PACIS 2023 Proceedings

Pacific Asia Conference on Information Systems (PACIS)

7-8-2023

Micro-Fulfillment Center Inventory Policies for Digital Grocery Ecosystem

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Completed Research Paper

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Abstract

As a new phenomenon of grocery business digital transition, the micro-fulfillment center (MFC) requires further exploration of its management issues. This study aims to address the MFC assortment and inventory decision problem for the digital grocery ecosystem. With the goal of maximizing the profit, we first propose an MFC inventory decision framework based on the Markov decision process. Under this decision framework, we analyze various inventory decision scenarios, including single-period, multi-period, deterministic demand, stationary demand distribution, and varying demand distribution cases. To solve the problem under these scenarios, we propose several effective heuristics and algorithms. Experimental results show that the proposed heuristic policies outperform the benchmark significantly. Meanwhile, based on the critical findings, we also provide management insights for the MFC inventory problem. This study contributes to the research and practice in the field of grocery business digital transformation and digital ecosystems.

Keywords: Micro-fulfillment center, last-mile delivery, inventory policy, omnichannel retail, digital grocery ecosystem

Introduction

Amid the steady rise of e-commerce and changing customer behaviors, traditional brick-and-mortar retailers are increasingly adopting omnichannel customer services, such as buy online pick up in-store (BOPS) and buy online return in-store (BORS). The COVID-19 crisis has accelerated this trend dramatically over the past three years. According to the Quarterly E-Commerce Report Historical Data,[1] e-commerce sales as a share of total retail sales in the third quarter of 2022 were 32% higher than before the pandemic (in the third quarter of 2019). For physical retailers, adopting online channels has become a survival requirement. Omnichannel transformation not only helps retailers attract new customers from different channels (Gao & Su, 2017b), but it also allows retailers to integrate all channels into one seamless experience, which can increase customer loyalty and repeat purchases (Lazaris & Vrechopoulos, 2014; Bell et al., 2018; Kumar et al., 2019).

[1] https://www.census.gov/retail/ecommerce/historic_releases.html

With the rising of digital transformation in grocery business, online channels also benefit from brick-and-mortar stores in the race for same-day and instant delivery, such as one-hour or 30-minute delivery. Faster delivery is becoming the new norm, with 90% of consumers expecting 2-to-3 day shipping as a baseline expectation, and 30% expecting same-day delivery (McCabe & Custard, 2022). The World Economic Forum reports that same-day and instant delivery already account for over 10% of parcel deliveries in China, and are projected to reach an aggregate online retail share of around 15% in the United States by 2025 (World Economic Forum, 2020). This shift is changing the role of physical stores in e-commerce. Retailers like Walmart, Target, and Best Buy are utilizing their brick-and-mortar stores instead of central distribution centers (CDCs) as fulfillment centers to fulfill online orders and ship from stores closer to customers, reducing shipping time and costs (Burns et al., 2022).

While omnichannel retailers are utilizing brick-and-mortar stores for last-mile delivery, they face new challenges in fulfilling orders. One of the main obstacles is the lack of inventory visibility across channels. For example, a product may appear in stock to an online customer, when it has already been added to an in-store customer's shopping cart. Retailers estimate that their inventory accuracy is only 65%, according to Zebra (2018). This unreliable inventory visibility can lead to stockouts and shipping delays, resulting in lower customer satisfaction and fewer repeat purchases (Schwartz, 1966). Another challenge is picking costs. Unlike CDCs, stores are not designed for efficient order fulfillment, which can result in low picking efficiency. According to McKinsey (Barbee et al., 2021), in-store picking can cost 1.5 to 2 times more than picking at CDCs.

Micro-fulfillment is an increasingly popular solution to overcome obstacles in the online order fulfillment process. Micro-fulfillment centers (MFCs) are small-scale warehouses placed in densely populated urban areas closer to consumers (Ladd, 2022). They can be installed inside stores, leveraging unproductive space, or placed as separate warehouse sites in urban areas. MFCs offer several advantages. First, they can improve inventory visibility by separating the MFC inventory from the store's inventory, reducing the likelihood of selling out of stock and avoiding shipping delays. Second, MFCs only store the items that customers want most in their limited space, which can make it quicker and easier to pick products and reduce labor costs. Third, as MFCs are located closer to customers, they can drive down the last-mile delivery costs and shipping time. Walmart has already piloted MFCs and has started to scale in dozens of stores (Ward, 2021).

While using MFCs can offer several benefits for the online order fulfillment process, there is a critical challenge that needs to be addressed: MFC inventory decision problem. Due to the limited space in an MFC, omnichannel retailers can only select a portion of their Stock Keeping Units (SKUs) to be housed there. While selecting the most popular items is one way to approach this, retailers can further improve MFC performance by considering other SKU-specific factors as well. For example, two items with the same level of customer demand may have different volumes and picking costs. By selecting a product with a smaller volume and higher in-store picking cost to be placed in the MFC, retailers can expect to get higher profits. Furthermore, it is also important to quantify the demand uncertainty of online channels to improve the performance of the MFC. In a word, the MFC, as a new phenomenon of grocery business digital transformation, requires further exploration of its management issues. Therefore, in this paper, we endeavor to propose an MFC inventory decision framework and derive inventory policies for MFCs by taking into account the demand uncertainty.

This paper contributes to the research and practice in the field of IS on digital transformation and digital ecosystems. First, we provide an MFC inventory decision framework for the digital grocery ecosystem. Specifically, we formulate the MFC inventory decision problem as a dynamic decision problem under the Markov decision process (MDP) framework. In the formulation, we recognize and define the new cost and profit, which differ from prior inventory studies. Second, we derive and propose several MFC inventory policies under different scenarios. The proposed policies can effectively avoid the curse of dimensionality, making them more applicable to practical scenarios. We also provide algorithms to implement the proposed policies. Experiment results show that our policies outperform the benchmark significantly. Third, this paper provides several important findings that contribute valuable management insights to the digital transformation practice in the grocery business field. Fourth, the study also contributes to the research on inventory management, as we propose an inventory decision framework and several heuristics under a new scenario in grocery business digital transformation.

Related Literature

Inventory management for omnichannel retailers has become a growing research area (Hübner et al., 2022). Most works in this field focus on determining optimal target inventory levels and inventory replenishment policies under omnichannel scenarios. For instance, Gao & Su (2017a) propose an in-store inventory decision model with a random demand for a single product case to determine optimal inventory levels. Meanwhile, Zhang et al. (2018) derive optimal pricing and inventory decisions under both the online-only and omnichannel strategies for an online retailer. Shi et al. (2018) determine the optimal inventory level by considering an optimal price discount for a single durable product under the BOPS strategy with pre-orders. Gabor (2022) designs an inventory model considering a CDC and multiple stores to propose an online discounts policy. Studies have also explored omnichannel settings with independent in-store and online demands. For example, Lu et al. (2020) explore the optimal inventory level for fulfilling dual-channel demands. Saha & Bhattacharya (2021) propose a Markov model to derive optimal inventory policy by assuming online and in-store demands follow independent Poisson processes. Govindarajan et al. (2021) propose a heuristic to determine how much inventory to keep at each location and where to fulfill each online order from for a seasonal product with multiple facilities (i.e., stores and CDCs). Abouelrous et al. (2022) extend this work with a stochastic optimization approach and scenario clustering. However, these studies are limited to one product and do not consider assortment planning.

In practice, managers often have to make inventory decisions for a wide range of products under limited resource conditions, such as limited shelf space or warehouse capacity. In such scenarios, retailers need to decide which products to stock in different channels, in order to maximize sales while minimizing inventory costs. For example, Gao & Su (2017b) reveal the types of products that are suitable for implementing a BOPS strategy. Geunes & Su (2020) use a consumer choice model to determine which products should be available in-store, online, or through both channels and their stock levels. Hense & Hübner (2022) consider customer demand interactions across channels to investigate the assortment and inventory problem for an omnichannel retailer. Building on this work, Schäfer et al. (2023) further analyze the demand effects across channels to optimize assortment decisions. However, in this stream, previous research mostly focuses on assortment and inventory decisions for different channels (e.g., online and in-store) under a single-period decision scenario.

Our work is also related to the literature on omnichannel inventory replenishment. For instance, Xu et al. (2017) develope a dynamic programming algorithm to address the problem of a single product with a given demand. Xu & Cao (2019) propose a finite horizon, periodic review inventory model that takes into account stochastic demand. They also suggest an optimal myopic policy for a single product. In contrast, Li (2020) examines the multi-period inventory model for the replenishment of the CDC and multiple nearby stores under the ship-from-store-to-store strategy. The study is conducted for a single product with exogenous demand, and aimed to address the problem of replenishing inventory at multiple locations. Goedhart et al. (2022) develope a Hierarchical Markov Process-based model to address the replenishment problem for a brick-and-mortar store that serves both online and in-store demand. Nevertheless, these models are restricted to the scenario where only one product is considered.

Distinguished from prior studies, this paper focus on addressing the MFC assortment and inventory decision problem in the context of grocery business digital transformation. Multiple items, uncertain demand, and multiple decision periods are considered. This paper proposes novel heuristic policies for optimizing MFC inventory decision problem, which effectively address the curse of dimensionality. By recognizing and defining different decision factors under the new digital grocery scenario, this paper offers several insights on inventory optimization that distinguishes it from prior research.

Problem Formulation

In this section, we begin by presenting the process of fulfilling online orders for a brick-and-mortar store that utilizes an MFC. Then, we proceed to formulate the assortment and inventory problem under the Markov decision process framework (Puterman, 2014).

The Online Order Fulfillment Process with an MFC

An MFC is a small warehouse that hold a small assortment of frequently purchased products for rapid shipping. MFCs can utilize robotic mobile fulfillment systems to speed up the picking process (Hein et al., 2022). Hence, picking items from the MFC is much faster than picking items from the sales floor. In practice, retailers transfer products into an MFC periodically (e.g., every week) from their in-store stockrooms to fulfill online orders effectively.

Figure 1 depicts the online order fulfillment process for a brick-and-mortar store with an MFC. After an online order arrives, a picker or automation bot will try to pick items from the MFC. If an item in the order is not available in the MFC, the picker will pick it from the sales floor with incurring a higher picking cost and a longer picking time. All picked products will be sent to the packing workstation, which is mostly located in the MFC, to be packaged for a delivery or customer pick-up.

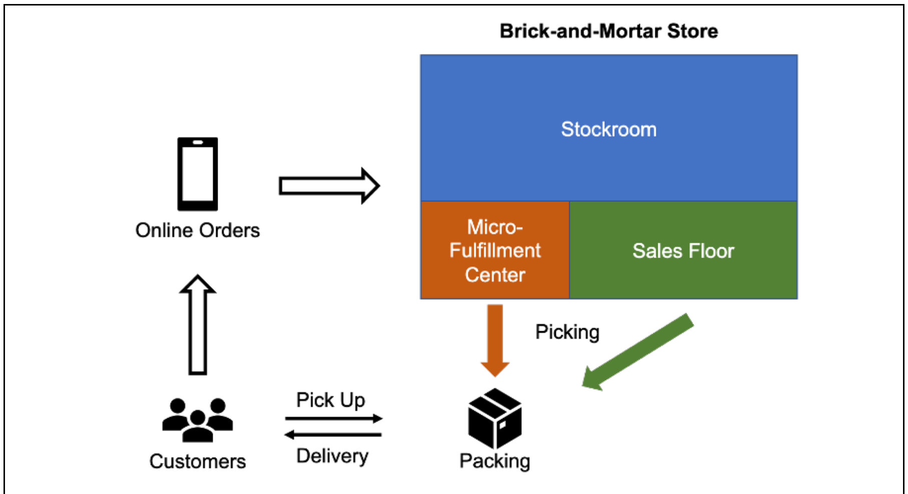

Figure 1. The Online Order Fulfillment Process

The MFC Inventory Decision Framework

We formulate the MFC inventory decision problem based on the MDP framework. In the setting, managers make the assortment and inventory decisions periodically (e.g., every week) with real-time inventory levels, demand distributions, recognized costs, and profits. As shown in Figure 2, at each decision point, Managers can use the information about forecasted online demand, current inventory level, the free space of MFC, and the volume, extra profit, transfer cost of SKUs to determine which SKUs to transfer into or out from the MFC, as well as how many units to transfer. Table 1 summarizes the notations used in this paper.

Under the time horizon T, the *system state* at decision period $t \in [0, T]$ is denoted as $\boldsymbol{y_t} = (y_{1,t}, y_{2,t}, \cdots, y_{i,t}, \cdots, y_{I,t})$, where $y_{i,t} \geq 0$ is the *current inventory level* of SKU $i \in \{1,2,\cdots,I\}$ at period t. The system state space is $\boldsymbol{Y}$. At the beginning of each decision period, the manager needs to make the assortment and inventory decisions for the MFC, that is, determine which set of SKUs and their quantities to transfer in or out the MFC to maximize the profit. We denote the *transfer action* as $\boldsymbol{x_t} = (x_{1,t}, x_{2,t}, \cdots, x_{i,t}, \cdots, x_{I,t})$, where $x_{i,t} \in [-y_{i,t}, +\infty)$ is the transfer quantity of SKU i at period t. $x_{i,t} < 0$ means transferring out from the MFC, which happens when there is a need to make space for other products at current period. $x_{i,t} > 0$ means transferring into the MFC. The transfer action space is $\boldsymbol{X}$. We optimize the MFC assortment and inventory problem by deriving the optimal transfer actions for each decision period. We also define a *post-action state*, which is denoted as $\boldsymbol{z_t} = (z_{1,t}, z_{2,t}, \cdots, z_{i,t}, \cdots, z_{I,t})$, $z_{i,t} = y_{i,t} + x_{i,t}$ is the *post-action inventory level*. The post-action inventory level can be utilized to aggregate system states and transfer actions, as different current inventory levels and transfer actions can result in the same post-action inventory level.

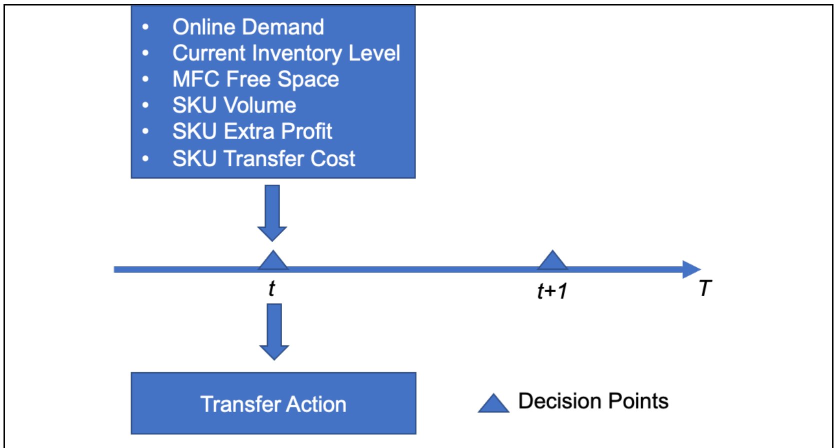

Figure 2. Dynamic Decision Process

Notations	Description
T	Time horizon.
t	Decision period, $t \in [0, T]$.
$\boldsymbol{y_t}$	System state at decision period t, $\boldsymbol{y_t} = (y_{1,t}, y_{2,t}, \cdots, y_{i,t}, \cdots, y_{I,t})$.
$y_{i,t}$	Current inventory level of SKU i, $i \in \{1,2, \cdots, I\}$, at period t.
$\boldsymbol{x_t}$	Transfer action at period t, $\boldsymbol{x_t} = (x_{1,t}, x_{2,t}, \cdots, x_{i,t}, \cdots, x_{I,t})$.
$x_{i,t}$	Transfer quantity of SKU i at period t, $x_{i,t} \in [-y_{i,t}, +\infty)$.
$\boldsymbol{z_t}$	Post-action state at period t, $\boldsymbol{z_t} = (z_{1,t}, z_{2,t}, \cdots, z_{i,t}, \cdots, z_{I,t})$.
$z_{i,t}$	Post-action inventory level of SKU i at period t, $z_{i,t} = y_{i,t} + x_{i,t}$.
$\boldsymbol{d_t}$	Customer demand at period t, $\boldsymbol{d_t} = (d_{1,t}, d_{2,t}, \cdots, d_{i,t}, \cdots, d_{I,t})$.
$d_{i,t}$	Customer demand of SKU i at period t, $d_{i,t} \in \mathbb{N}^0$.
$\boldsymbol{\Theta_t}$	Customer demand distribution at period t, $\boldsymbol{\Theta_t} = (\boldsymbol{\theta_{1,t}}, \boldsymbol{\theta_{2,t}}, \cdots, \boldsymbol{\theta_{i,t}}, \cdots, \boldsymbol{\theta_{I,t}})$.
$\boldsymbol{\theta_{i,t}}$	Customer demand distribution parameter of SKU i at period t.
S	Total space of an MFC.
s_i	Unit volume of SKU i.
c_t^{tr}	Transfer cost at period t.
a_i	Unit transfer cost of SKU i.
q_i	Unit extra profit of SKU i.
g_t	Extra profit at period t.
r_t	Single-period profit at period t.
R_t	Total system discounted profit at period t.
γ	Discount rate.

Table 1. Notations

The system state transition depends on current inventory level, transfer action, and customer demand during the period. Let $\boldsymbol{d_t} = (d_{1,t}, d_{2,t}, \cdots, d_{i,t}, \cdots, d_{I,t})$ denote the customer demand at period t, where $d_{i,t} \in$

$\mathbb{N}$ is the customer demand of SKU i at period t. $d_{1,t}, d_{2,t}, \cdots, d_{i,t}, \cdots, d_{I,t}$ are independent but not necessarily identically distributed random variables. Their distributions are denoted as $\boldsymbol{\Theta_t} = (\boldsymbol{\theta_{1,t}}, \boldsymbol{\theta_{2,t}}, \cdots, \boldsymbol{\theta_{i,t}}, \cdots, \boldsymbol{\theta_{I,t}})$, where $\boldsymbol{\theta_{i,t}}$ is a vector of demand distribution parameters of SKU i. Because retailers mostly put popular products in the MFC, demand information is crucial for the MFC decision problem. Our model is designed to be adaptable and flexible in utilizing different types of forecasted demand information, such as deterministic demand, demand distribution, and mixture demand distribution, which can be forecasted by machine learning algorithms (Carbonneau et al., 2008; Huber & Stuckenschmidt, 2020).

Then, the inventory level transition of SKU i from current period t to the next period $t+1$ can be written as

$$y_{i,t+1} = \max\{y_{i,t} + x_{i,t} - d_{i,t}, 0\}, \tag{1}$$

and the system state transition probability is

$$p(\boldsymbol{y_{t+1}}|\boldsymbol{y_t}, \boldsymbol{x_t}) = p(\boldsymbol{d_t} = \boldsymbol{y_t} + \boldsymbol{x_t} - \boldsymbol{y_{t+1}}|\boldsymbol{\Theta_t}), \tag{2}$$

where $p(\boldsymbol{d_t} = \boldsymbol{y_t} + \boldsymbol{x_t} - \boldsymbol{y_{t+1}}|\boldsymbol{\Theta_t}) = \prod_{i=1}^{I} p(d_{i,t} = y_{i,t} + x_{i,t} - y_{i,t+1}|\boldsymbol{\theta_{i,t}})$.

Considering the limited storage capacity of the MFC, we also take into account the size of the products in the decision-making process. Suppose the space of an MFC is S, the unit volume of products is $\boldsymbol{s} = (s_1, s_2, \cdots, s_i, \cdots, s_I)$, where s_i is the unit volume of SKU i. Then, we have the constraint that $\boldsymbol{s}' \cdot (\boldsymbol{y_t} + \boldsymbol{x_t}) \leq S$, which can help us obtain a finite system state space and a finite action space.

In the framework, differing from inventory problems in prior studies, costs and profits in the MFC problem should be defined by comparing two different online order fulfillment processes. Comparing with fulfilling an online order by picking items from a store shelf, we recognize the following cost and profit for the MFC: (1) *Transfer cost*. At the beginning of each period, selected products are either transferred into the MFC or transferred out from the MFC, which can incur the transfer cost. We denote the unit transfer cost as $\boldsymbol{a} = (a_1, a_2, \cdots, a_i, \cdots, a_I)$, where a_i is the unit transfer cost of SKU i. Then, we assume the transfer cost of period t takes the following form:

$$c_t^{tr}(\boldsymbol{x_t}) = \sum\nolimits_{i=1}^{I} a_i |x_{i,t}|. \tag{3}$$

(2) *Extra profit*. As we introduced, picking items from the MFC can lead to lower picking costs, shorter picking time, and faster order fulfillment than picking them from the sales floor. It can also improve inventory visibility, reducing the likelihood of selling out of stock. Given these benefits, we introduce a unit extra profit $\boldsymbol{q} = (q_1, q_2, \cdots, q_i, \cdots, q_I)$, where q_i is the unit extra profit incurred by picking a unit of SKU i from the MFC instead of the sales floor. The expected extra profit at decision period t can be written as

$$\mathbb{E}[g_t(\boldsymbol{y_t}, \boldsymbol{x_t})] = \sum_{i=1}^{I} \left[\sum_{d_{i,t}=0}^{y_{i,t}+x_{i,t}} p(d_{i,t}|\boldsymbol{\theta_{i,t}}) d_{i,t} q_i + \sum_{d_{i,t}=x_{i,t}+y_{i,t}+1}^{\infty} p(d_{i,t}|\boldsymbol{\theta_{i,t}}) (x_{i,t} + y_{i,t}) q_i \right], \tag{4}$$

where the first part inside the square brackets represents the extra profit generated when the demand is lower than or equal to the inventory level, while the second part represents the extra profit generated when the demand exceeds the inventory level.

Note that our model does not include the cost and profit of a product itself, as picking it from different channels (i.e., MFC and sales floor) does not change its cost and profit. We focus on the relative cost and benefit of using the MFC compared to picking items from sales floor.

Then, at period t, the *single-period profit function* is

$$r_t(\boldsymbol{y_t}, \boldsymbol{x_t}) = g_t(\boldsymbol{y_t}, \boldsymbol{x_t}) - c_t^{tr}(\boldsymbol{x_t}). \tag{5}$$

The *total discounted profit* takes the form:

$$R_t(\boldsymbol{y_t}, \boldsymbol{x_t}) = \sum\nolimits_{k=t}^{T} \gamma^{k-t} r_k(\boldsymbol{y_k}, \boldsymbol{x_k}), \tag{6}$$

where γ is the *discount factor*, $0 \leq \gamma \leq 1$. Our objective is to obtain an optimal policy π^*:

$$\pi^* = \underset{\pi}{\operatorname{argmax}}\, R_t^{\pi}(\boldsymbol{y_t}), \tag{7}$$

where a policy, $\pi(\boldsymbol{y}) = \boldsymbol{x}$ for all $\boldsymbol{y} \in \boldsymbol{Y}$ and $\boldsymbol{x} \in \boldsymbol{X}$, is a function that maps each inventory state to a transfer action.

The Bellman equation is given by

$$R_t^*(\boldsymbol{y_t}) = \max_{\boldsymbol{x_t}} \left[r_t(\boldsymbol{y_t}) + \gamma \sum_{\boldsymbol{y_{t+1}} \in \boldsymbol{Y}} p(\boldsymbol{y_{t+1}} | \boldsymbol{y_t}, \boldsymbol{x_t}) R_{t+1}^*(\boldsymbol{y_{t+1}}) \right]. \tag{8}$$

In practice, there are thousands of different products that should be placed into the MFC. Due to the curse of dimensionality, it is extremely difficult, if not impossible, to solve the MFC inventory problem using dynamic programming methods. In next section, we propose different inventory policies for different scenarios to address this problem.

Micro-Fulfillment Center Inventory Policies

This section shows our critical analytical findings of the inventory policy structure and value function properties. Based on these findings, we propose several policies and algorithms under different decision scenarios.

The Deterministic Demand Case

We first consider an MFC inventory problem with deterministic demand to explore the structure of the optimal policy. Given the deterministic demand, it is unnecessary for the inventory level to exceed the customer demand at each period, and by the end of each period, all products in the MFC should be sold out. In this scenario, the MFC inventory problem can be simplified as:

$$\begin{aligned} &\max && \textstyle\sum_{i=1}^{I} (q_i - a_i) x_i \\ &\text{subject to} && \textstyle\sum_{i=1}^{I} s_i x_i \leq S \\ & && 0 \leq x_i \leq d_i,\, i \in \{1,2,\cdots,I\}. \end{aligned} \tag{9}$$

This problem can be approximately regarded as a fractional knapsack problem, which can be solved by the greedy solution (Magazine et al., 1975). Inspired by this, we propose a greedy MFC inventory policy for the deterministic demand case.

Deterministic Demand Policy (DD). *There is a greedy policy for the MFC inventory problem with deterministic demand, that is, at each period, iteratively selecting the SKU with the greatest profit-to-volume ratio,* $(q_i - a_i)/s_i$.

Algorithm 1. (The algorithm for solving DD)

Step 1. Sort the SKUs with the profit-to-volume ratio $(q_i - a_i)/s_i$.

Step 2. Check if the MFC has available space. If the available space is 0, terminate. Otherwise, go to Step 3.

Step 3. Transfer the SKU with the highest profit-to-volume ratio into the MFC until $x_i = d_i$ or the available space of the MFC becomes 0. Return to Step 2.

The Single-Period MFC Inventory Problem

In the following, we consider the MFC scenarios with random demand. For a single product, the expected extra profit function can be written as:

$$\mathbb{E}[g_t(y_t, x_t)] = \sum_{d_t=0}^{y_t+x_t} p(d_t|\boldsymbol{\theta_t}) d_t q + \sum_{d_t=x_t+y_t+1}^{\infty} p(d_t|\boldsymbol{\theta_t})\,(x_t + y_t) q, \tag{10}$$

and the single-period profit function is given by

$$r_t(y_t) = g_t(y_t, x_t) - c_t^{tr}(x_t) \tag{11}$$

where $c_t^{tr}(x_t) = a|x_t|$.

In this scenario, we obtain the following findings:

Lemma 1. *For a single product, the expected extra profit at the current period $\mathbb{E}(g_t)$ is increasing in the post-action inventory level z_t, $z_t = y_t + x_t$. That is, given the current inventory level y_t, g_t is increasing in the transfer action x_t. And, given a fixed transfer action x_t, g_t is also increasing in system state y_t. As z_t increases, $\mathbb{E}(g_t)$ gradually approaches $\mathbb{E}(d_t)q$.*

Proof: The expected extra profit at the current period $\mathbb{E}(g_t)$ is

$$\mathbb{E}[g_t(z_t)] = \sum_{d_t=0}^{z_t} p(d_t|\boldsymbol{\theta_t})d_t q + \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta_t})\, z_t q, \tag{12}$$

and

$$\mathbb{E}(d_t)q = \sum_{d_t=0}^{+\infty} p(d_t|\boldsymbol{\theta_t})d_t q\,. \tag{13}$$

Then, we can get:

$$\begin{aligned}\mathbb{E}(d_t)q - \mathbb{E}(g_t) &= \sum_{d_t=0}^{+\infty} p(d_t|\boldsymbol{\theta_t})d_t q - \sum_{d_t=0}^{z_t} p(d_t|\boldsymbol{\theta_t})d_t q - \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta_t})\, z_t q \\ &= \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta_t})\, d_t q - \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta_t})\, z_t q \\ &= \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta_t})\,(d_t - z_t)q. \end{aligned} \tag{14}$$

The Eq. (14) indicates that, as z_t increases, $\mathbb{E}(d_t)q - \mathbb{E}(g_t)$ will decrease and approach 0. Hence, we conclude that, for a single product, the expected extra profit at the current period $\mathbb{E}(g_t)$ is increasing in the post-action inventory level z_t, $z_t = y_t + x_t$. That is, given the current inventory level y_t, g_t is increasing in the transfer action x_t. And, given a fixed transfer action x_t, g_t is also increasing in system state y_t. As z_t increases, $\mathbb{E}(g_t)$ gradually approaches $\mathbb{E}(d_t)q$. □

Lemma 2. *Given current inventory level y_t, when $x_t < 0$, that is, we transfer out the product from the MFC, the single-period profit r_t is increasing in x_t. When $x_t \geq 0$, that is, we transfer the product into the MFC, if the cumulative possibility $F(d_t = y_t + x_t) \leq 1 - a/q$, r_t is increasing in x_t, otherwise, r_t is decreasing in x_t. Meanwhile, given a fixed transfer action x_t, r_t is increasing in y_t.*

Proof: Given current inventory level y_t, when $x_t < 0$, based on Eq. (11), we can obtain

$$r_t(y_t) = g_t(y_t, x_t) + a x_t. \tag{15}$$

According to Lemma 1, given y_t, $g_t(y_t, x_t)$ is increasing in x_t, so the single-period profit r_t is increasing in x_t.

When $x_t \geq 0$, we have

$$r_t(y_t) = g_t(y_t, x_t) - a x_t. \tag{16}$$

Let $g_t(y_t, x_t + 1) - g_t(y_t, x_t)$:

$$\begin{aligned} g_t(y_t, x_t+1) - g_t(y_t, x_t) &= p(d_t = z_t + 1|\boldsymbol{\theta}_t)(z_t+1)q + \sum_{d_t=z_t+2}^{+\infty} p(d_t|\boldsymbol{\theta}_t)\,(z_t+1)q \\ &\quad - \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta}_t)\,z_t q \\ &= \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta}_t)\,(z_t+1)q - \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta}_t)\,z_t q \\ &= \sum_{d_t=z_t+1}^{+\infty} p(d_t|\boldsymbol{\theta}_t)\,q \\ &= [1 - F(d_t = z_t)]q, \end{aligned} \tag{17}$$

where $F(d_t)$ is the cumulative possibility function, and $z_t = y_t + x_t$.

Then, the marginal profit is

$$[1 - F(d_t = z_t)]q - a. \tag{18}$$

when the marginal profit is 0, z_t is optimal and $F(d_t = z_t) = 1 - a/q$.

Therefore, when $x_t \geq 0$, if the cumulative possibility $F(d_t = y_t + x_t) \leq 1 - a/q$, r_t is increasing in x_t, otherwise, r_t is decreasing in x_t. Meanwhile, given a fixed transfer action x_t, r_t is increasing in y_t. ☐

Lemma 1 and Lemma 2 indicate that, for a single product, we should put it into the MFC as many as possible to gain extra profit. But, when we take into account the transfer cost to maximize the total profit of a decision period, there is an optimal threshold of the inventory level. Based on this finding, we provide the following MFC inventory policy:

Single-Period Single-Product Random Demand Policy (SSR). *There is a heuristic for the single-period single-product MFC inventory problem with random demand. That is, given the demand distribution of period t, there is an inventory level z_t^* that satisfies $F(d_t = z_t^*) = 1 - a/q$. When the current inventory level is equal to or greater than z_t^*, the transfer action $x_t^* = 0$, otherwise, $x_t^* = z_t^* - y_t$.*

The SSR policy is not able to address the MFC inventory problem well in practice, because there are thousands of products. However, it can help us develop policies for the multi-product case, where different products share the limited space of the MFC.

For the multi-product scenario, we propose a two-step decision process:

Step 1. Transfer-in Step. Given the current inventory level and the remaining available space, we make the assortment decision on which SKU to transfer into the MFC until the available space in the MFC is fully utilized.

Step 2. Replacement Step. When the MFC space is fully utilized, we decide whether to transfer out items and which items to transfer out from the MFC to free up space for other products.

Inspired by policies DD and SSR, we derive a heuristic for the single-period multi-product case based on the two-step decision process.

Single-Period Multi-Product Random Demand Policy (SMR). *There is a heuristic for the single-period multi-product random demand scenario:*

(1) Transfer-in step. The SKU i with the highest marginal profit-to-volume ratio, $\{[1 - F(d_i = z_i)]q_i - a_i\}/s_i$, should be prioritized for placement in the MFC until its inventory level reaches the threshold z_i^* that satisfies $F(d_i = z_i^*) = 1 - a_i/q_i$ or the MFC space is fully utilized.

(2) Replacement step. The SKU i should be transferred into the MFC and The SKU j should be transferred out from the MFC only when a) The SKU i has the highest marginal profit-to-volume ratio and its inventory level is lower than the threshold z_i^*. b) The SKU j has the lowest transfer-

out loss $\sum_{d_j=z_j-min\,(\frac{s_i}{s_j},z_j)}^{z_j-1}[1-F(d_j)]q_j + a_j s_i/s_j$. *c)* $\sum_{d_j=z_j-min\,(\frac{s_i}{s_j},z_j)}^{z_j-1}[1-F(d_j)]q_j + a_j s_i/s_j < [1-F(d_i = z_i)]q_i - a_i$.

The algorithm to implement the heuristic is provided as follows:

Algorithm 2. (The algorithm for solving SMR)

Step 1. Check if the MFC has available space. If the available space is 0, go to Step 4. Otherwise, go to Step 2.

Step 2. Sort the SKUs with the marginal profit-to-volume ratio $\{[1-F(d_i = z_i)]q_i - a_i\}/s_i$. Go to step 3.

Step 3. Transfer the SKU with the highest marginal profit-to-volume ratio into the MFC until $z_i = z_i^*$ that satisfies $F(d_i = z_i^*) = 1 - a_i/q_i$ or the MFC space is fully utilized. Return to Step 1.

Step 4. Sort the SKUs with the transfer-out loss $\sum_{d_j=z_j-min\,(\frac{s_i}{s_j},z_j)}^{z_j-1}[1-F(d_j)]q_j + a_j s_i/s_j$. Go to Step 5.

Step 5. Find the SKUs that satisfy $z_i < z_i^*$, and then sort them with the marginal profit-to-volume ratio $\{[1-F(d_i = z_i)]q_i - a_i\}/s_i$. Go to Step 6.

Step 6. Compare the lowest transfer-out loss (SKU *j*) with the marginal profit $[1-F(d_i = z_i)]q_i - a_i$ of the product (SKU *i*) having the highest marginal profit-to-volume ratio. If the lowest transfer-out loss is lower than the marginal profit, transfer out `round` (s_i/s_j) units j, and transfer one unit *i* in to the MFC. Return to Step 4. Otherwise, terminate.

The Multi-Period MFC Inventory Problem

According to the proposed MFC inventory decision framework, maximizing total discount profit requires considering the impact of current-period decision on future-period system state and profit. In this section, we derive the MFC inventory policy under the multi-period decision scenarios.

First, we consider a case where there is one product with stationary demand distribution, that is, the demand distribution of the product is the same across different periods. In this case, the Bellman equation takes the following form:

$$R_t^*(y_t) = \max_{x_t}\left[r_t(y_t) + \gamma \sum_{y_{t+1}\in Y} p(y_{t+1}|y_t, x_t) R_{t+1}^*(y_{t+1})\right]. \quad (19)$$

We can obtain the following findings:

Lemma 3. *At the decision period t, the total discounted profit $R_t^*(y_t)$ is increasing in the system state (current inventory level) y_t.*

Proof: When there is only one SKU, at each decision period, we do not need to consider transferring out products to free up inventory space. In addition, based on Lemma 2, Eq. (1), and Eq. (2), given the customer demand information, if there is a lager current inventory level, we can achieve the same extra profit g_t with a smaller transfer cost. Therefore, at the decision period t, the total discounted profit $R_t^*(y_t)$ is increasing in the system state (current inventory level) y_t. □

Lemma 4. *Given the system state y_t, there is an optimal transfer action x_t^* that satisfies $F(d_t = y_t + x_t^*) = 1 - (\alpha - \gamma a)/q$.*

Proof: As we noted in the proof of Lemma 3, if there is only one SKU, at each decision period, we do not need to consider transferring out products to free up inventory space, that is, $x_t \geq 0$. Given the system state y_t, let $R_t(z_t + 1) - R_t(z_t)$:

$$\begin{aligned} R_t(z_t + 1) - R_t(z_t) &= [1 - F(d_t = z_t)]q - a + \gamma\mathbb{E}[R_{t+1}^*(y_{t+1})|z_t + 1] - \gamma\mathbb{E}[R_{t+1}^*(y_{t+1})|z_t] \\ &= [1 - F(d_t = z_t)]q - a + \gamma a. \end{aligned} \quad (20)$$

Hence, $R_t(z_t+1) - R_t(z_t)$ is decreasing in z_t. As $z_t = y_t + x_t$, Given y_t, when $R_t(z_t+1) - R_t(z_t) = 0$, that is, $F(d_t = z_t) = 1 - (a - \gamma a)/q$, x_t is optimal. In other words, given the system state y_t, there is an optimal transfer action x_t^* that satisfies $F(d_t = y_t + x_t^*) = 1 - (a - \gamma a)/q$. ☐

Lemma 4 indicates a stationary policy for the multi-period single-product stationary demand distribution case (MSS).

Based on these findings, we next analyze the case where there are multiple products with stationary demand distributions. Unlike the single-period multi-product random demand case in the last section, we find that there is no need to take into account the replacement step in the decision process under the stationary demand distribution case.

Lemma 5. *Under the multi-period multi-product stationary demand distribution scenario, items in the MFC need not to be transferred out at any decision period.*

Proof: Based on Lemma 4, under the multi-period case, each product has an optimal inventory level threshold. When multiple products share the inventory space, the inventory level of each product will always be lower than this optimal threshold. This is because when the inventory level of a product approaches this threshold, the marginal profit approaches 0, and before reaching 0, it will be replaced by other products with higher marginal profits. When the demand distribution is stationary, product *i* has the same optimal post-action state z_i^* across decision periods, and its inventory level will never exceed z_i^* at the beginning of each decision period. Therefore, items in the MFC need not to be transferred out at each decision period. ☐

We can also obtain a stationary policy in this case.

Multi-Period Multi-Product Stationary Demand Distribution Policy (MMS). *There is a heuristic for the multi-period multi-product stationary demand distribution case, that is, at each decision period t, the product i with the highest multi-period marginal profit-to-volume ratio,* $\{[1 - F(d_i = z_{i,t})]q_i - a_i + \gamma a_i\}/s_i$*, should be transferred into the MFC first until the inventory level of product i reaches the optimal threshold* $x_{i,t}^*$ *that satisfies* $F(d_i = y_{i,t} + x_{i,t}^*) = 1 - (a_i - \gamma a_i)/q_i$ *or the MFC space is fully utilized.*

Then, we consider the case where there are multiple products with varying demand distributions.

In this case, as the demand distribution of the same product differ across periods, the MFC inventory problem becomes more challenging. First, it is hard to forecast the demand information for many future periods and the forecasted demand information of the period after a long time is unreliable. Second, even we know the varying demand distributions for all future periods, the curse of dimensionality can make the MDP problem extremely challenging.

To make the problem solvable, we consider the following assumption.

Future Demand Assumption: *At decision period t, given* $x_{i,t}^*$ *in policy MMS, the post-action inventory level* $z_{i,t}^* = y_{i,t} + x_{i,t}^*$*, we assume that* $z_{i,t}^* - \mathbb{E}(d_{i,t}) \le z_{i,t+1}^*$ *for all SKUs.*

The future demand assumption guarantees that the future demand fluctuations of SKUs remain within reasonable bounds. Actually, based on policy MMS, for product *i* at each decision period *t*, the post-action inventory level $z_{i,t}$ will approach but never reach the threshold $z_{i,t}^*$, that is, $z_{i,t} < z_{i,t}^*$. This is because when the post-action inventory level of a product *i* approaches the threshold, the marginal profit of that product gradually approaches zero. When the marginal profit of product *i* becomes lower than that of other products, we will choose to transfer other products into the MFC. Given $z_{i,t} < z_{i,t}^*$, then we can obtain $\mathbb{E}(y_{i,t+1}) = z_{i,t} - \mathbb{E}(d_{i,t}) < z_{i,t}^* - \mathbb{E}(d_{i,t}) \le z_{i,t+1}^*$, which indicates that, in practice, the current inventory level $y_{i,t+1}$ at decision period *t*+1 is unlikely to reach the threshold $z_{i,t+1}^*$. Therefore, the future demand assumption is very reasonable.

Lemma 6. *If the Future Demand Assumption holds, the heuristic for the multi-period multi-product varying demand distribution case (MMV) will be the same as the MMS policy.*

Proof: Under the multi-period multi-product varying demand distribution case, for product *i*, given the system state $y_{i,t}$, let $R_{i,t}(z_{i,t}+1) - R_{i,t}(z_{i,t})$, where $z_{i,t}+1 \le z_{i,t}^*$, we can get:

$$R_{i,t}(z_{i,t}+1)-R_{i,t}(z_{i,t})=[1-F(d_{i,t}=z_{i,t})]q_i-a_i+\gamma\mathbb{E}[R^*_{i,t+1}(y_{i,t+1})|z_{i,t}+1]$$
$$-\gamma\mathbb{E}[R^*_{i,t+1}(y_{i,t+1})|z_{i,t}], \quad (21)$$

where

$$\mathbb{E}[R^*_{i,t+1}(y_{i,t+1})|z_{i,t}+1]=R^*_{i,t+1}[\mathbb{E}(y_{i,t+1})=z_{i,t}+1-\mathbb{E}(d_{i,t})], \quad (22)$$

$$\mathbb{E}[R^*_{i,t+1}(y_{i,t+1})|z_{i,t}]=R^*_{i,t+1}[\mathbb{E}(y_{i,t+1})=z_{i,t}-\mathbb{E}(d_{i,t})]. \quad (23)$$

If the Future Demand Assumption holds, that is, $z^*_{i,t}-\mathbb{E}(d_{i,t})\le z^*_{i,t+1}$, then, $\mathbb{E}(y_{i,t+1})\le z^*_{i,t+1}$, which means we will never need to transfer products out of the MFC, that is $x_{i,t}\ge 0$ for each decision period t. Then,

$$R_{i,t}(z_{i,t}+1)-R_{i,t}(z_{i,t})=[1-F(d_{i,t}=z_{i,t})]q_i-a_i+\gamma a_i, \quad (24)$$

which indicates the multi-period marginal profit of product i in policy MMS still holds.

Therefore, if the Future Demand Assumption holds, the heuristic in policy MMS is still effective under the multi-period multi-product varying demand distribution case. □

Lemma 6 indicates that under the Future Demand Assumption, whether the demand follows a stationary distribution or variable distributions across decision periods, we should use the MMS policy, which we redefine as multi-period multi-product policy (MM). The algorithm to implement the heuristic is provided as follows:

Algorithm 3. (The algorithm for solving MM)

Step 1. Check if the MFC has available space. If the available space is 0, terminate. Otherwise, go to Step 2.

Step 2. Sort SKUs with the multi-period marginal profit-to-volume ratio $\{[1-F(d_i=z_{i,t})]q_i-a_i+\gamma a_i\}/s_i$. Go to Step 3.

Step 3. Transfer the SKU with the highest multi-period marginal profit-to-volume ratio into the MFC if its inventory level is lower than the threshold $x^*_{i,t}$ that satisfies $F(d_i=y_{i,t}+x^*_{i,t})=1-(a_i-\gamma a_i)/q_i$. Return to Step 1.

To summarize, we derive several heuristic policies and provide algorithms for the MFC inventory problem in different scenarios. The policies are designed to avoid the curse of dimensionality that arises in the MDP framework, making them more feasible in practice. Specifically, the policies include:

- Deterministic Demand Policy (DD).
- Single-Period Single-Product Random Demand Policy (SSR).
- Single-Period Multi-Product Random Demand Policy (SMR).
- Multi-period Single-product Stationary Demand Distribution Policy (MSS).
- Multi-Period Multi-Product Stationary and Varying Demand Distribution Policy (MM).

These policies provide a framework for making MFC inventory decisions and optimizing online order fulfillment process, with the goal of maximizing profits.

Experimental Results

In this section, we compare and evaluate the proposed policies by numerical simulations. In the simulation, we suppose the manager make the MFC inventory decision every week. We vary the time horizon T from 1 to 10 weeks, and consider there are three types of products. In the experiment, we assume that the customer demand of SKU i (d_i) is a random variable following a Poisson distribution with the arrival rate λ_i (Forsberg, 1997). We set their demand distribution with arrival rates 15, 20, 30, unit volumes as 2, 6, 4, unit extra profit as 6, 4, 2, and unit transfer cost as 0.6, 1, 0.4. We suppose the initial inventory of the products at the first period is 0. The total space of the MFC is 200. The discount factor is 0.99.

We compare the proposed policies with the benchmark: demand sorting policy (DS). Under this policy, the manager places the products into the MFC in accordance with the sorting of product demand. It is a common practice in industry to transfer products into the MFC based on their expected sales performance.

As the single-product policies serve only as the foundation for our multi-product policies, we focuse on comparing the performance of the multi-product policies (i.e., DD, SMR, and MM), which are more practical than single-product policies.

The experimental results depicted in Figure 3 clearly demonstrate the superiority of our proposed policies (i.e., DD, SMR, MM). As shown, the total discounted profit generated by our policies are consistently higher than the benchmark (DS) across different time horizons, indicating its robustness and effectiveness. Table 2 summarizes the performance improvement of our proposed policies compared to the benchmark. DD, SMR, and MM can improve the total discounted profit by 76.6%, 81.4%, 82.5%, respectively.

SMR and MM exhibit comparable performance as they utilize similar decision-making criteria and principles. SMR is based on the marginal profit-to-volume ratio $\{[1 - F(d_i = z_i)]q_i - a_i\}/s_i$, while MM is based on the multi-period marginal profit-to-volume ratio, $\{[1 - F(d_i = z_{i,t})]q_i - a_i + \gamma a_i\}/s_i$. This shows that each unit of product i contributes to the future profit only by an amount of γa_i. Therefore, when the customer demand distribution is stationary or fluctuates within a certain range across different periods, considering the impact of current transfer actions on future returns does not have a significant impact on the total discounted profit.

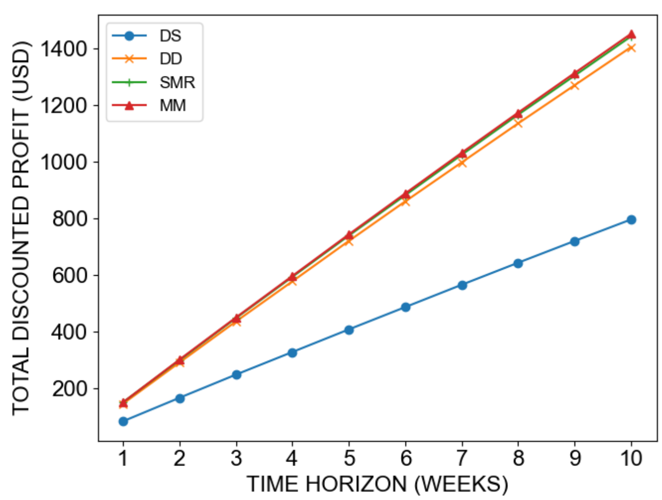

Figure 3. Policy Comparison

Policies	Profit Improvement
DS	–
DD	76.6%
SMR	81.4%
MM	82.5%

Table 2. Profit Improvement

Conclusion

As a new phenomenon of grocery business digital transition, the MFC requires further exploration of its management issues. This study aims to address the MFC assortment and inventory decision problem for the digital grocery ecosystem. With the goal of maximizing the profit, we first propose an MFC inventory decision framework based on the Markov decision process. Under this decision framework, we analyze several inventory decision scenarios, such as single-period cases, multi-period cases, deterministic demand case, stationary demand distribution case, and varying demand distribution case. We solve the MFC inventory problem under these scenarios and propose effective heuristics, such as DD, SMR, and MM. Simulation results show that the proposed heuristic policies outperform the benchmark significantly.

This paper contributes to the research and practice in the field of grocery business digital transformation and digital ecosystems. First, we provide an MFC inventory decision framework for the digital grocery ecosystem. Specifically, we formulate the MFC inventory decision problem as a dynamic decision problem under the Markov decision process (MDP) framework. In the formulation, we recognize and define the new cost and profit, which differ from prior inventory studies. Second, we derive and propose several MFC inventory heuristic policies under different scenarios. The proposed policies can effectively avoid the curse of dimensionality, making them more applicable to practical scenarios. We also provide algorithms to implement the proposed policies. Third, this paper provides several important findings that contribute valuable management insights to the digital transformation practice in the grocery business field. Fourth, the study also contributes to the research on inventory management, as we propose an inventory decision framework and several heuristics under a new scenario in grocery business digital transformation. This paper provides a foundation for future research on the MFC management problem.

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