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Big Bang Nucleosynthesis Initial Conditions: Revisiting Wagoner et al. (1967)

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Published May 2021 · © 2021. The Author(s). Published by the American Astronomical Society.

[Research Notes of the AAS, Volume 5, Number 5](#)

Citation Charlie Sharpe et al 2021 Res. Notes AAS 5 106

DOI 10.3847/2515-5172/abf9ac

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Abstract

We revisit Wagoner et al., a classic contribution in the development of Big Bang Nucleosynthesis. We demonstrate that it presents an incorrect expression for the temperature of the early universe as a function of time in the high temperature limit, $T \gtrsim 10^{10}$ K. As this incorrect expression has been reproduced elsewhere, we present a corrected form for the initial conditions required for calculating the formation of the primordial elements in the Big Bang.

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1. Introduction

Big Bang Nucleosynthesis (BBN) is considered an outstanding success and pillar of modern cosmology, explaining the abundance of primordial elements within the universe (Tytler et al. 2000; Barnes & Lewis 2020). Calculating the output of BBN requires integrating a series of coupled differential equations representing the sources and sinks of various elements, encompassing cross-sections that are temperature dependent. Such integrals, of course, depend upon the initial conditions.

The structure of this research note is as follows; in Section 2, we discuss the process Wagoner et al. (1967) take in deriving the initial conditions for the integration of BBN in the early universe. We then discuss the issue with their results in Section 3 before concluding in Section 4.

2. BBN Initial Conditions

In their classic paper, Wagoner et al. (1967) derive the dependence of the temperature of the early universe on time, in the high temperature limit $T \gtrsim 10^{10}$ K. To do this, they first derive the work-energy equation for the Friedmann–Lemaître–Robertson–Walker metric. The baryon mass density ρ_b scales as R^{-3} and the neutrino energy density ρ_ν scales as R^{-4} , where R is the scale factor of the universe. After some algebra, they arrive at the following first-order differential equation for the expansion $R(T)$,

$$\frac{dR}{dT} = \frac{-R}{3[\rho_e(T) + \rho_r(T)/c^2]} \frac{d\rho_e(T)}{dT} \quad (1)$$

where $\rho_e = \rho_e + \rho_r$ is the sum of the energy densities of electrons and radiation, and $p_e = p_e + p_r$ is the sum of the pressures of electrons and radiation. They then combine this equation with the Friedmann equation to arrive at a differential equation for $T(t)$,

$$\frac{dT}{dt} = \mp (24\pi G\rho)^{1/2} \left[\rho_e(T) + \frac{p_e(T)}{c^2} \right] \left[\frac{d\rho_e(T)}{dT} \right]^{-1} \quad (2)$$

Now, at such high temperatures ($T \gtrsim 10^{10}$ K), electrons are highly relativistic with energy density $\frac{7}{4}\rho_e$, the same as neutrinos. The coefficient of $\frac{7}{4}$ comes from the number of species and allowed spin states of each respective particle. This gives $\rho = \frac{9}{2}\rho_e$ and $p_e = \frac{11}{4}\rho_e$. Hence, assuming an equation of state of $\frac{1}{3}$ for highly relativistic species, they finally arrive at a relationship between the temperature of the universe and time (their Equation (A15)),

$$T_9 = (12\pi G a_* c^{-2})^{1/4} t^{-1/2} = 10.4 t^{-1/2} \quad (3)$$

Here, T_9 is the temperature, G is Newton's gravitational constant, $a_* = 4\sigma/c$ is the radiation density constant, $\sigma = 2\pi^5 k_B^3 / 15h^3 c^2$ is the Stefan–Boltzmann constant, and c is the speed of light. The right-hand expression gives the temperature of the universe in units of 10^9 K when the time, t , is measured in seconds from $t = 0$.

There are some interesting observations to be made about Equation (3), but perhaps the most important is that while the second equality, $T_9 = 10.4t^{-1/2}$, is correct, the first equality is *incorrect*. Most obviously, the dimensions are incorrect. Working in cgs units, the central expression in Equation (3) has units of $s^{-1} K^{-1}$, which is clearly inconsistent with T_9 having units of kelvin.

3. Correcting the Initial Conditions

Wagoner et al. (1967) state that details that are relevant to their derivation of Equation (3) are provided by Alpher et al. (1953). However, this earlier paper finds that temperature follows $T_9 = 15.2t^{-1/2}$, which has a different numerical factor from Equation (3). This discrepancy arises from the fact that Alpher et al. (1953) do not consider the presence of electrons or neutrinos at early times, and hence use $\rho = \rho_e = \rho_r$.

Directly solving Equation (2) we find that, for time in seconds,

$$T_9 = (48\pi a_* c^{-2} G)^{-1/4} t^{-1/2} = 10.4t^{-1/2}. \quad (4)$$

This expression is dimensionally correct, with the central equation having units K, and yields the same numerical factor as presented in Wagoner et al. (1967).

4. Conclusions

Normally, such a minor typographical error in a paper has no significant consequences and so can pass unnoted. However, the incorrect expression presented in Wagoner et al. (1967) has been propagated into other key publications, and numerical codes for calculating BBN. We highlight two of these below;

1. Kawano (1992) presents a fortran implementation of the integration of the coupled BBN equations named NUC123.³This directly reproduces the expression from Wagoner et al. (1967) as the initial condition for the integration (their Equation (D.6)). However, an examination of the fortran source code reveals that the initial time is set to be `t = 1/(const1*t9)**2 !Initial time (Ref 1) where PARAMETER (const1 = 0.09615) !Relation between time and temperature, so only the numerical aspect of Equation (3) is employed.`
2. Arbey et al. (2020) presents a C implementation of BBN known as AlterBBN.⁴They too cite Equation (3) in determining the initial conditions of their integration. Exploring the source code, the initial time is set to `double t = sqrt(12.*pi*G*sigma_SB)/pow(Ti,2.);` and so directly implements the incorrect form of the initial time. We note that in the natural (energy) units employed in AlterBBN results in a very early starting time, well into the high temperature limit, and so does not impact the resulting integration. We have discussed this with the AlterBBN author(s) and the initial conditions will be corrected in a forthcoming update (Arbey 2021, private communication).

This work was undertaken as part of Charlie Sharpe's honors project at the University of Sydney's School of Physics.

Footnotes

³ <https://github.com/ckald/KAWANO-sterile>

⁴ <https://alterbbn.hepforge.org>

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