

MASTER

Design of a distributed supply chain for spare parts

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Design of a distributed supply chain for spare parts

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Executive summary

In this master thesis, the need for multiple warehouses in a distributed network for the spare parts of Lely is researched. The aim is to increase availability, fasten response time, decrease costs and improve sustainability. Currently, all customers are served from a single warehouse. Only customers in Europe are considered. Only parts for which all data needed for the analysis was available. They also need to have the status active in 2020 and 2021, meaning that it was sold during those years and not in the introduction phase or phasing out.

The problem sketched is the location transportation problem. Decisions on where to put warehouses, how much stock they need to carry and transportation decisions are taken simultaneously. No model fully corresponds to Lely's situation. The model of Ghorbani & Jokar (2016) is selected as the base model because it resembles Lely's situation and it can be solved heuristically within two hours for large instances.

Important input for the model is demand. The demand for spare parts is uncertain. To capture the demand uncertainty, demand distributions are fit to historical data. The mathematical model cannot handle 2,723 parts. Hierarchical clustering is used to reduce the number of parts in the mathematical model. The parts are clustered on their criticality level, lead time, part value, demand frequency, average usage per year, and volume. Performing hierarchical clustering resulted in 24 clusters for the analysis. Afterwards, a set of distributions is fit to the aggregated demand of 2020 and 2021 for each cluster through maximum likelihood estimation. The five distributions fitted are the normal distribution, the gamma distribution, the Poisson distribution, the negative binomial distribution and the compound Poisson distribution. Based on the AIC, the distribution with the best fit is selected. The clusters that did not get a good fit on any distribution are, if necessary, separated based on the size of demand in case it is positive.

A few adaptations are made to the base model of Ghorbani & Jokar (2016) to make it in line with the case of Lely. First, the shipment policies are changed. The routing part is omitted from the model. Instead, direct shipment between warehouses and customers is considered. Additionally, the adapted model restricts a supplier to deliver to one warehouse only because this supports the consolidation of shipments. The introduction of transshipment between warehouses allows for goods exchange between warehouses. Furthermore, the capacity levels of the warehouses are omitted because they do not pose a restriction.

The biggest adaptation is the introduction of stochasticity by including demand distributions. Instead of calculating the inventory levels and backlog quantities in each period, the inventory control policy is translated to calculating base stock levels for a certain fill rate. Gzara et al. (2014) give an expression for the fill rate for demand under a Poisson distribution. Given the mean demand λ_{jp} and stock levels S_{jc} for each part p and each warehouse j , the service level can be defined as $\beta(\lambda_{jp}, S_{jc}) = \Pr(\tilde{\lambda}_{jc} \leq S_{jc} - 1)$. The expected on-hand inventory is computed by taking the base stock level from which the expected demand during lead time plus half a review period is subtracted.

The model is solved with a Simulated Annealing (SA) algorithm because it has a strong local search ability (Ghorbani & Jokar, 2016). Hence, it can provide good

solutions with a stochastic approach which allows for the continuation of the search to a neighbouring state even if the move brings a worse solution. This feature avoids getting trapped in a local optimum. Two methods for calculating base stock levels are used. The first method is taking the fraction a part accounts for from the cluster base stock level to calculate the base stock level per part. The second method is applying a correction factor to the base stock levels from the model to correct for the clustering effect. Applying a correction is necessary because the safety stock was now calculated per cluster, while in reality a safety stock is calculated for each part individually. The actual demand for 2022 is used for the simulation. A different dataset than the training period of the built model is used to avoid overfitting. The simulation shows what the actual costs and availability levels would be when the results of the model are implemented.

Results

The results showed where DCs should be placed and their corresponding base stock levels. The model gives two options for placing a second DC, locations 50 and 51, next to the option of not adding a second warehouse. The option with three warehouses was discarded because the costs were too high. A simulation is conducted using the output from the heuristic in terms of placement of warehouses, required base stock levels, and assignment of suppliers and customers to warehouses. Data from 2022 is used as input for the simulation because it is different data on which the model is built. The simulation shows that, at the moment, it is not necessary to open a new DC because this would lead to a higher cost for achieving the same availability.

The number of warehouses included in the model influences the total sum of the base stock levels and, therefore, the holding cost. A higher base stock level leads to more holding costs. The availability increases with the base stock level because the safety stock follows the same trend. The emission of CO₂ followed the same trend as the transportation cost. An increase in the number of warehouses decreases the number of kilometres driven and, hence, a lower CO₂ emission.

Recommendations

At the moment, Lely should not invest in a second warehouse. With two warehouses, the currently achieved availability cannot be reached at a lower cost. However, this decision can be subject to change as it is a trade-off between holding and transportation costs. At the moment, the holding costs are too high for a second warehouse. A change in factors determining the holding cost, for example, squared metre price and labour cost, could change the decision to invest in a second warehouse. The same holds for factors that influence transportation costs, for example, fuel prices.

Secondly, it is recommended to gather more data on the criticality level, weight and volume of parts. These features were sometimes missing in the data leading to the discarding of parts for this research. Additionally, Lely should gather more detailed data about the squared metre price and labour costs for the warehouses in Europe of their current third-party logistics provider. More detailed data would lead to a more accurate analysis of the distributed network.

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1 Introduction

This master thesis project is conducted at the supply chain operations division of Lely. This department is responsible for the planning of the spare parts such that the local warehouses (also referred to as Lely Centers) can perform maintenance and respond to failures quickly. Currently, the Lely Centers are replenished from a single warehouse. This thesis project aims to investigate the need for multiple warehouses in a distributed network to increase availability, fasten response time, decrease costs and improve sustainability (Nahmias & Olsen, 2015). Based on literature research, an existing model that resembles Lely's situation is selected as the base model. Afterwards, this model is adapted to fully correspond to the context of Lely. The model is solved with a heuristic to determine if and where warehouses should open and what their base stock levels should be. Finally, a simulation is applied to test the solutions from the model and the sensitivity of the model. This introduction further describes the project setting, after which the problem statement is posited. Next, the scope is defined, followed by the formulation of the research questions.

1.1 Project setting

1.1.1 Lely International

Lely International is a production company for robots and data systems for dairy farmers. The company was founded in 1948 by the brothers Arij and Cornelis van der Lely with the invention of the finger wheel rake. Over the years, Lely continued facilitating farmers' life with innovations. In 1992, this led to a prototype of the first milking robot, the Astronaut A1, which was marketed in 1995. Over the years, the focus of Lely shifted more and more towards automation, which led to the introduction of manure robots and feeding robots. In 2017, the forage harvesting machine division was sold to fully focus on the dairy industry. As of 2022, the majority of the order book consists of milking robots. At the moment, Lely produces the fifth generation of its milking robots, the Astronaut A5, which is depicted in Figure 1.



Figure 1: Astronaut A5

Lely is one of the leading companies in the field of dairy farm automation. Its headquarter is based on the Lely campus in Maassluis. One of the two production locations is on this campus, while the other is in Pella in the USA. The annual sales in 2020 were 611 million euros. Lely has 2,100 employees worldwide and serves farmers in 45 countries.

Lely's goal is to create innovative solutions to help their farmers excel in sustainable milk production. Sustainability, enjoyability and profitability are key factors for achieving this goal. The vision is to have fully robotised farms that are operative 24/7, which is important as cows can get infections (in the worst case they can even die) if they do not get milked on time. Therefore, spare parts for critical components should be available quickly in case of a malfunction to keep the robots operative.

1.1.2 Lely Customer Care

The Lely Customer Care division supports a network of Lely Centers. Some Lely Centers are owned by Lely, but most are franchise holders. The Lely Centers are responsible for delivering spare parts and services for its farmers worldwide. Therefore, these Lely Centers are the customer for Lely International when considering spare parts. The Lely Centers deliver original spare parts guaranteeing high-quality maintenance for the machines. Fast and reliable performance is important in ensuring maximum utilization of the robots. The Lely Centers ship the spare parts to the farmer through their service technicians, who repair the robots.

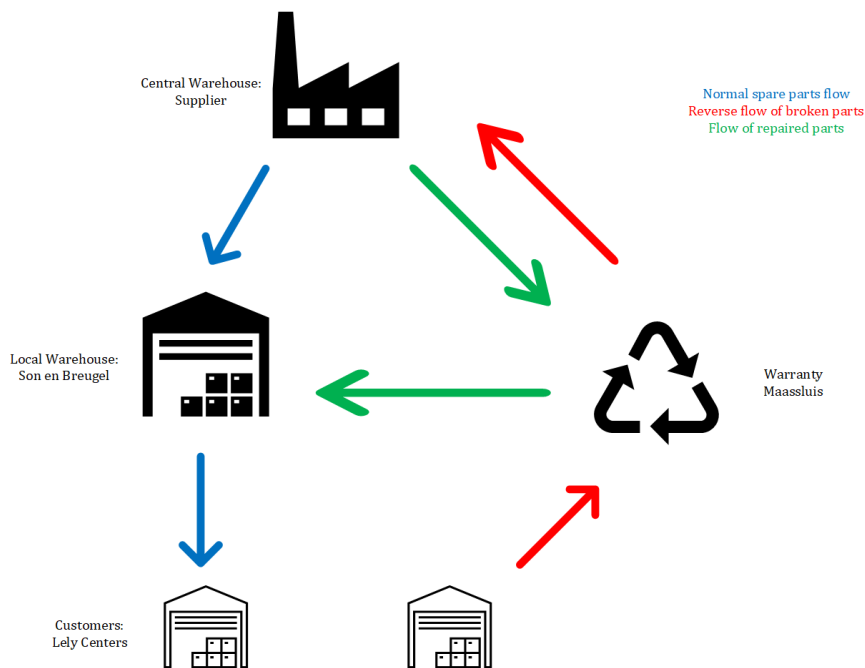


Figure 2: Current spare parts supply chain Lely

1.1.3 Current supply chain design

Figure 2 displays the current spare part supply chain. Almost all spare parts follow the forward flow. For a stock keeping unit (SKU), the supplier delivers it to the local warehouse. From there, the spare parts are shipped to the Lely Centers, which are responsible for further distribution to the farmer. A small subset of parts can be sent for repair when broken. They are sent to the warranty department in Maassluis for a check. The part is scrapped if the warranty department deems it unrepairable. Parts that are deemed repairable are sent back to the supplier for repair. It is repaired to an as-good-as-new state and labelled as s-part (repaired part). Afterwards, it is shipped back

to the warranty department in Maassluis for a final check. Afterwards, it is transferred to the warehouse and ready for distribution.

1.1.4 Inventory management

The spare parts are divided into 36 categories based on criticality, cost and usage. Parts can have criticality 1, 2, or 3. Criticality level 1 means the part is not critical for the machine to be operative. Criticality level 2 means the part is important, but the malfunction will not disrupt the operation of the machine in the short term. The part should be replaced within a week. Criticality level 3 means the machine cannot run without it and immediate replacement is necessary. Some parts are not classified in the criticality system and are labelled as non-critical (level 1). Concerning the cost, a part can be cheap (under 5 euros), middle-priced (between 5 and 150 euros) and expensive (above 150 euros). For usage, a part can be ordered rarely (under 5 times per year), occasionally (between 5 and 150 times per year) or frequently (above 150 times per year). One order can include more than one part of the same SKU. It is also possible that a part is not ordered at all. An overview of the classification of all 12,404 parts can be found in Tables 1 to 4. Specific parts can be purchased to order (PTO) to reduce the number of stock locations needed in the warehouse. This means that parts are only ordered at the supplier after demand comes in. The exact number of parts demanded can be ordered and there is no need for stock. Parts can only be PTO if they are non-critical, have a lead time of no more than four weeks and have demand fewer than five times per year.

Table 1: Critical

Price \ Frequency	>150	6-150	1-5	0	
> 150	45	85	6	1	137
10-150	163	205	28	4	400
< 10	146	195	8	7	356
	354	485	42	12	893

Table 2: Medium critical

Price \ Frequency	>150	6-150	1-5	0	
> 150	9	43	18	3	73
10-150	136	326	79	12	553
< 10	279	705	120	24	1.128
	424	1.074	217	39	1.754

Table 3: Non critical

Price \ Frequency	>150	6-150	1-5	0	
> 150	9	139	134	179	461
10-150	50	676	579	803	2.108
< 10	87	987	950	2.840	4.864
	146	1.802	1.663	3.822	7.433

Table 4: PTO

Price \ Frequency	>150	6-150	1-5	0	
> 150	1	6	45	218	270
10-150	10	48	318	665	1.041
< 10	13	33	268	779	1.093
	24	87	631	1.662	2.404

Lely uses a continuous review policy for order placement. The orders trigger out of the material requirements planning (MRP) system based on the safety stock, forecast and lead time. The method of forecasting differs per classification group. The economic order quantity (EOQ) is used to determine the order quantity for replenishment. It is possible to order multiple amounts of the EOQ. Sometimes this quantity is increased because the minimum order quantity (MOQ) is higher than the EOQ. In that case, the MOQ is ordered instead of the EOQ.

1.2 Problem statement

Currently, the network of spare parts is centrally organized. All spare parts suppliers deliver their goods to a logistic service provider in the Netherlands. For the remainder of this research project, this warehouse is referred to as warehouse X. From this central location, the spare parts are shipped to customers worldwide. The shipments within Europe are road shipments, while emergency shipments can, depending on the location of the customer, be shipped by plane. For North America and Australia, the spare parts first go to the central locations in these countries by boat before they are distributed to the customers. Again, emergency shipments are distributed by plane.

Lely supply chain aspires to become more flexible by servicing the increasingly demanding customers in a distributed network. Hence, a study into what a distributed network will look like and what parameters are important to make decisions about forward stocking locations is required. The lead times towards customers vary depending on how far away they are located from the logistic service provider. At the moment, all the shipments go directly to the customer. However, for shipment, it may be interesting to consolidate customer orders per region and decouple them later on in the supply chain (Nahmias & Olsen, 2015). To do this, Lely wants to investigate the possibility of placing more distribution centres (DCs) between the suppliers and the Lely Centers. These DCs will be in the same tier as the current logistic service provider and can transfer goods between

them. A DC receives shipments for all Lely Centers in the area, after which the DC serves the customer. The goal is to reduce the total cost in the supply chain, which consists of operating costs, ordering costs, holding costs and transportation costs. It may not only influence the costs but also reduce CO2 emissions.

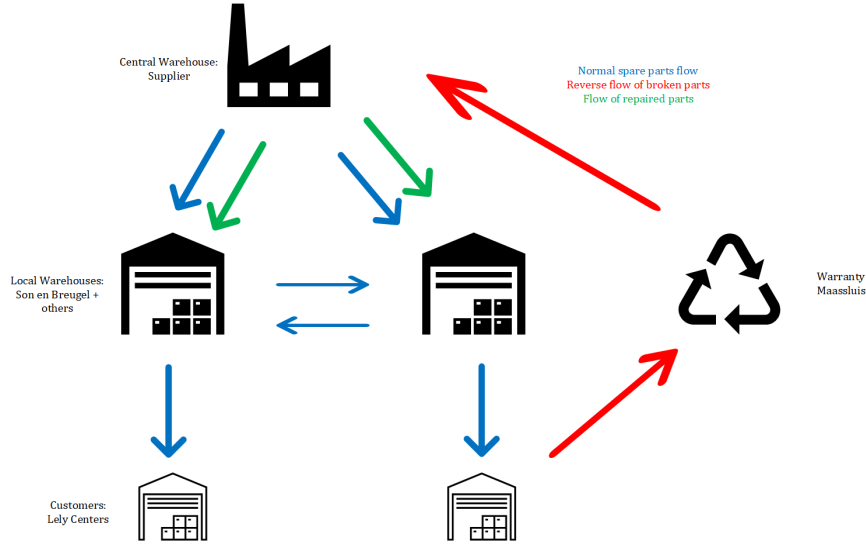


Figure 3: Desired supply chain

To design the future supply chain, first, the desired supply chain is sketched. Figure 3 displays the desired supply chain for the spare parts for the future, which is a distributed network. The supplier of a SKU now delivers the part to the closest local warehouse, which does not necessarily have to be the current warehouse. The local warehouses are allowed to perform lateral transshipment. This means that warehouses are allowed to share stock in case of shortages. The delivery to the Lely Center and the reverse flow do not change. After repair at the supplier, the part is not sent back to the warranty department in Maassluis but is sent directly to the closest local warehouse from where it can be delivered to the customer again. The reverse flow is brought back to a number of repaired parts that enter the system again.

1.3 Scope

The scope of the research is determined such that the research questions can be answered within the available amount of time and that the provided model includes the most important requirements for Lely. The following decisions or assumptions are made to scope down the project:

- The focus will be on the customers within Europe. Intercontinental customers are excluded from the project because of low volumes and time constraints for this project.
- Parts that have not been sold since 2020 are considered to be inactive or phased out and therefore left out of scope. The year 2020 is chosen, because by then the new warehouse, which opened in 2019, was fully up and running.

- Parts with criticality level 0 are not taken into account, because these parts are not classified on their importance.
- Parts should have the status active for the years 2020 and 2021, meaning that they are not in an introduction phase or phasing out during those years.
- The supply chain from the logistic service provider up to the customer (Lely Center) is considered. The shipment from the Lely Center to the farmer is not taken into account, because the Lely Centers are not owned by Lely International and therefore Lely International does not have enough influence on them.
- The reverse flow is too small to take into account for the design of the supply chain. However, the number of parts returning into the flow after repair can be important.

1.4 Research questions

A lot of factors need to be taken into account when introducing new DCs in the supply chain, such as their placement, the radius in which they should serve the Lely Centers and how much stock they should carry. The key performance indicators (KPIs) are availability, cost and sustainability. This leads to the following research question:

How does the introduction of multiple distribution centres influence cost, availability and sustainability?

To answer this question, several sub-questions are formulated. First, the performance of the current supply chain will be mapped as a reference point for comparison, which leads to the following research question:

RQ1: What is the performance of the current supply chain concerning cost, availability and sustainability?

Given the current situation, there is uncertainty in the demand for spare parts. Hence, a way method should be proposed to capture this uncertainty. Therefore, the second research question is:

RQ2: How to capture the uncertain demand for each SKU for each Lely Center?

After a method for capturing the uncertainty has been established, it can be used for modelling. The next step is to design the desired supply chain. This includes where to put new DCs and determining their base stock levels if new DCs are required. Hence, the third research question is:

RQ3: How many new DCs should be placed where and what should their stock levels be?

The performance of the desired supply chain can be sensitive to its input. A change in input parameters will influence availability, costs and sustainability. Therefore a sensitivity analysis can show how the model reacts to changing input parameters. The fourth research question is:

RQ4: What is the sensitivity of the model with respect to different input parameters?

The remainder of this thesis is organised as follows: Chapter 2 contains a short literature review of the model selection. The method to handle the uncertain demand is discussed in Chapter 3. Afterwards, Chapter 4 explains the mathematical model and the adaptations made to it. The case study conducted at Lely and its results are described in Chapter 5, which is followed by the conclusion and discussion in Chapter 6.

2 Literature review

This literature review addresses two topics that are relevant to this thesis project. The first part concerns the location-transportation problem for network design. The second part discusses clustering methods.

2.1 Location-transportation problem

For a long time, the location of warehouses and the determination of vehicle routes have been addressed separately. With the progression of new optimization techniques, it is now possible to integrate the problems into one as a location-transportation problem (Prodhon & Prins, 2014). This problem involves decisions at multiple levels. The strategic level decision includes the placement of warehouses, whereas inventory and transportation decisions are taken on the tactical and operational levels. Salhi & Rand (1989) were the first to quantitatively show that the location of depots and vehicle routes are interdependent. Solving the two problems separately leads to suboptimal solutions. Therefore, the Lely case concentrates on this location-transportation problem.

Cooper (1972) was the first to research the location-transportation problem. The problem can also be referred to as the transshipment-location problem or the path location-routing problem (Nagy & Salhi, 2007). In this problem, the amounts to ship from origins to destinations are determined simultaneously with the optimal locations of facilities selected from a known set of destinations. This implies that the approach should address the interrelation of the location and transportation aspects of the problem for it to belong to the location-transportation problem. The objective is to minimize the combination of both transportation and investment costs (Ogryczak et al., 1989). Although this approach is mainly used for consumer goods and parcels, it can also be used in other applications such as healthcare, military and communications (Nagy & Salhi, 2007). Schittekat & Sörensen (2009) demonstrate that the location-transportation problem can also be applied to the distribution of spare parts through third-party logistic service providers.

Mara et al. (2021) developed an extensive taxonomy for the problem which classifies 19 features that can be included in the model. For the Lely case, the most important features are a two-echelon system, stochastic data, the presence of inventory decisions and multiple parts. Stochastic data is in the form of a probability distribution where the data is represented by random variables. The decision on how much inventory each warehouse should hold can be included. Besides optimizing the location cost and the transportation cost, the inventory cost is minimized. The stochastic location-transportation problem was introduced by Holmberg & Tuy (1999). They linearized a concave cost term in the objective to obtain a linear problem. This was later extended to a multi-period stochastic location-transportation by Klibi et al. (2010). They formulated it as a two-stage stochastic program. As this problem belongs to the NP-hard problems, exact solving for larger instances is not possible within a reasonable amount of time. They show that choosing efficient heuristics can yield good results.

The stochastic location-transportation can also be extended by including multiple parts, which is demonstrated by Gzara et al. (2014). They propose two models, one with a part-warehouse inventory structure and one with a part-specific structure. Both are widely observed in practice. By exploiting specific properties of nonlinear constraints, an

equivalent linear problem can be formulated. The results obtained demonstrate that this approach is effective both in terms of computation time and solution quality.

Considering both multiple time periods and multiple parts yields another class of models, the multi-period multi-part location-transportation problem. Ghorbani & Jokar (2016) introduced a mixed-integer programming formulation for this particular problem, also including routing decisions. They found an efficient algorithm to evaluate this problem. This model was extended by Jalal et al. (2022), who added multi-modality to the model. Their results on a real-life case in the pharmaceutical industry show its effectiveness as it outperforms the solver CPLEX in solution quality and solution time.

Over the last few years, robust optimization models have become more popular. Sun et al. (2019) use robust optimization for the location-transportation problem for recharging batteries from electric vehicles. They state the importance of choosing the right uncertainty set such that the solutions are immune to demand uncertainty. The prespecified interval used is the range forecast centred at the point forecast to capture the uncertain flow demand. They reformulate the robust counterpart from a min/max inner optimization problem to an integer program which preserves the computation tractability of the robust approach.

Robust optimization can be extended to a multi-echelon setting, as demonstrated by Tirkolaee et al. (2019). They use the method of Bertsimas & Sim (2003) in which the objective function is a minimization function and both the objective function and constraints contain uncertainty coefficients. Marandi & van Houtum (2020) have extended the robust optimization method for the location-transportation problem to a problem with integer-valued demands. They have combined this with a simplex-type method that uses the convexity principle. They applied their model in the field of a distribution network in e-commerce. They show that a multi-stage location-transportation problem with integer-valued demands can be solved through adjustable robust optimization. Additionally, they demonstrate that the multi-stage problem is computationally tractable with a specific budget uncertainty set.

Lately, more attention has been paid to the closed loop supply chain design due to the impact on the environment, which can be both in reduction of CO_2 as well as the reuse of resources. Fareeduddin et al. (2015) have created a model that optimizes both the economic and environmental aspects. Amin & Zhang (2012) created a two-stage model for the closed-loop supply chain to determine where to locate warehouses and refurbishing centres and the number of parts and parts in each section of the network. Additionally, it also considers the selection of suppliers. Pazhani et al. (2021) have developed two further models for the closed loop supply chain network design. One has six echelons and separates the forward and reverse flow, while the second one has four echelons and combines the forward and reverse flow at hybrid facilities.

The model of Ghorbani & Jokar (2016) is used as a base model for the Lely case. It includes multiple parts, which matches the Lely case. What plays a significant role is that this model can be solved heuristically within two hours for large instances. Additionally, the quality of the results is high as the gap towards the optimal solution for small instances is smaller than 0.5%. Since the Lely case deals with large instances, this model is opted for.

2.2 Clustering method

An increase in the number of parts affects the solvability of the model of Ghorbani & Jokar (2016). Lely distributes 12,484 parts to its customers in Europe. This number is too large for solving the mathematical model. Therefore, a reduction in the number of parts is needed. To achieve this, parts can be clustered into groups for which the main characteristics are similar. Machine learning can be exploited for this (Ahuja et al., 2020). The two most widely used machine learning techniques for clustering are K-means clustering and hierarchical clustering. Each is discussed in more detail below.

K-means Clustering

K-means clustering is an unsupervised learning technique (Ahuja et al., 2020). The algorithm acts without supervision and no labelled or classified data is used. K-means clustering aims to find a predefined number of groups denoted by k . First, any k points are selected as initial centroids. Afterwards, based on a given set of features, each data point is allocated to one of the k groups for which the centroid is closest based on the Euclidean distance. Next, the centroid positions are recalculated and the points again are allocated to the closest centroid. The algorithm stops when no improvements are observed. The advantages of this method are its easy implementation and the fast running time for a high number of instances with small values for k . The disadvantages include its inability to work with categorical or non-contiguous data, getting stuck in local optima and deciding on the initial value of k , because no algorithm calculates it upfront.

Hierarchical Clustering

Hierarchical clustering is an unsupervised learning technique that aims to cluster unlabeled data points (Ahuja et al., 2020). Hierarchical clustering can be performed Agglomerative or Divisive. The Agglomerative method starts by considering all data points as one cluster and then breaking it into smaller clusters until each data point forms its own cluster. The Divisive method uses a bottom-up approach in which all data points make their own cluster, after which clusters are merged until one cluster with all data points remains. The clusters are formed based on the distances between the data points. The result is a dendrogram (Figure 4a), a visualisation of the clusters and their distances. The y-axis shows the distance metric between clusters. The x-axis shows the clusters. The main advantages of hierarchical clustering are its easy implementation and hierarchical output. On the other hand, the method does not work well for large data sets and is very sensitive to outliers.

To classify the parts into different clusters, hierarchical clustering is selected, because this approach calculates the appropriate number of clusters in the algorithm. Furthermore, the algorithm always yields the same result, which is not necessarily true for k-means clustering. Hierarchical clustering has already been applied to spare parts by Raja et al. (2016). Several clustering approaches are available to merge the clusters, namely single linkage, maximum linkage, weighted linkage and ward method (Pezer, 2017). For the purpose of interpretation, it is common practice to select Ward's approach, because it results in a dendrogram which helps to define clusters for the classification of inventory parts (Narkhede & Rajhans, 2020). The agglomeration coefficient determines when to stop combining clusters. A large increase in the coefficient suggests combining two rather different clusters (Chavez et al., 2010). To visualise the agglomeration coefficient, an elbow plot (Figure 4b) can be used. This example shows an optimal number of 3 clusters.

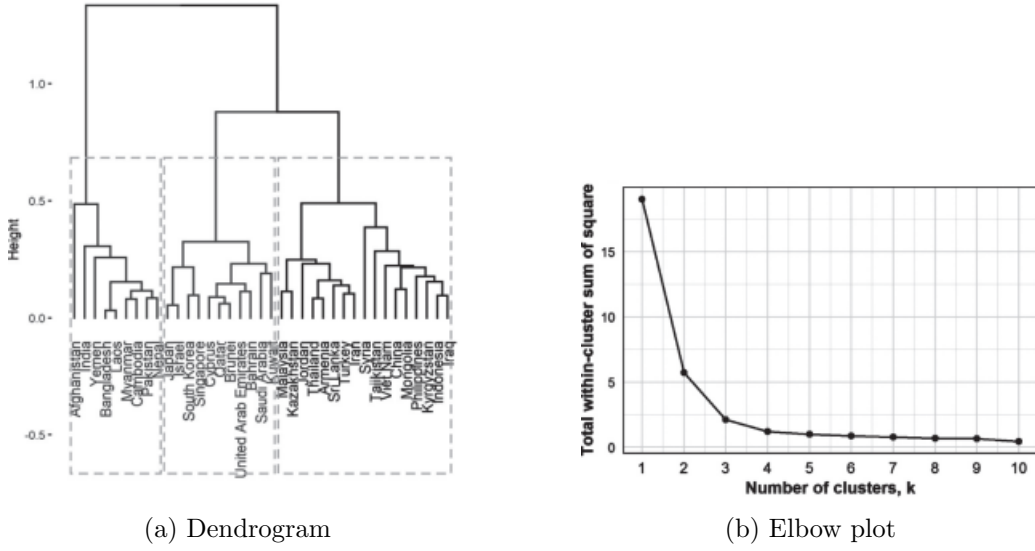


Figure 4: Dendrogram and Elbow plot (Saifuddin & Hassan, 2021)

Feature selection

The first step is to select the appropriate features for clustering. Raja et al. (2016) list eleven features they use to cluster on and Bacchetti et al. (2010) consider three additional ones. All features can be found in Table 5. Not all of them are relevant to this research. Below, each of them is discussed shortly to explain why the feature is included or not.

Table 5: Clustering features

1. Part value	8. Size
2. Average usage per year	9. Current inventory policy
3. Number of suppliers	10. Current maximum inventory level
4. Lot size of purchasing	11. User of the part
5. Lead time	12. Sales cycle phase
6. Type of material	13. Demand frequency
7. Volume	14. Criticality

A few features are selected for the purpose of data cleaning only. The sales cycle phase should be equal for all parts for fitting demand distributions. Hence only parts that have had an active status for the whole of 2020 and 2021 are kept for the analysis. Furthermore, only parts with a known criticality are considered. Concerning criticality, the parts are split manually based on expert knowledge, because criticality is measured on a nominal scale. Hence, parts are separated on criticality before applying the clustering algorithm.

The features selected for clustering are part value, average usage per year, lead time, volume and demand frequency. Part value is an important feature for determining the order-up-to level, as there are more costs for expensive parts. Hence this feature is important for inventory control. Average usage per year is an important feature for fitting demand distributions to the groups. The same goes for lead time and demand frequency. For the fitting of demand distributions, the lead time and frequency of demand

must not vary too much within the clusters. Volume is selected as a feature that can be of importance in inventory control as bigger parts consume more space in the warehouse.

The remaining features are not selected. The number of suppliers is not considered because each part is supplied by one supplier only. The lot size of purchasing is not included as Lely wants to make them subject to change rather than as a given feature. The type of material is irrelevant as all materials are treated the same. The size is considered to be a similar feature as volume and is therefore not included. Current inventory policy and current maximum inventory level are left out because of the creation of a new situation. Lastly, the user of the part is not considered, because the only customers are Lely Centers and therefore the user group is homogeneous.

Normalization

Each of the selected features is measured on a different scale. To properly cluster the data, all features are required to be on the same scale. This is achieved by normalizing the data, as previously demonstrated by Raja et al. (2016). For demand data, normalization is necessary to ensure that the clusters are formed based on the shape of the demand pattern, even though some might have high overall demand while others have low overall demand (Steuer et al., 2018). Normalization is done through the Z-score. This method uses the mean and standard deviation to rescale the data such that all features have a zero mean and unit variance (Singh & Singh, 2020).

Multicollinearity

One of the assumptions of hierarchical clustering is the absence of interdependencies among the features. Interdependence poses a problem because it means that multiple features measure the same variance. A multicollinearity test can be applied to check for independence. A formal test for multicollinearity is the variance inflation factor (VIF). A VIF of five or higher indicates a problem with multicollinearity (Yu et al., 2015). The VIF scores for the three criticality groups are shown in Table 6. All scores are well below the threshold value of five, so multicollinearity is not a problem.

Table 6: VIF scores

Feature	Criticality 1	Criticality 2	Criticality 3
Lead time	1.37	1.29	1.37
Demand frequency	1.31	1.22	1.21
Average usage per year	1.01	1.02	1.03
Part value	1.26	1.32	1.81
Volume	1.18	1.24	1.59

3 Demand uncertainty

This chapter concerns demand uncertainty. The first part focuses on scaling down the number of parts. The mathematical model of Ghorbani & Jokar (2016) cannot handle 2,723 parts because the computing time would be too long. Therefore, the number of parts is reduced through clustering. Afterwards, the uncertainty of the demand for those groups needs to be captured. Hence, demand distributions are fitted on the demand data of the groups that resulted from the clustering analysis, similar to work by Turrini & Meissner (2019).

3.1 Clustering results

Before clustering is applied, for each part the demand data is aggregated per week, which makes it easier to fit demand distributions in a later stage. The hierarchical clustering algorithm considers each feature to have equal weight. However, it is possible to adapt this and assign weights to emphasize the importance of each feature (Murtagh & Contreras, 2012). All features play a role in inventory control, while lead time and demand frequency are the two most important features for demand fitting. The weights are determined based on expert knowledge from the company and me. Lead time is a major factor in determining the base stock policy because parts with a long lead time need more safety stock than parts with a short lead time. Therefore, lead time got assigned a weight of 2. Demand frequency can distinguish between frequently occurring demand and intermittent demand. Hence, demand frequency receives a weighting of 1.5. Since volume is less important than the other features in inventory control as it only influences the space needed, which in turn influences the holding cost, it got assigned a weighting of 0.5. Average usage per year and part value receive the regular weighting of 1.

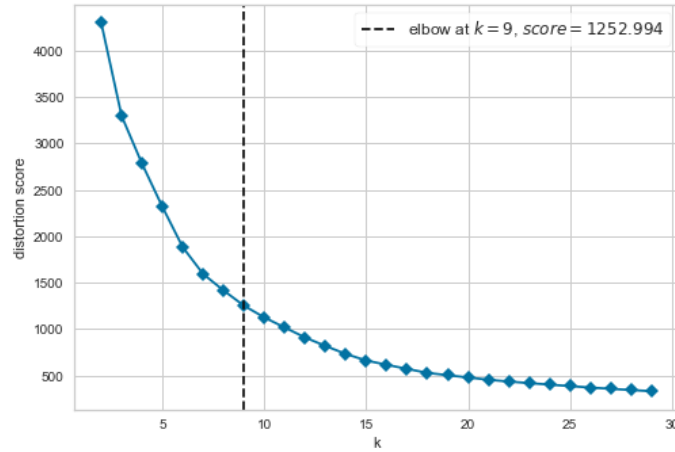


Figure 5: Elbow plot criticality 3 parts

Clustering is performed twice. Once for the optimal number of groups per criticality level and once for fifteen clusters per criticality level. The purpose of creating fifteen clusters is to check what the influence of providing more clusters is on the fitting of distributions as the ranges are expected to become smaller. The clustering results for parts with criticality 3 are depicted in Figure 5 and Figure 6 in form of an elbow plot and a dendrogram. The elbow plot shows that the optimal number of clusters is nine. This is represented by the dotted lines in the figures. Table 7 shows the number of parts per cluster for the model with

the optimal number of clusters. For the model with fifteen clusters per criticality level, the number of parts per cluster is represented by the numbers on the x-axis of the dendrogram. Figure 6 shows which clusters were split in order to get from nine to fifteen clusters. The parts in cluster four of the optimal model remain one cluster and become cluster six in the model with fifteen clusters per criticality level. Cluster nine of the optimal model is split into two to get clusters fourteen and fifteen for the model with fifteen clusters per criticality level. Some clusters are split twice such as cluster five of the optimal model, which becomes clusters seven, eight and nine in the model with fifteen clusters per criticality level. The results for parts with criticality 1 and 2 can be found in Appendix A.

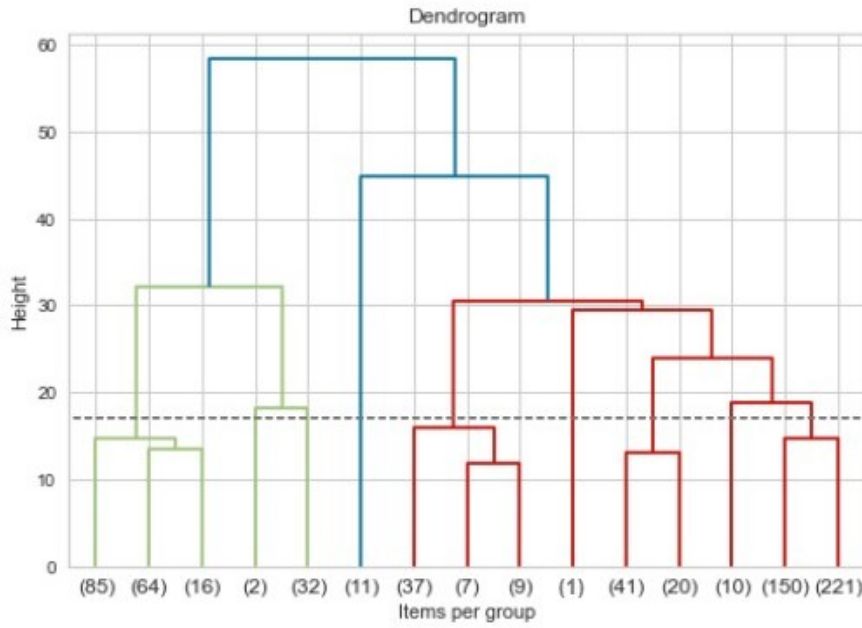


Figure 6: Dendrogram criticality 3 parts

Table 7: Number of parts per cluster with optimal number of cluster for criticality 3

Cluster	1	2	3	4	5	6	7	8	9
Number of parts	165	2	32	11	53	1	61	10	371

Table 8: Ranges clusters criticality 3 parts

C	Lmin	Lmax	Pmin	Pmax	Imin	Imax	Qmin	Qmax	Vmin	Vmax
1	35	80	0.01X	157.21X	1.00	5.00	0.28	174.46	1	32584
2	205	210	0.05X	1.03X	1.13	1.57	4.44	20.37	1	22
3	76	140	0.02X	103.29X	1.00	3.06	0.62	116.51	1	31660
4	10	40	0.10X	86.79X	18.20	34.00	0.03	7.14	12	15157
5	5	65	38.54X	236.86X	1.00	10.78	0.10	45.29	1	145025
6	35	35	0.85X	0.85X	1.00	1.00	2931.67	2931.67	53	53
7	5	49	0.02X	44.50X	2.51	16.00	0.05	59.33	1	6670
8	10	65	0.15X	14.66X	1.00	5.33	183.06	1279.11	6	2000
9	4	35	0.00X	56.25X	1.00	4.04	0.19	290.68	1	16505

Table 8 displays the ranges for each feature for the clusters for the model with nine clusters. L denotes the lead time in days, P denotes the part value in euros, I denotes the demand interval in weeks (a high number means low frequency and vice versa), Q denotes average usage per year in the number of parts, and V denotes volume in cm^3 . For confidentiality reasons, the real part values are not given. The minimum price of cluster one serves as the default and is set to 0.01X. All part values are scaled accordingly. A value of 0.00X means the part has a low value. The results in Table 8 show that, as a result of the applied weightings, the ranges for lead time are the smallest while the ranges for volume are the largest. Clusters might focus on a specific feature. Cluster eight has parts with a high average usage per year, while cluster five includes parts with a high part value. However, some ranges can be extensive, such as the lead time for clusters three and eight, or the price of cluster one. Table 9 displays the ranges for each feature for the clusters for the model with fifteen clusters. When comparing the ranges in Table 9 to the ranges in Table 8, it can be noted that some ranges are split. This is the case for the lead time of clusters fourteen and fifteen of the model with fifteen clusters for criticality three compared to cluster nine of the model with nine clusters for criticality level three. Smaller ranges improve the representability of a group for the parts in it. The results for parts with criticality 1 and 2 can be found in Appendix A.

Table 9: Ranges clusters criticality 3 parts

C	Lmin	Lmax	Pmin	Pmax	Imin	Imax	Qmin	Qmax	Vmin	Vmax
1	35	61	0.01X	72.44X	1.00	5.00	0.28	74.69	1	23891
2	60	70	0.11X	18.67X	1.00	2.81	0.84	174.46	1	4701
3	40	80	54.73X	157.22X	1.01	4.43	0.29	11.40	38	32584
4	205	210	0.05X	1.03X	1.13	1.57	4.44	20.37	1	22.00
5	76	140	0.02X	103.38X	1.00	3.06	0.63	116.51	1	31660
6	10	40	0.10X	86.79X	18.20	34.00	0.03	7.14	12	15157
7	10	30	38.54X	136.78X	1.00	6.80	0.18	38.48	1	56086
8	5	30	166.38X	236.86X	1.00	3.85	0.33	45.29	9198	45721
9	10	65	105.25X	174.00X	1.02	10.78	0.10	10.30	61029	145025
10	35	35	0.85X	0.85X	1.00	1.00	2931.67	2931.67	53	53
11	5	30	0.02X	21.24X	2.51	8.67	0.18	59.33	1	6224
12	10	49	0.09X	44.50X	6.80	16.00	0.05	0.50	1	6670
13	10	65	0.15X	14.66X	1.00	5.33	183.06	1279.11	6	2000
14	4	20	0.00X	56.25X	1.00	3.96	0.82	290.68	1	16505
15	20	35	0.00X	48.61X	1.00	4.04	0.19	168.34	1	15771

In Table 8, clusters one, three, five and eight have a large range for lead time. The clustering algorithm opts to split clusters one and five to decrease the range in lead time for those clusters in the model with fifteen clusters per criticality level. In Table 9, clusters one, two and three originate from cluster one from Table 8. Apart from lead time, part value is considered for the split. Clusters three and eight remain intact and become clusters five and thirteen in the model with fifteen clusters per criticality level. These clusters do not have priority to be split and the algorithm decides to split clusters one and five from the optimal model. Except for the part value in cluster one, no clear reason is found for why these specific clusters are prioritised for splitting. For this thesis, it is out of scope to find this reason, but future research can dig deeper into an explanation for why priority is given to prioritising certain clusters above others while they have similar ranges for features.

One direction could be research into a quantitative method for calculating weights for features, which was done based on the judgement of experts, but not on a quantitative method. Another direction could be research into other methods than applying weights to emphasise certain features.

3.2 Demand fitting method

In a context where demand is uncertain, it is common practice to fit a distribution to the known historical demand data. Turrini & Meissner (2019) list the five most widely used distributions for spare parts: normal distribution, Poisson distribution, Negative binomial distribution (NB), gamma distribution and the compound Poisson (CP) distribution. Boon et al. (2020) describe how maximum likelihood estimation (MLE) can be applied for the estimation of the parameters of these distributions. MLE makes use of the maximum likelihood function which is also called the likelihood of the data. For a function with one parameter, the derivative of the likelihood function is taken and set equal to zero to find the estimator. For distributions that depend on more than one parameter, the partial derivatives with respect to all parameters need to be set to zero. The Kolmogorov-Smirnov (KS) test can be used to determine if there is a good fit or not. A p-value larger than 0.05 indicates a strong fit. A p-value larger than 0.01 but smaller than 0.05 indicates a good fit (Turrini & Meissner, 2019). The Akaike Information Criterion (AIC) and Bayesian information criterion (BIC) can be used to select the best fitting probability distribution (Gupta et al., 2020). Both the AIC and BIC minimize the negative likelihood. The lower the value for AIC the better the model fits the data. The same goes for BIC.

3.3 Demand fitting results

The demand of each part in a cluster is summed to get the total aggregated demand for each week per cluster. The lead time, price and volume of a part are calculated as a weighted average over the total demand. A part with a high overall demand for that cluster has a bigger influence on these features than a part with a low overall demand. To illustrate, if there are two parts in a cluster with one having a demand of 8 and lead time of 4 and the other having a demand of 2 and lead time of 6, the lead time for that cluster is calculated to be $\frac{8}{10} * 4 + \frac{2}{10} * 6 = 4.4$.

The normal distribution, gamma distribution, Poisson distribution, negative binomial distribution and compound Poisson distribution are fitted to the clusters. These distributions are chosen, because they are most commonly used for spare parts (Turrini & Meissner, 2019). Appendix B contains the formulation of each of the demand distributions. Below you can find the results for the clusters with criticality level three. Table 10 and Table 12 display the AIC values for the fitted distributions. The numbers in black indicate a strong fit for the data. The numbers in blue indicate a good fit, whereas the numbers in red indicate no fit. For the gamma distribution, it was not always possible to fit a distribution with MLE. In some cases, it was possible to fit one with the method of moments. When this is the case, the AIC is shown in green if there was a good fit. In some other cases, it was not possible to fit a gamma distribution, because the dataset contains values of zero. The gamma distribution is only defined for values greater than zero. The distribution can afterwards be corrected for by parameter $\hat{q} = \frac{n_0}{n}$, which is the probability of a zero value in the data (Husak et al., 2007). If the number of zero values was less than

25% of the data, the zeros are removed and MLE is used to fit the gamma distribution. When this is the case, the AIC is shown in grey if there was a good fit. Since data is omitted for this approach, this distribution will only be picked when no other distribution fits. When more than 25% of the data consists of zero values, no gamma distribution could be fitted. The bold number in each row shows which distribution fits best because it has the lowest AIC. Tables 11 and 13 display the parameters of the fitted distribution for each cluster. The results for the clusters with criticality levels 1 and 2 can be found in Appendix C.

Cluster four contains a lot of zero demand values, several small demand values and a few large ones. No distribution could be fitted because of the odd values as shown in Table 11. The same goes for cluster six in Table 13, because it is the same cluster as shown in the dendrogram in Figure 6. It was also not possible to fit a demand distribution to cluster twelve. Together with cluster eleven, it originates from cluster seven from the nine-cluster model. Cluster seven is split on demand interval and average quantity. Cluster twelve contains parts that do not occur frequently and if they do, demand is low.

Table 10: AIC fitting models criticality 3 clusters

Cluster	Normal	Gamma	Poisson	Negative binomial	Compound Poisson
1	1540.09	1558.93	9532.39	1588.69	1551.63
2	996.30	859.60	2820.95	892.12	910.56
3	1307.12	1313.63	4289.87	1313.34	1309.40
4	1136.02	-	5927.47	349.29	1085.41
5	1159.87	1161.16	2239.61	1160.94	1158.48
6	1828.04	1371.86	75007.86	1821.33	1827.28
7	1539.77	1358.36	36467.59	1359.14	1363.96
8	1815.55	1824.79	40309.05	1824.75	1819.61
9	1839.90	1856.48	34765.59	1856.44	1855.86

Table 11: Parameters fitted distribution criticality 3 clusters

Cluster	Distribution	Parameter 1	Parameter 2	Parameter 3
1	Normal	$\mu = 1758.57$	$\sigma = 363.97$	
2	Negative binomial	$p = 1.09$	$r = 0.0440$	
3	Normal	$\mu = 452.83$	$\sigma = 119.87$	
4				
5	Compound Poisson	$p = 1.5$	$\mu = 257.67$	$\phi = 0.9643$
6	Negative binomial	$p = 3.68$	$r = 0.0013$	
7	Gamma	$k = 0.7104$	$\beta = 0.0029$	
8	Normal	$\mu = 5076.71$	$\sigma = 1349.48$	
9	Normal	$\mu = 7742.44$	$\sigma = 1515.38$	

Table 12: AIC fitting models criticality 3 for 15 clusters

Cluster	Normal	Gamma	Poisson	Negative binomial	Compound Poisson
1	1442.89	1455.80	16718.82	1455.52	1449.32
2	1388.20	1397.91	5687.69	1397.56	1391.72
3	836.20	836.61	951.19	836.01	833.70
4	996.30	859.60	2820.95	892.12	910.56
5	1307.12	1313.63	4289.87	1313.34	1309.40
6	1136.02	-	5927.47	349.29	1085.41
7	1075.68	1081.96	1714.65	1081.14	1104.49
8	981.27	977.22	1698.02	976.73	974.53
9	769.70	734.12	920.98	736.83	762.52
10	1828.04	1822.38	75007.86	1821.33	1827.28
11	1539.90	1351.24	36933.07	1352.38	1357.16
12	646.89	253.77	797.19	508.62	554.94
13	1815.55	1824.79	40309.05	1824.75	1819.61
14	1743.58	1751.20	24463.74	1751.16	1751.52
15	1680.37	1699.27	17646.87	1699.16	1696.18

Table 13: Parameters fitted distribution criticality 3 for clusters

Cluster	Distribution	Parameter 1	Parameter 2	Parameter 3
1	Normal	$\mu = 1002.47$	$\sigma = 228.81$	
2	Normal	$\mu = 706.79$	$\sigma = 176.36$	
3	Compound Poisson	$p = 1.3$	$\mu = 49.32$	$\phi = 1.11$
4	Negative binomial	$p = 1.09$	$r = 0.0440$	
5	Normal	$\mu = 452.83$	$\sigma = 119.87$	
6				
7	Normal	$\mu = 175.44$	$\sigma = 39.82$	
8	Compound Poisson	$p = 1.6$	$\mu = 63.54$	$\phi = 0.96$
9	Gamma	$k = 4.63$	$\beta = 0.2548$	
10	Negative binomial	$p = 3.68$	$r = 0.0013$	
11	Gamma	$k = 0.6685$	$\beta = 0.0028$	
12				
13	Normal	$\mu = 5076.71$	$\sigma = 1349.48$	
14	Normal	$\mu = 4286.40$	$\sigma = 957.95$	
15	Normal	$\mu = 3456.04$	$\sigma = 708.95$	

The purpose of the demand distribution is to calculate a base stock level. Groups with no demand distribution will get a different approach to calculating a base stock level. Therefore, clusters that do not have a demand distribution are further analysed on individual part level. In these clusters, there might be some parts with higher demand than others. Therefore these clusters are split into multiple groups if necessary to create groups that have the same demand pattern in terms of quantity. Hence, cluster four is split into two groups, one with items that have a high demand if it occurs and one with a low demand if it occurs. For the clusters in the other two criticality levels, cluster nineteen is split into two groups as well based on the same logic, while cluster twenty-three is split into

three groups because the differences were larger there. Cluster eleven remains one cluster. The model with fifteen clusters per criticality level has a total of twelve groups for which no demand distribution could be fitted. This makes it much more difficult to calculate reliable base stock levels in the mathematical model. Therefore, these results are not used in the mathematical model and the model that displays the optimal number of clusters per criticality level is opted for.

4 Mathematical model

This chapter addresses the mathematical model used to solve the network design problem. First, the base model is presented, after which some adaptations are established to obtain the final model, so it is in line with the case of Lely. Next, the solution method to solve the mathematical model is posed. The last section describes the simulation model to evaluate the performance of the results of the model.

4.1 Model description

The model of Ghorbani & Jokar (2016) is used as a basis (Appendix D). They provide a mixed-integer programming formulation for the multi-part, multisource location inventory routing problem. Customers demand multiple parts. The parts are grouped into clusters for the mathematical model. Therefore, clusters are considered and customers demand clusters. The customers are numbered 1 to K and \mathcal{K} denotes the set of customers. The clusters are numbered 1 to c and \mathcal{C} denotes the set of clusters. Demand is served from a warehouse selected from a set of candidate warehouses. The candidate warehouses are numbered from 1 to J and \mathcal{J} denotes the set of candidate warehouses. Customer demand is fulfilled from stock. Each warehouse, therefore, has a base stock level for all clusters. The base stock level can take the value of an integer in the range from 0 to S_{max} and \mathcal{S} denotes the set of base stock levels.

External suppliers supply the warehouses. An external supplier can deliver to one warehouse only. External suppliers are always assumed to have enough stock available for replenishment of the warehouses. The transportation cost per cluster per kilometre between suppliers and warehouses is denoted by Cs_{jc} , where the distance is denoted by ds_{jc} . The other warehouses are replenished through transshipments between warehouses. The transportation costs between two warehouses are not cluster-dependent and are only per kilometre denoted by $Cw_{jj'}$, where the distance is denoted by $dw_{jj'}$. On top of that, a fixed cost for driving between warehouses is incurred, which is expressed by $ew_{jj'}$. A fixed order cost A is incurred for every order placed at a supplier or another warehouse in case of transshipment. Clusters transhipped from one warehouse to the next need to be unloaded and loaded. Handling costs ha_{jc} are incurred for this.

Customers place their orders every period. Hence, the review period (R_{jc}) is one period. The average periodic demand of customers for a cluster is denoted by d_{kc} . Binary parameter g_{kc} states whether this periodic demand is positive (with value 1 if periodic demand is positive). After lead time t_{jc} , the replenishment order arrives at the warehouse. The transportation cost per cluster per kilometre between warehouses and customers is denoted by Cd_{jkc} , where the distance is denoted by dd_{jk} .

Apart from order costs, handling costs and transportation costs, a periodic holding cost h_{jp} is incurred for the on-hand inventory. The on-hand inventory follows from the base stock level. The base stock level is set such that the required availability level α_{jc} is reached. Furthermore, the base stock level depends on the demand distribution and its parameters for the cluster. The input to get the unique solution for a base stock level is denoted by $\eta_s(\alpha_{jc})$. The on-hand inventory is the base stock level minus the average demand during lead time minus half the average demand during the review period.

The company needs to make several decisions. The first decision is whether a warehouse is opened at candidate location j , which is modelled with binary decision variable y_j (with value 1 if the warehouse is opened). The second decision is on which warehouse a supplier is assigned to, which is modelled with binary decision variable Z_{jc} (with value 1 if the supplier of cluster c delivers to warehouse j). The third decision is on which warehouse a customer is assigned to, which is denoted with binary decision variable B_{jk} (with value 1 if customer k is served by warehouse j). The fourth decision is on the base stock levels of the warehouses, which is denoted by binary decision variable V_{jcs} (with value 1 if the base stock level equals s). The final decisions are on three types of transportation movements. The first type is transportation between the supplier of cluster c and warehouse j , which is modelled by binary decision variable ts_{jc} (with value 1 if transportation is required). The second type is transportation between warehouse j and customer k , which is modelled by binary decision variable td_{jk} (with value 1 if transportation is required). The third type is transportation between warehouses, which is modelled with decision variable $tw_{jj'}$ (with value 1 if transshipment between warehouses occurs).

Besides the model, the CO₂ emission is calculated based on the driven distance and the emission factor CO. Table 14, Table 15 and Table 16 summarise the notation used.

Table 14: Sets adapted model

$\mathcal{J} = \{1, 2, \dots, J\}$	Set of warehouses
$\mathcal{K} = \{1, 2, \dots, K\}$	Set of customers
$\mathcal{C} = \{1, 2, \dots, c\}$	Set of clusters
$\mathcal{S} = \{0, 1, \dots, S_{max}\}$	Set of base stock levels

Table 15: Parameters adapted model

h_{jc}	Inventory holding cost for cluster c at warehouse j per time period	(€/cluster)
ha_{jc}	Handling cost of cluster c at warehouse j in case of transshipment from warehouse j to any other warehouse	(€/cluster)
A	Fixed cost for placing an order at supplier or at other warehouse in case of transshipment between warehouses	(€/order)
ds_{jc}	Distance between the supplier of cluster c and warehouse j	(kilometres)
dd_{jk}	Distance between warehouse j and customer k	(kilometres)
$dw_{jj'}$	Distance between warehouse j and warehouse j'	(kilometres)
α_{jc}	Required availability level of cluster c at warehouse j	
Cs_{jc}	Transportation cost of cluster c from the supplier of cluster c to warehouse j	(€/cluster/kilometre)
Cd_{jkc}	Transportation cost of cluster c from warehouse j to customer k	(€/cluster/kilometre)
$Cw_{jj'}$	Transportation cost from warehouse j to warehouse j'	(€/kilometre)

$ew_{jj'}$	Fixed transport cost from warehouse j to warehouse j'	(€/shipment)
d_{kc}	Average periodic demand of cluster c from customer k	(clusters)
t_{jc}	Lead time of cluster c for warehouse j	(weeks)
$\eta_s(\alpha_{jc})$	input data for unique solution of fill rate $\beta(\zeta_c, s) = \alpha_{jc}$, where ζ_c denotes the input parameters for the necessary distribution for cluster c	
R_{jc}	Review period for cluster c at warehouse j	(weeks)
g_{kc}	binary parameter indicating if cluster c is demanded by customer k	
CO	CO ₂ emission factor	(kilogram/kilometre)
M	Big value	

Table 16: Decision variables adapted model

Z_{jc}	$\begin{cases} 1 & \text{if cluster } c \text{ is supplied to warehouse } j \\ 0 & \text{otherwise} \end{cases}$
y_j	$\begin{cases} 1 & \text{if warehouse } j \text{ is opened} \\ 0 & \text{otherwise} \end{cases}$
B_{jk}	$\begin{cases} 1 & \text{if customer } k \text{ is assigned to warehouse } j \\ 0 & \text{otherwise} \end{cases}$
ts_{jc}	$\begin{cases} 1 & \text{if transport happens between the supplier of cluster } c \text{ and warehouse } j \\ 0 & \text{otherwise} \end{cases}$
td_{jk}	$\begin{cases} 1 & \text{if transport happens between warehouse } j \text{ and customer } k \\ 0 & \text{otherwise} \end{cases}$
$tw_{jj'}$	$\begin{cases} 1 & \text{if transport happens between warehouse } j \text{ and warehouse } j' \\ 0 & \text{otherwise} \end{cases}$
V_{jcs}	$\begin{cases} 1 & \text{if base-stock level is } s \text{ for cluster } c \text{ at warehouse } j \\ 0 & \text{otherwise} \end{cases}$

4.2 Adaptations

A couple of changes are proposed. First, the inventory control policy changes from calculating inventory levels and backlog quantities to base stock levels. Additionally, the deterministic model turns to a stochastic model by including demand distributions instead of using demand realisations. Furthermore, the shipment policies are changed. In the base model, suppliers may supply multiple warehouses. The adapted model restricts a supplier to deliver to one warehouse only because this supports consolidation of shipments. The introduction of transshipment between warehouses allows for goods exchange between warehouses. Additionally, the capacity levels of the warehouses are omitted. Furthermore, the routing part is omitted from the model. Each of the changes is explained in more detail.

4.2.1 Transport & capacity levels

Contradictory to the model of Ghorbani & Jokar (2016), suppliers may deliver the clusters to one open warehouse only instead of all opened ones. Constraints (1) occur for all suppliers and warehouses j , where Z_{jc} indicates if the supplier of cluster c is assigned to supply to warehouse j .

$$\sum_{j \in \mathcal{J}} Z_{jc} = 1 \quad \forall c \in \mathcal{C} \quad (1)$$

Customers can only be served by one warehouse. The warehouse where the supplier delivers the cluster might not necessarily be the warehouse from which every customer is served. Therefore, transshipments between warehouses are allowed, which results in the consolidation of shipments. Hence, there will be three sorts of transportation movements instead of two, namely from the supplier of cluster c to warehouse j , from warehouse j to customer k and from warehouse j to warehouse j' . The variables ts_{jp} , td_{jk} and $tw_{jj'}$ are introduced to denote if transport occurs between two nodes. They will be one if transport occurs between the two nodes and zero otherwise. The cost of transportation between suppliers and warehouses, as well as the cost of transportation between warehouses and customers, is cluster-dependent and distance-dependent. The transportation cost between warehouses is distance-dependent only, as there are special agreements about a fixed price for setting up a shuttle service between warehouses. Last, each warehouse is large enough to carry all inventory. Therefore, capacity levels are dropped from the model as they do not pose a restriction.

4.2.2 Stochasticity

Instead of calculating the inventory levels and backlog quantities in each period, the inventory control policy is translated to calculating base stock levels for a certain fill rate. Gzara et al. (2014) give an expression for the fill rate for demand under a Poisson distribution. Given the mean demand λ_{jc} and stock levels S_{jc} for each cluster c and each warehouse j , the service level can be defined as $\beta(\lambda_{jc}, S_{jc}) = \Pr(\tilde{\lambda}_{jc} \leq S_{jc} - 1)$. In case of a continuous review policy, such as used by Gzara et al. (2014), λ_{jc} denotes the lead time demand for cluster c at warehouse j . However, this research considers replenishment only once a week. Hence, a review period of one week is used. Therefore, the expression is converted to make it suitable for a periodic review by letting λ_{jc} denote the demand during lead time plus review period (Van Donselaar & Broekmeulen, 2008). A fill rate of α_{jc} for each cluster at each warehouse can be achieved when stock availability during lead time is strictly larger than demand at least α_{jc} . This results in a non-linear expression:

$$S_{jc} \leq S_{max} * y_j, \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \quad (2)$$

$$\Pr(\tilde{\lambda}_{jc} \leq S_{jc} - y_j) \geq \alpha_{jc}, \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \quad (3)$$

This non-linear expression can be reformulated in a linear one to make it suitable for a mixed integer program. The fill rate function is equal to the probability density function, which is:

$$\beta(\lambda, S) = e^{-\lambda} \sum_{k=0}^{S-1} \frac{\lambda^k}{k!}, \quad \lambda \in [0, \infty), \quad S \in \{1, 2, \dots\} \quad (4)$$

The fill rate equals one when the demand rate is zero and the fill rate tends to zero when the demand rate tends to ∞ . Additionally, the fill rate is strictly monotonically decreasing with respect to the demand rate, because the derivative is negative for $\lambda \in (0, \infty)$, which is displayed in equation (5). Equivalently, the cumulative distribution function must be strictly monotonically increasing.

$$\frac{d\beta(\lambda, S)}{d\lambda} = -e^{-\lambda} \frac{\lambda^{S-1}}{S-1!} < 0, \quad \lambda \in [0, \infty), \quad S \in \{1, 2, \dots\} \quad (5)$$

This means that there is a unique solution for any $\alpha \in (0, 1)$ for $\beta(\lambda, S) = \alpha$, which can be denoted by $\lambda(S, \alpha)$. For $\alpha = 0.9$ and $S = 1, 2, \dots, 6$, this is demonstrated in Figure 7. Hence, the following linear expression is formulated to replace the expressions in Equations (2) and (3):

$$\sum_{s \in \mathcal{S}} V_{jcs} \leq y_j, \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C}, \quad (6)$$

$$\lambda_{jp} \leq \sum_{s \in \mathcal{S}} \lambda_s(\alpha_{jc}) * V_{jcs}, \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C}, \quad (7)$$

where the binary variable V_{jcs} takes the value one if the base stock level for cluster c in warehouse j is s and zero otherwise. Equation (6) ensures that only one base stock level is selected for each cluster at each warehouse. Equation (7) sets the binary variable V_{jcs} at one for the lowest base stock level that ensures the fill rate α_{jc} for demand during lead time plus review period λ_{jc} , which is $\lambda_s(\alpha_{jc})$.

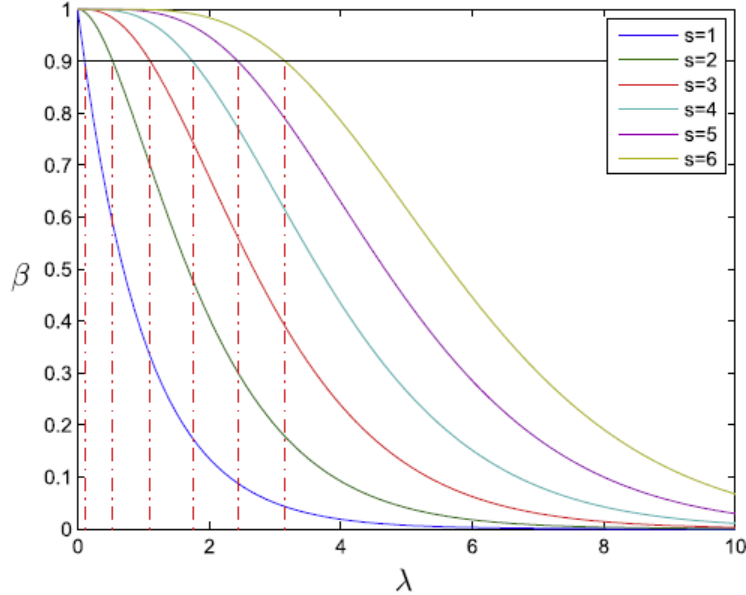


Figure 7: Graph of $\beta(\lambda, S)$ with respect to λ for $S = 1, 2, \dots, 6$ and $\alpha = 0.9$ (Gzara et al., 2014)

This method can also be applied to other demand distributions as long as their equivalent fill rate function has the same properties. The cumulative distribution function must be strictly monotonically increasing, which is the case as long as you have a distribution where

the probability density function is not 0 because $\Pr(X \leq x) = \Pr(X \leq y) + \Pr(y \leq X \leq x)$ for $y \leq x$. Montgomery & Runger (2010) show that this holds for discrete distributions such as the Poisson distribution and the negative binomial distribution and continuous distributions such as the normal distribution, gamma distribution and compound Poisson distribution. Therefore, the cumulative distribution function is monotonically strictly increasing and hence the fill rate distribution is monotonically strictly decreasing.

The derivation for Equation (7) is from the Poisson distribution. Because the model includes multiple distributions, we generalize the constraints. Let d_{jc} denote the average demand for cluster c realised in warehouse j . λ_{jc} is generalised to d_{jc} , which denotes the average demand for a product at a warehouse. So, $d_{jc} = \sum_{k \in \mathcal{K}} d_{kc} * B_{jk}$. $\sum_{k \in \mathcal{K}} d_{kc}$ is calculated from the data. The demand distributions from chapter 3 are calculated for all warehouses combined. To get d_{jc} , the distributions need to be scaled per warehouse. Let the scaling factor for each warehouse and each cluster be denoted by θ_{jc} , which equals $\frac{d_{jc}}{\sum_{k \in \mathcal{K}} d_{kc}}$. For the Poisson distribution, both the mean and variance are denoted by λ . Therefore the parameter can be scaled to $\theta_{jc} * d_{kc}$ to get the scaled distribution. For the other distributions, the scaling is not so straightforward and more computations are needed. Appendix B explains the derivations for the scaled parameters for each distribution.

From the expression in Equation (7), $\lambda_s(\alpha_{jc})$ is also generalised to make it suitable for more distributions. It is denoted by $\eta_s(\alpha_{jc})$ which is the unique solution of fill rate $\beta(\zeta_c, s) = \alpha_{jc}$, where ζ_c denotes the input parameters for the necessary distribution for cluster c . Hence, Equation (7) can be translated to Equation (8) to make it applicable to all used distributions.

$$\sum_{k \in \mathcal{K}} d_{kp} * B_{jk} \leq \sum_{s \in \mathcal{S}} \eta_s(\alpha_{jc}) * V_{jcs} \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \quad (8)$$

Equation (8) cannot be used for clusters that do not have a demand distribution. For those clusters, the safety stock is set to two times the average in case of a positive demand value (Q). Two times Q is opted for because two standard deviations correspond with a fill rate of more than 95% for the normal distribution. Therefore, S becomes $\lambda_{jc} + 2 * Q$. When this results in a decimal number, it is rounded up to the next integer.

4.3 Final model

Applying the changes explained before to the model of Ghorbani & Jokar (2016), the following mixed-integer linear program is obtained:

4.3.1 Objective

The previously defined sets, parameters and decision variables are used to construct the objective function. The overall objective is to minimise the total costs of the network. The objective function consists of eight components. Each of them is discussed separately.

The base stock level contains items in the warehouse and in the pipeline, which means they have already been ordered but have not yet arrived in the warehouse because of the lead time. Holding costs will only be paid for the items physically present in the warehouse. Therefore, the on-hand inventory needs to be calculated from the base stock levels. In case

of a fill rate close to 100%, the expected on-hand inventory ($E[I^{OH}]$) can be computed by taking the base stock level from which the expected demand during lead time plus half a review period is subtracted (Equation (9) (Van Donselaar & Broekmeulen, 2019)). The fill rates aimed for by Lely are close to 100%, so this method can be applied.

$$E[I^{OH}] = s * V_{jcs} - (t_{jc} + 0.5 * R_{jc}) * \sum_{k \in \mathcal{K}} d_{jk} * B_{jk} \quad (9)$$

Holding costs are only charged for the items that are physically present in the warehouse and not for the entire base stock level. Equation (10) shows the expression for the total holding cost.

$$\sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}} \sum_{j \in \mathcal{J}} h_{jc} * (s * V_{jcs} - (t_{jc} + 0.5 * R_{jc}) * \sum_{k \in \mathcal{K}} d_{kc} * B_{jk}) \quad (10)$$

Equation (11) displays the total order cost for placing orders at the suppliers. This objective is also referred to as *order cost 1*.

$$\sum_{j \in \mathcal{J}} \sum_{c \in \mathcal{C}} A * Z_{jp} * ts_{jp} \quad (11)$$

Equation (12) displays the total order cost for making a distribution order for clusters transported between warehouses. This objective is also referred to as *order cost 2*.

$$\sum_{j \in \mathcal{J}} \sum_{j' \in \mathcal{J}} \sum_{c \in \mathcal{C}} A * Z_{jc} * tw_{jj'} \quad (12)$$

Equation (13) displays the total handling cost at the first warehouse for clusters that require transport between warehouses.

$$\sum_{j \in \mathcal{J}} \sum_{j' \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{C}} ha_{jc} * Z_{jc} * tw_{jj'} * d_{kc} * B_{j'k} \quad (13)$$

Equation (14) displays the transportation cost for transport between suppliers and warehouses. The expected amount to be transported per period is the average demand per period. This objective is also referred to as *transportation cost 1*.

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{C}} ds_{jc} * Cs_{jc} * d_{kc} * Z_{jc} \quad (14)$$

Equation (15) displays the transportation cost for transport between warehouses and customers. This objective is also referred to as *transportation cost 2*.

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{C}} dd_{jk} * Cd_{jkc} * d_{kc} * B_{jk} \quad (15)$$

Equation (16) displays the distance-dependent transportation cost for transport between two warehouses. This objective is also referred to as *transportation cost 3*.

$$\sum_{j \in \mathcal{J}} \sum_{j' \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{C}} dw_{jj'} * Cw_{jj'c} * tw_{jj'} \quad (16)$$

Equation (17) displays the fixed cost for transportation between two warehouses. This objective is also referred to as *transportation cost 4*.

$$\sum_{j \in \mathcal{J}} \sum_{j' \in \mathcal{J}} ew_{jj'} * tw_{jj'} \quad (17)$$

4.3.2 Constraints

Below, each of the constraints is addressed separately. Constraints (18) ensure that each customer is assigned to one warehouse only.

$$\sum_{j \in \mathcal{J}} B_{jk} = 1 \quad \forall k \in \mathcal{K} \quad (18)$$

Constraints (19) ensure that a customer can only be assigned to a warehouse if this warehouse is opened.

$$B_{jk} \leq y_j \quad \forall j \in \mathcal{J}, \quad \forall k \in \mathcal{K} \quad (19)$$

Constraints (20) ensure that each supplier can supply to one warehouse only.

$$\sum_{j \in \mathcal{J}} Z_{jc} = 1 \quad \forall c \in \mathcal{C} \quad (20)$$

Constraints (21) ensure that each warehouse can place orders with suppliers only if it is opened.

$$Z_{jc} \leq y_j \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \quad (21)$$

Constraints (22) allow positive stock levels for open warehouses only.

$$\sum_{s \in \mathcal{S}} V_{jps} \leq y_j \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \quad (22)$$

Constraints (23) ensures that only one base stock level can be selected for each cluster at each warehouse such that the required availability level is respected.

$$(t_{jc} + R_{jc}) * \sum_{k \in \mathcal{K}} d_{kc} * B_{jk} \leq \sum_{s \in \mathcal{S}} \eta_s(\alpha_{jp}) * V_{jcs} \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \quad (23)$$

Constraints (24) tracks if transport occurs between the supplier of cluster c and warehouse j .

$$\begin{aligned} \sum_{k \in \mathcal{K}} g_{kc} * Z_{jc} &\leq M * ts_{jc} \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \\ - \sum_{k \in \mathcal{K}} g_{kc} * Z_{jc} &\leq M * (1 - ts_{jc}) \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \end{aligned} \quad (24)$$

Constraints (25) tracks if transport occurs between warehouse j and customer k .

$$\begin{aligned} \sum_{c \in \mathcal{C}} g_{kc} * B_{jk} &\leq M * td_{jk} \quad \forall j \in \mathcal{J}, \quad \forall k \in \mathcal{K} \\ - \sum_{c \in \mathcal{C}} g_{kp} * B_{jk} &\leq M * (1 - td_{jk}) \quad \forall j \in \mathcal{J}, \quad \forall k \in \mathcal{K} \end{aligned} \quad (25)$$

Constraints (26) tracks if transport occurs between warehouse j and warehouse j' .

$$\begin{aligned} \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}} g_{kc} * B_{j'k} * Z_{jc} &\leq M * tw_{jj'} \quad \forall j \in \mathcal{J}, \quad \forall j' \in \mathcal{J} \\ - \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}} g_{kc} * B_{j'k} * Z_{jc} &\leq M * (1 - tw_{jj'}) \quad \forall j \in \mathcal{J}, \quad \forall j' \in \mathcal{J} \end{aligned} \quad (26)$$

Constraints (27) impose the integrality restrictions.

$$Z_{jc}, y_j, B_{jk}, ts_{jc}, td_{jk}, tw_{jj'}, V_{jcs} \in \{0, 1\} \quad \forall j \in \mathcal{J}, \quad \forall j' \in \mathcal{J}, \quad \forall c \in \mathcal{C}, \quad \forall s \in \mathcal{S} \quad (27)$$

4.3.3 Correction base stock level

Applying a correction factor serves as the third method for the calculation of base stock levels. The higher the number of parts in a cluster, the higher the correction factor needs to be, because the aggregation effect increases with the number of parts. The variability of the distribution is also considered. The variability is represented by the coefficient of variation, which is the standard deviation divided by the mean. A distribution with a large coefficient of variation needs more correction than a distribution with a small coefficient of variation. It seems intuitive to include the target availability, where a cluster with a higher target availability needs a stronger correction. Two correction models were applied, one including the target availability and one that did not. The model that does not consider availability performed better because the target availability remains unchanged.

$$CF_{cu} = 1 + \frac{(X_c - 1)}{X_{av}} * (cf_u) * CV_c \quad (28)$$

Equation (28) displays the calculation for the correction factor. The one at the start of the equation ensures that each cluster starts with its original base stock level as a basis. X_c represents the number of SKUs in a cluster. It is subtracted by one because there is no need for a correction for a cluster existing of only one SKU. X_{av} is the average number of SKUs per cluster. The factor cf_u represents the correction needed for a model with u open warehouses, which is the fraction of the sum of the base stock levels when calculated individually per SKU and the base stock levels that resulted from the model without correction. CV_c is the coefficient of variation of a cluster. For the base stock calculation with correction, the base stock level is calculated as $CF_{cu} * s * V_{jcs}$. The constraints in Equation (23) therefore changes to the expression in Equation (29).

$$CF_{cu} * ((t_{jc} + R_{jc}) * \sum_{k \in \mathcal{K}} d_{kc} * B_{jk} \leq \sum_{s \in \mathcal{S}} \eta_s(\alpha_{jc}) * V_{jcs}) \quad \forall j \in \mathcal{J}, \quad \forall c \in \mathcal{C} \quad (29)$$

The correction factor also needs to be included in the objective function for the holding cost. Therefore, the objective in Equation (10) changes to the expression in Equation (30).

$$\sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}} \sum_{j \in \mathcal{J}} h_{jc} * (CF_{cu} * s * V_{jcs} - (t_{jc} + 0.5 * R_{jc}) * \sum_{k \in \mathcal{K}} d_{kc} * B_{jk}) \quad (30)$$

4.3.4 Sustainability

Equation (31) displays a side objective is constructed to keep track of the CO₂ emission, which is calculated per driven kilometre. This objective equation is not minimised.

$$CO * \sum_{j \in \mathcal{J}} \sum_{c \in \mathcal{C}} ds_{jc} * ts_{jc} + CO * \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} dd_{jk} * td_{jk} + CO * \sum_{j \in \mathcal{J}} \sum_{j' \in \mathcal{J}} dw_{jj'} * tw_{jj'} \quad (31)$$

4.4 Solution method

This problem belongs to the NP-hard problems. Therefore, large-scale instances cannot be solved efficiently. The model is solved with a Simulated Annealing (SA) algorithm because it has a strong local search ability (Ghorbani & Jokar, 2016). Hence, it can provide good solutions with a stochastic approach which allows for the continuation of the search to a neighbouring state even if the move brings about a worse solution. This feature avoids getting trapped in a local optimum. At the start, this feature allows the algorithm to

jump over peaks in the cost function, while later on, it focuses on finding solutions close to the so-far found optimum. Figure 8 visually represents the algorithm. At the start, there exists a possibility of accepting a solution that is worse than the current one to avoid getting trapped in a local optimum. The possibility of accepting a worse solution decreases with the temperature. Table 17 describes the parameters of the algorithm.

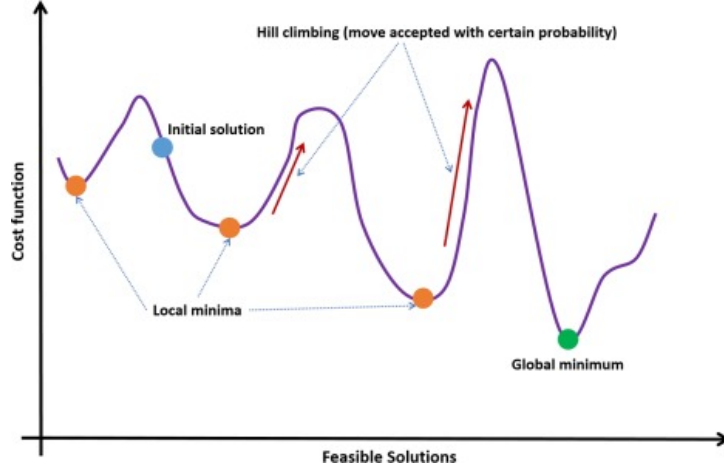


Figure 8: SA algorithm (Blocho, 2020)

Table 17: parameters SA algorithm

T_0	Initial temperature
T	Current temperature
CS	Decreasing rate of current temperature
FT	Freezing temperature
$MNSA$	Maximum number of accepted solutions at each temperature
cSA	Counter for number of accepted solutions at each temperature
X	Current solution

The algorithm contains the following steps. First, the initial solution X_0 is obtained according to the steps of section 4.4.1 and also set as best solution X_{best} . This solution is the basis for generating a new solution X_{new} according to one of the five methods described in section 4.4.2. It is accepted and set as the best solution if it is better than the obtained best solution. It can also be accepted if it is close to but not better than the current solution. The closer it is, the higher the chance of acceptance. This process continues until the maximum number of solutions at a temperature is reached. At that point, the temperature is decreased. The algorithm runs until the stopping criterion is reached. Figure 9 shows a flowchart of the process. The pseudocode for this algorithm is displayed below, where C is the total cost of the solution and $U(0,1)$ is a uniform distribution between zero and one.

- Step 1: Obtain initial solution X_0 according to section 4.4.1 and set $X_{best} = X_0$ and $X = X_0$.
- Step 2: Generate new solution X_{new} in the neighbourhood of X according to section 4.4.2.

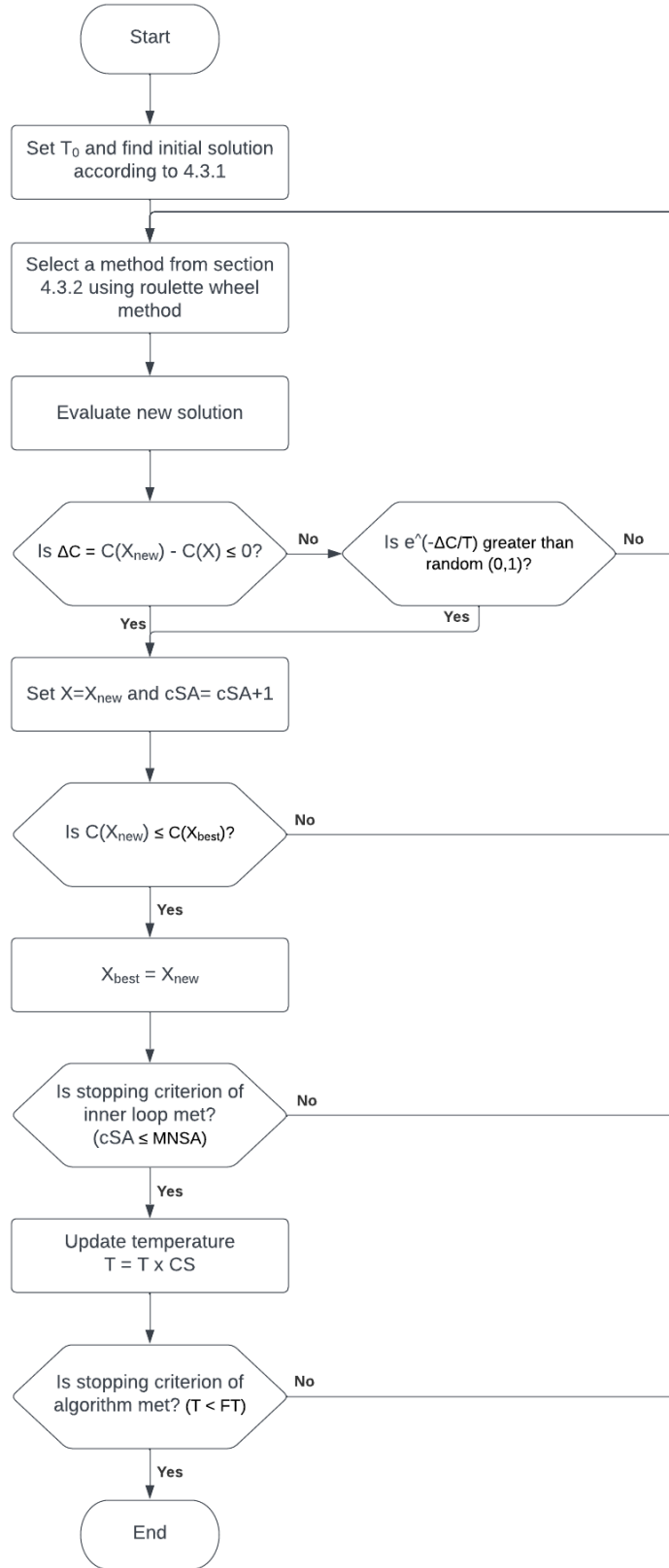


Figure 9: Flowchart heuristic

- Step 3: Let $\Delta C = C(X_{new}) - C(X)$.
if $\Delta C \leq 0$, $X = X_{new}$. if $C(X_{new}) \leq C(X_{best})$, $X_{best} = X$ and $cSA = cSA + 1$.
if $\Delta C > 0$, $y \leftarrow U(0,1)$, $w = e^{-\frac{\Delta C}{T}}$, if $y < w$, $X = X_{new}$ and $cSA = cSA + 1$.
- Step 4: Is the number of iterations in temperature T not greater than MNSA? If yes, go to Step 2; else go to Step 5.
- Step 5: Update temperature $T = T * CS$, where T starts at T_0 .
- Step 6: Is the stopping criterion $T < FT$ met? if yes, stop; else go to Step 2.

4.4.1 Determine initial solution

First, the number of warehouses to be opened is determined, denoted by u . Afterwards, some warehouses are opened randomly until the number of opened warehouses equals u . There exist two variants of choosing which warehouses to open. The first one is to select them randomly. The second one requires warehouse X to open first, after which the remaining warehouses are selected randomly. Next, customers are assigned to an open warehouse. There are two variants of assigning customers to open warehouses. They can be assigned randomly or to the closest open warehouse. Hereafter, the base stock level for each cluster at each open warehouse is calculated based on the demand distribution and required availability level. Next, suppliers are assigned to an open warehouse. There are three variants of assigning suppliers to an open warehouse. They can be assigned randomly, they can be assigned to the closest open warehouse, or all suppliers must deliver to warehouse X . This option is only available in case warehouse X needs to be selected. The next step is to calculate the on-hand inventory of each cluster at each open warehouse from the earlier computed base stock levels by subtracting average demand during lead time plus half a review period from the base stock level. The final steps concern transportation. First, for each supplier and each open warehouse, it is determined if transportation occurs between the supplier and warehouse. Second, for each open warehouse and each customer, it is determined whether transportation occurs between the warehouse and customer. Third, for all open warehouses, it is determined whether transportation occurs between two warehouses.

- Step 1: Let u be the number of warehouses to open.
- Step 2: Open u warehouses.
- Step 3: Assign each customer to an open warehouse.
- Step 4: Calculate base stock levels of each cluster at each warehouse according to the demand distribution and required availability level of the cluster. In other words, determine for which s V_{jcs} should be one for all combinations of open warehouses and suppliers and set base stock level S to that value.
- Step 5: Assign each supplier to an open warehouse.
- Step 6: Calculate on-hand inventory of each cluster at each warehouse:
 $IOH_{jc} = S_{jc} - (t_{jc} + 0.5 * R_{jc}) * \sum_{k \in \mathcal{K}} d_{kc} * B_{jk}$
- Step 7: Determine if transport occurs between supplier and warehouse for each supplier and open warehouse:
 $ts_{jc} = 1$, if $\sum_{k \in \mathcal{K}} g_{kc} * Z_{jc} > 0$; else 0.

- Step 8: Determine if transport occurs between warehouse and customer for each open warehouse and each customer:
 $td_{jk} = 1$, if $\sum_{c \in \mathcal{C}} g_{kc} * B_{jk} > 0$; else 0.
- Step 9: Determine if transport occurs between two warehouses for all open warehouses:
 $tw_{jj'} = 1$, if $\sum_{k \in \mathcal{K}} \sum_{c \in \mathcal{C}} g_{kc} * B_{j'k} * Z_{jc} > 0$; else 0.

4.4.2 Generate neighbourhood solution

A solution in the neighbourhood of the last solution is generated. This can be done according to one of the following five solution methods.

- All decisions are made according to section 4.4.1.
- Randomly select two warehouses and exchange their assigned customers and their base stock levels. Afterwards continue at Step 5 of section 4.4.1.
- Randomly select a closed and an opened warehouse. The opened warehouse is closed and its customers and base stock levels are assigned to the new one. Afterwards continue at Step 5 of section 4.4.1.
- Randomly a closed warehouse is opened and an opened one is closed. The customers assignment and calculation of base stock levels is redone. Continue at Step 3 of section 4.4.1.
- The opened warehouses remain unchanged. The customers assignment and calculation of base stock levels is redone. Continue at Step 3 of section 4.4.1. This option cannot be chosen if customers and suppliers are assigned to the closest warehouse because it will yield the same solution.

A roulette wheel method based on the method of Martins et al. (2019) is used to select one of the five solution methods. Let the set of solution methods be denoted by $\mathcal{F} = \{1,2,3,4,5\}$. At the start, each solution method has an equal chance of being chosen. Every 50 iterations, the possibilities of selecting the methods are updated using Equation (32), where Φ_f denotes the probability of solution method f being selected and ρ_f denotes the weight of solution method f . As a result, solution methods that provide good solutions have a higher chance of being chosen as the algorithm progresses.

$$\Phi_f = \frac{\rho_f}{\sum_{f \in \mathcal{F}} \rho_f} \quad (32)$$

After 50 iterations, the weights ρ_f are updated according to Equation (33) and the scores are reset to zero for the round of iterations. Here, Θ_f denotes how often solution method f is selected. The parameter ν denotes the reaction factor that controls how much the weights influence the performance. The higher ν , the more emphasis on the most recent observations. a low ν focuses more on the past values of the weights.

$$\rho_f = (1 - \nu)\rho_f + \nu * \frac{\Psi_f}{\max(1, \Theta_f)} \quad (33)$$

The starting value of the weights is one. During the algorithm, the weights are updated dynamically. Each solution method f has a score Ψ_f that is updated each time the method is applied:

$\Psi_f + v_1$, if the method generates a new best solution

$\Psi_f + v_2$, if the method generates a accepted solution

4.5 Simulation

A discrete event simulation is used to evaluate the performance of the results of the mathematical model. The opened warehouses with their base stock levels, supplier assignments to warehouses and customer assignments to warehouses are the input for the simulation. Additionally, the clusters are ungrouped and each part is treated individually. Let the parts be numbered from 1 to p and \mathcal{P} denotes the set of parts. There are three methods to calculate the base stock level of a single part. The first method takes the fraction of demand the part accounts for in the cluster from the base stock level to get the base stock level for an individual part. In case of a decimal number, rounding to the nearest integer is applied. A base stock level can become zero due to rounding. In that case, it is rounded to one because it is undesirable to have an availability level of zero. Equation (34) shows this calculation, where S_{jp} denotes the base stock level of an individual part p at warehouse j and κ_{pc} denotes the fraction of demand of part p within cluster c .

$$S_{jp} = \max(\text{round}(S_{jc} * \kappa_{pc}), 1) \quad (34)$$

The second method calculates the new base stock levels according to the demand distribution of the part and its required availability level as described in step 4 of the section 4.4.1, but now for each part individually. This results in much higher base stock levels as the safety stocks are now calculated for every single part and not for each cluster combined. The costs and availability are expected to be higher with this second method.

The third method is applying the correction factor to calculate the new base stock levels for individual parts per warehouse. Equation (35) can be used to calculate the base stock levels for individual SKUs by multiplying the cluster base stock level by the correction factor and the fraction of demand for the SKU within the cluster.

$$S_{jp} = \max(\text{round}(S_{jc} * CF_{cu} * \kappa_{pc}), 1) \quad (35)$$

The actual demand for 2022 is used for the simulation. A different dataset than the training period of the built model is used to avoid overfitting. The simulation shows what the actual costs and availability levels would be when the results of the model are implemented. The simulation runs on weekdays. Therefore, demand occurring at the weekend is treated as if it occurs on the Friday before. Warehouses can order from the suppliers once a week on Thursday. Thursday is opted for as this is five days after the start of the simulation. For each part, an order is placed if the inventory position is below base stock level S . The order quantity is such that the inventory position is raised to the base stock level S . The order arrives at the warehouse after the lead time has passed. Once a week, a transshipment between two warehouses takes place if required. Customer orders are served from stock. In case of shortages, the order is split into two orders. The first order's quantity is as high as the available stock and is served immediately. The second order's quantity consists of the remaining unserved demand and is served as soon as new stock arrives at the warehouse.

5 Case study

This chapter describes the case study conducted at Lely. The first section explains the values for the parameters. Next, the mathematical model of Chapter 4 is solved to determine the locations of the warehouses, their base stock levels and the assignment of suppliers and customers to the warehouses. Two different methods for the calculation of base stock levels are applied. The results from the case study are presented and discussed. Afterwards, a simulation evaluates the results obtained from the model. First, the simulation is validated, after which the three settings for calculating the base stock levels are used for the simulation. A sensitivity analysis of the different calculation methods for the base stock level is performed, and the simulation results are discussed.

5.1 Parameter estimation

For confidentiality reasons, the actual values of the parameters are not presented. The inventory holding cost (h_{jp}) has three components. The first component concerns the storage cost and depends on the squared metre price that differs per region. The squared metre price for warehouse X is taken as a basis and converted for the location of the potential warehouse according to the prices stated by Beyerle & Lieser (2021). The second component concerns all costs that have to do with personnel. The wage in the country where warehouse X is located is taken as a basis and converted per country (Clark, 2022). The third component exists of the remaining costs that are the same for all warehouses. The handling cost (ha_{jp}) also includes personnel and hence is based on the wages of the country where the potential warehouse is located. The fixed costs for placing an order (A) are calculated based on the average time for a planner to make an order and the wage of a planner.

The distances between all nodes (ds_{jp} , dd_{jk} , $dw_{jj'}$) are calculated with the haversine method (Prasetya et al., 2020), which corrects for the curvature of the Earth. The haversine method can be found in Appendix E. The transportation cost per item per kilometre (Cs_{jp} , Cd_{jkp}) are calculated based on the weight of a product and a standard tariff per kilogram. The transportation cost between warehouses consists of two components. The kilometre price between warehouses ($Cw_{jj'}$), which is independent of the weight and number of items and a fixed transport cost ($ew_{jj'}$) for transportation between warehouses. Both the kilometre price and the fixed transportation cost are based on the current tariffs for transportation between the current warehouse and the factory.

The target availability levels (α_{jp}) are based on the current availability levels, which are between 90% and 99.9%. The lead times (t_{jp}) correspond with the agreed lead times for each part. The review period (R_{jp}) is one week for all items. The average periodic demand (d_{kp}) and unique solution of the fill rate ($\eta_s(\alpha)$) follow from the distributions of the clusters.

5.2 Heuristic results

Eight scenarios of the model are considered, which differ in the number of warehouses and the assignment of suppliers and customers. Additionally, some include the restriction that it is compulsory to open warehouse X. This allows for sensitivity analysis of how the model reacts to different assignment methods for suppliers and customers. The notation is in the format AB_c . The first character (A) denotes if warehouse X is required to be opened

(Y) or not (N). The second character (B) denotes if the supplier is assigned randomly (R), to the closest warehouse (C) or to warehouse X (X). The customers follow the same assignment rule, except when the supplier is assigned to warehouse X. In that case, the customer is assigned to the closest warehouse. The subscript (c) denotes the number of open warehouses. Table 18 shows an overview of the scenarios.

Table 18: Scenarios

Scenario name	# warehouses	Warehouse X required	Supplier assignment	Customer assignment
NC ₁	1	No	Closest	Closest
NR ₂	2	No	Random	Random
NC ₂	2	No	Closest	Closest
YR ₂	2	Yes	Random	Random
YC ₂	2	Yes	Closest	Closest
YX ₂	2	Yes	Closest	Warehouse X
YC ₃	3	Yes	Closest	Closest
YX ₃	3	Yes	Closest	Warehouse X

First, an initial investigation of the model is performed by using the base stock levels without the correction factor. Because of a mistake in one of the demand distributions, the numbers are not correct. However, they do provide some first insights into the scenarios. The results of the initialization phase can be found in Appendix F. The initialization shows that the scenarios with random assignment (NR₂ and YR₂) do not yield good results because the model chooses two warehouses close to the centre of supply and demand because of the random assignment. On average, warehouses at the centre of supply and demand will do better at random assignment. Warehouses not located in the centre might suffer from bad random assignments from customers and suppliers. Another reason for not finding good results under random assignment is that the SA algorithm does not converge within the time frame.

Scenarios NR₂ and YR₂ are discarded as their objectives are the highest because the SA algorithm does not converge well under random assignment. Hence, the solutions from scenarios NC₁, NC₂, YC₂, YX₂, YC₃ and YX₃ are selected to continue with. For those scenarios, the heuristic is performed with the correction factor for the base stock levels and the right demand distributions. The results are presented in Table 19 and Table 20 in terms of the weekly cost of the system. Because of confidentiality reasons, the warehouses are represented by a number. Total S denotes the sum of all base stock levels. For confidentiality reasons, the real costs are not mentioned. Scenario NC₁ is the benchmark and therefore the total cost of scenario NC₁ is set to 100%. All costs in all scenarios are scale accordingly.

The costs in Table 19 and Table 20 cannot be compared with the realised costs in 2022. There are differences because of the assumptions made. The realised costs include omitted parts and customers from outside of Europe. Scenario NC₁ serves as a benchmark because it only includes warehouse X and hence represents the current situation.

The holding cost increases with the number of warehouses because each warehouse carries its own safety stock. As a result, the sum of all base stock levels increases with the number

Table 19: Results heuristic with correction factor scenarios NC₁, NC₂ and YC₂

Scenario	NC ₁	NC ₂	YC ₂
Opened warehouses	X	X & 51	X & 51
Holding cost	35.85%	40.04%	40.04%
Order cost 1	0.02%	0.02%	0.02%
Order cost 2	-	0.02%	0.02%
Handling cost	-	3.53%	3.53%
Transportation cost 1	7.27%	7.27%	7.27%
Transportation cost 2	57.13%	51.33%	51.33%
Transportation cost 3	-	1.75%	1.75%
Transportation cost 4	-	0.91%	0.91%
Total cost	100%	104.87%	104.87%
Total S	1,095,639	1,337,038	1,337,038
CO ₂	1,290 kg	1,141 kg	1,141 kg

Table 20: Results heuristic with correction factor scenarios YX₂, YC₃ and YX₃

Scenario	YX ₂	YC ₃	YX ₃
Opened warehouses	X & 50	X & 51 & 55	X & 51 & 55
Holding cost	39.79%	42.95%	42.95%
Order cost 1	0.02%	0.02%	0.02%
Order cost 2	0.02%	0.03%	0.03%
Handling cost	3.97%	7.61%	7.61%
Transportation cost 1	7.27%	7.27%	7.27%
Transportation cost 2	49.97%	45.83%	45.83%
Transportation cost 3	1.50%	2.84%	2.84%
Transportation cost 4	0.91%	1.82%	1.82%
Total cost	103.44%	108.47%	108.47%
Total S	1,339,759	1,514,806	1,514,806
CO ₂	1,119 kg	1,014 kg	1,014 kg

of warehouses. The ordering cost increases with the number of warehouses as more orders are placed between warehouses. The number of orders at the supplier stays the same as the supplier may only deliver at one warehouse. The handling cost increases with the number of warehouses because the number of parts transported between warehouses increases. The transportation costs from supplier to warehouse and warehouse to customer decrease as the number of warehouses increases. Because of the assignment to the closest warehouse, the number of kilometres driven decreases when the number of warehouses increases. Instead, transport between warehouses is introduced. The number of transportation movements between warehouses increases with the number of warehouses. However, the combined costs for transportation get lower with the number of warehouses. Simultaneously, the emission of CO₂ decreases because the total number of kilometres driven decreases.

The scenario with one warehouse is the cheapest, followed by the scenarios with two warehouses. Even though it is not compulsory to choose warehouse X in scenario NC₂, the model selects it as the best option. Therefore, scenarios NC₂ and YC₂ have the

same solution. The scenarios with three warehouses also yield the same solution because the scenario YC_3 opts for a combination of warehouses where all suppliers are closest to warehouse X. For both the scenarios with two and three warehouses, the model yields the best results when all suppliers need to supply to warehouse X. This is a result of the current situation where most suppliers are geographically close to warehouse X. The scenarios with three warehouses perform worse in terms of total cost than the scenarios with two warehouses. Therefore, the simulation is conducted for scenarios with only one or two warehouses. Since scenarios NC_2 and YC_2 yield the same results, they are referred to as scenario NC_2/YC_2 for the simulation.

5.3 Results simulation

First, the simulation is conducted with demand sampled from the demand distributions to validate the simulation. For each criticality level, the target availability level is the weighted average of the availability level per cluster. The simulation is validated when it reaches the target availability levels. It is unnecessary to prove this for every scenario because they all behave similarly regarding availability. Therefore, only scenario NC_1 is used for validation because it yields the least cost in the model without the correction factor. Five simulations with one year of demand (short runs) and one with four years of demand (long run) are conducted to omit the effects of good or bad luck during sampling. Table 21 shows the results of the simulation runs. For both the short runs and the long run, criticality 3 reaches its target. For the short runs, the target is also reached for criticality 1. Criticality 2 and the total combination of criticalities do not reach their target. For the short runs, the total combination of criticalities is close. Simulation data is only available for short runs of one year, where all targets except the ones for criticality 2 and the total combination of criticalities are reached. Overall, the targets are reached or close enough to the target to continue with the simulation. Hence, the simulation is validated.

Table 21: Availability validation simulation

Scenario YX_2	short runs	long run	Target
Opened warehouses	X	X	X
Criticality 3	99.06%	98.39%	98.97%
Criticality 2	98.40%	98.01%	99.19%
Criticality 1	99.03%	98.44%	98.91%
All criticalities	98.80%	98.18%	99.02%

The validation allows us to proceed to the first run of simulations, which uses the first base stock calculation method. The base stock level for each SKU is the fraction of the cluster base stock level that resulted from the heuristic solution. Again, this is an initial investigation of the simulation because of a mistake in one of the demand distributions, the numbers are not correct. However, they do provide some first insights. The results of the initialization phase can be found in Appendix G. The availability levels are far below the targets. Therefore, a second initial investigation simulation is conducted with the second method for base stock calculation. The base stock levels are calculated according to the demand distribution of the part and its required availability level. The results of the second initialization phase can be found in Appendix G. The availability levels are much closer to the targets now. The availability level of criticality two items is lower than the other ones. This is due to the mistake in the demand distribution of cluster fifteen, which belongs to

criticality level 2. The results show that a correction is needed. Therefore, a third method for base stock calculation that uses the correction factor CF_c to recalculate the base stock levels from the heuristic is applied. As is the case for the heuristic, this method is performed with the right demand distributions. The results can be found in Table 22 and the availability levels in Table 23. Again, scenario NC_1 is the benchmark and therefore the total cost of scenario NC_1 is set to 100%. All costs in all scenarios are scale accordingly.

Table 22: Results simulation with third base stock calculation method

Scenario	NC_1	YR_2/YC_2	YX_2
Opened warehouses	X	X & 51	X & 50
Holding cost	38.15%	42.60%	42.50%
Order cost 1	0.62%	0.60%	0.59%
Order cost 2	-	0.25%	0.30%
Handling cost	-	2.80%	3.60%
Transportation cost 1	6.57%	6.57%	6.57%
Transportation cost 2	54.66%	48.60%	47.62%
Transportation cost 3	-	1.28%	1.10%
Transportation cost 4	-	0.67%	0.67%
Total cost	100%	103.37%	102.97%
Total S	1,095,649	1,337,249	1,339,910
CO ₂	69,610 kg	63,080 kg	64,557 kg

The total costs sketch the same image as the results from the heuristic without correction. Scenario NC_1 gets the lowest total cost followed by scenario YX_2 and scenarios NC_2/YC_2 give the highest cost. As expected, the holding costs are higher with two warehouses. The sum of the base stock levels is higher because it is necessary to carry more safety stock. Therefore, the total sum of base stock levels in scenarios NC_2/YC_2 and YX_2 are 22.05% and 22.29% higher. The holding cost in scenarios NC_2/YC_2 and scenario YX_2 increases by 11.66% (42.60/38.15) and 11.40% (42.50/38.15) when compared to the holding cost of scenario NC_1 . Additionally, the handling cost for parts transported between warehouses is incurred in a situation with more than one warehouse. However, the overall transportation cost decreases with the addition of a second warehouse. The transportation cost for scenarios NC_2/YC_2 and scenario YX_2 decreases by 6.71% (57.12/61.23) and 8.61% (55.86/61.23) compared to the overall transportation cost of scenario NC_1 . Order cost 1 is much higher in the simulation than in the mathematical model because all parts are ordered individually instead of per cluster.

Table 23: Availability simulation with third base stock calculation method

Scenario	NC_1	YR_2/YC_2	YX_2
Opened warehouses	X	X & 51	X & 50
Criticality 3	89.09%	90.06%	90.09%
Criticality 2	96.84%	97.26%	97.15%
Criticality 1	96.68%	96.03%	96.12%
All criticalities	93.43%	94.01%	93.97%

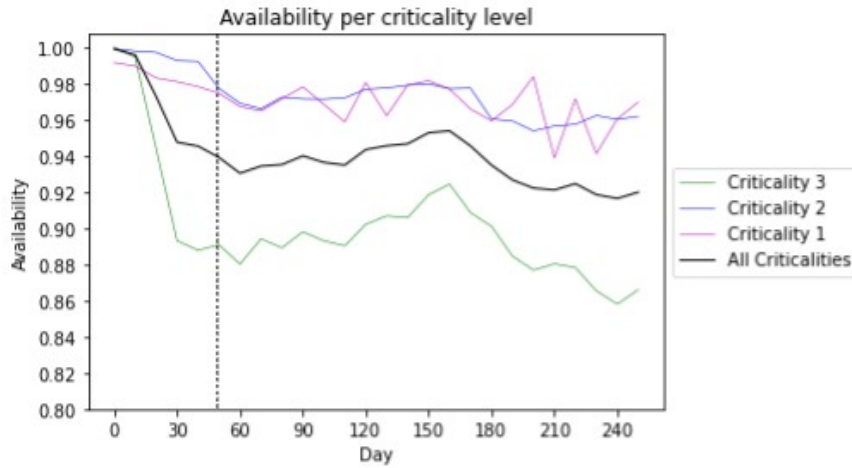


Figure 10: Availability warehouse X with third base stock calculation method

The warm-up period for the simulation is 50 days, denoted by the vertical dotted line in Figure 10. Hence, the period after the warm-up is considered for the annual holding cost and availability levels. The availability plot over time for the other scenarios looks similar. The availability levels in Table 10 are a bit lower than the target availability levels from Table 21. They are also lower than the realised availability of 2022, which was 96.73%. A slight deviation from the realised availability is possible because the realised availability also includes the orders that were out of scope for this thesis project. Since the correction factor is a general estimation to calculate new base stock levels, it is not surprising that the availability levels are lower than their targets. The base stock level might be a bit higher than necessary for some SKUs, while for other SKUs, the base stock levels might be a bit lower than desired. Scenarios YR_2/YC_2 and YX_2 have a 0.58% and 0.54% higher availability than the scenario with only one warehouse.

Table 24 displays the target and realised availability levels per cluster. The clusters with their number displayed in green are the clusters that reached their target availability level, while the clusters with their number displayed in red did not. Only nine clusters reached their target availability. The clusters with a normal distribution or gamma distribution perform well. Six of them reached their target and three of them are not far from it. Only cluster eight has a low realised availability level. The clusters with a negative binomial distribution or compound Poisson distribution perform poorly. Especially for the negative binomial distribution, the correction factor is too weak. All except one cluster do not reach their target availability. The correction factor partially functions, but there is still room for improvement. The clusters with no distribution do not perform well, which could be due to the alternative base stock calculation method. The type of distribution seems to have a big influence on the performance in inventory control because each distribution performs differently. Another reason for some parts not to reach the target availability level could be a change in demand pattern. When the demand pattern behaves differently in the tested year 2022 compared to the demand in 2020 and 2021 on which the model was built, differences in availability levels occur.

Overall, the simulation paints the picture that, at the moment, investment in a second warehouse is not financially viable because a similar availability is achieved at a higher

cost. The decision on how many warehouses to open is always a trade-off between holding costs and transportation costs. Other parameter settings might lead to a different result in future work/simulations.

Table 24: Availability per cluster with third base stock calculation method

Cluster	1	2	3	4.1	4.2	5	6
Distribution	Normal	NB	Normal	None	None	CP	NB
Target α_{jp}	95%	97.5%	99%	96%	96%	97.5%	99%
Realised α_{jp}	95.85%	55.96%	96.20%	57.14%	50.00%	76.86%	100%
Cluster	7	8	9	10	11	12	13
Distribution	Gamma	Normal	Normal	Normal	Gamma	None	NB
Target α_{jp}	99%	99%	99.9%	98%	95%	96%	95%
Realised α_{jp}	99.08%	40.02%	99.91%	98.63%	93.61%	100%	49.35%
Cluster	14	15	16	17	18	19.1	19.2
Distribution	Gamma	NB	Normal	Normal	NB	None	None
Target α_{jp}	90%	99.5%	99.5%	98%	95%	96%	96%
Realised α_{jp}	90.13%	72.66%	99.68%	96.89%	86.34%	46.67%	58.62%
Cluster	20	21	22	23.1	23.2	23.3	24
Distribution	NB	CP	Normal	None	None	None	NB
Target α_{jp}	99%	95%	99%	96%	96%	96%	99%
Realised α_{jp}	98.96%	81.69%	98.56%	97.73%	91.11%	51.85%	95.91%

6 Conclusion & discussion

This section contains the discussion and conclusion of this research project. First, the conclusion per research question is discussed. Afterwards, the limitations of this research project are stated. Next, recommendations for Lely are discussed. The final section states some future research directions.

6.1 Conclusion

This thesis aimed to investigate if Lely needs to invest in new warehouses for its European supply chain. Currently, they serve all customers from one central warehouse. This research investigates the possibility of a distributed network with multiple warehouses. The research question to answer was

How does the introduction of multiple distribution centres influence cost, availability and sustainability?

Four sub-questions were formulated to answer this question. The answers to each of the sub-research questions are given

RQ1: What is the performance of the current supply chain concerning cost, availability and sustainability?

Currently, Lely delivers all demand from one central warehouse. In 2022, the availability was 96.73%. The realised costs in 2022 also include costs made for parts and customers that were out of scope. Therefore, a comparison between different models is more telling, especially because model one resembles the current situation and can serve as a benchmark for comparison.

RQ2: How to capture the uncertain demand for each SKU for each Lely Center?

Hierarchical clustering was used to reduce the number of parts in the mathematical model. Demand data for 2020 and 2021 and data on features of parts served as input. Only active parts from which all the necessary data was available were included. The parts were clustered into 24 groups based on their criticality level, lead time, part value, demand frequency, average usage per year, and volume. Afterwards, a set of distributions was fit to the aggregated demand of 2020 and 2021 for each cluster through maximum likelihood estimation. The five distributions fitted were the normal distribution, the gamma distribution, the Poisson distribution, the negative binomial distribution and the compound Poisson distribution. Based on the AIC, the distribution with the best fit was selected. Four of the five distributions were used. The Poisson distribution could not be fit to any cluster. The clusters that did not get a good fit on any distribution were, if necessary, further separated based on the size of demand in case it is positive.

RQ3: How many new DCs should be placed where and what should their stock levels be?

The demand distributions from the previous research question were used as input for the adapted version of the location-transportation model of Ghorbani & Jokar (2016).

The major adaptations were changing the transportation procedure, removing the capacity restrictions and introducing stochasticity. Stochasticity was introduced using the demand distributions to calculate the base stock levels necessary to achieve the target fill rate. A simulated annealing heuristic was applied to solve the model. The results showed where DCs should be placed and their corresponding base stock levels. The model gave two options for placing a second DC, locations 50 and 51, next to the option of not adding a second warehouse. The option with three warehouses was discarded because the costs were too high. A simulation was conducted using the output from the heuristic in terms of placement of warehouses, required base stock levels, and assignment of suppliers and customers to warehouses. Data from 2022 was used as input for the simulation because it is different data on which the model was built. The simulation shows that, at the moment, it is not necessary to open a new DC because this would lead to a higher cost for achieving the same availability. The holding costs increase with the number of warehouses, while the transportation costs decrease. At the moment, the increase in holding cost is larger than the decrease in transportation cost when a warehouse is added.

RQ4: What is the sensitivity of the model with respect to different input parameters?

The number of warehouses included in the model influences the total sum of the base stock levels. The base stock level directly influences the on-hand inventory and, therefore, the holding cost. A higher base stock level leads to more holding costs. More inventory is needed when the number of warehouses increases because each warehouse carries its own safety stock. Hence, the holding costs increase with the number of warehouses. The availability increases with the base stock level because the safety stock follows the same trend. The emission of CO₂ followed the same trend as the transportation cost. An increase in the number of warehouses decreases the number of kilometres driven and, hence, a lower CO₂ emission.

6.2 Limitations

The exact data for the squared metre price and labour costs at the third-party logistics service provider was not known. Therefore, the holding cost is based on estimations of the squared metre price and labour costs across Europe from other data sources. Hence, the numbers can be slightly inaccurate.

Additionally, parts were omitted if data on at least one of the necessary features was unavailable. Parts that are very influential because of high demand or lead time could have been discarded for this thesis. As a result, the model's accuracy might be harmed.

Furthermore, supply disruptions were not taken into account. It was assumed the supplier could always deliver the right amount within the lead time. However, especially in the last couple of years, disruptions prove to be an increasing problem in supply chain management because of poor raw material availability and major disruption events such as the COVID-19 pandemic and the war in Ukraine.

Lastly, the model was built on only two years of data because of major changes in inventory management at Lely in 2019 due to the movement from the previous warehouse to the

current one. Ideally, a larger data set is used to construct the model to reduce the bias in the model. Consequently, this would provide the opportunity to use a dataset consisting of multiple years of demand data for the simulation.

6.3 Recommendations

At the moment, Lely should not invest in a second warehouse in Europe. With two warehouses, the currently achieved availability cannot be reached at a lower cost. However, this decision can be subject to change as it is a trade-off between holding and transportation costs. At the moment, the holding costs are too high for a second warehouse. A change in factors determining the holding cost, for example, squared metre price and labour cost, could change the decision to invest in a second warehouse. The same holds for factors that influence transportation costs, for example, fuel prices.

Secondly, it is recommended to gather more data on the criticality level, weight and volume of parts. These features were sometimes missing in the data leading to the discarding of parts for this research. Additionally, Lely should gather more detailed data about the squared metre price and labour costs for the warehouses in Europe of their current third-party logistics provider. This would lead to a more accurate analysis of the distributed network.

6.4 Future research

The first future research direction concerns the clustering algorithm. It is unclear why certain clusters are prioritised over others for splitting when they have similar ranges for all features. One direction could be research into a quantitative method for calculating weights for features, which was done based on the judgement of experts, but not on a quantitative method. Another direction could be research into other methods than applying weights to emphasise certain features.

A further future research direction focuses on demand fitting. More research on the compound Poisson distribution could be conducted. In this thesis project, the compound Poisson distribution did not perform well, while theoretically, it seems to have the potential for intermittent demand. Another method to cover demand uncertainty that could be considered is robust optimization. Compared to stochastic models, it is more stable when input parameters change.

Another direction for future research focuses on the decomposition of base stock levels from clusters into base stock levels for SKUs. The same correction factor was applied to all four distributions (normal, gamma, negative binomial and compound Poisson). This correction factor did not capture all properties of the distributions. A different correction factor could be found for each distribution to more accurately estimate the base stock level for each SKU.

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Appendix A: Results clustering

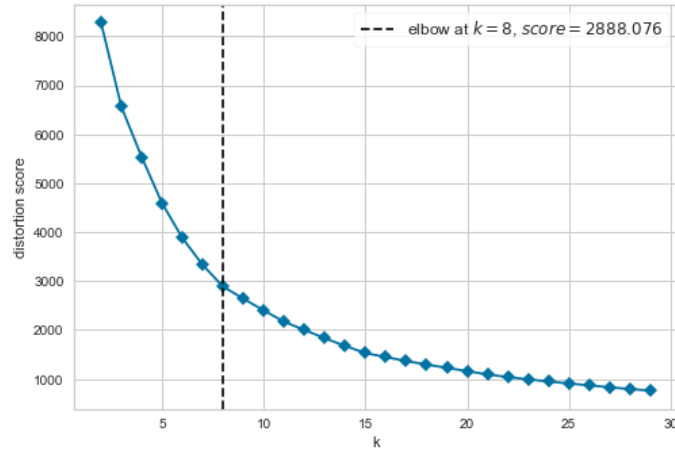


Figure 11: Elbow plot criticality 2 parts

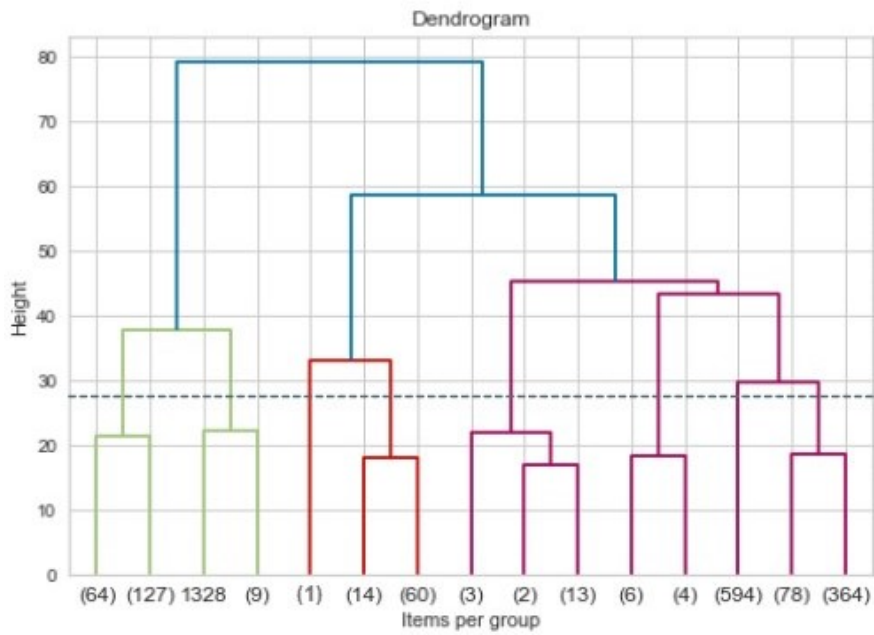


Figure 12: Dendrogram criticality 2 parts

Table 25: Number of parts per cluster with optimal number of cluster for criticality 2

Cluster	10	11	12	13	14	15	16	17
Number of parts	191	10	4	78	18	10	594	442

Table 26: Ranges clusters criticality level 2

C	Lmin	Lmax	Pmin	Pmax	Imin	Imax	Qmin	Qmax	Vmin	Vmax
10	45	100	0.00X	53.22X	1.00	18.40	0.07	498.90	1	167725
11	112	335	0.10X	129.47X	1.00	11.00	0.09	27.88	2	96460
12	20	90	0.31X	5.35X	60.00	95.00	0.03	0.10	45	295
13	5	90	0.00X	59.24X	12.25	41.50	0.03	0.65	1	276762
14	10	80	45.04X	337.35X	1.02	32.33	0.03	6.11	4900	1295600
15	15	60	0.01X	0.47X	1.00	2.56	603.89	2112.28	1	1440
16	4	35	0.00X	20.82X	1.00	9.60	0.06	561.8	1	46136
17	5	50	0.01X	70.04X	1.00	12.00	0.02	233.21	1	219960

Table 27: Ranges clusters criticality level 2

C	Lmin	Lmax	Pmin	Pmax	Imin	Imax	Qmin	Qmax	Vmin	Vmax
16	65	100	0.01X	32.06X	1.00	11.44	0.09	497.90	1	7786
17	45	70	0.00X	53.22X	1.00	18.40	0.07	100.34	1	167725
18	112	205	0.10X	129.47X	1.00	11.00	0.09	27.88	2	96460
19	335	335	4.38X	4.38X	4.90	4.90	0.35	0.35	27	27
20	20	90	0.31X	5.35X	60.00	95.00	0.03	0.10	45	295
21	11	90	0.16X	19.81X	25.25	41.50	0.03	0.24	4	5489
22	5	48	0.00X	59.24X	12.25	26.00	0.03	0.65	1	276762
23	30	35	45.04X	148.32X	10.40	32.33	0.03	0.10	957600	1295600
24	16	51	285.51X	337.35X	1.78	16.75	0.04	0.76	46080	337980
25	10	80	93.69X	187.06X	1.02	13.43	0.12	6.11	4900	418015
26	20	60	0.01X	0.48X	1.00	1.06	606.89	1173.39	1	1440
27	15	40	0.01X	0.29X	1.00	2.56	1187.70	2112.28	25	116
28	4	35	0.00X	20.82X	1.00	9.60	0.06	561.80	1	46136
29	10	30	0.01X	29.94X	6.00	12.00	0.03	19.85	1	219960
30	5	50	0.01X	70.04X	1.00	9.80	0.03	233.21	1	37145

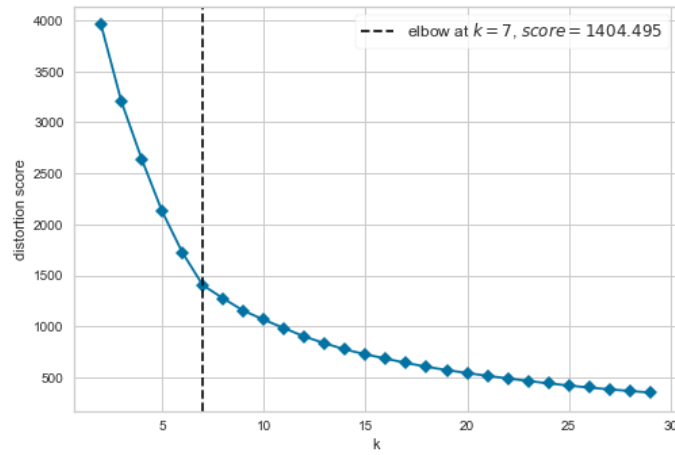


Figure 13: Elbow plot criticality 1 parts

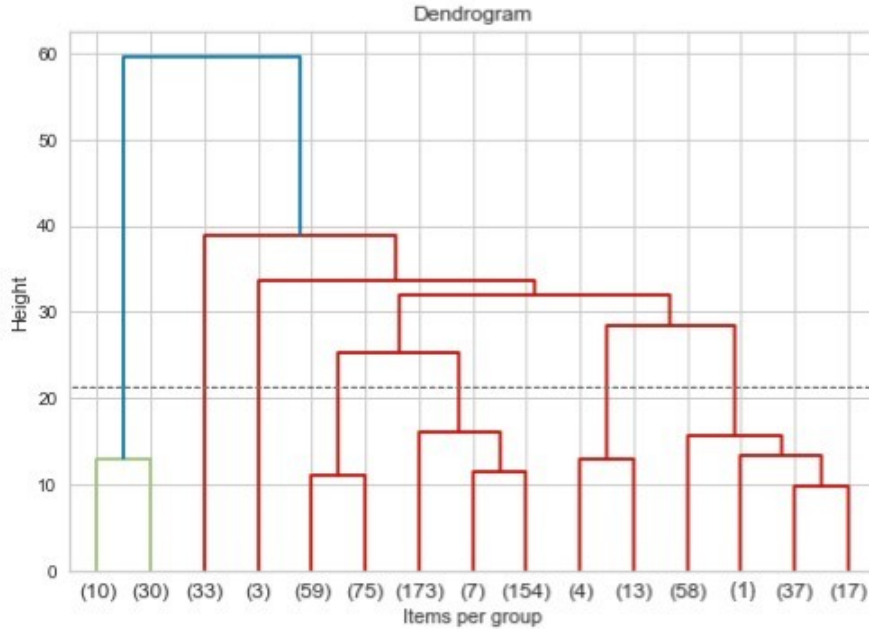


Figure 14: Dendrogram criticality 1 parts

Table 28: Number of parts per cluster with optimal number of cluster for criticality 1

Cluster	18	19	20	21	22	23	24
Number of parts	40	33	3	134	334	17	113

Table 29: Ranges clusters criticality level 1

C	Lmin	Lmax	Pmin	Pmax	Imin	Imax	Qmin	Qmax	Vmin	Vmax
18	50	100	0.01X	41.32X	1.00	15.33	0.04	81.82	1	829847
19	2	50	0.01X	13.25X	27.00	51.50	0.03	0.40	1	14589
20	15	15	0.00X	0.41X	1.00	5.88	394.49	1870.57	4	189936
21	5	25	0.00X	22.94X	8.00	26.00	0.03	16.93	1	19742
22	2	25	0.00X	40.05X	1.00	11.44	0.03	631.28	1	347861
23	2	40	45.15X	164.39X	1.46	23.00	0.03	4.70	225	1140480
24	15	49	0.01X	45.08X	1.00	34.00	0.03	79.55	1	2700000

Table 30: Ranges clusters criticality level 1

C	Lmin	Lmax	Pmin	Pmax	Imin	Imax	Qmin	Qmax	Vmin	Vmax
31	90	100	0.01X	11.38X	1.66	13.67	0.08	4.99	1	1524
32	50	90	0.01X	41.32X	1.00	15.33	0.04	81.82	1	829847
33	2	50	0.01X	13.25X	27.00	51.50	0.03	0.40	1	14589
34	15	15	0.00X	0.41X	1.00	5.88	394.49	1870.57	4	189936
35	7	15	0.00X	7.20X	8.00	15.80	0.03	16.93	1	19742
36	5	25	0.00X	22.94X	12.29	26.00	0.03	1.95	1	16402
37	2	12	0.00X	40.05X	1.00	11.44	0.05	194.34	1	48725
38	10	20	0.01X	0.39X	1.00	7.42	51.98	631.28	1	169
39	14	25	0.00X	32.39X	1.00	10.78	0.03	107.88	1	347861
40	5	40	32.11X	164.39X	1.46	10.56	0.10	3.16	12285	1140480
41	2	40	45.15X	97.50X	3.22	23.00	0.03	4.70	225	939170
42	25	40	0.01X	27.14X	1.00	12.88	0.04	79.55	1	78408
43	20	20	9.34X	9.34X	19.20	19.20	0.03	1.15	2700000	2700000
44	15	35	0.01X	17.76X	6.38	19.80	0.03	0.30	1	96481
45	15	49	0.01X	45.08X	12.13	34.00	0.05	0.05	1	865800

Appendix B: Demand distributions

Normal distribution

Normal distribution behaves identically left and right of the mean with parameters μ and σ denoting the mean and variance. The parameters, expected value and variance are:

$$\text{Parameter 1: } -\infty < \mu < \infty$$

$$\text{Parameter 2: } \sigma^2 > 0$$

$$\text{Expected Value} = \mu$$

$$\text{Variance} = \sigma^2$$

Let θ be the parameter for which the distribution is scaled, where $\theta > 0$. μ must be scaled by θ and becomes $\theta * \mu$ to get the scaled expected value. To get the scaled variance, first σ must be squared before it can be multiplied by θ . Afterwards the square root of this can be taken get the scaled value for σ . The scaled value becomes $\sqrt{\theta * \sigma^2}$. The scaled parameters, expected value and variance are:

$$\text{Parameter 1: } -\infty < \theta * \mu < \infty$$

$$\text{Parameter 2: } \sqrt{\sigma^2 * \theta} > 0$$

$$\text{Expected Value} = \theta * \mu$$

$$\text{Variance} = \theta * \sigma^2$$

Gamma distribution

The gamma distribution is a non-negative distribution with shape parameter α . and rate parameter β , where $\beta = \frac{1}{\lambda}$. The parameters, expected value and variance are:

$$\text{Parameter 1: } \alpha > 0$$

$$\text{Parameter 2: } \lambda > 0$$

$$\text{Expected Value} = \frac{\alpha}{\lambda}$$

$$\text{Variance} = \frac{\alpha}{\lambda^2}$$

Let θ be the parameter for which the distribution is scaled, where $\theta > 0$. For both the expected value and the variance, α can be scaled by θ to get the scaled expected value and scaled variance. Hence, α becomes $\alpha * \theta$. The scaled parameters, expected value and variance are:

$$\text{Parameter 1: } \alpha * \theta > 0$$

$$\text{Parameter 2: } \lambda > 0$$

$$\text{Expected Value} = \frac{\alpha * \theta}{\lambda}$$

$$\text{Variance} = \frac{\alpha * \theta}{\lambda^2}$$

Negative binomial distribution

This negative binomial distribution counts the number of independent Bernoulli experiments with equal success probability p needed to arrive at a prespecified number r of successful experiments. The parameters, expected value and variance are:

$$\text{Parameter 1: } 0 \leq p \leq 1$$

$$\text{Parameter 2: } r = 1, 2, \dots$$

$$\text{Expected Value} = \frac{r}{p}$$

$$\text{Variance} = \frac{r(1-p)}{p^2}$$

Let θ be the parameter for which the distribution is scaled, where $\theta > 0$. For both the expected value and the variance, r can be scaled by θ to get the scaled expected value and scaled variance. Hence, r becomes $r * \theta$. The scaled parameters, expected value and variance are:

$$\text{Parameter 1: } 0 \leq p \leq 1$$

$$\text{Parameter 2: } r * \theta = 1, 2, \dots$$

$$\text{Expected Value} = \frac{r * \theta}{p}$$

$$\text{Variance} = \frac{(r * \theta)(1-p)}{p^2}$$

Compound Poisson distribution

The compound Poisson model does not assume independence between the cases compared to the Poisson distribution. It uses a two-level counting system. Power parameter p being larger than 1 but smaller than 2 indicates it is compound Poisson. Parameters μ and ϕ denote the mean and dispersion. The parameters, expected value and variance are:

$$\text{Parameter 1: } 1 < p < 2$$

$$\text{Parameter 2: } 0 < \mu < \infty$$

$$\text{Parameter 3: } \phi > 0$$

$$\text{Expected Value} = \mu$$

$$\text{Variance} = \phi * \mu^p$$

Let θ be the parameter for which the distribution is scaled, where $\theta > 0$. μ must be scaled by θ and becomes $\theta * \mu$ to get the scaled expected value. To get the scaled value of ϕ , the new expression $\mu * \theta$ is filled in in the variance and afterwards the equation $\phi * \mu^p = \theta * \phi_{new} * (\mu * \theta)^p$ can be solved to obtain the scaled value of ϕ . The scaled parameters, expected value and variance are:

$$\text{Parameter 1: } 1 < p < 2$$

$$\text{Parameter 2: } 0 < \theta * \mu < \infty$$

$$\text{Parameter 3: } \frac{\phi}{\theta} * \frac{\mu^p}{(\mu * \theta)^p} > 0$$

$$\text{Expected Value} = \mu * \theta$$

$$\text{Variance} = \frac{\phi}{\theta} * \frac{\mu^p}{(\mu * \theta)^p} * (\mu * \theta)^p$$

Appendix C: Distribution parameters

Table 31: AIC fitting models criticality 2 clusters

Cluster	Normal	Gamma	Poisson	Negative binominal	Compound Poisson
10	1700.46	1702.42	22175.02	1702.40	1702.41
11	988.92	968.18	1574.06	968.83	984.62
12	351.65	-	178.04	62.61	118.40
13	799.70	676.86	1328.00	711.79	725.03
14	783.05	750.05	926.80	752.76	768.44
15	2061.18	2052.23	164380.80	2050.21	2068.44
16	1994.61	2007.90	83483.20	2006.50	2002.03
17	1663.40	1672.26	16311.40	1672.18	1670.24

Table 32: Parameters fitted distribution criticality 2 clusters

Cluster	Distribution	Parameter 1	Parameter 2	Parameter 3
10	Normal	$\mu = 3071.39$	$\sigma = 780.13$	
11	Gamma	$k = 8.32$	$\beta = 0.1159$	
12				
13	Negative binominal	$r = 1.18$	$p = 0.1150$	
14	Gamma	$k = 5.71$	$\beta = 0.2663$	
15	Negative binominal	$r = 7.1$	$p = 6.096 * 10^{-4}$	
16	Normal	$\mu = 13545.80$	$\sigma = 3165.78$	
17	Normal	$\mu = 3046.59$	$\sigma = 653.90$	

Table 33: AIC fitting models criticality 1 clusters

Cluster	Normal	Gamma	Poisson	Negative binominal	Compound Poisson
18	1224.39	1208.27	4293.75	1208.03	1227.33
19	709.04	-	1106.13	426.34	606.11
20	1916.77	1890.85	124741.90	1890.84	1893.55
21	1326.55	1063.65	11411.04	1066.00	1070.99
22	1892.50	1894.69	71244.16	1894.11	1894.80
23	972.69	699.89	2500.81	722.91	755.15
24	1290.14	1286.87	5487.93	1286.10	1297.17

Table 34: Parameters fitted distribution criticality 1 clusters

Cluster	Distribution	Parameter 1	Parameter 2	Parameter 3	
18	Negative binominal	r = 5.23	p = 0.0290	$\phi = 1.71$	
19					
20	Negative binominal	r = 3.10	$p = 8.112 * 10^{-4}$		
21	Compound Poisson	p = 1.9	$\mu = 46.23$		
22	Normal	$\mu = 5826.23$	$\sigma = 1946.76$		
23	Negative binominal	r = 5.42	p = 0.0204		
24					

Table 35: AIC fitting models criticality 2 for 15 clusters

Cluster	Normal	Gamma	Poisson	Negative binominal	Compound Poisson
16	1662.62	1654.474	22312.97	1654.472	1667.1
17	1451.89	1456.37	6864.79	1456.24	1453.20
18	264.29	-	176.16	155.79	182.05
19	988.91	968.22	1578.32	968.87	984.34
20	351.65	-	178.04	62.61	118.40
21	540.56	-	502.82	251.35	527.39
22	782.82	649.63	1250.73	689.29	705.13
23	271.84	-	246.20	248.20	280.05
24	769.70	734.12	920.98	736.83	762.52
25	783.07	747.29	937.76	750. 26	779.16
26	1900.40	1896.34	80788.98	1896.09	1895.63
27	1991.44	1969.48	148439.20	1969.34	1971.76
28	1994.61	2007.90	83483.20	2006.50	2002.03
29	1360.61	1078.34	14562.80	1080.61	1090.48
30	1656.50	1664.73	15592.44	1664.64	1661.74

Table 36: Parameters fitted distribution criticality 2 for 15 clusters

Cluster	Distribution	Parameter 1	Parameter 2	Parameter 3
16	Negative Binominal	$r = 9.95$	$p = 0.0049$	
17	Normal	$\mu = 1030.91$	$\sigma = 238.84$	
18				
19	Gamma	$k = 8.24$	$\beta = 0.1154$	
20				
21				
22	Negative binominal	$r = 1.14$	$p = 0.1245$	
23				
24				
25	Gamma	$k = 5.26$	$\beta = 0.2575$	
26	Compound Poisson	$p = 1.8$	$\mu = 5328.99$	$\phi = 1.04$
27	Negative binominal	$r = 4.28$	$p = 6.791 * 10^{-4}$	
28	Normal	$\mu = 13545.80$	$\sigma = 3165.78$	
29	Compound Poisson	$p = 1.9$	$\mu = 49.49$	$\phi = 1.74$
30	Compound Poisson	$p = 1.6$	$\mu = 2987.94$	$\phi = 0.9647$

Table 37: AIC fitting models criticality 1 for 15 clusters

Cluster	Normal	Gamma	Poisson	Negative binominal	Compound Poisson
31	780.57	650.74	1224.54	698.15	715.95
32	1222.05	1204.36	4374.06	1204.03	1221.96
33	709.04	-	1106.13	426.34	606.11
34	1916.77	1890.85	124741.90	1890.84	1893.55
35	1319.71	981.26	11521.54	995.74	1022.74
36	1009.43	726.91	2928.09	762.74	806.30
37	1781.31	1739.51	63870.72	1737.15	1744.82
38	1785.83	1792.09	57006.25	1789.43	1783.49
39	1605.73	1597.12	20875.82	1597.11	1597.42
40	670.15	416.89	841.06	510.18	561.06
41	969.00	568.38	2618.762	615.62	695.11
42	1286.04	1280.59	5393.83	1279.85	1289.44
43	728.16	543.05	1100.31	628.32	662.50
44	485.52	-	406.15	325.09	346.29
45	-22.82	-	42.45	44.44	41.55

Table 38: Parameters fitted distribution criticality 1 for 15 clusters

Cluster	Distribution	Parameter 1	Parameter 2	Parameter 3
31	Negative binominal	$r = 1.25$	$p = 0.1297$	
32	Negative binominal	$r = 4.81$	$p = 0.0282$	
33				
34	Negative binominal	$r = 3.10$	$p = 8.112 * 10^{-4}$	
35	Compound Poisson	$p = 1.9$	$\mu = 31.99$	$\phi = 1.92$
36	Compound Poisson	$p = 1.8$	$\mu = 10.58$	$\phi = 1.64$
37	Negative binominal	$r = 3.41$	$p = 0.0018$	
38	Compound Poisson	$p = 1.8$	$\mu = 2635.91$	$\phi = 1.04$
39	Negative binominal	$r = 6.53$	$p = 0.0051$	
40				
41				
42	Negative binominal	$r = 5.41$	$p = 0.0210$	
43	Compound Poisson	$p = 1.6$	$\mu = 6.58$	$\phi = 1.77$
44				
45				

Appendix D: Mathematical model Ghorbani & Jokar (2016)

Index sets

Table 39: Sets base model

I	Set of suppliers
J	Set of warehouses
K	Set of customers
N	Set of capacity levels
H	Merged set of customers and warehouses, i.e. $K \cup J$
T	Set of time periods
P	Set of items
V	Set of vehicles

Parameters

Table 40: Parameters base model

Ca_n	Capacity associated with capacity level n for each depot	(items)
VC	Maximum vehicle capacity	(items)
h_{jp}	Per period inventory holding cost for product p in warehouse j	(€/item)
A_{ij}	Fixed cost for placing an order from warehouse j to supplier i	(€/order)
π_{kp}	Backlog cost of product p for customer k	(€/item)
F_j^n	Periodic operating cost for warehouse j with capacity level n	(€/warehouse)
Pr_p^i	Cost of purchasing product p from supplier i	(€/item)
ds_{ij}	Distance between supplier i and warehouse j	(kilometres)
dd_{hk}	Distance between node h and k	(kilometres)
G_p	Volume of product p	(m ²)
C_{ijp}	Transportation cost of product p from supplier i to warehouse j	(€/item/kilometre)
Cr_{hk}	Transportation cost from node h to node k	(€/kilometre)
D_{kpt}	Demand of customer k for product p in period t	(items)
U_{ipt}	Capacity of supplier i for product p in period t	(units)
α	Allowable backlog percentage for each customer for each product	
M	Big value	

Decision variables

Table 41: Decision variables base model

S_{jpt}	Final inventory level of product p in warehouse j in period t	(items)
Or_{ijpt}	Order quantity of warehouse j to supplier i for product p in period t	(items)
$E_{k\vartheta pt}$	Backlog quantity of product p for customer k on the route of vehicle ϑ in period t	(items)
L_{jkpt}	Backlog quantity of product p for customer k assigned to warehouse j in period t	(items)
Z_{ijpt}	$\begin{cases} 1 & \text{if supplier } i \text{ is assigned to warehouse } j \text{ in order to supply product } p \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$	
A_j^n	$\begin{cases} 1 & \text{if warehouse } j \text{ is opened with capacity level } n \\ 0 & \text{otherwise} \end{cases}$	
B_j^k	$\begin{cases} 1 & \text{if customer } k \text{ is assigned to warehouse } j \\ 0 & \text{otherwise} \end{cases}$	
$X_{hk\vartheta t}$	$\begin{cases} 1 & \text{if } h \text{ precedes } k \text{ immediately in route of vehicle } \vartheta \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$	
$M_{k\vartheta t}$	Auxiliary non-negative variable used for sub-tour elimination in the route of vehicle ϑ in period t	

Objective

$$\begin{aligned}
\text{Min: } & \sum_{t \in T} \sum_{p \in P} \sum_{j \in J} h_{jp} * S_{jpt} + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} A_{ij} \min \left\{ \sum_{p \in P} Z_{ijpt}, 1 \right\} \\
& + \sum_{t \in T} \sum_{j \in J} \sum_{n \in N} A_j^n * F_j^n * T + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} ds_{ij} * C_{ijp} * Or_{ijpt} \\
& + \sum_{\vartheta \in V} \sum_{t \in T} \sum_{k \in H} \sum_{h \in H} dd_{hk} * Cr_{hk} * X_{hk\vartheta t} + \sum_{t \in T} \sum_{p \in P} \sum_{j \in J} \sum_{i \in I} pr_p^i * Or_{ijpt} \\
& + \sum_{\vartheta \in V} \sum_{t \in T} \sum_{k \in K} \sum_{p \in P} \pi_{kp} * E_{k\vartheta pt}
\end{aligned} \tag{36}$$

The first component displays the holding cost of products over all periods, and the second component concerns the total ordering cost. The third component considers the total operating cost of depots over all periods according to their capacity levels. The fourth component shows the total transportation cost from suppliers to warehouses over all periods, while component five displays the routing cost from warehouses to customers over all periods. Component six considers the purchasing cost of products over all periods. The

seventh component shows the total backlog cost of products over all periods.

Constraints

$$\sum_{j \in J} B_{jk} = 1 \quad \forall k \in K \quad (37)$$

$$B_{jk} \leq \sum_{n \in N} A_j^n \quad \forall j \in J, \quad \forall k \in K \quad (38)$$

$$Z_{ijpt} \leq \sum_{n \in N} A_j^n \quad \forall j \in J, \quad \forall i \in I, \quad \forall p \in P, \quad \forall t \in T \quad (39)$$

$$\sum_{\vartheta \in V} \sum_{h \in H} X_{hk\vartheta t} = 1 \quad \forall k \in K, \quad \forall t \in T \quad (40)$$

$$\sum_{n \in N} \sum_{k \in K} X_{hk\vartheta t} \leq 1 \quad \forall \vartheta \in V, \quad \forall t \in T \quad (41)$$

$$\sum_{n \in N} A_j^n \quad \forall j \in J \quad (42)$$

$$\sum_{j \in J} Or_{jpt} \leq U_{ipt} \quad \forall p \in P, \quad \forall i \in I, \quad \forall t \in T \quad (43)$$

$$Or_{jpt} \leq M * Z_{ijpt} \quad \forall p \in P, \quad \forall i \in I, \quad \forall t \in T \quad (44)$$

$$\sum_{p \in P} L_{jkpt} \leq M * B_{jk} \quad \forall j \in J, \quad \forall t \in T, \quad \forall k \in K \quad (45)$$

$$L_{jkpt} \leq \alpha D_{kpt} \quad \forall p \in P, \forall j \in J, \quad \forall t \in T, \quad \forall k \in K \quad (46)$$

$$\sum_{j \in J} L_{jkpt} = \sum_{\vartheta \in V} E_{k\vartheta pt} \quad \forall k \in K, \forall p \in P, \quad \forall t \in T \quad (47)$$

$$\sum_{p \in P} E_{k\vartheta pt} \leq M * \sum_{h \in H} X_{hk\vartheta t} \quad \forall \vartheta \in V, \quad \forall t \in T, \quad \forall k \in K \quad (48)$$

$$S_{jpt} = S_{jp(t-1)} + \sum_{i \in I} Or_{ijpt} - \sum_{k \in K} D_{kpt} * B_{jk} - \sum_{k \in K} L_{jkp(t-1)} + \sum_{k \in K} L_{jkpt} \quad (49)$$

$$\forall p \in P, \quad \forall j \in J, \quad \forall t \in T$$

$$\sum_{p \in P} \left(\sum_{k \in K} D_{kpt} * B_{jk} \right) G_p \leq \sum_{n \in N} Ca_n * A_j^n \quad \forall j \in J, \quad \forall t \in T \quad (50)$$

$$\sum_{p \in P} \sum_{k \in K} \left(D_{kpt} * G_p \sum_{h \in H} X_{hk\vartheta t} \right) - \sum_{p \in P} \sum_{k \in K} E_{k\vartheta pt} + \sum_{p \in P} \sum_{k \in K} E_{k\vartheta p(t+1)} \leq VC \quad (51)$$

$$\forall \vartheta \in V, \quad \forall t \in T$$

$$\sum_{k \in H} X_{hk\vartheta t} - \sum_{k \in H} X_{kh\vartheta t} = 0 \quad \forall \vartheta \in V, \forall h \in H, \quad \forall t \in T \quad (52)$$

$$\sum_{h \in H} X_{hk\vartheta t} + \sum_{h \in H} X_{kh\vartheta t} - B_{jk} \leq 1 \quad \forall \vartheta \in V, \quad \forall j \in J, \forall k \in K, \quad \forall t \in T \quad (53)$$

$$M_{k\vartheta t} - M_{h\vartheta t} + |K| * X_{kh\vartheta t} \leq |K| - 1 \quad \forall h \in H, \quad \forall k \in K, \quad \forall \vartheta \in V, \quad \forall t \in T \quad (54)$$

$$S_{jp0} = 0, L_{jkp0} = 0, E_{k\vartheta p0} = 0 \quad \forall p \in P, \quad \forall j \in J, \quad \forall k \in K, \quad \forall \vartheta \in V \quad (55)$$

$$L_{j\vartheta pt}, E_{k\vartheta pt}, S_{jpt}, Y_{ijpt} \geq 0 \quad \forall i \in I, \quad \forall p \in P, \quad \forall j \in J, \quad \forall k \in K, \quad \forall \vartheta \in V, \quad \forall t \in T \quad (56)$$

$$Z_{ijpt}, Y_{ip}, A_{jn}, B_{jk}, X_{hk\vartheta t} \in \{0, 1\} \quad \forall i \in I, \quad \forall p \in P, \quad \forall j \in J, \quad \forall t \in T \quad (57)$$

Constraints (37) ensure that each customer is assigned only to one depot. Constraints (38) ensure that a customer can only be assigned to a depot if this depot is opened. Constraints (39) make sure that each depot could place orders with suppliers only if it is opened. Constraints (40) make sure that each customer is placed on exactly one vehicle route per period. Constraints (41) ensure that only one depot is included in each route per period. Constraints (42) imply that each depot can be assigned to at most one capacity level. Constraints (43) are the capacity constraints associated with the suppliers for each product per period. Constraints (44) are added to ensure linearisation. Constraints (45) allow a depot to backlog a fraction of the demand of any customer assigned to it. Constraints (46) ensure that the backlog quantity of each product for each customer per period is less than his allowable backlog quantity. Constraints (47) state that the backlog quantity of a customer assigned to a depot is equal to that of this customer assigned to the vehicle starting its route from this depot. Constraints (48) ensure that the backlog quantity of a customer for each product per period on a vehicle route could be positive if this route exists. Constraints (49) impose the inventory equality between consecutive periods. Constraints (50) are the capacity constraints associated with the depots. Constraints (51) are the vehicle capacity ones ensuring that the total deliveries of customers on each vehicle route do not exceed the vehicle capacity. Constraints (52) are flow conservation ones ensuring that whenever a vehicle enters a node in the network, it must leave it again, so that the routes remain circular. Constraints (53) link the allocation and routing components in each period: customer k is assigned to depot j if vehicle visiting customer k starts its trip from depot j per period. Constraints (54) are the sub-tour elimination ones. Constraints (55) state that the backlog quantities of products for all customers are equal to zero as well as the final inventory level of all products in all depots. Constraints (56) and (57) impose the non-negativity and integrality restrictions, respectively.

Appendix E: Haversine method

The haversine formula for distance calculation is displayed in Equation (58). Here ϕ_1 denotes the x-coordinate of node one, ϕ_2 denotes the x-coordinate of node two, $\Delta\phi$ denotes the difference between the x-coordinates of the two nodes, $\Delta\lambda$ denotes the difference between the y-coordinates of the two nodes and R is the radius of the earth, which is 6371km. The factor 1.3 represents the correction factor needed because the haversine formula considers straight-line distance instead of road distance.

$$\begin{aligned} a &= \sin^2\left(\frac{\Delta\phi}{2}\right) + \cos(\phi_1) * \cos(\phi_2) * \sin^2\left(\frac{\Delta\lambda}{2}\right) \\ c &= 2 * \operatorname{atan2}(\sqrt{a}, \sqrt{1-a}) \\ \text{distance} &= R * c * 1.3 \end{aligned} \tag{58}$$

Appendix F: Results initial investigation heuristic

Table 42: Results heuristic scenarios NC₁-YR₂

Scenario	NC ₁	NR ₂	NC ₂	YR ₂
Opened warehouses	X	2 & 4	X & 57	1 & X
Holding cost	7.79%	9.38%	9.53%	9.32%
Order cost 1	0.02%	0.02%	0.02%	0.02%
Order cost 2	-	0.02%	0.02%	0.02%
Handling cost	-	6.52%	2.95%	5.29%
Transportation cost 1	7.02%	6.97%	6.02%	7.00%
Transportation cost 2	53.10%	53.17%	43.75%	53.09%
Transportation cost 3	-	0.10%	2.54%	0.03%
Transportation cost 4	-	0.91%	0.91%	0.91%
Total cost	67.92%	77.10%	65.74%	75.67%
Total S	384,674	417,968	412,702	417,877
CO ₂	1,290 kg	1,292 kg	1,141 kg	1,290 kg

Table 43: Results heuristic scenarios YC₂-YX₃

Scenario	YC ₂	YX ₂	YC ₃	YX ₃
Opened warehouses	X & 57	X & 70	X & 57 & 71	X & 20 & 70
Holding cost	9.53%	9.32%	11.08%	10.88%
Order cost 1	0.02%	0.02%	0.02%	0.02%
Order cost 2	0.02%	0.02%	0.03%	0.03%
Handling cost	2.95%	2.77%	2.98%	6.49%
Transportation cost 1	6.02%	7.02%	6.02%	7.02%
Transportation cost 2	43.75%	41.97%	40.40%	36.33%
Transportation cost 3	2.54%	1.81%	5.74%	2.84%
Transportation cost 4	0.91%	0.91%	1.82%	1.82%
Total cost	65.74%	63.82%	68.09%	65.44%
Total S	412,702	412,208	428,714	441,116
CO ₂	1,141 kg	1,144 kg	1,124 kg	969 kg

Appendix G: Results initial investigation simulation

Table 44: Results simulation with first base stock calculation method

Scenario	NC ₁	YR ₂ /YC ₂	YX ₂
Opened warehouses	3	3 & 57	3 & 70
Holding cost	10.03%	11.99%	11.73%
Order cost 1	0.62%	0.60%	0.60%
Order cost 2	-	0.26%	0.24%
Handling cost	-	2.73%	2.61%
Transportation cost 1	6.57%	6.89%	6.57%
Transportation cost 2	53.83%	43.89%	42.54%
Transportation cost 3	-	1.68%	1.32%
Transportation cost 4	-	0.67%	0.67%
Total cost	71.05%	68.71%	66.28%
Total S	384,769	413,280	412,836
CO ₂	72,200 kg	67,753 kg	66,187 kg

Table 45: Availability simulation with first base stock calculation method

Scenario	NC ₁	YR ₂ /YC ₂	YX ₂
Opened warehouses	3	3 & 57	3 & 70
Criticality 3	77.32%	77.30%	77.35%
Criticality 2	69.28%	69.51%	69.54%
Criticality 1	79.79%	78.48%	78.67%
All criticalities	73.53%	73.54%	73.59%

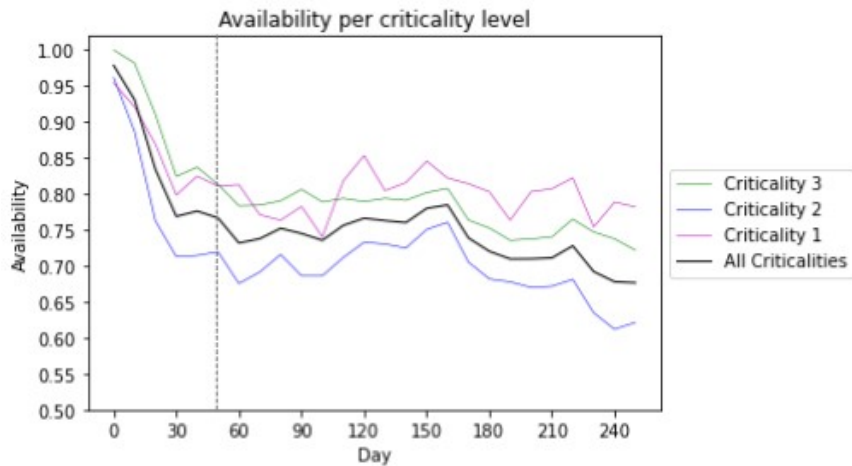


Figure 15: Availability warehouse X with first base stock calculation method

Table 46: Results simulation with second base stock calculation method

Scenario	NC ₁	YR ₂ /YC ₂	YX ₂
Opened warehouses	X	X & 57	X & 70
Holding cost	45.22%	67.68%	66.00%
Order cost 1	0.62%	0.60%	0.60%
Order cost 2	-	0.26%	0.26%
Handling cost	-	2.73%	2.61%
Transportation cost 1	6.57%	6.89%	6.57%
Transportation cost 2	54.45%	44.48%	43.10%
Transportation cost 3	-	1.70%	1.32%
Transportation cost 4	-	0.67%	0.67%
Total cost	106.86%	125.02%	121.14%
Total S	924,431	1,229,748	1,229,748
CO ₂	69,142 kg	65,054 kg	63,493 kg

Table 47: Availability simulation with second base stock calculation method

Scenario	NC ₁	YR ₂ /YC ₂	YX ₂
Opened warehouses	X	X & 57	X & 70
Criticality 3	97.10%	96.98%	96.99%
Criticality 2	93.22%	93.56%	93.53%
Criticality 1	98.01%	97.77%	97.77%
All criticalities	95.25%	95.35%	95.34%

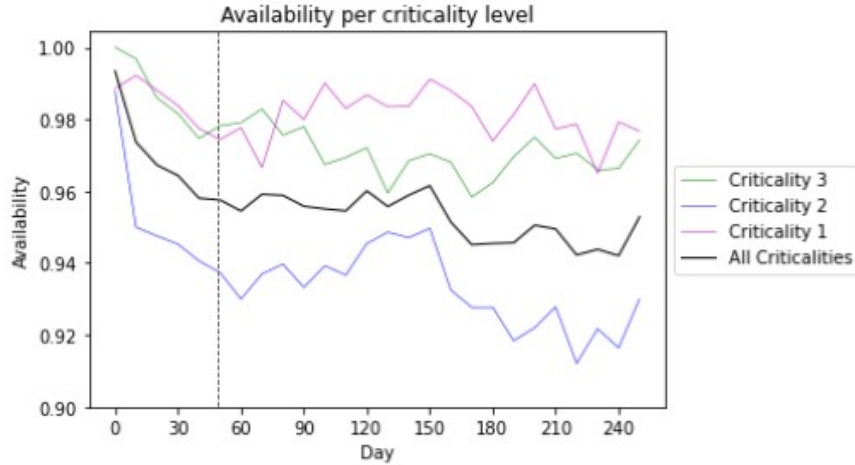


Figure 16: Availability warehouse X with second base stock calculation method

Table 48: Availability per cluster with second base stock calculation method

Cluster	1	2	3	4.1	4.2	5	6
Distribution	Normal	NB	Normal	None	None	CP	NB
Target α_{jp}	95%	97.5%	99%	96%	96%	97.5%	99%
Realised α_{jp}	96.62%	66.97%	100%	50.00%	50.00%	68.53%	100%
Cluster	7	8	9	10	11	12	13
Distribution	Gamma	Normal	Normal	Normal	Gamma	None	NB
Target α_{jp}	99%	99%	99.9%	98%	95%	96%	95%
Realised α_{jp}	96.86%	99.96%	99.98%	99.36%	95.95%	50.00%	46.45%
Cluster	14	15	16	17	18	19.1	19.2
Distribution	Gamma	NB	Normal	Normal	NB	None	None
Target α_{jp}	90%	99.5%	99.5%	98%	95%	96%	96%
Realised α_{jp}	96.63%	2.28%	99.89%	94.21%	99.18%	40.00%	55.17%
Cluster	20	21	22	23.1	23.2	23.3	24
Distribution	NB	CP	Normal	None	None	None	NB
Target α_{jp}	99%	95%	99%	96%	96%	96%	99%
Realised α_{jp}	99.83%	37.34%	99.63%	97.73%	66.67%	57.40%	99.83%