Design of Event-Triggered Asynchronous H∞ Filter for Switched Systems Using the Sampled-Data Approach

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RESEARCH ARTICLE

Design of Event-Triggered Asynchronous $\mathcal{H}_\infty$ Filter for Switched Systems Using the Sampled-Data Approach

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ABSTRACT The design of networked switched systems with event-based communication is attractive due to its potential to save bandwidth and energy. However, ensuring the stability and performance of networked systems with event-triggered communication and asynchronous switching is challenging due to their time-varying nature. This paper presents a novel sampled-data approach to design event-triggered asynchronous $\mathcal{H}_\infty$ filters for networked switched systems. Unlike most existing event-based filtering results, which either design the event-triggering scheme only or co-design the event-triggering condition and the filter, we consider that the event-triggering policy is predefined and synthesize the filter. We model the estimation error system as an event-triggered switched system with time delay and non-uniform sampling. By implementing a delay-dependent multiple Lyapunov method, we derive sufficient conditions to ensure the global asymptotic stability of the filtering error system and an $\mathcal{H}_\infty$ performance level. The efficacy of the proposed design technique and the superiority of the filter performance is illustrated by numerical examples and by comparing the performance with a recent result.

INDEX TERMS Event-triggered sampling, asynchronous switching, $\mathcal{H}_\infty$ filtering, non-uniformly sampled systems.

I. INTRODUCTION

Switched dynamical systems are hybrid systems that involve several subsystems and a switching rule; orchestrating the switching among the subsystems. Switched dynamical systems find applications in a variety of engineering fields, e.g., robotic systems [1], [2], automobile systems [3], DC-to-DC converters [4], [5], chemical reactors, oscillators, and chaos generators [6], [7], [8].

Networked control is a control scheme where measurement feedback and control actions route through a communication network. Such a scheme offers many advantages, such as low installation cost, reduced maintenance cost, and flexible system structure [9]. Networked control of switched dynamical systems is an active research area, with many researchers addressing problems in this area, see for example [10] and [11]. Existing results on networked control of switched systems either consider problems in the continuous time or use a periodic sampling scheme [12]. However, periodic sampling often results in the wastage of network resources. An alternative to periodic sampling is event-based sampling, which saves energy and communication resources [13]. Recent approaches to event-triggered control use adaptive event-triggering schemes that offer a further reduction in the usage of network resources [14], [15], [16].

Filtering or state estimation problem is an important one in control systems. A reason for that is many advance control algorithms are based on state feedback. However, not all state variables can be measured. A filter or a state estimator then provides an estimate of the state variables for feedback.

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Monitoring systems also use an estimate of the state variables. Two popular algorithms for filtering or state estimation in control systems are Kalman filtering and $\mathcal{H}_\infty$ filtering [17]. Conventional implementation of these algorithms use periodic sampling. This work is focused on event-based $\mathcal{H}_\infty$ filtering or state estimation.

Event-based filtering for networked switched systems has been addressed in [6], [18], [19], [20], and [21]. In [6], an asynchronous finite-time stable filter is devised for continuous-time switched systems. In [18], an event-based $\mathcal{H}_\infty$ filtering problem is addressed for networked switching systems in continuous time domain. Finite-time stability and $\mathcal{H}_\infty$ performance of the filtering error system are proved by using a delay-dependent Lyapunov functional. In [19], an event-triggered $\mathcal{H}_\infty$ filter is designed to ensure stability in the presence of packet disorders and maintain an $\mathcal{H}_\infty$ performance level. In [20], an event-triggered fault detection $\mathcal{H}_\infty$ filter is designed. A discrete-time event-triggered $\mathcal{H}_\infty$ filter is designed in [21]. By using multiple Lyapunov function approach, sufficient conditions are derived for exponential stability and $\mathcal{H}_\infty$ performance of the error system. It is pertinent to note that all these works follow the co-design framework where the event-triggering conditions and filter parameters are designed simultaneously [22]. However, there are situations where an event-generator is predefined, such as hardware-based event triggers, and it is not possible to redesign them. In such situations, the goal is to design the filter parameters such that the filtering error system remains stable and satisfies the given performance criterion. To the author's best knowledge, this problem has not been addressed in the literature.

To fill this research gap, we propose a technique for the event-based $\mathcal{H}_\infty$ filter design where an event-triggering policy is pre-defined. This type of filter design problem can be addressed using the sampled-data systems framework [23]. In particular, we view the event-based filter as a sampled-data system with non-uniformly sampled measurements [24]. The filtering error system can then be modeled as a switched dynamical systems with time delay to cater for the non-uniform sampling of measurement. The stability and performance of the error system can be analysed using the tools developed for switched time-delay systems.

The main contributions of this paper are as follows.

1) A novel technique to design an event-triggered $\mathcal{H}_\infty$ filter is proposed for networked switched systems when an event-triggering policy is predefined. We show that the estimation error system can be modelled as switched system with time delay and non-unform sampling.
2) We show utilization of a delay-dependent multiple Lyapunov functional to develop sufficient exponential stability criterion of the error system and the $\mathcal{H}_\infty$ performance.

The rest of this paper is organized as follows. In the next section, the problem is formulated. In the following section, the key results are presented to analyse filter performance and design it. Then, two simulation examples are given to show the efficacy of the presented approach. Finally, the paper is concluded in the last section.

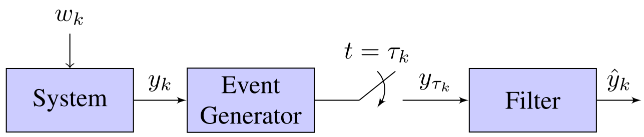

FIGURE 1. Event-based filtering for switched systems.

Notation: In this paper, $\mathbb{R}$ denotes the set of real numbers and $\mathbb{Z}$ denotes the set of integers. $\mathbb{R}^n$ is the n-dimensional Euclidean space. For a matrix $P \in \mathbb{R}^{n\times n}$, $P > 0$ means P is positive definite, P^T denotes the transpose of matrix P, and $l_2[0, \infty)$ represents sequences with finite 2-norm $\|.\|_2$. For $x \in \mathbb{R}^n$ and $S \in \mathbb{R}^{n\times n}$, $\|x\|_S^2 = x^T Sx$ is the weighted norm of x. The acronym LTI stands for linear, time-invariant.

II. PROBLEM FORMULATION

Consider the filtering scenario as shown in Figure 1. The components of the filtering system are described as follows.

A. SYSTEM

The system is described by a discrete LTI model

$$\begin{aligned} x_{k+1} &= A_{\theta_k} x_k + B_{\theta_k} w_k \\ y_k &= C_{\theta_k} x_k + D_{\theta_k} w_k \end{aligned} \tag{1}$$

This model is obtained by sampling the original continuous-time state-space model of the switched system at sampling instants t_k, where $t_k = kh$, $k \in \mathbb{Z}$, $x_k = x(t_k) \in \mathbb{R}^n$ is the state of the system, $y_k = y(t_k) \in \mathbb{R}^q$ is the output of the system, and $w_k = w(t_k) \in \mathbb{R}^p$ is the disturbance signal. We assume that $w_k \in l_2[0, \infty)$, where l_2 is the space of finite-energy sequences. A sequence $\{f_k\}_{k\geq 0}$ belongs to l_2 if

$$\|f_k\|_2 = \sum_{k=0}^{\infty} f_k^T f_k < \infty.$$

$\theta_k : [0, \infty) \rightarrow \mathcal{I}_N = \{1, 2, 3, \ldots, N\}$ represents the switching rule and $N \geq 1$ represents the number of subsystems. At any instant t_k, θ_k depends on k or x_k or some other switching rule. For all $\theta_k \in \mathcal{I}_N$, we have the following matrices A_{θ_k}, B_{θ_k}, C_{θ_k} and D_{θ_k} with compatible dimensions. Furthermore, the matrices corresponding to $\theta_k = i \in \mathcal{I}_N$ are denoted by $A_i := A_{\theta_k=i}$, $B_i := B_{\theta_k=i}$, $C_i := C_{\theta_k=i}$ and $D_i := D_{\theta_k=i}$.

In this paper, we study switched systems of the form in (1) with Average Dwell Time (ADT) switching. The system can switch from one mode to another arbitrarily, but the switching is slow. The dwell time between mode switches is no less than the ADT.

Definition 1: For $k > k_0$, let $N_\theta(k, k_0)$ denotes the number of switchings of θ_k over the interval $[k_0, k)$. If there exist

$N_0 > 0$ and $\tau_a > 0$ such that

$$N_\theta(k, k_0) \leq N_0 + \frac{k - k_0}{\tau_a}, \tag{2}$$

then N_0 and τ_a are called the chatter bound and average dwell time of the switching signal θ_k.

B. EVENT-TRIGGERED SAMPLING

The measurement y_k is sampled at sampling instants t_k and is fed to the event-generator. The event-generator uses a predefined criterion to determine whether to communicate the measurement or not. Let $\{\tau_k\}_{k\geq 0}$, $k \in \mathbb{Z}$ be the instants when the measurement is transmitted. The time instants τ_k are called event triggering instants. The event triggering instants are decided according to a given event triggering policy $\mathcal{L}(x_k, y_k, k)$.

$$\tau_{k+1} := \min_{k\geq 0}\{k > \tau_k | \mathcal{L}(x_k, y_k, k) \geq 0 \text{ or } k - \tau_k >= M\} \tag{3}$$

with $\tau_0 = 0$ and $M \in \mathbb{Z}$. The constant M is chosen in such a way that $M \leq \tau_a$, i.e., there is at most one switching in event-triggering instant. Notice that τ_k forms an increasing sequence, that is $\tau_0 < \tau_1 < \tau_2 < \ldots < \tau_k$, therefore, the Zeno phenomenon does not occur. We assume that

Assumption 1: $\tau_k = mt_k$ where $m, k \geq 0$ and $m, k \in \mathbb{Z}$.

Assumption 2: τ_{k+1} - $\tau_k \leq (\phi_k + 1)h$ where $\phi_k \in \mathcal{I}_\phi = \{0, 1, \ldots, M-1\}$.

Assumption 1 implies that the event-triggering instants commensurate with the measurement sampling instants. This is a realistic assumption because the event-generator activates at each measurement sampling instants to decide whether to discard or transmit the measurement. Assumption 2 implies that the measurement from the process will be transmitted at least after M periods. Event-triggered sampling with an upper bound on the sampling interval has already been used in the literature, see for example [21]. This sampling technique is a hybridization of the two sampling techniques; the event triggered-sampling and self-triggered sampling [25]. It provides a reasonable compromise between the utilization of network bandwidth and system performance by ensuring that fresh data is communicated to the filter after M sampling intervals.

The event-triggering policy $\mathcal{L}(x_k, y_k, k)$ could be a function of state, output, or time. Various event-triggered sampling policies have been proposed in the literature [22]. Some examples are:

Example 1: $\mathcal{L}(x_k, y_k, k) = \gamma(\|e_k\|) - \sigma\alpha(\|x_k\|)$ where $0 < \sigma < 1$ is a scalar, $e_k = x_{\tau_k} - x_k(\text{ or } y_{\tau_k} - y_k)$ is the error since last transmission, α, and γ are suitable functions.

Example 2: $\mathcal{L}(x_k, y_k, k) = \eta_k + \theta(\sigma\alpha(\|x_k\|) - \gamma(\|e_k\|))$ where $\eta_{k+1} = -\beta\eta_k + \sigma\alpha(\|x_k\|) - \gamma(\|e_k\|)$ with a suitable function β.

C. FILTER

We take the filter structure to be

$$\begin{aligned} \hat{x}_{k+1} &= A_{f_{\theta'_k}}\hat{x}_k + B_{f_{\theta'_k}}\bar{y}_k \\ \hat{y}_k &= C_{f_{\theta'_k}}\hat{x}_k + D_{f_{\theta'_k}}\bar{y}_k \end{aligned} \tag{4}$$

where $\hat{x}_k \in \mathbb{R}^n$ is the state of the filter, $\bar{y}_k \in \mathbb{R}^p$ is the input to the filter and $\hat{y}_k \in \mathbb{R}^q$ is the filter output. The matrices $A_{f_{\theta'_k}}$, $B_{f_{\theta'_k}}$, $C_{f_{\theta'_k}}$, and $D_{f_{\theta'_k}}$ are the filter design parameters and θ'_k is the switching signal of the filter. Again, for the sake of brevity, we denote the filter parameters $A_{f_{\theta'_k}}$, $B_{f_{\theta'_k}}$, $C_{f_{\theta'_k}}$, and $D_{f_{\theta'_k}}$ with A_{f_i}, B_{f_i}, C_{f_i}, and D_{f_i}, respectively, when $\theta'_k = i \in \mathcal{I}_N$. The switching signals θ_k and θ'_k are in general not synchronized. There is generally some lag between the switching of the system mode and the corresponding filter. Let $\mathcal{T}_{\max}$ be the maximum lag between θ_k and $\theta_{k'}$, that is,

$$\theta'_k = \theta_{k-\mathcal{T}_{max}}.$$

A larger value of $\mathcal{T}_{\max}$ means that the activated system and filter modes are not synchronised for longer time. Also, due the event-triggered sampling, the input to the filter is intermittent and non-uniform. That is

$$\bar{y}_k = y_k, \quad \text{when} \quad t_k = \tau_k. \tag{5}$$

A longer interval may cause the estimation error to diverge due to the presence of disturbance and asynchronous switching. This inherently presents a compromise between the duration of the event-triggering interval and disturbance attenuation to maintain the state estimation quality at a desired level. From the sampled-data control theory point of view, the event-triggered filtering problem with a specified event-triggering policy can be viewed as a filtering problem with non-uniform sampling [24].

D. ERROR SYSTEM

Let $\tilde{z} = y_k$ - $\hat{y}_k$ denotes the estimation error. From assumption 2, the input to the filter during each transmission interval will be

$$\bar{y}_k = y_{k-\phi_k} \quad \phi_k \in \mathcal{I}_\phi, k \in [\tau_k, \tau_{k+1}). \tag{6}$$

The transmission interval can be partitioned as $[\tau_k, \tau_{k+1}) = [\tau_k, \tau_k + \mathcal{T}_{\max}) \cup [\tau_k + \mathcal{T}_{\max}, \tau_{k+1})$. Different system and filter modes are activated when $k \in [\tau_k, \tau_k + \mathcal{T}_{\max})$. Let $\theta_k = i$, $\theta'_k = j$, and $\phi_k = 0$, the dynamics of the filtering error system will be

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} &= \begin{bmatrix} A_i & 0 \\ 0 & A_{f_j} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{f_j}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} \\ &\quad + \begin{bmatrix} B_i & 0 \\ 0 & B_{f_j}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix} \\ \tilde{z}_k &= \begin{bmatrix} C_i & -C_{f_j} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_{f_j}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} \\ &\quad + \begin{bmatrix} D_i & -D_{f_j}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix} \end{aligned}$$

When $k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1})$, the system and filter modes will be synchronized. Let $\theta_k = \theta_k' = i$, and $\phi_k = 0$, the dynamics of the filtering error system will be

$$\begin{aligned}\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} &= \begin{bmatrix} A_i & 0 \\ 0 & A_{f_i} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{f_i}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} \\ &\quad + \begin{bmatrix} B_i & 0 \\ 0 & B_{f_i}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix} \\ \tilde{z}_k &= \begin{bmatrix} C_i & -C_{f_i} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_{f_i}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} \\ &\quad + \begin{bmatrix} D_i & -D_{f_i}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix}\end{aligned}$$

When $\phi_k = 1$, $\theta_k = i$, and $\theta_k' = j$, the dynamics of the error system will be

$$\begin{aligned}\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} &= \begin{bmatrix} A_i & 0 \\ 0 & A_{f_j} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{f_j}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} \\ &\quad + \begin{bmatrix} B_i & 0 \\ 0 & B_{f_j}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix} \\ \tilde{z}_k &= \begin{bmatrix} C_i & -C_{f_j} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_{f_j}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} \\ &\quad + \begin{bmatrix} D_i & -D_{f_j}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix}\end{aligned}$$

Similarly, when $\phi_k = 1$, $\theta_k = \theta_k' = i$, the dynamics of the error system will be

$$\begin{aligned}\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} &= \begin{bmatrix} A_i & 0 \\ 0 & A_{f_i} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{f_i}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} \\ &\quad + \begin{bmatrix} B_i & 0 \\ 0 & B_{f_i}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix} \\ \tilde{z}_k &= \begin{bmatrix} C_i & -C_{f_i} \end{bmatrix}\begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_{f_i}C_i & 0 \end{bmatrix}\begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} \\ &\quad + \begin{bmatrix} D_i & -D_{f_i}D_i \end{bmatrix}\begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix}\end{aligned}$$

In general, the dynamics of the error system can be written as

$$\begin{aligned}\eta_{k+1} &= \bar{A}_{ij}\eta_k + \bar{A}_{d_{ij}}\eta_{k-\phi_k} + \bar{B}_{\phi_k,ij}\bar{w}_k \\ \tilde{z}_k &= \bar{C}_{ij}\eta_k + \bar{C}_{d_{ij}}\eta_{k-\phi_k} + \bar{D}_{\phi_k,ij}\bar{w}_k, \\ &\quad k \in [\tau_k, \tau_k + \mathcal{T}_{\max}) \\ \eta_{k+1} &= \bar{A}_i\eta_k + \bar{A}_{d_i}\eta_{k-\phi_k} + \bar{B}_{\phi_k,i}\bar{w}_k \\ \tilde{z}_k &= \bar{C}_i\eta_k + \bar{C}_{d_i}\eta_{k-\phi_k} + \bar{D}_{\phi_k,i}\bar{w}_k, \\ &\quad k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1}) \end{aligned} \tag{7}$$

where $\eta_k = [x_k^T, \hat{x}_k^T]^T$, $\bar{w}_k = [w_k^T, w_{k-1}^T, \ldots, w_{k-M+1}^T]^T$, and

$$\bar{B}_{\phi_k,ij} = \begin{cases} \begin{bmatrix} B_i & 0 & \cdots & 0 \\ B_{f_j}D_i & 0 & \cdots & 0 \end{bmatrix} & \phi_k = 0 \\ \vdots & \vdots \\ \begin{bmatrix} B_i & 0 & \cdots & 0 \\ 0 & 0 & \cdots & B_{f_j}D_i \end{bmatrix} & \phi_k = M-1 \end{cases}$$

$$\bar{B}_{\phi_k,i} = \begin{cases} \begin{bmatrix} B_i & 0 & \cdots & 0 \\ B_{f_i}D_i & 0 & \cdots & 0 \end{bmatrix} & \phi_k = 0 \\ \vdots & \vdots \\ \begin{bmatrix} B_i & 0 & \cdots & 0 \\ 0 & 0 & \cdots & B_{f_i}D_i \end{bmatrix} & \phi_k = M-1 \end{cases}$$

$$\begin{aligned}\bar{A}_{ij} &= \begin{bmatrix} A_i & 0 \\ 0 & A_{f_j} \end{bmatrix}, \quad \bar{A}_{d_{ij}} = \begin{bmatrix} 0 & 0 \\ B_{f_j}C_i & 0 \end{bmatrix} \\ \bar{A}_i &= \begin{bmatrix} A_i & 0 \\ 0 & A_{f_i} \end{bmatrix}, \quad \bar{A}_{d_i} = \begin{bmatrix} 0 & 0 \\ B_{f_i}C_i & 0 \end{bmatrix} \\ \bar{C}_{ij} &= \begin{bmatrix} C_i & -C_{f_j} \end{bmatrix}, \quad \bar{C}_{d_{ij}} = \begin{bmatrix} -D_{f_j}C_i & 0 \end{bmatrix} \\ \bar{C}_i &= \begin{bmatrix} C_i & -C_{f_i} \end{bmatrix}, \quad \bar{C}_{d_i} = \begin{bmatrix} -D_{f_i}C_i & 0 \end{bmatrix}\end{aligned}$$

$$\bar{D}_{\phi_k,ij} = \begin{cases} \begin{bmatrix} D_i - D_{f_j}D_i & 0 & \cdots & 0 \end{bmatrix} & \phi_k = 0 \\ \vdots & \vdots \\ \begin{bmatrix} D_i & 0 & \cdots & -D_{f_j}D_i \end{bmatrix} & \phi_k = M-1 \end{cases}$$

$$\bar{D}_{\phi_k,i} = \begin{cases} \begin{bmatrix} D_i - D_{f_i}D_i & 0 & \cdots & 0 \end{bmatrix} & \phi_k = 0 \\ \vdots & \vdots \\ \begin{bmatrix} D_i & 0 & \cdots & -D_{f_i}D_i \end{bmatrix} & \phi_k = M-1 \end{cases}$$

Notice that the error system is a switched time-delay system where the time-dependent delay ϕ_k takes values in the interval $\mathcal{I}_\phi$.

E. PROBLEM STATEMENT

In this paper, we consider the following problem:

Given the system in (1) and event-triggered sampling in (3), satisfying assumptions 1 and 2, design the filter parameters in (4), such that the error system in (7) is

- globally uniformly asymptotically stable with $\bar{w}_k = 0$,
- when initially relaxed, the $\mathcal{H}_\infty$-norm of the error system is less than γ, where $\gamma > 0$ is a positive scalar.

Definition 2 [26]: When $\bar{w}_k = 0$, the system in (7) is said to be globally uniformly asymptotically stable if, for all switching signals θ_k and θ_k', its solutions satisfy

$$\|\eta_k\| \leq \beta \|\psi_l\|, \qquad \forall k \geq k_0$$

for any initial condition $(\psi_l, k_0,)$, where $\psi(l) = \eta(l)$ for $l = k_0 - M, k_0 - M + 1, \ldots, k_0$ and $\|\psi_l\| = \sup_{k_0 - M \leq l \leq k_0} \|\psi(l)\|$.

Definition 3: The error system in (7) is said to have $\mathcal{H}_\infty$ performance level $\gamma > 0$, if it is globally asymptotically stable and under zero initial conditions, the l_2 gain satisfies

$$\|\tilde{z}_k\|_2^2 \le \gamma^2 \|\bar{w}_k\|_2^2 .$$

When there is no event-triggered sampling, we can write the system in (7) as

$$\begin{aligned}
\eta_{k+1} &= f_{\theta_k}(\eta_k, w_k) = (\bar{A}_{ij} + \bar{A}_{d_{ij}})\eta_k + \bar{B}_{ij}w_k \\
\tilde{z}_k &= h_{\theta_k}(\eta_k, w_k) = (\bar{C}_{ij} + \bar{C}_{d_{ij}})\eta_k + \bar{D}_{ij}w_k, \\
&\quad k \in [\tau_k, \tau_k + \mathcal{T}_{\max}) \\
\eta_{k+1} &= f_{\theta_k}(\eta_k, w_k) = (\bar{A}_i + \bar{A}_{d_i})\eta_k + \bar{B}_i w_k \\
\tilde{z}_k &= h_{\theta_k}(\eta_k, w_k) = (\bar{C}_i + \bar{C}_{d_i})\eta_k + \bar{D}_i w_k, \\
&\quad k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1})
\end{aligned} \tag{8}$$

The stability and $\mathcal{H}_\infty$ performance of this system can be analyzed by using the following results from [26] and [5].

Lemma 1: Consider the error system in (8) with $w_k = 0$ and given $0 < \alpha < 1$, $\mu \ge 1$, and $\beta \ge 0$. Let there be $\mathcal{C}^1$ functions $V_{\theta_k} : \mathbb{R}^n \to \mathbb{R}$, $\theta_k \in \mathcal{I}_N$, and κ_1 and κ_2 belonging to class $\mathcal{K}_\infty$, $\forall \theta_k = i \in \mathcal{I}_N$

$$\kappa_1(\|\eta_k\|) \le V_i(\eta_k) \le \kappa_2(\|\eta_k\|)\|$$

$$\Delta V_i(\eta_k) \le \begin{cases} \beta V_i(\eta_k), & \forall k \in [\tau_k, \tau_k + \mathcal{T}_{\max}) \\ -\alpha V_i(\eta_k), & \forall k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1}) \end{cases}$$

for all $\theta_{\tau_k + \mathcal{T}_{\max}} = i, \theta_{\tau_k} = j \in \mathcal{I}_N \times \mathcal{I}_N, i \ne j$

$$V_i(\eta_{\tau_k}) \le \mu V_j(\eta_{\tau_k})$$

then the estimation error system in (8) is globally uniformly asymptotically stable for any θ_k with ADT τ_a given as

$$\tau_a > \tau_a^* = -\frac{\mathcal{T}_{\max}[\ln(1+\beta) - \ln(1-\alpha)] + \ln \mu}{\ln(1-\alpha)}.$$

Lemma 2: Consider the error system in (8) and let $0 < \alpha < 1$, $\mu \ge 1$, and $\beta \ge 0$ be given constants. Suppose there exist positive definite functions $\mathcal{C}^1$ functions $V_{\theta_k} : \mathbb{R}^n \to \mathbb{R}$, $\theta_k \in \mathcal{I}_N$ with $V_{\theta_{k_0}}(\eta_{k_0}) \equiv 0$ such that $\forall (i,j) \in \mathcal{I}_N \times \mathcal{I}_N, i \ne j$, $V_i(\eta_{\tau_k}) \le \mu V_j(\eta_{\tau_k})$ and $\forall \in \mathcal{T}$,

$$\Delta V_i(\eta_k) \le \begin{cases} \beta V_i(\eta_k) - \Gamma(\tilde{z}_k, w_k), \\ \qquad \forall k \in [\tau_k, \tau_k + \mathcal{T}_{\max}) \\ -\alpha V_i(\eta_k) - \Gamma(\tilde{z}_k, w_k), \\ \qquad \forall k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1}) \end{cases}$$

where $\Gamma(\tilde{z}_k, w_k) \equiv \tilde{z}_k^T \tilde{z}_k - \gamma_i^2 w_k^T w_k$, then the switched system is globally asymptotically stable for any θ_k satisfying (2) and has an l_2-gain no greater that $\gamma^* = \max\{\sqrt{\Theta^{\mathcal{T}_{\max}-1}}\gamma_i\}$, where $\Theta = (1+\beta)/(1-\alpha)$.

Note that the Lyapunov function can increase with a bounded rate during the asynchronous periods. This allows to capture the impact of asynchronous switching between the system and filter modes on the error system stability and $\mathcal{H}_\infty$ performance. Lemma 1 and Lemma 2 consider the impact of asynchronous switching, but do not consider the impact of event-triggered sampling. Our goal in this manuscript is to present improved filter design criteria that incorporate both the aforementioned phenomena.

III. MAIN RESULTS

In this section, we present the main results. First, we develop the conditions under which the filtering error system will be globally asymptotically stable and will have the desired $\mathcal{H}_\infty$ performance level. This is accomplished by using a delay-dependent multiple Lyapunov functional. Next, we convert these analysis conditions to synthesis conditions by appropriately partitioning the Lyapunov matrices.

A. $\mathcal{H}_\infty$ FILTER PERFORMANCE ANALYSIS

Theorem 1: Consider $0 < \alpha < 1$, $\mu \ge 1$, and $\beta \ge 0$ are specified, the error dynamics in (7) are globally uniformly asymptotically stable and has $\mathcal{H}_\infty$ performance γ^*, where $\gamma^* = \max\{\sqrt{\Theta^{\mathcal{T}_{\max}-1}}\gamma_i\}$, if one can find matrices $P_i > 0$ and $Q_{r,i} > 0$ for all $i, j \in \mathcal{I}_N$, $i \ne j$, and $r \in \mathcal{I}_\phi$ such that $P_i \le \mu P_j$, $Q_{r,i} \le \mu Q_{r,j}$ and the conditions in (9) and (10) hold

$$\begin{bmatrix} -P_i & 0 & P_i\tilde{A}_{ir} & P_i\tilde{B}_{ir} \\ * & -I & \tilde{C}_{ir} & \tilde{D}_{ir} \\ * & * & \Lambda_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \le 0 \tag{9}$$

$$\begin{bmatrix} -P_i & 0 & P_i\tilde{A}_{ijr} & P_i\tilde{B}_{ijr} \\ * & -I & \tilde{C}_{ijr} & \tilde{D}_{ijr} \\ * & * & \Lambda_{ijr} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \le 0 \tag{10}$$

where

$$\begin{aligned}
\tilde{A}_{ir} &= \begin{bmatrix} \bar{A}_i + \epsilon_0\bar{A}_{d_i} & \epsilon_1\bar{A}_{d_i} & \cdots & \epsilon_r\bar{A}_{d_i} \end{bmatrix} \\
\tilde{A}_{ijr} &= \begin{bmatrix} \bar{A}_{ij} + \epsilon_0\bar{A}_{d_{ij}} & \epsilon_1\bar{A}_{d_{ij}} & \cdots & \epsilon_r\bar{A}_{d_{ij}} \end{bmatrix} \\
\tilde{B}_{ir} &= \begin{bmatrix} B_i & 0 & \dots & 0 \\ \epsilon_0 B_{f_i} D_i & \epsilon_1 B_{f_i} D_i & \dots & \epsilon_r B_{f_i} D_i \end{bmatrix} \\
\tilde{B}_{ijr} &= \begin{bmatrix} B_i & 0 & \dots & 0 \\ \epsilon_0 B_{f_j} D_i & \epsilon_1 B_{f_j} D_i & \dots & \epsilon_r B_{f_j} D_i \end{bmatrix} \\
\tilde{C}_{ir} &= \begin{bmatrix} \bar{C}_i + \epsilon_0\bar{C}_{d_i}\epsilon_1\bar{C}_{d_i} & \dots & \epsilon_r\bar{C}_{d_i} \end{bmatrix} \\
\tilde{C}_{ir} &= \begin{bmatrix} \bar{C}_{ij} + \epsilon_0\bar{C}_{d_{ij}}\epsilon_1\bar{C}_{d_{ij}} & \dots & \epsilon_r\bar{C}_{d_{ij}} \end{bmatrix} \\
\tilde{D}_{ir} &= \begin{bmatrix} D_i - \epsilon_0 D_{f_i} D_i & -\epsilon_1 D_{f_i} D_i & \dots & -\epsilon_r D_{f_i} D_i \end{bmatrix} \\
\tilde{D}_{ijr} &= \begin{bmatrix} D_i - \epsilon_0 D_{f_j} D_i & -\epsilon_1 D_{f_j} D_i & \dots & -\epsilon_r D_{f_j} D_i \end{bmatrix} \\
\Lambda_{ir} &= \begin{bmatrix} Q_{1,j} - \bar{\alpha}P_i & 0 & \cdots & 0 \\ 0 & Q_{2,j} - \bar{\alpha}Q_{1,i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\bar{\alpha}Q_{r,i} \end{bmatrix} \\
\Lambda_{ijr} &= \begin{bmatrix} Q_{1,j} - \bar{\beta}P_i & 0 & \cdots & 0 \\ 0 & Q_{2,j} - \bar{\beta}Q_{1,i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\bar{\beta}Q_{r,i} \end{bmatrix} \\
\epsilon_r &= \begin{cases} 1 & r = \phi_k \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

$$\bar{\alpha} = 1-\alpha, \bar{\beta} = 1+\beta$$

Proof: First of all the asymptotic stability of (7) is proved by considering the following switched Lyapunov function

$$V_{\theta_k}(\eta_k) = V_{1\theta_k}(\eta_k) + V_{2\theta_k}(\eta_k, \cdots, \eta_{k-M+1}) \tag{11}$$

where

$$V_{1\theta_k}(\eta_k) = \eta_k^T P_{\theta_k}\eta_k$$
$$V_{2\theta_k}(\eta_k, \cdots, \eta_{k-M+1}) = \sum_{r=1}^{M-1} \eta_{k-r}^T Q_{r,\theta_k}\eta_{k-r}$$

With $\bar{w}_k = 0$, the system in (7) becomes

$$\begin{aligned} \eta_{k+1} &= \bar{A}_{ij}\eta_k + \bar{A}_{d_{ij}}\eta_{k-\phi_k} \\ \tilde{z}_k &= \bar{C}_{ij}\eta_k + \bar{C}_{d_{ij}}\eta_{k-\phi_k}, \quad k \in [\tau_k, \tau_k + \mathcal{T}_{\max}) \\ \eta_{k+1} &= \bar{A}_{i}\eta_k + \bar{A}_{d_{i}}\eta_{k-\phi_k} \\ \tilde{z}_k &= \bar{C}_{i}\eta_k + \bar{C}_{d_{i}}\eta_{k-\phi_k}, \quad k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1}) \end{aligned}$$

For $\theta_k = i$ and $\theta_{k+1} = j$, taking the increment of the Lyapunov function $\Delta V_i(\eta_k) = V_i(\eta_{k+1}) - V_i(\eta_k)$ along the state path of the estimation error system, we obtain

$$\Delta V_{1i} = \eta_{k+1}^T P_j \eta_{k+1} - \eta_k^T P_i \eta_k$$
$$\Delta V_{2i} = \sum_{r=1}^{M-1} \eta_{k+1-r}^T Q_{r,j}\eta_{k+1-r} - \sum_{r=1}^{M-1} \eta_{k-r}^T Q_{r,i}\eta_{k-r}$$

Then, for $k \in [\tau_k, \tau_k + \mathcal{T}_{\max})$

$$\Delta V_i - \beta V_i = \|\eta_{k+1}\|_{P_j}^2 - \bar{\beta}\,\|\eta_k\|_{P_i}^2 + \sum_{r=1}^{M-1} \|\eta_{k+1-r}\|_{Q_{r,j}}^2 - \bar{\beta}\sum_{r=1}^{M-1}\|\eta_{k-r}\|_{Q_{r,i}}^2$$

and, for $k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1})$

$$\Delta V_i + \alpha V_i = \|\eta_{k+1}\|_{P_j}^2 - \bar{\alpha}\,\|\eta_k\|_{P_i}^2 + \sum_{r=1}^{M-1} \|\eta_{k+1-r}\|_{Q_{r,j}}^2 - \bar{\alpha}\sum_{r=1}^{M-1}\|\eta_{k-r}\|_{Q_{r,i}}^2$$

With further manipulation, we can write

$$\begin{cases} \Delta V_i - \beta V_i = \zeta_k^T\left(\tilde{A}_{ijr}^T P_j \tilde{A}_{ijr} + \Lambda_{ir}\right)\zeta_k, \\ \quad k \in [\tau_k, \tau_k + \mathcal{T}_{\max}) \\ \Delta V_i + \alpha V_i = \zeta_k^T\left(\tilde{A}_{ir}^T P_j \tilde{A}_{ir} + \Lambda_{ir}\right)\zeta_k, \\ \quad k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1}) \end{cases} \tag{12}$$

where $\zeta_k = \begin{bmatrix}\eta_k^T & \eta_{k-1}^T & \cdots & \eta_{k-M+1}^T\end{bmatrix}^T$. If the inequalities in (9) and (10) hold, then $\tilde{A}_{ijr}^T P_j \tilde{A}_{ijr} + \Lambda_{ir} \le 0$ and $\tilde{A}_{ijr}^T P_j \tilde{A}_{ijr} + \Lambda_{ir} \le 0$ for $i, j \in \mathcal{I}_N$ and $r \in \mathcal{I}_\phi$. Therefore, the error system will be asymptotically stable. Now for $k \in [\tau_k, \tau_k + \mathcal{T}_{\max})$

$$\begin{aligned} \tilde{z}_k^T\tilde{z}_k - \gamma_i^2\bar{w}_k^T\bar{w}_k = \zeta_k^T\tilde{C}_{ijr}^T\tilde{C}_{ijr}\zeta_k + \zeta_k^T\tilde{C}_{ijr}^T\tilde{D}_{ijr}\bar{w}_k \\ + \bar{w}_k^T\tilde{D}_{ijr}^T\tilde{C}_{ijr}\zeta_k + \bar{w}_k^T\tilde{D}_{ijr}^T\tilde{D}_{ijr}\bar{w}_k \\ - \bar{w}_k^T\gamma_i^2\bar{w}_k \end{aligned} \tag{13}$$

For $k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1})$

$$\begin{aligned} \tilde{z}_k^T\tilde{z}_k - \gamma_i^2\bar{w}_k^T\bar{w}_k = \zeta_k^T\tilde{C}_{ir}^T\tilde{C}_{ir}\zeta_k + \zeta_k^T\tilde{C}_{ir}^T\tilde{D}_{ir}\bar{w}_k \\ + \bar{w}_k^T\left(\tilde{D}_{ir}^T\tilde{D}_{ir} - \gamma_i^2 I\right)\bar{w}_k \\ + \bar{w}_k^T\tilde{D}_{ir}^T\tilde{C}_{ir}\zeta_k \end{aligned} \tag{14}$$

Using (12)-(14), we can write

$$\begin{cases} \Delta V_i - \beta V_i + \tilde{z}_k^T\tilde{z}_k - \gamma_i^2\bar{w}_k^T\bar{w}_k = \xi_k^T\Gamma_{ijr}\xi_k \\ \Delta V_i + \alpha V_i + \tilde{z}_k^T\tilde{z}_k - \gamma_i^2\bar{w}_k^T\bar{w}_k = \xi_k^T\Gamma_{ir}\xi_k \end{cases} \tag{15}$$

where $\xi_k = \begin{bmatrix}\zeta_k^T & \bar{w}_k^T\end{bmatrix}^T$ and

$$\Gamma_{ijr} = \begin{bmatrix}\tilde{A}_{ijr} & \tilde{B}_{ijr} \\ \tilde{C}_{ijr} & \tilde{D}_{ijr}\end{bmatrix}^T \begin{bmatrix}P_j & 0 \\ 0 & I\end{bmatrix} \begin{bmatrix}\tilde{A}_{ijr} & \tilde{B}_{ijr} \\ \tilde{C}_{ijr} & \tilde{D}_{ijr}\end{bmatrix} + \begin{bmatrix}\Lambda_{ir} & 0 \\ 0 & -\gamma^2 I\end{bmatrix}$$
$$\Gamma_{ir} = \begin{bmatrix}\tilde{A}_{ir} & \tilde{B}_{ir} \\ \tilde{C}_{ir} & \tilde{D}_{ir}\end{bmatrix}^T \begin{bmatrix}P_j & 0 \\ 0 & I\end{bmatrix} \begin{bmatrix}\tilde{A}_{ir} & \tilde{B}_{ir} \\ \tilde{C}_{ir} & \tilde{D}_{ir}\end{bmatrix} + \begin{bmatrix}\Lambda_{ir} & 0 \\ 0 & -\gamma_i^2 I\end{bmatrix}$$

If (9) and (10) hold, then from (15)

$$\Delta V_i(\eta_k) \le \begin{cases} \beta V_i(\eta_k) + \tilde{z}_k^T\tilde{z}_k - \gamma_i^2\bar{w}_k^T\bar{w}_k, \\ \quad \forall k \in [\tau_k, \tau_k + \mathcal{T}_{\max}) \\ -\alpha V_i(\eta_k) + \tilde{z}_k^T\tilde{z}_k - \gamma_i^2\bar{w}_k^T\bar{w}_k, \\ \quad \forall k \in [\tau_k + \mathcal{T}_{\max}, \tau_{k+1}) \end{cases}$$

This completes the proof. ∎

The inequalities in (9) and (10) involves the matrix product of different system modes. It becomes hard to convert them into design conditions. This difficulty can be overcome by using the approach given in [27].

Lemma 3: The filtering error dynamics in (7) are asymptotically stable and have $\|\tilde{z}_k\|_2 \le \gamma^*\|\bar{w}_k\|_2$ where $\gamma^* = \max\{\sqrt{\Theta^{\mathcal{T}_{\max}-1}}\gamma_i\}$, if one can find $P_i > 0$ and $Q_{r,i} > 0$, R_i for all $i, j \in \mathcal{I}_N$, $i \ne j$ and $r \in \mathcal{I}_\phi$ such that $P_i \le \mu P_j$, $Q_{r,i} \le \mu Q_{r,j}$ and the conditions in (16) and (17) hold:

$$\begin{bmatrix} P_i - R_i - R_i^T & 0 & R_i\tilde{A}_{ir} & R_i\tilde{B}_{ir} \\ * & -I & \tilde{C}_{ir} & \tilde{D}_{ir} \\ * & * & \Lambda_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \le 0 \tag{16}$$

$$\begin{bmatrix} P_i - R_j - R_j^T & 0 & R_j\tilde{A}_{ijr} & R_j\tilde{B}_{ijr} \\ * & -I & \tilde{C}_{ijr} & \tilde{D}_{ijr} \\ * & * & \Lambda_{ijr} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \le 0 \tag{17}$$

Proof: We prove (16) only. The proof of (17) is similar. Note that

$$\begin{aligned} (P_i - R_i)^T P_i^{-1}(P_i - R_i) &\ge 0, \\ (I - R_i^T P_i^{-1})(P_i - R_i) &\ge 0, \\ P_i - R_i^T - R_i + R_i^T P_i^{-1} R_i &\ge 0, \\ P_i - R_i - R_i^T &\ge -R_i P_i^{-1} R_i^T \end{aligned}$$

If the inequality in (17) holds, then

$$\begin{bmatrix} -R_iP_i^{-1}R_i^T & 0 & R_i\tilde{A}_{ir} & R_i\tilde{B}_{ir} \\ * & -I & \tilde{C}_{ir} & \tilde{D}_{ir} \\ * & * & \Lambda_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} < 0$$

Pre- and post- multiplying the above equation with $\text{diag}\{R_i^{-1}, I, I, I\}$ and $\text{diag}\{R_i^{-T}, I, I, I\}$ and then pre- and post- multiplying with $\text{diag}\{P_i, I, I, I\}$ and $\text{diag}\{P_i, I, I, I\}$, yields (9), thus the proof is concluded. ■

B. $\mathcal{H}_\infty$ FILTER DESIGN

In this section, we provide a theorem that establishes conditions such that a solution to the $\mathcal{H}_\infty$ filtering problem exists, and a filter can be designed.

Theorem 2: Consider $0 < \alpha < 1$, $\mu \geq 1$, and $\beta \geq 0$ for the system in (1) be given, if one can matrices $P_{1i} > 0$, $P_{3i} > 0$, $Q_{1r,i} > 0$, $Q_{3r,i} > 0$, and P_{2i}, $Q_{2r,i}$, U_i, Y_i, W_i, A_{F_i}, B_{F_i}, C_{F_i}, and D_{F_i}, $\forall i, j \in \mathcal{I}_N$, $i \neq j$, and $r \in \mathcal{I}_\phi$ such that the matrix inequalities given below

$$\begin{bmatrix} \Pi_{ij}^1 & 0 & \Pi_{ij}^2 & \Pi_{ij}^3 \\ * & -I & \Pi_{ij}^4 & \Pi_{ij}^5 \\ * & * & \Upsilon_{ijr} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \Pi_{i}^1 & 0 & \Pi_{i}^2 & \Pi_{i}^3 \\ * & -I & \Pi_{i}^4 & \Pi_{i}^5 \\ * & * & \Upsilon_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} < 0 \tag{19}$$

hold, where

$$\Pi_{ij}^1 = \begin{bmatrix} P_{1i} - U_j - U_j^T & P_{2i} - Y_j - W_j^T \\ * & P_{3i} - Y_j - Y_j^T \end{bmatrix}$$

$$\Pi_{i}^1 = \begin{bmatrix} P_{1i} - U_i - U_i^T & P_{2i} - Y_i - W_i^T \\ * & P_{3i} - Y_i - Y_i^T \end{bmatrix}$$

$$\Pi_{ij}^2 = \begin{bmatrix} U_jA_i + \epsilon_0 B_{F_j}C_i & A_{F_j} & \epsilon_1 B_{F_j}C_i & 0 & \dots & \epsilon_r B_{F_i}C_i & 0 \\ W_jA_i + \epsilon_0 B_{F_j}C_i & A_{F_j} & \epsilon_1 B_{F_j}C_i & 0 & \dots & \epsilon_r B_{F_i}C_i & 0 \end{bmatrix}$$

$$\Pi_{i}^2 = \begin{bmatrix} U_iA_i + \epsilon_0 B_{F_i}C_i & A_{F_i} & \epsilon_1 B_{F_i}C_i & 0 & \dots & \epsilon_r B_{F_i}C_i & 0 \\ W_iA_i + \epsilon_0 B_{F_i}C_i & A_{F_i} & \epsilon_1 B_{F_i}C_i & 0 & \dots & \epsilon_r B_{F_i}C_i & 0 \end{bmatrix}$$

$$\Pi_{ij}^3 = \begin{bmatrix} U_jB_i + \epsilon_0 B_{F_j}D_i & \epsilon_1 B_{F_j}D_i & \dots & \epsilon_r B_{F_j}D_i \\ W_jB_2 + \epsilon_0 B_{F_j}D_i & \epsilon_1 B_{F_j}D_i & \dots & \epsilon_r B_{F_j}D_i \end{bmatrix}$$

$$\Pi_{i}^3 = \begin{bmatrix} U_iB_i + \epsilon_0 B_{F_i}D_i & \epsilon_1 B_{F_i}D_i & \dots & \epsilon_r B_{F_i}D_i \\ W_iB_2 + \epsilon_0 B_{F_i}D_i & \epsilon_1 B_{F_i}D_i & \dots & \epsilon_r B_{F_i}D_i \end{bmatrix}$$

$$\Pi_{ij}^4 = \begin{bmatrix} C_2 - \epsilon_0 D_{F_j}C_i & -C_{F_j} & -\epsilon_1 D_{F_j}C_i & 0 & \dots & \epsilon_r D_{F_i}C_i & 0 \end{bmatrix}$$

$$\Pi_{i}^4 = \begin{bmatrix} C_2 - \epsilon_0 D_{F_i}C_i & -C_{F_i} & -\epsilon_1 D_{F_i}C_i & 0 & \dots & \epsilon_r D_{F_i}C_i & 0 \end{bmatrix}$$

$$\Pi_{ij}^5 = \begin{bmatrix} D_i - \epsilon_0 D_{F_j}D_i & -\epsilon_1 D_{F_j}D_i & \dots & -\epsilon_r D_{F_j}D_i \end{bmatrix}$$

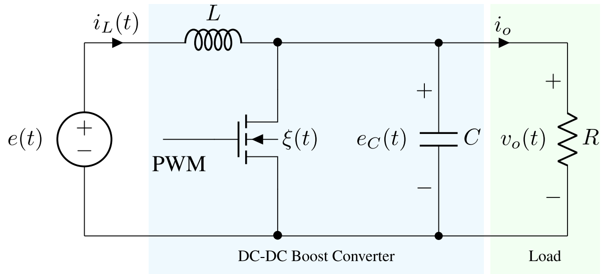

FIGURE 2. PWM-driven DC-DC boost converter.

$$\Pi_i^5 = \Pi_{i5} = \begin{bmatrix} D_i - \epsilon_0 D_{F_i}D_i & -\epsilon_1 D_{F_i}D_i & \dots & -\epsilon_r D_{F_i}D_i \end{bmatrix}$$

Υ_{ijr} and Υ_{ir} are partitions of Λ_{ijr}, Λ_{ir}, and ϵ_r, defined in (9). Then, the error system in (7) will be asymptotically stable with $\mathcal{H}_\infty$ performance level $\gamma^* = \max\{\sqrt{\Theta^{\mathcal{T}_{\max}-1}}\gamma_i\}$. The corresponding filter parameters are given by

$$A_{f_i} = Y_i^{-1}A_{F_i}, B_{f_i} = Y_i^{-1}B_{F_i}C_{f_i} \quad = C_{F_i}, D_{f_i} = D_{F_i}.$$

Proof: Take matrices P_i, R_i and $Q_{r,i}$ in (16) as

$$P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix}, \; R_i = \begin{bmatrix} U_i & Y_i \\ W_i & Y_i \end{bmatrix}, \; Q_{r,i} = \begin{bmatrix} Q_{1r,i} & Q_{2r,i} \\ Q_{2r,i}^T & Q_{3r,i} \end{bmatrix}$$

Inserting them in (16), we get

$$P_i - R_i - R_i^T = \begin{bmatrix} P_{1j} - U_i - U_i^T & P_{2j} - Y_i - W_i^T \\ P_{2j}^T - W_i - Y_i^T & P_{3j} - Y_i - Y_i^T \end{bmatrix}$$

$$R_i\tilde{A}_{ir} = \begin{bmatrix} U_iA_i + \epsilon_0 Y_iB_{f_i}C_i & Y_iA_{f_i} & \epsilon_1 Y_iB_{f_i}C_i & 0 & \cdots & \epsilon_r Y_iB_{f_i}C_i & 0 \\ W_iA_i + \epsilon_0 Y_iB_{f_i}C_i & Y_iA_{f_i} & \epsilon_1 Y_iB_{f_i}C_i & 0 & \cdots & \epsilon_r Y_iB_{f_i}C_i & 0 \end{bmatrix}$$

Define $B_{F_i} = Y_iB_{f_i}$, and $A_{F_i} = Y_iA_{f_i}$, then

$$R_i\tilde{A}_{ir} = \begin{bmatrix} U_iA_i + \epsilon_0 B_{F_i}C_i & A_{F_i} & \epsilon_1 B_{F_i}C_i & 0 & \cdots & \epsilon_r B_{F_i}C_i & 0 \\ W_iA_i + \epsilon_0 B_{F_i}C_i & A_{F_i} & \epsilon_1 B_{F_i}C_i & 0 & \cdots & \epsilon_r B_{F_i}C_i & 0 \end{bmatrix}$$

$$R_i\tilde{B}_{ir} = \begin{bmatrix} U_iB_i + \epsilon_0 B_{F_i}D_i & \epsilon_1 B_{F_i}D_i & \cdots & \epsilon_r B_{F_i}D_i \\ W_iB_i + \epsilon_0 B_{F_i}D_i & \epsilon_1 B_{F_i}D_i & \cdots & \epsilon_r B_{F_i}D_i \end{bmatrix}$$

$$\tilde{C}_{ir} = \begin{bmatrix} C_i - \epsilon_0 D_{F_i}C_i & -C_{F_i} & -\epsilon_1 D_{F_i}C_i & 0 & \cdots & -\epsilon_r D_{F_i}C_i & 0 \end{bmatrix}$$

$$\tilde{D}_{ir} = \begin{bmatrix} D_i - \epsilon_0 D_{F_i}D_i & -\epsilon_1 D_{F_i}D_i & \cdots & -\epsilon_r D_{F_i}D_i \end{bmatrix}$$

Inserting these expressions, we get (18). The inequality in (19) can be derived similarly. This completes the proof. ■

IV. SIMULATION RESULTS

In this section, we give two examples to show the utility of the proposed technique. Example 1 demonstrate the effectiveness of the technique in reducing the utilization of network bandwidth while Example 2 compares the performance of the proposed technique with an existing result.

A. EXAMPLE 1: EVENT-TRIGGERED FILTER DESIGN FOR PWM-DRIVEN DC-DC BOOST CONVERTER

Consider the Pulse Width Modulation (PWM)-driven DC-DC boost converter studied in [4], [28], and [21]. A schematic of the converter circuit is shown in Figure 2. A PWM signal drives the switch $\xi(t)$ with a period of T seconds. The circuit has a source voltage $e(t)$, resistance R, capacitance C, and inductance L. Also, $i_L(t)$ is the current through the inductor, $e_C(t)$ is the voltage across the capacitor, $i_o(t)$ is the current through the resistor, and $v_o(t)$ is the voltage across the resistor. The dynamics of this circuit are governed by the following differential equations.

$$\begin{aligned}\dot{e}_C(s) &= -\frac{1}{RC_1}e_C(s) + (1-\xi(s))\frac{1}{C_1}i_L(s)\\ \dot{i}_L(s) &= -(1-\xi(s))\frac{1}{L_1}e_C(s) + \xi(s)\frac{1}{L_1}e(t)\end{aligned} \tag{20}$$

where $s = t.T$, $C_1 = CT^{-1}$, and $L_1 = LT^{-1}$. The dynamics in (20) can be written in switched form as

$$\dot{x}(t) = A^c_{\theta(t)}x(t)$$

where $x(t) = \begin{bmatrix} e_C(t) & I_L(t) & I\end{bmatrix}$, and

$$A^c_1 = \begin{bmatrix} -\frac{1}{RC1} & \frac{1}{C_1} & 0\\ -\frac{1}{L_1} & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ A^c_2 = \begin{bmatrix} -\frac{1}{RC1} & 0 & 0\\ 0 & 0 & -\frac{1}{L_1}\\ 0 & 0 & 0\end{bmatrix}$$

The switching signal is defined as

$$\theta(t) = \begin{cases} 1, & \xi(t) = 0(\text{OFF})\\ 2, & \xi(t) = 1(\text{ON})\end{cases}$$

Using similar approach as in [28], we write matrices A^c_1 and A^c_2 in the normalized form.

$$A^c_1 = \begin{bmatrix} -1 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ A^c_2 = \begin{bmatrix} -1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 0 & 0\end{bmatrix}$$

Both A^c_1 and A^c_2 are not Hurwitz. Since, we are considering the filtering problem, they should be Hurwitz. Therefore, as in [28], we take $B^c_1 = B^c_2 = \begin{bmatrix}-0.1 & 0.4 & 0.5\end{bmatrix}^T$, state feedback gain matrices $F_1 = \begin{bmatrix}-6.61 & -1.07 & -9.32\end{bmatrix}$, and $F_2 = \begin{bmatrix}-5.37 & -12.42 & -10.07\end{bmatrix}$ to get the closed-loop matrices

$$\bar{A}^c_1 = \begin{bmatrix} -0.3 & 1.1 & 0.9\\ -3.7 & -0.4 & -3.7\\ -3.3 & -0.5 & -4.7\end{bmatrix},$$
$$\bar{A}^c_2 = \begin{bmatrix} -0.5 & 1.1 & 1\\ -2.2 & -5 & -3\\ -2.7 & -6.2 & -5\end{bmatrix}$$

By taking the sampling period $h = T/10$, this system can be put in the form as given in (1) with $N = 2$ and the following parameters

$$A_1 = \begin{bmatrix} 0.94 & 0.10 & 0.06\\ -0.30 & 0.95 & -0.30\\ -0.25 & -0.06 & 0.63\end{bmatrix},\quad B_1 = \begin{bmatrix}-0.30\\ 0.20\\ 0.10\end{bmatrix}$$

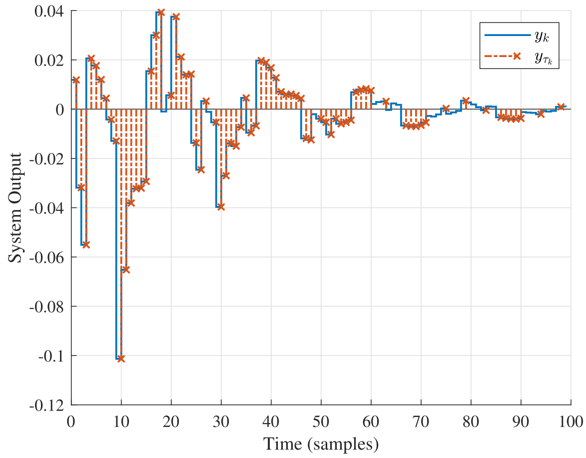

FIGURE 3. Event-triggered sampling of the measurement.

$$C_1 = \begin{bmatrix}-0.10 & 0.40 & 0.40\end{bmatrix},\quad D_1 = 0.10$$
$$A_2 = \begin{bmatrix} 0.93 & 0.08 & 0.07\\ -0.14 & 0.66 & -0.20\\ -0.16 & -0.40 & 0.66\end{bmatrix},\quad B_2 = \begin{bmatrix}0.70\\ -1.0\\ 0.30\end{bmatrix}$$
$$C_2 = \begin{bmatrix}0.70 & -1.0 & 0.30\end{bmatrix},\quad D_2 = 0.10$$

Using Theorem 2 with $\alpha = 0.02$, $\beta = 0.01$, $\mu = 1.02$, and $\mathcal{T}_{\max} = 2$, we can design the following filter parameters.

$$A_{f_1} = \begin{bmatrix} 0.91 & 0.05 & 0.04\\ -0.52 & 0.72 & -0.55\\ -0.52 & -0.32 & 0.30\end{bmatrix},\ B_{f_1} = \begin{bmatrix}-0.01\\ 0.01\\ 0.02\end{bmatrix}$$
$$C_{f_1} = \begin{bmatrix}-0.11 & 0.09 & -0.12\end{bmatrix},\ D_{f_1} = 0.01$$
$$A_{f_2} = \begin{bmatrix} 0.91 & -0.05 & 0.04\\ -0.41 & 0.51 & -0.48\\ -0.44 & -0.44 & 0.38\end{bmatrix},\ B_{f_2} = \begin{bmatrix}0.01\\ 0.01\\ 0.01\end{bmatrix}$$
$$C_{f_2} = \begin{bmatrix}-0.21\ q0.17 & -0.08\end{bmatrix},\ D_{f_2} = 0.01$$

The parameter α controls the rate of decrease of the Lyapunov during the matched (synchronous) period and β controls the rate of increase during the mismatch (asynchronous) period. For slow switching, α and β have small values. The parameter μ controls the increase in Lyapunov function at switching instants. We choose μ to ensure that the value of multiple Lyapunov functional forms a decreasing sequence. The attained value of $\mathcal{H}_\infty$ performance level is $\gamma^* = 2.4689$. The event-triggered sampling policy is taken as

$$\mathcal{L}(x_k, y_k, k) = \|e_k\| - \eta\left\|y_{\tau_k}\right\|$$

where $e_k = y(k) - y(\tau_k)$ and $\eta = 0.2$. Let the disturbance input w_k be

$$w_k = 0.1\exp(-0.04k)\sin(0.1\pi k).$$

Figure 3 shows event-triggered sampling of the system measurement. The corresponding event-triggering instants are shown in Figure 4. As seen in Figure 4, the measurement may be transmitted after one sampling period or two sampling periods, if the event-triggering condition is satisfied, However, it is certainly transmitted after three sampling periods.

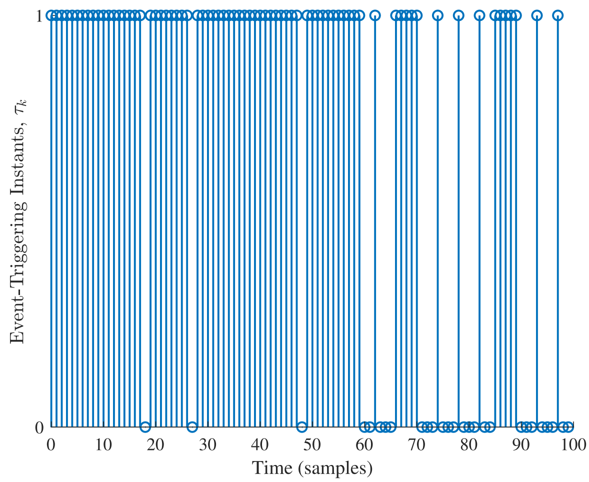

FIGURE 4. Event-triggering instants.

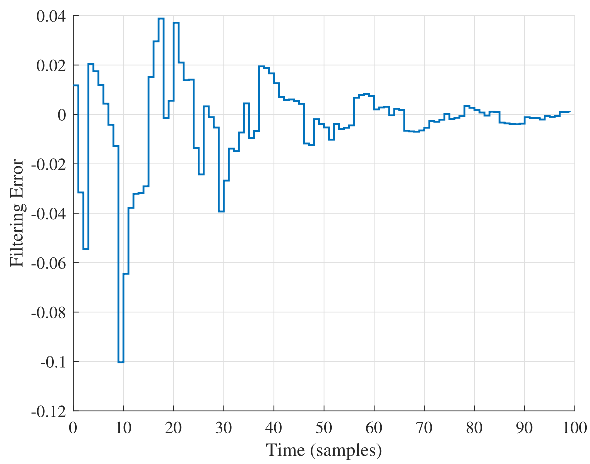

FIGURE 5. Filtering error.

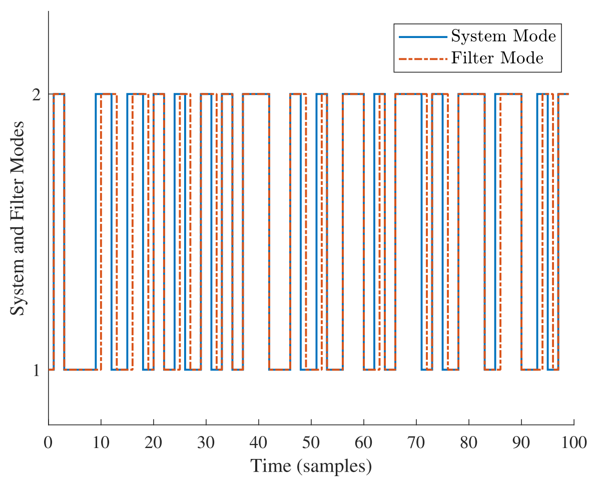

FIGURE 6. System and filter modes.

Also, as expected, the number of measurement transmissions is reduced as the filtering error converges to zero. There are 73 data transmissions during 100 sampling intervals. That means that the data transmission is reduced by 27%. The estimation error response is shown in Figure 5. Clearly, the error converges to zero. The corresponding system and filter modes are shown in Figure 6. There is a maximum lag of two sampling intervals between the system and filter modes. The switching signal has an average dwell time of 2. The convergence of the error under average dwell time switching and the significant reduction in data transmission rate demonstrate that the approach is effective.

B. EXAMPLE 2: COMPARISON WITH THE EXISTING TECHNIQUE

Consider the switched system with parameters

$$A_1 = \begin{bmatrix} 0.40 & -0.50 & -0.10 \\ 0.10 & 0.40 & -0.02 \\ 0.40 & 0.01 & -0.50 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.70 \\ 1.30 \\ 0.60 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 0.20 & 0.10 & 0.20 \end{bmatrix}, \quad D_1 = 0.20$$

and

$$A_2 = \begin{bmatrix} 0.50 & 0.20 & -0.20 \\ -0.40 & 0.40 & -0.10 \\ 0.60 & -0.10 & 0.20 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.20 \\ 1.40 \\ -0.50 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} 0.30 & 0.40 & -0.20 \end{bmatrix}, \quad D_2 = 0.30$$

This system was considered in [21] where the authors designed a switched $\mathcal{H}_\infty$ filter for this system with $M = 3$ (cf. $\tau_d = 3$ in [21]). Their attainable weighted $\mathcal{H}_\infty$ performance level was $\gamma = 6.8761$.

For the same system, using Theorem 2 with $\alpha = 0.02$, $\beta = 0.25$, $\mu = 1.02$, and $\mathcal{T}_{\max} = 0$, we design the filter parameters

$$A_{f_1} = \begin{bmatrix} 0.27 & -0.37 & -0.23 \\ -0.40 & 0.23 & 0.07 \\ 0.49 & -0.21 & -0.43 \end{bmatrix}, \quad B_{f_1} = \begin{bmatrix} 0.22 \\ 0.90 \\ 0.03 \end{bmatrix}$$
$$C_{f_1} = \begin{bmatrix} 0.56 & 0.55 & 0.55 \end{bmatrix}, \quad D_{f_1} = 0.19$$
$$A_{f_2} = \begin{bmatrix} 0.31 & 0.17 & -0.15 \\ -1.19 & 0.39 & -0.19 \\ 1.08 & -0.65 & 0.12 \end{bmatrix}, \quad B_{f_2} = \begin{bmatrix} 0.23 \\ 1.03 \\ -0.23 \end{bmatrix}$$
$$C_{f_2} = \begin{bmatrix} 0.55 & 0.55 & 0.86 \end{bmatrix}, \quad D_{f_2} = 0.27$$

with minimum $\mathcal{H}_\infty$ performance level $\gamma = 4.1116$. Clearly, the filter designed by the proposed technique can attain 40% better $\mathcal{H}_\infty$ performance. Let the disturbance input w_k be

$$w_k = 2\exp(-0.1k)$$

and the same event-triggered sampling policy as in [21].

$$\mathcal{L}(x_k, y_k, k) = e_k^T \Phi_i e_k - y_k^T \Psi_i y_k$$

with

$$\Phi_1 = 1.4690, \quad \Phi_2 = 1.4989$$
$$\Psi_1 = 0.5096, \quad \Psi_2 = 0.5001$$

Fig. 7 shows the actual system output and outputs estimated by the proposed filter and the filter in [21]. It can be seen that the output estimated by the proposed filter is closer to the actual system output.

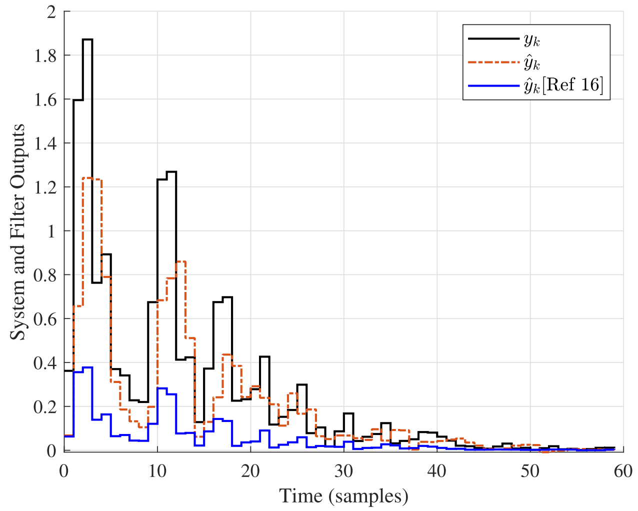

FIGURE 7. System and filter outputs.

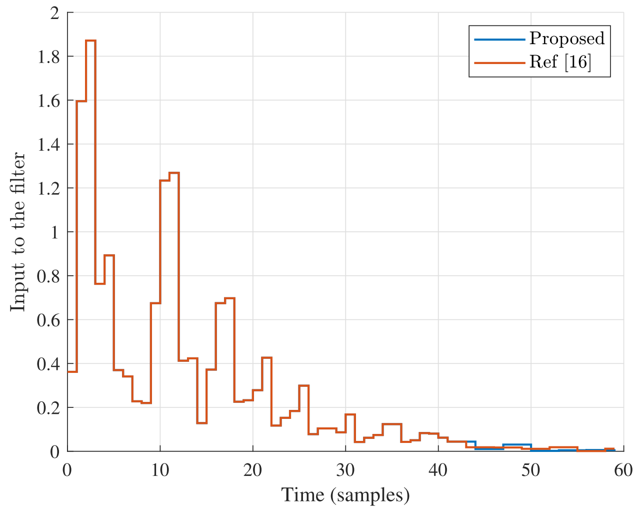

FIGURE 8. Input to both filters.

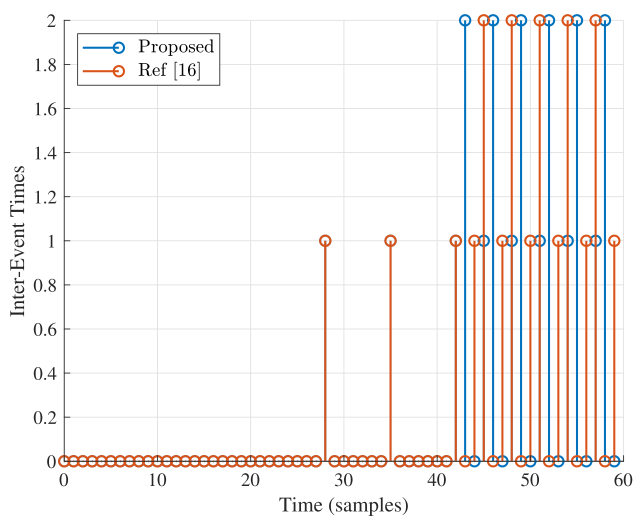

FIGURE 9. Inter-event times.

The inputs to the filters is shown in Figure 8. The duration of inter-event data transmission intervals are shown in Fig. 9.

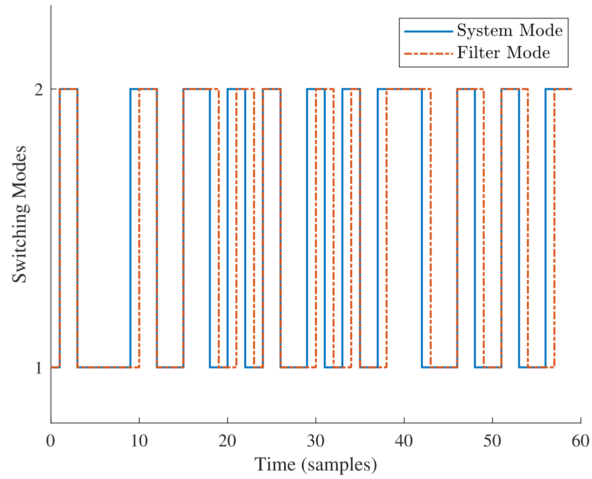

FIGURE 10. Switched system modes.

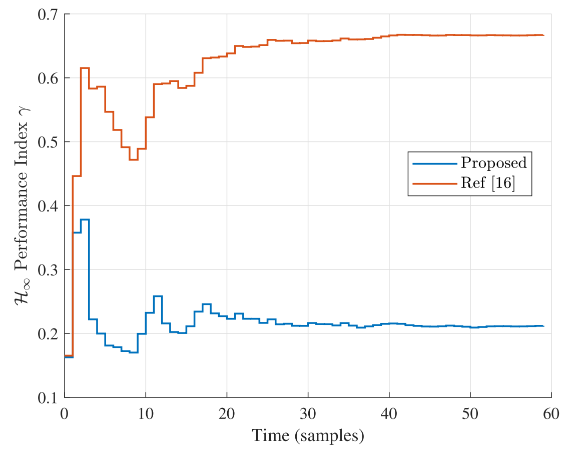

FIGURE 11. Computed $\mathcal{H}_\infty$ performance index.

The duration varies from 0 to 2 indicating that the measurement may be transmitted after one, two, or three sampling periods, if the event-triggering condition is satisfied or if the system is self-triggered. Also, as expected, the number of measurement is reduced as the actual and filtered outputs converge. The corresponding system modes are shown in Figure 10.

To quantify the $\mathcal{H}_\infty$ performance of both filters, we compute the performance index as

$$\gamma_k = \sqrt{\frac{\sum_{i=0}^{k} \tilde{z}_i^T \tilde{z}_i}{\sum_{i=0}^{k} w_i^T w_i}}$$

Figure 11 shows a plot of the $\mathcal{H}_\infty$ performance index γ for both filters. The performance index of the proposed filter is about half of the performance index of the filter in [21]. The computed performance indices of both filter in 60 samples simulation are shown in Table 1. Both filters have performance indices smaller than their computed worst-case bounds; however, the computed bound for the proposed filter is much small than the filter in [21]. These results clearly demonstrate that the filter designed using the proposed tech-

TABLE 1. Computed $\mathcal{H}_\infty$ performance indices of both filters.

$\mathcal{H}_\infty$ Performance	Filter in [21]	Proposed Filter
γ_k	0.6671	0.2118

nique exhibits improved performance as compared to the existing results.

V. CONCLUSION

We have presented a technique for event-triggered $\mathcal{H}_\infty$ filtering when an event-triggering policy already exists or is prespecified. We have shown that this problem can be viewed as a state estimation problem with non-uniform sampling. Thus, the filtering error system is modelled as a delay-dependent switched system with non-uniformly sampled measurements. By using the multiple Lyapunov method, the filter design conditions are given as linear matrix inequalities that can be easily solved using modern solvers. In the end, two numerical examples are given to show the efficacy of the proposed approach and to compare results with the existing ones in the literature. The approach is applicable to event-triggered systems where an event-generator is predefined, such as systems with hardware-based event-generators. A limitation of the method is that the parameter M is chosen manually to adapt to the behavior of the event-generator. In the future, we plan to extend this work to systems with modeling uncertainties and automatic selection of the parameter M.

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