Nomograms: history, properties and applications

G. Mottola[1], M. Strozzi[1], R. Rubini[1], M. Cocconcelli[1]

[1]*Dept. of Sciences and Methods for Engineering, Univ. of Modena and Reggio Emilia, Italy*
E-mail: gmottola@unimore.it, mstrozzi@unimore.it, rrubini@unimore.it, mcocco@unimore.it

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ABSTRACT

Introduction

Nomograms are a graphical computing tool for solving a mathematical equation through a graphical procedure that requires no other tools than straightedge, pencil and paper. For almost a century after their introduction by French mathematician M. d'Ocagne at the turn of the 19th century, nomograms have been applied in a number of fields, from medicine to engineering: indeed, before the advent of personal computers, nomograms were often the most immediate method of solving a mathematical problem without tedious calculations by hand. While largely surpassed nowadays in practical use, they still find niche applications in several fields, due to their ease of use and flexibility; moreover, the study of nomography is an interesting window in the history of the engineering profession. In this work, we discuss the role of nomography in the history of graphical computing, together with the basic rules for drawing a nomogram for a given problem. Moreover, we present example nomograms (realized through pyNomo, a modern and flexible nomographic software) for introducing basic concepts in Mechanics of Machines and Vibrations to students in a didactically effective manner.

History

One of the foundations of modern engineering is the application of mathematical tools to physical models. From said models, one can derive equations of different complexity, which must be solved to obtain the parameters defining the final design. Thus, methods for solving equations quickly and reliably have always had clear interest for our profession. Exact *analytical* methods are often insufficient (or impractical) for finding a solution, which in engineering practice can often be approximate: thus, *numerical* methods are commonly applied. Before the invention and widespread use of electronic computing devices, these methods frequently relied on tedious, error-prone hand calculations; *graphical methods*, on the other hand, are based on a geometrical construction, in which both the parameters of the problem to solve and its final solution correspond to graphical elements, such as segments or angles. Graphical methods offer a procedure which is quick, easy to check, and which intrinsically offers an immediate visualization of the results; thus, these methods became a staple of engineering studies.

Among graphical methods, *nomograms* embed a mathematical equation as a graphical relationship, such as the collinearity of three or more points. This way, the solution of the equation can be found with no other tools than straightedge, pencil and paper.

While general works are available on nomography and its history [1–5], little have been written on its role in mechanical engineering; here, we will summarize these topics. We will restrict our discussion to methods that only require a straightedge, disregarding more complex mechanical computers such as abacuses, astrolabes, slide rules and slide charts.

The term nomogram was first proposed by French mathematician and engineer M. d'Ocagne in the late 19th century, to highlight his original contribution, in his seminal work [6]; the name derives from Greek words νόμος ("law") and γραμμή ("line"). Early concepts in graphical calculus, from which nomograms were ultimately derived, were the graphs presented by French manufacturer L.-E. Pouchet. Essentially, Pouchet reported the curves (called *isopleths*) corresponding to constant values of a given function $F = F(x, y)$ on a Cartesian coordinate system: by searching for the intersection of these curves with lines parallel to the coordinate axes, it is immediate to find the value of one between x, y, and $F(x, y)$, when the other two are known. This method, however, requires drawing complex curves, which was time-consuming without computers. An improvement was then offered by French engineer L.-L. Lalanne, who suggested using nonlinear scales for the axes; this way, the curves can be transformed into straight lines. These methods already display some interesting advantages of nomograms:

- any variables can be the input, and direct and inverse problems are equivalently easy;
- the approximation of the solution is directly visible, giving a clear feedback on the error;
- errors due to misreading are unlikely, if meaningful ranges are chosen for the variables;
- their usage is immediate, even for less-skilled operators.

Finally, M. d'Ocagne proposed two essential improvements over these early concepts. The first was using general scaled lines in the plane instead of orthogonal axes, which gives greater flexibility in the design of the nomogram and can also make it easier to read. Also, d'Ocagne applied *projective geometry*, a branch of mathematics by then recently developed by French mathematician J.-V. Poncelet: using the principle of duality of points and lines, d'Ocagne replaced curves (all passing through the same point) with points (all being on the same line), thus obtaining graphs that were less cluttered and easier to draw. Thus, the solution is given by aligning three points on their corresponding scales; the third such point, knowing the remaining two, can be found with a straightedge. This construction corresponds to the equation

$$\det \begin{bmatrix} f_{1x}(v_1) & f_{1y}(v_1) & 1 \\ f_{2x}(v_2) & f_{2y}(v_2) & 1 \\ f_{3x}(v_3) & f_{3y}(v_3) & 1 \end{bmatrix} = 0 \tag{1}$$

where the i-th line is defined as $\mathcal{L}_i = \left[f_{ix}(v_i), f_{iy}(v_i) \right]^T$ in the Cartesian plane. In conclusion, any function that can be written in the form (1) can be computed with a nomogram [7]. Two nomograms can also be combined, by having a scale that is common to both, in a single, more complex graph; this allows us to solve equations in more than three variables.

Another contribution of d'Ocagne was devising applications for his new tool: in his works, translated in 14 languages, he presented more than 200 nomograms [8], on such diverse topics as physics, hydraulics, topography, navigation, aviation and accounting. By then, nomograms had caught the attention of pure mathematicians; for instance, in his famous list of 23 open problems, German mathematician D. Hilbert asked whether the roots of a 7th-degree polynomial can be found with a nomogram. The problem of determining and classifying the equations that can be solved in this way proved to be a complex and stimulating problem; a complete solution was finally found by Polish mathematician M. Warmus [7] in 1959.

Nomograms began to decline in the 1960s as computers spread; nevertheless, they kept being used (and, to a smaller extent, still are) in engineering and medicine, due to the advantages above, especially with approximate relationships that are condensed from numerical data.

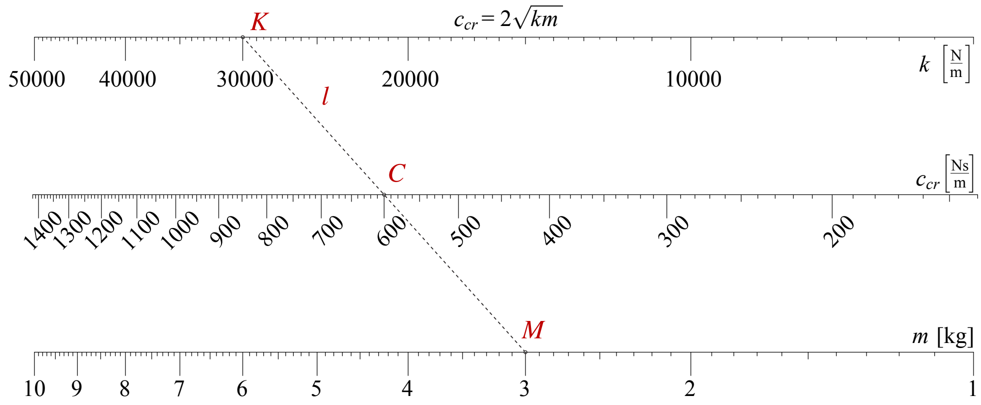

Figure 1. Nomogram for the critical damping coefficient c_{cr} of an harmonic oscillator; M, C and K, aligned on l, are defined by mass m and damping and stiffness coefficients c and k.

Applications

Nomograms have found several applications in mechanical engineering, including internal combustion engine design, manufacturing processes, hydraulics and soil mechanics; in these areas, a quick method to obtain reasonably accurate results even under field conditions is desirable. Another topic of interest is the design of planar [9–12] or spatial [13] mechanisms, where the resulting equations are usually too complex to be solved by hand; also, nomograms make it convenient to explore design solutions with different input parameters. Nomographic approaches were also proposed for gear train design [14, 15] and vibration analysis [16].

Creating nomograms

While nomograms are immediate to use, their design requires both skill and creativity [5]. Recently, new programs for digital nomography have been presented: among these, we have chosen to use the Python package pyNomo [17], which is free, flexible and easy to use.

We have thus designed a few nomograms to illustrate basic concepts in Machine Mechanics, as detailed in our previous work [18]. An example is in Fig. 1, for computing the critical damping coefficient in a 1-DoF mass-spring-damper system. This is a Type 1 nomogram, using the classification in [17]: among the most common, Type 1 nomograms are composed of three parallel lines and allow us to compute a variety of functions in two variables. The isopleth in the figure shows an example computation, with $c_{cr} = 2\sqrt{km} = 2\sqrt{30000\ \frac{\mathrm{N}}{\mathrm{m}} \cdot 3\ \mathrm{kg}} = 600\ \frac{\mathrm{N\,s}}{\mathrm{m}}$.

We have also developed other nomograms on topics that are of interest for students in a course of Mechanical Engineering, for instance to compute the stiffness of two springs in series or the displacement of a swash-plate piston pump [18]. It is expected that these nomograms can be effective in presenting complex equations to the students, just like other graphical methods for kinematics and statics are still in use today due to their pedagogical value.

Conclusions

While largely surpassed by modern technology, nomograms have had for decades an important place in engineering practice and education. In our discussion, we presented their history and development within the larger field of graphical calculus, together with their role and applications in different areas of mechanical engineering. We also present a few examples of nomograms developed with a modern software tool, that greatly simplifies the otherwise laborious drawing process. In the past, computers caused the decline of nomography; it is then possible that, in the future, computers will also bring new life into this topic.

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