Geometrically nonlinear buckling analysis of truss with length imperfection subjected to mechanical and thermal load using hybrid FEM

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Abstract. This paper presents a novel hybrid FEM-based approach to establish the mathematical model for solving the nonlinear buckling problem of truss systems with length imperfection under mechanical and thermal load. due to constant temperature change-based hybrid FEM. The proposed approach deals with establishing hybrid types of truss elements, including perfect truss elements without thermal deformation and truss elements with length imperfection and thermal deformation. The equilibrium equation of both truss elements is established based on compatibility relationships considering geometric nonlinearity. The hybrid global equilibrium equations of truss systems are developed by assembling constructed perfect truss elements without thermal deformation and truss elements with length imperfection and thermal deformation. The incremental-iterative algorithm based on the arc-length method is used to establish calculation programs to solve the hybrid global equilibrium equation for investigating the geometrically nonlinear buckling behavior of the truss system. The numerical test is presented to investigate the buckling and post-buckling behavior of truss systems having some elements with length imperfection under thermal and mechanical load.

1 Introduction

The geometric imperfection of truss elements as a result of the manufacturing, transporting, and handling processes significantly influences the buckling behavior of the truss system, especially in nonlinear buckling analysis. In recent years, many research works have been published and addressed the influence of geometrical imperfection on the behavior of truss structures [1-3]. In many practical cases, the designing truss system with initial length imperfection needs to consider thermal deformation due to temperature load affected to the truss system. Research works for nonlinear analysis of buckling behavior of truss under temperature load can be found in some modern publishing papers [4,5]. The thermal load will affect the behavior of the truss system and increase the difficulty of establishing an algorithm for solving the geometrically nonlinear buckling problem of the truss system with initial length imperfection. The approaches and techniques for solving geometrically nonlinear problems of structures such as truss systems based on FEM have been developed

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decades ago. Using the displacement-based finite element method, the establishing algorithm for solving a geometrically nonlinear problem of the truss system with initial length imperfection under thermal loading requires imposing length imperfection and temperature deformation constraint depending on the incremental element length to the master stiffness equation by using mathematical methods for constrained optimization such as penalty augmentation method or Lagrange multiplier adjunction method [6-7]. The mathematical technique for treating initial length imperfection and thermal load considerably increases the difficulty of developing an incremental-iterative algorithm for solving the geometrically nonlinear buckling problem of the truss system. Based on mixed finite element formulation, the author introduced an approach for establishing the mathematical model for solving the geometrically nonlinear buckling problem of truss with initial length imperfection [8] and under thermal load [9]. Using a mixed model shows a significant advantage over the displacement-based formulation model but there is not without a disadvantage that deals with increasing the dimension of the solving system. The hybrid finite element formulation [10-11] is widely utilized in solving the nonlinear mechanical contact problem [12] and it can be used as a novel approach to overcoming difficulties in the mathematical treatment of length imperfection and thermal load in geometrically nonlinear buckling analysis of truss system. The main idea of hybrid FEM formulation is based on discretizing the truss system into different types of truss elements including perfect truss elements without thermal deformation and length imperfection truss elements with thermal deformation. Both common truss elements are established based on compatibility relationships considering geometric nonlinearity. The hybrid global equilibrium equations are developed by assembling constructed perfect truss elements without thermal deformation and truss elements with length imperfection and thermal deformation. The incremental-iterative algorithm based on the arc-length method is used to establish calculation programs to solve the hybrid global equilibrium equation for investigating the geometrically nonlinear buckling behavior of the truss system. The numerical test is presented to investigate the buckling and post-buckling behavior of truss systems having some elements with length imperfection under thermal and mechanical load.

2 Equilibrium equations for the truss elements considering large displacements

The hybrid finite element model of the truss system consists of two types of truss elements (shown in fig.1), including (e_I) - the first type element is a perfect truss element without temperature change; (e_{II}) - the second type element is a truss element having initial length imperfection Δ_i and thermal expansion ΔL_T due to a constant temperature change ΔT (the truss element subjected to uniform thermal loads.

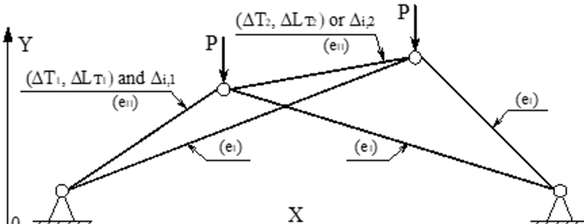

Fig. 1. Types of truss elements.

Consider two-node truss elements (e_I) and (e_{II}) in the global coordinate system (X0Y) as shown in Fig.2.

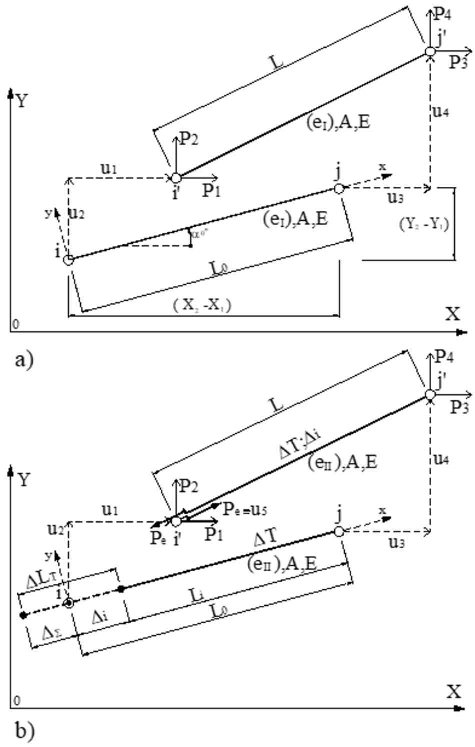

Fig. 2. Truss elements before and after deformation: a) truss element (e_I) ; b) truss element (e_{II}) .

For establishing the finite element equation, designating the followings:

- $\{X_1,Y_1\},\{X_2,Y_2\}$: ith and jth nodal coordinates in the global coordinate system before and after deformation;

- L : distance between ith and jth node after deformation; L_i and Δ_i initial length (manufactured length) and length imperfection of the truss element (e_{II}) ;

- α : linear thermal expansion coefficient; $\Delta L_T = \alpha.\Delta T.L_0$: the change in length due to thermal expansion [13] in case of temperature change ΔT of the truss element (e_{II}) ;

- u_1, u_2, u_3, u_4 and P_1, P_2, P_3, P_4 : nodal displacements and forces in global coordinates; P_e : resultant external force at the ith cross-section after deformation;

- $u_5 \equiv P_e = N$: resultant external force at the i' cross-section of the truss element (e_{II}) after deformation;

- A: cross-sectional area of truss element; E: elastic modulus of material; N: an axial load of truss element;

The length of the truss element after deformation is defined as follows

$$L = \sqrt{(X_2 - X_1 + u_3 - u_1)^2 + (Y_2 - Y_1 + u_4 - u_2)^2} \tag{1}$$

The axial deformation of the truss elements can be computed by the expression

$$\begin{cases} (e_I):\ \Delta L^{(e_I)} = L - L_0 = L - \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} \\ (e_{II}):\ \Delta L^{(e_{II})} = L - (L_i + \Delta L_T) = L - \left((L_0 - \Delta_i) + \Delta L_T\right) \\ \qquad\qquad = L - \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} + \Delta_i - \Delta L_T \\ \qquad\qquad = L - \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} - \Delta_\Sigma \\ \text{where } (\Delta_\Sigma = \Delta L_T - \Delta_i) \end{cases} \tag{2}$$

Work of internal axial force can be calculated for both truss elements as below

$$\begin{cases} (e_I): \delta V^{(e_I)} = -\int \sigma_x \delta\varepsilon_x dV = -\int_A \sigma_x dA \int_0^{L_0} \delta\varepsilon_x dx = -N\delta\Delta L = -N\left\{\sum_{i=1}^{4} \frac{\partial \Delta L}{\partial u_i}\delta u_i\right\} \\ (e_{II}): \delta V^{(e_{II})} = -\int \sigma_x \delta\varepsilon_x dV = -\int_A \sigma_x dA \int_0^{L_i} \delta\varepsilon_x dx \\ \qquad = -N\delta\Delta L = -N\left\{\sum_{i=1}^{4} \frac{\partial \Delta L}{\partial u_i}\delta u_i + \frac{\partial \Delta L}{\partial \Delta_\Sigma}\delta\Delta_\Sigma\right\} \end{cases} \tag{3}$$

Neglecting the body forces the virtual external work can be defined for each truss element

$$\begin{cases} (e_I):\ \delta\overline{V}^{(e_I)} = \mathrm{P}_1\,\delta u_1 + \mathrm{P}_2\delta u_2 + \mathrm{P}_3\delta u_3 + \mathrm{P}_4\delta u_4 = \sum_{i=1}^{4} \mathrm{P}_i\,\delta u_i \\ (e_{II}): \delta\overline{V}^{(e_{II})} = \mathrm{P}_1\,\delta u_1 + \mathrm{P}_2\delta u_2 + \mathrm{P}_3\delta u_3 + \mathrm{P}_4\delta u_4 - \mathrm{P}_e\,\delta\Delta_\Sigma = \sum_{i=1}^{4} \mathrm{P}_i\,\delta u_i - \mathrm{P}_e\,\delta\Delta_\Sigma \end{cases} \tag{4}$$

The total work done by internal and external forces for each truss element is obtained by summing Eq. (3) and Eq. (4), getting

$$\begin{cases} (e_I):\ \delta V^{(e_I)} + \delta\overline{V}^{(e_I)} = -N\left\{\sum_{i=1}^{4} \frac{\partial \Delta L}{\partial u_i}\delta u_i\right\} + \sum_{i=1}^{4} \mathrm{P}_i\,\delta u_i = \sum_{i=1}^{4}\left\{-N\frac{\partial \Delta L}{\partial u_i} + \mathrm{P}_i\right\}\delta u_i = 0 \\ (e_{II}): \delta V^{(e_{II})} + \delta\overline{V}^{(e_{II})} = -N\left\{\sum_{i=1}^{4} \frac{\partial \Delta L}{\partial u_i}\delta u_i + \frac{\partial \Delta L}{\partial \Delta_\Sigma}\delta\Delta_\Sigma\right\} + \left\{\sum_{i=1}^{4} \mathrm{P}_i\,\delta u_i - P_e\delta\Delta_\Sigma\right\} \\ \qquad = \sum_{i=1}^{4}\left\{-N\frac{\partial \Delta L}{\partial u_i} + \mathrm{P}_i\right\}\delta u_i + \left\{-N\frac{\partial \Delta L}{\partial \Delta_\Sigma} - P_e\right\}\delta\Delta_\Sigma = 0 \end{cases} \tag{5}$$

Applying the principle of virtual work to establish governing equation for each truss element, from equation (5) obtaining

$$\begin{cases} (e_I):\left\{\ -N\frac{\partial \Delta L}{\partial u_i} + \mathrm{P}_i = 0 \quad (i = 1,2,3,4)\right. \\ (e_{II}):\begin{cases} -N\frac{\partial \Delta L}{\partial u_i} + \mathrm{P}_i = 0 \quad (i = 1,2,3,4) \\ -N\frac{\partial \Delta L}{\partial \Delta_\Sigma} - P_e = 0 \end{cases} \end{cases} \tag{6}$$

Expressing axial force through deformation and adding axial deformation from equation (2) to equation (6), getting the system

$$\begin{cases} (e_I): \begin{cases} q_i^{(e_I)}(\mathbf{u}) = \dfrac{EA}{L_0}(L-L_0)\dfrac{\partial L}{\partial u_i} = \mathrm{P}_i \quad ; & or \begin{cases} q_i^{(e_I)}(\mathbf{u}^{(e_I)}) = \mathrm{P}_i^{(e_I)} \\ i = 1,2,3,4 \end{cases} \\ \mathbf{u}^{(e_I)} = \{u_1, u_2, u_3, u_4\}^T \end{cases} \\ (e_{II}): \begin{cases} q_i^{(e_{II})}(\mathbf{u}, \Delta_\Sigma) = \dfrac{EA}{L_i}(L - L_0 - \Delta_\Sigma)\dfrac{\partial L}{\partial u_i} = \mathrm{P}_i \\ q_5^{(e_{II})}(\mathbf{u}, \Delta_\Sigma) = \dfrac{EA}{L_i}(L - L_0 - \Delta_\Sigma) - P_e = 0 \end{cases} ; or \begin{cases} q_k^{(e_{II})}(\mathbf{u}^{(e_{II})}, \Delta_\Sigma) = \mathrm{P}_k^{(e_{II})} \\ k = 1,2,..,5 \\ \mathbf{u}^{(e_{II})} = \{u_1, u_2, u_3, u_4, u_5 \equiv P_e\}^T \end{cases} \end{cases} \tag{7}$$

The incremental equilibrium equation for each truss element is obtaining by applying incremental loading [14-15] into equation (7), expressing in compact matrix format as follows

$$\begin{cases} (\mathrm{e}_I):\ \mathbf{k}^{(e_I)}(\mathbf{u})\delta\mathbf{u} = (\mathbf{P}^{(e_I)} + \Delta\mathbf{P}^{(e_I)}) - \mathbf{q}^{(e_I)}(\mathbf{u}) \\ (\mathrm{e}_{II}):\ \mathbf{k}^{(e_{II})}(\mathbf{u}, \Delta_\Sigma)\delta\mathbf{u} = (\mathbf{P}^{(e_{II})} + \Delta\mathbf{P}^{(e_{II})}) - \mathbf{q}^{(e_{II})}(\mathbf{u}, \Delta_\Sigma) \end{cases} \tag{8}$$

Where $\begin{cases} (\mathrm{e}_I): \mathbf{k}^{(e_I)}(\mathbf{u}^{(e_I)}) = \dfrac{\partial \mathbf{q}^{(e_I)}(\mathbf{u}^{(e_I)})}{\partial \mathbf{u}^{(e_I)}}; \\ (\mathrm{e}_{II}): \mathbf{k}^{(e_{II})}(\mathbf{u}^{(e_{II})}, \Delta_\Sigma) = \dfrac{\partial \mathbf{q}^{(e_{II})}(\mathbf{u}^{(e_{II})}, \Delta_\Sigma)}{\partial \mathbf{u}^{(e_{II})}} \end{cases}$ are the tangent stiffness matrices in equation (8)

The change in length due to thermal expansion length imperfection ΔL_T and length imperfection Δ_i is included in the matrix $\mathbf{k}^{(e_{II})}(\mathbf{u}^{(e_{II})}, \Delta_\Sigma)$.

3 Incremental finite element equation of global truss system based on hybrid formulation

The incremental equation of the truss system (9) is constructed by assembling all perfect truss element temperature change and length imperfection truss elements with temperature change, expressed by equation

$$\mathbf{K}(\mathbf{u})\delta\mathbf{u} = (\mathbf{P} + \Delta\mathbf{P}) - \mathbf{q}(\mathbf{u}) \tag{9}$$

Assembling the force vector the tangent stiffness matrix is developed as below

$$\begin{cases} \mathbf{u} = \{u_1, u_2, ..., u_n\}^T ; \mathbf{q}(\mathbf{u}) = \{\mathrm{q}_1(\mathbf{u}), \mathrm{q}_2(\mathbf{u}), ..., \mathrm{q}_n(\mathbf{u})\}^T ; \mathbf{P} = \{\mathrm{P}_1, \mathrm{P}_2, ..., \mathrm{P}_n\}^T \\ \mathrm{q}_i(\mathbf{u}) = \sum\limits_{e_I, e_{II}=1}^{m} \{\mathrm{q}_i^{(e_I)}(\mathbf{u}); \mathrm{q}_i^{(e_{II})}(\mathbf{u}, \Delta_\Sigma)\}; \mathrm{P}_i = \sum\limits_{e_I, e_{II}=1}^{m} \{\mathrm{P}_i^{(e_I)}; \mathrm{P}_i^{(e_{II})}\};\ (i = 1,2,..,n) \\ \mathbf{K}_{ij}(\mathbf{u}) = \sum\limits_{e_I, e_{II}=1}^{m} \{\mathrm{k}_{ij}^{(e_I)}(\mathbf{u}); \mathrm{k}_{ij}^{(e_{II})}(\mathbf{u}, \Delta_\Sigma)\} \ ; (i, j = 1,2,...,n) \end{cases}$$

Where: "m" is a number of truss elements and "n" is number of unknowns;

Using arc length technique [16-17] the incremental-iterative algorithm is established for solving nonlinear system (Shown in Fig.3). Based on proposed incremental-iterative algorithm, the calculation program for solving geometrically nonlinear buckling problem of truss system is written using Matlab software.

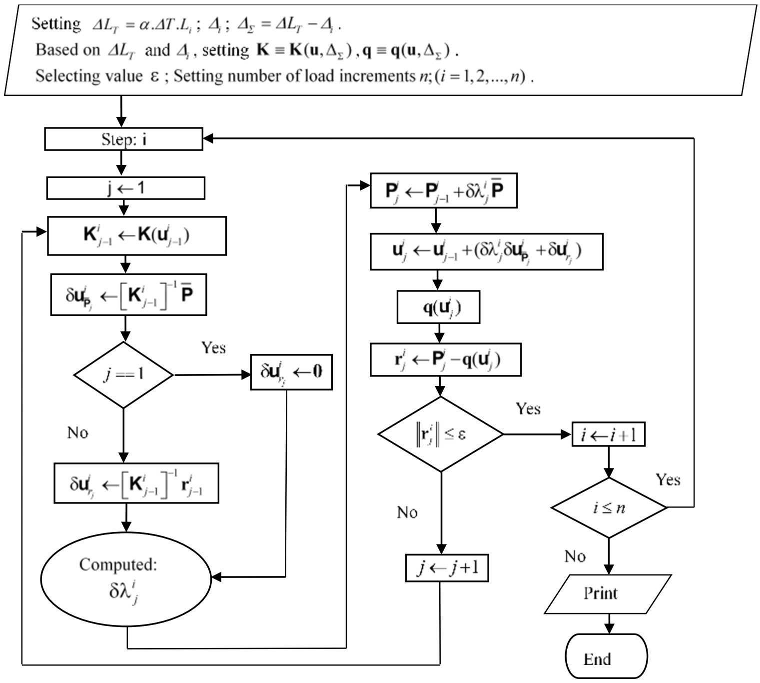

Fig. 3. Block diagram of the incremental-iterative algorithm for solving geometrically nonlinear buckling problem of truss system based on arc length method.

4 Numerical investigation

Investigate the truss system shown in Fig. 4, the 1st, 3rd, and 5th truss element having initial length imperfection and temperature change. All of the truss bars made of the same material and have the same cross-sectional area.

The parameters are given $\begin{cases} E = 2.10^4 kN/cm^2; A = 4cm^2;\ \alpha = 11.10^{-6}\,(^oC)^{-1} \\ \Delta_{i,(1)} = -2cm; \Delta_{i,(2)} = 0cm; \Delta_{i,(3)} = -2cm; \Delta_{i,(4)} = 0cm; \Delta_{i,(5)} = 2cm \\ \Delta T_{(2),(4)} = 0^oC;\ \Delta T_{(1),(3),(5)} \equiv \Delta T = 0^oC; 100^oC; 200^oC; 300^oC \end{cases}$

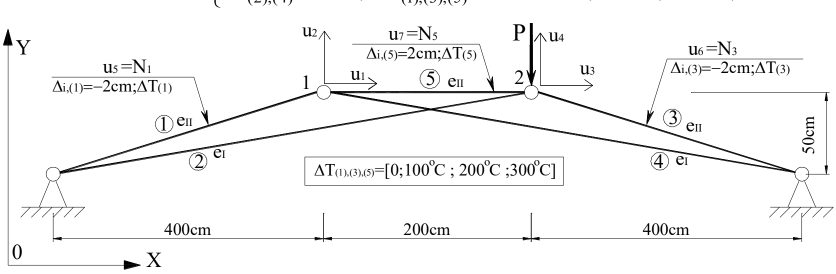

Fig. 4. Investigated truss system.

The hybrid model of the truss system is assembled from 1st type elements and 2nd type elements. The unknowns of the hybrid truss model are designated as shown in Fig. 4, including (u_1, u_2, u_3, u_4) - nodal displacement unknowns and $(u_5 = N_1;\ u_6 = N_3;\ u_7 = N_5)$ - axial force unknowns.

The calculation results are load-displacement and load-internal force equilibrium path in different cases of temperature change shown in Fig. 5 & 6.

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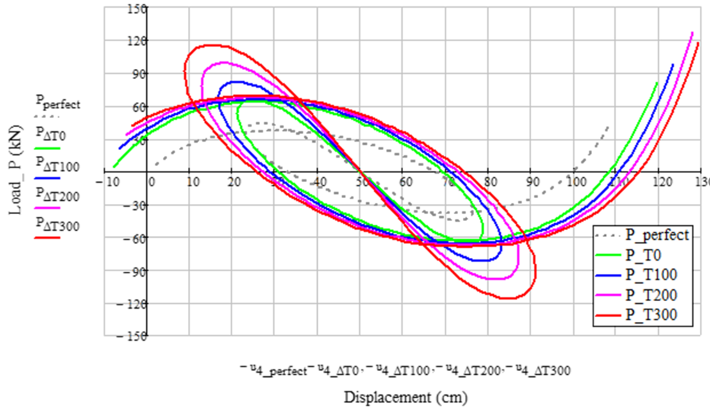

Fig. 5. Load-displacement equilibrium path $(\boldsymbol{P}-\boldsymbol{u}_4)$ in cases $\boldsymbol{\Delta T}_{(5)}=\mathbf{0}^oC;\mathbf{100}^oC;\mathbf{200}^oC;\mathbf{300}^oC$.

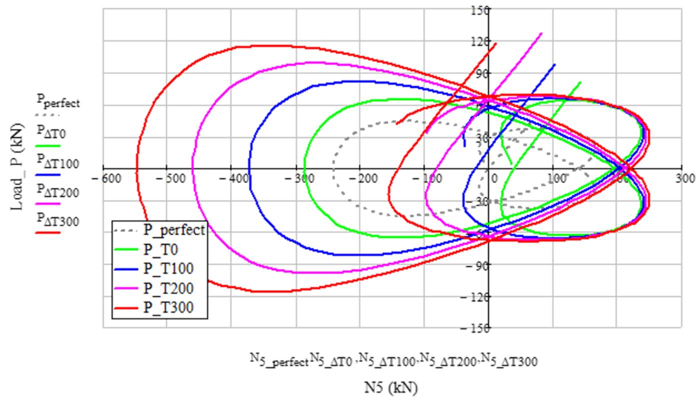

Fig. 5. Load-displacement equilibrium path $(\boldsymbol{P}-\boldsymbol{N}_5)$ in cases $\boldsymbol{\Delta T}_{(5)}=\mathbf{0}^oC;\mathbf{100}^oC;\mathbf{200}^oC;\mathbf{300}^oC$.

Comment: The calculation results show the significant influence of length imperfection and temperature change on the equilibrium path making the critical load value into both negative and positive sides.

5 Conclusion

The hybrid model for solving geometrically nonlinear buckling problem of truss system with length imperfection subjected to mechanical and thermal had been built in this research. The proposed approach has a significant advantage of indirectly inserting length imperfection and thermal deformation in the truss element matrix considering geometrical nonlinearity. The hybrid finite element model of the truss system overcomes the mathematical difficulty in constructing the solving algorithm associated with the displacement-based finite element model and decreases the unknowns of the equilibrium equations in comparison with the mixed-based formulation. The proposed hybrid model and established algorithm can be effectively used to determine the equilibrium path and investigate the buckling and post-buckling behavior of the truss system with length imperfection subjected to thermal load.

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