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Using penalty method in identification of elastic fixed stiffness of frame structure

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Abstract. This paper studies a method to identify the elastic fixed stiffness of the frame structure. The model of the problem is three dimensional structure, linear elastic deformation, pile - soil link is replaced by elastic fixed with stiffness. The problem will be solve by the penalty function method - the minimum of the objective function (which is the total squared error between the measured value and the calculated values particular) - combined with the finite element method. The numerical calculations show that the model, algorithm and calculation program are reliable. The program can be used to identify the elastic fixed stiffness of the frame structure in three dimension, serving to determine the actual working state of the structure, to propose solutions for reinforcement, repairing, improving bearing capacity, prolonging the life of the structure.

1 Introduction

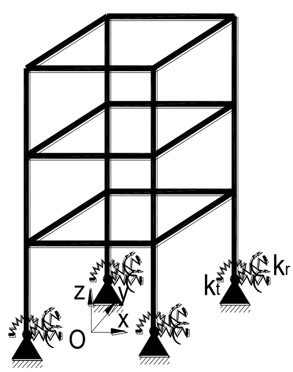

Fig. 1. Model of frame - pile

Pile foundation is a type of structure that used a lot in construction, transportation, irrigation, and offshore constructions… In the calculation, there are many different models

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can be used to describe the pile-soil link, such as: equivalent fixed depth [1-3], elastic spring on the whole pile, elastic fixed,…. In this paper, the author uses the model of pile – soil link which is an elastic fixed at the soil surface, the stiffness of elastic fixed can be determined according to Venkataramana [5]. However, during use and extraction, the piles may be reduced in connection with the ground over time, so the stiffness of elastic fixed is changed. Identification the stiffness of elastic fixed (torsion springs, rotation springs) corresponding to the actual working state of the pile (based on the specific vibration frequencies measured in the field) to determine the technical state of the structure is very important and necessary.

The problem of structural identification has been mentioned by many scientists. Chan Ghee Koh [6] evaluated the hardness index of each floor to diagnose damage of frame structure. Narkis Y. [7] locates cracks in the beam structure. Hassiotis S. [8] use the global planning method with finite element method to solve the problem of structural identification. J.K.Sinha [9, 10, 11], M.I.Friswell [12], M.I. Friswell [13] studied the problem of diagnosing structural damage, stiffness link by the penalty function method.

Petersen [14], Davide Balatti [15], Gardner [16] studied the problem of diagnosing structural damage, stiffness linked by the penalty function method. Mousa Rezaee [17] studied damage detection and structural health monitoring with the autoregressive moving average (ARMA) model and fuzzy classification. N.X.Bang [4] determined the Equivalent Fixd Depth of 3D frame by the penalty function method. Xu M [18] using Bayesian methods to identify the damage of structural.

2 Methods

2.1. The equations of motion of the frame structural system in three-dimensional frame

Investigation of frame structural system in the form of three-dimensional frame under dynamic load effect (Figure 2) in the coordinates Oxyz.

Recognize the following assumptions:

- Pile-soil link is replaced by elastic fixed with torsion springs, rotation springs on x, y direction at ground.
- The strain of the frame structural system is linear and small.

The analysis model of the structure is shown in Figure 3.

To build the equation of the motion of the frame structural system, the finite element method (FEM) will be used.

The equations of motion of the frame structural system according to FEM method [4, 19], after applying boundary conditions to the system, can be formulated as follows:

$$\mathbf{M}\ddot{\mathbf{U}}(t)+\mathbf{C}\dot{\mathbf{U}}(t)+\mathbf{K}\mathbf{U}(t)=\mathbf{P}(t)$$

Where $\mathbf{U}(t), \dot{\mathbf{U}}(t), \ddot{\mathbf{U}}(t)$ respectively are the displacement vector, velocity and node acceleration of the frame - pile structural system.

$\mathbf{M}, \mathbf{K}, \mathbf{C}$ respectively are mass, stiffness, and damping matrices of the structural system.

$\mathbf{P}(t)$ is the nodal load vector of the structural system.

The damping matrix of structural system can be calculated according to the mass matrix and stiffness matrix as:

$$\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}$$

Where α, β are the factors depend on the specific vibration frequencies of the system and the viscous damping factors of the material.

α_1, α_2 are *Rayleigh* damping coefficient, determined by the lowest individual frequencies of the structure ω_1, ω_2; and the corresponding damping ratios ζ_1, ζ_2 according to the formula:

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix} = \begin{Bmatrix} \dfrac{2\left(\zeta_1\omega_2 - \zeta_2\omega_1\right)\omega_1\omega_2}{\omega_2^2 - \omega_1^2} \\ \dfrac{2\left(\zeta_2\omega_2 - \zeta_1\omega_1\right)}{\omega_2^2 - \omega_1^2} \end{Bmatrix}$$

Where ω_1, ω_2 are 1st and 2nd individual frequencies of the structural system.

ζ_1, ζ_2 are damping ratios depend on the structural material and characteristic of work of the system.

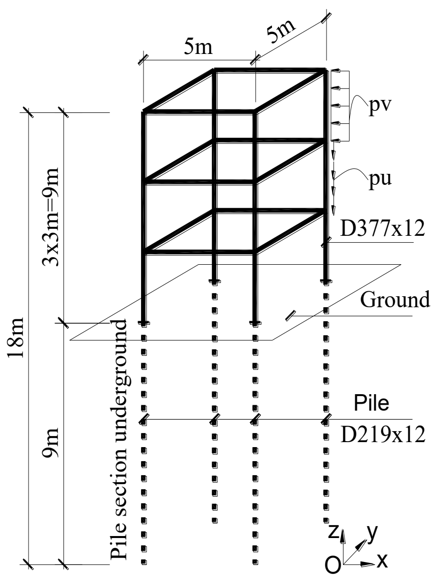

Fig. 2. Real model of frame - pile in the thre dimensional.

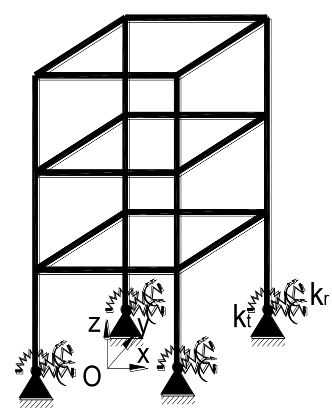

Fig. 3. Structural analysis model Frame - pile in the three- dimensional.

The matrices of the whole system in equation (1) can be built from the matrices of FEM in the system by the "direct stiffness" method [19]. The following are the matrices of FEM for the three- dimensional frame structural system.

In order to establish the overall matrix of structural system $\mathbf{M}, \mathbf{K}$ and nodal load vector $\mathbf{P}$, it is necessary to define mass matrices $\mathbf{m}$, stiffness matrices $\mathbf{k}$, and nodal load vector $\mathbf{p}$ of element in local coordinate system [19].

2.2. The problem of identification of the elastic fixed stiffness and the solution method

Investigate the Frame - pile structural system in the form of existing space frame at the site. The problem here is to identify the elastic fixed stiffness of each pile on the basis of the specific vibration frequencies measured by dynamic testing of structures at the site.

To solve the problem, we will apply the penalty function method of the FEM update model in structural dynamics [4, 19, 20], whereby the identification parameters of the problem are determined on the basis of minimizing the penalty function - is the sum of squares of errors between measured values and calculated values.

Symbols: $\boldsymbol{\theta}=[\theta_1,\theta_2,..,\theta_j,...,\theta_p]^T=[k_{t1},k_{r1},...,k_{tj},k_{rj}...,k_{tp},k_{rp}]^T$ is the vector of the identification parameters has an unknown value; $\mathbf{z}_e=[\lambda_{e1},\lambda_{e2},..,\lambda_{ei},...,\lambda_{eN}]^T$ is the vector of the first N values obtains from measurement when dynamically testing the structure at the site; $\mathbf{z_c}=[\lambda_{c1},\lambda_{c2},...,\lambda_{ci},...,\lambda_{cN}]^T$ is the vector of the first N values receives from the analysis, depending on the identification parameters, $\mathbf{z}_c=\mathbf{z}_c(\theta)$.

$\boldsymbol{\varepsilon}=(\mathbf{z}_e-\mathbf{z}_c(\theta))=\boldsymbol{\varepsilon}(\theta)$ is errors vector between measured values and calculated values.

The penalty function $J(\theta)$ has the form

$$J(\theta)=\|\boldsymbol{\varepsilon}(\theta)\|^2=(\mathbf{z}_e-\mathbf{z}_c(\theta))^T\mathbf{W}_\varepsilon(\mathbf{z}_e-\mathbf{z}_c(\theta))=\sum_{i=1}^{N}W_{\varepsilon i}(\lambda_{ei}-\lambda_{ci}(\theta))^2$$

where $\mathbf{W}_\varepsilon=diag(W_{\varepsilon1},W_{\varepsilon2},..,W_{\varepsilon i},...,W_{\varepsilon N})$ is diagonal matrix is positive and is usually the inverse matrix of the variance of the eigenvalues measurement data.

The functions $\varepsilon(\theta)$ and $J(\theta)$ usually high-level nonlinear functions of updated parameters θ. Therefore, the solution θ of the minimization problem of the aforementioned penalty function is hard to get in closed form by the precise analysis method. In this paper, instead of using the correct method, we use the iterative method, called the penalty function method. Below are set up the equations and algorithms according to this iterative method.

Develop Taylor error vector $\varepsilon(\theta)$ according to certain identification parameters in a given vector $\theta=\theta_k$, obtaining

$$\varepsilon_k(\theta)\approx(z_e-z_c(\theta_k))+[\frac{\partial(z_e-z_c(\theta))}{\partial\theta}]_{\theta=\theta_k}\delta\theta_k=\delta z_k-S_k\delta\theta_k=\varepsilon_k(\delta\theta_k)$$

Where $\delta\theta_k$ is the increment of the identification parameter; δz_k is errors vector between eigenvalues measured and eigenvalues calculated when $\theta=\theta_k$.

$$\delta z_k=z_e-z_{c,k}$$

$$z_{c,k}=z_c(\theta_k)=[\lambda_{c1,k},\lambda_{c2,k},...,\lambda_{ci,k},...,\lambda_{cN,k}]^T$$

$\mathbf{S}_k$ is sensitivity matrix - the first derivative of eigenvalues calculated according to the identification parameters at $\theta=\theta_k$.

$$\mathbf{S}_k=\left[\frac{\partial\mathbf{z}_c(\theta)}{\partial\theta}\right]_{\theta=\theta_k}=\begin{bmatrix} S_{11,k} & S_{12,k} & ... & S_{1j,k} & ... & S_{1p,k}\\ S_{21,k} & S_{22,k} & ... & S_{2j,k} & ... & S_{2p,k}\\ . & . & . & . & . & .\\ S_{i1,k} & S_{i2,k} & . & S_{ij,k} & . & S_{ip,k}\\ . & . & . & . & . & .\\ S_{N1,k} & S_{N2,k} & . & S_{Nj,k} & . & S_{Np,k}\end{bmatrix}_k$$

with:

$$S_{ij,k}=\frac{\partial \lambda_{ci,k}}{\partial \theta_j}$$

The penalty function $J(\theta)$ is in this form

$$J_k(\theta)=\varepsilon_k^T(\delta\theta_k)W_\varepsilon\varepsilon_k(\delta\theta_k)=(\delta z_k-S_k\delta\theta_k)^T W_\varepsilon(\delta z_k-S_k\delta\theta_k)=J_k(\delta\theta_k)$$

The solution of equation (10) is obtained by minimizing the function $J_k(\delta\theta_k)$ follow $\delta\theta_k$, whereby

$$\frac{\partial J_k(\delta\theta_k)}{\partial \delta\theta_k}=0$$

Replace $J_k(\delta\theta_k)$ from (10) to (11), we get

As of (5), minimize (4) according to $\delta\theta_k$ have the result:

$$\delta\theta_k=\left[S_k^T W_\varepsilon S_k\right]^{-1} S_k^T W_\varepsilon \delta z_k$$

Because the function (5) is a linear approximation function θ, to get as close to the exact value of the problem as iterative. If performed:

$$\delta\theta_k=\theta_{k+1}-\theta_k$$

from (12) may write:

$$\theta_{k+1}=\theta_k+\left[S_k^T W_\varepsilon S_k\right]^{-1} S_k^T W_\varepsilon \delta z_k$$

or:

$$\theta_k=\theta_{k-1}+\left[S_{k-1}^T W_\varepsilon S_{k-1}\right]^{-1} S_{k-1}^T W_\varepsilon \delta z_{k-1}$$

Here (k-1), k, (k+1) index indicate the interation steps.

The looping process ends when the solution of the problem converges with the required accuracy.

The elements of a sensitive matrix S can be obtained from partial vibration differential equation of the structure:

$$\left[\mathbf{K}-\lambda\mathbf{M}\right]\boldsymbol{\varphi}=0$$

where: λ and $\boldsymbol{\varphi}$ are the normalized eigenvalues and eigenvectors of the structure.

The result we have:

$$S_{ij}=\frac{\partial \lambda_{ci}}{\partial \theta_j}=\boldsymbol{\varphi}_i^T\left[\frac{\partial \mathbf{K}}{\partial \theta_j}-\lambda_i\frac{\partial \mathbf{M}}{\partial \theta_j}\right]\boldsymbol{\varphi}_i$$

With: λ_i, φ_i are Normalized eigenvalues and eigenvectors i of the structure.

At the iteration k the above quantity has the form:

$$S_{ij,k}=\frac{\partial \lambda_{ci,k}}{\partial \theta_j}=\varphi_{i,k}^T\left[\frac{\partial \mathbf{K}}{\partial \theta_j}-\lambda_i\frac{\partial \mathbf{M}}{\partial \theta_j}\right]_{\theta=\theta_k}\varphi_{i,k}$$

Where: $\varphi_{i,k}$ is Normalized vector of the structure corresponding to the i value at the k iteration (or at $\theta=\theta_k$).

According to the iterative algorithms established above, the content of calculating the updated parameters is conducted in the following order:

Based on the received algorithms, the author has built the **UFEM** program to solve the problem of identifying the elastic restraint stiffness of the frame - pile structure working according to three-dimensional model in MATLAB language [21], [22]. **UFEM** has been tested for reliability [19].

3 Results

This numerical calculations are performed to check the reliability of established algorithms and programs. Identify elastic fixed stiffness of the structure (Figure 2, Figure 3), with elastic rotation stiffness in x, y direction are equal ($k_{r,x} = k_{r,y} = k_r$), therefore, there are 02 identification parameters (k_r, k_t).

* *Starting data:*

- Structure of frame - pile made of steel pipes, the size shown in figure 2, horizontal-section of frames on the ground type ϕ377x12mm; pile section in soil 9m long, horizontal-section type ϕ219x12mm; elastic modulus of steel: E=2.1.10^{-7} T/m^2; Specific gravity of steel: $\gamma = \rho / g$ =7.8 T/m^3.

- Choose the allowable error: $\varepsilon = 0.5\%$.

- Measurement vector (assumed):

$$\boldsymbol{\lambda}_e = \begin{bmatrix} \lambda_{e1} & \lambda_{e2} & \lambda_{e3} & \lambda_{e4} \end{bmatrix} = [37,25 \quad 4031,9 \quad 4813,1 \quad 5276]^T \tag{19}$$

- The assumed eigenvalues here are not actually true measurements, it is eigenvalues calculate for the frame-pile structure with a given elastic restraint stiffness:

$\mathbf{k}_{t,r} = [4.10^{11} \quad 10^7\,]^T$ (kN.m)

* *Calculation results:*

- Select the identification parameters as the elastic fixed stiffness (02 parameters): $\boldsymbol{\theta}_0 = \mathbf{k}_{t,r} = [k_t \quad k_r\,]^T$.

- Discrete FEM for structure.

- Select initial parameters: $\boldsymbol{\theta}_0 = \mathbf{k}_0 = [k_{t,0},\ k_{r,0}\,]^T = [3,9.10^{11} \quad 9,9.10^6\,]^T$ (kN.m).

- The solution of the problem when calculating iteration needs to focus on the values of vector (24) with allowed errors $\varepsilon = 0.5\%$. The results of calculating the value of the identification parameters according to the iterative calculation steps are shown in figure 4.

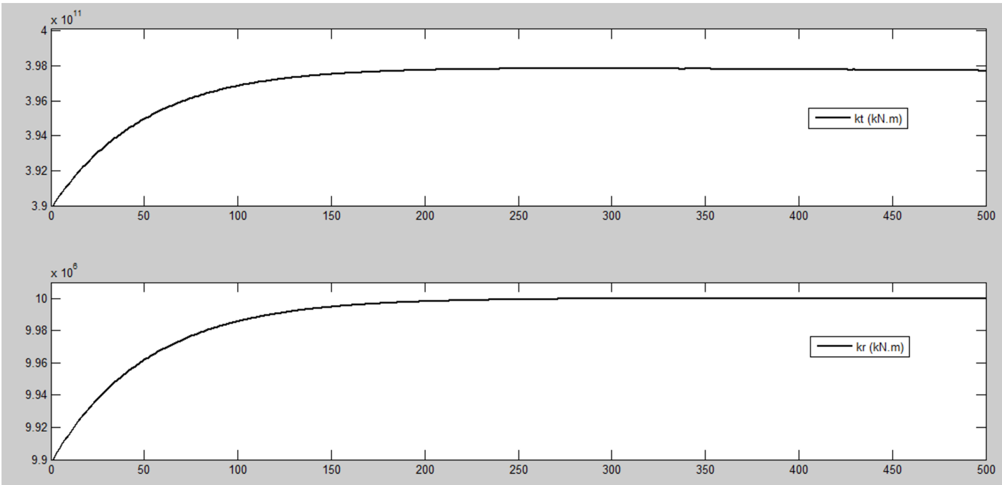

Fig. 4. Identification parameter graph by calculation step.

The results of the iterative calculation step by step gradually converge to the initial parameters, solution of convergence problems after 300 steps calculated with the results:

$$\mathbf{\theta} = \mathbf{k}_{t,r} = [k_t \ \ k_r]^T = [3{,}98.10^{11} \quad 9{,}99.10^{6}]^T \text{ (kN.m)}$$

4 Conclusions

Replacing the pile - soil link with the elastic fixed, it will simplify the calculation model, and thus will reduce the time for calculation, but still ensure the accuracy and close to the actual working structure. When the connection parameters of the pile - soil change (due to incorrect design assumptions or during of using the structure), here is the stiffness of the elastic fixed, which will change the state of the structure. Therefore identification of elastic fixed stiffness is very important and necessary.

The solutions of the above problem converge on the values to be sought, the numerical analytical data demonstrate reability of studied results, proving that the established algorithms and programs (UFEM) can be used to identify the elastic fixed stiffness of the structure in three-dimensional model.

After identifying the stiffness of the elastic fixed, the actual working state of the structure will be determined, in order to propose solutions to strengthen, repair, improve the bearing capacity, and prolong the life of the structure.

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