Correlation between Paris function parameters to crack velocity for Alumina ceramics

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Abstract:

The question about the existence of correlation between the parameters A and m of the Paris function is re-examined theoretically for brittle material such as alumina ceramic (Al_2O_3) with different grain size. Investigation about existence of the exponential function which fit a good approximation to the majority of experimental data of crack velocity versus stress intensity factor diagram. The rate theory of crack growth was applied for data of alumina ceramics samples in region I and making use of the values of the exponential function parameters the crack growth rate theory parameters were estimated.

**Key words: Paris parameters, Crack velocity, Stress intensity factor, Correlation, Alumina ceramics, Grain size.**

Introduction:

Creep crack growth data for brittle materials   are usually presented   in terms of   the crack velocity , da/dt , and the stress- intensity factor , K_1 . At present, it is a common   practice to describe   the process of creep crack growth by a logarithmic da/dt vs. K diagram(see Fig.1)[1]. Three regions are   generally   recognized   on   this diagram for a wide collection of experimental results. The first region corresponds   to   stress − intensity factor near a lower threshold value , K_{th} , below   which   no   crack propagation takes place . This region of the diagram is usually referred to as Region I , or the near - threshold region . The second linear portion of the diagram defines a power - law relationship   between   the   crack velocity   and   the   stress − intensity factor   and   is usually referred to as Region II . Finally, when K_{max} tends to the critical stress - intensity factor, K_{IC}, rapid crack   propagation   takes place   and   crack   growth instability occurs (Region III) [2]. In Region I, II

and IΠ the Paris equation   provides a good approximation to the majority of experimental data for brittle material [3] .

$$V = \frac{da}{dt} = A * K^m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1-1)$$

Where , A, and , m, are empirical constants   usually referred to Paris function parameters(PFP).
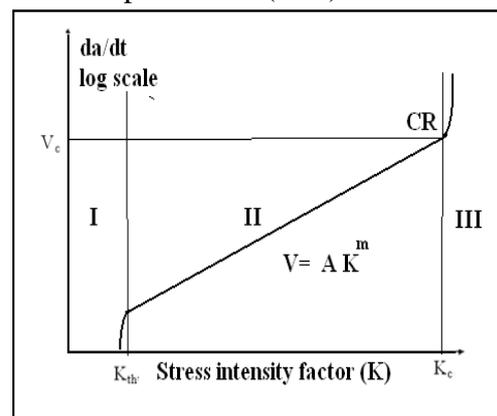
Fig(1) Scheme of the typical crack propagation [1].

In present work are studied the correlation between (PFP) for brittle material such as   alumina ceramics

(Al_2O_3) and the rate theory of crack growth was applied for experimental data of alumina ceramics samples in region I.

Theory :

The simplest example of a least-squares approximation is fitting a straight line to set paired observations: $(X_1,Y_1),(X_2,Y_2),….,(X_n,Y_n)$. The mathematical expression for the straight line is [4]:

$$Y= a_0+a_1 X…………(2-1)$$

Where a_0 and a_1 are coefficients representing the intercept and slope, there are given by the equations below[4] :

$$a_1 = \frac{N\sum X_i Y_i - \sum X_i \sum Y_i}{N\sum X_i^2 - (\sum X_i)^2} …………………(2-2)$$

$$a_0 = \frac{\sum Y_i - a_1 \sum X_i}{N} …………………(2-3)$$

To estimate the errors of the experimental data (X_i,Y_i) from straight line in equation (1-2) , we calculate the Correlation Coefficient or linear regression Coefficient ,r, whch given by equation below[4] :

$$r = \sqrt{\frac{\sum(Y_i - \tilde{y})^2 - \sum(Y_i - a_0 - a_1 X_i)}{\sum(Y_i - \tilde{y})^2}} …………………(2-4)$$

Where :

$$\tilde{y} = \frac{\sum Y_i}{N} …………………………….(2-5)$$

Where N is number of experimental data (X_i,Y_i) . the crack velocity versus stress intensity factor was investigated , two fitting method of comparable degree of fit were found ,the Paris function is given by equation (1-1) and exponential function is given by equation below[3]:

$$V = da/dt = \beta \exp^{\alpha K} …………… (2-6)$$

Where β and α are exponential function parameters (EFP) . Eq(2-6) can be linearizated by taking its natural logarithm to yield[4] .
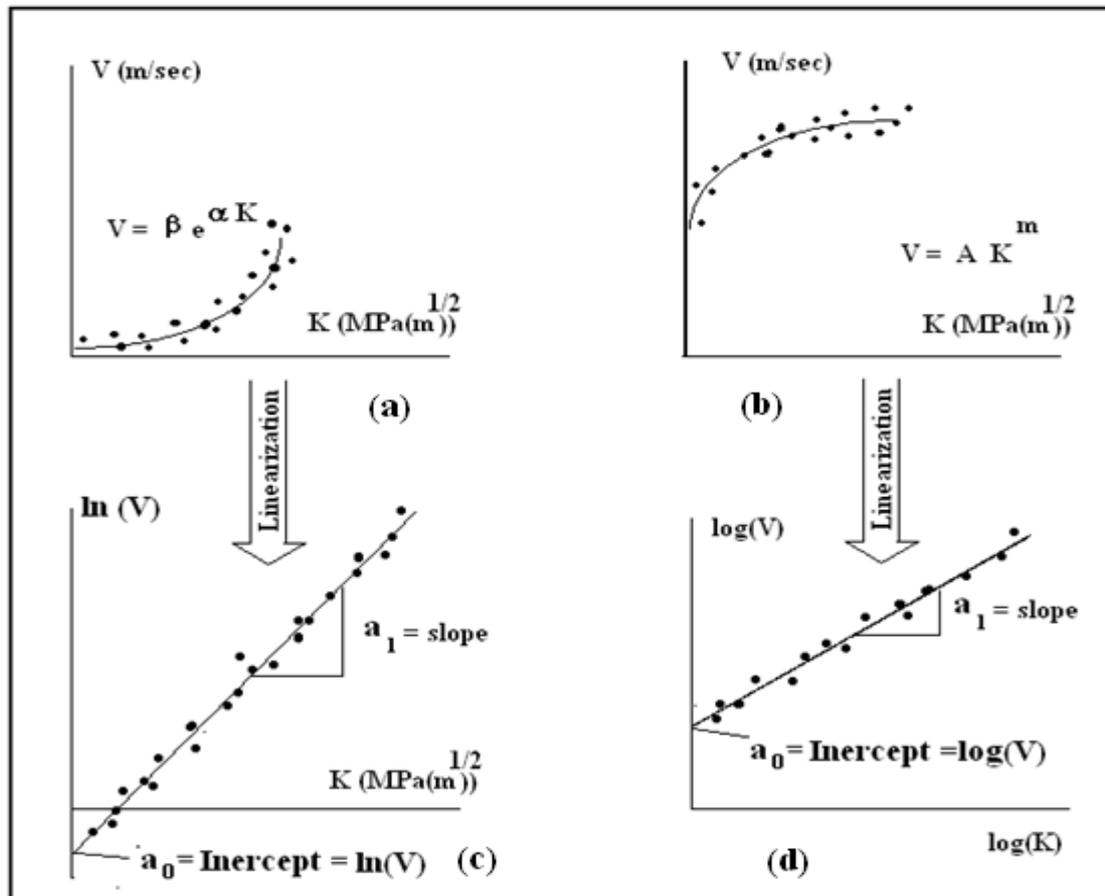
$$\ln V = \ln \beta + \alpha K ………………(2-7)$$

To calculate EFP , we redraw the graph (ln(V),K) plot , as shown in fig(2-a,c)[4] , by comparative Eq(2-7) with Eq(2-1) , we get :

$$Y=\ln V, a_0= \ln \beta, a_1= \alpha, X = K…(2-8)$$

To calculate PFP ,we redraw the graph (log(V),log(K)) plot , as shown in fig(2-b,d) [4] , by comparative Eq(1-1) with Eq(2-1) , we get :

$$Y = \log(V) , a_0 = \log(A) , a_1 = m , X = \log(K)……(2-9)$$

Fig(2)(a)the exponential function (b) the Paris Function (c) , (d) are linearized versions of these function that result from simple transformations [4].

We derive a correlation between the PFP similar to that in Eq. (1-1) on the basis of the condition of crack growth instability[1] . In fact, the crack propagation rate, da/dt, is not only a function of the stress-intensity factor, K , but also on the condition of instability of the crack growth when the maximum stress-intensity factor approaches its critical value for the material. Focusing our attention on this dependence, the crack propagation rate must tend to infinity when $K \rightarrow K_{IC}$[5] , i.e.

$$lim_{K \rightarrow Kc} \frac{da}{dt} \approx \infty \quad …….(2\text{-}10).$$

This rapid increase in the crack propagation rate is then responsible for the fast deviation from the linear part of the Region II in the creep plot (see Fig.1). Considering the transition point labeled CR in Fig. (1) between Region II and Region III, the following relationship between the crack growth rate and the stress-intensity factor range can be derived according to the PF[5]:

$$(\frac{da}{dt})_{CR} = V_{CR} = A * (K_{CR})^m …….(2\text{-}11).$$

Where K_{CR} denotes the value of the stress-intensity factor at the point CR. Due to the fact that a rapid variation in the crack propagation rate takes place when the onset of crack instability is reached , it is a reasonable assumption to consider CR $K \cong$ KIC . As a consequence, if we take the logarithmic to both side of eq,(2-2). We get the final formation of the relationship between ,A, and , m ,such as equation below:

LOG(A)=LOG(V_{CR})–m*LOG(K_{CR}) …..(2-12).

In the Rate theory form(RTF), the crack velocity ,V, can be expressed as [6].

$$V = \frac{da}{dt} = L \; \frac{K_B \, T}{h} \; exp\left[-\frac{G_b - \delta K}{K_B \, T}\right] \dots\dots\dots\dots\dots(2-13)$$

Where L is creep crack length per one step of activation energy , G_b is apparent activation energy for creep crack propagation ,δ is work constant ,K_b Boltzmann's constant , h is Plank's constants (K_b=1.38E-23JK^{-1} , h=6.62E-34JS) , T is absolute temperature . If we compare (EF) with (RTF), we get to formulas of (EFP) are given by the equation below :

$$\beta = L \frac{K_B \, T}{h} exp\left[-\frac{G_b}{K_B \, T}\right] \dots\dots\dots\dots(2-14)$$

$$\alpha = \delta/K_B \, T \dots\dots\dots\dots..(2-15)$$

In region I, the activation energy (ΔG_b) closed to zero where the stress intensity factor equal to threshold stress intensity factor (K =K_{th})[6]

$$\Delta G_b \, (W) = G_b - \delta K \dots (2-16)$$

There for , the apparent activation energy for creep crack propagation is given by equation below[6] :

$G_b = \delta \, K_{th} \dots\dots\dots\dots..(2-17)$

We get to formulas of parameters (L ,δ , G_b) from Eq(2-14) ,EQ(2-15) and Eq(2-17) , they are given by the forms [6]:

$\delta = \alpha \; k_B \, T \dots\dots\dots\dots\dots.(2-18)$

$G_b = \delta \; K_{th} \dots\dots\dots\dots\dots\dots.(2-19)$

$L = (\beta \; h/k_B \, T) \; exp(G_b/k_B \, T) \dots(2-20)$

Results:

Fig 3[7] show *V–K* diagram of alumina ceramics in air with three different grain sizes 1.9μm ,4.8μm ,12.4μm, obtained by the relaxation test with the double torsion technique [7], we get the experimental data of crack velocity and stress intensity factor via GetData Graph Digitizer(GDGD) ,its a program for digitizing graphs, plots and maps. we redraw this data in the Graf Program(GP) to calculate (PFP)as

shown in fig (4), and calculate (EFP) as shown in fig (5) .
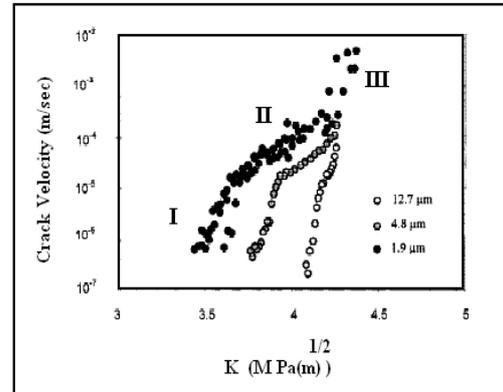
Fig(3) *V–K* plots of alumina ceramics[7].

We calculate PFP and EFP from Eq(2-9) and Eq(2-8) respectively. Table (1) shows the linear regression coefficients, r, which calculated by Eq(2-4) to PF & EF of three region crack propagation in fig (4) & fig(5) respectively. This table explain two fitting functions provide majority of the experimental data of (V-K)plot , there are PF & EF .

Table (1) shows the linear regression coefficients, r , to PF & EF.

G.S(μm)	Region	(r) for PF	(r) for EF
1.9	I	0.99687	0.9963
1.9	II	0.99748	0.9971
1.9	III	0.94133	0.9398
4.8	I	0.98641	0.9872
4.8	II	0.99133	0.99236
4.8	III	0.97945	0.97903
12.4	I	0.97974	0.97994
12.4	II	0.99847	0.99859
12.4	III	0.92246	0.92246

fig(6) shows the correlation between (m,log(A)),PFP of creep crack propagation in alumina ceramics. Correlation coefficients, a_0 and a_1, are calculated from Eq(2-2) ,Eq(2-3), respectively (see fig(6)) .There are given by the values below :

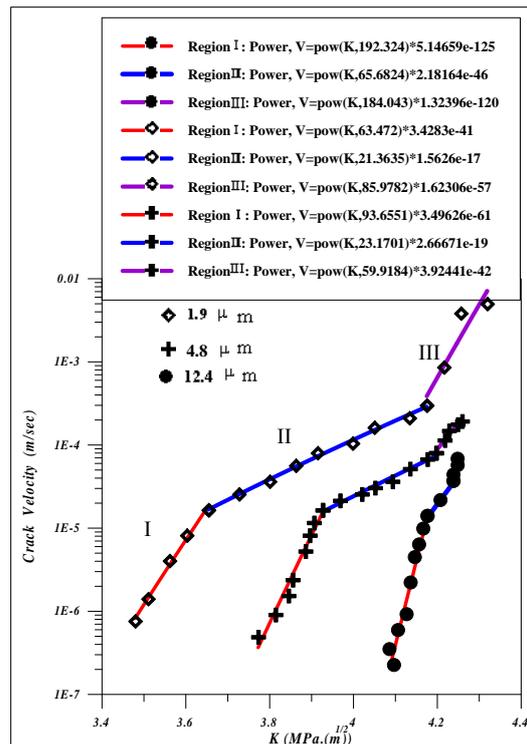a_0 = -3.32845 a_1 = -0.644698

Fig (4) explains fitting the experimental data of (V-K)plot with Paris function of alumina ceramics .
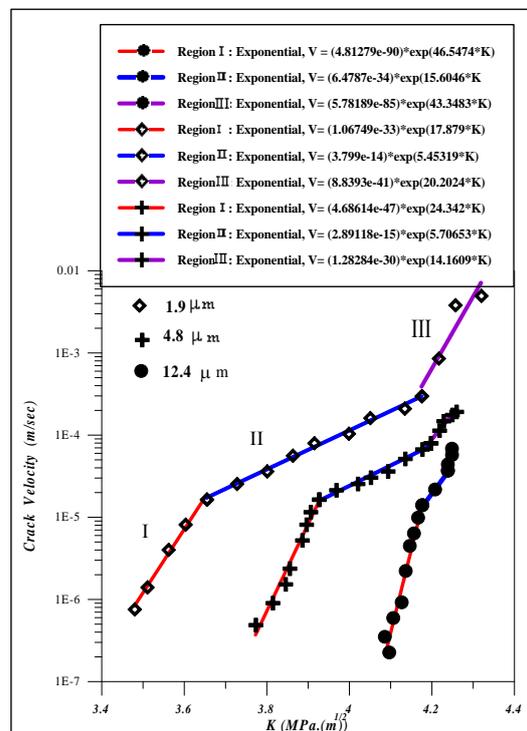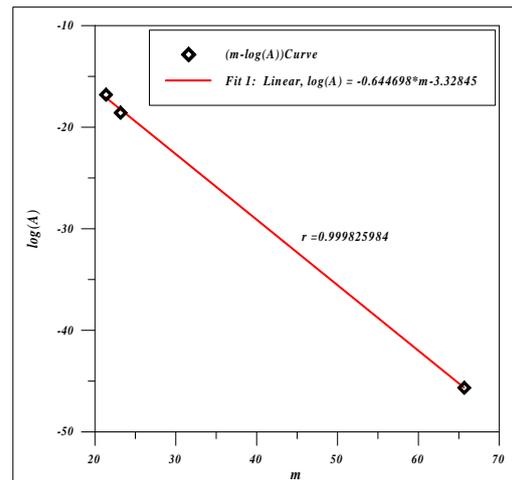

Fig (5) explains fitting the experimental data of (V-K)plot with exponential function of alumina ceramics .

Fig(6) explain the correlation equation to PFP in region II for the alumina ceramics.

We try to estimate the critical values of stress intensity factor and crack velocity from Eq(2-12) via comparative between this equation with correlation equation in fig(6), we get $a_0 = \log(V_c)$ and $a_1 = \log(K_c)$, we estimate V_c=4.783E-4m/sec, and K_c=4.419MPa√m from correlation equation in fig(6), but the experimental data to V_c>1E-4m/s and K_c=4.3MPa√m[2] . Linear regression coefficients of correlation in fig(6) has been calculated by using Eq(2-4) and its closed to (r =0.9998) because fewness points number in curve of fig(6). As consequence, we found a good agreement between the theoretical predictions obtained using this correlations and experimental data has been achieved. Alberto and Marco [1] (2007) found the correlation between PFP in critical point of (V-ΔK) curve of fatigue crack propagation , they estimated V_c and ΔK_c from the correlation coefficients $a_0 = \log(V_c)$ and $a_1 = \log(\Delta K_c)$ for Al alloys and steels alloys . Table (2) shows the Rate theory function parameters(RTFP) values ((L ,δ , G_b) in region I , we calculate these parameters from Eq(2-18), Eq(2-19) and Eq(2-20) with supposing this experiment is executed at room temperature degree

(T=300K).We note in this table , both of parameters δ and G_b increase with increase a grain size of the specimens because these parameters refer to energy terms' and break-bond energy increase with increase a grain size of the specimens duo to increase break-bond number per unit volume. We note, creep crack length (L) decrease with increase a grain size of the specimens because this parameter refer to move crack in the space of the crystal of material and a grain boundary space per unit volume decrease with increase a grain size of the specimens .

Table(2) shows RTFP values with different a grain size of Al_2O_3 in region I.

RTFP in region I	Grain size of specimens(μm)		
	1.9	2.4	12.4
L(m)	1.3070673E-19	5.832455E-20	5.5441016E-20
δ(J/MPa√m)	7.364646E-20	1.0077588E-19	1.932326E-19
G_b(J)	2.5627789E-19	3.802707E-19	7.898566E-19

Conclusions:

To shed light on the results about the existence of a correlation between the Paris' constants for alumina ceramics. The main consequence of this correlation is that parameters V_c and K_c are estimated theoretically. A good agreement between the theoretical prediction obtained using this correlations and experimental data has been achieved . Both of Rate theory and the correlation between that the Paris function parameters have been profit ably used. The rate theory of crack growth was applied for experimental data of alumina ceramics samples. the crack growth rate theory

parameters were estimated from the values of the EFP . Energy terms (δ and G_b) of RTFP increase when the a grain size of the samples increase too, but the creep crack length (L) decrease with increase the grain size of the samples.

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الترابط بين معلمات باريس لسرعة نمو الشق لسيراميك الالومينا

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الخلاصة :

تم اختبار وجود ترابط بين m, A , معلمات باريس نظريا بالنسبة للمواد الهشة مثل سيراميك الألومينا (Al_2O_3) باختلاف الحجم الحبيبي, والتحقق من وجود دالة أسية توائم أغلب القيم التجريبية لمخطط سرعة نمو الشق مع عامل شدة الإجهاد. وتم تطبيق نظرية معدل التغير لنمو الشق على نماذج من سيراميك الالومينا في المنطقة I وتم تخمين معلمات نظرية معدل التغير من معلمات الدالة الأسية .