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New Operational Matrices of Seventh Degree Orthonormal Bernstein Polynomials

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Abstract:

Based on analyzing the properties of Bernstein polynomials, the extended orthonormal Bernstein polynomials, defined on the interval [0, 1] for n=7 is achieved. Another method for computing operational matrices of derivative and integration D_b and R_{n+1}^B respectively is presented. Also the result of the proposed method is compared with true answers to show the convergence and advantages of the new method.

Keywords: The Bernstein Basis and Bezier Curves, Gram-Schmidt Orthonormalization Process, Numerical Solution of Optimal Control of Time-varying Singular via Operational Matrices.

Introduction:

We already know that orthogonal polynomials play a central role in the solution of least-squares problems. The main characteristic of this technique is to reduce the problems related to those of solving a system of algebraic equations. The polynomials determined in the Bernstein basis [1],enjoy considerable popularity in many different applications. For example in the solution of integral equations, differential equations and approximation theory, see e.g.],[2],[3]. On the other hand, recently the method of operational matrix of integration was proposed as an effective tool for processing of singular integrals of Abel type using one –step procedure. Example, Legendre Wavelet was used [4], [5].Further, Singh et al. [6] derived the operational matrices of Bernstein polynomials, which have certain advantages for the considered problem in the case of smooth transformed functions. Due to the increasing interest on Bernstein polynomials, the question arises of how to describe their properties in terms of their coefficients when they are given in the Bernstein basis. Recently Yousefi and Behoozifar derived the operational matrices of Bernstein polynomials [7].In this work we proposed a method to give the operational matrix of derivative D_b and integration R_{n+1}^Brespectively such that:

$$D_b = \frac{d}{dx} b(x) = D_b\, B(x)$$

$$and \int_0^t B(x)dx = R_{n+1}^B\, B(t)$$

where $b(x)$ $[b_{07}(x), b_{17}(x), b_{27}(x), b_{37}(x), b_{47}(x), b_{57}(x), b_{67}(x), b_{77}(x)]$

And $B_{i7}, i = 0,1,2, \dots 7$ are the basis Bernstein polynomials.

The remainder of this paper is organized as follows. In section2, we describe the formulation of the Bernstein polynomials (BP), fundamental relations andwe give approximate function for BP. In section4, a class of orthonormal polynomials for n=7 are given. In section5 wecalculate the operational matrix of derivative. In section6 we briefly describe calculating the operational matrix of integration. Finally, in section7 we demonstrate the accuracy of the proposed numerical scheme by numerical example.

Bernstein polynomials (BP) and Fundamental Relations

From the binomial theorem we have for any n:

$$1 = ((1-t)+t)^n = \sum_{i=1}^{n} \binom{n}{i} (1-t)^{n-i} t^i$$

The Bernstein basis polynomials of degree n are defined on the interval [0, 1] as [8]:

$B_{in}(x) = \binom{n}{i} x^i (1-x)^{n-i}$, For$i = 0,1,2 \dots . n$ ... (1)

The set of Bernstein basis polynomials $B_{0n}(x), B_{1n}(x), \dots, B_{nn}(x)$ formsas basis of the vector space of polynomials of real coefficients and degree no more than n.

For convenience, we set $B_{in}(x) = 0$ if $i < 0$ or $i > n$.

By using the binomial expansion of $(1-x)^{n-i}$, we have

$\binom{n}{i} x^i (1-x)^{n-i} = \sum_{k=0}^{n-i} (-1)^k \binom{n}{i} \binom{n-i}{k} x^{i+k} \dots$ (2)

A function $f \in L^2[0,1]$ may be written as in the following expansion:

$f(x) = \lim_{n\to\infty} \sum_{i=0}^{n} c_{in} b_{in} \dots$ (3)

Here $c_{in} = < f, b_{in} >$ where $< . >$ is the inner product over $L^2[0,1]$.

If the series is truncated at$n = m$, then denote:

$$f(x) \approx \sum_{i=0}^{n} c_{im} b_{im}(x) = C^T B(x) \dots (4)$$

Where $C = [c_{0m}, c_{1m}, \dots, c_{mm}]^T$,

$B(x) = [b_{0m}, b_{1m}, \dots, b_{mm}]^T \dots$ (5)

$H = L^2[0,1]$ is a Hilbert space with the inner product that is defined by $(f,g) = \int_0^1 f(x)g(x)dx$ and .Let $S_n = span\,\{B_{0n}, B_{1n} \dots B_{nn}\}$ is a finite dimensional and closed subspace, therefore S_n is a complete subset of$H$, so, f has the unique best approximation out of S_n such as $s_0 \in S_n$, that is;$\exists s_0 \in S_n$ s.t $\forall s \in S_n ||f - s_0|| \leq ||f - s||$, this implies that:

$\forall s \in S_n (f - s_0, s) = 0$ ... (6)

Therefore, exist the coefficients $c_0, c_1, \dots, c_n$ such that

$s_0(x) = C^T \emptyset(x) \approx f,$ ... (7)

Where $C^T = [c_0, c_1, \dots, c_n\ ]$. By eq.s (6)

$\left(f - C^T \emptyset(x), B_{in}(x)\right) = 0$, $i = 0,1, \dots n$ ... (8)

For simplicity, we write:

$$C^T\left(\emptyset(x), \emptyset(x)\right) = \left(f, \emptyset(x)\right),$$

Where

$$\left(f, \emptyset(x)\right) = \int_0^1 f(x)\emptyset^T(x)dx$$

$= [(f, B_{0n}), (f, B_{1n}), \dots, (f, B_{nn})] \dots$ (9)

We define the matrix$D = (\emptyset(x), \emptyset(x))$ is an $(n+1) \times (n+1)$which is called the dual matrix of $\emptyset_n(x)$.

Let $D = \left(\emptyset(x), \emptyset(x)\right) = A\,[\int_0^1 T_n(x) T_n^T(x)dx\,]\,A^T = AHA^T$ ... (10)

Where H is a Hilbert matrix and we can obtain the elements of D as:

$D_{i+1,j+1} = \int_0^1 B_{in}(x)\, B_{jn}(x)\, dx = \frac{\binom{n}{i}\binom{n}{j}}{(2n+1)\binom{2n}{i+j}}$, Where $i, j = 0,1, \dots, n$.

Generation of the Orthonormal Polynomials

Let us first define the inner product in the functional space for two functions $f(x)$ and $g(x)$ defined over the domain $D \in R^n$ by:

$$(f, g) = \int_D w(x)f(x)g(x)dD \quad \dots (11)$$

Where $w(x)$ the suitable chosen weight function .The is induced norm of a function using above inner product is, therefore, given as

$$||f||^2 = \int_D w(x)f^2(x)\, dD \quad \dots (12)$$

To generate an orthogonal sequence, we can start with the set:

$$\{f_i(x)\} = \{B_{i7}(x)\}, \quad i = 0,1,\dots 7 \quad \dots (13)$$

Where $B_{i7}(x)$ are the linearly independent Bernstein polynomials over the domain [0, 1].

To generate an orthogonal sequence $\emptyset_{i7}$, we apply the well-known Gram-Schmidtprocess, on $\{B_{i7}\}_{i=0}^{7}$, which is given as:

$$\emptyset_{07} = B_{07} \quad \dots (14)$$

$$\emptyset_{i7} = B_{i7} - \sum_{j=1}^{i-1} c_{ij}\emptyset_{j7} \quad , i = 1,2,\dots 7 \quad \dots (15)$$

Where

$$c_{ij} = (B_{i7}, \emptyset_{j7})/(\emptyset_{j7}, \emptyset_{j7}) \quad \dots (16)$$

By dividing each $\emptyset_{i7}$ by its norm, we obtain a class of orthonormal polynomials from Bernstein polynomials,

Namely $b_{07}, b_{17}, b_{27}, b_{37}, b_{47}, b_{57}, b_{67}, b_{77}$.

And they are given by:

$$b_{07} = \sqrt{15}\,(1-t)^7$$

$$b_{17} = 2\sqrt{13}[7t(1-t)^6 - \frac{1}{2}(1-t)^7]$$

$$b_{27} = \frac{26\sqrt{11}}{7}[21 t^2(1-t)^5 - 7t(1-t)^6 + \frac{7}{26}(1-t)^7]$$

$$b_{37} = \frac{132}{7}[35t^3(1-t)^4 - \frac{63}{2}t^2(1-t)^5 + \frac{63}{11}t(1-t)^6 - \frac{7}{44}(1-t)^7]$$

$$b_{47} = \frac{66}{\sqrt{7}}[35t^4(1-t)^3 - 70t^3(1-t)^4 + 35t^2(1-t)^5 - \frac{14}{3}t(1-t)^6 + \frac{7}{66}(1-t)^7]$$

$$b_{57} = 12\sqrt{5}[21t^5(1-t)^2 - \frac{175}{2}t^4(1-t)^3 + 100t^3(1-t)^4 - \frac{75}{2}t^2(1-t)^5 + \frac{25}{6}t(1-t)^6 - \frac{1}{12}(1-t)^7]$$

$$b_{67} = 12\sqrt{3}[7t^6(1-t) - 63t^5(1-t)^2 + \frac{315}{2}t^4(1-t)^3 - 140t^3(1-t)^4 + 45t^2(1-t)^5 - \frac{9}{2}t(1-t)^6 + \frac{1}{12}(1-t)^7]$$

$$b_{77} = 8[t^7 - \frac{49}{2}t^6(1-t) + 147t^5(1-t)^2 - \frac{1225}{4}t^4(1-t)^3 + 245t^3(1-t)^4 - \frac{147}{2}t^2(1-t)^5 + 7t(1-t)^6 - \frac{1}{8}(1-t)^7]$$

The explicit representation for the orthonormal, in general product of a factorable polynomial and non – factorable polynomial. For the factorable, there exists a pattern of the form $(\sqrt{2(n-i)+1})(1-x)^{n-i}$,$i = 0,1,\dots,n$. and the pattern in the non-factorable part can be determined by analyzing the binomial coefficients present in Pascal's triangle. In this way we have determined this formula

$$\emptyset_{i,n}(x) = (\sqrt{2(n-i)+1})(1-x)^{n-i}\sum_{k=0}^{i}(-1)^k\binom{2n+1-k}{i-k}\binom{i}{k}x^{i-k}.$$

The Operational Matrix of Derivative for Orthonormal Polynomials

In this section, orthonormal Bernstein operational matrix of derivative will be derived; before we derive we need the following theorem.

Theorem: [9]

The first derivatives of nth degree generalized Bernstein basis polynomials can be written as a linear combination of the generalized Bernstein basis polynomials of degree n

$$\frac{d}{dx}B_{in}(x) = (n-i+1)B_{i-1,n}(x) + (2i-1)B_{i,n}(x) - (i+1)B_{i+1,n}(x) \quad \dots (17)$$

From this formula, there is a relation between Bernstein basis polynomials matrix and their derivatives.

The matrix relationwhich obtained is given by:

$$N = \begin{bmatrix} -7 & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -5 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & -3 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -6 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7 & 7 \end{bmatrix}$$

$$\dot{B}(x) = B(x)N \quad \dots (18)$$

Such that

B(x) = $[B_{07}(x), B_{17}(x), B_{27}(x), B_{37}(x), B_{47}(x) B_{57}(x), B_{67}(x) B_{77}(x)]$

$$\dot{B}(x) = \left[\dot{B_{07}}(x), \dot{B_{17}}(x), \dot{B_{27}}(x), \dot{B_{37}}(x), \dot{B_{47}}(x), \dot{B_{57}}(x), \dot{B_{67}}(x), \dot{B_{77}}(x)\right]$$

Now, we introduce a new method for deriving operational matrix of derivative for orthonormal Bernstein polynomials of degree seven. The idea of the technique depends on the following derivative property of the basis vector $\emptyset(x)$

$$\frac{d\Psi(x)}{dx} = D\ \emptyset(x) \quad \dots (19)$$

Where $\Psi(x)$are the orthogonal Bernstein polynomials of the degrees even and $\emptyset(x)$be the Bernstein polynomials respectively defined by:

$\Psi(x) = [b_{07}, b_{17}, b_{27}, b_{37}, b_{47}, b_{57}, b_{67}, b_{77}]^T$

And

$\emptyset(x) = [B_{07}, B_{17}, B_{27}, B_{37}, B_{47}, B_{57}, B_{67}, B_{77}]^T \quad \dots (20)$

Where D is the 8×8 operational matrix of derivative defined as follow

$$\begin{bmatrix} -27.110883 & -3.872983 & 0 & 0 & 0 & 0 & 0 & 0 \\ 75.716577 & -32.449961 & -14.422205 & 0 & 0 & 0 & 0 & 0 \\ -109.448618 & 132.191188 & -12.318892 & -36.956676 & 0 & 0 & 0 & 0 \\ 129 & -243.857143 & 148.285714 & 66 & -75.428571 & 0 & 0 & 0 \\ -134.933317 & 329.962985 & -340.923955 & 24.945655 & 224.510897 & -124.728276 & 0 & 0 \\ 127.455875 & -365.117957 & 495.129338 & -201.246118 & -293.244343 & 415.908644 & -160.996894 & 0 \\ -105.655099 & 332.306319 & -522.584472 & 323.646065 & 239.023011 & -613.145986 & 478.046023 & -145.492268 \\ 63 & -207 & 348 & -252 & -126 & 462 & -468 & 252 \end{bmatrix}$$

Orthonormal Bernstein Operational Matrix of Integration

The main objective of this section is derived the orthonormal Bernstein polynomials matrix of integration, to achieve this, integrating the orthonormal base eight function from 0to t as given i.e.

$$\int_0^t B(x)\,dx = \int_0^t [b_{07}(x), b_{17}(x), b_{27}(x), b_{37}(x), b_{47}(x), b_{57}(x), b_{67}(x), b_{77}(x)]^T$$

$= [\Gamma_0(x), \Gamma_1(x), \Gamma_2(x), \Gamma_3(x), \Gamma_4(x), \Gamma_5(x), \Gamma_6(x), \Gamma_7(x)]^T$

$= R^B_{7+1}\ B(t) \quad \dots (21)$

Where $\Gamma_i(x),\ i = 0,1,\dots,7$ are defined as follows:

$$\Gamma_i(x) \approx \sum_{j=0}^{n} c_{j7}^{i}\, B_{j7}(x)$$
$$= [c_{07}^{i}, c_{17}^{i}, \dots, c_{77}^{i}]\, B(x),$$
$$0 \le t < 1, \quad \dots (22)$$

For $n = 7$, the explicit expressions for R_8^B via eight orthonormal polynomials for eqs. (21) is given as

$$\begin{bmatrix} 0.117188 & 0.225464 & 0.199751 & 0.181746 & 0.160050 & 0.135339 & 0.104805 & 0.060520 \\ -0.007273 & 0.101563 & 0.200194 & 0.166224 & 0.149872 & 0.125658 & 0.097724 & 0.056271 \\ 0.000956 & -0.013346 & 0.085938 & 0.173405 & 0.131835 & 0.117904 & 0.088817 & 0.052245 \\ -0.000200 & 0.002786 & -0.017938 & 0.070313 & 0.144690 & 0.096875 & 0.084880 & 0.045218 \\ 0.000059 & -0.000819 & 0.005273 & -0.020670 & 0.054688 & 0.113444 & 0.061839 & 0.045724 \\ -0.000023 & 0.000315 & -0.002026 & 0.007941 & -0.021009 & 0.039063 & 0.078670 & 0.026786 \\ 0.000010 & -0.000146 & 0.000942 & -0.003690 & 0.009764 & -0.018155 & 0.023438 & 0.037588 \\ -0.000005 & 0.000066 & -0.000423 & 0.001657 & -0.004385 & 0.008152 & -0.010525 & 0.007813 \end{bmatrix}$$

Solving variational problem

In this section, we solved the problems of finding the minimum of the time-varying functional by using the operational matrix of derivative

Algorithm 1via BP

Consider the first order functional extremal

$$J(t) = \int_0^1 [\dot{t}^2(x) + 2\,x\,\dot{t}(x) + t^2(x)]\,dx \qquad \dots (23)$$

With two fixed boundary conditions

$$t(0) = 2 \quad , \dot{t}(1) = -1 \qquad \dots (24)$$

In this case, the exact solution is $t(x) = c_1 e^{x} = c_2 e^{-t} + 1$, Where $c_1 = e^1 - 1/e^1 + e^{-1}$, $c_2 = 1 + e^1/e^1 + e^{-1}$ … (25)

Approximate the variable $t(x)$ using (OBP)

$$t(x) = c^T\, b(x) \qquad \dots (26)$$

Differentiated eq. (26), we get

$$\dot{t}(x) = c^T \dot{b}(x) = c^T\, Db\ b(x) \quad \dots \quad (27)$$

Where $c = [c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7]^T$,

$b = [b_{07}, b_{17}, b_{27}, b_{37}, b_{47}, b_{57}, b_{67}, b_{77}]$

Substituting eqs. (26) and (27) in eq. (23) , yields

$$J(t) = \int_0^1 [c^T\, \dot{b}(x)\dot{b}^T(x)\, c + c^T\, x\, \dot{b}(x) + c^T\, b(x)b^T(x)\, c]\,dx \quad \dots (28)$$

The quadratic programming problem in eq. (28) can be simplified to

$$J(t) = 1/2\, c^T\, H\, c + d^T\, C \dots (29)$$

Subject to

$F_1\, c - b_1 = 0$,

Where

$$F_1 = \begin{pmatrix} b^T(0) \\ b^T(1) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7 & 7 \end{pmatrix},\ b_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$d^T = 2\int_0^1 x\,\dot{b}^T(x)\,dx = \begin{pmatrix} \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{7}{4} \end{pmatrix}$$

$$H = 2\int_0^1 [b(x)\,\dot{b}^T(x) + b(x)b^T(x)]\,dx$$

$$= \begin{pmatrix} 7.6718 & -3.7026 & -2.0252 & -1.0152 & -0.4522 & -0.1699 & -0.0486 & -0.0081 \\ -3.7026 & 4.1837 & 1.0818 & -0.1958 & -0.4977 & -0.3851 & -0.1857 & -0.0486 \\ -2.0252 & 1.0818 & 1.4294 & 0.7343 & -0.0163 & -0.3990 & -0.3851 & -0.1699 \\ -1.0152 & -0.1958 & 0.7343 & 1.0334 & 0.6595 & -0.0163 & -0.4977 & -0.4522 \\ -0.4522 & -0.4977 & -0.0163 & 0.6595 & 1.0334 & 0.7343 & -0.1958 & -1.0152 \\ -0.1699 & -0.3851 & -0.3990 & -0.0163 & 0.7343 & 1.4294 & 1.0818 & -2.0252 \\ -0.0486 & -0.1857 & -0.3851 & -0.4977 & -0.1958 & 1.0818 & 4.1837 & -3.7026 \\ -0.0081 & -0.0486 & -0.1699 & -0.4522 & -1.0152 & -2.0252 & -3.7026 & 7.6718 \end{pmatrix}$$

The optimal values of unknown parameters c^* can be obtained using Lagrange multiplier technique as

$$c^* = -H^{-1}\, c + H^{-1} F_1^T (F_1 H^{-1} F_1^T)^{-1} (F_1 H^{-1}\, c + b_1),$$

$$c^* = [2 \;\; 1.798622 \;\; 1.621053 \;\; 1.460581 \;\; 1.311684 \;\; 1.169471 \;\; 1.029317 \;\; 0.886460]$$

Table (1) shows comparison between exact and approximate solution by using the operational matrix of derivative of BP of degree 8

x	Exact solution	BP	$\lvert$Exact - B_{i7} $\rvert$
0	2	2	0
0.1	1.863804265845607	1.863804265845607	0.000000000000000
0.2	1.736253775770209	1.736253775770209	0.000000000000000
0.3	1.616071961185921	1.616071961185921	0.000000000000000
0.4	1.502056000789419	1.502056000789419	0.000000000000000
0.5	1.393064785622723	1.393064785622723	0.000000000000000
0.6	1.288007495877680	1.288007495877680	0.000000000000000
0.7	1.185832682072615	1.185832682072615	0.000000000000000
0.8	1.085517743229661	1.085517743229661	0.000000000000000
0.9	0.986058694681254	0.986058694681254	0.000000000000000
1	0.886460118134294	0.886460118134294	0.000000000000000

Algorithm 2via OBP

Consider the first order functional extremal

$$J(t) = \int_0^1 [\dot{t}^2(x) + 2\, x\, \dot{t}(x) + t^2(x)]\, dx \quad \dots (23)$$

With two fixed boundary conditions

$$t(0) = 2 \quad , \dot{t}(1) = -1 \quad \dots (24)$$

In this case, the exact solution is $t(x) = c_1 e^{x} = c_2 e^{-t} + 1,$ Where $c_1 = e^1 - 1/e^1 + e^{-1},$ $c_2 = 1 + e^1/e^1 + e^{-1}$ … (25)

Approximate the variable $t(x)$ using (OBP)

$$t(x) = c^T\, b(x) \quad \dots (26)$$

Differentiated eq. (26), we get

$$\dot{t}(x) = c^T \dot{b}(x) = c^T\, Db\;\, b(x) \dots (27)$$

Where$c = [c_0,\, c_1,\, c_2,\, c_3,\, c_4,\, c_5,\, c_6,\, c_7]^T$, $b = [b_{07},\, b_{17},\, b_{27},\, b_{37},\, b_{47},\, b_{57},\, b_{67},\, b_{77}]$

Substitutingeqs. (26) and (27) in eq. (23), yields

$$J(t) = \int_0^1 [c^T\, \dot{b}(x) \dot{b}^T(x)\, c + c^T\, x\, \dot{b}(x) + c^T\, b(x) b^T(x)\, c]\, dx \quad \dots (28)$$

The quadratic programming problem in eq. (28) can be simplified to

$$J(t) = 1/2\, c^T\, H\, c + d^T\, C \quad \dots (29)$$

Subject to

$$F_1\, c - b_1 = 0,$$

Where

$$F_1 = \begin{pmatrix} b^T(0) \\ \dot{b}^T(1) \end{pmatrix} = \begin{pmatrix} \sqrt{15} & -\sqrt{13} & \sqrt{11} & -3 & \sqrt{7} & -\sqrt{5} & \sqrt{3} & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -145.49227 & 252 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix},\; d^T = 2 \int_0^1 x\, \dot{b}^T(x)\; dx = \begin{pmatrix} \frac{-\sqrt{15}}{4} & \frac{-\sqrt{13}}{4} & \frac{-\sqrt{11}}{4} & \frac{-3}{4} & \frac{-\sqrt{7}}{4} & \frac{-\sqrt{5}}{4} & \frac{-\sqrt{3}}{4} & \frac{63}{4} \end{pmatrix}$$

$$H = 2 \int_0^1 [b(x)\, \dot{b}^T(x) + b(x) b^T(x)]\, dx$$

$$= \begin{pmatrix} 115.0769 & -210.5378 & 178.5757 & -162.6653 & 143.4573 & -121.2436 & 93.9149 & -54.2218 \\ -210.5378 & 509.8182 & -530.3447 & 449.7106 & -400.6545 & 338.6148 & -262.2899 & 151.4332 \\ 178.5757 & -530.3447 & 744.9231 & -695.8882 & 567.2957 & -489.4691 & 379.1411 & -218.8972 \\ -162.6653 & 449.7106 & -695.8882 & 848.4675 & -730.9833 & 548.7950 & -446.8691 & 258 \\ 143.4573 & -400.6545 & 567.2957 & -730.9833 & 856 & -659.2203 & 359.4108 & -269.8666 \\ -121.2436 & 338.6148 & -489.4691 & 548.7950 & -659.2203 & 837.7143 & -503.4878 & -26.8328 \\ 93.9149 & -262.2899 & 379.1411 & -446.8961 & 395.4108 & -503.4878 & 1268 & -1447.9945 \\ -54.2218 & 151.4332 & -218.8972 & 258 & -269.6866 & -26.8328 & -1447.9945 & 2900 \end{pmatrix}$$

The optimal values of unknown parameters c^* can be obtained using Lagrange multiplier technique as

$$c^* = -H^{-1}\, c + H^{-1} F_1^T (F_1 H^{-1} F_1^T)^{-1} (F_1 H^{-1}\, c + b_1),$$

$$c^* = [0.897145 \;\; 0.710459 \;\; 0.566356 \;\; 0.450951 \;\; 0.355758 \;\; 0.274292 \;\; 0.198797 \;\; 0.110808]$$

Table (2) shows comparison between exact and approximate solution by using the operational matrix of derivative of OBP of degree 8

x	Exact solution	OBP	|Exact - b_{i7} |
0	2	2.000000000000002	0.000000000000002
0.1	1.863804265845607	1.863804265845606	0.000000000000001
0.2	1.736253775770209	1.736253775770209	0.000000000000000
0.3	1.616071961185921	1.616071961185920	0.000000000000001
0.4	1.502056000789419	1.502056000789417	0.000000000000002
0.5	1.393064785622723	1.393064785622723	0.000000000000000
0.6	1.288007495877680	1.288007495877678	0.00000000000002
0.7	1.185832682072615	1.185832682072615	0.000000000000000
0.8	1.085517743229661	1.085517743229658	0.00000000000003
0.9	0.986058694681254	0.986058694681250	0.000000000000004
1	0.886460118134294	0.886460118134288	0.00000000000006

Conclusion:

In this paper the properties of the combination for (OBP) and Bernstein polynomials themselves defined on the interval [0, 1] are analyzed. We derived $8*8$ Bernstein polynomials operational Matrices for derivative and integration in details directly. The orthonormal Bernstein operational matrix is used to reduce the variational problems to solve a system of linear algebraic equations. The above example supports this claim.

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مصفوفات العمليات الجديدة من الدرجة السابعة لمتعددات حدود برنشتن المتعامدة

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الخلاصة:

استنادا" الى تحليل خصائص متعددات حدود برنشتن ، تم توسيع متعددات حدود برنشتن المتعامدة المعرفة على الفترة [0,1] للدرجة السابعة، تم تقديم طريقة حسابية اخرى لمصفوفات العمليات للمشتقة D_b وللتكامل R_{n+1}^{B} على التوالي .كذلك قارنا نتيجة للطريقة المقترحة مع الاجابات الحقيقية لأظهار التقارب ومزايا الطريقة الجيدة

الكلمات المفتاحية: أساس بيرنشتاين ومنحنيات بيزيه، عملية عزام شميت المتعامدة، الحل العددي للسيطرة المثلى الوقت المتغاير المفرد بأستخدام المصفوفات التنفيذية.