Application of a Multivariate Process Control Technique for Set-Up Dominated Low Volume Operations

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Abstract. In traditional high-volume manufacturing applications, the timing of control adjustments to processes is based on parametric Statistical Process Control (SPC) methods, such as Shewhart X & R charts. In high-value, high-complexity and low-volume industries, where production runs are in the order of tens rather than thousands, traditional SPC approaches are not easily applicable. A manufactured component's complexity, with multiple critical features to monitor, increases the difficulty for a process operator to maintain all of them within their design tolerances. In response to this, this paper presents a framework of non-parametric SPC, called multivariate Set-Up Process Algorithm (mSUPA), for managing control adjustment when required. mSUPA uses a simple to interpret traffic light system for alerting process operators when an adjustment is required. mSUPA is underpinned by multivariate statistics and probability theory for validating a process set up. The case of mSUPA application to a real industry process is discussed.

Keywords. SPC, Pre-control, Multivariate, SUPA.

1. Introduction

The development and deployment of Advanced Manufacturing Technologies (AMT) in high-value, high-precision manufacturing environments has led to production of smaller batch sizes of more complex product [1,2]. These processes are defined as set-up dominant [3]. They are typically stable part to part, with the major source of variation between batches of the same part. This is a result of the AMT being used for different jobs in-between batches of the same part, significantly changing the operating environment. This makes timely intervention to correct the AMT set-up, to produce parts near the design target, critical. For example, an operator setting up a batch on a machine tool process adjusts the machine tool's offsets to steer the process on-target.

Traditionally, in high-volume manufacturing environments statistical process control (SPC) techniques have become prevalent in assisting operators identify a statistically significant change in the process. Examples of SPC technique are mean and range charts, and individuals and moving range charts [4]. These SPC techniques are

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not valid in a low-volume situation, because the production run may have finished before control limits have been established. In a low-volume environment defect prevention methods, with control limits derived from tolerances, have been shown to be advantageous [5]. Examples of defect prevention methods are Pre-Control [6] and Pre-Control variants, including 10 unit Pre-Control [7] and Set-Up Process Algorithm (SUPA) [5].

Although defect prevention methods address the issue of assisting the identification of an off-target process in low-volume manufacturing, they do not address the issue of product complexity. These defect prevention methods are univariate, i.e. they address one critical-to-quality (CtQ) feature at a time. It is common for in-excess of 30 CtQ features to be machined on the part by a single multi-axis Computer Numerical Control (CNC) machining centre process. This would require the machine tool operator to manage over 30 separate SUPA charts for a single part and process combination. There have been multivariate Pre-Control variants published in the literature [8,9] to address this. However, these techniques derive their control limits from statistical analysis rather than from tolerances. This means they are in fact SPC methods that are not suitable for a low-volume application.

This paper aims to address the control of a low-volume, high complexity manufacturing process. This is achieved by outlining a multivariate SUPA (mSUPA) method. Then its application to real production data is demonstrated.

2. mSUPA

In this section the procedure for applying the mSUPA method is described. This provides the framework for the method to be applied to a real process in the subsequent section.

The principle behind the mSUPA technique is to classify each part that is manufactured as either: green, yellow or red. This is in much the same way as univariate Pre-Control methods, the difference being that univariate approaches are describing the conformance of an individual CtQ whereas, mSUPA is describing the conformance of all CtQs attached to a part. Therefore, mSUPA is describing the overall conformity of a part.

Consecutive parts are sampled and their measured CtQ design features are categorized as green, yellow or red. If a sampled part is red it signals that the process is out-of-tolerance. Two consecutive parts in the same yellow zone signal an off-target process. Five consecutive green parts demonstrate the process is capable with 98% confidence and is allowed to continue without further checks. A full derivation of the calculation of the probability of qualifying a process using a Pre-Control derived method can be found in San Matias *et al.* [10].

In order to perform mSUPA in a manner that maintains a nonparametric approach, a series of sampling rules needs to be established. Therefore, this requires that consecutive parts are sampled from a process after set-up. The CtQ features of these parts are measured and recorded as the CtQ vector, x_i. If a feature of the CtQ vector is outside its design tolerance, the part is scrapped.

Consider the simplified case of a part with two CtQ design features, i.e. $x = [x_1, x_2]^T$. If x_1 and x_2 have the same specified design tolerances of $U = 250$ and $L = 50$, the tolerance boundary can be represented as a box, as in figure 1. Taking the philosophy of linking a red zone to design tolerance from SUPA, this box can be used in mSUPA as a boundary between the red and yellow zones. Let two measured parts $k = 1$ and $k = 2$ be collected with CtQ design vectors of $x(k = 1) = x(1) = [200,100]^T$ and $(k = 2) = x(2) = [40,200]^T$. These points are plotted on figure 1, where it is shown there that $x(1)$ is within and $x(2)$ is outside the design tolerance.

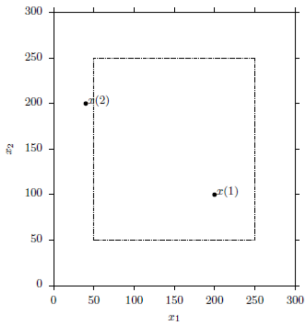

Figure 1. Tolerance boundary of x and positions of CtQ design vectors $x(1)$ and $x(2)$.

Although this information tells a user if a part is in or out-of-tolerance, it does not give any indication of how close a part is to the design target. However, to formulate a green zone, and therefore an mSUPA chart, a minimum acceptable capability level ($C_{p(i)}$) must be defined f or each CtQ feature (x_i). This is driven by the criticality of the feature, i.e. a functionally critical feature may require a $C_p = 2.0$, whereas, an aesthetic feature may only require a $C_p = 1.33$. This results in a capability vector, as follows:

$$C_p = \begin{bmatrix} C_{p(1)} \\ C_{p(2)} \\ \vdots \\ C_{p(n)} \end{bmatrix}, \tag{1}$$

where, n is the total number of i CtQ features. If the monitored process then meets the required minimum capabilities defined, the probability of a green part been generated should be $P(g) = 0.94$.

For a part with n CtQ features, the minimum variation in each x_i for $i = 1, \cdots, n$ can be represented by σ_i^2. Rearranging the formula for C_p as follows:

$$\sigma = \frac{U-L}{6C_p}, \tag{2}$$

where, U and L are the features upper and lower design tolerances. Using the values from (1) in (2) allows the calculation of σ_i^2 values. This allows the definition of the target covariance matrix (S) as:

$$S = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \sigma_n^2 \end{bmatrix}. \tag{3}$$

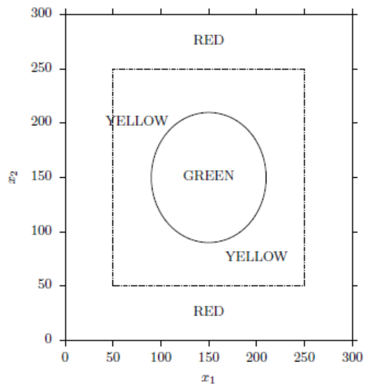

Figure 2. 2-dimensional mSUPA chart showing green, yellow and red zones.

The S matrix only contains diagonal elements. This reflects the fact that no assumptions are made about correlations between CtQ design features. With S, the multivariate chart in figure 1 is refined by using this as a scale of maximum variation acceptable in the process. S is used with the measured CtQ design vector, x, and the process target vector (T) to calculate the Mahalanobis distance [11], between x and T as follows:

$$(x - T)^T S^{-1} (x - T)^T < H^2, \tag{4}$$

where H^2 is a pre-selected constant. This allows the definition of a multidimensional green zone, which is the set of those points that have a Mahalanobis distance less than H^2 from T. The left hand side of (4) has the property of following a $\chi^2_{n,\varepsilon}$ distribution, where n is the degrees of freedom, which is equal to number of CtQ design features and ε is the probability of a sample from a population that is on-target falling outside the green zone.

In the case of univariate SUPA the green zone is defined so that a part produced by an on-target process has a minimum probability of falling in the green zone of $P(g) = 0.94$. Hence, extending this to the multivariate case results in $\varepsilon = 0.06$. This results in the sphere shown in figure 2. Using this chart, a decision about whether a process is off-target or not, is still made by following the SUPA rules described.

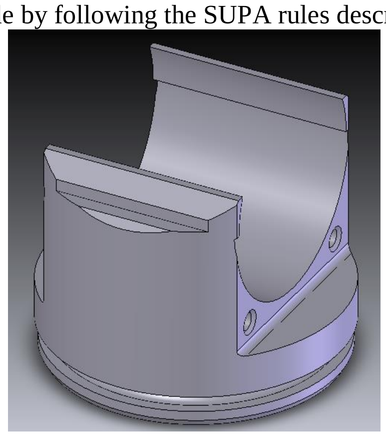

Figure 3. Hydraulic piston machined in a single operation with 20 features.

3. Case Study

This section demonstrates the application of the mSUPA technique to real process data. The data used in this study was collected from a machine tool operation at Rotary Power that produced pistons for hydraulic motors from a stock bar. An example of the piston is shown in figure 3. This operation produces 20 features on the piston. The application of the mSUPA method in this case focuses on four CtQ features.

The four features monitored had an upper tolerance, $U = [13.90,46.35,45.13,5.13]^T$, lower tolerance, $L = [13.60,46.30,44.87,4.87]^T$, and minimum $C_p = [1.33,1.33,1.33,1.33]^T$. By using these values in (2) results in the following target S matrix:

$$S = \begin{bmatrix} 0.00140 & 0 & 0 & 0 \\ 0 & 0.00004 & 0 & 0 \\ 0 & 0 & 0.00106 & 0 \\ 0 & 0 & 0 & 0.00106 \end{bmatrix}. \tag{5}$$

Consecutive units were then sampled from production. The results of these sampled units are shown in figure 4. The first unit sampled, $x(1) = [13.76,46.30,44.95,5.08]^T$ and the second unit sampled, $x(2) = [13.75,46.30,44.95,5.10]^T$, fell in the yellow zone. This zone is determined by comparing the Mahalanobis distane $x(1)$ and $x(2)$ using (4) against $\chi^2_{4,0.06} = 9.044$.

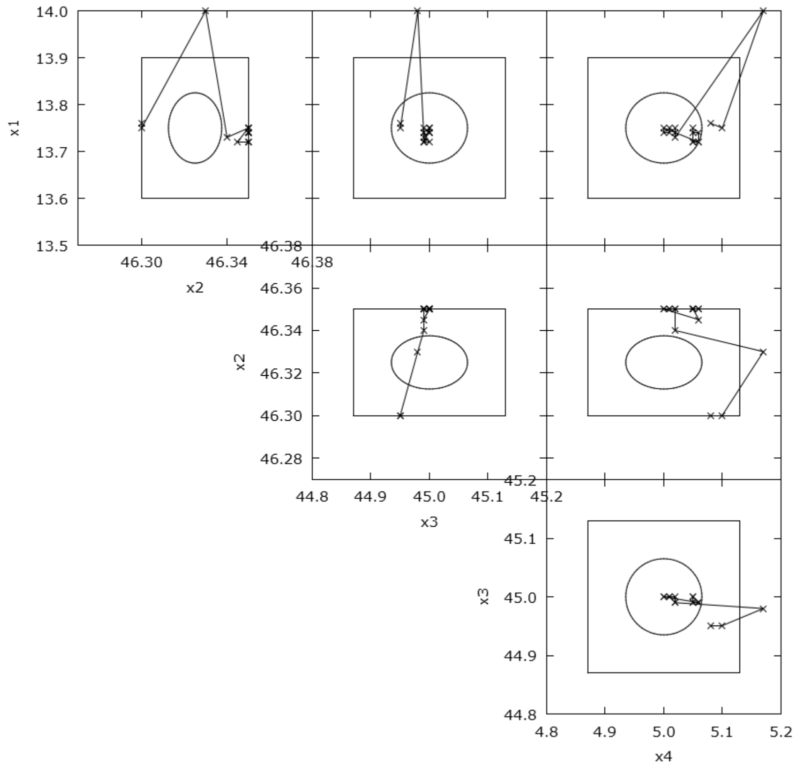

Figure 4. mSUPA plot of 4 CtQ features in the piston manufacture process.

Both $x(1)$ and $x(2)$ have Mahalanobis distances greater than $\chi^2_{4,0.06} = 9.044$ and do not fall in the green zone. However, both $x(1)$ and $x(2)$ are within $x(1)$ and $x(2)$ which indicates a yellow zone. These two units were not scrapped, but they indicated a change to the process offsets was required.

After a process change the third sample, $x(3) = [14.00,46.33,44.98,5.17]^T$, fell in a red zone. Although this sample was closer to target in $x_2(3)$ and $x_3(3)$, it was scrapped as it was out of tolerance in features $x_1(3)$ and $x_4(3)$. This highlights the difficulty in accurately adjusting process offsets were one offset can effect multiple features. This red result led to a further process change.

From samples $x(4)$ onwards, CtQ features x_1, x_3 and x_4 all fall within their individual green zones but fall in a global yellow zone due to feature x_2. The operator allowed this process to run in this state as it produced in-tolerance components with the greatest number of CtQ features in their individual green zone.

4. Conclusion

In this paper, a new nonparametric process control tool was introduced known as multivariate Set-Up Process Algorithm (mSUPA). mSUPA was designed to monitor multivariate set-up dominant processes; in order, to identify when the process mean is not on the global design target. This extends the univariate SUPA method into the multivariate case. By doing this, an easy to digestive traffic-light system is maintained that is linked to tolerance boundaries. Unlike using Pre-Control, the mSUPA charts can be adjusted to suit a range of required process capabilities. Although, the presentation of results through a simple global traffic-light to inform the operator that the process is on- or off-target is easy to digestive, it requires more complex calculations to decide which zone a part falls into. Therefore, it is recommended that this approach is implemented through an online software interface to perform the required calculations.

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