

MASTER

On a multiple item replenishment problem in the presence of carbon emissions

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ILGIN EFE ŞENYUVA

ON A MULTIPLE ITEM REPLENISHMENT PROBLEM

Bilkent University 2022

ON A MULTIPLE ITEM REPLENISHMENT
PROBLEM IN THE PRESENCE OF CARBON
EMISSIONS

A Master's Thesis

by
ILGIN EFE ŞENYUVA

Department of
Management
İhsan Doğramacı Bilkent University Ankara
September 2022

To Sinem, my undeterred supporter

ON A MULTIPLE ITEM REPLENISHMENT PROBLEM IN THE
PRESENCE OF CARBON EMISSIONS

The Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

by

ILGIN EFE ŞENYUVA

In Partial Fulfilment of the Requirements for the Degree of
MASTER OF SCIENCE

THE DEPARTMENT OF
MANAGEMENT
İHSAN DOĞRAMACI BİLKENT UNIVERSITY
ANKARA

September 2022

A MULTI-ITEM REPLENISHMENT PROBLEM IN THE PRESENCE OF CARBON EMISSIONS
By Ilgin Efe Şenyuva

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science in Business Administration.

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ABSTRACT

ON A MULTIPLE ITEM REPLENISHMENT PROBLEM IN THE PRESENCE OF CARBON EMISSIONS

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September 2022

Motivated by the increasing popularity of sustainability issues, and the opportunities to create positive value through collaboration, the purpose of the study is to investigate a joint replenishment model in the presence of carbon emissions. The joint replenishment problem here is one where there is a deterministic demand rate over an infinite time horizon where there are major fixed, minor fixed, holding, and per unit costs and emissions. Since extensive research has been done on carbon caps and carbon tax, in order to differentiate the thesis from the existing work, a novel objective function is implemented. Through this, the optimality of the objective function has been investigated and the obtained results prove the concavity of the objective function for the common order interval and joint concavity for the item-specific order intervals. Relationships between the parameters of the variables are obtained to decrease the computational complexity. Lastly, basic numerical analysis is conducted to compare the performance of the objective function to traditional objective functions.

Keywords: Collaboration, joint replenishment, carbon emission, sustainability

ÖZET

KARBON SALINIMI ORTAMINDA BİR TOPLU SİPARİŞ PROBLEMİ

Şenyuva, Ilgın Efe

Yüksek Lisans, İşletme Bölümü

Tez Yöneticisi: Doç. Dr. Emre Berk

Eylül 2022

Bu çalışmada, dünya çapında artan sürdürülebilirlik bilincine bağlı olarak azaltılmaya çalışılan karbon emisyonu ve firmalar arasındaki işbirliğinin operasyon yönetimine yapabileceği potansiyel olumlu katkıdan yola çıkılarak bir toplu sipariş problemi, karbon emisyonlarıyla birlikte değerlendirilmiştir. Bu toplu sipariş probleminde sonsuz vadeli zamanda bilinen ve sürekli bir talebin yanısıra siparişle ortaya çıkan büyük ve küçük sabit, depolama, ve ürün başı masraf ve karbon salınımı vardır. Literatürde karbon salınım limiti ve karbon salınım vergisi üzerine çok sayıda araştırma olduğundan, bu çalışmayı benzerlerinden ayırtmak adına yeni bir amaç fonksiyonu, yatırım getirisi, üzerinden çalışmalar sürdürüldü. Bu doğrusal olmayan çok değişkenli amaç fonksiyonu göz önüne alınarak en iyileme üzerine analizler gerçekleştirildi ve ulaştığımız sonuçlar her iki değişken tipimizin ayrı ayrı kendi içinde konkav olduğunu gösterdi. Problemimizi basitleştirecek özel durumlar üzerinde duruldu. Son olarak, basit sayısal analizler üzerinden belirlediğimiz amaç fonksiyonunun daha geleneksel amaç fonksiyonlarıyla kıyaslaması yapıldı.

Anahtar Kelimeler: İşbirliği, toplu sipariş, karbon salınımı, sürdürülebilirlik

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CHAPTER 1

INTRODUCTION

The worldwide greenhouse gas emissions have been increasing steadily since the beginning of the 20th century while carbon dioxide emissions being the most prominent one (*Global Greenhouse Gas Emissions Data*, 2022). This increase in greenhouse gas emissions, especially the carbon-dioxide emission, has a significant effect on the average temperature of our world such that it has increased more than a centigrade degree over the last 150 years (*NOAA National Centers for Environmental Information, State of the Climate: Monthly Global Climate Report for Annual 2020*, 2022).

The changes that are foreseen as a result of this global warming include rise in sea levels, loss of coastal land, alterations in precipitation patterns, increased risk of floods and droughts, and an increased threat to biodiversity (Kasotia, 2007). Thus, global warming has become one of the most urgent and forceful problems that the world currently faces. That is why, the word “sustainability” is thrown around more than ever in today’s day and age. Firms are looking to

differentiate themselves by acting in accordance with corporate social responsibility (Ioannou & Serafeim, 2022), the governments are forcing the businesses to obey agreed upon codes of conduct (A. B. Carroll, 2016), and the altruistic customers are considering the environmental impact of their purchases (Onel & Mukherjee, 2015).

Corporate social responsibility (CSR), as it is defined by Kotler & Lee (2011) is a mindset that encourages businesses to support or initiate philanthropic, activist, or charitable practices has been a concern of businesses for hundreds of years (A. Carroll, 1999). However, the issue has begun to be discussed in the literature in 1930's when the role of the executives also appeared in the literature (A. Carroll, 1999). Sustainability, that is a crucial part of CSR mindset, is defined as the act of meeting the needs of the present without compromising the ability of future generations to meet their own needs by the United Nations in their report (Imperatives, 1987). Therefore, by this definition, we know that the company that operates by taking sustainability into account desires the well-being of the current and future generations. Here, we can define the current and next generations as any stakeholder, actual and potential, that is affected by the operations and decisions of the companies. Apart from the CSR mindset, that is mostly a self-regulated set of practices used by the company, the governments and global organizations enforce certain rules and regulations to ensure the sustainability of the operations as well as the stakeholders of the company. Among these rules and regulations, one significant pioneer is Kyoto Protocol that is signed in 1997 by 194 countries in Japan through the the United Nations Framework Convention on Climate Change (*The Kyoto Protocol Mechanisms*, 2022). Under the Kyoto Protocol, countries are given certain emission quotas for a given time period and are obliged to control the emission that is assigned to them. This protocol has enabled the carbon trading mechanism and market to emerge as the countries that underused their quotas were able to sell their leftovers to countries that need

the extra carbon emission right to operate. Moreover, through Clean Development Mechanism, Kyoto Protocol enables the countries that approve it to conduct projects reducing emissions in developing countries. These projects helps companies acquire extra carbon emission capacity. Moreover, Joint Implementation Mechanisms enable two approving parties to conduct projects together while one becomes more sustainable, and the other gains extra carbon emission capacity (*Kyoto Protocol*, 2021).

A more recent development is the Paris Agreement that was initiated by the European Commission in 2015 during the United Nations Climate Change Conference (COP21). Paris Agreement aims to reduce emissions and create transparency regarding the environmental impact of the parties involved in order to battle the environmental problems that we are experiencing. The main goal that the Paris Agreement has focused on is to limit global warming to a level well below 2°C, preferably down to 1.5°C. Carbon neutrality is desired to be achieved by the participating countries by the middle of this century. The governments that have agreed to convene every 5 years to assess the work that they have done in the meantime, compare the results with the objectives, and adjust and manifest their plans. They agree to maintain a system that is transparent such that it also provides a certain level of accountability for the participating countries. Another sustainability practice that is highlighted is the possible cooperation between the regions, countries, cities, and local authorities, which relate closely to our topic (*Paris Agreement*, 2022).

According to International Energy Agency (IEA), about 37% of the carbon dioxide emission from the end-use sectors comes from transportation (*Transport – Topics*, 2021). Moreover, the report that the United States Environmental Protection Agency has published shows transportation among the top five sources of greenhouse gas emissions by accounting for 27% of the total greenhouse gas emissions (*Sources of Greenhouse Gas Emissions*, 2022).

Establishing the issue as a significant and immediate concern, our focus will be

on the logistics operations of the companies since transportation accounts for a large percentage of the total emissions according to these various sources. Since an average of 20% of all road transports running with empty vehicles in Europe in 2020 (Hara et al., 2022), it could be argued that the current logistics system is at least questionable in terms of efficiency and could benefit from collaboration of firms that would help increase the load rates and the efficiency of the operations. Thus, the firms collaborating could be a potential solution to inefficiencies and increase sustainability. In order to create a competitive advantage, companies engage in horizontal collaboration, that is the collaboration of firms that are in the same echelon in a supply chain (Pan et al., 2019; Ferrell et al., 2020). The horizontal collaboration could potentially help firms to tackle the problems related to underused capacities.

Therefore, in order to create sustainability, create efficiency, and encourage firms to collaborate, we need the retailers in the system to have a reason. Since the research that focuses on penalizing the firms through taxation or quotas is widely studied, we wanted to look at the issue from a different angle. The revenue scheme that we have used is through an environmentally sensitive customer. An environmentally sensitive customer purchases a product while taking the carbon footprint into consideration. Through this, a customer, that we can define as "the altruistic customer" or "the environmentally sensitive customer", makes the decision to purchase a product or not. It was shown that around 30% of the customers were willing to pay a price premium in The Global Sustainability Study conducted by Simon-Kucher & Partners among 10,000 people over 17 countries (Pope, 2022). Therefore, it is not uncommon for the firms to charge extra for the products that adhere to higher sustainability standards. According to studies in the areas of psychology, sociology, and consumer behavior, these environmentally sensitive customers are most likely well-educated young adults with high socioeconomic status (Fisher et al., 2012). The environmentally conscious consumers adopt sustainable practices for their

everyday activities such as opting to purchase green products while considering the carbon footprint of the product that they purchase (Sönmez, 2015). Thus, the consumer in our system is powerful enough to dictate a price based on how sustainable the product is.

With the desire to observe and increase the sustainability of the operations related to transportation through collaboration of firms, we consider the well known Joint Replenishment Problem (JRP) that is derived from the Economic Order Quantity (EOQ) policy. The EOQ enables a single retailer to minimize the total cost rate of replenishment. JRP is the problem where multiple items are considered at once to further benefit from economies of scale and share the costs of replenishment. In JRP, the decision variables are the order cycle time of the system consisting of multiple items that determines how frequent there will be an order placed, and the integer multipliers for all retailers that enable these retailers to decide how frequently they will participate in the replenishment of the coalition. Since we are considering consumers that are able to manipulate the market, if we are to minimize the costs or maximize the profits, we would be essentially taxing the firms based on their carbon emissions. Therefore, in order to differentiate the environmentally sensitive customer from a government taxation, we shall maximize the profit/cost ratio.

With the discussed motivation and aim, we shall first introduce the related literature in Chapter 2, provide details regarding the environment and the model that we are considering in Chapter 3, discuss our preliminary and main structural results in chapters 4 and 5, present the special cases in Chapter 6, conduct numerical analysis in Chapter 7, point out the societal impact that the thesis may have in Chapter 8, and finally make our concluding remarks in Chapter 9.

CHAPTER 2

LITERATURE REVIEW

In this chapter, we review the literature related to our research. The existing literature will be summarized in sub-topics of (i) joint replenishment problem, (ii) carbon emissions in inventory models (iii) and novel objective functions.

2.1 Joint Replenishment Problem

Joint replenishment settings enable the retailers replenish multiple products at once. By benefiting from economies of scale, sharing the fixed costs of ordering, the solution to the joint replenishment problems provides a minimum total cost rate that is always less than the sum of individual minimum cost strategies.

The fixed costs of ordering are divided into two: the major cost of ordering that is incurred by every order, and the fixed cost per item (Sengupta et al., 2017; Salameh et al., 2014).

The JRP may be considered for both deterministic, and stochastic demands.

The problems that have stochastic demands are called stochastic joint replenishment problems. Stochastic joint replenishment was first considered by (Balintfy, 1964) where the continuous review policy was discussed. In continuous review policies, it is assumed that the inventory level in the system is always known. In this model, there is a stochastic demand. This policy determines a lower bound of inventory for each item, and once an item hits the lower bound, an order is placed to replenish the item to a pre-determined inventory level. Based on another lower bound per item, some of the other items are replenished to their respective inventory levels as well. With this method, the existence of an optimum reorder range is proven.

Because of the complexity of this policy due to its requirement of high computational power, there have been many heuristics in stochastic JRP (Silver, 1965 1974; Federgruen et al., 1984; Schultz & Johansen, 1999). There have also been alternative policies such as accumulating the orders up to a certain level, and then replenishing each item as in Balintfy (1964) (Nielsen & Larsen, 2005). Another policy checks the accumulated orders and the time elapsed after the last order to check for the replenishment needs (Özkaya et al., 2006).

2.1.1 Deterministic Joint Replenishment Problem

Inventory models that have deterministic demands are generally either controlled by a periodic, or continuous review policy.

Periodic review policy models check the inventory of the products in the system in periods and the inventory levels are not constantly known. On the other hand, we shall be assuming a continuous review policy where the inventory levels are known at any given time.

In order to understand the JRP that we shall be considering, we need to first be able to understand the economic order quantity setting, specifically its emergence, structure, and objective. EOQ setting has been first introduced in

1913 by Ford Harris, and has been widely studied ever since (Erlenkotter, 1990). Although there is a stream of literature focusing on a stochastic demand that varies overtime, we shall only be considering a demand that is stable throughout the infinite planning horizon.

The EOQ total cost rate formula consists of three different sources of cost that the purchasing party shall consider. These are per unit cost of purchasing, holding cost, and fixed cost of ordering. Per unit cost of purchasing is simply the price of the product. Holding cost is the cost associated with keeping and maintaining the said product in the warehouse as an inventory. Finally, the fixed cost of ordering is the cost that is directly incurred by ordering that exists irrespective of number of units. In this model, the purchasing party aims to minimize its total cost rate. When c is the per unit cost, h is the holding cost, D is the constant demand, and K is the fixed cost of ordering, with an order cycle denoted by T , the total cost rate is stated as follows:

$$TC(T) = \frac{K}{T} + \frac{hDT}{2} + cD \quad (2.1)$$

The EOQ setting assumes that the demand is known and constant at D , there are no shortages, the lead time of the delivery is zero, the order batches arrive at once, the shelf-life of the items are infinite, the cost parameters are not time-dependent, and there is an infinite planning horizon (Sengupta et al., 2017).

The EOQ policy assumes the ability of the system to check the inventory continuously and makes an order whenever the inventory becomes zero. The total cost rate is minimized and an optimal quantity is obtained under the given conditions.

The total cost rate of the deterministic joint replenishment with different items $i = 1, 2, \dots, V$ can be stated as follows where the newly added k_i is the fixed cost per item, and $n_i \forall i \in L$ is the integer multiplier for each item that determines

the individual order interval where $\mathbf{n} = \{n_i \forall i \in L\}$:

$$TC(\mathbf{n}, T) = \frac{K}{T} + \sum_i^V \left(\frac{k_i}{n_i T} + \frac{h_i D_i n_i T}{2} + c_i D_i \right) \quad (2.2)$$

We use a cyclic joint replenishment policy where there is an order every T time units, and any retailer $i \in L$ decides to be a part of this order every n_i 'th replenishment. This is denoted as the $(T, n_1, n_2, \dots, n_V)$. The optimal policy is found by minimizing the total cost rate over these introduced variables.

To the extent of our knowledge, the first time the deterministic JRP was considered is in the book *Inventory Control* (Starr & Miller, 1962). There has been many iterative near optimal solutions that aim to tackle the problem joint replenishment. However, they have all worked for relatively small number of items and deemed not usable for practical problems (Starr & Miller, 1962; Jackson et al., 1985; Roundy, 1985).

S. K. Goyal (1974) found the first algorithm that finds the optimal solution. He first determines the minimum and maximum values that the frequency can take under a given number of items. Then, orders the items from the most frequent to the least frequent. Each item is considered one by one in the calculated range of integer values, and the frequency that gives the lowest value for the multiplication of the holding costs and fixed costs is selected. However, this algorithm only works for a small number of items as well.

As the optimal solution worked for a small number of items, the heuristics were the main focus point in the research (Sengupta et al., 2017).

Silver (1976) provided a simple approach that results in a near-optimal solution. The objective is to minimize the total costs by deciding on the time interval between two orders, and the integer multipliers of the order cycle time for each retailer. Silver first finds the optimal value of the order cycle time for a given set of item-specific integer multipliers. Then, he inserts this value in the cost function to determine the set of integer multipliers that minimizes this expression. Ignoring the integer constraints, Silver finds the item that is

replenished with every given order. Then, determining this newly found item-specific order interval as the one that is equal to 1, and deriving a relationship between different order intervals that minimize the function, other multipliers are found and rounded to the nearest integer. Although this does not provide an optimal solution, it has laid the foundation to many heuristics as it provided lower and upper bounds for the values while obtaining relationships between the integer multipliers that enable ordering the values.

S. Goyal & Belton (1979), Kaspi & Rosenblatt (1983), Eynan & Kropp (1998), Fung & Ma (2001)) and many others have used and improved on the algorithm of Silver (1976) algorithm in the later years.

2.2 Carbon Emissions in Inventory Models

According to Dekker (2012) modern society requires the sustainability of corporations. The authors deem operations research as a possible contributor to sustainability and green logistics by its various dimensions and areas of research. They emphasize that one technique to increase the efficiency of the transportation operations is the consolidation of shipments that is achieved through collaboration between items that have smaller demands. Another important issue that the authors emphasize is the selection of performance metrics that we shall be considering when searching the optimal solution through a novel objective function, and applying different policies in carbon allocation.

In recent years, classical problems have been re-considered taking this relatively new concern of sustainability into account. The companies who are looking to improve their competitiveness in today's market by increasing logistics efficiency to gain economic, environmental, and social sustainability consider logistics collaboration to be one of the most effective mechanisms (Aloui et al., 2021; Vanovermeire et al., 2014)

The topic of horizontal collaboration and sustainability is getting more popular in recent years (Aloui et al., 2021). Aloui et al. (2021) address five potential gaps in the literature of sustainable operations management. One of them is the fact that there is less attention paid to inventory management, and another one is that almost all of the existing literature focuses on suppliers collaborating but not retailers. We aim to fill the literature in this area that is mentioned.

Horizontal collaboration, that is the activity of collaboration within members at different levels in a supply chain, could also be viewed as a type of resource pooling since the firms benefit from economies of scale. Pan et al. (2019)) and Ballot & Fontane (2010) found that pooling sources in supply chains is an effective way of decreasing carbon emissions through investigating the French retail system. Bonney & Jaber (2011) found that inventory planning is crucial to the environmental performance of the company by focusing on an EOQ model with carbon emissions.

Moreover, X. Chen et al. (2019) investigate an EOQ model and observe that by modifying the order quantity, carbon emission can be effectively reduced without increasing the costs too much.

Let us briefly explain the most prominent carbon emission control mechanisms. Carbon taxation is simply penalizing the firm directly by the amount of carbon emitted. Strict carbon caps disallow firms to exceed a certain level of emissions. Flexible carbon caps are the ones which the firm can exceed provided that they will be penalized by doing so. In some settings, the firms are rewarded for being below a cap. Carbon offsets are the projects that a firm may invest in to neutralize the negative effects of their businesses and gain extra carbon caps. Benjaafar et al. (2012) investigate a single firm with fixed, purchasing, holding, and shortage costs and emissions in an EOQ which is the same as the structure that we use in our analysis. The objective of the model is to find the optimal order cycle time that provides the minimum total cost rate. They implement different carbon control techniques such as carbon cap, carbon taxation, a

non-strict cap and carbon-offsets. Additionally, they investigate the same scenarios with the added techniques by allowing firms to benefit from JRP. They find that the greatest value created in terms of cost and emission minimization is when the firms collaborate. The difference that they have with our thesis is the fact that they have a different objective function, they do not assume the effect of the environmentally sensitive consumer, as well as the difference that various allocation policies make.

2.2.1 Carbon Emissions Allocation

Since multiple retailers are collaborating when solving a JRP, the carbon emissions need to be allocated to the respective items somehow. There has been many propositions on how to allocate the carbons emitted by the system to individual items or retailers.

Zeng et al. (2022) consider a class of inventory games in a JRP setting where the firms are trying to minimize their joint inventory cost through collaboration. The major cost of the coalition is allocated to a retailer i through the ratio between the square of the minimum cost of item i calculated through an EOQ, and sum of all items' EOQ costs. The authors also consider a JRP with carbon taxation, and strict and flexible carbon caps.

Elomri et al. (2012) assume an environment where the savings of the coalition is allocated to each member based on the profit rate of the coalition. The profit rate of the coalition is the difference between the sum of individual minimum costs and the total cost of the coalition divided by the sum of individual minimum costs. The algorithm first find the coalition that has the highest profit rate among all retailers, set these aside and iterate. Profit rate of the coalition multiplied by the demand of retailer i gives us the total savings of that retailer by forming a coalition.

Sunar & Plambeck (2016) work on the allocation of emissions among co-products and its implications for procurement and climate policy. Co-products are defined as the multiple outputs apart from the primary product as a result of the same production process. There are three allocation protocols that are considered. Value-based allocation considers the monetary value of the products. Mass-based allocation considers the mass of the product. Finally, the system expansion considers per unit emission of the whole product and extracts per unit emission for individual co-products as they are the ones that would have caused carbon emissions if they were produced but not obtained as a co-product.

2.2.2 Environmentally Sensitive Demand

According to Kotler & Armstrong (2010), a purchasing activity has five steps. These are, the recognition of the problem, the search for information about solutions, the evaluation of different solutions, the decision, and post purchase behavior. At the evaluation phase, the customers compare and analyze the products based on different attributes. At this stage, we shall assume that the consumer will decide on the price that they are willing to pay for a product based on its sustainability performance.

A pricing model is a way to determine what prices to charge for a company's products and/or services. The model typically takes into account factors such as the cost of the product, the type of the product, the performance, and the value of the product to the customer. With our approach, the sustainability of the company can be viewed as both the value and the performance of the product. The sustainability that creates value to the customer, and the sustainability performance of the firm are determines the price in this system. Performance based pricing has two main advantages. One is that it incentivizes the seller to not underperform. The other is that the buyer will have the sense of having the

chance to pay what the product deserves as there is a specifically laid out arrangement (Shapiro, 2002).

(Bunch et al., 1993) investigate how much more the customers are willing to pay for a car that emits less carbon. It is found that Californians are willing to pay \$9,000 more on average, and the New Yorkers are willing to pay an average green premium of \$6,000. Drozdenko et al. (2011) document that 56% of the consumers in the United States and Canada would be unwilling to pay more for an electric or hybrid vehicle.

Griskevicius et al. (2010) investigate the drivers behind an environmentally conscious consumer and find that most of the purchasing activity is the results of having a better public image. Chua et al. (2010) find a similar result among hybrid car drivers who mostly value the "green image" that they have in public. However, it seems that although the intentions are questionable, people are becoming more environmentally conscious. Studies have shown that the consumers are willing to pay a premium price for a green product since they have a gain out of purchasing the more sustainable product. Specifically, 75% of the consumers in the European Union indicated that they would be willing to pay this premium (Schlegelmilch et al., 1996). Chitra (2007) further states that with an increased environmental awareness, people become more willing to pay the premium.

Adenso-Díaz et al. (2017) conduct an analysis on the effects of dynamic pricing of perishable products on revenue and waste. The demand of a product decreases as the age of the product approaches to the maximum shelf life. Moreover, price is another function of age and maximum shelf life. The authors use a bi-variate objective function to maximize the profits while minimizing the waste products. Through pricing, they aim to create more sustainable businesses, that is similar to our case.

2.3 Novel Objective Function

Recently, there are papers in operations management literature that consider novel objective functions.

The novelty that is to be added to the literature heavily relies on the structure of the objective function that is quite different from minimizing costs or maximizing profits. This way, we are able to treat the pricing scheme as something different than simply taxing the carbon emissions.

The profit percentage is the ratio of the total profit rate to the total cost rate. The return that the company obtains as a result of its costs are measured. As a result, we obtain the profit per unit cost incurred. The profit/cost ratio is a direct indicator of the operational efficiency of a firm as it is dependent on the ability of the firm to obtain as much profit as it can with every unit that it spends.

According to Pando et al. (2020), maximizing the profit to cost ratio handled in inventory is the same as maximizing the return on investment (ROI). Since we are maximizing the ratio of the total profit rate to total cost rate, that are the system-wide profits and costs given an infinite time horizon, profit to cost ratio maximization, and therefore, the ROI maximization relates to our problem as well.

Schroeder & Krishnan (1976) are the first to consider an ROI maximization in an inventory problem as an alternative to EOQ policy. Also, Rosenberg (1991) makes benefit of both ROI and profit maximization in inventory management and compares the results from two policies. Trietsch (1995) suggests that using the maximization of ROI instead of EOQ could potentially help companies better deal with market volatility.

Revelle & Laporte (1996) replace maximizing ROI as the objective in industrial applications. Otake et al. (1999) use the ROI to determine the optimal policy when there exists the possibility to invest in setup operations. C.-K. Chen

(2001) maximizes ROI to determine the optimal quantity, price, and quality. More recently, C.-K. Chen & Liao (2014) used the maximization of ROI in a setting where there are deteriorating items.

Since we want to be able to observe a setting with an environmentally sensitive customer, we had to distinguish from carbon taxation literature and minimizing costs or maximizing profits would result in this. Therefore, through our objective function, not only do we diverge from the existing literature on carbon emissions, but consider a novel objective function that values the efficiency of the company in creating profits by every unit cost that it incurs.

CHAPTER 3

THE MODEL

In this chapter, we first introduce our basic setting and the optimization model. Further, we provide a list of notation, and introduce carbon allocation policies that we use in the numerical analysis.

3.1 Basic Setting

In our analysis, we assume that there are V number of items that are to be jointly replenished. The items are denoted as $i \in L$ where L is the set of all items.

The demand for the items, denoted as D_i for all $i \in L$ are deterministic and constant. The leadtime of the orders is assumed to be zero, and shortages are not allowed.

We can break down our cost structure into two main parts, the fixed costs, and the variable costs. There are two kinds of fixed costs. The major fixed cost,

denoted as K , is the fixed cost of the replenishment that is incurred no matter how many items are ordered or how many items are replenished. The minor fixed cost, denoted as k_i for all $i \in L$, is incurred whenever an item is included in a replenishment.

The variable costs are holding and per unit costs. Holding costs, denoted as h_i for all $i \in L$ are the material holding costs that are associated with maintaining the inventory. Per unit costs, denoted as c_i for all $i \in L$, are costs per item purchased.

The carbon emissions that we consider have a similar structure to the total cost rate function 3.1. The total sources of carbon emissions are a major fixed K_e , item-specific minor fixed k_{ei} for all $i \in L$, holding h_{ei} for all $i \in L$, and per unit c_{ei} for all $i \in L$ emissions.

The revenue structure that we consider is one where there is an emission penalty, β_i for all $i \in L$, imposed on the emission allocated to item i . Price of the items before the penalty deduction is denoted as p_{0i} .

The carbon emission per item i for any $i \in L$, $em_i(\mathbf{n}, T)$, is allocated through our allocation policies. We introduce these policies in detail in section 3.4.

There are three parameters that help us determine the allocated carbon in each policy. S_i enables us to allocate the total emission of the joint replenishment to item i . M_i helps us allocate the major fixed emission of the joint replenishment specifically. Calculating the S_i and M_i , we use the demand of item i , and the distance of item i to the supplier, that is denoted as d_i . $L_i \subseteq L$ enables us to choose which items' emissions to include to the allocation calculation of item i .

There are four policies that we use in the numerical analysis in allocating the carbon emissions to items. The first policy is to allocate the total emission of the joint replenishment based solely on the demand of the item. The second policy allocates the item-specific minor fixed, holding, and per unit emissions only to respective items and allocate the major emission using the demand of the item. The third policy allocates the total emission to an item weighing the

distance covered for the item, denoted as d_i for all $i \in L$, by the demand of that item. The fourth policy allocates the item-specific emissions such as the minor fixed, holding, and per unit emissions to respective items and only uses distance weighted demand when allocating the major fixed emission.

The ordering policy is that there is going to be an order at every T time units where any item $i \in L$ will be replenished at every n_i 'th order. T is the order cycle time, and n_i for all $i \in L$ is the item-specific order interval. Moreover, we let $\mathbf{n} = \{n_1, n_2, \dots, n_V\}$ and denote our policy structure as (T, \mathbf{n}) .

3.2 Optimization Model

In this section, we define the optimization problem that we consider throughout the thesis. The total cost rate of the system is given as follows.

$$TC(\mathbf{n}, T) = \frac{K}{T} + \sum_{i \in L} \left(\frac{k_i}{n_i T} + \frac{h_i D_i n_i T}{2} + c_i D_i \right) \quad (3.1)$$

Similarly, the total emission rate of the system is as follows.

$$em(\mathbf{n}, T) = \frac{K_e}{T} + \sum_{i \in L} \left(\frac{k_{ei}}{n_i T} + \frac{h_{ei} D_i n_i T}{2} + c_{ei} D_i \right) \quad (3.2)$$

Total revenue rate of the system, $r(\mathbf{n}, T)$, is calculated by the following function:

$$r(\mathbf{n}, T) = \sum_{i \in L} (p_{0i} D_i - \beta_i em_i(\mathbf{n}, T)) \quad (3.3)$$

where $em_i(\mathbf{n}, T)$ is the emission rate allocated to retailer i . The specific rules for determining the emission rate allocated to each retailer (S_i , M_i and L_i) are discussed separately in section 3.4. Emission that is allocated to retailer i is of the following form:

$$em_i(\mathbf{n}, T) = S_i \left[\frac{M_i K_e}{T} + \sum_{j \in L_i} \left(\frac{k_{ej}}{n_j T} + \frac{h_{ej} D_j n_j T}{2} \right) \right] \quad (3.4)$$

The total emission rate of the system is as follows:

$$em(\mathbf{n}, T) = \sum_{i \in L} em_i(\mathbf{n}, T) \quad (3.5)$$

The total profit rate for the system is as follows:

$$\pi(\mathbf{n}, T) = r(\mathbf{n}, T) - TC(\mathbf{n}, T) \quad (3.6)$$

In our analysis we maximize the ratio of the profit rate to the total cost rate as follows:

$$\max_{(\mathbf{n}, T)} \frac{\pi(\mathbf{n}, T)}{TC(\mathbf{n}, T)} \quad (3.7)$$

Observing that the above function can be written as:

$$\frac{\pi(\mathbf{n}, T)}{TC(\mathbf{n}, T)} = \frac{r(\mathbf{n}, T)}{TC(\mathbf{n}, T)} - 1$$

maximizing the ratio of the profit rate to the total cost rate is the same as maximizing the ratio of the total revenue to the total cost rate. Therefore, we consider the maximization of the ratio of the total revenue rate to the total cost rate throughout the analysis as follows:

$$\max_{(\mathbf{n}, T)} z(\mathbf{n}, T) = \max_{(\mathbf{n}, T)} \frac{r(\mathbf{n}, T)}{TC(\mathbf{n}, T)} \quad (3.8)$$

The reason that we are considering this objective function is due to our need to distinguish ourselves from the carbon taxation literature. If we were to simply maximize the total profit rate or minimize the total cost rate, we would be treating the deduction from the price the same as a taxation imposed by the government. However, we want to have an environmentally sensitive customer imposing its power over the firms. That is why, we maximize the ratio of the total revenue rate to the total cost rate, making the price deduction become only part of the total revenue rate.

Throughout the analysis, we make the following assumptions.

Assumption 1. *Each retailer has a non-negative revenue rate, i.e.*

$$\forall i \in L : p_{0i}D_i \geq \beta_i em_i(\mathbf{n}, T)$$

Assumption 2. *The values of item-specific order intervals do not have to be integers, that is $\forall i \in L : n_i \in R^+ \setminus (0, 1)$.*

3.3 List of Notation

In this section, we list the notation that we use throughout the thesis for convenience.

T = Order cycle time of the joint replenishment.

n_i = Integer multiplier of the order cycle time for retailer i .

L = The set containing all members of the system.

L_i = The set of retailers included in the emission allocation calculation for retailer i .

\mathbf{n} = The vector that contains all n_i for $i \in L = \{1, 2, \dots, V\}$

p_{0i} = Price of retailer i without any emission penalty.

$em(\mathbf{n}, T)$ = The function that gives the total emission rate of the system.

$em_i(n_i, T)$ = Total emission rate of the retailer i .

β_i = The coefficient of emission by which per unit price decreases for retailer i .

S_i = The coefficient that enables us to allocate whole emission of the coalition to retailer i at once.

M_i = The coefficient that enables us to allocate the major fixed emission to retailer i .

k_i = Fixed cost of ordering for retailer i

h_i = Holding cost per unit for retailer i

c_i = Purchasing cost per unit for retailer i

d_i = Distance of retailer i to the supplier

k_{ei} = Fixed emission of ordering for retailer i

h_{ei} = Holding emission per unit for retailer i

c_{ei} = Purchasing emission per unit for retailer i

3.4 Carbon Allocation Policies

There are four allocation methods that we use in the numerical analysis. Here, we provide a brief introduction to each of these policies.

3.4.1 Policy I

The decision that is to be made is directly affected by how the retailers will allocate the total carbon emission rate of the coalition. The first policy is a basic one. We simply allocate the total emission rate of the system based on the demands of each retailer. The function for the emission allocated to retailer i becomes:

$$em_i(\mathbf{n}, T) = \frac{D_i}{\sum_{i \in L} D_i} \left(\frac{K_e}{T} + \sum_{j \in L} \left(\frac{k_{ej}}{n_j T} + \frac{h_{ej} D_j n_j T}{2} + c_{ej} D_j \right) \right) \quad (3.9)$$

Observe that our allocation coefficients S_i and M_i , and subset L_i under this policy becomes as follows for all $i \in L$:

$$S_i = \frac{D_i}{\sum_{i \in L} D_i}$$

$$M_i = 1$$

$$L_i = L$$

3.4.2 Policy II

The second policy allocates the emission caused by the minor fixed, holding, and per unit sources to the respective retailers, and divide the fixed emission of ordering among the retailers based on their individual demands. Then, the

emission allocated to retailer i becomes as follows:

$$em_i(\mathbf{n}, T) = \frac{D_i}{\sum_{i \in L} D_i} \frac{K_e}{T} + \frac{k_{ei}}{n_i T} + \frac{h_{ei} D_i n_i T}{2} + c_{ei} D_i \quad (3.10)$$

Observe that our allocation coefficients S_i and M_i , and subset L_i for this policy becomes as follows for all $i \in L$:

$$S_i = 1$$

$$M_i = \frac{D_i}{\sum_{i \in L} D_i}$$

$$L_i = \{i\}$$

3.4.3 Policy III

The actual transportation activity is one of the major sources of carbon emissions, and therefore, we need to consider the distance that the supplier is expected to cover. Assuming that the delivery is made to a retailer's location, and that the distance covered by the supplier increases with each retailer joining a coalition, additional retailers will cause additional distance travelled.

Moreover, with the increasing quantity, the resulting carbon emissions would increase. Therefore, we are hereby considering an allocation where the distance covered for the retailer is weighted by the demand of that particular retailer.

The structure is similar to that of Policy I in terms of allocating the emission of the system directly using a ratio. The resulting carbon emission allocated to retailer i then becomes:

$$em_i(\mathbf{n}, T) = \frac{d_i D_i}{\sum_{i \in L} d_i D_i} \left(\frac{K_e}{T} + \sum_{j \in L} \left(\frac{k_{ej}}{n_j T} + \frac{h_{ej} D_j n_j T}{2} + c_{ej} D_j \right) \right) \quad (3.11)$$

Observe that our allocation coefficients S_i and M_i , and subset L_i for this policy becomes as follows for all $i \in L$:

$$S_i = \frac{d_i D_i}{\sum_{i \in L} d_i D_i}$$

$$M_i = 1$$

$$L_i = L$$

3.4.4 Policy IV

Similar to Policy II, we allocate the individual emission caused by minor fixed, holding, and per unit sources to each retailer and only allocate the fixed emission of ordering to the retailers through the distance weighted demand method as follows:

$$em_i(\mathbf{n}, T) = \frac{d_i D_i}{\sum_{i \in L} d_i D_i} \frac{K_e}{T} + \frac{k_{ei}}{n_i T} + \frac{h_{ei} D_i n_i T}{2} + c_{ei} D_i \quad (3.12)$$

Observe that our allocation coefficients S_i and M_i , and subset L_i for this policy becomes as follows for all $i \in L$:

$$S_i = 1$$

$$M_i = \frac{d_i D_i}{\sum_{i \in L} d_i D_i}$$

$$L_i = \{i\}$$

CHAPTER 4

PRELIMINARY STRUCTURAL RESULTS

4.1 Structural Results On Total Cost Rate

In this section, we provide some structural results regarding the total cost rate function.

4.1.1 Structural Results On Total Cost Rate With Respect To Common Order Intervals

Define the following notation for conciseness:

$$B_1^2 = \sum_{i \in L} \frac{h_i D_i n_i}{2}$$

$$B_2^2 = \sum_{i \in L} c_i D_i$$

$$B_3^2 = K + \sum_{i \in L} \frac{k_i}{n_i}$$

We know that $\forall j \in \{1, 2, 3\} : B_j^2 > 0$ since all item-specific parameters are

positive. Using the above notation, we can state our total cost rate as follows:

$$TC(\mathbf{n}, T) = B_1^2 T + B_2^2 + B_3^2 T^{-1} \quad (4.1)$$

Lemma 1. *For a given vector \mathbf{n} , total cost rate is a strictly positive convex function in T .*

Proof. The second order derivative of the total cost rate with respect to T is as follows:

$$\frac{d^2 TC(\mathbf{n}, T)}{dT^2} = 2B_3^2 T^{-3} \quad (4.2)$$

Since the above function has all positive parameters, for any $T > 0$, the second order derivative is positive. Thus, the total cost rate is a strictly positive convex function as we have previously assumed $\forall i, j \in \{1, 2, 3\} : B_i^j$ to be positive. \square

Lemma 2. *The common order interval value that minimizes the total cost rate is as follows:*

$$T^{TC*} = \sqrt{\frac{B_3^2}{B_1^2}}$$

Proof. The first order derivative of the total cost rate is as follows:

$$\frac{\partial TC(\mathbf{n}, T)}{\partial T} = B_1^2 - B_3^2 T^{-2} \quad (4.3)$$

Therefore, the first order condition of the total cost rate becomes:

$$B_1^2 = B_3^2 T^{-2}$$

Solving for T , the result follows. \square

4.1.2 Structural Results On Total Cost Rate With Respect To Item Specific Order Intervals

Lemma 3. For a given order cycle time T , and given vector

$\hat{n} = \{n_i : \forall i \in L \setminus p\}$ the total cost rate is strictly positive convex in an n_p for any $p \in L$.

Proof. The second order derivative of the total cost rate with respect to n_p is stated as follows:

$$\frac{\partial^2 TC(\mathbf{n}, T)}{\partial n_p^2} = \frac{k_p}{n_p^3 T} \quad (4.4)$$

Since all of the parameters are positive, for a $T > 0$ and n_p , the second order derivative is always positive. Therefore, the total cost rate is a strictly positive convex function. \square

Lemma 4. The item-specific order interval that minimizes the total cost rate, denoted as n_r^{TC*} is as follows:

$$n_r^{TC*} = \sqrt{\frac{2k_r}{h_r D_r T}}$$

Proof. Let us consider the following first order condition of the cost rate with respect to n_r :

$$\frac{\partial r(\mathbf{n}, T)}{\partial n_r} = -\frac{k_r}{n_r^2 T} + \frac{h_r D_r T}{2} = 0$$

In order for the above statement to be equal to zero, we need the following equality:

$$\frac{k_r}{n_r^2 T} = \frac{h_r D_r T}{2}$$

Solving for n_r , we obtain the result. \square

4.2 Structural Results On Total Revenue Rate

In this section, we shall provide some structural results regarding our total revenue rate function.

4.2.1 Structural Results On Total Revenue Rate With Respect To Common Order Intervals

Defining the following notation for conciseness:

$$\begin{aligned} B_1^1 &= \sum_{i \in L} \beta_i S_i \left(\sum_{j \in L_i} \frac{h_{ej} D_j n_j}{2} \right) \\ B_2^1 &= \sum_{i \in L} p_{0i} D_i - \beta_i S_i \left(\sum_{j \in L_i} c_{ej} D_j \right) \\ B_3^1 &= \sum_{i \in L} \beta_i S_i \left(M_i K_e + \sum_{j \in L_i} \frac{k_{ej}}{n_j} \right) \end{aligned}$$

By Assumption 1, we already know that B_2^1 is positive. Additionally, since emission parameter are positive, we know that $\forall j \in \{1, 2, 3\} : B_j^1 \geq 0$. Using the above notation, we can denote our total revenue rate as follows:

$$r(\mathbf{n}, T) = -B_1^1 T + B_2^1 - B_3^1 T^{-1} \quad (4.5)$$

Lemma 5. *For a given vector \mathbf{n} , the total revenue rate is a non-negative concave function in T .*

Proof. The second order derivative of the total revenue rate with respect to T is:

$$\frac{d^2 r(\mathbf{n}, T)}{dT^2} = -2B_3^1 T^{-3} \quad (4.6)$$

Since all of the parameters are positive, for a $T > 0$, the second order derivative is always negative. Therefore, the results follows. \square

4.2.2 Structural Results On Total Revenue Rate With Respect To Item Specific Order Intervals

Lemma 6. For a given order cycle time T , and given vector

$\hat{n} = \{n_i : \forall i \in L \setminus p\}$ the total revenue rate is non-negative concave in n_p for any $p \in L$.

Proof. The second order derivative of the total revenue rate with respect to an arbitrary n_p for some $p \in S$ is as follows:

$$\frac{\partial^2 r(\mathbf{n}, T)}{\partial n_p^2} = \sum_{i \in L} -\beta_i S_i\left(\frac{k_{ep}}{n_p^3 T}\right) \quad (4.7)$$

Since the above function has all positive parameters, for any $T > 0$ and n_p , the second order derivative is negative. Thus, the revenue rate is a non-negative concave function. \square

Lemma 7. The item specific order interval that maximizes the total revenue rate, denoted as n_r^{r*} is as follows:

$$n_r^{r*} = \sqrt{\frac{2k_{er}}{h_{er}D_rT}}$$

Proof. Let us consider the following first order condition of the cost rate with respect to n_r :

$$\frac{\partial r(\mathbf{n}, T)}{\partial n_r} = \sum_{i \in L} -\beta_i S_i\left(-\frac{k_{er}}{n_r^2 T} + \frac{h_{er}D_rT}{2}\right) = 0$$

In order for the above statement to be equal to zero, we need the following equality:

$$\frac{k_{er}}{n_r^2 T} = \frac{h_{er}D_rT}{2}$$

Therefore, solving for n_r , we obtain the result. \square

$$z = \frac{-B_1^1 T + B_2^1 - B_3^1 T^{-1}}{B_1^2 T + B_2^2 + B_3^2 T^{-1}}$$

CHAPTER 5

STRUCTURAL RESULTS ON THE OBJECTIVE FUNCTION

In this chapter, we present structural results on the objective function when the vector \mathbf{n} is given, common order cycle time T is given, .

5.1 Structural Results On The Objective Function With Respect To Common Order Interval

Here, we shall present structural results on the objective function when the vector \mathbf{n} is given. From Chandra (1972) we have:

Lemma 8. *(Chandra, 1972) For an $f(x)$ that is non-negative concave, and $g(x)$ that is strictly positive convex, $h(x) = \frac{f(x)}{g(x)}$ is a strong pseudoconcave function.*

By Lemma 1, we know that our total cost rate is a strictly positive convex function. By Lemma 5, we know that our total revenue rate is a non-negative

concave function. Therefore, by Lemma 8, our objective function is a strong pseudoconcave function in T . We state this results in the following Lemma.

Lemma 9. *For a given vector \mathbf{n} , the objective function is a strong pseudoconcave function in T .*

Since strong pseudoconcave functions share the property of concave functions stating that the value that makes the first order derivative equal to zero is the global optimum (Todd, 2003), we can search for the global optimum for T by setting the first order derivative of the objective function with respect to T equal to zero.

Define the following notation:

$$d = -(B_1^1 B_2^2 + B_2^1 B_1^2)$$

$$e = 2(B_3^1 B_1^2 - B_1^1 B_3^2)$$

$$f = B_3^1 B_2^2 + B_2^1 B_3^2$$

Lemma 10. *For a given vector \mathbf{n} , the optimal order cycle time has a unique, positive, real value given by:*

$$\frac{-e - \sqrt{e^2 - 4df}}{2d}$$

Proof. The first order derivative of the objective function with respect to T is stated as follows:

$$\begin{aligned} \frac{\partial z(\mathbf{n}, T)}{\partial T} = & \frac{(-B_1^1 + B_3^1 T^{-2})(B_1^2 T + B_2^2 + B_3^2 T^{-1})}{(B_1^2 T + B_2^2 + B_3^2 T^{-1})^2} \\ & - \frac{(-B_1^1 T + B_2^1 - B_3^1 T^{-1})(B_1^2 - B_3^2 T^{-2})}{(B_1^2 T + B_2^2 + B_3^2 T^{-1})^2} \end{aligned} \quad (5.1)$$

At optimality, we need the above function to be equal to zero. When we rearrange the statement and multiply it with T^2 at optimality, we obtain the following:

$$-(B_1^1 B_2^2 + B_2^1 B_1^2)T^2 + 2(B_3^1 B_1^2 - B_1^1 B_3^2)T + (B_3^1 B_2^2 + B_2^1 B_3^2) = 0 \quad (5.2)$$

Observe that the above function can be written as:

$$dT^2 + eT + f = 0$$

Since the above is a quadratic function, the roots are known and are as follows:

$$\hat{T}_1 = \frac{-e - \sqrt{e^2 - 4df}}{2d}$$

$$\hat{T}_2 = \frac{-e + \sqrt{e^2 - 4df}}{2d}$$

First, observing $e^2 - 4df$, we can see that the value is always positive since d is negative and f and e^2 are positive. Thus, we know that the roots are both real.

Moreover, we know that $\sqrt{e^2 - 4df} \geq e$ holds by the positivity of $-4df$.

Therefore, we know that $-e - \sqrt{e^2 - 4df}$ is negative. Since d is a negative parameter, \hat{T}_1 is a positive real root. Thus, the objective has a positive, real, and unique root that is given by \hat{T}_1 . \square

Lemma 11. *For a given vector \mathbf{n} , when T is at the optimal value, the objective function is concave in T .*

Proof. The first order derivative of the objective function with respect to T is as follows:

$$\frac{\partial z(\mathbf{n}, T)}{\partial T} = \frac{\frac{\partial r(\mathbf{n}, T)}{\partial T}}{TC(\mathbf{n}, T)} - \frac{z(\mathbf{n}, T) \frac{\partial TC(\mathbf{n}, T)}{\partial T}}{TC(\mathbf{n}, T)^2} \quad (5.3)$$

Let the following notation for conciseness:

$$z'_T = \frac{\partial z(\mathbf{n}, T)}{\partial T}$$

$$z''_T = \frac{\partial^2 z(\mathbf{n}, T)}{\partial T^2}$$

$$r'_T = \frac{\partial r(\mathbf{n}, T)}{\partial T}$$

$$r''_T = \frac{\partial^2 r(\mathbf{n}, T)}{\partial T^2}$$

$$TC'_T = \frac{\partial TC(\mathbf{n}, T)}{\partial T}$$

$$TC''_T = \frac{\partial^2 TC(\mathbf{n}, T)}{\partial T^2}$$

The second order derivative of the objective function with respect to T is as

follows:

$$z_T'' = \frac{r_T''}{TC(\mathbf{n}, T)} - \frac{r_T' TC_T'}{(TC(\mathbf{n}, T))^2} - \frac{z_T' TC_T' + z(\mathbf{n}, T) TC_T''}{TC(\mathbf{n}, T)} + \frac{z TC_T' TC_T'}{(TC(\mathbf{n}, T))^2} \quad (5.4)$$

Arranging the above statement, we obtain the following form for the second order derivative of the objective function with respect to T :

$$z_T'' = \frac{r_T'' - z(\mathbf{n}, T) TC_T'' - 2z_T' TC_T''}{TC(\mathbf{n}, T)}$$

At optimality, when $z_T' = 0$, the above statement becomes the following:

$$z_T'' = \frac{r_T'' - z(\mathbf{n}, T) TC_T''}{TC(\mathbf{n}, T)}$$

Stating the above equation explicitly, we get:

$$z_T'' = \frac{(-B_3^1 T^{-1}) - z(\mathbf{n}, T)(B_3^2 T^{-3})}{TC(\mathbf{n}, T)}$$

Observing the above statement, we can see that it is negative for a positive T , total cost rate, and objective function value. Since we know that there exists a positive, unique, real T by Lemma 10, our objective function is concave. \square

5.2 Structural Results On The Objective Function With Respect To Item-Specific Order Intervals

In this section, we present our structural results on the objective function when the common order interval T is given.

5.2.1 Optimality Results For A Single Item's Order Interval

In this section, we assume that in addition to the common order interval being given, all item-specific multipliers in vector \mathbf{n} except for one are given. A similar problem has been investigated by Berk & Ayas (2022) for a single item setting in the presence of operational emissions.

Lemma 12. *For a given order cycle time T , and given vector*

$\hat{n} = \{n_i : \forall i \in L \setminus p\}$ the objective function is strong pseudoconcave in n_p for any $p \in L$.

Proof. By Lemma 3, we know that the total cost rate is a strictly positive convex function in n_p . By Lemma 6, we know that the total revenue rate is a non-negative concave function in n_p . Therefore, by Lemma 8, we know that our objective function is a strong pseudoconcave function in n_p . \square

Since strong pseudoconcave functions have a global optimum when the first order derivative is equal to zero (Todd, 2003), we can search for the global optimum for n_p by setting the first order derivative of the objective function with respect to n_p equal to zero.

Define the following notation for conciseness:

$$\sum_{i \in L} \beta_i S_i = \beta S$$

$$TC_R = \frac{K}{T} + \sum_{i \in L \setminus p} \left(\frac{k_i}{n_i T} + \frac{h_i D_i n_i T}{2} + c_i D_i \right) + c_p D_p$$

$$TR_R =$$

$$\sum_{i \in L \setminus p} \left(p_{0i} D_i - \beta_i S_i \left(\frac{M_i K_e}{T} + \sum_{j \in L \setminus p} \left(\frac{k_{ej}}{n_j T} + \frac{h_{ej} D_j n_j T}{2} + c_{ej} D_j \right) \right) \right) + p_{0p} D_p - \beta_p S_p c_{ep} D_p$$

Using this notation, the functions the following functions can be stated as:

$$\frac{\partial r(\mathbf{n}, T)}{\partial n_p} = -\beta S \left(\frac{-k_{ep}}{n_p^2 T} + \frac{h_{ep} D_p T}{2} \right)$$

$$TC(\mathbf{n}, T) = \frac{k_p}{n_p T} + \frac{h_p D_p T}{2} + TC_R$$

$$r(\mathbf{n}, T) = \frac{-k_p}{n_p^2 T} + \frac{h_p D_p T}{2}$$

$$\frac{\partial TC(\mathbf{n}, T)}{\partial n_p} = TR_R - \beta S \left(\frac{k_{ep}}{n_p T} + \frac{h_{ep} D_p n_p T}{2} \right)$$

Further define the following notation:

$$\begin{aligned} a_p &= 2(\beta S k_{ep} T C_R + k_p T R_R) \\ b_p &= 2\beta S D_p T (k_{ep} h_p - k_p h_{ep}) \\ c_p &= -T^2 D_p (h_{ep} T C_R + h_p D_p T R_R) \end{aligned}$$

Lemma 13. *For a given order cycle time T , given vector $\hat{n} = \{n_i : \forall i \in L \setminus p\}$, the optimal n_p , that is n_p^* , has a unique, real, positive value given by:*

$$\frac{-b_p - \sqrt{b_p^2 - 4a_p c_p}}{2c_p}$$

Proof. The first order condition for optimality of an arbitrary n_p can be stated as follows:

$$\left\{ \frac{\partial r(\mathbf{n}, T)}{\partial n_p} \right\} [TC(\mathbf{n}, T)] = [r(\mathbf{n}, T)] \left\{ \frac{\partial TC(\mathbf{n}, T)}{\partial n_p} \right\} \quad (5.5)$$

Stating the function 5.5 using the notation that we have just introduced, we obtain the following equality:

$$\begin{aligned} \left\{ -\beta S \left(\frac{-k_{ep}}{n_p^2 T} + \frac{h_{ep} D_p T}{2} \right) \right\} \left[\frac{k_p}{n_p T} + \frac{h_p D_p T}{2} + T C_R \right] \\ = \left[\frac{-k_p}{n_p^2 T} + \frac{h_p D_p T}{2} \right] \left\{ T R_R - \beta S \left(\frac{k_{ep}}{n_p T} + \frac{h_{ep} D_p n_p T}{2} \right) \right\} \end{aligned} \quad (5.6)$$

Equalizing the denominators in the above statement at $2n_p^2 T$, we obtain the following first order condition:

$$\begin{aligned} \frac{2(\beta S k_{ep} T C_R + k_p T R_R) + 2\beta S D_p T (k_{ep} h_p - k_p h_{ep}) n_p}{2n_p^2 T} \\ - \frac{T^2 D_p (h_{ep} T C_R + h_p D_p T R_R) n_p^2}{2n_p^2 T} = 0 \end{aligned} \quad (5.7)$$

Using the previously introduced notation and letting the numerator be equal to zero, we obtain the following first order condition for n_p :

$$c_p n_p^2 + b_p n_p + a_p = 0$$

Then, the quadratic function has two roots that are given as follows:

$$\begin{aligned} n_{p1}^* &= \frac{-b_p + \sqrt{b_p^2 - 4a_p c_p}}{2c_p} \\ n_{p2}^* &= \frac{-b_p - \sqrt{b_p^2 - 4a_p c_p}}{2c_p} \end{aligned}$$

Observing the above roots, since c_p is negative and a_p is positive valued, $4a_p c_p$ is negative and $b_p^2 - 4a_p c_p$ is always positive. Thus, we have a real root here from both of these equations. Moreover, since $b_p^2 - 4a_p c_p > b_p^2$ with $4a_p c_p < 0$, we further have $\sqrt{b_p^2 - 4a_p c_p} \geq b_p$. Therefore, we know that n_{p1}^* is negative and n_{p2}^* is positive. Since we are searching for the optimal n_p in positive real values, we know that there is a positive, real, and unique root. So, we can denote

$$n_{p2}^* = n_p^*. \quad \square$$

Lemma 14. *For a given order cycle time T , given vector $\hat{n} = \{n_i : \forall i \in L \setminus p\}$, and when n_p for some $p \in L$ is at its optimal value, the objective function is concave.*

Proof. The first order derivative of the objective function with respect to an arbitrary n_p is as follows:

$$\frac{\partial z(\mathbf{n}, T)}{\partial n_p} = \frac{\frac{\partial r(\mathbf{n}, T)}{\partial n_p}}{TC(\mathbf{n}, T)} - \frac{z(\mathbf{n}, T) \frac{\partial TC(\mathbf{n}, T)}{\partial n_p}}{TC(\mathbf{n}, T)^2} \quad (5.8)$$

The second order derivative of the objective function with respect to n_p is as follows:

$$\frac{\partial^2 z(\mathbf{n}, T)}{\partial n_p^2} = \frac{\frac{\partial^2 r(\mathbf{n}, T)}{\partial n_p^2} - z(\mathbf{n}, T) \frac{\partial^2 TC(\mathbf{n}, T)}{\partial n_p^2} - 2 \frac{\partial z(\mathbf{n}, T)}{\partial n_p} \frac{\partial TC(\mathbf{n}, T)}{\partial n_p}}{TC(\mathbf{n}, T)} \quad (5.9)$$

Let the following notation for conciseness:

$$\begin{aligned} z'_{n_p} &= \frac{\partial z(\mathbf{n}, T)}{\partial n_p} \\ z''_{n_p} &= \frac{\partial^2 z(\mathbf{n}, T)}{\partial n_p^2} \\ r'_{n_p} &= \frac{\partial r(\mathbf{n}, T)}{\partial n_p} \\ r''_{n_p} &= \frac{\partial^2 r(\mathbf{n}, T)}{\partial n_p^2} \\ TC'_{n_p} &= \frac{\partial TC(\mathbf{n}, T)}{\partial n_p} \end{aligned}$$

$$TC''_{n_p} = \frac{\partial^2 TC(\mathbf{n}, T)}{\partial n_p^2}$$

Then, we can write the second order derivative of the objective function with respect to n_p as follows:

$$z''_{n_p} = \frac{r''_{n_p} - z(\mathbf{n}, T)TC''_{n_p} - 2z'_{n_p}TC''_{n_p}}{TC(\mathbf{n}, T)}$$

When n_p is at the optimal value, i.e. $z'_{n_p} = 0$, we obtain the following statement:

$$z''_{n_p} = \frac{r''_{n_p} - z(\mathbf{n}, T)TC''_{n_p}}{TC(\mathbf{n}, T)}$$

Stating the above equation explicitly, we get:

$$\frac{\partial^2 z(\mathbf{n}, T)}{\partial n_p^2} = \frac{\sum_{i \in L} -\beta_i S_i(\frac{k_{ep}}{n_p^3 T}) - z(\mathbf{n}, T)(\frac{k_p}{n_p^3 T})}{TC(\mathbf{n}, T)} \quad (5.10)$$

Observing the above equation, we can see that the second order derivative is negative for a positive T , n_p , total cost rate, and objective function value, which are previously assumed. Therefore, our objective function is concave when n_p is at its optimal value. \square

Lemma 15. *If $\frac{k_{er}}{h_{er}} = \frac{k_r}{h_r}$, for some $r \in L$, then the optimal value for n_r , is found by:*

$$n_r^* = \sqrt{\frac{2k_r}{h_r DT}} = \sqrt{\frac{2k_{er}}{h_{er} DT}}$$

Proof. Since we have assumed that $\frac{k_{er}}{h_{er}} = \frac{k_r}{h_r}$, we know from our Lemmas 4, and 7, that the value that minimizes both the total cost rate and the total revenue rate are equal to each other. Let us denote this value as follows:

$$n_r^{**} = \sqrt{\frac{2k_r}{h_r DT}} = \sqrt{\frac{2k_{er}}{h_{er} DT}}$$

Then, let us look at the first order derivative of the objective function and let

$n_r = n_r^{**}$ as follows:

$$\frac{\partial z(n_r = n_r^{**} | \mathbf{n} \setminus n_r, T)}{\partial n_r} = \frac{\frac{\partial r(n_r = n_r^{**} | \mathbf{n} \setminus n_r, T)}{\partial n_r}}{TC(n_r = n_r^{**} | \mathbf{n} \setminus n_r, T)} - \frac{r(n_r = n_r^{**} | \mathbf{n} \setminus n_r, T) \frac{\partial TC(n_r = n_r^{**} | \mathbf{n} \setminus n_r, T)}{\partial n_r}}{(TC(n_r = n_r^{**} | \mathbf{n} \setminus n_r, T))^2} \quad (5.11)$$

Observing the above statement, we know that the first order derivative of the revenue rate and the first order derivative of the total cost rate with respect to n_r are both zero. Therefore, the first order derivative of the objective function with respect to n_r becomes zero as well, satisfying the first order condition for optimality. Therefore, since n_r^{**} is a positive, unique, and real value, we can let:

$$n_r^* = \sqrt{\frac{2k_r}{h_r DT}} = \sqrt{\frac{2k_{er}}{h_{er} DT}}$$

□

5.2.2 Pairwise Results of Item-Specific Order Intervals

In this section, we try to derive a relationship between the elements of the vector \mathbf{n} .

Assumption 3. $\forall p \in L : \frac{k_{ep}}{k_p} \neq \frac{h_{ep}}{h_p}$.

Lemma 16. *Under Assumption 3, when both n_p and n_r are at their optimal values for some $p, r \in L$, the relationship between these two variables can be given by:*

$$n_p^2 = \frac{4(k_{ep}k_r - k_{er}k_p) - 2D_r(k_{ep}h_r - k_ph_{er})n_r^2T^2}{2D_p(k_rh_{ep} - k_{er}h_p)T^2 - D_pD_r(h_{ep}h_r - h_{er}h_p)n_r^2T^4}$$

Proof. The first order derivative of the objective function with respect to an

n_p for some $p \in L$ for a given T can be stated as follows:

$$z'_{n_p} = \frac{r'_{n_p} TC(\mathbf{n}) - TC'_{n_p} r(\mathbf{n})}{(TC(\mathbf{n}))^2}$$

At the optimal point, we know that the above equation should be equal to zero, which means that the numerator must be equal to zero. Setting the numerator equal to zero, we obtain the following statement:

$$r'_{n_p} TC(\mathbf{n}) = TC'_{n_p} r(\mathbf{n}) \quad (5.12)$$

Arranging the function 5.12, we know that the following first order condition holds:

$$\frac{r'_{n_p}}{TC'_{n_p}} = \frac{r(\mathbf{n})}{TC(\mathbf{n})}$$

The above statement for an arbitrary n_p also holds for some other arbitrary n_r for some $r \in L$ as well. Therefore, we know that the following equation holds at the optimality for each of them:

$$\frac{r'_{n_p}}{TC'_{n_p}} = \frac{r'_{n_r}}{TC'_{n_r}} \quad (5.13)$$

Here, we shall continue with the condition that Assumption 3 holds. The case where Assumption 3 does not hold was discussed in Lemma 15. Now, let us write down the first order derivatives of the revenue rate and total cost rate functions with respect to an arbitrary n_p for some $p \in L$ for a given T .

$$r'_{n_p} = \left[\sum_{i \in L} \beta_i S_i \right] \left(\frac{k_{ep}}{n_p^2 T} - \frac{h_{ep} D_p T}{2} \right) \quad (5.14)$$

$$TC'_{n_p} = -\frac{k_p}{n_p^2 T} + \frac{h_p D_p T}{2} \quad (5.15)$$

Inserting the above statements in 5.13, we can state the first order conditions as

follows explicitly:

$$\frac{\frac{k_{ep}}{n_p^2 T} - \frac{h_{ep} D_p T}{2}}{-\frac{k_p}{n_p^2 T} + \frac{h_p D_p T}{2}} = \frac{\frac{k_{er}}{n_r^2 T} - \frac{h_{er} D_r T}{2}}{-\frac{k_r}{n_r^2 T} + \frac{h_r D_r T}{2}}$$

Rearranging the above statement, we can obtain the following equation.

$$\begin{aligned} & -4(k_{ep}k_r - k_{er}k_p) + 2D_r(k_{ep}h_r - k_ph_{er})n_r^2 T^2 \\ & + 2D_p(k_r h_{ep} - k_{er} h_p)n_p^2 T^2 - D_p D_r (h_{ep} h_r - h_{er} h_p)n_p^2 n_r^2 T^4 = 0 \end{aligned} \quad (5.16)$$

Further rearranging, we can obtain the following relationship between n_p and n_r .

$$n_p^2 = \frac{4(k_{ep}k_r - k_{er}k_p) - 2D_r(k_{ep}h_r - k_ph_{er})n_r^2 T^2}{2D_p(k_r h_{ep} - k_{er} h_p)T^2 - D_p D_r (h_{ep} h_r - h_{er} h_p)n_r^2 T^4} \quad (5.17)$$

□

5.2.3 Joint Analysis of the Item-Specific Order Intervals

Here, we consider the possible joint concavity of the elements of the vector \mathbf{n} containing the integer multipliers of T for each retailer i when common order cycle time T is given.

Lemma 17. *For a given T , the members of the vector \mathbf{n} are jointly concave when all of them are at their respective optimal points.*

Proof. We have previously argued that the second order derivative of the objective function with respect to an arbitrary n_p for any $p \in L$ is negative in 5.10. Therefore, the following Hessian matrix for all $i = 1, 2, \dots, V$ is a negative

diagonal matrix.

$$\begin{bmatrix} z''_{n_1} & 0 & \cdots & 0 & 0 \\ 0 & z''_{n_2} & 0 & \cdots & 0 \\ \vdots & \cdots & z''_{n_3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & z''_{n_V} \end{bmatrix}$$

Since all the elements in the diagonal of the matrix are known to be negative, the matrix is negative definite, making the eigenvalues of the matrix negative. Since we can say that a function is concave if and only if the Hessian is negative semi-definite, and the negative definiteness also ensures negative semi-definiteness, the function is jointly concave for the members of the vector **n**. □

CHAPTER 6

SPECIAL CASES

6.1 Special Cases for the Common Order Interval Analysis

Here in this chapter, we discuss special cases and present the optimality analysis of the common order interval, treating vector \mathbf{n} as a given parameter.

Lemma 18. *When there is no holding costs or emissions in the system, the optimal order cycle time approaches infinity.*

Proof. Observe that when there is no holding costs or emissions, then, $B_1^1 = B_1^2 = 0$. Therefore, the objective function under these circumstances can be stated as:

$$z(\mathbf{n}, T) = \frac{B_2^1 - B_3^1 T^{-1}}{B_2^2 + B_3^2 T^{-1}}$$

The first order derivative of the objective function becomes:

$$\frac{\partial z(\mathbf{n}T)}{\partial T} = \frac{(B_3^1 T^{-2})(B_2^2 + B_3^2 T^{-1}) + (B_3^2 T^{-2})(B_2^1 - B_3^1 T^{-1})}{(B_2^2 + B_3^2 T^{-1})^2}$$

Setting the numerator of the function equal to zero, we obtain the following statement:

$$(B_3^1 B_2^2 + B_2^1 B_3^2) T^{-2} = 0$$

In order for the statement here to hold, observe that we need $T \rightarrow \infty$. The intuition here is that when there are no holding costs or emissions to be considered, the order cycle time is as large as possible since there are no downside to keeping an inventory and the only source of cost and emission is the fixed cost and emission of ordering. Thus, the system forces T to approach infinity in order not to incur this major cost or emission of ordering. \square

Lemma 19. *When we set per unit revenue, that is*

$p_{0i} D_i - \beta_i S_i (\sum_{i \in L_i} c_{ei} D_i) \forall i \in L$, and per unit costs, $c_i \forall i \in L$, equal to zero, the optimal T approaches infinity.

Proof. Observe that under these described circumstances, $B_2^1 = B_2^2 = 0$.

Rewriting the objective function under this scenario, we get:

$$z(\mathbf{n}, T) = \frac{-B_1^1 T - B_3^1 T^{-1}}{B_1^2 T + B_3^2 T^{-1}}$$

The first order derivative of the function becomes:

$$\frac{\partial z(\mathbf{n}, T)}{\partial T} = \frac{(-B_1^1 + B_3^1 T^{-2})(B_1^2 T + B_3^2 T^{-1}) - (B_1^2 - B_3^2 T^{-2})(-B_1^1 T - B_3^1 T^{-1})}{(B_1^2 T + B_3^2 T^{-1})^2}$$

Then, similarly setting the numerator of the above statement equal to zero, we get the following equality at optimality:

$$(B_3^1 B_1^2 - B_1^1 B_3^2) T^{-1} = 0$$

Then, we need $T \rightarrow \infty$ at optimality, of course assuming that $B_3^1 B_1^2 - B_1^1 B_3^2$ is not zero. The intuition here is that when there are no revenues gained from ordering inventory, the retailers delays purchasing as much as possible, forcing

to not operate. □

Lemma 20. *When there is no fixed costs or emissions associated with ordering, optimal order cycle time becomes zero.*

Proof. Observing Lemma 10, we can see that the numerator of the root that we have found becomes zero, making the optimal T equal to zero. The intuition here is that when there are no costs or emissions associated with ordering, the retailer wants to minimize its order cycle time so that it can minimize holding costs and emissions. Thus, it maximizes the frequency of the order cycle time, making it approach the lower boundary that we set. So, the optimal solution for the order cycle time approaches zero. □

6.2 Special Cases for Pairwise Relationship of Item-Specific Order Intervals

In this section, we study special cases of the relationship given in statement 5.17. We have four statements in equation 5.17 that we can set equal to zero, that are:

(i) $k_{ep}k_r - k_{er}k_p$

(ii) $k_{ep}h_r - k_p h_{er}$

(iii) $k_r h_{ep} - k_{er} h_p$

(iv) $h_{ep}h_r - h_{er}h_p$

Looking at these four different statements, we have different scenarios that we can generate.

6.2.1 Scenario I

Let us assume that we have the following equality that lets the statement (i) become equal to zero:

$$\frac{k_{ep}}{k_p} = \frac{k_{er}}{k_r}$$

Then, the relationship depicted in statement 5.17 between the n_p and n_r becomes as follows:

$$n_p^2 = \frac{-2D_r(k_{ep}h_r - k_ph_{er})n_r^2}{2D_p(k_rh_{ep} - k_{er}h_p) - D_pD_r(h_{ep}h_r - h_{er}h_p)n_r^2T^2} \quad (6.1)$$

6.2.2 Scenario II

Let us assume that we have the following equality that sets the statement (ii) become equal to zero:

$$\frac{k_{ep}}{k_p} = \frac{h_{er}}{h_r}$$

Then, our equality in 5.17 stating the the relationship between the n_p and n_r becomes as follows:

$$n_p^2 = \frac{4(k_{ep}k_r - k_{er}k_p)}{2D_p(k_rh_{ep} - k_{er}h_p)T^2 - D_pD_r(h_{ep}h_r - h_{er}h_p)n_r^2T^4} \quad (6.2)$$

6.2.3 Scenario III

Let us assume that we have the following equality that sets the statement (iii) become equal to zero.

$$\frac{k_{er}}{k_r} = \frac{h_{ep}}{h_p}$$

Then, our equality in 5.17 stating the the relationship between the n_p and n_r becomes as follows:

$$n_p^2 = \frac{4(k_{ep}k_r - k_{er}k_p) - 2D_r(k_{ep}h_r - k_p h_{er})n_r^2 T^2}{-D_p D_r (h_{ep}h_r - h_{er}h_p)n_r^2 T^4} \quad (6.3)$$

6.2.4 Scenario IV

Let us assume that we have the following equality that sets the statement (iv):

$$\frac{h_{ep}}{h_{er}} = \frac{h_p}{h_r}$$

By the above statement, we know that $h_{ep}h_r = h_{er}h_p$. Then, the relationship between the n_p and n_r at 5.17 becomes as follows:

$$n_p^2 = \frac{4(k_{ep}k_r - k_{er}k_p) - 2D_r(k_{ep}h_r - k_p h_{er})n_r^2 T^2}{2D_p(k_r h_{ep} - k_{er}h_p)T^2} \quad (6.4)$$

6.2.5 Scenario V

Now, let us assume that multiple statements hold at once. First, let us assume that the following statement holds, making (i) and (ii) equal to zero:

$$\frac{k_{ep}}{k_p} = \frac{k_{er}}{k_r} = \frac{h_{er}}{h_r}$$

6.2.6 Scenario VI

Now, let us assume that statements (i) and (iii) are both equal to zero, that is, the following statement holds:

$$\frac{k_{ep}}{k_p} = \frac{k_{er}}{k_r} = \frac{h_{ep}}{h_p}$$

6.2.7 Scenario VII

Now, let us assume that statements (i) and (iv) are both equal to zero, that is, the following statements hold:

$$\frac{k_{ep}}{k_p} = \frac{k_{er}}{k_r}$$

$$\frac{h_{ep}}{h_p} = \frac{h_{er}}{h_r}$$

Then, the statement 5.17 becomes as follows:

$$n_p^2 = \frac{D_r(k_p h_{er} - k_{ep} h_r) n_r^2}{D_p(k_r h_{ep} - k_{er} h_p)} \quad (6.5)$$

6.2.8 Scenario VIII

Now, let us assume that statements (ii) and (iii) are both equal to zero, that is, the following statements hold:

$$\frac{k_{ep}}{k_p} = \frac{h_{er}}{h_r}$$

$$\frac{k_{er}}{k_r} = \frac{h_{ep}}{h_p}$$

Then, the statement 5.17 becomes as follows:

$$n_p^2 = \frac{4(k_{ep} k_r - k_{er} k_p)}{D_p D_r (h_{er} h_p - h_{ep} h_r) n_r^2 T^4} \quad (6.6)$$

6.2.9 Scenario IX

Let us assume that the statements (iii) and (iv) are equal to zero, making the following statement hold:

$$\frac{k_{er}}{k_r} = \frac{h_{ep}}{h_p} = \frac{h_{er}}{h_r}$$

6.2.10 Scenario X

If we were to let any 3 of the statements (i) – (iv) become equal to zero, the fourth one automatically becomes equal to zero as well. Therefore, we would not have an equality to present.

6.3 Discussion of Special Cases for Pairwise Relationship of Item Specific Order Intervals

We can observe the scenarios stemming from equation 5.17 under three categories: the ones that can never be observed as they violate Assumption 3, the ones that may hold for only a pair of items without violating Assumption 3, and the ones that can hold true for an entire set of items without violating Assumption 3.

6.3.1 Special Cases Where $\frac{k_{er}}{k_r} \neq \frac{h_{er}}{h_r}$ Never Holds

The scenarios discussed in sections 6.2.5, 6.2.6, 6.2.9, and 6.2.10 belong to the first group as they immediately violate Assumption 3. Observe that under these assumptions, we obtain:

$$\frac{k_{er}}{k_r} = \frac{h_{er}}{h_r}$$

for some $r \in L$. Using our Lemma 15, the first order derivative of the total cost rate with respect to n_r becomes zero. Thus, Assumption 3 is violated, and we cannot continue our analysis under this scenario.

6.3.2 Special Cases Where $\frac{k_{er}}{k_r} \neq \frac{h_{er}}{h_r}$ Only Holds For A Pair Of Items

There are three scenarios that fall under this category. First, let us observe scenario II, previously introduced in section 6.2.2. If the condition introduced in

scenario II were to apply to a whole set, we would have obtained for all pairs of $i, j \in L$ where $j \neq i$:

$$\frac{k_{ei}}{k_i} = \frac{h_{ej}}{h_j} \quad (6.7)$$

Then, observing the above condition, let us demonstrate the problem caused by having more than two items, setting $V = 3$:

$$(a) \frac{k_{e1}}{k_1} = \frac{h_{e2}}{h_2}$$

$$(b) \frac{k_{e1}}{k_1} = \frac{h_{e3}}{h_3}$$

$$(c) \frac{k_{e2}}{k_2} = \frac{h_{e3}}{h_3}$$

$$(d) \frac{k_{e2}}{k_2} = \frac{h_{e1}}{h_1}$$

Observe that our condition dictates the above relationships for a case with three items. Therefore, looking at (a), (b), and (c) consecutively, we can see that

$\frac{k_{e1}}{k_1} = \frac{k_{e2}}{k_2}$. Further observing (d), one can easily see that we have:

$$\frac{k_{e1}}{k_1} = \frac{h_{e1}}{h_1}$$

Therefore, Assumption 3 is violated, and we cannot further continue our analysis. However, our condition under this scenario is still relevant since the following relationship can exist for a determined pair of $p, r \in L$:

$$\frac{k_{ep}}{k_p} = \frac{h_{er}}{h_r}$$

$$\frac{k_{er}}{k_r} \neq \frac{h_{ep}}{h_p}$$

Now, let us move to scenario III, previously shown in section 6.2.3. When the condition in that section is generalized to all of the items in a set, we obtain the following condition for all $i, j \in L$:

$$\frac{k_{ei}}{k_i} = \frac{h_{ej}}{h_j} \quad (6.8)$$

It is easy to observe that the results that we have found for scenario II holds

since the above condition is the same as the condition in equation 6.7.

Now, we can move on to the last scenario that we have under this section, scenario VIII that was previously introduced in section 6.2.8. Observe that we need the following ratio to be positive in order to use the equation 6.6:

$$\frac{k_{ep}k_r - k_{er}k_p}{h_{er}h_p - h_{ep}h_r}$$

In order to make the above ratio positive, we need either of the two following conditions:

$$(i) (a) k_{ep}k_r < k_{er}k_p \text{ and } (b) h_{er}h_p < h_{ep}h_r$$

$$(ii) (c) k_{er}k_p < k_{ep}k_r \text{ and } (d) h_{ep}h_r < h_{er}h_p$$

Now, observe that we have the following equalities from our assumption at the beginning of this special case:

$$k_{ep}h_r = k_p h_{er}$$

$$k_{er}h_p = k_r h_{ep}$$

Observe that by these assumptions, we have the following equalities as well:

$$k_{ep} = \frac{k_p h_{er}}{h_r}$$

$$k_r = \frac{k_{er} h_p}{h_{ep}}$$

First, let us observe the condition (i) that makes our ratio positive. Replacing the values in part (a) of condition (i) with the above equalities that we have, we obtain the following:

$$h_{er}h_p < h_{ep}h_r$$

Observe that the above equation is part (b) of condition (i). Thus, under the assumption that we have made, condition (i) is self-affirmative. Observing condition (ii) in a similar manner and replacing the values in part (c) this time, we obtain the following:

$$h_{ep}h_r < h_{er}h_p$$

Observe that the above equation is part (d) of condition (ii), which makes this condition self-affirmative as well. Therefore, our ratio is always positive.

Proving the positivity of the ratio, we are able to obtain the following equation:

$$n_p = \sqrt{\frac{4(k_{ep}k_r - k_{er}k_p)}{D_p D_r (h_{er}h_p - h_{ep}h_r)}} n_r^{-1} T^{-2} = B_p n_r^{-1} T^{-2} \quad (6.9)$$

It is easy to observe that the results we have found under scenario II holds if we were to assume that for all $i, j \in L$:

$$\frac{k_{ei}}{k_i} = \frac{k_{ej}}{k_j}$$

However, it is possible to have the following relationship so that the condition under this case could work for a determined pair of $p, r \in L$ such that:

$$\frac{k_{ep}}{k_p} = \frac{h_{er}}{h_r}$$

$$\frac{k_{er}}{k_r} = \frac{h_{ep}}{h_p}$$

$$\frac{k_{ep}}{k_p} \neq \frac{h_{ep}}{h_p}$$

$$\frac{k_{er}}{k_r} \neq \frac{h_{er}}{h_r}$$

So, what is the benefit that this scenario brings to us? For example, if the above statements hold true for a specific pair of $p, r \in L$, then, we can treat these two variables as one. We may remove the item p from the objective function and simply replace it with $B_p n_r^{-1} T^{-2}$. By pairing the items that have the above relationship, we can decrease the number of items that will be considered in the objective function, decreasing the computational time.

6.3.3 Special Cases Where $\frac{k_{er}}{k_r} \neq \frac{h_{er}}{h_r}$ Always Holds

There are three scenarios that are under this category of special cases. Observe that scenario I, introduced under section 6.2.1, and scenario II introduced under

section 6.2.2 only concern the minor fixed costs and emissions, and holding costs and emissions respectively. There are no cross-item-terms that draw any equalities between a minor fixed cost of an item with the holding cost of another one. Therefore, since those relationships are not depicted, it is not possible to violate Assumption 3 with these conditions.

Further observe that under scenario VII, introduced in section 6.2.7, there is only a relationship between holding costs and emissions of different items, and the minor fixed costs and emissions of different items. Therefore, there are no conditions that could potentially violate Assumption 3. For the rest of the analysis, we shall be focusing on scenario VII since it is the most promising one. First, let us present our case explaining why the conditions in scenario VII are realistic.

Observing the first condition regarding the fixed costs and emissions, one can argue that it is a fair assumption to be made. The fixed cost of ordering is incurred by the retailer regardless of how many items have been ordered. In our setting, it is the cost that the firm faces just by making the supplier deliver to them and the cost associated with receiving the items. Focusing on the delivery aspect, it is plausible to assume that the fixed costs are related to how far the retailer is to the supplier. As the distance increases, the supplier would be charging more to deliver to those retailers. Similarly, as the distance between the retailer and the supplier increases, the emission caused by making a stop at a given location increases as well. Therefore, we can argue that the fixed costs and emissions of a retailer could have a certain ratio.

Observing the second condition regarding the holding costs and emissions, one can argue that this also is a fair assumption. For example, a retailer that needs to manipulate the natural conditions of its warehouse such as the temperature would not only spend more money, but would also emit more carbon during this process. The damaged or spoiled goods are another source of cost that the retailer faces as a result of the handling activities. These damaged and spoiled

goods become waste that creates the need for disposal, and additional items need to be purchased to satisfy the demand, which increases carbon emissions. Observe that we need the following ratio to be positive in order to use equation 6.5:

$$\frac{k_p h_{er} - k_{ep} h_r}{k_r h_{ep} - k_{er} h_p}$$

Thus, we need either of the two following scenarios to hold for the positivity:

(i) (a) $k_{ep} h_r < k_p h_{er}$ and (b) $k_{er} h_p < k_r h_{ep}$

(ii) (c) $k_p h_{er} < k_{ep} h_r$ and (d) $k_r h_{ep} < k_{er} h_p$

Now, observe that we have the following equalities from our assumption at the beginning of this special case:

$$k_{ep} k_r = k_{er} k_p$$

$$h_{ep} h_r = h_{er} h_p$$

Observe that by our assumptions, we have the following equalities as well:

$$k_p = \frac{k_{ep} k_r}{k_{er}}$$

$$h_{er} = \frac{h_{ep} h_r}{h_p}$$

Now, let us first observe the condition (i) that makes our ratio positive.

Replacing the values in the part (a), that is $k_{ep} h_r < k_p h_{er}$, with the above equalities stemming from our initial assumption, we have:

$$k_r h_{ep} > k_{er} h_p$$

Observe that the above inequality is the part (b) of the condition (i). Thus, we can see that under our initial assumptions, condition (i) is self-affirmative.

Secondly, let us observe condition (ii) in a similar manner. Replacing the values in the part (c) with the equalities stemming from our initial assumption, we have:

$$k_r h_{ep} < k_{er} h_p$$

Observe that the above inequality is the part (d) of the condition (ii). Thus, we can see that under our initial assumptions, condition (ii) is self-affirmative as well. Thus, our ratio is always positive. Proving the positivity of the ratio, we

are able to obtain the following equation:

$$n_p = \sqrt{\frac{D_r(k_p h_{er} - k_{ep} h_r)}{D_p(k_r h_{ep} - k_{er} h_p)}} n_r \quad (6.10)$$

Assumption 4. For $i, j \in L$,

$$\frac{k_{ei}}{k_i} = \frac{k_{ej}}{k_j}$$

$$\frac{h_{ei}}{h_i} = \frac{h_{ej}}{h_j}$$

Then, we can state all of the members of the vector \mathbf{n} using a single member.

Let n_r be some known member of the vector for some $r \in L$. Then $\forall p \in L \setminus r$:

$$n_p = \sqrt{\frac{D_r(k_p h_{er} - k_{ep} h_r)}{D_p(k_r h_{ep} - k_{er} h_p)}} n_r = A_p n_r \quad (6.11)$$

Now, we can state our objective function as a bivariate one by only using the optimal order cycle time T and integer multiplier n_r as follows:

$$z(n_r, T) = \frac{\sum_{i \in L} \{p_{0i} D_i - \beta_i S_i [\frac{K_e}{T} + \sum_{j \in L} (\frac{k_{ej}}{A_j n_r T} + \frac{h_{ej} D_j A_j n_r T}{2} + c_{ej} D_j)]\}}{\frac{K}{T} + \sum_{i \in L} (\frac{k_i}{A_i n_r T} + \frac{h_i D_i A_i n_r T}{2} + c_i D_i)}$$

Now, let us define the following notation so that we can state our bivariate objective function in the most concise way:

$$J_1^1 = \sum_{i \in L} \{\beta_i S_i [\sum_{j \in L_i} (\frac{h_{ej} D_j A_j}{2})]\}$$

$$J_2^1 = \sum_{i \in L} \{p_{0i} D_i - \beta_i S_i [\sum_{j \in L_i} (c_{ej} D_j)]\}$$

$$J_3^1 = \sum_{i \in L} \{\beta_i S_i [\sum_{j \in L_i} (\frac{k_{ej}}{A_j})]\}$$

$$J_4^1 = \sum_{i \in L} \{\beta_i S_i [K_e]\}$$

$$J_1^2 = \sum_{i \in L} (\frac{h_i D_i A_i}{2})$$

$$J_2^2 = \sum_{i \in L} (c_i D_i)$$

$$J_3^2 = \sum_{i \in L} (\frac{k_i}{A_i})$$

$$J_4^2 = K$$

Then, we can write our bivariate objective function as follows:

$$z(n_r, T) = \frac{-J_1^1 n_r T + J_2^1 - J_3^1 n_r^{-1} T^{-1} - J_4^1 T^{-1}}{J_1^2 n_r T + J_2^2 + J_3^2 n_r^{-1} T^{-1} + J_4^2 T^{-1}} \quad (6.12)$$

Using Assumption 3, the first order condition for the above function for the variable n_r can be stated as follows:

$$\frac{-J_1^1 T + J_3^1 n_r^{-2} T^{-1}}{J_1^2 T - J_3^2 n_r^{-2} T^{-1}} = \frac{r(n_r, T)}{TC(n_r, T)} \quad (6.13)$$

Assumption 5. $\frac{\partial TC(n_r, T)}{\partial T} \neq \sqrt{\frac{B_3^2}{B_1^2}}$

Under Assumption 5, by Lemma 2, we know that the first order derivative of the total cost rate with respect to T is not zero. Therefore, the first order condition of the objective function for the variable T can be stated as follows:

$$\frac{-J_1^1 n_r + J_3^1 n_r^{-1} T^{-2} + J_4^1 T^{-2}}{J_1^2 n_r - J_3^2 n_r^{-1} T^{-2} - J_4^2 T^{-2}} = \frac{r(n_r, T)}{TC(n_r, T)} \quad (6.14)$$

Then, when both of the variables are at their optimal points, we can write the following relationship:

$$\frac{-J_1^1 T + J_3^1 n_r^{-2} T^{-1}}{J_1^2 T - J_3^2 n_r^{-2} T^{-1}} = \frac{-J_1^1 n_r + J_3^1 n_r^{-1} T^{-2} + J_4^1 T^{-2}}{J_1^2 n_r - J_3^2 n_r^{-1} T^{-2} - J_4^2 T^{-2}}$$

By cross-multiplication and multiplying both sides by T , we obtain the following statement:

$$(J_1^1 J_4^2 - J_4^1 J_1^2) + (J_4^1 J_3^2 - J_3^1 J_4^2) n_r^{-2} T^{-2} = 0$$

Further arranging the above function, we obtain the following relationship:

$$n_r^2 T^2 = \frac{J_4^1 J_3^2 - J_3^1 J_4^2}{J_4^1 J_1^2 - J_1^1 J_4^2}$$

Assumption 6. $\frac{J_4^1 J_3^2 - J_3^1 J_4^2}{J_4^1 J_1^2 - J_1^1 J_4^2} > 0$

Finally, we can obtain the following statement:

$$n_r T = \sqrt{\frac{J_4^1 J_3^2 - J_3^1 J_4^2}{J_1^1 J_4^2 - J_4^1 J_1^2}} = A_J \quad (6.15)$$

At the optimality of the variable T , we know that the following first order condition must hold:

$$\frac{r(n_r, T)}{TC(n_r, T)} = \frac{\frac{\partial r(n_r, T)}{\partial T}}{\frac{\partial TC(n_r, T)}{\partial T}}$$

Writing the above first order condition, and replacing n_r with $\frac{A_J}{T}$, we obtain the following:

$$\frac{-J_1^1 A_J + J_2^1 - J_3^1 A_J^{-1} - J_4^1 T^{-1}}{J_1^2 A_J + J_2^2 + J_3^2 A_J^{-1} + J_4^2 T^{-1}} = \frac{-J_1^1 A_J T^{-1} + J_3^1 A_J^{-1} T^{-1} + J_4^1 T^{-2}}{J_1^2 A_J T^{-1} - J_3^2 A_J^{-1} T^{-1} - J_4^2 T^{-2}} \quad (6.16)$$

By cross multiplication, we obtain the following equation for the optimal common order interval that is denoted as T^* :

$$T^* = \frac{2J_1^1 J_4^2 A_J - 2J_4^1 J_1^2 A_J - J_4^1 J_2^2 - J_2^1 J_4^2}{-2J_1^1 J_3^2 - J_2^1 J_1^2 A_J + J_2^1 J_3^2 A_J^{-1} + 2J_3^1 J_1^2 - J_1^1 J_2^2 A_J + J_3^1 J_2^2 A_J^{-1}} \quad (6.17)$$

Moreover, since $n_r^* = \frac{A_J}{T^*}$ at the optimality of both variables, we have the following equation as well:

$$n_r^* = \frac{-2J_1^1 J_3^2 A_J - J_2^1 J_1^2 A_J^2 + J_2^1 J_3^2 + 2J_3^1 J_1^2 A_J - J_1^1 J_2^2 A_J^2 + J_3^1 J_2^2}{2J_1^1 J_4^2 A_J - 2J_4^1 J_1^2 A_J - J_4^1 J_2^2 - J_2^1 J_4^2} \quad (6.18)$$

Moreover, by equation 6.11, we know the following for all $p \in L$ such that $p \neq r$:

$$n_p^* = A_p \frac{-2J_1^1 J_3^2 A_J - J_2^1 J_1^2 A_J^2 + J_2^1 J_3^2 + 2J_3^1 J_1^2 A_J - J_1^1 J_2^2 A_J^2 + J_3^1 J_2^2}{2J_1^1 J_4^2 A_J - 2J_4^1 J_1^2 A_J - J_4^1 J_2^2 - J_2^1 J_4^2} \quad (6.19)$$

Therefore, we have obtained a closed form expression of the optimal common order cycle time, and the optimal item specific order intervals under the assumptions 3, 4, 5 and 6 using only the parameters that the problem provides.

6.4 Identical Items

In this section, we assume that the items are identical. That is, $\beta_i = \beta$, $S_i = S$, $p_{0i} = p$, $k_{ei} = k_e$, $h_{ei} = h_e$, $k_i = k$, $h_i = h$, $D_i = D \forall i \in L$.

Lemma 21. *For identical items, under Assumption 3, for a given order cycle time, the optimal n_i values are equal to each other for all $i \in L$.*

Proof. Using the notation that we have previously defined, we rearrange the statement 5.13 and obtain the following for some $p, r \in S$:

$$\frac{\frac{k_e}{n_p^2 T} - \frac{h_e D T}{2}}{\frac{-k}{n_p^2 T} + \frac{h D T}{2}} = \frac{\frac{k_e}{n_r^2 T} - \frac{h_e D T}{2}}{\frac{-k}{n_r^2 T} + \frac{h D T}{2}}$$

Rearranging the upper equality, we obtain the equality of $n_p = n_r$. Therefore, we can say that, when there are identical parameters, and at the optimal solution: $\forall i, j \in S : n_i = n_j$. □

Let us denote $n_i = n \forall i \in L$. Assuming that there are V number of items, since we are able to decrease our variables to only n and T , we can state our total cost rate, total revenue rate, and objective functions as bivariate ones as follows:

$$TC(n, T) = \frac{K}{T} + V\left(\frac{k}{nT} + \frac{hDnT}{2} + cD\right) \quad (6.20)$$

$$r(n, T) = V\left[pD - \beta S\left(\frac{K_e}{T} + V\left\{\frac{k_e}{nT} + \frac{h_e D n T}{2} + c_e D\right\}\right)\right] \quad (6.21)$$

$$z(n, T) = \frac{V\left[pD - \beta S\left(\frac{K_e}{T} + V\left\{\frac{k_e}{nT} + \frac{h_e D n T}{2} + c_e D\right\}\right)\right]}{\frac{K}{T} + V\left(\frac{k}{nT} + \frac{hDnT}{2} + cD\right)} \quad (6.22)$$

Lemma 22. *When all items are identical, and if $\frac{Kk_e - K_e k}{Kh_e - K_e h} \geq 0$, then the optimal n and the optimal T have the following relationship:*

$$n^* T^* = \sqrt{\frac{Kk_e - K_e k}{(Kh_e - K_e h)D}}$$

Proof. The first order condition for the order cycle time is as follows:

$$\frac{r'_T}{TC'_T} = \frac{r(n, T)}{TC(n, T)}$$

Moreover, the first order condition for the integer multiplier is as follows:

$$\frac{r'_n}{TC'_n} = \frac{r(n, T)}{TC(n, T)}$$

Merging the two above statements, we obtain the following equation:

$$\frac{r'_T}{TC'_T} = \frac{r'_n}{TC'_n}$$

Stating the above equation explicitly, we obtain the following first order condition when both of the variables are at their optimal:

$$\frac{\frac{K_e}{T^2} + V\left(\frac{k_e}{nT^2} - \frac{h_e Dn}{2}\right)}{\frac{-K}{T^2} + V\left(\frac{-k}{nT^2} + \frac{hDn}{2}\right)} = \frac{\frac{k_e}{n^2T} - \frac{h_e DT}{2}}{\frac{-k}{n^2T} + \frac{hDT}{2}}$$

Rearranging the above statement, we get the following relationship:

$$n^2T^2 = \frac{2(Kk_e - K_e k)}{(Kh_e - K_e h)D}$$

Then, the result follows. □

CHAPTER 7

NUMERICAL ANALYSIS FOR TWO ITEMS

In this chapter, we conduct a brief numerical analysis with two items. We generate 64 scenarios by manipulating six of the parameters. We further observe the scenarios under the four policies that we have previously presented. While choosing the values for the parameters and detecting which ones to manipulate when obtaining alternative scenarios, we have used X. Chen et al. (2013) as a reference to determine these in a joint replenishment problem. Therefore, a total of 256 scenarios are analyzed. The following are the vectors containing the data of the parameters that are used:

$$\beta_i = \{1, 5\} \text{ for } i \in L = \{1, 2\}$$

$$D_1 = \{100, 500\}$$

$$D_2 = \{200, 1000\}$$

$$K = \{200, 100\}$$

$$K_e = \{10, 20\}$$

$$k_1 = \{10, 5\}$$

$$k_2 = \{8, 4\}$$

$$k_{e1} = \{1\}$$

$$k_{e2} = \{2\}$$

$$h_1 = \{0.1\}$$

$$h_2 = \{0.2\}$$

$$h_{e1} = \{0.02\}$$

$$h_{e2} = \{0.05\}$$

$$c_1 = \{1\}$$

$$c_2 = \{2\}$$

$$c_{e2} = \{1\}$$

$$c_{e1} = \{2\}$$

$$p_1 = \{10\}$$

$$p_2 = \{20\}$$

$$d_1 = \{2\}$$

$$d_2 = \{5\}$$

Observe that D_1 , D_2 , K , K_e , k_1 , and k_2 have two different values. Using all possible scenario combinations stemming from these alternatives, we obtain 64 scenarios.

We used Microsoft Excel Solver with a non-linear solver in order to obtain the optimization results. When we have searched for the optimal T for our objective function by providing the values for \mathbf{n} , our results have been consistent with the results that we have found in Lemma 10. When we have given a T and one of the n_i parameters as a given and searched for the other optimal item-specific order interval, the optimal solution is also the same as the one we have found in Lemma 13.

In addition to the objective function that we have analyzed throughout the thesis, we consider the minimization of the total cost rate given in 3.1, maximization of the total revenue rate given in 3.3, and the maximization of the total profit rate given in 3.6. Considering these four functions, we obtain 1024

optimization problems to be solved.

7.1 Findings

In this chapter, we provide the analysis and brief discussion related to our numerical study. First observe the instances and their manipulated parameters that are enumerated in Table 1.

Instance	K	k_1	k_2	D_1	D_2	K_e
1	200	10	8	100	200	10
2	100	5	4	500	1000	10
3	100	5	4	500	1000	20
4	200	10	8	100	200	20
5	100	10	8	100	200	10
6	100	5	8	100	200	10
7	100	5	4	100	200	10
8	100	5	4	500	200	10
9	100	5	4	500	200	20
10	100	10	4	500	1000	20
11	100	10	8	500	1000	20
12	100	10	8	100	1000	20
13	100	10	8	100	200	20
14	100	5	4	500	1000	20
15	100	10	8	100	1000	10
16	200	5	4	500	1000	20
17	200	5	4	500	1000	10
18	200	5	4	500	200	20
19	200	5	4	100	200	10
20	200	5	4	500	200	10

21	200	5	8	100	200	10
22	200	10	4	100	200	10
23	200	10	8	500	200	10
24	200	10	8	100	1000	10
25	200	5	8	500	200	10
26	200	5	8	100	1000	10
27	200	5	8	100	200	20
28	200	10	8	100	200	20
29	200	10	4	100	200	20
30	200	10	4	500	200	10
31	200	10	4	500	1000	20
32	200	5	8	500	1000	20
33	200	5	4	100	1000	20
34	100	10	4	100	1000	20
35	100	10	4	500	200	20
36	100	10	4	500	1000	10
37	100	5	8	100	1000	20
38	100	5	8	500	200	20
39	100	5	8	500	1000	20
40	100	5	4	100	1000	10
41	200	10	8	500	1000	20
42	200	10	4	100	1000	20
43	200	10	4	500	200	20
44	200	10	4	500	1000	10
45	200	5	8	100	1000	20
46	200	5	8	500	200	20
47	200	5	8	500	1000	10
48	200	5	4	100	200	20

49	200	5	4	100	1000	10
50	100	10	8	500	200	20
51	100	10	8	500	1000	10
52	100	10	4	100	200	20
53	100	10	4	100	1000	10
54	100	10	4	500	1000	10
55	100	5	8	100	200	20
56	100	5	8	100	1000	10
57	100	5	8	500	200	10
58	200	10	8	500	200	20
59	200	10	8	500	1000	10
60	100	5	4	100	1000	20
61	100	5	8	500	1000	20
62	100	10	4	100	200	10
63	100	10	8	500	200	10
64	100	10	8	100	1000	20

Table 1: All the Instances Created

One may observe the optimal solutions and performance measures for these instances under different policies and objective functions through tables 2-65.

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.767	2440.51	3088.45	647.93	521.33	1	1	2.775
	Profit Maximization	4.752	2443.18	3094.31	651.13	519.73	1	1	2.377
	Revenue Maximization	4.529	2417.00	3101.90	684.90	517.66	1	1	1.472
	Cost Minimization	4.764	2437.91	3085.56	647.65	522.12	1	1	2.953
Policy II	Profit / Cost Ratio Maximization	4.338	2163.64	2811.79	648.16	521.09	1	1	2.718
	Profit Maximization	4.321	2167.20	2819.77	652.57	519.39	1	1	2.282
	Revenue Maximization	4.072	2134.74	2829.59	694.85	517.73	1	1	1.354
	Cost Minimization	4.334	2159.43	2807.08	647.65	522.12	1	1	2.953
Policy III	Profit / Cost Ratio Maximization	4.230	2093.68	2741.81	648.12	521.12	1	1	2.725
	Profit Maximization	4.216	2096.77	2748.74	651.98	519.52	1	1	2.319
	Revenue Maximization	4.025	2071.89	2756.79	684.90	517.66	1	1	1.472
	Cost Minimization	4.227	2089.83	2737.48	647.65	522.12	1	1	2.953
Policy IV	Profit / Cost Ratio Maximization	4.334	2161.06	2809.17	648.12	521.13	1	1	2.727
	Profit Maximization	4.319	2164.30	2816.41	652.11	519.49	1	1	2.311
	Revenue Maximization	4.112	2137.89	2824.83	686.94	517.67	1	1	1.446
	Cost Minimization	4.331	2157.18	2804.82	647.65	522.12	1	1	2.953

Table 2: Results of Instance 1

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.74	12947.67	15681.27	2733.59	2541.47	1	1	0.902
	Profit Maximization	5.73	12949.58	15685.14	2735.56	2540.42	1	1	0.816
	Revenue Maximization	5.71	12940.64	15688.51	2747.87	2539.50	1	1	0.658
	Cost Minimization	5.74	12946.12	15679.57	2733.45	2541.94	1	1	0.934
Policy II	Profit / Cost Ratio Maximization	5.24	11597.14	14330.89	2733.75	2541.28	1	1	0.888
	Profit Maximization	5.24	11600.25	14337.29	2737.04	2540.10	1	1	0.784
	Revenue Maximization	5.20	11586.86	14342.56	2755.70	2539.63	1	1	0.606
	Cost Minimization	5.24	11594.11	14327.56	2733.45	2541.94	1	1	0.934
Policy III	Profit / Cost Ratio Maximization	5.12	11253.80	13987.48	2733.68	2541.35	1	1	0.893
	Profit Maximization	5.11	11256.01	13992.03	2736.02	2540.30	1	1	0.805
	Revenue Maximization	5.09	11247.65	13995.51	2747.87	2539.50	1	1	0.658
	Cost Minimization	5.12	11251.49	13984.95	2733.45	2541.94	1	1	0.934
Policy IV	Profit / Cost Ratio Maximization	5.24	11589.40	14323.09	2733.69	2541.34	1	1	0.893
	Profit Maximization	5.24	11591.83	14328.07	2736.24	2540.25	1	1	0.800
	Revenue Maximization	5.21	11582.50	14331.91	2749.41	2539.50	1	1	0.646
	Cost Minimization	5.24	11586.97	14320.42	2733.45	2541.94	1	1	0.934

Table 3: Results of Instance 2

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.72	12906.94	15640.40	2733.46	2552.62	1	1	0.926
	Profit Maximization	5.72	12907.03	15640.58	2733.55	2552.57	1	1	0.907
	Revenue Maximization	5.72	12906.77	15640.70	2733.94	2552.54	1	1	0.876
	Cost Minimization	5.72	12906.85	15640.30	2733.45	2552.64	1	1	0.934
Policy II	Profit / Cost Ratio Maximization	5.23	11555.41	14288.92	2733.51	2552.58	1	1	0.914
	Profit Maximization	5.23	11555.93	14289.95	2734.02	2552.54	1	1	0.871
	Revenue Maximization	5.22	11554.58	14290.59	2736.01	2552.72	1	1	0.805
	Cost Minimization	5.23	11554.84	14288.29	2733.45	2552.64	1	1	0.934
Policy III	Profit / Cost Ratio Maximization	5.10	11205.21	13938.68	2733.46	2552.61	1	1	0.924
	Profit Maximization	5.10	11205.32	13938.89	2733.57	2552.56	1	1	0.905
	Revenue Maximization	5.10	11205.08	13939.01	2733.94	2552.54	1	1	0.876
	Cost Minimization	5.10	11205.09	13938.54	2733.45	2552.64	1	1	0.934
Policy IV	Profit / Cost Ratio Maximization	5.22	11540.73	14274.20	2733.47	2552.61	1	1	0.923
	Profit Maximization	5.22	11540.87	14274.48	2733.61	2552.56	1	1	0.900
	Revenue Maximization	5.22	11540.54	14274.64	2734.10	2552.54	1	1	0.867
	Cost Minimization	5.22	11540.56	14274.02	2733.45	2552.64	1	1	0.934

Table 4: Results of Instance 3

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.75	2426.91	3074.71	647.80	525.08	1	1	2.821
	Profit Maximization	4.74	2428.26	3077.62	649.36	524.29	1	1	2.536
	Revenue Maximization	4.67	2420.23	3080.52	660.29	523.49	1	1	1.958
	Cost Minimization	4.75	2425.49	3073.14	647.65	525.51	1	1	2.953
Policy II	Profit / Cost Ratio Maximization	4.32	2149.62	2797.58	647.96	524.91	1	1	2.767
	Profit Maximization	4.31	2151.65	2802.06	650.41	524.06	1	1	2.435
	Revenue Maximization	4.21	2140.28	2806.35	666.07	523.58	1	1	1.801
	Cost Minimization	4.32	2147.02	2794.66	647.65	525.51	1	1	2.953
Policy III	Profit / Cost Ratio Maximization	4.21	2077.24	2725.15	647.90	524.97	1	1	2.784
	Profit Maximization	4.20	2078.77	2728.51	649.74	524.19	1	1	2.496
	Revenue Maximization	4.14	2071.23	2731.52	660.29	523.49	1	1	1.958
	Cost Minimization	4.20	2075.16	2722.80	647.65	525.51	1	1	2.953
Policy IV	Profit / Cost Ratio Maximization	4.31	2144.62	2792.52	647.90	524.97	1	1	2.785
	Profit Maximization	4.30	2146.24	2796.06	649.82	524.17	1	1	2.488
	Revenue Maximization	4.24	2138.28	2799.22	660.93	523.50	1	1	1.938
	Cost Minimization	4.31	2142.50	2790.15	647.65	525.51	1	1	2.953

Table 5: Results of Instance 4

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.09	2489.22	3097.94	608.72	518.74	1	1	2.084
	Profit Maximization	5.08	2490.19	3099.97	609.79	518.19	1	1	1.877
	Revenue Maximization	5.03	2484.94	3101.90	616.96	517.66	1	1	1.472
	Cost Minimization	5.09	2488.30	3096.93	608.63	519.02	1	1	2.173
Policy II	Profit / Cost Ratio Maximization	4.64	2214.67	2823.49	608.82	518.63	1	1	2.047
	Profit Maximization	4.63	2216.16	2826.69	610.53	518.03	1	1	1.802
	Revenue Maximization	4.56	2208.59	2829.59	621.00	517.73	1	1	1.354
	Cost Minimization	4.64	2212.95	2821.57	608.63	519.02	1	1	2.173
Policy III	Profit / Cost Ratio Maximization	4.52	2143.65	2752.43	608.78	518.67	1	1	2.060
	Profit Maximization	4.52	2144.75	2754.79	610.04	518.12	1	1	1.849
	Revenue Maximization	4.47	2139.83	2756.79	616.96	517.66	1	1	1.472
	Cost Minimization	4.52	2142.29	2750.92	608.63	519.02	1	1	2.173
Policy IV	Profit / Cost Ratio Maximization	4.63	2211.30	2820.08	608.79	518.67	1	1	2.059
	Profit Maximization	4.63	2212.50	2822.65	610.15	518.10	1	1	1.838
	Revenue Maximization	4.57	2207.06	2824.83	617.77	517.67	1	1	1.446
	Cost Minimization	4.63	2209.88	2818.50	608.63	519.02	1	1	2.173

Table 6: Results of Instance 5

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.11	2492.00	3098.39	606.39	518.62	1	1	2.043
	Profit Maximization	5.10	2492.87	3100.21	607.34	518.12	1	1	1.849
	Revenue Maximization	5.06	2488.33	3101.90	613.57	517.66	1	1	1.472
	Cost Minimization	5.11	2491.17	3097.47	606.30	518.87	1	1	2.126
Policy II	Profit / Cost Ratio Maximization	4.66	2217.59	2824.06	606.48	518.52	1	1	2.007
	Profit Maximization	4.65	2218.96	2827.00	608.04	517.97	1	1	1.775
	Revenue Maximization	4.58	2212.29	2829.59	617.31	517.73	1	1	1.354
	Cost Minimization	4.65	2216.00	2822.30	606.30	518.87	1	1	2.126
Policy III	Profit / Cost Ratio Maximization	4.54	2146.48	2752.92	606.44	518.56	1	1	2.020
	Profit Maximization	4.53	2147.47	2755.04	607.57	518.07	1	1	1.822
	Revenue Maximization	4.49	2143.22	2756.79	613.57	517.66	1	1	1.472
	Cost Minimization	4.54	2145.26	2751.56	606.30	518.87	1	1	2.126
Policy IV	Profit / Cost Ratio Maximization	4.65	2214.15	2820.60	606.44	518.55	1	1	2.019
	Profit Maximization	4.65	2215.24	2822.91	607.67	518.05	1	1	1.811
	Revenue Maximization	4.60	2210.52	2824.83	614.31	517.67	1	1	1.446
	Cost Minimization	4.65	2212.87	2819.17	606.30	518.87	1	1	2.126

Table 7: Results of Instance 6

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.13	2494.26	3098.74	604.48	518.53	1	1	2.010
	Profit Maximization	5.12	2495.05	3100.39	605.35	518.07	1	1	1.826
	Revenue Maximization	5.08	2491.05	3101.90	610.85	517.66	1	1	1.472
	Cost Minimization	5.13	2493.50	3097.90	604.40	518.75	1	1	2.088
Policy II	Profit / Cost Ratio Maximization	4.67	2219.95	2824.52	604.57	518.43	1	1	1.975
	Profit Maximization	4.67	2221.23	2827.23	606.01	517.93	1	1	1.753
	Revenue Maximization	4.61	2215.24	2829.59	614.35	517.73	1	1	1.354
	Cost Minimization	4.67	2218.48	2822.88	604.40	518.75	1	1	2.088
Policy III	Profit / Cost Ratio Maximization	4.55	2148.78	2753.31	604.53	518.47	1	1	1.988
	Profit Maximization	4.55	2149.68	2755.23	605.55	518.02	1	1	1.801
	Revenue Maximization	4.51	2145.94	2756.79	610.85	517.66	1	1	1.472
	Cost Minimization	4.55	2147.66	2752.06	604.40	518.75	1	1	2.088
Policy IV	Profit / Cost Ratio Maximization	4.67	2216.47	2821.00	604.53	518.46	1	1	1.987
	Profit Maximization	4.66	2217.46	2823.11	605.65	518.00	1	1	1.790
	Revenue Maximization	4.62	2213.29	2824.83	611.54	517.67	1	1	1.446
	Cost Minimization	4.67	2215.29	2819.69	604.40	518.75	1	1	2.088

Table 8: Results of Instance 7

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.74	5966.34	7006.41	1040.07	930.34	1	1	1.554
	Profit Maximization	6.74	5966.35	7006.42	1040.08	930.34	1	1	1.544
	Revenue Maximization	6.74	5966.31	7006.43	1040.12	930.33	1	1	1.517
	Cost Minimization	6.74	5966.34	7006.41	1040.07	930.34	1	1	1.556
Policy II	Profit / Cost Ratio Maximization	6.17	5378.77	6418.85	1040.08	930.33	1	1	1.537
	Profit Maximization	6.17	5378.96	6419.24	1040.28	930.34	1	1	1.474
	Revenue Maximization	6.16	5377.96	6419.61	1041.65	930.56	1	1	1.340
	Cost Minimization	6.17	5378.63	6418.70	1040.07	930.34	1	1	1.556
Policy III	Profit / Cost Ratio Maximization	5.97	5168.91	6208.98	1040.07	930.34	1	1	1.552
	Profit Maximization	5.97	5168.92	6208.99	1040.08	930.34	1	1	1.541
	Revenue Maximization	5.97	5168.89	6209.01	1040.12	930.33	1	1	1.517
	Cost Minimization	5.97	5168.90	6208.98	1040.07	930.34	1	1	1.556
Policy IV	Profit / Cost Ratio Maximization	6.16	5367.62	6407.69	1040.07	930.34	1	1	1.555
	Profit Maximization	6.16	5367.62	6407.69	1040.07	930.34	1	1	1.549
	Revenue Maximization	6.16	5367.62	6407.70	1040.08	930.33	1	1	1.538
	Cost Minimization	6.16	5367.62	6407.69	1040.07	930.34	1	1	1.556

Table 9: Results of Instance 8

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.74	5966.34	7006.41	1040.07	930.34	1	1	1.554
	Profit Maximization	6.74	5966.35	7006.42	1040.08	930.34	1	1	1.544
	Revenue Maximization	6.74	5966.31	7006.43	1040.12	930.33	1	1	1.517
	Cost Minimization	6.74	5966.34	7006.41	1040.07	930.34	1	1	1.556
Policy II	Profit / Cost Ratio Maximization	6.17	5378.77	6418.85	1040.08	930.33	1	1	1.537
	Profit Maximization	6.17	5378.96	6419.24	1040.28	930.34	1	1	1.474
	Revenue Maximization	6.16	5377.96	6419.61	1041.65	930.56	1	1	1.340
	Cost Minimization	6.17	5378.63	6418.70	1040.07	930.34	1	1	1.556
Policy III	Profit / Cost Ratio Maximization	5.97	5168.91	6208.98	1040.07	930.34	1	1	1.552
	Profit Maximization	5.97	5168.92	6208.99	1040.08	930.34	1	1	1.541
	Revenue Maximization	5.97	5168.89	6209.01	1040.12	930.33	1	1	1.517
	Cost Minimization	5.97	5168.90	6208.98	1040.07	930.34	1	1	1.556
Policy IV	Profit / Cost Ratio Maximization	6.16	5367.62	6407.69	1040.07	930.34	1	1	1.555
	Profit Maximization	6.16	5367.62	6407.69	1040.07	930.34	1	1	1.549
	Revenue Maximization	6.16	5367.62	6407.70	1040.08	930.33	1	1	1.538
	Cost Minimization	6.16	5367.62	6407.69	1040.07	930.34	1	1	1.556

Table 10: Results of Instance 9

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.71	12901.38	15640.15	2738.76	2552.69	1	1	0.945
	Profit Maximization	5.71	12901.55	15640.48	2738.93	2552.60	1	1	0.919
	Revenue Maximization	5.71	12901.06	15640.70	2739.65	2552.54	1	1	0.876
	Cost Minimization	5.71	12901.23	15639.98	2738.75	2552.73	1	1	0.955
Policy II	Profit / Cost Ratio Maximization	5.22	11549.54	14288.36	2738.82	2552.64	1	1	0.932
	Profit Maximization	5.22	11550.22	14289.73	2739.50	2552.54	1	1	0.882
	Revenue Maximization	5.21	11548.37	14290.59	2742.22	2552.72	1	1	0.805
	Cost Minimization	5.22	11548.80	14287.54	2738.75	2552.73	1	1	0.955
Policy III	Profit / Cost Ratio Maximization	5.09	11199.63	13938.40	2738.77	2552.68	1	1	0.942
	Profit Maximization	5.09	11199.83	13938.79	2738.96	2552.59	1	1	0.915
	Revenue Maximization	5.09	11199.37	13939.01	2739.65	2552.54	1	1	0.876
	Cost Minimization	5.09	11199.41	13938.15	2738.75	2552.73	1	1	0.955
Policy IV	Profit / Cost Ratio Maximization	5.21	11535.11	14273.88	2738.77	2552.67	1	1	0.941
	Profit Maximization	5.21	11535.35	14274.36	2739.01	2552.58	1	1	0.911
	Revenue Maximization	5.21	11534.77	14274.64	2739.87	2552.54	1	1	0.867
	Cost Minimization	5.21	11534.83	14273.58	2738.75	2552.73	1	1	0.955

Table 11: Results of Instance 10

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.70	12896.98	15639.90	2742.92	2552.75	1	1	0.959
	Profit Maximization	5.70	12897.22	15640.38	2743.16	2552.62	1	1	0.928
	Revenue Maximization	5.70	12896.49	15640.70	2744.21	2552.54	1	1	0.876
	Cost Minimization	5.70	12896.76	15639.66	2742.90	2552.82	1	1	0.972
Policy II	Profit / Cost Ratio Maximization	5.21	11544.89	14287.87	2742.99	2552.69	1	1	0.946
	Profit Maximization	5.21	11545.71	14289.53	2743.82	2552.54	1	1	0.891
	Revenue Maximization	5.20	11543.40	14290.59	2747.18	2552.72	1	1	0.805
	Cost Minimization	5.21	11543.99	14286.89	2742.90	2552.82	1	1	0.972
Policy III	Profit / Cost Ratio Maximization	5.08	11195.20	13938.13	2742.93	2552.74	1	1	0.956
	Profit Maximization	5.08	11195.48	13938.68	2743.21	2552.61	1	1	0.924
	Revenue Maximization	5.08	11194.80	13939.01	2744.21	2552.54	1	1	0.876
	Cost Minimization	5.08	11194.88	13937.78	2742.90	2552.82	1	1	0.972
Policy IV	Profit / Cost Ratio Maximization	5.20	11530.65	14273.58	2742.94	2552.73	1	1	0.955
	Profit Maximization	5.20	11530.98	14274.25	2743.27	2552.60	1	1	0.920
	Revenue Maximization	5.20	11530.16	14274.64	2744.48	2552.54	1	1	0.867
	Cost Minimization	5.20	11530.27	14273.17	2742.90	2552.82	1	1	0.972

Table 12: Results of Instance 11

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.75	8713.66	11035.70	2322.04	2149.16	1.328	1	1.020
	Profit Maximization	4.75	8714.02	11036.42	2322.41	2149.01	1.250	1	0.986
	Revenue Maximization	4.75	8712.96	11036.90	2323.94	2148.90	1.066	1	0.938
	Cost Minimization	4.75	8713.21	11035.20	2321.99	2149.27	1.361	1	1.039
Policy II	Profit / Cost Ratio Maximization	4.60	8347.81	10669.87	2322.06	2149.15	1.382	1	1.012
	Profit Maximization	4.59	8348.26	10670.79	2322.53	2149.02	1.399	1	0.968
	Revenue Maximization	4.59	8346.58	10671.36	2324.79	2148.93	1.103	1	0.907
	Cost Minimization	4.59	8347.14	10669.13	2321.99	2149.27	1.361	1	1.039
Policy III	Profit / Cost Ratio Maximization	4.56	8262.82	10584.86	2322.04	2149.16	1.326	1	1.019
	Profit Maximization	4.56	8263.18	10585.60	2322.43	2149.00	1.246	1	0.985
	Revenue Maximization	4.56	8262.14	10586.08	2323.94	2148.90	1.066	1	0.938
	Cost Minimization	4.56	8262.31	10584.30	2321.99	2149.27	1.361	1	1.039
Policy IV	Profit / Cost Ratio Maximization	4.59	8343.61	10665.65	2322.04	2149.17	1.378	1	1.016
	Profit Maximization	4.59	8343.94	10666.34	2322.39	2149.04	1.385	1	0.977
	Revenue Maximization	4.59	8342.52	10666.78	2324.26	2148.91	1.082	1	0.925
	Cost Minimization	4.59	8343.10	10665.09	2321.99	2149.27	1.361	1	1.039

Table 13: Results of Instance 12

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.06	2471.53	3080.17	608.64	523.59	1	1	2.142
	Profit Maximization	5.06	2471.63	3080.37	608.74	523.53	1	1	2.075
	Revenue Maximization	5.06	2471.30	3080.52	609.22	523.49	1	1	1.958
	Cost Minimization	5.06	2471.43	3080.05	608.63	523.62	1	1	2.173
Policy II	Profit / Cost Ratio Maximization	4.61	2196.50	2805.18	608.68	523.56	1	1	2.109
	Profit Maximization	4.61	2196.84	2805.87	609.04	523.50	1	1	1.992
	Revenue Maximization	4.60	2195.80	2806.35	610.54	523.58	1	1	1.801
	Cost Minimization	4.61	2196.07	2804.70	608.63	523.62	1	1	2.173
Policy III	Profit / Cost Ratio Maximization	4.49	2122.50	2731.15	608.65	523.58	1	1	2.134
	Profit Maximization	4.49	2122.61	2731.38	608.77	523.53	1	1	2.066
	Revenue Maximization	4.48	2122.31	2731.52	609.22	523.49	1	1	1.958
	Cost Minimization	4.49	2122.34	2730.97	608.63	523.62	1	1	2.173
Policy IV	Profit / Cost Ratio Maximization	4.60	2190.11	2798.76	608.65	523.58	1	1	2.131
	Profit Maximization	4.60	2190.25	2799.04	608.79	523.52	1	1	2.056
	Revenue Maximization	4.59	2189.88	2799.22	609.34	523.50	1	1	1.938
	Cost Minimization	4.60	2189.93	2798.56	608.63	523.62	1	1	2.173

Table 14: Results of Instance 13

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.72	12906.94	15640.40	2733.46	2552.62	1	1	0.926
	Profit Maximization	5.72	12907.03	15640.58	2733.55	2552.57	1	1	0.907
	Revenue Maximization	5.72	12906.77	15640.70	2733.94	2552.54	1	1	0.876
	Cost Minimization	5.72	12906.85	15640.30	2733.45	2552.64	1	1	0.934
Policy II	Profit / Cost Ratio Maximization	5.23	11555.41	14288.92	2733.51	2552.58	1	1	0.914
	Profit Maximization	5.23	11555.93	14289.95	2734.02	2552.54	1	1	0.871
	Revenue Maximization	5.22	11554.58	14290.59	2736.01	2552.72	1	1	0.805
	Cost Minimization	5.23	11554.84	14288.29	2733.45	2552.64	1	1	0.934
Policy III	Profit / Cost Ratio Maximization	5.10	11205.21	13938.68	2733.46	2552.61	1	1	0.924
	Profit Maximization	5.10	11205.32	13938.89	2733.57	2552.56	1	1	0.905
	Revenue Maximization	5.10	11205.08	13939.01	2733.94	2552.54	1	1	0.876
	Cost Minimization	5.10	11205.09	13938.54	2733.45	2552.64	1	1	0.934
Policy IV	Profit / Cost Ratio Maximization	5.22	11540.73	14274.20	2733.47	2552.61	1	1	0.923
	Profit Maximization	5.22	11540.87	14274.48	2733.61	2552.56	1	1	0.900
	Revenue Maximization	5.22	11540.54	14274.64	2734.10	2552.54	1	1	0.867
	Cost Minimization	5.22	11540.56	14274.02	2733.45	2552.64	1	1	0.934

Table 15: Results of Instance 14

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.77	8761.15	11083.50	2322.34	2138.85	1.381	1	0.981
	Profit Maximization	4.77	8763.96	11089.35	2325.39	2137.59	1.416	1	0.871
	Revenue Maximization	4.74	8753.59	11093.76	2340.17	2136.64	1.443	1	0.693
	Cost Minimization	4.77	8757.82	11079.81	2321.99	2139.65	1.361	1	1.039
Policy II	Profit / Cost Ratio Maximization	4.62	8395.91	10718.36	2322.45	2138.77	1.439	1	0.973
	Profit Maximization	4.61	8399.14	10725.12	2325.99	2137.50	1.584	1	0.855
	Revenue Maximization	4.58	8387.14	10730.13	2342.98	2136.66	1.489	1	0.671
	Cost Minimization	4.61	8391.75	10713.74	2321.99	2139.65	1.361	1	1.039
Policy III	Profit / Cost Ratio Maximization	4.58	8312.65	10635.05	2322.40	2138.80	1.383	1	0.977
	Profit Maximization	4.58	8315.51	10641.07	2325.56	2137.56	1.417	1	0.867
	Revenue Maximization	4.55	8305.34	10645.51	2340.17	2136.64	1.443	1	0.693
	Cost Minimization	4.58	8308.94	10630.93	2321.99	2139.65	1.361	1	1.039
Policy IV	Profit / Cost Ratio Maximization	4.61	8393.67	10716.09	2322.42	2138.79	1.436	1	0.974
	Profit Maximization	4.61	8396.69	10722.42	2325.73	2137.55	1.574	1	0.860
	Revenue Maximization	4.58	8385.72	10727.03	2341.31	2136.64	1.462	1	0.684
	Cost Minimization	4.61	8389.73	10711.72	2321.99	2139.65	1.361	1	1.039

Table 16: Results of Instance 15

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.54	12805.19	15628.70	2823.51	2555.81	1	1	1.244
	Profit Maximization	5.53	12808.23	15634.95	2826.72	2554.10	1	1	1.117
	Revenue Maximization	5.49	12792.56	15640.70	2848.14	2552.54	1	1	0.876
	Cost Minimization	5.53	12802.61	15625.88	2823.26	2556.58	1	1	1.293
Policy II	Profit / Cost Ratio Maximization	5.05	11448.25	14272.00	2823.76	2555.51	1	1	1.224
	Profit Maximization	5.05	11453.01	14281.94	2828.93	2553.62	1	1	1.073
	Revenue Maximization	5.00	11430.42	14290.59	2860.17	2552.72	1	1	0.805
	Cost Minimization	5.05	11443.42	14266.68	2823.26	2556.58	1	1	1.293
Policy III	Profit / Cost Ratio Maximization	4.93	11102.04	13925.70	2823.66	2555.61	1	1	1.231
	Profit Maximization	4.93	11105.56	13933.05	2827.49	2553.91	1	1	1.100
	Revenue Maximization	4.89	11090.87	13939.01	2848.14	2552.54	1	1	0.876
	Cost Minimization	4.93	11098.23	13921.49	2823.26	2556.58	1	1	1.293
Policy IV	Profit / Cost Ratio Maximization	5.05	11436.99	14260.65	2823.66	2555.61	1	1	1.231
	Profit Maximization	5.05	11440.71	14268.38	2827.66	2553.87	1	1	1.097
	Revenue Maximization	5.01	11425.17	14274.64	2849.47	2552.54	1	1	0.867
	Cost Minimization	5.05	11433.11	14256.37	2823.26	2556.58	1	1	1.293

Table 17: Results of Instance 16

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.55	12835.89	15659.62	2823.73	2547.38	1	1	1.226
	Profit Maximization	5.54	12842.14	15672.76	2830.62	2543.79	1	1	1.045
	Revenue Maximization	5.41	12788.73	15688.51	2899.78	2539.50	1	1	0.658
	Cost Minimization	5.54	12830.97	15654.23	2823.26	2548.85	1	1	1.293
Policy II	Profit / Cost Ratio Maximization	5.06	11479.80	14303.89	2824.09	2546.92	1	1	1.204
	Profit Maximization	5.05	11488.34	14322.06	2833.73	2543.06	1	1	1.003
	Revenue Maximization	4.91	11421.72	14342.56	2920.84	2539.63	1	1	0.606
	Cost Minimization	5.06	11471.77	14295.04	2823.26	2548.85	1	1	1.293
Policy III	Profit / Cost Ratio Maximization	4.94	11139.06	13963.09	2824.03	2546.98	1	1	1.207
	Profit Maximization	4.94	11146.44	13978.83	2832.40	2543.35	1	1	1.020
	Revenue Maximization	4.83	11095.73	13995.51	2899.78	2539.50	1	1	0.658
	Cost Minimization	4.94	11131.74	13955.00	2823.26	2548.85	1	1	1.293
Policy IV	Profit / Cost Ratio Maximization	5.06	11474.00	14298.02	2824.02	2546.99	1	1	1.208
	Profit Maximization	5.05	11481.73	14314.43	2832.69	2543.28	1	1	1.016
	Revenue Maximization	4.94	11427.82	14331.91	2904.10	2539.50	1	1	0.646
	Cost Minimization	5.06	11466.62	14289.88	2823.26	2548.85	1	1	1.293

Table 18: Results of Instance 17

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.40	5908.79	7002.78	1093.99	932.03	1	1	2.117
	Profit Maximization	6.40	5909.45	7004.18	1094.77	931.38	1	1	1.972
	Revenue Maximization	6.33	5900.38	7006.43	1106.06	930.33	1	1	1.517
	Cost Minimization	6.40	5908.42	7002.38	1093.96	932.22	1	1	2.155
Policy II	Profit / Cost Ratio Maximization	5.86	5317.53	6411.59	1094.06	931.89	1	1	2.086
	Profit Maximization	5.85	5319.18	6415.07	1095.88	931.01	1	1	1.872
	Revenue Maximization	5.75	5303.33	6419.61	1116.28	930.56	1	1	1.340
	Cost Minimization	5.86	5316.40	6410.36	1093.96	932.22	1	1	2.155
Policy III	Profit / Cost Ratio Maximization	5.67	5110.15	6204.18	1094.03	931.94	1	1	2.097
	Profit Maximization	5.67	5111.21	6206.40	1095.19	931.20	1	1	1.925
	Revenue Maximization	5.61	5102.95	6209.01	1106.06	930.33	1	1	1.517
	Cost Minimization	5.67	5109.37	6203.33	1093.96	932.22	1	1	2.155
Policy IV	Profit / Cost Ratio Maximization	5.85	5309.16	6403.19	1094.02	931.95	1	1	2.100
	Profit Maximization	5.85	5310.17	6405.29	1095.12	931.23	1	1	1.932
	Revenue Maximization	5.80	5302.61	6407.70	1105.08	930.33	1	1	1.538
	Cost Minimization	5.85	5308.44	6402.40	1093.96	932.22	1	1	2.155

Table 19: Results of Instance 18

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.79	2444.46	3089.30	644.84	521.10	1	1	2.720
	Profit Maximization	4.78	2447.00	3094.86	647.86	519.58	1	1	2.337
	Revenue Maximization	4.57	2423.11	3101.90	678.79	517.66	1	1	1.472
	Cost Minimization	4.79	2442.00	3086.57	644.57	521.84	1	1	2.891
Policy II	Profit / Cost Ratio Maximization	4.36	2167.77	2812.82	645.05	520.87	1	1	2.665
	Profit Maximization	4.34	2171.18	2820.42	649.25	519.26	1	1	2.243
	Revenue Maximization	4.11	2141.38	2829.59	688.21	517.73	1	1	1.354
	Cost Minimization	4.36	2163.77	2808.34	644.57	521.84	1	1	2.891
Policy III	Profit / Cost Ratio Maximization	4.25	2097.75	2742.77	645.02	520.90	1	1	2.673
	Profit Maximization	4.24	2100.68	2749.33	648.65	519.38	1	1	2.281
	Revenue Maximization	4.06	2078.00	2756.79	678.79	517.66	1	1	1.472
	Cost Minimization	4.25	2094.11	2738.67	644.57	521.84	1	1	2.891
Policy IV	Profit / Cost Ratio Maximization	4.36	2165.14	2810.15	645.01	520.91	1	1	2.674
	Profit Maximization	4.34	2168.22	2817.01	648.78	519.35	1	1	2.272
	Revenue Maximization	4.15	2144.11	2824.83	680.72	517.67	1	1	1.446
	Cost Minimization	4.35	2161.46	2806.03	644.57	521.84	1	1	2.891

Table 20: Results of Instance 19

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.41	5919.13	7013.15	1094.02	927.20	1	1	2.101
	Profit Maximization	6.40	5920.56	7016.21	1095.66	925.77	1	1	1.888
	Revenue Maximization	6.19	5887.95	7022.56	1134.61	922.80	1	1	1.140
	Cost Minimization	6.41	5918.36	7012.32	1093.96	927.58	1	1	2.155
Policy II	Profit / Cost Ratio Maximization	5.87	5328.14	6422.26	1094.12	926.97	1	1	2.069
	Profit Maximization	5.86	5330.87	6428.10	1097.23	925.19	1	1	1.794
	Revenue Maximization	5.61	5289.81	6437.62	1147.81	922.90	1	1	1.040
	Cost Minimization	5.87	5326.34	6420.30	1093.96	927.58	1	1	2.155
Policy III	Profit / Cost Ratio Maximization	5.68	5124.91	6219.02	1094.11	926.99	1	1	2.072
	Profit Maximization	5.67	5127.24	6224.00	1096.76	925.33	1	1	1.818
	Revenue Maximization	5.49	5096.98	6231.59	1134.61	922.80	1	1	1.140
	Cost Minimization	5.68	5123.29	6217.25	1093.96	927.58	1	1	2.155
Policy IV	Profit / Cost Ratio Maximization	5.87	5323.88	6417.97	1094.09	927.02	1	1	2.076
	Profit Maximization	5.86	5326.14	6422.77	1096.63	925.38	1	1	1.826
	Revenue Maximization	5.68	5298.47	6429.86	1131.39	922.81	1	1	1.169
	Cost Minimization	5.87	5322.36	6416.32	1093.96	927.58	1	1	2.155

Table 21: Results of Instance 20

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.78	2442.70	3088.92	646.22	521.20	1	1	2.744
	Profit Maximization	4.77	2445.30	3094.62	649.32	519.65	1	1	2.355
	Revenue Maximization	4.55	2420.40	3101.90	681.50	517.66	1	1	1.472
	Cost Minimization	4.78	2440.18	3086.12	645.95	521.97	1	1	2.919
Policy II	Profit / Cost Ratio Maximization	4.35	2165.93	2812.36	646.44	520.97	1	1	2.689
	Profit Maximization	4.33	2169.40	2820.14	650.73	519.31	1	1	2.261
	Revenue Maximization	4.09	2138.43	2829.59	691.16	517.73	1	1	1.354
	Cost Minimization	4.35	2161.83	2807.78	645.95	521.97	1	1	2.919
Policy III	Profit / Cost Ratio Maximization	4.24	2095.93	2742.34	646.41	521.00	1	1	2.696
	Profit Maximization	4.23	2098.93	2749.07	650.14	519.45	1	1	2.298
	Revenue Maximization	4.05	2075.29	2756.79	681.50	517.66	1	1	1.472
	Cost Minimization	4.24	2092.20	2738.14	645.95	521.97	1	1	2.919
Policy IV	Profit / Cost Ratio Maximization	4.35	2163.32	2809.72	646.40	521.01	1	1	2.698
	Profit Maximization	4.33	2166.47	2816.74	650.27	519.42	1	1	2.290
	Revenue Maximization	4.13	2141.35	2824.83	683.48	517.67	1	1	1.446
	Cost Minimization	4.34	2159.55	2805.49	645.95	521.97	1	1	2.919

Table 22: Results of Instance 21

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.78	2442.26	3088.82	646.57	521.23	1	1	2.751
	Profit Maximization	4.76	2444.87	3094.56	649.68	519.67	1	1	2.360
	Revenue Maximization	4.55	2419.72	3101.90	682.18	517.66	1	1	1.472
	Cost Minimization	4.77	2439.72	3086.01	646.29	522.00	1	1	2.926
Policy II	Profit / Cost Ratio Maximization	4.35	2165.47	2812.25	646.78	520.99	1	1	2.695
	Profit Maximization	4.33	2168.96	2820.06	651.10	519.33	1	1	2.265
	Revenue Maximization	4.09	2137.69	2829.59	691.90	517.73	1	1	1.354
	Cost Minimization	4.34	2161.35	2807.64	646.29	522.00	1	1	2.926
Policy III	Profit / Cost Ratio Maximization	4.24	2095.48	2742.23	646.75	521.02	1	1	2.702
	Profit Maximization	4.23	2098.50	2749.01	650.51	519.46	1	1	2.302
	Revenue Maximization	4.04	2074.61	2756.79	682.18	517.66	1	1	1.472
	Cost Minimization	4.24	2091.72	2738.01	646.29	522.00	1	1	2.926
Policy IV	Profit / Cost Ratio Maximization	4.34	2162.86	2809.61	646.74	521.03	1	1	2.704
	Profit Maximization	4.33	2166.03	2816.67	650.64	519.43	1	1	2.294
	Revenue Maximization	4.13	2140.65	2824.83	684.18	517.67	1	1	1.446
	Cost Minimization	4.34	2159.07	2805.36	646.29	522.00	1	1	2.926

Table 23: Results of Instance 22

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.39	5914.33	7012.49	1098.16	927.51	1	1	2.144
	Profit Maximization	6.38	5915.84	7015.72	1099.89	926.00	1	1	1.924
	Revenue Maximization	6.15	5880.06	7022.56	1142.51	922.80	1	1	1.140
	Cost Minimization	6.39	5913.52	7011.61	1098.09	927.92	1	1	2.201
Policy II	Profit / Cost Ratio Maximization	5.85	5323.03	6421.29	1098.26	927.27	1	1	2.112
	Profit Maximization	5.84	5325.90	6427.43	1101.53	925.39	1	1	1.827
	Revenue Maximization	5.57	5281.15	6437.62	1156.46	922.90	1	1	1.040
	Cost Minimization	5.85	5321.15	6419.24	1098.09	927.92	1	1	2.201
Policy III	Profit / Cost Ratio Maximization	5.66	5119.87	6218.12	1098.25	927.29	1	1	2.114
	Profit Maximization	5.65	5122.33	6223.40	1101.07	925.53	1	1	1.851
	Revenue Maximization	5.45	5089.08	6231.59	1142.51	922.80	1	1	1.140
	Cost Minimization	5.66	5118.16	6216.25	1098.09	927.92	1	1	2.201
Policy IV	Profit / Cost Ratio Maximization	5.84	5318.86	6417.09	1098.24	927.32	1	1	2.119
	Profit Maximization	5.83	5321.25	6422.19	1100.93	925.58	1	1	1.858
	Revenue Maximization	5.64	5290.77	6429.86	1139.08	922.81	1	1	1.169
	Cost Minimization	5.84	5317.25	6415.34	1098.09	927.92	1	1	2.201

Table 24: Results of Instance 23

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.60	8654.67	11058.29	2403.62	2144.29	1.021	1	1.326
	Profit Maximization	4.59	8662.72	11075.73	2413.01	2140.53	1.115	1	1.105
	Revenue Maximization	4.47	8609.25	11093.76	2484.50	2136.64	1.443	1	0.693
	Cost Minimization	4.60	8645.52	11048.11	2402.59	2146.49	1	1	1.441
Policy II	Profit / Cost Ratio Maximization	4.45	8287.01	10690.88	2403.87	2144.07	1.066	1	1.312
	Profit Maximization	4.44	8295.97	10710.42	2414.44	2140.26	1.249	1	1.084
	Revenue Maximization	4.31	8238.22	10730.13	2491.91	2136.66	1.489	1	0.671
	Cost Minimization	4.44	8275.75	10678.34	2402.59	2146.54	1	1	1.444
Policy III	Profit / Cost Ratio Maximization	4.41	8205.42	10609.20	2403.79	2144.13	1.024	1	1.317
	Profit Maximization	4.40	8213.67	10627.26	2413.59	2140.41	1.119	1	1.097
	Revenue Maximization	4.28	8161.00	10645.51	2484.50	2136.64	1.443	1	0.693
	Cost Minimization	4.41	8195.21	10597.80	2402.59	2146.49	1	1	1.441
Policy IV	Profit / Cost Ratio Maximization	4.45	8285.32	10689.16	2403.84	2144.09	1.065	1	1.314
	Profit Maximization	4.44	8294.04	10708.16	2414.11	2140.33	1.245	1	1.088
	Revenue Maximization	4.31	8239.50	10727.03	2487.53	2136.64	1.462	1	0.684
	Cost Minimization	4.44	8274.59	10677.18	2402.59	2146.49	1	1	1.441

Table 25: Results of Instance 24

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.40	5916.98	7012.86	1095.87	927.33	1	1	2.120
	Profit Maximization	6.39	5918.45	7016.00	1097.55	925.87	1	1	1.904
	Revenue Maximization	6.17	5884.44	7022.56	1138.12	922.80	1	1	1.140
	Cost Minimization	6.40	5916.20	7012.00	1095.81	927.73	1	1	2.176
Policy II	Profit / Cost Ratio Maximization	5.86	5325.86	6421.83	1095.97	927.11	1	1	2.088
	Profit Maximization	5.85	5328.65	6427.80	1099.15	925.28	1	1	1.809
	Revenue Maximization	5.59	5285.96	6437.62	1151.66	922.90	1	1	1.040
	Cost Minimization	5.86	5324.02	6419.83	1095.81	927.73	1	1	2.176
Policy III	Profit / Cost Ratio Maximization	5.67	5122.66	6218.62	1095.96	927.13	1	1	2.091
	Profit Maximization	5.66	5125.05	6223.73	1098.69	925.42	1	1	1.833
	Revenue Maximization	5.48	5093.47	6231.59	1138.12	922.80	1	1	1.140
	Cost Minimization	5.67	5121.00	6216.81	1095.81	927.73	1	1	2.176
Policy IV	Profit / Cost Ratio Maximization	5.86	5321.63	6417.58	1095.95	927.16	1	1	2.095
	Profit Maximization	5.85	5323.96	6422.51	1098.56	925.47	1	1	1.840
	Revenue Maximization	5.67	5295.05	6429.86	1134.81	922.81	1	1	1.169
	Cost Minimization	5.85	5320.08	6415.89	1095.81	927.73	1	1	2.176

Table 26: Results of Instance 25

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.61	8659.37	11059.46	2400.09	2144.04	1	1	1.313
	Profit Maximization	4.60	8667.11	11076.19	2409.08	2140.43	1	1	1.101
	Revenue Maximization	4.47	8614.25	11093.76	2479.50	2136.64	1.443	1	0.693
	Cost Minimization	4.61	8650.53	11049.63	2399.10	2146.16	1	1	1.424
Policy II	Profit / Cost Ratio Maximization	4.45	8291.60	10691.91	2400.31	2143.83	1	1	1.302
	Profit Maximization	4.44	8300.18	10710.66	2410.48	2140.14	1	1	1.082
	Revenue Maximization	4.31	8243.22	10730.13	2486.91	2136.66	1.489	1	0.671
	Cost Minimization	4.45	8281.17	10680.27	2399.10	2146.16	1	1	1.424
Policy III	Profit / Cost Ratio Maximization	4.42	8210.15	10610.40	2400.25	2143.89	1	1	1.305
	Profit Maximization	4.41	8218.08	10627.71	2409.64	2140.31	1	1	1.093
	Revenue Maximization	4.29	8166.00	10645.51	2479.50	2136.64	1.443	1	0.693
	Cost Minimization	4.42	8200.29	10599.38	2399.10	2146.16	1	1	1.424
Policy IV	Profit / Cost Ratio Maximization	4.45	8289.89	10690.17	2400.28	2143.86	1	1	1.303
	Profit Maximization	4.44	8298.25	10708.41	2410.16	2140.21	1	1	1.086
	Revenue Maximization	4.32	8244.50	10727.03	2482.53	2136.64	1.462	1	0.684
	Cost Minimization	4.45	8279.69	10678.79	2399.10	2146.16	1	1	1.424

Table 27: Results of Instance 26

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.76	2428.96	3075.05	646.09	524.99	1	1	2.791
	Profit Maximization	4.75	2430.24	3077.80	647.57	524.24	1	1	2.515
	Revenue Maximization	4.68	2422.78	3080.52	657.74	523.49	1	1	1.958
	Cost Minimization	4.76	2427.61	3073.56	645.95	525.39	1	1	2.919
Policy II	Profit / Cost Ratio Maximization	4.33	2151.76	2798.01	646.24	524.83	1	1	2.738
	Profit Maximization	4.32	2153.72	2802.29	648.58	524.01	1	1	2.415
	Revenue Maximization	4.23	2143.06	2806.35	663.29	523.58	1	1	1.801
	Cost Minimization	4.33	2149.27	2795.22	645.95	525.39	1	1	2.919
Policy III	Profit / Cost Ratio Maximization	4.22	2079.33	2725.52	646.19	524.88	1	1	2.756
	Profit Maximization	4.21	2080.78	2728.70	647.93	524.15	1	1	2.476
	Revenue Maximization	4.15	2073.79	2731.52	657.74	523.49	1	1	1.958
	Cost Minimization	4.22	2077.35	2723.30	645.95	525.39	1	1	2.919
Policy IV	Profit / Cost Ratio Maximization	4.32	2146.72	2792.90	646.19	524.88	1	1	2.756
	Profit Maximization	4.32	2148.25	2796.26	648.00	524.13	1	1	2.468
	Revenue Maximization	4.25	2140.86	2799.22	658.35	523.50	1	1	1.938
	Cost Minimization	4.32	2144.70	2790.65	645.95	525.39	1	1	2.919

Table 28: Results of Instance 27

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.75	2426.91	3074.71	647.80	525.08	1	1	2.821
	Profit Maximization	4.74	2428.26	3077.62	649.36	524.29	1	1	2.536
	Revenue Maximization	4.67	2420.23	3080.52	660.29	523.49	1	1	1.958
	Cost Minimization	4.75	2425.49	3073.14	647.65	525.51	1	1	2.953
Policy II	Profit / Cost Ratio Maximization	4.32	2149.62	2797.58	647.96	524.91	1	1	2.767
	Profit Maximization	4.31	2151.65	2802.06	650.41	524.06	1	1	2.435
	Revenue Maximization	4.21	2140.28	2806.35	666.07	523.58	1	1	1.801
	Cost Minimization	4.32	2147.02	2794.66	647.65	525.51	1	1	2.953
Policy III	Profit / Cost Ratio Maximization	4.21	2077.24	2725.15	647.90	524.97	1	1	2.784
	Profit Maximization	4.20	2078.77	2728.51	649.74	524.19	1	1	2.496
	Revenue Maximization	4.14	2071.23	2731.52	660.29	523.49	1	1	1.958
	Cost Minimization	4.20	2075.16	2722.80	647.65	525.51	1	1	2.953
Policy IV	Profit / Cost Ratio Maximization	4.31	2144.62	2792.52	647.90	524.97	1	1	2.785
	Profit Maximization	4.30	2146.24	2796.06	649.82	524.17	1	1	2.488
	Revenue Maximization	4.24	2138.28	2799.22	660.93	523.50	1	1	1.938
	Cost Minimization	4.31	2142.50	2790.15	647.65	525.51	1	1	2.953

Table 29: Results of Instance 28

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.76	2428.55	3074.98	646.44	525.00	1	1	2.797
	Profit Maximization	4.75	2429.84	3077.77	647.93	524.25	1	1	2.519
	Revenue Maximization	4.68	2422.27	3080.52	658.25	523.49	1	1	1.958
	Cost Minimization	4.76	2427.19	3073.48	646.29	525.42	1	1	2.926
Policy II	Profit / Cost Ratio Maximization	4.33	2151.33	2797.92	646.59	524.85	1	1	2.744
	Profit Maximization	4.32	2153.30	2802.25	648.95	524.02	1	1	2.419
	Revenue Maximization	4.23	2142.50	2806.35	663.85	523.58	1	1	1.801
	Cost Minimization	4.32	2148.82	2795.11	646.29	525.42	1	1	2.926
Policy III	Profit / Cost Ratio Maximization	4.22	2078.91	2725.45	646.53	524.90	1	1	2.761
	Profit Maximization	4.21	2080.37	2728.67	648.29	524.15	1	1	2.480
	Revenue Maximization	4.15	2073.27	2731.52	658.25	523.49	1	1	1.958
	Cost Minimization	4.21	2076.91	2723.20	646.29	525.42	1	1	2.926
Policy IV	Profit / Cost Ratio Maximization	4.32	2146.30	2792.83	646.53	524.90	1	1	2.762
	Profit Maximization	4.31	2147.85	2796.22	648.37	524.14	1	1	2.472
	Revenue Maximization	4.25	2140.35	2799.22	658.87	523.50	1	1	1.938
	Cost Minimization	4.32	2144.26	2790.55	646.29	525.42	1	1	2.926

Table 30: Results of Instance 29

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.40	5916.45	7012.78	1096.33	927.37	1	1	2.125
	Profit Maximization	6.39	5917.92	7015.94	1098.02	925.89	1	1	1.908
	Revenue Maximization	6.17	5883.57	7022.56	1139.00	922.80	1	1	1.140
	Cost Minimization	6.40	5915.66	7011.92	1096.27	927.77	1	1	2.181
Policy II	Profit / Cost Ratio Maximization	5.86	5325.29	6421.72	1096.43	927.14	1	1	2.093
	Profit Maximization	5.85	5328.10	6427.73	1099.63	925.30	1	1	1.813
	Revenue Maximization	5.59	5285.00	6437.62	1152.62	922.90	1	1	1.040
	Cost Minimization	5.86	5323.44	6419.71	1096.27	927.77	1	1	2.181
Policy III	Profit / Cost Ratio Maximization	5.67	5122.10	6218.52	1096.42	927.16	1	1	2.096
	Profit Maximization	5.66	5124.50	6223.67	1099.17	925.44	1	1	1.837
	Revenue Maximization	5.47	5092.59	6231.59	1139.00	922.80	1	1	1.140
	Cost Minimization	5.67	5120.43	6216.69	1096.27	927.77	1	1	2.181
Policy IV	Profit / Cost Ratio Maximization	5.85	5321.08	6417.48	1096.41	927.19	1	1	2.100
	Profit Maximization	5.84	5323.41	6422.45	1099.03	925.49	1	1	1.844
	Revenue Maximization	5.66	5294.19	6429.86	1135.66	922.81	1	1	1.169
	Cost Minimization	5.85	5319.51	6415.78	1096.27	927.77	1	1	2.181

Table 31: Results of Instance 30

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.53	12800.56	15627.92	2827.36	2556.02	1	1	1.258
	Profit Maximization	5.52	12803.77	15634.55	2830.77	2554.21	1	1	1.127
	Revenue Maximization	5.48	12786.85	15640.70	2853.85	2552.54	1	1	0.876
	Cost Minimization	5.53	12797.84	15624.95	2827.11	2556.83	1	1	1.308
Policy II	Profit / Cost Ratio Maximization	5.05	11443.39	14271.01	2827.62	2555.70	1	1	1.237
	Profit Maximization	5.04	11448.37	14281.42	2833.05	2553.71	1	1	1.082
	Revenue Maximization	4.99	11424.21	14290.59	2866.38	2552.72	1	1	0.805
	Cost Minimization	5.05	11438.34	14265.45	2827.11	2556.83	1	1	1.308
Policy III	Profit / Cost Ratio Maximization	4.92	11097.31	13924.84	2827.53	2555.81	1	1	1.244
	Profit Maximization	4.92	11101.03	13932.62	2831.59	2554.01	1	1	1.109
	Revenue Maximization	4.88	11085.16	13939.01	2853.85	2552.54	1	1	0.876
	Cost Minimization	4.92	11093.29	13920.40	2827.11	2556.83	1	1	1.308
Policy IV	Profit / Cost Ratio Maximization	5.04	11432.24	14259.76	2827.52	2555.81	1	1	1.244
	Profit Maximization	5.04	11436.17	14267.94	2831.76	2553.97	1	1	1.106
	Revenue Maximization	5.00	11419.40	14274.64	2855.24	2552.54	1	1	0.867
	Cost Minimization	5.04	11428.15	14255.26	2827.11	2556.83	1	1	1.308

Table 32: Results of Instance 31

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.53	12801.48	15628.08	2826.60	2555.98	1	1	1.255
	Profit Maximization	5.52	12804.66	15634.63	2829.97	2554.19	1	1	1.125
	Revenue Maximization	5.48	12787.99	15640.70	2852.71	2552.54	1	1	0.876
	Cost Minimization	5.53	12798.79	15625.14	2826.34	2556.78	1	1	1.305
Policy II	Profit / Cost Ratio Maximization	5.05	11444.36	14271.21	2826.85	2555.66	1	1	1.234
	Profit Maximization	5.04	11449.29	14281.52	2832.23	2553.69	1	1	1.080
	Revenue Maximization	4.99	11425.45	14290.59	2865.13	2552.72	1	1	0.805
	Cost Minimization	5.05	11439.35	14265.70	2826.34	2556.78	1	1	1.305
Policy III	Profit / Cost Ratio Maximization	4.93	11098.25	13925.01	2826.76	2555.77	1	1	1.241
	Profit Maximization	4.92	11101.94	13932.71	2830.77	2553.99	1	1	1.107
	Revenue Maximization	4.89	11086.30	13939.01	2852.71	2552.54	1	1	0.876
	Cost Minimization	4.93	11094.27	13920.62	2826.34	2556.78	1	1	1.305
Policy IV	Profit / Cost Ratio Maximization	5.04	11433.19	14259.94	2826.76	2555.77	1	1	1.241
	Profit Maximization	5.04	11437.08	14268.03	2830.95	2553.95	1	1	1.104
	Revenue Maximization	5.00	11420.55	14274.64	2854.09	2552.54	1	1	0.867
	Cost Minimization	5.04	11429.14	14255.48	2826.34	2556.78	1	1	1.305

Table 33: Results of Instance 32

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.60	8626.33	11023.12	2396.79	2151.88	1	1	1.330
	Profit Maximization	4.59	8630.01	11030.89	2400.89	2150.20	1	1	1.183
	Revenue Maximization	4.56	8615.63	11036.90	2421.27	2148.90	1.066	1	0.938
	Cost Minimization	4.60	8621.71	11017.98	2396.28	2152.98	1	1	1.411
Policy II	Profit / Cost Ratio Maximization	4.45	8258.17	10655.10	2396.93	2151.74	1	1	1.320
	Profit Maximization	4.44	8262.45	10664.25	2401.80	2150.02	1	1	1.164
	Revenue Maximization	4.40	8245.68	10671.36	2425.69	2148.93	1.103	1	0.907
	Cost Minimization	4.44	8252.44	10648.71	2396.28	2152.98	1	1	1.411
Policy III	Profit / Cost Ratio Maximization	4.41	8175.17	10572.04	2396.87	2151.80	1	1	1.324
	Profit Maximization	4.41	8178.92	10580.04	2401.12	2150.15	1	1	1.178
	Revenue Maximization	4.37	8164.81	10586.08	2421.27	2148.90	1.066	1	0.938
	Cost Minimization	4.41	8170.03	10566.31	2396.28	2152.98	1	1	1.411
Policy IV	Profit / Cost Ratio Maximization	4.44	8254.86	10651.75	2396.89	2151.78	1	1	1.323
	Profit Maximization	4.44	8258.86	10660.28	2401.42	2150.09	1	1	1.171
	Revenue Maximization	4.40	8243.72	10666.78	2423.06	2148.91	1.082	1	0.925
	Cost Minimization	4.44	8249.46	10645.74	2396.28	2152.98	1	1	1.411

Table 34: Results of Instance 33

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.76	8717.82	11035.96	2318.14	2149.11	1.350	1	1.004
	Profit Maximization	4.76	8718.09	11036.52	2318.43	2148.99	1.262	1	0.977
	Revenue Maximization	4.76	8717.23	11036.90	2319.67	2148.90	1.066	1	0.938
	Cost Minimization	4.76	8717.47	11035.58	2318.10	2149.19	1.387	1	1.020
Policy II	Profit / Cost Ratio Maximization	4.60	8352.07	10670.23	2318.16	2149.10	1.404	1	0.997
	Profit Maximization	4.60	8352.41	10670.92	2318.51	2149.01	1.413	1	0.959
	Revenue Maximization	4.60	8350.99	10671.36	2320.38	2148.93	1.103	1	0.907
	Cost Minimization	4.60	8351.57	10669.67	2318.10	2149.19	1.387	1	1.020
Policy III	Profit / Cost Ratio Maximization	4.57	8266.98	10585.13	2318.15	2149.10	1.347	1	1.003
	Profit Maximization	4.57	8267.26	10585.70	2318.44	2148.98	1.258	1	0.976
	Revenue Maximization	4.56	8266.41	10586.08	2319.67	2148.90	1.066	1	0.938
	Cost Minimization	4.57	8266.60	10584.70	2318.10	2149.19	1.387	1	1.020
Policy IV	Profit / Cost Ratio Maximization	4.60	8347.82	10665.96	2318.14	2149.11	1.398	1	1.000
	Profit Maximization	4.60	8348.06	10666.45	2318.39	2149.02	1.398	1	0.968
	Revenue Maximization	4.60	8346.85	10666.78	2319.94	2148.91	1.081	1	0.925
	Cost Minimization	4.60	8346.61	10664.80	2318.19	2149.34	1.387	1	1.049

Table 35: Results of Instance 34

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.72	5963.12	7006.37	1043.25	930.36	1	1	1.587
	Profit Maximization	6.72	5963.13	7006.40	1043.26	930.35	1	1	1.568
	Revenue Maximization	6.71	5963.02	7006.43	1043.42	930.33	1	1	1.517
	Cost Minimization	6.72	5963.11	7006.36	1043.25	930.37	1	1	1.592
Policy II	Profit / Cost Ratio Maximization	6.15	5375.34	6418.61	1043.26	930.35	1	1	1.569
	Profit Maximization	6.15	5375.60	6419.12	1043.52	930.33	1	1	1.496
	Revenue Maximization	6.14	5374.23	6419.61	1045.38	930.56	1	1	1.340
	Cost Minimization	6.15	5375.17	6418.41	1043.25	930.37	1	1	1.592
Policy III	Profit / Cost Ratio Maximization	5.95	5165.67	6208.92	1043.25	930.36	1	1	1.584
	Profit Maximization	5.95	5165.69	6208.97	1043.27	930.34	1	1	1.562
	Revenue Maximization	5.95	5165.59	6209.01	1043.42	930.33	1	1	1.517
	Cost Minimization	5.95	5165.65	6208.90	1043.25	930.37	1	1	1.592
Policy IV	Profit / Cost Ratio Maximization	6.14	5364.40	6407.65	1043.25	930.36	1	1	1.587
	Profit Maximization	6.14	5364.42	6407.68	1043.26	930.35	1	1	1.571
	Revenue Maximization	6.14	5364.37	6407.70	1043.33	930.33	1	1	1.538
	Cost Minimization	6.14	5364.39	6407.64	1043.25	930.37	1	1	1.592

Table 36: Results of Instance 35

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.73	12941.36	15680.27	2738.90	2541.74	1	1	0.921
	Profit Maximization	5.72	12943.50	15684.63	2741.12	2540.56	1	1	0.829
	Revenue Maximization	5.69	12933.05	15688.51	2755.46	2539.50	1	1	0.658
	Cost Minimization	5.72	12939.62	15678.37	2738.75	2542.26	1	1	0.955
Policy II	Profit / Cost Ratio Maximization	5.23	11590.51	14329.58	2739.07	2541.53	1	1	0.906
	Profit Maximization	5.23	11593.92	14336.62	2742.70	2540.21	1	1	0.796
	Revenue Maximization	5.19	11578.61	14342.56	2763.96	2539.63	1	1	0.606
	Cost Minimization	5.23	11587.19	14325.94	2738.75	2542.26	1	1	0.955
Policy III	Profit / Cost Ratio Maximization	5.11	11247.36	13986.36	2739.00	2541.61	1	1	0.912
	Profit Maximization	5.10	11249.85	13991.49	2741.65	2540.42	1	1	0.817
	Revenue Maximization	5.08	11240.05	13995.51	2755.46	2539.50	1	1	0.658
	Cost Minimization	5.11	11244.78	13983.53	2738.75	2542.26	1	1	0.955
Policy IV	Profit / Cost Ratio Maximization	5.23	11582.91	14321.92	2739.01	2541.60	1	1	0.911
	Profit Maximization	5.23	11585.63	14327.50	2741.87	2540.37	1	1	0.812
	Revenue Maximization	5.20	11574.76	14331.91	2757.15	2539.50	1	1	0.646
	Cost Minimization	5.23	11580.21	14318.96	2738.75	2542.26	1	1	0.955

Table 37: Results of Instance 36

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.76	8718.26	11036.15	2317.89	2149.07	1	1	1.019
	Profit Maximization	4.76	8718.50	11036.63	2318.13	2148.96	1.014	1	0.986
	Revenue Maximization	4.76	8717.96	11036.90	2318.94	2148.90	1.066	1	0.938
	Cost Minimization	4.76	8717.86	11035.72	2317.85	2149.16	1	1	1.041
Policy II	Profit / Cost Ratio Maximization	4.60	8352.07	10669.99	2317.92	2149.04	1	1	1.012
	Profit Maximization	4.60	8352.51	10670.88	2318.37	2148.93	1.033	1	0.968
	Revenue Maximization	4.60	8351.58	10671.36	2319.79	2148.93	1.103	1	0.907
	Cost Minimization	4.60	8351.45	10669.30	2317.85	2149.14	1	1	1.037
Policy III	Profit / Cost Ratio Maximization	4.57	8267.43	10585.32	2317.89	2149.06	1	1	1.018
	Profit Maximization	4.57	8267.66	10585.81	2318.14	2148.96	1.015	1	0.985
	Revenue Maximization	4.57	8267.14	10586.08	2318.94	2148.90	1.066	1	0.938
	Cost Minimization	4.57	8267.14	10584.99	2317.85	2149.13	1	1	1.034
Policy IV	Profit / Cost Ratio Maximization	4.60	8347.87	10665.78	2317.90	2149.05	1	1	1.015
	Profit Maximization	4.60	8348.19	10666.43	2318.24	2148.94	1.023	1	0.977
	Revenue Maximization	4.60	8347.52	10666.78	2319.26	2148.91	1.081	1	0.925
	Cost Minimization	4.60	8347.40	10665.26	2317.85	2149.14	1	1	1.037

Table 38: Results of Instance 37

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.72	5963.76	7006.38	1042.62	930.36	1	1	1.580
	Profit Maximization	6.72	5963.77	7006.40	1042.63	930.35	1	1	1.563
	Revenue Maximization	6.72	5963.68	7006.43	1042.76	930.33	1	1	1.517
	Cost Minimization	6.72	5963.75	7006.37	1042.62	930.36	1	1	1.585
Policy II	Profit / Cost Ratio Maximization	6.16	5376.02	6418.66	1042.63	930.34	1	1	1.562
	Profit Maximization	6.16	5376.27	6419.15	1042.88	930.34	1	1	1.492
	Revenue Maximization	6.15	5374.98	6419.61	1044.63	930.56	1	1	1.340
	Cost Minimization	6.16	5375.86	6418.47	1042.62	930.36	1	1	1.585
Policy III	Profit / Cost Ratio Maximization	5.96	5166.31	6208.93	1042.62	930.36	1	1	1.578
	Profit Maximization	5.96	5166.33	6208.97	1042.64	930.34	1	1	1.558
	Revenue Maximization	5.95	5166.25	6209.01	1042.76	930.33	1	1	1.517
	Cost Minimization	5.96	5166.30	6208.92	1042.62	930.36	1	1	1.585
Policy IV	Profit / Cost Ratio Maximization	6.15	5365.04	6407.66	1042.62	930.36	1	1	1.580
	Profit Maximization	6.15	5365.05	6407.68	1042.63	930.35	1	1	1.566
	Revenue Maximization	6.15	5365.02	6407.70	1042.68	930.33	1	1	1.538
	Cost Minimization	6.15	5365.04	6407.66	1042.62	930.36	1	1	1.585

Table 39: Results of Instance 38

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.71	12902.49	15640.20	2737.71	2552.67	1	1	0.941
	Profit Maximization	5.71	12902.64	15640.50	2737.86	2552.59	1	1	0.916
	Revenue Maximization	5.71	12902.20	15640.70	2738.50	2552.54	1	1	0.876
	Cost Minimization	5.71	12902.35	15640.05	2737.70	2552.71	1	1	0.951
Policy II	Profit / Cost Ratio Maximization	5.22	11550.71	14288.48	2737.77	2552.63	1	1	0.928
	Profit Maximization	5.22	11551.36	14289.77	2738.42	2552.54	1	1	0.880
	Revenue Maximization	5.21	11549.61	14290.59	2740.98	2552.72	1	1	0.805
	Cost Minimization	5.22	11550.00	14287.70	2737.70	2552.71	1	1	0.951
Policy III	Profit / Cost Ratio Maximization	5.09	11200.75	13938.46	2737.72	2552.66	1	1	0.938
	Profit Maximization	5.09	11200.92	13938.81	2737.89	2552.58	1	1	0.913
	Revenue Maximization	5.09	11200.51	13939.01	2738.50	2552.54	1	1	0.876
	Cost Minimization	5.09	11200.54	13938.24	2737.70	2552.71	1	1	0.951
Policy IV	Profit / Cost Ratio Maximization	5.21	11536.23	14273.95	2737.72	2552.66	1	1	0.937
	Profit Maximization	5.21	11536.45	14274.39	2737.94	2552.57	1	1	0.909
	Revenue Maximization	5.21	11535.93	14274.64	2738.72	2552.54	1	1	0.867
	Cost Minimization	5.21	11535.98	14273.68	2737.70	2552.71	1	1	0.951

Table 40: Results of Instance 39

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.79	8770.59	11084.87	2314.27	2138.56	1.036	1	0.965
	Profit Maximization	4.79	8773.06	11089.99	2316.94	2137.45	1.163	1	0.860
	Revenue Maximization	4.76	8764.36	11093.76	2329.39	2136.64	1.443	1	0.693
	Cost Minimization	4.79	8767.70	11081.66	2313.96	2139.25	1	1	1.019
Policy II	Profit / Cost Ratio Maximization	4.63	8405.13	10719.51	2314.38	2138.46	1.046	1	0.956
	Profit Maximization	4.63	8408.10	10725.71	2317.61	2137.32	1.185	1	0.844
	Revenue Maximization	4.60	8398.10	10730.13	2332.03	2136.66	1.489	1	0.671
	Cost Minimization	4.63	8401.36	10715.32	2313.96	2139.25	1	1	1.019
Policy III	Profit / Cost Ratio Maximization	4.60	8322.13	10636.45	2314.32	2138.51	1.041	1	0.961
	Profit Maximization	4.59	8324.64	10641.73	2317.08	2137.42	1.168	1	0.856
	Revenue Maximization	4.57	8316.12	10645.51	2329.39	2136.64	1.443	1	0.693
	Cost Minimization	4.60	8318.90	10632.87	2313.96	2139.25	1	1	1.019
Policy IV	Profit / Cost Ratio Maximization	4.63	8402.85	10717.21	2314.36	2138.48	1.044	1	0.958
	Profit Maximization	4.63	8405.62	10722.98	2317.36	2137.37	1.177	1	0.850
	Revenue Maximization	4.60	8396.57	10727.03	2330.46	2136.64	1.462	1	0.684
	Cost Minimization	4.63	8399.30	10713.26	2313.96	2139.25	1	1	1.019

Table 41: Results of Instance 40

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.52	12796.89	15627.30	2830.42	2556.19	1	1	1.269
	Profit Maximization	5.52	12800.24	15634.21	2833.98	2554.31	1	1	1.134
	Revenue Maximization	5.47	12782.28	15640.70	2858.42	2552.54	1	1	0.876
	Cost Minimization	5.52	12794.06	15624.21	2830.15	2557.03	1	1	1.321
Policy II	Profit / Cost Ratio Maximization	5.04	11439.53	14270.21	2830.68	2555.86	1	1	1.248
	Profit Maximization	5.04	11444.68	14281.00	2836.32	2553.79	1	1	1.089
	Revenue Maximization	4.98	11419.25	14290.59	2871.34	2552.72	1	1	0.805
	Cost Minimization	5.04	11434.31	14264.46	2830.15	2557.03	1	1	1.321
Policy III	Profit / Cost Ratio Maximization	4.92	11093.55	13924.14	2830.59	2555.97	1	1	1.254
	Profit Maximization	4.91	11097.44	13932.27	2834.83	2554.09	1	1	1.116
	Revenue Maximization	4.88	11080.59	13939.01	2858.42	2552.54	1	1	0.876
	Cost Minimization	4.92	11089.37	13919.52	2830.15	2557.03	1	1	1.321
Policy IV	Profit / Cost Ratio Maximization	5.04	11428.47	14259.05	2830.59	2555.97	1	1	1.255
	Profit Maximization	5.03	11432.57	14267.58	2835.01	2554.05	1	1	1.113
	Revenue Maximization	4.99	11414.79	14274.64	2859.86	2552.54	1	1	0.867
	Cost Minimization	5.04	11424.21	14254.37	2830.15	2557.03	1	1	1.321

Table 42: Results of Instance 41

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.59	8621.92	11022.26	2400.34	2152.06	1.007	1	1.344
	Profit Maximization	4.59	8625.81	11030.49	2404.68	2150.29	1.035	1	1.190
	Revenue Maximization	4.55	8610.63	11036.90	2426.27	2148.90	1.066	1	0.938
	Cost Minimization	4.59	8617.05	11016.85	2399.80	2153.23	1	1	1.428
Policy II	Profit / Cost Ratio Maximization	4.44	8253.81	10654.31	2400.51	2151.92	1.051	1	1.331
	Profit Maximization	4.43	8258.34	10663.98	2405.63	2150.12	1.160	1	1.168
	Revenue Maximization	4.39	8240.68	10671.36	2430.69	2148.93	1.103	1	0.907
	Cost Minimization	4.44	8251.24	10651.24	2400.00	2152.49	1	1	1.376
Policy III	Profit / Cost Ratio Maximization	4.40	8170.75	10571.17	2400.43	2151.98	1.008	1	1.338
	Profit Maximization	4.40	8174.71	10579.64	2404.93	2150.23	1.036	1	1.185
	Revenue Maximization	4.36	8159.81	10586.08	2426.27	2148.90	1.066	1	0.938
	Cost Minimization	4.40	8165.32	10565.12	2399.80	2153.23	1	1	1.428
Policy IV	Profit / Cost Ratio Maximization	4.44	8250.52	10650.98	2400.47	2151.96	1.049	1	1.334
	Profit Maximization	4.43	8254.76	10660.01	2405.25	2150.19	1.150	1	1.176
	Revenue Maximization	4.39	8238.72	10666.78	2428.06	2148.91	1.081	1	0.925
	Cost Minimization	4.44	8245.70	10645.51	2399.81	2153.03	1	1	1.414

Table 43: Results of Instance 42

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.39	5906.23	7002.53	1096.30	932.15	1	1	2.141
	Profit Maximization	6.38	5906.93	7004.01	1097.08	931.46	1	1	1.991
	Revenue Maximization	6.32	5897.08	7006.43	1109.35	930.33	1	1	1.517
	Cost Minimization	6.39	5905.83	7002.10	1096.27	932.35	1	1	2.181
Policy II	Profit / Cost Ratio Maximization	5.85	5314.80	6411.17	1096.37	932.00	1	1	2.110
	Profit Maximization	5.84	5316.53	6414.81	1098.28	931.07	1	1	1.890
	Revenue Maximization	5.73	5299.60	6419.61	1120.01	930.56	1	1	1.340
	Cost Minimization	5.85	5313.62	6409.88	1096.27	932.35	1	1	2.181
Policy III	Profit / Cost Ratio Maximization	5.66	5107.50	6203.84	1096.34	932.05	1	1	2.121
	Profit Maximization	5.65	5108.62	6206.20	1097.58	931.27	1	1	1.943
	Revenue Maximization	5.60	5099.65	6209.01	1109.35	930.33	1	1	1.517
	Cost Minimization	5.66	5106.67	6202.94	1096.27	932.35	1	1	2.181
Policy IV	Profit / Cost Ratio Maximization	5.84	5306.52	6402.85	1096.33	932.07	1	1	2.124
	Profit Maximization	5.84	5307.60	6405.10	1097.50	931.29	1	1	1.949
	Revenue Maximization	5.78	5299.36	6407.70	1108.33	930.33	1	1	1.538
	Cost Minimization	5.84	5305.76	6402.02	1096.27	932.35	1	1	2.181

Table 44: Results of Instance 43

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.54	12830.94	15658.52	2827.59	2547.68	1	1	1.240
	Profit Maximization	5.53	12837.38	15672.09	2834.70	2543.98	1	1	1.055
	Revenue Maximization	5.40	12781.14	15688.51	2907.37	2539.50	1	1	0.658
	Cost Minimization	5.54	12825.87	15652.98	2827.11	2549.19	1	1	1.308
Policy II	Profit / Cost Ratio Maximization	5.06	11474.61	14302.56	2827.96	2547.20	1	1	1.218
	Profit Maximization	5.05	11483.38	14321.26	2837.88	2543.22	1	1	1.013
	Revenue Maximization	4.90	11413.46	14342.56	2929.10	2539.63	1	1	0.606
	Cost Minimization	5.06	11466.36	14293.47	2827.11	2549.19	1	1	1.308
Policy III	Profit / Cost Ratio Maximization	4.94	11133.95	13961.85	2827.90	2547.27	1	1	1.220
	Profit Maximization	4.93	11141.56	13978.10	2836.55	2543.51	1	1	1.030
	Revenue Maximization	4.81	11088.14	13995.51	2907.37	2539.50	1	1	0.658
	Cost Minimization	4.94	11126.41	13953.52	2827.11	2549.19	1	1	1.308
Policy IV	Profit / Cost Ratio Maximization	5.06	11468.87	14296.76	2827.89	2547.28	1	1	1.221
	Profit Maximization	5.05	11476.84	14313.68	2836.84	2543.45	1	1	1.026
	Revenue Maximization	4.92	11420.08	14331.91	2911.83	2539.50	1	1	0.646
	Cost Minimization	5.05	11461.27	14288.38	2827.11	2549.19	1	1	1.308

Table 45: Results of Instance 44

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.59	8622.80	11022.43	2399.63	2152.02	1	1	1.342
	Profit Maximization	4.59	8626.64	11030.56	2403.92	2150.27	1	1	1.190
	Revenue Maximization	4.55	8611.36	11036.90	2425.54	2148.90	1.066	1	0.938
	Cost Minimization	4.59	8617.98	11017.08	2399.10	2153.18	1	1	1.424
Policy II	Profit / Cost Ratio Maximization	4.44	8254.57	10654.35	2399.78	2151.89	1	1	1.331
	Profit Maximization	4.43	8259.02	10663.88	2404.85	2150.09	1	1	1.171
	Revenue Maximization	4.39	8241.27	10671.36	2430.10	2148.93	1.103	1	0.907
	Cost Minimization	4.44	8248.61	10647.71	2399.10	2153.18	1	1	1.424
Policy III	Profit / Cost Ratio Maximization	4.41	8171.62	10571.34	2399.72	2151.95	1	1	1.336
	Profit Maximization	4.40	8175.53	10579.70	2404.16	2150.22	1	1	1.185
	Revenue Maximization	4.36	8160.54	10586.08	2425.54	2148.90	1.066	1	0.938
	Cost Minimization	4.40	8166.26	10565.36	2399.10	2153.18	1	1	1.424
Policy IV	Profit / Cost Ratio Maximization	4.44	8251.28	10651.02	2399.74	2151.92	1	1	1.334
	Profit Maximization	4.43	8255.45	10659.92	2404.47	2150.16	1	1	1.179
	Revenue Maximization	4.39	8239.40	10666.78	2427.38	2148.91	1.081	1	0.925
	Cost Minimization	4.44	8245.67	10644.77	2399.10	2153.18	1	1	1.424

Table 46: Results of Instance 45

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.39	5906.74	7002.58	1095.84	932.13	1	1	2.136
	Profit Maximization	6.39	5907.43	7004.05	1096.61	931.45	1	1	1.987
	Revenue Maximization	6.32	5897.74	7006.43	1108.69	930.33	1	1	1.517
	Cost Minimization	6.39	5906.35	7002.15	1095.81	932.33	1	1	2.176
Policy II	Profit / Cost Ratio Maximization	5.85	5315.35	6411.26	1095.91	931.98	1	1	2.105
	Profit Maximization	5.84	5317.06	6414.86	1097.80	931.06	1	1	1.886
	Revenue Maximization	5.74	5300.34	6419.61	1119.27	930.56	1	1	1.340
	Cost Minimization	5.85	5314.17	6409.98	1095.81	932.33	1	1	2.176
Policy III	Profit / Cost Ratio Maximization	5.66	5108.03	6203.91	1095.88	932.03	1	1	2.116
	Profit Maximization	5.66	5109.14	6206.24	1097.10	931.25	1	1	1.939
	Revenue Maximization	5.60	5100.31	6209.01	1108.69	930.33	1	1	1.517
	Cost Minimization	5.66	5107.21	6203.02	1095.81	932.33	1	1	2.176
Policy IV	Profit / Cost Ratio Maximization	5.84	5307.05	6402.92	1095.87	932.05	1	1	2.119
	Profit Maximization	5.84	5308.11	6405.14	1097.03	931.28	1	1	1.946
	Revenue Maximization	5.78	5300.01	6407.70	1107.68	930.33	1	1	1.538
	Cost Minimization	5.84	5306.29	6402.10	1095.81	932.33	1	1	2.176

Table 47: Results of Instance 46

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.54	12831.92	15658.74	2826.82	2547.62	1	1	1.237
	Profit Maximization	5.53	12838.33	15672.22	2833.89	2543.94	1	1	1.053
	Revenue Maximization	5.40	12782.66	15688.51	2905.86	2539.50	1	1	0.658
	Cost Minimization	5.54	12826.88	15653.23	2826.34	2549.12	1	1	1.305
Policy II	Profit / Cost Ratio Maximization	5.06	11475.64	14302.83	2827.19	2547.15	1	1	1.215
	Profit Maximization	5.05	11484.36	14321.42	2837.05	2543.19	1	1	1.011
	Revenue Maximization	4.90	11415.11	14342.56	2927.45	2539.63	1	1	0.606
	Cost Minimization	5.06	11467.44	14293.79	2826.34	2549.12	1	1	1.305
Policy III	Profit / Cost Ratio Maximization	4.94	11134.97	13962.10	2827.13	2547.21	1	1	1.218
	Profit Maximization	4.93	11142.53	13978.25	2835.72	2543.48	1	1	1.028
	Revenue Maximization	4.82	11089.66	13995.51	2905.86	2539.50	1	1	0.658
	Cost Minimization	4.94	11127.47	13953.81	2826.34	2549.12	1	1	1.305
Policy IV	Profit / Cost Ratio Maximization	5.06	11469.89	14297.01	2827.12	2547.22	1	1	1.218
	Profit Maximization	5.05	11477.81	14313.83	2836.02	2543.41	1	1	1.024
	Revenue Maximization	4.92	11421.63	14331.91	2910.28	2539.50	1	1	0.646
	Cost Minimization	5.06	11462.34	14288.68	2826.34	2549.12	1	1	1.305

Table 48: Results of Instance 47

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.77	2430.61	3075.32	644.71	524.91	1	1	2.767
	Profit Maximization	4.76	2431.83	3077.95	646.12	524.20	1	1	2.498
	Revenue Maximization	4.70	2424.82	3080.52	655.69	523.49	1	1	1.958
	Cost Minimization	4.77	2429.32	3073.89	644.57	525.30	1	1	2.891
Policy II	Profit / Cost Ratio Maximization	4.34	2153.49	2798.35	644.85	524.76	1	1	2.715
	Profit Maximization	4.33	2155.38	2802.48	647.10	523.98	1	1	2.398
	Revenue Maximization	4.25	2145.28	2806.35	661.07	523.58	1	1	1.801
	Cost Minimization	4.34	2151.09	2795.66	644.57	525.30	1	1	2.891
Policy III	Profit / Cost Ratio Maximization	4.23	2081.02	2725.81	644.80	524.81	1	1	2.732
	Profit Maximization	4.22	2082.40	2728.86	646.46	524.11	1	1	2.460
	Revenue Maximization	4.17	2075.83	2731.52	655.69	523.49	1	1	1.958
	Cost Minimization	4.23	2079.12	2723.69	644.57	525.30	1	1	2.891
Policy IV	Profit / Cost Ratio Maximization	4.33	2148.41	2793.21	644.80	524.81	1	1	2.733
	Profit Maximization	4.33	2149.88	2796.42	646.54	524.09	1	1	2.452
	Revenue Maximization	4.27	2142.93	2799.22	656.29	523.50	1	1	1.938
	Cost Minimization	4.33	2146.48	2791.05	644.57	525.30	1	1	2.891

Table 49: Results of Instance 48

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.61	8663.21	11060.45	2397.24	2143.82	1	1	1.301
	Profit Maximization	4.60	8670.75	11076.76	2406.01	2140.31	1	1	1.093
	Revenue Maximization	4.48	8620.02	11093.76	2473.73	2136.64	1.443	1	0.693
	Cost Minimization	4.61	8654.57	11050.84	2396.28	2145.90	1	1	1.411
Policy II	Profit / Cost Ratio Maximization	4.46	8295.50	10692.96	2397.46	2143.62	1	1	1.290
	Profit Maximization	4.45	8303.90	10711.28	2407.38	2140.03	1	1	1.074
	Revenue Maximization	4.33	8249.18	10730.13	2480.95	2136.66	1.489	1	0.671
	Cost Minimization	4.46	8285.30	10681.57	2396.28	2145.90	1	1	1.411
Policy III	Profit / Cost Ratio Maximization	4.43	8214.01	10611.41	2397.40	2143.68	1	1	1.293
	Profit Maximization	4.42	8221.75	10628.29	2406.54	2140.19	1	1	1.085
	Revenue Maximization	4.30	8171.78	10645.51	2473.73	2136.64	1.443	1	0.693
	Cost Minimization	4.42	8204.38	10600.66	2396.28	2145.90	1	1	1.411
Policy IV	Profit / Cost Ratio Maximization	4.46	8293.78	10691.21	2397.43	2143.65	1	1	1.292
	Profit Maximization	4.45	8301.95	10709.01	2407.06	2140.09	1	1	1.078
	Revenue Maximization	4.33	8250.34	10727.03	2476.69	2136.64	1.462	1	0.684
	Cost Minimization	4.46	8283.81	10680.09	2396.28	2145.90	1	1	1.411

Table 50: Results of Instance 49

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.70	5960.57	7006.31	1045.74	930.39	1	1	1.613
	Profit Maximization	6.70	5960.60	7006.37	1045.77	930.36	1	1	1.587
	Revenue Maximization	6.70	5960.38	7006.43	1046.05	930.33	1	1	1.517
	Cost Minimization	6.70	5960.55	7006.29	1045.74	930.40	1	1	1.619
Policy II	Profit / Cost Ratio Maximization	6.14	5372.63	6418.39	1045.76	930.37	1	1	1.594
	Profit Maximization	6.14	5372.94	6419.01	1046.07	930.33	1	1	1.514
	Revenue Maximization	6.12	5371.25	6419.61	1048.36	930.56	1	1	1.340
	Cost Minimization	6.14	5372.42	6418.16	1045.74	930.40	1	1	1.619
Policy III	Profit / Cost Ratio Maximization	5.94	5163.10	6208.85	1045.74	930.38	1	1	1.609
	Profit Maximization	5.94	5163.15	6208.93	1045.79	930.36	1	1	1.579
	Revenue Maximization	5.94	5162.95	6209.01	1046.05	930.33	1	1	1.517
	Cost Minimization	5.94	5163.07	6208.81	1045.74	930.40	1	1	1.619
Policy IV	Profit / Cost Ratio Maximization	6.13	5361.86	6407.60	1045.74	930.39	1	1	1.612
	Profit Maximization	6.13	5361.88	6407.65	1045.77	930.36	1	1	1.587
	Revenue Maximization	6.13	5361.77	6407.70	1045.93	930.33	1	1	1.538
	Cost Minimization	6.13	5361.84	6407.57	1045.74	930.40	1	1	1.619

Table 51: Results of Instance 50

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.72	12936.38	15679.45	2743.07	2541.97	1	1	0.936
	Profit Maximization	5.71	12938.71	15684.20	2745.49	2540.67	1	1	0.840
	Revenue Maximization	5.68	12926.97	15688.51	2761.54	2539.50	1	1	0.658
	Cost Minimization	5.72	12934.50	15677.40	2742.90	2542.53	1	1	0.972
Policy II	Profit / Cost Ratio Maximization	5.22	11585.27	14328.52	2743.25	2541.74	1	1	0.921
	Profit Maximization	5.22	11588.93	14336.08	2747.15	2540.31	1	1	0.806
	Revenue Maximization	5.18	11572.00	14342.56	2770.56	2539.63	1	1	0.606
	Cost Minimization	5.22	11581.73	14324.63	2742.90	2542.53	1	1	0.972
Policy III	Profit / Cost Ratio Maximization	5.10	11242.27	13985.45	2743.18	2541.82	1	1	0.926
	Profit Maximization	5.09	11244.98	13991.05	2746.07	2540.53	1	1	0.827
	Revenue Maximization	5.07	11233.97	13995.51	2761.54	2539.50	1	1	0.658
	Cost Minimization	5.10	11239.48	13982.38	2742.90	2542.53	1	1	0.972
Policy IV	Profit / Cost Ratio Maximization	5.22	11577.79	14320.98	2743.19	2541.81	1	1	0.926
	Profit Maximization	5.22	11580.74	14327.04	2746.30	2540.48	1	1	0.822
	Revenue Maximization	5.19	11568.58	14331.91	2763.34	2539.50	1	1	0.646
	Cost Minimization	5.22	11574.87	14317.77	2742.90	2542.53	1	1	0.972

Table 52: Results of Instance 51

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.08	2473.50	3080.28	606.78	523.56	1	1	2.110
	Profit Maximization	5.08	2473.57	3080.42	606.85	523.52	1	1	2.054
	Revenue Maximization	5.07	2473.35	3080.52	607.17	523.49	1	1	1.958
	Cost Minimization	5.08	2473.42	3080.19	606.77	523.58	1	1	2.135
Policy II	Profit / Cost Ratio Maximization	4.62	2198.58	2805.39	606.81	523.54	1	1	2.078
	Profit Maximization	4.62	2198.85	2805.96	607.11	523.50	1	1	1.972
	Revenue Maximization	4.61	2198.02	2806.35	608.32	523.58	1	1	1.801
	Cost Minimization	4.62	2198.22	2804.99	606.77	523.58	1	1	2.135
Policy III	Profit / Cost Ratio Maximization	4.50	2124.48	2731.26	606.78	523.55	1	1	2.103
	Profit Maximization	4.50	2124.56	2731.42	606.87	523.52	1	1	2.047
	Revenue Maximization	4.50	2124.35	2731.52	607.17	523.49	1	1	1.958
	Cost Minimization	4.50	2124.37	2731.14	606.77	523.58	1	1	2.135
Policy IV	Profit / Cost Ratio Maximization	4.61	2192.10	2798.89	606.79	523.55	1	1	2.101
	Profit Maximization	4.61	2192.20	2799.09	606.89	523.51	1	1	2.037
	Revenue Maximization	4.61	2191.94	2799.22	607.27	523.50	1	1	1.938
	Cost Minimization	4.61	2191.97	2798.74	606.77	523.58	1	1	2.135

Table 53: Results of Instance 52

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.78	8766.01	11084.44	2318.43	2138.65	1.405	1	0.965
	Profit Maximization	4.78	8768.58	11089.79	2321.21	2137.50	1.433	1	0.860
	Revenue Maximization	4.75	8759.36	11093.76	2334.39	2136.64	1.443	1	0.693
	Cost Minimization	4.78	8762.94	11081.04	2318.10	2139.38	1.387	1	1.020
Policy II	Profit / Cost Ratio Maximization	4.62	8400.88	10719.41	2318.52	2138.57	1.463	1	0.956
	Profit Maximization	4.62	8403.85	10725.62	2321.77	2137.41	1.604	1	0.844
	Revenue Maximization	4.59	8393.10	10730.13	2337.03	2136.66	1.489	1	0.671
	Cost Minimization	4.62	8397.03	10715.13	2318.10	2139.38	1.387	1	1.020
Policy III	Profit / Cost Ratio Maximization	4.59	8317.53	10636.02	2318.48	2138.60	1.406	1	0.961
	Profit Maximization	4.58	8320.16	10641.52	2321.37	2137.46	1.434	1	0.856
	Revenue Maximization	4.56	8311.12	10645.51	2334.39	2136.64	1.443	1	0.693
	Cost Minimization	4.59	8314.12	10632.22	2318.10	2139.38	1.387	1	1.020
Policy IV	Profit / Cost Ratio Maximization	4.62	8398.60	10717.10	2318.50	2138.59	1.460	1	0.958
	Profit Maximization	4.62	8401.37	10722.89	2321.52	2137.46	1.593	1	0.850
	Revenue Maximization	4.59	8391.57	10727.03	2335.46	2136.64	1.462	1	0.684
	Cost Minimization	4.62	8394.97	10713.08	2318.10	2139.38	1.387	1	1.020

Table 54: Results of Instance 53

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.73	12941.36	15680.27	2738.90	2541.74	1	1	0.921
	Profit Maximization	5.72	12943.50	15684.63	2741.12	2540.56	1	1	0.829
	Revenue Maximization	5.69	12933.05	15688.51	2755.46	2539.50	1	1	0.658
	Cost Minimization	5.72	12939.62	15678.37	2738.75	2542.26	1	1	0.955
Policy II	Profit / Cost Ratio Maximization	5.23	11590.51	14329.58	2739.07	2541.53	1	1	0.906
	Profit Maximization	5.23	11593.92	14336.62	2742.70	2540.21	1	1	0.796
	Revenue Maximization	5.19	11578.61	14342.56	2763.96	2539.63	1	1	0.606
	Cost Minimization	5.23	11587.19	14325.94	2738.75	2542.26	1	1	0.955
Policy III	Profit / Cost Ratio Maximization	5.11	11247.36	13986.36	2739.00	2541.61	1	1	0.912
	Profit Maximization	5.10	11249.85	13991.49	2741.65	2540.42	1	1	0.817
	Revenue Maximization	5.08	11240.05	13995.51	2755.46	2539.50	1	1	0.658
	Cost Minimization	5.11	11244.78	13983.53	2738.75	2542.26	1	1	0.955
Policy IV	Profit / Cost Ratio Maximization	5.23	11582.91	14321.92	2739.01	2541.60	1	1	0.911
	Profit Maximization	5.23	11585.63	14327.50	2741.87	2540.37	1	1	0.812
	Revenue Maximization	5.20	11574.76	14331.91	2757.15	2539.50	1	1	0.646
	Cost Minimization	5.23	11580.21	14318.96	2738.75	2542.26	1	1	0.955

Table 55: Results of Instance 54

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.08	2473.99	3080.30	606.31	523.55	1	1	2.102
	Profit Maximization	5.08	2474.06	3080.43	606.37	523.52	1	1	2.049
	Revenue Maximization	5.08	2473.86	3080.52	606.66	523.49	1	1	1.958
	Cost Minimization	5.08	2473.93	3080.23	606.30	523.57	1	1	2.126
Policy II	Profit / Cost Ratio Maximization	4.63	2199.10	2805.44	606.34	523.53	1	1	2.070
	Profit Maximization	4.63	2199.36	2805.98	606.62	523.49	1	1	1.967
	Revenue Maximization	4.62	2198.58	2806.35	607.77	523.58	1	1	1.801
	Cost Minimization	4.63	2198.75	2805.06	606.30	523.57	1	1	2.126
Policy III	Profit / Cost Ratio Maximization	4.50	2124.98	2731.29	606.31	523.55	1	1	2.096
	Profit Maximization	4.50	2125.05	2731.43	606.39	523.52	1	1	2.042
	Revenue Maximization	4.50	2124.86	2731.52	606.66	523.49	1	1	1.958
	Cost Minimization	4.50	2124.88	2731.18	606.30	523.57	1	1	2.126
Policy IV	Profit / Cost Ratio Maximization	4.62	2192.60	2798.92	606.31	523.55	1	1	2.093
	Profit Maximization	4.62	2192.69	2799.10	606.41	523.51	1	1	2.032
	Revenue Maximization	4.61	2192.46	2799.22	606.76	523.50	1	1	1.938
	Cost Minimization	4.62	2192.48	2798.78	606.30	523.57	1	1	2.126

Table 56: Results of Instance 55

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.78	8765.74	11083.93	2318.19	2138.76	1.019	1	0.981
	Profit Maximization	4.78	8768.44	11089.55	2321.11	2137.55	1.149	1	0.871
	Revenue Maximization	4.75	8758.59	11093.76	2335.17	2136.64	1.443	1	0.693
	Cost Minimization	4.78	8762.63	11080.48	2317.85	2139.50	1	1	1.037
Policy II	Profit / Cost Ratio Maximization	4.62	8400.17	10718.47	2318.30	2138.65	1.028	1	0.973
	Profit Maximization	4.62	8403.39	10725.22	2321.83	2137.41	1.170	1	0.855
	Revenue Maximization	4.59	8392.14	10730.13	2337.98	2136.66	1.489	1	0.671
	Cost Minimization	4.62	8396.14	10713.99	2317.85	2139.50	1	1	1.037
Policy III	Profit / Cost Ratio Maximization	4.59	8317.25	10635.49	2318.24	2138.71	1.023	1	0.977
	Profit Maximization	4.58	8320.00	10641.28	2321.28	2137.51	1.154	1	0.867
	Revenue Maximization	4.56	8310.34	10645.51	2335.17	2136.64	1.443	1	0.693
	Cost Minimization	4.59	8313.78	10631.64	2317.85	2139.50	1	1	1.037
Policy IV	Profit / Cost Ratio Maximization	4.62	8397.92	10716.20	2318.28	2138.68	1.026	1	0.975
	Profit Maximization	4.62	8400.94	10722.52	2321.58	2137.45	1.163	1	0.860
	Revenue Maximization	4.59	8390.72	10727.03	2336.31	2136.64	1.462	1	0.684
	Cost Minimization	4.62	8393.78	10711.64	2317.86	2139.57	1	1	1.042

Table 57: Results of Instance 56

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.73	5977.51	7020.15	1042.64	923.93	1	1	1.559
	Profit Maximization	6.73	5977.97	7021.09	1043.13	923.49	1	1	1.456
	Revenue Maximization	6.69	5972.15	7022.56	1050.42	922.80	1	1	1.140
	Cost Minimization	6.73	5977.27	7019.89	1042.62	924.05	1	1	1.585
Policy II	Profit / Cost Ratio Maximization	6.17	5390.07	6432.74	1042.68	923.84	1	1	1.540
	Profit Maximization	6.16	5391.13	6434.94	1043.81	923.26	1	1	1.392
	Revenue Maximization	6.10	5382.15	6437.62	1055.47	922.90	1	1	1.040
	Cost Minimization	6.17	5389.38	6432.00	1042.62	924.05	1	1	1.585
Policy III	Profit / Cost Ratio Maximization	5.97	5185.73	6228.40	1042.66	923.87	1	1	1.546
	Profit Maximization	5.97	5186.46	6229.90	1043.44	923.37	1	1	1.424
	Revenue Maximization	5.93	5181.17	6231.59	1050.42	922.80	1	1	1.140
	Cost Minimization	5.97	5185.23	6227.85	1042.62	924.05	1	1	1.585
Policy IV	Profit / Cost Ratio Maximization	6.16	5384.41	6427.06	1042.66	923.88	1	1	1.549
	Profit Maximization	6.16	5385.06	6428.40	1043.34	923.40	1	1	1.433
	Revenue Maximization	6.13	5380.59	6429.86	1049.27	922.81	1	1	1.169
	Cost Minimization	6.16	5383.97	6426.59	1042.62	924.05	1	1	1.585

Table 58: Results of Instance 57

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.38	5904.20	7002.32	1098.13	932.25	1	1	2.160
	Profit Maximization	6.37	5904.93	7003.87	1098.94	931.53	1	1	2.006
	Revenue Maximization	6.30	5894.44	7006.43	1111.99	930.33	1	1	1.517
	Cost Minimization	6.38	5903.78	7001.87	1098.09	932.46	1	1	2.201
Policy II	Profit / Cost Ratio Maximization	5.84	5312.64	6410.84	1098.20	932.09	1	1	2.129
	Profit Maximization	5.83	5314.42	6414.60	1100.18	931.12	1	1	1.904
	Revenue Maximization	5.72	5296.61	6419.61	1123.00	930.56	1	1	1.340
	Cost Minimization	5.84	5311.41	6409.50	1098.09	932.46	1	1	2.201
Policy III	Profit / Cost Ratio Maximization	5.65	5105.40	6203.57	1098.17	932.14	1	1	2.139
	Profit Maximization	5.64	5106.57	6206.04	1099.47	931.32	1	1	1.956
	Revenue Maximization	5.58	5097.01	6209.01	1111.99	930.33	1	1	1.517
	Cost Minimization	5.65	5104.53	6202.62	1098.09	932.46	1	1	2.201
Policy IV	Profit / Cost Ratio Maximization	5.83	5304.42	6402.59	1098.16	932.16	1	1	2.142
	Profit Maximization	5.83	5305.55	6404.94	1099.39	931.35	1	1	1.963
	Revenue Maximization	5.77	5296.76	6407.70	1110.93	930.33	1	1	1.538
	Cost Minimization	5.83	5303.62	6401.71	1098.09	932.46	1	1	2.201

Table 59: Results of Instance 58

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.53	12827.01	15657.65	2830.64	2547.91	1	1	1.251
	Profit Maximization	5.52	12833.61	15671.54	2837.94	2544.12	1	1	1.063
	Revenue Maximization	5.38	12775.06	15688.51	2913.45	2539.50	1	1	0.658
	Cost Minimization	5.53	12821.82	15651.97	2830.15	2549.46	1	1	1.321
Policy II	Profit / Cost Ratio Maximization	5.05	11470.49	14301.51	2831.02	2547.43	1	1	1.228
	Profit Maximization	5.04	11479.44	14320.61	2841.17	2543.36	1	1	1.021
	Revenue Maximization	4.89	11406.86	14342.56	2935.71	2539.63	1	1	0.606
	Cost Minimization	5.05	11462.08	14292.23	2830.15	2549.46	1	1	1.321
Policy III	Profit / Cost Ratio Maximization	4.93	11129.90	13960.86	2830.96	2547.49	1	1	1.231
	Profit Maximization	4.92	11137.69	13977.52	2839.83	2543.65	1	1	1.037
	Revenue Maximization	4.80	11082.06	13995.51	2913.45	2539.50	1	1	0.658
	Cost Minimization	4.93	11122.18	13952.33	2830.15	2549.46	1	1	1.321
Policy IV	Profit / Cost Ratio Maximization	5.05	11464.80	14295.75	2830.95	2547.51	1	1	1.232
	Profit Maximization	5.04	11472.95	14313.08	2840.13	2543.58	1	1	1.033
	Revenue Maximization	4.91	11413.90	14331.91	2918.02	2539.50	1	1	0.646
	Cost Minimization	5.05	11457.03	14287.18	2830.15	2549.46	1	1	1.321

Table 60: Results of Instance 59

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.77	8722.41	11036.39	2313.99	2149.01	1	1	1.004
	Profit Maximization	4.77	8722.57	11036.72	2314.15	2148.94	1.024	1	0.977
	Revenue Maximization	4.77	8722.23	11036.90	2314.67	2148.90	1.066	1	0.938
	Cost Minimization	4.77	8722.19	11036.15	2313.96	2149.06	1	1	1.019
Policy II	Profit / Cost Ratio Maximization	5.05	11470.49	14301.51	2831.02	2547.43	1	1	1.228
	Profit Maximization	5.04	11479.44	14320.61	2841.17	2543.36	1	1	1.021
	Revenue Maximization	4.89	11406.86	14342.56	2935.71	2539.63	1	1	0.606
	Cost Minimization	4.61	8355.85	10669.82	2313.96	2149.06	1	1	1.019
Policy III	Profit / Cost Ratio Maximization	4.57	8271.58	10585.57	2313.99	2149.01	1	1	1.003
	Profit Maximization	4.57	8271.74	10585.90	2314.16	2148.94	1.025	1	0.976
	Revenue Maximization	4.57	8271.41	10586.08	2314.67	2148.90	1.066	1	0.938
	Cost Minimization	4.57	8271.34	10585.30	2313.96	2149.06	1	1	1.019
Policy IV	Profit / Cost Ratio Maximization	4.61	8352.07	10666.07	2314.00	2149.00	1	1	1.000
	Profit Maximization	4.61	8352.30	10666.54	2314.24	2148.93	1.033	1	0.968
	Revenue Maximization	4.61	8351.85	10666.78	2314.94	2148.91	1.081	1	0.925
	Cost Minimization	4.61	8351.72	10665.68	2313.96	2149.07	1	1	1.020

Table 61: Results of Instance 60

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.71	12902.49	15640.20	2737.71	2552.67	1	1	0.941
	Profit Maximization	5.71	12902.64	15640.50	2737.86	2552.59	1	1	0.916
	Revenue Maximization	5.71	12902.20	15640.70	2738.50	2552.54	1	1	0.876
	Cost Minimization	5.71	12902.35	15640.05	2737.70	2552.71	1	1	0.951
Policy II	Profit / Cost Ratio Maximization	5.22	11550.71	14288.48	2737.77	2552.63	1	1	0.928
	Profit Maximization	5.22	11551.36	14289.77	2738.42	2552.54	1	1	0.880
	Revenue Maximization	5.21	11549.61	14290.59	2740.98	2552.72	1	1	0.805
	Cost Minimization	5.22	11550.00	14287.70	2737.70	2552.71	1	1	0.951
Policy III	Profit / Cost Ratio Maximization	5.09	11200.75	13938.46	2737.72	2552.66	1	1	0.938
	Profit Maximization	5.09	11200.92	13938.81	2737.89	2552.58	1	1	0.913
	Revenue Maximization	5.09	11200.51	13939.01	2738.50	2552.54	1	1	0.876
	Cost Minimization	5.09	11200.54	13938.24	2737.70	2552.71	1	1	0.951
Policy IV	Profit / Cost Ratio Maximization	5.21	11536.23	14273.95	2737.72	2552.66	1	1	0.937
	Profit Maximization	5.21	11536.45	14274.39	2737.94	2552.57	1	1	0.909
	Revenue Maximization	5.21	11535.93	14274.64	2738.72	2552.54	1	1	0.867
	Cost Minimization	5.21	11535.98	14273.68	2737.70	2552.71	1	1	0.951

Table 62: Results of Instance 61

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	5.11	2491.44	3098.30	606.86	518.65	1	1	2.051
	Profit Maximization	5.10	2492.33	3100.16	607.83	518.14	1	1	1.855
	Revenue Maximization	5.05	2487.65	3101.90	614.25	517.66	1	1	1.472
	Cost Minimization	5.10	2490.59	3097.37	606.77	518.90	1	1	2.135
Policy II	Profit / Cost Ratio Maximization	4.65	2217.00	2823.95	606.95	518.54	1	1	2.015
	Profit Maximization	4.65	2218.40	2826.94	608.54	517.98	1	1	1.780
	Revenue Maximization	4.58	2211.55	2829.59	618.04	517.73	1	1	1.354
	Cost Minimization	4.65	2215.39	2822.16	606.77	518.90	1	1	2.135
Policy III	Profit / Cost Ratio Maximization	4.54	2145.91	2752.83	606.91	518.58	1	1	2.028
	Profit Maximization	4.53	2146.93	2754.99	608.07	518.08	1	1	1.828
	Revenue Maximization	4.49	2142.54	2756.79	614.25	517.66	1	1	1.472
	Cost Minimization	4.53	2144.66	2751.43	606.77	518.90	1	1	2.135
Policy IV	Profit / Cost Ratio Maximization	4.65	2213.58	2820.49	606.92	518.58	1	1	2.027
	Profit Maximization	4.64	2214.69	2822.86	608.17	518.06	1	1	1.817
	Revenue Maximization	4.59	2209.83	2824.83	615.00	517.67	1	1	1.446
	Cost Minimization	4.65	2212.26	2819.04	606.77	518.90	1	1	2.135

Table 63: Results of Instance 62

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	6.71	5974.05	7019.82	1045.76	924.09	1	1	1.592
	Profit Maximization	6.71	5974.56	7020.88	1046.31	923.59	1	1	1.482
	Revenue Maximization	6.66	5967.76	7022.56	1054.80	922.80	1	1	1.140
	Cost Minimization	6.71	5973.79	7019.53	1045.74	924.22	1	1	1.619
Policy II	Profit / Cost Ratio Maximization	6.15	5386.41	6432.21	1045.80	923.99	1	1	1.572
	Profit Maximization	6.15	5387.57	6434.62	1047.05	923.34	1	1	1.416
	Revenue Maximization	6.07	5377.34	6437.62	1060.28	922.90	1	1	1.040
	Cost Minimization	6.15	5385.65	6431.39	1045.74	924.22	1	1	1.619
Policy III	Profit / Cost Ratio Maximization	5.96	5182.16	6227.95	1045.79	924.02	1	1	1.578
	Profit Maximization	5.95	5182.97	6229.64	1046.66	923.45	1	1	1.447
	Revenue Maximization	5.91	5176.79	6231.59	1054.80	922.80	1	1	1.140
	Cost Minimization	5.95	5181.60	6227.34	1045.74	924.22	1	1	1.619
Policy IV	Profit / Cost Ratio Maximization	6.15	5380.86	6426.64	1045.78	924.03	1	1	1.581
	Profit Maximization	6.14	5381.60	6428.16	1046.56	923.49	1	1	1.456
	Revenue Maximization	6.10	5376.31	6429.86	1053.54	922.81	1	1	1.169
	Cost Minimization	6.15	5380.36	6426.10	1045.74	924.22	1	1	1.619

Table 64: Results of Instance 63

		$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$	n_1	n_2	T
Policy I	Profit / Cost Ratio Maximization	4.75	8713.66	11035.70	2322.04	2149.16	1.328	1	1.020
	Profit Maximization	4.75	8714.02	11036.42	2322.41	2149.01	1.250	1	0.986
	Revenue Maximization	4.75	8712.96	11036.90	2323.94	2148.90	1.066	1	0.938
	Cost Minimization	4.75	8713.21	11035.20	2321.99	2149.27	1.361	1	1.039
Policy II	Profit / Cost Ratio Maximization	4.60	8347.81	10669.87	2322.06	2149.15	1.382	1	1.012
	Profit Maximization	4.59	8348.26	10670.79	2322.53	2149.02	1.399	1	0.968
	Revenue Maximization	4.59	8346.58	10671.36	2324.79	2148.93	1.103	1	0.907
	Cost Minimization	4.59	8347.14	10669.13	2321.99	2149.27	1.361	1	1.039
Policy III	Profit / Cost Ratio Maximization	4.56	8262.82	10584.86	2322.04	2149.16	1.326	1	1.019
	Profit Maximization	4.56	8263.18	10585.60	2322.43	2149.00	1.246	1	0.985
	Revenue Maximization	4.56	8262.14	10586.08	2323.94	2148.90	1.066	1	0.938
	Cost Minimization	4.56	8262.30	10584.29	2321.99	2149.27	1.361	1	1.040
Policy IV	Profit / Cost Ratio Maximization	4.59	8343.61	10665.65	2322.04	2149.17	1.378	1	1.016
	Profit Maximization	4.59	8343.94	10666.34	2322.39	2149.04	1.385	1	0.977
	Revenue Maximization	4.59	8342.52	10666.78	2324.26	2148.91	1.081	1	0.925
	Cost Minimization	4.59	8343.10	10665.09	2321.99	2149.27	1.361	1	1.039

Table 65: Results of Instance 64

7.1.1 Findings on Different Objective Functions

In this chapter, we observe the performance of different objective functions over the instances. The average performances of profit/cost ratio maximization, total profit rate maximization, total revenue rate maximization and total cost rate minimization under different policies are presented in tables 66, 67, 68, and 69 respectively. In Table 70, we first obtain the average performance metrics of each objective function through four policies. Then, we compare the average difference that each objective function makes in percentages.

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Policy I	5.449	7665.22	9405.77	1740.56	1568.42
Policy II	5.043	7040.03	8788.79	1748.76	1574.49
Policy III	4.919	6808.28	8548.92	1740.64	1568.31
Policy IV	5.031	6984.88	8725.53	1740.64	1568.31

Table 66: Performance of Profit/Cost Ratio Maximization in Various Policies

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Policy I	5.444	7667.17	9409.90	1742.73	1567.32
Policy II	5.037	7042.94	8794.99	1752.06	1573.23
Policy III	4.914	6810.51	8553.70	1743.19	1567.19
Policy IV	5.026	6987.23	8730.53	1743.30	1567.17

Table 67: Performance of Profit Maximization in Various Policies

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Policy I	5.380	7652.71	9414.36	1761.66	1566.06
Policy II	4.956	7023.21	8801.39	1778.18	1572.28
Policy III	4.859	6796.76	8558.41	1761.66	1566.06
Policy IV	4.969	6973.03	8735.41	1762.37	1566.07

Table 68: Performance of Revenue Maximization in Various Policies

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Policy I	5.448	7663.34	9403.71	1740.36	1568.95
Policy II	5.035	6988.46	8728.83	1740.37	1568.94
Policy III	4.918	6805.77	8546.13	1740.36	1568.95
Policy IV	5.030	6982.30	8722.67	1740.36	1568.95

Table 69: Performance of Cost Minimization in Various Policies

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Profit/Cost Maximization	0.38%	0.09%	0.00%	-0.34%	0.07%
Profit Maximization	0.28%	0.12%	0.06%	-0.19%	0.00%
Revenue Maximization	-0.99%	-0.10%	0.12%	0.99%	-0.07%
Cost Minimization	0.33%	-0.12%	-0.19%	-0.47%	0.01%

Table 70: Performance of Objective Functions Compared to the Average

Observing Table 70, one can see that profit/cost ratio maximization, profit maximization, revenue maximization and cost minimization do not always result in better performance in every metric that we use. However, one can observe that while profit maximization is not better in terms of total emission rate, it is not worse either while achieving better results than the average in all other categories. Profit/cost ratio maximization performs better in terms of total cost rate compared to the total profit rate. Profit maximization performs better in terms of maximizing the revenue rate and minimizing the emission rate. However, one could state that these differences are negligible.

	min n_1^*	n_1^*	max n_1^*	min n_2^*	n_2^*	max n_2^*	min T^*	T^*	max T^*
Profit/Cost Ratio Maximization	1	1.03	1.46	1	1	1	0.89	1.61	2.82
Profit Maximization	1	1.04	1.60	1	1	1	0.78	1.45	2.54
Revenue Maximization	1	1.06	1.49	1	1	1	0.61	1.13	1.96
Cost Minimization	1	1.03	1.39	1	1	1	0.93	1.68	2.95

Table 71: Percent Differences in Values for Different Objective Functions

Observing Table 71, we can first see that n_2^* has never been different than 1. This means that the second item becomes a part of every order in every solution. However, observing the average n_1^* , we can see that it is slightly different than 1 under different objective functions. We can observe that in certain instances n_1^* resulted in much less frequent orders by obtaining values as high as 1.46, 1.60, 1.49 and 1.39 through objective functions profit/cost ratio maximization, profit maximization, revenue maximization, and cost minimization respectively. The average T^* values are the highest in cost minimization due to the major fixed cost being much higher than other parameters. On the other hand, the average T^* is the lowest in revenue maximization due to fixed emission not

being that major of a source compared to other emission sources such as holding and minor fixed emissions. One key takeaway here could be from observing the differences in optimal order cycle times between profit/cost ratio maximization and profit maximization. Since the minimum, maximum, and average optimal order cycle times are higher for profit/cost ratio maximization, one could argue that as the major fixed emission increases, profit/cost ratio maximization could potentially result in lower total emission rate than the profit maximization.

7.1.2 Findings on Allocation Policies

In this chapter, we observe the performance of different allocation policies over our instances. One can observe the performances of different allocation policies given various objective functions in tables 72, 73, 74, and 75.

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	5.449	7665.22	9405.77	1740.56	1568.42
Maximize Profit	5.444	7667.17	9409.90	1742.73	1567.32
Maximize Revenue	5.380	7652.71	9414.36	1761.66	1566.06
Minimize Cost	5.448	7663.26	9403.63	1740.37	1568.96

Table 72: Performance of Policy I with Various Objectives

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	5.043	7040.03	8788.79	1748.76	1574.49
Maximize Profit	5.037	7042.94	8794.99	1752.06	1573.23
Maximize Revenue	4.956	7023.21	8801.39	1778.18	1572.28
Minimize Cost	5.042	7037.00	8785.43	1740.37	1575.19

Table 73: Performance of Policy II with Various Objectives

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	4.919	6808.28	8548.92	1740.64	1568.31
Maximize Profit	4.914	6810.51	8553.70	1743.19	1567.19
Maximize Revenue	4.859	6796.76	8558.41	1761.66	1566.06
Minimize Cost	4.918	6805.67	8546.04	1740.37	1568.96

Table 74: Performance of Policy III with Various Objectives

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	5.031	6984.88	8725.53	1740.64	1568.31
Maximize Profit	5.026	6987.23	8730.53	1743.30	1567.17
Maximize Revenue	4.969	6973.03	8735.41	1762.37	1566.07
Minimize Cost	5.030	6982.30	8722.67	1740.37	1568.95

Table 75: Performance of Policy IV with Various Objectives

Looking at tables 72, 73, 74, and 75, Policy I has the best performance on average when we consider maximizing the profit/cost ratio, total profit rate, and the total revenue rate. Since maximizing the total revenue rate is equivalent to minimizing the total emission rate, Policy I also has the least average minimum total emission rate among these four policies. However, observing the total cost rate minimization, it can be seen that policies I, III, and IV are tied for the lowest average minimum cost.

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Policy I	6.65%	7.59%	6.07%	-0.16%	-0.10%
Policy II	-1.42%	-1.20%	-0.88%	0.44%	0.29%
Policy III	-3.71%	-4.44%	-3.59%	-0.15%	-0.10%
Policy IV	-1.52%	-1.96%	-1.60%	-0.14%	-0.10%

Table 76: Performance of Policies Compared to the Average

By obtaining the average of every policy under every objective function in Table 76, we demonstrate how much better, or worse, is a policy than the average. Observing Table 76, one can see that using Policy I results 6.65%, 7.59% and 6.07% more on our objective function, total profit rate, and total revenue rate respectively. Moreover, Policy I creates a decrease of 0.16% on costs and a decrease of 0.10% on total emissions that is tied with policies III and IV. Therefore, one could argue that when all objective functions are considered, Policy I is the best option.

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Policy I	6.62%	7.59%	6.07%	-0.12%	-0.09%
Policy II	-1.32%	-1.19%	-0.88%	0.35%	0.29%
Policy III	-3.75%	-4.44%	-3.59%	-0.12%	-0.10%
Policy IV	-1.55%	-1.96%	-1.60%	-0.12%	-0.10%

Table 77: Profit/Cost Ratio Maximization Performance of Policies Compared to the Average

One can observe Table 77 to see how much better or worse than the average the policies are when we only consider the maximization of our objective function.

When the maximization of the profit/cost ratio is considered, it is obvious Policy I performs the best in the objective function value, the total profit rate, and the total revenue rate. The lowest total cost rate is a three-way tie between policies I, III, and IV. The lowest emission rate is another tie between policies III and IV. Policy II has the worst cost rate and emission rate.

	$\min n_1^*$	n_1^*	$\max n_1^*$	$\min n_2^*$	n_2^*	$\max n_2^*$	$\min T^*$	T^*	$\max T^*$
Policy I	1	1.03	1.41	1	1	1	0.90	1.62	2.82
Policy II	1	1.04	1.46	1	1	1	0.89	1.60	2.77
Policy III	1	1.03	1.41	1	1	1	0.89	1.61	2.78
Policy IV	1	1.03	1.46	1	1	1	0.89	1.61	2.78

Table 78: Optimal Solutions for Profit/Cost Ratio Maximization under Four Policies

Table 78 summarizes the average optimal value of the variables n_1 , n_2 , and T take when we maximize the profit/cost ratio under each policy. Observing Table 78, we can see that the n_2 values average 1 over the four policies. Thus, we do not have a scenario where the respective item-specific order interval is different than 1. However, in certain cases we can see that n_1 is forced to become different than 1. When we observe the different values that n_1 and T take over different policies, we can see that they are only incrementally different than each other.

	$\min n_1^*$	n_1^*	$\max n_1^*$	$\min n_2^*$	n_2^*	$\max n_2^*$	$\min T^*$	T^*	$\max T^*$
Policy I	1	1.03	1.41	1	1	1	0.82	1.48	2.54
Policy II	1	1.04	1.46	1	1	1	0.78	1.42	2.43
Policy III	1	1.03	1.41	1	1	1	0.81	1.46	2.50
Policy IV	1	1.03	1.46	1	1	1	0.80	1.45	2.49

Table 79: Optimal Solutions for Profit Maximization under Four Policies

In Table 79, you may see the minimum, maximum and average value of the variables n_1 , n_2 , and T take when we maximize the profit rate under each policy. Observing Table 79, one can see that the differences between variable values in different policies are negligible yet again. However, one can clearly observe that the optimal order cycle times result in more frequent orders on average than profit/cost ratio maximization. Therefore, with increased major fixed emissions, profit/cost ratio maximization could potentially yield lower total emission rates than profit maximization.

	$\min n_1^*$	n_1^*	$\max n_1^*$	$\min n_2^*$	n_2^*	$\max n_2^*$	$\min T^*$	T^*	$\max T^*$
Policy I	1	1.06	1.44	1	1	1	0.66	1.16	1.96
Policy II	1	1.06	1.49	1	1	1	0.61	1.06	1.80
Policy III	1	1.06	1.44	1	1	1	0.66	1.15	1.96
Policy IV	1	1.06	1.46	1	1	1	0.65	1.15	1.94

Table 80: Optimal Solutions for Revenue Maximization under Four Policies

In Table 80, one may observe the minimum, maximum, and average values of the variables n_1 , n_2 , and T take when we maximize the revenue rate under each policy. As it can be observed, revenue maximization results in a decrease in the especially optimal common order cycle times. Since revenue rate maximization only considers the revenue rate of the system, it could be argued that the orders become more frequent since the major fixed emission is not as large as major fixed cost. Since the emission allocation policy has no effect on the total cost rate, we shall not discuss the minimization of total cost rate under different policies.

7.1.3 Findings on Changing Parameters

Here in this subsection, we observe how the solutions and results change with respect to a changing parameters. Starting with K , we discuss how the change in parameters change the performance measures and optimal solutions. When K increases from 100 to 200, the changes depicted in Table 81 occur in the optimal solution under different objective functions:

	n_1	n_2	T
Maximize Profit/Cost Ratio	-3.74%	0.00%	23.85%
Maximize Profits	-4.42%	0.00%	18.90%
Maximize Revenue	0.00%	0.00%	0.00%
Minimize Costs	-3.51%	0.00%	25.96%

Table 81: Percent Differences in Optimal Values When K Changes - Objective Function Based

When K changes from 100 to 200, we can see that the values for T are clearly affected. When profit/cost ratio, profit rate, and costs are being maximized, there is a 23.85%, 18.90% and 25.96% increase in the optimal T value. Since the major fixed cost is increasing by 100%, it makes sense to increase the value of T that makes the orders less frequent.

However, when we look at Table 82 that depicts the performance measures changing under different objective functions, we can see that the changes in the performance measures are much less with the adjusted optimal order cycle times:

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	-3.38%	-2.66%	-1.95%	0.89%	-2.52%
Maximize Profits	-3.50%	-2.62%	-1.88%	1.06%	-2.61%
Maximize Revenue	-5.36%	-2.94%	0.00%	2.63%	0.00%
Minimize Costs	-3.36%	-2.68%	-1.98%	0.82%	-2.48%

Table 82: Percent Differences in Performance Measures When K Changes - Objective Function Based

Let us observe what happens when K_e changes. Let us start with observing the

change different optimal solutions under different objective functions when K_e increases from 10 to 20.

	n_1	n_2	T
Maximize Profit/Cost Ratio	-0.48%	0.00%	9.08%
Maximize Profits	1.25%	0.00%	3.76%
Maximize Revenue	7.41%	0.00%	-15.76%
Minimize Costs	0.00%	0.00%	0.00%

Table 83: Percent Differences in Optimal Solutions When K_e Changes - Objective Function Based

Observe that, when maximizing the profit/cost ratio, there is a slight increase in T with the increasing fixed emission as expected. Moreover, when revenue is being maximized, T decreases which contradicts our intuition and previous findings. However, we observe that n_1 increases by 7.41% which could be happening in order to balance for the increased common order cycle time.

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	-1.98%	-13.42%	-13.18%	-12.18%	-13.55%
Maximize Profits	-2.08%	-13.39%	-13.12%	-12.02%	-13.63%
Maximize Revenue	-3.59%	-13.69%	-13.05%	-10.49%	-13.73%
Minimize Costs	-2.02%	-13.47%	-13.21%	0%	-13.50%

Table 84: Percent Differences in Performance Measures When K_e Changes - Objective Function Based

One may observe Table 85 on performance measure on what happens to them when D_1 decreases from 500 to 200.

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	-21.27%	-47.77%	-42.21%	-21.81%	-22.24%
Maximize Profits	-21.32%	-47.74%	-42.17%	-21.75%	-22.25%
Maximize Revenue	-21.54%	-47.79%	-42.15%	-21.75%	-22.24%
Minimize Costs	-21.34%	-47.84%	-42.25%	-21.75%	-22.21%

Table 85: Percent Differences in Optimal Solutions When D_1 Changes - Objective Function Based

Observe that all the performance measures under different objective functions decrease when the demand decreases, as expected. Observe the changes in

optimal solutions in Table 86:

	n_1	n_2	T
Maximize Profit/Cost Ratio	6.06%	0.00%	12.80%
Maximize Profits	7.59%	0.00%	11.22%
Maximize Revenue	10.67%	0.00%	11.38%
Minimize Costs	5.48%	0.00%	14.33%

Table 86: Percent Differences in Optimal Solutions When D_1 Changes - Objective Function Based

Observing Table 86, we can see that as demand increases the common order time and the item-specific order interval for n_1 increases. Intuitively, this means that when the demand decreases, the holding costs and emissions become less of a problem.

One may observe the set of similar results for the parameter D_2 as it would be repetitive to go over the similar results and intuition when D_2 decreases from 1000 to 200.

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	8.91%	-107.44%	-112.89%	-137.47%	-147.94%
Maximize Profits	8.90%	-107.45%	-112.91%	-137.49%	-147.98%
Maximize Revenue	8.42%	-107.55%	-112.91%	-136.61%	-148.06%
Minimize Costs	8.95%	-107.39%	-112.87%	-137.55%	-147.95%

Table 87: Percent Differences in Performance Measures When D_2 Changes - Objective Function Based

	n_1	n_2	T
Maximize Profit/Cost Ratio	-6.45%	0.00%	39.44%
Maximize Profits	-8.24%	0.00%	40.27%
Maximize Revenue	-11.95%	0.00%	39.01%
Minimize Costs	-5.80%	0.00%	39.04%

Table 88: Percent Differences in Optimal Solutions When D_2 Changes - Objective Function Based

Observe that the decrease in performance measures is much more drastic for D_2 as it is simply the larger value of the two. Moreover, one can see that n_1 decreases as T increases. In the case where D_1 decreases, n_1 increased but since now we are dealing with the other demand value, the decrease in n_1 is expected.

Finally, let us look at how the changes in minor fixed costs affect the performance measures and optimal solutions. However, observing Table 89 related to changes in k_1 , we can say that any changes related to this fixed cost does not have a prominent effect on performance measures:

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	-0.03%	3.94%	3.82%	3.32%	3.69%
Maximize Profits	-0.03%	3.94%	3.81%	3.30%	3.69%
Maximize Revenue	0.00%	3.96%	0%	3.21%	0%
Minimize Costs	-0.05%	3.93%	3.82%	3.34%	3.69%

Table 89: Percent Differences in Performance Measures When k_1 Changes - Objective Function Based

However, looking at the optimal solutions, one can see that there is a notable change in n_1 when we maximize the profit/cost ratio or profit rate:

	n_1	n_2	T
Maximize Profit/Cost Ratio	5.68%	0.00%	-3.12%
Maximize Profits	5.50%	0.00%	-3.19%
Maximize Revenue	0%	0.00%	0%
Minimize Costs	5.48%	0.00%	-3.15%

Table 90: Percent Differences in Optimal Solutions When k_1 Changes - Objective Function Based

This is intuitively due to the fact that the increasing fixed cost for the specific item results in the desire for the retailer to order that item less frequently.

Finally, let us observe the parameter k_2 . Observe that the findings for the changes in k_1 regarding the performance measures hold for this parameter as well:

	$z(\mathbf{n}, T)$	$\pi(\mathbf{n}, T)$	$r(\mathbf{n}, T)$	$TC(\mathbf{n}, T)$	$em(\mathbf{n}, T)$
Maximize Profit/Cost Ratio	1.24%	3.76%	3.70%	3.48%	3.73%
Maximize Profits	1.25%	3.76%	3.70%	3.48%	3.73%
Maximize Revenue	1.26%	3.75%	0%	3.51%	0%
Minimize Costs	1.22%	3.75%	3.70%	3.52%	3.73%

Table 91: Percent Differences in Performance Measures When k_2 Changes - Objective Function Based

However, looking at Table 92, one may observe the change in optimal solutions as k_2 decreases from 8 to 4.

	n_1	n_2	T
Maximize Profit/Cost Ratio	0.92%	0.00%	-6.56%
Maximize Profits	1.08%	0.00%	-6.40%
Maximize Revenue	0%	0.00%	0%
Minimize Costs	0.93%	0.00%	-7.01%

Table 92: Percent Differences in Optimal Solutions When k_2 Changes - Objective Function Based

Observe that the values for n_1 remains relatively the same but the values for T decreases. This is due to the fact that with the decreasing fixed cost to be incurred, the frequency at which the item will be order is expected to increase. Since the system only manipulates the value of n_1 and not n_2 , the change in T is the result of the change in k_2 .

CHAPTER 8

SOCIETAL IMPACT

The carbon emissions is one of the most urgent problems that our world is facing with its depleting resources and intensifying global warming. The results of this global warming could potentially be devastating. The governments take initiative in the battle against global warming through agreements such as the Kyoto Protocol and Paris Agreement. Yearly conferences are conducted to better deal with the problem global warming. The firms are being taxed and capped in order to limit their ability to conduct business. Thus, we have asked ourselves how the consumer can play an active part in these regulations and control mechanisms.

The problem that we aim to tackle creates benefits towards achieving environmental sustainability. Throughout the thesis, we have focused on taking advantage from the economies of scale through joint replenishment to reduce the carbon emissions and costs of the systems while managing the inventory. The thesis aims to show that when the consumers have high environmental

awareness and consciousness, with the power that they can assert over the market, they can change the way that the firms operate. The consumer then has the option to pay what they think is fair for a product. By forcing firms to collaborate, the resources are utilized more efficiently as the replenishment for various items are done together. This directly affects the carbon that is emitted by the system.

In addition, we have discussed in the introduction chapter that on average, one fifth of every road freight is empty in Europe. This means that an extremely carbon intensive source is being used nowhere near its full capacity. We believe that joint replenishment of items is a viable and effective solution to that issue as proven by Ballot & Fontane (2010) in their study of the French retail supply chain.

We have seen that when the only concern is the total cost rate, the emission rate increased as expected in the numerical analysis. However, through the environmentally sensitive consumer, profit/cost ratio maximization and profit maximization have proven to be effective at reducing emission and keeping a respectable total cost rate. Moreover, we have observed that profit/cost ratio maximization performs better than profit maximization in terms of total cost rate. Thus, different than the classic carbon taxation scheme, our pricing enables the retailer to incur less cost.

With a new perspective and objective we have provided to a crucial issue, we believe that this area of research is worth considering and conducting further analysis on.

CHAPTER 9

CONCLUSION

With the motivation to provide further insight in a globally trending concern, carbon emissions, we introduce an environmentally sensitive customer to the environment of a well-known joint replenishment problem. We aim to diverge from the existing literature of carbon emissions and joint replenishment by making the carbon emissions penalty a part of the revenue function while introducing a relatively new objective function, the profit/cost ratio.

What we expect to achieve through this new objective function is to first be able to introduce the environmentally sensitive consumer to depict how the consumers in a system can affect how businesses operate. Since maximizing the profits or minimizing the costs would mean that we would be treating these environmentally sensitive consumers as a carbon tax. However, through our objective function which is the ratio of the total revenue rate to the total cost rate, the environmentally sensitive customer only affects the total revenue rate and cannot be treated as a cost such as carbon taxation.

We assume a joint replenishment setting where there are fixed costs of ordering, fixed costs per item, holding costs, and unit purchasing costs over an infinite time horizon with no backlogging. Moreover, we introduce the same sources for the carbon emissions and deduct a penalty from the base price of each item based on the carbon emissions associated with it.

We present four different policies to allocate the carbon emissions. In these policies, we first allocate the carbon emission rate of the system based on the demand of each retailer, then, allocate only the fixed major emission by the demand and the other sources of emissions directly to the respective retailers. Further, we investigate the same two policies by weighing the demands with the distances of the retailers since we believe distance to be an important source of carbon emissions.

One of the significant findings is that when the ratio of the holding cost and emission is the same as the ratio of the minor fixed cost and emission, the total revenue rate and the total cost rate are minimized at the same optimal solution. This gives us the opportunity to immediately maximize the values for the items for which this condition holds.

We find that the maximization of the profit/cost ratio maximization problem that we have is strong pseudoconcave for the common order intervals when the item-specific order intervals are given, and vice versa. Additionally, we find that when the item-specific order intervals are given, the common order intervals can be found via a quadratic function that always provides a unique, positive, and real value. Moreover, when the common order interval and all item-specific order intervals except for one are given, we observe that we could obtain the left item-specific order interval via a quadratic function which again provides a unique, positive, and real value.

Using these, we find that at the optimum point for the common order interval, when the item-specific order intervals are given, the function is concave, and vice versa. In addition, we observe that when the common order interval is

given, the function is concave over the vector containing all item-specific order intervals.

Then, we investigate certain special cases first on the common order interval.

We observe that when there is no holding costs or emissions, the order cycle time approaches infinity, increasing the value between orders infinitely as the system wants not to incur holding costs or emissions.

When there is no revenue gained from selling an item, and no per unit cost, the optimal cycle time approaches to infinity as the system does not wish to replenish that item due to not obtaining any benefits from its sale.

Lastly, we observe that when we omit major and minor fixed costs or emissions, the optimal order cycle time approaches zero as the system wants to order as frequent as possible to not incur the holding costs or emissions associated with ordering.

After observing the special cases for the common order interval, we move on to the special cases for the item-specific order intervals. In these scenarios, we have used the relationship between two item-specific order intervals when both are optimal.

We assume different relationships between parameters of different items in order to obtain these special cases. To obtain these relationships, we have the condition that states that the total cost rate is not minimized at the same point the objective function is maximized for each item.

We have three categories of relationships between these parameters that provide us with our special cases. The relationships in the first category are the ones that immediately violate the previously mentioned assumption, which makes them not feasible. The second category of relationships can only hold for a select two items at most as the addition of the third item violates the assumption. The third set of relationships never violate this assumption no matter what the number of items is.

Then, we show that when we consider one of the special cases under the second

category, we can immediately state one of the item-specific order intervals in terms of the other by simply multiplying the variable with a parameter.

Detecting the variables that have this relationship could potentially decrease the number of variables, and decrease the computational power and time required to obtain the optimal solution.

When we consider the relationships that do not violate the assumption for any given number of items, we can observe that one special case enables us to obtain a bivariate objective function, for which we have a linear solution on both variables. Thus, the complexity of the problem could decrease drastically as the bivariate objective function requires much less time to find the solution to.

Under the identical items assumption, we observe that the item-specific order intervals are optimal at the same value. Further, under a given condition, we obtain a linear solution to both the common order interval and the item-specific order interval.

Continuing with the numerical analysis, we have obtained the optimal solution for maximizing the profit/cost ratio to 64 instances with two items for our objective function under the 4 policies previously discussed. Moreover, we analyze these instances under different objective functions considering maximizing the total revenue rate and the total profit rate, and minimizing the total cost rate.

Through the numerical analysis, we expected to find operations that have a better balance between their total cost rates and total profit rates. We expected the retailers to not only value the profits that they obtain, but also the cost they incur in order to obtain those profits. Since the retailers will maximize every unit of profit that they obtain by every unit of cost that they incur, the operations are expected to be more cautious when spending. Moreover, since the sources of the emissions are similar to the sources of the total cost rate, we expected the operations to be more cautious when emitting carbon as well. Since more frequent orders result in higher carbon emissions and total cost

rates, and we are observing the ratio between these, we expected less frequent orders, and a decrease in total emission rate when compared to a profit maximization solution.

We first compare the objective functions based on their overall performance in other performance measures that we have determined that are profit/cost ratio, total profit rate, total revenue rate, and total cost rate. We then see that profit maximization always performs better than the average of all objective functions in all these measures. profit/cost ratio maximization performs worse in total emission rate than the average but results in a decreased total cost rate when compared to the profit maximization. We expected to see a decrease in the total cost rate, however, we also expected a decrease in the total emission rate.

Having said that, the total emission rate is only 0.07% higher when we compare profit/cost ratio maximization and profit maximization, which is a negligible difference

Then, we compare the optimal values for our decision variables under different objective functions. We observe that on average, profit/cost ratio maximization and cost minimization yield significantly larger T values, revenue maximization yields significantly smaller values for T . Therefore, by profit/cost ratio maximization, we achieve less frequent orders than profit maximization. If the major fixed emissions increase, profit/cost ratio maximization could potentially result in lower total emission rates than profit maximization. The less frequent orders that we have expected before conducting the numerical analysis seems to hold.

We further compare the policies based on their performances in profit/cost ratio, total profit rate, total revenue rate, and total cost rate. We observe that when all of the objective functions are considered together, Policy I achieves the best performance on average when we consider profit/cost ratio, total profit rate, and total revenue rate, and total cost rate. Minimum total emission rate is a three-way tie between policies I, III, and IV. Thus, one could easily conclude

that Policy I has the best system results among the proposed policies. Moreover, when we maximize profit/cost ratio, Policy I again performs similarly by achieving the highest profit/cost ratio, total profit rate, total revenue rate and the lowest total emission rate. Total cost rate is a three-way tie between policies I, III, and IV.

When we observe how the policies and objective functions behave under alternatives of parameters, our results are consistent with intuition. We have observed that the changes in parameters effect the performance measures and optimal solutions relatively the same when we compare the effects on different policies and objective functions. It is clear that the demand is the most significant parameter to have an effect on the performance measures and optimal solutions.

For further contribution to the literature, we would like to recommend conducting more in-depth numerical analysis in order to be able to more effectively compare the employed objective functions and policies since our numerical analysis is limited to two items. Moreover, increasing the number of instances and variables and making use of special cases, one can precisely compute how much computational time is actually saved by these special cases. One important limitation of this thesis was the assumption of non-integer values for item-specific order intervals. Therefore, further analysis on rounding the optimal item-specific order intervals to the nearest integers in the numerical analysis could prove to be beneficial for practical reasons.

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