be constructed by adding anti D3-branes. The vacuum energy is proportional to the number of anti branes and its exponentially small due to the warping of their tension. In order for the configuration to correspond to a state in the same gauge theory, the asymptotic D-brane charges must remain unchanged. This can be achieved in our model by simultaneously adding N_c anti D3-branes and jumping the NS flux along the B cycle by one unit

$$\int_B G_3 - \frac{i}{g_s} k \longrightarrow \int_B G_3 - \frac{i}{g_s} (k+1) \,. \tag{20}$$

Notice that we cannot introduce a smaller number of anti D3-branes while preserving the asymptotic D3-brane charge. In order to do so, we would have to add them in D3 − $\overline{\text{D3}}$ pairs, which would perturbatively annihilate.

Due to the F_5 background, the anti D3-branes are attracted to the tip of the geometry, where presumably all of them are dissolved as gauge flux into the N_c D5-branes

$$\int_{\mathscr{C}} \mathscr{F} = -N_c .$$ (21)

The single deformation brane on node 4 plays an important role in our model, at least in the field theory regime, by introducing an extra flavor with mass given by a stringy instanton. Being just one brane, it remains as a probe and our gravity analysis does not capture its effect.

The Meta-Stable Vacuum: Gravity versus Field Theory

The gravity realization of the meta-stable vacuum that we have proposed matches nicely with the field theory analysis. In particular, we have seen that it is impossible to add $1, \ldots, N_c - 1$ anti D3-branes. We have argued that the meta-stable state candidate corresponds to N_c anti branes. This agrees with the field theory analysis, where the only known meta-stable state has a vacuum energy $\sim N_c$ in units of the dynamical scale.

We have checked the stability of the non-SUSY vacuum from a field theory point of view. However, we have not been able to prove the stability of its proposed counterpart in the gravity regime yet. In the KPV construction [1], a meta-stable vacuum only exists when the number of anti D3-branes is small. More concretely, their number has to be smaller than approximately 8% of the RR 3-form flux driving the cascade. Beyond this point, the anti D3-branes polarize via the Myers effect into NS5-branes that classically unwrap the S^3 at the bottom of the geometry and annihilate against flux. In our case, both the number of anti D3-branes and the RR 3-form flux are equal to N_c. Thus, we might wonder whether we could suffer a similar instability. An important difference between our setup and KPV is the presence of N_c explicit D5-branes. We heuristically expect that the system might become stable due to the attraction between these branes and the anti D3's. As N_c grows, there are more attracting D5-branes. At the same time, the S^3 (whose size goes as $\sqrt{g_s N_c}$) becomes larger and then the gradient of the Myers potential becomes small. Under these circumstances, it is in principle possible that the attracting force overcomes the effect that tends to polarize the anti D3-branes, and a meta-stable state is attained.

CONCLUSIONS

We have engineered a gauge theory with interesting properties using D-branes at a simple Calabi-Yau singularity, an orbifold of the conifold. We have shown that it has an ISS-like meta-stable SUSY breaking vacuum. All the necessary mass terms are generated dynamically and thus are naturally small. An important role is played by a stringy instanton. This model, or similar ones, might be useful in phenomenological constructions such as models with direct mediation.

We have also discussed a gravity dual describing a cascading gauge theory terminating in our model at the IR. We have argued that a meta-stable state can be engineered by adding anti D3-branes and jumping NS fluxes. Interestingly, the resulting state has a vacuum energy that goes as N_c times the dynamical scale associated with the anti D-brane tension. This matches the field theory result and supports the identification of this state as the extrapolation to large 't Hooft coupling of the ISS vacuum. It would be interesting to understand the stability of the gravity configuration and the backreaction of the wrapped D5-branes in more detail.

Moving beyond the application discussed in this note, (generalized) conifolds provide a simple, yet extremely rich, class of geometries and associated gauge theories in which various issues related to SUSY breaking can be investigated. For example, it is in this context that it has been shown that superpotential perturbations generated by D-brane instantons can also be interpreted as arising from ordinary gauge theory effects in cascading gauge theories [9]. Another interesting application is in the construction of simple SUSY breaking models that do not involve non-abelian gauge dynamics [10].

ACKNOWLEDGMENTS

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