Creep of an inhomogeneous polymeric cylindrical shell under heating

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Abstract. The article presents the calculation of a polymer thick-walled cylindrical shell, taking into account creep under the action of a uneven temperature field. The relevance of the work lies in the widespread introduction of polymer pipes in the repair and construction of pipelines for heating, sewerage, and water supply systems. The problem is solved in an axisymmetric formulation under plane deformation conditions. The calculation is based on the non-linear Maxwell-Gurevich equation, which is widely used in the calculations of polymer structures.
In the problem under consideration, the cylindrical shell is in a flat deformed state. It is also believed that temperature is a function of radius and time.

Keywords: heterogeneity; stress state; polymer, cylindrical shell, flat state

1 Introduction

In the problem under consideration, the cylindrical shell is in a flat deformed state. It is also believed that temperature is a function of radius r and time t. Relaxation and elastic characteristics, strongly dependent on temperature, will be functions of the coordinate and time; temperatures $T(a) = T_a$ and $T(b) = T_b$ act on the inner and outer surfaces of the shell. Here a and b
respectively, the radii of the inner and outer surfaces of the shell. Taking into account the axial symmetry, the problem in the geometric formulation is one-dimensional (all functions depend only on r), and in relation to the influence of temperature, which in some cases can change over time, it is considered as quasi-stationary.

2 Physical relationships for a viscoelastic material

The problems of viscoelasticity of continuously inhomogeneous bodies considered in this article are related to the problems of polymer mechanics. Below are physical relations in differential form, which are valid for polymers and composites [1], which can be considered quite general and, in particular cases, applicable to other materials [2, 3]. Let us consider the equations describing the actual viscoelastic behavior of the material, i.e. defining the

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term $\overset{*}{\varepsilon}_{jk}$. Due to the micro-heterogeneity of the structure at the molecular and supramolecular levels, the polymer can have a spectrum of relaxation times, therefore, in the general case; highly elastic deformations are the sums of individual components, each of which corresponds to a certain member of the spectrum:

$$\overset{*}{\varepsilon}_{jk} = \sum_{s} (\overset{*}{\varepsilon}_{jk})_s \tag{1}$$

Usually, when solving problems of mechanics in relation to rigid polymers in quasi-static modes, we consider it sufficient to take into account one or two components of highly elastic deformation.
For the rate of components of highly elastic deformation, there is an unambiguous dependence on the parameters of the deformation process. As analytical expressions of these dependencies (coupling equations), the generalized nonlinear Maxwell equation is valid. This equation was derived in [4] based on molecular concepts of the behavior of materials. Subsequently, it was repeatedly tested in solving various problems of the mechanics of polymers and composites [2, etc.]:

$$\frac{\partial(\overset{*}{\varepsilon}_{jk})_s}{\partial t} = \left[\frac{3}{2}(\sigma_{jk} - \sigma_{\text{cp}}\delta_{jk}) - E_{\infty s}(\overset{*}{\varepsilon}_{jk})_s\right]\frac{1}{\overset{*}{\eta}_s}; \quad (j,k = \xi,\eta,\zeta). \tag{2}$$

The function $\overset{*}{\eta}_s$ in (2) is the coefficient of relaxation viscosity, which is invariant with respect to the coordinate system, has the form:

$$\overset{*}{\eta}_s = \overset{*}{\eta}_{0s} \exp\left\{-\frac{1}{\overset{*}{m}_s}\left[\overset{*}{\gamma}_s\sigma_{\text{cp}} + \left|\frac{3}{2}(\sigma_{ll} - \sigma_{\text{cp}}) - E_{\infty s}(\overset{*}{\varepsilon}_{ll})_s\right|_{\max}\right]\right\}. \tag{3}$$

In (2) and (3) the following designations are accepted: $E_{\infty s}$ - modulus of high elasticity $(\sigma = E_{\infty s} \cdot \varepsilon_{\infty s})$; $\overset{*}{m}_s$ - modulus of strain rate; $\overset{*}{\gamma}_s$ - volumetric coefficient; $\overset{*}{\eta}_{0s}$ - is the coefficient of the initial relaxation viscosity of the s-component of the highly elastic deformation.

The index ll in (3) denotes the principal stresses and strains for which the equalities are valid:

$$3\sigma_{\text{cp}} = \sum_{l=1}^{3} \sigma_{ll}; \;\; 3\varepsilon_{\text{cp}} = \sum_{l=1}^{3} \varepsilon_{ll} . \tag{4}$$

The coefficient of relaxation viscosity $\overset{*}{\eta}_s$, defined by equality (3), is related to the relaxation time T_s^* by the relation

$$\eta_s^* = 3G(1 + G_\infty / G)T_s^* . \tag{5}$$

3 Derivation of resolving equations

As in the problems of the theory of elasticity, the problems of the theory of creep can be solved in stresses and in displacements. The following is a way to solve in displacements. The basic formulas of the theory of creep formally coincide with the equations of the theory of elasticity.

For deformations ε_z we have $\varepsilon_z = \varepsilon_z^0 + \varepsilon_z^* + \alpha T$.

Assuming $\varepsilon_z = 0$ from the plane strain condition, we obtain: $\varepsilon_z^0 = -\varepsilon_z^* - \alpha_T T$. Then the elastic component of the average strain ε_{cp}^0 will be equal to

$$\varepsilon_{cp}^0 = \frac{1}{3}(\varepsilon_r^0 + \varepsilon_\theta^0 + \varepsilon_z^0) = \frac{1}{3}\left[\frac{\partial u}{\partial r} + \frac{u}{r} - (\varepsilon_r^* + \varepsilon_\theta^* + \varepsilon_z^*) - 3\alpha_T T\right] = \frac{1}{3}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \alpha_T T. \quad (6)$$

The last transformation was made on the basis of the hypothesis that the bulk creep strain is zero.

Using the Cauchy relations and Hooke's law for elastic deformations, we obtain:

$$\varepsilon_r^0 = \frac{1}{E}(\sigma_r - \nu\sigma_\theta); \quad \varepsilon_\theta^0 = \frac{1}{E}(\sigma_\theta - \nu\sigma_r), \quad (7)$$

whence follow the expressions

$$\varepsilon_r^0 = \frac{\partial u}{\partial r} - \varepsilon_r^* - \alpha_T T; \quad \varepsilon_\theta^0 = \frac{u}{r} - \varepsilon_\theta^* - \alpha_T T.$$

Substituting (6) and (7) into Hooke's law in the Lame form, we obtain

$$\begin{aligned} \sigma_r &= \lambda\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2\mu\frac{\partial u}{\partial r} - 2\mu\varepsilon_r^* - 3K\alpha_T T; \\ \sigma_\theta &= \lambda\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2\mu\frac{u}{r} - 2\mu\varepsilon_\theta^* - 3K\alpha_T T. \end{aligned} \quad (8)$$

We obtain the resolving equation in displacements by substituting expressions for stresses (8) into the equilibrium equation:

$$\frac{\partial^2 u}{\partial r^2} + \varphi(r,t)\frac{\partial u}{\partial r} + \psi(r,t)\frac{u}{r} = f(r,t). \quad (9)$$

In this equation:

$$\begin{aligned} &\psi(r,t) = \frac{1}{r^2} + \frac{1}{r(\lambda+2\mu)}\frac{\partial\lambda}{\partial r}; \quad \varphi(r,t) = \frac{1}{r} + \frac{1}{\lambda+2\mu}\frac{\partial(\lambda+2\mu)}{\partial r}; \\ &f(r,t) = \frac{1}{\lambda+2\mu}\left[3\alpha_T\frac{\partial(KT)}{\partial r} + 2\mu\left(\frac{\partial\varepsilon_r^*}{\partial r} + \frac{\varepsilon_r^* - \varepsilon_\theta^*}{r}\right) + 2\frac{\partial\mu}{\partial r}\varepsilon_r^*\right]. \end{aligned} \quad (10)$$

4 Solution method

The resolving equation (9) is a second-order partial differential equation with variable coefficients. Due to the nonlinearity and complexity of the coefficients, the analytical solution of these equations cannot be found even with significant simplifications.

If we assume that the temperature field and force loads change slowly with time, then creep problems can be considered as quasi-stationary. One of the first papers in which a "layered" method for solving quasi-stationary creep problems was proposed was [5]. Subsequently, this method was used in [6, 7, etc.].

Let us explain the essence of the "layered integration" method by the example of solving equation (9) under the action of a temperature field. At the zero stage (at $t=0$), the problem of determining the temperature field $T=T_0(r)$ is first solved and the dependences of the elastic and relaxation parameters of the material on temperature and coordinates are found:

$$E=E_0[T_0(r)], \quad \nu=\nu_0[T_0(r)], \quad E_{\infty s}=E_{\infty s,0}[T_0(r)],...$$

If we consider the loading to be instantaneous, then at the moment of time $t=0$ the initial conditions will be valid:

$$t=0; \;\; \varepsilon^*_{r,0}=\varepsilon^*_{\theta,0}=0\,. \tag{11}$$

Thus, at the zero stage, we arrive at an elastic problem. In this case, in equation (9), the partial derivatives with respect to the radius can be replaced by ordinary ones

$$u''_0+\varphi(r,0)u'_0+\psi(r,0)u_0=f(r,0)\,, \tag{12}$$

where

$$\varphi(r,0)=\frac{1}{r}+\frac{\lambda'_0+2\mu'_0}{\lambda_0+2\mu_0}; \qquad \psi(r,0)=\frac{1}{r^2}-\frac{\lambda'_0}{r(\lambda_0+2\mu_0)};$$
$$f(r,0)=\frac{3\alpha_T}{\lambda_0+2\mu_0}\left(K_0T_0\right)'. \tag{13}$$

Here, the dependences of the mechanical characteristics on the radius are due to the initial (in this case, temperature) inhomogeneity of the material.

Equation (12) with boundary conditions $t=0; \quad \varepsilon^*_{r,0}=\varepsilon^*_{\theta,0}=0$ is a two-point boundary value problem, which, due to the complexity of the coefficients of the equation, must be solved numerically. One of the effective methods for solving such boundary value problems is the sweep method [8, 9, etc.].

Having determined at the zero stage all the necessary quantities (displacements, strains and stresses), from the corresponding physical equations (for example (2)) it is possible to find the creep strain rates

$$\left(\frac{\partial\varepsilon^*_r}{\partial t}\right)_0, \left(\frac{\partial\varepsilon^*_\theta}{\partial t}\right)_0$$

Assuming that the time step Δt can be arbitrarily small, it is possible to carry out a linear approximation in time and calculate the creep strains on the next "time layer" $t=\Delta t$:

$$t = t_1 = \Delta t;\ \ \varepsilon^*_{r,1} = \left(\frac{\partial \varepsilon^*_r}{\partial t}\right)_0 \cdot \Delta t;\ \ \varepsilon^*_{\theta,1} = \left(\frac{\partial \varepsilon^*_\theta}{\partial t}\right)_0 \cdot \Delta t\ .$$

By means of numerical differentiation, the derivatives of creep deformations along the radius, which are included in the right side of equation (9), are also found. In a non-stationary thermal process, it is also necessary to determine analytically or by linear approximation in time a new temperature distribution and new dependences of mechanical characteristics on temperature, and, consequently, on the radius. Having thus formed the right side of equation (9), we again come to an elastic problem with a new function $f(r)$ and, in the general case, with new coefficients $\varphi(r)$ and $\psi(r)$.

Also, solving this problem numerically, we obtain the solution at the first stage. Continuing the process up to an arbitrary point in time, it is possible to determine stresses, deformations and displacements at any point of the body.

In step methods, the question of their convergence remains open. One way to analyze convergence is to compare the results obtained at different values. In long processes, when the creep rate decreases, a non-uniform (increasing) time step can be used. There are some other possibilities for analyzing and improving the convergence of the method, based, for example, on comparing the results at two successive stages of the iterative process with subsequent adjustment of the previous time step.

5 Creep of an unevenly heated cylinder

A characteristic feature of temperature problems for structural elements made of polymeric materials is the strong dependence of all mechanical characteristics on temperature. Thus, even with relatively small temperature field gradients, it is necessary to solve the problem of mechanics, taking into account the inhomogeneity of the material.

Consider the problem of calculating a thick-walled cylinder located in an axisymmetric temperature field determined by the following boundary and initial conditions ((*a,b*) - respectively, the radii of the inner and outer surfaces of the cylinder).

$$\begin{aligned} &t = 0, && T(r,0) = T_0 = const; \\ &0 < t < t_1, && T(a,t) = T_0 + \beta t; \quad T(b,t) = T_0; \\ &t > t_1, && T(a,t) = T_0 + \beta t_1; \quad T(b,t) = T_0. \end{aligned} \tag{14}$$

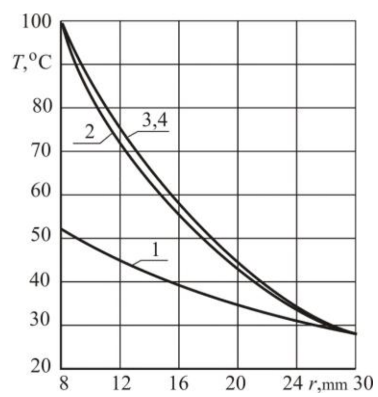

Fig. 1. Temperature distribution in cylinder at different times. :
1 - 0,4 h.; 2 - 1,2 h.; 3 - 3,6 h.; 4 - 100 h.

In accordance with (14), a constant temperature T_0 is maintained on the outer surface of the cylinder during the entire time, and the inner surface is first heated (during time t_1), and then its temperature is also maintained constant. If you set the final temperature of the inner surface of the cylinder T_1, then the heating time t_1 can be determined by the formula $t_1 = (T_1 - T_0)/\beta$, where β is the heating rate.

The process is non-stationary until some point in time $t_2 > t_1$, until the final temperature distribution along the radius is established. On fig. 1 shows the results of the numerical solution of the equation

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$$\lambda \nabla^2 T - c\rho \frac{\partial T}{\partial t} + W = 0,$$

obtained under the assumption of a constant thermal diffusivity with the following initial data: $a = 8\text{mm}$; $b = 28\text{mm}$; $T_0 = 28^\circ\text{C}$; $T_1 = 100^\circ\text{C}$; $\beta = 60$ °C/h. From the above dependences, we can conclude that the stabilization of the temperature field occurs at a time $t_2 = 3.6$ hours. Assuming a sufficiently long cylinder, we will assume that a plane deformed state occurs in it, for which the resolving equation in displacements (9) is valid.

Considering a short-term process (up to 100 hours), we limit our calculations to only the “senior” component of the highly elastic deformation.

Below are the results of solving the problem of creep of a cylinder made of EDT-10 epoxy resin. For this material, the dependences of mechanical characteristics on temperature were studied in fundamental work [10] Given the slight change in the considered temperature range of the Poisson's ratio ν and the coefficient of linear thermal expansion α, we will assume them constant and equal to $\nu = 0{,}3$; $\alpha = 8 \cdot 10^{-5}$ 1/deg. In the mentioned work, for EDT-10, the following empirical dependences on temperature of the modulus of elasticity and relaxation characteristics corresponding to the leading component of the highly elastic deformation are given:

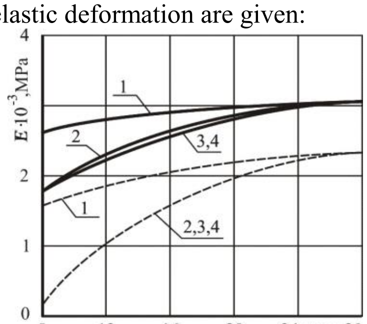

Fig. 2. Changing a module elasticity E (—–) and modulus of high elasticity E_∞ (- - -) in cylinder for various points in time

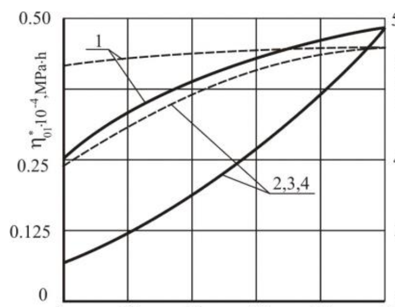

Fig. 3. Change in the coefficient of initial relaxation viscosity η^*_{01} (—–) and velocity modulus m^*_1 (- - -) in the cylinder for different moments of time

$E = (8302 - 1{,}75T_K)\text{MPa};$

$E_{\infty 1} = (11340 - 30T_K)\text{MPa};$

$m_1^* = (7{,}75 - 0{,}011T_K)\text{MPa};$

$\eta_{01}^* = 685 \cdot 10^8 \exp(-0{,}0275T_K)\text{MPa} \cdot \text{sec}.$

Hear T_K – temperature, *K*.

On fig. 2 and 3 show the change along the radius of these characteristics for some time points. Numerical designations in these and subsequent figures are the same as in Fig.1. It should be noted that, with the exception of the modulus of elasticity, all other characteristics of the material are already stabilized by the end of the heating of the inner surface of the cylinder at $t =$ 1.2 hours.

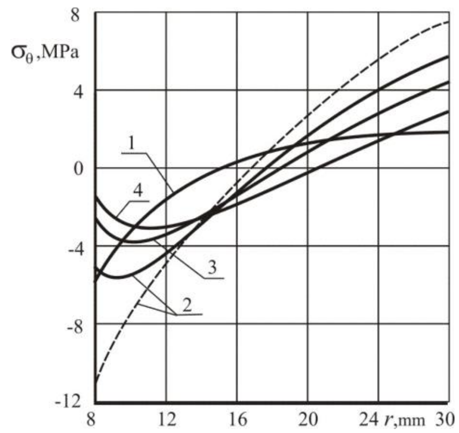

Fig. 4. Distribution of stresses σ_θ in the cylinder for various moments of time;
- - - elastic solution.

As will be presented below, taking into account in the calculations a relatively small density compared to the whole time is essential, since the process of increase in the period of temperature increase increases the creep significantly from the very beginning. The calculation was carried out by the numerical method described in Section 3. Figure 4 shows transfer diagrams σ_θ for some points in time.

In the same place, for comparison, the stress diagram obtained by the elastic solution and corresponding to the end time of heating (t=1.2 hours) is shown. The following results should be noted. In the initial period, during the heating process, the stresses increase, which is natural, since the temperature loads increase. Then, during the creep of the cylinder, a significant relaxation of stresses occurs both in the stretched and in the compressed zones. In this case, if at t >3.6 hours the temperature distribution along the radius remains unchanged (see Fig. 1), then the relaxation process continues, which leads to an even greater decrease in stresses.

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6 Conclusion

The article shows that the solution of viscoelasticity problems based on the nonlinear Maxwell-Gurevich equation using differential equations is quite simple. Several papers [11 – 13, etc.] have been published on a related topic. Note that the method of successive loadings was used in the article to solve the nonlinear problem. You can also use the method of successive approximations [14], which can be a development of the topic under consideration.

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