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A Fuzzy Dynamic Programming for the Optimal Allocation of Health Centers in some Villages around Baghdad

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Abstract:

The Planning and Resource Development Department of the Iraqi Ministry of Health is very interested in improving medical care, health education, and village training programs. Accordingly, and through the available capabilities of the ministry, itdesires to allocate seven health centers to four villages in Baghdad, Iraq therefore the ministry needs to determine the number of health centers allocated to each of these villages which achieves the greatest degree of the overall effectiveness of the seven health centers in a fuzzy environment. The objective of this study is to use a fuzzy dynamic programming(DP) method to determine the optimal allocation of these centers, which allows the greatest overall effectiveness of these health centers to be achieved, which is the expected increase in the average life years in the village population in a fuzzy environment. The results of this studyareproved after a real-life problem was solved that the proposed method is an effective mathematical model for making a series of related decisions, and it provides us with a systematic procedure to determine the optimal combination of decisions.

Keywords: Dynamic Programming, Fuzzy Set,Optimal Allocation Problem,Ranking Function.

Introduction:

The Ministry of Health is one of the largest ministries in Iraq aiming to provide health and medical services to every Iraqi citizen during the natural and emergency conditions of the country. The health reality of the villages needs a set of important data, which in any case cannot be isolated from the fuzzy of the economic, social, environmental, educational, and cultural realities of these remote places in particular and for the country in general.The Iraqi environment is characterized by its being fuzzy in different aspects, which calls for use of the fuzzy logic in addressing instances of uncertainty imposed by the fuzzy environment.Using the fuzzy sets theory is for achieving the goals inspired from this study which it is hoped to contribute in providing proposed acceptable solutions to solve the problem of allocation these centers to find the greatest expected increase in the average of expectancy life in years for the residents of Baghdad's villages in this complex and successive fuzzy environment.

The Ministry of Health is very interested in improving medical care, health education, and village training programs. On this basis, and within the capabilities available to the ministry, it wishes to allocate seven health centers to four villages so that it achieves a great deal of overall effectiveness of the seven health centers in a fuzzy environment through the use of an effective quantitative method which is the dynamic programming method.

The dynamic programming (DP) method was developed by Richard Bellman in the 1950s and it has been applied in various fields, from aerospace engineering to economics. DPis a quantitative method that converts one large problem with many decision variables into a series of small problems with a few decision variables. Consequently, a large problem that is difficult to solve can be transformed into a series of small problems, which can be solved easily. As for the meaning of the word "programming" in the dynamic programming style, it refers to the mathematical concept of choosing the best allocation of resources, whereas the word dynamic refers to decisions taken in several stages. Daily, Weekly, etc.

Many studies have proposeda DP algorithm for different optimization problems.

Yakowitz 1982[1]conducted a survey whose main objective is to review the dynamic programming models of water source problems and to test the computational methods used to obtain solutions to these problems. The problem locations surveyed here include channel design, irrigation system control, project development, water quality maintenance well as tank operations analysis, innovative numerical methods for applying dynamic programming methods applied to water source problems.

Elmaghraby 1993[2] supposed that there is a relationship between the quantity of the source allocated to the activity and its duration. The incentive for this study is a recent development in the procedure that results in a smaller number of nodes that will be reduced in network activities.

Elmaghraby also proposed an elementary approximating procedure that provides the maximum optimization mathematically more economically. Chin 1995[3] proposed a new algorithm using fuzzy dynamic programming to determine the best location and size of compensation shunt capacitors for distribution systems with harmonic distortion.

Esogbue1999[4] presented how obscure dynamic programming and neural networks can be used to extend the process to other stages by linking the four stages of the medical hypothesis, physician's observation, preliminary analysis, and final analysis inan adroit approach.

Mahdavi and et al.2009[5] presented a dynamic programming method to find the fuzzy shortest chain in a graph with fuzzy distance for every edge using the appropriate ranking method. By using MATLAB, two descriptive examples are worked out to prove the suggested algorithm.Gagula-Palalic and Can 2012[6] used Bellman and Zadeh’s method of fuzzy dynamic programming. The problem studied the management of a company that needs to close down a specific plant within a certain time interval. Therefore manufacturing stages should be reduced to zero as smoothly as possible and the stock level at the end of the planning phase should be as low as possible. The demand is supposed to be deterministic. Yu S et al.2016[7] suggested using a discrete dynamic programming method to give an active solution to decide on the treatment project investment. Furthermore, a case study including the Laojuntang coal Mine in China was carried out on the investment problem in the processing project using the proposed model. The results indicated that the proposed model is active and appropriate for making environmental investment decisions at a perfect coal mine in terms of reducing the total losses.

Khalaf and Halim 2018[8] used the fuzzy dynamic programming method to find the fuzzy maximum flow for Imam Al-Kazim’s (AS) visitors on Rajab 25th on the anniversary of his martyrdom.The research problem emerged through a clear difference in the numbers of visitors during the same day and the clear increase in the number of visitors during the period of the visit, which culminated in the days 22nd-24th of Rajab. The ranking function was used to eliminate fuzzy in the number of visitors per day or three days, also, to using triangular membership function to find out the peak period of flow based on the days and times of the visit.

Nagalakshmi and Uthra 2018[9]used a new fuzzy method to find a fuzzy optimal subdivision problem where the positive quantity which is to be separated was taken as a trapezoidal fuzzy number. The result was obtained by the method of mathematical induction. The particular feature of this suggested method is that the fuzzy and inaccuracy in the optimal subdivision models is easily eliminated by fuzzy dynamic programming.Ahmadov and Gardashova 2019[10] suggested the fuzzy dynamic programming method to the multiphase control problem of the flash evaporator system. The essential feature of this method lies in its ability to control any kind of constraint.

Yazdi and et al 2019[11]proposed an approach for maintenance planning which assistedin increasing the reliability of the components and safety implementation in process facilities. This approachassistances design an optimal safety maintenance investment plan by combining the optimization methods and a fuzzy dynamic risk-based method.

Mohanaselvi and Suparna Mondal 2019[12] proposed an explanation to generalized trapezoidal fuzzy least cost route problem using fuzzy dynamic programming method. Utilizing the fuzzy forward and backward recursive equations, the fuzzy optimal solution is improved without converting them into an equivalent crisp problem. Using appropriate ranking and arithmetic operation in terms of parametric form, the minimized fuzzy cost is attained.

Wang and Zhang 2019[13] provided a novel method to develop the order-picking process which is the most time- and labour-concentrated activity in the procedure of order execution. Wang and Zhang built the mathematical model as an integer programming and improved a dynamic programming method to solve it. To assess the suggested method, a computational experiment is performed and the results are reported.

Shiono and et al2019[14]provided the optimization model with the aim of designing a pipe network. An underground pipe network for geographically must be prepared concerning the many possible roads below which pipes are put. The layout and pipe sizes must then be specified to reduce the cost of pipe network construction. Subsequently, they improved an exact algorithm depending on dynamic programming. The obtained results for a practical gas distribution network demonstrate that our approach offers an effective solution.

Jenkins and et al2020[15] specified the high-quality DPR plans that develop the performance of United States Army MEDEVAC systems and finally raise the fight victim survivability rate. A deducted, unlimited-horizon MDP model of the MEDEVAC DPR problem is designed and solved via an approximate dynamic programming (ADP) approach that uses a propped vector regression value function approximation scheme within an estimated plan iteration algorithmic framework. Eventually, this study notifies the improvement and application of future strategies, techniques, and procedures for military MEDEVAC procedures.

Summersand et al 2020[16] built the dynamic weapon target assignment model as a Markov decision process and used a simulation-based, approximate dynamic programming (ADP) method to solve model patterns based on a representative scenario. The aim ofefficient air defense is to determine the firing policy for interceptor allocation to incoming missiles that reduce the predicted entire damage to protect assets over a series of engagements.

The scarcity of the specialized health centers in the villages and the loss of the specialized medical staff led to a decrease in the level of medical and health awareness in these villages and the prevalence of ignorance and illiteracy among their inhabitants, and this helped to some extent to control the underdeveloped methods of dealing with diseases and the prevalence of concepts far away from science and the reality of health status and satisfactory among its inhabitants. The people of our homeland in villages and distant agricultural places, as well as places of a Bedouin nature, have been subjected to many pre-Islamic illusions, which have become somewhat unshakable convictions, but have reached the level of firm belief and thus the position of knowledge and knowledge of hostility and mockery of them.

Since the current Iraqi environment is characterized by a state of high uncertainty due to the conditions that the country is going through, therefore the research problem can be illustrated in the context of great problems that the Ministry of Health suffers from and which it has sometimes had to work in a fuzzy environment. The following questions can express the basic dimensions of the research's problem:

1. Does applying the fuzzy sets theory contribute to limiting high volatility in finding the optimal allocation of health centers to some villages in Baghdad?
2. Is it possible to reduce the levels of uncertainty for the ministry and ensure higher accuracy in estimating the greatest overall effectiveness of health centers for some villages in Baghdad when applying the fuzzy sets theory?
3. Did the use of the ranking function and its calculations contribute to defuzzification to determine the optimal allocation of health centers for some villages in Baghdad?
4. Does the use of the DPmethod contribute to determining the optimal allocation of these centers, which allows achieving the greatest overall effectiveness of these health centers, which is the expected increase in the average age of years for the residents of the village?

This research aims to find the optimal allocation of seven health centers to four villages in a fuzzy environment which contributes to achieving the greatest overall effectiveness of these health centers represented by the expected increase in the average life (in years) for residents of the four villages, where the theory of fuzzy sets will be applied to reduce the ministry's levels of uncertainty and ensure higher accuracy by using a DPmethod to build a very effective mathematical model for making a series of related decisions.

The proposed methods

This research is based on a set of assumptions, the most important of which are:

1. The ability to allocate a group of health centers (allocations) to a group of administrative units (villages) after removing the blurring of the greatest degree of effectiveness for health centers and these health centers can only take integer numbers.
2. The greatest degree of effectiveness of health centers in a fuzzy environment is related to the optimal allocation of health centers designated to some villages after removing mistiness from them.
3. The stages of work in allocating health centers are related to the number of villages so that

health centers allocated based on villages number, one village, two villages, and so on until seven health centersare allocated to achieve the greatest measure of their overall effectiveness.

4. The population of the four villages was 2500, 2950, 3270, and 4345, respectively.

Fuzzy set

Zadeh presentedthe fuzzy set theory in1[17]. The theory provided a mathematical method for dealing with imprecise notions and problems that have many feasible solutions. The following definitions of the fuzzy numbers and some essential arithmetic operations on them may be useful.

A fuzzy number $\tilde{A}$ is a Triangular-Fuzzy number represented by $(a_1, a, a_2)$and it's a membership function $\mu_{\tilde{A}}(x)$is given as follows [17,18]:

$$\mu_A(x, a_1, a, a_2) = \begin{cases} 0 & if \quad x < a_1 \\ \frac{x-a}{a-a_1} & if \quad a_1 \le x \le a \\ \frac{a_2-x}{a_2-a_1} & if \quad a \le x \le a_2 \\ 0 & if \quad x > a_2 \end{cases} \quad (1)$$

The α- cut off the triangular fuzzy number$\tilde{A}$= (a_1, a, a_2) is the closed interval $A_\alpha = [a_\alpha^L, a_\alpha^R] = [a_1 + (a - a_1)\alpha\, , a_2 + (a - a_2)\alpha], \alpha \in (0,1]$.Another representation of the triangular fuzzy number is $\tilde{A} = (a - \alpha, a, a + \beta)$. Where $a - \alpha = a_1$ and $a + \beta = a_2$ and $\tilde{A}$ can also be written in another formula as $\tilde{A} = (a, \alpha, \beta)$.

let $\tilde{A} = (a_1, \alpha_1\, , \beta_1)$ and $\tilde{B} = (a_2, \alpha_2\, , \beta_2)$ be two triangular fuzzy numbers. Then 1965(17),1990(18):

$$\tilde{A} \oplus \tilde{B} = (a_1, \alpha_1\, , \beta_1) \oplus (a_2, \alpha_2\, , \beta_2) = (a_1 + a_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$$

$$\tilde{A} \ominus \tilde{B} = (a_1, \alpha_1\, , \beta_1) \ominus (a_2, \alpha_2\, , \beta_2) = (a_1 - a_2, \alpha_1 + \beta_2, \beta_1 + \alpha_2)$$

Ranking Functions

An appropriate method for comparinfuzzy number is by using a ranking function [19,20]. A ranking function $\Re: F(R) \to R$, where $F(R)$ (a set of all fuzzy numbers defined on a set of real numbers), maps each fuzzy number into a real number of $F(R)$.

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers in $F(R)$, then

$(i)\tilde{a} \ge_{\Re} \tilde{b}$ if and only if $\Re(\tilde{a}) \ge \Re(\tilde{b})$

$(ii)\tilde{a} >_{\Re} \tilde{b}$ if and only if $\Re(\tilde{a}) > R(\tilde{b})$

$(iii)\tilde{a} =_{\Re} \tilde{b}$ if and only if $\Re(\tilde{a}) = \Re(\tilde{b})$

Let $\Re$ be any linear ranking function. Then,

$\tilde{a} \ge_{\Re} \tilde{b}$ if and only if $\tilde{a} - \tilde{b} \ge_{\Re} \tilde{0}$ if and only if $-\tilde{b} \ge_{\Re} - \tilde{a}$.

if$\tilde{a} \ge_{\Re} \tilde{b}$ and $\tilde{c} \ge_{\Re} \tilde{d}$, then $\tilde{a} + \tilde{c} \ge_{\Re} \tilde{b} + \tilde{d}$.

For a triangular fuzzy number $\tilde{a} = (a_1, a, a_2)$ or (a, α, β), the ranking function is given by $\Re(\tilde{a}) = \frac{1}{2}\int_0^1 (inf\ a_\alpha + sup\ a_\alpha)\ d_\alpha$, where a_α is α −cut on $\tilde{a}$. This reduces to:

$$\Re(\tilde{a}) = \frac{1}{4}(a_1 + 2a + a_2) or \left\{ a + \frac{(\beta - \alpha)}{4} \right\} \quad (2)$$

Then triangular fuzzy number $\tilde{a} = (a, \alpha, \beta)$ and $\tilde{b} = (b, y, \theta)$, have

$\tilde{a} \ge_{\Re} \tilde{b}$ if and only if

$$\Re(\tilde{a}) = \left[a + \frac{(\beta - \alpha)}{4}\right] \ge \left[b + \frac{(\theta - y)}{4}\right] = \Re(\tilde{b})$$

Characteristic of the DPmodel

The basic features that characterize the DP problems are presented here[21, 22]:

1. The problem can be divided into stages, with a policy decision required at each stage.
2. Each stage has a number of states associated with the beginning of that stage.
3. The effect of the policy decision at each stage is to transform the current state to a state associated with the beginning of the next stage (possibly according to a probability distribution).
4. The solution procedure is designed to find an optimal policy for the overall problem, i.e., a prescription of the optimal policy decision at each stage for each of the possible states. The solution procedure constructs a table for each stage (n) that prescribs the optimal decision (X_n^*) for each possible state (s).
5. Given the current state, an optimal policy for the remaining stages is independent of the policy decisions adopted in previous stages. Therefore, the optimal immediate decision depends on only the current state and not on how you got there. This is the principle of optimality for dynamic programming.
6. The solution procedure begins by finding the optimal policy for the last stage. The optimal policy for the last stage prescribes the optimal policy decision for each of the possible states at that stage.
7. A recursive relationship that identifies the optimal policy for stage n, given the optimal policy for stage $n + 1$, is available.

The advantages and disadvantages of the proposed method

The main advantages and disadvantages of the proposed method are as follows:

Advantages

a. Dynamic programming determines the optimal solution to a multivariate problem by dividing the problem into stages, with each stage having a sub-problem that aims to find the optimal value for only one variable. The characteristic feature is due to dealing with only one variable and is much easier to mathematically than dealing with all variables at the same time. The model consists of a set of consecutive equations that link the different stages of the original problem in a way that ensures that the best possible solution to the original problem ultimately includes all possible optimal solutions that were obtained when solving sub-problems of different stages.

b. For the various problems in areas such as the distribution of effort problem, scheduling employment levels, inventory, chemical engineering design, and control theory, DP is the only technique used to solve these problems.

c. It is well suited for multi-stage or multi-point or sequential decision process.

d. It is suitable for linear or non-linear problems, discrete or continuous variables, deterministic and probabilistic problems.

e. DP tries to exploit the properties (optimal substructure and overlapping subproblems) to give a reasonable solution that is better than exponential and factorial ones.

Disadvantages

a. There does not exist a standard mathematical formulation of the DP problems. Rather, DP is a general type of approach to problem solving, and the particular equations used must be developed to fit each situation.

b. Dividing the main big problem into sub-problems then finding the optimal solution for each sub-problem where the optimum solution of one sub problem is used as an input to the next sub problem, these procedures may consume the memory in the computer.

A Real Practical Example

The Department of Planning and Resource Development in the Ministry of Health wishes to allocate seven of the health centers to four of the villages to improve health care, health culture, and training programs. Therefore, the ministry needs to specify the number of health centers allocated (if possible) for each of those villages, which achieves the greatest amount of the overall effectiveness of the seven health centers.

The ministry is responsible for providing security and safety for all doctors and employees of health centers, and the number allocated from health centers to these villages must be an integer. The performance measure of these centers is a fuzzy number, and it represents the number of years of life added to a person (for a particular village, this scale equals the expected fuzzy increasein the years of average agefor the village's residents). Table 1shows the fuzzy number of additional estimated years of life from each possible allocation of health centers for residents of each of these villages. (Taking into consideration the population density, the social and economic level of the village's residents), knowing that the population of the four villages was 2500, 2950, 3270, and 4345 people respectively.

Table 1. Fuzzy number of additional years (in thousands) of estimated life for residents of each of these villages as a result of the possible allocation from their health centers

Health Centers	fuzzy number of additional estimated years of life			
	Villages			
	1	2	3	4
0	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
1	(4,5,6)	(2,3,4)	(3,4,5)	(5,6,7)
2	(7,8,9)	(4,5,6)	(5,6,7)	(8,9,10)
3	(9,10,11)	(7,8,9)	(6,7,8)	(11,12,13)
4	(11,12,13)	(8,9,10)	(10,11,12)	(12,13,14)
5	(13,14,15)	(15,16,17)	(13,14,15)	(14,15,16)
6	(14,15,16)	(16,17,18)	(17,18,19)	(18,19,20)
7	(15,16,17)	(17,18,19)	(18,19,20)	(19,20,21)

The basic terminology of the fuzzyDP method

The following terms need to be clarified as follows:

N = Total number of stages is (4) and represents the number of villages when formulating the fuzzy DP model.

n = Label for current stage($n = 1, 2, .., N$).

s_n = Current state for stage n, which represents the appropriate choice of "state of the system," which is the number of health centers that are still available for allocation to the remaining villages $(n, \dots, 4)$and so on, in stage 1 (village (1)) where all of these villages are taken into consideration for allocation and:

$s_1 = 7$

As for the value ofs_n for stage 2, 3, or 4 (village (2), (3), or (4)) it will be as follows:

$s_1 = 7, s_2 = 7 - x_1,\ s_3 = s_2 - x_2,\ s_4 = s_3 - x_3$

x_n= decision variable for stage n where ($n = 1,2, \dots, 7$) is the number of health centers assigned to stage (village) n.

$\tilde{p}_i(x_i)$ = Measure of the fuzzy performance of the allocation$x_i$from health centers to village i, i.e. the number of estimated additional hazy years of life from each possible allocation of health centers to the population of each of those villages.

x_n^* =Optimum decision for the current state s_n(note that $s_n$is given)

$\tilde{f}_n(s_n,\ x_n)$ = Fuzzy contribution of stages $(n, n + 1, \dots, N)$ to objective function if the system starts in the state $s_n$at stage n, an immediate decision is $x_n$and optimal decisions are made thereafter

$\tilde{f}^*{}_n(s_n)$ =$\tilde{f}_n(s_n, x_n^*)$

The recursive relationship will always be of the form:

$\tilde{f}^*{}_n(s_n) = \max_{x_n=0,1,\ldots,s_n} \tilde{f}_n(s_n, x_n)$

The proposed mathematical model

The goal of building a fuzzy mathematical model is to find the path from the initial state 7 (beginning of stage 1) to the last state 0 (after stage 4) which maximizes the sum of the fuzzy contributions along the path.

$$\left.\begin{aligned} &Maximize\tilde{Z} = \sum_{i=1}^{4} \tilde{p}_i(x_i) \\ &\text{S.to.} \\ &\sum_{i=1}^{n} x_i = 7 \\ &x_i \geq 0 \text{ and integers.} \end{aligned}\right\} \quad (3)$$

and $\tilde{f}_n(s_n, x_n)$ is:

$\tilde{f}_n(s_n, x_n)$ = $\tilde{p}_n(x_n) \oplus \sum_{i=n+1}^{4} \tilde{p}_n(x_i)$(4)

Since:

$$\tilde{f}^*{}_n(s_n) = \max_{x_n=0,1,\ldots,s_n} \tilde{f}_n(s_n, x_n) \quad (5)$$

$$\tilde{f}_n(s_n, x_n) = \tilde{p}_n(x_n) \oplus \tilde{f}^*{}_{n+1}(s_n - x_n) \quad (6)$$

Hence fuzzy iterative relationships associated with functions$\tilde{f}^*{}_1$,$\tilde{f}^*{}_2$, $\tilde{f}^*{}_3$, $\tilde{f}^*{}_4$for this problem are:

$$\tilde{f}^*{}_n(s_n) = \max_{x_n=0,1,\ldots,s_n} \{\tilde{p}_n(x_n) \oplus \tilde{f}^*{}_{n+1}(s_n - x_n)\} \quad (7)$$

for $n = 1, 2, 3, 4$

As for the fuzzy recursive relationshipof the last stage (n = 4)

$$\tilde{f}^*{}_4(s_4) = \max_{x_4=0,1,\ldots,s_4} \tilde{p}_4(x_4) \quad (8)$$

Solving the Fuzzy DP model

The steps for solving the fuzzy DP model are as follows:

1. Find the fuzzy number of additional years of live for the residents ofthe village (4), taking into account the number of possible health centers allocated to the village, and this is done after n = 4 as in (8), (this stepstart from the last stage where n = 4), notice that the fuzzy values$\tilde{p}_4(x_4)$are given in the last column of Table 1 and these values start with a fuzzy increase as the move is toward the bottom of the column. Therefore, with $s_4$health centers still designated tothe village (4), the fuzzy optimization$\tilde{p}_4(x_4)$is possible if it is obtained automatically by allocating all health centers s_4, sox_4^*=s_4 and$\tilde{f}^*{}_4(s_4)$ = $\tilde{p}_4(s_4)$as shown in Table 2:

Table 2. Fuzzy number of additional years of life for the residents of the village (4)

s_4	$\tilde{f}^*{}_4(s_4)$	x_4
0	(0,0,0)	0
1	(5,6,7)	1
2	(8,9,10)	2
3	(11,12,13)	3
4	(12,13,14)	4
5	(14,15,16)	5
6	(18,19,20)	6
7	(19,20,21)	7

2. Depend on the arithmetic operations of triangular-fuzzy number by applying Eq. (1), the fuzzy numbers of additional years of life are found for the residents of village (3) taken into consideration the number of potential health centers allocated to village (3) and what was allocated to village (4) by applying the fuzzy recursive relationship forDP method after applying n = 3in Eq. (6) as shown in Table 3.

Table 3. Fuzzy number of additional years of life for residents of the village (3)

	$\tilde{f}_3(s_3, x_3)$= $\tilde{p}_3(x_3) \oplus \tilde{f}^*{}_4(s_3 - x_3)$							
x_3 / s_3	0	1	2	3	4	5	6	7
0	(0,0,0)							
1	(5,6,7)	(3,4,5)						
2	(8,910)	(8,10,12)	(5,6,7)					
3	(11,12,13)	(11,13,15)	(10,12,14)	(6,7,8)				
4	(12,13,14)	(14,16,18)	(13,15,17)	(11,13,15)	(10,11,12)			
5	(14,15,16)	(15,17,19)	(16,18,20)	(14,16,18)	(15,17,19)	(13,14,15)		
6	(18,19,20)	(17,19,21)	(17,19,21)	(17,19,21)	(18,20,22)	(18,20,22)	(17,18,19)	
7	(19,20,21)	(21,23,25)	(19,21,23)	(18,20,22)	(21,23,25)	(21,23,25)	(22,24,26)	(18,19,20)

3. Find the fuzzy number of the optimal additional years and the optimal fuzzy number of potential health centers for the allocation of village (3) by applying the fuzzy recursive relationship ofthe DP method as in Eq. (7), as shown in Table 4.

Table 4. Fuzzy number of additional years and the optimal fuzzy number of potential healthcenters for the allocation of the village (3)

	$\tilde{f}^*{}_3(s_3) = \max_{x_3=0,1,\dots,s_3} \{\tilde{p}_3(x_3) \oplus \tilde{f}^*{}_{3+1}(s_3-x_3)\}$								$\tilde{f}_3(s_3)$	x_3*
x_3 / s_3	0	1	2	3	4	5	6	7		
0	(0,0,0)								(0,0,0)	0
1	(5,6,7)	(3,4,5)							(5,6,7)	0
2	(8,910)	(8,10,12)	(5,6,7)						(8,10,12)	1
3	(11,12,13)	(11,13,15)	(10,12,14)	(6,7,8)					(11,13,15)	1
4	(12,13,14)	(14,16,18)	(13,15,17)	(11,13,15)	(10,11,12)				(14,16,18)	1
5	(14,15,16)	(15,17,19)	(16,18,20)	(14,16,18)	(15,17,19)	(13,14,15)			(16,18,20)	2
6	(18,19,20)	(17,19,21)	(17,19,21)	(17,19,21)	(18,20,22)	(18,20,22)	(17,18,19)		(18,20,22)	4 or 5
7	(19,20,21)	(21,23,25)	(19,21,23)	(18,20,22)	(21,23,25)	(21,23,25)	(22,24,26)	(18,19,20)	(22,24,26)	6

4. Use the ranking function to convert the fuzzy number of additional years of life for residents of the village (3) as well as converting the fuzzy number of optimal additional years and the fuzzy optimal number of health centers from its allocation to village (3) to the crisp number by applying Eq. (2) as shown in Table 5.

Table 5. Fuzzy and crisp numbers of the optimal additional years of life for residents of the village (3) and the optimal fuzzy number of potential health centers for allocation to the same village.

	$\tilde{f}^*{}_3(s_3) = \max_{x_3=0,1,\dots,s_3} \{\tilde{p}_3(x_3) \oplus \tilde{f}^*{}_{3+1}(s_3-x_3)\}$								$\tilde{f}_3(s_3)$	x_3*
x_3 / **s_3**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**		
0	(0,0,0) *Rank=0*								(0,0,0) *Rank=0*	0
1	(5,6,7) *Rank=6*	(3,4,5) *Rank=4*							(5,6,7) *Rank=6*	0
2	(8,910) *Rank=9*	(8,10,12) *Rank=10*	(5,6,7) *Rank=6*						(8,10,12) *Rank=10*	1
3	(11,12,13) *Rank=12*	(11,13,15) *Rank=13*	(10,12,14) *Rank=12*	(6,7,8) *Rank=7*					(11,13,15) *Rank=13*	1
4	(12,13,14) *Rank=13*	(14,16,18) *Rank=16*	(13,15,17) *Rank=15*	(11,13,15) *Rank=13*	(10,11,12) *Rank=11*				(14,16,18) *Rank=16*	1
5	(14,15,16) *Rank=15*	(15,17,19) *Rank=17*	(16,18,20) *Rank=18*	(14,16,18) *Rank=16*	(15,17,19) *Rank=17*	(13,14,15) *Rank=14*			(16,18,20) *Rank=18*	2
6	(18,19,20) *Rank=19*	(17,19,21) *Rank=19*	(17,19,21) *Rank=19*	(17,19,21) *Rank=19*	(18,20,22) *Rank=20*	(18,20,22) *Rank=20*	(17,18,19) *Rank=18*		(18,20,22) *Rank=20*	4 or 5
7	(19,20,21) *Rank=20*	(21,23,25) *Rank=23*	(19,21,23) *Rank=21*	(18,20,22) *Rank=20*	(21,23,25) *Rank=23*	(21,23,25) *Rank=23*	(22,24,26) *Rank=24*	(18,19,20) *Rank=19*	(22,24,26) *Rank=24*	6

5. Find the fuzzy numbers of additional years of life for the residents of village (2) taken into consideration the number of potential health centers allocated to village (2) and what was allocated to village (3) by applying the fuzzy recursive relationship forDP method after applying n = 2 in Eq. (6) as shown in Table 6.

Table 6. Fuzzy number of additional years of life for residents of village (2)

x_2 / s_2	$\tilde{f}_2(s_2, x_2)= \tilde{p}_2(x_2)\oplus\tilde{f}^*{}_3(s_2 - x_2)$ 0	1	2	3	4	5	6	7
0	(0,0,0)							
1	(5,6,7)	(2,3,4)						
2	(8,10,12)	(7,9,11)	(4,5,6)					
3	(11,13,15)	(10,13,16)	(9,11,13)	(7,8,9)				
4	(14,16,18)	(13,16,19)	(12,15,18)	(12,14,16)	(8,9,10)			
5	(16,18,20)	(16,19,22)	(15,18,21)	(15,18,21)	(13,15,17)	(15,16,17)		
6	(18,20,22)	(18,21,24)	(18,21,24)	(18,21,24)	(16,19,22)	(20,22,24)	(16,17,18)	
7	(22,24,26)	(20,23,26)	(20,23,26)	(21,24,27)	(19,22,25)	(23,26,29)	(21,23,25)	(17,18,19)

Repeat step (3, 4) to find the number of additional years of life for the residents of village (2) as well as the number of optimal additional years and the optimal number of health centers from the allocation ofvillage (2)as shown in Table 7.

Table 7. Fuzzy and crisp numbers of the optimal additional years of life for residents of the village (2) and the optimal fuzzy number of potential health centers for allocation to the same village.

x_2 / s_2	$\tilde{f}^*{}_2(s_2) = \max_{x_2=0,1,\dots,s_2}\{\tilde{p}_2(x_2) \oplus\tilde{f}^*{}_{2+1}(s_2-x_2)\}$ 0	1	2	3	4	5	6	7	$f_2^*(s_2)$	x_2^*
0	(0,0,0) *Rank=0*								(0,0,0) *Rank=0*	0
1	(5,6,7) *Rank=6*	(2,3,4) *Rank=3*							(5,6,7) *Rank=6*	0
2	(8,10,12) *Rank=10*	(7,9,11) *Rank=9*	(4,5,6) *Rank=5*						(8,10,12) *Rank=10*	0
3	(11,13,15) *Rank=13*	(10,13,16) *Rank=13*	(9,11,13) *Rank=11*	(7,8,9) *Rank=8*					(11,13,15) *Rank=13*	0 or 1
4	(14,16,18) *Rank=16*	(13,16,19) *Rank=16*	(12,15,18) *Rank=15*	(12,14,16) *Rank=14*	(8,9,10) *Rank=9*				(14,16,18) *Rank=16*	0 or 1
5	(16,18,20) *Rank=18*	(16,19,22) *Rank=19*	(15,18,21) *Rank=18*	(15,18,21) *Rank=18*	(13,15,17) *Rank=15*	(15,16,17) *Rank=16*			(16,19,22) *Rank=19*	1
6	(18,20,22) *Rank=20*	(18,21,24) *Rank=21*	(18,21,24) *Rank=21*	(18,21,24) *Rank=21*	(16,19,22) *Rank=19*	(20,22,24) *Rank=22*	(16,17,18) *Rank=17*		(20,22,24) *Rank=22*	5
7	(22,24,26) *Rank=24*	(20,23,26) *Rank=23*	(20,23,26) *Rank=23*	(21,24,27) *Rank=24*	(19,22,25) *Rank=22*	(23,26,29) *Rank=26*	(21,23,25) *Rank=23*	(17,18,19) *Rank=18*	(23,26,29) *Rank=26*	5

Table 8 of village (1) is calculated in similar aforementioned steps.

Table 8. Fuzzy and crisp numbers of the optimal additional years of life for residents of the village (1) and the optimal fuzzy number of potential health centers for allocation to the same village.

	$\tilde{f}^*_1(s_1) = \max_{x_1=0,1,\ldots,s_1} \{\tilde{p}_1(x_1) \oplus \tilde{f}^*_{1+1}(s_1 - x_1)\}$								$f_1^*(s_1)$	x^*_1
x_1	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**		
s_1										
7	(23,26,29) *Rank=26*	(24,27,30) *Rank=27*	(23,27,31) *Rank=27*	(23,26,29) *Rank=26*	(22,25,28) *Rank=25*	(21,24,27) *Rank=24*	(19,21,23) *Rank=21*	(15,16,17) *Rank=16*	(24,27,30) *Rank=27*	1or 2

Figure 1 illustrates aDP method for determining the optimal allocation of health centers through the optimal decision policy shown in dark black lines, which allows the greatest overall effectiveness of these centers to be achieved and it is the expected increase in the average age in years for the inhabitants of the four villages after eliminating the blurring by using the ranking function.

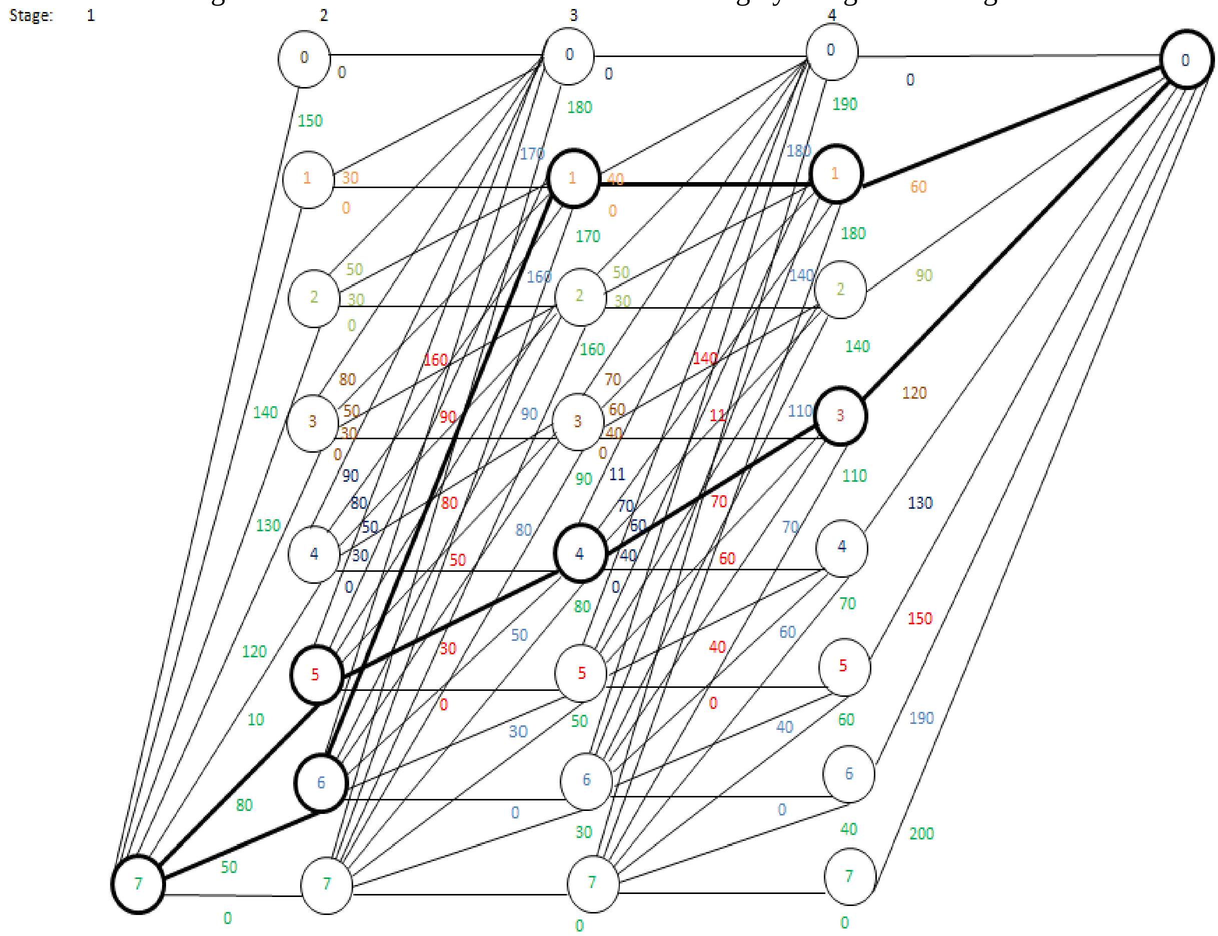

Fig.1. Solving the problem of allocating seven health centers to the four villages and the optimal paths

Discussion of Results:

Achieving the greatest overall effectiveness of the seven health centers is by determining the optimal number of health centers assigned to each of these villages, where the measure of performance of these centers is a fuzzy number and represents the number of years of life added to a person (for a particular village, this scale is equal to the expected fuzzy increase in life expectancy in years). The optimal solution for the first stage in Table 8 requires that one or two health centers be allocated, i.e.$X_1^* = 1$ or 2, which indicates that an alternative optimal solution exists as shown in Table 9.

If the optimal allocation is one health center, the fuzzy number of years added to the lives of the residents of village (1) will be (4000, 5000, 6000), after using the ranking function as in Eq. (2) to eliminate fuzziness, the addition will be (5000) years of life for the residents of village (1) i.e. adding 2 years for every person of its population. The rest of the health centers in the second stage is $S_2 = 7 - 1 = 6$, so the optimal allocation of health centers for the second stage is five health centers

($X_2^* = 5$), (see Table 7).As a result of this allocation the fuzzy number of years added to the lives of the residents of village (2) will be (15000,16000,17000), after applying Eq. (2) the added years of life for the inhabitants of village (2) will be 16,000, i.e. adding 5.4 years for each person of its population.

Accordingly, the remaining health centers in the third stage is$S_3 = 6 - 5 = 1$. Therefore, the optimal allocation of health centers to the third stage and after eliminating fuzziness is$X_3^* = 0$ (see Table 5), i.e. no health center is allocated in this stage, as a result of this allocation the fuzzy number of years added to the lives of the residents of village (2) will be (0, 0, 0), after applying Eq. (2). The added years of life for the inhabitants of village (3) will be 0.

The remaining health centers in the fourth stage is$S_4 = 1 - 0 = 1$, so the optimal allocation of health centers for the fourth stage is$X_4^* = 1$ (see Table 2), i.e. allocating one health center to the fourth stage and the result of this allocation after applying Eq. (2) is an additional 6000 years of life for the inhabitants of village (4), i.e. adding 1.4 years for each person of its population, so the sum of the additional years of life for the population of the four villages is 27,000 additional years of life, which represents the optimal allocation as shown in Table 10.

Table 9. The alternative optimal allocation of health centers tovillages in addition to the added years of life for residents of thosevillages as a result of this allocation

An alternative optimal solution				
The stage	Optimal allocation From health centers for the stages	the fuzzy number of years added to the lives of the residents of the village	The number of years added to the lives of the residents of the village after eliminating the fuzziness	Added years for each person from village residents
First stage Village (1)	2	(7000, 8000, 9000)	8000	3.2
Second stage Village (2)	1	(2000,3000,4000)	3,000	1
Third stage Village (3)	1	(3000,4000,5000)	4000	1.2
Fourth stage Village (4)	3	(11000,12000,13000)	12000	2.7

Table 10. The optimal allocation of health centers tovillages in addition to the added years of life for residents of thosevillages as a result of this allocation

The optimal allocation				
The stage	Optimal allocation from health centers for the stages	the fuzzy number of years added to the lives of the residents of the village	The number of years added to the lives of the residents of the village after eliminating the fuzziness	Added years for each person from village residents
First stage Village (1)	1	(4000, 5000, 6000)	5000	2
Second stage Village (2)	5	(15000,16000,17000)	16,000	5.4
Third stage Village (3)	0	(0,0,0)	0	0
Fourth stage Village (4)	1	(5000,6000,7000)	6000	1.4

Conclusion:

The lack of specialized health centers in the villages led to a decrease in the level of medical and health awareness in these villages and the prevalence of ignorance and illiteracy among their residents and the prevalence of concepts far away from science and the reality of health status.In addition, the current Iraqi environment is characterized by a state of high uncertainty due to the conditions that the country is going through, therefore the research problem can be illustrated in the context of great problems that the Ministry of Health suffers which is sometimes forced to work in a fuzzy environment. Applying the fuzzy sets theory is to reduce the levels of uncertainty for the Ministry of Health and ensure higher accuracy in estimating the greatest overall effectiveness of health centers for some villages in Baghdad. The greatest degree of effectiveness of health centers in a fuzzy environment is related to the optimal allocation of health centers designated to some villages after the removal of mistiness from them. Ranking function and its calculations contribute todefuzzification to determine the optimal allocation of health centers for some villages in Baghdad. The fuzzy dynamic programming method is proposed to find the optimal allocation of seven health centers to four of the villages, which allowed the greatest overall effectiveness of these centers to be achieved, which is the expected increase in the average life years in the village population in a fuzzy environment.

The results of this research proved that the proposed (DP) method is a very effective mathematical model for making a series of related decisions, and it provides us with a systematic procedure for determining the optimal combination of decisions.The DP method gave two optimal allocations.These two optimal allocations achieve the greatest overall effectiveness of the seven health centers, so the DP method can be considered one of the important mathematical methods in determining the optimal allocation.The first is represented by giving village (1) one health center, village (2) five health centers, not to give village (3) any health center, while assigned one health center to the village (4). As for the alternative optimal allocation, it is represented by giving village (1) two health centers, village (2) one health center, village (3) one health center, while allocating three health centers tovillage (4).

Utilization of theDP method to find the optimal allocation helped to add the following:
2 years of life for each person in village (1), 5.4 years of life for every person in village (2), no year of life is added to the residents of village (3) and 1.4 years of life for every person in village (4). As for the optimal alternative allocation, it helped to add the following: 3.2 years of life for each person in village (1), 1 year of life for every person in village (2), 1.2 years of life for every person in village (3) and 2.7 years of life for every person in village (4).

As a future research direction, we can conclude that the proposed method can be applied in the important field such as the Iraqi armed forces, for example:

- Maximize the performance of the Iraqi armed forces through the dynamic programming method.
- Determine the optimal path of the Iraqi armed forces using the dynamic programming method in a fuzzy environment.
- Determine the size of the Iraqi Special Armed Forces be allocated to each stage of the battle.

Author's declaration:

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Besides, the Figures and images, which are not mine, have been given the permission for re-publication attached with the manuscript.
- The author has signed an animal welfare statement.
- Author sign on ethical consideration’s approval
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors' contributions statement:

Wakas Saad Khalaf contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript

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البرمجة الديناميكية الضبابية للتخصيص الأمثل للمراكز الصحية لبعض القرى في بغداد

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الخلاصة:
دائرة التخطيط وتنمية الموارد في وزارةالصحة مهتمة جدا فيتحسينالرعاية الطبية، الثقافة الصحية وبرامج التدريب للقرى وعلى هذا الأساس وضمن الإمكانيات المتاحة للوزارة فإنها ترغب في تخصيص سبعة مراكز صحية لاربعقريفي بغداد، العراق، لذلك فإن الوزارةتحتاج إلتحديد عددالمراكز الصحية المخصصة لكل واحدة من تلك القريوالذي يحقق أعظم قدر من الفعاليةالإجمالية للمراكز الصحية السبعة في بيئة ضبابية.الهدفمن هذه الدراسةهو استعمال أسلوب البرمجة الديناميكية الضبابية لتحديدالتخصيص الأمثل لهذه المراكز والذي يسمح بتحقيق أعظم قدرمن الفعاليةالإجمالية لهذهالمراكز الصحية والمتمثلة بالزيادة المتوقعة في متوسط العمر بالسنوات لسكان القريةفي بيئة ضبابية. وقدأثبتت نتائج الدراسة بعد أن تم حلمشكلة من واقع الحياة أن الأسلوب المقترحهو نموذجحسابيفعّال جدالصنعسلسلة منالقراراتالمترابطة، كماأنه يزودنا بإجراءمنهجيلتحديدالتوليفة المثلىمنالقرارات.

الكلمات المفتاحية: مشكلة التخصيص الأمثل، البرمجة الديناميكية، المجموعات الضبابية، دالة الرتب.