Local Dependence for Bivariate Weibull Distributions Created by Archimedean Copula

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Abstract:

In multivariate survival analysis, estimating the multivariate distribution functions and then measuring the association between survival times are of great interest. Copula functions, such as Archimedean Copulas, are commonly used to estimate the unknown bivariate distributions based on known marginal functions. In this paper the feasibility of using the idea of local dependence to identify the most efficient copula model, which is used to construct a bivariate Weibull distribution for bivariate Survival times, among some Archimedean copulas is explored. Furthermore, to evaluate the efficiency of the proposed procedure, a simulation study is implemented. It is shown that this approach is useful for practical situations and applicable for real datasets. Moreover, when the proposed procedure implemented on Diabetic Retinopathy Study (DRS) data, it is found that treated eyes have greater chance for non-blindness compared to untreated eyes.

Keywords: Archimedean copula, Bivariate distribution, Local Dependence, Survival Analysis, Weibull distribution.

Introduction:

A copula function is a rule which gathers or couples one-dimensional marginal distribution functions into a form of multivariate distribution function. In the last century, copulas had an important role in several areas of statistics. Fisher (1), discussed the importance of copula precisely in his transcripts in the Encyclopedia of Statistical Sciences, "Copulas are of interest to statisticians for two main reasons; first, as a way of studying scale-free measures of dependence; and secondly, as a starting point for constructing families of bivariate distributions". For more details about copula models see (2).

One of the most popular families of copulas is the Archimedean Copula, which is an easy function to handle, simple and closed-form expression (3, 4, 5, 6). Furthermore, over the years, it has been successfully applied in many fields of research studies (7, 8, 9, 10).

In order to identify the most appropriate copula model, tail dependence coefficient has been applied by many researchers (11, 12, 13, 14, 15 and 16). It is a simple technique used for measuring the dependence between variables (associated pairs of data) in the tail of the multivariate distribution. Actually, it measures only the dependence at the extreme data values and ignores the others. To overcome the deficiency of the tail dependence, Esa and Dimitrov (17) introduced a new technique called local dependence in which the dependency is measured at every point on the distribution surface including extreme points.

In this paper, to drive a bivariate Weibull function, Archimedean copulas are used. By using Weibull marginal distributions three different bivariate Weibull functions are constructed. These functions can be used for analyzing multi-dimensional problems such as survival or reliability analysis. In order to identify the best copula, a correlate Weibull random variable is generated to compute the Local Dependence for these copulas. Hence, the results can provide a clear guideline for selecting the best copula model and then a proper bivariate Weibull distribution.

Bivariate Weibull Distribution

The Archimedean copula is a convenient method to model a bivariate distribution due to its simple form and a variety of dependence

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structures. Therefore in this section three different models of Archimedean have been considered to derive bivariate Weibull distribution (BWD); Gumbel copula, Clayton copula (aka Cook and Johnson's copula) and Independent (or Product) copula, with association parameter θ which is given by Kendal tau (τ) (2). These Archimedean copulas C(u, v;θ) are defined as follows:

Gumbel's copula: $C(u, v; \theta) = \exp\{-[(-\log u)^{\theta} + (-\log v)^{\theta}]^{1/\theta}\}$ where $\tau = \frac{1}{1-\theta}$

Clayton's copula: $C(u, v; \theta) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$ where $\tau = \frac{\theta}{1-\theta}$

and **Independent copula:** $C(u, v, \theta) = u.v$

Now, to construct bivariate Weibull functions, let T_1, T_2 be two Weibull random variables with $\lambda_1, \alpha_1, \lambda_2, \alpha_2$ scale and shape parameters respectively. Then the marginal distribution functions are

$$F_1(t_1) = 1 - Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right] \quad \& \quad F_2(t_2) = 1 - Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right] \quad 0 < t_1, t_2 < \infty$$

and survival distribution functions are

$$S_1(t_1) = Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right] \quad \& \quad S_2(t_2)$$
$$= Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right] \quad 0 < t_1, t_2 < \infty$$

Then, the BWFs are derived from:

1- Gumbel's formula, the BWD is defined as:

$$F(t_1, t_2) = \exp\{-[(-\log(1 - Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right]))^{\theta} + (-\log(1 - Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right]))^{\theta}]^{\frac{1}{\theta}}\}$$

and bivariate survival is defined as

$$S(t_1, t_2) = \exp\{-[(-\log(Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right]))^{\theta} + (-\log(Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right]))^{\theta}]^{\frac{1}{\theta}}$$

2- Clayton's formula, the BWD and bivariate survival are defined as:

$$F(t_1, t_2) = ((1 - Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right])^{-\theta} + (1 - Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right])^{-\theta} - 1)^{\frac{-1}{\theta}}$$

$$S(t_1, t_2) = ((Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right])^{-\theta} + (Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right])^{-\theta} - 1)^{\frac{-1}{\theta}}$$

3- The Independent, BWD and survival distribution are

$$F(t_1, t_2) = \left(1 - Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right]\right)\left(1 - Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right]\right)$$

$$S(t_1, t_2) = \left(Exp\left[-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\right]\right)\left(Exp\left[-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\right]\right).$$

The following figures (Fig. 1) show Weibull distributions and their copulas for W (2, 1.5), W (3,1.7)

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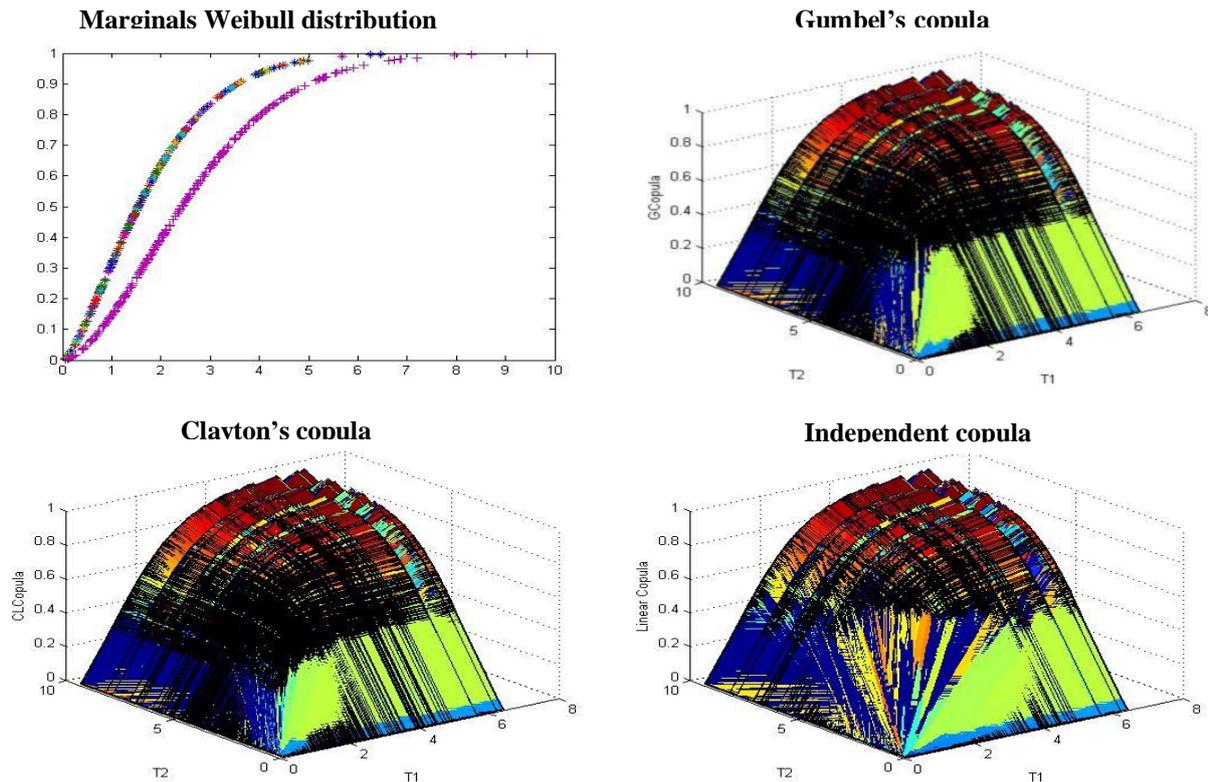
Figure 1. Marginal Weibull distributions and their bivariate Weibull distributions

Local Dependence

The measure of local dependence can be derived from the sources of probability theory. Esa and Dimitrov (17) have developed the idea of how probability tools can be used to measure strength of dependence between random events and then defined regression coefficients for measuring the magnitude of local dependence between random variables.

The most informative measures of dependence between random events are two regression coefficients defined by:

Definition1. Regression coefficient $r_B(A)$ of the event A with respect to the event B is the difference between the conditional probability for the event A given the event B, and the conditional probability for the event A given the complementary event B, namely

$$r_B(A) = Pr(A\backslash B) - P(A\backslash B^C)$$
$$= \frac{\Pr(A \cap B) - \Pr(A)\Pr(B)}{\Pr(B)(1 - pr(B))}$$

Similarly the regression coefficient $r_A(B)$ of the event B with respect to the event A, given

$$r_A(B) = Pr(B\backslash A) - P(B\backslash A^C)$$
$$= \frac{\Pr(A \cap B) - \Pr(A)\Pr(B)}{\Pr(A)(1 - pr(A))}$$

Definition2. Local Correlation coefficient between two events A and B is defined by

$$\rho_{AB} = \pm\sqrt{r_B(A).r_A(B)}$$

Moreover, these measures allow studying the behavior of interaction between any pair of numeric random variables (T_1, T_2) throughout the sample space.

Let the joint distribution function be $F(t_1,t_2)=P(T_1<t_1, T_2<t_2)$, and the marginal distribution function is defined as $F_1(t_1)=P(T_1 < t_1)$, $F_2(t_2)=P(T_2<t_2)$.

Let introduce the events A= $\{t_1 \leq T_1 \leq t_1 + \Delta_1 t_1\}$, B= $\{t_2 \leq T_2 \leq t_2 + \Delta_2 t_2\}$, for any $t_1, t_2 \in (-\infty, \infty)$.

Then the measures of dependence between events A and B turn into a measure of local dependence between the pair of r.v.'s T_1 and T_2 on the rectangle D=$[t_1, t_1 + \Delta_1 t_1] \times [t_2, t_2 + \Delta_2 t_2]$. Naturally, they are computed as follows:

$$r_{T_2}((T_1, T_2) \in D)$$
$$= \frac{\Delta_D F(t_1, t_2) - [F_1(t_1 + \Delta_1 t_1) - F_1(t_1)][F_2(t_2 + \Delta_2 t_2) - F_2(t_2)]}{[F_1(t_1 + \Delta_1 t_1) - F_1(t_1)]\{1 - [F_1(t_1 + \Delta_1 t_1) - F_1(t_1)]\}}$$

Similarly for

$$r_{T_2}((T_1, T_2) \in D)$$
$$= \frac{\Delta_D F(t_1, t_2) - [F_1(t_1 + \Delta_1 t_1) - F_1(t_1)][F_2(t_2 + \Delta_2 t_2) - F_2(t_2)]}{[F_2(t_2 + \Delta_2 t_2) - F_2(t_2)]\{1 - F_2(t_2 + \Delta_2 t_2) - F_2(t_2)]\}}$$

Where

$$\Delta_D F(t_1, t_2) = F(t_1 + \Delta_1 t_1, t_2 + \Delta_2 t_2)$$
$$- F(t_1 + \Delta_1 t_1, t_2)$$
$$- F(t_1, t_2 + \Delta_2 t_2) + F(t_1, t_2)$$

Hence, from Definition2, the Local Correlation will be as follow:

$$\rho_{T_1 T_2} = \pm \sqrt{r_{T_2}\big((T_1, T_2) \in D\big) . r_{T_1}\big((T_1, T_2) \in D\big)}$$

Simulation and Results
Generate the correlate bivariate Weibull random variable

In multivariate analysis, there are always difficulties in generating the correlated random variables for most types of distribution functions, except for normal function. Although the Exponential and Weibull distributions have important characteristics in lifetimes (survival or reliability) analysis, but their multivariate distribution functions cannot be directly defined.

Therefore, to construct a multivariate distribution function for Exponential or Weibull random variables copulas are mostly used. For testing such functions, Nelsen (2) has discussed several methods for generating numbers from copulas. Whilst, these approaches are not the best methods for testing their performance as the data obtained from a particular copula will favor that copula over the others. In this paper, a simple technique is used to generate a correlated Weibull random variable via bivariate Normal random variable. The main procedure is described below:

If $X \sim N(0,1)$ then $X^2 \sim \chi^2_{(1)}$

Now, if X_1, X_2 are two independent Normal variables then it can be shown that Nelsen (2):

$$T = \frac{X_1^2 + X_2^2}{2} \sim Exp(1) \ , \ T_1 = \lambda_1 T^{\frac{1}{\alpha_1}} \sim Web(\lambda_1, \alpha_1)$$

Similarly, T_2 can be derived from two independent normal variables $Y_1, Y_2 \sim N(0,1)$ Then, $T_2 \sim Web(\lambda_2, \alpha_2)$. Therefore, if (X_1, Y_1) and (X_2, Y_2) are generated from bivariate Normal distribution $((X_1, Y_1), (X_2, Y_2)) \sim N(\mu, \Sigma)$ where $\mu = (0,0)$ & $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ for any $0 < \rho < 1$, then a bivariate Weibull random variable (T_1, T_2), with some kind of dependence, can be obtained. To generate bivariate Weibull random variables with $\lambda_1, \alpha_1, \lambda_2, \alpha_2$ from $BivN(\mu, \Sigma)$, different sizes (250, 500, 1000) are considered. By using Matlab 10, the following results, in Table 1, are obtained.

Table 1. Bivaraite N(μ, Σ) random variables

Size (n)	BivN(μ, Σ)	
	$\mu = (0,0)$	$\Sigma = \begin{pmatrix} 1 & .9 \\ .9 & 1 \end{pmatrix}$
250	-0.0121, 0.0287	1.000 .919 .919 1.000
	0.0353, 0.0420	1.000 .897 .897 1.000
500	-0.0158, 0.0141	1.000 .903 .903 1.00
	0.0075, -0.0289	1.000 .889 .889 1.000
1000	-0.0057, -0.0140	1.000 .907 .907 1.000
	-0.0480, -0.0344	1.000 .904 .904 1.000

Then, the correlate Exponential and Weibull R.V. s with correlation can be generated. The main results are shown in Table2.

Table 2. Exponential and Weibull Random variables

Size (n)	Exponential R.V.		Weibull R. V.		
	$\lambda = (1,1)$	Σ	$\lambda_1 = 2, \alpha_1 = 1.5$	$\lambda_1 = 3, \alpha_1 = 2.5$	Σ
250	0.9830, 0.9862	1.000 .786 .786 1.000	2.0027 1.4501	2.9834 2.3492	1.000 .794 .794 1.000
500	0.9911, 1.0345	1.000 .829 .829 1.000	2.0121 1.5006	2.9473 2.4413	1.000 .796 .796 1.000
1000	1.0089, 0.9992	1.000 .799 .799 1.000	2.0046 1.5455	3.0118 2.5567	1.000 .781 .781 1.000

To show that the generated random samples are distributed as Weibull, PPplot is applied to test these samples for different sample sizes (250, 500 and 1000), The main outcomes are displayed in Fig.2.
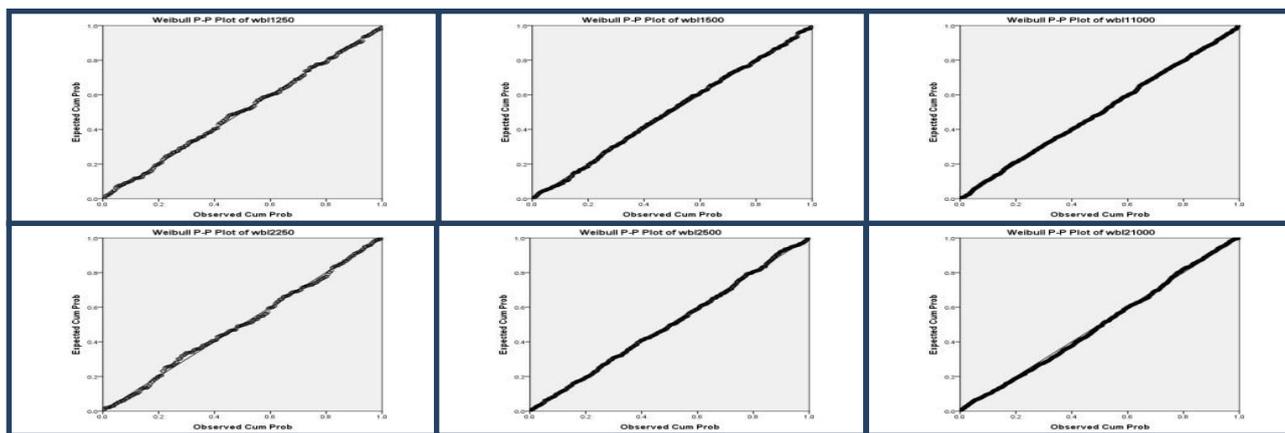
Figure 2. PPplot for W1 and W2 R.Vs for size 250, 500, 1000

Local Dependence for bivariate Weibull distribution

In this part, when the Weibull R.V.s are generated, and $\Delta_1 t_1, \Delta_2 t_2$ discussed in section 3, are computed where $\Delta_1 t_1 = \frac{range\ of\ R.V\ of\ T_1}{10}$ and

$\Delta_2 t_2 = \frac{range\ of\ R.V\ of\ T_2}{10}$, the local $\rho_{xy}(x,y)$ for each copula (Gumbel, Clayton and Independent) is determined, then the range of local correlation $\rho_{xy}(x,y)$ is evaluated. The main results are shown in Table3:

Table 3. Correlate Weibull R.V. with sample Size=250 for different sets of Parameters

$\lambda_1, \lambda_2,$ α_1, α_2	$\hat{\lambda}_1, \hat{\lambda}_2,$ $\hat{\alpha}_1, \hat{\alpha}_2$	Tau	Interval Correlation		
			Gumbel's	Clayton's	Independence
2, 3 , 0.5, 0.7	1.963, 2.971, 0.463, 0.656	0.7135	1.4e-5<p<0.7568	7.2e-6<p<0.5865	0<p<2.3e-13
2, 3, 0.5, 2	1.99, 3.005, 0.4736, 1.896	0.6997	1.07e-5<p<0.747	1.4e-5<p<0.5995	0<p<2.07e-13
2, 3, 1.5, 0.7	1.95, 2.89, 1.539, 0.7007	0.6837	4.8e-6<p<0.5667	2.73e-7<p<0.7376	0<p<1.064e-12
2, 3, 1.5, 1.7	1.89, 2.85, 1.65, 1.81	0.6518	6.6e-6<p<0.4288	5.6e-7<p<0.7069	0<p<1.14e-13
1, 1, 1, 1	0.85, 0.903, 0.918, 0.908	0.6763	2.01e-5<p<0.6507	6.16e-6<p<0.6953	0<p<1.39e-13
3, 2, 1.5, 0.7	2.7, 1.7, 1.45, 0.706	0.6526	1.11e-7<p<0.4921	2.24e-6<p<0.5747	0<p<9.41e-14
2, 2, 1.5, 1.5	1.949, 1.948, 1.47, 1.49	0.6718	4.7e-7<p<0.5433	9.64e-6<p<0.8067	0<p<2.63e-14

Table 3 shows the results for the estimated value of the parameters, the Kendall tau; Pearson correlation and the range of local correlation, for each set of parameters. The results conclude that the range of local dependence is changing according to the value of the parameter as well as the copula methods.

However, to support the results in Table 3, the same procedures have been repeated 100 times for sample size 250, with two sets of parameters, the results are given in Table 4.

Table 4. Correlate Weibull R. V. and same size 250 with replicated it 100 times

$\lambda_1, \lambda_2,$ α_1, α_2	Mean Tau	Interval Correlation		
		Gumbel's copula	Clayton's copula	Independence copula
2, 3 , 1.5, 1.7	0.6907	6.23e-6<ρ<0.4891	4.08e-6< ρ <0.7051	0< ρ <9.90e-14
2, 3, 0.5, 0.7	0.6875	8.35e-6< ρ <0.7161	3.9e-6< ρ <0.5539	0< ρ <1.44e-13

Finally, for uncorrelated Weibull's random variables, when the local correlation is estimated for the same set of parameters in Table 3, the computed

value of $\rho_{xy}(x,y)$ are approximately zero for all cases. These are explained in Table 5.

Table 5. Uncorrelated Weibull R.V. for sample Size=250 and difference sets of Parameters.

$\lambda_1, \lambda_2,$ α_1, α_2	$\hat{\lambda}_1, \hat{\lambda}_2,$ $\hat{\alpha}_1, \hat{\alpha}_2$	Tau	Interval Correlation		
			Gumbel's	Clayton's	Independence
2, 3, 0.5, 0.7	2.01, 3.08, 0.517, 0.726	-0.018	2.3e-8<p<0.0108	1.99e-8<p<0.0125	0<p<2.95e-14
2, 3, 0.5, 2	2.04, 2.83, 0.51, 1.954	-0.012	6.8e-9<p<0.0045	6.56e-8<p<0.0051	0<p<1.97e-13
2, 3, 1.5, 0.7	1.95, 3.13, 1.45, 0.700	0.0221	1.83e-8<p<0.0176	1.02e-8<p<0.0237	0<p<1.35e-14
2, 3, 1.5, 1.7	2.05, 3.28, 1.51, 1.80	-0.034	1.87e-7<p<0.0257	2.21e-7<p<0.0275	0<p<4.31e-14
1, 1, 1, 1	0.96, 1.08, 1.03, 1.04	-0.039	1.17e-7<p<0.0381	2.14e-8<p<0.0243	0<p<1.26e-13
3, 2, 1.5, 0.7	2.73, 1.89, 1.41, 0.710	0.0059	4.33e-8<p<0.0048	1.9e-8<p<0.005	0<p<1.04e-13
2, 2, 1.5, 1.5	2.26, 1.87, 1.51, 1.44	-0.028	1.918e-7<p<0.0158	1.11e-7<p<0.0235	0<p<4.76e-14

Application

In this section, in order to select the most efficient bivariate distribution among the three constructed functions from each copula (Gumbul's, Clayton's and Independent) the Local Dependence Procedure is applied. To test this procedure, a data set from the Diabetic Retinopathy Study (DRS) (18) for pairs of right-censored failure time data is implemented. DRS were conducted by the National Eye Institute to measure the effect of laser photocoagulation in delaying the blindness in the patients with diabetic retinopathy. The study was consisted of 197 high risk patients where each patient had one eye randomized to laser treatment and the other eye received no treatment. For each patient, the times to blindness in both eyes were recorded in months when censoring caused by withdrawal, death or end of the study.

In this application let T1, T2 denotes the time to the blindness of the treated and untreated eye respectively. To estimate the parameters, suppose that the times are distributed as univariate Weibull distribution, the marginal Survival distribution functions of T1 and T2 can be written as follows:

$$S_1(t_1) = \text{Exp}\{-\left(\frac{t_1}{\lambda_1}\right)^{\alpha_1}\} \qquad \text{and}$$

$$S_2(t_2) = \text{Exp}\{-\left(\frac{t_2}{\lambda_2}\right)^{\alpha_2}\}$$

Hence,

$$\log\left(-\log(S_1(t_1))\right) = \alpha_1 \log(t_1) - \alpha_1 \log(\lambda_1)$$

$$\log\left(-\log(S_2(t_2))\right) = \alpha_2 \log(t_2) - \alpha_2 \log(\lambda_2)$$

From that if $S_1(t_1)$ and $S_2(t_2)$ are known then the linear models given above can easily be fitted to estimate the Scale and Shape parameters. Specifically, when $\hat{S}_k$ (k=1,2) denote the Kaplan–Meier (KM) estimate of S_k based on the DRS data.

Table 6 shows the results obtained to the Scale and shape parameters of T1 and T2. The estimated values (especially Scale parameters) indicate that the treatment did seem to significantly delay the blinded in the patients.

Table 6. Estimated value of Scale and Shape parameters

Times	Scale Parameter λ	Shape Parameter α
T_1	114.583	0.948
T_2	58.596	0.896

To check the compatibility of the above estimation procedures, the estimated marginal Weibull Survival functions and K-M Survival functions of T1 and T2 are plotted. It is shown in Fig.3.
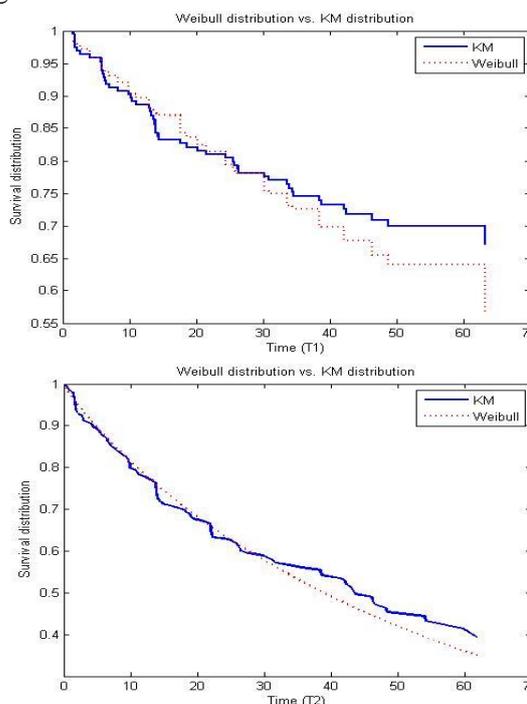
Figure 3. K-M Survival distribution and Weibull Survival distribution of T_1 and T_2.

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In Figure 3 two survival distributions are displayed; the first curve is for T1 (treated eyes) in which non-parametric survival distribution (K-M) and parametric survival distribution (Weibull distribution) are plotted while the second curve represents the non-parametric survival distribution (K-M) and parametric survival distribution (Weibull distribution) for T2 (untreated eyes). Moreover, it is shown that there is a significant difference between the survival distributions for T1 and T2 at any time scale; it is found that the probability of survival T1 is always higher than the probability of survival T2. For example, the probability of survival of T1 after 60 month time is between .65 to .7 (Survival Rate 65% to 70%) while for T2 at the same time is between .35 to .4, in other words, the treated eyes have greater chance for non-blindness Compared to untreated eyes.

Finally the Local Dependence technique is applied to each Copula (Gumbel, Clyton and Independent) and the results are displayed in Figure 4 and Table 7. This indicated that, at the beginning of the DRS there exists a high relationship between the two times T_1 and T_2 while the local dependence gradually decreasing to zero for each of Gumbel's and Clyton's Copula at the end of the study. While for independent copula the value of local dependence becomes zero everywhere. Furthermore, the data explain that Clyton Copula shows higher value of local dependence, this, implies that Clyton Copula forms an efficient bivariate distribution for a bivariate right censored data T_1 and T_2.

Local Dependence of Gumbel Copula

Local Dependence Clyton Copula

Local Dependence of Independent Copula
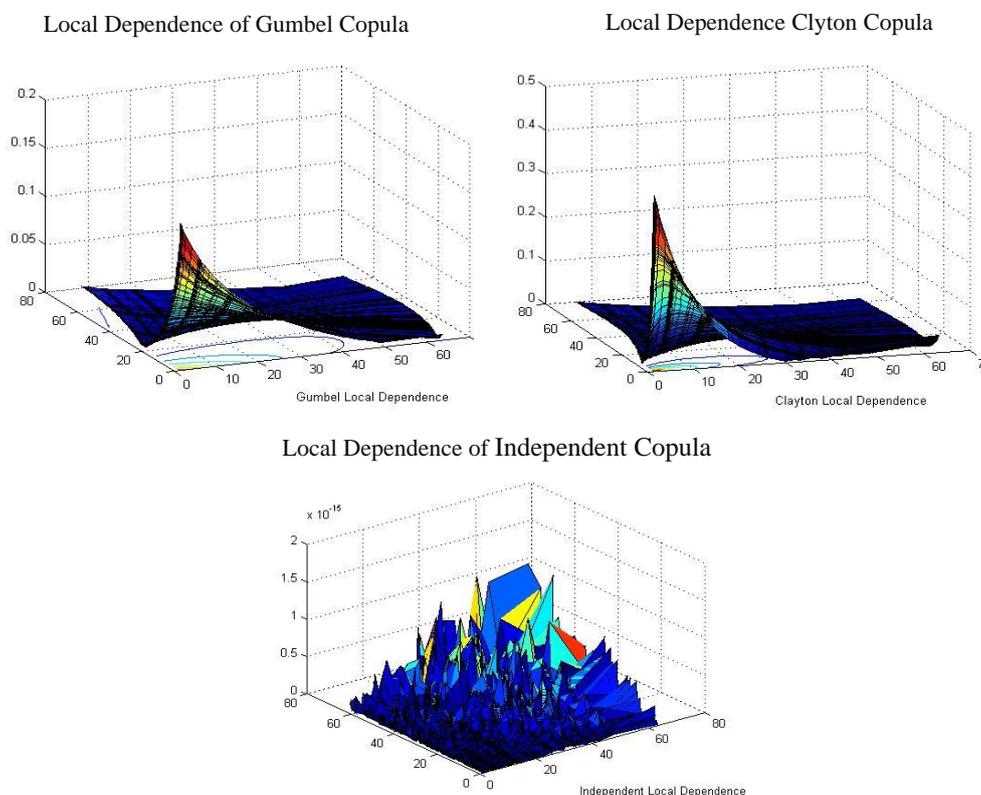
Figure 4. Local Dependence function of the copulas

Table 7. shows the range of Local Dependence for each Copula

	Gumbel Copula	Clayton Copula	Independent Copula
Local Dependence	6.31e-6 $<\rho_{T_1,T_2}<$ 0.1522	9.565e-6$<\rho_{T_1,T_2}<$0.4033	0 $<\rho_{T_1,T_2}<$ 1.3706e-15

Conclusion:

In this paper, the local dependence is used to identify an efficient bivariate Weibull distribution among the three Archimedean Copula Models using Weibull marginal; these bivariate functions are commonly used in survival and reliability analysis. The main results concluded that it is possible to apply Local Dependence to identify the best estimated bivariate distribution. Moreover, for a bivariate Weibull distribution, it is shown that the values of the parameters have great effect on the strength of dependence for each method.

Authors' declaration:

- Conflicts of Interest: None.

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- Ethical Clearance: The project was approved by the local ethical committee in University of Salahaddin.

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ارتباط الموضعي على توزيعات ويبل ثنائية المتغيرات التي بنيت باستخدام ارخميديان كوبيلا

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الخلاصة:

في تحليل البقاء على قيد الحياة متعدد المتغيرات ، يعد تقدير دالة التوزيع متعدد المتغيرات و من ثم قياس علاقة و ارتباط بين أوقات البقاء ذات أهمية كبيرة. تُستخدم دالات الكوبيلا ، مثل ارخميديان كوبيلا، بشكل شائع لتقدير توزيعات المتغيرات غير المعروفة بناءً على الدوال الهامشية المعروفة. في هذا البحث تم استكشاف جدوى استخدام فكرة ارتباط الموضعي لتحديد افضل نموذج الكوبيلا و الأكثر كفاءة ، والذي يستخدم لبناء دالة ويبل ثنائي المتغير كدالة وقت البقاء ثنائي المتغير، من بين بعض انواع الآرخميديان كوبيلا. لتقييم كفاءة طريقة المقترحة ، تم تنفيذ دراسة محاكاة، وقد ثبت أن هذا طريقة مفيد للحالات العملية وقابل للتطبيق على مجموعات البيانات الحقيقية. و عند تنفيذ الإجراء المقترح، على بيانات فعلية، على بيانات دراسة اعتلال الشبكية السكري وجد أن العيون المعالجة لديها فرصة أكبر لعدم فقدان البصر مقارنة بالعين غير المعالجة.

الكلمات المفتاحية: ارتباط الموضعي، توزيع ثنائي المتغير، ارخميديان كوبيلا، تحليل البقاء، توزيع ويبل.

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