MASTER

A Simulation Model of Empty Transport Item Repositioning in a Parcel Delivery System

Hoogenboom, Derek M.

Award date:
2023

A Simulation Model of Empty Transport Item Repositioning in a Parcel Delivery System

Master Thesis Report

Department of Industrial Engineering and Innovation Sciences

Student:
D.M. (Derek) Hoogenboom
1015038

University Supervisors:
First assessor: Dr. C. (Christina) Imdahl
Second assessor: Dr. W.L. (Willem) van Jaarsveld
Third assessor: Dr. A.H. (Albert) Schrotenboer

Company Supervisor:
Dr. J.C. (Jan) de Munck

Eindhoven, 30th April 2023

TU/e School of Industrial Engineering

Series Master Thesis Operations Management and Logistics

Keywords: Empty repositioning, parcel delivery system, roll cages, simulation, transport optimization.

PREFACE

Dear reader,

This document contains the research from my master thesis project for the degree in Operations Management & Logistics from Eindhoven University of Technology. It has been an enriching journey, with ups and downs, in which I was supervised in the best way possible. I want to thank Dr. Christina Imdahl (Eindhoven University of Technology) and Dr. Jan de Munck (PostNL) a million times for this. Again, you are the best Also, I want to thank Dr. Willem van Jaarsveld and Dr. Albert Schrotenboer, who were excellent second and third supervisors from Eindhoven University of Technology.

In particular, I would like to thank Jan for his help in modelling the collection and inter transport processes of PostNL's problem, which, in my opinion, resulted in a nice model.

I hope you enjoy this research as much as I do.

Derek Hoogenboom

SUMMARY

Introduction

PostNL is a large parcel delivery company in Benelux. Their main business activity is delivering parcels from origin to destination every day. First parcels are collected at customer sites and brought to nearby sorting centers, called depots. There the parcels are sorted and transported to depots nearby the parcel destinations, called inter transport. PostNL uses roll cages, which are reusable transport, to efficiently carry and move parcels in this process. However, the parcel deliveries are imbalanced. I.e. some regions in PostNL's network have larger parcel inflow than parcel outflow, or the opposite. As a result, in the long run, some locations run out of roll cages and some develop excess roll cage inventory. Therefore, PostNL rebalances the system each day by determining the number of empty roll cages that flow between each pair of locations, aiming at minimizing transport costs, minimizing (roll cage) overage at depots and ensuring sufficient roll cage supply at locations.

Finding the optimal solution to this problem is difficult for several reasons. Firstly, demand at customers is random. Also, PostNL uses two types of transport orders for repositioning. Immediate orders are expensive but executed the same day. Structural orders are cheap, but need to planned three weeks ahead. Furthermore, in case of shortage at a location, costly emergency transport is arranged from a nearby location. Lastly, the number of customer locations is large. In the current control policy, customers are replenished using structural orders. Each day, first this structural planning is followed as much as possible before determining the rebalancing between depots, for which immediate orders are used. In each time period and at each depot, only a very small number of overage roll cages is allowed in this policy. This number is also called the maximum surplus level or S. There are buffer locations in the network, which have large inventory (and no overage restriction) to ensure that this control policy always works. This control of the between-depot repositioning can potentially be improved. Namely, there is a pattern of decreasing roll cage demand at depots from Monday until Sunday. Therefore, when choosing low S, roll cages must be repositioned to buffers throughout the week, and brought back on Monday, which is costly. Concluding, the optimal solution is affected by several factors and there is reason to believe that the current control policy can be improved. Therefore, in this work, we aim to develop a simulation model that expresses the long term repositioning costs and inventory levels at depots as a function of the control policy. This model is used to compare the S-level in the current control policy to other S-levels in terms of resulting costs and inventory levels.

Decision process model

The main idea of the model is the following. We first mathematically express the roll cage movements between locations in one arbitrary period (day), leading to the starting inventory of next period. That is, we define the inventory evolution. These movements are a result of the repositioning decision and random parcel delivery demands (corresponding to PostNL's collection and inter transport processes). We then define the costs corresponding to these movements and the objectives, which are the expected per period costs and the expected mean inventory level at normal (non-buffer) depots. By providing the set of rules (decision policy) by which PostNL determines the repositioning decision in each period, we model the current decision policy. In this way we obtain a model that evaluates the long term repositioning costs and inventory levels as a function of a decision policy and random events for a provided time horizon.

We formalize the current decision policy as follows. Against some costs, PostNL plans structural orders, which make the repositioning fee between two locations smaller on a specific weekday (Monday, Tuesday,..., Sunday) for a long period of time. We assume the set of structural orders are fixed for each weekday. PostNL only repositions roll cages to a customer when the customer has a planned structural order. The company employs a heuristic base stock policy, including a fixed safety stock margin, and specific corrective rules in case the order origin has insufficient inventory to execute the order. PostNL bases the repositioning between depots on each day on the starting inventory, the determined orders to customers,

the roll cage inflow from collection, the inter transport outflow (demand), and a set-up inventory for the sorting machine. For each depot, given these quantities, they calculate the overage and restrict this overage to become between zero and S by repositioning between depots, aiming at minimizing costs. Currently this is done manually, which make it hard to replicate. We translate this to a mathematical optimization problem and solve optimally with commercial software Gurobi.

Simulation experiment with company data

By establishing the probability distributions for the random events (collection quantities at customers), and implementing the model in a Python program, we can simulate the process for a number of periods and obtain results. We include 28 depots and 1694 customers in our model. To obtain probability distribution functions (PDFs) of the collection quantity for each customer we use seven months of company data of the collection process: the number of roll cages that were collected on each date, from which customer, to which depot. To prevent distributions that have little data for the whole range of possible outcomes, we do not fit a distribution for each individual customer. Instead, we divide the data in five groups based on how large customers are in terms of total collection quantity, and assume that all customers of a group show the same collection behaviour. For each of these customer groups, we fit probability distributions of the collection quantity for each weekday. Hence, we obtain PDFs for the collection quantity at each customer on each weekday. Also, for each customer, the fraction of its collection that is brought to each depot is derived from this data. Then we need events for the number of roll cages (loaded with parcels) that flow between depots. The outflow at a depot in a period mostly depends on how many parcels are received from collection. We use data of these loaded roll cage flows between depots to determine the roll cage outflow at each depot in each time period as a function of how much collection this depot received. Also, the fraction of roll cage outflow that flows into each other depot is derived from this data.

To obtain results on the long term expected costs and long term expected inventory levels at depots, we simulate the system multiple times in which we sample random events using the PDFs we fitted. We simulate the system $m = 42$ periods to obtain one outcome for each objective, and we perform $n = 10$ of such runs to obtain multiple independent outcomes for the objectives. Based on these $n = 10$ outcomes we can provide an estimate for each objective, including approximate confidence interval.

Discussion

We establish a model of the decision process. We show that our simulation experiment yields several outcomes of using the current decision policy. Repositioning flows of individual depots, per period costs of the several costs components, and inventory levels at locations are generated for specific points in time in the simulation and as long term expected values. The power of our tool is primarily in the ability to compare different decision policies. To illustrate this strength, we test alternative decision policies, which differ in the chosen S-level. We show that the expected inventory at depots is increasing in S, and the expected per period costs are decreasing in S. E.g., by changing S from 180 to 360, PostNL could reduce its total repositioning costs by 4.0% against an increase in inventory at depots of 7.9%, which is a significant cost saving for the company. Moreover, the model has a broad scope of the process, enabling the analysis of several other parameters like the effect of the safety stock level at customers and the set of structural orders on repositioning costs. Future work could, for instance, aim at optimizing the structural planning as the largest part of the repositioning is planned ahead and structural orders are significantly cheaper than immediate orders.

A SIMULATION MODEL OF EMPTY TRANSPORT ITEM REPOSITIONING IN A PARCEL DELIVERY SYSTEM

D.M. (Derek) Hoogenboom

1 INTRODUCTION

Companies in the parcel delivery industry, a branch of the logistic service industry, provide the service to transport parcels from origin to destination for customers. Customers of parcel delivery companies are mostly e-commerce companies that need to send their products to their customers, which mostly are consumers. The main challenge for parcel delivery companies is processing high volumes of parcels and at the same time realizing short delivery time, reliable delivery and attractive pricing, as these are key components in customer decision making. The market is therefore characterized by high pressure on operational excellence, especially because of increased competition resulting from the enormous growth of e-commerce in the last decade.

Operating a parcel delivery system involves the physical movement of a large number of parcels from several origins to several destinations in an area. To efficiently carry and move the parcels, roll container trolleys, a.k.a. roll cages, are used to stack parcels on. These roll cages carry multiple parcels at once, and are easily moved into trucks or within facilities. A common problem of using such (reusable) transport items in a logistic system, is that they need to be repositioned empty after they have been used for a loaded shipment, because the loaded shipments are imbalanced. That is, areas in the network have more outgoing loaded shipments than incoming loaded shipments (or the opposite), which leads to either accumulation or shortage of these transport items. Operators of the logistic service must determine how empty transport items are transported between locations to ensure availability and minimize costs. This problem occurs in many domains of the logistic service industry, e.g. in freight transport with trucks (Erera et al. 2009) and in sea transport with containers (Song and Dong 2015). In this research we study the design of a model for roll cage repositioning in a parcel system. This research is done in close collaboration with PostNL, a large courier in Benelux, which has expressed the need for tools to design and evaluate new control policies for the repositioning of roll cages.

PostNL's roll cage repositioning process is a daily repeating process consisting of the steps ”collection”, ”inter transport” and ”repositioning”. First, parcels are collected at customer sites using trucks. Here, these parcels are stacked on either roll cages or on other transport items, like pallets, before they are moved into trucks. The trucks bring the parcels to nearby sorting centers, called depots. Then, during the inter transport step, the parcels are unloaded from the transport items and fed to a sorting machine. This machine sorts the parcels based on destination area, whereafter they are all placed on roll cages. These sorted parcels on roll cages are moved into trucks and transported to a depot in the destination area. Hence, in the inter transport phase, a depot first sends roll cages (with sorted parcels) to all depots, and then receives roll cages from all depots. In the repositioning step, first the parcels are unloaded from the roll cages and the parcels are placed in a van for the last-mile delivery to the destination. Then, PostNL repositions the empty roll cages from the depots to customers and to other depots to rebalance the system. Namely, in this process it is important that there are enough roll cages at the customer sites to enable the collection process and at depots to enable the inter transport process. However, in the long run, customers naturally run out of roll cages and depots may have increasing or decreasing roll cage inventory because of an imbalance in the inflow and outflow of roll cages. Figure 1 provides an overview of this process.

This repositioning is done by placing transport orders: booking a truck to transport a number of empty roll cages from an origin to a destination at some point in time. There are two main types of transport orders, abbreviated as orders: immediate orders and structural orders. An immediate order can be booked

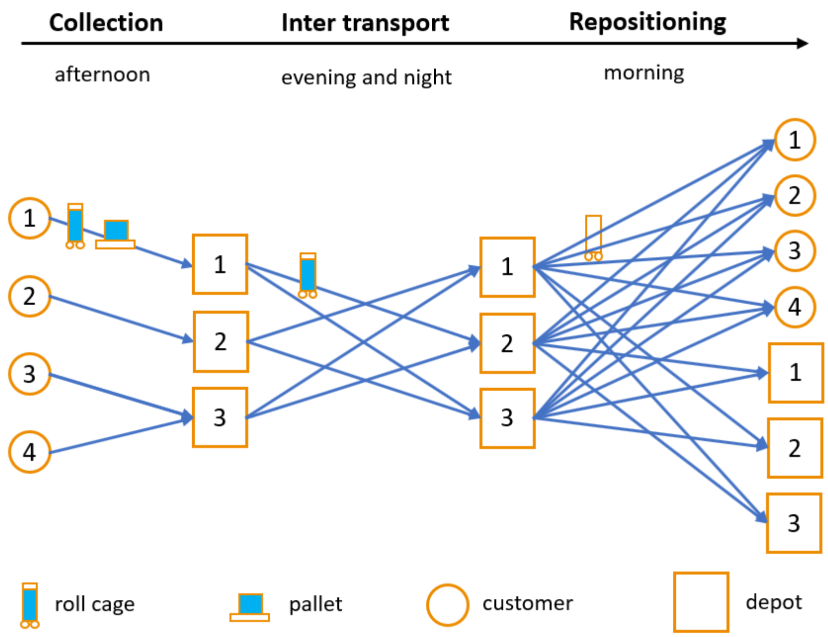


Figure 1: The roll cage repositioning process of PostNL.

for the same day and is relatively expensive. A structural order needs to booked one week in advance, recur for a minimum of three weeks and is relatively cheap. Furthermore, the costs of an order increase in the travel distance (fuel costs), travel time and the number of roll cages in the order. The company aims at minimizing repositioning costs, minimize redundant roll cages at depots and to meet process requirements, such as ensuring enough roll cage inventory at locations. Hence, they face a sequential decision problem. Each day they must decide, how much roll cages are transported from where to where and with which order type. Next to the cost structure and different order types, there are several complicating factors in this problem. The number of roll cages that are needed at customers for collection is typically not known in advance. Therefore, future inventories at locations are uncertain and each day the repositioning decision is subject to these inventories. Moreover, when a location is not sufficiently supplied with roll cages (e.g., a customer needs more roll cages for the collection of parcels than it has on stock), emergency transport is arranged from a nearby location. Also, there are over a thousand customers, which complicates the planning.

Currently, PostNL repositions roll cages to customers using structural orders, because there is a large number of customers and structural orders are relatively cheap. Each day PostNL sticks to this planning as much as possible and creates immediate orders to customers if this structural planning is not feasible. Then, given these customer orders, PostNL checks how much roll cages are left at the depots and creates immediate orders such that each depot ends up with enough roll cages for the upcoming inter transport process. Each day period and at each depot, only a very small number of overage roll cages is allowed in this policy. This number is also called the maximum surplus level or S. There are buffer locations in the network, which have large inventory (and no overage restriction), to ensure that this control policy always works.

Future needs at depots are not considered in this control policy, and allowing only a small surplus at depots may increase future repositioning costs if future needs are different from current needs. Namely, PostNL experiences a pattern of decreasing roll cage demand at depots from Monday until Sunday. Therefore, when choosing for low S, roll cages are repositioned to buffers throughout the week, and brought back on Monday, which is costly. Moreover, it is unlikely that this control policy minimizes costs, because it does not consider several opportunities for decreasing the costs of repositioning on an arbitrary day. For example, it might be beneficial to deviate from the planning of orders to customers because it happens to be the case that a depot nearby a customer has excess roll cages (through randomness of the system). Another reason why analyzing the control policy could be beneficial is that by first determining the orders to customers each day before determining orders between depots, the orders between depots might become more expensive, because some depots might need supply that would not be needed if the depot need was taken into account right away. Hence, PostNL wonders how the current control and design of the process can be evaluated and possibly be improved. Therefore, they aim to design a mathematical model of the system.

The literature on empty repositioning is rich: numerous application domains and problem types are addressed. Some, mostly early, works focus on the (basic) setting of deterministic supply and demand, and a single decision moment for a fixed planning horizon. They utilize linear programming techniques to finding cost optimal assignments for transport items to origin-destination (O/D) pairs in a transport network. Examples of such studies are White and Bomberault (1969), Shen and Khoong (1995), and (Vogel et al. 2014; Neumann-Saavedra et al. 2016) who consider rail car repositioning, sea container repositioning, and repositioning in bike sharing systems, respectively. A large stream of research concentrates on settings with stochasticity in supply and demand, and dynamic decision environments as these characteristics are often encountered in real-world problems. Most of the studies with this focus consider problems in which there are per-unit costs associated with unmet demand, and (holding) overage. Several approaches are proposed. Jordan and Turnquist (1983) formulate a stochastic transportation problem in a railway wagon repositioning setting, in which decisions need to be made in a rolling time horizon, using the mean and variance of demand and supply. They solve the formulation using a Frank-Wolfe algorithm. Crainic et al. (1993) develop a Two-Stage approach for a dynamic, stochastic inland (maritime) container distribution problem. In the first stage, the problem is solved assuming deterministic parameters. In the second stage, the decisions are adjusted such that expected costs related to random realizations of demand and supply are accounted for. Yan et al. (2017) study repositioning in a bike sharing system. They generate a number of demand scenarios and solve a multi-scenario mathematical program.

In these studies, unmet demand at one location does not imply (emergency) transport from other locations, which is the case in our roll cage repositioning problem. Also, it is hard to use above models, because we deal with a large number of customers and transport options of different prices. To deal with modeling specific environment characteristics, simulation models have been very popular to assess overall impact of control policies in problems in practice (Dejax and Crainic 1987). One such example is Ghosh et al. (2017), who define a bike repositioning problem as Markov Decision Process and obtain solutions by repeatedly solving an MILP. They test the performance of this method using a simulation model. Dong and Song (2009) propose a simulation model for a maritime logistic system to test a customized repositioning policy. Since we deal with several special features, we choose to design a simulation model of the roll cage repositioning problem to provide a practical and adequate tool.

The aim of this work is to design a simulation model of this decision process that can support the optimization of the process design and control. Specifically, we aim to model the long term repositioning costs and long term inventory levels at depots as a function of the control policy and the process design. The ultimate objective is to minimize both, but the costs objective has priority over the inventory objective as precise costs of inventory is out of scope. To this end, we use our model to analyze the effect of the maximum surplus level (S) in the control policy on both objectives.

2 DECISION PROCESS MODEL

In this section, the roll cage repositioning decision process is elaborated in more detail and a model of it is introduced. We want to model the effect of control policies on long term repositioning costs and roll cage inventories at depots in this process. Therefore, the main idea of the model is the following. Firstly, roll cages move between locations in each period of a time horizon, explained in Subsection 2.1. Secondly, we mathematically express the roll cage movements (i.e. inventory evolution) in the different steps of a single period in Subsection 2.2. These include movements resulting from random events and movements resulting from the repositioning decision. We then define the repositioning costs corresponding to these movements and the objective values of this model (Subsection 2.3). Lastly, in Subsection 2.4, we define how decisions are currently taken, which we will call the current decision policy. In this way we obtain a model that can evaluate the long term repositioning costs and inventory levels as a function of a decision policy for this decision process. A list of the notation that we introduce in this section can be found in appendix A.

2.1 Model structure

In this process roll cages move over a network of locations at various points in time. Let the set of locations i be defined as $\mathscr{L}$. This set consists of a set of customers $\mathscr{C}$ and a set of depots $\mathscr{D}$. At some depots, there is additional storage space for roll cages. These depots are also called buffer locations and let the set of buffer location be denoted by $\mathscr{B}$, where $\mathscr{B} \subseteq \mathscr{D}$. The depots that are not buffer locations ($\mathscr{D} \setminus \mathscr{B}$) are also called normal depots.

On a high level, the roll cage repositioning process consists of three steps repeating every 24 hours, in which roll cages move over the network and costs are incurred. These steps are repositioning, collection and inter transport, which roughly take place between 08.00-12.00 o'clock, 12.00-23.00 o'clock and 20.00-08.00 o'clock, respectively. Hence, one iteration of the process takes place over two calendar days. It is convenient to define time buckets such that all events of one iteration of the process belong to a single time period. Hence, in the model a period starts at 08.00 just before repositioning and ends at 07.59 after inter transport. Let $\mathscr{T}$ denote the set $\{0, 1, 2, ..., |\mathscr{T}| - 1\}$ of periods t. The movement of roll cages is subject to a week pattern, driven by parcel demand dynamics. Hence, we distinct between different days in the week, which is modelled by the set $\mathscr{W} = \{Mo, Tu, We, Th, Fr, Sa, Su\}$ of weekdays w. Figure 2 shows the difference between the real world (calendar) time scheme and the modelled scheme of time.

By modelling the roll cage movements between locations in each step, we model the evolution of inventory in a single period. Specifically, the process starts at $t = 0$ and $w = Mo$. The roll cage inventory at the beginning of period t at location i is denoted by r_i^t. Then, the inventory at locations is updated in each step, whereafter the process moves to the next period $t + 1$, and the next weekday and thus we obtain r_i^{t+1}. The process stops when $t = |\mathscr{T}|$. Also, we define the costs in each of the steps as a function of these roll cage movements. Repeating this for multiple periods, we obtain a model for long term inventory levels and costs. In the repositioning step a decision is made using a decision policy, aiming at reaching objectives.

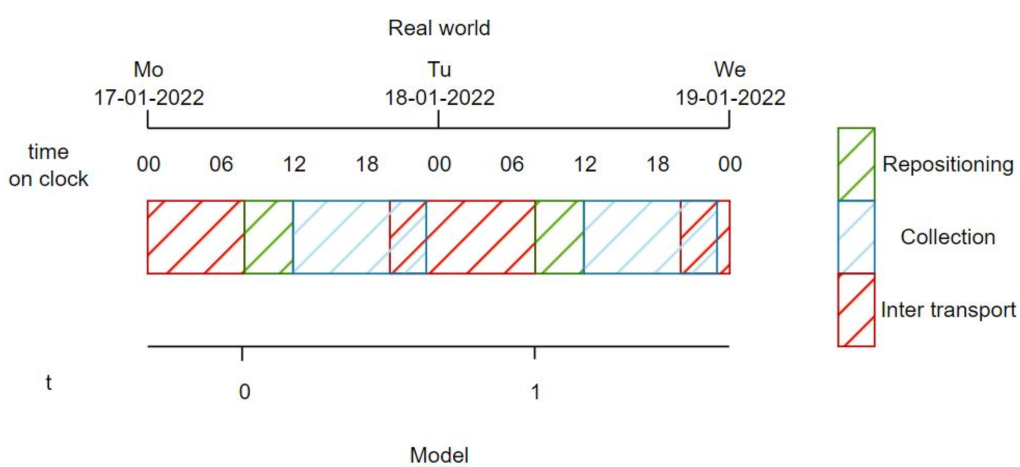


Figure 2: The difference between the real world time scheme and the time scheme of the model.

2.2 Inventory evolution

2.2.1 Collection

Every day from 12.00 - 23.00 o'clock, parcels are collected at pick-up locations and brought to depots using trucks. Most of these pick-up locations are customer sites, others are hubs to which small (web) shops and individuals bring their parcels themselves. In this paper all these pick-up locations will be viewed and referred to as customers. To efficiently carry and move the parcels into the truck, customers use big roll cages, pallets, small roll cages, no transport items (loose loaded) or a combination of these. However, most customers only use one type of transport item. In this study, big roll cages are referred to as roll cages, since we focus on this type of equipment. All other transport items will be called pallets, since we only need to distinguish between the use of roll cages and the use of other transport items.

In each period, at each customer a number of roll cages is collected. Let $c\_out\_rc_i^t$ denote the roll cage collection quantity of customer i in period t. We also need to model the flow of pallet collection, because this determines how many parcels flow into depots. This, in turn, determines the roll cage need at depots since roll cages are needed to process those parcels after the sorting. The collection of pallets is measured in number of roll cage equivalents. Let $c\_out\_pal_i^t$ denote the pallet collection quantity of customer i in period t. The variables $c\_out\_rc_i^t$ and $c\_out\_pal_i^t$ are typically not known in advance and depend on factors like the day of the week, discounts at the customer, season and competitor prices. Therefore, we model these variables as random variables with known probability distribution function (PDF). In Section 3 we elaborate on how we establish probability distributions for these variables for PostNL's problem instance.

All collected parcels are transported to one of the depots, which causes an inflow of roll cages and pallets. This is modelled by the variables $c\_in\_rc_i^t$ and $c\_in\_pal_i^t$, which denote the roll cage collection inflow at depot i in period t and the pallet collection inflow at depot i in period t, respectively. Usually, the collection of a customer i is brought to the nearest depot to keep the associated transportation costs low. This is not always the case, because other objectives, such as ensuring equal spread of the collection inflow over the depots, also play a role. We assume that the roll cage collection $c\_out\_rc_i^t$ at customer i flows into each depot j according to a fixed fraction (independent of the weekday). The same goes for the fraction of $c\_out\_pal_i^t$ at customer i that flows into each depot j. Hence, when we have the collection outflow variables for each customer in period t and these fractions, we obtain the collection inflow variables $c\_in\_rc_i^t$ and $c\_in\_pal_i^t$ for each depot in period t. In Section 3, we elaborate on how we obtain these fractions for each customer/depot pair for both roll cages and pallets, for the problem instance of PostNL.

Since $c\_out\_rc_i^t$ are random variables, it can occur that customers run out of roll cages. If that happens, an emergency order is initiated from the nearest depot that has sufficient roll cage inventory to supply the deficit at the customer. Let e_{ij}^t denote the emergency order quantity from location i to location j in period t. Such a shortage is revealed at the start of the collection step, since at that moment all $c\_out\_rc_i^t$ are revealed. Hence, e_{ij}^t is determined after the repositioning step in period t and thus not part of the repositioning decision.

Figure 3 provides an overview of the model of the collection process. It shows that customer i has roll cage collection and pallet collection in each period t and these are sampled from from a probability distribution. For both roll cages and pallets, according to a fraction for each depot, these are divided over the depots. Using this notation of the roll cage flows, the roll cage inventory change in the collection step is given by the expression

$$-c\_out\_rc_i^t + c\_in\_rc_i^t + \sum_{j \in \mathscr{L}} (e_{ji}^t - e_{ij}^t) \qquad \forall i \in \mathscr{L}, \forall t \in \mathscr{T}.$$

2.2.2 Inter transport

In the inter transport step roll cages move between depots, because at each depot collected parcels are sorted, placed on a roll cage and transported to a destination depot. The sorting of the parcels roughly takes

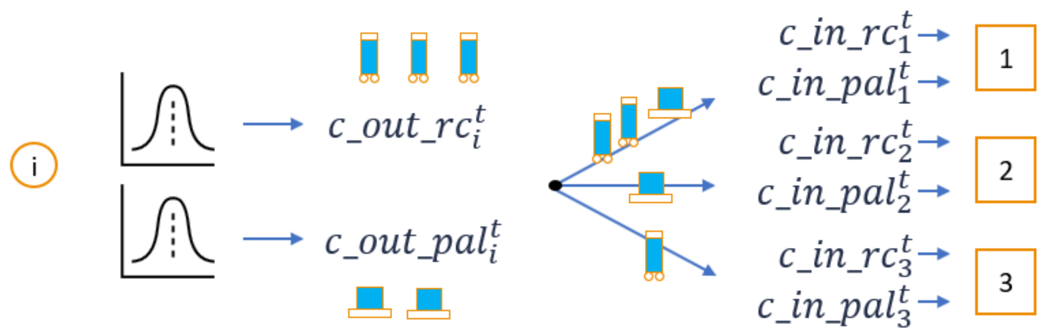


Figure 3: The model of the collection process.

place between 20.00-23.00 o'clock, in which collected parcels are unloaded from their transport item and fed to the sorting machine. The sorting machine redirects the parcels to the outflow arm corresponding to the destination depot of the parcel. At the end of these outflow arms, the parcels are all placed on roll cages. When a roll cage is full it is moved into a truck and replaced by a new empty roll cage. This new roll cage is taken from the roll cage inventory at the depot, which partly consists of the roll cage inflow from the collection process. Note that there is a minimally required inventory of roll cages at a depot to start the sorting process, since at each outflow arm of the machine empty roll cages are needed. This sorting machine set-up inventory is called M. From 20.00-08.00 the roll cages are moved into trucks and transported to the destination depot of the roll cages. In some cases the destination depot is the collection depot, in this case no transport is needed.

Again, we aim to model the roll cage flows to enable the tracking of inventories at locations. Let $d\_out_i^t$ be the number of roll cages that flow out of depot i in period t and $d\_in_i^t$ be the number of roll cages that flow into depot i in period t as a result of the inter transport process. $d\_out_i^t$ depends on the parcel volume that needs to be sorted at depot i in period t. The parcel volume that needs to be sorted on a day is subject to the collected parcel volume of that day and of the collected parcel volume of the day before. Namely, sometimes not all parcels that are collected on a day can be sorted, so they are sorted the day after. Therefore, we assume that $d\_out_i^t$ at depot i in period t is a linear function of the collection inflow of that period ($c\_in\_rc_i^t$ and $c\_in\_pal_i^t$) and the collection inflow of previous period ($c\_in\_rc_i^{t-1}$ and $c\_in\_pal_i^{t-1}$). Hence,

$$d\_out_i^t = \beta_0 + \beta_1 \cdot c\_in\_rc_i^t + \beta_2 \cdot c\_in\_pal_i^t + \beta_3 \cdot c\_in\_rc_i^{t-1} + \beta_4 \cdot c\_in\_pal_i^{t-1}.$$

In Section 3, we explain how we fit values for $\beta_0, \beta_1, \beta_2, \beta_3$ and β_4 for the problem instance of PostNL, using their data. Each of the $d\_out_i^t$ roll cages at depot i flows into a certain depot j according to a fixed fraction for each pair of depots i and j. For the problem instance of PostNL, these fractions are derived from data, which we explain in Section 3. Hence, with all $d\_out_i^t$ variables and these fractions we obtain all $d\_in_i^t$ variables in each period t. Then, using the established roll cage flows in the inter transport step, the inventory change of this step is formulated as:

$$-d\_out_i^t + d\_in_i^t \qquad \forall i \in \mathscr{L}, \forall t \in \mathscr{T}.$$

2.2.3 Repositioning

In the repositioning step, PostNL moves empty roll cages between locations to ensure the availability of roll cages for the upcoming collection and inter transport processes. Around 08.00 o'clock all trucks from the preceding inter transport process have arrived at the destination depot, where the roll cages are unloaded to sort the parcels a second time for last-mile delivery in vans (without roll cages). At this moment a team of supply chain planners observe the roll cage inventory at each location in the network and makes a plan for the repositioning of empty roll cages. That is, the system is rebalanced. Specifically, the planners decide how many roll cages flow between each pair of locations by creating orders. Let x_{ij}^t denote the number of roll cages that are repositioned from location i to location j in period t, also called the order quantity. The inventory change in the repositioning step then is

$$\sum_{j\in\mathscr{L}} (x_{ji}^t - x_{ij}^t) \qquad \forall i \in \mathscr{L}, \forall t \in \mathscr{T}.$$

2.2.4 Summary

Using the formulated inventory changes of each step, the inventory evolution in period t is formulated as

$$r_i^{t+1} = r_i^t + \sum_{j\in\mathscr{L}} (x_{ji}^t - x_{ij}^t) - c\_out\_rc_i^t + c\_in\_rc_i^t + \sum_{j\in\mathscr{L}} (e_{ji}^t - e_{ij}^t) - d\_out_i^t + d\_in_i^t \qquad \forall i \in \mathscr{L}, \forall t \in \mathscr{T}.$$

In this equation we observe that, for each location, the the starting inventory of the next period r_i^t is obtained by taking the starting inventory of current period r_i^t, processing the orders from the repositioning step x_{ij}^t, subtracting the collection outflow $c\_out\_rc_i^t$ and adding the collection inflow $c\_in\_rc_i^t$, processing the emergency orders e_{ij}^t, which occur during collection, subtracting the inter transport outflow $d\_out_i^t$, and adding the inter transport inflow $d\_in_i^t$.

However, negative inventory is not allowed throughout the whole process. Therefore, the inventory evolution formula only holds if certain criteria are met. Namely, x_{ij}^t and e_{ij}^t must ensure the criterion

$$r_i^t + \sum_{j\in\mathscr{L}} (x_{ji}^t - x_{ij}^t) - c\_out\_rc_i^t + \sum_{j\in\mathscr{L}} (e_{ji}^t - e_{ij}^t) \geq 0 \qquad \forall i \in \mathscr{L}, \forall t \in \mathscr{T} \tag{1}$$

to avoid negative inventory during repositioning and during collection. If this does not hold, a location would send more roll cages to other locations than the sum of its starting inventory and roll cages it receives from other locations. Moreover, the following criterion must be met to avoid negative inventory at depots during inter transport.

$$r_i^t + \sum_{j\in\mathscr{L}} (x_{ji}^t - x_{ij}^t) - c\_out\_rc_i^t + c\_in_i^t + \sum_{j\in\mathscr{L}} (e_{ji}^t - e_{ij}^t) - d\_out_i^t \geq 0 \qquad \forall i \in \mathscr{L}, \forall t \in \mathscr{T}. \tag{2}$$

2.3 Objectives

The main goal in this decision process is to minimize the costs of the repositioning process. Another goal is to minimize roll cage inventories at normal depots ($\mathscr{D} \setminus \mathscr{B}$). Specifically, we aim to minimize expected costs per period and expected mean roll cage inventory at normal depots per period. These objectives are subject to the policy with which repositioning decisions are made, which we elaborate in a later subsection. To model these objectives, in this subsection we introduce the order costs and emergency order costs. Using these, we define the mean total costs per period over an arbitrary time horizon. Furthermore, we define the after-repositioning roll cage inventory, and with that, we define the mean after-repositioning inventory at normal depots per period over an arbitrary time horizon. With these definitions we define the objectives of this model.

Costs in this process originate from orders. We distinguish three order types in this model, the immediate order, the structural order and the emergency order. The immediate order and the structural order are part of the repositioning decision. The emergency order is not part of the repositioning decision; it is created during the collection process. As said, an immediate order can be created the same day the transport is needed. A structural order needs to be created at least one week in advance and it needs to recur at the same weekday for minimally three weeks. Moreover, only the origin and destination of a structural order are fixed, the order quantity can still be chosen the same day (where quantity zero implies an order cancellation). Let $\mathscr{P}_d$ denote the set of structural orders (i,j) on weekday w. Usually structural orders are fixed for a relatively long period, hence we assume that the structural orders on each weekday are fixed.

Let $O = \{I,S,E\}$ denote the set of order types k, representing immediate, structural and emergency orders, respectively. Let f_k denote a cost factor of type k. The costs of an order depend on several more factors. They depend on the travel time in minutes between i and j, denoted by d_{ij} and the process time (loading/unloading from truck) per roll cage, denoted by d. Also, the costs depend on the labor costs of a truck driver per minute, denoted by c. Lastly, g denotes the gasoline costs per traveled kilometer and α denotes the travel distance per travel minute (thus, we assume the travel distance is linearly related with the travel time). A truck has a capacity of K, and the number of trucks needed for order quantity x_{ij}^t is denoted by $y_{ij}^t = \lceil x_{ij}^t/K \rceil$. The costs of order quantity x_{ij}^t is denoted by $costs_{ij}^t$ and given by

$$costs_{ij}^t = f_k \cdot c(y_{ij}^t \cdot d_{ij} \cdot (1 + \frac{\alpha \cdot g}{c}) + x_{ij}^t \cdot d).$$

The costs of an emergency order $em\_costs_{ij}^t$ are calculated in similar fashion:

$$em\_costs_{ij}^t = f_E \cdot c(z_{ij}^t \cdot d_{ij} \cdot (1 + \frac{\alpha \cdot g}{c}) + e_{ij}^t \cdot d),$$

where $z_{ij}^t = \lceil e_{ij}^t/K \rceil$.

2.3.1 Total costs

Let the mean total costs per period over time horizon $\mathscr{T}$ be denoted by C. The main objective is to minimize its expected value:

$$\min_{x_{ij}^t} E[C], \tag{3}$$

$$\text{where } C = |\mathscr{T}|^{-1} \sum_{i,j \in L, t \in \mathscr{T}} costs_{ij}^t + em\_costs_{ij}^t.$$

The total costs per period may be divided into three subcomponents: the depot order costs, the customer order costs and the emergency order costs. The mean depot order costs per period over time horizon $\mathscr{T}$ is denoted by C^d and given by

$$C^d = |\mathscr{T}|^{-1} \sum_{i \in \mathscr{L}, j \in \mathscr{D}, t \in \mathscr{T}} costs_{ij}^t.$$

The mean customer order costs per period over time horizon $\mathscr{T}$ is denoted by C^c and given by

$$C^c = |\mathscr{T}|^{-1} \sum_{i \in \mathscr{L}, j \in \mathscr{C}, t \in \mathscr{T}} costs_{ij}^t.$$

The mean emergency order order costs per period over time horizon $\mathscr{T}$ is denoted by C^e and given by

$$C^e = |\mathscr{T}|^{-1} \sum_{i,j \in \mathscr{L}, t \in \mathscr{T}} em\_costs_{ij}^t.$$

Then, C equals the sum of its subcomponents: $C = C^d + C^c + C^e$.

2.3.2 After-repositioning inventory

We aim to minimize the roll cage inventories at normal depots as floor space is limited at these depots. We will choose a very large starting inventory (r_i^0) for buffer locations and remember that the total inventory in the system is constant. Specifically, the inventory right after the repositioning step is of interest as practitioners judge the floor occupancy based on this number. Let $q_i^t = r_i^t + \sum_{j \in \mathscr{L}} x_{ji}^t - x_{ij}^t$ denote the after-repositioning roll cage inventory at location i in period t. Let Q denote the mean after-repositioning inventory at normal depots per period over time horizon $\mathscr{T}$. We aim to minimize its expected value:

$$\min_{x_{ij}^t} E[Q], \tag{4}$$

$$\text{where } Q = (|\mathscr{T}||\mathscr{D} \setminus \mathscr{B}|)^{-1} \sum_{i \in \mathscr{D} \setminus \mathscr{B}, t \in \mathscr{T}} q_i^t.$$

In the remainder of the report, we may leave out the words "at normal depots" in this name, because when we discuss Q, we are always interested in the mean over the normal depots. We choose to model the mean after-repositioning inventory, as this is preferred over the total after-repositioning inventory by practitioners.

2.4 Decision policy

We define the term decision policy as a method to obtain all x_{ij}^t, being in period t. A decision policy aims at optimizing the objectives (3) and (4). Precise costs of inventory is out of scope of this research, but we know that less inventory yields less costs. Hence, we aim to obtain insight in the effect of a policy on both objectives, rather than finding a single optimum considering both objectives. The policy should be defined such that it generates feasible solutions. I.e., conditions (1) and (2) must hold. e_{ij}^t is a decision variable, but not part of the repositioning decision (in the repositioning step). Namely, e_{ij}^t is determined after the repositioning step when the collection quantities at customers are realized, to avoid negative inventory.

This problem involves a decision policy rather than a classical mathematical optimization problem, where the decision variables x_{ij}^t are determined at once, because decisions can be made in each period t and the problem involves random variables. $c\_out\_rc_i^t$, $c\_out\_pal_i^t$, $c\_in\_rc_i^t$, $c\_in\_pal_i^t$, $d\_out_i^t$, $d\_in_i^t$ are random variables with known PDF for all t. These values get revealed after the repositioning decision in period t. Being in period t, we assume we can make a perfect prediction of $c\_in\_rc_i^t$, $c\_in\_pal_i^t$, $d\_out_i^t$, $d\_in_i^t$, i.e. they are given. Hence, in each period, when making a decision, we do not know the realisation of the specific collection at customers in that period, but we do know the collection inflow and inter transport at depots in that period. For future periods, all of those variables are random. This is a realistic assumption, because each day PostNL has accurate forecasts for roll cage and pallet collection inflow for that day.

To reach those goals, a policy is exploited, which will be called the current policy. In the following, a model of the current policy will be provided. In the current policy, the problem is divided into determining orders to customers and determining orders to depots. First orders to customers are determined, then these orders serve as input in the problem of determining the orders to depots. This is done, because each customer has at least one planned structural order per week and roll cages are repositioned to customers only on these days. In general, PostNL does not have planned structural orders between depots. So, giving priority to orders to customers is a heuristic method to minimize costs, because structural orders have a lower price than immediate orders. Orders to customers will be referred to as customer orders and orders to depots will be referred to as depot orders.

2.4.1 Customer order policy

We consider the situation that the process is in period t. In determining the customer orders, first the planners observe to which customers there are planned structural orders in t. These are the to be replenished customers in t. For each customer they determine the replenishment quantity using a heuristic base stock method. In this method, the replenishment quantity is determined such that the inventory level increases to the expected roll cage collection until next planned structural order to this customer plus a safety stock. This safety stock is a constant percentage of the expected roll cage collection until next structural order (same for all customers). cov_i^t denotes the number of periods until the next structural order to customer i, being in period t, also called the coverage. Let the replenishment quantity in period t of customer i be denoted by qty_i^t. Let the safety factor be denoted by s. Then, the formula to calculate qty_i^t is

$$qty_i^t = \max(0, \sum_{x=1}^{x=cov_i^t} (E[c\_out\_rc_i^{t+x-1}]) \cdot (1+s) - r_i^t).$$

Then, the to be replenished customers that are the destination in only one structural trip are observed. We call these customers the single source customers. For each depot, the planners make a list of all single source customers that have a planned structural order with that depot as origin. This list is then sorted based on replenishment quantity, from large to small. Then, one by one, a customer order is created from that depot for each customer in that list. This is done until all customers from the list are supplied or until the depot would end up with negative inventory by creating an order to the customer.

To be replenished customers that are the destination of multiple planned structural orders in a period are called multi-source customers and are considered after single source customers. The replenishment of these customers involves an extra decision: the portion of roll cages that comes from each source (depot). The orders to these customers are determined by sorting all planned structural orders to these customers based on travel duration, from small to large. Then, one by one, for each planned structural order, an order is created to the customer with the largest possible quantity. The largest possible quantity is the maximum of the inventory at the origin depot and the portion of qty_i^t that is not yet replenished by another source.

After determining customer orders in this way, there might be customers to which there are planned structural orders but not yet determined/created orders, as the depot from which there is a planned structural order does not have sufficient inventory to supply this customer. A list of these customers is created and sorted by replenishment quantity. For each customer in this list, starting with the customer that has the largest replenishment quantity, an (immediate) order is created from the closest depot that has sufficient inventory to supply the customer.

Using this procedure, the orders (order quantity, origin and destination) for each customer that has a planned structural order are determined. All other x_{ij}^t to customers are set to zero.

2.4.2 Depot order policy

After the customer orders are determined, some x_{ij}^t are fixed, and the planners know how much of the inventory (r_i^t) at depots can still be used for between-depot rebalancing. They use rules to ensure depots end up with enough roll cages, but not too much. The net roll cage need for inter transport is defined as $d\_out_i^t - c\_in\_rc_i^t$, because $d\_out_i^t$ is the inter transport outflow and $c\_in\_rc_i^t$ is the roll cage inflow from the collection process. Specifically, the planners must choose the remainder of the x_{ij}^t variables such that each depot can execute the upcoming inter transport process. That is,

$$\begin{array}{ll} r_i^t + \sum_{j\in\mathscr{L}}(x_{ji}^t - x_{ij}^t) \geq \max(d\_out_i^t - c\_in\_rc_i^t,\ M) & \forall i \in \mathscr{D} : d\_out_i^t > 0 \\ r_i^t + \sum_{j\in\mathscr{L}}(x_{ji}^t - x_{ij}^t) \geq 0 & \forall i \in \mathscr{D} : d\_out_i^t = 0 \end{array} \tag{5}$$

If the inter transport outflow ($d\_out_i^t$) at a depot is positive, the current inventory (r_i^t) plus the repositioning ($\sum_{j\in\mathscr{L}}(x_{ji}^t - x_{ij}^t)$) must be larger than the net roll cage need for inter transport ($d\_out_i^t - c\_in\_rc_i^t$) and larger than sorting machine set-up inventory (M). If the inter transport outflow ($d\_out_i^t$) at a depot is zero, we do not need roll cages, because there is no inter transport process.

In order to avoid large inventories at normal depots (objective 4), the planners stick to the following rule: the after-repositioning inventory should not exceed the net roll cage need for inter transport by more than S. If M is larger than the net roll cage need, the after repositioning inventory should not exceed M. I.e., the inventory overage must be equal to or below S. However, if a normal depot has an inter transport outflow of zero, inventory is not a burden, so the after-repositioning inventory upper bound becomes the current inventory. That is,

$$\begin{array}{ll} r_i^t + \sum_{j\in\mathscr{L}}(x_{ji}^t - x_{ij}^t) \leq \max(d\_out_i^t - c\_in\_rc_i^t,\ M) + S & \forall i \in \mathscr{D}\setminus\mathscr{B} : d\_out_i^t > 0 \\ r_i^t + \sum_{j\in\mathscr{L}}(x_{ji}^t - x_{ij}^t) \leq r_i^t & \forall i \in \mathscr{D}\setminus\mathscr{B} : d\_out_i^t = 0 \end{array} \tag{6}$$

In practice, given the customer orders, the intermediate evaluation of

$$r_i^t + \sum_{j\in\mathscr{L}} (x_{ji}^t - x_{ij}^t) - \max(c\_in\_rc_i^t - d\_out_i^t, M)$$

is calculated for each (normal) depot. If the result is outside the interval $[0,S]$, it shows how much roll cages must be repositioned out of this depot. The planners manually find a solution to this problem aiming at solving this as cheaply as possible, i.e. they manually solve

$$\min_{x_{ij}^t} \qquad \sum_{i,j\in\mathscr{D}} f_I \cdot c(y_{ij}^t \cdot d_{ij} \cdot (1 + \frac{\alpha \cdot g}{c}) + x_{ij}^t \cdot d),$$

subject to the already determined customer orders, conditions 5 and 6, and $x_{ij}^t \geq 0 \qquad \forall i,j \in \mathscr{D}$.

The planners use their experience and knowledge of the distances between locations to solve the problem, they do not formalize the method. Hence, their precise method is hard to model. Therefore, in this paper we assume that the planners solve this problem by feeding this mathematical problem to Gurobi, a commercial optimization problem solver. Gurobi solves this problem optimally. We have now defined the current policy, with which all x_{ij}^t are determined in each period t.

3 SIMULATION EXPERIMENT WITH COMPANY DATA

In this section, we apply the developed model of Section 2 to the specific problem of PostNL to obtain the desired results. First, we describe the most important characteristics of the problem instance of PostNL and fill the model parameters with company data. Then we present the set-up of the simulation experiment which we use to obtain results on the defined objectives.

3.1 Problem instance description

We include 1694 customers and 28 depots, spread over The Netherlands and a small part of Belgium. Figure 4 shows all depot locations on the map. Remember that we model the collection quantities $c\_out\_rc_i^t$ and $c\_out\_pal_i^t$ as random variables with known PDFs (Section 2). To create these PDFs, we examine PostNL data of all collection trips (trucks collecting parcels at customer and bringing it to a depot) of the first seven months of 2021. The data consists of the date, origin (customer), destination (depot), collection quantity and transport item type of collection trips in that period. First, the data is split in roll cage collection and pallet collection (since we want to obtain separate PDFs for $c\_out\_rc_i^t$ and $c\_out\_pal_i^t$). Creating distributions for each individual customer is not practical, since we would need data with a long time span to obtain smooth distributions for each customer, while the collection of parcels at customers changes over long periods of time. Hence, the data is divided in five groups based on the total collection quantity of customers, and we assume that all customers in a group exhibit the same collection behaviour. This is done by ranking the customers in terms of total collection quantity over the whole time span and using the quintiles to split the customers in groups. Then, for each group the data is grouped by weekday. In this way, we obtain a PDF for the collection of roll cages and a PDF for the collection of pallets for each day of the week

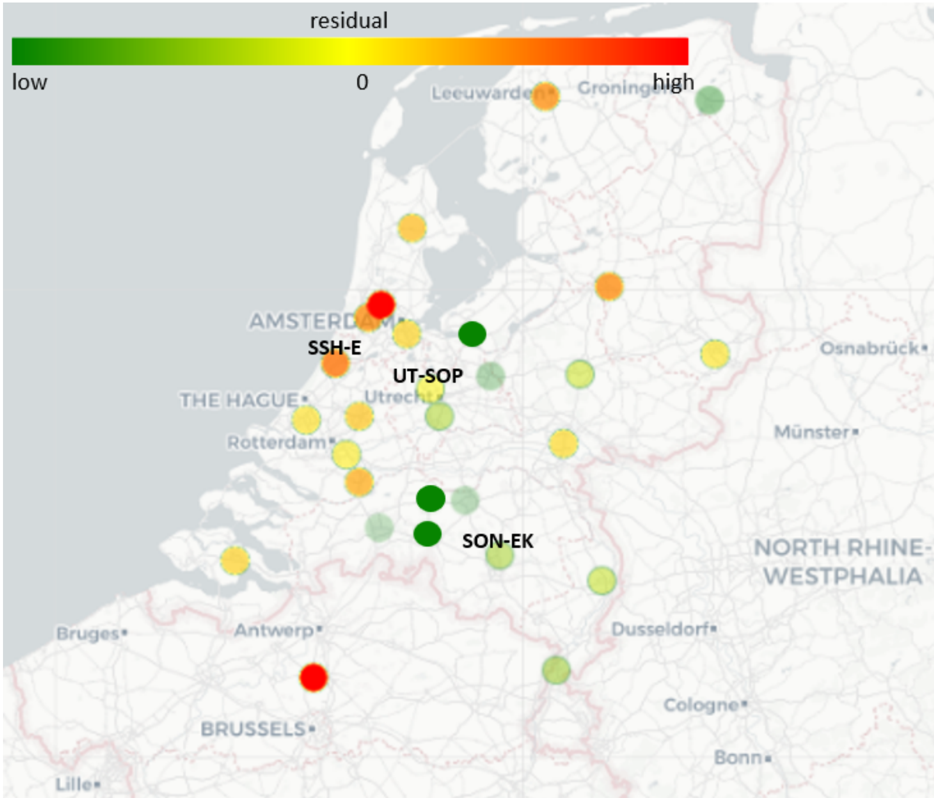


Figure 4: The depot locations in Benelux with residual value (inter transport inflow minus inter transport outflow).

for each customer. Figure 5 provides an overview of this procedure. Also, the fraction of $c\_out\_rc_i^t$ and $c\_out\_pal_i^t$ at customer i that goes to each depot is derived from this data. Namely, looking at all roll cage collection occurrences of a customer, a certain fraction of the total quantity flows to each depot. We adopt this fraction in our model. We do the same for pallet collection. With these fractions we obtain $c\_in\_rc_i^t$ and $c\_in\_pal_i^t$.

Figure 6a displays the expected total collection flow for both roll cages (number of roll cages) and pallets (number of roll cage equivalents) for each weekday. One can see that there is a week pattern in the total collection quantity: The collection volume is the largest on Monday and it decreases steadily until Friday. During the weekend, the collection quantity is relatively small.

During the inter transport process, before roll cages are moved into trucks to send them to destination depots, they are scanned to register the inter transport flow of roll cages from depot to depot. This scan-event data of the first seven months of 2021 is used to fit the linear coefficients $\beta_0, \beta_1, \beta_2, \beta_3$ and β_4 to obtain $d\_out_i^t$ for each depot. Note that these coefficients therefore differ per depot. This is desired, because the shape of parcels and the quality of filling the roll cages differ per depot. This data is also used to fit the fraction of the outflow $d\_out_i^t$ at depot i that flows to each depot j. Since the inter transport outflow on depots is a function of the collection quantities of that period and the previous period, we see a similar week pattern in the inter transport quantity, visualized by Figure 6b.

Rebalancing flows between depots depend strongly on the result of the inter transport inflow minus the inter transport outflow, which we will call the residual. To illustrate, a high residual indicates that a depot receives more roll cages than it sends during inter transport. Therefore such a depot is likely to supply other locations during repositioning. We visualized the expected weekly residual for each depot in Figure

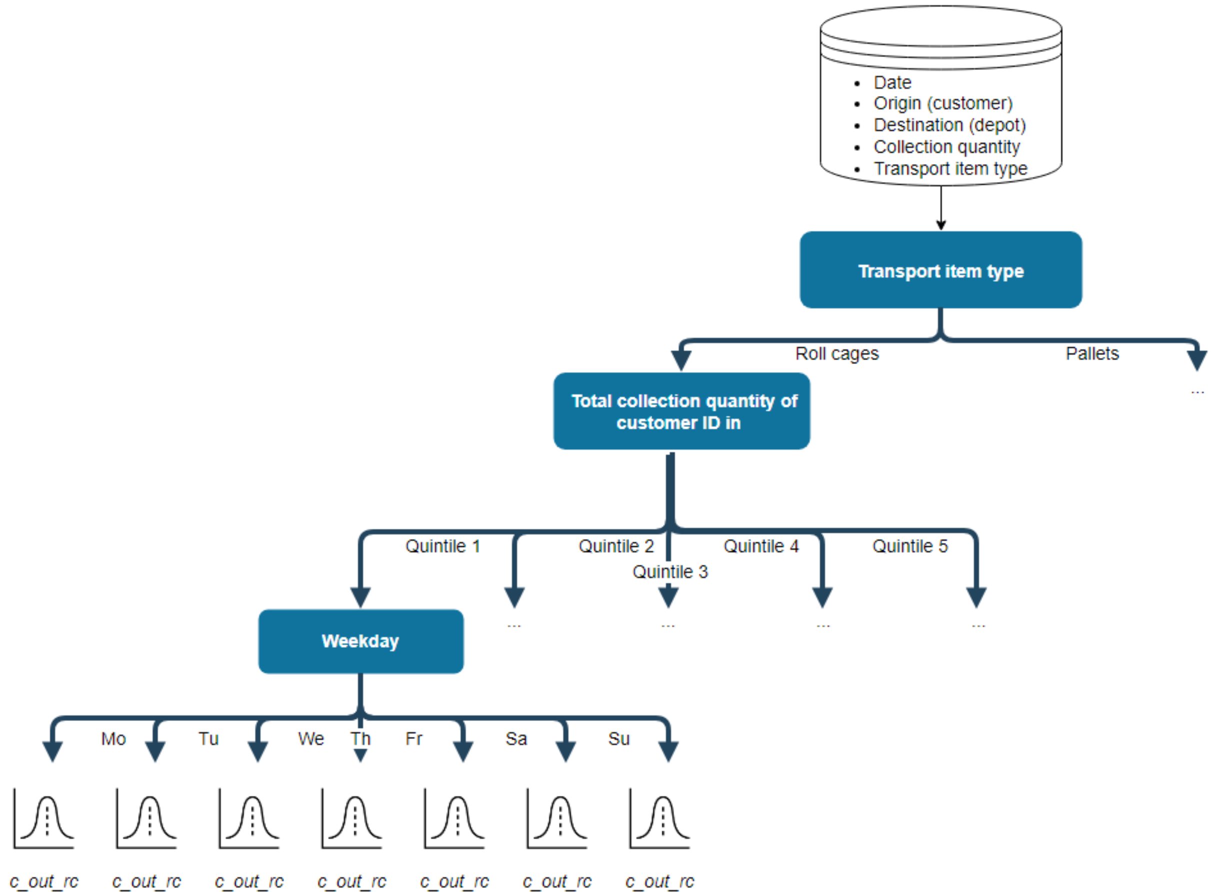


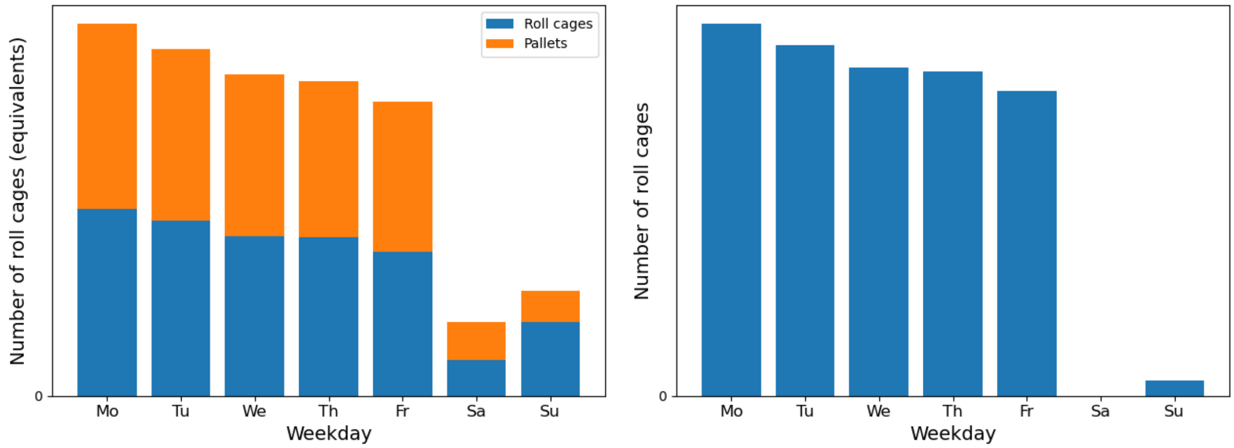
Figure 5: Grouping collection data to obtain probability distributions per transport item type, customer size, and weekday.

4, which provides an overview of the imbalance in roll cage flows between locations. In Section 4 we provide rebalancing flow outcomes of the simulation, reflecting on the residual values shown here.

3.2 Simulation experiment

We are interested in obtaining the long term mean inventory at depots (Q), total costs per period (C) and its subcomponents (C_d, C_c, C_e). That is, we want to obtain the expected value for these objectives for an arbitrary time horizon $\mathscr{T}$ and corresponding random events. To this end, we conduct a simulation experiment to obtain estimates for these expected values. In the following, this simulation set-up is described. In Section 4, we present results which we obtained using this simulation set-up.

By simulating the process for $m = 42$ number of periods in which we randomly generate values for the random collection parameters, we obtain one realization for each objective (using expressions 3 and 4). We perform $n = 10$ of these runs to obtain a sample of these realizations. We let the mean of this sample be the estimate of the expected value of the corresponding objective variable. Furthermore, adopting the method from Boon et al. (2020), the expression $(\bar{Z} - \Phi(0.975)\sqrt{\frac{S^2}{n}}, \bar{Z} + \Phi(0.975)\sqrt{\frac{S^2}{n}})$ provides an approximate 95% confidence interval for these estimates of the objectives. Here $\bar{Z}$ is the sample mean, $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and S^2 is the sample variance (degrees of freedom = 1).



(a) The expected total collection flow per weekday. (b) The expected total inter transport flow per weekday.

Figure 6: The total collection and inter transport flows over the week.

In each simulation run of 42 periods, we have a warm up period of seven periods (one week), since these periods differ slightly from later periods. That is, we leave the first seven periods out of the calculation of the realizations of the objective values of a run. I.e.: we remove the first seven periods, when calculating the objectives. In Figure 9, one can see why we do this. The figure shows the costs of the different cost components for the first 35 periods. One can see that the first week deviates from later weeks. The cause of this deviation is that we use slightly biased (too low) starting inventories at locations, which results in larger costs in the first week.

4 RESULTS

This section presents the results of applying the model of Section 2 to the company data of PostNL (described in Section 3). First, we present the results of the objectives when choosing $S = 180$, as the company currently does. Then, we present the results of the objectives of different S.

4.1 Current decision policy

Rebalancing flows are of interest to process owners to evaluate the model and process dynamics. For instance, when a depot turns out to have large net rebalancing inflow, the process owner might obtain reduced repositioning costs by decreasing the collection inflow to that depot. The net depot order inflow of depot i is defined as $\sum_{j\in\mathscr{D}} x_{ji} - x_{ij}$. Hence, we provide results about the net depot order inflow for depots SON-EK, SSH-E, UT-SOP to demonstrate that the simulation model is capable of supporting such analysis. Figure 7 shows the net depot order inflow of the three depots for twenty one subsequent periods of one arbitrary simulation run. One can see that in general, SON-EK has a positive, and relatively large net depot order inflow, so SON-EK receives more roll cages than it supplies in the between-depot repositioning. This is not surprising as this depot has a relatively low residual (see Figure 4). In general, SSH-E has a negative and low net depot order inflow, meaning that this depot supplies other depots more than it receives from depots. This also makes sense since SSH-E in general has a relatively high residual (Figure 4). UT-SOP in general has a net depot order inflow close to zero, which was to be expected as the depot in general has a residual close to zero (Figure 4). Figure 7 also shows the long term mean of the net depot order inflow for the three depots, represented by the horizontal lines, and the corresponding confidence interval, represented by the error bars at the at the left side of the plot.

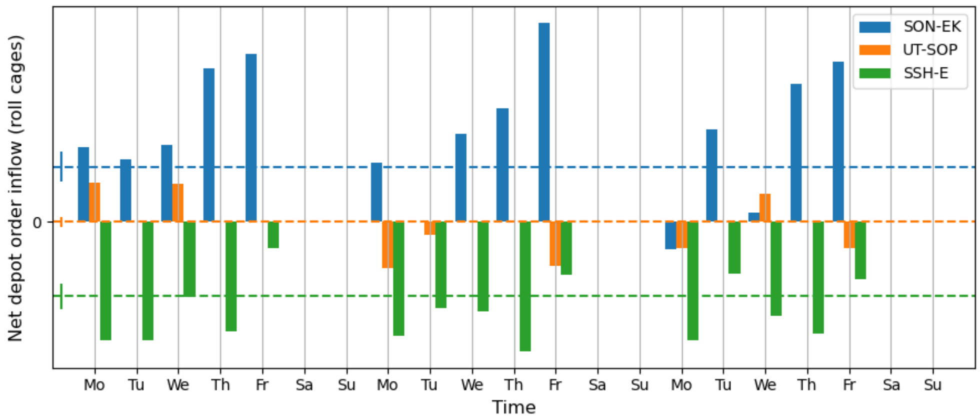


Figure 7: The net depot order inflow over time for three depots.

Figure 8 shows the mean after-repositioning inventory for the first 35 periods of one arbitrary simulation run (the first run). Also, the expected mean after-repositioning inventory per period $E[Q]$ is depicted by the horizontal line and the corresponding confidence interval is depicted at the left side of the figure. Note that the measurements are normalized (by expressing them as a percentage of the maximum mean after-repositioning inventory of this first run) to keep the absolute values confidential. In the remainder of the report, when presenting results on the inventory levels, the normalization is based on this maximum, so that all results on inventory levels can be compared. Clearly, there is a week pattern. In general, from Mondays to Fridays, the mean after-repositioning inventory decreases. The reason for this is a combination of two system behaviours. Firstly, the inter transport flow decreases over these days, and hence less roll cages are needed right before the inter transport process. Secondly, the current S-level forces the company to remove most of the inventory overage to the buffer locations. These two together result in a decreasing mean after-repositioning inventory from Monday until Friday. In the weekends there is a remarkable peak. This is caused by the fact that the repositioning process is almost completely shut off during the weekend. Therefore, almost no customer orders are performed and thus all roll cages from the inter transport of Friday remain at the depots during the weekend (where they stand still). On Sunday there are a few depots that do have an inter transport process, hence we see some minor changes from Saturday to Sunday.

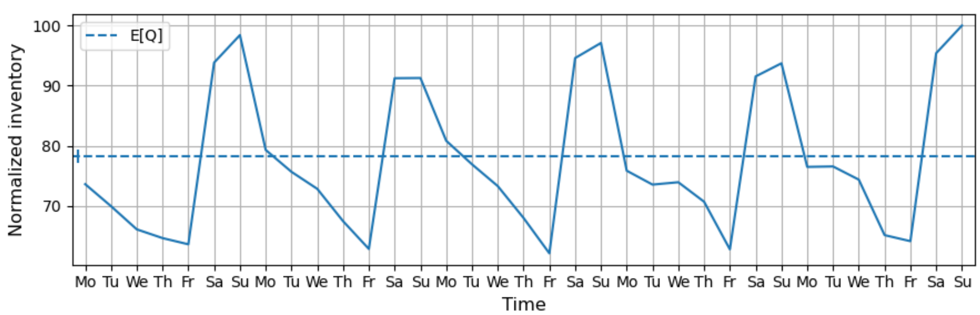


Figure 8: The mean after-repositioning inventory over time and $E[Q]$.

Figure 9 shows the depot order costs, the customer order costs and the emergency order costs for the first 35 periods of one arbitrary simulation run. Also, $E[C]$ is depicted by the horizontal line, accompanied by its confidence interval at the left side of the figure. Again, the quantities are normalized. To do this, we use the maximum total cost per period over the periods in this simulation run. In the remainder of this report, this maximum cost is used to normalize results on costs. The customer order costs is the largest cost component, which can be explained by the fact that the number of customer locations is way larger than the number of depots, thus the number of customer orders is relatively large. Moreover, one can observe a peak in the depot order costs on Mondays and Fridays. On Fridays, the normal depots receive a relatively large number of roll cages from buffer locations, because throughout the week, roll cages are repositioned from normal depots to buffer locations (as a result of the current S level), but the customer orders are still average in size. On Mondays, normal depots are slightly packed, because the customers received their roll cages for Monday and Tuesday mostly already on Friday. This contributes to a larger stream of roll cages to buffer locations on Monday.

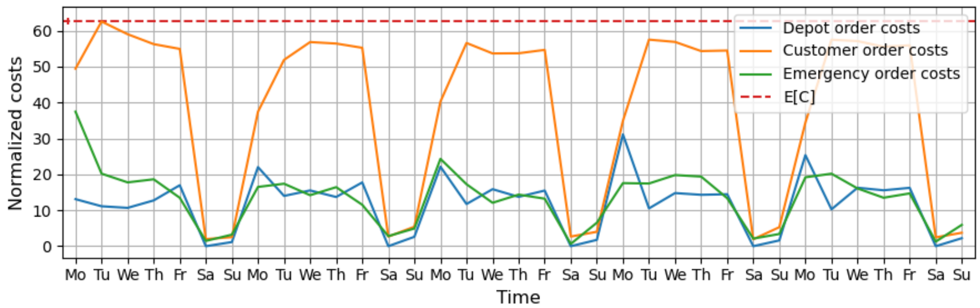


Figure 9: The different cost components over time and $E[C]$.

4.2 Alternative policies: effect of S

In this section, we present the results on the effect of the S-level on $E[C]$, its subcomponents $E[C^d]$, $E[C^c]$, $E[C^e]$ and the expected mean after-repositioning inventory $E[Q]$. In Figure 10 the mean after-repositioning inventory for the first 35 periods of one arbitrary simulation run is given for two levels of max surplus level S. Note that these thus are two different runs, but they have the same set of random events. Also, $E[Q]$ is shown for both S-levels including confidence intervals. It can be seen that the larger S, the larger $E[Q]$. It is worthwhile noting that especially from Monday until Thursday a less steep decrease in inventory occurs. This is probably caused by less incentive in the optimization problem to remove redundant inventory at normal depots to buffer locations.

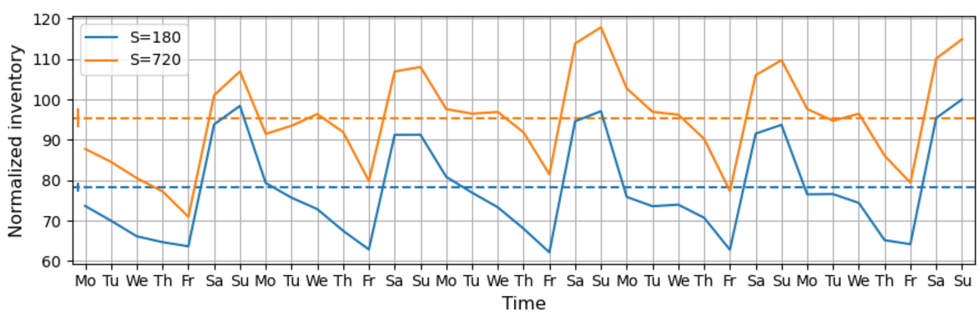


Figure 10: The effect of S on the after-repositioning inventory.

To show the effect of S on the repositioning costs we show a similar plot for the depot order costs in Figure 11. We observe that when $S = 720$, the depot order costs are smaller in any period compared to when $S = 180$. This is also reflected by the $E[C^d]$ for both S levels. A reduction of 32.4% in the $E[C^d]$ is obtained by changing the S level to 720. Since the depot order costs is just one subcomponent of the total costs, we now focus on $E[C]$. In Figure 12 we show $E[C]$ for several values of S, including confidence intervals. One can see that the larger S, the smaller $E[C]$. However, we see that the decrease in $E[C]$ decreases as S gets larger. Hence, the benefit per extra roll cage S is decreasing in S. Table 1 provides a summary of the effect of S on the objectives. It shows the value for the different (subcomponents of the) objectives in a percentage to the value one obtains by using current S level. Worthwhile noting is that by changing the S level from 180 to 360, the total repositioning costs can be reduced by 4.0%, which is a significant saving for PostNL.

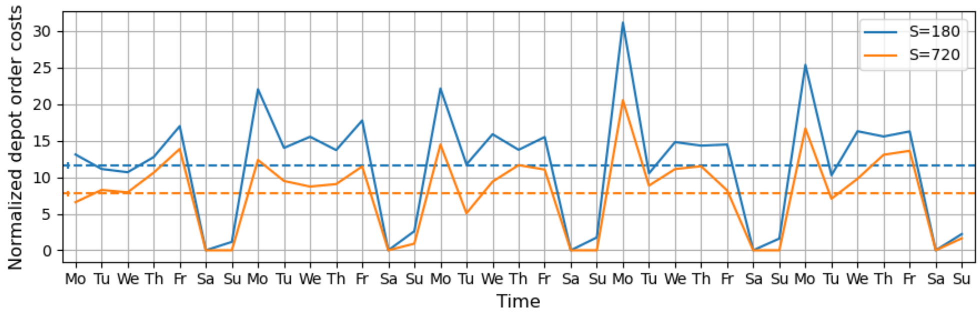


Figure 11: The effect of S on the depot order costs during a simulation run.

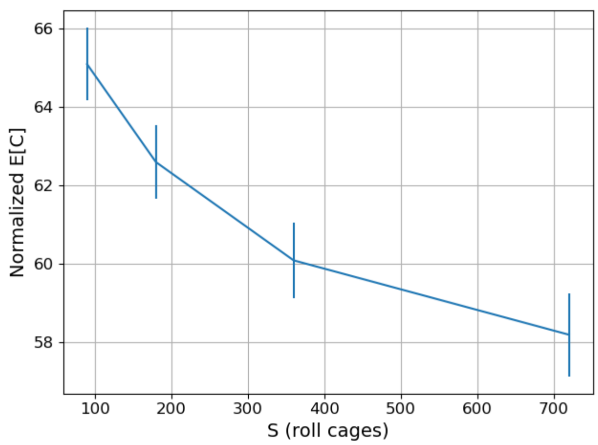


Figure 12: The effect of S on $E[C]$.

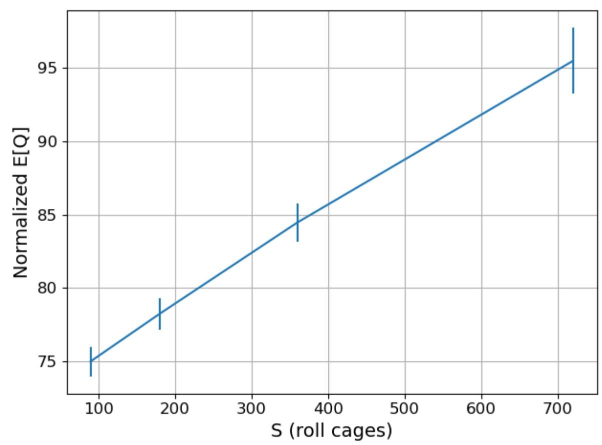


Figure 13: The effect of S on $E[Q]$.

Table 1: The objective values for several levels of S (in percentages of the objective value when $S = 180$, which PostNL currently uses).

S	$E[Q]$	$E[C^d]$	$E[C^c]$	$E[C^e]$	$E[C]$
90	95.9	113.2	100.2	107.1	104.0
180	100.0	100.0	100.0	100.0	100.0
360	107.9	82.6	99.9	96.4	96.0
720	122.0	67.6	99.7	95.6	93.0

5 DISCUSSION

We developed a simulation model that supports optimization of decision making in empty transport item repositioning in a parcel delivery system. Our model accounts for randomness in the system, by modelling the demand at customers for roll cages, our transport items, as random variables. Also, the relation between transport item inflow of several types (roll cages and pallets) and roll cage demand at depots is modelled. Furthermore, the model accounts for the structural planning, which are repositioning transport options against a lower fee. The main idea of the model is to formulate the inventory evolution in the system subject to the repositioning decision, the random demands, and emergency transport. Also, the repositioning costs subject to this inventory evolution is formulated. In this way, the model simulates the repositioning process for an arbitrary control policy, hence several decision policies and algorithms can be evaluated in terms of costs and inventory levels. Moreover, we modelled the current control policy, called decision policy, of PostNL. Currently, PostNL manually solves the problem of rebalancing roll cages between depots to obtain inventories that lay between bounds. We translate this problem to a mathematical optimization problem and solve it with commercial software Gurobi.

We showed that the model is capable of analyzing various characteristics of the process. First we examined the current control policy of the company. We showed that (long term and temporal) repositioning flows at individual locations can be analyzed. Furthermore, we obtained long term per period repositioning costs of the system and mean inventory levels at depots. Secondly, we compared the current control policy, in which the company has very small tolerance for inventory overage at depots, to the situation of allowing larger inventory overage at depots. We clarified the relation between repositioning costs and this overage tolerance and we show that significant cost savings can be achieved by allowing larger inventory overage at depots. Specifically, choosing $S = 360$ results in an increase in the mean inventory at depots of 7.9 %, but a reduction in the total repositioning costs of 4% (17.4% reduction in depot order costs), which is a significant saving for the company.

A limitation of this model is that costs of inventory is not accounted for explicitly. This makes the claim that total costs of PostNL will decrease when increasing inventory at normal depots debatable. Namely, storage space (e.g. storage trucks) might be needed when PostNL increases inventory at depots. PostNL has ample inventory at buffer locations for the largest part of the year, so the the number of roll cages that PostNL owns does not need to be increased, when increasing S. Hence, increased capital costs will probably not be a problem. Furthermore, the model on its own does not optimize the control policy, it only evaluates control policies that are provided to the model. This is, however, typical for simulation models (Dejax and Crainic 1987). Moreover, we make some simplifying assumptions. For instance, we assume that we perfectly know upcoming (inter transport) demand at depots, but in reality we do not have perfect predictions. This leads to the occurrence of shortage at depots in reality, with corresponding emergency costs. In practice this issue is handled by using a safety stock for demand at depots. Hence by assuming that we have perfect predictions, it is likely that cost savings through larger S are even bigger in reality, because increased inventory also reduces such shortage costs. Moreover, this assumption can easily be dropped by just taking the expected demand at a depot in a certain period and including the emergency trips in the same way we do with customers.

Future work could investigate optimization of the structural planning in a parcel delivery system. Namely, a large part of repositioning to customers can be planned ahead based on forecasts, and customer replenishments are the biggest cost component. Challenges in that problem are the large number of customers and the fixed charge of planning structural orders, which is an issue as the system is random. Also, future work can focus on modelling the impact of transport item inventory at depots on the system performance. E.g.: making the storage costs explicit or setting a maximum value on the inventory at depots. Furthermore, scientists could focus on developing control policies that consider the supply and demands of depots of multiple periods in the future, instead of myopically repositioning the transport items (which is done currently).

REFERENCES

Boon, M., M. van der Boor, J. van Leeuwaarden, B. Mathijsen, J. van der Pol, and J. Resing. 2020. "Stochastic Simulation using Python Lecture Notes". Department of Mathematics and Computer Science Eindhoven University of Technology.

Crainic, T. G., M. Gendreau, and P. Dejax. 1993. "Dynamic and stochastic models for the allocation of empty containers". *Operations research* 41(1):102–126.

Dejax, P. J., and T. G. Crainic. 1987. "Survey Paper—A Review of Empty Flows and Fleet Management Models in Freight Transportation". *Transportation Science* 21(4):227–247.

Dong, J.-X., and D.-P. Song. 2009. "Container fleet sizing and empty repositioning in liner shipping systems". *Transportation Research Part E: Logistics and Transportation Review* 45(6):860–877.

Erera, A. L., J. C. Morales, and M. Savelsbergh. 2009. "Robust Optimization for Empty Repositioning Problems". *https://doi.org/10.1287/opre.1080.0650* 57(2):468–483.

Ghosh, S., P. Varakantham, Y. Adulyasak, and P. Jaillet. 2017. "Dynamic repositioning to reduce lost demand in bike sharing systems". *Journal of Artificial Intelligence Research* 58:387–430.

Jordan, W. C., and M. A. Turnquist. 1983. "A Stochastic, Dynamic Network Model for Railroad Car Distribution". *Source: Transportation Science* 17(2):123–145.

Neumann-Saavedra, B. A., T. G. Crainic, B. Gendron, D. C. Mattfeld, and M. Römer. 2016. "Service network design of bike sharing systems with resource constraints". In *International Conference on Computational Logistics*, 352–366. Springer.

Shen, W., and C. Khoong. 1995. "A DSS for empty container distribution planning". *Decision Support Systems* 15(1):75–82.

Song, D. P., and J. X. Dong. 2015. "Empty container repositioning". *International Series in Operations Research and Management Science* 220:163–208.

Vogel, P., B. A. Neumann Saavedra, and D. C. Mattfeld. 2014. "A hybrid metaheuristic to solve the resource allocation problem in bike sharing systems". *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)* 8457 LNCS:16–29.

White, W. W., and A. M. Bomberault. 1969, apr. "A network algorithm for empty freight car allocation". *IBM Systems Journal* 8(2):147–169.

Yan, S., J.-R. Lin, Y.-C. Chen, and F.-R. Xie. 2017. "Rental bike location and allocation under stochastic demands". *Computers & Industrial Engineering* 107:1–11.

APPENDIX

A List of model symbols

Sets	
$\mathscr{D}$	Set of depots.
$\mathscr{C}$	Set of customers.
$\mathscr{L} = \mathscr{D} \cup \mathscr{C}$	Set of locations.
$\mathscr{B} \subseteq \mathscr{D}$	Set of buffers.
$\mathscr{T}$	Set of periods.
$\mathscr{W}$	Set of weekdays.
$\mathscr{P}_d$	Set of planned structural orders (i, j) on $d \in \mathscr{W}$.
$\mathscr{O}$	Set of orders types.
Parameters	
d_{ij}	Travel time between $i \in L$ and $j \in L$ (minutes).
d	Process time per roll cage (minutes).
c	Labor cost of truck driver per minute (euro/minutes).
g	Gasoline costs per traveled kilometer (euro/kilometer).
α	Travel distance per minute travel time (kilometer/minutes).
f_k	Cost factor of order type k.
K	Capacity of a truck (number of roll cages).
M	Sorting machine set-up inventory (number of roll cages).
S	Maximum surplus level (number of roll cages).
cov_i^t	The number of number of periods until next structural order at location i in period t.
$c\_out\_rc_i^t$	Roll cage collection at location i, in period t (number of roll cages).
$c\_out\_pal_i^t$	Pallet collection at location i, in period t (number of roll cage equivalents).
$c\_in\_rc_i^t$	Roll cage collection inflow at location i, in period t (number of roll cages).
$c\_in\_pal_i^t$	Pallet collection inflow at location i, in period t (number of roll cages).
$d\_out_i^t$	Inter transport outflow at location i, in period t (number of roll cages).
$d\_in_i^t$	Inter transport inflow at location i, in period t (number of roll cages).
Variables	
x_{ij}^t	Order quantity from location i to location j in period t (number of roll cages).
e_{ij}^t	Emergency order quantity from location i to location j in period t (number of roll cages).
y_{ij}^t	Number of trucks from location i to location j in period t (number of trucks).
z_{ij}^t	Number of emergency trucks from location i to location j in period t (number of trucks).
r_i^t	Starting inventory at location i in period t (number of roll cages).
q_i^t	After-repositioning inventory at location i in period t (number of roll cages).
qty_i^t	Replenishment quantity at location i in period t.
$E[C]$	Expected repositioning costs per period (euro).
$E[C^d]$	Expected depot order costs per period (euro).
$E[C^c]$	Expected customer order costs per period (euro).
$E[C^e]$	Expected emergency order costs per period (euro).
$E[Q]$	Expected mean after repositioning inventory at normal depots (number of roll cages).