MASTER

Inventory Control under Supply Disruption and Yield Uncertainty
a Jumbo Supermarkten case study

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Abstract

We study a multi-echelon, multi-product, multi-period retailing inventory system with stochastic demand, where supply is subject to yield uncertainty and random disruptions. The distribution centres (DCs) and retailers follow a periodic order-up-to policy where orders are placed in multiples of a fixed base quantity. Three mitigation strategies in the form of lateral transshipments between the DCs, dual sourcing under demand substitution and inventory buffers are proposed to protect against supply risk. We model the system as a two-stage stochastic program. In the first stage, the order-up-to levels are decided for both echelons. In the second stage, lateral transshipments can be used as a recourse action. Furthermore, we present a framework for the modeling of disruptions and yield uncertainty.

Since the original mathematical formulation proves to be intractable even for very small instances, we propose a continuous approximation of the fixed base quantities order logic. We approximate the objective value of the resulting two-stage stochastic program using Sample Average Approximation. The Mixed-Integer Linear Programs that follow from this method are solved using the Progressive Hedging Algorithm. For the continuous order logic, our numerical experiments show that under supply uncertainty the proposed mitigation strategies lead to substantial cost and inventory reductions under fixed service levels.

To establish the practical value of our work, we conduct a case study at Jumbo Supermarkten. Slight savings are found under the continuous order logic. We present a heuristic to translate the continuous approximation to the fixed quantity order logic. For the fixed quantity order logic, our proposed policy achieves significant savings in costs and inventory while keeping service level constant. We conclude that lateral transshipments and dual sourcing under demand substitution are effective mitigation strategies to protect against disruptions and yield uncertainty.

Summary

In recent times, practitioners and researchers alike have increasingly more focused on supply chain risk and disruption management. Disruptions such as the Tohoku Earthquake and the COVID-19 pandemic demonstrate that supply risk should explicitly be considered by decision makers. In this study, supply chain risk consists of both disruptions and yield uncertainty. Disruptions occur when supply is subject to partial or complete failure, which means that the capacity of a supplier becomes less than normal capacity. Yield uncertainty refers to the uncertainty of deliveries of suppliers, where the quantity received might be different from the quantity ordered (Tinani & Kandpal, 2017).

We propose three mitigation strategies to protect against supply risk: lateral transshipments, dual sourcing under demand substitution and inventory buffers. To test the effectiveness of these strategies, we consider a multi-echelon, multi-product, multi-period retailing inventory system with stochastic demand and where supply can be disrupted and is subject to yield uncertainty. We study a retailing system since product availability is key and since the company that instigated this case study is a retailer.

The network is a standard retail system consisting of external suppliers, distribution centres (DCs) and retailers. Both the DCs and retailers follow a periodic order-up-to policy where orders are placed in multiples of a fixed base quantity. We propose to model the network as a two-stage stochastic program where in the first stage, the order-up-to levels are decided for a longer period. In the second stage, lateral transshipments can be utilized as a recourse action when the uncertainties realize.

We present a methodology to model the uncertainty in supply. For this purpose, we first introduce the re-parameterized Beta distribution that takes a mean and precision as arguments.

Disruptions at an external supplier arrive according to a Poisson process. A disruption is represented by a hit that has a certain intensity modeled by the re-parameterized Beta distribution. A higher intensity means a higher impact on the capacity. The disruption also has a duration, where the length of the disruption depends on the severity of the hit which is modeled through the impact-duration function. Throughout the duration, the capacity gradually recovers back to the undisrupted capacity by means of a recovery function. We present some useful functions to model this behaviour.

We propose to model the yield uncertainty as a combination of the Bernoulli and re-parameterized Beta distribution.

The uncertainty in demand and supply is modeled through the use of scenarios. A very large number of scenarios are needed to completely model the stochastic nature of the real

world. Since the model then becomes intractable, we use the Sample Average Approximation by Shapiro (2008) to approximate the objective value by solving the two-stage stochastic program with a representative set of scenarios. We present a method to determine the statistical gap corresponding to a sample of scenarios. We deem a sample representative if the gap is sufficiently small.

However, solving the model for a small instance and few scenarios already proved to be impossible. Hence, we present a continuous approximation of the order logic. Specifically, orders need not be in fixed base quantities anymore, but can be any size to bring the inventory position precisely to the order-up-to level.

Since the deterministic MILPs that follow from the SAA are still intractable under the continuous approximation, we adopt the Progressive Hedging Algorithm by Rockafellar & Wets (1991) to solve the two-stage stochastic programs. This algorithm decomposes the two-stage stochastic program horizontally by relaxing the problem using the method of Lagrange multipliers, meaning that each scenario can be solved individually which greatly decreases the computational burden. By iteratively solving these individual scenarios, the PHA converges to equal order-up-to levels that aim to minimize costs over all scenarios. For the final objective value, the full model with the representative sample of scenarios is solved with fixed order-up-to levels that result from the PHA.

In our numerical experiments, we first show with a base case that the quality of our solution is acceptable. Specifically, the statistical gap of the SAA is 2.4% when solving the model 5 times with 50 scenarios. Furthermore, the PHA optimality gap is on average 3% for these five solves. We therefore conclude that we can draw meaningful conclusions from the numerical experiments with our proposed solution methodology.

Furthermore, we perform a sensitivity analysis on the input parameters of the base case. We draw conclusions about the impact of the input parameters on the solution.

Lastly, we conduct experiments where we solve our model with the continuous approximation for 25 different test beds. We find that our proposed policy that combines the mitigation strategies performs well under supply uncertainty, achieving cost savings in the range of 4.5 to 7% and inventory reductions in the range of 15 to 20% while keeping the service level almost constant. Lateral shipments mainly show cost savings and inventory reductions at the DC, whereas demand substitution helps in saving inventory at the retailer. Moreover, disruptions moderate the performance of the mitigation strategies. Yield uncertainty seems to have limited influence on the savings. The proposed policy is however more computationally intensive.

We conduct a case study at Jumbo Supermarkten. Since demand for the products in scope is non-stationary concerning the weekdays, we adapt the model and define three safety stock levels (low, medium, high) that correspond to certain weekdays. We use the demand fitting procedure by Adan et al. (1995) to fit discrete distributions on the historical demand. For the continuous order logic, we draw similar conclusions as for the numerical experiments, although the savings are less substantial. Furthermore, we present a heuristic that translates the continuous order logic back to the fixed base quantity order logic. The heuristic ensures that each period the order is large enough to cover at least the continuous order amount while taking into account whether there was an overshoot because of the fixed size in the previous period. For the original order logic, significant cost and inventory savings are found under almost equal service levels. The proposed policy is however computationally intensive at a total runtime of 150 hours.

Based on the findings of the case study, we advice Jumbo to investigate two possible improvements. The first improvement is to incorporate forecasting, for which they already have existing processes, into the model. The second improvement concerns the designing of a general lateral transshipment policy that also incorporates demand substitution.

Moreover, we recommend Jumbo to use the disruption and yield uncertainty modeling framework to support decision making at various strategic levels, such as supplier selection or assortment decisions. Lastly, we advice to collect and store certain delivery data to be able to more adequately model disruptions and yield uncertainty.

Preface

This Master's Thesis, conducted at Jumbo Supermarkten, marks the final step in obtaining a Master of Science degree in Operations Management and Logistics at the Eindhoven University of Technology (TU/e). I have thoroughly enjoyed the whole journey of my six years as a student, developing myself to become the person I am today. Many people have supported me throughout this period, for which I would like to give my thanks.

First and foremost, I want to express my immense gratitude to my mentor and first supervisor, Melvin Drent, for his guidance and support. Your always valuable feedback, insights and time investment have helped to achieve what we have. I have only positive feelings regarding our collaboration. Furthermore, I would like to thank Albert Schrotenboer, my second supervisor, for his help during the project. Your input and knowledge has taken this research to the next level. My thanks go to Zümbül Atan for completing the assessment committee.

Secondly, I would like to extend my gratitude to Erik Kievit, my supervisor at Jumbo. Already from the fist moment we met online, I immediately felt at home. Your enthusiasm, practical knowledge and the way you gave me freedom to conduct my research have positively impacted the thesis. Moreover, I would like to thank my colleagues at Jumbo for the pleasant working environment and the many games of table football.

Thirdly, I would like to thank my friends for the fun we had and the memories we created together. You have made my student life as enjoyable as it was. The Thesiscircle deserves my gratitude for all their help, both professionally and socially, that enabled me to successfully complete this project.

Lastly, my thanks go to my family for their continuous love and support throughout my time as a student. I would not have been where I am without you. Thank you.

Patrik Roest

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DC	Distribution Centre
CDC	Central Distribution Centre
IP	Inventory Position
ISA	In-store Availability
KPI	Key Performance Indicator
LT	Lateral Transshipment
MER	Multi-Echelon Replenishment
MILP	Mixed-Integer Linear Program
PHA	Progressive Hedging Algorithm
RDC	Regional Distribution Centre
SAA	Sample Average Approximation

Chapter 1

Introduction

In recent times, practitioners and researchers alike have increasingly more focused on supply chain risk and disruption management. Real-world examples that catalyzed the interest into this domain are plenty. Well-known is the disruption caused by the Tohoku Earthquake, which impacted the supply chain of Toyota greatly. After the earthquake, it took them three months to get back to pre-disaster levels. The main issues were identifying missing parts within the supply chain and having almost no backup suppliers or different sourcing options (Matsuo, 2015). More recently, the COVID-19 pandemic affected approximately 94% of Fortune 1000 companies (Sherman, 2020). Other examples include the case of a fire at a semi-conductor plant of Phillips (Norrman & Jansson, 2004), the 9/11 terrorist attacks which suspended air traffic for several days within the US and floods in Thailand that disrupted global supply chains (Abe, 2014). These cases illustrate that decision makers have to take into account supply risks explicitly when making decisions.

In this study, supply risk consists of both disruptions and yield uncertainty. We include both as separate concepts since otherwise, significant cost increases can arise when evaluating the system (Chopra, Reinhardt & Mohan, 2007). Disruptions occur when supply is subject to partial or complete failure, which means that the capacity of a supplier becomes less than normal capacity. Yield uncertainty refers to the uncertainty of deliveries of suppliers, where the quantity received might be different from the quantity ordered (Tinani & Kandpal, 2017).

This study analyzes three possible mitigation strategies to protect against supply risk: lateral transshipments, dual sourcing under demand substitution and higher safety stock levels. Lateral transshipment refers to sharing stocks between entities in the same echelon. The second strategy uses multiple suppliers to ensure ample product availability where one product is sourced at one supplier and demand substitution can take place between those products to still fulfill demand in case of stock-out. Note that this differs slightly from the usual definition of dual sourcing in the literature since we do not explicitly place for example emergency orders at a different supplier, but we only use demand substitution between products sourced at different suppliers to hedge the risk of supply uncertainty. Lastly, holding more inventory is a possible mitigation strategy. Throughout the study, this is done implicitly by setting the order-up-to levels.

Motivated by a retailer that instigated this research, we study the effects of those mitigation strategies on the inventory control in a retail setting. Availability of their products is of utmost importance. We note however that our model formulation is sufficiently general that it can be applied to different settings after minor changes. An example is the blood supply chain, where availability of products is also key and demand substitution is a major factor (Salehi, Mahootchi & Husseini, 2019).

We model a multi-echelon, multi-product, multi-period retailing network where the supply can be influenced by disruptions and is subject to yield uncertainty. The network is a standard retail system consisting of external suppliers, distribution centers (DCs) and retailers. Both the DCs and retailers follow a periodic order-up-to policy where orders are placed in multiples of a fixed base quantity which is a common policy for retailers (van Donselaar & Broekmeulen, 2014). When modeling this supply chain, there are two decisions to be made: setting the order-up-to level and deciding on lateral transshipments. Since these decisions are made at different time granularities, we propose to model the system with a two-stage stochastic program. The order-up-to levels are set for a longer period, whereas the lateral transshipment decisions are made after the realization of uncertainties as a recourse action.

To account for the uncertainty in supply, we introduce a methodology that models both disruptions and yield uncertainty. For supply disruptions, we adapt the framework of Klibi & Martel (2012) to fit our general modeling approach. Disruptions occur following a Poisson process, where a single disruption is represented by a hit intensity and a duration. After a hit, the capacity of the external supplier gradually recovers back towards the base capacity. We present useful distributions and functions that adequately model this behaviour.

We propose to model the yield uncertainty by a combination of the Bernoulli distribution and the re-parameterized Beta distribution. This combination allows for the modeling of a broad range of different cases of yield uncertainty. Note that it is also possible to use other distributions if those better fit the situation.

The uncertainty is modeled by scenarios that represent the stochastic nature of the real world. Solving the original model for a small instance and few scenarios already proves to be impossible. We therefore propose a continuous approximation of the problem. However, even then the two-stage stochastic with a very large number of scenarios as input is intractable. Hence, we follow Shapiro (2008) and adapt the Sample Average Approximation (SAA) to approximate the objective value by solving the model for a representative sample of scenarios. We determine whether a sample is representative by calculating a statistical gap corresponding to that sample. We deem the sample representative if the gap is sufficiently small.

The Mixed-Integer Linear Programs (MILPs) that follow from the SAA are then solved using the Progressive Hedging Algorithm (PHA) by (Rockafellar & Wets, 1991). This algorithm decomposes the two-stage stochastic program horizontally by relaxing the problem using the method of Lagrange multipliers, meaning that each scenario can be solved individually which greatly decreases the computational burden. By iteratively solving these individual scenarios, the PHA converges to equal order-up-to levels that aim to minimize costs over all scenarios. For the final objective value, the full model with the representative sample of scenarios is solved with fixed order-up-to levels that result from the PHA. Chapter 4 describes the method of analysis in more detail.

The main contributions of this study are summarized as follows:

1. We are the first to present a mathematical formulation of a multi-echelon, multi-product, multi-period inventory system where supply can be both disrupted and subject to yield uncertainty, ordering is done periodically in fixed base quantities and mitigation strategies in the form of lateral transshipments, dual sourcing under demand substitution and inventory buffers through the setting of order-up-to levels are incorporated.

2. We present a generic and robust methodology to model both supply disruptions and yield uncertainty.

3. Our solution method which combines the SAA and the PHA allows for the solving of difficult two-stage stochastic programs within acceptable runtimes. Furthermore, since the original mathematical formulation proves to be intractable, we present a continuous approximation of the ordering logic that makes the model solvable.

4. The numerical experiments show that for the continuous order logic, our proposed policy that combines the mitigation strategies performs well under supply uncertainty, achieving cost savings in the range of 4.5 to 7% and inventory reductions in the range of 15 to 20% while keeping the service level almost constant.

5. We draw similar conclusions for the continuous order logic in the case study, although the savings are less substantial. Furthermore, we present a straightforward heuristic that translates the continuous approximation back to the original order logic with fixed quantities. For the original order logic, our proposed policy shows considerable savings, reducing costs and inventory significantly while keeping service level constant.

The remainder of this study is structured as follows. In Chapter 2, we review the literature related to this study. We present the modeling approach in Chapter 3. Chapter 4 describes the solution methodology. The results of the numerical experiments are presented in Chapter 5. In Chapter 6, we show the results of a case study at Jumbo Supermarkten. Lastly, we present a discussion and conclusion in Chapter 7.

Chapter 2

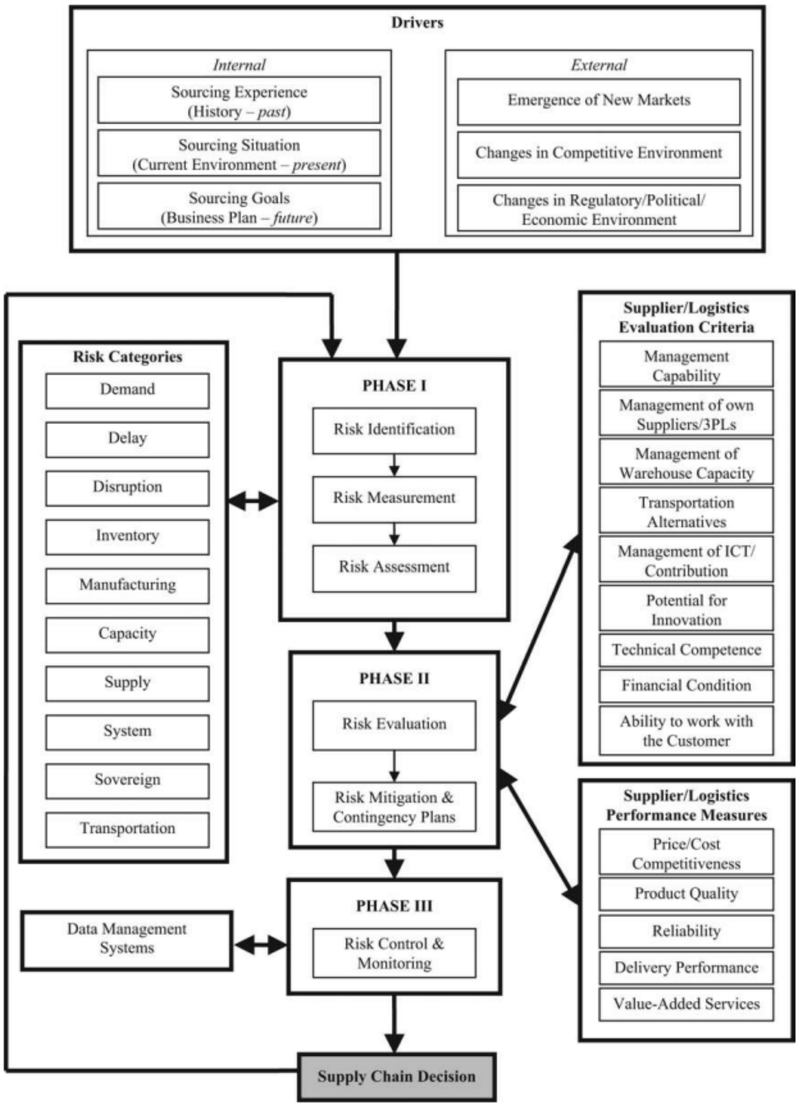
Literature review

This chapter presents the relevant literature for this study. Firstly, in Section 2.1, we give relevant definitions and a framework of supply chain risk and its management. Secondly, we discuss identifying, measuring and assessing those risks in Section 2.2. Next, we present relevant literature on single-echelon inventory control under supply risk in Section 2.3. Lastly, we discuss multi-echelon inventory management in Section 2.4. Specifically, we discuss dual sourcing in Subsection 2.4.1, lateral transshipment in Subsection 2.4.2, two-stage stochastic programming in Subsection 2.4.3 and the progressive hedging algorithm in Subsection 2.4.4.

2.1 Supply chain risk management

There is no clear consensus on a definition of both supply chain risk and its management within the literature (Ho, Zheng, Yildiz & Talluri, 2015). Aqlan & Lam (2016) define supply chain risk management as a 'systematic approach for identifying, assessing, mitigating, and monitoring potential disruptions in the supply chain in order to reduce the negative impact of these disruptions on supply chain operation'. Samvedi, Jain & Chan (2013) give four dimensions that constitute supply chain risk: supply, demand, environmental and process risks. We follow this classification where the focus is on supply risk. They define supply risk as the risk emanating from the problems in a smooth flow from the downstream side. An overview of definitions concerning supply chain risk can be found in e.g. Tang (2006) and Ho et al. (2015). Furthermore, different frameworks can be followed when managing supply risk. Examples are Franck (2007) and Bandaly, Satir, Kahyaoglu & Shanker (2012). Following other papers within this field, we use the framework by Tummala & Schoenherr (2011). It consists of three phases (see Figure 2.1). The first phase consists of three steps: risk identification, risk measurement and risk assessment. The next phase concerns evaluation and mitigation. Lastly, phase three consists of the continual monitoring and controlling of the risks. Our study concerns the first two phases.

Figure 2.1: Supply chain risk management framework by Tummala & Schoenherr (2011)



2.2 Identify, measure and assess supply risk

In this section, we present different methods of risk analysis in supply chains. Following the risk framework of Tummala & Schoenherr (2011), this corresponds with the first phase: risk identification, risk measurement and risk assessment. The boundaries between the three sub-phases can however be vague and are therefore taken together. Tran, Dobrovnik & Kummer (2018) distinguish between qualitative, semi-quantitative, quantitative and mixed risk assessment methods. We focus on quantitative methods, which are used to accurately estimate risk probability and other indicators, using observations as well as laws and theories, since they fit the environment best. More specifically, we are interested in methods that use simulation. Other methods are e.g. mathematical modeling (Bogataj & Bogataj, 2007; Sawik, 2017), Bayesian networks (Amundson, Brown, Grabowski & Badurdeen, 2014; Lawrence, Ibne

Hossain, Jaradat & Hamilton, 2020) and mean-variance analysis (Ray & Jenamani, 2016; Mukherjee & Padhi, 2022) The reader is referred to Ho et al. (2015), Tran et al. (2018) and Choudhary, Singh, Schoenherr & Ramkumar (2022) for a more complete overview.

Schmitt & Singh (2009, 2012) start with identifying types of disruption and estimating the likelihood and duration of these with interviews and discussions with various stakeholders within the researched supply chain. This is used as input for a Monte Carlo simulation, which leads to the distribution of interarrival times and durations of disruptions. Risk profiles are then built per facility. Findings are used to build a discrete event simulation of the whole network that can now also incorporate disruptions. Other types of simulation used are e.g. system dynamics (Mehrjoo & Pasek, 2016) and Petri-nets (Khilwani, Tiwari & Sabuncuoglu, 2011). We follow Klibi & Martel (2012), who apply Monte Carlo simulation to generate plausible future scenarios. The simulation is based on descriptive models that describe three event types and their characterization: random, hazardous and deeply uncertain events. Findings can be used for practitioners to evaluate their supply chain network and as input to other modeling techniques such as stochastic programming. The framework as applied in this study is described in more detail in Section 3.2.

2.3 Single-echelon inventory control under supply risk

As described in the Introduction and following the classification of Silver, Pyke & Peterson (1998), our study follows an (R, s, nQ)-policy for inventory control. The policy was first introduced by Morse (1959). The first exact search was presented by Zheng & Chen (1992), who find that the cost-performance is insensitive to the choice of Q. For a system with lost sales, van Donselaar & Broekmeulen (2013) present some useful approximations for the fill rate, helping managers to determine the amount of safety stock needed (higher safety stock being a higher s) in their systems to reach a certain fill rate.

In what follows, we present relevant papers on both supply disruptions and yield uncertainty. For a more comprehensive overview, see Tinani & Kandpal (2017).

Under both supply disruptions and yield uncertainty, Parlar & Perry (1995) study the (R, s, nQ)-policy where the availability of suppliers is modeled using a two-state continuous Markov chain. One state corresponds to the supplier being available (ON state) and the other state refers to the supplier being unavailable (OFF state). The yield uncertainty is characterized by a Beta distribution. The objective function as the long-run average cost is constructed by the renewal reward theorem. The reorder point, the order quantity during the ON state and how long to wait for placing an order when the system is in the OFF state are the decision variables that are found. They test the findings using a numerical sensitivity analysis.

Another paper that combines both disruption and yield uncertainty is Chopra et al. (2007). First, they present a case to make clear that disruptions and yield uncertainty should be analyzed separately when looking at supply variance. They then investigate a situation where there is one primary supplier that is unreliable and one backup supplier that is reliable while being more expensive. The unreliability of the unreliable supplier is modeled by both disruptions and yield uncertainty, where the supplier can be disrupted by a probability p and the yield uncertainty follows a density function $f()$. In a single-period setting with deterministic demand, they derive closed-form solutions for the two decision variables (order quantity at the primary supplier and reservation quantity at the backup supplier). By comparing the costs of their solution with the solution where disruptions and yield uncertainty are not looked at separately, they find that the latter leads to significant cost increases.

Schmitt & Snyder (2012) stress the importance of looking at sufficient long time when analyzing both disruptions and yield uncertainty since single-period models underestimate the risk of both factors and lead to sub-optimal solutions. Disruptions are modeled by either a simple discrete-time Markov process or from a more general supply process. The yield is modeled as additive random quantity with a general distribution, independent of the order size. They distinguish between two cases, one with only one unreliable supplier and a second case where also a more expensive supplier is available. By determining the optimal order and reserve qualities and comparing those with findings of a single-period approximation, they find that there is an increase in cost for the latter situation.

Jeon, Lim, Peng & Rong (2021) look into an inventory system where two products are partially substitutable and have different levels of supply reliability, where both disruptions and yield uncertainty are used to model the reliability. Disruptions are modeled by ON and OFF states, following exponential distributions. Yield uncertainty is modeled by a general probability density function $f()$. Findings include the fact that under higher disruption risk, order sizes for both products are increased, but that under higher yield uncertainty, only the order sizes for one product increase.

2.4 Multi-echelon inventory control under supply risk

The nature of multi-echelon systems makes it difficult to study these, especially larger networks, analytically since models either become intractable or have to be vastly simplified (Snyder & Shen, 2006). One example of a paper that takes a more analytical point of view is Atan & Snyder (2012). They study a One-Warehouse Multiple-Retailer (OWMR) network where supply can be disrupted and find optimal or near-optimal stocking levels for all locations. They do not take any of our mitigation strategies into account.

We focus here on other solving methods. Specifically, we discuss two-stage stochastic programming, as that is also the approach chosen in this research. Overviews that also discuss other methods can be found in e.g. Tang (2006), Ho et al. (2015), Heckmann, Comes & Nickel (2015), Snyder et al. (2016) and Xu, Zhang, Feng & Yang (2020). We first discuss literature concerning the mitigation strategies in a multi-echelon environment. Thereafter, we discuss the literature on our modeling and solution method. The literature in this section thus corresponds to the second phase of the risk framework: risk evaluation and mitigation.

2.4.1 Dual sourcing under demand substitution

In this subsection, we discuss the literature concerning dual sourcing with unreliable suppliers (either because of disruptions, yield uncertainty or both). We are indebted to Svoboda et al. (2021) for supplying a comprehensive overview of the literature on dual and multiple sourcing. The reader is referred to their paper for a complete overview. Another interesting overview is given by Golmohammadi & Hassini (2020), who show an overview of supplier diversification under uncertain demand and supply. We focus on the case where dual scouring is used in combination with demand substitution. In a retail setting, the idea is that product substitution, where demand for one product is fulfilled by another product, takes place based on either the assortment, the inventory or the price (Shin, Park, Lee & Benton, 2015). We model the demand substitution following Yücel et al. (2009), who assume that substitution takes place when stock-out happens. They assume that for every substitution step, a part of the original demand is substituted by another product and a part is lost sales.

Tomlin (2009) studies three different mitigation strategies (dual sourcing, backup supply and demand switching) in the context of a two-product newsvendor. An interesting finding is that demand switching works if the set of suppliers is different. Moreover, a firm that single-sources products but uses different suppliers for each product is somewhat diversified as a failure at one supplier does not disrupt its entire product portfolio (dedicated single source). They present the optimal choice of strategy based on nine different attributes and find that in general, demand switching is the least effective.

Lu, Huang & Shen (2011) consider a supply chain with two downward substitutable products (i.e. a higher and lower grade product) and two suppliers, where one is reliable but expensive, and one is unreliable. They find that product substitution is an effective way to mitigate supply chain disruptions and find that the higher grade product is always strictly preferred in the optimal sourcing policy.

Lastly, Wu, Gong, Peng, Yan & Wu (2020) study the situation where product substitution is combined with dual sourcing and production lines of suppliers can partly fail, instead of full disruption which is usually the case in this line of research. They present optimal policies for both deterministic and stochastic demand. The effectiveness of their solutions is proved in a real-world case study.

We extend the previously mentioned studies by also incorporating lateral transshipments as a possible mitigation strategy. We discuss the literature on that topic in the next section.

2.4.2 Lateral transshipment

Lateral transshipment is the last mitigation strategy that we discuss. It concerns sharing stocks between entities in the same echelon. We focus on proactive transshipment, where the transshipment takes place before the realization of demand. Proactive transshipment is most useful in a retail environment, since handling costs are often the most dominant cost and proactive transshipment can be organized in such a way that those costs are minimized. Interesting literature reviews on lateral transshipment are given by Kumari, Wijayanayake & Niwunhella (2021), who focus on a retail environment, and Paterson, Kiesmüller, Teunter & Glazebrook (2011).

A first influential paper on lateral transshipments is a study by Gross (1963), who analytically studies a two-location network with negligible lead time and derives the optimal policy, which is dependent on the starting inventory and cost parameters, when transshipment and order decisions are taken at the same time.

When the transshipment is done between order moments, Tagaras & Vlachos (2002) find that the performance of the system depends on the setting of this redistribution timing, where this decision depends on characteristics such as the distribution of demand. Furthermore, they note that transshipments can be advantageous, but mostly under a setting of highly variable demand.

Feng, Fung & Wu (2017) study a centralized, one warehouse multiple retailers system and find that using proactive shipments is valuable compared to having no transshipment, arriving at the solution by a sorting heuristic based on dynamic programming.

Dynamic programming is also utilized by Meissner & Senicheva (2018). They find the near-optimal solution for a multi-location multi-period setting where a policy is formed that guides the decisions on transshipments.

Avci (2019) study a typical retail system with multiple DCs and retailers under lateral transshipment and expedited shipments. They present a general lateral transshipment policy and find ordering policy parameters for the retailers by using simulation-based optimization. They conclude that lateral transshipment is a cost-efficient stock-out risk mitigation strategy.

None of the studies mentioned in this section have included dual sourcing under demand substitution as a possible mitigation strategy. Our study adds to the literature by also incorporating demand substitution.

2.4.3 Two-stage stochastic programming

This section describes relevant literature on our modeling approach. Firoozi (2018) investigates a capacitated multi-echelon supply chain with both stochastic demand and possible disruptions in the supply chain where a periodic (s,S)-policy is applied at the various stages. Uncertainty is included through scenarios based on the risk framework presented by Klibi & Martel (2012), which we also follow in this research, in combination with Monte Carlo simulation. Two mitigation strategies in the form of multi-sourcing and lateral transshipments are incorporated in the second stage model, which is a MILP. Since there is an infinite amount of scenarios possible, the Sample Average Approximation by Shapiro (2008) is used, where the optimal solution is approximated by sampling techniques. By applying the proposed solving method in a case study of a large retailer in France, it was found that the approach fits the environment and that significant cost decreases can be achieved by including lateral transshipment and multi-sourcing.

Jalali, Tavakkoli-Moghaddam, Ghomi-Avili & Jabbarzadeh (2017) study a closed-loop supply chain where supply disruptions can take place, where the uncertainty is taken into account through various scenarios. The decisions concerned in the model are the location of the facilities, the quantity of products ordered and the amount of lost sales. The mitigation strategies proposed are inventory buffers and lateral transshipments where it is found that the latter significantly decreases costs.

Snoeck, Udenio & Fransoo (2019) use two-stage stochastic programming to test different mitigation strategies within a supply chain to protect against disruptions. Interesting in their approach is the fact that they distinguish between business as usual and disruption periods, which leads to a decrease of approximately 85% in terms of model size and computational power. A second interesting fact is that they include conditional value at risk in their objective function, by which the risk aversion of decision makers can be modeled. The model is successfully applied in a case study of a chemical supply chain.

Sanci, Daskin, Hong, Roesch & Zhang (2021) apply a multi-stage stochastic programming model to a case study at Ford, where they test multiple possible mitigation strategies related to multiple sourcing and inventory buffers. Risks are incorporated through input of decision makers, where it should be noted that precise estimations are not necessary for good results which is a strength of the model. Investing in the mitigation strategy that is proposed by the model decreases costs significantly compared to what would be optimal in a no-disruption environment.

This study differs from the previously mentioned works in that we solve our two-stage stochastic program by combining the Sample Average Approximation and the Progressive Hedging Algorithm. In the next section, we present relevant literature on the algorithm.

2.4.4 Progressive hedging

As mentioned in the introduction, we use the Progressive Hedging Algorithm (PHA) by Rockafellar & Wets (1991) to solve the two-stage stochastic program (see Algorithm 2 for our implementation). For convex, continuous cases, it is proven that the PHA converges to the optimal solution (Rockafellar & Wets, 1991). Even though it does not necessarily converge for mixed-integer models, it nevertheless can be used as an effective heuristic (Watson & Woodruff, 2011; Løkketangen & Woodruff, 1996). Determination of the penalty factor ρ is in this regard important to ensure proper convergence within tractable computational times (Torres, Li, Apap & Grossmann, 2022). Within the literature, there is no definitive way to set this factor (Listes & Dekker, 2005). From their experiments, Listes & Dekker (2005) find that values between 50 and 100 work best. Contrary to that, Haugen, Løkketangen & Woodruff (2001) use a ρ of 0.8 for their computations in stochastic lot-sizing. Watson & Woodruff (2011) propose a variable ρ setting strategy based on the cost function of the decision variable where the penalty factor is not one scalar value but can be different per component. They find substantial improvement in the performance of the PHA. Kaisermayer, Muschick, Horn & Gölles (2021) expand this strategy by introducing an adaptive update scheme where the ρ can change per iteration, successfully implementing this in their experiments.

As stated before, in a mixed-integer setting, the PHA does not necessarily converge. However, we still want to assess the quality of the solution. For this purpose, we follow Gade et al. (2016), who propose constructing lower bounds based on the dual prices of the standard PHA (in our notation, the dual prices, which we call weights, are denoted by λ). They show that the lower bounds indeed converge towards the optimal value.

The PHA is successfully used in multiple supply chain related studies. Kim, Wu & Huang (2015) study a multi-period, two-echelon newsvendor problem with non-stationary demand and find that the PHA can find good quality solutions in relatively low computational times. Poudel, Quddus, Marufuzzaman, Bian & Burch (2019) use the PHA in the biomass supply chain where there is uncertainty in supply to solve their stochastic programming model. Ghorashi Khalilabadi, Zegordi & Nikbakhsh (2020) look at supply chain risk with demand substitution as possible mitigation strategy and propose an improvement on the PHA for solving larger instances based on their specific problem setting.

Chapter 3

Modeling approach

In this chapter, we present the mathematical formulation of our problem. Secondly, we present our modeling approach for the disruptions and yield uncertainty.

3.1 Model formulation

We model a multi-echelon, multi-product, multi-period network with yield uncertainty and supply disruptions that can take place. The network consists of external suppliers, distribution centres (DCs) $w \in W := \{1, 2, ..., |W|\}$ where W denotes the set of distribution centres and retailers $n \in N := \{1, 2, ..., |N|\}$ where N denotes the set of retailers. Each echelon is fed from the higher echelon. We consider products $p \in P := \{1, 2, ..., |P|\}$ where P denotes the set of products. Stock-out based demand substitution can take place between two products. Stock-out based demand substitution means that when a customer finds his/her preferred item to be out of stock, he/she will decide to either choose a different similar product or not purchase anything at all, in which case a lost sale takes place. Motivated by the case study that instigated this study, we assume that for a product p, a retailer is assigned to only one DC. It is possible to store all products at all locations, without inventory capacity limitations. A lateral transshipment option exists between the DCs which means that shipments within the same echelon are possible.

We consider a time horizon $\tau \in T := \{1, 2, ..., |T|\}$ that is partitioned in planning periods $\psi \in \Psi := \{1, 2, ..., |\Psi|\}$ of equal size. If not otherwise specified, a period refers to $\tau \in T$. The corresponding periods of each planning period $\psi \in \Psi$ are denoted by $T^{\psi} := \{|T|/|\Psi|(\psi - 1) + 1, ..., (|T|/|\Psi|)\psi\}$. In the remainder of this study, we assume $|T|/|\Psi| \in \mathbb{N}$. If for example we have 2 planning periods and 28 periods, T^2 would be the set $\{15, 16, ..., 28\}$.

The timing of the events in a period τ is as follows, where for ease of exposition we omit all indices related to time and product:

1. A shipment F (a shipment from supplier to DC) and/or R (either a lateral transshipment or a shipment from a DC to a retailer) arrive at the start of the period and can be used to fulfill orders or demand in that time period

2. The actual demand at the retailers of that period realizes and is either fulfilled or lost based on the available inventory

3. The retailer can place an order of size Q at the DC where the retailer is assigned to based on the ordering logic that we explain in the next paragraph
4. The DC can place an order at the supplier and/or use lateral transshipment
5. An order arrives after a certain deterministic leadtime L where we assume that the leadtime is at least two because when a retailer (DC) orders at the DC (supplier), it can be used to fulfill demand the earliest in two days because of handling and transportation times.

The two echelons in control of the decision maker (DCs and retailers) follow an (R, s, nQ)-policy which means that every review period, it is checked whether the inventory position IP is lower than the order-up-to level s. If that is the case, a replenishment order is placed of size $n \cdot Q$ where Q is the fixed base quantity (e.g. a case pack holds 12 consumer units) and n the minimum integer which is needed to bring the inventory position after ordering back to or above the order-up-to level s. In the remainder of this study, we review our inventory every time period. Note that we use different notation than Silver et al. (1998) in our model: fq is used for fixed quantity and Q for the total order quantity. See Table 3.1 for more information on the notation. For now, we omit all subscripts for ease of exposition. For the DCs, the inventory position is defined as the inventory on-hand I plus the scheduled receipts (orders that are in transit) F and incoming lateral transshipments minus the backorders BO. Since we model lost sales for the retailer, the inventory position for them is defined as the inventory on-hand I plus the scheduled receipts R. The retailers face uncertain random demand. We discuss the modeling of uncertainty in a later paragraph.

The actual shipment F to the DC depends on the available capacity of the supplier which can be lower than normal because of disruptions. Because it is difficult to judge the capacity of an external supplier, we assume that the supplier is uncapacitated in undisrupted periods. To make a clear distinction between disruptions and yield uncertainty, we assume that the former is known at the moment of ordering, whereas the latter gets known when the shipment arrives. For a retailer, the actual shipment depends on the available on-hand inventory at the DC.

As mentioned in the paragraph before, the actual shipment can be lower than the order quantity. When this is the case between a retailer and DC, the quantity that cannot be shipped is backordered. As previously mentioned, customers do not backorder at the retailers since we assume that either the sale is lost or stock-out based demand substitution takes place.

Within the model, there are three variables that we assume to be random. Again, we omit the indices for ease of exposition. The first random variable is the demand D for a product at each retailer per period. Note that the demand can follow any distribution and can also be non-stationary. Demand across periods is independently distributed, but need not be identical. The second random variable is the capacity K of a supplier which is random because of disruptions that take place. These capacities are modeled through the framework of Klibi & Martel (2012). In this choice we follow Firoozi (2018), who successfully apply this framework in a two-stage stochastic program in a similar setting. Lastly, we assume that the yield uncertainty γ is random. Furthermore, we assume that yield uncertainty is independent of order size, multiplicative (i.e. a fraction of the order quantity) and only becomes known when the shipment arrives (contrary to uncertainty in the capacity, which is known at the order moment) since this fits the retailing environment and makes the modeling of the uncertainty straightforward. More details on our modeling approach concerning disruptions and yield uncertainty are given in Section 3.2. This is a generally applicable modeling frame work that has merit on itself beyond this specific study.

We model the uncertainty in the form of scenarios. These scenarios are meant to represent the stochastic nature of the real world. A scenario ω has a probability $p(\omega)$ of occurrence. Section 4.1 describes our procedure to generate scenarios.

The decisions of the model are the the order-up-to levels for both the DCs and the retailers, and the usage of lateral transshipment. The order-up-to levels are set for a longer period of time. The lateral transshipments however can be used as a flexibility option when the safety levels set by the order-up-to levels are not sufficient after uncertainty in supply and demand realizes. Thus, these decisions happen at different time granularities. We therefore propose to model the problem as a two-stage stochastic program. Specifically, in the first stage, the order-up-to levels are set for a planning period ψ and decided upon based on the expectation of uncertainties, thus before the actual realization of the variables. On the other hand, in the second stage, lateral transshipment decisions are made every period τ based on the realization of the uncertainties. We study a static problem, i.e. we do not consider any roiling horizon and solve the problem for the planning periods at one point in time.

The goal of the two-stage stochastic program is to minimize the costs associated with those decisions. Specifically, the costs consist of holding costs c^{H} per unit, internal backorder costs c^{B} per unit, fixed and variable lateral transshipment costs (c^{TF} per shipment and c^{TV} per unit respectively), substitution costs c^{SC} per unit and a lost sales penalty c^{L} per unit. The internal backorder costs are included because of the situation at the company of the case study that instigated this research. At the company of the case study, retailers are entrepreneurs themselves and expect a certain delivery performance from the DC. A penalty has to be paid if the delivery performance is insufficient. Hence the inclusion of internal backorder costs. We denote the optimal solution of the first stage variables as $\mathbf{s}^*$, which is a vector containing all order-up-to levels, the optimal solution of the second stage variables as $\mathbf{y}^*$, which is a vector containing all decision (lateral transshipments) and auxiliary (e.g. orders, shipments) variables and the objective value where the costs are minimized by z^*.

In the remainder of the section, we firstly present the notation in Table 3.1. Secondly, we introduce the mathematical formulation of the model. Lastly, we explain the objective function and the different constraints in more detail.

Table 3.1: Notation

Sets	
N	set of retailers $n \in N$ where N = {1,2,3,...}
W	set of DCs $w \in W$ where W = {1,2,3,...}
P	set of products $p \in P$ where P = {1,2,3,...}
N_{wp}	set of retailers assigned to DC w for product p where $N_{wp} \subseteq N$
T	set of periods $\tau \in T$ where T = {1,2,3,...}
Ψ	set of planning periods $\psi \in \Psi$ where Ψ = {1,2,3,..}
T^{ψ}	set of periods $\tau \in T^{\psi}$ where $T^{\psi} = \{\lvert T\rvert/\lvert\Psi\rvert(\psi-1)+1, ..., (\lvert T\rvert/\lvert\Psi\rvert)\psi\}$
Ω	set of scenarios $\omega \in \Omega$ where Ω = {1,2,3,...}
Cost parameters	
c^{H}_{ip}	holding cost per period for location $i \in \{W, N\}$ and product p
c^{L}_{p}	lost sales cost of product p
c^{SC}_{p}	penalty cost of substitution from product p to another product
$c^{\mathrm{TF}}_{ww'p}$	fixed cost of a transshipment between w and w' for product p
$c^{\mathrm{TV}}_{ww'p}$	variable cost for a transshipment between w and w' per unit for product p
c^{B}_{p}	internal backorder cost per period for product p
Parameters	
t	a planning period
τ	a period
$p(\omega)$	probability of scenario ω happening
$D_{np\tau\omega}$	demand at retailer n for product p in period τ in scenario ω
$\gamma_{wp\tau\omega}$	fraction of order delivered at DC w of product p in period τ in scenario ω
$K_{p\tau\omega}$	capacity at the supplier for product p in period τ in scenario ω
w_{kp}	substitution rate, i.e. proportion of customers whose preference is product p that substitute product p with product k where $k \in P \setminus p$
w^{l}_{p}	proportion of customers whose preference is product p and that refuse to substitute product p with any other product

Decision variables	
$s_{ip\psi}$	order-up-to level of location $i \in \{W, N\}$ for product p in planning period ψ
$R_{ww'p\tau\omega}$	lateral transshipment quantity arrived in period τ of product p at w' shipped from w in scenario ω where $w \neq w'$
Auxiliary variables	
$I_{ip\tau\omega}$	the inventory on-hand of location $i \in \{W, N\}$ for product p in period τ in scenario ω
$BO_{np\tau\omega}$	backorders of retailer n at the DC for product p in period τ in scenario ω
$IP_{ip\tau\omega}$	the inventory position of location $i \in \{W, N\}$ for product p in period τ in scenario ω
$O_{i\tau\omega}$	binary variable that takes value 1 if $i \in \{W, N\}$ orders in period τ in scenario ω
$Q_{ip\tau\omega}$	quantity ordered of product p by $i \in \{W, N\}$ in period τ in scenario ω
$R_{ijp\tau\omega}$	quantity arrived in period τ of product p at $j \in \{W \setminus i, N\}$ shipped from $i \in W$ in scenario ω
$F_{wp\tau\omega}$	quantity arrived in period τ of product p at w shipped from an external supplier in scenario ω
$q_{ip\tau\omega}$	integer multiple of the order quantity for location $i \in \{W, N\}$ for product p in period τ in scenario ω
$x0_{np\tau\omega}$	amount of satisfied demand of product p without substitution in period τ at retailer n (that is, demand satisfied by product p when it is available in stock) in scenario ω
$xs_{nkp\tau\omega}$	amount of product $k \in P \setminus p$ used to satisfy substitution from product p in period τ at retailer n (if p is out of stock in τ) in scenario ω
$xl_{np\tau\omega}$	lost sales of product p in period τ (if p is out of stock in τ) at retailer n in scenario ω
$l_{ww'p\tau\omega}$	binary variable equal to 1 if a lateral transshipment takes place between DCs w and $w' \neq w$ for product p in period τ for scenario ω
Constants	
M	large number
L_i	deterministic leadtime after which an order of $i \in \{W, N\}$ arrives
fq_{ip}	fixed base quantity for product p if you order as $i \in \{W, N\}$

$$\min \sum_{\omega\in\Omega} p(\omega)[\sum_{\tau\in T} [\sum_{p\in P} [\sum_{w\in W}[c^{\mathrm{H}}_{wp} I_{wp\tau\omega}] + \sum_{n\in N}[c^{\mathrm{H}}_{np} I_{np\tau\omega} + c^{\mathrm{B}}_{p} BO_{np\tau\omega} + \sum_{k\in P\setminus k}[c^{\mathrm{SC}}_{p} xs_{nkp\tau\omega}] + c^{\mathrm{L}}_{p} xl_{np\tau\omega}] + \sum_{w\in W} \sum_{\substack{w'\in W\\ w'\neq w}}[c^{\mathrm{TF}}_{ww'p} l_{ww'p} + c^{\mathrm{TV}}_{ww'p} R_{ww'p(\tau)\omega}]]]] \tag{3.1}$$

$$\text{s.t.}\quad I_{np(\tau-1)\omega} + R_{wnp(\tau)\omega} - x0_{np\tau\omega} - \sum_{k\in P\setminus p} xs_{npk\tau\omega} = I_{np(\tau)\omega} \qquad \forall n\in N, p\in P, \tau\in T, \omega\in\Omega \tag{3.2}$$

$$I_{wp(\tau-1)\omega} + \gamma_{wp\tau\omega} F_{wp(\tau)\omega} - \sum_{n\in N_{wp}} R_{wnp(\tau+L_n)\omega} + \sum_{\substack{w'\in W\\ w'\neq w}} R_{w'wp(\tau)\omega} - \sum_{\substack{w'\in W\\ w'\neq w}} R_{ww'p(\tau+L_{ww'})\omega} = I_{wp(\tau)\omega} \qquad \forall w\in W, p\in P, \tau\in T, \omega\in\Omega \tag{3.3}$$

$$IP_{np\tau\omega} = I_{np(\tau)\omega} + \sum_{x\in 1\ldots(L_n-1)} R_{wnp(\tau+x)\omega} \qquad \forall n\in N, p\in P, \tau\in T, \omega\in\Omega \tag{3.4}$$

$$IP_{wp\tau\omega} = I_{wp(\tau)\omega} + \sum_{x\in 1\ldots(L_w-1)} F_{wp(\tau+x)\omega} + \sum_{\substack{w'\in W\\ w'\neq w}} R_{w'wp(\tau)\omega} - \sum_{n\in N} BO_{np\tau\omega} \qquad \forall w\in W, p\in P, \tau\in T, \omega\in\Omega \tag{3.5}$$

$$IP_{ip\tau\omega} - M(1-O_{ip\tau\omega}) \leq s_{ip\psi} \qquad \forall i\in W\cup N, p\in P, \psi\in\Psi, \tau\in T^{\psi}, \omega\in\Omega \tag{3.6}$$

$$IP_{ip\tau\omega} + MO_{ip\tau\omega} > s_{ip\psi} \qquad \forall i\in W\cup N, p\in P, \psi\in\Psi, \tau\in T^{\psi}, \omega\in\Omega \tag{3.7}$$

$$q_{ip\tau\omega} fq_{ip} + M(1-O_{ip\tau\omega}) \geq s_{ip\psi} - IP_{ip\tau\omega} \qquad \forall i\in W\cup N, p\in P, \psi\in\Psi, \tau\in T^{\psi}, \omega\in\Omega \tag{3.8}$$

$$q_{ip\tau\omega} fq_{ip} - M(1-O_{ip\tau\omega}) < s_{ip\psi} - IP_{ip\tau\omega} + fq_{ip} \qquad \forall i\in W\cup N, p\in P, \psi\in\Psi, \tau\in T^{\psi}, \omega\in\Omega \tag{3.9}$$

$$q_{ip\tau\omega} fq_{ip} = Q_{ip(\tau)\omega} \qquad \forall i\in W\cup N, p\in P, \psi\in\Psi, \tau\in T^{\psi}, \omega\in\Omega \tag{3.10}$$

$$R_{ww'p(\tau)\omega} \leq l_{ww'p\tau\omega} M \qquad \forall w\in W, w'\in W\setminus w, p\in P, \tau\in T, \omega\in\Omega \tag{3.11}$$

$$l_{ww'p\tau\omega} \leq R_{ww'p(\tau)\omega} M \qquad \forall w\in W, w'\in W\setminus w, p\in P, \tau\in T, \omega\in\Omega \tag{3.12}$$

$$l_{ww'p\tau\omega} \neq l_{w'wp\tau\omega} \qquad \forall w\in W, w'\in W\setminus w, p\in P, \tau\in T, \omega\in\Omega \tag{3.13}$$

$$x0_{np\tau\omega} + \sum_{k\in P\setminus p} xs_{nkp\tau\omega} + xl_{np\tau\omega} = D_{np\tau\omega} \qquad \forall n\in N, p\in P, \tau\in T, \omega\in\Omega \tag{3.14}$$

$$xs_{nkp\tau\omega} \leq (d_{np\tau\omega} - x0_{np\tau\omega}) w_{kp} \qquad \forall n\in N, p\in P, k\in P\setminus p, \tau\in T, \omega\in\Omega \tag{3.15}$$

$$F_{wp(\tau+L_w)\omega} = \min(Q_{wp\tau\omega}, K_{p\tau\omega}) \qquad \forall w\in W, p\in P, \tau\in T, \omega\in\Omega \tag{3.16}$$

$$\sum_{n\in N_{wp}} R_{wnp(\tau+L_n)\omega} = \min(\sum_{n\in N_{wp}} Q_{np\tau\omega}, (I_{wp\tau\omega} + \sum_{n\in N_{wp}} R_{wnp(\tau+L_n)\omega})) \qquad \forall w\in W, p\in P, \tau\in T, \omega\in\Omega \tag{3.17}$$

$$R_{wnp(\tau+L_n)\omega} \leq Q_{np\tau\omega} \qquad \forall n\in N, p\in P, \tau\in T, \omega\in\Omega \tag{3.18}$$

$$BO_{np\tau\omega} = Q_{np\tau\omega} - R_{wnp(\tau+L_n)\omega} \qquad \forall n\in N, p\in P, \tau\in T, \omega\in\Omega \tag{3.19}$$

$$O_{ip\tau\omega} \in \{0,1\} \qquad \forall i\in W\cup N, p\in P, \tau\in T, \omega\in\Omega \tag{3.20}$$

$$l_{ww'p\tau\omega} \in \{0,1\} \qquad \forall w\in W, w'\in W\setminus w, p\in P, \tau\in T, \omega\in\Omega \tag{3.21}$$

$$q_{ip\tau\omega} \in \mathbb{Z} \qquad \forall i\in W\cup N, p\in P, \tau\in T, \omega\in\Omega \tag{3.22}$$

$$IP_{ip\tau\omega} \in \mathbb{R} \qquad \forall i\in W\cup N, p\in P, \tau\in T, \omega\in\Omega \tag{3.23}$$

$$I_{wp0\omega}, I_{np0\omega} = 0 \qquad \forall w\in W, n\in N, p\in P, \omega\in\Omega \tag{3.24}$$

$$I_{wp(\tau)\omega}, I_{np(\tau)\omega}, R_{wnp(\tau)\omega}, F_{wp(\tau)\omega}, Q_{wp\tau\omega}, Q_{np\tau\omega}, BO_{np\tau\omega}, x0_{np\tau\omega}, xl_{np\tau\omega}, xs_{nkp\tau\omega}, R_{ww'p(\tau)\omega}, s_{ip\psi} \geq 0 \tag{3.25}$$

The objective function (3.1) consists of the holding costs, internal backorder costs, substitution costs, lost sales costs and both fixed and variable costs of lateral transshipments.

Constraint (3.2) is the inventory on hand balance equation for a retailer. Lost sales are assumed, so if the demand is higher than the previous inventory on hand, plus what is delivered because of an order a leadtime ago, the inventory on hand becomes 0 and no backorders are logged. Inventory on-hand either decreases because demand for the product itself is filled ($x0_{np\tau\omega}$) or the product is used as a substitute for another product ($xs_{npk\tau\omega}$).

Constraint (3.3) is the inventory on hand balance equation for a DC. In this case, backorders are assumed. The inventory of period τ depends on the inventory of the previous period, plus deliveries from the supplier with a yield uncertainty factor, which is a fraction smaller than 1 taken as value from scenario ω, minus the amount that is shipped to the retailers. Furthermore, the outbound lateral transshipments to other DCs and inbound lateral transshipments from other DCs are included. Note that there is no specific lateral transshipment logic and that if there is available on-hand inventory, the model is allowed to decide whether to transship or not.

Constraint (3.4) determines the inventory positions of the retailer. It is defined as the inventory on hand plus the scheduled receipts. (3.5) determines the inventory position of the DC. It depends on the inventory on hand, the amount of stock in transit including lateral transshipments and the outstanding backorders.

Constraints (3.6) until (3.10) determine the order quantity for either the DC or retailer. The binary variable $O_{ip\tau\omega}$ has been introduced to ensure linearity of the order logic. It is forced to 1 when the inventory position drops below the order-up-to level s, i.e. when an order is placed. If an order is placed, the order quantity is forced to be an integer q of the fixed base quantity fq. The last constraint then sets $q_{ip\tau\omega} fq_{ip}$, which has been used to ensure the fixed quantity, to $Q_{i\tau\omega}$, the amount ordered.

Constraints (3.11) and (3.12) exist to be able to model fixed costs when a lateral transshipment takes place. The binary variable $l_{ww'p\tau\omega}$ is forced to 1 if a shipment takes place between DC w and DC w' where $w \neq w'$. Constraint (3.13) makes it impossible for DCs to let transshipments arrive at each other in the same period.

Constraints (3.14) and (3.15) model demand fulfillment and the substitution effect following Yücel et al. (2009). In constraint (3.14), it is stated that the demand for a product at a retailer should either be filled by the product itself ($x0_{np\tau\omega}$) or by substitution of a different product ($xs_{nkp\tau\omega}$) or it is lost ($xl_{np\tau\omega}$). The substitution is then modeled in constraint (3.15). The substitution demand is less or equal to the amount of demand not in the first hand filled by the original product times the substitution rate. Contrary to Yücel et al. (2009), we leave the lost sales unconstrained and only look at one level of substitution. The lost sales are unconstrained to be able to model the case where the possible substitution product is also out of stock. One level of substitution is enough in our situation because the substitution amounts get significantly lower when looking at higher levels of substitution and one level of substitution is conceptually easier to understand and use.

Constraint (3.16) ensures that the actual quantity shipped by the supplier towards the DC does not exceed the capacity of the supplier. If the capacity is sufficient, this constraint makes the actual quantity shipped equal to the order quantity.

Constraint (3.17) ensures that the quantity shipped from a DC to the retailers is the minimum of the on-hand inventory of the DC and the total quantity ordered by the retailers (either the DC can ship all orders, or the DC ships all available on-hand inventory). Constraint (3.18) ensures that each actual flow from the DC to a retailer does not exceed the total quantity ordered.

The backorders are given by constraint (3.19) and are defined as the difference between what the retailer has ordered and what is shipped by the DC. Clearing backorders is not incorporated into the model. Therefore, the backorders are calculated for each period without carrying over backorders from the previous period. This is realistic because of the lost-sales situation at the retailer, which means that backorders cannot exceed the order-up-to level and can thus be recalculated each time period.

The last constraints model the type (real, binary or integer), non-negativity and the initialization of the different variables.

3.2 Modeling disruptions and yield uncertainty

As explained in the section before, the capacity of the supplier and the yield are uncertain in our model. In this section, we propose an approach of modeling disruptions and yield uncertainty. Note that the framework presented here is sufficiently general and can be used in many other applications. We first introduce the re-parameterized Beta distribution (Ferrari & Cribari-Neto, 2004), since we use it for both uncertainties. The distribution takes a mean $\mu \in (0, 1)$ and precision $\phi > 0$ as arguments, where for a fixed μ, a larger value of ϕ leads to a smaller variance. We denote the distribution by $BETA(\mu, \phi)$. The probability density function is obtained by redefining the α and β variables of the standard Beta distribution as follows: $\alpha = \phi\mu$ and $\beta = \phi(1 - \mu)$. The probability density function of the re-parameterized Beta distribution is given in Equation 3.26:

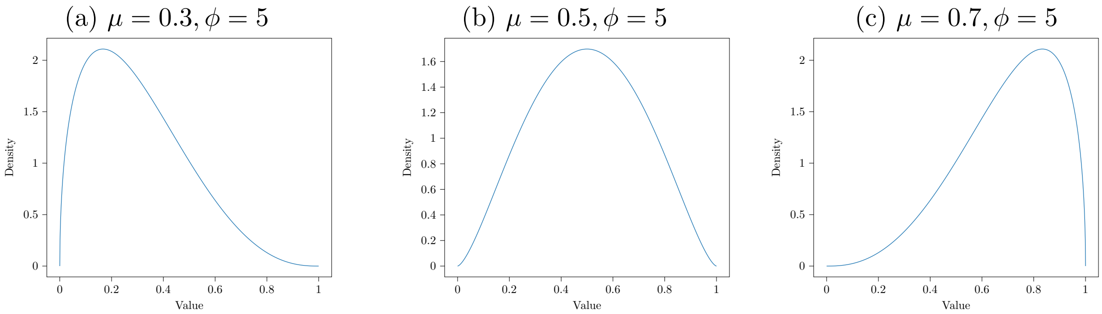
$$f_X(x : \mu, \phi) = \frac{\Gamma(\phi)}{\Gamma(\phi\mu)\Gamma(\phi(1-\mu)} x^{\phi\mu-1}(1-x)^{\phi(1-\mu)-1} \tag{3.26}$$

where $\Gamma(r)$ denotes the Gamma function:

$$\Gamma(r) = \int_0^\infty t^{r-1}e^{-t}dt \tag{3.27}$$

We use this distribution because it has two desirable properties. Firstly, the distribution is bounded between 0 and 1, which is useful for both the disruptions and yield uncertainty. Secondly, with the two parameters presented above, several distributions can be constructed. For example, a μ of 0.5 and ϕ of 2 will give a uniform distribution between 0 and 1. By setting ϕ to 5 and varying μ, distributions with different skewness can be made, see Figure 3.1. This distribution can therefore be easily applied to many cases.

Figure 3.1: Different probability density plots of the re-parameterized Beta distribution



3.2.1 Modeling disruptions

For modeling the disruptions, we present an adaptation of the framework of Klibi & Martel (2012). Our approach is sufficiently generic and robust, which makes it usable in many environments where disruptions are concerned. The goal of the framework is to model the effect of disruptions on the capacity of the external supplier over time. Examples of disruptions are the failure of a production line or unanticipated shortage of production material. A disruption at the supplier is caused by a hit that has a certain intensity. A higher intensity means a higher impact on the capacity. The disruption also has a duration, where the length of the disruption depends on the severity of the hit. Throughout the duration, the capacity gradually recovers back to the undisrupted capacity.

In the remainder of this subsection, we present functions and distributions that model the behaviour of the disruptions as described before.

Following the general approach in the literature, we assume a Poisson process for the disruptions taking place which means that the time between two disruptions happening is modeled by an exponential distribution with $\frac{1}{\lambda}$ as parameter, where λ is the rate of arrivals per time unit and $\frac{1}{\lambda}$ is thus the mean inter-arrival time between two disruptions taking place (Klibi & Martel, 2012).

A disruption that takes place has a hit intensity $\beta \in [0, 1]$ which models the impact of the hit. A higher hit intensity means that the negative impact on the capacity of the supplier is higher. For example, a hit with intensity 0.8 will decrease the initial capacity by 80%.

Furthermore, a disruption has a duration θ, measured in time periods. The duration θ depends on the hit intensity β by the so-called impact-duration function. Let this function be given by f. Hence $\theta = f(\beta)$. This function maps the intensity to a duration, that is:

$$f : [0, 1] \to \mathbb{N}$$

If the length of a duration is for example linearly related to the hit intensity, the duration can be modeled by the following function: $\theta = \lfloor \eta\beta \rfloor$ where η is a positive constant greater than zero and $\lfloor x \rfloor$ is the floor function that takes as input x and returns the greatest integer less than x.

The hit intensity and duration are linked to the capacity by a recovery function, where gradually over the length of the disruption the supplier recovers towards their normal capacity. This function is not necessarily linear. For example, after a disruption, it can take some time for countermeasures to come into place. Note that since we source at external suppliers, we do not have influence on these measures. After implementation of these measures at the suppliers, the capacity will gradually grow back towards normal levels. Let r_p be the recovery function of a supplier supplying product p which takes the hit intensity β and time period τ as input, having the following property:

$$r_p : [0,1] \times \mathbb{N} \to \mathbb{R}$$

An example of a recovery function is the following (recall that the duration θ depends solely on β through the impact-duration function):

$$r_p(\beta, \tau) = \begin{cases} 1 - \beta & \tau = 1, .., \lceil 0.25\theta \rceil \\ 1 - \frac{\theta - \tau + 1}{\lfloor 0.75\theta \rfloor}\beta & \tau = \lceil 0.25\theta \rceil + 1, .., \theta \end{cases} \tag{3.28}$$

For this specific example, it takes 25% of the duration for countermeasures to come into place. After the countermeasures, the capacity linearly grows back to the undisrupted capacity. One point of the recovery function is thus a fraction of the initial capacity. Hence if a disruption happens in time τ', the capacity in each period $\tau = \tau'+1, \tau'+2, \ldots, \theta$ is given by the following formula:

$$K_\tau = r_p(\beta, \tau - \tau' + 1) K_{base} \tag{3.29}$$

where K_{base} is the capacity of the supplier in an undisrupted period. Since we assume an uncapacitated supplier for undisrupted periods, we let K_{base} be equal to the maximum order quantity of a DC at the supplier in a model without any disruptions or yield uncertainty. Note that it is possible for double hits to happen. However, for notational clarity, we only regard one hit in our formulas. After the first example, we explain the mechanisms involved for the occurrence of a double hit by means of an extension of the example.

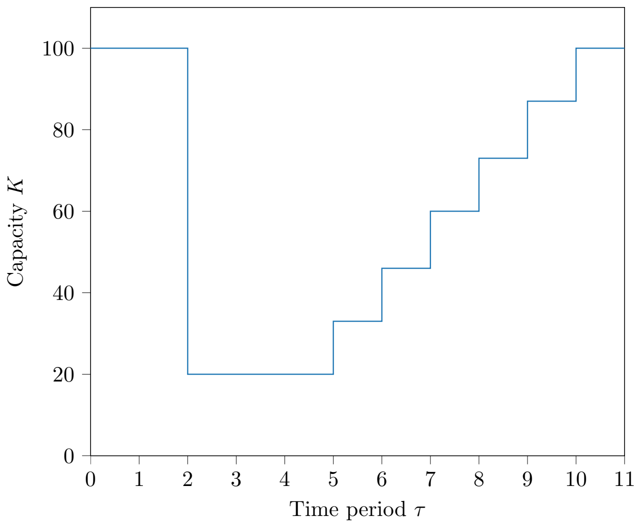
We now discuss an example. Let a hit happen at a supplier with a base capacity of 100 at time period 2 (τ') with an intensity of 0.8 (β). We assume a linear impact-duration function with a constant of 10, i.e. $\theta = \lfloor 10\beta \rfloor = 8$. The recovery function is the same as the example given in Equation 3.28. The capacity K of the supplier over time is given in Table 3.2 and shown in Figure 3.2. We observe that after the hit in period 2, it takes three periods for countermeasures to come into place. Thereafter, the capacity linearly grows back to the base capacity. Note that we round down the capacity to the nearest integer since that is necessary for the implementation of our model. However, it is not necessary for applying this framework.

Table 3.2: Capacity over time for the example

τ	1	2	3	4	5	6	7	8	9	10	11
K	100	20	20	20	33	46	60	73	87	100	100

The framework allows for double hits to happen. When calculating the capacity after the second hit, the base capacity K_{base} in Equation 3.29 should be replaced by the capacity from the first hit. Using the same example as before, let a second hit happen at $\tau' = 5$ with a hit intensity of 0.1. The duration θ is then 1. Using Equation 3.28 and Equation 3.29 with K_{base} set to 33, the capacity at time 5 becomes 29. The rest of the capacities remains the same as reported in Table 3.2.

Figure 3.2: Capacity recovery after a hit for the example



Furthermore, it is worth mentioning that it is possible to model different types of disruptions at the same time. Per disruption type, a different $\frac{1}{\lambda}$, hit intensity distribution, impact-duration function or recovery function can be set to best fit the environment. One can for example distinguish between low impact, high frequency disruptions and high impact, low frequency disruptions. The former can be modeled with a lower $\frac{1}{\lambda}$ and a β distribution that is more skewed towards lower values compared to the latter.

3.2.2 Modeling yield uncertainty

In this subsection, we describe our method for modeling yield uncertainty using a combination of the Bernoulli and the re-parameterized Beta distribution.

As explained in Section 3.1, we assume that the yield uncertainty is independent of order size, multiplicative and gets known only when the shipment arrives. The first two properties make the modeling of the yield uncertainty straightforward. The yield uncertainty is modeled as a fraction of an order that gets delivered and is thus between 0 and 1 since this fits the retailing environment.

We propose to use a combination of distributions. Based on our experience in the retail sector, we believe that this combination enables us to adequately model the yield uncertainty as it is seen in practice. First, we run a Bernoulli trial where with a chance p, no yield uncertainty takes place and thus everything that is ordered, gets delivered. With a chance $1-p$, only a fraction of the order is delivered. This fraction is then modeled by a re-parameterized Beta distribution with parameters μ and ϕ which have the same properties as described before. The yield uncertainty $\gamma \in [0,1]$ is thus given by:

$$\gamma_\tau = \begin{cases} 1 & \text{w.p. } p \\ BETA(\mu, \phi) & \text{w.p. } 1-p \end{cases} \tag{3.30}$$

To give an example, let p be 0.8, μ 0.5 and ϕ 5 (see Figure 3.1 for the shape of the density of this re-parameterized Beta distribution). Intuitively, this means that for about 20% of the deliveries, approximately 50% is delivered. On average, for 80% of the cases, the delivery is done in full. Table 3.3 gives a possible sample based on the aforementioned parameters.

Table 3.3: Yield uncertainty example

τ	1	2	3	4	5	6	7	8	9	10
γ	1	0.68	1	1	1	1	0.43	1	1	0.55

For example, if the leadtime is 2 and the DC ordered 100 units at time period 5, the actual delivery will be 43 in time period 7.

Chapter 4

Solution methodology

In this chapter, we present the solution methodology of this study. Firstly, we discuss the Sampling Average Approximation and scenario generation in Section 4.1. Secondly, we present a continuous approximation of our model since the original mathematical formulation proves to be intractable even for small instances. Lastly, we introduce the Progressive Hedging Algorithm to be able to solve the two-stage stochastic program in acceptable runtimes.

4.1 Sample Average Approximation

In this section, we present the methodology of the Sample Average Approximation (SAA). Since the SAA uses scenarios, we first explain how we generate a scenario. Subsequently, we explain the SAA method where our main goal is to determine how many scenarios are sufficient to be able to adequately approximate the objective value of our two-stage stochastic program. Lastly, we rigorously define the steps of the methodology in Algorithm 1.

Recall that we have three random variables that are represented by scenarios: demand D, capacity K and yield uncertainty γ. The generation of a scenario is straightforward. For the demand, we sample from a distribution for each combination of retailer, product and time period. Depending on the problem at hand, this distribution can be different per retailer, product or time period.

The capacities over time for one scenario are generated following the framework presented in Section 3.2. Firstly, the timing of the hits and their intensity are sampled using the aforementioned distributions. Secondly, the duration is calculated using the impact-duration function. Thirdly, using the hit intensity, the values of the recovery function are calculated over the duration period using Equation 3.28. Lastly, using Equation 3.29, the capacities from this hit per time period are calculated.

The yield uncertainties are sampled for each combination of DC, product and time period following the distribution as presented Equation 3.30. Similar to the sampling of the demand, the parameters of this distribution can differ between DCs, products and time periods.

However, to be able to draw meaningful conclusions, one scenario is not a good enough representation of the real world. A very large number of scenarios is necessary to entirely shape the demand and supply uncertainty. However, this makes the model intractable. We therefore follow Shapiro (2008), who propose to model the two-stage stochastic program as a deterministic MILP where the latter is solved with a sample of scenarios that is representative of the real world.

The SAA uses Monte-Carlo scenario sampling method to approximate the objective value of our two-stage stochastic program. Monte-Carlo sampling takes statistical information as basis and uses this information to generate future scenarios that can occur. One scenario is generated as described in the previous paragraph. The Monte-Carlo method then generates N independent scenarios that have equal probability of occurring. The probability $p(\omega)$ as mentioned in the objective of our model (see (3.1)) is thus equal to $1/N$. A high N however makes the model intractable. We therefore solve M deterministic MILPs with N independently generated scenarios to approximate the objective value of our two-stage stochastic program.

Furthermore, the framework by Shapiro (2008) proposes a methodology to compute the statistical gap that belongs to a combination of M and N. If the computed gap is deemed to be acceptable, the corresponding combination of M and N can be used to approximate the objective value of our two-stage stochastic program. The steps of the gap determination of the SAA are as follows, adapted from Firoozi (2018):

Algorithm 1 Gap determination of the SAA

1: Generate M samples of size N using Monte-Carlo sampling.

2: For all M samples, solve the model as formulated in Chapter 3. Let z_m^N be the objective value corresponding to sample m with size N and let $\mathbf{s}_m^N$ be the solution of the order-up-to levels, i.e. $\mathbf{s}_m^N$ is a vector containing all s_{ipt}.

3: Compute the mean and variance of the M models by the following two equations:

$$\bar{z}_{M,N} = \frac{1}{M}\sum_{m=1}^{M} z_m^N \tag{4.1}$$

$$\hat{\sigma}^2_{M,N} = \frac{1}{M(M-1)}\sum_{m=1}^{M}(z_m^N - \bar{z}_{M,N})^2 \tag{4.2}$$

4: Using the mean and variance computed in the previous Step, calculate a $100(1-\alpha)\%$ confidence upper bound based on a t-distribution, where θ is the α-critical value with $M-1$ degrees of freedom.

$$U_{M,N} = \bar{z}_{M,N} + \theta_{\alpha,M-1}\hat{\sigma}_{M,N} \tag{4.3}$$

5: Generate a sample of size N', where $N' >> N$.

6: Solve the model for all scenarios of N' with $\bar{\mathbf{s}}^N$ as fixed input values where $\bar{\mathbf{s}}^N$ is the average solution of the order-up-to levels among the M samples. Let $\hat{z}_{N'}$ be the objective value.

7: Solve the model for all individual scenarios $\omega \in N'$ with $\bar{\mathbf{s}}^N$ as fixed input values. Let $\hat{z}_\omega$ be the objective value for one scenario.

8: Calculate the $100(1-\alpha)\%$ confidence lower bound for the expectation of z^* in Equation 4.5 based on the variance calculated by Equation 4.4.

$$\hat{\sigma}^2_{N'} = \frac{1}{N'(N'-1)}\sum_{\omega=1}^{N'}(\hat{z}_\omega - \hat{z}_{N'})^2 \tag{4.4}$$

$$L_{N'} = \hat{z}_{N'} - z_\alpha \hat{\sigma}_{N'} \tag{4.5}$$

9: Calculate the optimality gap with the following Equation:

$$gap_{N,M,N'} = \frac{U_{M,N} - L_{N'}}{U_{M,N}} \times 100\% \tag{4.6}$$

If the gap is acceptable, terminate. Otherwise, increase N and/or M and return to Step 1.

4.2 Continuous approximation

In this section, we present a continuous approximation of our model which is necessary because of computational issues with the original mathematical formulation. As previously mentioned, solving the model in the extensive form, i.e. giving the model a very large number of scenarios as input and solving it as a deterministic MILP, is impossible due to the combinatorial complexity of the many possible scenarios. However, for our problem, solving a small instance for one scenario and a short time horizon already proves to be a challenge. We refer to an instance with two DCs and two retailers per DC. Even without disruptions and yield uncertainty, under a realistic parameter setting, our linear solver still reports an optimality gap of 26% after 6 hours for a time horizon of 18 time periods (τ) and 1 planning period (ψ). We can already conclude that solving the model for multiple planning periods is unrealistic. Furthermore, it is clear that the solving time of the model needs to be greatly decreased to be able to run meaningful experiments.

It is generally known that binary and integer variables increase the difficulty of Linear Programming problems. We therefore propose a continuous approximation of our ordering logic. Specifically, we approximate the (R, s, nQ) inventory system by an (R, s) model. This means that in the new situation, every review period R, an order is placed that brings the inventory position exactly to the order-up-to level s. For the model, this means that constraints (3.6) - (3.10) are replaced by the following constraint:

$$Q_{ip\tau\omega} = s_{ip\psi} - IP_{ip\tau\omega} \qquad \forall i \in \{W, N\}, p \in P, \psi \in \Psi, \tau \in T^{\psi}, \omega \in \Omega \tag{4.7}$$

We compare the continuous approximation with a straightforward heuristic that transforms the order quantities back to multiples of a fixed base quantity in the case study in Chapter 6.

Under the same setting as before, we now find acceptable runtimes where the model solves to optimality within a few seconds. However, solving the reduced MILPs resulting from the SAA method is still not possible within acceptable runtimes. We therefore use a heuristic called the Progressive Hedging Algorithm (PHA). We explain the heuristic in more detail in the next section.

4.3 Progressive Hedging Algorithm

In this section, we discuss the Progressive Hedging Algorithm (PHA). We first give a high-level explanation of the algorithm. Secondly, we discuss the PHA in more detail by presenting new notation for our problem and a pseudo-code of the algorithm. Lastly, we construct a lower bound on the solution of the heuristic and discuss possible improvements of the classical PHA.

The most desirable property of the PHA in our situation is the fact that it decomposes the problem horizontally. This means that we do not have to solve the model with all the scenarios at the same time, but that we can solve each scenario individually. Of course it is still necessary to enforce equal order-up-to levels over all scenarios. Therefore, when iteratively solving the individual scenarios, we extend the objective function of each sub-problem with two factors that penalize deviation from a common solution. After several iterations, the order-up-to levels then converge to a common value which is the end solution of the PHA. Lastly, the model is solved using these order-up-to levels to get the final objective value. Note that the model with fixed order-up-to levels and all scenarios is solvable in acceptable runtimes.

The PHA is based on the split-variable formulation of our two-stage stochastic program (Kaisermayer et al., 2021), which in short can be written as

$$\mathbf{s}^* = \operatorname{argmin}_{\overline{\mathbf{s}}} \sum_{\omega \in \Omega} p(\omega)(c_\omega \cdot \mathbf{y}_\omega) \tag{4.8}$$

$$\text{s.t. } (\mathbf{s}_\omega, \mathbf{y}_\omega) \in \mathcal{Q}_\omega, \forall \omega \in \Omega \tag{4.9}$$

$$\mathbf{s}_\omega - \overline{\mathbf{s}} = 0, \forall \omega \in \Omega \tag{4.10}$$

where c_ω denotes the cost coefficients for vector $\mathbf{y}_\omega$. Vector $\mathbf{y}_\omega$ contains all second-stage decision and auxiliary variables. The decision variables are the variables that concern lateral transshipments, whereas the auxiliary variables refer to all other variables in the model (e.g. order and substitution variables). Note that here, $\mathbf{s}$ refers to the vector containing the order-up-to levels per entity and product. Furthermore, $(\mathbf{s}_\omega, \mathbf{y}_\omega) \in \mathcal{Q}_\omega$ means that the solution should be implementable, i.e. it must satisfy all constraints of each scenario. Equation 4.10 enforces the non-anticipativity of the solution by setting all $\mathbf{s}_\omega$ equal to the common solution $\overline{\mathbf{s}}$. Since this is the constraint that prohibits the horizontal decomposition of the scenarios, the PHA applies the method of Lagrange multipliers to relax this constraint. The Lagrange multipliers which we call weights are denoted by λ_ω. A penalty factor ρ and termination threshold ϵ are input parameters for the PHA. The steps of the PHA are shown Algorithm 2, adapted from Watson & Woodruff (2011).

Algorithm 2 Progressive Hedging Algorithm

1: $k \leftarrow 0$
2: For all $\omega \in \Omega$, $\mathbf{s}_\omega^{(k)} \leftarrow \operatorname{argmin}_{\mathbf{s},\mathbf{y}_s}(c_s \cdot \mathbf{y}_s) : (\mathbf{s}, \mathbf{y}_s) \in \mathcal{Q}_\omega$
3: $\overline{\mathbf{s}}^{(k)} \leftarrow \sum_{\omega \in \Omega} p(\omega)\mathbf{s}_\omega^{(k)}$
4: For all $\omega \in \Omega$, $\lambda_\omega^{(k)} \leftarrow \rho(\mathbf{s}_\omega^{(k)} - \overline{\mathbf{s}}^{(k)})$
5: $k \leftarrow k + 1$
6: For all $\omega \in \Omega$, $\mathbf{s}_\omega^{(k)} \leftarrow \operatorname{argmin}_{\mathbf{s},\mathbf{y}_s}(\lambda_\omega^{(k-1)}\mathbf{s} + \rho/2 \left\| \mathbf{s} - \overline{\mathbf{s}}^{(k-1)} \right\|^2 + c_s \cdot \mathbf{y}_s) : \mathbf{s}, \mathbf{y}_s) \in \mathcal{Q}_\omega$
7: $\overline{\mathbf{s}}^{(k)} \leftarrow \sum_{\omega \in \Omega} p(\omega)\mathbf{s}_\omega^{(k)}$
8: For all $\omega \in \Omega$, $\lambda_\omega^{(k)} \leftarrow \lambda_\omega^{(k-1)} + \rho(\mathbf{s}_\omega^{(k)} - \overline{\mathbf{s}}^{(k)})$
9: $g^{(k)} \leftarrow \sum_{\omega \in \Omega} p(\omega) \left\| \mathbf{s}_\omega^{(k)} - \overline{\mathbf{s}}^{(k)} \right\|$
10: If $g^{(k)} > \epsilon$, then go to Step 5. Otherwise, or when the iteration limit is reached, terminate.

The first step of the PHA is the initialization of the iteration variable k. In Step 2, the individual scenarios are solved. The aggregate solution $\overline{\mathbf{s}}$ is then calculated. Step 4 updates the weights based on the penalty factor ρ. After these initialization steps, the algorithm follows more or less the same steps as in the initialization until termination; only now, in Step 6, two terms are added to the objective function compared to the previous function. Firstly, the weights λ are included to influence the first-stage decision on $\mathbf{s}$. Secondly, a proximal term is included in the form of the L_2-norm to penalize deviation of the scenario solution from the aggregate solution $\overline{\mathbf{s}}$. The algorithm terminates when all $\mathbf{s}_\omega^{(k)}$ have converged sufficiently to $\overline{\mathbf{s}}$ (i.e. $g^{(k)}$ is under a certain threshold ϵ, which we set to 0.001 for all our experiments) or when an iteration limit has been reached. The final objective value is then calculated by solving the model for all scenarios with s fixed to $\bar{s}$. Note that the duration of this last calculation is included when reporting runtimes of the algorithm.

At termination of the algorithm, we can use the weights $(\lambda_\omega^{(k)})$ of the last iteration to construct a lower bound on our solution following the method of Gade et al. (2016). Recall that our problem has an optimal value which we denote by z^*. Let $\sum_{\omega\in\Omega} p(\omega)\lambda_\omega = 0$ (componentwise). Note that this holds for all steps of the PHA (Gade et al., 2016). Furthermore, let

$$\Phi(\lambda_\omega) := \min_{(\mathbf{s}_\omega,\mathbf{y}_\omega)\in Q_\omega} (c_\omega \mathbf{y}_\omega + \lambda_\omega \mathbf{s}_\omega) \tag{4.11}$$

Then

$$\Phi(\lambda) := \sum_{\omega\in\Omega} p(\omega)\Phi(\lambda_\omega) \leq z^* \tag{4.12}$$

For the complete proof and numerical experiments that show that this lower bound indeed converges to the optimal solution, we refer the reader to Gade et al. (2016). Thus, by comparing our solution to the lower bound $\Phi(\lambda)$, we can assess the quality of the solution by defining the following gap, where z is the final objective value calculated through the PHA:

$$gap_{PHA} = \frac{|\Phi(\lambda) - z|}{z} \times 100\% \tag{4.13}$$

We use this gap as metric to make claims about the quality of the solution of the PHA. In words, the gap means that the solution of the PHA cannot be improved by more than that percentage.

As described in the Literature Review in Subsection 2.4.4, it is important to properly set the penalty factor ρ. Since runtimes are an issue for this research, we use a relatively high penalty factor to force convergence. We show in our experiments that this still leads to acceptable gaps.

Furthermore, we experimented with improvement suggestions on the classical PHA as presented in the literature. Firstly, we implemented bundling of scenarios, where instead of solving one scenario in Step 2 and 6 in Algorithm 2, multiple scenarios are solved. This is known to increase solution quality, although at greater computational burden (Gade et al., 2016). Preliminary experiments however show for our problem that the extra computational burden is not worth the increase in quality. For some settings, the complete PHA can be finished for multiple scenarios within the time it takes the model to even solve the first bundle of two scenarios in Step 2. We therefore do not use any scenario bundling.

Secondly, Kaisermayer et al. (2021) propose to use a different penalty function. The classical PHA uses the L_2-norm as penalty function. Since this makes the problem quadratic, the computational complexity for the solver greatly increases. They therefore suggest to use either the L_1-norm or the L_∞-norm. However, when implementing the first norm for our model, the problem became either unbounded or infeasible. Therefore, for our experiments, we use the classical PHA as it is presented in Algorithm 2.

Chapter 5

Numerical experiments

In this chapter, we present the numerical experiments. The goal of the experiments is to assess the quality of our solution as well as to draw robust conclusions about our proposed mitigation strategies for the continuous approximation.

Firstly, we discuss the objective and design of the experiments. Secondly, we assess the quality of our solutions by studying the PHA and SAA on one test bed. Thirdly, we conduct a sensitivity analysis on the same test bed. Lastly, we present the results of the numerical experiments for twenty-five randomly generated test beds.

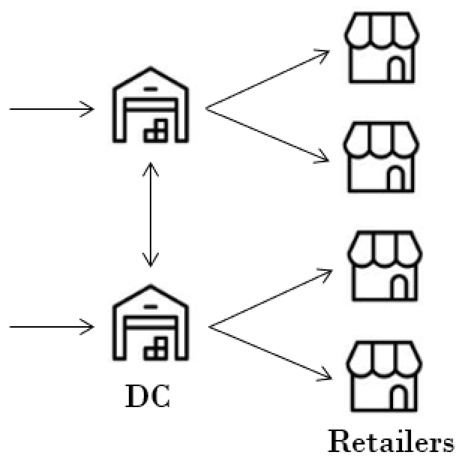
5.1 Objective and design of the experiments

In our numerical experiments, we consider a supply chain with two DCs, two retailers per DC, and two products. This supply chain is sufficiently rich to draw meaningful conclusions yet not to large to run into computational issues. The supply chain is depicted in Figure 5.1. We assume identical retailers and products. Both the leadtime between a supplier and a DC and between a DC and a retailer is 2. Demand at a retailer for a product follows a Negative Binomial distribution. We use a Negative Binomial distribution to model demand because that distribution fits the retail environment well (Agrawal & Smith, 2015). The other random variables are modeled as described in Section 4.1. We consider a time horizon of 18 days with 1 planning period. Note that this models exactly two weeks, since the first two (initialization of the system) and last two days (no orders placed anymore, since they will not be delivered on time) are non-standard. Setting of the parameters is done per experiment and explained in the respective section. Throughout the numerical experiments, we assume a linear impact-duration function with a constant of 8, i.e. $\theta = \lfloor 8\beta \rfloor$. For the recovery function, we follow the example of Klibi & Martel (2012) as given in Equation 3.28, that is:

$$r_p(\beta,\tau) = \begin{cases} 1-\beta & \tau = 1,..,\lceil 0.25\theta \rceil \\ 1-\frac{\theta-\tau+1}{\lfloor 0.75\theta \rfloor}\beta & \tau = \lceil 0.25\theta \rceil + 1,..,\theta \end{cases}$$

The objective of the experiments is to measure the effect of lateral transshipments and dual sourcing under demand substitution on the total costs, inventory levels and service level, which we denote by the fill rate (% of demand that is filled). Furthermore, we are interested in how this effect is moderated by disruptions and yield uncertainty. For this purpose, we design

Figure 5.1: Model instance



a baseline policy and compare three other policies to this benchmark. The baseline policy takes neither lateral transshipment or substitution into account. The three other policies do incorporate lateral transshipments (LT) and/or demand substitution in the model. The policies are given in Table 5.1.

Table 5.1: Policies of the experiments

Policy	LT	Substitution
Base		
NoSub	x	
NoLT		x
Both	x	x

Note that not taking substitution into account means that first you solve the model without substitution which gives a certain decision concerning the order-up-to levels. The objective value reported is then calculated by using these values as input for the model where substitution is incorporated again. This methodology enables one to state that the decision is $x\%$ worse if one does not take substitution into account. If one would only report the objective value of the model where substitution is not included, one can only state that one judges the costs $x\%$ worse and not state anything about the quality of your solution.

We evaluate the performance of the three policies ('Both', 'NoLT' or 'NoSub') by comparing the solution of a policy with the solution of the benchmark policy under the same experimental setting. Comparison of the policies is done on costs, the average inventory and the fill rate. Since we are interested in the moderating effect of disruptions and yield uncertainty, we define three levels for both. Disruptions either do not take place, occur at a high frequency with a low impact or occur at a low frequency with a high impact. For the yield uncertainty, we define no, low and high yield uncertainty. The specific parameters per level can be found in Table 5.3. We then compare the policies under the same level of disruption and yield uncertainty for every combination of the different levels by calculating the savings they achieve compared to the benchmark policy. We define the savings as

$$SAVE = \frac{k_{base} - k}{k_{base}} \times 100\% \tag{5.1}$$

where k is the performance indicator (either the costs, average inventory or fill rate). Note that for the fill rate, the numbers in the nominator are switched (i.e. $k - k_{base}$) since a higher fill rate is a saving, whereas e.g. for the costs a saving is reached when the policy has a lower cost.

The fill rate is defined as the percentage of demand that is filled. We calculate it over the entire system, meaning that we divide the total demand for all products for all retailers over the complete time horizon minus all lost sales by the total demand. Substituted demand is seen as filled demand. The average inventory is calculated per time period per entity type over the whole system. The results of the experiments are reported in Section 5.4.

The experiments were conducted on the supercomputer Snellius, a cluster computer available for Dutch universities and research institutes (SURFSara, 2022). We ran our experiments on 'thin' nodes, where each node contains two AMD EPYC 7H12 (2nd gen. Rome) multicore-CPUs, each consisting of 64 cores with 2.6 GHz and 8 memory channels. Each experiment runs on a single core. We use Python version 3.10.4 and CPLEX version 22.1.1. Since no consistent improvement could be found by changing parameters of CPLEX, we use the default settings. Each solve within the iterations of the PHA has a time limit of five minutes. When solving the full model, the time limit is one hour.

5.2 Solution quality

Firstly, we determine the number of scenarios we have to run to get acceptable SAA gaps using the methodology described in Section 4.1. Secondly, we assess the quality of our PHA using the lower bound and gap defined in Section 4.3. We look at a base case with parameters as given in Table 5.2. Note that μ and ϕ refer to the parameters of the re-parameterized Beta distribution as explained in Section 3.2.

Setting $M = 5$, $N = 50$, i.e. solving the model with 50 scenarios 5 times, leads to a SAA gap of 2.43% following the steps of Algorithm 1. We find this gap acceptable. Note that the SAA usually assumes that the model is solved to optimality. However, because of the long runtimes, we used the PHA to solve the models in Step 2 of the SAA.

With $M = 5$ and $N = 50$, we find that on average the PHA has a gap of 3.09% with a runtime of 10.75 minutes. Note that with the inclusion of lateral transshipment, substitution, yield uncertainty and disruptions, this is one of the more difficult cases for the model. Since the PHA finds an acceptable gap here, it is reasonable to assume that the quality of our solution is good across all cases.

Since the PHA finds good quality solutions and we have a small SAA gap, we conclude that we can use the PHA in combination with running 50 scenarios 5 times to draw meaningful conclusions from the experiments.

Table 5.2: Base case input

Variable/Parameter	Value/Distribution
Demand	NB(5, 0.5)
Disruptions	$\lambda = \frac{1}{10}$, $\mu = 0.5$, $\phi = 5$
Yield uncertainty	$p = 0.7$, $\mu = 0.5$, $\phi = 5$
c_w^{H}	1
c_r^{H}	3
c^{B}	5
c^{SC}	3
c^{L}	16
c^{TF}	20
c^{TV}	2
w_{kp}	0.3
Policy	Both
ρ	1000

5.3 Sensitivity analysis

In this section, we discuss the sensitivity analysis. By varying various variables and parameters of the base case shown in Table 5.2, we try to isolate the influence of the input of the model. Note that a full factorial test bed is not possible due to the many combinations of input variables and parameters, which would lead to intractable total computation times. However, the analysis presented in this section combined with the usage of randomly generated test beds in Section 5.4 gives sufficient insight in the robustness of the model.

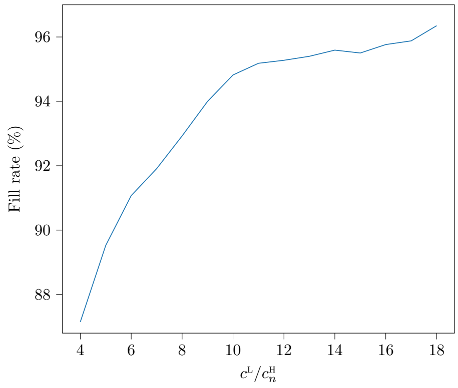
Because of the long runtimes, we could only conduct these experiments with few scenarios. For the plots, we vary the input parameter on the x-axis, while on the y-axis, we show the resulting costs or other interesting performance measure. The costs are normalized based on the objective value corresponding to the lowest x-axis value in the plot.

Firstly, we analyze the lost sales cost. In Figure 5.2a and Figure 5.2b, we plot on the x-axis the ratio between the lost sales cost and the retail holding cost and on the y-axis the fill rate and the total costs respectively. We see that it gets increasingly more costly to fill demand. Specifically, with approximately 50% more costs, only 5% more demand gets filled. Increasing the fill rate to 96% proves to be very costly. These plots show a clear trade-off between costs and service level. A decision maker should therefore use these plots to monitor this trade-off and carefully set the lost sales cost, especially since high service levels are usually desired in the retail sector.

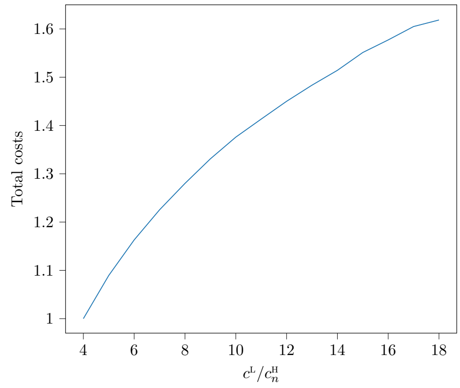
Secondly, we investigate the effect of the lateral transshipment costs. The sensitivity analysis is shown in Figure 5.3 and Figure 5.4. The LT shipment percentage is calculated by dividing the total lateral transshipments by the total deliveries from the suppliers. We observe that the effect of the lateral transshipment costs on the total costs gradually gets less. This can be explained by Figure 5.3b and Figure 5.4b, where it is visible that lateral transshipments get utilized less as the costs increase. However, it is interesting to note that this relationship is not linear. Even under high lateral transshipment costs, there is still some usage of lateral

Figure 5.2: Sensitivity analysis of the lost sales cost

(a) Fill rate vs. lost sales cost ratio



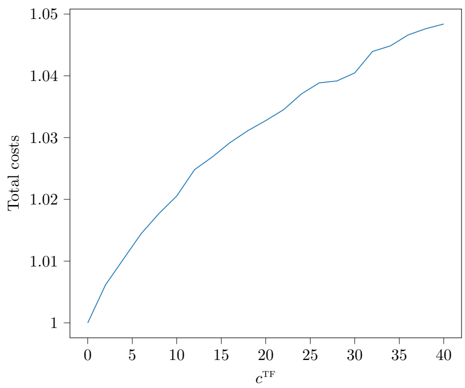
(b) Total costs vs. lost sales cost ratio



transshipment. An explanation is that in a period with disruptions and/or yield uncertainty and high demand, it is still beneficial to use lateral transshipment to minimize the internal backorder costs and especially the lost sales costs, even if lateral transshipment is costly. The effect on the total cost size of both cost parameters is however limited, as the costs only increase about 5% under high parameter setting.

Figure 5.3: Sensitivity analysis of the LT fixed cost

(a) Total costs vs. fixed LT cost



(b) Total LT shipments (%) vs. fixed LT cost

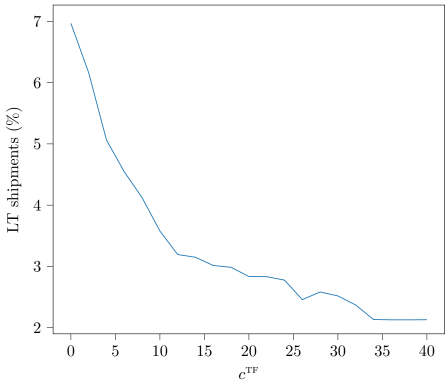
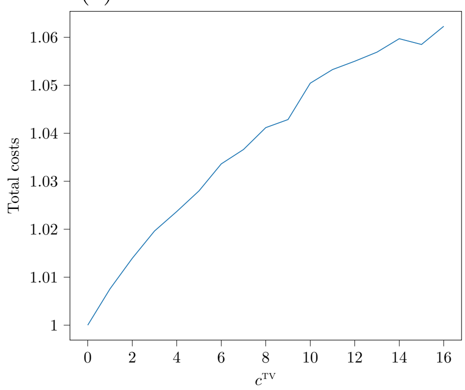
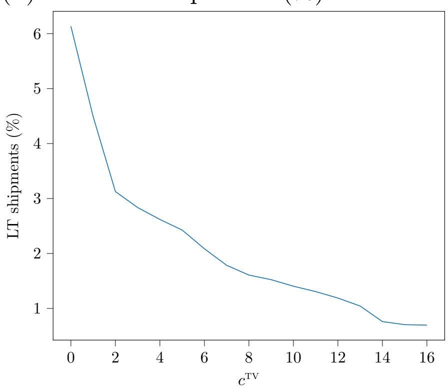


Figure 5.4: Sensitivity analysis of the LT variable cost

(a) Total costs vs. variable LT cost



(b) Total LT shipments (%) vs. variable LT cost



Thirdly, we examine the substitution parameters in Figure 5.5 and Figure 5.6. Substituted demand is given as a percentage of the total demand. We observe a linear relationship for both the substitution cost and the substitution rate with the two performance measures. Especially the substitution rate has a high influence on the total costs and substituted demand. Relatively, the substitution cost has a low influence.

Figure 5.5: Sensitivity analysis of the substitution cost

(a) Total costs vs. substitution cost (b) Substituted demand (%) vs. substitution cost

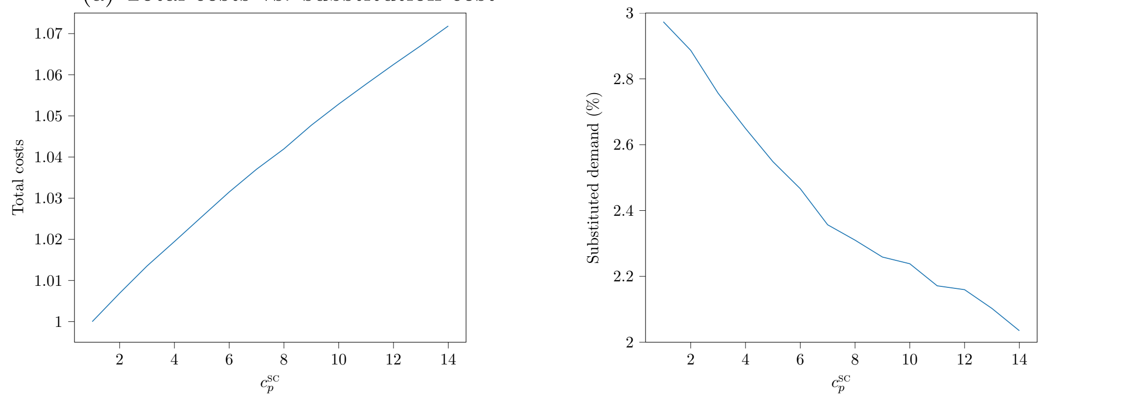
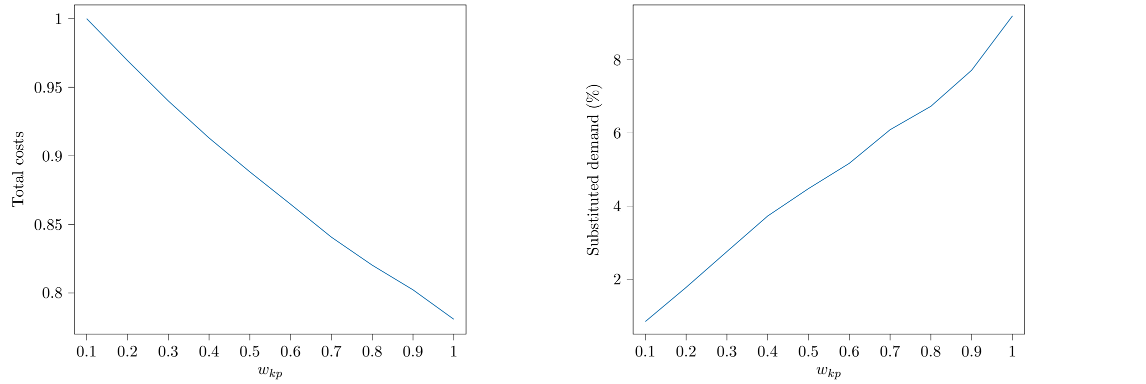


Figure 5.6: Sensitivity analysis of the substitution rate

(a) Total costs vs. substitution rate (b) Substituted demand (%) vs. substitution rate

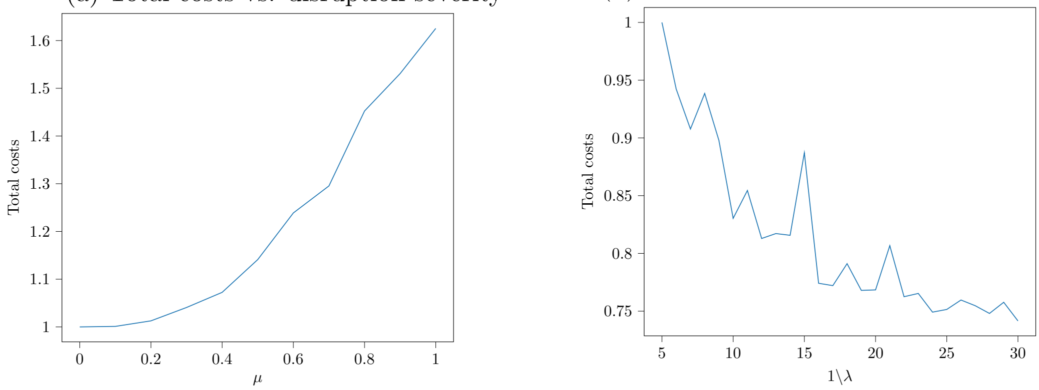


Fourthly, we are interested in the effect of varying input parameters concerning the disruptions. The graphs are shown in Figure 5.7. Varying the μ, which models the hit intensity through the re-parameterized Beta distribution, has a large influence on the costs from a certain threshold. As shown in Figure 5.7a, it seems that μ values smaller than 0.3 have almost no influence on the costs. This can be explained by the manner in which we set the base capacity. Recall that we set K_{base} equal to the highest order amount in a setting without disruptions or yield uncertainty. In a lot of the time periods, the order quantity is not close to that number. Hence a small disruption that only reduces the capacity by for example 10% will usually not have a large influence. However, as the hit intensity increases, the effect of disruptions increases as well, which subsequently leads to higher costs.

The sensitivity analysis on the arrival rate of disruptions is shown in Figure 5.7b. Note that we plot $1/\lambda$, which denotes the average time between disruptions. The line is not smooth because the experiments here are done with few scenarios, which is inadequate to model the random process of disruption arrivals. We do observe that the costs decrease as the time between disruptions increases. This effect seems to gradually lessen as the time between disruptions increases.

Figure 5.7: Sensitivity analysis of the disruptions

(a) Total costs vs. disruption severity (b) Total costs vs. time between disruptions

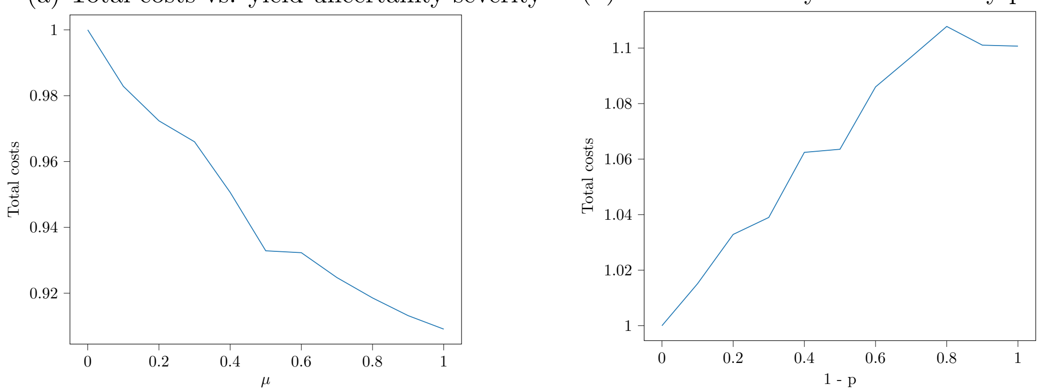


Lastly, we analyze the parameters concerning the yield uncertainty in Figure 5.8. From Figure 5.8a, we can conclude that varying the μ only has a marginal influence on the total costs. The decrease in costs from a μ value where often hardly anything is delivered to a value where almost everything is delivered, is only approximately 8%. An explanation can be that the probability that yield uncertainty occurs is relatively low in this setting, thereby limiting its influence.

The influence of the probability of yield occurring also seems limited on the total costs as can be seen in Figure 5.8b. Even when in every period a fraction of the order gets delivered, the total costs seem to only increase by approximately 10%. Based on these observations concerning the yield uncertainty, we tentatively conclude that the effect of yield uncertainty is limited and/or that our proposed mitigation strategies are effective in mitigating the risk of yield uncertainty.

Figure 5.8: Sensitivity analysis of the yield uncertainty

(a) Total costs vs. yield uncertainty severity (b) Total costs vs. yield uncertainty probability



5.4 Results for randomly generated test beds

In this section, we discuss the results of the numerical experiments. Scenarios and parameter values for a total of 25 test beds were generated according to Table 5.3. These values are based on our experience within the retail sector, the sensitivity analysis and are in accordance with the literature. Note that we model discrete demand even though the order-up-to levels are continuous but we expect that this will have no significant impact. We let $u(a, b)$ denote the uniform distribution with upper bound a and lower bound b. Note that we use a fixed lost sales cost because we are only interested in the ratio between the retail holding costs and the lost sales cost, and therefore randomizing both is not logical.

Table 5.3: Experimental input

Variable/Parameter	Value/Distribution
Demand	$NB(5,\, 0.5)$
High frequency, low impact disruptions	$\lambda = \frac{1}{10}$, $\mu = 0.5$, $\phi = 5$
Low frequency, high impact disruptions	$\lambda = \frac{1}{18}$, $\mu = 0.8$, $\phi = 5$
Low yield uncertainty	$p = 0.8$, $\mu = 0.7$, $\phi = 5$
High yield uncertainty	$p = 0.7$, $\mu = 0.5$, $\phi = 5$
c_w^{H}	1
c_r^{H}	$u(2,\, 4)$
c^{B}	$u(3,\, 7)$
c^{SC}	$u(2,\, 4)$
c^{L}	16
c^{TF}	$u(15,\, 30)$
c^{TV}	$u(1.5,\, 3)$
w_{kp}	$u(0.2,\, 0.4)$
ρ	1000

The results are reported in Table 5.4, Table 5.5 and Table 5.6. The two max columns give the maximum solution and maximum runtime respectively over all test beds. The values for one test bed are calculated by the average of the 5 (M) models with 50 (N) scenarios. The other columns show the average over the 25 test beds. For the base policy, the absolute values are reported. For the other three policies, the values in the tables refer to the savings percentage as defined in Equation 5.1. The time columns refer to the total runtime of the 5 models, i.e. one test bed.

The main observations drawn from the numerical results can be summarized as follows:

- The policy 'Both' performs well in all settings. Cost savings under disruption range from 4.5 to 7% with almost equal fill rates. Savings of total inventory range from 15 to 20%. The highest savings are achieved under high frequency, low impact disruptions. Without disruptions, cost savings amount to approximately 1-2% with slightly lower fill rates.
- By examining the 'NoSub' policy, we observe that lateral transshipments seem to have the most benefit regarding cost and inventory reductions, especially at the DC. The 'NoLT' policy shows that dual sourcing under demand substitution seems to mostly

Table 5.4: Results for no disruptions

Policy	Yield	Costs	Max costs	I (DC)	I (retailer)	I (total)	Fill rate	Time (min)	Max (min)
Base	No	2761.56	3078.11	3.96	2.93	3.28	91.72	46.92	83.59
	Low	2810.32	3130.65	4.38	2.97	3.44	91.71	43.51	70.23
	High	2941.18	3279.81	5.49	3.00	3.83	91.54	36.70	54.15
NoSub	No	0.37	0.18	3.03	1.37	2.13	0.18	83.33	153.52
	Low	0.64	0.41	5.94	2.36	4.07	0.23	108.97	317.03
	High	1.66	1.36	12.39	3.67	7.83	0.31	112.32	222.59
NoLT	No	0.89	1.34	-3.28	9.22	4.27	-1.33	72.75	136.70
	Low	0.79	1.08	-3.42	9.43	4.07	-1.29	55.09	109.08
	High	0.54	0.76	-4.55	9.00	2.35	-1.36	40.18	63.54
Both	No	1.18	1.38	-0.25	10.24	6.10	-1.16	69.38	116.16
	Low	1.39	1.42	2.28	11.45	7.56	-1.09	82.51	152.57
	High	2.25	2.12	7.83	12.33	10.18	-0.94	128.93	277.01

Table 5.5: Results for high frequency, low impact disruptions

Policy	Yield	Costs	Max costs	I (DC)	I (retailer)	I (total)	Fill rate	Time (min)	Max (min)
Base	No	3250.79	3574.61	7.73	3.16	4.68	90.17	35.27	48.39
	Low	3243.87	3570.67	7.96	3.12	4.73	90.36	35.26	62.82
	High	3377.67	3713.00	9.06	3.14	5.11	90.13	32.16	40.40
NoSub	No	4.76	4.16	23.54	4.43	14.96	0.89	78.40	193.81
	Low	4.49	3.76	23.12	4.17	14.59	1.09	75.40	114.78
	High	4.95	4.25	21.52	5.10	14.68	1.09	98.20	226.12
NoLT	No	-0.07	-0.13	-4.79	9.81	1.71	-1.25	30.37	35.99
	Low	0.10	0.13	-3.39	9.62	2.33	-1.03	30.96	36.91
	High	0.09	0.22	-4.08	9.24	1.37	-1.19	31.24	35.49
Both	No	4.97	4.34	19.53	13.29	16.67	-0.37	83.77	181.21
	Low	4.59	4.14	19.60	12.82	16.70	-0.18	91.43	279.82
	High	5.17	4.69	17.88	13.69	16.05	-0.12	96.47	212.27

Table 5.6: Results for low frequency, high impact disruptions

Policy	Yield	Costs	Max Costs	I (DC)	I (retailer)	I (total)	Fill rate	Time (min)	Max (min)
Base	No	3676.01	4003.89	11.32	3.21	5.91	89.13	26.93	32.19
	Low	3823.75	4152.47	12.83	3.23	6.43	88.81	29.67	34.51
	High	3908.88	4233.90	14.27	3.21	6.90	89.03	32.39	43.09
NoSub	No	6.96	6.10	26.33	4.36	18.27	1.29	51.33	90.89
	Low	7.02	6.26	23.15	4.64	16.95	1.11	48.17	65.40
	High	6.35	5.83	17.03	4.67	13.19	1.53	50.63	64.95
NoLT	No	0.02	-0.08	-2.92	9.66	1.52	-1.25	28.70	31.29
	Low	0.06	0.03	-1.87	9.60	1.87	-1.59	29.08	31.55
	High	0.11	0.03	-2.10	9.03	1.45	-1.25	25.13	28.60
Both	No	7.03	6.18	23.50	13.71	19.80	0.06	37.70	67.79
	Low	5.66	6.32	21.98	14.24	19.44	-0.88	34.99	47.93
	High	6.58	6.10	15.49	13.71	14.93	0.31	42.52	84.83

ensure lower inventory levels at the retailer with the same costs as the base policy, while having a small decrease in fill rate. By combining lateral transshipments and demand substitution, the 'Both' policy captures the benefits of the individual policies, leading to substantial cost and inventory savings at both echelons with equal fill rates compared to the base policy.

- The performance of the policies is moderated by disruptions. Under disruptions, savings increase compared to the setting with no disruptions. The lower savings for the setting with no disruptions can be explained by the fact that we aim to protect against supply risk. Since this setting has no disruptions, the policy only mitigates the effect of yield and demand uncertainty, thus having a lower influence. Furthermore, the policies protect especially well against low frequency, high impact disruptions. An explanation is that even high safety stock levels are usually not sufficient to prevent stock-outs from occurring when a high impact disruption takes place. When these stock-outs then occur, the proposed mitigation strategies are highly effective.

- It seems that the influence of yield uncertainty on the savings is limited. There is no clear pattern visible in the savings for the three levels of yield uncertainty. The absolute values of the costs however increase approximately 6% between no and high yield uncertainty.

- On average, the solution method is able to find good quality solutions in 25-130 minutes of computational time. Policies with lateral transshipment have a higher average solving time. This can be explained by the fact that the inclusion of lateral transshipment in the model adds some binary variables which usually makes a model more difficult to solve. Moreover, in general, the setting with low frequency, high impact disruptions has a lower solving time. Lastly, we observe that the maximum computational time can sometimes be as high as three times the average.

Based on these observations, we conclude that the proposed mitigation strategies are effective in mitigating the risk of supply uncertainty, especially when implemented together. Significant cost and inventory savings are achieved while keeping the service level constant. However, these improvements come at a higher computational cost.

Chapter 6

Case study

In this chapter, we discuss the application of our model in a case study at Jumbo Supermarkten. Firstly, we introduce the company and its supply chain. Secondly, we discuss the goal and scope of the case study. Thirdly, we present the parameter setting and scenario generation. Lastly, we report and discuss the results.

6.1 Company introduction

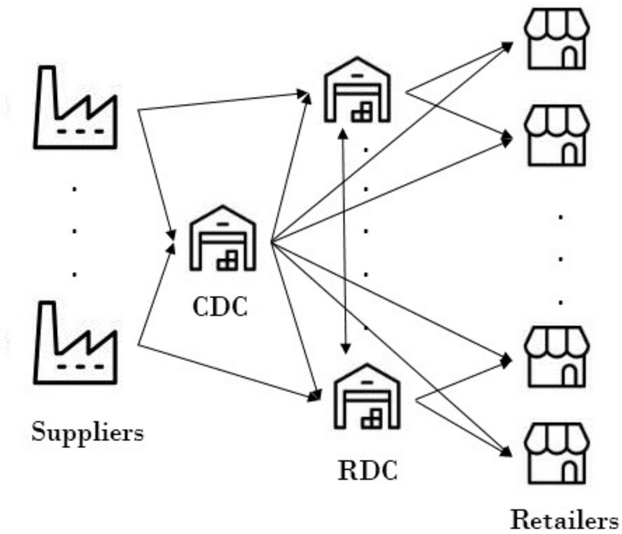
Jumbo Supermarkten is the second biggest chain of supermarkets within the Netherlands with over 700 stores and a market share of 21.8% at the end of 2021, having only Albert Heijn in front of them. Total revenue in 2021 was €9.91 billion. The headquarters, from which all operations are run, is located in Veghel. Outside of Veghel, Jumbo has 8 distribution centres with the newly opened, fully automated centre in Nieuwegein drawing the most attention. There are five different types of stores that are part of Jumbo Supermarkten: Jumbo Supermarkets (their core type of supermarkets), Jumbo City (smaller, urban supermarkets for short shopping), Jumbo Foodmarkt (a full experience, with professional chefs and fresh produce), Jumbo.com (their online department) and Jumbo Golf & Hockey. Since recent times, they are also active in Belgium, having opened almost 30 stores. These Belgian stores are supplied from the DCs in the Netherlands (Jumbo, 2021). The focus of this case study is on the normal supermarkets.

6.2 Supply chain

Jumbo has a general retailer network. It consists of external suppliers, distribution centres (DCs) and retail locations, which in this case are the supermarkets. Suppliers can be from all over the world, although the main sourcing locations are within the Netherlands, and partly Belgium. There are DCs all over the Netherlands, having various functions. For example, Jumbo's DC for frozen products is located in Raalte. DCs of interest for this research are the four regional DCs (RDC), where roughly speaking the top 1200 fast movers are placed. From those DCs, the various stores across the country are replenished. Note that one store is always assigned to one RDC and can only be supplied by that RDC. The DCs can also act as cross-docking locations, which means that the deliveries from the suppliers are not added to the inventory, but immediately loaded on new transportation to go directly to the stores.

These type of shipments are also possible between the CDC (central distribution centre) and the RDC. Cross-docking is out of scope for this research. The general supply chain structure is given in Figure Figure 6.1.

Figure 6.1: The general supply chain of Jumbo



For their replenishment of non-perishable goods, Jumbo uses what they call Multi-Echelon Replenishment (MER). This system uses the forecasts of demand of the stores as basis. Forecasting is not in scope for this case study. For one product, the sales forecast of one store is translated into an order forecast (OF) for that store, which is then aggregated for all stores into an AOF (aggregated order forecast) on DC level. This AOF is then used to determine how much the DC has to order from the suppliers based on the inventory policy.

As inventory control policy for both the stores and the DCs, Jumbo uses a standard order-up-to policy. Recall that this means that every review period R, it is checked whether the inventory position (IP) is below the order-up-to level s. When this is indeed the case, an order of size n times Q is placed, where n is an integer and Q is the order quantity (e.g. a certain product can only be ordered in multiples of 5), to take the inventory position to or above the order-up-to level. We denote this policy as an (R, s, nQ) inventory model. The order-up-to level is based on two factors: first, the forecasted demand in periods $R + L$ and second, a safety stock based on certain safety factors. One safety factor is based on the standard deviation of the AOF, where based on the normal distribution a factor (based on the desired service level) times the standard deviation is added to protect against demand fluctuations. The second factor is based on a risk profile or a combination of profiles that a product can have. It is sufficient to know that these profiles add extra safety stock. Our model sets one order-up-to level which represents the combination of these factors.

Jumbo measures the performance within the supply chain by various measures. The goal is to minimize the costs while still achieving certain service levels. The service level is given by both the ISA (in-store availability), defined as the percentage of items in the store that still have positive inventory on hand before a delivery moment, and the fill rate, defined as the percentage of demand that is fulfilled. As in the numerical experiments, we use the fill rate as the performance measure.

Furthermore, internally, Jumbo strives to deliver the retailers from the DC with a certain performance. They measure this by the OTIF (on time, in full) percentage, which is defined as the percentage of orders that arrive in the quantity that was ordered (in full), on the time that the order had to arrive (on time). Within our model, we set the internal backorder costs in such a way that this norm for this performance measure is approximately achieved.

6.3 Goal and scope

In recent times, suppliers of Jumbo have more and more difficulty supplying in a reliable way. Disruptions such as the COVID-19 pandemic and the Russian invasion of Ukraine are a major factor in this. Furthermore, yield uncertainty is of influence. Yield uncertainty can for example happen when the quality of a product is not deemed sufficient and can therefore not be taken as inventory, leading to a delivery quantity that is less than expected. We have identified three mitigation strategies, which are the same as we have used throughout the research, to mitigate supply risk. The first strategy is having more safety stock. Secondly, the possibility of lateral transshipments between DCs is identified as possible mitigation strategy. Lastly, dual sourcing in combination with demand substitution is explored as an option. The goal of this case study is to investigate whether and how much it is beneficial for Jumbo to use those mitigation strategies to protect against supply risk.

Unfortunately, because of the runtime issues already explained before, it is not possible to precisely model the inventory system and supply chain of Jumbo as it is described in the previous section. The challenge of this case study mainly lies in converting the analysis of the continuous approximation towards relevant findings for the realistic situation of fixed order quantities. For this purpose, we propose a heuristic where we accumulate the orders until it is a multiple of the fixed order size.

We use the same network as in the numerical experiments, i.e. we have four identical retailers and two DCs. We investigate products within the seasonal category 'kruidnoten'. These are identified by Jumbo as key products that need high availability. The costs of lateral transshipments can therefore be worthwhile for this product category. Furthermore, there are two brands that are sourced from different suppliers. Lastly, there is a large possibility of demand substitution between those brands. To not increase the computational burden, we only focus on two products. One is the own brand of Jumbo, while the other is a premium brand.

Lastly, we look at a period of 18 days for the same reasoning as in the numerical experiments.

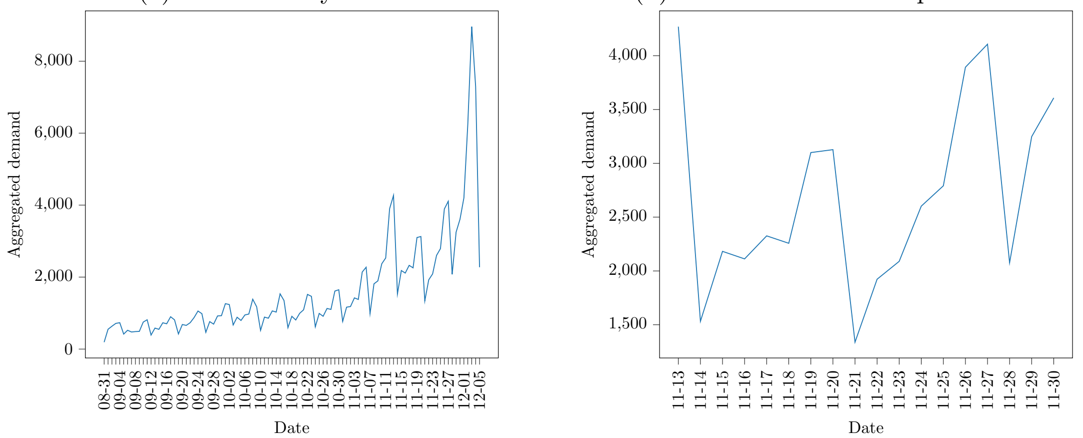
6.4 Parameter setting and scenario generation

As already mentioned before, these products fall within a seasonal category. When Sinterklaas (a national holiday in the Netherlands, rather similar to Christmas) gets closer, more kruidnoten will be sold. Furthermore, for retailers, there can be significant variations in sales across different weekdays. Both of these factors are clearly visible in the plot in Figure 6.2a.

It is clear that we can not use a stationary distribution like the Negative Binomial distribution of the numerical experiments to represent the demand a retailer faces for the complete

Figure 6.2: Aggregated demand for the premium brand per day

(a) For the full year 2021 (b) For the chosen time period in 2021



time period. We can however use the available historical demand data to fit different distributions on different days. More specifically, we take a period of 18 days just before the end of Sinterklaas because then the supply chain of Jumbo is most pressurised, and fit a different distribution per day. We take 13-11 until 30-11 for 2021. Note that we want the weekdays to align between the years, so for 2020 we look at the period of 14-11 until 01-12. The data of 2022 was deemed unrepresentative because of a large number of stock-outs. We can fit a distribution for each individual day, such as the first Saturday, Sunday, and so on, thereby accounting for both the variability across weekdays and the non-stationarity of demand. Note that we have a slight bias in our data since in case of a stock-out, lost demand does not get registered. Hence demand in reality is probably slightly higher than our reported values. However, since Jumbo strives for a fill rate of 95%, we believe that the impact of this is low.

For the fitting of the distributions, we use the procedure by Adan, Eenige & Resing (1995) who use the first two moments to fit a discrete probability distribution on a set of real numbers. The procedure works as follows. Let $\mathbf{D}$ be the vector of historical demand points as described before, e.g. belonging to the first Saturday. Based on the expectation $E(\mathbf{D})$ and coefficient of variation $c_{\mathbf{D}}$, either a Poisson distribution, a mixture of two Binomial distributions, a mixture of two Negative Binomial distributions or a mixture of two Geometric distributions is fitted. This is based on the the value a, where $a := \frac{c_{\mathbf{D}}^2 - 1}{E(\mathbf{D})}$. The eventual choice for the best distribution and the corresponding parameters is done based on the description in Appendix A, where Y is the final fitted variable that matches the first two moments of D. We refer the reader to Adan et al. (1995) for proofs and for showing the effectiveness of the procedure.

For most days, the procedure fits the Negative Binomial distribution with slightly different parameters. For a few days, the Geometric distribution is fitted. However, since the Geometric distribution is a special case of the Negative Binomial distribution, we can conclude that the demand can in general be modeled by a Negative Binomial distribution. Note that since we look only at a small period, there is at most a small trend in the data as can be seen in Figure 6.2b.

For the capacities of the suppliers and the yield uncertainties, the parameters of the distributions are set together with an expert panel within Jumbo because relevant data is not available for the products in scope. Based on their experience, we concluded that disruptions take place relatively often, but that the intensity is not that high. For the yield uncertainty, in the time period we are focusing on, we determined that the full amount is delivered quite often and that if it is not the full amount, the amount is distributed around 50-70% of the order.

Together with the same expert panel as mentioned before, the other parameters are set. The choices are made with the goal to achieve different norms Jumbo has for their KPIs and to let the model show desired behaviour. The final values of the parameters can be found in Table 6.1. If there is no subscript, the parameters are equal for both products. Otherwise, 1 refers to the premium brand and 2 to the Jumbo brand. The parameters of the SAA are equal to that of the numerical study, i.e. $M = 5$ and $N = 50$. The functions regarding the disruption modeling framework are set in the same way as in the numerical experiments. Hence, $\theta = \lfloor 8\beta \rfloor$ and the recovery function is equal to the example of Klibi & Martel (2012) as given in Equation 3.28. We need to set a higher penalty factor to ensure convergence because for the new model, three instead of one s values have to converge. The next paragraph explains this in more detail. The penalty factor ρ is thus set to 5000.

Table 6.1: Case study input

Variable/Parameter	Value/Distribution
Demand	Fit by Adan et al. (1995)
Disruptions	$\lambda = \frac{1}{10}$, $\mu = 0.6$, $\phi = 5$
Yield uncertainty	$p = 0.7$, $\mu = 0.65$, $\phi = 5$
c_w^{H}	1
c_r^{H}	3
c^{B}	5
c^{SC}	8
c_1^{L}, c_2^{L}	24, 36
c^{TF}	40
c^{TV}	2
w_{12}, w_{21}	0.2, 0.4
fq_{n1}, fq_{n2}, fq_{w1}, fq_{w2}	10, 12, 20, 24
ρ	5000

In addition to the generation of scenarios and the setting of parameters, it is also necessary to modify the model for the case study. Recall that we concluded that demand is seasonal per weekday. Intuitively, setting one order-up-to level for the whole time horizon does not make sense since the level will probably be too high for the days with lower demand (e.g. Monday, Tuesday) but too low for other days (e.g. Friday, Saturday). It proved to be computationally impossible to set a different s per day. We therefore propose to define three levels for the order-up-to level, i.e. s_{ip} becomes s_{ipl} where l is either low (Sunday, Monday, Tuesday), medium (Wednesday, Thursday) or high (Friday, Saturday). The order at time τ then depends on expected demand, e.g. if τ is a Monday and all leadtimes are equal to two, the retailer places an order based on the medium order-up-to level ($\tau + L_n$ is a Wednesday) and the DC based on the high order-up-to level ($\tau + L_n + L_w$ is a Friday). Note that the s for each time period still needs to be equal across all scenarios.

Furthermore, in this case study, we are interested in translating our continuous approximation back to an inventory model where orders are placed in multiples of a fixed base quantity. We propose a simple heuristic that accumulates the orders from the continuous model into multiples of the fixed quantity. In what follows, we omit all subscripts for ease of exposition. Let $\mathbf{Q}$ be a vector of orders placed over the time horizon for one entity, Q an element of that vector, fq the fixed quantity and q the integer multiple of the fixed quantity. Furthermore, we define a variable E that denotes how much the fixed quantity overshoots the orders of the continuous model. In words, the heuristic ensures that each period the order is large enough to cover at least the continuous order amount while taking into account whether there was an overshoot because of the fixed size in the previous period.

Let E be initialized to zero. The new order size Q_{new} is equal to qfq where q is the smallest integer such that qfq is greater than or equal to $Q - E$. The overshoot is then updated to $E = E - Q + Q_{new}$. By following these steps for the whole time horizon, we calculate the new order quantities. After using this heuristic, we recalculate the objective value with the order sizes set to the newly calculated values Q_{new}. An example of the application of the heuristic is given in Table 6.2, where the fixed quantity is 20. Note that we let the orders be integers for this example for straightforward computations, but that this is usually not the case for the order quantities that follow from the continuous approximation.

Table 6.2: Example of order heuristic

Time (τ)	1	2	3	4	5	6	7	8
Continuous (R, s)	2	3	12	6	6	9	7	15
Fixed quantity (R, s, nQ)	20	0	0	20	0	0	20	0

The results of the experiments for the continuous approximation are shown in Table 6.3 and for the fixed base quantity order model in Table 6.4. The savings shown for the 'NoSub', 'NoLT' and 'Both' policies are calculated by Equation 5.1.

Table 6.3: Results for the continuous approximation

Policy	Costs	I (DC)	I (retailer)	I (total)	Fill rate p1	Fill rate p2	Time (h)
Base	5066.59	7.53	5.37	6.09	89.07	90.79	3.23
NoSub	0.44	11.29	2.61	6.08	0.97	1.12	35.09*
NoLT	0.49	-2.39	7.82	3.61	-2.57	-0.18	7.25
Both	1.60	8.234	11.17	10.01	-1.79	0.90	151.28*

* The PHA did not converge fully within the iteration limit for eight of the cases. However, since the convergence value is very close to the termination value, we believe this does not impact the solution quality greatly.

Table 6.4: Results for the fixed base quantity order model

Policy	Costs	I (DC)	I (retailer)	I (total)	Fill rate p1	Fill rate p2
Base	5378.60	15.15	7.29	9.91	93.64	97.71
NoSub	6.37	9.17	10.70	9.89	1.27	0.79
NoLT	3.88	9.04	3.70	6.36	-0.54	-0.01
Both	8.65	13.80	13.58	13.72	0.06	0.69

For the continuous order logic, we can draw similar conclusions as for the numerical experiments, although the savings are less substantial. The 'Both' policy seems to perform well with slight cost savings, significant inventory reductions, a slightly worse fill rate for the premium brand but a slightly better fill rate for the Jumbo brand. Regarding costs, both individual strategies have equal savings. We observe the same patterns as before regarding the inventory and fill rate. The 'NoSub' policy has inventory reductions for both echelons, whereas the 'NoLT' policy has substantial inventory reductions at the retailer. The combination of the two mitigation strategies takes the advantages of both policies and thereby performs well.

An explanation for the lower cost savings compared to the continuous experiments is the fact that the cost parameters regarding the mitigation strategies are set at a much higher level compared to the numerical experiments. The lateral transshipment fixed costs are set at such a high level because Jumbo does not have any standard redistribution of stocks among its warehouses, which makes lateral transshipments very costly. The substitution costs are set at this level because for this product category, customer satisfaction is key and substitution should thus be minimal.

Even though the 'Both' policy achieves the best savings, the computational burden is very high at a total computational time of 150 hours, which is significantly more than the other three policies. The higher computation times can be explained by the fact that three s values have to be optimised compared to only one for the numerical experiments. Furthermore, incorporating both lateral transshipments and demand substitution makes the model in each iteration of the PHA more difficult to solve.

For the fixed base quantity order model, we observe more significant savings for the three policies. First however it is interesting to note that compared to the continuous order system, the heuristic has less costs for all policies except the base policy at higher fill rates, although total inventory is higher. Upon closer inspection of the output, an explanation seems to be that the higher order quantities sometimes act as a buffer against heavy disruptions, thereby offsetting the substantially higher inventory costs by lower lost sales and internal backorder costs.

Again, the 'Both' policy performs the best of all policies. Significant cost and inventory savings are achieved while keeping service level constant. The other two policies also exhibit considerable savings. 'NoSub' outperforms the 'NoLT' policy which is as expected since the benefit of the 'NoLT' policy, setting the order-up-to levels exactly to the right level considering substitution, is lessened because precise order-up-to levels have less influence because of the ordering heuristic. However, the combination of lateral transshipment and substitution still achieves a better performance than the other two policies.

From the results of the case study, we conclude that the 'Both' policy could lead to significant savings for Jumbo.

Chapter 7

Conclusion and discussion

In this study, we have considered a multi-echelon, multi-product, multi-period retailing network where the supply can be influenced by disruptions and is subject to yield uncertainty. To protect against supply risk, we have evaluated three mitigation strategies in the form of lateral transshipments, dual sourcing under demand substitution and inventory buffers. We have modeled the system as a two-stage stochastic program, where in the first stage, the order-up-to levels are decided for a planning period and in the second stage, lateral transshipments can be used as a recourse action. Furthermore, we have presented a generic and robust methodology to model both supply disruptions and yield uncertainty that has merit in itself beyond this specific application in the retailing sector.

Since the original mathematical formulation where orders are placed in fixed base quantities was intractable, we have proposed a continuous approximation of the order logic that is solvable. The objective value of the resulting formulation was approximated by the SAA. The deterministic MILPs that resulted from the SAA were solved by the PHA. The numerical experiments show that considerable cost and inventory savings can be achieved under equal service levels when both mitigation strategies are implemented.

In this research, we have focused on the retailing environment. However, the generality of the model allows for the modeling of similar two-echelon inventory systems within other environments. One example is the blood supply chain, where availability is also key and substitution between different products takes place often (Salehi et al., 2019). Moreover, the model itself is also adaptable to other situations. For example, instead of lost sales at the lowest echelon, backorders can be implemented. Assumptions such as that one retailer is assigned to one warehouse can be relaxed to fit other supply chains.

There are several directions in which this research can be extended. The first direction of future research concerns the PHA. One of the main advantages of this algorithm is the possibility of parallelization. Recall that the PHA decomposes the problem horizontally, making each scenario individually solvable. Per iteration, each of these individual solves can be done in parallel, which can greatly help in reducing the computation times.

Secondly, it can be interesting to not only include supply disruptions, but also disruptions throughout the supply chain. Examples can be a disruption at the DC or at the transport lanes between any of the entities. The proposed mitigation strategies could protect against these risks as well.

Lastly, it can be worthwhile to implement a lateral transshipment policy. Currently, the model can freely decide on the lateral transshipments, which can lead to great fluctuations depending on different inputs and situations. However, this is not desirable in practice. A policy, such as the three parameter lateral transshipment policy presented by Avci (2019), can make the proposed lateral transshipment mitigation strategy more robust and implementable in practice. The three parameter policy defines two thresholds per entity. If the inventory position of an entity drops below the lower threshold, the entity will request a lateral transshipment from another entity whose inventory position is above the second, higher threshold.

We have conducted a case study at Jumbo Supermarkten to show the benefits of the proposed mitigation strategies in practice. For the continuous order logic, similar conclusions as for the numerical experiments were drawn, although the savings were less substantial. Furthermore, we presented a heuristic to translate the continuous order logic to the original order logic where orders are placed in multiples of fixed base quantities. For the fixed order quantity model, considerable savings were found. Costs and inventory were substantially reduced under almost equal fill rates. We therefore advise Jumbo to use this study as a basis to implement the mitigation strategies into their inventory system. Specifically, they should investigate two ways that could improve the model in their situation.

The first improvement is to incorporate forecasting, for which they already have processes in place as explained in Section 6.2, into the model. In our study, the order-up-to levels are given as a single number. However, these can also be defined as forecast plus some safety stock. The model could then use the forecast as input and optimise the safety stock setting, thereby adequately using information that is already available within Jumbo.

The second improvement has previously been proposed as topic of future research. Following the same reasoning, we believe that to be able to successfully implement lateral transshipments in practice, Jumbo should investigate designing a general lateral transshipment policy. Such a policy should also incorporate demand substitution.

Furthermore, we believe that the disruption and yield uncertainty modeling framework can have benefits for Jumbo in other applications as well. One example is supplier selection. When deciding between two suppliers that have different risk profiles, the framework can help in giving insight into the supply risk for Jumbo concerning these suppliers, thereby supporting the decision to source at one or both of the suppliers. Another example could be product assortment decisions.

A last remark for Jumbo regards their data collection. Currently, only dynamic delivery data is stored. Dynamic in this sense means that when an employee of Jumbo moves the delivery date because a supplier has notified them beforehand that a delivery will not be successful, the delivery is still counted as in full in time if the supplier delivers on the new date. However, to properly estimate disruption and yield uncertainty parameters, the static delivery data can be more insightful. For the static delivery data, the promised delivery date is not moved, and if the order is not delivered on that date, it is counted as too late. We therefore recommend that Jumbo also collects and stores the static delivery data to be able to more adequately model disruptions and yield uncertainty.

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Appendix A

Demand fitting procedure

1. If $-1/k \leq a \leq -1/(k+1)$ for certain $k = 1, 2, 3, \ldots$, then

$$Y = \begin{cases} BIN(k,p) & \text{w.p. } q, \\ BIN(k+1,p) & \text{w.p. } 1-q, \end{cases}$$

where

$$q = \frac{1 + a(1+k) + \sqrt{-ak(1+k) - k}}{1+a}, p = \frac{E(\mathbf{D})}{k+1-q}.$$

2. If $a = 0$, then $Y = POIS(\lambda)$ with $\lambda = E(\mathbf{D})$.
3. If $1/(k+1) \leq a \leq 1/k$ for certain $k = 1, 2, 3, \ldots$, then

$$Y = \begin{cases} NB(k,p) & \text{w.p. } q, \\ NB(k+1,p) & \text{w.p. } 1-q, \end{cases}$$

where

$$q = \frac{(1+k)a - \sqrt{(1+k)(1-ak)}}{1+a}, p = \frac{E(\mathbf{D})}{k+1-q+E(\mathbf{D})}.$$

4. If $a \geq 1$, then

$$Y = \begin{cases} GEO\,(p_1) & \text{w.p.} \quad q_1 \\ GEO\,(p_2) & \text{w.p.} \quad q_2 \end{cases}$$

where

$$p_1 = \frac{E(\mathbf{D})\left[1 + a + \sqrt{a^2-1}\right]}{2 + E(\mathbf{D})\left[1 + a + \sqrt{a^2-1}\right]}, \quad q_1 = \frac{1}{1 + a + \sqrt{a^2-1}},$$

$$p_2 = \frac{E(\mathbf{D})\left[1 + a - \sqrt{a^2-1}\right]}{2 + E(\mathbf{D})\left[1 + a - \sqrt{a^2-1}\right]}, \quad q_2 = \frac{1}{1 + a - \sqrt{a^2-1}}.$$