Estimation of Confidence-Interval for Yearly Electricity Load Consumption Based On Fuzzy Random Auto-Regression Model

Riswan Efendi[1,2,*], Nureize Arbaiy[1], Mustafa Mat Deris[1]

[1]Faculty of Computer Science and Information Technology
University Tun Hussein Onn Malaysia, Batu Pahat 86400, Malaysia

[2]Mathematics Department, UIN Sultan Syarif Kasim Riau, Indonesia
riswan@uthm.edu.my;nureize@uthm.edu.my;mmustafa@uthm.edu.my

Abstract. Many models have been implemented in the energy sectors, especially in the electricity load consumption ranging from the statistical to the artificial intelligence models. However, most of these models do not consider the factors of uncertainty, the randomness and the probability of the time series data into the forecasting model. These factors give impact to the estimated model's coefficients and also the forecasting accuracy. In this paper, the fuzzy random auto-regression model is suggested to solve three conditions above. The best confidence interval estimation and the forecasting accuracy are improved through adjusting of the left-right spreads of triangular fuzzy numbers. The yearly electricity load consumption of North-Taiwan from 1981 to 2000 are examined in evaluating the performance of three different left-right spreads of fuzzy random auto-regression models and some existing models, respectively. The result indicates that the smaller left-right spread of triangular fuzzy number provides the better forecast values if compared with based line models.

Keywords: Fuzzy random variable, auto-regression model, left-right spread, triangular fuzzy number, forecasting error, electricity.

1 Introduction

The decision makers and researchers should pay attention seriously to enhance the studies in organizing and managing the electricity load demand and consumption, respectively. The output of these studies is very determinative for energy planning and power management. Additionally, load forecasting helps an electric utility to make important decisions including decisions on purchasing and generating electric power, load switching, and infrastructure development [1].

Forecasting is a predictive analytical technique that deals with estimation the future, generally by considering the past data sets and models. It can be applied in various domains of management, finance-economic, energy, engineering, computer science, and others. In electricity forecasting, among the models frequently used for electricity forecasting are autoregressive integrated moving average (ARIMA), regression time series, time series, genetic algorithm (GA), artificial neural network (ANN), and particle swarm optimization (PSO) [1]. In this decade, the implementation of fuzzy theories with regression and time series are frequently used to forecast the electricity load consumptions by researchers [2-8].

In the electricity forecasting models, the accuracy of forecasted values is still in issue and very important. Because, not easy to get the historical data accurately and many factors may influence the behavior of electricity load data. Moreover, the randomness and fuzziness of these data play the important role. To solve both conditions, the fuzzy random regression and auto-regression models and its applications have been introduced [9, 10, 15, 16].

From [9, 10], we are interested to modify some aspects such as the formatting of fuzzy data and the left-right spreads (LRS) of TFN in this paper. Both aspects are very essential to be considered in improving of the estimated confidence interval (CI) performance and the forecasting accuracy of fuzzy random auto-regression (FR-AR) model. The rest of paper is organized as follows: In Section 2, the theories of fuzzy random variable (FRV) and fuzzy random auto-regression (FR-AR) are described. The proposed ideas are presented in Section 3. In Section 4, the empirical analysis of electricity load consumption are discussed. In the end of this paper, the conclusion is mentioned briefly.

2 Fundamental Theories of Fuzzy Random Variable and Fuzzy Random Auto-Regression Model

In this section, there are two fundamental theories, namely, fuzzy random variable and fuzzy random auto-regression. Both theories are very important in building the proposed procedure of LRS of TFN for FR-AR model as described in Sections 2.1 and 2.2.

2.1 *Fuzzy Random Variables*

Suppose some universe Γ, let *Pos* be a possibility measure that is defined on the power set Ƥ(Γ) of Γ. Let R be the set of real numbers. A function $Y : \Gamma \rightarrow R$ is said to be a fuzzy variable defined on Γ [11]. The possibility distribution μ_Y of Y is defined by $\mu_Y(t) = Pos\{Y = t\}, t \in R$, which is the possibility of event $\{Y = t\}$. For fuzzy variable Y, with possibility distribution μ_Y, the possibility, necessity, and credibility of event $\{Y \leqslant r\}$ are given as follows:

$$Pos\,\{Y \leq r\} = sup\,\mu_Y(t), t \leq r, \quad (1)$$

$$Nec\,\{Y \leq r\} = 1 - \sup\mu_Y(t), t \geq r, \quad (2)$$

$$Cr\,\{Y \leq r\} = \frac{1}{2}\,(1 + sup\ t \leq r\ \mu_Y(t) - sup\ t \geq r\ \mu_Y(t)). \quad (3)$$

The credibility measure is an average of the possibility and the necessity measures from Eq. (3), i.e., ., $Cr\{.\} = \frac{Pos\{.\} + Nec\{.\}}{2}$. The motivation behind the introduction of the credibility measure is to develop a certain measure, which is a sound aggregate of the two extreme cases, such as the possibility (which expresses a level of overlap and highly optimistic in this sense) and necessity (that articulates a degree of inclusion and is pessimistic in its nature). Based on credibility measure, the expected value of fuzzy variable is presented as follows.

Definition 1. Expected value of fuzzy variable [12]
Let Y be a fuzzy variable. The expected value of Y is defined as:

$$E(Y) = \int Cr\{Y \geq r\}\, dr - \int Cr\{Y \leq r\}\, dr, \tag{4}$$

under the condition that the two integral are finite. Assume that $Y = [a^l, c, a^r]_{\mathrm{T}}$ is triangular fuzzy variable (TFV = TFN) whose possibility distribution is given by

$$\mu_Y(t) = \begin{cases} \frac{x-a^l}{c-a^l}, if\ a^l \leq x \leq c \\ \frac{a^r-x}{a^r-c}, if\ c \leq x \leq a^r, \\ 0, \quad otherwise \end{cases} \tag{5}$$

Making use of Eq. (4), the expected value of Y can be written as

$$E(Y) = \frac{(a^l + 2c + a^r)}{4}. \tag{6}$$

Definition 2. Fuzzy random variable [13]
Suppose that (Ω, Σ, *Pr*) is a probability space and F_v is a collection of fuzzy variables defined on possibility space (Γ, $\mathcal{P}$(Γ), *Pos*). a fuzzy random variable is a mapping $X : \Omega \rightarrow F_v$ such that for any Borel subset B of R, $Pos\{X(\omega) \in B\}$ is a measurable function of ω.
Let X be a fuzzy random variable on Ω. From the previous definition, we know, for each ω ϵ Ω, that X(ω) is a fuzzy variable. Moreover, a fuzzy random variable X is said to be positive if, for almost every ω, fuzzy variable X(ω) is positive almost surely. For any fuzzy random variable X on Ω, for each ω ϵ Ω, the expected value of the fuzzy variable X(ω) is denoted by $E(X(\omega))$, which has been proved to be a measurable function of ω [13], i.e., it is random variable. Given this, the expected value of the fuzzy random variable X is defined as the mathematical expectation of the random variable $E(X(\omega))$.

Definition 3. Expected value of fuzzy random variable [13]
Let X be a fuzzy random variable defined on probability space (Ω, Σ, *Pr*). Then, the expected value of X and variance of X are defined as

$$E(X) = \int \Omega \left[\int Cr\{\xi(\omega) \geq r\}\, dr - \int Cr\{ \xi(\omega) \leq r\}\, dr \right] Pr(\omega), \tag{7}$$

$$Var(X) = E(X - e)^2, \quad (8)$$

where $e = E(X)$ is given by Eq. (7).

2.2 *Fuzzy Random Auto-regression* (FR-AR) *Model*

In time series, autoregressive or AR(p) model can be written as [14]:

$$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots + \emptyset_p Y_{t-p} + e_t, \quad (9)$$

where $\emptyset_1, \dots, \emptyset_p$ are coefficients of $Y_{t-1}, \dots, Y_{t-p}$, respectively, e_t is an error models at time-t. From [10], the fuzzy random auto-regression (FR-AR) model can be defined as input and output data Y_{t-p} for all $p = 0, 1, 2, \dots, n$ are fuzzy random variables, which are written as:

$$Y_t = \cup_{i=1}^{n} \left[\left(Y_{it}^{l}, Y_{it}^{c}, Y_{it}^{r} \right)_T, P_{it} \right], \quad (10)$$

where Y_t is a time series data at time-t and its formatted as a triangular fuzzy number [left, l; center, c; ; right, r]. From Eq. (10), all values given as fuzzy numbers with probabilities, P_{it}. These data, Y_t also can be presented in Table 1.

Table 1. Fuzzy Random Input-Output Time Series Data

Time/Sample	Output	Input			
0	Y_t	Y_{t-1}	Y_{t-2}	…	Y_{t-k}
1	Y_{t-1}	Y_{t-2}	Y_{t-3}	…	$Y_{t-(k+1)}$
2	Y_{t-2}	Y_{t-3}	Y_{t-4}	…	$Y_{t-(k+2)}$
…	…	…	…		…
n	Y_{t-n}	$Y_{t-(n+1)}$	$Y_{t-(n+2)}$	…	$Y_{t-(k+n)}$

Let a simple FR-AR model with coefficients $[\emptyset_1^l, \emptyset_1^r]$ and $[\emptyset_2^l, \emptyset_2^r]$ can be written as:

$$Y_t = [\emptyset_1^l, \emptyset_1^r] Y_{t-1} + [\emptyset_2^l, \emptyset_2^r] Y_{t-2} + [e_t^l, e_t^r], \quad (11)$$

To estimate CI of both coefficients in Eq. (11) can be derived by following steps:

Step 1: Provide the real time series data in the fuzzy data format [min, max] per interval time-t, such as, per one week, per one month, etc. For example, week-1; [3020, 3050], week-2; [3000, 3057], etc.

Step 2: Divide the fuzzy data into the fuzzy random data [min, center, right] with probabilities. For example, week-1; FRD1 = [3020, 3030, 3040], Pr-1 = 0.4 and FRD2 = [3030, 3040, 3050], Pr-2 = 0.6.

Step 3: Calculate the expected value (EV) and standard deviation ($Std.Dev$) of fuzzy random data (FRD) in Step 2, respectively.

EV = $E(Y)$ = (Center of FRD1× Pr-1) + (Center of FRD2 × Pr-2)
= (3030 × 0.4) + (3040 × 0.6)
= 3036

Variance(Y) = $E(Y – e)^2$

Standard deviation ($Std.Dev$) = $s(Y)$ = √Variance(Y) = 7.4

Step 4: Determine the confidence interval (CI) of FRD. For example,
Week -1 : [(EV – $Std.Dev$), (EV + $Std.Dev$)] = [3028.6, 3043.4]

Step 5: Estimate CI for each coefficient model by using linear programming (LP) approach.

Objective function: $\min J(\emptyset) = \sum_{i=1}^{n}(\emptyset_i^r - \emptyset_i^l)$,

Subject to

$$\emptyset_i^r \geq \emptyset_i^l$$

$$a_1\emptyset_{11}^l + \left(a_1 + \frac{1}{3}l\right)\emptyset_{12}^l \leq E_1(Y) - Std.Dev_1(Y)$$
$$a_2\emptyset_{21}^l + \left(a_2 + \frac{1}{3}l\right)\emptyset_{22}^l \leq E_2(Y) - Std.Dev_2(Y)$$
$$\vdots$$
$$a_n\emptyset_{n1}^l + \left(a_n + \frac{1}{3}l\right)\emptyset_{n2}^l \leq E_n(Y) - Std.Dev_n(Y)$$

and

$$\left(a_1 + \frac{2}{3}l\right)\emptyset_{11}^r + (b_1)\emptyset_{12}^r \geq E_1(Y) + Std.Dev_1(Y)$$
$$\left(a_2 + \frac{2}{3}l\right)\emptyset_{21}^r + (b_2)\emptyset_{22}^r \geq E_2(Y) + Std.Dev_2(Y)$$
$$\vdots$$
$$\left(a_n + \frac{2}{3}l\right)\emptyset_{n1}^r + (b_n)\emptyset_{n2}^r \geq E_n(Y) + Std.Dev_n(Y)$$

Step 6: From Step 5, define the estimated confidence-interval (CI) for each coefficient model.

$$\hat{Y}_t = [\hat{\emptyset}_1^l, \hat{\emptyset}_1^r]Y_{t-1} + [\hat{\emptyset}_2^l, \hat{\emptyset}_2^r]Y_{t-2}$$

3 Proposed LRS of TFN in Estimating Confidence-Interval of FR-AR Model

In fuzzy random auto-regression model, the left-right spreads (LRS) of TFN are very important to be considered, because their contributions are very significant in reducing the length of confidence-interval (CI) and the forecasting error. In this paper, the main motivation is to investigate the effect of various LRS in achieving the high forecasting accuracy and to introduce a new formatting of fuzzy data which not clearly described in the previous studies. The forecasting procedure can be derived by following steps:

Step 1: Define the new data format. We suggest to transform the real data into TFN by using various L-R spreads (± k).
Real data → TFN : $Y_t \rightarrow [Y_t - k, Y_t, Y_t + k]$, $k = 5, \ldots, 10$
$3000 \rightarrow [2990, 3000, 3010]$, if $k = 10$

Step 2: Define the real data in Step 1 as new fuzzy data $[Y_t - k, Y_t + k]$.
Year-t: 3000 → [2990, 3010]

Step 3: Divide fuzzy data (FD) into FRD1 and FRD2 as described in Section 2.
FRD1: [2990, 2996.66, 3003.33], FRD2: [2996.66, 3003.33, 3010]

Step 4: Calculate EV and $Std.Dev$ of FRD

Step 5: Determine CI of FRD.

Step 6: Estimate coefficients FR-AR(p) model using LP.

Step 7: Determine the estimated CI for each coefficient model.

Step 8: Change k = 6, 7, .., 10 and repeat Steps 1 – 7.

Step 9: Find and state the best coefficients model based on various k.

The effect of LRS (k) to the forecasting accuracy can be explained as follows:
Since $k_1 < k_2 < k_3 < \cdots < k_n$. Thus, the area of triangles can be written as:

$$A_1 < A_2 < \cdots < A_n, \tag{12}$$

By using Eq. (12), $E(Y)$ and $Var(Y)$, of the fuzzy random variables can be written as:

$$E_1(Y) > E_2(Y) > \cdots > E_n Y, \tag{13}$$

and

$$Var_1(Y) > Var_2(Y) > \cdots > Var_n(Y), \tag{14}$$

Thus, the confidence intervals of fuzzy random variables (FRDs) can be written as:

$$\left[\left(E_1(Y) - \sqrt{Var_1(Y)},\ E_1(Y) + \sqrt{Var_1(Y)},\right)\right], \ldots, \left[\left(E_n(Y) - \sqrt{Var_n(Y)},\ E_1(Y) + \sqrt{Var_n(Y)},\right)\right], \tag{15}$$

From Eq. (15), the range of FRDs can be denoted as:

$$R_1(FRD) < R_2(FRD) < \cdots < R_n(FRD), \tag{16}$$

By using (16), the range of FRD decrease gradually by following values of k (LRS). Therefore, the smaller k will produces the better coefficients of FR-AR model. From this equation, we can claim that the adjusting of LRS is very important in improving of forecasting accuracy.

4 Empirical Analysis

In this section, the various LRS of TFN are examined to investigate the best CI of the yearly electricity load consumption of North-Taiwan, the period 1981 to 2000 [6,7] which are used as model building. By using the proposed algorithm given in Section 3, the estimated CI for model's coefficients can be calculated as follows:

Step 1: Transform the yearly electricity load consumption into TFN format as shown in Table 2. In this paper, we examine k = 10, 8, 5.

Table 2. Actual and TFN electricity load data with k = 10

Year	Actual data	TFN data
1981	3388	[3378, 3388, 3398]
1982	3523	[3513, 3523, 3533]
1983	3752	[3742, 3752, 3762]
…	...	…
2000	12924	[12914, 12924, 12934]

Step 2: Define the fuzzy data using TFN in Step 1 as shown in Table 3.

Table 3. Yearly electricity load of fuzzy data

Year	Fuzzy Data
1981	[3378, 3398]
1982	[3513, 3533]
…	…
2000	[12914, 12934]

Step 3: Divide fuzzy data (FD) in Step 2 into FRD1 and FRD2 with probabilities as shown in Table 4.

Table 4. FRD of electricity load consumption

Year	FRD-1	Pr-1	FRD-2	Pr-2
1981	[3378, 3384.6, 3391.3]	0.4	[3384.6, 3391.3, 3398]	0.6
1982	[3513, 3519.6, 3526.3]	0.2	[3519.6, 3526.3, 3533]	0.8
…	…	…	…	…
2000	[12914, 12920.6, 12927.3]	0.1	[12920.6, 12927.3, 12934]	0.9

Step 4: Calculate EV and SD of FRD of electricity load consumption as shown in Table 5.

Table 5. EV and SD of FRD-1 and FRD-2

Year	Expected Value	Standard Deviation
1981	3386.67	4.3
1982	3523.00	4.6
…	…	…
2000	12926.67	3.6

Step 5: Determine CI of FRD as presented in Table 6.

Table 6. CI of FRD1 and FRD2

Year	Confidence Intervals
1981	[3382.4, 3390.9]
1982	[3518.4, 3527.6]
…	…
2000	[12923, 12930]

Step 6: Estimate coefficients FR-AR(p) model using LP.

Min = $((\alpha_t)_{T,r} - (\alpha_t)_{T,l}) + ((\delta_t)_{T,r} - (\delta_t)_{T,l})$, $(\alpha_t)_{T,r} \geq (\alpha_t)_{T,l}$, $(\delta_t)_{T,r} \geq (\delta_t)_{T,l}$.

Subject to

Inequalities of Left-LP:

$3378(\alpha_t)_{T,l} + 3384.6(\delta_t)_{T,l} \leq 3382.4$
$3513(\alpha_t)_{T,l} + 3519.0(\delta_t)_{T,l} \leq 3518.4$
… … …
$12914(\alpha_t)_{T,l} + 12920(\delta_t)_{T,l} \leq 12923$

Inequalities of Right-LP:

$3391.3(\alpha_t)_{T,r} + 3398(\delta_t)_{T,r} \leq 3390.9$
$3526.3(\alpha_t)_{T,r} + 3533(\delta_t)_{T,r} \leq 3527.6$
… … …
$12927.3(\alpha_t)_{T,r} + 12934(\delta_t)_{T,r} \leq 12930$

$(\alpha_t)_{T,l} \geq 0$, $(\alpha_t)_{T,r} \geq 0$, $(\delta_t)_{T,l} \geq 0$, $(\delta_t)_{T,r} \geq 0$

Step 7: Write the estimated CI for each model with k = 5, 8, 10 in Table 8.

Table 8. The estimated of model coefficients

k	Model-1
10	$\alpha = (\alpha_t)_{T,l} = (\alpha_t)_{T,r} = 0.598$ $\delta = (\delta_t)_{T,l} = (\delta_t)_{T,r} = 0.401$
k	Model-2
8	$\alpha = (\alpha_t)_{T,l} = (\alpha_t)_{T,r} = 0.615$ $\delta = (\delta_t)_{T,l} = (\delta_t)_{T,r} = 0.384$
k	Model-3
5	$\alpha = (\alpha_t)_{T,l} = (\alpha_t)_{T,r} = 0.624$ $\delta = (\delta_t)_{T,l} = (\delta_t)_{T,r} = 0.375$

Remark: $(\alpha_t)_{T,l} = (\alpha_t)_{T,r} = [\hat{\emptyset}_1^l, \hat{\emptyset}_1^r]$, $(\delta_t)_{T,l} = (\delta_t)_{T,r} = [\hat{\emptyset}_2^l, \hat{\emptyset}_2^r]$

Mathematically, the predicted Models 1 - 3 can be written as:

$$Y_{1t} = 0.598Y_{t-1} + 0.401Y_{t-2}, \quad (12)$$

$$Y_{2t} = 0.615Y_{t-1} + 0.384Y_{t-2}, \quad (13)$$

$$Y_{3t} = 0.624Y_{t-1} + 0.305Y_{t-2}, \quad (14)$$

Step 9: Find and state the best coefficients model based on various k. By using Eq. (12 – 14), the comparison of forecasting errors are measured using mean square error (MSE) from three different models can be shown in Table 9.

Table 9. Actual, Forecasted Values and MSE using M1 – M3 models

Year	North	M1	M2	M3
1981	3388	3384.0	3384.0	3384.2
1982	3523	3518.8	3518.9	3519.1
1983	3752	3747.6	3747.6	3747.8
1984	4296	4291.0	4291.1	4291.3
1985	4250	4245.1	4245.1	4245.3
1986	5013	5007.3	5007.4	5007.6
1987	5745	5738.6	5738.6	5738.8
1988	6320	6313.0	6313.1	6313.3
1989	6844	6836.5	6836.5	6836.7
1990	7613	7604.7	7604.8	7605.0
1991	7551	7542.8	7542.8	7543.0
1992	8352	8343.0	8343.0	8343.2
1993	8781	8771.6	8771.6	8771.8
1994	9400	9389.9	9390.0	9390.2

1995	10254	10243.1	10243.1	10243.3
1996	11222	11210.1	11210.2	11210.4
1997	10719	10707.6	10707.7	10707.9
1998	11642	11629.7	11629.7	11629.9
1999	11981	11968.4	11968.4	11968.6
2000	12924	12910.4	12910.5	12910.7
MSE		78.6	77.9	74.6

From Table 9, Model-3 (M3) indicates the smaller MSE as compared with M1 and M2 in term of forecasting accuracy. Through this model, the estimated of CI with $k = 5$ is better than $k = 10$ and $k = 8$, respectively. The decreasing of k contributes to reduce the forecasting error, thus, the forecasting accuracy can be improved significantly. Moreover, the time series plot between actual electricity load consumption and its forecasted values are also illustrated in Figure 1 by using Models 1 – 3.

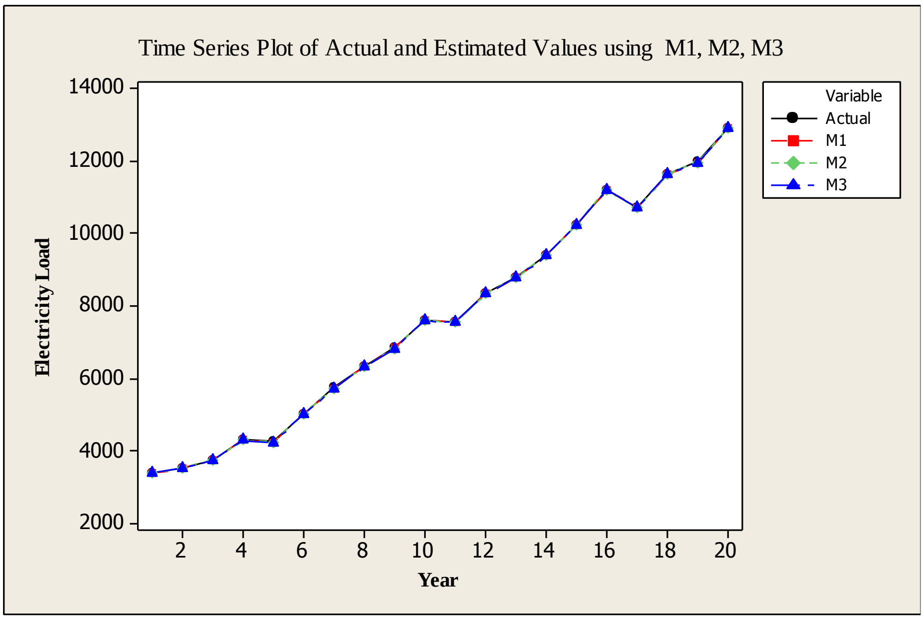

Fig. 1. Actual and Forecasted Values Using Models 1-3

Figure 1 shows the forecasted values which derived by M1, M2 and M3 are not too much different. Thus, the graphs of actual and models look like similar in this figure. Furthermore, the comparison of mean absolute percentage error (MAPE) is also presented with the existing models in Table 10.

Table 10. Comparison MAPE between FR-AR and the Existing Models

Model	MAPE (%)
Support Vector Regression (SVR-CAS)	1.30
SVR-CGA	1.35
SVR-CPSO	1.31
Artificial Neural Network (ANN)	1.06
Regression	2.46
Fuzzy Time Series (FTS)	1.42
FR-AR (Proposed LRS with k = 5)	0.10^{**}

** : smallest MAPE.

Table 10 indicates the proposed LRS with k = 5 has smaller MAPE as compared with existing models. Our proposed LRS is able to achieve the higher level forecasting significantly. From this table, the contribution of smaller LRS is very satisfactory in reducing the forecasting error of FR-AR model.

5 Conclusion

The new formatting of the real time series data into the fuzzy data has been introduced in this paper clearly. Moreover, in achieving the higher forecasting accuracy of FR-AR model, we adjusted the left-right spreads of TFN. The smaller of LRS in TFN is a promising procedure to achieve the best estimated confidence-interval (CI) which shown by MSE of three different models (M1, M2, M3). Furthermore, the comparison MAPE with existing models is also done in this paper, the result indicates the forecasting error which obtained by proposed LRS is better than others. From this study, the increasing of LRS in TFN will increase the forecasting error also. Finally, the further study should be completely investigated with various k and others time series data in determining the smaller LRS of TFN.

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