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Reducing utilized storage capacity for non-stationary demand incorporating honeycombing

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Eindhoven University of Technology

Department of Industrial Engineering and Innovation Sciences
Operations Management and Logistics

Reducing utilized storage capacity for non-stationary demand incorporating honeycombing

Master Thesis

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Abstract

In practice, many companies experience non-stationary customer demand. Moreover, because of the increasing importance of e-commerce and emerging economy resulting in a high variety of products, warehouse storage locations are becoming more scarce as one type of SKU is stored per warehouse location. Because in practice often incoming shipments are stored in a new location and not merged with an already occupied location by the same SKU, honeycombing occurs. In this study, a multi-item inventory model under honeycombing and non-stationary demand restricted by warehouse capacity storage locations is presented. Three models are formulated in which the first model represents the current situation. The second model modifies the reorder level per SKU with as objective to minimize expected inventory costs (containing ordering, holding and shortage costs) restricted by a minimum fill rate, weighted fill rate and occupied warehouse storage locations. This model is solved using a MILP. The third model is formulated in the same manner as the second and modifies, besides the reorder level, the MOQ and IOQ per SKU. This model is solved using a combination of a greedy heuristic and a MILP. To cope with non-stationary demand, the horizon is divided into phases. The phases are set in such a way that within a phase the majority of the demand of the SKUs is stationary. A method for smooth transition between the phases is proposed to ensure there is enough available stock (when changing to a higher demand phase) and inventory is not unnecessarily built up (when changing to a lower demand phase). The results of the models (using mathematical formulas and simulation) show how the number of used warehouse storage locations and inventory costs can strongly be decreased while still maintaining the fill rate. Moreover, the smooth transition between phases results in increased inventory and used locations while having lower total inventory costs.

Executive Summary

Introduction

This master thesis project is conducted in cooperation with Coolbue, a Dutch e-commerce company. The retailer offers a broad product assortment and is active in the Netherlands, Belgium and Germany. The scope of this project is limited to mature products with the size classification corresponding to product group Parcel Large, ordered via planned or regular replenishment and have been sold at least in the last three years.

A non-stationary product demand is observed with a seasonal pattern with, for most of the SKUs, a high peak towards the end of the year. The inventory is stored at one stocking point (warehouse in Tilburg). Moreover, the context is characterized by lost sales, positive lead times and multi-period.

Problem statement

Coolblue has recently undergone significant growth and achieved record sales. Besides, due to expansion to Germany, acquisition of German version products is necessary. Therefore, to meet customer demand, an increasing number of products must be stored in the warehouse as products can only be sold when they are available in the warehouse. Unfortunately, due to experience, incorrect forecasts, increased sales, and uncertain supplier lead times, supply planners order higher product quantities. However, when sales are not as high as expected resulting in excess inventory. The high number of SKUs and product quantities lead to a high warehouse utilization resulting in too few available locations for other SKUs. This results in unnecessary stock-outs, high inventory costs, and less available cash to make investments. All in all, the number of SKUs (assortment) and product quantities are increasing while the warehouse still has the same size, resulting in capacity problems and a need to use the available space more efficiently.

Currently, to determine the quantity and timing of the order, for some SKUs, no logic is taken into account concerning service level, costs, margin, value and, for all SKUs, used warehouse space, or size. Accordingly, this research focuses on designing a system to control inventories for Parcel Large products and especially takes into account product characteristics and warehouse space capacity. A system to control inventories in which less inventory is needed can significantly reduce the needed storage locations since the products within Parcel Large are of significant size. Moreover, better balancing the available warehouse space among products can reduce the number of stock-outs. Therefore, the main research question is formulated as follows:

How to design a system to control inventories in which expected inventory costs are minimized and take into account warehouse space capacity?

Model design

To design an inventory control system for this context, assumptions are made regarding lost sales and the inventory system used. Lost sales are assumed but can be relaxed using a backorder model because of the calculated number of outstanding orders and relative demand uncertainty during lead time and review period. The used inventory system is an (R,s,nQ) inventory control policy.

In this study, three models are designed. The first proposed model, referred to as current situation, sets the reorder level based on the target service level. This model does not take into account the capacity restriction and is used as a reflection of the current situation.

The second model (hereafter ss-model) minimizes expected inventory costs per time unit restricted by service level constraints and a warehouse capacity constraint to restrict the average number of used locations per time unit considering honeycombing. Expected inventory costs are modeled as the sum of the holding, ordering and shortage costs. The service level constraints refer to a minimum and (volume-based) weighted target fill rate. Based on the decision variable reorder levels per SKU, a safety stock can be calculated per SKU. This model is solved using a MILP.

In the third model (hereafter IOQ/MOQ-model) uses instead of an (R,s,nQ) inventory control policy, an (R,s, MOQ,IOQ) inventory control policy as space is the problem and solely ordering full pallets is not preferable. The IOQ/MOQ-model determines, besides the reorder level, the IOQ and MOQ per SKU by minimizing the expected inventory costs constrained by warehouse capacity, minimum and (volume-based) weighted target fill rate. A combination of a greedy heuristic and MILP is used to solve this model.

Because of observed non-stationary stochastic demand, the horizon is divided into phases. For each phase per SKU is the expected demand determined and used as input for the model. The changeover between phases is of high importance because of the possibility of having too high or low inventory levels. Therefore, a modified reorder level is used to smoothen the transition to a new phase. When the lead time and review period exceeds the length of the phase, the modified reorder level is calculated as the sum of the demand needed for the remaining days in the current phase, the demand for the remaining days before the order will arrive in the next phase and the maximum safety stock in the current phase or the weighted safety stock.

Results

The models are compared using theoretical mathematical formulas and a simulation. A discrete-event simulation is built to evaluate the models performance taking into account non-stationary demand. A full year is simulated on a daily level including the distinction between week and weekend days.

Using mathematical formulas, the safety stock model shows a 4% cost decrease and the IOQ/-MOQ model a 37% cost decrease compared to the modeled current situation. Using the simulation, the safety stock model shows a cost decrease of 3% and the IOQ/MOQ a decrease of 20%. Implementing a modified reorder level to smoothen this transition results in even higher costs decreases (3 % for the safety stock model and 22% for the IOQ/MOQ model) relative to the current situation (with no modified reorder levels). The computational time of the IOQ/MOQ model is higher than for the safety stock model but still possible to implement in practice (6053 s for 868 SKUsvs 3045 s for 868 SKUs). Therefore, from a cost and location perspective, the IOQ/MOQ model is the most preferable method to control inventories.

Conclusion

Through the utilization of mathematical formulas and simulations, the models demonstrate that it is possible to significantly reduce the number of warehouse storage locations and inventory costs without compromising the fill rate. Additionally, the transition between phases leads to higher inventory and storage utilization rates, while simultaneously lowering costs. In conclusion, the redesigned inventory control model follows an (R,s,MOQ,IOQ) inventory policy with dynamic values for s, MOQ and IOQ. These values change every phase and are set using the designed IOQ/MOQ model. This model minimizes expected inventory costs restricted by the number of storage locations and fill rate constraints. Honeycombing is taking into account to not overestimate the available warehouse locations.

The company is advised to implement the IOQ/MOQ model for the products in scope as this has been shown to have high potential. A disadvantage of this model is the increase in inventory on hand which can be problematic in terms of risk as is also agreed on with inventory experts from Coolblue. Comparing the inventory levels of the model with the inventory levels in practice still shows a significant reduction. The model is demonstrated for a subset of Parcel Large products but can also be implemented for other products.

Preface

This Master's Thesis Project is the final step in completing the Master Operations Management and Logistics at Eindhoven University of Technology. Hereby, I would like to express my sincere gratitude towards everyone who supported me during this period.

First, I would like to thank Karel van Donselaar, my mentor throughout this academic adventure. His experience and critical view taught me how to select relevant concepts in the big complex world of inventory control and brought the research to a higher level. Besides, his enthusiasm and passion for inventory control within retail motivated me and made the meetings memorable. Also, I would like to thank Nico Dellaert for his time and advice on how to improve my graduation project. I greatly appreciated his critical questions, feedback and insights to the 'real' world which improved the quality of this research and report

I would like to thank my company supervisor, Elroy van Seventer for providing me with all relevant information and for our valuable discussions that helped me to understand the operations at Coolblue. His experience and knowledge of the operations contributed to the quality of my research. Moreover, I would like to thank all other colleagues from the Purchasing team for answering all my questions and for the many enjoyable moments.

Finally, I would like to thank my boyfriend Bart, friends and family for their unconditional support. Your company has made my student life unforgettable and your support and endless trust in my abilities have helped me to complete this project.

Studying at Eindhoven University of Technology has been a great experience, and I am proud of the outcome of this master thesis. I hope you will enjoy reading it.

Daphne Mey

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List of Abbreviations

SKU	Stock Keeping Unit
CoV	Coefficient of Variation
DOS	Duration of Stay
MOQ	Minimum Order Quantity
IOQ	Incremental Order Quantity
Q	Fixed Replenishment Quantity
s	Reorder Level
OS	Order Size
ss	Safety Stock
L	Lead Time
R	Review Period
IP	Inventory Position
KPIs	Key Performance Indicators
MILP	Mixed-Integer Linear Programming
FIFO	First in, First out

Chapter 1

Introduction

An increase in the importance of filling orders within 24 hours in the new global economy, has increased the importance of warehouse operations and inventory management (Gagliardi et al., 2008). When the inventory is lower than demand and thus not having products in stock, stock-outs occur. This may result in loss of customer satisfaction and delays in other parts of the system (Boulaksil et al., 2009) (Brito and de Almeida, 2012). Alternatively, customers may decide to buy their products from the competitor leading to lost sales and a decrease in market share (Zinn and Liu, 2001) (Gruen et al., 2002). Besides demand uncertainties, a company can be exposed to other uncertainties. Therefore, the role of inventory is serving as the buffer between demand and supply important (Williams and Tokar, 2008). When there is insufficient warehouse space, ordering trade-offs have to be made among products; which products need to be available in stock and what quantities are needed regarding minimizing costs, minimizing used space, maximizing service level, and maximizing possible yields.

Inventory management and control is a broad research domain and closely influences other areas of the operations management domain (Silver, 1981). The main focus is on the trade-off between costs and profitability of the operations while meeting customer demand. According to Silver (1981), the three key questions are: *'(i) How often should the inventory status be determined, that is, what is the review interval? (ii) When should a replenishment order be placed? (iii) How large should the replenishment order be?'*. Within this field, a division is often made between single-item and multi-item inventory models. Single-item inventory models do not take into account the dependencies between stock keeping units (SKUs) while multi-item inventory models do take into account these dependencies. Item interdependencies can have different causes such as budget or space constraints, complementary products, substitution or needed coordination to save costs.

Products are often stored in warehouses. According to Heragu et al. (2005), a warehouse has two primary functions; (i) temporary storage and protection of goods and (ii) value-added services like packaging, repairs, and inspection. Due to the increasing importance of e-commerce and emerging economy resulting in a high variety of products in small quantities under uncertain demand, warehouse operations have become more complex (De Koster et al., 2017). According to De Koster et al. (2017), optimizing the utilization of space is one of the main goals in warehouse design and operations. Within a warehouse, the largest standardized material handling unit is a pallet (Bartholdi and Hackman, 2008). Due to storing in one location one specific SKU, honeycombing can occur. Honeycombing refers to loss of space because of an empty but unusable position. While Bartholdi and Hackman (2008) refers to the occurrence of honeycombing within pallet lanes,Van Donselaar and Broekmeulen (2022) generalize the effect to a direct accessible location.

This research is in the intersection of inventory management and warehouse management. Because of warehouses not having an infinite size and resulting product dependencies, there is an emphasis on multi-item inventory models constrained by capacity. This research aims to incorporate warehouse capacity and the occurrence of honeycombing in the to-be-made inventory decisions. In the remainder of this section, the cooperating company is introduced. Moreover, the problem is defined and corresponding research questions are formulated. Then the scope is discussed followed by the methodology.

1.1 Company description

The project is conducted in cooperation with team Purchasing of Coolblue. The overall goal of this team is *'to create and implement easy to use, cost-efficient and sustainable solutions to ensure the stock required to realize commercial goals'* (Coolblue, 2021). This is done by balancing the right products on stock, efficient processing of orders, and efficient use of capital and warehouse space. In this section, an introduction is given to the company, the warehouse and the process of ordering products.

1.2 Company Coolblue

Coolblue was founded in 1999 by Pieter Zwart, Paul de Jong, and Bart Kuijpers. The company is a Dutch retailer and offers a broad assortment of products ranging from washing machines to laptops and solar panels to telescopes in the business-to-business market as well as business-to-customer market. They offer their services online and have an in-house delivery service to deliver specific packages by electric bike (CoolblueFietst) or van (CoolblueBezorgt). The remainder of the packages is delivered by distribution partners. Besides the webshop, customers can buy products in their 22 physical stores. Coolblue aims to amaze the customer and surpass the customer's expectations (*'Anything for a smile'*). This can be seen in providing the best possible customer journey. The retailer is active in the Netherlands, Belgium, and since July 2020 Germany. In 2022, the company realized annual sales of 2.35 billion euros and employed more than 6000 people with more than 83 nationalities (Coolblue, 2022). The company itself is headquartered in Rotterdam.

1.2.1 Warehouse

Coolblue has one warehouse located in Tilburg with a size of 88,000 m^2. In the warehouse, shipments are received and stored, customer orders are made ready to send and returned products are received and checked. Also, the physical stores are replenished from the warehouse. The process of receiving a shipment and sending of a customer order is visualized using BPMN language in Appendix A.

The products are divided into four product groups - Autostore, Parcel Large, Parcel XL, and 'Whitegoods' based on product size and weight (see Table 1.1 for criteria). Product group Autostore includes for example headphones, and screen protectors. Product group Parcel Large includes products such as microwaves and vacuum cleaners. Product group Parcel XL includes products like lawnmowers, and BBQs. Product group 'Whitegoods' includes amongst others washing machines and dryers, with no set size or weight limit.

Table 1.1: Overview of product groups based on product size and weight

Product group	**Size class**	**Maximum size (lxwxh) in cm**	**Maximum weight (in kg)**
Autostore	S	31x21.5x2.8	1.8
	M	60x40x40	30
Parcel Large	L1	100x70x58	30
	L2	175x78x58	30
XL	XL	400x90x60	35
	XXL	max. 1.1 m^3	108
	XXXL		170

The warehouse contains designated hall(s) for each product group. However, in exceptional cases such as high demand due to discounts, exceptions can be made. Only one specific product type is stored at each location and the locations are not permanent for unique product types. Note that Parcel Large is used to refer to the product group as well as the inbound location.

Because of the high utilization of Parcel Large products and the expectation of encountering future problems within this group due to future growth, this research focuses on Parcel Large products. For this group, the significant product size may result in more locations needed per SKU compared to smaller sized products. On September 16 2022, the Parcel Large product group contains 7684 SKUs, occupying a total of 17859 locations, out of which 14051 locations are currently in use storing 194706 products. The locations can be categorized into active and reserve racking, denoting pick area and storage area locations respectively. Additionally, there are exceptional locations, such as those for bulk products. The height of locations ranges from 150 cm to 230 cm, with the number of locations subject to change over time due to modifications in the warehouse structure. Table 1.2 presents the current situation of Parcel Large products, including the occupied locations, location utilization, stored SKUs, number of products, and average stored products per occupied location, all as of September 16, 2022.

Table 1.2: Specification of number of locations within Parcel Large hall

Inbound location	**Location**	**Total locations**	**Number occupied locations**	**Location utilization (in %)**	**Stored SKUs**	**Number stored products**	**Average product/ occupied location**	**Average product/ SKU**
Parcel Large	Active (pick area)	3171	2901	91.5	2592	20985	7	8
	Reserve (storage area)	14450	11079	76.7	5560	169078	15	30
	Floor(bulk)	46	44	95.7	44	4600	105	105
	Upright	192	27	14.1	22	43	2	2

1.2.2 Ordering of products

Coolblue has category teams responsible for the sales of products. The aim of the supply planner is to maximize availability while minimizing costs. There are three ways of ordering products; manual, semi-manual, and automatic. For a visualization of this process and more information see Appendix A.

Semi-manual and automatic orders are triggered by a replenishment proposal based on a replenishment logic. Based on a target fill rate, an order moment (using equation 2.3 in the article by Van Donselaar and Broekmeulen (2014)) and order quantity (using EOQ formula) is calculated every day. This may result in a replenishment proposal. The supply planner can modify this proposal.

Manual orders are made in cases of volume discounts (deals), planned marketing campaigns, custom-made products, limited stock at the supplier, specific inventory planners' past experiences, and uncertain lead times. Planned replenishment orders are planned instantly or for a term of approximately 1 until 3 months and can be modified intermediate because of no purchase obligation. Products that are most of the time ordered as planned replenishment are referred to as planned replenishment products (this also applies to regular replenishment products). Planned replenishment orders are manually decided and based on experience and an Excel file. This Excel file gives an overview of the forecast, sold items, on-hand inventory, and expected on-hand inventory. When planning a longer period, the available 7-day forecast is used.

1.3 Problem description

Coolblue has recently undergone significant growth and achieved record sales. Customer satisfaction is of utmost importance and products can only be sold when available in the warehouse. Therefore, to meet customer demand, an increasing number of products must be stored in the central warehouse. Unfortunately, due to the COVID-19 pandemic, incorrect forecasts, increased sales, and uncertain supplier lead times have led supply planners to order higher quantities of products in the hope of ensuring timely delivery. However, customer demand was not as high as expected resulting in excess inventory. Moreover, Coolblue has expanded to Germany, necessitating the acquisition of German version products.

The number of SKUs (assortment) and incoming product quantities are increasing while the warehouse still has the same size, resulting in capacity problems and a need to use the available space more efficiently. This is especially for 'Whitegoods', Parcel Large and XL products. When future periods are forecasted, the number of required storage locations is more than the available storage locations. The capacity utilization of the warehouse for 'Whitegoods', Parcel Large and XL products is in general around 90-95% while this has to be around 85% for flexibility reasons. For the Parcel Large hall, the number of occupied locations is higher than the set maximum at certain points in time and even after changing the warehouse structure the number of used locations exceeds the (new) maximum (Figure 1.1). Moreover, because of labor shortages, a diminished number of incoming shipments can be handled.

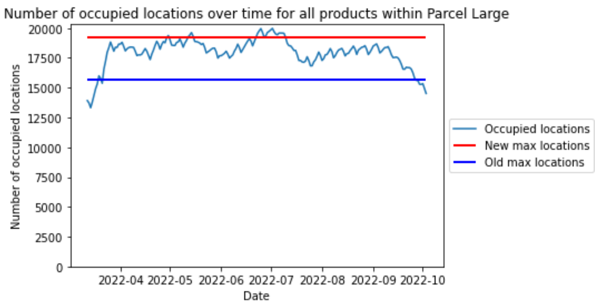


Figure 1.1: Occupied locations over time for Parcel Large hall

This high warehouse utilization and labor shortages are a problem and result in the stopping of ordering of certain products with lost sales as a result. Other consequences are high inventory costs, and less available cash to make investments. Furthermore, around September 2022, a queue of several days exists to handle incoming products from the supplier. This results in a problem because normally a product could be delivered at the warehouse with a duration of the supplier lead time but now there is an additional uncertain lead time. This may result in a response time spiral; longer lead times lead to inaccuracy of forecast (Hopp and Spearman, 2011). Therefore, safety stock is needed to cope with variability and uncertainty resulting in high inventory. Due to this high inventory, there is less space available for other products and the queue for inbound handling increases. This in turn increases the lead time and this spiral can go on.

Planned replenishment products have the highest value of total stock in the warehouse relative to other order types. When converting the stock values to weighted average stock in days (by dividing the number of products on stock by the forecast per day for each product and taking the average of these numbers weighted on price), it is remarkable that this value is especially high for planned replenishment compared to regular replenishment products. Moreover, to determine how much to order for planned replenishment products no logic is taken into account concerning service level, costs, margin, used warehouse space, value, size, or labor capacity.

All in all, the high warehouse utilization is a problem. Moreover, planned replenishment products have a higher stock and no logic regarding service level, warehouse space, size and costs is taken into account. Furthermore, because of not taking into account the restrictions of the storage locations, important products cannot be stored resulting in unnecessary stock-outs. The context is characterized by positive lead times, stochastic demand and lost sales in a setting in which the number of storage locations is restricted. Accordingly, this research focuses on designing a system to control inventories for Parcel Large products and especially takes into account product characteristics and warehouse space capacity. A system to control inventories in which less inventory is needed can significantly reduce the needed storage locations since the products within Parcel Large are of significant size. Moreover, better balancing the available warehouse space among products can reduce the number of stock-outs.

1.4 Research objective and questions

Based on the described problem, the research objective and questions are formulated. The goal of this research is to *design a system to control inventories in which expected inventory costs are minimized and take into account warehouse space capacity.* Within literature, the majority of multi-item inventory models correspond to minimizing the expected inventory costs. The company aims at reaching high sales in combination with high revenue and profit, which is in line with the objective of minimizing inventory costs. This to-be-designed system will set a reorder level per SKU such that all products within the assortment will fit within the warehouse while minimizing costs and maintaining a service level. Based on the described problem description, and research objective, the following main research question is formulated:

How to design a system to control inventories in which expected inventory costs are minimized and take into account warehouse space capacity?

This to-be-designed system (order control policy and its settings) is in particular for products that have a large size. To answer the main research question, sub-research questions are formulated. The first sub-research question refers to defining objective(s), constraints, decision variables, and parameters.

What are the objective(s), constraints, decision variables, and parameters within the system to control inventories?

Based on the previous sub-research question and the performed literature review, an adequate model will be formulated. To solve this model, several solution alternatives may exist or be formulated. First, existing methods will be explored and possibly modified to solve the model. Based on the second sub-research question, a method to solve the proposed model will be developed.

Which solution methods can be used to solve the proposed system to control inventories?

The developed solution methods in the previous sub-research questions can perform differently in terms of among others computational effort and objective value. Therefore, the last sub-research question aims to compare the different solution methods and choose the best solution method.

What is the best method to solve the proposed system to control inventories in terms of performance?

1.5 Scope

The focus of this research is to provide a systematic approach to determine reorder levels for SKUs with a significant size taking into account a limited number of storage locations. Because of the high utilization of storage locations for Parcel Large products and the expectation of encountering future problems within this group, the research focuses on this product group and corresponding storage locations.

Products are seen as Parcel Large when according to their order information the inbound location was Parcel Large and the product is classified as Large (Table 1.1). Only mature products within the assortment and sold to customers are within scope. Phasing-in and phasing-out SKUs are out of scope. Since this project does not take into account commercial planning, market data and market share, choices regarding assortment are not the goal of this project and therefore a fixed assortment is assumed. Only products that are most often (in terms of quantity) ordered as planned or regular replenishment are included. Products that are stored on upright, bulk or trampoline locations are excluded as these products only have a subset of specific locations that can be used resulting in a total of 2651 SKUs. In this study, 868 SKUs are used as for these SKUs the sales are known of the last three years which is found to be necessary to investigate the seasonal pattern and possible sales disruptions because of COVID-19.

As indicated by several articles, demand forecasting influences inventory management (Silver, 1981) (Goltsos et al., 2021). Within Coolblue, forecasting is done using a two-step forecasting method. Using a machine learning algorithm, first, a statistical forecast is generated based on historical data. As second, adjustments to the forecast can be made by analysts to take into account exceptional circumstances that the algorithm is not able to consider. Because the forecast relies heavily on human judgment and improving the forecast is not the main goal of this research, forecasting is out of scope.

Since the shortage in labor within the warehouse of Coolbue is assumed to be incidental and not structural, it has no added value to take this into account within the project. Therefore, labor capacity constraints are assumed to be out of scope.

Capacity extension of the warehouse is often taken into account in cases of non-stationary demand and decision sizing models (Lee and Elsayed, 2005). In this project, the size of the warehouse is taken as fixed and therefore capacity extension is not considered in the models. Reflection on the binding storage capacity constraint is done in the sensitivity analysis.

Several storage policies within a warehouse can be distinguished such as dedicated, shared, random, and turnover-based policies (Graves et al., 1977) (Gu et al., 2007). The chosen storage policy can result in reductions in order picking time and thus costs (Gu et al., 2010). The storage policies are out of scope since it is closely related to the warehouse floor map which is another team's responsibility.

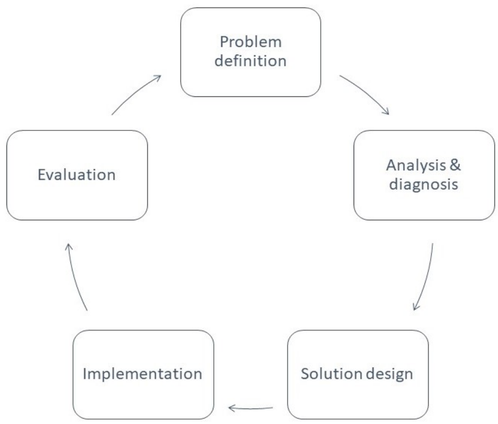


Figure 1.2: The problem solving cycle (adopted from Van Aken and Berends (2018))

1.6 Methodology

Based on the research questions, a methodology is proposed. Since this research aims to develop solutions for a field problem in a structured manner, the research paradigm, problem solving cycle of Van Aken and Berends (2018) will be used in this research (see Figure 1.2 for a visualization). In this cycle the following steps can be distinguished; problem definition, analysis and diagnosis, solution design, implementation and evaluation.

In the first step of the cycle, problem definition, is the problem context and research problem precisely formulated including scope (which is done in this chapter).

In the second step, analysis and diagnosis, a comprehensive overview of the current situation is established using both qualitative and quantitative methods. Qualitative data is gathered through informal conversations and brainstorms. Because of the informality of these conversations, no reports or transcriptions are made of these conversations. Quantitative methods are used to analyze data, providing insight into the current situation (e.g. warehouse occupancy, supplier performance, product characteristics). The findings are discussed with experts to gain a better understanding. This information, along with research literature, will be used to answer the first sub-research question: *"What are the objective(s), constraints, decision variables and parameters within the system to control inventories?"*.

Based on literature, an appropriate model will be formulated, which is the third step, design of the solution. To solve this designed model, several solution methods can be used. This corresponds to the second sub-research question *Which solution methods can be used to solve the proposed system to control inventories?*.

The fourth step, implementation, aims to implement the designed solution method. The system to control inventories will be implemented in a simulation because of the limited time span and simulation can imitate years in a relatively short time. Simulation is especially useful in cases of high uncertainty and to compare solution alternatives which is here the case(Hillier and Lieberman, 2015).

Finally, the system is evaluated in the last step, evaluation. First, performance indicators are defined corresponding to existing literature and the problem context. With the indicated performance indicators, the designed system is evaluated. This corresponds to answering the final sub-research question, *What is the best method to solve the proposed system to control* *inventories in terms of performance?*. Moreover, a sensitivity analysis will be performed to analyze how the system will react to varying input parameters.

Chapter 2

Literature

This chapter will provide an overview of literature regarding inventory management and other related topics that are part of this study. Before the start of this study, an extensive literature study was conducted with a focus on multi-item inventory models and capacity constraints. In this section, first, some background is provided on warehouse management and inventory management literature. Finally, literature on the modeling of constraints and objectives within multi-item inventory models is presented.

2.1 Warehouse operations

With the emerging economy and increasing importance of e-commerce, the complexity of warehouse operations has increased because of a high variety of products in small quantities under fluctuating demand patterns (De Koster et al., 2017). One of the major challenges to capacity planning, allocation and inventory management is demand fluctuations (Kembro et al., 2018). According to Gu et al. (2010), when assessing capacity requirements, seasonality, storage policy and order characteristics need to be considered since these factors impact the obtainable storage efficiency.

Honeycombing
The largest standardized material handling unit within a warehouse is a pallet (Bartholdi and Hackman, 2008). One of the simplest methods to store pallets is in lanes. The lane depth refers to the number of pallets stored back to back away from the pick lane. The height of a lane is measured as the number of pallets on top of each other and depends on amongst others fragility and pallet weight. Pallet racking is often used for storage and to support full case picking. In one location is often one specific SKU stored in order to keep track of where products are stored. Therefore, honeycombing can happen which refers to a loss of space because of an empty but unusable position (Bartholdi and Hackman, 2008). The unusable position can be used when the remaining items in the same aisle and/or column are removed. Especially in deep lanes, this occurs often. The magnitude of honeycombing depends on individual product withdrawal rate and lane depth (Gu et al., 2010). Van Donselaar and Broekmeulen (2022) generalizes honeycombing; it can also happen within an one unit location since the remaining units consume the full space of the location. For example, half a pallet arrives of a specific product and this is stored in a new location instead of merging it with a location that already contains this specific product and has sufficient space for the remaining products.

2.2 Inventory management

Having enough inventory on hand is of high importance because the supply chain is exposed to several uncertainties (e.g. lead time and demand). When the inventory is lower than demand, stock-outs can occur. This may cause emergency shipments, loss of customer goodwill, lost sales and delays in other parts of the system (Boulaksil et al., 2009), (Brito and de Almeida, 2012). Especially, customer's reaction to stock-outs in a retail setting is complex. According to Gruen et al. (2002), when there is a stock-out approximately 40% of demand is lost. Moreover, customers often substitute than delay the purchase of a product (Zinn and Liu, 2001) (Gruen et al., 2002). Therefore, it is of high importance to have the necessary amount of buffer.

For inventory models, a division is often made between single-item and multi-item inventory models. Other factors that are used to make a distinction in methods are among others type of demand, uncertainty in lead times and single or multiple period(s). The main difference between single and multiple period(s) is the fact that in multiple periods the unsold stock needs to be taken into account in the next period, which results in extra complexity regarding order quantity. Another often-made assumption in inventory models is how the customer will respond to an out-of-stock; backorders or lost sales (Williams and Tokar, 2008). All these factors result in different inventory methods. The combination of different parameters, factors and limitations in a changing environment makes it challenging to find a proper solution.

2.2.1 Lost sales and backorders

How customers react to a stock-out is of high importance for an inventory model as this assumption is important for the model's applicability in practice. When the customer demand cannot be fulfilled from inventory on hand, the demand can be backordered, in other words demand will be satisfied in the future. Assuming backorders for unmet demand is not realistic in many retail environments (Bijvank and Vis, 2011). The other option is that the demand will be lost. This adds complexity to the model since lost sales can be unobserved (Bijvank and Vis, 2011). This study is in the context of an online retailer in which unmet demand is lost since customers are unable to place backorders.

When unmet demand is backordered, the average sales are higher compared to a lost sales model because the backordered demand is still sold while the lost demand is not sold. This results in a higher average inventory on hand in a lost sales model relative to a backorder model. Because the actual sales within a lost sales model are less than in a backorder system per period, the service level for a lost sales system is at least as high as in the backorder system. Accordingly, due to the lower sales, it is possible to achieve the same or higher service level with the same amount of inventory in a lost sales system. Therefore, cost deviations may occur.

Moreover, within a backorder model, the inventory position is used as a main indicator of the inventory status. The inventory position is decreased when demand takes place and increased when an order is placed. Within the lost sales model, when the SKU is out of stock and thus demand is lost the inventory position does not decrease (Hadley and Whitin, 1963). Therefore, it is not possible to treat changes in the inventory position independent of the inventory level for a lost sales model. This makes it more difficult to approach the case of a lost sale as it is needed to take explicit account of available inventory on hand and outstanding orders (that have not yet arrived) (Bijvank and Vis, 2011). For lost sales models, when the lead time is higher than the review period, approximations are available for the fill rate while for other key performance indicators simulation is often suggested.

In some cases, it is possible to assume backorders while the model is clearly in an environment with lost sales. According to Van Donselaar and Broekmeulen (2013), in situations with a high target service level, the backorder model can overestimate the needed amount of safety stock.

This is especially the case when the number of outstanding orders is large and the relative demand uncertainty during lead time and review period is small. The number of outstanding orders (nOO) can be calculated as the expected demand during lead time divided by a simple approximation of the expected order size. The relative demand uncertainty during the lead time and review period can be measured by the coefficient of variation of demand during the lead time and review period. When the target fill rate is at least 0.95, $c_{L+R} < 0.5$ and $nOO \geq 1$, the service level using a backorder approximation will result in an actual service level in a lost sales system which deviates by at least 1% from the target service level (Van Donselaar and Broekmeulen, 2013).

2.2.2 Inventory control systems

Inventory control systems can be classified (for an overview of inventory systems see Table 2.1) based on replenishment quantity and review period.

Table 2.1: Inventory system classification adopted from Van Donselaar and Broekmeulen (2014)

	Periodic review	Continuous review
Fixed base replenishment quantity	(R, s, nQ)	(s, nQ)
Variable replenishment quantity	(R, s, S)	(s, S)

In a continuous system, the inventory is reviewed at all times while in a periodic review system, every review period, R, the inventory is reviewed. R is the time between two review moments. Within a continuous system, the safety stock only needs to cover the demand during lead time since there is no review period and thus an order is placed when the inventory position falls below the reorder level. In a periodic review system, the safety stock needs to cover the demand during lead time and review period since the inventory is only reviewed at a review moment. Because of achieving the same customer service with less safety stock and thus costs, continuous systems perform better (Van Donselaar and Broekmeulen, 2014).

The choice for a fixed base replenishment quantity results in ordering with a quantity of the largest multiple of the order quantity (Q) that will bring the inventory position after ordering to or above the reorder level (s) when the inventory position at a review moment is below the reorder level, s. A variable order quantity results in the ordering of a variable number of units to bring the inventory position back to order up to level S. There is only ordered at a moment when the inventory position is below the reorder level. In general batch systems, (s,nQ) as well as (s,S) can be used. In an (s,S) policy, the decision variables are the reorder point and order up to level. In periodic review settings, this becomes an (R,s,S) model (Van Donselaar and Broekmeulen, 2014).

According to Sani and Kingsman (1997), the continuous (s,S) system is from a theoretical perspective the best to manage products with low and intermittent demand. However, this continuous order up-to-level system is often not preferred in practice. Due to the usage of standard packaging quantities such as fixed case packs or pallet layers, the replenishment quantity in practice is often greater than one. Inventory planners of Coolblue need to order in a multiple of a fixed base replenishment quantity often set by the supplier. This enables efficient handling of goods; handling of one product or a pallet requires the same amount of time (Van Donselaar and Broekmeulen, 2014). Therefore, this order-up-to-level system is not applicable to Coolblue.

Due to periodic review and fixed case packs, the (R,s,nQ) inventory control system is preferred at Coolblue. To take into account, besides a predetermined order quantity, a minimal order quantity (MOQ), there is also a modification of the (R,s,nQ) inventory control system, namely the (R,s,MOQ, IOQ) inventory control system. When the inventory position at a review moment is below the reorder level, s, an order is placed with a quantity of the largest multiple of the incremental order quantity (IOQ) but not that the inventory position exceeds s-1+ MOQ. In other words, the order up to level is s-1+ MOQ. For example, when the s is 14, the IOQ 2, the MOQ is 20 and at a review moment the inventory position is 6. The inventory position is below s and therefore an order is placed with quantity 26 (maximum inventory position is $s - 1 + MOQ = 14 - 1 + 20 = 33$, resulting in an order of $33 - 6 = 27$, converting to multiples of IOQ results in 26).

2.2.3 Reorder level

In this section is zoomed in on the reorder level. The reorder level should ensure that the number of stock-outs during the lead time and review period is minimal and can be determined using a general formula (Nahmias and Olsen, 2015).

$$s = \mu_{R+L}^{\text{demand}} + ss \tag{2.1}$$

This formula reflects that the reorder level should ensure that the demand during lead time and review period is covered including an additional safety stock to cope with uncertainties (Williams and Tokar, 2008). A balance needs to be found in setting the reorder level, since too high results in high inventory costs while too low results in stock-outs. Safety stock often depends on the set service level and can be calculated using the formula $ss = k \cdot \sigma_{L+R}^{\text{demand}}$. The k refers to the safety factor and $\sigma_{L+R}^{\text{demand}}$ to the standard deviation of the demand during the review period and lead time. The safety factor depends on the service level and grows with the service value (Axsäter, 2015). For cases with non-stationary demand, the reorder level is set based on the forecasted demand for the next lead time and review period and additional safety stock (Van Donselaar and Broekmeulen, 2014). This safety stock depends on the standard deviation of the forecast error for the next lead time periods and next lead time and review period periods. Within this calculation of the safety stock, capacity constraints and costs are not taken into account.

2.2.4 Demand pattern

As mentioned previously, the type of demand, especially its fluctuations, influences inventory control models and is even seen as one of the major challenges of inventory management (Kembro et al., 2018).

Demand can be classified into stationary or non-stationary and deterministic or stochastic. Most demand patterns are non-stationary in practice (Silver et al., 2016). The changing variance of non-stationary demand can occur because of the occurrence of a seasonal pattern and trend. A repeating pattern at fixed intervals, such as every week or month, is referred to as a season (Nahmias and Olsen, 2015). Ignoring the seasonal pattern within the demand data may be destructive to optimal controlling inventory (Ehrenthal et al., 2014). A trend can be described as a downward or upward movement or a stable increasing or decreasing pattern (Silver et al., 2016) (Nahmias and Olsen, 2015).

The non-stationary demand pattern leads to an irregular pattern of how much and when to order (Pauls-Worm et al., 2014). The exact computation of reorder levels in non-stationary demand systems is often complicated and heuristics are rather used (Silver et al., 2016). Determining the reorder level at every review moment can be a solution (Van Donselaar and Broekmeulen, 2014). Another option is to divide the planning horizon into different phases and assume within each phase stationary demand (Neale and Willems, 2009). Neale and Willems (2009) apply this in a multi-stage system to minimize safety stock holding costs in all periods and stages. For each phase, the reorder levels can be determined based on the characteristics of that phase. The phases do not have to be of equal length. Chen and Chang (2007) also divide their planning horizon in phases with a duration of one month. Each month has constant demand and at the beginning of every month is decided whether or not to place an order. Besides dividing the planning horizon into phases, non-stationarity can also be implemented using a rolling horizon (Ettl et al., 2000) or a two-stage process as proposed by Tarim and Kingsman (2004) Bookbinder and Tan (1988); first determining the replenishment periods and as second determining the modifications for the planned orders. In this study, the planning horizon will be divided into phases (see section 5.3).

Tunc et al. (2011) investigate the costs of approximating a non-stationary policy using a stationary policy. Depending on the magnitude of the variability of demand, using a stationary policy can be expensive. Their recommendations involve periodically updating policy parameters to take into account variability in non-stationary demand. The timing of updating depends on the balance between the mean and standard deviation. Shorter intervals stabilize the mean but reduce data points, giving exceptional points more impact. Longer intervals reduce abnormalities' impact but increase the likelihood of non-stationary demand.

2.2.5 Service level

Within inventory control, the fill rate is most commonly used as service level and refers to the fraction of demand that is directly fulfilled from on-hand stock (Teunter et al., 2017). Guijarro et al. (2012) indicate the difficulty of obtaining or calculating the expected unfulfilled demand during a replenishment cycle which is used to calculate the fill rate. When only sales are registered and not demand, the achieved fill rate cannot be measured (Van Donselaar and Broekmeulen, 2014). Meistering and Stadtler (2017) indicate besides the fill rate, non-stock-out probability (fraction of periods without stock-outs). The fill rate is more often used since it also takes into account the backorder size while this is not the case for the non-stock-out probability (Tempelmeier, 2007). Besides fill rate, Axsäter (2015) considers two other service level definitions; the probability that an order arrives on time before the on-hand stock is finished and ready rate. The ready rate refers to the portion of time during which the on-hand stock is positive. Other relevant performance indicators are expected inventory on hand, expected number of backorders and discrete ready rate (Van Donselaar and Broekmeulen, 2014). The discrete ready rate refers to the probability of having positive on-hand inventory before a potential delivery moment. The discrete ready rate is in settings with small review periods and large lot sizes similar to the fill rate. When there are long review periods and at the end an order is delivered, the discrete ready rate does not reflect the true product availability through the period (since at the end of the period the inventory can be 0 which results in a discrete ready rate of 0%) (Van Donselaar and Broekmeulen, 2014). Product availability is measured continuously over time while for the discrete ready rate this is only before a potential delivery moment (from the perspective of incoming orders at the warehouse). Due to taking into account the number of backorders and reflecting the product availability through the period, the fill rate will be used to indicate the service level. Besides, the fill rate has already been used within the company.

Individual fill rates can be aggregated using several methods as distinguished by Van Donselaar et al. (2021); general weights, volume-based weights, turnover-based weights and profit-weighted (weights are proportional to the average demand multiplied by the profit margin). As shown by Van Donselaar et al. (2021), the benefits of service differentiation depend on the definition of the weights. The reported benefits for the volume-based weights were the greatest. A disadvantage is that for medium to high-priced SKUs, the demand needs to be backordered while turnover-based weights resulted in higher inventory costs. Due to the reported benefits, the volume-based weights will be used to aggregate the individual fill rates.

2.2.6 Multi-item inventory models

In multi-item inventory models, dependencies between items are taken into account. Based on system characteristics, an optimal policy can be generated (De Schrijver et al., 2013). Significant cost reductions can be observed when applying a system approach instead of an item approach. Multi-item inventory problems can be identified in three categories; independent items, network of items and shared supply chain processes (De Schrijver et al., 2013). This research is in the context of the first category; independent items.

Objective
For articles in which demand is stochastic and a stochastic programming modeling technique is used, the objective is often set to optimize an expected value of a function (Hillier and Lieberman, 2015). In most reviewed articles, a model is proposed in which the costs are minimized. Other objectives that are often set are maximize fill rate (Bijvank and Vis, 2012a), minimize used capacity (Bijvank and Vis, 2012a), maximize expected profit (Taleizadeh et al., 2010) and maximize order fulfillment (Yang et al., 2020). In some models, two objectives are taken into account such as minimizing total system costs and costs of used space (Najafi et al., 2018). Pasandideh and Keshavarz (2015) propose a model maximizing the service level and simultaneously minimizing costs.

Modelling of costs
Various sorts of costs are encountered in articles, namely penalty, purchase, ordering, holding, shortage (backorder and/or lost sales), (individual) set-up, production, machine, rework, leftover inventory, fixed joint replenishment, warehouse space, handling and salvage costs. The total inventory costs often are a combination of holding, ordering and purchase costs. In some models, an additional shortage cost is added. Remarkable is that often handling costs are not explicitly taken into account since there can be a big difference in time for handling an order of 600 items or 10 orders of each 60 items. This distinction has not been found in the studied articles. Moreover, as indicated by Gu et al. (2010), travel time depends on used storage policy and sort of aisle system and therefore it is complex to model handling costs. Accordingly, Van Donselaar and Broekmeulen (2022) indicates the influence of honeycombing on handling costs which are right now not taken into account. Holding costs are influenced by the inventory level and are often modeled independently of space. In most articles holding costs are linear modeled dependent on the (average) inventory level (Janakiraman et al., 2018) (Fan and Wang, 2018).

Service constraints
In the selected articles, service is most often incorporated as fill rate. Examples of other implementations are individual(/class) service level per cycle (Das et al., 2019), probability of not having a shortage (Taleizadeh et al., 2010), requiring a positive net inventory at the end of period (Tarim and Kingsman, 2004) and to use an objective function to maximize order fulfillment (Yang et al., 2020). An alternative for service level is shortage/penalty/backorder costs (Axsäter, 2015). These costs depend on the real costs incurred by shortages. Because of difficulties in determining these costs, a service level is set. As indicated by Van Donselaar et al. (2021), two types of models deal with balancing the fill rates and inventory levels. On the one hand, models in which holding costs are minimized take into account a fill rate constraint, and, on the other hand, models in which inventory holding and shortage costs are minimized.

Capacity constraints
Capacity in multi-item inventory models can be modeled as constraint or objective. As capacity constraint, it is modeled as safety stock space in numbers (Das et al., 2019), capacity in time (Najafi et al., 2018), total needed space for inventory (Pasandideh and Keshavarz, 2015; Taleizadeh et al., 2010). As objective it is often taken into account to minimize needed storage space (Mousavi et al., 2016; Sarkar et al., 2022), used capacity (reorder level and order quantity) (Bijvank and Vis, 2012a) and costs of used space (Najafi et al., 2018). Solving capacity constraints is more complex and provides less flexibility. Tight capacity can result in changes in stock-outs, costs and replenishment. Budget constraints in which the total value of inventory is restricted, can also be used to consider capacity constraints (Bera et al., 2009; Taleizadeh et al., 2010; Najafi et al., 2018).

Because of honeycombing, it can be that more space is needed since locations will not be as efficient as possible filled with products. Therefore, it is remarkable that the honeycombing effect is not considered. Van Donselaar and Broekmeulen (2022) derive expressions for multiple key performance indicators in an unit-load warehouse (including the honeycombing effect) as well as propose a model for optimizing reorder levels and minimum order quantities simultaneously taking into account warehouse capacities. When enlarging the minimum order quantity of expensive products, a balance exists between decreasing the number of unit load arrivals (which can decrease the needed storage locations and boost availability) and financial assets invested in inventory. Conducted experiments show the importance of taking into account stochastic demand and the honeycombing effect when deciding warehouse size in a decision support model. Including the honeycombing effect when estimating warehouse occupancy results in a warehouse occupancy of three times more compared to models with stochastic demand and no honeycombing. Only receiving and storing full unit loads results in efficient use of warehouse capacity and high investment in inventories. When using service differentiation and optimizing corresponding inventory variables, warehouse occupancy can be reduced. In short, honeycombing can have large effects and therefore needs to be included in models for unit-load warehouses.

2.3 Gap in literature

This literature section provides a short overview of the modeling of constraints and objectives in multi-item inventory models. In the remainder of this section, unexplored perspectives in literature are identified which the thesis aims to address.

Only a few articles could be found with multi-item inventory models incorporating both capacity and service level constraints. The article by Van Donselaar and Broekmeulen (2022) incorporates honeycombing in the key performance indicators and capacity constraints. No articles have been found in which honeycombing is included as a constraint in the context of a single location, cost minimization objective and multi-item inventory models, while neglecting honeycombing shows the overestimation of the available warehouse space.

According to Tarim and Kingsman (2004), while the stationary demand assumption is well known, non-stationary demand is often not considered. It is inappropriate to use constant inventory parameters in cases of non-stationary demand. Neale and Willems (2009) and Chen and Chang (2007) divide the planning horizon into phases to handle non-stationary demand. These approaches have a lot of similarities with a rolling horizon approach as updated demand data can be used when planning the next cycle (Narayanan and Robinson, 2010). A disadvantage of the rolling horizon approach is the possibility of adjusting decisions (Tunc et al., 2013) which may result in a loss in planning confidence (Van der Sluis, 1993). Moreover, as adjusting decisions is unwanted, smooth changeovers between phases must be included when dividing the horizon into phases to handle non-stationary demand. To the best of the author's knowledge, smooth changeovers in a rolling horizon for inventory control without adjusting inventory decisions are unexplored.

Lastly, unexplored yet relevant within literature is the influence of incorporating honeycombing within the context of non-stationary demand as most demand patterns in practice are non-stationary (Silver et al., 2016). It is especially relevant how inventory control policies perform in a setting with smooth transitions and honeycombing regarding required capacity, stock-outs and costs. Businesses with seasonal demand frequently face limitations on capacity due to cost considerations of maintaining peak capacity (Neale and Willems, 2009).

Concluding, the integration of honeycombing within cost minimization multi-item inventory models, smooth changeovers between phases without order modification and the performance of inventory control policies including honeycombing under non-stationary demand are novel and relevant contributions to literature.

Chapter 3

Current situation

In this section, the current situation is analyzed. Within this project, sales will be measured in quantity and revenue and profit in monetary terms. It is specified when is done otherwise. Product characteristics, exceptional cases, and achieved results are discussed in this section regarding products in scope. The aim of this section is to understand the products in scope and achieved results in the current situation. Therefore, the uncleaned data is used and the findings are discussed with practitioners. Finally, the first research question *What are the objective(s), constraints, decision variables, and parameters within the system to control inventories?* will be answered.

3.1 Descriptive products in scope

The most occurring product categories within the products in scope are monitors, garden tools, and kitchen appliances. To understand the products in scope, data descriptives are described in Table 3.1. Because of difficulties in lead time (as will be explained in the remainder of this section), the lead time is taken per supplier. Moreover, because the review period is not strictly fixed within Coolblue, the review period is approximated as the time between the arrival of orders of the same supplier (for more information see the remainder of this section).

Table 3.1: Data descriptives of all SKUs within scope

	L (in working days)	**R (in working days)**	**Demand (in quantity per day per SKU)**	**CoV**	**Order quantity (per SKU)**	**Order quantity/ daily demand**	**IOQ (in quantity)**	**MOQ (in quantity)**	K^C	**Purchase price (in €per SKU)**
mean	7.19	4.76	1.01	1.58	3.31	9.34	2	4.28	37	197.13
std	3.71	3.39	3.30	0.67	4.66	13.75	2	6.92	60	209.44
min	2	1	0	0.44	1	0.02	1.0	1	2	4.27
25%	5	3	0	1.31	1	1.75	1.0	1	15	79.86
50%	6	4	1	1.55	2	4.41	1.0	3	24	127.94
75%	8	5	1	1.73	4	10.81	1.0	5	36	238.51
max	23	38	157	2.05	50	188.97	28.0	121	1080	1784.31

CoV = coefficient of variation of daily sales; K^C = maximum number of products on a pallet; DOS = Duration of stay

The median order quantity is 2 products while the median capacity of a pallet is 24 showing that often partially fulfilled pallets are ordered. This shows the potential to increase the MOQ to order more products to fill the locations more resulting in less frequent ordering and less needed locations. Increasing the MOQ can be a method to decrease the number of used locations.

The average purchasing price and average daily demand per SKU is higher for planned replenishment relative to regular replenishment products. Following the logic of Camp, with for both groups the same fixed ordering costs, the expected order quantity of the planned replenishment products is expected to be higher which is the case. From the perspective of median values, the purchase price and demand of planned replenishment products is higher also resulting in a higher order quantity, as expected.

Lead time

At Coolblue, lead time is the sum of ordering, supplier, inbound, and operational lead time. Ordering lead time is the duration between the start of the order creation to the sending to the supplier. Ordering lead time is mostly influenced by manual checking of orders for exceptional sizes but is neglected as mean and median lead time is approximately 0. Inbound and operational lead time depends on Coolblue's inbound capacity and is neglected due to labor shortages (which are denoted as incidental). Supplier lead time is analyzed only for regular replenishment products as it is unknown and meaningless for planned replenishment products since orders are sent with a delivery date in the future (resulting in a supplier lead time of 0). The lead times are analyzed in working days as the majority of suppliers do not deliver during weekends.

The supplier lead time shows high variation between January 2020 and January 2022 (figure 3.1), which coincides with the outbreak of COVID-19 (based on the timeline of corona measures of RIVM). This increasing variation was observed across different suppliers, regardless of their size.

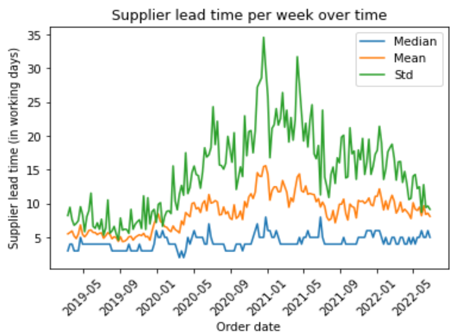


Figure 3.1: Supplier lead time of orders over time

Order characteristics, such as whether the order is delivered by post and whether the supplier is a distributor, were examined to determine the influences on supplier lead time at Coolblue. An independent t-test was used to compare group differences. The assumptions for this test are independence of observations, normality of data and homogeneity of variance. As all orders are recorded, the observations are assumed to be independent. Normality assumption does not need to hold as the sample size was over 200 Hair Jr (2014), and homogeneity of variance was not assumed, therefore Welch's t-test was used. The mean supplier lead time for shipments delivered by post was significantly higher than for shipments not delivered by post. Shipping by post is common for orders with a quantity between two and ten. Distributor suppliers had a lower mean supplier lead time than non-distributor suppliers. No significant differences in lead times were found based on other order characteristics.

Review period

Inventory planners are able to review the inventory every day but it is unknown whether this also happens. Therefore, is the review period approximated by analyzing the time between shipments of the same supplier as orders are often consolidated by the supplier or carrier. The time between arriving of shipments of the same supplier is on average 4.76 working days, in other words, each supplier delivers on average once a week. 50% of the suppliers deliver every 4 working days. When investigating the times a supplier comes per week based on week numbers, the results are in line with the days between the delivered shipments. On average suppliers deliver 1.8 times a week and the median is once a week. When investigating the time between shipments of a product, on average there are 16.7 working days between shipments of the same product. The median is 10 working days between incoming shipments of the product. This is longer than the days between shipments calculated per supplier as it is not usual to order and receive the same products every week.

Demand

The demand cannot be determined exactly because of not recording lost sales because products that are out of stock are not sold to customers. Because of the set target fill rate of 96.9% for regular replenishment products and inventory planners act similarly for planned replenishment products, there is assumed that the demand is equal to the observed sales. This assumption is in line with the made assumption in Van Donselaar and Broekmeulen (2022).

Instead of solely investigating the sales of the scoped products, the sales of all Parcel Large products are also investigated as these sales influence the available warehouse capacity throughout the year. The sales of all the products within Parcel Large have high peaks between February 2020 and 2022 which can partially be explained by the closing of stores due to COVID-19 (see Figure 3.3). Besides, the sales is decomposed in a trend, seasonal and residual components using time series decomposition. The multiplicative model is used because of increasing and decreasing trend and seasonal variation. From this decomposition, the clear seasonal pattern of higher sales within the end of year and lower sales in the beginning of the year can be observed. Some category teams have sales peaks during different periods of the year than the decomposed seasonal pattern such as category team Team Garden, Tools & Climate Control. The observed patterns are in accordance with the experience of practitioners and inventory planners.

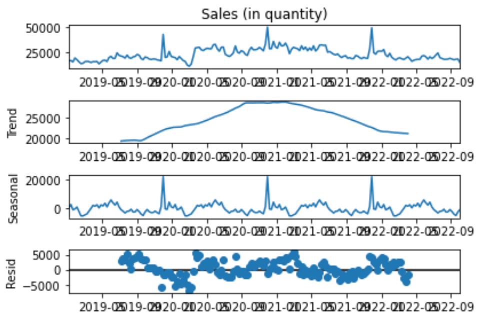
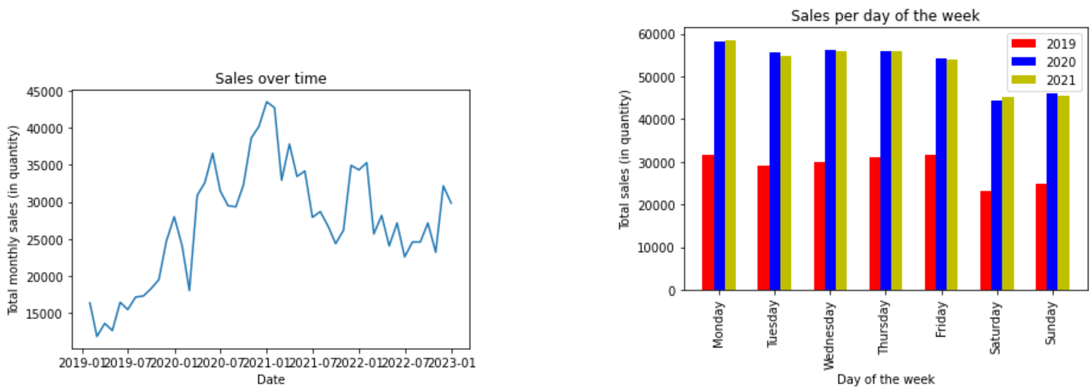


Figure 3.2: Decomposition of sales using time series decomposition for all Parcel Large products

Zooming in on the products in scope also shows relatively higher peaks between February 2020 and 2022 which can partially be explained by COVID-19. Because of the occurrence of trend and seasonality (higher peaks towards the end of the year), the demand can be classified as non-stationary. Furthermore, during the weekend days, the sales are less than during the week.



(a) Monthly sales over time

(b) Sales per day

Figure 3.3: Overview of sales (in quantity) for SKUs in scope

Order quantity

An order contains on average 40 products while 50 % of all orders contain 10 or less products. An order consists of several order lines which correspond to an order per SKU. The mean order line quantity is 22 and the median is 8 products in the last 3 years. Solely for 2022, the mean order line quantity is 3 and the median is 2. For planned replenishment products, the average order quantity is approximately 5 times higher than for regular replenishment products because regular replenishment products are ordered more often resulting in smaller quantities.

MOQ
Most suppliers have set a minimal order value in euros. The mean minimal order value is €305,32 for all products within scope. Internally Coolblue has set a minimal order value of €100,00 meaning that when an order is below this value, the order will not be placed at the supplier.

The minimal order value in euros can be converted to minimal order amount in quantity. This is done by dividing the minimal order value by mean product purchasing price. The mean minimal order amount is 4.28 products and a median of 3 products.

Maximum number of products on a pallet
The maximum number of products on a pallet is known for some SKUs. The maximum pallet quantity, that is missing, is calculated using the volume of a pallet and SKU. This can be seen as an approximation since it can be difficult to fully fulfill a pallet calculated using volume. The average number of products that fit on a pallet is 37 and the median is 24. The average order quantity in terms of pallets is 1.38 and 50% of the orders have an order size of 0.4 pallets.

Pallet layers will not be discussed as it is unknown and it can be the case that a supplier stacks the pallet in such a way that products of the same SKU are not on the same layer of the pallet.

Some suppliers have determined the number of products that fit within a box. The size and volume of this box are not standardized. For 75% of the SKUs (of which the box quantity is known), the number of products that fit within a box is 1 or less.

Purchase price
When products are bought, a purchase price is known. The net purchase price considers subsequent agreements (e.g. rebates and listing fees). This net purchase price is used within this project since the 'standard' purchase price is not realistic. For clarity reasons, when referring to the purchase price, the net purchase price is used.

A clear linear relationship has been observed between the selling price and purchase price; a relatively high selling price corresponds with a relatively high purchase price. No clear relationships can be seen between the selling price and profit margin or the purchase price and profit margin.

3.2 KPIs current situation

In this section, first, the observable results of the current situation are discussed. As second, the inventory levels and warehouse utilization are analyzed.

The results regarding inventory on hand, the value of the inventory, number of orders, order quantity, and used locations are summarized in Table 3.2. The values are based on the period between February and November 2022. The values for the KPIs are calculated for the SKUs in scope.

3.2.1 Inventory

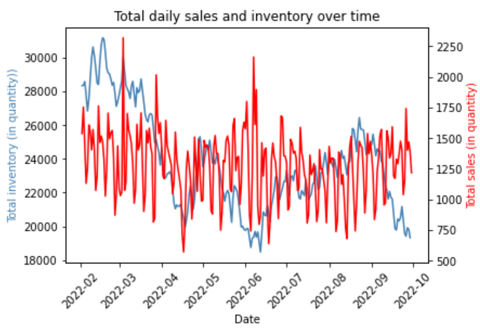
When comparing the total daily sales with the total daily inventory, it is notable that the total inventory fluctuates and increases over time while the sales do not increase and fluctuate that much over time (Figure 7.2a).

Table 3.2: Overview of achieved results in current situation (per day)

	Total inventory (in units)	Total inventory (in €)	Used locations (in number)
mean	23 254	4 224 182.36	2056
std	3193	609 599.62	180
min	16 380	3 038 656.73	1637
25%	21 173	3 777 812.76	1924
50%	22 971	4 173 396.87	2071
75%	24 848	4 535 855.75	2184
max	31 160	5 622 429.48	2437

As expected, the inventory of planned replenishment products has been observed to be higher as these orders have a higher order quantity. The inventory levels of the regular replenishment products are lower, the products are more frequently ordered and the difference between sales and inventory levels are lower. This can also be seen in the inventory levels of a planned and regular replenishment product (Figure 7.2c and 7.2d).

Within Coolbue, ABC classifications are set for products. While this classification to helps identify important products, it is not used to set a target service level. Therefore, the ABC classification will not explicitly be taken into account within the to-be-designed models.



(a) Sales and total inventory over time

(b) Inventory and sales over time for a planned replenishment product

(c) Inventory and sales over time for a regular replenishment product

Figure 3.4: Overview of inventory levels

3.2.2 Warehouse

On average, the 868 SKUs within scope occupy 2056 locations daily with a maximum of 2437 locations. Of these locations, the majority are active racking locations. Other distinctions that can be made within the locations are based on height. Approximately 74 % of the products occupy locations with a height of 230 cm and the rest of the products occupy locations with a height of 150 cm. The number of used locations fluctuates over time. Especially more locations are used within Augustus and September which can partly be explained by sales peaks in these months (Figure 7.2a). Unfortunately, because of access to limited data, it cannot be discussed whether more locations are needed near the end of the year because of the higher sales. It can be observed that within September and October the number of used locations decrease which can be explained by the removal of excess stock and making the warehouse ready for the high number of needed products for the end of the year. The number of products per location fluctuates over time between the 6.8 and 8.2.

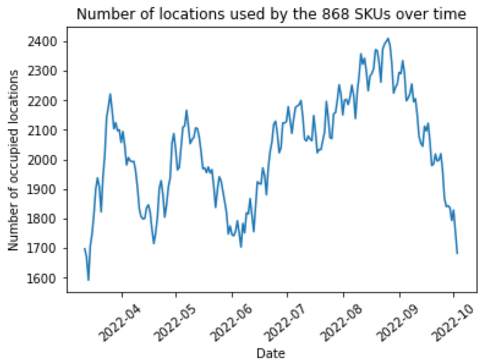


Figure 3.5: Occupied locations over time

3.3 Conclusion

To summarize, the context is characterized by positive lead times and non-stationary stochastic demand. Moreover, requirements are set on the order quantity as this needs to be at least the MOQ and in multiples of IOQ (if this is specified by the supplier). There are high inventory levels, especially for products ordered via planned replenishment. The inventory is stored at one stocking point (warehouse in Tilburg). Within the warehouse, the number of occupied locations fluctuates and is often near the set maximum bound.

All in all, within this context, the decision variable is the reorder level per SKU. Moreover, the possibility of increasing the order quantity is shown which can be seen as an additional potential decision variable. The number of storage locations is a restricting constraint. Furthermore, customer satisfaction is highly important resulting in the need of setting service level constraints. Finally, the objective must clearly reflect the objective of the inventory planner and Coolblue which on the one hand is to have maximum customer satisfaction and on the other hand minimizing of costs.

Chapter 4

Data preparation

This project relies on quantitative data as input for the to-be-designed models. The pre-processing of data is an important process to improve the quality of the raw data. When the data is not well prepared, it will produce misleading output, according to the phenomena of garbage in is garbage out (Rahm and Do, 2000). In this section, the pre-processing and cleaning of the data are described.

4.1 Data gathering

The relevant data is available in Looker and exported to .csv files. Used data in this research are daily inventory levels (in units), sales per SKU per customer order, occupied storage locations within the warehouse per SKU per day, SKU characteristics, and orders made by inventory planners. All data is gathered in the time period January 2019 until October 2022. The sales are gathered from January 2019 up to and including December 2022.

Filters are used to scope down to products in scope. Filters are set to solely analyze arrived product orders. The sales per SKU per customer per day are aggregated to daily sales per SKU.

4.2 Data cleaning

To improve the quality of raw data, the data is cleaned. The goal of data cleaning is to identify and remove inconsistencies and errors. this is done by detecting and correcting missing data and outliers. It is important to understand the data by graphically examining (Hair Jr, 2014). Within this section, there will be further zoomed in on missing data, outlier detection, and correctness of the data points.

Missing data
Missing data is a result of a systematic event (e.g. data entry errors) or dependent on the respondent (like refusing to answer) (Hair Jr, 2014). Within the inventory and sales data, no missing data or duplicates are found.

In the data concerning product characteristics, the box quantity or maximum number of products on a pallet was missing for some SKUs. The reasons for this missing data can partly be explained by the suppliers and inventory planners not knowing this information. Therefore, the values are approximated using the volume of an Europallet and an SKU. The height of the location is 230 cm but the maximum possible height is 220 cm since the pallet needs to fit within a location with a height of 230 cm. This results in a possible tolerated height on which SKUs can be placed of

$220 - 14.4$ (since the pallet consists of a frame). This is an approximation since boxes can have extraordinary forms resulting in not fitting perfectly to use the whole pallet. Since the entering of the product characteristics is done manually, sometimes mistakes are made when entering the data. Another possible reason for mistakes in the maximum number of pallets can be explained by the change in product package size over time. Therefore, the given maximum number of products on a pallet is checked by calculating the highest possible number of products that can fit on a pallet based on the pallet and product volume. If the calculated number is lower than specified in the data set, the number is modified. The missing values regarding the maximum number in a box cannot be calculated and are set to 1.

Other incorrectness found within the data are negative sales and inventory levels. These values are set to 0 since the inventory levels or sales cannot be negative. The occurrence of these negative values can be explained by data entry errors.

Outlier detection
Outliers are usually judged as unusually high or low on a variable or combination of variables in comparison with other values. According to Hair Jr (2014), outliers are "observations with a unique combination of characteristics identifiable as distinctly different from the other observations". An outlier must be investigated concerning the representatives of the population. Based on the source of the unique observation, an outlier can be classified; procedural errors, extraordinary events, extraordinary observations, and outliers unique in their combination of values across variables. The first class originates from mistakes in the gathering of data or data entry errors and is eliminated during the previous step.

Extraordinary events
Outliers in this category occur as a result of an extraordinary event (Hair Jr, 2014). During the period of COVID-19, extremely high and fluctuating sales patterns have been observed. The first signals of COVID-19 were in January 2020 and the working from-home advice was given by the government in March 2020. Especially, since this event, the total sales doubled in March 2020 in comparison with February 2020. This can be explained by the closing of stores and working from home advice as Coolblue is (mainly) an online retailer and sells electronic devices. The total sales of February 2022 decreased by 25% relative to January 2022. Besides this, the mid of February COVID-19 restrictions have been modified and this can partly explain the decrease in total sales. The sales points during COVID-19 are eliminated as the COVID-19 situation is an extraordinary event and not representative of the normal situation. In short, the data points between March 2020 and January 2022 are deleted from the data set. The values of the month of January 2022 will be modified because the simulation aims to replicate the year 2022 based on the cleaned sales data. Because of seasonality, the values of January 2022 are modified using a combination of the seasonal indices of 2019 and the median.

Because of the increase in sales, also an increase in ordered products by inventory planners can be observed. Moreover, the mean supplier lead time is relatively higher and the standard deviation shows high deviations in this period. This can be explained by the negative impact on suppliers resulting in longer lead times and shortages. The data points during COVID-19 (between March 2020 and January 2022) are deleted from the data set.

Extraordinary observations
This class contains observations for which there is no explanation. The promotional calendar is unavailable and therefore it is unknown whether an outlier is because of a promotion or another reason. Therefore, will these promotion outliers be detected with the method described in this section.

A method to detect univariate outliers is to examine the distribution of the observations of the variables and select outliers that fall outside the ranges of the distribution (Hair Jr, 2014). This is done by converting the values to standard scores with a mean of 0 and a standard deviation of 1. The formula to convert the values is $x' = \frac{x-\mu}{\sigma}$. For a number of observations smaller than 81, the standard score must be 2.5 or greater to be detected as an outlier. For a larger number of observations, the cut-off value is 4. The value of detected outliers will be set to the median value. The median value (without outliers) is chosen since the median is less prone to outliers relative to the mean or mode. This results in a lower number of incidental sales points.

Another extraordinary observation is no registration of sales when the inventory is 0 because there is no inventory on hand available. These values are substituted with the median demand value because of the possibility of having demand when the inventory level was positive instead of 0.

The number of changed sales data points is 2%. According to Hair Jr (2014) any of the imputation methods can be applied when the to be substituted data points are below 10%. This cleaned data will be used as input for the models.

Chapter 5

Design of model

To determine when and how much to order, or in other words, the reorder level and order quantity, a model is formulated incorporating the constraints in line with the current situation. The context is characterized by lost sales, stochastic demand, positive lead times, multi-period and capacity constraints. In this chapter, three models are designed and discussed. The first two models are in line with the current situation while the third model reflects the possibilities of modifying the MOQ and IOQ. First, the inventory system, assumptions and the used notation and concepts are discussed. Finally, the models are formulated, and the solution methods are provided.

5.1 Inventory system

As could be seen in Table 2.1 in Chapter 2, inventory systems can be classified into several classes. Within Coolblue, a periodic review is used since inventory can be reviewed every weekday. An advantage of using periodic review is that it is known in which time interval, orders are potentially created. Because of this, regular ordering is preferred for the inventory planner and supplier.

Because of this and a fixed base replenishment quantity, the preferable inventory control system is an (R_j,s_j,nQ_j) inventory control policy within this context. The index j indicates the uniqueness of the variables for SKU j.

As is shown in Section 3, there are possibilities to modify the order quantity in order to decrease the number of used storage locations. This together with the minimum order quantity (set by the supplier and Coolblue), an (R_j,s_j,MOQ_j, IOQ_j) inventory control system will be used in model 3. In the remainder of this chapter, assumptions and models will be discussed and formulated to determine reorder levels.

5.2 Assumptions

Lead time
When an order is placed at the supplier, there is assumed that the supplier can always deliver the order in full with a duration of the supplier specific lead time.

Time periods
Because of the possibility of selling products every day and ordering products every weekday, the time period within the model is set to daily time periods. This has no impact on the lead

time and review period in the models which is in days.

Prices
In reality, the purchase and sales prices fluctuate. Within the model, the sales and purchase prices are taken as fixed per product and independent of quantity.

Storage locations
As mentioned in Chapter 3 and 1, several distinctions can be made within storage locations. The location distinction based on height is relaxed and not taken into account within the models as the height of an incoming pallet/shipment is unknown. Moreover, the distinction between pick and storage locations is unnecessary as within practice an empty pick location is immediately filled with inventory from a storage location.

Sales and backorders
As discussed in Section 3.1, the average demand is assumed to be equal to the average sales. Substitution behavior is neglected as a high fill rate will be set resulting in a small part of the demand lost and minimal substitution behavior.

The model is in the context of an online retailer in which unmet demand is lost since customers are not able to place backorders. As described in Section 2.2.1, in some cases it is possible to assume backorders while the model is clearly in an environment with lost sales. For the products within scope, the median nOO is 1.4 and the median c_{L+R} is 0.49. As for the majority of the products (86%) $c_{L+R} < 0.5$ and $nOO \geq 1$ holds, lost sales can be ignored, and backorders are assumed since the actual service level will not deviate much based on the outcome of the variables c_{L+R} and nOO.

5.3 Notation and concepts

The following concepts, logic and formulas are based on Van Donselaar and Broekmeulen (2014, 2022). In this section, it will be explained in detail.

The demand is stochastic, fluctuates over time and a seasonal pattern can be indicated, in short, the demand is non-stationary. As the determination of the reorder level at each review moment for each SKU will be too time-consuming to implement in practice, the choice has been made to divide the planning horizon into phases and assume that within a phase demand is stationary. For each phase, a reorder level per SKU is set. At the start of a phase, the reorder level will be calculated based on the characteristics of that phase and the proposed models. The demand within a phase is assumed to be independent and identically distributed.

Dividing the planning horizon into phases is reflected within the variables since the variables are for a specific SKU and a specific phase denoted with j and g. The indication of a specific phase g is not always denoted due to readability reasons. To formulate the models, the following variables are distinguished;

(Potential) Decision variables

$s_{g,j}$	Reorder level for phase g and SKU j
$ss_{g,j}$	Safety stock in phase g for SKU j
$MOQ_{g,j}$	Minimum order quantity for phase g and SKU j
$IOQ_{g,j}$	Incremental order quantity for phase g and SKU j
$s_{g,j}^{mod}(x^{cur})$	Modified reorder level for phase g and SKU j at time x^{cur}

Input	
J	Set of all SKUs in assortment
G	Set of all phases
L_j	Lead time of SKU j in time units
R_j	Review period of SKU j in time units
$IOQ_j^{current}$	Current IOQ of SKU j in quantity
$MOQ_j^{current}$	Current MOQ of SKU j in quantity
$Q_j^{current}$	Current order quantity of SKU j
$h_{g,j}$	Holding costs for SKU j in phase g per product per time unit
$b_{g,j}$	Shortage costs for SKU j in phase g per product
$v_{g,j}$	Value of SKU j in phase g
K	Ordering costs
C_j	Maximum number of products of SKU j on pallet in [products/pallet]
W^{max}	Maximum number of storage locations within warehouse
P_2	Target fill rate
P_2^{min}	Minimum target fill rate
$E[D_{1,g}]_j$	Expected demand in phase g for SKU j per time unit
x^{cur}	Current time
T_g	Length of phase g in time units
$IP_{j,x^{cur}}$	Inventory position of SKU j at time x^{cur}

To further analyze the relationship between the decision variables and other distinguished variables, first, the dynamics of the inventory system are further analyzed. Within this project, a (R_j, s_j, nQ_j) policy is used in model 1 and 2 for replenishment and (R_j, s_j, MOQ_j, IOQ_j) policy in model 3. The MOQ_j is a limit on the minimum order size that can be ordered while the step size is determined by the IOQ_j (Broekmeulen et al., 2017). When the MOQ_j is increased, a lower number of the expected number of order lines can be observed since the quantity of an order will increase. The inventory position is defined as the sum of the inventory on hand and planned deliveries minus the number of backorders.

To be able to formulate a model, expressions for the system are derived at an arbitrary review moment (τ) (without loss of generality) for an SKU. The time between two review moments is R. If an order is placed at review period τ, the order will arrive after the fixed lead time (L) has passed at $\tau + L$. At $\tau + R$ is the next review period and if an order is placed, it will arrive at $\tau + R + L$. Moments $\tau + L$ and $\tau + R + L$ are potential delivery moments. Potential since not at every review moment, an order is placed. The potential delivery cycle is the time interval $\tau + L$, $\tau + R + L$. The inventory on hand is highest after a potential delivery $\tau + L$ and referred to as $I^{OH}(L)$. Before the next potential delivery moment ($\tau + R + L$), the inventory on hand is at its lowest indicated by $I^{OH}(R + L)$.

Expected inventory on hand
In the case of an (R,s,nQ) replenishment logic, the expected inventory on hand after a potential delivery (at the beginning of the potential delivery cycle, $I^{OH}(L)$) and before a potential delivery (at the end of the potential delivery cycle, $I^{OH}(R + L)$) can be calculated. Because backorders are assumed, the expected inventory on hand after a potential delivery (at moment τ+L) is reflected by the inventory position at τ and demand during lead time. The inventory placed in transit at moment τ arrives in the warehouse at $\tau+L$ and is already part of the inventory position at moment τ. Moreover, during time interval $(\tau, \tau + L)$, the inventory is decreased because of demand during this time interval. The inventory on hand cannot be negative. Therefore, the inventory on hand at $\tau + L$, can be written as $I^{OH}(\tau + L) = (IP(\tau) - D(\tau, \tau + L))^+$.

An expression for the inventory on hand just before a potential delivery ($I^{OH}(\tau + R + L)$ can be formulated using the same logic. The order has not arrived yet at moment $\tau + R$ (since it will arrive at $\tau+R+L$). Therefore, the inventory on hand at moment $\tau+R+L$ is similar to the inventory at moment $\tau+L$ minus the demand in time interval $(\tau+L, \tau+R+L)$. As inventory on hand cannot be negative, the inventory on hand before the next potential delivery moment at time $\tau+R+L$ is similar to $I^{OH}(\tau+R+L) = (I^{OH}(\tau+L) - D(\tau+L, \tau+R+L))^{+} = (IP(\tau) - D(\tau, \tau+R+L))^{+}$.

Besides, the inventory position after a potential ordering moment is always between s-1 and s-1+Q because the inventory position must be smaller than s to be able to order. The quantity to order will bring the inventory position back to or above s with a maximum of s-1+Q. This maximum is reflected as maximum in the second sum sign since demand higher than this will result in backorders. The expected inventory depends on the probability of a certain value of the demand (lower than s-1+Q) and Q.

$$\begin{aligned} E\left[I^{OH}(t)\right]_j &= E\left[\{IP(\tau) - D_j(\tau, \tau+t)\}^{+}\right] \\ &= \frac{1}{Q}\sum_{i=0}^{Q-1}\sum_{d=0}^{s+i-1}\{s+i-d\}P\left(D_t = d\right) \end{aligned} \tag{5.1}$$

The expected inventory on hand at moment $\tau+L$ and $\tau+R+L$ can be calculated using formula 5.1. In this formula, t can be replaced with $t = L$ or $t = L+R$ to reflect the expected inventory on hand at time t. Moreover, D_t reflects the demand during t.

In the case of an (R_j,s_j,MOQ_j, IOQ_j) replenishment logic, the inventory position ranges between s and s-1+MOQ. Because it is unknown whether at an arbitrary review moment just after the potential ordering moment the inventory position follows a discrete uniform distribution, an additional probability for the inventory position is included. These distributions can be derived using Adan's Fitting procedure based on the mean demand and standard deviation. These changes result in the following formulas 5.2 and 5.3.

$$E[I^{OH}(L)] = \sum_{i=s}^{s-1+MOQ}\sum_{d=0}^{i}(i-d)\cdot P(IP=i)\cdot P\left(D_L = d\right) \tag{5.2}$$

$$E[I^{OH}(R+L)] = \sum_{i=s}^{s-1+MOQ}\sum_{d=0}^{i}(i-d)\cdot P(IP=i)\cdot P\left(D_{R+L} = d\right) \tag{5.3}$$

The average expected value of the inventory on hand is similar to the average of the expectation of $I^{OH}(L)$ and $I^{OH}(R+L)$.

$$E[I^{OH}] = \frac{E[I^{OH}(L)] + E[I^{OH}(R+L)]}{2} \tag{5.4}$$

Fill rate

The fill rate refers to the fraction of demand that is directly fulfilled from on-hand stock. The fill rate can be expressed as the expected sales during the potential delivery cycle divided by the expected demand during the potential delivery cycle (Van Donselaar and Broekmeulen, 2022). The expected sales during a potential delivery cycle are the difference between $I^{OH}(L)$ and $I^{OH}(R+L)$. The expected demand during the potential delivery cycle is similar to the expected demand per time unit times R. This results in the following formula for fill rate per SKU per phase (note $E[D_1]$ reflects the expected demand during a time unit);

$$P_2 = \frac{E[I^{OH}(L)] - E[I^{OH}(R+L)]}{R * E[D_1]} \tag{5.5}$$

Stock-outs occur when the demand is more than the inventory on hand. The expected number of backorders during a time unit is when the expected demand is more than the expected inventory during the time unit. This can be approximated by multiplying the expected demand for a time unit with one minus the expected fill rate.

$$E[X^-] = (1 - P_2(s)) \cdot E[D_1] \tag{5.6}$$

Expected number of occupied locations

Constraints are often set on the maximum number of stored products within a warehouse, which is often calculated by multiplying the average inventory on hand by used space (Taleizadeh et al., 2010; Pasandideh and Keshavarz, 2015). Within Coolblue, these approximations are incorrect since the incoming stock is not added to an already-in-use storage location, but a new storage location is used resulting in more storage locations occupied (and thus space) than approximated.

The maximum used space during a potential delivery cycle is when the inventory on hand is at its maximum. This is just after a potential delivery at moment $\tau + L$. Since the inventory of incoming SKUs is not merged with inventory already in the warehouse, the maximum number of occupied locations (in the number of whole storage locations or equivalent whole pallets) for an SKU is the sum of the number of unit-load arrivals at a potential delivery moment and the number of occupied locations before the arrival (Van Donselaar and Broekmeulen, 2022).

$$E[USL(L)] = E[USL(R+L)] + E[UA] \tag{5.7}$$

The number of occupied locations before arrival can be estimated by the inventory on hand at $\tau + R + L$ divided by the replenishment order size. The maximum replenishment order size is the maximum number of products on a pallet (C). The replenishment order size will be approximated with the expected order size ($E[OS]$) and cannot exceed the maximum number of products on a pallet. This results in the expected number of occupied locations before arrival for the (R,s,MOQ,IOQ) logic as described in formula 5.8. To calculate, the expected number of occupied locations before arrival for the (R,s,nQ) logic, the specified IOQ and MOQ need to be replaced with Q in formula 5.8.

$$E[USL(R+L)] = \sum_{i=s}^{s-1+MOQ} \sum_{d=0}^{i} \left\lceil \frac{i-d}{\min(E[OS], C)} \right\rceil \cdot P(IP = i) \cdot P(D_{R+L} = d) \tag{5.8}$$

The number of unit-load arrivals at a potential delivery moment is similar to the order size divided by the maximum number of products on a pallet (rounding this number up). Taking the expectation of this number results in the expected number of unit-load arrivals at a potential delivery moment (formula 5.9).

An order is created when the inventory position is below the reorder level s_j. For an (R,s,MOQ,IOQ) logic, the maximum order size is $s_j - 1 + MOQ_j$ because the order is made to increase the inventory position to a maximum of $s_j - 1 + MOQ_j$. This results in an order when $D_R > IP - s$, reflected in formula 5.10. The expected order size and expected number of unit load arrivals can then be calculated using formulas 5.11 and 5.12.

$$E[UA] = E[\lceil OS/C \rceil] \tag{5.9}$$

$$OS = \begin{cases} \lfloor (s - 1 + MOQ - IP + D_R)/IOQ \rfloor \cdot IOQ & \text{if } D_R > IP - s \\ 0 & \text{otherwise} \end{cases} \tag{5.10}$$

$$E[OS] = \sum_{i=s}^{s-1+MOQ} \sum_{d-i-s+1}^{\infty} \left\lfloor \frac{(s-1+MOQ-i+d)}{IOQ} \right\rfloor \cdot \text{IOQ} \cdot P(IP=i) \cdot P\left(D_R=d\right) \quad (5.11)$$

$$E[UA] = \sum_{i=s}^{s-1+MOQ} \sum_{d=i-s+1}^{\infty} \left\lceil \left\lfloor \frac{(s-1+MOQ-i+d)}{IOQ} \right\rfloor \cdot \frac{IOQ}{C} \right\rceil \cdot P(IP=i) \cdot P(D_R=d) \quad (5.12)$$

For an (R,s,nQ), the maximum order size is $s_j - 1 + Q_j$. To formulate the formula of expected order size, the logic described by (Van Donselaar and Broekmeulen, 2022) is used. An order is placed when at moment $\tau + R$, the inventory position is below the reorder level. During the time interval $(\tau, \tau + R)$, the inventory position only changes because of demand. Summarily, an order is made when the difference in the inventory position after a potential replenishment order (at time τ) and demand during the review period is less than the reorder level ($IP - D_R < s$ can be rewritten as $D_r > IP - s$). The order size will be equal to converting IOQ and MOQ to Q in formula 5.10. The expected order size is similar to converting IOQ and MOQ to Q in formula 5.11. The expected number of unit-load arrivals at a potential delivery moment is calculated by filling the expected order size in in formula 5.9 resulting in formula 5.13.

$$E[UA] = \sum_{i=s}^{s-1+Q} \sum_{d=i-s+1}^{\infty} \left\lceil \left\lfloor \frac{(s-1+Q-i+d)}{Q} \right\rfloor \cdot \frac{Q}{C} \right\rceil \cdot P(IP=i) \cdot P(D_R=d) \quad (5.13)$$

Lastly, the expected number of unit-storage locations at a moment in time ($E[USL]$) is calculated by taking the mean of $E[USL(L)]$ and $E[USL(R+L)]$ resulting in $E[USL(R+L)] + E[UA]/2$.

5.4 Models

Based on the discussed notation and concepts, three models are formulated. Within the literature, the reorder level is often based on a target service constraint or inventory costs (Axsäter, 2015). The main goal of this project is to design a system to control inventories in which expected inventory costs are minimized. The model will result in a value for the decision variables per phase. The parameter setting is discussed in Section 6.1.2.

The first proposed model sets the reorder level based on the target service level. This model does not take into account the capacity restriction and is used as a reflection of the current situation. Within the second model, the reorder level is set using a general formula consisting of expected demand during lead time and review period and safety stock. The model minimizes the expected inventory costs restricted by warehouse storage locations. The third model optimizes, besides the reorder levels, also the incremental order quantity and minimum order quantity by taking into account the expected inventory costs and warehouse storage locations. For an overview of the decision variables per model see Table 5.1. Model 2 and 3 are subject to an aggregrate fill rate constraint allowing different service targets for different models. Therefore, to enable fair comparison and discussion of differences among the models, all models have a set aggregated fill rate constraint. The models and solution methods are further explained below.

Table 5.1: Overview of decision variables per model

Model	Decision variables per SKU j and phase g
1. Current situation	Reorder level (/safety stock)
2. Safety stock model (SS-model)	Reorder level (/safety stock)
3. IOQ and MOQ model (IOQ/MOQ-model)	Reorder level (/safety stock) , IOQ and MOQ

5.4.1 Model 1: current situation

As this model represents the current situation, the (R,s,nQ) replenishment logic is used. One method to determine the SKU reorder level is by setting a service level. The DoBr-tool, developed by Van Donselaar & Broekmeulen, determines the reorder level of an SKU based on a stochastic inventory model and the input parameters; review period, standard deviation of time unit demand, mean time unit demand, lead time, IOQ, demand distribution and a target service level in the form of fill rate or discrete ready rate. The lead time, review period and IOQ are known and constant for each SKU. The goal of this current situation model is to provide a method to mimic the current situation on setting the current reorder levels. Moreover, the results of these set reorder levels can later be compared to the reorder levels set by the other models regarding performance.

The minimum reorder levels that are calculated do not take into account the warehouse capacity constraint nor can individual products compensate fill rates for the aggregated target fill rate. When each SKU is set to the same service level, the aggregate service level will also reach that value. The tool shows exact results in situations with backorders, non-perishable products and stationary demand. Because of not taking into account the warehouse capacity, these calculated reorder levels can be seen as an approximation of the base situation.

In combination with a search procedure, in the case of discrete demand, the following formula of the fill rate is used to determine the reorder level per SKU per phase.

$$\begin{aligned} P_2 &= 1 - \frac{E\left[\{D(\tau,\tau+R+L)-IP(\tau)\}^+\right]-E\left[\{D(\tau,\tau+L)-IP(\tau)\}^+\right]}{E[D(\tau+L,\tau+R+L)]} \\ &= 1 - \frac{\frac{1}{Q}\sum_{i=0}^{Q-1}\sum_{d=s+i+1}^{\infty}[P(D_{L+R}=d)-P(D_L=d)]\{d-s-i\}}{E[D_R]}. \end{aligned} \tag{5.14}$$

Here, $E\left[D_R\right]$ is the demand during the review period and P represents a probability.

Besides the reorder levels, the DoBr tool provides the expected inventory on hand for two moments in time. These two moments are after a potential delivery (at the beginning of the potential delivery cycle, $I^{OH}(L)_j$) reflecting the highest inventory level, and before a potential delivery (at the end of the potential delivery cycle, $I^{OH}(R+L)_j$). This corresponds to formula 5.1 .

5.4.2 Model 2: determining safety stock

Besides setting a service level or using an optimization model to minimize costs, the reorder levels can also be determined using a general formula as described in Section 2.2.3. The reorder level should ensure that the demand during lead time and review period is covered including an additional safety stock to cope with uncertainties. As this second model reflects an improvement on the current situation, the (R,s,nQ) replenishment logic is used. The current situation will be improved as this model allows different service targets for different products based on expected inventory costs.

There is expected that there will be a higher safety stock for products with lower uncertainty and/or lower value (which results in a lower holding cost and backorder cost making it more interesting to stock more of this specific product compared to products with a relatively high value). When there is a capacity constraint, reorder levels depend on each other. The aggregate fill rate will be met by stocking also other products that cost less in euros and/or space. The model will contain the following objective and constraints.

Objective
The objective is set to minimize the expected total inventory costs per time unit expressed in euros. The total inventory costs per time unit contains the holding costs per time unit, the ordering costs incurred when an order is placed and shortage costs when demand cannot be satisfied and is thus backordered (as backorders are assumed). The company aims at reaching high sales in combination with high revenue and profit, which is in line with the objective of minimizing inventory costs (containing the elements of holding costs and shortages). This objective ensures that stock can be built up to deal with uncertainty while balancing shortage and holding costs and adhering to the set constraints. Moreover, because of the holding costs, the objective will penalize overstocking which is one of the main problems in the current situation.

According to Axsäter (2015), due to the holding of stock there is an opportunity cost for capital tied up in inventory. These costs are the dominating part of the holding costs. Coolblue owns its warehouse and therefore these fixed warehouse costs will not be taken into account within the model. Holding costs will be determined as a percentage of the unit value (Axsäter, 2015). Multiplying the average expected inventory on hand with the holding costs of an item and the length of the period results in the expected holding costs incurred per SKU per period.

As there will be ordered from an external supplier, fixed ordering costs will be taken into account. These costs reflect the costs of replenishing independent of the batch size. The expected times that will be ordered within a phase g is the expected total demand of the phase ($T_g \cdot E[D_1]_{g,j}$) divided by the expected order size (Ghiani et al., 2004). The expected order size will be calculated using formula 5.11 (including converting IOQ and MOQ to Q). The expected total ordering costs within a phase are the number of expected times that will be ordered multiplied by the ordering costs. To calculate the ordering costs per time unit, the total ordering costs are divided by the number of time units within a phase which is equal to the length of the phase (T_g).

$$\begin{aligned} E[\text{Ordering costs}]_j &= \tfrac{(T_g \cdot E[D_1]_{g,j}/E[OS]_{g,j}) \cdot K}{T_g} \\ E[\text{Ordering costs}]_j &= (E[D_1]_{g,j}/E[OS]_{g,j}) \cdot K \end{aligned} \tag{5.15}$$

The last distinguished cost component is shortage costs. These costs reflect the associated costs of having stock-outs. Shortage costs only appear when there is a stock-out and this happens when the demand is more than the inventory on hand. The shortage costs are incurred per backordered item. The expected number of backordered items of an SKU during a period can be approximated by multiplying the expected demand for a period by 1 minus the expected fill rate of that SKU. The total resulting shortage costs for an SKU for a period can be calculated by multiplying the expected number of backordered items of an SKU by the shortage costs incurred per item.

$$E[X^-]_j = (1 - P_{2,j}(s_j)) \cdot E[D_1]_j \tag{5.16}$$

Summing all cost components results in the expected inventory costs per period per SKU. Dividing this sum by the length of the period results in the average expected costs per time unit for an SKU.

Constraints
A constraint is set on the aggregated fill rate. The fill rate is calculated for an SKU as in formula

5.5. Individual fill rates can be aggregated using several methods; general weights, volume-based weights, turnover-based weights and profit-weighted average fill rate (Van Donselaar et al., 2021). Therefore, the aggregated fill rate is modelled as the sum of weight times the individual fill rate. This aggregate constraint is set to make sure that the fill rate of the assortment is at least sufficient to serve the customer demand.

Besides an aggregated fill rate constraint, a minimum fill rate constraint is set. This ensures that all SKUs have sufficient fill rates including the expensive and highly uncertain demand SKUs. Otherwise, for SKUs with high holding costs will the reorder levels be lower in comparison to SKUs with lower holding costs. Since the products are decided to be within the assortment, it is needed that these products are most of the time available.

As the model is used to set a reorder level consisting of the sum of the demand during lead time and review period and a safety stock. The reorder level must be at least the expected demand during the lead time and review period. For some SKUs, this constraint is already satisfied because of the minimum fill rate constraint.

Another constraint will be set on the used capacity in the number of storage locations. This constraint cannot be exceeded. Therefore, the maximum inventory on hand can be used which is after a potential delivery and at moment $\tau + L$. Unfortunately, the inventory of incoming SKUs cannot be merged with inventory already in the warehouse when there is still enough space in the already occupied location. Therefore, a 'new' location will be occupied for the incoming shipments. The expected number of occupied locations cannot be exceeded and therefore will a maximum be set on $E[USL]$. Using $E[USL(L)]$ to set a maximum is not realistic as not all SKUs will occupy at the same moment the maximum number of locations. Within the model, the reorder level will be changed resulting in different reorder levels and potential changes in the number of occupied locations. The word potential is used since it can be the case that the expected number of used locations will not change since the used location is not fully occupied.

This objective and these constraints result in the formulated model described in 5.17. T_g refers to the length of the phase. The model will result in the safety stock and reorder levels as output. The expected inventory on hand, expected order size and expected number of backorders depend on the reorder level which is the sum of the expected demand during the lead time and review period and safety stock.

$$
\begin{array}{llcll}
\text{Min} & \multicolumn{4}{l}{\sum_{j\in J}(h_j \cdot E[I^{OH}]_{g,j} + (E[D_{1,g}]_j / E[OS]_{g,j}) \cdot K + b_j \cdot E[X^-]_{g,j})} \\
\text{s.t.} & \sum\limits_{j\in J} w_{g,j} P_{2,g,j} & \geq & P_2^{\star} & \\
 & P_{2,g,j} & \geq & P_2^{\min} & \forall j \in \{1,..,N\} \\
 & \sum\limits_{j\in J} E[USL]_{g,j} & \leq & W^* & \\
 & s_{g,j} & \geq & E[D_{R_j+L_j}]_{g,j} & \forall j \in \{1,..,N\} \forall g \in \{1,..,G\} \\
 & s_{g,j} & \in & \mathbb{N}^+ & \forall j \in \{1,..,N\} \forall g \in \{1,..,G\}
\end{array}
\tag{5.17}
$$

Solving of model

The model will be approached as a knapsack problem. Using a knapsack approach to solve a model like this is already done for example by Schwarz (2008), who solve a knapsack problem to determine the number of products in storage by balancing the limited resources. Another example is by Bijvank and Vis (2012a) in which inventory control variables are determined within an environment with limited storage capacity. This limited storage capacity is expressed in the number of bins and within a bin fit at most a specific amount of units. To solve the model, in this project, two solution methods are used; a greedy heuristic and a mixed integer linear program (MILP). Because the MILP has a lower computational time and shows better results than the greedy heuristic, there will be a focus on the MILP. The greedy heuristic is described in Appendix B including its performance.

MILP

To solve the model using a mixed integer linear program (MILP), first, all possible reorder levels are determined for an SKU. The minimum reorder level is determined as the demand during the lead time and review period. When this minimum reorder level results in a fill rate lower than the minimum fill rate constraint, the minimum reorder level is adjusted. The maximum reorder level is determined as the reorder level when the fill rate is higher than 0.999 as a higher fill rate results in very high costs. For every reorder level, in the range the minimum and maximum reorder level are the expected required locations, the expected inventory costs and the expected fill rate calculated.

All possible reorder levels of an SKU form the set Z. Using the following MILP, one reorder level per SKU is determined. The values of $E[I^{OH}]_{j,z}$, $E[OS]_{j,z}$, $E[X^-]_{j,z}$,$P_{2,j,z}$ and $E[USL]_{j,z}$ are for SKU j and combination z. An additional restriction is included to make sure that only one combination is selected per SKU. All in all, this results in the following MILP.

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{j \in J} \sum_{z \in Z} x_{j,z} \cdot (h_j \cdot E[I^{OH}]_{j,z} + (E[D_{1,g}]_j / E[OS]_{j,z}) \cdot K + b_j \cdot E[X^-]_{j,z}) \\
\text{s.t.} \quad \sum_{j \in J} \sum_{z \in Z} x_{j,z} \cdot w_j \cdot P_{2,j,z} \quad & \geq \quad P_2^\star \\
\sum_{z \in Z} x_{j,z} \cdot P_{2,j,z} \quad & \geq \quad P_2^{\min} & \forall j \in \{1,..,N\} \\
\sum_{j \in J} \sum_{z \in Z} x_{j,z} \cdot E[USL]_{j,z} \quad & \leq \quad W^* \\
\sum_{z \in Z} x_{j,z} \quad & = \quad 1 & \forall j \in \{1,..,N\} \\
x_{j,z} \quad & \in \quad \{0,1\} & \forall j \in \{1,..,N\}
\end{aligned}
\tag{5.18}
$$

5.4.3 Model 3: modifying IOQ and MOQ

As shown in Section 3, the ordered quantity could be increased as the ordered quantity is relatively low compared to the maximum pallet quantity. By also incorporating the $IOQ_{g,j}$ and $MOQ_{g,j}$ as decision variables, there is expected to be a positive result in expected inventory costs. This model is based on the model proposed by Van Donselaar and Broekmeulen (2022). By also optimizing the $MOQ_{g,j}$ and $IOQ_{g,j}$ (e.g. setting the $MOQ_{g,j}$ and $IOQ_{g,j}$ higher), the ordered quantity will increase resulting in fewer times ordering and filling locations can more be filled resulting in fewer locations needed. This can solve the perceived current location deficit.

The objective is to minimize the expected inventory costs and similar to the objective of model 2. The warehouse capacity and (minimum) fill rate constraints are similar to model 2. Additional constraints are set to ensure that all products within the set have an incremental and minimal order quantity that is at least the value of the current. Moreover, the set $MOQ_{g,j}$ must be a multiple of the set $IOQ_{g,j}$. The new set $IOQ_{g,j}$ must be a multiple of the current $IOQ_{g,j}^{Current}$ if the $MOQ_{g,j}$ is set smaller than the maximum pallet capacity. When the $MOQ_{g,j}$ is set to the maximum pallet capacity, it can be beneficial to set the $IOQ_{g,j}$ also to the maximum pallet quantity as it will facilitate the most efficient use of storage locations. The maximum of the new set MOQ is the maximum pallet capacity as this is one unit load. Incorporating these additional constraints result in the model presented below.

$$
\begin{array}{lrcll}
\text{Min} & \multicolumn{4}{l}{\sum_{j\in J}(h_j \cdot E[I^{OH}]_{g,j} + (E[D_{1,g}]_j/E[OS]_{g,j}) \cdot K + b_j \cdot E[X^-]_{g,j})} \\
\text{s.t.} & \sum_{j\in J} w_{g,j}P_{2,g,j} & \geq & P_2^\star & \\
& P_{2,g,j} & \geq & P_2^{\min} & \forall j \in \{1,..,N\} \\
& \sum_{j\in J} E[USL]_{g,j} & \leq & W^* & \\
& MOQ_j^{\text{Current}} & \leq & MOQ_{g,j} \leq C_j & \forall j \in \{1,..,N\} \\
& \text{IOQ}_{g,j} & & \begin{cases} = k \cdot IOQ_j^{\text{Current}} & \text{if } MOQ_{g,j} < C_j \\ \in \left\{\text{IOQ}_j^{\text{Current}}, C_j\right\} & \text{if } MOQ_{g,j} = C_j \end{cases} & k \in \mathbb{N}^+, \forall j \in \{1,..,N\} \\
& MOQ_{g,j} & = & k \cdot IOQ_{g,j} & k \in \mathbb{N}^+, \forall j \in \{1,..,N\} \\
& s_{g,j} & \geq & E[D_{R_{g,j}+L_j}]_{g,j} & \forall j \in \{1,..,N\} \forall g \in \{1,..,G\} \\
& s_{g,j}, MOQ_{g,j}, IOQ_{g,j} & \in & \mathbb{N}^+ & \forall j \in \{1,..,N\} \forall g \in \{1,..,G\}
\end{array} \tag{5.19}
$$

Solving of model
To solve this model, three solution methods are used; the two-step heuristic of Van Donselaar and Broekmeulen (2022), a modified version of the two-step heuristic and a MILP. Because the results and computational time of the modified version of the two-step heuristic is satisfactory, the results of the two-step heuristic and MILP are discussed in Appendix B.2. In the remainder of this section is the modified version of the two-step heuristic (hereafter IOQ/MOQ model) described.

The IOQ/MOQ model determines first the target fill rate per SKU as a first step. In the second step, all combinations of MOQ_j, IOQ_j and s_j of an SKU that meet this fill rate are sought. With all these possible combinations, the best values for MOQ_j, IOQ_j and s_j for all SKUs are determined using a MILP.

In the first, the target fill rate is determined using a greedy heuristic. The starting reorder level of each SKU will be calculated as the demand during the lead time and review period. If this resulting reorder level is lower than the minimum fill rate constraint, the reorder level of this SKU will be increased until the minimum fill rate is met. The objective here is the minimization of the expected inventory costs. The ratio for each SKU is calculated as the change in objective (expected inventory costs) when the reorder level is increased by 1 divided by the (potential) change in the expected number of used locations when the reorder level is increased with one unit. This ratio is used since the main goal is to investigate the SKUs that yield the most (in controversy have the lowest costs) while not consuming a lot of space. The increased reorder level is assigned to the item with the lowest ratio. Increasing the reorder level of SKUs is done until the aggregated fill rate is satisfied.

As second, all combinations of MOQ_j, IOQ_j and s_j of an SKU that meet the fill rate (determined in the first step) are sought. In this MILP, the expected inventory costs are minimized and restricted by the number of locations and weighted fill rate. The combination of $MOQ_{j,f}$, $IOQ_{j,f}$ and $s_{j,f}$ for all SKUs form the set of combinations F. For each combination f and SKU j, the values of $E[I^{OH}]_{j,f}$, $E[USL(L)]_{j,f}$, $E[X^-]_{j,f}$,$P_{2,j,f}$ and $E[USL(L)]_{j,f}$ are calculated. These found combinations are the input for the MILP described in formula 5.20 with constraints on the weighted fill rate, warehouse capacity and selecting one combination per SKU.

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{j\in J}\sum_{f\in F} x_{j,f}\cdot(h_j\cdot E[I^{OH}]_{j,f} + (E[D_{1,g}]_j/E[OS]_{j,f})\cdot K + b_j\cdot E[X^-]_{j,f}) && \\
\text{s.t.} \quad & \sum_{j\in J}\sum_{f\in F} x_{j,f}\cdot w_j\cdot P_{2,j,f} \geq P_2^{\star} && \\
& \sum_{f\in F} x_{j,f}\cdot P_{2,j,f} \geq P_2^{\min} && \forall j\in\{1,..,N\} \\
& \sum_{j\in J}\sum_{f\in F} x_{j,f}\cdot E[USL]_{j,f} \leq W^{*} && \\
& \sum_{f\in F} x_{j,f} = 1 && \forall j\in\{1,..,N\} \\
& x_{j,f} \in \{0,1\} && \forall j\in\{1,..,N\}
\end{aligned}
\tag{5.20}
$$

5.5 Transition between phases

Since between the phases, the expected demand can change significantly, handling the transition to the next phase is needed. This transition is important because it needs to make sure that there is enough available stock when the phase starts in cases of transitioning from a phase with low to high demand. While in the case of changing from a phase with high to low demand, the transition needs to make sure that the inventory is not unnecessarily built up in the last days of the current phase as this can result in excess stock in the next phase.

To change the ordered quantity, the reorder level will be modified. The reorder level is modified at a review period resulting in a modified reorder level ($s^{mod}_{g,j}$). Because of the dependencies among the SKUs due to the capacity constraint, the reorder level is only modified when the sum of the review period and lead time exceeds the remaining duration of the current phase. Otherwise, the model could best be executed at every review moment which can be too computationally expensive. Because of modifying the reorder level in transitions from a low to high demand and vice versa, there is assumed that the capacity constraint will not be exceeded as the start of preparing for the transition from a high to low demand phase starts early.

There are several possibilities when the changeover of a phase is nearby in the future. When in the next phase the demand and thus corresponding reorder level is more than in the current phase, there needs to be made sure that at the end of the current phase, enough inventory is built up otherwise it will result in stock-outs at the start of the next phase. Therefore, before the end of the current phase, an additional quantity needs to be ordered. This is reflected in formula 5.21 in which the modified reorder level at a day (x^{cur}) is the sum of the demand needed for the remaining days in the current phase, the demand for the remaining days before the order will arrive in the next phase and the maximum safety stock in the current phase or the weighted safety stock. x^{cur} refers to the number of the day in the current phase. It is necessary to stock this additional stock for the next phase as late as possible because of the holding costs. Therefore, the reorder level will only be modified when the sum of the current time, lead time and review period are more than the end date of the phase. Using this modified reorder level will result in an order with a higher quantity. This results in advantages such as no additional ordering costs and decreasing the chance of being too late resulting in stock-outs.

Another possibility is that the demand in the next phase is expected to be lower than in the current phase. To not have a significant amount of stock leftovers, there needs to be made sure that especially near the end of the phase, only the amount is ordered that is needed within this phase. This correction is reflected in the modified reorder level. This modified reorder is set in the same manner as for the case when the demand is expected to be higher than in the current phase. The reorder level is only modified when the sum of the current time and review period and lead time exceeds the duration of the phase, otherwise, it will not be logical as the normal reorder level can be used.

$$s_{g,j}^{mod}(x^{cur}) = (T_g - x^{cur}) \cdot E[D_{1,g}]_j + (L_j + R_j - T_g + x^{cur}) \cdot E[D_{1,g+1}]_j + \tfrac{T_g - x^{cur}}{L_j + R_j} \cdot ss_{g,j} + \tfrac{L_j + R_j - T_g + x^{cur}}{L_j + R_j} \cdot ss_{g+1,j} \tag{5.21}$$

Chapter 6

Model settings & Simulation

Within the project, the expected performance of each model will be approximated. Besides, a simulation will be built to test the models including the transition between phases. As stated before, the horizon of a year will be divided into phases. Within this section, will first be explained how these phases are determined whereafter the parameter setting within the models will be discussed. Finally, how the simulation is built and the simulation specific parameters are discussed.

6.1 Parameter setting

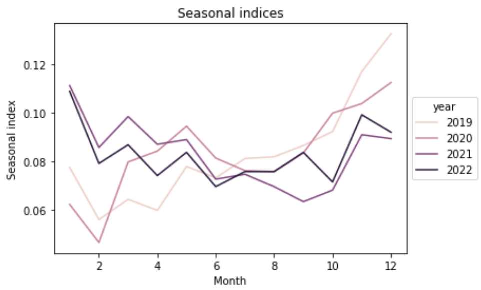
To be able to determine the phases within a year, the demand pattern of the products needs to be understood. Within this section, first, the demand pattern will be investigated where after the phases and other parameters are determined.

6.1.1 Demand

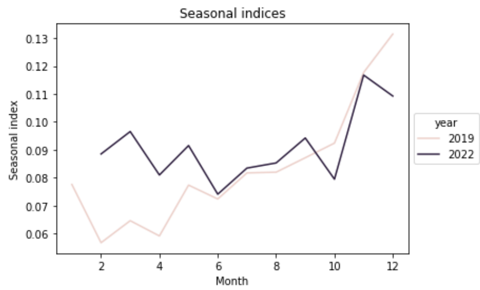
Seasonality
Recall that the recorded sales are assumed to be the demand as explained in Section 3. Because of the occurrence of a trend and seasonal pattern in the demand data, the demand can be classified as non-stationary.

To be able to understand the sales throughout the year, seasonal patterns within a year are investigated. The seasonal indices of all products within scope within a year are drawn from 2019 to 2022 for monthly periods (figure 6.1). While the sales in 2021 are not representable for the current situation, the seasonal indexes are still drawn for this year to show the occurrence of a seasonal pattern. As expected the seasonal pattern shows an increase towards the end of the year (for all years) which can be explained by Black Friday and the Christmas period. Moreover, at the start of the year (especially in 2019 and 2020), the sales decease where after it increases again. This can partly be explained by the increase in temperature and the corresponding 'needed' products such as air coolers, barbecues, and garden appliances in the summer period (as can be seen in the total daily sales of garden tools and climate control products in Figure 6.1c). The seasonal indices are inconsistent over time. The sales within 2021 and 2022 are more evenly spread compared to 2019 and 2020. To examine the seasonal pattern on an SKU level, ACF plots are used since the relationship between lagged values of a times series is measured by autocorrelation. This is done for some SKUs and a seasonal pattern has been observed.

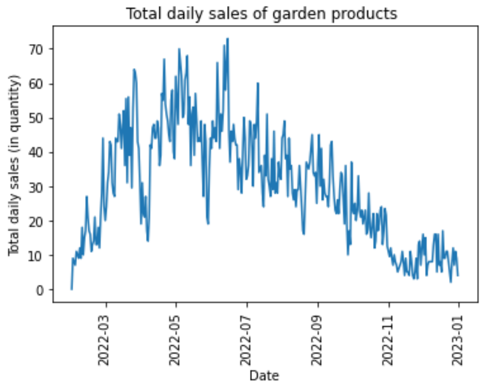
Determining length of phases
There are several options to cope with non-stationary demand in an inventory control situation



(a) Seasonal indices original data



(b) Seasonal indices cleaned data



(c) Daily sales of garden tools and climate control products over time

Figure 6.1: Seasonality

as described in Section 2.2.4. Within this project, the planning horizon will be divided into phases and in these phases, the demand is assumed to be stationary as described in Section 5.3. As a starting point, the duration of the phases is set to a month as is done by Chen and Chang (2007).

Within a phase is assumed that demand is stationary. To test the stationary of the SKUs within a phase, the Augmented Dickey-Fuller (AD-F) test is used. This test checks whether the time series is stationary by setting a null hypothesis; the time series is non-stationary. The null hypothesis can be rejected when the $p - value$ is below the 0.05 significance level. This test is done per SKU per month per year using the cleaned sales data as described in Section 4. The duration of phases are varied. On average most SKUs have a stationary demand when the duration of the phases is set to 13 weeks (Table 6.1). Looking at the seasonal indices of the cleaned data of 2022, there was expected that two phases will be most beneficial; the first phase having a duration of months 1 to 10 and the second phase of solely months 11 and 12. This is not the case and can be explained by the underlying seasonal patterns of SKUs for example garden SKUs which have a higher demand between April and June. Therefore, there will be 4 phases with a length of 13 weeks.

Table 6.1: Overview of results AD-F test

Duration of phase	**Ratio stationary SKUs (in %)**
1 month	88.75
4 weeks	87.24
2 months	93.61
8 weeks	91.23
13 weeks	96.03
Months 1-10 and months 11-12	95.39

Demand distribution
Using the data of the year 2022 and the set duration (13 weeks) of the phase, a demand distribution will be fit on the demand per SKU. Because of actual discrete demand values, it is possible to fit a discrete distribution (Axsäter, 2013). This is also preferred as there is discrete demand in practice and there is a relatively low average demand (Van Donselaar and Broekmeulen, 2014). Therefore, a discrete demand distribution will be used.

A theoretical distribution will be fitted based on the standard deviation and mean of the demand using the method of Adan et al. (1995). The standard deviation and mean demand are calculated based on the cleaned demand data per SKU and per phase of the year 2022. Based on the outcome of the variable a (calculated using formula 6.1), a discrete probability distribution function is chosen.

$$a = \frac{\sigma^2/\mu - 1}{\mu} \tag{6.1}$$

For $a = 0$ the Poisson distribution is chosen, for $-1 < a < 0$ the Binomial distribution, for $0 < a < 1$ the negative binomial distribution, and for $a \geq 1$ the geometric distribution. As the theoretical distribution is based on a limited set of historical data, the distribution is an estimate of the true probability function.

6.1.2 Other parameters

Within this section, the setting of other parameters is discussed. The setting of parameters is based on literature and analysis of the current situation.

Lead time
As explained in section 3, the lead time per supplier instead of per SKU will be used as lead time. This is estimated by calculating the supplier lead time of all regular replenishment SKUs (excluding data points in the COVID-19 period) and taking the average of this per supplier.

Review period
Within Coolblue, the review period is not strictly reported as inventory planners can review the inventory every weekday. As indicated by experts within Coolblue, the placed orders are possibly consolidated at the supplier. This means that reviewing the inventory every day and placing small orders may not result in also receiving the order with a duration of the lead time as the supplier can consolidate the orders. Therefore, the review period is estimated as the time between the delivery of shipments by the same supplier.

MOQ
The minimum order quantity will be set as the required minimum order value set by the supplier divided by the purchasing price of an SKU. If no minimum order value is set by the supplier, €100,00 is used as the minimum order value to calculate the minimum order quantity.

IOQ
The incremental order quantity is used in the third model and will be set as defined by the supplier. If nothing is defined for an SKU, the IOQ will be set to 1.

Q
To mimic a realistic current situation the used ordered quantity per SKU in the first and second model is set to the average order quantity of an SKU.

Maximum number of products on pallet
The maximum number of products on a pallet are calculated as described in Section 4.2.

Holding costs
Within Coolblue, the precise holding costs are unknown and therefore an approximation is used. Teunter et al. (2017) models the holding costs of an SKU per time unit as the inventory holding charge times purchasing price of the SKU. The inventory holding charge is assumed to be constant for all items. This approach is in line with the common assumption that the holding cost parameter is constant (Berling, 2008). Moreover, according to Azzi et al. (2014) cost of capital makes up most of the costs. Unfortunately, no standard inventory holding rate are found within literature. The used of costs of capital at Coolblue is 23% and therefore is the annual holding cost rate set at 23% (resulting in 23/365 % as daily holding cost rate used in model). This 23 % is compared to other articles using a holding cost rate. As within most textbooks, the percentages range between 12 and 34 % (Berling, 2008). Therefore, 23% as the holding cost rate is appropriate.

Shortage costs
Due to having stock-outs, a company will have a loss of sales and thus revenue. Unfortunately, stock-outs may also result besides the loss in sales, in loss of customer satisfaction and decrease in market share (Boulaksil et al., 2009) (Zinn and Liu, 2001) (Gruen et al., 2002).

Quantifying these losses due to stock-outs numerically is difficult. Solely using product profit margin cannot be done as some products have a low profit margin while these products are important, for example, as a complementary product or for attracting people to the website.

Because of these reasons, the lost product profit margin does not reflect the impact of a lost sale. Therefore, the shortage costs are set using the result of the classical Newsvendor problem as also done by Broekmeulen and Van Donselaar (2009). The Newsvendor problem is a one-period model in which is determined how much to stock under uncertain demand knowing that leftover stock cannot be sold anymore. The result is that in the classical Newsvendor problem, the service level needs to be equal to the division of the underage cost by the sum of the overage and underage cost, which is used to determine the costs associated with lost sales. The holding costs and the shortage costs are balanced such that it is unfavorable to either hold more inventory nor backorder more. Within this study, the overage costs are equal to the holding costs (assuming that for excess stock, Coolblue removes the product for a price similar to the purchasing price and depreciation). The underage costs are equal to the lost sales costs. The fill rate is set equal to the aggregated target fill rate. The shortage cost per SKU are determined using the following formula;

$$b_j = h_j \cdot \frac{P_2}{1 - P_2} \tag{6.2}$$

Fill rate
As explained in Section 5.4.2, a minimum and target fill rate is used. The minimum fill rate is set to 0.83 as decided in consultation with practitioners of Coolblue. Bijvank and Vis (2012b) set a minimum fill rate in the range between 0.75 and 0.99. As the set minimum fill rate lies within this range, the minimum fill rate is found to be appropriate.

The target fill rate is set to 0.95 reflecting the commercial goals of Coolblue for these products. This set target fill rate is within the range (0.8 - 0.99) of often set fill rates within literature and therefore denoted as sufficient (Pauls-Worm et al., 2014; Van Donselaar et al., 2021; Teunter et al., 2017).

Ordering costs
The ordering costs will be set to €10,00 per placed order per SKU. This value is based on the costs of creating, and verifying orders, planning the orders, and invoice handling. Moreover, an additional part is included regarding handling delivery conflicts as the duration of this process

is high resulting in high costs relative to the normal flow of placing and receiving orders. In addition, a part is included that mirrors the costs when small order quantities are ordered to penalize high frequent ordering. Orders with small order quantities at the same supplier are delivered on a pallet with multiple SKUs (so-called mixed pallets). These mixed pallets need to be decomposed and sealed again resulting in additional time needed. Moreover, it also results in extra traveling time as a forklift can only bring one pallet per time to a storage location. As the duration of these processes depends on a high number of factors, the mean process duration is approximated.

Warehouse capacity
In 2022, the mean number of storage locations for the SKUs in scope used a day is 2056, the minimum is 1637, the median is 2071 and the maximum is 2437. In 2022, the number of used locations was at some points too high and therefore the warehouse capacity cannot be set based on the achieved results in 2022. The 868 SKUs were approximately 10% of the sales and 12% of the volume of all SKUs stored in Parcel Large locations. The numbers are relatively low which can be explained by other Parcel Large products that have high sales because of deals and the fact that a lot of products within Coolblue have a short product life cycle. In this study, products with sales of at least 3 years are in scope (because of being able to investigate seasonal patterns). Based on these numbers the preferred number of occupied warehouse locations by these SKUs was between the 1436 and 1821 storage locations (including a 15% buffer of storage locations). The warehouse capacity is set to 1725 as this is the 75% point between 1426 and 1821 and found to be sufficient by Coolblue practitioners.

6.2 Simulation

This study uses simulation to explore the performance of parameter changes and the designed models in a realistic situation. The MILP part of the models are solved using Gurobi Optimizer 9.5.1 in Python 3.9. The main goal of the simulation is to imitate the behavior of the actual inventory system and evaluate the impact of the designed models and transition between phases based on performance indicators. A discrete system is used because these systems are only observed at regular points in time which is in line with the system under study (because of reviewing periodically). Discrete-event simulation with an objective-oriented approach is used because the system state changes due to randomly occurring events (e.g. customer demand). First, the distinguished events are discussed whereafter the simulation specific parameters are explained.

6.2.1 Simulation procedure

In principle, the working of the simulation is as follows; first, an event is selected from the Future Event set. As second, the time is set corresponding to the event's time. Then the event is handled based on the specific type according to the event handler. This is done until the time exceeds the running time. The events that are distinguished are:

- *Check incoming customer demand*: Because of customer demand, a decrease in the inventory position and inventory on hand can be observed.
- *Review inventory*: Every R time unit, the inventory position of an SKU is reviewed and potentially an order is placed.
- *Receive incoming order*: A placed order is received resulting in an increase in the inventory on hand.
- *Start of new phase*: This event starts a new phase and sets the parameters per SKU to the right values (corresponding with the new phase).

The order of events on a day is, first inventory is reduced by occurring demand, whereafter a replenishment decision is possibly made whereafter a potential order is delivered (Van Donselaar and Broekmeulen, 2014). Demand per SKU is sampled using a discrete distribution based on past data using the fitting procedure of Adan et al. (1995). When demand occurs, demand is fulfilled from a location using the FIFO principle. As the simulation reflects as best as possible the current situation, demand is lost when available inventory is less than the demand.

The placing of an order increases the inventory position immediately and the order is received after the lead time. The receiving of the order increases the inventory on hand. When the order is received, the products of the new order will be placed on new storage locations instead of filling the already-used locations of the specific SKU. This reflects honeycombing and is in line with the current operations within the warehouse.

Within the simulation, a distinction is made between week and weekend days. On weekend days, it is not possible to place orders or receive incoming orders. Therefore, incoming orders scheduled on Saturday are delivered on Friday and scheduled on Sunday are delivered on Monday. This reduces high inventory fluctuations by dividing the incoming shipments planned in the weekend on two days instead of solely one day. Moreover, the reorder level is corrected for weekend days by considering the number of total days until the next lead time and review moment.

When the event start of a new phase occurs, the parameters of the SKUs are set to the parameters within this phase (i.e. fill rate weight, standard deviation, and mean demand) and the reorder levels and safety stocks are calculated using one of the designed models. The transition between phases (as described in Section 5.5) is implemented by modifying the reorder level at a review moment when the conditions (the sum of the current day, lead time and review period exceed the length of the phase) are met.

An overview of the flow between the events can be seen in Appendix C. In the simulation, a day represents a single time step. At the start of the simulation, the events of incoming customer demand, review inventory, and the start of a new phase are triggered. To better reflect the situation in practice, the review inventory events for all SKUs are scheduled using a schedule to make sure that not all review inventory moments are at the same moment. During the handling of these events, the next event of these types is scheduled.

6.2.2 Simulation parameters

Besides model parameters, also values for specific simulation parameters need to be defined. These values are specified using the simulation of the current situation model as this situation shows high variance. A detailed discussion of the settings is provided in Appendix D. As the simulation will start with inventory levels of 0, the warm-up period is of high importance. The warm-up period will be set to 1000 time units (see Figure D.1 in Appendix D), the replication length to one year and the number of runs to 12 (based on the method described by Boon et al. (2020)) .

6.2.3 Validation simulation

The simulation model needs to be verified and validated before it can be used as a tool to gather outputs. To determine whether a simulation program performs as intended is referred to as verification (Kleijnen, 1995). Validation refers to whether the simulation is a correct representation of the system in practice.

First, the model is verified. This is done by verifying intermediate output with theory and context of the system. Moreover, as objective-oriented programming is used, modular testing is done.

Validation is done by comparing the simulation results to the results of analytical formulas. In this way the validation of the simulation is guided by knowledge of theoretical models with known solutions as is asserted by Kleijnen (1995) when studying real systems. The values of analytical formulas are calculated using the DoBr tool. Based on the results (as shown in Appendix E), the simulation can be used to gather outputs.

Chapter 7

Results

The performance of the described models and solutions methods can be compared using two methods. On the one hand, mathematical formulas can be used to approximate the expected performance of the model. On the other hand, expected performance can be approximated by implementing the models in a simulation. A disadvantage of the first method is that no distinction can be made between week and weekend days. First, the results using mathematical formulas are discussed. As second, obtained results using simulation are discussed and the sensitivity of the models is tested.

7.1 Mathematical formulas

Using the formulas described in Section 5, the expected performance of the models is approximated. The KPIs that are of interest are the expected total costs, average expected inventory per time unit, expected number of required locations, and expected order size. These KPIs are calculated per phase. The mean value for the KPIs of each model is compared. The overview of the results using approximation is documented in Table 7.1.

Table 7.1: Overview of results using mathematical formulas

	Current	**SS-model**	**Difference current and SS-model**	**IOQ/MOQ-model**	**Difference current and IOQ/MOQ**
Average expected inventory on hand per time unit	6596.4	6192.2	-6%	8675.9	32%
Average expected order size	6.1	6.1	0%	14.8	142%
Average expected backorders per time unit	0.05	0.05	0%	0.05	-10%
Average expected used locations per time unit	1745.9	1514.1	-13%	1015.5	-42%
Maximum expected used locations per time unit	2011.2	1779.4	-12%	1144.6	-43%
Total expected costs per time unit	2102.30	2025.07	-4%	1329.95	-37%
Fill rate	0.95	0.95	0%	0.95	0%
Expected holding costs per time unit	721.29	618.85	-14%	778.25	8%
Expected ordering costs per time unit	1295.29	1295.29	0%	449.24	-65%
Expected backorder costs per time unit	85.73	110.93	29%	102.46	20%
Ratio holding costs (in %)	34%	31%		59%	
Ratio ordering costs (in %)	62%	64%		34%	
Ratio backorder costs (in %)	4%	5%		8%	
Computational time (in s)	2.4	3045		6053	

When comparing the current situation with the safety stock model, especially the 4% cost decrease is interesting. This can partly be explained by the fact that the MILP sets the reorder levels in such a way that it perfectly fits within the capacity constraint while considering costs. It results in holding less expensive products (this is reflected in the decrease of expected inventory on hand and expected holding costs). In the current situation, all products have the same fill rate resulting in a situation in which no service differentiation can take place, while service differentiation is done in the MILP as a weighted target fill rate is set. This results in setting higher reorder levels for less expensive SKUs.

The holding costs are not close to the ordering costs. While this was expected since in the EOQ formula, an optimal Q is given by balancing the ordering and holding costs. In this situation, the holding and ordering costs are not perfectly balanced but can be explained by the set capacity constraint and the used Q from practice. The expected order quantity based on the EOQ formula is $Q^* = \sqrt{\frac{2AD}{vr}} = \sqrt{\frac{2\cdot 10\cdot 0.44}{127.94\cdot 0.23/365}} = 10.4$ which is higher than the used Q within these models. This shows possibilities to further improve by setting the IOQ and MOQ based on the current values as is done in the IOQ/MOQ model.

As expected, the IOQ/MOQ model results in a 37% expected inventory costs decrease compared to the current situation. Within this model, the IOQ and MOQs are set in such a way that the locations are more fulfilled reflected in the higher expected inventory and expected order size. This results in higher holding costs and lower ordering costs (because of higher inventory levels, there needs to be ordered less frequently). The IOQ is most often set to 1 as this results in the most flexibility while placing an order. The MOQ is set to a value between the current MOQ and max pallet capacity. The expected order size relies heavily on the set MOQ. Moreover, the inventory on hand is often similar to half of the expected order size. This can be explained by when there is ordered at least the MOQ needs to be ordered. The expected order size is not similar to the calculated Q^* resulting from the EOQ formula. This can be explained by the set capacity constraints and lower bounds of the MOQ and IOQ.

In all models, the backorder costs are a small proportion of the total costs which can be explained by the setting of a minimum fill rate and a weighted fill rate. As these fill rates are already relatively high, the expected shortages are low resulting in a low backorder costs.

7.2 Simulation model results

In this section, the models are implemented in the simulation as described in Chapter 6. The models are tested for two situations: 1) using the modified reorder level and 2) not modifying the set reorder level.

No modified ROL
The results for not modifying the reorder level between phases are described in this section. As expected based on the previous section, the IOQ/MOQ model outperforms the other two models regarding costs. Especially with the current situation, a cost decrease of 20% can be observed. Also in line with the approximation results, the inventory and average order size increased. Moreover, the IOQ/MOQ model is the only model that always use less locations than the set maximum which can be explained by the increased order size. Moreover, the median number of used locations is 22% lower compared to the current situation.

The total inventory costs of the safety stock model is 3% lower than the current situation model. This can be explained by the decrease in holding and ordering costs (which is also reflected in the slightly higher total inventory and average order size). What is especially interesting is that within the simulation, the ratio ordering and holding costs are closer than within the mathematical formulas for the current situation and safety stock model. This can be explained by the transitions between phases and the increased order sizes because of taking into account weekend days.

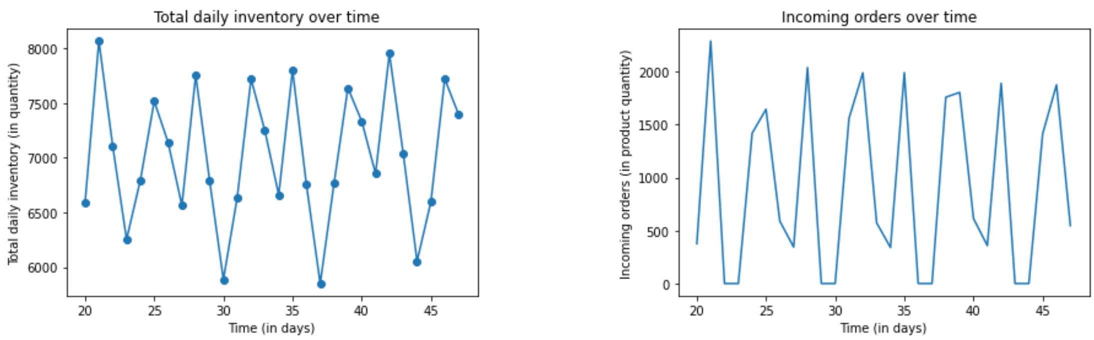
Approximation vs simulation
When comparing the approximation and simulation results, the higher inventory is remarkable. This can be explained by the building up of inventory throughout the year. When a product goes from high to low demand for several phases, it takes a long time to decrease the inventory as this depends on the demand. An example is air coolers, these products are sold during the summer period and the rest of the year is the demand exceptional low. Therefore, first, the inventory is built up whereafter it takes a long time to decrease the inventory. In line with the

Table 7.2: Overview of results using simulation

	Current	SS-model	Difference between current and SS	IOQ/MOQ model	Difference between current and IOQ/MOQ
Total inventory	3071562	3109040	1%	4158948	35%
Average order size	7.9	8.0	2%	17.0	116%
Total shortages	41168	40985	-0.4%	41580	1%
Total times ordered	36913	36694	-1%	16287	-56%
Maximum occupied locations	1831	1785	-3%	1367	-25%
Maximum % of occupied locations	1.06	1.04	-3%	0.79	-25%
Median % of occupied locations	0.87	0.88	1%	0.67	-22%
Fill Rate	0.950	0.950	0%	0.950	0%
Total inventory costs	797662.20	775605.59	-3%	639497.64	-20%
Total holding costs	338234.39	309095.44	-9%	372090.83	10%
Total ordering costs	369133.51	366940.00	-1%	162877.99	-56%
Total shortage costs	87743.26	99570.15	13%	104528.81	19%
Ratio holding costs	42%	40%		58%	
Ratio ordering costs	46%	47%		25%	
Ratio shortage Costs	11%	13%		16%	

higher inventory, also the number of used locations is building up throughout the phases which can be explained by the same reasoning. This situation frequently occurs in practice.

The fluctuating pattern of the inventory levels can be explained by solely receiving orders on week days (figure 7.1). Because of this, two days worth of orders are received on the Friday and Monday resulting in high peaks in the incoming orders with as result high inventory levels. The low inventory level peaks are on Sunday because no orders are received in the weekend.



(a) Daily inventory over time for a period of the year (b) Incoming orders over time for period of the year

Figure 7.1: Overview of inventory levels SS-model

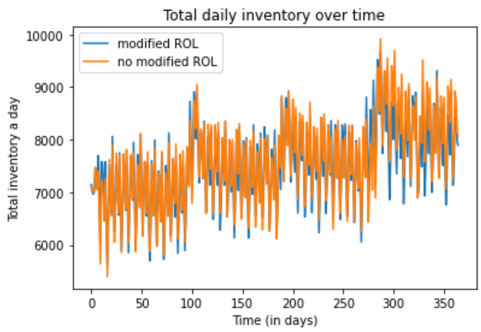
No modified reorder vs modified reorder level

The reorder level is modified to ensure that at the start of the next phase, there is enough inventory available when the demand increases or to ensure that unnecessary stock is not built up. In the days before the start of a new phase, the number of locations used by the system with the modified reorder level is higher because more products are ordered due to increased demand in the next phase (Figure 7.2). When zooming in on day 183 (the day at which phase 3 starts), the number of used locations before this day in the case of modifying the reorder level is higher than not modifying the reorder level. This can be explained by the building up of inventory. The effect of modifying the reorder level resulting in a lower order quantity cannot clearly be seen as this effect is relatively small compared to the corresponding increase in order quantity.

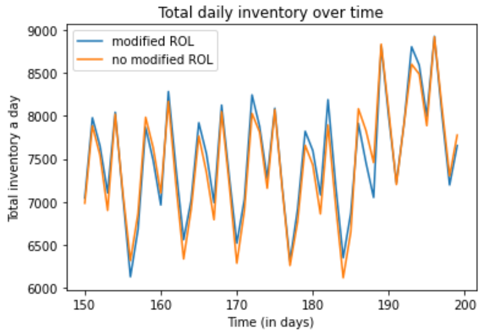
Modified reorder level

Modifying the reorder level influences the inventory on hand and costs. Because of ordering the needed products for the next phase earlier, the inventory on hand and number of maximum used locations increased. Because of this, the number of times ordered decreased due to already heightening the orders earlier resulting in less needed orders. Therefore, from a cost perspective, modifying is interesting.

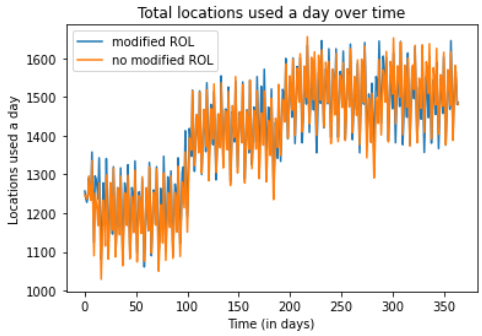
In the SS-model, there is ordered slightly more (reflected in the order size) and ordered less frequently compared to the current situation. This results in a decrease in the maximum used locations. This can be explained by on the one hand having less inventory and order less



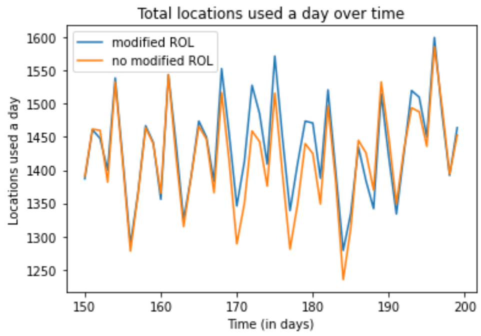
(a) Daily inventory



(b) Daily inventory zoomed in



(c) Used locations



(d) Used locations zoomed in

Figure 7.2: Differences in inventory and used locations for modified and not modified reorder level

frequently. The focus of the SS-model, minimizing costs, can best be seen in the increase in total inventory while having a decrease in holding costs. The shortage costs have increased remarkably which can partly be explained by the transition between phases. When there is a transition from a low to high demand phase, there is still a possibiltiy that the order is too late resulting in shortages of relatively expensive products in the first few days.

The differences between models 1) modifying the reorder level or 2) not are highest for the IOQ/MOQ model. Less inventory is needed and costs decrease when the reorder level is modified. This can be explained by lowered holding and ordering costs (because of higher order size) which outbalances the increased shortage costs.

All in all, the IOQ/MOQ model outperforms the other models on the most important objective, inventory costs but the model is not necessarily an optimum solution on all KPI's as the inventory increases greatly.

Table 7.3: Overview of results using simulation with s^{mod}

	Current	SS-model	Difference between current and SS	IOQ/MOQ-model	Difference between current and IOQ/MOQ
Total inventory	3050231	3048513	0%	3980755	31%
Average order size	7.89	7.96	1%	17.07	116%
Total shortages	41323	41286	0%	41286	0%
Total times ordered	37010	36787	-1%	16416	-56%
Maximum occupied locations	1803	1762	-2%	1385	-23%
Maximum % of occupied locations	1.05	1.02	-2%	0.75	-28%
Median % of occupied locations	0.86	0.87	4%	0.67	-22%
Fill rate	0.950	0.950	0%	0.950	0%
Total costs	795786.33	772285.40	-3%	624020.43	-22%
Total holding costs	338826.21	302927.38	-11%	356067.79	5%
Total ordering costs	370100.00	367870.00	-1%	164163.31	-56%
Total shortage costs	86860.12	101488.02	17%	103789.33	19%
Ratio holding costs	43%	39%		57%	
Ratio ordering costs	47%	48%		26%	
Ratio shortage Costs	11%	13%		17%	

7.3 Sensitivity

The performance of the different models depends on the input parameters. Therefore, in this section, the input model parameters are changed regarding weekend days, ordering costs, review period, storage locations and fill rate. Moreover, also the influence of shortage costs is analyzed.

As described in Section 7.2, the fluctuating pattern of the inventory levels can be explained by receiving orders only on week days. When also enabling the simulation to receive orders on weekend days, the costs decrease with 0.2% in the case of the SS-model. Especially, the high decrease in order size and more frequent ordering resulting in a 13% decrease in inventory and 11% increase in ordering costs. The costs for the IOQ/MOQ decreases with 4% when also receiving orders on weekend days. This can also be explained by the decrease in order size and more frequent ordering.

The set MOQ depends on the set ordering costs. When the ordering costs are 50% decreased, the mean order size decreases with 24% while ordering occurs more frequent. Moreover, as ordering becomes less expensive, it becomes preferable to order more frequent and have lower inventory levels (10% decrease). As expected, when increasing the ordering costs, an increase in the inventory and order size and decrease in the times ordered occurs.

According to the formula of the reorder level, when the review period is decreased, the reorder level will decrease. Decreasing the review period with 25% results in a decrease in inventory which can be explained by reviewing more frequently leading to needing less inventory on stock. Moreover, it results in more frequent ordering and lowered order sizes. Because of more frequent ordering, the ordering costs increase which do not balance the decrease in holding costs with as result a cost increase of 10% of the SS-model.

Increasing the storage locations, results for the IOQ/MOQ not in other outcomes as not the full capacity is used. For the SS-model, costs decrease (by 3%). This can be explained by more frequent ordering and the decrease in holding costs outbalances the increase in ordering costs resulting in lower costs.

As expected, increasing the fill rate results in both models in increases in inventory, costs and used locations. For the SS-model, increasing the fill rate to 0.98 results in 38% more inventory, 27% more occupied locations, and 14 % costs increase (because of a decrease of 38% shortage costs). For the IOQ/MOQ model, increasing the fill rate to 0.98 results in 29% more inventory, 24% more occupied locations, and 15 % costs increase.

Within literature, two types of models are distinguished regarding fill rates and inventory levels; models in which costs are minimized restricted by a fill rate constraint and models in which inventory and shortage costs are minimized. In this study in both models, the shortage costs and a fill rate constraint are set. Based on the results, the shortage costs are a small proportion of the total costs. When the shortage costs are not part of the cost minimization, the difference in results regarding inventory and costs are small (less than 3%). Moreover, the number of shortages slightly changes but especially the shortage costs increase when not taking into account the shortage costs as part of the costs. This can be explained by having shortages of products that are of higher value. Therefore, adding the shortage costs as part of the inventory costs minimization and setting fill rate constraints show advantages.

Chapter 8

Conclusion

This chapter describes the main findings of this study. Research questions are answered and managerial insights are presented. Moreover, the scientific contribution is discussed followed by the limitations of the study and directions for future research.

8.1 Answer to research questions

The research's objective is to *design a system to control inventories in which expected inventory costs are minimized and take into account warehouse space capacity.* Therefore, the following main research question was formulated:

How to design a system to control inventories in which expected inventory costs are minimized and take into account warehouse space capacity?

To answer the main research question, three sub-research questions were formulated. The first sub-research question focused on defining objective(s), constraints and decision variables. The context of this study is characterized by lost sales, non-stationary stochastic demand, positive lead times, multi-period, one stocking point and capacity constraints. Also, the model is constrained by warehouse storage locations and fill rate. The decision variable is the reorder level/safety stock per SKU. The MOQ and IOQ are potential decision variables.

Based on the performed literature review, an adequate model is formulated. The objective of the model is to minimize total expected inventory costs per time unit modeled as the sum of the holding, ordering and shortage costs. The model is restricted using a warehouse capacity constraint to restrict the average number of used locations per time unit considering honeycombing. Besides, a minimum and weighted target fill rate are set.

To design an inventory control system for the situation, assumptions are made regarding lost sales and the inventory system used. Lost sales are assumed but can be relaxed using a backorder model because of the calculated number of outstanding orders and relative demand uncertainty during lead time and review period. The used inventory system is an (R,s,nQ) inventory control policy.

Because of non-stationary stochastic demand, the horizon is divided into phases to cope with non-staionary demand. For each phase per SKU is the expected demand determined and used as input for the model. The changeover between phases is of high importance due to the possibility of having too high or low inventory levels. Therefore, a modified reorder level is used to smoothen the transition to a new phase resulting in lower inventory costs.

The second sub-research question focused on determining solution methods to solve the proposed system to control inventories. Combining the objective and constraints, a safety stock model is formulated in which the costs are minimized restricted by warehouse storage locations and fill rate constraints. Based on the set reorder levels per SKU, the set safety stock can be calculated per SKU. The model is solved using a MILP (and greedy heuristic). As space is the problem and solely ordering full pallets is not preferable, another model is designed in which besides the reorder level (/safety stock), also the IOQ and MOQ are determined per SKU. This model is solved using a combination of a greedy heuristic and a MILP. The combination of honeycombing and minimization of expected inventory costs in a model is novel.

The third sub-research question focuses on comparing the solution methods. The methods are compared based on mathematical formulas and a simulation. Using mathematical formulas, the safety stock model shows a 4% cost decrease and the IOQ/MOQ model a 37% cost decrease compared to the modeled current situation. Using the simulation, the safety stock model shows a cost decrease of 3% and the IOQ/MOQ a decrease of 20%. Implementing a modified reorder level to smoothen this transition results in even higher costs decreases (3% for the safety stock model and 22% for the IOQ/MOQ model) relative to the current situation (with no modified reorder levels). The computational time of the IOQ/MOQ model is higher than for the safety stock model but still possible to implement in practice (6053 seconds for 868 SKUs vs 3045 seconds for 868 SKUs). Therefore, from a cost and location perspective, the IOQ/MOQ model is the most preferable method to control inventories. The combination of smoothening the transition between phases and the designed models (without order modification) are a novel contribution and shows a way of handling non-stationary demand in inventory control.

In conclusion, the redesigned inventory control model follows an (R,s,MOQ,IOQ) inventory policy with dynamic values for s, MOQ and IOQ. These values change every phase and are set using the designed IOQ/MOQ model. This model minimizes expected inventory costs restricted by the number of storage locations and fill rate constraints. Honeycombing is taking into account to not underestimate the needed warehouse locations.

8.2 Managerial insights

This research provides relevant findings for the company in various ways. Firstly, the company is advised to take into account a system approach in order to set SKU reorder levels. On the one hand by including a weighted fill rate. On the other hand, taking into account the warehouse capacity. This results in decreased inventory costs and not extremely exceeding the capacity constraints.

Moreover, the company is advised to optimize SKU's MOQ and IOQ values as this is shown to result in decreased inventory costs and required locations. Moreover, because of increased order sizes and less frequent ordering, the handling time of incoming orders in the warehouse will decrease because as expected less mixed pallets will arrive. A potential downside of increasing the order size, is the increased inventory.

In addition, there is no clear rule regarding reviewing inventory. In practice, this is often done when the inventory is low or sales has extremely increased or decreased (but this also depends on the method of the inventory planner). Right now, it is unknown what the review period is and the review period is thus approximated. When data is collected regarding the review period, the models can better reflect the situation. Moreover, as shown in the sensitivity analyses for the SS-model more often reviewing the inventory results in more frequent ordering, needing more locations and a cost decrease. Therefore, setting the review period not too low is advised.

All in all, the company is advised to implement the IOQ/MOQ model for the products in scope as this has been shown to have high potential. A disadvantage of this model is the increase in inventory on hand which can be problematic in terms of risk as is also agreed on with inventory experts from Coolblue. Comparing the inventory levels of the model with the inventory levels in practice still shows a significant reduction (10906 vs 23254). Moreover, the overall goal of the team is *'to create and implement easy to use, cost-efficient and sustainable solutions to ensure the stock required to realize commercial goals'*. The models are perceived to be easy to use as the outcome of the model is explainable and is in the same range of complexity of other used models within Coolblue. Moreover, the proposed models are found to be cost-efficient because of the focus on cost minimization while on the other hand still enabling Coolblue to realize commercial goals by the set fill rate constraints. Finally, the models are found to be sustainable as it is easy to modify input parameters to reflect the changing environment.

The model is demonstrated for a subset of Parcel Large products but can also be implemented for, for example, Autostore products. For the Autostore case, the maximum number of products on a pallet needs to be modified to number of products that fit within a bin. Moreover, the occurrence of honeycombing is not a problem as it is possible to store multiple SKUs within a bin. Other cases to which the models can be applied to (without modifications) is to groups of locations with specific characteristics (such as upright locations) and Parcel XL products. Especially the case for Parcel XL products is expected to be promising because of the significant size and also perceived location deficit. Implementing the model for other products within the assortment can be done based on the forecasted demand instead of expected demand.

8.3 Scientific contribution

This research has contributed to existing literature in several ways. Firstly, the mixed integer linear program designed (for the SS-model) in this research is, to the best of the researcher's knowledge, innovative in the sense that it includes honeycombing while minimizing costs. The safety stock model results in decreased costs while not increasing the inventory levels significantly. This makes it applicable to a broad specter of industries. Taking into account honeycombing and minimizing inventory costs and number of used locations without significantly increasing inventory levels shows a different application of the honeycombing formulas and is a contribution of this study.

Similarly, the designed IOQ/MOQ model with a cost minimization objective is solved using the two-step heuristic and a modified version of the two-step heuristic. The combination of honeycombing and minimization of expected inventory costs in a model and solving using the modified version of the two-step heuristic is novel and is shown to outperform existing solution methods.

Finally, coping with non-stationary demand is done by dividing the horizon into phases. The changeover between phases is of high importance because of the possibility of having too high or low inventory levels. A modified reorder level is designed to smoothen the transition to a new phase resulting in lower inventory costs. The combination of non-stationary demand, smoothening the transitions without order modification in inventory control is a contribution of this research.

8.4 Limitations

This section discusses the limitations. The limitations are threefold. The input for the models is based on provided data. Some product characteristics have been approximated using a realistic rule of thumb since some values were missing. This influences the performance of the models and therefore sensitivity of the parameters has been checked. While namely simulated values are compared, instead of values from the situation in practice, the comparison sketches a realistic view.

Secondly, a discrete-event simulation is built to investigate the performance of the designed models in a context of varying demand. The demand within this simulation is sampled using a discrete distribution based on the mean and standard deviation of historical demand. Because a lot of (unknown) factors influence real demand, the difference between simulated and real demand cannot be specified. Moreover, the price setting within the company but also of the competitor influences the demand. This cannot be taken into account when simulating the demand. Additionally, the simulation is based on a predefined order of events. The customer demand is sampled in one time while it can be the case that it happens throughout the whole day. Moreover, receiving of orders happens after customer demand at a day while in practice this may occur simultaneously. Moreover, assumptions are made regarding decreasing of inventory on locations. Within the simulation is FIFO assumed while in practice a location may be picked randomly or situations with the lowest number of SKUs are chosen to decrease inventory from. This assumption has a high influence on the number of locations used. All models are run based on these same assumptions and therefore these limitations will most likely only affect absolute performance.

Finally, a MILP is used to solve the different models. While a MILP results in optimal solutions based on the given combinations, it can take a lot of computational effort to find all combinations. Therefore, is solving the models using a MILP only be preferable when the number of unique SKUs is relatively low. Because of this, are in the appendix other solution methods discussed. These solution methods are less computational expensive for a high number of SKUs while still showing improvements compared to the current situation.

8.5 Future research

There are several suggestions for future research based on the model design and key findings of this research. Firstly, the modified reorder level should be investigated more closely in future research. The current modified reorder level only shows modifications to the reorder level when the review period and lead time exceed the duration of the phase. Investigating when to stop ordering and modeling this in a systematic way is of scientific and practical relevance.

Secondly, the impact of promotion and changed prices on customer demand as well as fluctuating purchase prices are suggested to be included. Especially, promotions will cause peaks in the inventory levels and it is unknown how the model will react to this.

Thirdly, incorporating order advancement within the model is suggested as research direction. This can lead to more stable inventory levels and incoming orders a day. This can have positive results for the labor capacity and number of required locations.

Incorporating human judgment within the model is suggested as last research direction. This will make the model more realistic as replenishment decisions often involve a high degree of human judgment and decision-making (Bijvank and Vis, 2012a).

Chapter 9

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Appendix A

Processes

In this chapter the process of reordering, handling of incoming shipment and fulfilling customer orders are described using the BMPN language. Because of clarity reasons, not all flows and roles are modelled.

A.1 Reordering

The reorder process is mostly executed by a supply planner and consists of several steps (see figure A.1 for a visualization). There are several triggers which can be seen as the start of the reordering process; low stock, discount opportunities, planning of future demand or other. The triggers are received by the supply planner and form the start of the reordering process. Based on the trigger, a manual or semi-manual order is proposed and made. After creating the order, the order is verified based on automatic verification rules. When the order is not confirmed, the order will be verified manually by a supply specialist. If the order is not verified, it needs to be changed by the supply planner. When the order is verified, it is send to the supplier for confirmation via Electronic Data Interchange (EDI). After order acceptance by the supplier, the order is planned in the inbound schedule taken into account labour and space capacities. Since these capacities are taken into account after placing of the order, a queue can be formed because of not having enough capacity. After this, delivery details are sent to the supplier. Creating of an Advanced Shipping Notice (ASN) marks the end of the reordering process. For the ASN, the location of the delivery is specified.

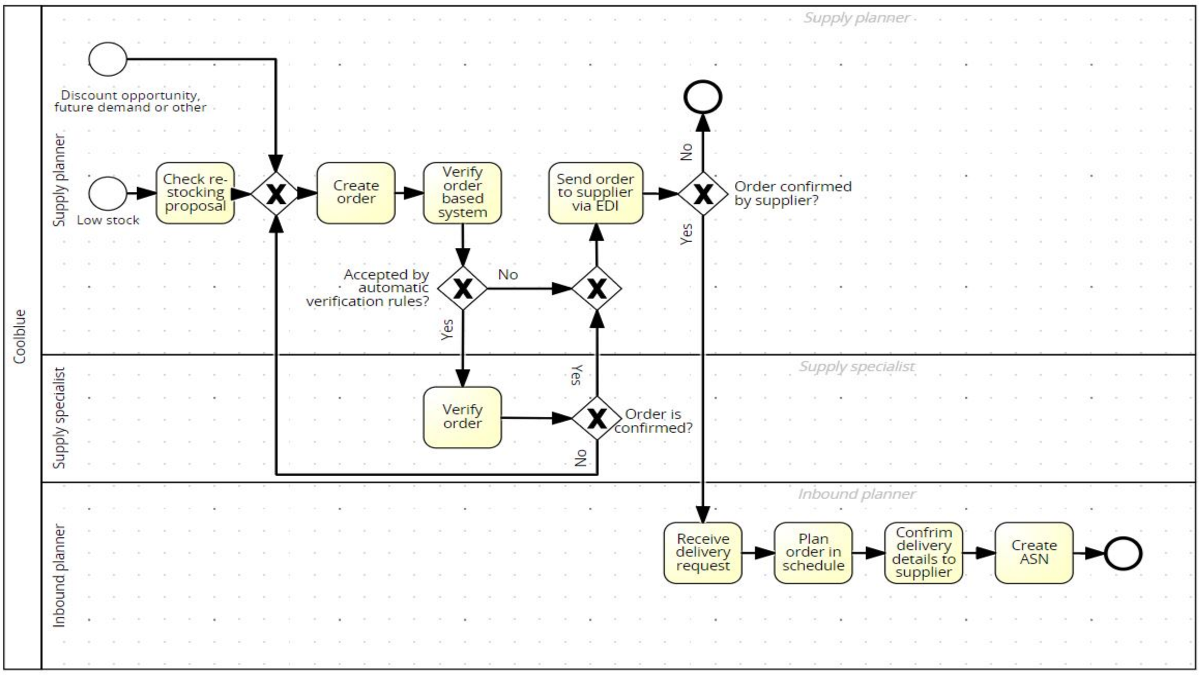


Figure A.1: Visualization of process of reordering

A.2 Handling of a shipment

The start of handling a shipment starts when there is an incoming shipment (for visualization see figure A.2). Within the ASN is specified where the supplier needs to deliver the ordered shipment. There are distinguished inbound locations; Whitegoods, Parcel XL, Parcel large and Autostore. In each location is a docking master who informs the truck driver to which dock to go. First is checked whether the supplier is on time. When this is not the case the shipment can be rejected but this depends on the inbound planning and the degree in which the shipment is needed. When the shipment is on time, the shipment and package slip is checked. When the shipment is not accepted because of damages it will be sent back. Then the truck is unloaded and the products are placed on the floor. The products are again checked. At this point there are several reasons why a shipment is not accepted such as damaged products, amount of the order line is missing (manco) or missing, unknown or unscanable barcode. When the shipment is not accepted, the troubleshooter will decide what will happen to the shipment. When the shipment is accepted, the order in the system needs to be adjusted and the finance team needs to send a credit/purchase invoice. After this, the products will be brought to the correct location. As can be seen, a distinction is made between the aforementioned groups of products.

The *Autostore* is an automatic storing and retrieving system with bins. The bin is automatically brought to a location within the Autostore.

In*Parcel Large*, there is a division between easy and hard to reach locations. When an easy to reach location is (almost) empty, it will be refilled by a specialized team. This team first needs to pick the products from an hard to reach location after which the products needs to be unpacked. Then the unpacked products are brought to an easy to reach locations. The product locations are flexible.

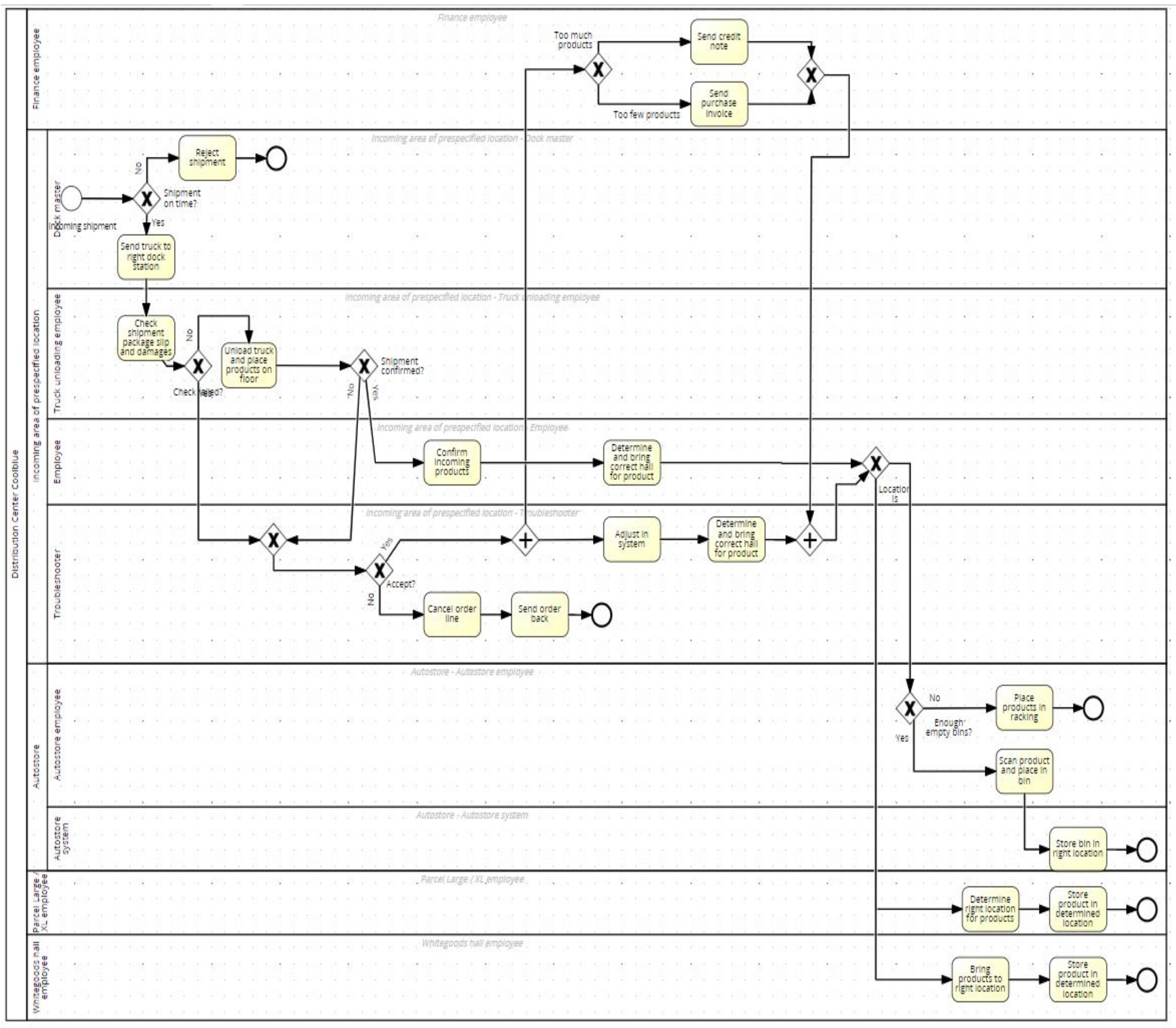


Figure A.2: Visualization of process of handling an incoming shipment

A.3 Handling of customer order

Orders can be for customers or stores. An order can contain multiple products. As mentioned before there are four distinguished product categories based on size. For each location, a product is handled differently (see figure A.3 for visualization). In the Autostore, the packaging method and retrieving of the bin is done simultaneously while in the other locations this is done in serie. For store orders, the products are first collected where after it is placed in a crate. For Parcel Large, Parcel XL and Whitegoods the products are picked manually. Only for Whitegoods and Parcel XL products, a customer can buy extras for which a manual action is needed, for example adjusting the door rotation direction of a fridge.

When the products are ready to be shipped, several methods are available. When the shipping adress is near a CoolblueFietst hub, the product will be shipped via CoolblueFietst and will have a special packaging (zak). For Autostore and Parcel Large products, products can be sent via delivery partners or physical stores. Whitegoods and Parcel XL products are sent to the customer via CoolblueDelivery, for which the package is first sent to a hub whereafter it will be sent to the customer.

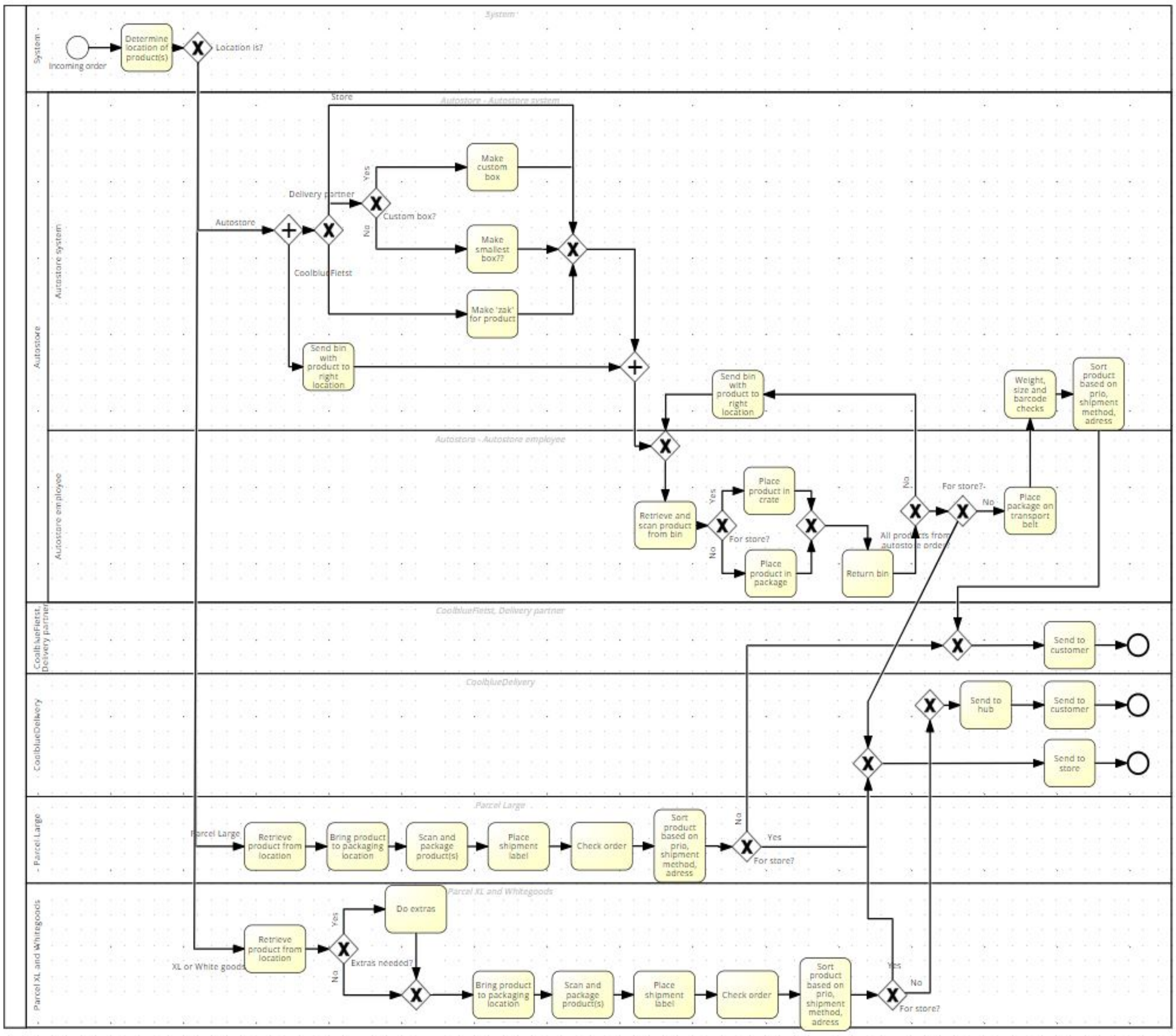


Figure A.3: Visualization of process of fulfilling customer order

Appendix B

Additional solution methods

In this chapter, methods are described which where found to perform worse in comparison with other solution methods to solve a certain model. One of these solution methods is the greedy heuristic which is used to solve the safety stock model.

B.1 Safety stock model - greedy heuristic

A possible solution method for the safety stock model is a greedy heuristic. In each step, the next element of the solution is chosen that results in the best immediate benefit (Silver, 2004). An add heuristic is used in which the variables are set to the lowest value and when the objective value improves by adding a specific variable, this variable is added. The starting reorder level of each SKU will be calculated as the demand during lead time and review period. If this resulting reorder level is lower than the minimum fill rate constraint, the reorder level of this SKU will be increased until the minimum fill rate is met. The objective here is the minimization of the expected inventory costs. The ratio for each SKU is calculated as the change in objective (expected inventory costs) when the reorder level is increased by 1 divided by the (potential) change in the expected number of used locations when the reorder level is increased with one unit. This ratio is used since the main goal is to investigate the SKUs that yield the most (in controversy have the lowest costs) while not consuming a lot of space. Another ratio that can be used is based on the fill rate and expected number of used storage locations, as the fill rate is also used as a constraint within this model. The increased reorder level is assigned to the item with the highest ratio. The decrease in expected inventory costs is calculated using the objective of the model. The expected number of used locations is calculated using formula 5.7 and the fill rate using formula 5.5.

Increasing the reorder level of SKUs is done until the aggregated fill rate is satisfied or all warehouse capacity is occupied. There can be a point at which the constraints are not met but no empty locations are left. When this is the case, the greedy heuristic is modified to choose to increase the reorder level of SKUs with the highest ratio for which no increase in required locations is needed.

Results

Based on the approximation results, the greedy heuristic (using the costs and locations ratio and the fill rate and location ratio) was found to perform less than the MILP. This can be explained by the fact that a MILP takes all possible combinations into account while a greedy heuristic chooses an SKU reorder level to increase based on a specific moment instead of taking into account all possible combinations of reorder levels of all SKUs. An advantage of the greedy heuristic is that it can be relatively fast regarding computational time for a high number of SKUs.

Within the computational time of the MILP, the making of the combinations is included.

The MILP has the lowest expected inventory costs. This can be explained by the decrease in expected inventory on hand and corresponding holding costs. Moreover, calculating the average holding per unit results in slightly lower holding cost per unit of the MILP compared to the greedy heuristics. The greedy heuristic focuses on the decrease in costs and therefore increases the reorder level of expensive SKUs with high uncertain demand resulting in lower average backorder costs per unit.

When comparing the greedy heuristic with the ratio regarding the fill rate and the costs, it can be noted that the computational time of the greedy heuristic with the fill rate ratio is slightly higher. Moreover, the expected inventory costs are slightly lower for the heuristic with the cost ratio. This is because of the higher expected inventory on hand as this is more balanced in the costs focused ratio while in the fill rate focused ratio is solely focused on increasing the reorder level of SKUs with the highest impact on the fill rate.

Table B.1: Overview of approximation results for safety stock model

	SS- CL	**SS- FL**	**SS - MILP**
Average expected inventory on hand per time unit	6514.2	6528.1	6192.2
Average expected order size	6.1	6.1	6.1
Average expected backorders per time unit	0.1	0.1	0.1
Average expected used locations per time unit	1725.0	1725.0	1514.1
Maximum expected used locations per time unit	1856.6	1856.9	1779.4
Total expected costs per time unit	2090.35	2091.89	2025.07
Fill rate	0.950	0.950	0.950
Expected holding costs per time unit	699.20	701.16	618.85
Expected ordering costs per time unit	1295.29	1295.29	1295.29
Expected backorder costs per time unit	95.86	95.45	110.93
Ratio holding costs	33.4 %	33.5 %	30.6 %
Ratio ordering costs	62.0 %	61.9 %	64.0 %
Ratio backorder costs	4.6%	4.6%	5.5%
Computational time (in s)	6037	6127	3045

B.2 IOQ/MOQ model

The IOQ/MOQ model can be solved using, besides the modified version of the two-step version, the two-step heuristic developed by Van Donselaar and Broekmeulen (2022).

The IOQ/MOQ model can be solved using the two-step heuristic developed by Van Donselaar and Broekmeulen (2022). First, the target fill rate per SKU is determined. In the second step, all combinations of MOQ_j, IOQ_j and s_j of an SKU that meet this fill rate are sought. With all these possible combinations, the best values for MOQ_j, IOQ_j and s_j for all SKUs are determined using a MILP.

In the first, the target fill rate is determined using a greedy heuristic. The starting reorder level of each SKU will be calculated as the demand during the lead time and review period. If this resulting reorder level is lower than the minimum fill rate constraint, the reorder level of this SKU will be increased until the minimum fill rate is met. The objective here is the minimization of the expected inventory costs. The ratio for each SKU is calculated as the change in objective (expected inventory costs) when the reorder level is increased by 1 divided by the (potential) change in the expected number of used locations when the reorder level is increased with one unit. This ratio is used since the main goal is to investigate the SKUs that yield the most (in controversy have the lowest costs) while not consuming a lot of space. The increased reorder level is assigned to the item with the lowest ratio. Increasing the reorder level of SKUs is done until the aggregated fill rate is satisfied.

As second, all combinations of MOQ_j, IOQ_j and s_j of an SKU that meet the fill rate (determined in the first step) are sought. In this MILP, the expected inventory costs are minimized and restricted by the number of locations and weighted fill rate. The combination of $MOQ_{j,f}$, $IOQ_{j,f}$ and $s_{j,f}$ for all SKUs form the set of combinations F. For each combination f and SKU j, the values of $E[I^{OH}]_{j,f}$, $E[USL(L)]_{j,f}$, $E[X^{-}]_{j,f}$,$P_{2,j,f}$ and $E[USL(L)]_{j,f}$ are calculated. These found combinations can be used as input in the MILP described in formula 5.20 with constraints on the weighted fill rate, warehouse capacity and selecting one combination per SKU.

Results
Based on the approximation results, the modified two-step heuristic was found to perform the best regarding expected inventory costs compared to the two-step heuristic. This can be explained by lower expected holding costs as a result of the focus on expected inventory costs in the first step. The expected ordering costs have decreased and the expected order size have increased showing ordering less frequently while ordering higher quantities (also reflected in the higher expected inventory).

Table B.2: Overview of results using mathematical formulas for IOQ/MOQ model

	CL IOQ/MOQ-MILP	**Two-step**
Average expected inventory on hand per time unit	8675.9	8554.0
Average expected order size	14.8	14.7
Average expected backorders per time unit	0.0	0.0
Average expected used locations per time unit	1015.5	1021.5
Maximum expected used locations per time unit	1144.6	1151.9
Total expected costs per time unit	1329.95	1357.00
Fill rate	0.950	0.950
Expected holding costs per time unit	778.25	798.24
Expected ordering costs per time unit	449.24	469.05
Expected backorder costs per time unit	102.46	67.53
Ratio holding costs (in %)	58.5%	58.8%
Ratio ordering costs (in %)	33.8%	34.6%
Ratio backorder costs (in %)	7.7%	5.0%

Appendix C

Simulation procedure

Within the discrete-event simulation, several events are used as described in section 6.2.1. In figure C.1, the interactions and the handling of events are visualized.

The inventory is reviewed every R_j time unit. In the simulation, week and weekend days are taken into account as a result that sometimes the next review moment is further away than R_j or L_j time unit. When the inventory position of the SKU is lower than the reorder level, the quantity to order is determined. The determination of quantity to order is based on the (R,s,nQ)-policy for models 1 and 2 or (R,s, MOQ,IOQ) policy for the third model. If this quantity to order is positive, the order is placed resulting in immediately increasing the SKU inventory position. After the completion of L_j time units, the inventory on hand is increased. When the transition between phases needs to be smoothed (in other words when section 5.5 is used within the simulation), a modified reorder will be calculated at the review moment when the set conditions are met.

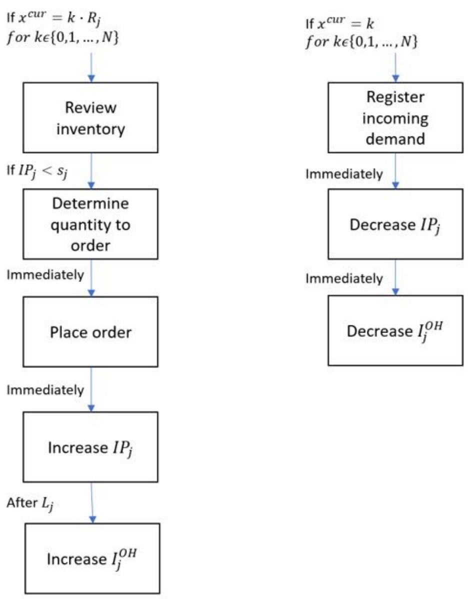


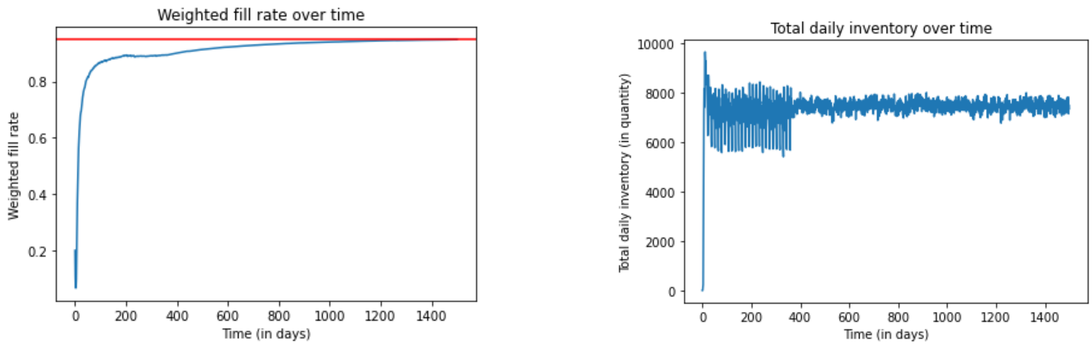
Figure C.1: Visualization of simulation

Appendix D

Setting of simulation parameters

Before collecting results, the simulation will run a warm-up period that enables the simulation aspects to reach typical running conditions in the considered environment (Boon et al., 2020). As the simulation will start with an initial inventory of 0 and the building up of inventory takes time, the warm-up period is of high performance. The warm-up period is set based on a visual inspection of the fill rate and inventory levels per day (figure D.1). After approximately 1000 time units, the simulation reaches a steady state.

Because the goal of the simulation is to replicate one year, the replication length is set to one year corresponding with 365 days.



(a) Weighted fill rate over time

(b) Total daily inventory over time

Figure D.1: Weighted fill rate and total daily inventory over time to set warm-up period

To estimate the number of runs, the method as described by Boon et al. (2020) is used. The number of runs has to satisfy the formula D to obtain a (1-α) confidence interval. In line with (Van Donselaar and Broekmeulen, 2022), an accuracy of three digits and 95% confidence intervals is preferred. Therefore, the ε is set to 10^{-3} and $Z_{a/2}$ is 1.96. Because no initial guess for σ is available, an initial guess is made for σ with a simulation setup of a warm-up period set to 1000, running length to 365, and the number of runs to 10. The σ of the fill rate across is 0.0017. This results in at least 12 runs (Equation D). All in all, the warm-up period is set to 1000, the replication length to 365, and the number of runs to 12.

$$n > \left(\frac{z_{\alpha/2} \cdot \sigma}{\varepsilon}\right)^2 \tag{D.1}$$

$$11.1 > \left(\frac{1.96 \cdot 0.0017}{10^{-3}}\right)^2 \tag{D.2}$$

Appendix E

Validation of simulation

The simulation model needs to be verified and validated before it can be used as a tool to gather outputs. In this appendix, is the process of validation in more detail discussed.

Validation is done by computing the values for different KPIs using analytical formulas and comparing these values with the simulated results for the same inputs. To calculate the values of different KPIs, the DoBr tool is used. The used analytical formulas within this tool are also used within this research several times.

Before being able to validate the simulation, the simulation needs to be slightly modified. Within the DoBr tool, it is not possible to distinguish between week and weekend days and therefore this is modified within the simulation. Moreover, the inventory position is decreased when there are backorders. The simulation is validated for a stationary situation (i.e. one phase) with three SKUs. The input parameters are defined in table E.1 and the same simulation parameters are used as defined in section 6.2.2.

Table E.1: Input parameters for validation

Parameters	**SKU 1**	**SKU 2**	**SKU 3**
μ_j	0.30	0.22	2.68
σ_j	0.53	0.49	2.08
R_j	2	5	3
L_j	6	7	6
C_j	15	12	24
P_j^2	0.95	0.95	0.95
Q_j	5	3	24
s_j	4	5	25

The results of the approximation formulas and simulation output can be seen in table E.2. When comparing the values for the KPIs, mostly similar results are observed in both situations. As the validation situation is with stationary demand, deterministic lead times, non-perishable products, and backordering, the value of the analytical formulas is exact except for the value of the expected number of locations (as this value was given in blue in the DoBr-tool). Especially, marginal differences are observed in the expected number of locations. All in all, the simulation can be used to gather output.

Table E.2: Results theoretical formulas and simulation

	KPI	SKU 1	SKU 2	SKU 3
Simulation	$Fillrate$	0.96	0.96	0.95
	$E[OS]_j$	5.0	3	24
	$E[I^{OH}(L_j)]$	4.2	4.5	20.4
	$E[I^{OH}(R_j + L_j)]$	3.6	3.4	12.8
	$E[USL(L_j)]$	1.4	1.9	1.4
DoBr	$Fillrate$	0.96	0.96	0.95
	$E[OS]_j$	5.0	3	24
	$E[I^{OH}(L_j)]$	4.2	4.5	20.4
	$E[I^{OH}(R_j + L_j)]$	3.6	3.4	12.8
	$E[USL(L_j)]$	1.2	1.8	1.3