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Fuzzy Autoregressive Time Series Model Based on Symmetry Triangular Fuzzy Numbers

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The symmetry triangular fuzzy number has been developed to build fuzzy autoregressive models by using various approaches such as low-high data, integer number, measurement error, and standard deviation data. However, most of these approaches are not simulated and compared between ordinary least square and fuzzy optimization in parameter estimation. In this paper, we are interested in implementation of measurement error and standard deviation data in construction symmetry triangular fuzzy numbers. Additionally, both types of triangular fuzzy numbers are deployed to build a fuzzy autoregressive model, especially the second order. The simulation result showed that the fuzzy autoregressive model produced the smaller mean square error and average width if compared with the ordinary autoregressive model. In the implementation, the high accuracy was also achieved by the fuzzy autoregressive model in consumer goods stock prediction. From the simulation and implementation, the proposed fuzzy autoregressive model is a competent approach for upper and lower forecasts.

Keywords: Fuzzy autoregressive; symmetry triangular fuzzy number; measurement error; standard deviation; narrow interval.

1. Introduction

The most commonly applied parameter estimation approach for simple autoregressive time series models is the ordinary least square (OLS).[1] However, this approach

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is not compatible when data input is presented in fuzzy forms. Various fuzzy time series models have been developed by previous works to handle the time series from any sectors.[2–7] These works have been focused on fuzzy logical relationships, weighted fuzzy relations, and fuzzy rules. Fuzzy optimization has been introduced into regression and autoregressive models in parameter investigation,[8–10] especially for fuzzy forms with symmetry triangular fuzzy numbers (TFN). Meanwhile, the spread values for the left-right TFN are not well considered by previous works.[8–13] Thus, very limited standard procedures can be followed on building symmetry TFN.

The low-high data was introduced to construct symmetry TFN in the prediction stock markets.[8–13] However, some time series data are not measured and provided in these forms. In the latest studies, the symmetry TFN has been developed by researches[14,15] using measurement error and standard deviation (SD) approaches. Both approaches assumed the confidence level during data collection cannot be achieved by 100%. A lot of errors may occur such as human or material errors. However, both studies do not employ fuzzy optimization in parameter estimation. Motivated by these studies, we are interested to simulate symmetry TFN in building a fuzzy autoregressive (F-AR) model using fuzzy optimization and to compare with the ordinary autoregressive methods, especially mean square error (MSE) and average width.

The rest of paper is organized as follows. In Sec. 2, the fundamentals of TFN and fuzzy autoregression model are described. The simulation procedure and result to handle data preparation using symmetry TFN are discussed in Sec. 3. In Sec. 4, the empirical analysis using the daily consumer good stocks is presented. Finally, the conclusion is presented in Sec. 5.

2. Fundamental Concepts

2.1. *Concept of triangular Fuzzy number*

Definition: Let a, b, and c be real numbers with $a < b < c$. Then the TFN, $A = (a, b, c)$ is the fuzzy number (FN) with membership function.[16]

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b], \\ \dfrac{c-x}{c-b}, & x \in [b,c], \\ 0, & x < a \quad \text{and} \quad x > c. \end{cases} \tag{1}$$

Thus, Eq. (1) is interpreted as membership functions, as shown in Fig. 1.

Based on Eq. (1), a TFN can be defined as

$$\text{TFN} = y = [\alpha_l, c, \alpha_r]. \tag{2}$$

Based on Eq. (2), if TFN is symmetry, $\alpha_2 - \alpha_1 = \alpha_3 - \alpha_2$, then y is denoted as

$$y = [c, \alpha], \tag{3}$$

where a is spread of TFN and$y$ is a non-fuzzy number if $a = 0$.

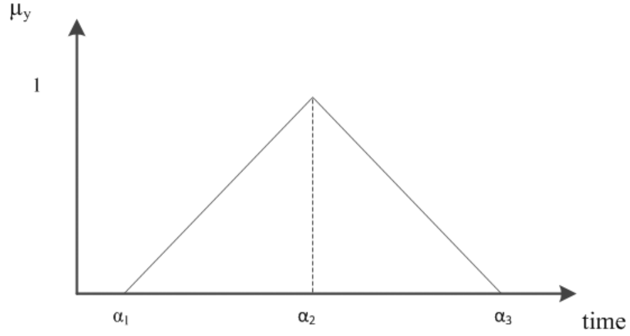

Fig. 1. Triangular FN $A = (\alpha_1, \alpha_2, \alpha_3)$.

2.2. *Concept of F-AR model*

AR(p) model predicts future behavior based on past behavior.[1] It is used for forecasting when there are some correlations between values in time series and their lead and successful values. In the AR model, the value of result (Y) at some point t in time directly related to variable predicator (X) where simple linear regression and AR model different is that Y depends on X and previous values of Y. Additionally, AR(p) is a model where specific lagged values of y_t are used as predictor variable. Lag is where results from a one-time period affect the following periods. The value "p" is called order.

$$y_t = c + \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \cdots + \emptyset_p y_{t-p} + e_t. \quad (4)$$

In Eq. (4), the order can be $1, 2, 3, \ldots, p$. For example, AR(2) model is written mathematically:

$$Z_t = \emptyset_1 Z_{t-1} + \emptyset_2 Z_{t-2} + a_t \quad (5)$$

Based on Eq. (5), the parameters $\emptyset_1$ and $\emptyset_2$ are widely estimated using OLS approach. However, this equation and approach are not appropriate to be applied if time series data in fuzzy forms. Some previous studies have been introducing and proposing integration between fuzzy approach into time-series data, such as fuzzy time series,[2–7] fuzzy regression,[8,9] and F-AR[8–13] models. Thus, Eq. (4) can be represented in F-AR(p) as follows:

$$\dot{Z}_t = \tilde{\phi}_1 \dot{Z}_{t-1} + \tilde{\phi}_2 \dot{Z}_{t-2} + \cdots + \tilde{\phi}_p \dot{Z}_{t-p} + a_t. \quad (6)$$

Based on Eq. (6), the second-order, F-AR(2) can be written as follows:

$$\tilde{Z}_t = (\phi_0^l, \phi_0^r) + (\phi_1^l, \phi_2^r)\tilde{Z}_{t-1} + (\phi_2^l, \phi_2^r)\tilde{Z}_{t-2} + a_t \quad (7)$$

From Eq. (7), the fuzzy parameters $\phi_0^l, \phi_0^r, \ldots, \phi_2^l, \phi_2^r$ are estimated using fuzzy optimization with a linear programming approach.

3. Building F-AR(p) Model Using Symmetry TFN

In recent works, the implementation of measurement errors (MEs) and SD has been introduced by previous researchers[14,15] in developing the symmetry TFN for the autoregressive model, especially the first order, AR(1). However, the estimation parameters of this model have still focused on the OLS method by following the Box–Jenkins procedure. Meanwhile, the inputs of time series data are represented and formed in TFN. Through this paper, we are interested in building the autoregressive model using both TFN with a fuzzy optimization approach in predicting parameters. Here, the procedure on the model building is detailed out in Secs. 3.1 and 3.2. At the end of the study, both values derived by OLS and fuzzy optimization approaches are compared, respectively.

3.1. *Building F-AR(p) model using symmetry TFN based on measurement error*

The procedure on the construction of fuzzy-AR(p) using symmetry TFN based on ME is given by the following steps[15]:

Step 1. Transform a single point of time series data into symmetry TFN using ME approach, such as $\alpha = 0.01; 0.03; 0.05$, respectively, as presented in Table 1.

Based on Table 1, the single point of time series data is represented in symmetry TFN-ME as follows:

$$\text{TFN}_\alpha = [Z_t - (Z_t \cdot \alpha), Z_t, Z_t + (Z_t \cdot \alpha)]; \quad t = 1, 2, 3, \ldots, n \tag{8}$$

By using Eq. (8), all single-point data can be transformed using symmetry TFN-ME. This transformation is also illustrated in Fig. 2.

Step 2. Determine the objective functions for left-right sides, respectively. In this case, we consider AR(2) model.

Table 1. Transformation single point data into symmetry TFN using ME approach.

Single point (Center)	Left boundary	Right boundary
Z_1	$Z_1 - (Z_1 \cdot \alpha)$	$Z_1 + (Z_1 \cdot \alpha)$
Z_2	$Z_2 - (Z_2 \cdot \alpha)$	$Z_2 + (Z_2 \cdot \alpha)$
Z_3	$Z_3 - (Z_3 \cdot \alpha)$	$Z_3 + (Z_3 \cdot \alpha)$
$\vdots$	$\vdots$	$\vdots$
Z_n	$Z_n - (Z_n \cdot \alpha)$	$Z_n + (Z_n \cdot \alpha)$

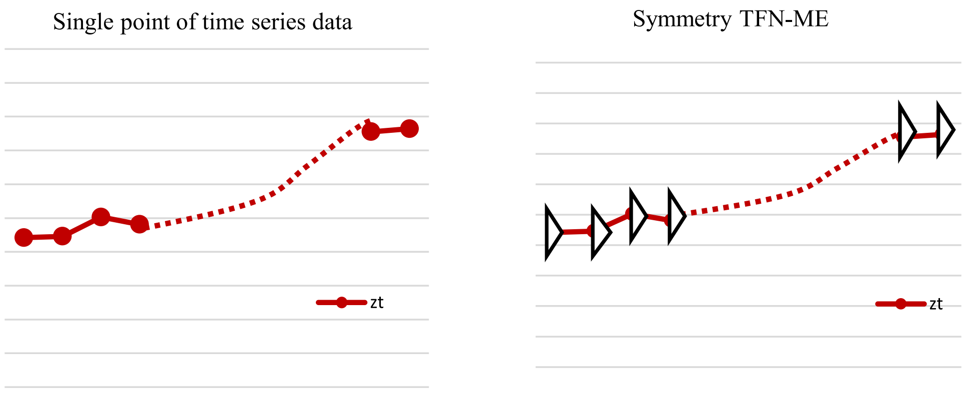

Fig. 2. Data transformation into symmetry TFN-ME.

(1) The objective function and its constraints for the left side

$$\text{Min.} \quad S = \sum_{i=1}^{p}\sum_{t=1}^{k} c_i|\varphi_{ii}||Z_{t-i(\text{left})}|$$
$$= \sum_{t=1}^{k} c_1|\varphi_{11}||Z_{t-1(\text{left})}| + \sum_{t=1}^{k} c_2|\varphi_{22}||Z_{t-2(\text{left})}| \tag{9}$$

with constraints;

$$\sum_{i=1}^{p}\alpha_i Z_{t-i(\text{left})} + a_t + (1-h)\left(\sum_{i=1}^{p} c_i|Z_{t-i(\text{left})}| + a_t\right) \geq Z_{t(\text{left})},$$
$$\sum_{i=1}^{p}\alpha_i Z_{t-i(\text{left})} + a_t - (1-h)\left(\sum_{i=1}^{p} c_i|Z_{t-i(\text{left})}| + a_t\right) \leq Z_{t(\text{left})},$$
$$c_i \geq 0, \quad i = 1,2,3,\ldots,p, \quad t = 1,2,3,\ldots,k.$$

(2) The objective function and its constraints for the right side

$$\text{Min.} \quad S = \sum_{i=1}^{p}\sum_{t=1}^{k} c_i|\varphi_{ii}||Z_{t-i(\text{right})}|$$
$$= \sum_{t=1}^{k} c_1|\varphi_{11}||Z_{t-1(\text{right})}| + \sum_{t=1}^{k} c_2|\varphi_{22}||Z_{t-2(\text{right})}| \tag{10}$$

with constraints;

$$\sum_{i=1}^{p} \alpha_i Z_{t-i(\text{right})} + a_t + (1-h)\left(\sum_{i=1}^{p} c_i |Z_{t-i(\text{right})}| + a_t\right) \geq Z_{t(\text{right})},$$
$$\sum_{i=1}^{p} \alpha_i Z_{t-i(\text{right})} + a_t - (1-h)\left(\sum_{i=1}^{p} c_i |Z_{t-i(\text{right})}| + a_t\right) \leq Z_{t(\text{right})},$$
$$c_i \geq 0, \quad i = 1, 2, 3, \ldots, p \quad t = 1, 2, 3, \ldots, k.$$

Step 3. Solve the fuzzy optimization problems in Step 2 using the linear programming approach to find out the values of a and c. In this case, we consider the value of $h = [0, 1]$.

Step 4. Construct AR(p) models based on parameters obtained from Step 3. Since we focus on and start with $p = 2$, the estimated models are written mathematically as follows:

(1) F-AR(2) model for the left side

$$\tilde{Z}_{t(\text{left})} = (\phi_1, c_1)\tilde{Z}_{t-1(\text{left})} + (\phi_2, c_2)\tilde{Z}_{t-2(\text{left})} + a_t. \tag{11}$$

(2) F-AR(2) model for the *right* side

$$\tilde{Z}_{t(\text{right})} = (\phi_1, c_1)\tilde{Z}_{t-1(\text{right})} + (\phi_2, c_2)\tilde{Z}_{t-2(\text{right})} + a_t. \tag{12}$$

By using Eqs. (11) and (12), the boundary AR(2) models for both sides can be written as follows:

For the left side:

$$\tilde{Z}_{t(\text{left})} = (\phi_1 - c_1)Z_{t-1(\text{left})} + (\phi_2 - c_2)Z_{t-2(\text{left})}$$
$$\tilde{Z}_{t(\text{left})} = (\phi_1 + c_1)Z_{t-1(\text{left})} + (\phi_2 + c_2)Z_{t-2(\text{left})}$$

For the right side:

$$\tilde{Z}_{t(\text{right})} = (\phi_1 - c_1)Z_{t-1(\text{right})} + (\phi_2 - c_2)Z_{t-2(\text{right})}$$
$$\tilde{Z}_{t(\text{right})} = (\phi_1 + c_1)Z_{t-1(\text{right})} + (\phi_2 + c_2)Z_{t-2(\text{right})}.$$

Step 5. Evaluate MSEs for each model from Step 4, respectively. The lowest MSE indicates the best model. The final model is written as

$$\tilde{Z}_t = (\phi_0^l, \phi_0^r) + (\phi_1^l, \phi_2^r)\tilde{Z}_{t-1} + (\phi_2^l, \phi_2^r)\tilde{Z}_{t-2}. \tag{13}$$

3.2. *Building F-AR(p) model using symmetry TFN based on SD*

The main difference between Procedures 3.1 and 3.2 is the type of symmetry TFN (data input) on data preprocessing. In this procedure, every single point of time series data is transformed into a fuzzy form by considering a SD data as a spread value of TFN. SD data have been considered as a spread because of the nature of SD itself, the deviation data that can be left or right spreading from the center. Thus, based on Eq. (8), the TFN form for SD is rewritten as follows[14]:

$$\tilde{Z}_t^s = [Z_t - s, Z_t, Z_t + s] = \text{TFN}; \quad t = 1, 2, 3, \ldots, n. \tag{14}$$

where $\tilde{y}_t^s$ is a fuzzy time series data at time, t with TFN form. The spread of TFN uses SD, s of y_t. y_t is a series of data at time, $t(t = 1, 2, \ldots, n)$ By using the same steps given in Sec. A, the final form of Fuzzy-AR(2) model is also the same with Eq. (13).

4. Simulation and Empirical Analysis

First, two different symmetry TFNs should be examined to generate data before proceeding to implementation by using a simulation technique. In this simulation, the main objective is to evaluate the performance of Procedures 3.1, 3.2 and Box–Jenkins in the building of AR(2) models, especially on parameter estimation. In Procedures 3.1 and 3.2, a linear programming approach is used for estimating intercept and slopes model, while OLS is considered in the Box–Jenkins procedure. Since they are different in determining parameters, the obtained parameters from the OLS approach can also be revealed in interval values. Thus, the width intervals among two different approaches can be compared, respectively, in Sec. 4.1. While some previous studies[14,15] are not yet concerned to simulate both types of TFN in building F-AR(p) with fuzzy optimization. TFN data cannot be derived with the OLS method in parameter estimation of AR(p) model.

4.1. *Simulation result*

The simulation stage is the sequence of steps should be provided, such as choosing an AR(2) process (model), generating error model, considering the number of data and experiment, parameter estimation using Procedures 3.1, 3.2 and Box–Jenkins, and checking the best model based on MSE of data training and testing, respectively. In this section, we only present the simulation result based on Procedure 3.1. By following the steps above, we obtain the best TFN with ME = 1% and fuzzy parameters $a_1 = 0, a_2 = 0.1533, c_1 = 0.2791, c_2 = 0.1104, h =$ and $h = 0.4$. In this case, the data have been generated around 100 (sample size) with 500 experiments. We illustrate the first experiment of training data with symmetry TFN-ME 1% and $h = 0$, 0.4 in Fig. 3.

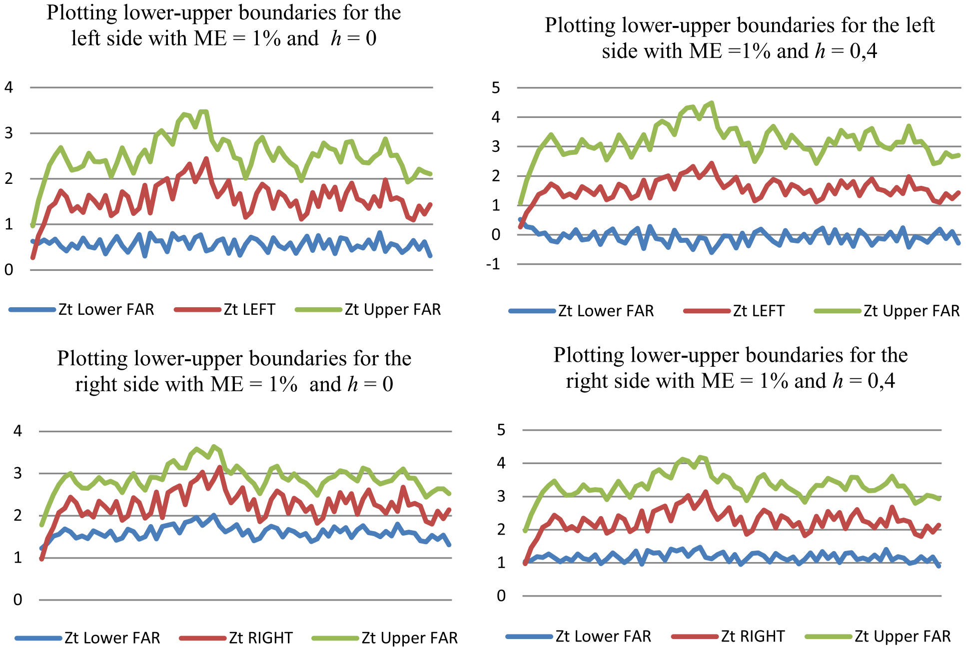

Fig. 3. Plotting of training data with symmetry TFN-ME 1%.

Figure 3 shows that actual data (center) are fluctuated in between lower-upper boundaries for both sides (left-right) of F-AR(2) model. Additionally, both boundaries are also known as lower-upper training data. While MSE for both testings is presented in Table 2.

Based on Fig. 3 and Table 2, the best MSE is achieved when $h = 0$ for Experiment 1. Moreover, it also indicates that the average width is also smaller if compared with $h = 0.4$. On the other hand, the rest of the experiment and its comparative MSE between AR(2) and F-AR(2) TFN-ME 1% models are presented in Table 3.

Based on all simulation results, the comparison of MSE testing among procedures can be completely presented in Table 4.

Table 4 shows the MSE testing values for lower and upper boundaries from three different procedures with 20% data testing. F-AR(2) with TFN-ME (1%) has

Table 2. MSE of testing data for $h = 0$ and 0.4 (Experiment 1).

h	MSE of testing data					
	Lower boundary			Upper boundary		
	Left	Right	TFN	Left	Right	TFN
0	1.907	1.599	1.753*	1.845	2.587	2.216*
0.4	4.280	6.631	5.456	4.025	5.684	4.855

Table 3. Comparative simulation result between AR(2) and F-AR(2) TFN-ME 1%.

Experiment no.	AR(2) model		F-AR(2) TFN-ME 1% model	
	Lower boundary	Upper boundary	Lower boundary	Upper boundary
1	2.074	3.186	1.753	2.216
2	3.784	4.762	2.986	3.530
3	3.067	2.998	2.164	1.935
4	6.748	5.573	4.766	3.915
⋮	⋮	⋮	⋮	⋮
500	3.755	4.547	3.549	3.974

Table 4. MSE testing for lower and upper boundaries using a different procedure.

Procedure or model	MSE testing (Lower boundary)	MSE testing (Upper boundary)	Average of width
F-AR(2) TFN-ME (A)	3.2207*	4.3819*	7.419*
F-AR(2) TFN-SD (B)	3.3220	4.4415	7.512
AR(2) (Box–Jenkins)	3.9152	9.6609	9.661

a smaller MSE if compared with F-AR(2)-SD and ordinary AR(2) models. The error performance is not significantly different between both F-AR(2) procedures. The implementation of measurement error and SD data can improve the capability of symmetry TFN in building F-AR(2) models as well. Additionally, the average width is also an indicator that fuzzy models are better than AR(2). There are some aspects which can be highlighted between OLS and fuzzy optimization from simulation results as presented in Table 5, especially in parameter estimation of AR(2) model.

Based on Table 5, the main difference between OLS and fuzzy optimization has been shown experimentally in investigating the intercept and coefficient of AR(2) and F-AR(2) models, respectively. The result indicates that the ME and SD are significant approaches in constructing symmetry TFN. Thus, this study is also different from some current studies[14,15] in terms of parameter estimation.

4.2. *Empirical analysis*

By following the steps given in Procedures 3.1, 3.2 and Box–Jenkins, the OLS and fuzzy optimization are examined into the daily consumer goods stock from 01 May 2017 to 01 August 2017. Based on three different procedures, the forecasting models can be investigated as follows:

(a) OLS with Box–Jenkins Procedure

Table 5. Comparison between OLS and fuzzy optimization in building AR(2) model.

Aspect	OLS	Fuzzy optimization
Data input and type	Single point and time series	Symmetry TFN based on ME and SD
Data preprocessing	No need	Data transformation from a single point into TFN or fuzzy form
Estimated parameter	$\phi_0, \phi_1, \phi_2, \ldots, \phi_n$	$[(\phi_0^l, \phi_0^r), (\phi_1^l, \phi_1^r), (\phi_2^l, \phi_2^r), \ldots, (\phi_n^l, \phi_n^r)]$
Objective	To minimize the sum quadratic error	To minimize the spread of the midpoint of FN toward objective function and its constraints
Simulation results	Experiment number: 500	Experiment number: 500
	AR(2) Model MSE data testing for LB: 3.9152 MSE data testing for UB: 9.6609	**F-AR(2) TFN-ME Model** MSE data testing for LB: 3.2207. MSE data testing for UB: 4.3819.
	LB: lower boundary UB: Upper boundary	**F-AR(2) TFN-SD Model** MSE data testing for LB: 3.3220. MSE data testing for UB: 4.4415.

By using the daily stock data and procedure above, the final AR(2) model is obtained as follows:

$$\hat{Z}_t = 21.926 + 0.74916Z_{t-1} + 0.2420Z_{t-2}. \tag{15}$$

Equation (15) has been verified by following statistical tests, such as p-value for parameter estimation, residual tests, and data training and testing, respectively.

(b) Fuzzy optimization with Procedure 3.1.

Procedure 3.1 is attempted to predict the daily stock model. However, selected steps are only shown as follows:

Table 6 shows the single stock data (actual) transformed into symmetry TFN-ME 1%. For example, TFN-ME for Day 1 is (2337.29, 2384.51). This fuzzy form is used until Day 77, respectively. Moreover, the objective functions and its constraints for the left-right sides are as follows:

Table 6. Transformation actual data into symmetry TFN-ME 1%.

Day	Left	Center (Actual)	Right
1	2337.29	2360.90	2384.51
2	2390.56	2414.71	2438.85
3	2369.73	2393.67	2417.61
⋮	⋮	⋮	⋮
77	2494.18	2519.37	2544.57

The objective function and its constraints for the left-side:

$$\begin{aligned}\text{Minimum} \quad S &= \sum_{i=1}^{p}\sum_{t=1}^{k} c_i|\varphi_{ii}||W_{t-i}| \\ &= 0.8989\sum_{t=1}^{77} C_1|W_{t-1}| + 0.2339\sum_{t=1}^{77} C_2|W_{t-2}| \\ &= 164877.3C_1 + 42867.92C_2.\end{aligned}$$

Inequality constraints;

$$\begin{gathered}2390.5a_1 + 2337.2a_2 + (1-h)(2390.52c_1 + 2337.2c_2) \geq 2369.7 \\ 2369.7a_1 + 23905.1a_2 + (1-h)(2369.72c_1 + 2390.5c_2) \geq 2368.7 \\ \vdots \\ 2552.0a_1 + 2528.8a_2 + (1-h)(2552.02c_1 + 2528.8c_2) \geq 2520.0\end{gathered}$$

and

$$\begin{gathered}-2390.5a_1 - 2337.2a_2 + (1-h)(2390.52c_1 + 2337.2c_2) \geq -2369.7 \\ -2369.7a_1 - 2390.5a_2 + (1-h)(2369.72c_1 + 2390.5c_2) \geq -2368.7 \\ \vdots \\ -2552.0a_1 - 2528.8a_2 + (1-h)(2552.02c_1 + 2528.8c_2) \geq -2520.0.\end{gathered}$$

The objective function and its constraints for the right-side:

$$\begin{aligned}\text{Minimum} \quad S &= \sum_{i=1}^{p}\sum_{t=1}^{k} c_i|\varphi_{ii}||W_{t-i}| \\ &= 0.8989\sum_{t=1}^{77} C_1|W_{t-1}| + 0.2339\sum_{t=1}^{77} C_2|W_{t-2}| \\ &= 168208.1C_1 + 43733.94C_2.\end{aligned}$$

Inequality constraints;

$$\begin{gathered}2438.8a_1 + 2384.5a_2 + (1-h)(2438.82c_1 + 2384.5c_2) \geq 2417.6 \\ 2417.6a_1 + 2438.8a_2 + (1-h)(2417.62c_1 + 2438.8c_2) \geq 2416.5 \\ \vdots \\ 2603.6a_1 + 2579.9a_2 + (1-h)(2603.62c_1 + 2579.9c_2) \geq 2570.9\end{gathered}$$

and

$$-2438.8a_1 - 2384.5a_2 + (1-h)(2438.82c_1 + 2384.5c_2) \geq -2417.6$$
$$-2417.6a_1 - 2438.8a_2 + (1-h)(2417.62c_1 + 2438.8c_2) \geq -2416.5$$
$$\vdots$$
$$-2603.6a_1 - 25799a_2 + (1-h)(2603.62c_1 + 2579.9c_2) \geq -2570.9.$$

In this step, all fuzzy parameters, such as a_1, a_2, c_1, c_2 are determined using a linear programming approach with $h = 0$, $h = 0.3$, $h = 0.6$, and $h = 0.8$, respectively, as presented in Table 7.

By using the parameters obtained in Table 7, the left-right side models of F-AR(2) can be presented mathematically in Tables 8 and 9.

Based on Tables 8 and 9, the intercept and coefficient of Z_{t-1} are almost the same for each h while the coefficient of Z_{t-2} varies for both sides, respectively. For each h value and side, the couple boundary models are constructed. Moreover, the best model is considered by using MSE of data training and testing. We obtained the smaller MSE of 52.270 with $h = 0$ and TFN-ME 1%. The final proposed model can be presented in Table 10.

From Table 10, both boundary models can be combined to build the final F-AR(2) model for customer goods stock prediction as follows:

$$\hat{\tilde{Z}}_t = [21.707; 22.145] + [0.7491]\tilde{Z}_{t-1} + [0.2205; 0.2635]\tilde{Z}_{t-2}. \tag{16}$$

Table 7. Estimated fuzzy parameters using linear programming.

Constrains		$h = 0$	Constrains		$h = 0.3$
Left side	a_1	0.0000	Right side	a_1	0.0000
	a_2	0.1533		a_2	0.1533
	c_1	0.2791		c_1	0.2791
	c_2	0.1104		c_2	0.1104
		$h = 0.6$			$h = 0.8$
	a_1	0.0000		a_1	0.0000
	a_2	0.1533		a_2	0.1533
	c_1	0.4113		c_1	0.4113
	c_2	0.3765		c_2	0.3765

Table 8. F-AR(2) models for the left side.

h	Lower boundary model	Upper boundary model
0	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2205)Z_{t-2}$	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2635)Z_{t-2}$
0.3	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2112)Z_{t-2}$	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2728)Z_{t-2}$
0.6	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.1881)Z_{t-2}$	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2959)Z_{t-2}$
0.8	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.1341)Z_{t-2}$	$\tilde{Z}_t = 21.7078 + (0.7491)Z_{t-1} + (0.3499)Z_{t-2}$

Table 9. F-AR(2) models for the right side.

h	Lower boundary model	Upper boundary model
0	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2205)Z_{t-2}$	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2635)Z_{t-2}$
0.3	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2112)Z_{t-2}$	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2728)Z_{t-2}$
0.6	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.1881)Z_{t-2}$	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2959)Z_{t-2}$
0.8	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.1341)Z_{t-2}$	$\tilde{Z}_t = 21.7078 + (0.7491)Z_{t-1} + (0.3499)Z_{t-2}$

Table 10. F-AR(2) models for both sides and its boundaries.

Side	Lower boundary (LB) model	Upper boundary (UB) model
Left	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2205)Z_{t-2}$	$\tilde{Z}_t = 21.707 + (0.7491)Z_{t-1} + (0.2635)Z_{t-2}$
Right	$\tilde{Z}_t = 22.145 + (0.7491)Z_{t-1} + (0.2205)Z_{t-2}$	$\tilde{Z}_t = 22.145 + (0.7491)Z_{t-1} + (0.2635)Z_{t-2}$

(a) Fuzzy optimization with Procedure 3.2

By following the procedure given on Sec. 3.2, the final F-AR(2) model is written mathematically as

$$\hat{\tilde{Z}}_t = [21.453; 22.399] + [0.7491]\tilde{Z}_{t-1} + [0.2200; 0.2640]\tilde{Z}_{t-2}. \tag{17}$$

(b) Model evaluation

Based on Procedures (a–c), the MSE of data training-testing and widths are compared to predict the consumer goods stock, respectively, in Table 11.

Table 11 shows the smaller MSE testing LB obtained through F-AR(2) TFN-ME 1%, while F-AR(2) TFN-SD performs in terms of MSE testing UB. Additionally, the average width of F-AR(2) TFN-ME 1% is also better than AR(2) and F-AR(2) TFN-SD. Interestingly, both F-AR(2) models produce smaller MSE and average widths if compared with classical AR(2). In this case, we illustrate the comparison between actual and data testing based on AR(2) and F-AR(2) TFN-SD in Figs. 3 and 4.

From Fig. 3, the upper-lower forecasts are gradually increasing using the ordinary AR(2) model. Thus, both types of forecasts also have huge widths gradually. Therefore, this model is not easy to capture nature stock data. On the other hand, F-AR(2) TFN-SD model is competent to improve the lower-upper forecasts, respectively in Fig. 4. Most of the actual stock data are close to their forecasts. This model

Table 11. Comparative MSE and average width between AR(2) and F-AR(2) models.

Component	AR(2)	F-AR(2) TFN-ME	F-AR(2) TFN-SD
MSE testing (LB)	69.36	42.58*	51.97
MSE testing (UB)	110.78	93.81	61.19*
Average Width	180.14	51.36*	111.05

Note: *The smaller value.

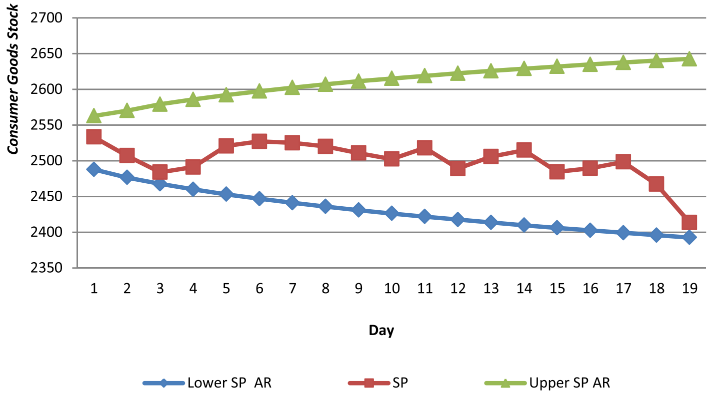

Fig. 4. Actual against data testing based on AR(2) model.

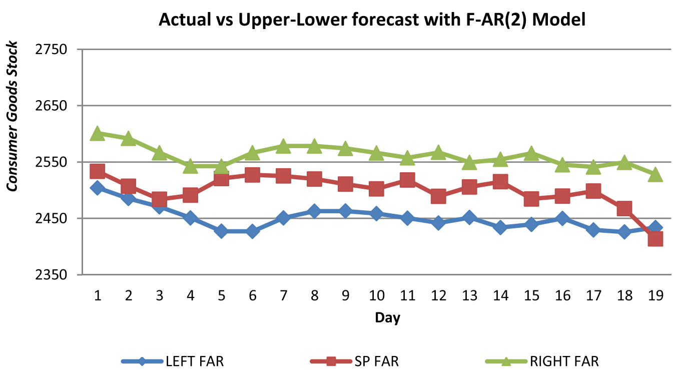

Fig. 5. Actual against data testing based on F-AR(2) model.

also produces tighter intervals of widths if compared with AR(2) model. Based on both figures, it can be claimed that the higher forecasting accuracy is achieved by F-AR(2) TFN-SD.

5. Conclusion

Two different types of TFN have been simulated and implemented to build F-AR(2) model. From simulation results (Table 4), both F-AR(2) models can produce the smaller MSE data testing if compared with the classical AR(2) model. In this case, we assumed that the measurement error and SD data are effective approaches in the construction of symmetry TFN if compared with some previous studies.[8–15] Additionally, F-AR(2) TFN-ME and F-AR(2) TFN-SD also perform better in terms of MSE data testing and average widths if compared with the ordinary AR(2) model. Based on the simulation and implementation results, both types of F-AR(2) are potential models in investigating and forecasting stationary time series data.

There are some benefits from both models, such as lower and upper forecasts are fluctuated values with narrow intervals, and lower-midpoint-upper forecasts can be determined at the same time. Finally, F-AR(2) TFN-ME and SD models produce a higher accuracy.

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Riswan Efendi <riswan.efendi@uin-suska.ac.id>

A manuscript number has been assigned to Fuzzy Autoregressive Time Series Model Based on Symmetry Triangular Fuzzy Numbers

2 messages

New Mathematics and Natural Computation (NMNC) <em@editorialmanager.com> 18 May 2020 at 09:27
Reply-To: "New Mathematics and Natural Computation (NMNC)" <nmnc@wspc.com>
To: riswan efendi <riswan.efendi@uin-suska.ac.id>

Dear Dr riswan efendi,

Your submission entitled "Fuzzy Autoregressive Time Series Model Based on Symmetry Triangular Fuzzy Numbers" has been been assigned the following manuscript number: NMNC-D-20-00054.

You will be able to check on the progress of your paper by logging on to Editorial Manager as an author.
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Thank you for submitting your work to this journal.

With kind regards

New Mathematics and Natural Computation

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In compliance with data protection regulations, you may request that we remove your personal registration details at any time. (Use the following URL: https://www.editorialmanager.com/nmnc/login.asp?a=r). Please contact the publication office if you have any questions.

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 18 May 2020 at 23:41
To: "New Mathematics and Natural Computation (NMNC)" <nmnc@wspc.com>

Thanks so much for your email.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia

[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id>

Submission Confirmation for Fuzzy Autoregressive Time Series Model Based on Symmetry Triangular Fuzzy Numbers

1 message

New Mathematics and Natural Computation (NMNC) <em@editorialmanager.com> 6 May 2020 at 14:59
Reply-To: "New Mathematics and Natural Computation (NMNC)" <nmnc@wspc.com>
To: riswan efendi <riswan.efendi@uin-suska.ac.id>

Dear Dr riswan efendi,

Your submission entitled "Fuzzy Autoregressive Time Series Model Based on Symmetry Triangular Fuzzy Numbers" has been received by journal New Mathematics and Natural Computation

You will be able to check on the progress of your paper by logging on to Editorial Manager as an author. The URL is https://www.editorialmanager.com/nmnc/.

Your manuscript will be given a reference number once an Editor has been assigned.

Thank you for submitting your work to this journal.

With kind regards

New Mathematics and Natural Computation

Riswan Efendi <riswan.efendi@uin-suska.ac.id>

NMNC 2150020 (1st proofs)

5 messages

Nithin ACES <nithin@acesworldwide.net> 24 March 2021 at 13:43
To: riswan.efendi@uin-suska.ac.id
Cc: Steven Patt <spatt@wspc.com>

Dear Prof. Riswan Efendi

Thank you for publishing your article with us.

Please take a look at the attached 1st-Reading proof. Please go over the proof carefully, including your affiliations, as well as each and every tables, figures and equations as they would have been reformatted to be in line with journal style. If you would like to make additional changes to the 1st-Reading proof, please let me know as well. You can send the changes marked in the PDF itself or in a separate file containing the changes to be incorporated.

I look forward to hearing from you by 26 March 2021.

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Regards
Nithin

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Riswan Efendi <riswan.efendi@uin-suska.ac.id> 26 March 2021 at 12:00
To: Nithin ACES <nithin@acesworldwide.net>
Cc: Steven Patt <spatt@wspc.com>

Dear Nithin Jayan
We already did some revisions and you may refer to our final version of the manuscript. there were 2 files attached.
1. Revision and changes.
2. full paper after final revision.

Thank you.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

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2 attachments

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Nithin ACES <nithin@acesworldwide.net> 26 March 2021 at 12:02
To: Riswan Efendi <riswan.efendi@uin-suska.ac.id>
Cc: Steven Patt <spatt@wspc.com>

Dear Professor

Thanks for the prompt response.
I will incorporate the changes and send you the updated proofs for approval asap.

Regards
Nithin
[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 26 March 2021 at 14:55
To: Nithin ACES <nithin@acesworldwide.net>
Cc: Steven Patt <spatt@wspc.com>

Great, thank you.
Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 25 April 2021 at 13:43
To: Nithin ACES <nithin@acesworldwide.net>

Would you like to send us a softcopy of the final script (online version)?
we are really interested.
Thank you.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia

https://efendiriswan.wordpress.com/

[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id>

Source file: NMNC-D-20-00054 - Fuzzy Autoregressive Time Series Model Based on Symmetry Triangular Fuzzy Numbers

2 messages

Steven Patt <spatt@wspc.com> 1 May 2021 at 20:24
To: Riswan Efendi <riswan.efendi@uin-suska.ac.id>
Cc: nithin - EXT <nithin@acesworldwide.net>, "Mordeson, John N" <JohnMordeson@creighton.edu>, "chen.shuheng@gmail.com" <chen.shuheng@gmail.com>

1 May 2021, 9.23 pm

Dear Prof

You have revised the sourcefiles and the new title is as above.
We've scheduled your above paper [Paper 8] for the Jul 2021 issue of NMNC.

My colleague, Nithin, will process the proofs of the paper and let you check soon.

Many tks for your patience.

Rgds.
Steven

From: Riswan Efendi <riswan.efendi@uin-suska.ac.id>
Sent: Saturday, 1 May 2021 12:33 PM
To: Steven Patt <spatt@wspc.com>
Subject: Re: Source file: NMNC-D-20-00054 - Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling

Dear Mr. Steven,
Our paper is already published online, would you like to send us a final softfile paper/script (NMNC-D-20-00054)? we are really interested.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

On Sat, 13 Mar 2021 at 21:48, Riswan Efendi <riswan.efendi@uin-suska.ac.id> wrote:

> Thanks for the fast response.
> Yes you are right, this is the final version from us.
> Really thanks for your consideration.
> all the best for NMNC.
>
> Best Regards,
>
> **Riswan Efendi, PhD**
> Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
> Mathematics Department, UIN Suska Riau, Indonesia
> https://efendiriswan.wordpress.com/

On Sat, 13 Mar 2021 at 20:26, Steven Patt <spatt@wspc.com> wrote:

13 Mar 2021, 9.26 pm

Dear **Prof Efendi**

Sure, we'll use this new version of the sourcefiles.

I hope it's the finalized version.

Pls don't keep changing and amending, It can be confusing.

Dear **SSKumar & Nithin**

This is for Paper 8 of the Jul 2021 issue:

Fuzzy Autoregressive Time Series Model

Based on Symmetry Triangular Fuzzy Numbers

Pls take note.

Tks to all.

Steven

From: Riswan Efendi <riswan.efendi@uin-suska.ac.id>

Sent: Friday, 12 March 2021 8:32 PM

To: Steven Patt <spatt@wspc.com>

Subject: Re: Source file: NMNC-D-20-00054 - Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling

Dear Mr. Steven,

We already revised some author's affiliations,

the new version of accepted manuscript (NMNC-D-20-00054) is attached.

Thank you.

Best Regards,

Riswan Efendi, PhD

Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling

Mathematics Department, UIN Suska Riau, Indonesia

https://efendiriswan.wordpress.com/

= = =

On Sun, 24 Jan 2021 at 11:53, Riswan Efendi <riswan.efendi@uin-suska.ac.id> wrote:

Ok Thanks for your email.

Noted.

Best Regards,

Riswan Efendi, PhD

Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling

Mathematics Department, UIN Suska Riau, Indonesia

https://efendiriswan.wordpress.com/

On Sat, 23 Jan 2021 at 20:33, Steven Patt <spatt@wspc.com> wrote:

23 Jan 2021, 9.33 pm

Dear Prof

Your paper is scheduled for the Jul 2021 issue of NMNC.
The Mar 2021 issue is already filled.

Tks again.
Steven

From: Riswan Efendi <riswan.efendi@uin-suska.ac.id>
Sent: Friday, 22 January 2021 1:08 PM
To: Steven Patt <spatt@wspc.com>
Subject: Re: Source file: NMNC-D-20-00054 - Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling

Dear Mr. Steven,
I would like to know the schedule of publication for our paper?
Thanks.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

On Sun, 14 Jun 2020 at 15:13, Steven Patt <spatt@wspc.com> wrote:

14 Jun 2020, 4.13 pm
Dear Prof Efendi

Many tks for the paper.
We'll schedule your paper for next year's issue of NMNC.
All the issues for this year has been filled.

Many tks again for your patience and effort.

Rgds.
Steven

From: Riswan Efendi <riswan.efendi@uin-suska.ac.id>
Sent: Saturday, 13 June 2020 1:23 PM
To: Steven Patt <spatt@wspc.com>
Subject: Source file: NMNC-D-20-00054

Dear Mr. Steven Patt,

Would you like to inform us about the source files above?
Here, full paper as attached.

Best Regards,
Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 1 May 2021 at 23:22
To: Steven Patt <spatt@wspc.com>

Thank you.
I Really appreciate it.
Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

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Riswan Efendi <riswan.efendi@uin-suska.ac.id>

Source file: NMNC-D-20-00054 - Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling

9 messages

Steven Patt <spatt@wspc.com> 14 June 2020 at 15:13
To: "riswan.efendi@uin-suska.ac.id" <riswan.efendi@uin-suska.ac.id>
Cc: Steven Patt <spatt@wspc.com>, "Mordeson, John" <JohnMordeson@creighton.edu>, "chen.shuheng@gmail.com" <chen.shuheng@gmail.com>

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Best Regards,
Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia

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Riswan Efendi <riswan.efendi@uin-suska.ac.id> 15 June 2020 at 17:24
To: Steven Patt <spatt@wspc.com>

Many thanks Mr. Steven..

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia

[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 22 January 2021 at 12:08
To: Steven Patt <spatt@wspc.com>

Dear Mr. Steven,
I would like to know the schedule of publication for our paper?
Thanks.

Best Regards,

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Mathematics Department, UIN Suska Riau, Indonesia
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[Quoted text hidden]

Steven Patt <spatt@wspc.com> 23 January 2021 at 20:33
To: Riswan Efendi <riswan.efendi@uin-suska.ac.id>
Cc: "Mordeson, John N" <JohnMordeson@creighton.edu>, "chen.shuheng@gmail.com" <chen.shuheng@gmail.com>

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Tks again.
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To: Steven Patt <spatt@wspc.com>
Subject: Re: Source file: NMNC-D-20-00054 - Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling

[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 24 January 2021 at 11:53
To: Steven Patt <spatt@wspc.com>

Ok Thanks for your email.
Noted.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 12 March 2021 at 19:32
To: Steven Patt <spatt@wspc.com>

Dear Mr. Steven,

We already revised some author's affiliations, the new version of accepted manuscript (NMNC-D-20-00054) is attached.
Thank you.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

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Steven Patt <spatt@wspc.com> 13 March 2021 at 20:26
To: Riswan Efendi <riswan.efendi@uin-suska.ac.id>, Shanmuga Kumar <sskumar@stallionpress.com>, nithin - EXT <nithin@acesworldwide.net>
Cc: Zafirah Binte Mohd Othman <zafirah@stallionpress.com>

13 Mar 2021, 9.26 pm

Dear **Prof Efendi**

Sure, we'll use this new version of the sourcefiles.

I hope it's the finalized version.

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Fuzzy Autoregressive Time Series Model
Based on Symmetry Triangular Fuzzy Numbers

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Tks to all.

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Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia

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Riswan Efendi <riswan.efendi@uin-suska.ac.id> 13 March 2021 at 21:48
To: Steven Patt <spatt@wspc.com>

Thanks for the fast response.
Yes you are right, this is the final version from us.
Really thanks for your consideration.
all the best for NMNC.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
https://efendiriswan.wordpress.com/

[Quoted text hidden]

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 1 May 2021 at 11:33
To: Steven Patt <spatt@wspc.com>

Dear Mr. Steven,
Our paper is already published online, would you like to send us a final softfile paper/script (NMNC-D-20-00054)?
we are really interested.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia
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Riswan Efendi <riswan.efendi@uin-suska.ac.id>

Your Submission Manuscript No. NMNC-D-20-00054

2 messages

New Mathematics and Natural Computation (NMNC) <em@editorialmanager.com> 13 June 2020 at 08:49
Reply-To: "New Mathematics and Natural Computation (NMNC)" <nmnc@wspc.com>
To: riswan efendi <riswan.efendi@uin-suska.ac.id>

CC: chen.shuheng@gmail.com, spatt@wspc.com.sg, mordes@creighton.edu

Ref.: Ms. No. NMNC-D-20-00054
Fuzzy Autoregressive Time Series Model Based on Symmetry Triangular Fuzzy Numbers
New Mathematics and Natural Computation

Dear Dr riswan efendi,

I am pleased to inform you that your work has now been accepted for publication in New Mathematics and Natural Computation.

If you wish to have your article published as Open Access, please note the Article-Processing Charge (APC) is USD2000. You may contact us or visit http://www.worldscientific.com/page/open for more details.

Please send the source files of your accepted paper to Steven Patt (spatt@wspc.com).

Thank you for submitting your work to this journal.

IMPORTANT NOTICE

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With kind regards

Shu-Heng Chen, PhD
Managing Editor
New Mathematics and Natural Computation

Comments from the Editors and Reviewers:

The authors prove some interesting results concerning fuzzy autoregressive time series models. A good approach for upper and lower forecasting is also provided.

Riswan Efendi <riswan.efendi@uin-suska.ac.id> 13 June 2020 at 11:13
To: "New Mathematics and Natural Computation (NMNC)" <nmnc@wspc.com>

Dear Dr. Shu-Heng Chen,
We are really happy and appreciative of your consideration of our manuscript for publication in NMNC.

Again thanks so much. We will send the source files asap.

Best Regards,

Riswan Efendi, PhD
Fuzzy Time Series and Fuzzy Random Auto-Regression Modelling
Mathematics Department, UIN Suska Riau, Indonesia

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