Theoretical status of B_s-mixing and lifetimes of heavy hadrons

Alexander Lenz [a]

[a] Institut für theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

We review the theoretical status of the lifetime ratios τ_{B^+}/τ_{B_d}, τ_{B_s}/τ_{B_d}, $\tau_{\Lambda_b}/\tau_{B_d}$ and τ_{B_c} and of the mixing quantities ΔM_s, $\Delta\Gamma_s$ and ϕ_s. ΔM_s and $\Delta\Gamma_s$ suffer from large uncertainties due to the badly known decay constants, while the ratio $\Delta\Gamma_s/\Delta M_s$ can be determined with almost no non-perturbative uncertainties, therefore it can be used perfectly to find possible new physics contributions in the mixing parameters. We suggest a very clear method of visualizing the bounds on new physics and demonstrate this by combining the latest experimental numbers on the mixing quantities quantities with theory - one already gets some hints for new physics contributions, but more precise experimental numbers are needed to draw some definite conclusions. We conclude with a ranking list of all the discussed quantities according to their current theoretical uncertainties and point out possible improvements.

1. Introduction

Inclusive decays (see e.g.[1] and references therein) and lifetimes of heavy mesons can be calculated within the framework of the heavy quark expansion (HQE) [2]. In this approach the decay rate is calculated in an expansion in inverse powers of the heavy b-quark mass: $\Gamma = \Gamma_0 + \Lambda^2/m_b^2\,\Gamma_2 + \Lambda^3/m_b^3\,\Gamma_3 + \ldots$. Γ_0 represents the decay of a free heavy b-quark, according to this contribution all b-mesons have the same lifetime. The first correction arises at order $1/m_b^2$, they are due to the kinetic and the chromomagnetic operator. At order $1/m_b^3$ the spectator quark gets involved in the weak annihilation and Pauli interference diagrams [2,3]. This contributions are numerically enhanced by a phase space factor of $16\pi^2$. Each of the Γ_i contains perturbatively calculable Wilson coefficients and non-perturbative parameters, like decay constants or bag parameters. This approach clearly has to be distinguished from QCD inspired models. It is derived directly from QCD and the basic assumptions (convergence of the expansion in α_s and Λ/m_b) can be simply tested by comparing experiment and theory for different quantities (see e.g. [4]).

2. Lifetimes

The lifetime ratio of two heavy mesons reads

$$\frac{\tau_1}{\tau_2} = 1 + \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right)$$

If one neglects small isospin or SU(3) violating effects one has no $1/m_b^2$ corrections [1] and a deviation of the lifetime ratio from one starts at order $1/m_b^3$. For the ratio τ_{B^+}/τ_{B_d} the leading term $\Gamma_3^{(0)}$ has been determined in [2,5]. For a quantitative treatment of the lifetime ratios NLO QCD corrections are mandatory - $\Gamma_3^{(1)}$ has been determined in [6]. Subleading effects of $\mathcal{O}(1/m_b)$ turned out to be negligible [7]. Updating the result from [6] with matrix elements from [8] and the values $V_{cb} = 0.0415$, $m_b = 4.63$ GeV and $f_B = 216$ MeV [9] we obtain a value, which is in excellent agreement with the experimental number [10,11]:

$$\frac{\tau(B^+)}{\tau(B_d^0)}_{\rm NLO} = 1.063 \pm 0.027, \frac{\tau(B^+)}{\tau(B_d^0)}_{\rm Exp} = 1.076 \pm 0.008.$$

To improve the theoretical accuracy further we need more precise lattice values, in particular of the appearing color-suppressed operators. In the lifetime ratio τ_{B_s}/τ_{B_d} a cancellation of weak annihilation contributions arises, that differ only by small SU(3)-violation effects. One expects a number that is close to one [5,6,12,13]. The experimental number [10,11] is slightly smaller

$$\frac{\tau(B_s)}{\tau(B_d)}_{\rm Theo} = 1.00 \pm 0.01, \frac{\tau(B_s)}{\tau(B_d)}_{\rm Exp} = 0.950 \pm 0.019.$$

[1] In the case of $\tau_{\Lambda_b}/\tau_{B_d}$ these effects are expected to be of the order of 5%.

Next we consider two hadrons, where the theoretical situation is much worse compared to the mesons discussed above. The lifetime of the doubly heavy meson B_c has been investigated in [14], but only in LO QCD.

$$\tau(B_c)_{\rm LO} = 0.52^{+0.18}_{-0.12}\,{\rm ps}, \tau(B_c)_{\rm Exp} = 0.460 \pm 0.066\,{\rm ps}$$

In addition to the b-quark now also the c-charm quark can decay, giving rise to the biggest contribution to the total decay rate. The current experimental number is taken from [15,10]. In the case of the Λ_b-baryon the NLO-QCD corrections are not complete and there are only preliminary lattice studies for a part of the arising matrix elements, see e.g. [16], so the theoretical error has to be met with some skepticism. Moreover there are some discrepancies in the experimental numbers [10,17].

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}_{\rm Theo} = 0.88 \pm 0.05 \quad , \frac{\tau(\Lambda_b)}{\tau(B_d)}_{\rm Exp} = 0.912 \pm 0.032\,.$$

3. Mixing Parameters

In this section we briefly investigate the status of the mixing parameters. For a more detailed review we refer the interested reader to [18].

The mixing of the neutral B-mesons is described by the off diagonal elements Γ_{12} and M_{12} of the mixing matrix. Γ_{12} stems from the absorptive part of the box diagrams - only internal up and charm quarks contribute, while M_{12} stems from the dispersive part of the box diagram, therefore being sensitive to heavy internal particles like the top quark or heavy new physics particles. The calculable quantities $|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \arg(-M_{12}/\Gamma_{12})$ can be related to three observables (see [18,19] for a detailed description):

- Mass difference $\Delta M \approx 2|M_{12}|$
- Decay rate difference $\Delta \Gamma \approx 2|\Gamma_{12}| \cos\phi$
- Flavor specific or semi-leptonic CP asymmetries: $a_{fs} = {\rm Im}\frac{\Gamma_{12}}{M_{12}} = \frac{\Delta\Gamma}{\Delta M} \tan\phi$.

Calculating the box diagram with internal top quarks one obtains

$$M_{12,q} = \frac{G_F^2}{12\pi^2} (V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

The Inami-Lim function $S_0(x_t = \bar{m}_t^2/M_W^2)$ [20] is the result of the box diagram without any gluon corrections. The NLO QCD correction is parameterized by $\hat{\eta}_B \approx 0.84$ [21]. The non-perturbative matrix element is parameterized by the bag parameter B and the decay constant f_B. Using the conservative estimate $f_{B_s} = 240 \pm 40$ MeV [18] and the bag parameter B from JLQCD [22] we obtain in units of ${\rm ps}^{-1}$ (experiment from [10,11,23])

$$\Delta M_s^{\rm Theo} = 19.3 \pm 6.4 \pm 1.9, \Delta M_s^{\rm Exp} = 17.77 \pm 0.12$$

The first error in the theory prediction stems from the uncertainty in f_{B_s} and the second error summarizes the remaining theoretical uncertainties. The determination of ΔM_d is affected by even larger uncertainties because here one has to extrapolate the decay constant to the small mass of the down-quark. The ratio $\Delta M_s/\Delta M_d$ is theoretically better under control since in the ratio of the non-perturbative parameters many systematic errors cancel, but on the other hand it is affected by large uncertainties due to $|V_{ts}|^2/|V_{td}|^2$. To be able to distinguish possible new physics contributions to ΔM_s from QCD uncertainties much more precise numbers for f_{B_s} are needed.

In order to determine the decay rate difference of the neutral B-mesons and flavor specific CP asymmetries a precise determination of Γ_{12} is needed, which can be written as

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right)$$

The leading term $\Gamma_3^{(0)}$ was determined in [24]. The numerical and conceptual important NLO-QCD corrections ($\Gamma_3^{(1)}$) were determined in [25, 19]. Subleading $1/m$-corrections, i.e. $\Gamma_4^{(0)}$ were calculated in [13,26] and even the Wilson coefficients of the $1/m^2$-corrections ($\Gamma_5^{(0)}$) were calculated and found to be small [27]. In [18] a strategy was worked out to reduce the theoretical uncertainty in Γ_{12}/M_{12} by almost a factor of 3, see Fig. (1) for an illustration. One gets

$$\frac{\Delta\Gamma_s}{\Delta M_s} = 10^{-4} \cdot \left[46.2 + 10.6\frac{B_S'}{B} - 11.9\frac{B_R}{B}\right]$$

The dominant part of $\Delta\Gamma/\Delta M$ can now be determined without any hadronic uncertainties (for

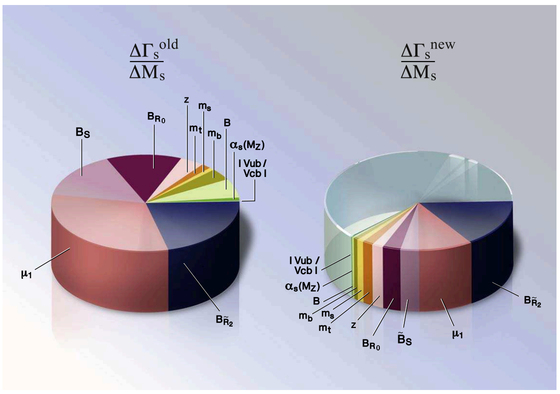

Figure 1. Error budget for the theoretical determination of $\Delta\Gamma_s/\Delta M_s$. Compared to previous approaches (left) the new strategy lead to a reduction of the theoretical error by almost a factor of 3.

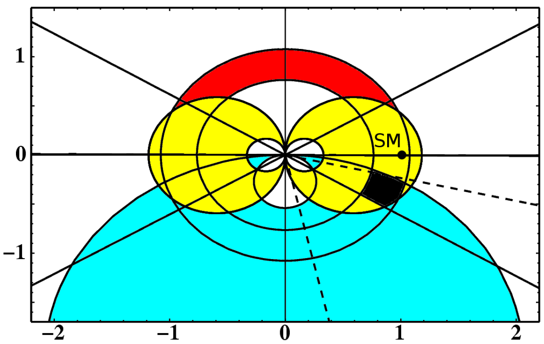

Figure 2. Current experimental bounds in the complex Δ_s-plane. The bound from ΔM_s is given by the red (dark-grey) ring around the origin. The bound from $\Delta\Gamma_s/\Delta M_s$ is given by the yellow (light-grey) region and the bound from a^s_{fs} is given by the light-blue (grey) region. The angle ϕ_s^Δ can be extracted from $\Delta\Gamma_s$ (solid lines) with a four fold ambiguity - one bound coincides with the x-axis! - or from the angular analysis in $B_s \to J/\Psi\phi$ (dashed line). If the standard model is valid all bounds should coincide in the point (1,0). The current experimental situation shows a small deviation, which might become significant, if the experimental uncertainties in $\Delta\Gamma_s$, a^s_{sl} and ϕ_s will go down in near future.

more details see [18])! Using the non-perturbative parameters from [22,28], we obtain the following final numbers (see [18] for the complete list of the numerical values of the input parameters and the very conservative ranges in which we varied them)

$$\Delta\Gamma_s = (0.096 \pm 0.039)\,\mathrm{ps}^{-1}, \; \frac{\Delta\Gamma_s}{\Gamma_s} = 0.147 \pm 0.060,$$
$$a^s_{fs} = (2.06 \pm 0.57) \cdot 10^{-5}, \; \frac{\Delta\Gamma_s}{\Delta M_s} = (49.7 \pm 9.4)\, 10^{-4},$$
$$\phi_s = 0.0041 \pm 0.0008 = 0.24^\circ \pm 0.04\,.$$

New physics (see e.g. references in [18]) is expected to have almost no impact on Γ_{12}, but it can change M_{12} considerably – we denote the deviation factor by the complex number Δ. Therefore one can write

$$\Gamma_{12,s} = \Gamma_{12,s}^{\mathrm{SM}}, M_{12,s} = M_{12,s}^{\mathrm{SM}} \cdot \Delta_s; \Delta_s = |\Delta_s| e^{i\phi_s^\Delta}$$

With this parameterisation the physical mixing parameters can be written as

$$\begin{aligned}
\Delta M_s &= 2|M_{12,s}^{\mathrm{SM}}| \cdot |\Delta_s|, \\
\Delta\Gamma_s &= 2|\Gamma_{12,s}| \cdot \cos\left(\phi_s^{\mathrm{SM}} + \phi_s^\Delta\right), \\
\frac{\Delta\Gamma_s}{\Delta M_s} &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\mathrm{SM}}|} \cdot \frac{\cos\left(\phi_s^{\mathrm{SM}} + \phi_s^\Delta\right)}{|\Delta_s|},
\end{aligned}$$

$$a^s_{fs} = \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\mathrm{SM}}|} \cdot \frac{\sin\left(\phi_s^{\mathrm{SM}} + \phi_s^\Delta\right)}{|\Delta_s|}. \tag{1}$$

Note that $\Gamma_{12,s}/M_{12,s}^{\mathrm{SM}}$ is now due to the improvements in [18] theoretically very well under control. Next we combine the current experimental numbers with the theoretical predictions to extract bounds in the imaginary Δ_s-plane by the use of Eqs. (1), see Fig. (2). The width difference $\Delta\Gamma_s/\Gamma_s$ was investigated in [29,11]. The semi-leptonic CP asymmetry in the B_s system has been determined in [11,30] (see [18] for more details). Therefore we use as experimental input

$$\Delta\Gamma_s = 0.17 \pm 0.09\,\mathrm{ps}^{-1}, \; \phi_s = -0.79 \pm 0.56.$$
$$a^s_{sl} = (-5.2 \pm 3.9) \cdot 10^{-3}\,.$$

4. Conclusion and outlook

We have reviewed the theoretical status of lifetimes of heavy hadrons and the measureable mixing quantities of the neutral B-mesons. Both classes of quantities can be described with the help of the HQE - a systematic expansion based simply on QCD. Inspired by the current theoretical uncertainties we put these quantities in 3 classes

1a: Gold: $\tau(B_s)/\tau(B_d)$, ϕ_q, a_{fs}^q

1b: Silver: $\tau(B^+)/\tau(B_d)$, $\Delta\Gamma/\Delta M$

2: Iron: $\Delta\Gamma$, ΔM, $\Delta M_s/\Delta M_d$, $\tau(B_c)$, $\tau(\Lambda_b)$

Gold and *silver* have similar theoretical uncertainties, but the *gold*-quantities are predicted to be small numbers, so a sizeable experimental result corresponds unambigously to some new effect.

Let's start with the worst. Iron: The theoretical uncertainty in the mixing parameters ΔM and $\Delta\Gamma$ is completely dominated by the decay constant. Here some progress on the non-perturbative side is mandatory. In $\Delta M_s/\Delta M_d$ the dominant uncertainty is given by $|V_{ts}/V_{td}|^2$. In $\tau(B_c)$ and $\tau(\Lambda_b)$ the important NLO-QCD are missing or are incomplete, moreover we have only preliminary lattice studies of the non-perturbative matrix elements.
Silver: Theoretical predictions of τ_{B^+}/τ_{B_d} are in excellent agreement with the experimental numbers. We do not see any signal of possible duality violations. To become even more quantitative in the prediction of τ_{B^+}/τ_{B_d} the non-perturbative estimates of the bag parameters - in particular of the color-suppressed ones - have to be improved. In [18] a method was worked out to reduce the theoretical error in $\Delta\Gamma/\Delta M$ considerably. For a further reduction of the theoretical uncertainty in the mixing quantities the unknown matrix elements of the power suppressed operators have to be determined. Here any non-perturbative estimate would be very desirable. A first step in that direction was performed in [31]. If accurate non-perturbative parameters are available one might think about NNLO calculations (α_s/m_b- or α_s^2-corrections) to reduce the remaining μ-dependence and the uncertainties due to the missing definition of the b-quark mass in the power corrections.
Gold: The improvements for $\Delta\Gamma/\Delta M$ apply to a_{fs} and Φ_q as well.

The relatively clean standard model predictions for the mixing quantities can be used to look for new physics effects in B_s-mixing. From the currently available experimental bounds on $\Delta\Gamma_s$ and a_{fs} one already gets some hints for deviations from the standard model. To settle this issue we are eagerly waiting for more data from TeVatron!

I would like to thank the organizers and convenors of HCP2007 for the invitation and the financial support, which enabled me to take part in this wonderful symposium and Uli Nierste for the pleasant collaboration.

Note added: There is sometimes a confusion between the mixing phases β_s and ϕ_s, which we would like to adress here. Both numbers are expected to be small in the standard model - $\phi_s = (0.24 \pm 0.04)^\circ$ and $2\beta_s = (2.2 \pm 0.6)^\circ (= (0.04 \pm 0.01)\mathrm{rad})$, but in view of the high future experimental precisions - in particular at LHCb [32]- a clear distinction might be useful.
$2\beta_s := -\arg[(V_{tb}V_{ts}^*)^2/(V_{cb}V_{cs}^*)^2]$ is the phase which appears in $b \to c\bar{c}s$ decays of neutral B-mesons taking possible mixing into account, so e.g. in the case $B_s \to J/\psi + \phi$. $(V_{tb}V_{ts}^*)^2$ comes from the mixing (due to M_{12}) and $(V_{cb}V_{cs}^*)^2$ comes from the ratio of $b \to c\bar{c}s$ decay and $\bar{b} \to \bar{c}c\bar{s}$ amplitudes. Sometimes β_s is approximated as $2\beta_s \approx -\arg[(V_{tb}V_{ts}^*)^2] \approx -\arg[(V_{ts}^*)^2]$ - the error due to this approximation is on the per mille level.
$\phi_s := \arg[-M_{12}/\Gamma_{12}]$ is the phase that appears e.g. in a_{fs}^s. In M_{12} we have again $(V_{tb}V_{ts}^*)^2$, while we have a linear combination of $(V_{cb}V_{cs}^*)^2$, $V_{cb}V_{cs}^*V_{ub}V_{us}^*$ and $(V_{ub}V_{us}^*)^2$ in Γ_{12}. Neglecting the latter two contributions - which is not justified - would yield the phase $2\beta_s$.

New physics alters the phase $-2\beta_s$ to $\phi_s^\Delta - 2\beta_s$ and the phase ϕ_s to $\phi_s^\Delta + \phi_s$. If the new physics contribution is sizeable, then in both cases only ϕ_s^Δ survives, since the standard model phases are very small.

REFERENCES

1. M. Battaglia *et al.*, arXiv:hep-ph/0304132; A. Lenz, U. Nierste and G. Ostermaier, Phys. Rev. D **56** (1997) 7228 [arXiv:hep-ph/9706501]; A. Lenz, U. Nierste and G. Ostermaier, Phys. Rev. D **59** (1999) 034008 [arXiv:hep-ph/9802202]; A. Lenz, arXiv:hep-ph/0011258.
2. M. A. Shifman and M. B. Voloshin, in: *Heavy Quarks* ed. V. A. Khoze and M. A. Shifman, Sov. Phys. Usp. **26** (1983) 387; M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. **41** (1985) 120 [Yad. Fiz. **41** (1985) 187]; M. A. Shifman and M. B. Voloshin, Sov. Phys. JETP **64** (1986) 698 [Zh. Eksp. Teor. Fiz. **91** (1986) 1180]; J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B **247** (1990) 399; I. I. Bigi, N. G. Uraltsev and A. I. Vainshtein, Phys. Lett. B **293** (1992) 430 [Erratum-ibid. B **297** (1992) 477]; I. I. Y. Bigi, M. A. Shifman, N. G. Uraltsev and A. I. Vainshtein, Phys. Rev. Lett. **71** (1993) 496 [arXiv:hep-ph/9304225]; B. Blok, L. Koyrakh, M. A. Shifman and A. I. Vainshtein, Phys. Rev. D **49** (1994) 3356 [Erratum-ibid. D **50** (1994) 3572] [arXiv:hep-ph/9307247]; A. V. Manohar and M. B. Wise, Phys. Rev. D **49** (1994) 1310 [arXiv:hep-ph/9308246].
3. B. Guberina, S. Nussinov, R. D. Peccei and R. Ruckl, Phys. Lett. B **89** (1979) 111; N. Bilic, B. Guberina and J. Trampetic, Nucl. Phys. B **248** (1984) 261; B. Guberina, R. Ruckl and J. Trampetic, Z. Phys. C **33** (1986) 297.
4. A. Lenz, arXiv:hep-ph/9906317; M. Beneke and A. Lenz, J. Phys. G **27** (2001) 1219 [arXiv:hep-ph/0012222]; A. Lenz and S. Willocq, J. Phys. G **27** (2001) 1207; A. Lenz, arXiv:hep-ph/0107033; A. Lenz, arXiv:hep-ph/0412007.
5. I. I. Y. Bigi, B. Blok, M. A. Shifman, N. Uraltsev and A. I. Vainshtein, arXiv:hep-ph/9401298; I. I. Y. Bigi, arXiv:hep-ph/9508408; M. Neubert and C. T. Sachrajda, Nucl. Phys. B **483** (1997) 339 [arXiv:hep-ph/9603202].
6. M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Nucl. Phys. B **639** (2002) 389 [arXiv:hep-ph/0202106]; E. Franco, V. Lubicz, F. Mescia and C. Tarantino, Nucl. Phys. B **633** (2002) 212 [arXiv:hep-ph/0203089].
7. A. Lenz and U. Nierste, unpublished, e.g. talk at Academia Sinica 2003; F. Gabbiani, A. I. Onishchenko and A. A. Petrov, Phys. Rev. D **70** (2004) 094031 [arXiv:hep-ph/0407004].
8. M. Di Pierro and C. T. Sachrajda [UKQCD Collaboration], Nucl. Phys. B **534** (1998) 373 [arXiv:hep-lat/9805028]; D. Becirevic, arXiv:hep-ph/0110124.
9. A. Gray *et al.* [HPQCD Collaboration], Phys. Rev. Lett. **95** (2005) 212001 [arXiv:hep-lat/0507015].
10. H. F. A. Group, arXiv:0704.3575 [hep-ex].
11. R. Lipton, these proceedings; D. Tsybychov, these proceedings; C. Bozzi, these proceedings.
12. Y. Y. Keum and U. Nierste, Phys. Rev. D **57** (1998) 4282 [arXiv:hep-ph/9710512].
13. M. Beneke, G. Buchalla and I. Dunietz, Phys. Rev. D **54** (1996) 4419 [arXiv:hep-ph/9605259].
14. C. H. Chang, S. L. Chen, T. F. Feng and X. Q. Li, Phys. Rev. D **64** (2001) 014003 [arXiv:hep-ph/0007162]; V. V. Kiselev, A. E. Kovalsky and A. K. Likhoded, Nucl. Phys. B **585** (2000) 353 [arXiv:hep-ph/0002127]. A. Y. Anisimov, I. M. Narodetsky, C. Semay and B. Silvestre-Brac, Phys. Lett. B **452** (1999) 129 [arXiv:hep-ph/9812514]. M. Beneke and G. Buchalla, Phys. Rev. D **53** (1996) 4991 [arXiv:hep-ph/9601249].
15. A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **97** (2006) 012002 [arXiv:hep-ex/0603027]. D0 Collaboration, D0 conference note 4539
16. C. Tarantino, arXiv:hep-ph/0702235.
17. A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **98** (2007) 122001 [arXiv:hep-ex/0609021]; D0-note 5263.
18. A. Lenz and U. Nierste, arXiv:hep-ph/0612167; A. Lenz, arXiv:hep-ph/0612176.

19. M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B **576** (2003) 173 [arXiv:hep-ph/0307344]; M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP **0308** (2003) 031 [arXiv:hep-ph/0308029].
20. T. Inami and C. S. Lim, Prog. Theor. Phys. **65** (1981) 297 [Erratum-ibid. **65** (1981) 1772].
21. A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347** (1990) 491.
22. S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. Lett. **91** (2003) 212001 [arXiv:hep-ph/0307039].
23. A. Abulencia *et al.* [CDF Collaboration], arXiv:hep-ex/0609040; A. Abulencia [CDF - Run II Collaboration], Phys. Rev. Lett. **97** (2006) 062003 [arXiv:hep-ex/0606027]; V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **97** (2006) 021802 [arXiv:hep-ex/0603029].
24. J.S. Hagelin, Nucl. Phys. **B193**, 123 (1981); E. Franco, M. Lusignoli and A. Pugliese, Nucl. Phys. **B194**, 403 (1982); L.L. Chau, Phys. Rep. **95**, 1 (1983); A.J. Buras, W. Słominski and H. Steger, Nucl. Phys. **B245**, 369 (1984); M.B. Voloshin, N.G. Uraltsev, V.A. Khoze and M.A. Shifman, Sov. J. Nucl. Phys. **46**, 112 (1987); A. Datta, E.A. Paschos and U. Türke, Phys. Lett. **B196**, 382 (1987); A. Datta, E.A. Paschos and Y.L. Wu, Nucl. Phys. **B311**, 35 (1988).
25. M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B **459** (1999) 631 [arXiv:hep-ph/9808385].
26. A. S. Dighe, T. Hurth, C. S. Kim and T. Yoshikawa, Nucl. Phys. B **624** (2002) 377 [arXiv:hep-ph/0109088].
27. A. Lenz and U. Nierste, to appear.
28. D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP **0204** (2002) 025 [arXiv:hep-lat/0110091].
29. R. Barate *et al.* [ALEPH Collaboration], Phys. Lett. B **486** (2000) 286; CDF collaboration, conference note 7925, http://www-cdf.fnal.gov; V. M. Abazov *et al.* [D0 Collaboration], arXiv:hep-ex/0702049; D. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **94** (2005) 101803 [arXiv:hep-ex/0412057]; A. Abulencia [CDF Collaboration], arXiv:hep-ex/0607021; V. M. Abazov *et al.* [D0 Collaboration], arXiv:hep-ex/0604046. R. Van Kooten, eConf **C060409** (2006) 031 [arXiv:hep-ex/0606005]; V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **98** (2007) 121801 [arXiv:hep-ex/0701012].
30. V. M. Abazov *et al.* [D0 Collaboration], arXiv:hep-ex/0701007; V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **74** (2006) 092001 [arXiv:hep-ex/0609014].
31. T. Mannel, B. D. Pecjak and A. A. Pivovarov, arXiv:hep-ph/0703244.
32. W. Bonivento, these proceedings; S. Bachmann, these proceedings.