MASTER

Optimizing a partial mixed bundling inventory system subject to an aggregate service constraint

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Optimizing a partial mixed bundling inventory system subject to an aggregate service constraint

Master Thesis

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Abstract

The classical trade-off in every inventory system is realizing the highest customer service level while the inventory costs must be as low as possible. Where conventional literature on multi-item inventory problems does not incorporate bundling strategies and bundling literature does not consider the perspective of the supply chain. Real world inventory systems following the mixed bundling strategy, such as at Van Geloven, face a lot of difficulties. This master thesis focuses on applying different approaches to study the effects of the mixed bundling principle on multi-item inventory problems and vice versa that can be applied to the inventory system of Van Geloven. The different approaches are analyzed via the DoBr-tool to obtain exact results for the KPI's of the mixed bundling multi-item inventory system. A system approach with a single aggregate fill rate constraint provides the best cost-optimal inventory system when considering an inventory model with independent items. In addition, a single item approach combined with the ATO principle is used to obtain feasible solutions when considering two inventory models with a network of items which follows the mixed bundling principle. The benefits of the mixed bundling principle are increased if negative demand correlations are present between end-products and if inventories of components can be pooled together.

Keywords: *Multi-item inventory system, Mixed bundling, Aggregate service level constraint, Fill rate, Periodic review, Demand correlation, Assemble-To-Order*

Preface

This Master thesis concludes the final phase of my master program 'Operations Management and Logistics' at Eindhoven University of Technology. This research is a cooperation between myself, Van Geloven B.V. and the University and lasted from July 2021 to May 2022. I would like to take the opportunity here to express my gratitude to several people who have been of amazing help, inspiration and support during this research project.

First of all, I would like to thank my university supervisor, Rob Broekmeulen, for his commitment, discussions and enthusiasm during the project. I would like to especially thank him for his readiness during the Covid-19 pandemic and feedback which contributed to the report as it lies before you. I am glad to have finished the project with a few face-to-face meetings which were more useful and productive when compared to Teams meetings.

In addition, I would like to thank my company supervisor, Helco Sala, for his kind support and pleasant cooperation during the entire project. I am especially grateful for the given freedom to work on my project, numerous interesting discussions and providing all the information I needed to complete this research. His ability to convince co-workers of new ideas and opportunities is exceptional. Furthermore, I want to thank Mark Feijt for his exceptional help during the end stages of this project. He only joined the company recently, but helped me the most with practical details, in-dept discussions and his enthusiasm about my research.

Finally, I would like to thank my family and friends for their inspiration and support during the hard times of the Covid-19 pandemic. I am very grateful for some close friends who contributed greatly by always making time for me if I wanted to have a discussion about my research.

Stefan Delfgou
May 24, 2022

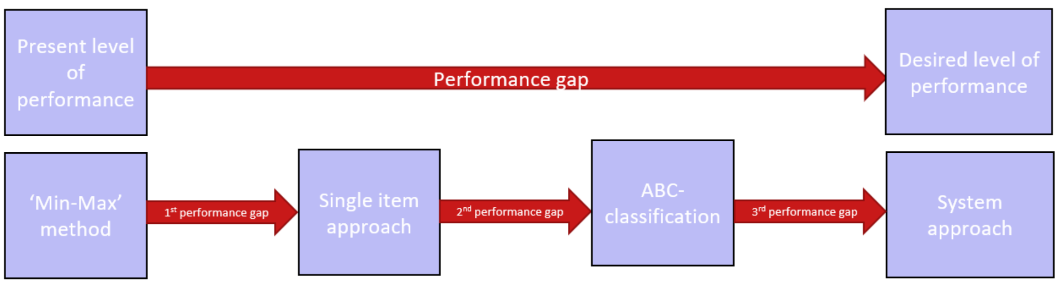
Management summary

Problem context

This master thesis describes the research on the implementation and effects of several approaches for multi-item inventory problems which use different bundling selling strategies. The objective is to minimize the inventory costs of a system while achieving the predetermined customer service level for inventory systems which follow the mixed bundling selling strategy. The complexity of the problem arises in the mixed bundling principle, as very little is known about how this principle affects the classical inventory trade-off between fill rates, inventory costs and other inventory dynamics. In addition, the mixed bundling strategy determines greatly how the supply chain is designed. The problem becomes even more complex if the inventory system is large, which is very common in the real world such as the inventory system of Van Geloven. The academic inventory literature does not include a lot of research about the negative and positive effects of the mixed bundling principle on multi-item inventory systems. Due to recent development at Van Geloven, they must minimize the total inventory costs of their multi-item inventory system subject to an aggregate service constraint. However, this brought to light that their current inventory policy and knowledge about standard inventory literature is not sufficient to realize this objective. This research expands the current literature on mixed bundling multi-item inventory problems and shows Van Geloven different approaches to optimize their inventory system.

Gap analysis

The gap analysis helps to reduce the performance gap of any system within a company. During this research, it is used to determine the present level of performance of the inventory system of Van Geloven, including an analysis on the presently applied policies and how the mixed bundling strategy currently affects their system. In addition, multiple approaches are implemented to reduce the performance gap of the inventory system of Van Geloven and study the effects of the mixed bundling principle. Finally, two reduced inventory models of the entire inventory system are created to study the effects of the mixed bundling principle in a larger part of the supply chain.

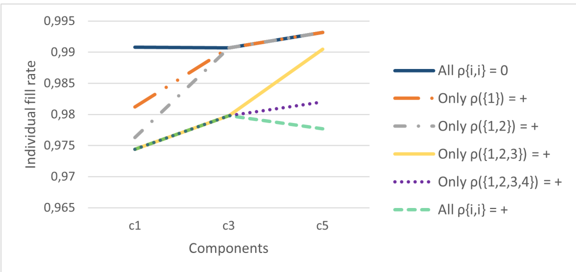


Results

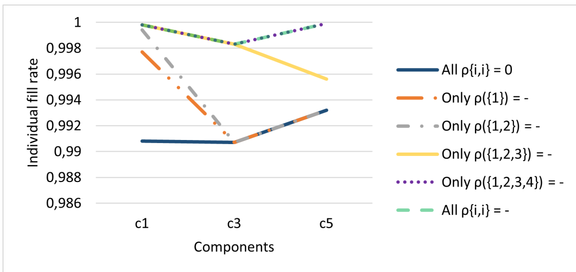
The results of the proposed approaches for the inventory model where we only consider the stock points of the SKU's at the central warehouse can be found in the table below. True demand data is used which is a little unfair as we determine the optimal reorders levels after demand is realized, but it shows the maximum potential of the approaches. If the forecast data would be used the cost savings are still significant. As far as the current situation is concerned, it can be stated that in terms of achieving the predetermined system service level Van Geloven is doing an reasonable job despite the difficulties faced by the Covid pandemic. However, taken into consideration the unawareness of possible improvements of all the system KPI's and the unwillingness to change, the current inventory performance should be taken with a grain of salt.

Inventory policy	*Total inventory costs*	*% cost improvement to current policy*	*% cost improvement to single item approach*
Current	€1.748.358	-	-
Single item	€1.195.562	31,62 %	-
ABC-classification	€1.177.467	32,65 %	1,51%
System - Sherbrooke	€1.066.743	38,99%	10,78 %

The results of both sub-inventory systems could unfortunately not be compared with the relative performance of the current situation, as the data of the local warehouse was very limited. While analyzing those systems the most promising finding is visualized below. Using the second subsystem with only a single stocking location, it is found that stronger negative demand correlations have a positive effect on component fill rates (b). On the other hand, positive demand correlations have a negative effect on the component fill rates (a).



(a) The impact of strictly positive correlations



(b) The impact of strictly negative correlations

Managerial Insights & recommendations

Despite some limitations of the research, multiple different general and research driven recommendations can be given based on the obtained results. General recommendations are needed to improve the efficiency of the supply chain and provide a solid foundation for the presented approaches. Most importantly, Van Geloven must change their current replenishment logic as soon as possible to the (R,s,nQ) policy. Furthermore, the speed of the implementation of the new information system must be increased, changing to a central decision making process regarding inventory management and become a more data-driven company. Once progress is made with these steps, it is recommended implement the system approach for the SKU's at the central warehouse to minimize the inventory costs while achieving the aggregate service level constraint.

Table of Contents

List of Figures

List of Tables

List of abbreviations

ATO	Assemble To Order
ATS	Assemble To Stock
BOM	Bill Of Material
CODP	Customer Order Decoupling Point
ELSP	Economic Lot Scheduling Problem
EOQ	Economic Order Quantity
FIFO	First-In-First-Out
GKD	Generalized Knapsack Duality
KPI	Key Performance Indicator
LIFO	Last-In-First-Out
MAPE	Mean Absolute Percentage Error
MAE	Mean Absolute Error
MTO	Make To Order
MTS	Make To Stock
OFR	Order Fill Rate
RMSE	Root Mean Square Error
RR	Ready Rate
SKU	Stock Keeping Unit
VFR	Volume Fill Rate

Chapter 1

Introduction

The first chapter comprises the project environment in which the research is conducted. In addition, some relevant background information is provided about the company to introduce the practical problem they are currently facing. Subsequently the research objective is defined with corresponding research questions. After that, the research phases are introduced and the scope is defined. Finally, the report outline is discussed.

1.1 Project environment

The research of this Master Thesis will be performed at Ad van Geloven B.V.. They yearly produce around 80 million kilograms of frozen snacks divided over roughly 850 active SKU's. Van Geloven has six factories located in the Netherlands and Belgium, but warehouse operations and outbound logistics are outsourced to a service provider. XPO Supply Chain Netherlands transports all products from the factories to the central warehouse, performs warehouse operations, handles the distribution to all the customers of Van Geloven and communicates closely with customers service department when out of stock situations occur. The current organizational situation is undergoing changes regarding inventory management. Previously, demand forecasters used to be in control of demand forecasting, inventory management and communication with the factories about production amounts. Master supply planners are recently introduced to be responsible for inventory management and being the communication link between the marketing department, demand forecasters, factory schedulers, and production operators at the factories in order to improve operations.

Van Geloven has recently been taken over by a Canadian multinational. McCain Foods Limited is the world's largest manufacturer of frozen potato products, who want to expand their business in different continents and enlarge their product portfolio. To achieve this, they bought Van Geloven, as it is one of the largest frozen snacks manufacturers in Europe and they already have an established supply chain in Western Europe. Due to the acquisition, new production lines are developed in the current factories to produce a different type of snack which is sold to customers of McCain located in Europe. All current products of Van Geloven are still produced for their 'own' customers.

1.2 Motivation and Problem Description

This research finds it's motivation in the current problems faced by Van Geloven in combination with a gap in the existing literature. Due to the recent takeover, every department within Van Geloven has to minimize costs and improve their efficiency. In addition, the Covid-pandemic resulted in large drops in customer service level and high inventory costs since lock-downs and other government regulations created large demand uncertainties and staff was limited. However, some employees of the company indicated that the customer service level had been unreliable before the Covid-pandemic and the cause was inexplicable. These problems exposes the inadequacy of the current applied inventory approach and inventory department of Van Geloven. At this moment, they determine all inventory decision on the average weekly coverage without incorporating the fill rate and resulting inventory costs in any way. This leads to a large problem, as the inventory managers of Van Geloven do not know how to minimize inventory costs and realize the predetermined aggregate system service level set by management, so they seek for a different inventory approach that is able to help them with this issue. The current design of the supply chain and the applied mixed bundling selling strategy both create additional complexity to their issue. The following two figures, Figure 1.1 and 1.2, visualizes the design of the supply chain and explains the concept of mixed bundling compared to other bundling strategies.

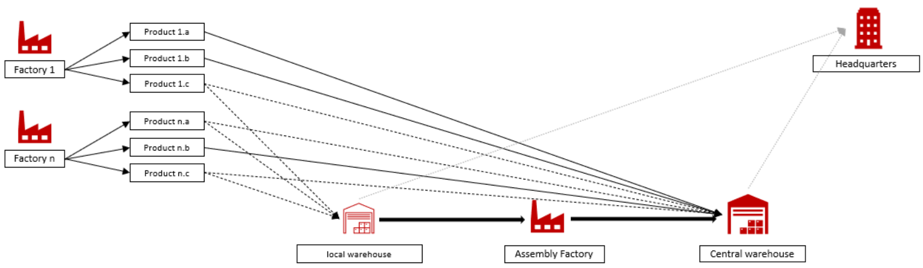


Figure 1.1: Visual representation of the supply chain of van Geloven

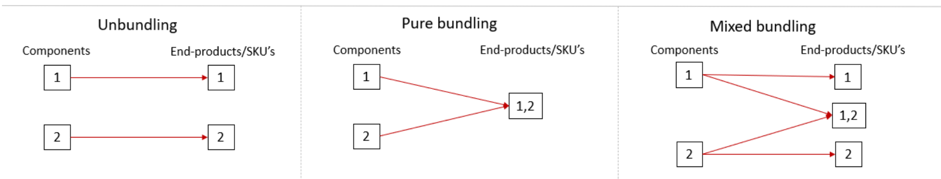


Figure 1.2: Types of bundling

Van Geloven uses the mixed bundling selling strategy to sell their products. This strategy holds that components can be sold individually as single-component SKU's or components could be bundled together and sold as multi-component SKU's. Components that are destined for multi-component SKU's have additional manufacturing processes, as can be seen in the visualization of the supply chain. This creates extra complexity, since component one is stored at the local warehouse waiting for it's assembly process and SKU's including component one are stored at the central warehouse. This creates at least three different stock points where component 1 is stored. Additionally, each stock point is treated individually and separately and has it's own packaging form which does not create the possibility to pool inventories of the three different stocking points together.

The problem addressed in this research is how to determine optimal inventory decisions for the components stocked at the local warehouse and of SKU's stored at the central warehouse. The underlying objective is to minimize total inventory costs in combination with achieving a predetermined aggregate system fill rate. Almost all classical inventory literature describes inventory approaches to tackle this problem for unbundling and pure bundling inventory system. These inventory systems have a lower complexity compared to the mixed bundling case, since fewer stock points are created. Only the research of Taleizadeh et al. (2020), Qiang (2012), Ernst and Kouvelis (1999) found some effects of mixed bundling on the inventory decision making process. However, they only used a system with three end-products/SKU's and having the objective to maximize profits instead of minimizing the total inventory costs subjected to an aggregate service constraint. Translating the problem statement into a model that is able to analyze and gain insights of the mixed bundling principle is of great interest to retailers and companies, as in a real market most of them handle dozens of SKU's.

1.3 Research objective

The main objective of this research is to find inventory approaches which can be applied for multi-item inventory systems following a mixed bundling selling strategy. These inventory strategies have to minimize total inventory costs of the system and achieve a predetermined customer service level which is often set by senior management. The secondary research objective is creating more insights on the effects of inventory dynamics and decisions while using the mixed bundling selling principle.

1.4 Research questions

Based on the research objective, the main research questions and multiple sub-questions can be defined. Answering all the research questions might lead to accomplishing the research goal.

Main research question: *What inventory approaches can be applied for minimizing total inventory costs while meeting the target service level in a multi-item inventory system which uses a mixed- or a non-mixed bundling strategy?*

The first two sub-questions comprises a lot of relevant background information found in the inventory control academic literature. This information is needed to fully understand the problem at hand, because the application, consequences and effects of mixed-or non-mixed bundling strategies is often not common knowledge for inventory managers.

- **SQ.1:** What inventory strategies can be used for a multi-item inventory system?
- **SQ.2:** What definitions of the aggregate fill rate constraint can be used for a food company with a production-inventory control system following a mixed bundling selling strategy?

The third sub-question comprehends the design of the current inventory system of Van Geloven. This includes the interaction between various departments for managing the inventory system, what the current inventory policy is together with control parameter settings and what the performance is of the system. A detailed analysis is performed using interviews, round-table discussions and most importantly data outputs of the information system.

- **SQ.3:** What is the current inventory strategy and performance of the multi-item inventory system of Van Geloven?

The fourth sub-question deals with the translation of the problem at hand to a mathematical inventory model. It is researched how the mixed bundling selling strategy for more than two end-product items consisting of at least three different components can be modelled regarding the trade-off between service level and inventory costs.

- **SQ.4:** What kind of inventory model could be developed that incorporates the mixed bundling principle and is able to minimize total inventory costs for a multi-item inventory problem?

The final sub-question comprises the relative effects that are found when the mixed bundling principle is applied for an inventory system.

- **SQ.5:** What are the effects on inventory decisions when the mixed bundling principle is implemented for a multi-item inventory system?

1.5 Research paradigms

The research of Van Aken and Berends (2018) described that there are two main research paradigms for graduation projects, namely the explanatory research paradigm and the design science research paradigm. The first paradigm is typically used in most social sciences, as students in this area aim to produce descriptive and explanatory knowledge. The second research paradigm, often referred as the problem-solving approach, aims to produce solutions for field problems in a structured manner and is the research paradigm for medical and engineering schools which is why it will be used in this Master Thesis. Van Aken and Berends (2018) defined the problem solving cycle in five steps which can be seen in Figure 1.3.

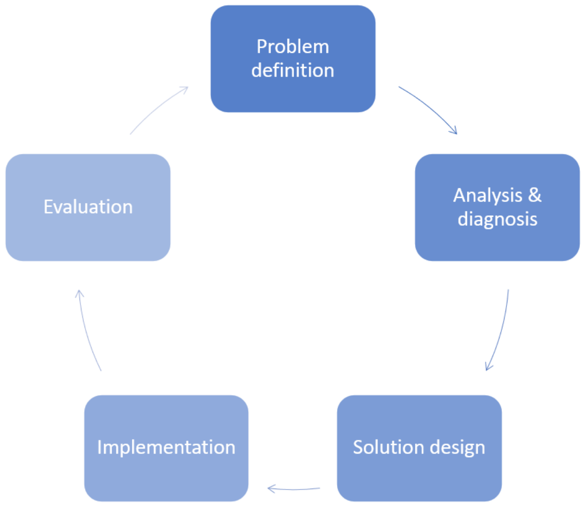


Figure 1.3: Problem solving cycle defined by Van Aken and Berends (2018)

1.6 Research methodology

Using the research paradigm of Van Aken and Berends (2018), the research process of this research can be described. The research process used in this report is divided into four phases and is visualized in Figure 1.4.

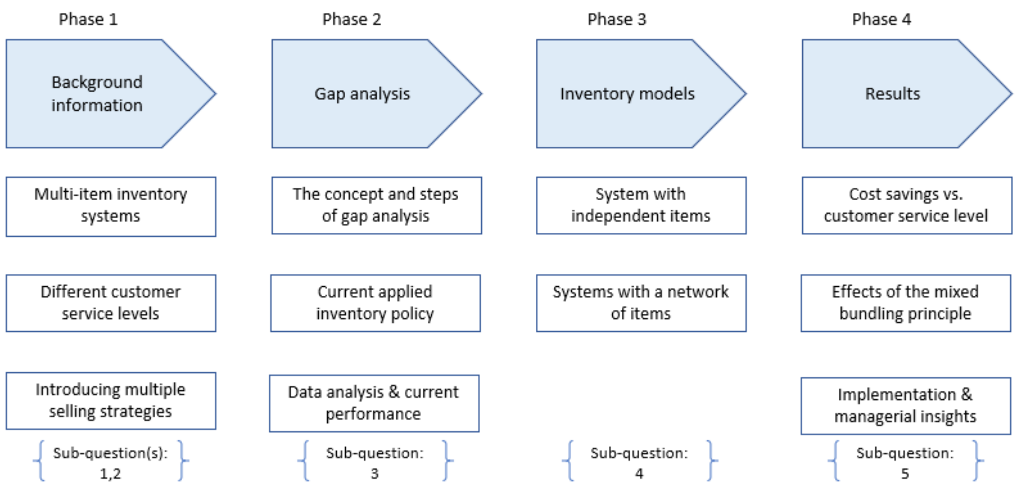


Figure 1.4: Research process

The first phase provides relevant background information for the research problem at hand. This phase includes a detailed literature review on the multi-item inventory problem. Different types of this phenomena are discussed with relevant inventory models and applicable strategies to achieve specific KPI's. The second part of this phase introduces different selling strategies and special attention is given to the mixed bundling principle. In addition, this phase gives an answer to sub-questions 1 and 2 of this research.

The second phase of this research is a detailed analysis of the current situation of Van Geloven regarding inventory management. A gap analysis is introduced to identify the performance gap of the present performance and the desired performance of the system. This includes a data analysis and determining the actual performance of the current inventory policy. Also, intermediate steps are discussed to reduce the possible performance gap over in several steps instead of all at once. Subsequently, this phase provides an answer to the third sub-question of this research.

The next phase, the third phase, comprises a detailed explanation of how the different inventory approaches presented in the gap analysis are going to be applied. A distinction will be made withing the inventory approaches, as the inventories of the components are shared or not shared. Subsequently, the DoBr-tool is used to obtain exact results for the different inventory approaches. In addition, this phase gives an answer to the fourth sub-question of this research.

The final phase, the fifth phase, is the results phase in which the final outcomes are presented in a structured overview for both types of inventory approaches. Based on the findings, various recommendations and managerial insights are given for Van Geloven. Finally, this phase provides an answer to the fifth sub-question of this research.

1.7 Scope of research

This research is part of a logistic cost savings project which focuses only on operations in the Netherlands. Therefore, we scope this research regarding the inventory decisions made for the central and local warehouses located in the Netherlands. Figure 1.1 shows a snapshot of current supply chain of Van Geloven. Multiple factories deliver end-products to the central warehouse and also deliver semi-finished products to intermediate warehouses which have to wait for their assembly process at a factory. The mixed bundling principle causes the split of the same semi-finished products (indicated by the dotted black lines) to the local and central warehouse. The inventory decision making process for all warehouses are made at the headquarters which receives daily information (indicated by the grey dotted line) about the stock levels of all end- and semi-finished products. During this research, we will focus first on the inventory decision making process at the central warehouse and after that we will include the local warehouses.

1.8 Outline report

In the remainder of this report, the project is described in more detail. Chapter 2, discusses theoretical background information on multi-item inventory problems and customer service levels. Besides, this chapter addresses some papers which performance research on the mixed bundling selling strategy in a multi-item inventory system environment. Chapter 3 discusses in more detail the problem for Van Geloven and the gap analysis is introduced to solve the problem at hand by proposing multiple different approaches. The presented approaches are further elaborated in chapter 4 and the results are discussed in chapter 5. Finally, chapters 6 and 7 contain the conclusion and implementation of this research.

Chapter 2

Literature Background

The second chapter, which is the first phase of this research, discusses theoretical background information which is necessary to fully understand the problem at hand. Prior to this research, an extensive literature review was written on several topics of the multi-item inventory problem. In this chapter, relevant sections of that report are discussed and in some cases extended in order to answer the first two sub-questions of this research. Those questions were stated as follows:

- **SQ.1:** *What inventory strategies can be used for a multi-item inventory system?*
- **SQ.2:** *What definitions of the aggregate fill rate constraint can be used for a food company with a production-inventory control system under an unbundling and mixed bundling selling strategy?*

2.1 Different types of multi-item inventory systems

In order to provide an answer to sub-question one, various types of multi-item inventory problems will be discussed including relevant inventory approaches and replenishment logic's. Classical inventory models regularly assume a single-item problem, however in practice it rarely occurs that a company only sells one product (Bhattacharya, 2005). There are several reasons why a company sells more than one item. According to Bulut et al. (2009), adding a second or third item to an inventory system favors the demand of the first product, it could expands their business, increase their market share power and due to efficiency reasons such as cost savings or quality improvements. These motives leads to the creation of a multi-item inventory problem. A distinction can to be made between different types of multi-item inventory problems. According to the paper of De Schrijver et al. (2013), three categories of multi-item inventory problems can be identified:

- Independent items
- Network of items
- Shared supply chain processes

The first category, independent items, describes inventory problems related to distinct supply chains and separate demand processes. In other words, different items do not share any supply or demand link and can therefore be treated individually (De Schrijver et al., 2013). This argument enables to propose one or several aggregate service constraints on the total set of individual items. The second category, a network of items, have a supply chain or demand relationship, such as a series system, a distribution system, an assemble system, or a general system (De Schrijver et al., 2013). The work of Axsäter (2003) provides an overview of how multi-echelon serial and distribution systems can be applied in a supply chain. The last category consist of systems where items completely share supply chain processes. The paper of Axsäter (2006) discusses the two most well known types of problems in this area. Firstly, the joint-replenishment problem states that a group of items should be replenished together as much as possible. Arguments for joint-replenishment can be: joint setup costs, coordinated transport or quantity discounts. The second discussed alternative in this paper is called the Economic Lot Scheduling Problem (ELSP). The ELSP looks at how the cyclic schedules for several items, with constant demand and no back ordering, can be planned.

2.1.1 Multi-item inventory problems with independent items

Within this category, several different approaches can be used to tackle the multi-item inventory problem. First, an overview is presented of different inventory approaches. Subsequently, several instances of this kind of inventory problem are explained including various replenishment policies.

Single item approach

Since items do not share any form of a supply or demand relationship, they can be treated individually and separately of each other in terms of ordering calculations, setting reorder levels and more (Wong et al., 2006). According to De Schrijver et al. (2013), this approach is often applied within companies as it is very simplistic. Furthermore, this paper suggests several reasons why companies or inventory managers still use a single item approach, despite the fact that using this approach realizes a loss in efficiency and an increase in costs. In their work, they suggest that managers have an unawareness of the existence of other approaches than the single item approach. If managers are familiar with other approaches, they assume it creates an unbridgeable extra complexity which is impossible to implement.

ABC-classification

Creating an effective and efficient supply chain when spending the amount of resources, time and money equally to all the SKU's can be very hard. Therefore, a different approach can be used in order to spend the amount of resources to SKU's based on it's relative importance. In practice, the general rule of Pareto holds in a lot of retail companies. This rules states, that the top 20% of the selling items accounts for roughly 80% of the total demand (Chu et al., 2008). The ABC-classification is an approach to divide SKU's into different classes based on a criteria that indicates the level of importance. The name obviously comes from it's design, as the SKU's are often classified into three classes: A - Very Important, B - Important and C - Least Important. According to Chu et al. (2008), the three classes can be defined as follows:

- A items: The top 15-20% of the sold items account for around 75-80% of the total annual inventory value.

- B items: The middle 30-40% of the sold items account for around 15% of the total annual inventory value.
- C items: The last 40-50% of the sold items account for around 10% of the total annual inventory value.

This general classification is not always correct for every company. However, this classification is found to be close to the actual occurrence in companies (Swamidass, 2000). The traditional classification criteria to divide SKU's into classes is only based on a single criteria (Teunter et al., 2010). The traditional criterion is often based on the turnover or demand volume of the SKU's. The turnover of an item is calculated by multiplying the price by the demand volume, while demand volume is the amount of goods sold per item. In practice, the traditional criteria are very popular as they are easy to understand and implement. The drawback of using either one of these criteria is that only a single inventory parameter is used.

Alternatively, the study of Zhang et al. (2001) proposes a classification method based on multiple inventory parameters. In their research, they include the demand, squared unit cost and lead time of a SKU to classify the SKU's into the groups. Secondly, a new criterion for the ABC-classification was developed by Teunter et al. (2010) which includes a fourth inventory parameter, namely shortage costs. This classification method significantly outperforms both traditional single criterion and the classification method of Zhang et al. (2001) on different service levels (99% and 95%) and demand distributions (Normal and Gamma) (Teunter et al., 2010). Lastly, Teunter et al. (2010) concluded that, the original demand volume and demand turnover classification methods performed poorly, as they both more than doubled the total safety stock costs in all used data sets. More specifically, the demand turnover criterion performed the worst based on the high safety stock costs.

The ABC-approach limits the number of optimizations of the target fill rate for an inventory system, as there are often 3 groups and at most 6 (Silver et al., 1998). Furthermore, a possibility is created to slightly differentiate between the service levels of the SKU's. The downside of this approach is that additional complexity is created, because it is has to be determined what the optimal number of groups is, how to divide the SKU's over the groups and what target fill rate must be set for each group to minimize the inventory costs.

System item approach

In practice, lots of inventory managers have to deal with system wide limitations and goals set by management, such as limited warehouse space, workforce constraints or an aggregate service level. Taking system constraints into account, an inventory manager must translate the system wide goals to individual or groups of SKU's. Instead of using a single item approach, a system approach could be used to reduce overall costs and have a more efficient inventory system. The current literature often describes the system approach as an aggregate constraint inventory system (De Schrijver et al., 2013, AUCAMP, 1990). If an aggregate service level constraint is applied within a company, an inventory manager has the possibility to differentiate between the service levels of the individual SKU's to minimize the costs. However, according to Teunter et al. (2017), De Schrijver et al. (2013), the predetermined aggregate system service level is in practice often simply copied to all individual SKU's which is in most cases far from optimal.

Various kinds of inventory problems

This subsection comprehends five different instances of inventory problems with independent items. The research of De Schrijver et al. (2013) reviewed multiple different inventory systems considering a single- or multi-item inventory setting with one or more aggregate constraints. For some instances, inventory policies are explained to handle the inventory problem. They discussed the following problem instances:

1. Deterministic lead time demand.
2. Newsvendor problem
3. Stationary inventory models: Base-stock models
4. Stationary inventory models: General batch systems (r,Q)
5. Stationary inventory models: General batch systems (s,S)

The first case, an inventory system with deterministic constant lead time demands, is known for being the most basic inventory model. The only decision variable in this model is the order quantity for each item. When there is only one item, the optimal order quantity to minimize the costs can be found through the economic order quantity (EOQ). The classical EOQ becomes unusable if the inventory system consists of multiple items and aggregate constraints are added. However, the paper of Starr and Miller (1962) developed a method called 'optimal policy curve' which is able to deal with multi-item systems and a single aggregate investment constraint. As research in this field continued, the work of Hadley and Whitin (1963) proved that the Lagrangian principle is able to incorporate multiple constraints together. Secondly, the research of Rosenblatt and Rothblum (1990), Haksever and Moussourakis (2005), Boctor (2010), developed several accurate approximations algorithms which were able to find good quality solutions for simple inventory models with deterministic constant lead time demands.

The second problem instance is the classical Newsvendor problem. This problem is identified as a single period model with stochastic demand and penalty costs for ordering too much or too little (De Schrijver et al., 2013). Moreover, the decision variable in this problem is the order quantity per item. The research of Hadley and Whitin (1963) analyzed this problem with a multi-item setting and a single aggregate constraint. They introduced a capacity constraint which can lead to negative order quantities and service levels if the constraint is very tight. The paper of Zhang et al. (2009) continued this research, as they created a solution algorithm which generated near-optimal solutions for continuous demand distributions and a reasonable approximation solution for discrete demand distributions. The work of Lau and Lau (1996), Niederhoff (2007), Özler et al. (2009) discussed newsvendor problems with multiple constraints. They concluded that, adding constraints enlarges the difficulty and complexity of the newsvendor problem but is desired to represent real-life situations more accurately.

The third category is a stationary inventory model with base-stock policies. The decision variable for this approach is the target stock level (s), with an order quantity (Q) equal to one and periodic review. This (r,Q) policy is often seen in spare part inventory systems, as the demand rates are low and the purchase costs are high. According to the inventory management literature, this policy can also be referred as the (R,S) inventory policy (de Kok, 1991). They state that this is one of the most widely used policies, as it is quite simple and intuitive. A definition of this strategy can be formulated as: *'Order an amount of items to ensure that the sum of physical stock and items in the pipeline minus the back orders is enough to cover demand from now until the next review period* (de Kok, 1991). This replenishment logic does not include lot-sizing, because the order quantity (Q) is equal to one and therefore it is a special case of the well-known (R,s,nQ) system.

The next problem instance of the multi-item inventory problem is the general approach of a (r, Q) policy with order size Q being larger than one. In contrast to the other evaluated instances, this approach has two decision variables, namely the reorder point and order quantity for each item. Additionally, this problem can be divided into two types, systems with or without marginal costs. Besides having two decision variables, it is also possible to include multiple aggregate service constraints. Despite the higher level of complexity, it turned out that the (r,Q) policy is very popular and also widely used (De Schrijver et al., 2013). In other academic papers, the (r,Q) system considered as an (R,s,nQ) replenishment logic. Reviewing the inventory system is done periodically (R) and if at a review moment the inventory position if below reorder level (s), an order is created with an integer multiple (n) of lot-size Q to bring the inventory position at least to reorder level (s). The research of Hadley and Whitin (1963) and Van Donselaar et al. (2021) use the (R,s,nQ) replenishment logic to study the multi-item inventory problem with an aggregate fill rate constraint and stochastic demand.

Another, more recent developed, special case of the (R,s,nQ) replenishment logic is presented in the research of Broekmeulen and Van Donselaar (2009). They developed the EWA policy which is a specially designed replenishment policy for perishable products and can lead to substantial cost reductions compared to base-stock policies that do not include the age of inventories. The only difference with the base policy (R,s,nQ) is using the estimated amount of outdating for the correcting the IP. The effects of this policy compared to the base policy are, the fill rate increases between 2-3%, the average inventory decreases between 4-10% and the average outdating decreased between 8-11% depending on the withdrawal policy (FIFO or LIFO) (Broekmeulen and Van Donselaar, 2009). In addition, the positive effects of the EWA-policy increase when products have a short remaining shelf life, customer withdrawal is LIFO, the lead-time is large, the review period is large and when outdating is expensive. The main advantage of this policy is it's simplicity, the ease to explain the logic behind it and the low complexity of parameter settings. The only drawback of the EWA-policy is that a detailed registration is needed of the age of all items.

The last problem instance is a stationary inventory model with a (s, S) batch policy. This approach also has two decision variables, the reorder level s and the order up to level S. An order is placed if the stock falls to or below the reorder point s, with an order size to reach the order-up-to level S (De Schrijver et al., 2013). Compared to the EWA, (R,s,nQ) and (R,S) replenishment policies, this approach has a variable replenishment quality. If the review moment of this policy is periodic, then it can be referred as the (R,s,S) replenishment policy.

2.1.2 Multi-item inventory problems with a network of items

This section describes the second category of multi-item inventory problems, namely a system with a network of items. Items in these kind of inventory systems have a supply-demand relationship. Two specific type of systems will be discussed, Assemble To Order (ATO) and Assemble to Stock (ATS) systems. Both are interesting for this research, as they can be related to the mixed bundling selling principle. How these concepts can be related to the mixed bundling strategy will be discussed at the end of this chapter.

An assemble to order system can be defined as a strategy for which standard individual parts or components are manufactured according to forecasts, while schedules for sub assemblies and final assembly are not executed until customer demand is realized (Wemmerlöv, 1984). Therefore, the overall manufacturing process is a mix of component procurement or production and an assembly process of end-products (Atan et al., 2017). In theory, an ATO system only keeps inventory of components and not of end-products, as by delaying the final assembly of end-products a company can benefit from inventory pooling of component inventories and reduce the costs of offering multiple end-products (Atan et al., 2017). There is one major difference between the ATO and ATS system. With an ATS system, the manufacturer must determine in advance how many components need to be assembled into end-products to satisfy future customer demands. The replenishment lead time of the components and process time to assemble the end-products must be taken into account (Peng Chew et al., 2006). In other words, the assemble process is not delayed until customer demand is realized, but the process immediately starts when components arrive on-site. The Customer Order Decoupling Point (CODP) is more downstream of the supply chain for an ATS system compared to an ATO system. Figure 2.1 shows a visual representation of both an ATO and ATS system. Finally, it is important to note that components can be used for more than one end-product.

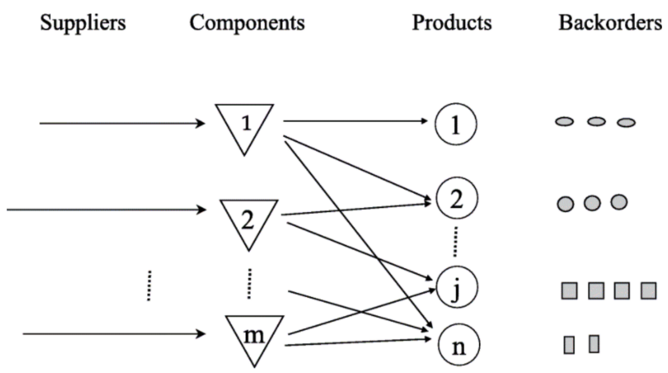


Figure 2.1: ATO & ATS system (Song and Zipkin, 2003)

According to ElHafsi et al. (2008), ATO systems are hard to analyze and manage because of the following difficulties: correlation of demand between the components, asymmetric component production lead times and dependency of the demand fulfillment on the availability of multiple components. When using an ATS system, the main operation issues are inventory planning, demand forecasting and lot-size determination. The research of Song and Zipkin (2003) concludes that both systems consists of an assembly and distribution system. The issue of an assembly system is the coordination of components, while the issue of a distribution system is the allocation of components among the products. The difference between both systems is visually displayed in Figure 2.2. An important piece of information is the Bill Of Material (BOM), which is a list of all materials, parts and sub-assemblies that are needed to produce the end-products (Wemmerlöv, 1984). Despite the challenges, Atan et al. (2017) indicates that over the last two decades more than 100 papers have been published about ATO systems. Their research provides a comprehensive overview of all kinds of ATO models for periodic and continuous review systems. For both review systems, they distinguish inventory systems for single and multiple periods including one or more end-products. They summarize and compare inventory models on replenishment policies, allocation policies, back order vs lost of sales principle and component lead times. On the other hand, the research performed on ATS systems is very limited.

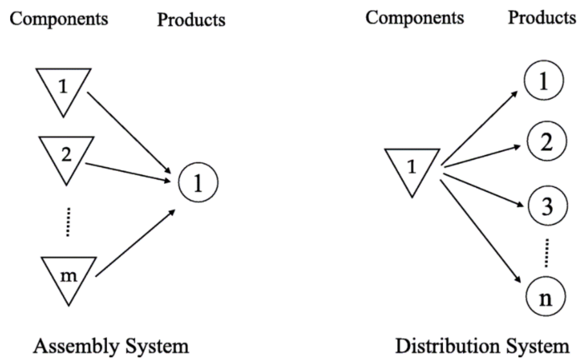


Figure 2.2: Assembly versus Distribution system (Song and Zipkin, 2003)

2.1.3 Multi-item inventory problems with shared supply chain processes

This section describes multi-item inventory systems with shared supply chain processes. A specific case of this kind of multi-item inventory system will be discussed. If a multi-item inventory system has multiple sets of different items that come from the same supplier, it is possible to replenish them jointly. This specific case is described as the Joint Replenishment Problem (JRP) by supply chain management literature (Cheung et al., 2016). The cost of placing an order at a supplier for a number of different products consists of two components: large ordering costs independent of the number of different products and minor ordering costs which does depend on the number of different products ordered (Khouja and Goyal, 2008). When you would replenish a set of items simultaneously, it can lead to high cost savings in the large ordering cost component (Poormoaied, 2021, Khouja and Goyal, 2008). When comparing joint replenishment with periodic or continuous base-stock policies the renewal reward theorem can't be utilized and therefore the fixed ordering cost component will increase significantly (Poormoaied and Atan, 2020, Poormoaied, 2021).

One of the most common types of joint replenishment policies for stochastic demand is the so-called 'can-order policy', or in mathematical terms (s_i, c_i, S_i) policy. According to Khouja and Goyal (2008), Balintfy (1964), this policy is very efficient for the JRP. This policy works as follows: if the inventory level of product i drops below the reorder point s_i, then a new order is placed to increase the inventory position to the level S_i. Subsequently, it is checked if other product's of the same supplier hit their can-order level c_i, as they can be ordered jointly. Moreover, a specific feature of an inventory system with a joint replenishment problem is when products are complementary and/or correlated. According to Poormoaied (2021), products are complementary if a change in one's product demand has immediately impact on the other product's demand. Additionally, if a stock-out situation occurs for either one of the product it can result in a completely lost sales situation (Kang and Gershwin, 2005). This can happen because if a customer demands the complementary products together and one of them is not available, they are not interested anymore (Thomopoulos et al., 2004, Kang and Gershwin, 2005). In the existing literature, there are a lot of papers that discuss different alternatives of joint replenishment policies for multi-item inventory systems with correlated complementary products. Multiple different articles can be found in the systematic literature study which was executed prior of this Master Thesis, only the most promising paper will be discussed.

The research of Feng et al. (2015) developed a policy which is an extension of the classical (s_i, c_i, S_i) can-order policy as it was orginally not designed to handle inventory systems with correlated demand arrivals. They designed a (s_i,c_i,d_i,S_i) policy which is more suited to minimize costs of multi-item inventory systems with correlated demand arrivals. The major difference is that the IP of products of the same supplier are not always increased to the order-up-to level S_i. The added variable d_i can be defined as a lower order up to level for a product that is included into the order based on it's can order value (c_i). They concluded that using this extended can-order policy instead of the traditional can-order policy leads to a decrease in holding and ordering costs. Figure 2.3 shows a visual representation between the basic can-order (s_i, c_i, S_i) and extended can-order (s_i,c_i,d_i,S_i) policies for a two-product system. It can be seen that for the (s_i, c_i, S_i) policy the inventory position is always increased to order-up-to level (S), while the (s_i,c_i,d_i,S_i) policy shows that the inventory level is increased somewhere between the minimal order up to level (d_i) and the maximal order up to level (S_i).

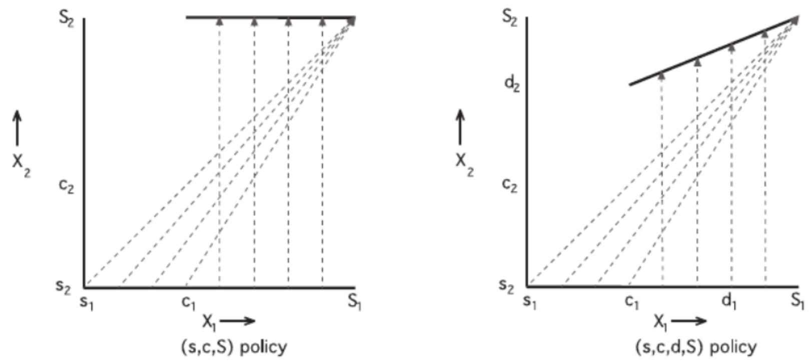


Figure 2.3: Visual comparison between (s,c,S) and (s,c,d,S) policy (Feng et al., 2015)

2.2 Introducing different selling strategies

This section will introduce different selling strategies of the bundling literature. Besides selling products individually of each other, it is a common practice that companies combine products into 'multiproduct packages' in order to increase their sales rates (Taleizadeh et al., 2020). According to (Ernst and Kouvelis, 1999), the main motivation behind this concept is to encourage product substitutions in stock out situations between the single-item products and multi-item products in order to increase customer satisfaction and enhanced profits. Similar conclusions about enhanced system profits are found by Yan and Bandyopadhyay (2011), McCardle et al. (2007) if product bundling was applied. The concept of combining several individual products into a combined end-product is classified under the general topic of 'bundling schemes' in the marketing and business strategy literature (Lilien, 1994). According to Bulut et al. (2009), there are three types of bundling strategies that can be applied in a business:

1. Pure bundling
2. Mixed Bundling
3. Unbundling

The first strategy 'pure bundling' is applied when a company only sell it's products in a bundled form. For example, a skating store can sell a bundle containing a helmet, small tools for repairs and protection pads for knees, shoulders and hands. The second strategy, 'mixed bundling', gives a company the opportunity to sell the products individually and thus independent of each other and in a combined bundle (Taleizadeh et al., 2020). The last strategy, unbundling, is a phenomena were the products are sold only independent and separate of each other. The implementation of either a pure or mixed bundling strategy for an organization brings potential benefits in terms of cost savings or increased revenue, but challenges can arise and the complexity of the supply chain increases (Bulut et al., 2009). For example, a company needs to specify how the bundles are composed in terms of size, which products to include, consideration of redesigning the supply chain, the prices of both the bundled products and separate independent products to maximize total profit.

Previous research on bundling in the marketing and economics literature revealed some interesting results on demand settings for which bundling strategy is most profitable. Jeuland (1984) conducted multiple experiments and found that some form of bundling is more profitable compared to a simple monopoly pricing of only selling products separately. Secondly, the paper of Schmalensee (1984) concluded that mixed bundling combined the benefits of both pure bundling and unbundling strategies and is the strategy that leads to the highest profit. Extensive literature can be found on important questions such as: how to determine the right pricing strategy or what the optimal number of items is to put into a bundle of products to maximize profits (Yue et al., 2006, Dassiou and Glycopantis, 2008, Bulut et al., 2009). However, for this research, it is more interesting to find the effects of using bundling strategies on the inventory decision making process and what inventory approaches and replenishment logic's can be used. According to the literature review in Bulut et al. (2009), the only paper that discusses this issue is the work of Ernst and Kouvelis (1999). Ernst and Kouvelis (1999) indicates that to the best of their knowledge, in the current literature, there has not been research on inventory cost-related benefits of mixed bundling practices or even a formal analysis on this topic in the presence of demand uncertainty. However, a long time has passed and recent studies of Taleizadeh et al. (2020), Qiang (2012) performed research on the mixed bundling principle and how it affects inventory decisions. More specifically, they both developed a two-product mixed bundling inventory model and determined optimal order quantities in combination with optimal prices to maximize profit of the system. To the best of our knowledge, this are the only papers in the current literature that describe how the mixed bundling strategy affects the inventory decision making process.

Within the mixed bundling principle there are some different alternatives to consider that can be applied for a multi-item inventory system. Firstly, the largest distinction within this principle, is the ability to use a full mixed bundling principle or a partial mixed bundling principle. The full mixed bundling principle prescribes that inventory systems must sell all the products separately of each other and combined in a bundle. The partial mixed bundling principle relaxes the restriction of selling all products individually of each other, as some does not need to be offered individually. Partial mixed bundling can also be referred as a tying strategy, as this terminology was used before the mixed bundling principle was introduced in the academic literature Bhargava (2014).

The second variation within the mixed bundling principle is the possibility to apply a non-sharing or sharing principle to handle the multi-item inventory problem. According to Qiang (2012), the non-sharing option is the most straightforward policy as inventories of the same product are not pooled and order quantities are determined separately for each product. On the other hand, the sharing principle enables to pool inventories together of the components to fulfill demands for all alternative products of the mixed bundle. Qiang (2012) states that the sharing policy, applied for their multi-item inventory problem, is a special case of the component commonality problem. The multi-item inventory problem they describe has two components and three end-products. In order to create a valid mixed bundling principle the products must follow the unique relationship showed in equation 2.1. The condition holds that the sum of prices of product p_1 and p_2 is greater than the price of the bundled product (p_{mb} and subsequently the price of the bundled product is greater than both individual prices of the separate components.

$$max(p_1, p_2) < p_{mb} < p_1 + p_2 \tag{2.1}$$

Lastly, bundling can further be distinguished between price or product bundling. According to Stremersch and Tellis (2002), price bundling can be described as a marketing practice by selling different products together without any physical integration. On the other hand, product bundling does involve physical integration between the products as bundles are created. Therefore, the product bundling principle requires additional manufacturing processes to create the bundles.

Ernst and Kouvelis (1999) developed a stylized model in order to analyze the nature of the optimal inventory decisions in an environment with two products. The two products are not direct substitutes of each other and are sold both independent of each other and as a bundle with one unit of each product. They showed that the usage of independent newsboy policies for both individual products and bundled product often leads to sub-optimal results regarding profitability. The newsboy policy tends to overstock the less profitable individual items and understock the bundled product. Taleizadeh et al. (2020) developed three different pricing-inventory models based on the three alternative bundling strategies. The models are build for two complementary items with correlation and the Economic Order Quantity (EOQ) model is used to determine the optimal order quantities and selling prices of the two products. The research of Qiang (2012) states that inventory decisions in the bundling literature are often not considered, because the market size is assumed to be fixed. Therefore, they tried to design an optimal mixed bundling strategy for a retailer with perishable products regarding pricing and inventory decisions under a stochastic market. To conclude on this research area, the papers of Taleizadeh et al. (2020), Qiang (2012) developed mathematical models which are designed for two products and one single bundled package. The objectives are maximizing profits by optimizing inventory and pricing decisions. This did not include the minimization of the total inventory costs and applying aggregate service constraints. It remains unknown what kind of inventory approaches and replenishment policies can be used to minimize inventory costs while achieving an aggregate system service level for multi-item inventory systems which apply the mixed bundling strategy.

Since both concepts of mixed bundling and ATO/ATS systems is explained, we shortly discuss the link between them. The bundling process of products can be seen as an assembly process according to (Qiang, 2012). Either a ATO or ATS principle could be applied to assemble the products into a bundle. An important difference between the ATO/ATS system and mixed bundling principle is the decision of where to stock inventories. The mixed bundling principle keeps inventory of products before and after the assembly process as the unbundled and bundled products can be sold separately of each other (Wemmerlöv, 1984). However, as discussed earlier, the ATO and ATS systems only store products before or after the assembly process. The mixed bundling principle uses both the characteristics of the ATO and ATS systems to fulfill demands of unbundled and bundled products.

2.3 Background information on aggregate service level constraints

The last section of this chapter discusses what types of service levels are available to measure the performance of an inventory system. After that, an evaluation is given between several definitions of the aggregate fill rate constraint applied in practice for multi-item inventory systems following an unbundling selling strategy.

2.3.1 Different type of service levels

A company can judge the performance of their inventory system by looking at the customer service levels. Targets of the service levels are set by management and drive the determination of safety stocks and thus the investments made on on-hand inventory (Teunter et al., 2017). The existing literature describes several different definitions of service levels which all evaluate the performance, but provide different insights. The most common service levels applied in practice and which are useful for this research are explained.

The fill rate is a popular service measure and is mostly used in companies, as it translate directly the achieved customer service level (Guijarro et al., 2012). This measure can be seen as a quantity-oriented performance measure, because the calculation is based on the expected amount of demand and back orders per period (Tempelmeier, 2000). In the literature, this service level is often indicated as the β-service level and can be defined as: *'the proportion of total demand that can be delivered immediately from stock on hand'* (Larsen and Thorstenson, 2008). The β-service level can be defined as:

$$\beta = 1 - \frac{E(Backorders\,per\,period)}{E(Demand\,per\,period)} \tag{2.2}$$

Two different type of fill rates are distinguished in the literature. Equation 2.2 describes the first option, which is the volume fill rate (VFR) (Silver et al., 1998). On the other hand, the order fill rate (OFR) (or line fill rate) calculates the fraction of orders that can be delivered directly from stock (Larsen and Thorstenson, 2008). This alternative provides insights in how many orders, irrespective of their size, can directly be delivered from on-hand stock. The second service level that can be used for analyzing inventory systems is the α-service-level.

This performance indicator is an event-oriented service measure, as it measures the probability that an arbitrary order can be fulfilled from stock on-hand (Tempelmeier, 2000). The inventory literature often refers this service measure as the Ready Rate (RR) (Larsen and Thorstenson, 2014, Alamri et al., 2017). The outcome of the ready rate can not directly translate the achieved customer service level, however in some situations the focus on complete order fulfillment can be very relevant (Hausman, 1969). The α-service-level can be defined as:

$$\alpha_c = P(\text{Period Demand} \leq \text{Physical Inventory}) \tag{2.3}$$

2.3.2 Definitions of the aggregate fill rate applied in practice

When applying an aggregate fill rate constraint for a set of SKU's, the main objective is to determine how to differentiate between the individual fill rates of the SKU's in order to meet an overall aggregate fill rate constraint that will reduces the total inventory costs. The system aggregate weighted fill rate is determined by the summation of the product between the individual fill rates and the corresponding weight of that fill rate. The weight given to an SKU indicates it's relative importance compared to the other SKU's. By using different definitions of those weights, a different definition of the aggregate system fill rate is created. The papers of Millstein et al. (2014), Silver et al. (2016), Teunter et al. (2017), Albrecht (2017) present different definitions of the weights that can be used for the system approach.

1. Generic weights
2. Volume based weights
3. Turnover based weights
4. Profit based weights

The first type, generic weights, is basically a single item approach as the target of the system's fill rate is set for all individual SKU's. Therefore, no service differentiation is created and according to Teunter et al. (2017) a loss in efficiency can be expected and the solution is far from optimal. Despite the inefficiency, this type of weight is applied a lot in practice as it basic and simple to implement and use (De Schrijver et al., 2013). The second weight is based on the average demand of SKU's and assigns higher weights to SKU's which have relative higher demands. The third type calculates weights based on the turnover value of the SKU's. The turnover of a SKU can be defined as the product between the demand and it's value. The fourth type calculates weights based on the profit margin and total demand of the SKU's. Higher weights are assigned to SKU's which generate higher profits. Several papers have performed empirical investigations on real-life data sets to investigate the effects of using the different weights for the aggregate system fill rate.

The research of Teunter et al. (2017) used three data sets with a large number of SKU's varying in fast to slow movers, expensive to inexpensive, order quantities and lead times. They found cost reductions between 10 and 50% by using the system approach instead of the single item approach. They used volume-based weights to define their aggregate fill rate constraint and imposed multiple minima for the individual SKU fill rate. Moreover, they showed that the system approach leads to higher cost savings compared to the ABC-classification approaches proposed by Teunter et al. (2010) and Zhang et al. (2001). The case study performed in the research of De Schrijver et al. (2013) concerned a wholesaler who supplied pharmacies of their orders. The wholesaler used the single item approach, but the ABC and system approach were applied to study the effects. The classification criterion to divide SKU's into classes was based on the turnover of an SKU. A reduction of 12% of inventory costs was realized and even a increase of 0.8% in the system fill rate was accomplished. In order to improve even more, a system approach was implemented. Besides the aggregate service constraint, the wholesaler faced a limitation in the available warehouse space. Despite using multiple aggregate constraints, an extra 5% reduction in inventory investment was realized while using the system approach. Lastly, the paper of Sherbrooke (2006) reported a huge saving of 46% on inventory investments for 1414 spare parts while keeping the availability of the spare parts constant. They used the marginal analysis technique (or 'greedy heuristic') to find the best solution. This technique raises the reorder levels for the SKU's in steps of one unit. For every step the SKU is selected with the highest ratio between the decrease in back orders divided by the increase in inventory. Once a SKU is selected, the reorder is increased by one unit. Subsequently, the ratio for all SKU's is calculated again and the highest ratio of an SKU selected. This iterative procedure is executed until the maximal budget for the inventories is reached.

The paper of Van Donselaar et al. (2021) found interesting results with their research regarding the different definitions of the aggregate fill rate. First of all, applying service differentiation with an aggregate constraint can reduce the costs of on-hand inventory up to 28.7% for volume-based weights and up to 9.0% for turnover-based weights. The impact of using one of those weights in order to determine the aggregate service constraint is large, but also different from each other. If a manager chooses to use volume-based weights, it can lead to low service levels for a major part of the assortment of a company. As a consequence, the inventory costs are also lower. In contrary to applying turnover-based weights, Van Donselaar et al. (2021) found that the inventory costs increase and less variation in the service level is achieved. The possible different impacts of both weights must studied by inventory managers before choosing how to define the aggregate service constraint in their company.

Chapter 3

Gap analysis

In this chapter, we start with introducing a specific methodology named the gap analysis. This methodology helps to define the gap between the present performance with the desired level of performance of a system. After that, it is explained what intermediate steps can be used in order to change the current inventory system to the desired inventory system. Thereafter, the current inventory system is analysed and evaluated based on the performance of the customer service level and total inventory costs. Subsequently, it is discussed how the intermediate steps of the gap analysis will be implemented for the inventory system. During this step, we also explain which data we use, and how this affects our research. In addition, this chapter answers the third sub-question of this research, which was defined as follows:

- **SQ.3:** *What is the current inventory strategy and performance of the multi-item inventory system of Van Geloven?*

3.1 The concept of a gap analysis

The gap analysis is a methodology which analysis the performance gap of a given system. The performance gap of a system can be defined as: the difference between the existing level of performance and the desired level of performance (Chevalier, 2010). The first step is to establish and analyze the present level of performance and what the desired or expected level of performance should be. The second step of the gap analysis is to determine ways how the systems could be improved to reach the desired performance. Figure 3.1 shows a visual representation of the gap analysis and how it is going to be applied for the inventory system of Van Geloven including intermediate steps. Intermediate steps are necessary to develop for this company, as the performance gap between the current inventory system and desired inventory system is too large to cover in one step. In addition, the unawareness among the employees of the inventory department is high regarding different ways to approach an inventory system including the potential benefits.

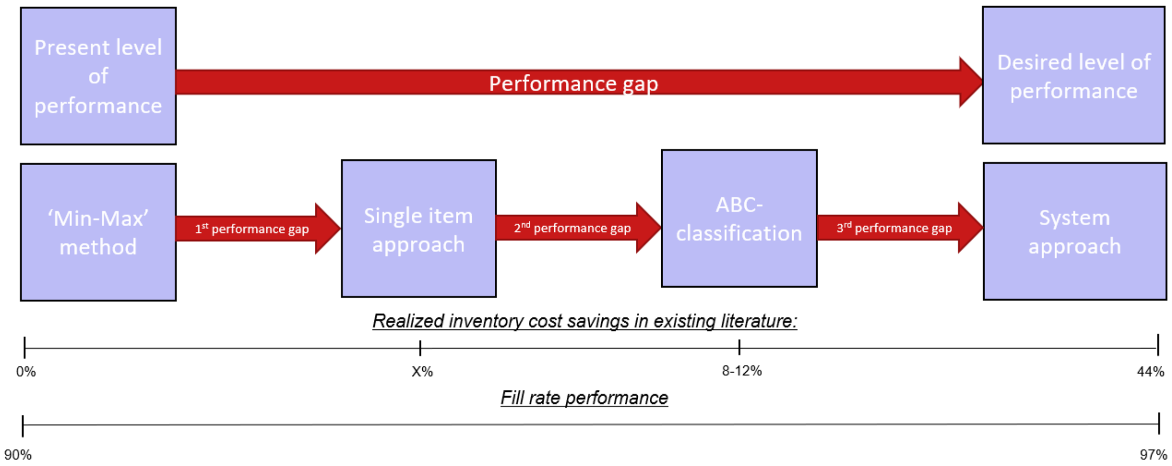


Figure 3.1: Visual representation of the gap analysis

3.1.1 Current inventory system vs. desired inventory system

The inventory management process of Van Geloven involves multiple stakeholders as they work together to achieve the predetermined service level of the system. Figure 3.2 shows a global representation of this process, how the different stakeholders work together and what the tasks are of each party. In the end, master planners are responsible to manage this process and are held accountable by senior management.

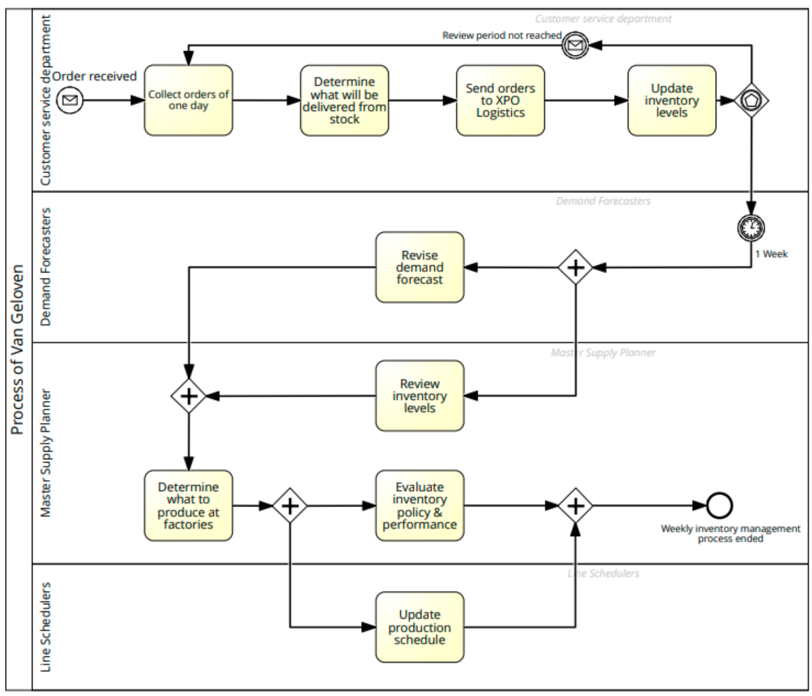


Figure 3.2: Global representation of inventory management process

The master planners are the most important member of this process, as they determine each week how much must be produced of each SKU to fulfill demand. They use an inventory methodology which can be classified as the 'min-max' method. Every week they review the inventory positions, which consist of the current inventory on-stock plus the amount of products in the pipeline, of all SKU's The unit of measure of the IP is average weekly coverage, meaning how many weeks of expected demand can be fulfilled with the current inventory position of that moment. In addition, the department sets an lower (min) and higher (max) bound of the average weekly coverage to evaluate if the current IP is too low or too high. So, the decision to create an order and the order size both depend on the IP (current weekly coverage) compared to the bounds. However, the lower and higher bound are set the same for all SKU's. All SKU's must have a weekly coverage interval between 6 and 10 weeks, different characteristics of the SKU's are not taken into account. To create a mathematical formula for this replenishment logic the following variables are introduced:

- $l_{i,t}$: lower bound of min-max coverage interval for SKU i on time t (weeks)
- $u_{i,t}$: upper bound of min-max coverage interval for SKU i on time t (weeks)
- $w_{i,t}$: current weekly coverage for SKU i on time t (IP, defined in weeks)
- O_i: order size for SKU i (units)
- Q_i: lot-size for SKU i (units)
- $\mu_{i,t}$: expected demand for SKU on time t i (units per week)

Compared to classical inventory literature notation, the $w_{i,t}$ and $l_{i,t}$ can be related to respectively the IP and reorder (s) since $w_i = \frac{IP}{\mu}$ & $l_i = \frac{s}{\mu}$. Equation 3.1 represents the current replenishment logic used by the inventory department of Van Geloven. If $w_{i,t}$ becomes smaller than $l_{i,t}$, an order (O_i) is created which is the minimal integer multiple of lot-size Q_i, to raise the w_i back to or above the lower bound ($l_{i,t}$) and lower than the upper bound ($u_{i,t}$) of the standard coverage interval.

$$O_i = \left\lceil \frac{(l_{i,t} - w_{i,t}) * \mu_{i,t}}{Q_i} \right\rceil * Q_i \qquad if \quad (w_{i,t} < l_{i,t}) \tag{3.1}$$

The inventory department adds the expected demand ($\mu_{i,t}$) to the equation, because otherwise the unit of measure in the fraction does not make sense. If you would remove $\mu_{i,t}$, then you would divide an number of weeks by the lot-size which is defined in units. By adding the expected demand per week, then you would divide units by units which leads to an logical order size. However, using this replenishment logic is wrong, since the expected demand changes constantly over time which leads to dynamic $l_{i,t}$, $u_{i,t}$ & $w_{i,t}$ during the potential delivery and review cycles. Secondly, it stands out that the upper bound of the average coverage interval is not even used by the equation. Before we elaborate further on this inventory policy, we will introduce two periodic inventory control policies.

The underlying principle of the current inventory policy of Van Geloven shows a lot of similarities with the (R,s,nQ) - and (R,s,S) periodic replenishment logic's, as most importantly the system is periodically. According to van Donselaar (2020) the first policy (R,s,nQ) works as follows: if during a review moment the IP drops to below reorder level (s), then an order n*Q is created to bring the IP above or equal to s. Secondly, an order during a review moment for the second policy (R,s,S) is created, if the IP drops to below s, then order up to the order-up-level (S). Since all lot-sizes are equal to the Minimal Order Quantity and Incremental Order Quantity (IOQ) and no upper order-up-to level is used, the (R,s,nQ) replenishment policy would be the best option to use for the inventory system for Van Geloven. Equation 3.2 shows how an order is created following the (R,s,nQ) periodic replenishment policy. All the variables of this equation have the same unit of measure, namely units.

$$O_i = \left\lceil \frac{s_i - IP_i}{Q_i} \right\rceil * Q_i \qquad if \quad (IP(i) < s_i) \tag{3.2}$$

Given the background information about correct periodic inventory control policies, they can be compared with the inventory policy used by Van Geloven. Although the inventory department mentions a higher bound (order-up-level S), they do not use it in the replenishment logic. Therefore, we will apply the (R,s,nQ) periodic replenishment policy during this research for improving the performance of the inventory system. The (R,s,nQ) periodic replenishment can also be dynamic since the reorder level (s) depends on the demand forecast made in period t for the demand for the next $R + L$ periods ($[t, t + R + L]$) and the safety stock. However, during that period the reorder will be static for the (R,s,nQ) policy which is the difference compared to the min-max policy used by Van Geloven.

Furthermore, the current inventory policy of Van Geloven does not consider two very important KPI's in any way. The fill rate and total inventory costs are completely neglected by the inventory managers. They do not have influence on the resulting costs or service levels and can only evaluate afterwards if too much or too little stock was set for a SKU. They do not have any information about the relation between the service level and the inventory costs of their SKU's. This is another important reason why the periodic (R,s,nQ) replenish logic could improve their inventory processes, as formula's can be derived which express the expected inventory on hand and the fill rate as a function of the reorder level. The inventory decision making process of the components at the local warehouse are not the responsibility of the master planners at the headquarters. Instead, the stock levels of the components are regulated by all the different factories independent of each other.

To summarize, the inventory department uses the same stock coverage interval as decision variable to determine all inventory decisions for each differently behaving SKU without using any formula's which relate to the service level and inventory costs. However, senior management evaluates the inventory system and the performance of the inventory department based on the realized system service level. The classical trade-off between achieving the highest possible customer service level and minimizing the total inventory costs is not investigated at all. Van Geloven does not have a clear vision of how they want to redesign their inventory system to accomplish the desired performance of the system. We do know that Van Geloven wants to minimize total inventory costs while achieving a predetermined aggregate system service level and involve the components to the inventory decision making process executed by the master planners. In order to achieve this strategic problem, a lot of tactical problems are encountered such as, when do we need to replenish, how much do we replenish and how to determine safety stocks and must be resolved.

3.1.2 Intermediate steps

The first intermediate step of the gap analysis is designed to focus especially on changing the replenishment policy of the current inventory system. The current min-max policy is basically a single item approach, since all SKU's are treated the same. Therefore, the (R,s,nQ) periodic replenishment policy will also follow the single item approach principle. The reorder level will be used as decision variable to determine all inventory decisions. Each SKU will still be treated individually and independent of each other, as the predetermined fill rate will be set for all SKU's. Based on the characteristics of the SKU's, such as demand average and standard deviation, different reorder levels will be calculated to achieve the fill rate target of each SKU. This is already a large difference compared to the current inventory policy of Van Geloven, as a new replenishment logic will be used: (R,s,nQ). We assume that this step will lead to significant improvements of the inventory system.

Once the replenishment logic of the multi-item inventory problem has been changed, the second intermediate step can be executed. In this step the single item approach will be relaxed and changed to the ABC-classification approach. This approach divides all SKU's into three classes and the classification of a SKU is based on the performance on a specific criterion (Teunter et al., 2010). Different fill rates are set for each class, but all SKU's within a class have the same fill rate. Since the fill rate is set equally for all SKU's within a class, they all receive the same 'treatment' in terms of the inventory decision making process. Compared to the single item approach, little service differentiation is now allowed. Secondly, if the same aggregate fill rate is applied the total inventory costs can be reduced between 8 and 12% (Teunter et al., 2017). On the other hand, additional complexity is created since it has to be determined how to divide the SKU's over the groups, what the ideal number of groups is and what the target fill rates of the groups must be to minimize the total inventory costs. How we deal with these issues during our research is discussed in chapter 4.

The final step of the gap analysis is to change the ABC-classification approach to the system approach. The system approach enables to set different fill rate targets for all SKU's which results in large service level differentiation. Compared to the ABC-classification, extra complexity is introduced as the number of classes will be equal to the number of SKU's and each class will have it's service level. According to the research of De Schrijver et al. (2013), Van Donselaar et al. (2021), Sherbrooke (2006), the system approach is able to reduce the inventory between 15 and 46%. The challenge lies in finding the optimal combination of service levels for which the inventory costs are minimized and the aggregate system service level is achieved. The next chapter, chapter 4, will explain in detail how the three proposed inventory approaches are going to be applied for Van Geloven.

3.2 Present performance of the inventory system

3.2.1 Data analysis

Before we can analyse the inventory system, we have to explain which data we are going to use and how it affects our research. Currently, Van Geloven does not work with a reliable information system where data is stored and retrieved by different departments. Departments all use different files and often they do not correspond with each other. Therefore, retrieving all the necessary data was a challenge and only the last two years could be used to analyse the current performance based on the realized costs and achieved service level. To provide additional insights for the inventory department, Appendix A compares the performance of the average weekly coverage realized by all presented approaches.

The current inventory system of Van Geloven can be defined as a mixed bundling multi-item inventory system. As discussed in chapter 2, there are three different bundling strategies and within each strategy several configurations are possible. Van Geloven has applied a partial mixed bundling selling strategy, since they offer end-products consisting of single and multiple component(s). The term partial is added, because not all components are sold individually as single-component end-products. Secondly, they apply a sharing principle instead of a non-sharing principle as inventories of components are shared partly. Thirdly, they use product bundling instead of price bundling, because components undergo physical integration and additional assembly manufacturing processes are needed.

Demand data cleaning

Before the data can be analysed it has to be cleaned from irregularities and outliers. Two main data sets are provided by Van Geloven, the first is an overview of all demand per week and the second data set is an overview of the weekly stock levels per week. The number of SKU's in both data sets did not correspond, so the first step is to only include SKU's which occur in both data sets. Secondly, we found that a lot of SKU's neither have any demand or products on stock for an entire year and are therefore removed. Lastly, a data cleaning action is needed to find the 'true' demand of the SKU's of each year. If a SKU is out of stock and an order arrives, the customer service department of Van Geloven contacts the client with the information that the requested item is not available and that they should order again in one or two weeks. This way of working comes close to the back-order principle, because orders which could not be fulfilled are not completely lost. However, instead of keeping track of the amount of undelivered demand and use this valuable information for inventory management, Van Geloven places the responsibility to order again at their clients. It often happens that a customer orders two or three times the same (or larger) quantity of a product in the advised time period when it is still unavailable. With the help of the information system and the customer service department, repeated order quantities in the advised period from the same customer are removed to find the 'true' demand of the SKU's. This policy of putting the responsibility of re-ordering at customers could lead to severe consequences. In the long term, they could lose customers or a possible bullwhip effect is created as clients of Van Geloven could be tempted to exaggerate their real needs when placing an order. This effect is called shortage gaming and according to (Lee et al., 1997) it is one of the four causes of the bullwhip effect.

Demand analysis

The next step is to find the demand distribution of the inventory system. This is an important step, because it affects the decisions that will need to be made for the inventory policies. It was not possible to find the 'true' demand data before 2019, so only the last three years are analyzed. Figure 3.3 shows the demand distribution of the last three years.

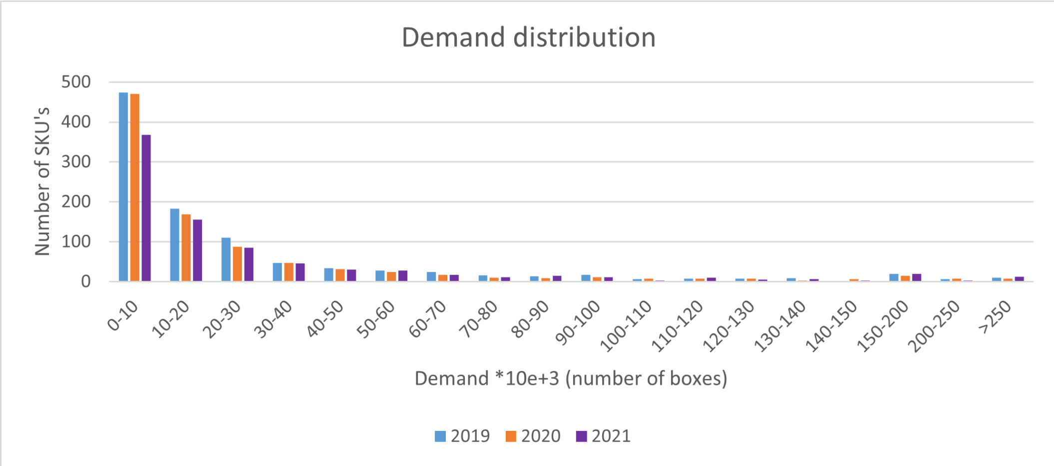


Figure 3.3: Demand distribution of the last years

Since the DoBr-tool will be used during this research one of the following distributions must be chosen: Normal, Discrete or Gamma. Based on Figure 3.3, it can be concluded that the demand clearly is not Normally distributed as it shows unsymmetrical bell curves and is skewed to the right (Robb and Silver, 1993, Tadikamalla, 1984). Additionally, based on the non-negative numbers, large amount of SKU's, huge spread of demand between SKU's and some very high demands a continuous distribution is a good approximation for the discrete data. Therefore, the Gamma distribution is selected for the demand data. Comparing the years with each other, it stands out that 2021 has fewer SKU's than the other years which is caused by the decrease of SKU's in the lowest demand interval.

Lastly, we will briefly show the effects of the Covid-19 pandemic on the demand. Table 3.1 shows the total demand for each year and distinguishes the demand for two markets. The influence of the pandemic is clearly visible, as the total demand has decreased with roughly four million (14,2%) between 2019 and 2020. This drop is explained by the decrease in the OOH market, as this market covers all companies which are not retailers and suffered from severe lock-downs during that year. The IH market, which covers all retailers, compensated the loss of the OOH only a little for both 2020 and 2021. For a small part of the SKU's of 2019 and 2020 it was unable to determine in which market they belonged, because a number of products were removed over the years and no data was available.

Table 3.1: Total demand comparison over the years

Year	*Total demand (boxes)*	*OOH market*	*% of total*	*IH market*	*% of total*	*Unknown*	*% of total*
2019	28.997.590	17.477.976	60.3%	10.729.108	37.0%	790.506	2.7%
2020	24.878.617	13.072.978	52.5%	11.732.678	47.2%	72.961	0.3%
2021	25.301.581	14.161.173	56.0%	11.140.411	44.0%	0	0%

2021 based on the first 49 weeks of the year

3.2.2 Current customer service level performance

The following table, Table 3.2, shows some KPI's of the inventory system. First of all, a difference can be seen between the total demand of tables 3.1 and 3.2. The total demand of Table 3.2 is lower, because the SKU's without information on the weekly stocking levels are excluded as the service level would be inaccurate.

Table 3.2: Fill rate and waste performance of recent years

Year	*Active SKU's*	*Total realized demand (boxes)*	*Total backorders (boxes)*	*Volume-based fill rate*	*Unweighted fill rate*	*Total waste (boxes)*
2020	920	22.407.772	1.351.711	93.97%	91.00%	107.595
2021	832	25.083.198	1.095.484	95.63%	93.17%	158.543

2020 based on the last 46 weeks of the year

2021 based on the first 49 weeks of the year

Total back orders during 2020 was a round 6% of total demand which was one % higher compared to 2021. Furthermore, two values for the system fill rate are given in this table, the volume based fill rate and the unweighted fill rate. The first definition is currently used at Van Geloven to judge the inventory system. Van Geloven calculates the volume-based fill rate of the system by dividing the delivered amount of products per period by the original requested demand per period. The unweighted fill rate scores lower in both years compared to the volume based fill rate which might indicate that multiple SKU's with small demands and low fill rates are present. Moreover, the higher realized fill rate during 2021 could be explained due to two factors. Firstly, management intentionally instructed to stock more inventory in order to realize a higher service level and keep the customer satisfaction high. Secondly, during 2021 there was less uncertainty as the pandemic had less influence and consequences on society. Lastly, Van Geloven performed well on the amount of waste during both years as it was less than 1% of the total realized demand.

A more in-depth analysis of the realized fill rate for both years is discussed to gain a more detailed insight on the current performance of the inventory system. Table 3.3 shows the distribution of the fill rate for all SKU's of the last two years. It can be seen that, a lot of SKU's during 2020 and 2021 have a very high fill rate as more than 500 SKU's have a fill rate above 99.9% and are responsible for respectively 50% and 70% of the total realized demand. Secondly, it is interesting to see that around 150 SKU's have a fill rate lower than 90% and are responsible for almost 14% of the total realized demand during 2020. This supports the previous assumption of the fill rate, regarding the difference between the volume-based and unweighted based average fill rate of the system. A lot of service differentiation is found among all SKU's, but most of the satisfied demand comes from SKU's with a very high fill rate. Compared to 2021, a change has happened as the amount of volume realized with a fill rate lower than 90% decreased and moved to the higher intervals.

Table 3.3: Distribution of the fill rate

Fill rate interval	*Spread of SKU's 2020*	*Demand volume 2020 (boxes)*	*% of total volume*	*Spread of SKU's 2021*	*Demand volume 2021 (boxes)*	*% of total volume*
<0,6	63	762.877	3,4%	44	891.078	3.6%
0,6-0,7	9	44.401	0,2%	10	57.168	0.2%
0,7-0,8	20	438.767	2,0%	15	85.530	0.3%
0,8-0,9	60	1.725.016	7,7%	38	356751	1.4%
0,9-0,95	100	2.968.387	13,2%	54	1.495.029	6.0%
0,95-0,99	164	5.906.557	26,4%	139	4.473.044	17.8%
>0,999	503	10.561.767	47,1%	532	17.724.598	70.7%

Customer service level performance of multi-component SKU's

As mentioned earlier, Van Geloven sells their products according to the mixed bundling principle which holds that a end-product can consist out a single or multiple components. The major part of the products which are sold consist only of one component. During this research we are very interested in the multi-component SKU's, so a brief analysis is performed on those SKU's. Table 3.4 shows the performance of the customer service level of the multi-component SKU's compared to the system. It can be seen that, for both years around 5% of the total SKU's consist of more than one component. Secondly, the multi-component SKU's realize a relative higher demand and fewer back orders compared to the whole system. As a consequence, both the volume-based fill rate and unweighted fill rate are increased compared to the system. Lastly, it is remarkable to see that, the amount of waste of the multi-component SKU's was very high compared to the total realized demand which could be a consequence of the higher realized fill rate.

Table 3.4: Performance of the fill rate for multi-component SKU's

Year	*Active SKU's*	*Total realized demand (boxes)*	*Total backorders (boxes)*	*Volume-based fill rate*	*Unweighted fill rate*	*Total waste (boxes)*
2020	920	22.407.772	1.351.711	93,97%	91,00%	107.595
2020	45	1.493.343	37.761	97,47%	94,11%	13.407
% of total	4,89%	6,66%	2,79%	-	-	12,46%
2021	840	25.083.198	1.136.847	95,47%	91,71%	158.543
2021	39	1.617.736	22.256	98,62%	94,24%	34.224
% of total	4,64%	6,45%	1,96%	-	-	21,59%

3.2.3 Current total inventory cost of the system

Since Van Geloven outsources logistic and warehouse operations, the costs of the supply chain consists of multiple components. First of all, Van Geloven pays the logistic service provider to transport the products from the factories to the central warehouse. If trucks arrive at the warehouse a fee is charged of €2,19 per pallet that enters the warehouse. This cost component is identified as the inbound cost of pallets. Thirdly, one of the largest cost component of the supply chain is the storage costs of the pallets at the central warehouse. The logistic service provider charges €1.77 for storing a pallet per week. Lastly, total value of the SKU's stocked at the central warehouse is relevant, since a percentage of that value is accounted as inventory carrying cost. Inventory carrying costs can be defined as the amount of interest a company loses out on the unsold stock value lying in the warehouse.

Table 3.5: Relevant inventory costs of supply chain

Year	*Active SKU's*	*Transport cost to warehouse*	*Inbound cost pallets*	*Total storage costs*	*Total stock value*	*Total waste value*
2020	920	€783.404	€294.960	€1.666.615	€168.791.899*	€686.343
2021	840	€948.915	€352.224	€1.977.294	€169.156.908	€947.823

*Including high starting inventories

Table 3.5 shows an increase of all cost comparing 2021 with 2020.This can be explained, since the realized sales and volume-based fill rate of the system increased. The total stock value is based on the amount of uniquely stored boxes during a year. The amount of unique boxes seems the same over the last two years, however a starting point had to be chosen for analyzing the inventory system. Therefore, the complete inventory of the first week of 2020 is considered as unique stock. This inventory account for almost 2.7 millions boxes which had a value of roughly 17.4 million euros. This indicates that the total stock value also increased during 2021. However, the total stock value also increased because the average value of a unit increased from €7,54 to €8,25. Additionally, the total waste costs did also increase during 2021 but is relatively low compared to the stock values of both years. Moreover, the total storage costs during 2021 of the multi-component SKU's was equal to €118.875 which is slightly over 6% of the total storage costs.

3.3 Implementing presented approaches

The presented approaches of the gap analysis will first be tested on the demand data set of 2021. In this way, the maximal potential of the approaches can be evaluated and compared to the current approach by Van Geloven. The following data cleaning actions are executed additionally on the demand data set of 2021 in order to remove outliers and irregularities.

- Ratio between total demand and lot-size < 1
- bp_i (number of units on a pallet) < 1
- Number of weeks with zero demand > 30
- Number of weeks with inventories > 49
- All SKU's ≠ MTS

Firstly, if the ratio between the total demand and lot-size is smaller than 1, it means that during a whole year (49 weeks) the product would be replenished only once. These kind of items are assumed to be MTO instead of MTS and are therefore not relevant for this research and are removed. Secondly, for a few SKU's it is unknown how many units are stacked on a pallet which makes it impossible to determine the inventory holding costs per unit and are therefore removed. Thirdly, SKU's which have more than 30 weeks of zero demand are removed as well. It is assumed that those products are also MTO. Due to an error in the information system, double inventory levels of some SKU's were stored for each week. It was not possible to determine which of the inventory level was the correct one, so these SKU's are removed. Finally, SKU's which are indicated by the information system as MTO products are removed. Table 3.6 shows a reduction of nearly 30% in number of SKU's. However, the removed SKU's are only responsible for roughly 5% of the total demand and inventory.

Table 3.6: Overview of demand-based selected data set

Data set	*Active SKU's*	*Total demand*	*Total inventory*	*Volume based fill rate*
Complete data set	832	25.083.198	165.772.442	95.6%
Partial data set	594	23.831.555	159.865.098	98.1%
Percentage	71.4%	95.0%	96.4%	-

After using the demand data to evaluate the maximum potential of the inventory approaches, the forecast data will be used as reorder levels and other inventory decisions have to be determined before demand is realized. The demand data set of 2021 will be cut in half and the first part will be used in combination with the forecast, made in week 28, to set the reorders levels and make other relevant inventory decisions for week 29 to 49. The actual performance of the second half of 2021 is known, so the performance of the inventory approaches can be compared to the performance of the current inventory policy. It is expected that the performance of the inventory approaches will relatively be lower since forecast data will be used. However, inventory decisions will be made before demand is realized which creates a more realistic view on possible improvements of the inventory system of Van Geloven.

Similar additional data cleaning actions are also performed on the forecast data set. Firstly, if the ratio between the total forecast demand and lot-size is smaller than one the SKU's are removed. Secondly, the same SKU's are removed with an unknown amount of units stocked on a pallet. The length of the forecast data set is 21 weeks, week 29 to week 49, and if more than 16 weeks have an expected demand of zero the SKU is removed. We assume that SKU's with this behaviour can be seen as MTO products. Thirdly, the SKU's indicated by the information system as MTO are removed from the data set. Finally, the remaining SKU's are checked with the demand data set and only the SKU's are selected which are in both data sets. Table 3.7 shows the results of the data cleaning actions. Again, nearly 30% of the total number of SKU's are removed from the data set, but the decrease in total forecast and realized demand is respectively 4% and 6% over a period of 21 weeks. The volume based fill rate increases with 2,3% to 98,2% for this period with the current selection.

- Ratio between total forecast and lot-size < 1

- $bp_i < 1$
- Number of weeks with zero expected demand > 16
- All SKU's $\neq$ MTS

Table 3.7: Overview of forecast-based selected data set

Data set	*Active SKU's*	*Total forecast*	*Realized demand*	*Volume based fill rate*
Complete data set	832	11.421.309	11.461.141	95.9%
Partial data set	596	10.993.232	10.742.840	98.2%
Percentage	71.6%	96.3%	93.7%	-

Empirical investigation of key variables

Since both data sets have been cleaned, an empirical investigation for some key variables will be executed. Two important key variables of the inventory system are fixed, the review period and lead time. The review period is equal to one for all SKU's, since the SKU's are periodically reviewed every period (week). The lead time for a SKU's depends on in which factories it is produced. For five of the six factories the lead time is four periods (weeks) and for the other factory it is five periods (weeks). The empirical analysis is performed on the following variables during week 29 and 49:

Key variables:

- $\mu_{i,D}$: average realized demand for SKU i
- $\mu_{i,F}$: average demand forecast for SKU i
- $\sigma_{i,D}$: standard deviation of realized demand for SKU i
- $\sigma_{i,F}$: standard deviation of demand forecast for SKU i
- h_i : inventory holding cost for SKU i
- bp_i : number of units placed on a pallet for SKU i
- Q_i : Lot-size for SKU i

Table 3.8 summarizes the descriptive statistics of the variables. First of all, the actual realized demand ($\mu_{i,D}$) is smaller for almost all quantiles compared to the demand forecast ($\mu_{i,F}$). This results in a lower average for the actual realized demand, but margins are relatively small. Based on the average demand it is concluded that the forecast performs quite well. Secondly, the standard deviation of the forecast ($\sigma_{i,F}$) is significantly lower for all quantiles compared to the realized standard deviation ($\sigma_{i,D}$). Moreover, the forecast data set reports 25 SKU's with a standard deviation of zero which does not occur for the demand data set. Based on this, it is concluded that the standard deviation created by the forecast is too low compared to the actual standard deviation. Furthermore, the holding cost (h_i) per

Table 3.8: Descriptive analysis on key variables

596 SKU's	$\mu_{i,D}$ *[units/week]*	$\mu_{i,F}$ *[units/week]*	$\sigma_{i,D}$ *[units/week]*	$\sigma_{i,F}$ *[units/week]*	h_i *[€]*	bp_i *[units]*	Q_i *[units]*
Min	24,1	75,1	20,6	0,0	0,17	40,0	120
25%	199,2	229,6	143,8	67,2	0,42	144,0	656
Median	415,6	464,2	255,6	159,9	0,49	192,0	1050
75%	1039,7	1032,0	520,5	398,6	0,62	225,0	1600
Max	9023,0	9623,0	4830,9	4733,1	3,14	420,0	21.384
Average	858,3	922,3	440,5	390,5	0,58	189,9	1.728
StDev	-	-	-	-	0,34	62,1	2.107

unit and number of units placed on a pallet (bp_i) show very little differentiation as average is close to the maximum value. Lastly, the average lot-size for a SKU has one very high value. The lot-size of 21 thousand units is correct, since the factory line produces this product between two to three straight days. Overall, it is concluded that most of the key variables show little differentiation and it is assumed that an estimate of the forecast standard deviation ($\sigma_{i,F}$) is needed to reduce the gap to the realized standard deviation $\sigma_{i,D}$.

The book of Silver et al. (2016) proposes a method to estimate the standard deviation for this kind of situations. They state that, several forecast accuracy's measures can be used to estimate the standard deviation. In their work, they show a relationship between the Mean Squared Error (MSE), Root Mean Square Error (RMSE) and the standard deviation which can be seen in equation 3.3.

$$\hat{\sigma} \approx \sqrt{MSE} = RMSE \tag{3.3}$$

Based on this relationship, we can determine another estimation for the standard deviation of the forecast by using the concept of power laws. A power law is a statistical relationship between two quantities, since a relative change in one of the quantities (x) creates a relative change in the other quantity (y). This interaction between two quantities is independent of the initial size of both of them, or in other words a quantity varies as a power of the other. The general form of power law can be described as:

$$y(x) = kx^n \tag{3.4}$$

The standard deviation of any Gamma distributed data can be related to the mean of that same Gamma distribution, as the shape (α) and scale (β) parameters are used to calculate both of them. The following relation holds for a Gamma distribution with respect of the mean and standard deviation:

- $\mu = \frac{\alpha}{\beta}$
- $\sigma = \frac{\alpha}{\beta^2}$

It is suspected that, the standard deviation and mean are related to each other by a power law relationship, but the values for k and n are unknown. Additionally, as we found earlier, the standard deviation of the forecast is too low and can not be used. However, based on the work of Silver et al. (2016), the standard deviation of a SKU can be estimated by the average RMSE of that SKU. It is possible to calculate the average RMSE per SKU, as Van Geloven stores all their individually made forecasts week by week. All individual forecast data sets from week 1 to week 28 are put in a single data set with the corresponding realized demand per week per SKU. Since the RMSE is equal to the $\sqrt{MSE}$, the RMSE for each SKU can be calculated by using equation 3.5.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2} \tag{3.5}$$

The MSE can be explained as the mean of the squares of errors where n is the number of sample data points, Y(i) is the observed value of the variable being predicted and $Y\hat{(i)}$ is the predicted value. Using equation 3.5, the RMSE can be calculated for each individual week for all SKU's.

The general idea is to convert the power law relation into a linear relationship between the RMSE and the mean. Subsequently, the slope and intercept of the linear relationship can be determined by using Ordinary Least Squares (OLS) regression. This can be used to determine the unknown values of k and n of the power law relation showed in equation 3.4. After the power relationship is established, any value for the μ_F (x) would result in the relative corresponding value of the RMSE (y). After that, the value of the RMSE can be used as estimator of the standard deviation, which will be denoted by $\hat{\sigma}_{i,pl}$.

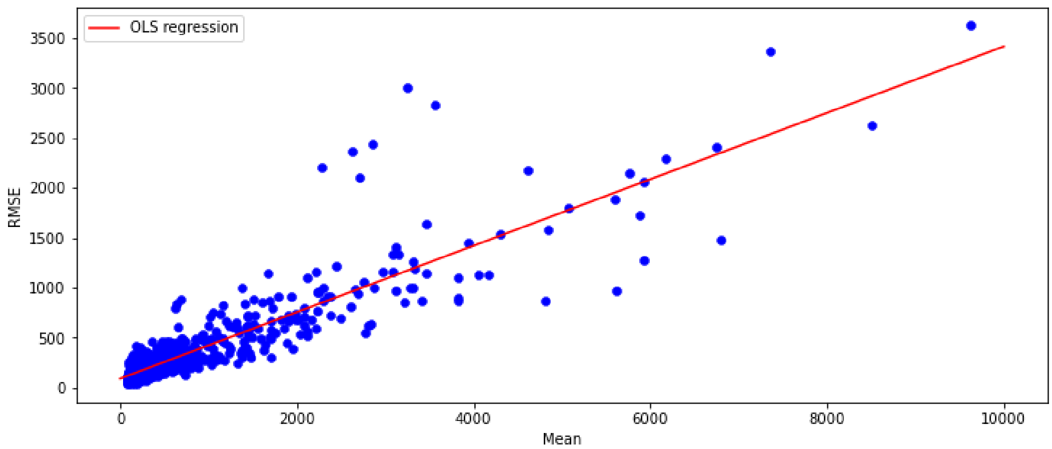


Figure 3.4: Relationship between RMSE and mean

Figure 3.4 shows the relationship between the mean and the average RMSE for all SKU's. Despite the fact that the relationship already looks linear we take the natural logarithm of both variables, because then it is converted into a linear relationship whose slope and intercept can be related to the unknown values of n and k. The values of x and y of equation 3.4 are replaced for the natural logarithms of the mean and RMSE to establish equation 3.6. This relationship is also plotted to see if indeed a linear relationship can be found between the log-transformed variables. The properties of logarithms allows us to rewrite the equation into a linear relationship, which is generally in the form of y(x) = a + bx

$$Ln(RMSE(\mu)) = Ln(k * \mu^n) = Ln(k) + n * Ln(x) \tag{3.6}$$

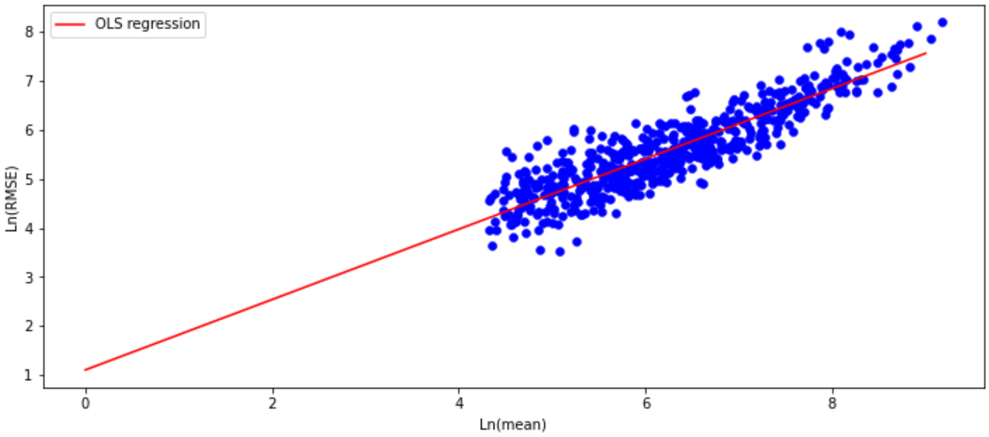


Figure 3.5: Relationship between Ln(RMSE) and Ln(mean)

Figure 3.5 shows a clear linear relationship between the two variables, the OLS regression method is used to find the slope and intercept of this linear relation. The intercept is equal to 1.1052 with a corresponding slope of 0.7173. Based on Figure 3.6, the intercept of the linear relationship equals Ln(k), which makes k equal to $e^{1,1052}$ and n being 0.7173. The following power relation between the RMSE and mean can be defined:

$$RMSE(\mu) = e^{1,1052} * \mu^{0.7173} \tag{3.7}$$

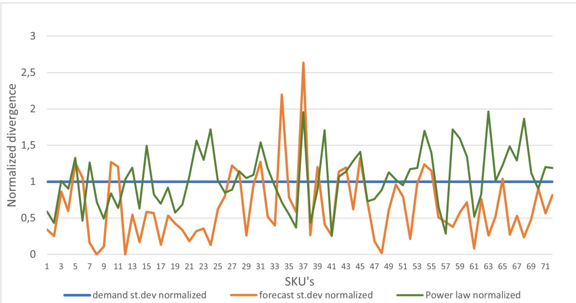
Using equation 3.7, standard deviation of each SKU can be estimated. Table 3.9 shows the resulting descriptive statistics of the actual standard deviation, the standard deviation based on the forecast, and the estimate of the standard deviation based on the power law. Additionally, the Mean Absolute Percentage Error (MAPE) is calculated for both the $\sigma_{i,F}$ and $\hat{\sigma}_{i,pl}$ compared to the $\sigma_{i,D}$. This measure indicates the accuracy of both standard deviations, the standard formula is shown by equation 3.8 where A_i is the actual value and F_i is the forecast or predicted value.

Table 3.9: Statistical analysis on estimators of the standard deviation

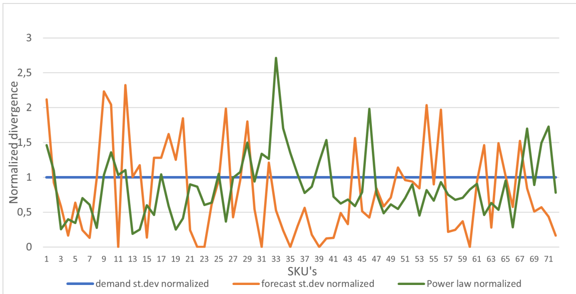
Type of standard deviation	*N*	*mean*	*0%*	*25%*	*50%*	*75%*	*100%*	*MAPE*
$\sigma_{i,D}$	596	440,5	20,6	143,8	255,6	520,5	4803,9	-
$\sigma_{i,F}$	596	390,5	0,0	67,2	159,9	398,6	4733,1	50,1%
$\hat{\sigma}_{i,pl}$	596	357,8	66,9	149,1	247,1	438,2	2173,7	36,9%

Table 3.9 indicates that $\sigma_{i,F}$ performs worse compared to $\hat{\sigma}_{i,pl}$, as it scores lower on most of the quantiles and has a larger score on the MAPE. On the other hand, the value for the

maximum standard deviation and average is more accurate for the $\sigma_{i,F}$ compared to $\hat{\sigma}_{i,pl}$. However, the maximum value for the$\sigma_{i,F}$ has probably a high influence on the average. The $\hat{\sigma}_{i,pl}$ gives a better performance for at least the first 75% of the SKU's. Figure 3.6 provides extra insights on the performance of the both standard deviations compared to the actual standard deviation. For both Figure 3.6a and 3.6b, 70 random SKU's are selected of the data set to review the performance of both standard deviations compared to the actual standard deviation. All three standard deviation, $\sigma_{i,D}$, $\sigma_{i,F}$ and $\hat{\sigma}_{i,pl}$, are normalized by dividing by the $\sigma_{i,D}$. This creates an straight line (blue) for the $\sigma_{i,D}$, since the value is equal to one for all SKU's. Both figures indicate that $\hat{\sigma}_{i,pl}$ (green line) moves closer around the blue line compared to the $\sigma_{i,F}$ (orange line). It is assumed that, this finding holds for the whole data set. Only 140 SKU's are plotted in Figure 3.6, because otherwise the figures would be unreadable and the difference between the standard deviations would be harder to notice. Appendix B shows the performance of the single item approach while using all three standard deviations.



(a) Random sample A



(b) Random sample B

Figure 3.6: Comparing the different standard deviations

$$MAPE = \frac{100\%}{n} * \sum_{i=1}^{n} \left| \frac{A_i - F_i}{A_i} \right| \tag{3.8}$$

Chapter 4

Inventory methodologies

The fourth chapter, which is the third phase of this research, discusses in detail how the presented inventory approaches of the gap analysis are implemented at Van Geloven to improve the current inventory system. Firstly, this chapter zooms in on the inventory dynamics of stocking items for a single-echelon periodic review system with lot-sizing, back ordering. Thereafter, it is discussed how the different inventory approaches are implemented and tested for the inventory system of Van Geloven. During this analysis, the aim is to minimize the total inventory costs of the SKU's stored at the central warehouse. After this, a small subset of the whole inventory system will be considered as we include the stocking levels of the components at the local warehouse. To reduce the complexity only the single item approach will be used and a smaller inventory system is selected. The aim of this analysis is researching the effects of the mixed bundling principle on the multi-item inventory system. During this analysis, we also study the effects of pooling inventories of the same component. In addition, this chapter provides an answer to the fourth sub-question of this research, which was defined as follows:

- **SQ.4:** *What kind of inventory model could be developed that incorporates the mixed bundling principle and is able to minimize total inventory costs for a multi-item inventory problem*

The following assumptions are used for all presented inventory approaches throughout this research:

- Demand substitution is not allowed between SKU's
- The requested order quantity can always be manufactured by the factories
- The frequency of ordering can be neglected

4.1 System with independent items

Before the inventory approaches are discussed in detail, we will introduce the main KPI's for evaluating the performances of the approaches compared to the currently applied approach. The achieved customer service level and the total inventory costs will be used to judge the performance. The fill rate will be used as service measure for determining the customer service level, because this measure is already applied at Van Geloven and it is able to directly translate the achieved customer service level of an inventory system (Guijarro et al., 2012). The total inventory costs of Van Geloven consists of three components, the storage costs charged by the external warehouse, the interest costs of the stock value and the potential risk costs for having waste. The storage costs is equal to €1.78 per pallet per week. The interest costs can be defined as a percentage times the average system stock value. The possible risk costs of wasting inventories will not be included in this research, as the yearly waste is below 1% of the total demand and is therefore assumed to be negligible. Lastly, the average weekly coverage of individual SKU's and of the entire inventory system is calculated. This is currently a very important KPI to the inventory department. The results for this KPI can be found in Appendix A. The following notation is introduced to define formula's for the important KPI's:

Main KPI's:

- FR_S : realized aggregate system fill rate of the system
- C_S : total inventory cost per period of the system

Variables:

- N : set of all active SKU's of the system
- ΔBO_i : back orders per time period for SKU i
- D_i : total demand per period for SKU i
- v_i : value per unit for SKU i
- sc_i : storage cost (per unit per period) for SKU i
- h_i : holding costs (per unit per period) for SKU i
- r : fixed interest rate
- $IOH_i(av)$: average inventory on-hand (per unit per period) for SKU i
- μ_i : average demand for SKU i
- s_i : reorder level for SKU i
- bp_i : number of units placed on a pallet for SKU i

Using the above notation, the formula's for the main KPI's can be defined. Equation 4.1 calculates the aggregate system fill rate, as the expected back orders per period are divided by the expected demand per period for all SKU's. The system fill rate is based on the relative volume of each SKU instead of taking the unweighted average of all individual fill rates. The second equation, equation 4.2, calculates the expected total inventory costs per period of the inventory system. The inventory cost of a SKU (h_i) consists of the storage costs (sc_i) and a percentage of the inventory value. The value of a SKU (v_i) is multiplied with a fixed interest rate (r). So, the inventory holding cost per period for SKU i is calculated by: $h_i = sc_i$ + r * v_i.

$$FR_S = 1 - (\frac{\sum_{i=1}^{N} E[\Delta BO_i]}{\sum_{i=1}^{N} D_i} \tag{4.1}$$

$$C_S = \sum_{i=1}^{N} h_i * E[IOH_i(av)]) \tag{4.2}$$

Figure 4.1 shows a snap shot of the current partial mixed bundling multi-item inventory system, including some relevant locations of the supply chain. The figure shows a small fraction of the inventory system, as three components are used to manufacturer four SKU's. Components, indicated by c_i, can be produced in one of the six factories of Van Geloven. A single component can be transported to the central warehouse where it is stocked and becomes a single-component SKU i. The component can also be transported to the local warehouse where it is stored and awaits it's assembly process. After the assembly process, the resulting end-product will be described as a multi-component SKU i and it transported to the central warehouse. The inventory department at the headquarters determines order quantities for both single- and multi-component SKU's based on the stocking levels of the SKU's at the central warehouse. Figure 4.1 also clearly indicates the inventory department does not evaluate the stocking levels of the components stored at the local warehouse. As mentioned earlier, this is regulated decentralized by the different factories. The approaches proposed by the gap analysis will be used to minimize the inventory costs for the central warehouse. The partial mixed bundling principle is confirmed, as the middle component is solely produced for multi-component SKU's and is not sold individually.

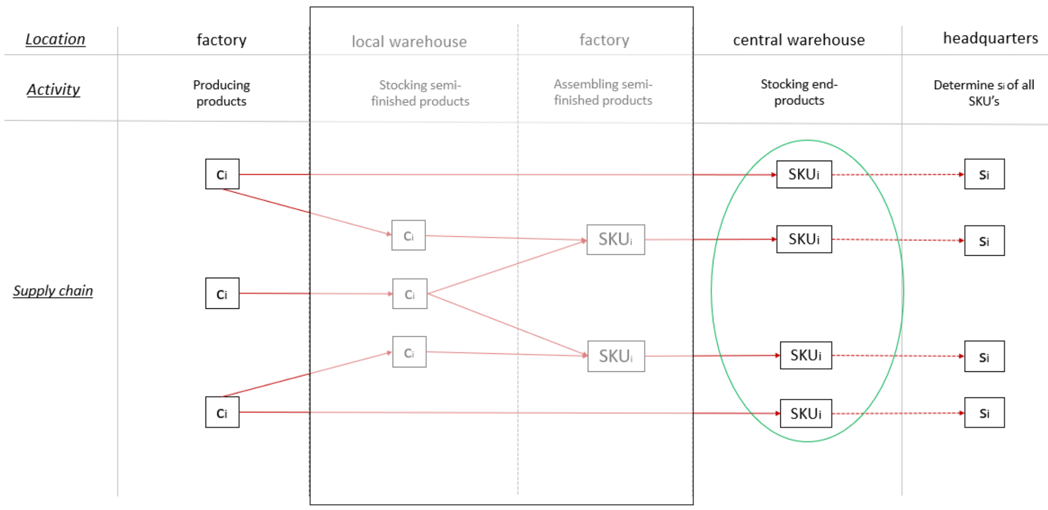


Figure 4.1: Snap shot of inventory system with independent items

The current design of the supply chain makes it impossible to share inventories in case of stock-out situations. For example, if the most upper SKU i is out of stock and demand is not completely fulfilled, then the inventory of it's relative component stored at the local warehouse can not be used. A new order is placed at the factory which can take either four or five weeks to arrive at the central warehouse. Furthermore, inventory decisions for the SKU's at the central warehouse are made independent and separately of each other. Despite the fact that they can consist out of the same component(s). So, the factory which produces the upper component can either receive an order to deliver the component to the local or central warehouse. A component stored at the local warehouse is packaged in large plastic bags with large volumes while the same component stored at the central warehouse is packaged into relatively small paper boxes which are ready for transport. The present inventory policy of Van Geloven determines inventory decisions, for both single-component and multi-component SKU's, based on the on-hand inventory level at the central warehouse plus possible outstanding order. Therefore, the proposed inventory approaches and analysis of this section will focus on the last two steps of the supply chain. By doing so, we try to optimize the mixed bundling multi-item inventory system by determining optimal reorder levels for each SKU using different inventory approaches. The physical integration of different components among the SKU's is neglected during this analysis, because around 90% of the total demand of Van Geloven is realized by SKU's which consist of a single component. The second part of this chapter does consider the stocking points of the components of the local warehouse.

4.1.1 Single item approach

The first step of the gap analysis is to change the current 'min-max' inventory policy of Van Geloven to a correct replenishment inventory control policy with periodic review, see Figure 3.1. The most important reason for this intermediate step is to change their incorrect replenishment policy and show how a reorder level affects the fill rate and total inventory costs. It is not the objective of this step to minimize the inventory costs as much as possible, but rather to change the replenishment logic for determining inventory decision.

To change this, a different replenishment logic will be applied where the reorder level is the decision variable. Both the fill rate and inventory on hand are a function of the reorder level. In this way, the inventory department could set different reorder levels for their SKU's in order to reach a 98% fill rate for each SKU. Besides the periodic reviewing of once per week, there are some more relevant inventory control parameters of the system. The lead time of the orders which is the time to manufacture products and transport them to the central warehouse. Secondly, all products have a Minimum Order Quantity (MOQ) or lot-size and if more needs to be ordered an integer number times the MOQ can to be ordered. Due to the design of the factory lines it only possible to order an integer number of the MOQ which means the MOQ is equal to the Incremental Order Quantity (IOQ). Given these parameters, a good inventory replenishment control strategy would be the RsnQ-policy which was already briefly discussed in chapter 3. This replenishment control policy is widely used despite the higher complexity compared to other policies (De Schrijver et al., 2013). Figure 4.2 shows the sample path of the (R,s,NQ) policy to illustrate it's functioning. In this figure, it can be seen that if the inventory position is below the reorder level an order is created which is the minimal integer multiple number of lot-size (Q) to raise the inventory position back to or above the reorder level. The inventory position of a SKU is equal to the inventory on-hand plus the outstanding orders minus the possible outstanding back orders. In addition, if an order is placed at a review moment it is delivered at t+L periods. The next review moment is t + R periods which is every three weeks, or in other words the review cycle. The other indicated period [t+L, t+R+L] is the potential delivery cycle, as it is not always the case that an order is generated as it could be possible that at a review moment the inventory position is still above the reorder point. For this reason, the word potential is added to this term and this happens at the second review moment of the sample path (at time [t+R]). The research of Van Donselaar et al. (2021), Donselaar, van and Broekmeulen

(2014) explains this principle and discusses in detail what definitions and expressions are used to calculate important KPI's of an inventory system. Based on these expressions, the authors have developed a tool to analyze all kinds of inventory problems encountered in practice. The DoBr-tool is a collection of Python modules capable of handling inventory systems with different demand distributions while calculating exact results for important KPI's for the (R,s,nQ), (R,s,S) and (R,s,S,nQ) inventory policies.

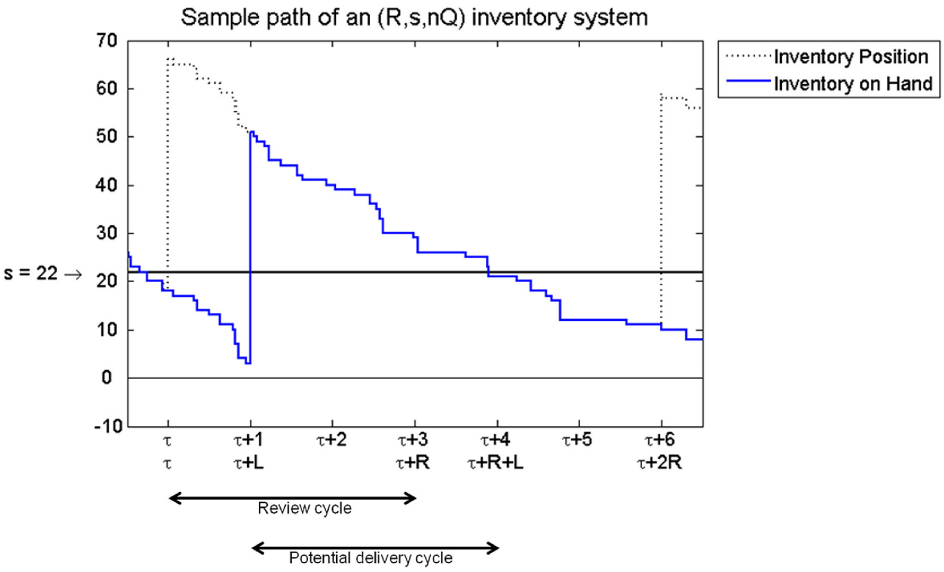


Figure 4.2: Example of an (R,s,nQ) inventory policy of a SKU with parameters R=3, L=1, μ=7.5, s=22, Q=48 (Van Donselaar et al., 2021)

The DoBr tool is able to handle Gamma distributed demand and applying the (R,s,nQ) policy, which is why this tool is used to optimize the inventory system of Van Geloven. The following KPI's can be calculated for each SKU by the DoBr-tool and are also relevant for our problem:

- P_2 : Fill rate
- $E[BO]$: Expected back orders
- $E[IOH]$: Expected inventory on-hand
- $E[OL]$: Expected number of order lines per review period

However, the next conditions must be satisfied in order to calculate exact values for the KPI's, stationary demand, deterministic lead-times, non-perishable goods and back ordering. All of those conditions are valid for the inventory system of Van Geloven, except for the non-perishable one. In chapter 2 is discussed that there is a possibility to choose a special case of the (R,s,nQ) policy, namely the EWA policy. This policy is specially designed for perishable products and can lead to significant cost reductions. The products that are manufactured by Van Geloven are officially perishable goods, however the product life of almost all products varies between 12 and 18 months as they are all frozen products. Because of this long product life and the small review cycle, the EWA-policy would not be very effective (Broekmeulen and Van Donselaar, 2009). In addition, the amount of waste is not really an issue for Van Geloven as it was not very high during the last two years. This can be explained, as products can still be sold with discount if they 'expire'. The customers of Van Geloven demands that 2/3 of the product life is left when the products are delivered. If this margin is not met, the products can often be sold for a reduced price. Based on these reasons, it is assumed the products fall under the non-perishable category which makes it possible to use the (R,s,nQ) policy and DoBr0-tool . The following overview introduces notation of the variables which are a direct input for the DoBr-tool in order to calculate the relevant KPI's:

Decision variable

- s_i : reorder level for SKU i

Input variables for the DoBr-tool

- μ_i : average demand per period for SKU i
- σ_i : standard deviation of the demand per period for SKU i
- L_i : lead-time for SKU i
- R_i : review period of SKU i
- Q_i : lot-size for SKU i
- $p^*_{2,i}$: fill rate target for SKU i

Other variables

- t : arbitrary review moment
- $P_{2,i}$: realized fill rate for SKU i
- $X_i^+(t+L)$: net stock right after a potential delivery moment for SKU i
- $X_i^+(t+R+L)$: net stock right before a potential delivery moment for SKU i
- $X_i^-(t+L)$: outstanding back orders after a potential delivery moment for SKU i
- $X_i^-(t+R+L)$: outstanding back right before a potential delivery moment for SKU i

In order to calculate the average inventory on-hand of a period, two variants of one function of the DoBr-tool must be used. The function to calculate the expected inventory on-hand can return both the expected inventory on-hand at the beginning and at the end of a potential delivery cycle. These specific moments of the potential delivery cycle are interesting, because at the start of the interval (t_i, L_i) the inventory on-hand is at its highest and at the end of the interval $(t_i + R_i + L_i)$ the inventory on-hand is at its lowest. According to Van Donselaar et al. (2021), van Donselaar (2020), the average inventory on-hand at an arbitrary point in time can be approximated by taking the average of the inventory on-hand at its lowest and highest possible moment. The formula which can be used to calculate the average is shown in equation 4.3. Similarly, the work of van Donselaar (2020) derived equation 4.4 to calculate the expected number of back orders during a period.

$$E[IOH_i(av)] \approx \frac{(E[X_i^+(t+L)] + E[X_i^+(t+L+])}{2} \tag{4.3}$$

$$E[\Delta BO_i] = E[X_i^-(t+L+R)] - E[X_i^-(t+L)] \tag{4.4}$$

4.1.2 ABC-classification

The next step of the gap analysis is the ABC-classification, as discussed in chapter 3. In order to keep the complexity of this approach low, the number of classes is limited to three. Since the target fill rate of the whole inventory system is known, we have to search for the optimal combination of fill rate targets for the different classes to minimize the total inventory costs. Almost all notation introduced for the single item approach can be used here, except the fill rate target for each individual SKU ($P^*_{2,i}$). This variable is expanded to three variables, as three classes are used. Besides the extra notation, two additional constraints are introduced as the target fill rate of the first group (A) must be greater or equal to the second group (B) which must subsequently be greater or equal to the target fill rate of group C. Using the extra notation an objective function with corresponding constraints can be derived and is denoted by problem (S).

Additional variables - ABC-approach:

- $P^*_{2,A}$: fill rate target for SKU's in group A
- $P^*_{2,B}$: fill rate target for SKU's in group B
- $P^*_{2,C}$: fill rate target for SKU's in group C
- $P^*_{2,L}$: target lower bound for the system aggregate fill rate

$$\begin{aligned} Min \sum_{i=1}^{N} h_i * E[IOH_i(av)] \\ \text{s.t.}: P^*_{2,A} \geq P^*_{2,B} \geq P^*_{2,C} \\ FR_S \geq P^*_{2,L} \end{aligned} \tag{S}$$

The first step of the ABC-approach is to appoint the right number of SKU's into the classes. The traditional demand based criteria will be used to classify the SKU's, because it reduces the inventory costs compared to the other traditional criteria (Teunter et al., 2010). Figure 4.3, shows the ranked SKU's with the cumulative % of total demand. The general rule of Pareto, which says that the top 20% of the SKU's account for roughly 80% of the total demand, does not hold for Van Geloven. The top 20% of the SKU only account for around 63% of the total demand. However, the last 50% of ranked SKU's account for around 10% of the total demand. Based on this, the classification used in the paper of Chu et al. (2008) will be used in this research as well. This means that the top 20% of SKU's will be put into class A, the next 30% of SKU's into class B and the remaining 50% will be in class C.

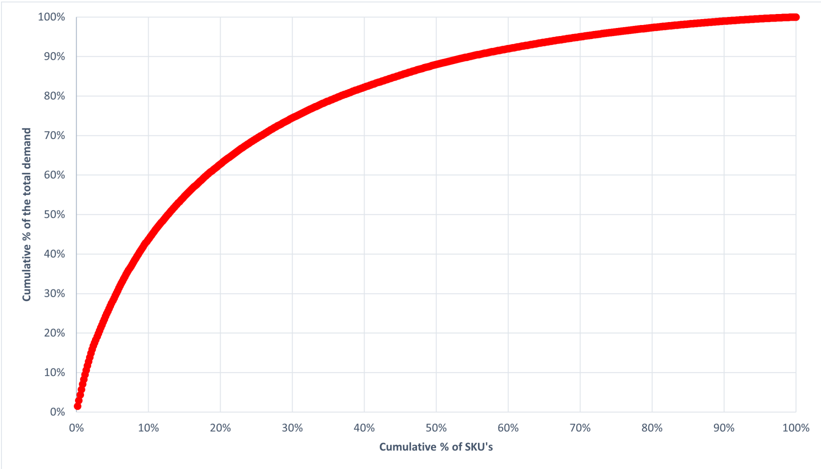


Figure 4.3: Distribution of total demand per SKU

The second step of the ABC-approach is to find the optimal fill rates for group A, B and C that minimizes the total inventory costs of the system. In order to get a feasible solution, we start with the lowest possible fill rate for group A and the highest possible fill rate for group B and C while satisfying both constraints. The fill rate has a natural interval between [0,100] percent, however testing all possible combinations for the three groups would be very time consuming and probably most of the possibilities would not even be feasible. The highest possible fill rate for groups B and C is set to 99.0%, as it is assumed that realizing a higher fill rate would lead to high unnecessary inventory costs. The target fill rate of inventory systems is often above 95%, thus we assume it is unlikely to realize this target with a fill rate for group A below 60%. With Python three for-loops are designed that iterate over this interval to find all possible combinations of fill rates for the three groups with respect to the two constraints. However, since the gap between 60% and 99% is quite large, the step size during the simulation is 1% as a smaller step size would result in a large running times as more combinations of fill rates for the classes will be tested. Subsequently, the DoBr tool is used during these for loops to calculate the resulting reorder levels and KPI's to evaluate the performance. Once a new lower bound for class A is found, the simulation is executed again with a smaller step size. This procedure is done multiple times until a step size of 0.01% can be used to find the optimal combination of fill rates for problem (S).

4.1.3 System approach

The final step of the gap analysis is the system approach and is the most complex of the presented approaches of the gap analysis, as each SKU's gets an individual fill rate assigned. By doing so, the cheaper and larger SKU's receive a higher fill rate since that results in larger safety stock levels as it relatively more convenient to stock more of those SKU's compared to more expensive and slow moving SKU's. This approach is already applied in numerous of papers resulting in significant decreases of inventory costs while attaining the same aggregate fill rate of the system compared to the other approaches of the gap analysis (Teunter et al., 2017, Sherbrooke, 2006, Van Donselaar et al., 2021, Thonemann et al., 2002). The additional notation stated below is introduced for the system approach. Compared to the ABC-approach the objective function remains the same, but the restrictions are changed, showed by problem (P). Volume-based weights are applied and the weight for a SKU is calculated by: $w_i = \frac{\mu_i}{\sum_{i=1}^{N} \mu_i}$. The last constraint of problem P indicates that the fill rate of an SKU i must at least be equal to the lower bound fill rate. Based on interviews with the inventory department the lower bound is set to 90%, as they would not prefer of having a lower fill rate.

Additional notation - system approach:

- ϵ : stepsize for increasing the reorder level (one unit)
- X_i^+ : Inventory on hand at an arbitrary moment in time for SKU i
- l_{p2} : Lower bound fill rate for all SKU's

$$\begin{aligned}
Min \sum_{i=1}^{N} h_i * E[IOH_i(av)] \\
\text{s.t.} \sum_{i=1}^{N} w_i * P_{2,i} \geq P_2^* \\
\sum_{i=1}^{N} w_i = 1 \\
P_{2,i} \geq l_{p2}
\end{aligned} \tag{S}$$

The book of Sherbrooke (2006) is highlighted as they aim to maximize the system availability for assembled products subject to a budget constraint. Sherbrooke uses the marginal analysis technique, which is already explained in chapter 2, to find the best solution. During this research a generalization of this marginal analysis technique is used to minimize the total inventory costs subject to an aggregate service constraint. The marginal analysis technique will be used as follows, first the reorder levels for all SKU's are set equal to the value that will meet the lower bound fill rate imposed on that SKU. After that, the reorder levels of the SKU's will be increased one by one in steps of one unit. During every step, the reorder level of the SKU with the highest ratio $\frac{w_i(P_{2,i}(s+\epsilon))-P_{2,i}(s)}{h_i(E[X_i^+(s+\epsilon)]-E[X_i^+(s)]}$) is increased with step size ϵ. This procedure is repeated until the target of the aggregate fill rate is reached. With the help of the DoBr-tool the relevant KPI's can be calculated. The results of all three approaches will be discussed in the next chapter.

4.2 Systems with a network of items

In this section the inventory system of Van Geloven will be converted to a smaller subsystem to investigate the effects of the mixed bundling principle on the inventory dynamics. The stock levels of the components at the local warehouse will be included in order to incorporate the physical integration of components of the SKU's. Two different models are presented to study the mixed bundling principle, but before those are introduced the main KPI's are revised and additional variables are introduced for the components.

Main KPI's:

- FR_{sub} : realized system aggregate fill rate of the subsystem
- C_{sub} : total inventory cost per period of the subsystem

Additional variables - subsystem:

- W : subset of the set N
- R : set of components of the subsystem
- ΔBO_j : back orders per time period for component j
- D_j : total demand per period for component j
- h_j : holding costs (per unit per period) for component j
- $IOH_j(av)$: average inventory on-hand (per unit per period) for component j
- $P_{2,j}$: realized fill rate for component j

The formula for calculating the fill rate of the subsystem does not change a lot compared to determining the fill rate of the complete system, as showed by equation 4.5. The formula for calculating the total inventory costs for the subsystem changes as an additional term is added. The components stored at the local warehouse must be taken into account as shown by equation 4.6. The subset R indicates which SKU's are present in the subsystem and R is part of the set N.

$$FR_{sub} = 1 - (\frac{\sum_{i=1}^{W} [\Delta BO_i]}{\sum_{i=1}^{W} D_i}) \tag{4.5}$$

$$C_{sub} = \sum_{i=1}^{W} h_i * E[IOH_i(av)] + \sum_{j=1}^{R} h_j * E[IOH_j(av)] \tag{4.6}$$

Furthermore, in this section we discuss two different configurations of the inventory system given the selected subset of SKU's and components. The first configuration is based on the current supply chain and inventory system of Van Geloven. However, we expand the inventory decision making process as the stock points of the component at the local warehouse are included. In addition, both subsystems use the characteristic of the ATO principle for the assembly process. The current inventory system of Van Geloven uses a hybrid form of both ATO and ATS principles, as both components and SKU's are stored at different locations. The second configuration of the subsystem redesigns the supply chain, as the local and central warehouse are combined into one location. For both subsystems it is assumed that the frequency of ordering can be neglected. In addition, as the complexity of both subsystems increases, only the single item approach will be used to investigate the effects of the mixed bundling principle. Again, the DoBr-tool will be used to calculate relevant KPI's for the components and SKU's. Therefore additional input variables for the DoBr-tool are introduced and to clarify the notation between components and SKU's. The notation used in the previous section for the complete system can still be used for the SKU's, however a

minor addition is made. The index i is slightly changed to indicated if an SKU consist of one or more components. The index i can be an element of of {1,2,b1,b2}, since we only consider two single-component and two multi-components SKU's in both subsystems.
Decision variable:

- s_j : reorder level for component j

Input variables for the DoBr-tool:

- μ_j : average demand per period for component j
- σ_j : standard deviation of the demand per period for component j
- L_j : lead-time for component j
- R_j : review period of component j
- Q_j : lot-size for component j
- $P^*_{2,j}$: fill rate target for component j

Other variables:

- $X^+_j(t+L)$: net stock right after a potential delivery moment for SKU j
- $X^+_j(t+R+L)$: net stock right before a potential delivery moment for SKU j
- $\lambda_{i,j}$: percentile contribution rate of component j to produce one unit of SKU i
- $\rho_{i,j}$: correlation value between SKU i and another SKU j (where $i \neq i$)
- $P_{2,j}$: realized fill rate for component j
- $P_{lower,i}$: lower bound fill rate for an assembled multi-component SKU i
- $P_{upper,i}$: upper bound fill rate for an assembled multi-component SKU i

4.2.1 Subsystem with multiple stock locations

The first configuration of the subsystem represents the current design of the supply chain and inventory system of Van Geloven. Figure 4.4 shows a visual representation of this subsystem. The major difference enabled by the subsystem is the extension of the inventory decision making process, as the stock levels of the components at the local warehouse are included. Furthermore, the visualization shows that the ATO principle is used for the assembly process. The components are stocked at the central warehouse and are assembled if customer orders arrive. The varying index of the SKU's can be noted, as this indicates which SKU's consists of one or more components.

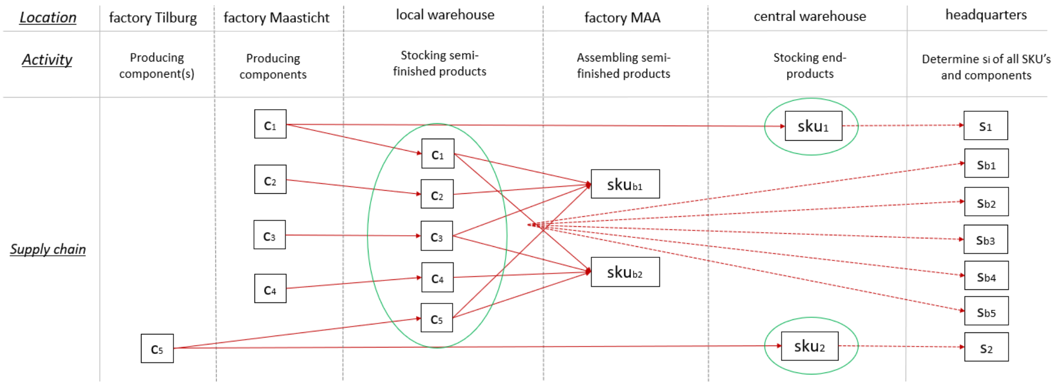


Figure 4.4: Visualization of the subsystem with multiple stock locations

Unfortunately, a lot of important information on the individual components was not available since the inventory department does not include the components in their inventory decision making process. The operations at the local warehouse are controlled by all factories independently of each other (decentralized). The factories did have some data about the transportation numbers from their facility to the local warehouse, but it was very incomplete for 2021. Therefore, the results of this subsystem can not be compared with the actual performance of the SKU's and component of the presented subsystem. Nevertheless, given the literature gap about mixed bundling, we are more interested in the positive and negative effects of the mixed bundling principle on the individual fill rates, system fill rate and total inventory costs. In addition, the following assumptions are made for some key variables as they are needed for the DoBr-tool and calculating the resulting inventory costs:

- The review period is one week
- The lot-size of $c_{\{2,3,4\}}$ are estimated based on similar components manufactured on the same production line
- The value per unit of $c_{\{2,3,4\}}$ is estimated based on the value of similar products
- The holding cost of the components at the local warehouse is based on the holding costs of storing inventory at the central warehouse
- The amount of units on a single pallet for all components is estimated based on the amount of kilograms that is stored on a pallet at the central warehouse
- The capacity of the packaging lines at the local warehouse is considered to be ample

The presented subsystem is an extension of the inventory systems used in the research of Qiang (2012), Ernst and Kouvelis (1999), because they consider a full mixed bundling system with two components and three end-products (SKU's). In addition, the subsystem of this research considers two multi-components SKU's instead of one. Since additional complexity is created by including more stoking points and using the partial mixed bundling principle, the single-component SKU's are treated independent and separately. The green circles indicate where reorder levels are determined and inventories are being kept. The dotted red lines are solely information flows from the local warehouse to the headquarters with information about the stock levels of the components. As mentioned earlier, the assembly process of the components executed according to the ATO principle. In practice, it could be allowed to store some multi-component SKU's at the central warehouse. This may only happen if something goes wrong during the assembly process. However, we assume that no inventories are being kept for those SKU's at the central resulting in zero inventory costs. Finally, only the single item approach will be used combined with the (R,s,nQ) replenishment logic to minimize the inventory costs.

In order to determine the reorder levels of the components the average demand (μ_j) and standard deviation (σ_j) must be determined. These values can be calculated based on the values of the demand and standard deviation of both multi-component SKU's. Table 4.1 shows which components are assembled into which SKU's. Secondly, the relative amount needed of each component to assemble a multi-component SKU is showed. For example, producing one kilogram of SKU_{b1} then you need 0.2195 kilograms of component c_i.

Table 4.1: Distribution of components to the multi-product SKU's

SKU	*Consists of components*	$\lambda_{i,j}$	*Value of multi-component SKU*	*Total weight of one unit (kg)*
b1	{1,2,3,5}	{21.95, 21.95, 31.71, 24.39} %	€7,04	2,9
b2	{1,2,4,5}	{21.95, 31.71, 21.95, 24.39} %	€7,06	2,8

Equation 4.7 shows how to calculate the demand per period for each component j. Determining the standard deviation of the components is not so simple as they can not be added together. Equation 4.7 resulted in a series of data points representing the demand per period of each component for it's assembled multi-component SKU. The standard deviation of two data series can not be added together since the covariance between those series has to be taken into account. Equation 4.8 shows the general formula for calculating the variance of the sum of two (random) variables including the covariance of two variables. It is showed that the correlation between two variables, in our case the demand between two multi-component SKU's, influences the covariance which has an effect on the variance of [X+Y]. The standard deviation of [X,Y] can be found by taking the root of the variance [X,Y]. According to Ernst and Kouvelis (1999), Qiang (2012), the correlation between end-products (SKU's) is very important to consider when a mixed bundling multi-item inventory system is used. Therefore, we will investigate what the effects are of using different correlations between the two bundled multi-component SKU's on the fill rates of the components.

$$D_j = \lambda_{b1,j} * D_{b1} + \lambda_{b2,j} * D_{b2} \tag{4.7}$$

$$\begin{gathered} VAR[X+Y] = VAR[X] + VAR[Y] + 2COV[X,Y] \\ COV[X,Y] = CORREL(X,Y) * (\sigma_x * \sigma_y) \end{gathered} \tag{4.8}$$

Allocation rule

In a valid mixed bundling environment, the condition explained by equation 2.1 must hold. This condition holds that selling one unit of the multi-component SKU is more profitable than selling one unit of either one of the single-component SKU's, but it will be less profitable than selling one unit of both single-component SKU's. The following allocation rule, based on the work of Qiang (2012), is applied for the subsystem to maximize profits and minimize possible left-over inventories of the components:

1. If the inventories of components {1,2,3,4,5} is more than the sum of demands requested by both bundles, then all requested multi-component SKU's can be assembled and profits are maximized. To increase the utility of the assembly manufacturing lines, it is advised to assemble all demand at once for a multi-component SKU. This minimizes possible set-up times and cleaning time.

2. If the available stock of at least one of the components is lower than it's requested demand for both multi-component SKU's, then the available stock will be allocated according to the equal fair share allocation scheme.

The fair share allocation method allocates a quantity of a component to a SKU based on the ratio between the total demand of that particular SKU compared to the total demand of all SKU's which need that specific component. The research of Agrawal and Cohen (2001) provides a detailed analysis on this allocation methodology. The authors conclude that in general this policy is not optimal, but it is intuitive and simple to implement. However, given our specific sub-inventory system the fair share allocation can be close to optimal. Table 4.1 shows that both multi-component SKU's are very similar in terms of value and weight per box. We assume that the profit margin of both SKU's is quite the same. Unfortunately, Van Geloven did not share the selling price and profit margin of both SKU's, as this is considered sensitive information. Given the fact that, selling either one of the multi-component SKU's results in the same profit, it does not matter which one is produced if the inventory is not sufficient to satisfy all demand.

Lastly, we will discuss the determination of the fill rate for the multi-component SKU's, as the DoBr-tool is not able to calculate this KPI exactly. The research of Song and Zipkin (2003) concludes that the determination of the fill rate of an ATO-system is often executed by complex evaluations methods which need a lot of computational time. Numerous papers present simple approximations in order to estimate the service level of an ATO-system (Hoen et al., 2011, Song, 1998, Iravani et al., 2004, Lu et al., 2003). During this research we will use the approximations developed by Song (1998) to estimate a lower and upper bound of the service level for the multi-component SKU's. Their inventory environment is similar to ours, since they consider a multi-component multi-endproduct inventory system with back-ordering. All components are controlled by an independent base-stock policy and the lead times are assumed to be constant. They used the following equations for determining a lower and upper bound for the fill rate:

$$
\begin{aligned}
P_{lower,i} &= \prod_{j=1}^{R} P_{2,j} \\
P_{upper,i} &= \bigwedge_{j=1}^{R} P_{2,j}
\end{aligned}
\tag{4.9}
$$

The lower bound of the fill rate is equal to the product of all component fill rates present in the assembled endproduct (Mamer and Smith, 1988, Song, 1998). They state that the fill rate of an assembled product can not be lower than this lower bound. On the other hand, these papers indicated that the upper bound of the fill rate is equal to the lowest component fill rate of all components present in the assembled the endproduct. The fill rate of the sub-inventory system is also estimated by using this principle, as some fill rates are known exactly (single-component SKU's) and some fill rates only have a lower and higher bound. This leads to a lower and upper bound of the subsystem fill rate which describe the worst and best case scenario of the subsystem fill rate. This last step is executed to compare and evaluate both subsystems to each other.

4.2.2 Subsystem with a single stock location

The second sub-inventory system considered in this research is visualized by Figure 4.5. This subsystem only stores the components at a single location and does not allow storage of the SKU's products. It is assumed that demand of single-component SKU's can be fulfilled immediately from stock without additional required manufacturing or packaging processes. The multi-components SKU's are assembled based on the ATO principle and the assembly process is executed at the central warehouse.

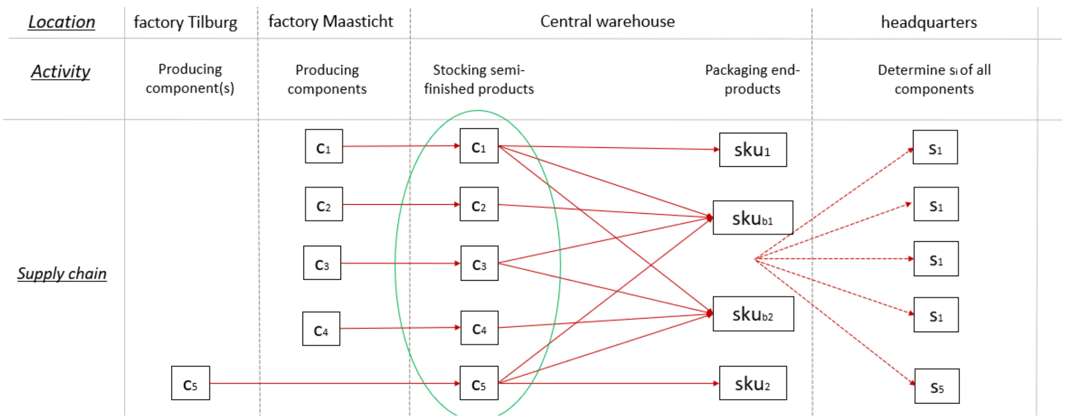


Figure 4.5: Visualization of the subsystem with a single stock location

Again, the green circle indicates at what place in the supply chain reorder levels are determined. The dotted lines from the warehouse to the headquarters contains information about the current stock levels and other relevant information. This subsystem is created to study the effects of the mixed bundling principle if inventories of the same components are shared. It is expected that this subsystem will outperform the other subsystem, as it benefits more of the inventory pooling effect. Equation 4.6 can again be used to calculate the total inventory costs per period, but one term is equal to zero as only inventories are kept of the component. Moreover, the following assumptions are made for this sub-inventory system:

- The lot-size of $c_{2,3,4}$ are estimated based on similar products which are manufactured on the same production line
- The value per unit of $c_{2,3,4}$ is estimated based on the value of similar products

- The holding cost per unit per time period is at the central warehouse is equal for all components
- The amount of kilograms put on a single pallet is assumed to be equal for all components
- The capacity of the packaging lines at the central warehouse is considered to be ample
- The packaging design of the inventories of the components are all the same

In order to determine the reorder levels of the components, the demand and standard deviation must be calculated based on the relative demand of the SKU's. The methodology explained for the previous subsystem is used again, but it must be expanded for components c_1 and c_5 to include the demand of the single-component SKU. This creates three demand flows to component c_1 and therefore equation 4.8 can not be used. However, it is extended to determine the standard variation of three random variables which is shown by equation 4.10.

$$\begin{aligned} VAR[X1+X2+X3] &= VAR[X1]+VAR[X2]+VAR[X3]+2COV[X1,X2]+ \\ &\quad 2COV[X1,X3]+2COV[X2,X3] \\ COV[X1,X2] &= CORREL(X1,X2)*(\sigma_{x1}*\sigma_{x2}) \end{aligned} \tag{4.10}$$

The allocation rule applied in this subsystem is also the equal fair share allocation method. It is expected that using this allocation technique does not result in the highest profit, because the single- and multi-component SKU's have different profit margins. However, is does provide a feasible solution and allows us to compared both subsystems with each other. At the end of this section, a new allocation method is proposed.

The determination of the fill rate for the assembled SKU's is again calculated by using equation 4.9. In addition, the fill rate of the subsystem is done in the same manner, a lower bound (worst-case) and upper bound (best-case) are determined. Also, for this subsystem the principle of the single item approach is used combined with the (R,s,nQ) replenishment logic. Comparing second subsystem to the first subsystem it has five relevant correlations between SKU's instead of only one. Figure 4.6 shows which correlation are relevant between the SKU's. During the analysis we investigate the global effects on the individual component fill rates, the lower and upper bound fill rates for the SKU's and on the total inventory costs. During the discussion of the results, the correlations indicated at the left-hand side in Figure 4.6 receive an abbreviation (indicated by the right-hand side) to create a clear overviews in following figures in chapter 5. Firstly, it is investigated what the effects are on the realized component fill rates ($P^*_{2,j}$) if all correlations are equal to zero. Subsequently, we investigate what the effects are on $P^*_{2,j}$ if only the first correlation $\rho\{1\}$ is moderately to strongly ($\rho = 0.7$) positive. After that, we keep adding positive correlations one by one to get an understanding of the effects on the inventory system. After this analysis, the same steps are executed but with moderately to strongly negative correlations.

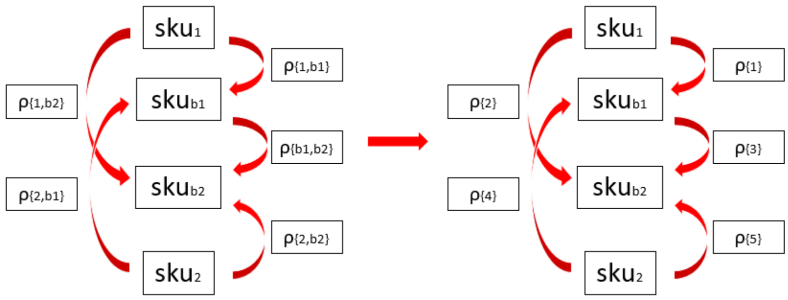


Figure 4.6: Relevant demand correlations in subsystem with a single stock location

The allocation rule developed in the research of Qiang (2012) is extended in order to find an allocation rule for our larger partial mixed bundling inventory system. Further research is however necessary for testing it on other partial mixed bundling systems. The general idea is based on the valid condition of the mixed bundling principle. Selling two single-item components is more profitable than selling one bundled endproduct and selling one bundled endproduct is better than selling one single-item product. Additional analysis is needed to compare this allocation rule to the fair share allocation rule, but that can be very complex and time consuming if the partial mixed bundling is large. As the number of end-products increases, the number of instances of different stock-out situations increases exponentially which are all needed to calculate the expected profit. The steps of the allocation rule are:

1. This step considers components which are destined for both single- and multi-item SKU's. Allocate the component with the highest difference between its demand and inventory to the single-item SKU, while the remaining inventory goes to the most profitable bundle and then to the next profitable bundle. Then, allocate the next component with the highest difference between demand and inventory to the bundles (highest profit margin first). Possible remaining stock can satisfy demand of the single-item SKU.

2. This step considers only components which have demand for at least two or more bundled SKU's. The demand is allocated to the bundle with the highest profit margin and then to the other bundle(s).

3. The next allocation steps considers the remaining components which are only used for one bundled SKU. Since no decision has to be made, it is a very straightforward allocation.

4. An additional check must be performed on the allocated components to the bundles. If components are left-over, then it must be checked if any demand of the single-item SKU of that component can be fulfilled additionally.

Chapter 5

Results

In this chapter, the results of the inventory approaches in chapter 4 are discussed and compared with the current performance of the inventory system of Van Geloven. The service levels and total inventory costs are successively compared. In addition, this chapter provides an answer to the last sub-question, which was stated as follows:

- **SQ.5:** *What are the effects on inventory decisions when the mixed bundling principle is implemented for a multi-item inventory system*

5.1 Inventory system with independent items

This paragraph discusses the results of the inventory approaches introduced by the gap analysis for the entire inventory system to optimize the stocking levels at the central warehouse. First, the true demand data is used to show the maximal potential of the approaches. After that, the results of the same approaches are discussed when forecast data is used. In addition, the results of several sensitive analysis are explained.

5.1.1 Inventory approaches based on demand data

The results in this subsection are based on the demand based data set of 594 SKU's from week 1 to 49 of 2021.

Customer service level

Table 5.1 shows the results of the fill rate of all three inventory approaches. The system volume based fill rate of the current inventory policy is equal to 98.1%, which means that the other approaches must realize the same system fill rate. The current inventory policy realized a zero percent fill rate for several SKU's, but more than half of the SKU's have a fill rate of 100% which probably leads unnecessary inventory costs. The standard deviation of this approach is relative high compared to the other approaches, but this can be explained due to the few very low fill rates. The service differentiation after the 25% quantile is almost equal to zero. The single item approaches realizes the same fill rate for each SKU, resulting in a volume based average of 98,1%. As expected, this approach does not have any service level differentiation. The following fill rates, [0.966, 0.980, 0.984], are found respectively for classes A, B and C of the ABC-approach which minimized the lowest total inventory costs and achieved a system fill rate of 98.1%. Compared to the single item approach, the standard deviation is higher because some service differentiation is allowed between the SKU's. Lastly, the system approaches is able to differentiate more between the service levels and realize the same volume based system fill rate. In addition, it can be seen that the lower bound of 90% is indeed realized for at least one SKU.

Table 5.1: Service level performance of all approaches - demand data set w1-w49

Inventory policy	FR_S	*St.dev*	*0%*	*25%*	*50%*	*75%*	*100%*
Current	98.1%	0,09	0%	99,0%	100,0%	100,0%	100,0%
Single item	98.1%	-	98,1%	98,1%	98,1%	98,1%	98,1%
ABC-classification	98.1%	0,008	96,6%	96,6%	97,3%	98,0%	98,4%
System - Sherbrooke	98,1%	0,02	90,0%	96,6%	98,0 %	98,8%	99,6%

Total inventory costs

Table 5.2 shows the expected inventory costs of all approaches. Compared to the current inventory approach, the system approach realizes the largest cost saving of almost 40%. Despite it's simplicity, the single item approach also realizes a significant cost saving compared to the current situation. As expected, the cost saving of the ABC-approach higher than the single item approach but lower than the system approach. However, the relative cost saving of the ABC-approach and system approach compared to the single item approach is lower than expected. Regarding the system approach, this could be explained by the allowed lower bound of individual fill rates (90%) for the SKU's.

Table 5.2: Total inventory costs comparison - demand data set w1-w49

Inventory policy	*Total inventory costs*	*% cost improvement to current policy*	*% cost improvement to single item approach*
Current	€1.748.358	-	-
Single item	€1.195.562	31,62 %	-
ABC-classification	€1.177.467	32,65 %	1,51%
System - Sherbrooke	€1.066.743	39,00 %	10,78 %

Sensitivity analysis

A sensitivity analysis is performed on the allowed lower bound of individual fill rates of the system approach. Table 5.3 shows the results of imposing a different lower bound for the inventory system. An additional cost savings of almost 1,7% is realized if the lower bound fill rate is decreased to 60%. Since these results are lower than expected, compared to the findings of the inventory literature, a second sensitivity analysis is executed. We suspect that both the low differentiation in holding cost per unit and average demand per unit limits the system approach of realizing higher cost savings. Therefore, a second sensitivity analysis is performed by increasing the average demand of 25% of the SKU's. At random, 25% of the SKU's are selected and are multiplied with a random factor between 20 and 40. This resulted in larger spread of average demand for the inventory system which is shown in Table 5.4. Using the new $\mu_{i,D(increased)}$, the cost savings is around 19% for a system fill rate of 98% while imposing a lower bound of 90%. In addition, Table 5.4 shows the effects of the new $\mu_{i,D(increased)}$ combined with different targets of the system fill rate and different lower bounds of individual fill rates.

Table 5.3: Different l_{p2} for system approach Sherbrooke, while FR_S = 98,1%

Lower bound (l_{p2})	*60%*	*70%*	*80%*	*90%*
Total inventory costs	€1.047.748	€1.048.501	€1.051.718	€1.066.743
% saving to single item	12,41%	12,30%	12,03%	10,78%

Table 5.4: Descriptive statistics of $\mu_{i,D}$ & $\mu_{i,D(increased)}$

594 SKU's	*Min*	*25%*	*Median*	*75%*	*Max*	Average
$\mu_{i,D}$	63	195	410	933	7.199	835
$\mu_{i,D(increased)}$	66	254	717	2.458	107.985	3.769

Table 5.5: Sensitivity on increased mu

	FR_S single item	*Total cost single*	*FR_S system Sherb*	*l_{p2} = 70%*	*l_{p2} = 80%*	*l_{p2} = 90%*
	95%	€1.244.425	95%	25,8%	22,4%	12,5%
$\mu_{i,D,increased}$	98%	€1.494.048	98%	23,6%	22,5%	19,1%
	99%	€1.670.594	99%	21,5%	21,0%	19,5%

5.1.2 Inventory approaches based on forecast data w29-w49

This section presents the results of the same inventory approaches while using forecast data based on weeks 29 to 49. Since a different data set is used during the analysis in this section it can not be compared to the performance of the inventory approaches based on the demand data set. In addition, two sensitive analysis will be executed and discussed.

Customer service level

Table 5.6 shows the customer service level performance of all approaches. Since the data set has slightly changed, the volume based system fill rate is equal to 98,2% which means all approaches must achieve this target. During this analysis, the expected demand (forecast data set) determines the height of the reorder level and after that the actual demand data is used to calculate the expected inventory on hand and resulting fill rate.The current situation during week 29 and 49 showed very little service level differentiation. Around 110 SKU's realized a fill rate below 100%, while all other SKU's realized a fill rate of 100% during the second half of 2021. The single item approach does not deviate a lot either. The principle of the single item approach states that all SKU's must receive the same treatment. In order to reach a 98,2% for the system, the target for each individual SKU's must be set on 99,97% because the forecast data is not equal to the realized demand during this period. The target fill rates for the classes of the ABC-approach equaled 99.98% for class A and 99.95% for both class B and C. This resulted in less SKU's that reached a 100% fill rate compared to the single item approach. The system approach provided the possibility to set different targets for the individual fill rates. The starting point for the marginal cost analysis was again the lower bound of 90% for all SKU's. However, for some SKU's the difference between the expected demand and realized demand was very large which resulted in a few fill rates below the lower bound. Using the system approached a greater differentiation among the fill rates is realized.

Table 5.6: Service level performance of all approaches - forecast data w29-w49

Inventory policy	FR_S	*0%*	*25%*	*50%*	*75%*	*100%*
Current	98.2%	0%	100,0%	100,0%	100,0%	100,0%
Single item	98.2%	67,0%	100,0%	100,0%	100,0%	100,0%
ABC-classification	98.2%	55.0%	99.1%	99,9%	100,0%	100,0%
System Sherbrooke	98.2%	65,4%	96,7%	98,1%	99,0%	100,0%

Total inventory costs

The total inventory costs of each approach can be seen in Table 5.7. The total inventory costs per period for the single item approach is almost 21% lower compared to the performance of the current inventory policy. Similar to the previous analysis based on demand data, the ABC-classification does not improve the inventory system by a lot. It stands out that the system approach realizes a very large cost saving of 48% compared to the current situation. The large difference of expected inventory costs between the single-item and system approach can be explained. The single-item approach sets the same target fill rate for each individual SKU in order to realize a system fill rate of 98,2%. It keeps increasing all individual fill rates until the target for the system is reached. That is why Table 5.6 shows a lot of SKU's having a fill rate of 100%. To confirm this, an additional analysis is performed. Table 5.8 shows the performance of both the single-item and system approach compared to the current situation while using the true demand data of week 29 to 49. This analysis shows that the single item approach realizes a very large saving and the system approach.

Table 5.7: Total inventory costs comparison - forecast data set w29-w49

Inventory policy	*Total inventory costs*	*% cost improvement to current policy*	*% cost improvement to single item approach*
Current	€1.762.512	-	-
Single item	€1.393.276	20,95 %	-
ABC-classification	€1.380.127	21,70 %	0,94%
System - Sherbrooke	€916.043	48,03 %	34,25 %

Table 5.8: Total inventory costs comparison - demand data w29-w49

Inventory policy	*Total inventory costs*	*% cost improvement to current policy*	*% cost improvement to single item approach*
Current	€1.762.512	-	-
Single item	€956.726	45,73 %	-
System - Sherbrooke	€880.540	50,04%	7,98%

Sensitivity analysis

The first sensitivity analysis tests multiple different service levels for the inventory system while using the single item approach (s.i.a). The realized aggregate fill rates and total inventory costs per period are showed in Table 5.9. Based on this table, it is found that the service level of the system can be increased to 99.0%, while realizing almost the same expected total inventory cost compared to the current situation.

Table 5.9: Sensitivity analysis - system fill rates

Key performance indicator	*S.I.A*	*S.I.A*	*S.I.A*	*S.I.A*
Aggregate system fill rate	97.5%	98.0%	98.5%	99.0%
Total inventory costs	€1.234.772	€1.336.132	€1.511.946	€1.767.534

The second sensitivity analysis tests different values for the average expected order lines per period (E[OL]) of the inventory system. During previous analysis the expected order lines per period was not taken into account, since we approached the inventory problem from the perspective of inventory department. However, Van Geloven produces the ordered quantities in her own factories, so it could be impossible to produce a certain product every other week as the production lines have limited capacity. Furthermore, if a production line switches frequently between different products a lot of time is lost due to cleaning. The single item approach realized an average expected order line of 0.43 during the previous analysis. This means that, on average there is a 43% chance an order is created for a SKU or in other words once every 2,3 periods. According to Donselaar, van and Broekmeulen (2014), the expected order line depends on the review period (R), average demand (μ), standard deviation (σ) and the lot-size (Q). To decrease the expected order lines per period, the best alternative is to increase the lot-sizes of the SKU's as it is the only variable that can changed relatively easily. During the sensitivity analysis, it is assumed that all lot-sizes of the SKU's can be increased equally. The work of Donselaar, van and Broekmeulen (2014) states that the E[OS] can be calculated by using the following relationship: $\mu_R = E[OS] * E[OL]$. If we decrease E[OL], while keeping the expected demand constant, it is expected to see an increase in the expected order size per period. Table 5.10 shows the effects of increasing the lot-sizes of the SKU's on the expected order lines and total inventory costs per period. The E[OL] ranges from 0.25 to 0.43, meaning that the lowest average of creating an order for an

SKU is equal to once every four periods. This results could be explained as follows: if the IP is below the reorder level (s), an order is created which is an integer number of Q to raise the IP to or above the s. However, if the expected order size increases it is more likely to end-up with a relative higher IP after an order is created. The higher IP could explain the increase of the total inventory cost.

Table 5.10: Scenario analysis - average expected order lines

Multiple Q by factor	*2,29*	*1,77*	*1,42*	*1,15*	*1*
Average E[OL]	0,25	0,3	0,35	0,40	0,43
Total inventory costs	€1.698.218	€1.570.547	€1.490.342	€1.431.754	€1.400.899

5.2 Inventory systems with a network of items

This section will discuss the results of both sub-inventory systems which were introduced in chapter 3. The subsystems are successfully compared on multiple KPI's. In addition, the effects of different demand correlations is discussed.

5.2.1 Subsystem with multiple stocking locations

The first step of analyzing the sub-inventory system with two stocking locations (see figure 4.4 is setting the same target fill rate for all stock points. Table 5.11 shows the resulting fill rate of the components and single-component SKU's including the expected inventory cost per period. Component 2 shows the largest drop in fill rate compared to the others, as the difference between the demand and forecast data is significant. Secondly, SKU 9930 shows a higher realized fill rate compared to it's target fill rate, since the expected demand of this SKU was higher than the actual demand. Table 5.12 shows the resulting lower and upper bound of the fill rate for both multi-component SKU's. The lower bound of SKU 9552 is significantly lower, as component two is only assembled into this SKU. The expected inventory costs is equal to €5.434 and the range of the aggregate subsystem fill rate (FR_{sub}) is equal to [80,92 - 91,78]%

Table 5.11: Performance of the components & single-component SKU's - $P_2^* = 98\%$

SKU	*Component(s)*	$P^*_{2,\{i,j\}}$	$P_{2,\{i,j\}}$	*Inventory cost per period*
-	c1	98,00%	93,20%	€761
-	c2	98,00%	88,67%	€395
-	c3	98,00%	92,22%	€591
-	c4	98,00%	98,96%	€538
-	c5	98,00%	94,25%	€774
1126	{1}	98,00%	93,50%	€1.180
9930	{5}	98,00%	99,70%	€1.195

Table 5.12: Performance of multi-component SKU's - $P_2^* = 98\%$

Assembled SKU's	*Components*	*Resulting $P_{lower,i}$*	*Resulting $P_{upper,i}$*
9552	{1,2,3,5}	71,82%	88,67%
9559	{1,3,4,5}	80,16%	92,22%

The range of the subsystem fill rate is considered too low. We perform additional analysis to achieve a subsystem fill rate of 98% for both the worst and best case scenario. First we will focus on achieving a subsystem fill rate of 98,0% using the upper bounds of the multi-component SKU's. An trail and error procedure is used to find the individual target fill rates of the components and SKU's to achieve this target. Table 5.13 shows the individual target fill rates ($P^*_{2,\{i,j\}}$) of the components and single-component SKU's to realize a 98,0% for every individual stock point. Compared to Table 5.11, the inventory cost for all components and single-component SKU's increases except for component 4 and SKU 9930. For those stock points the target fill rate could be lowed compared to previous analysis. In addition, Table 5.14 shows the resulting lower and upper bound of the fill rate for the multi-component SKU's. The upper bounds of the multi-components SKU's are at least 98,0% and also the lower bounds have increased significantly. The expected inventory costs is equal to €6.410 which is an increase of 18%. The range of the FR_{sub} has increased to [94,18 - 98,01]%. The scenario to achieve a minimal fill rate of at least 98,0% for the subsystem is discussed in Appendix C, as this alternative will not be compared to the other subsystem.

Table 5.13: Performance of components & single-component SKU's - $FR_{sub,upper} \geq 98,0\%$

SKU	*Component(s)*	$P^*_{2,\{i,j\}}$	$P_{2,\{i,j\}}$	*Inventory cost per period*
-	c1	99,60%	98,01%	€998
-	c2	99,90%	98,04%	€613
-	c3	99,67%	98,03%	€834
-	c4	96,80%	98,09%	€500
-	c5	99,50%	98,02%	€959
1126	{1}	99,33%	98,00%	€1.566
9930	{5}	94,83%	98,00%	€940

Table 5.14: Performance of multi-component SKU's - $FR_{sub,upper} \geq 98,0\%$

Assembled SKU's	*Components*	*Resulting $P_{lower,i}$*	*Resulting $P_{upper,i}$*
9552	{1,2,3,5}	92,33%	98,01%
9559	{1,3,4,5}	92,38%	98,01%

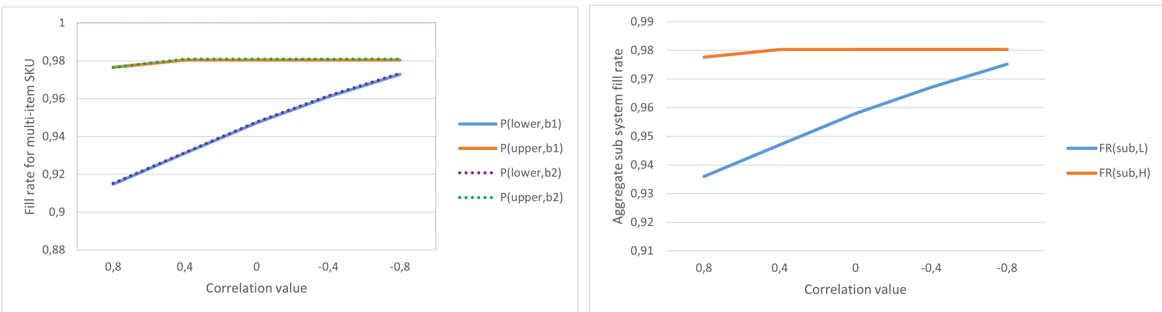
Lastly, we will discuss the effects of applying different correlations between both multi-component SKU's ($b1$&$b2$). The fill rate targets of the scenario where $FR_{sub,lower} \geq 98,0\%$ will be used to study the effects of different demand correlations. Since the single-component

SKU's are treated independent and separately, it is assumed that a different correlation between the bundles does not effect them. During the analysis, we investigate the effects if the correlation is either strongly negative, moderately negative, moderately positive, strongly positive or zero. Table 5.15 shows the effects on the realized individual fill rates of the components while using different correlations. If the correlation becomes negatively stronger, then the fill rate increases for the components that are assembled into multi-component SKU's. Components 2 and 4 are not influenced by the correlation, since they are assembled to only one multi-component SKU.

Table 5.15: Effect of correlation on component fill rates - $FR_{sub,upper} \geq 98,0\%$

Components	$P^*_{2,j}$	$P_{2,j}$ *if* $\rho_{\{1,2\}}$ = -0.80	$P_{2,j}$ *if* $\rho_{\{1,2\}}$ = -0.40	$P_{2,j}$ *if* $\rho_{\{1,2\}}$ = 0	$P_{2,j}$ *if* $\rho_{\{1,2\}}$ = +0.40	$P_{2,j}$ *if* $\rho_{\{1,2\}}$ = +0.80
c1	99,60%	99,72%	99,30%	98,88%	98,28%	97,73%
c2	99,90%	98,04%	98,04%	98,04%	98,04%	98,04%
c3	99,67%	99,88%	99,51%	98,98%	98,35%	97,65%
c4	96,80%	98,09%	98,09%	98,09%	98,09%	98,09%
c5	99,50%	99,62%	99,19%	98,73%	98,25%	97,77%

Subsequently, for each correlation the upper and lower bound fill rates are calculated for both multi-component SKU's. The resulting lower and upper bounds are plotted in Figure 5.1a. Based on these upper and lower bounds, a worst and best case scenario can be determined for FR_{sub} while using all different correlation values. Figure 5.1b plots the worst and best case scenario. Based on these figures, it is concluded that stronger negative correlations increases the lower bounds of the multi-component fill rates and improves the worst case scenario of the FR_{sub}.



(a) Correlation effect on the individual lower and upper bound fill rate for a SKU *i*

(b) Correlation effect on the aggregate subsystem fill rate (FR_{sub})

Figure 5.1: The effects of different correlations on the fill rate

The final analysis shows how inventory costs can be reduced if negative correlation is present. The left-hand side of Table 5.16 shows the original situation of the fill rate targets and realized fill rates for the components while $\rho_{\{1,2\}} = -0.40$. The negative correlation causes an increase in the realized fill rate of components c_1, c_3 and c_5 which improves the worst-case scenario of the multi-components SKU's and subsystem fill rate. On the other hand, the worst-case scenario's could not be improved and a cost reduction can be realized. The right-hand side of Table 5.16 decreases the fill rate targets of components c_1, c_3 and c_5 which results in a 98,0% fill rate for all components. This action decreases the expected inventory costs of the system with €382 (from €3.868 to €3.486).

Table 5.16: Effects of negative correlation on inventory costs - $FR_{sub,upper} \geq 98,0\%$

Component	*Initial* $P^*_{2,j}$	*Resulting* $P_{2,j}$	*Inventory costs per period*	*New* $P^*_{2,j}$	*Resulting* $P_{2,j}$	*Inventory costs per period*
c1	99,60%	99,30%	€988	99,03%	98,02%	€860
c2	99,90%	98,04%	€613	99,90%	98,04%	€613
c3	99,67%	99,51%	€816	98,95%	98,03%	€665
c4	96,80%	98,09%	€500	96,80%	98,09%	€500
c5	99,50%	99,19%	€951	98,91%	98,03%	€848

5.2.2 Subsystem with a single stock location

This subsection analysis the second sub-inventory system introduced in chapter 4 (see figure 4.5) This subsystem has a single stocking locations for keeping inventory of only the components. Table 5.17 shows the resulting component fill rates if the target are all equal to 98,0% including the expected inventory costs. In addition, Table 5.18 indicates the resulting lower and upper bounds of the fill rate for both single- and multi-component SKU's. The expected inventory costs of this system is equal to €4.193 with a corresponding fill rate range of [73,96 - 90,85]%.

Table 5.17: Performance of all components - $P^*_{2,j} = 98\%$

Component(s)	$P^*_{2,j}$	$P_{2,j}$	*Inventory cost per period*
c1	98,00%	91,28%	€1.420
c2	98,00%	88,67%	€396
c3	98,00%	92,23%	€591
c4	98,00%	98,96%	€538
c5	98,00%	96,70%	€1.248

Table 5.18: Performance of single- and multi-component SKU's - $P^*_2 = 98\%$

Assembled SKU's	*Component(s)*	*Resulting* $P_{lower,i}$	*Resulting* $P_{upper,i}$
1126 (1)	{1}	91,28%	91,28%
9552 (b1)	{1,2,3,5}	72,19%	88,67%
9559 (b2)	{1,3,4,5}	80,56%	91,28%
9930 (2)	{5}	96,70 %	96,70%

Just as for the other subsystem, we compute the component fill rates to reach a 98% of the upper bound of they system ($FR_{sub,upper}$). In this way, the performance of both subsystems can be compared in more detail. Table 5.19 shows the increased fill rate targets including the realized fill rates and inventory costs per component. Based on these numbers, the lower and uppers bounds of the fill rate for the SKU's can be computed. Table 5.20 shows an increase in the lower bound for each SKU and the 98% fill rate is achieved for the upper bounds. The expected inventory costs of the system is equal to €5.406 with a range of [94,21 - 98,02]% for the FR_{sub}.

Table 5.19: Performance of all components - - $FR_{sub,upper} \geq 98,0\%$

Component(s)	$P^*_{2,j}$	$P_{2,j}$	*Inventory cost per period*
c1	99,59%	98,04%	€2.073
c2	99,90%	98,04%	€613
c3	99,67%	98,03%	€834
c4	96,80%	98,09%	€500
c5	98,70%	98,02%	€1.386

Table 5.20: Performance of single- and multi-component SKU's - $FR_{sub,upper} \geq 98,0\%$

Assembled SKU's	*Component(s)*	*Resulting* $P_{lower,i}$	*Resulting* $P_{upper,i}$
1126 (1)	{1}	98,04%	98,04%
9552 (b1)	{1,2,3,5}	92,36%	98,02%
9559 (b2)	{1,3,4,5}	92,41%	98,02%
9930 (2)	{5}	98,02%	98,02%

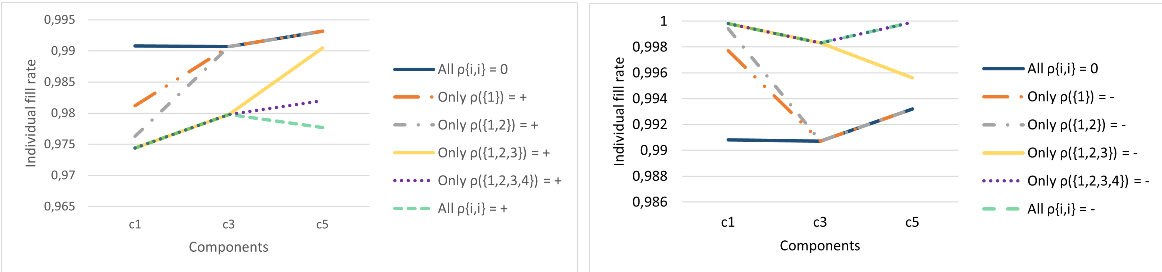
Table 5.21 summarizes the results of both sub-inventory systems. When imposing the same fill rate targets to both subsystems, the subsystem with multiple stock locations realizes a higher expected inventory costs and range of the subsystem fill rate compared to the single stock location subsystem. Since the range of FR_{sub} differs a lot it is a little unfair comparison. Therefore, for both systems it is calculated what the KPI's are when imposing the same target for the upper bound fill rate of the subsystem. It is concluded that the single location subsystem outperforms the other subsystem as almost the same range of FR_{sub} is achieved while a cost reduction of 15,7% is realized.

Table 5.21: Comparing both sub-inventory systems

Type of subsystem	*Condition*	*Range of* FR_{sub}	*Expected inventory cost per period*
Multiple stock locations	all $P^*_{2,j}$ = 98,0%	[80,92 - 91,78]%	€5.434
Single stock location	all $P^*_{2,j}$ = 98,0%	[73,96 - 90,85]%	€4.193
Multiple stock locations	all $P_{upper,i}$ = 98,0%	[94,18 - 98,00]%	€6.410
Single stock location	all $P_{upper,i}$ = 98,0%	[94,21 - 98,02]%	€5.406

Figure 5.2 shows the results of the impact on the realized component fill rates while applying different values for the correlations. The left-side, Figure 5.2a, shows the effect of strictly positive correlations. The blue line indicates that all correlations are equal to zero

and the dotted green line indicates that all correlations are moderately positive (0,5). It is found that, the more positive correlations are present in the inventory system, the lower the realized fill rates are for component 1,3 and 5. Components 2 and 4 are excluded from this figure, as it is concluded earlier that demand correlation does not effects these components. On the other hand, Figure 5.2b, shows the effect of strictly negative correlations. Again, the blue line indicates the case where all correlations are equal to zero and the dotted green line indicates that all correlations are negatively correlated. It seems that, the more negative correlations are present, the higher the realized fill rates are for components 1,3 and 5.



(a) The impact of strictly positive correlations

(b) The impact of strictly negative correlations

Figure 5.2: The impact of different correlations on the component fill rate

Furthermore, we zoom in on a specific configuration of correlations between the end-products to show what the effect is on the inventory costs per component. The same individual target fill rates are set for the components with a negative correlation of -0.5 for $\rho_{\{1,b1\}}$ and $\rho_{\{2,b2\}}$. Comparing the left-hand side of Table 5.22 with the resulting fill rate per component of Table 5.19, it can be seen that an increase is realized for components 1 and 5. In our inventory setting, an increase of those components leads only to an increase in the lower bound fill rates. The higher bounds are restricted by the lowest fill rates of the components, which are component c_2 and c_4. If increasing the lower bound fill rates of the multi-component SKU's would not be interesting, then two alternative options can be considered. First, as explained earlier, the target fill rates of component c_1 and c_5 can be lowered to achieve a 98% fill rate for the upper bound of the subsystem. The right-hand side of Table 5.22 shows a possible cost reduction of 15,6% and 5,7% for components c_1 and c_5. On the other hand, the design of the single location sub-inventory system allows to allocate 'excess' inventory of components c_1 and c_5 to the single-component SKU's if demand is not completely fulfilled. The equal fair share allocation does not hold anymore, since relatively more inventory would be allocated to the single-component SKU's.

Table 5.22: Effect of negative correlations on inventory costs

Component	*Initial* $P^*_{2,j}$	*Resulting* $P_{2,j}$	*Inventory costs*	*New* $P^*_{2,j}$	*Resulting* $P_{2,j}$	*Inventory costs*
c1	99,59%	99,32%	€2.062	99,11%	98,01%	€1.740
c2	99.90%	98,04%	€613	99,90%	98,04%	€613
c3	99,67%	98.03%	€834	99,67%	98,03%	€834
c4	96,80%	98,09%	€500	96,80%	98,09%	€500
c5	98.70%	98,59%	€1.382	98,34%	98,01%	€1.303

Chapter 6

Conclusion

The sixth chapter of this research starts with a discussion of the results in which all sub-questions are revisited. Thereafter, the conclusion is given of this research by providing an answer to the main research question and evaluating the research objective Finally, the limitations and future research possibilities of this research are discussed.

6.1 Discussion

Before an answer is formulated for the the main research question, the sub-questions are revisited and answered one by one. The first two sub-questions are answered together, since they are discussed in the same chapter, namely chapter 2. Both sub-questions were stated as follows:

- **SQ.1:** What inventory strategies can be used for a multi-item inventory system?
- **SQ.2:** What definitions of the aggregate fill rate constraint can be used for a food company with a production-inventory control system following a mixed bundling selling strategy?

The appropriate inventory strategy for an inventory system depends on what kind of inventory problem is faced. The academic inventory literature describes three categories of inventory problems, namely an inventory system with independent items, a network of items and shared supply chain processes.

The first category includes inventory problems where different items do not share any supply chain or demand processes and can be treated individually of each other. The most common applied inventory approach, in both research and practical instances, is the single item approach. This approach has a low complexity which makes it easy to understand and not hard to implement for an inventory system. However, the low complexity also has it's downside as the efficiency and potential cost savings are low compared to the ABC and system approach. The ABC-classification approach is popular in practice, since it can be used as a classification criteria to allocate the amount of resources based on the relative importance of items. Different service level targets can be determined for the classes, causing a slight service level differentiation among the SKU's. The system approach is an aggregate constraint inventory system, as it is able to incorporate one or multiple system wide limitations or goals and tries to find the most efficient solution while considering all items simultaneously. Instead of seeking local optima for each item, the system approach seeks to find the solution which is optimal for the system. While using either one of the above approaches various inventory problems can be encountered. The research of De Schrijver et al. (2013) provides a good overview of five general problem instances including inventory policies to handle the problem.

Inventory problems with a network of items is the second discussed category in this research. In this kind of systems items have a demand-supply relationship. Two specific type of networks are discussed during this research. ATO systems are strategies that produce and stock individual components which will only be assembled into end-products if customer demand is realized. By delaying the final assembly of end-products, the system improves since inventory pooling occurs for the components which can lead to a decrease in cost and increase in service level. On the other hand, ATS systems do not stock any components as the assembly process is already started before customer demand is realized. The CODP for the ATS system is located more downstream of the supply chain compared to an ATO system. The research of Atan et al. (2017) provides a comprehensive overview of all kinds of ATO models for both periodic and continuous systems while the research on ATS system is limited.

The third discussed category of inventory problems are systems with shared supply chain processes. An inventory system with shared supply chain process often means that multiple sets of items come from the same supplier. This creates a possibility or order jointly and save ordering costs. This problem instance is described by the academic literature as the joint replenishment problem. One of the most common joint replenishment policies for stochastic demand is the can-order policy (s,c,S). This policy takes into consideration the inventory position and reorders levels of all products ordered at the same supplier. However, this policy does not perform well for correlated demand arrivals. The work of Feng et al. (2015) developed an extension of the can-order policy, namely the (s,c,d,S)-policy which is designed for handling stochastic inventory systems with correlated demand arrivals.

In order to answer the second sub question of this research, the inventory literature was searched for different available definitions of the aggregate fill rate. The kind of weight given to the items in the inventory system determines what kind of definition of the aggregate system fill rate is applied. Several papers, Millstein et al. (2014), Teunter et al. (2017), Silver et al. (2016), indicate that the following type of weights are mostly used for research purposes: generic, volume-based, turnover-based and profit-based weights. Applying generic weights to the inventory system means that each fill rate is equally important, so the target of the aggregate system fill rate is set for all individual fill rates (single item approach). The other three weights determine the relative importance of a SKU compared to all SKU's in the system. The literature describes different effects for multi-item inventory system while using the different weights. Firstly, generic weights cause little variation in individual item fill rates of the inventory system and often realizes higher inventory cost of the system compared to other types of weights. The volume-based weights can lead to a very high degree of service level differentiation which leads to relatively lower inventory costs. On the other hand, the turn-over based weights have relatively higher inventory costs as the degree of service level differentiation is lower. Since there is very little research performed on multi-item inventory problems with a mixed bundling selling strategy, it is unknown how a system approach could even be applied for such a problem. However, it is concluded that the same definitions of the weights for the aggregate system fill rate could be applicable for a mixed bundling system. Now the first two sub-questions are answered, the third research question can be revisited, which was stated as follows:

- **SQ.3:** What is the current inventory strategy and performance of the multi-item inventory system of Van Geloven?

Before an answer can be given the this question, it is important to define what kind of inventory problem is faced combined with the configuration of the mixed bundling principle used by Van Geloven. It is concluded that Van Geloven faces a partial product mixed bundling multi-item inventory problem. They produce components which are solely used for multi-component SKU's and are not sold as single-component SKU's. Secondly, they apply product bundling as the SKU's share a physical integration of components and additional assembly processes applied. The inventory problem faced by van Geloven can be classified as a system with a network of items including a hybrid form of the ATO and ATS principle, as components and SKU's are stocked. However, Van Geloven approaches their problem as a system with independent items as the physical integration of components to their SKU's is not part of the inventory decision making process. In addition, they apply the single item approach and use an incorrect replenishment logic. The inventory department determines all inventory decisions on the actual weekly coverage of the SKU's compared to a very general coverage interval. The inventory department does not have any information about the relationship between the average weekly coverage and the resulting fill rate and inventory costs of their SKU's. Meanwhile, senior management evaluates the performance of the inventory system on the realized system fill rate during a year.

During the last few years Van Geloven experienced temporally inexplicable high drops in the customer service level across the inventory system and did not realize the target fill rate of the last years. In addition, the inventory system realized large differentiation among the individual fill rates of the SKU's, as around 500 SKU's realized a fill rate very close to 100% and dozens of SKU's realized a fill rate below 50%. This effects is unintended and inexplicable by the department as they try to achieve the same fill rate for each SKU. The resulting inventory costs are unnecessary high as the costs increases exponentially if the fill rate goes to 100%. It is concluded that, the inability of the inventory department by having both very little understanding of their own applied inventory strategy and the unawareness of all the existing inventory models, theories and formula's is one of the major causes of a far from optimal performing inventory system. However, there are also other factors contributing to the poor performance of the inventory system. First of all, the current information system is outdated and unable to support an integration between all departments and factories. Secondly, too many parties are involved of the inventory decision making process, as each factory has to authority to increase or decrease the determined order sizes of the inventory department. Lastly, the Covid-19 pandemic contributed to lower fill rates and higher inventory costs as demand was very uncertain and personnel was limited. Since the third research question is answered, the fourth research question can be revisited as it was stated as follows:

- **SQ.4:** What kind of inventory model could be developed that incorporates the mixed bundling principle and is able to minimize total inventory costs for a multi-item inventory problem?

The type of inventory model which is able to minimize the inventory costs as much as possible for a mixed bundling multi-item inventory system depends on how the mixed bundling principle is applied. During this research we consider two types of inventory model. Firstly we discuss the current inventory model considered by Van Geloven as it is the least complex configuration since all end-product are viewed independently and separately of each other. This brings it down to a classical multi-item inventory problem with independent items. Based on our research and the current academic literature, it is concluded that a system approach always realizes the best cost-optimal inventory system. However, the savings compared to the single item approach were lower than anticipated. One of the explanations could be the low variation of inventory holdings cost and average demand for the SKU's of the inventory system of Van Geloven. A sensitivity analysis was performed on the average demand, it was found that a larger variation of the average demand over the SKU's resulted in higher cost savings while using the system approach. The low variation in holding costs can be explained by the specific type of product that is manufactured by Van Geloven. The raw materials needed for all frozen snacks are almost completely the same, causing the value of the frozen snacks to be very similar. Although the inventory costs at the central warehouse are optimized, we think that considering the inventory problem of Van Geloven as an independent system of items is very inefficient for various reasons. The assembly process of the multi-component SKU's is completely neglected and all the benefits of pooling inventories are lost. Secondly, since the same components are present in numerous of end-products, the correlation between demands is highly important to consider. Thirdly, by determining inventory decisions separately for all end-products the opportunity of ordering or producing the same component at once for multiple SKU's is lost. Finally, the local warehouse where all components are stored is not even considered and thus not optimized. Based on these arguments, we conclude that optimizing the central warehouse could result in significant cost savings but does not lead to an efficient and cost-optimal supply chain.

During this research another different inventory model was considered. This model is a subset of the inventory system of Van Geloven and here we included inventory decisions for both the components and SKU's. Two different alternatives are created, one with separate stocking locations of the components and SKU's which is very similar to the current situation of Van Geloven. The second alternative subsystem reduced the supply chain by only allowing inventories of the components. The second model created a setting where more inventories are pooled together which leads to higher inventory cost savings. Both alternative subsystems considered the ATO principle for assembling the multi-components SKU's and the (R,s,nQ) replenishment logic was used to optimize both subsystems. Finally, the last research question can be revisited as it was stated as follows:

- **SQ.5:** What are the effects on inventory decisions when the mixed bundling principle is implemented for a multi-item inventory system?

We will discuss the effects of applying the mixed bundling principle for both inventory models representing the inventory problem of independent items and network of items. If an inventory problem with independent items is considered the mixed bundling principle does not effect the inventory decisions. All items (SKU's) are considered independently and separately of each other, so the physical integration of different components is not considered. Applying the mixed bundling principle and considering the inventory system as this problem instance does not lead to an optimal supply chain.

If the mixed bundling principle is applied for an inventory problem with a network of items, then several effects influence the inventory decision making process. First of all, how the inventory model has a large influence on the individual fill rates of the stocking points and expected inventory costs. The more inventories can be pooled together at the same location of the same component has positive impacts on the service levels and total inventory costs. In addition, the more stronger negative correlations are present in the inventory (sub)system, the higher the individual component fill rates will be and the expected inventory costs decreases. However, this effect only holds for the components which are assembled into multi-component SKU's. Higher component fill rates always increases the lower bound for the fill rate of multi-component SKU's and could increase the higher bound. Since the inventory costs remain the same and the fill rates are higher, the profitability of the system increases. This finding is in line with the results and conclusions of Qiang (2012), as they state that the expected profit increases if the demand correlations become less positive.

6.2 Conclusion

Since all sub-questions are answered and discussed, an answer can be formulated for the mean research question of this research, which was stated as follows:

Main research question: *What inventory approaches can be applied for minimizing total inventory costs while meeting the target service level in a multi-item inventory system which uses a mixed- or a non-mixed bundling strategy?*

The answer of the main research question is three-fold, because the type of inventory problem depends on the type of bundling strategy applied for the system. If a pure bundling strategy is used for the inventory system, then we classify the inventory problem as a system with a network of items. The inventory problem solely consists of an assembly process, which can either be approached with the ATO or ATS principles. If a non-bundling strategy (unbundling) is applied for an inventory system, then the inventory problem can be classified into a system with independent items. The system approach realizes the best inventory cost-optimal system for this problem. If the inventory system follows a mixed bundling principle, then the inventory problem can also be classified as a system with a network of items. During this research, the single item approach was used to determine reorder levels for the individual components. In addition, the ATO principle and equal fair share allocation rule are used for the bundled SKU's to get a feasible solution.

Based on the answer of the main research question, the main and secondary research objective of this paper can be revisited and discussed. The main objective is partially achieved, as this research confirms that using a system approach for inventory problems with independent items realizes the best cost optimal inventory system while meeting the predetermined target service level. In addition, feasible solution are found for inventory problems with a network of items while the mixed bundling principle is applied. We assume the solutions are not optimal, because generic weights are used to approximate the aggregate system fill rate. This approach can be used for other N-product mixed bundling inventory models to find feasible solutions. Subsequently, it can be concluded that the secondary research objective of this research is achieved. More insights are created on inventory dynamics and decisions while the mixed bundling principle is applied. However, that does not mean that all effects are found of this particular selling strategy. Additional research is definitely needed on multi-item inventory problems following the mixed bundling selling strategy.

6.3 Limitations

The limitations of this research are explained in this section and are described in Table 6.1. When interpreting the obtained results, the following limitations must be taken into account before generalizing the results.

Table 6.1: Limitations

Limitation	**Description**
Influence of Covid-19 pandemic	The global coronavirus had a major influences on the demand data from 2019 to 2021
Data on local warehouses	Multiple variables are estimated in order to calculate KPI's of components at the local warehouse
Uncertain factors	Several uncertain factors are not included during this research, such as constraints on the workforce, production breakdowns and availability of raw materials
Size of the sub-inventory systems	The size of both sub-inventory systems is limited, in practice some components are assembled in more multi-component SKU's
Different packaging sizes	The sub-inventory systems did not include different packaging sizes for the components
Frequency of ordering	During the analysis it is not considered how often, on average, an order may be created for an SKU

Firstly, the global Covid-19 pandemic had severe influences on the society from 2019 to 2021. Van Geloven experienced a lot of troubles, as the demand showed high fluctuations during multiple periods over the years. This must be taken into consideration when evaluating the realized system fill rates during 2020 and 2021, as probably more companies experienced lower customer service levels. However, that does not mean that the analysis and results of this research would not be representative. On the contrary, the same forecast data for our models was used which was also available for the inventory department of Van Geloven at the time.

Secondly, the available data on the weekly stock levels and other important variables of components stored at the local warehouse were not available. The information system of Van Geloven is outdated and multiple departments have stored important information on local databases. As discussed in chapter 4, the variables are estimated based on the known information of the central warehouse. As a result, the performance of the sub-inventory systems could not be compared to the actual performance of the system. Both these arguments can be described as a limitation of this research. Furthermore, several uncertainty factors could have been included in order to obtain more realistic results. For instance, Van Geloven experiences troubles with producing the ordered quantities by the inventory department as they have limited personnel, shortages of raw materials and breakdowns.

Additionally, as the complexity of the inventory models with a network of items would increase rapidly with more components and SKU's, it was chosen to restrict the size of the subsystem. However, this is considered a limitation of this research. Furthermore, the second sub-inventory system did not make a distinction between the packaging sizes stored at the local warehouse of the same component. This is a limitation of this research, since components arriving at the local warehouse are either packaged in small cases for a single-component SKU's or in large bulks for the assembly process.

Finally, the last limitation involves the allowed frequency of ordering for the components and SKU's. Since multiple components are manufactured on the same production line, it would be very inefficient if each component needs to be manufactured every week. However, a sensitivity analysis was performed to reduce the expected order lines per period and still significant savings were realized. Nevertheless, it is considered as a limitation of this research as the maximum allowed expected order line per period per SKU was not included.

6.4 Future research directions

This study can serve as the start for any further research within the area of mixed bundling and especially for inventory problems with more than two components and three end-products. Future research is required to obtain more substantial results, as the amount of research is very limited. The main topics of future research directions are listed in Table 6.2.

The first topic that can be investigated into further is developing a general N-product model for full-mixed bundling inventory systems. Although, the full sharing model is not used in this research, it is still a very important research area. A lot of retailers have inventory systems with dozens of products and since mixed bundling increases profitability it is very interesting to use.

In addition, it would be interesting to investigate the possibility to extend the general N-product model for full mixed bundling to a dynamic W-component & N-product model for partial-mixed bundling inventory systems. The model must be dynamic instead of general, since given a random set of W-components and N-products a lot of different inventory settings can be created. In practice, it is not unusual for a retailer to sell some additional bundled SKU's besides their single-component SKU's. Given a random configuration of a partial mixed bundling inventory problem, the dynamic model could help for making close to optimal inventory decisions.

Table 6.2: Future research directions

Directions of future research	Description
General N-product model	Developing a general N-product model for analysing full-mixed bundling inventory problems
Dynamic W-component, N-product model	Creating a dynamic model for analyzing and evaluating partial-mixed bundling inventory problems
Inclusion of demand substitution	The effect on inventory decisions while either using a full or partial mixed bundling strategy if demand substitution is present
Inclusion of the system approach	Researching the effects of applying system wide constraints or limitations for mixed bundling multi-item inventory problems
Allocation rule/algorithm	Developing a general allocation algorithm that leads to the highest profitability for both full and partial mixed bundling inventory problems

Furthermore, another aspect that could be researched in the future is the inclusion of demand substitution for both full and partial mixed bundling. It would be of great interest to find the effects of demand substitution in order to improve the inventory decision making process.

The inclusion of the system approach for full- and partial-mixed bundling systems would be another topic of future research. It would be interesting to research how to cope with multiple system wide limitations and constraint for this kind of inventory problems. It would be one of the first steps to improve the feasible solutions which are presented in this research.

Finally, the last direction of future research based on this study is developing an allocation rule or algorithm that is able to handle large full-and partial mixed bundling inventory problems. The objective of the allocation rule should be to maximize profits given a certain available stock of components. The additional allocation rule presented in this research could be used a starting point for more complex inventory systems.

Chapter 7

Implementation

The final chapter of this research starts with discussing the managerial insights and recommendations for Van Geloven based on the results and drawn conclusions of this research. After that, the managerial insights and recommendations are translated to a proper implementation plan to provide Van Geloven with a structured guideline to improve their inventory management processes.

7.1 Managerial insights & recommendations

Based on the current performance of the inventory system and results of this research in chapters 3 & 5, different managerial insights can be formulated which are listed in Table 7.1. Based on those insights, general and research driven recommendations for Van Geloven can be drawn up.

Based on the overall performance and processes of the inventory department, it can be concluded that they are not doing a very good job. If we would solely judge the performance of the inventory system on the achieved service level, then it could be said that the performance is not so bad. However, it also matters what the underlying processes are in order to reach your objectives and to look at other relevant KPI's. It is especially unfortunate, that the department is unaware of both the potential significant improvements and of all the available academic inventory literature about models, strategies, formula's and more. However, blaming only the inventory department would not be fair as higher management and other departments also play a vital role in the performance of the inventory system.

The first discussed recommendations are of general nature for the organization, as it is deemed necessary before any recommendations of the inventory approaches of this research can be given. In order to utilize the maximal potential of the presented inventory models and strategies it is crucial for Van Geloven to start with implementing the general recommendations.

The recommendations for the current situation is four-folded, firstly it is advised to change the current inventory policy used by Van Geloven. There are some flaws in the currently used replenishment logic by the inventory department. Their principle comes very close to the (R,s,nQ) replenishment logic, so it is recommended to use this policy instead of

Table 7.1: Managerial insights

Managerial insights	Description
Current situation	The inventory department of Van Geloven is a performing bad to moderate job, as they are especially unaware of the true potential and opportunities of their processes
Data driven decision making	In order to grow as an organization it is absolutely for Van Geloven to become a more data-driven company
Gap analysis	Implementing one of the approaches presented by the gap analysis would have large positive impacts on the inventory system
Redesign supply chain	Based on the willingness and ability to change and implement either one of the presented approaches, a redesign of the supply chain is advised

theirs. The second recommendation for the inventory department is to increase the present knowledge about inventory literature and basic statistics. This can be realized in two ways, either send the current employees to educational programs or hire new employees with the right knowledge to increase the level of expertise within the department.

The following two recommendations are directed to higher management, since their decisions effects multiple departments and could increase efficiency throughout the whole company. The current information system of Van Geloven is outdated and used incorrectly by most of the employees of different departments and factories. The system is unable to support an integration of information between all parties of the supply chain which would make the implementation of for example a system approach almost impossible. Currently, Van Geloven is aware of this fact that a new information system is needed and has planned to implement SAP within the company at 2024. However, as they are looking forward to this system it can be questioned if they fully understand the possible opportunities of such a system and how the maximum potential can be utilized. The ability of their current employees to work correctly with a new information system can be questioned as they have failed before with other systems. Currently, only a few employees receive training with the new system to fully understand it's functioning which is not ideal. As it is expected that a large number of employees will have daily interactions with the new system. It is therefore recommended, to engage more employees and increase the frequency and intensity of the training's to speed up the implementation. A good operational and integrated information system for all departments is essential to maximize the potential of the proposed inventory methodologies in this research. The second recommendation for management can be combined with the previous one. The current decision making process regarding the inventory system is decentralized. At the headquarters, inventory decisions are determined but when the orders arrive at the factories the order size and even the whole order can be changed by factory schedulers. They apply alterations for various reasons without having any form of communication with the master planners. It is recommended that the decision making process is centralized at the headquarters, since the master planners have more complete information regarding the complete inventory system. This can be combined with the new information system, as factory schedulers would not have the authority anymore of changing the orders without contacting the master planner. This will probably have positive impacts on the stocking levels, inventory costs and customer service level.

The next main recommendation for Van Geloven is becoming a data-driven organization where decision making processes are based on data instead of subjective opinions and intuition. They could start by hiring new employees which would be responsible for analyzing the supply chain including all operations of the local warehouses and multi-component SKU's. Furthermore, the analysis of the forecast department could be expanded by including forecasts on component level of the multi-component SKU's. Based on this and earlier research, it is necessary for Van Geloven to perform a correlation analysis on their SKU's as it can have major impact on the inventory decision making process. Finally, since an integrated system is created between all departments it is essential to accommodate optimal conditions for both the inventory department and manufacturing sites.

Once all of the above recommendation are put into practice, it is the right time for Van Geloven to start thinking about implementing one of the the approaches of the gap analysis as otherwise it would probably fail. It is recommended to implement the system approach, as it provides the best cost-optimal inventory system and the presented alternative in this research is not very complex. Since the major part of the demand of Van Geloven comes from the single-component SKU's, the largest cost savings and efficiency improvements are made by focusing on the bulk. However, it is recommended to apply more data analysis on the components stocked at the local warehouses to gain insights on the weekly stock levels.

Finally, the last recommendation concerns the possible redesigns of the supply chain depending on which inventory approach is chosen by Van Geloven. If Van Geloven chooses to neglect the results of this research, the same recommendations would create a more efficient supply chain. First of all, the multiple local warehouses for stocking components must be reduced to one large warehouse. It is recommended to create or build warehouse space next to the factory in Maastricht for the components, as all multi-component SKU's are assembled there. It would result in a decrease of transportation costs, lead time of the assembly process and could have positive impact on the customer service level since the stocking levels can be monitored more closely.

If in the near future, Van Geloven decides to include the components into their inventory decision making process it is recommended to move the assembly process of the multi-component SKU's to the central warehouse where all stock is kept. Although, in theory, the ATO principle does not allow to store inventories of components and SKU's it could be convenient to store all inventory in the same place. The inventory system would benefit from the positive effects of inventory pooling.

7.2 Implementation plan

The managerial insights and recommendations discussed in the previous section require a proper implementation plan to set up for Van Geloven. The implementation plan consists of multiple phases and is visualized in Figure 7.1. The implementation does not indicate what the time periods are for each phase, as it is hard to determine how long it would take to cause structural changes at Van Geloven.

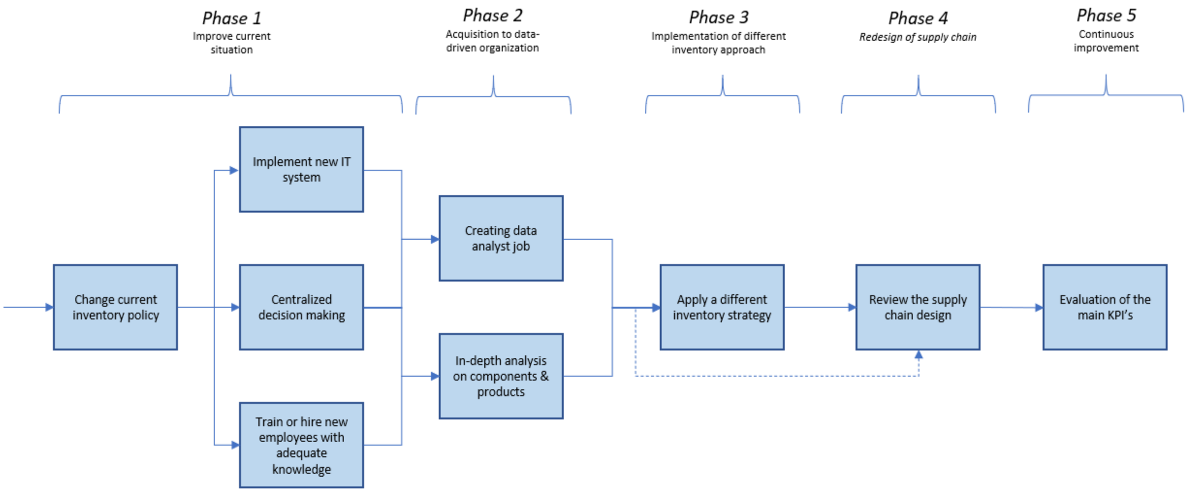


Figure 7.1: Implementation plan

The first phase of the implementation plan starts with improving the current situation. The first step is to change the applied inventory policy, as it is unable to support the inventory department to realize their objectives. After that, more general need to be implemented to realize a better performance of several processes within Van Geloven and creating a solid foundation for the discussed inventory approaches of this research. The implementation of the new information system must be completed faster in order to realize more efficient communication and coordinating of data between departments. Simultaneously, central decision making regarding the inventory process is important to let the master planners be in charge of the order sizes. The second phase includes the transition to a more data-driven organization. Additional jobs need to be created which analysis several processes of the organization. It is especially relevant for the inventory processes to get a good understanding of what the influences are of the mixed bundling principle on the organization.

After phase two, the following scenario's could unfold. If Van Geloven chooses not to implement any of the proposed inventory approaches then phase three can be skipped. However, if they wish to change their inventory approach they continue to phase three. Based on this choice, they can evaluate which inventory approach they would like to apply.

Phase four includes the redesign of the supply chain based on the choices made during the third phase. The previous section briefly discussed some supply chain designs, however a complete research could be dedicated to this phase. Finally, the last phase of the implementation plan includes continuous improvement of the chosen path of Van Geloven. It is desired to closely monitor the KPI's of the inventory system and to seek new opportunities for further improvements of the inventory system.

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Appendices

Appendix A

Average weekly coverage

This appendix will discuss a third Key Performance Indicator which is especially relevant for Van Geloven, as this is their current decision variable regarding the inventory decision making process. While using the presented approaches of the gap analysis for inventory system with independent items, the results are discussed here as they might provide additional insights for Van Geloven. First, the current performance of this KPI is discussed and how it can be calculated. Furthermore, the results while using demand and forecast data is discussed while using the approaches of the gap analysis.

Table A.1 shows the distribution of the stock coverage among the SKU's. It is remarkable to see that, only 265 SKU's during 2020 and 316 during 2021 are located in the interval which Van Geloven indicated as guideline for the min-max method. However, it has to be taken into account that this could be an consequence of the global COVID-19 pandemic. The SKU's in this interval are merely responsible for 38,5% and 45.1 % of the total realized demand. Secondly, roughly 350 SKU's are above the guideline interval and are accountable for almost 17% of satisfied demand. This would indicated that too much stock is kept for a lot of SKU's during 2020. Comparing these numbers to 2021 a difference can be noticed, all the stock coverage intervals above the guideline interval have fewer SKU's and more importantly the percentage of total volume decreased significantly. Only 237 SKU's are stocked with on average more than 10 weeks of coverage and account for around 8,4% of the total realized demand. During 2021, most of the SKU's shifted to lower stock coverage intervals or were completely removed out of the assortment.

Table A.1: Distribution of stock coverage

Stock coverage interval	*Spread of SKU's 2020*	*Demand volume 2020 (boxes)*	*% of total volume*	*Spread of SKU's 2021*	*Demand volume 2021 (boxes)*	*% of total volume*
0-6	288	9.971.961	44,5%	279	11.916.208	47.5%
6-10*	265	8.624.802	38,5%	316	10.958.467	43.7%
10-15	172	2.619.413	11,7%	141	1.905.520	7.6%
15-20	66	796.662	3,6%	39	161.921	0.6%
20-25	33	162.592	0,7%	24	105.377	0.4%
25-30	24	103.973	0,5%	9	12.260	0.05%
>30	57	128.369	0,6%	24	23.445	0.1%

* Used interval to determine all inventory decisions

Average weekly coverage entire system - demand based

Equation A.1 shows how the average weekly coverage of the inventory system can be calculated (AC_S). The numerator in this fraction is the summation of all individual average weekly coverage's per SKU (AC_i) and dividing this by the number of SKU's (N) gives the unweighted average weekly coverage of the inventory system. The performance of the inventory system, based on this KPI, will first be explained for the case if only demand data is used and after that for the case if forecast and demand data are used. For both analysis, the same cleaned data sets are used as described in chapter 3.

$$AC_S = \frac{\sum_{i=1}^{N} \left(\frac{E[IOH_i(av)]}{\mu_i}\right)}{N} \tag{A.1}$$

The results of the average weekly coverage of the different inventory approaches are shown in Table A.2. All three approaches realize a lower average weekly coverage of the system and have lower values for the quantiles compared to the current inventory policy. However, the maximum weekly coverage found for an SKU is larger compared to the current policy. This is probably caused by the combination of a relative high standard deviation and high lot-size for a few SKU's. Relatively high value for these variables could cause a high reorder level which subsequently can cause a high average inventory on-hand. Currently, the inventory department does not incorporate the standard deviation as a variable for the inventory decision making process, which could explain the differences between the presented and current approaches.

Table A.2: Average weekly coverage comparison between all approaches

Inventory policy	*Average*	*St.dev*	*0%*	*25%*	*50%*	*75%*	*100%*
Current	8,1	3,7	0,0	5,7	7,2	9,5	30,0
Single item	5,9	4,5	1,4	3,1	4,7	7,3	56,3
ABC-classification	5,6	4,0	1,4	3,0	4,5	7,0	49,9
System	5,9	4,4	1,5	3,1	4,7	7,6	47,3

Average weekly coverage entire system - forecast based

The performance of the inventory approaches on the average weekly coverage while using forecast data can be seen in Table A.3. As expected, all proposed inventory policies show a decrease in the average weekly coverage compared to the current policy. The standard deviation of the average weekly coverage increase, which can be explained due to the fact that the current policy of Van Geloven uses a fixed interval for each SKU despite it's different characteristics. The other proposed approaches calculate an individual specific interval for each and every SKU. The largest average weekly coverage for all approach can be explained by the specific behaviour of one SKU. This SKU shows a large difference between the realized average weekly demand ($\mu_{1033,D}$) and expected demand ($\mu_{1033,F}$), since they are respectively equal to 32,7 and 279,0 boxes per week. The relative high $\mu_{1033,F}$ sets a large reorder level for this SKU which results in a very high expected inventory on-hand.

Table A.3: Average weekly coverage comparison - forecast data set

Inventory policy	*Average*	*St.dev*	*0%*	*25%*	*50%*	*75%*	*100%*
Current	8,6	7.6	0,0	5,5	7.1	9,5	125
Single item	7,2	8,6	0,3	3.4	4,9	7,7	103
ABC-classification	6,7	7,6	0,3	3,3	4,8	7,3	91,6
System	7,1	8,6	0,3	3,3	4,9	7,7	105,8

Appendix B

Performance of different St.Dev

As discussed in chapter 3, alternative options are presented to replace the standard deviation of the forecast ($\sigma_{i,F}$). To find the different effects of the alternative options on the main KPI's, the other input variables ($\mu_{i,F}$, L_i, R_i, Q_i and $P^*_{2,i}$) for the DoBr-tool are kept constant. During this analysis, the alternatives of the standard deviation are only tested for the single item approach which means that for each SKU the individual target fill rate is the same. If we would use the actual average demand ($\mu_{i,D}$) and standard deviation ($\sigma_{i,D}$) of week 29 to 49, then the P^*_2 will be almost equal to the FR_S, as showed in 5. Therefore, we will implement the expected (forecast) average demand into the model ($\mu_{i,F}$) instead of the actual realized demand average ($\mu_{i,D}$) to find the effects on the FR_S when switching to forecast data. Table B.1 shows, in the first row, the performance of the inventory system while using the actual realized standard deviation ($\sigma_{i,D}$) and the average expected demand ($\mu_{i,F}$). A drop of around 0,5% is realized in the aggregate system fill rate (FR_S - P^*_2) with a total inventory costs of around 1.43 million. The drop of performance can be related to the difference between the actual average demand and the expected forecast average demand. The second row of this table shows the performance of the inventory model where the actual standard deviation ($\mu_{i,D}$) is replaced by the forecast standard deviation ($\mu_{i,F}$). Now, both parameters (μ and σ) are based on the forecast data set and the performance drop of the aggregate system fill rate can be determined.

The second row of the table shows that the aggregate system fill rate drops by an extra 4% to 93,59%. Obviously, this drop of performance for the aggregate system fill rate is too large and therefore we consider alternative options to estimate the $\sigma_{i,F}$. The first estimate, denoted by $\hat{\sigma}_{i,or}$, is based on previous sales data of a SKU(i) and it's performance is displayed at the third row of the table. The realized aggregate system fill rate (FR_S) drops from 98,2% to 97,32% with a corresponding total inventory costs of roughly 1.5 million. The second estimate, denoted by $\hat{\sigma}_{i,pl}$, is determined with the power law principle and it's performance is stated on the fourth row. The drop of FR_S is larger compared to the other estimate, with a also a lower resulting total inventory costs. The performance of the final estimate, 'the worst case scenario', is showed on the last row of the table. The performance drop is minimal, however that results in a total inventory costs that is significantly higher compared to the other alternatives. Based on these results, only the left-hand side of Table B.1, the last estimate would be the best option as the lowest drop in customer service level would be realized.

Table B.1: Performance of different alternatives of the standard deviation

Alternatives of st.dev	P_2^*	FR_S	*Total inventory costs*	*New* P_2^*	*New* FR_S	*New total inventory costs*
$\sigma_{i,D}$	98,20%	97,72%	€1.431.424	98,64%	98,20%	€1.504.463
$\sigma_{i,F}$	98,20%	93,59%	€1.345.744	99,996*%	98,20%	€2.508.849
$\hat{\sigma}_{i,or}$	98,20%	97,32%	€1.504.485	99,04%	98,20%	€1.678.098
$\hat{\sigma}_{i,pl}$	98,20%	94,01%	€1.209.289	99,97%	98,20%	€1.905.495
$\hat{\sigma}_{i,h}$	98,20%	98,19%	€1.716.354	98,24%	98,20%	€1.723.843

* An extra decimal is added on purpose, otherwise the fill rate would be rounded up to 100%

However, the total inventory costs is also an important KPI and is in the above argumentation not considered. Therefore, Table B.1 is extended and the right hand side shows the result of a second analysis. This analysis shows, for every alternative, how high the lower bound of the aggregate system fill ($P_{2,i}^*$) rate must be set to achieve a customer service level of 98,2% and it's resulting total inventory costs. In this way, we can evaluate which alternative minimizes the total inventory costs while achieving the same aggregate system fill rate as the actual performance of the inventory system during the last twenty weeks of 2021 (98,2%). It is found that the estimate based on the previous realized sales data $\hat{\sigma}_{i,or}$ (week 1 to 29) leads to the lowest total inventory costs while achieving the target aggregate system fill rate. This alternative will therefore be used for the analysis of the proposed inventory approaches the gap analysis.

Appendix C

Additional analysis on subsystem with multiple stocking locations

In order to achieve a minimal fill rate of 98,0% for the FR_{sub} of the first discusses subsystem, the lower bounds ($P_{lower,i}$) of the multi-component SKU's must be increased. Table C.1 shows the increased fill rate targets of the individual components to achieve a 98,0% fill rate at each stocking point. Table C.2 shows the resulting bounds for the multi-component SKU's. The inventory costs increases to €7.240 and the range of the fill rate of the subsystem equals [98,0 - 98,96]%.

Table C.1: Performance of components & single-component SKU's - $FR_{sub,lower} \geq 98,0\%$

SKU	*Component(s)*	$P^*_{2,\{i,j\}}$	$P_{2,\{i,j\}}$	*Inventory cost per period*
-	c1	99,93%	99,49%	€1.218
-	c2	99,99%	99,50%	€744
-	c3	99,94%	99,49%	€1.035
-	c4	98.89%	99,50%	€580
-	c5	99,91%	99,49%	€1.157
1126	{1}	99,33%	98,00%	€1.566
9930	{5}	94,83%	98,00%	€940

Table C.2: Performance of multi-component SKU's - $FR_{sub,lower} \geq 98,0\%$

Assembled SKU's	*Components*	*Resulting* $P_{lower,i}$	*Resulting* $P_{upper,i}$
9552	{1,2,3,5}	98,0%	99,49%
9559	{1,3,4,5}	98,0%	99,49%