MASTER

Comparison of three different pension contracts

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Comparison of three different pension contracts

Master Thesis

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Abstract

The goal of this report is to compare three different pension contracts. We want to compare the IRR-ambition-contract, suggested by Achmea, the current FTK-contract and the 2019-ambition-contract. Based on simulation, we will compare each contract using the certainty equivalent, adjustment factor and market value. The results show that no contract performs better for all three performance measures. The 2019-ambition-contract does best based on the certainty equivalent, so the highest overall utility is reached using this contract. However, the IRR-contract does better when we look at the adjustment factor and the market value. We saw that the IRR-contract has the desirable feature that the pension of the retirees is very stable, regardless this comes with a more volatile pension for the younger working class. We concluded that the 2019-ambition-contract outperforms the FTK-contract, but not necessarily the IRR-ambition-contract. Based on the objectives a fund wants to meet, the 2019-ambition-contract or IRR-ambition-contract will be preferred.

Contents

Chapter 1

Introduction

This master thesis is conducted at Achmea. Achmea is a well known Dutch company which provides financial services and which is specialized in insurances. It insures healthcare, damage of property and income of approximately 10 million Dutch citizens. But also provides travel insurance, life insurance and many other insurance products. Achmea focuses on simple insurances in which they handle premiums responsibly. In Figure 1.1 we can see the brands of Achmea which provide these products.

Figure 1.1: Brands of Achmea.

1.1 Achmea Pensioenservices

As briefly mentioned Achmea offers a lot of different types of insurance products to their customers. However in this report we will only focus on one product namely the insurance of old age income, which is also known as retirement pension. Within Achmea there are groups specialized on pensions services. These pension services include old age pension and survivor's pension. The group providing these pension services is called Achmea Pensioenservices. Achmea Pensioenservices can be divided in sub-components as can be seen in Figure 1.2. Each sub component has a specific function, we will look further into the component Strategy, Proposition and Advice.

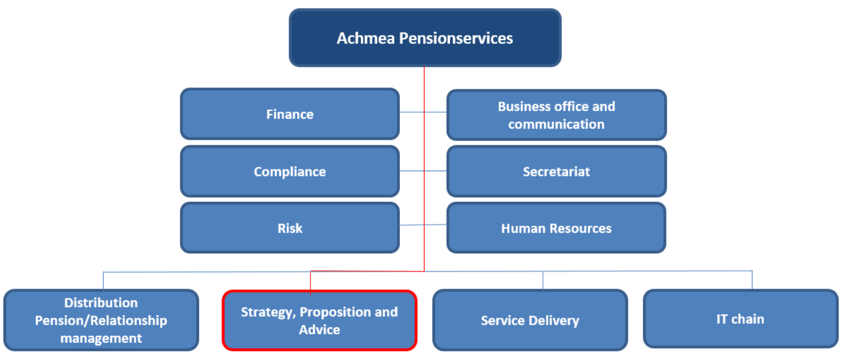


Figure 1.2: Organisation chart of Achmea Pensioenservices.

Strategy, Proposition and Advice helps with product management, product innovation, pension communication, legal affairs and many other pension related topics which can be seen in Figure 1.3. This thesis is conducted for the department which is called actuarial advice and ALM(Asset Liability Management). This department advises decision-makers such as pension funds and companies to make better informed decisions with as goal to make a balanced decision which protects the interest of participants within the pension fund. They also help managing the use of assets and cash flows to reduce the pension funds risk of loss.

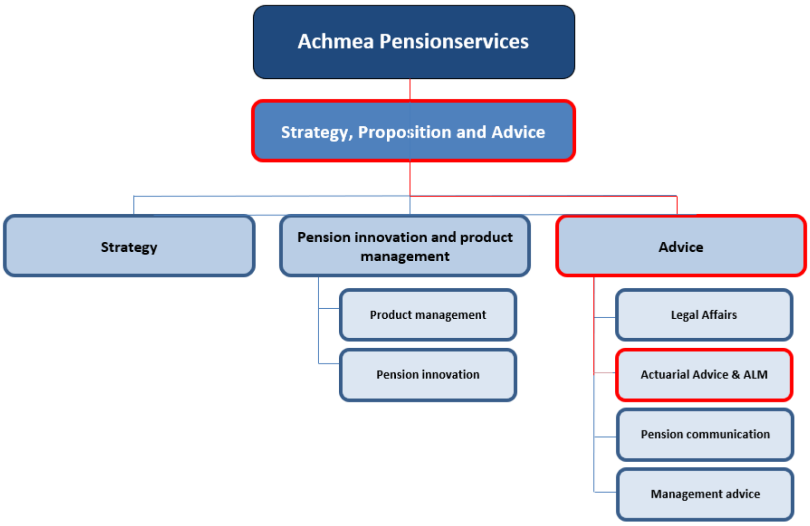


Figure 1.3: Zoomed in organisation chart of Achmea Pensioenservices.

1.2 Actuarial Advice and ALM

The department Actuarial Advice and ALM advises decision makers to make better informed decisions to protect the interest of participants within the pension fund. To do so, they have to follow certain guidelines and regulation. The guidelines and regulation can be found in the Dutch Pension Law[19]. The Dutch Pension Law states the holding regulations regarding pensions. A part of the Pension Law regards the Financial Assessment Framework, also known as FTK, based on the Dutch term "Financieel Toetsingskader". The Financial Assessment Framework can be seen as a contract, therefore we will often refer to it as the FTK-contact. The legal financial requirements of pension funds are recorded in the Financial Assessment Framework[18]. It states terms concerning the financial well-being of pension funds and is based on principles of market valuation, transparency and risk-based financial requirements. The goal of the FTK-contract is to create stability within pension funds in response to the large fluctuations within financial markets. So, having a well designed FTK-contract benefits the participants of the pension fund but also the fund itself. Therefore, it is interesting for Achmea to help improve the FTK-contract.

1.3 Problem description

The economic crisis of 2008 lead to a significant loss of capital by pension funds. Funding ratios dropped significantly. Pension funds ended up with a solvency problem, meaning the amount of assets of the fund were to low relative to the amount of liabilities. So, the payout of future pensions were in danger. This raised several questions about the pension funds, but also whether the pension contract was correct. Achmea Pensionservices works together with the government, universities and research groups like Netspar on new ideas for a better pension contract. The current guideline for pension systems in the Netherlands is the Financial Toetsingskader contract, also known as FTK-contract. However, the FTK-contract has a too strict regulation and shows vulnerabilities concerning the rising life-expectancy, changing labour market and financial markets. In 2019, social partners together with the government suggested a new pension agreement. We will call this the 2019-ambition-contract. However, many doubt that the 2019-ambition-contract will hold, including several people of Achmea Pensionservices. They think the agreement still has a too strict regulation. Providing more freedom will in their opinion benefit the funds, but more importantly the participants. Some people propose the IRR-ambition-contract, leading to this report.

In this report we will discuss the effect of each contract on the pension payments. We start by introducing the new IRR-ambition-contract. We compare the IRR-ambition-contract and the 2019-ambition-contract with the current FTK-contract. We compare based on three common used criteria. Namely:

1. The certainty equivalent as in [11], [5] and [13]. The certainty equivalent of a pension payment is a guaranteed value that someone would accept now, rather than taking a chance on a higher, but uncertain pension payment. This helps with comparing the value of the pension payments. The certainty equivalent can be designed such that it puts more weight on low pension payments in comparison to a high pension payment, depending on the risk-aversion parameter.

2. The adjustment factor as in [5], indicates how much the pension entitlements of a person change due to indexation. This is an important measure to check the stability of the pension of a retiree.

3. The market value, as used in [11], [16] and [15]. The market value indicates the shift in capital between generations, due to a change in pension contracts. This gives knowledge about possible classes of the society which benefit from the transition between pension contracts and possible classes which disadvantage from this transition.

We use these three comparison methods, because together they provide a thorough image of the distribution, value and stability of the pension payments given a pension contract. A more elaborate description of each comparison method will be given later on.

The report will be structured as follows. In Chapter 2 we explain the basic concepts of a pension fund. In Chapter 3 each contract will be introduced. Next, we will describe the assumptions and data required to simulate the process. This is done in Chapter 4. Chapter 5 will elaborate more on how a pension fund will work under the current FTK-contract and the 2019-ambition-contract. These contracts are mathematically very similar. The IRR-ambition-contract will be explained in Chapter 6. Chapter 7 shows the results of each contract, which we will discuss in Chapter 8. Furthermore, we added Appendix A and Appendix B to the report. Appendix A explains common used actuarial and financial terms. Appendix B contains large tables and more detailed pictures of the results. Appendix B also contains pictures which help to describe mathematical techniques.

Chapter 2

Basic pension fund model

Before we introduce the contracts and look at the mathematics needed, we will introduce some basic pension fund knowledge. We will start by looking at the most basic view of a pension fund. Next, we will expand this view, but look from the perspective of the pension fund. We will do this by providing a step by step explanation of how a pension fund works. Based on this explanation, we will introduce some simple mathematical formulas to clarify the concepts introduced. In this chapter, we will use actuarial terms, some of these terms might be uncommon to the reader. If so, we refer to Appendix A to clarify the meaning of these terms.

2.1 Basic pension fund

We start by looking at the most basic view of a pension fund. We do this by only taking into account several factors which influence the process from participant to the pension fund and vice versa. To do so, we will first look at Figure 2.1. In Figure 2.1 there is a division between three components, namely the participant, the pension fund and investment. This is the simplest way to describe a pension fund. Participants which are not yet retired pay premium to the pension fund. The pension fund will then invest his capital, which contains these premium payments, to receive a return on investment. This process increases the capital of the pension fund. However, the pension fund does also have to pay out pension payments to retired participants which decreases the capital of the pension fund. This gives the basic idea of how a pension fund works.

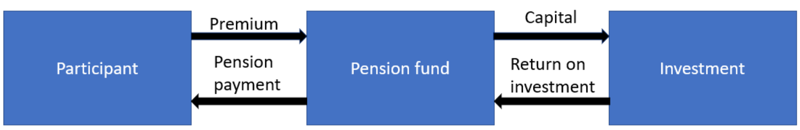
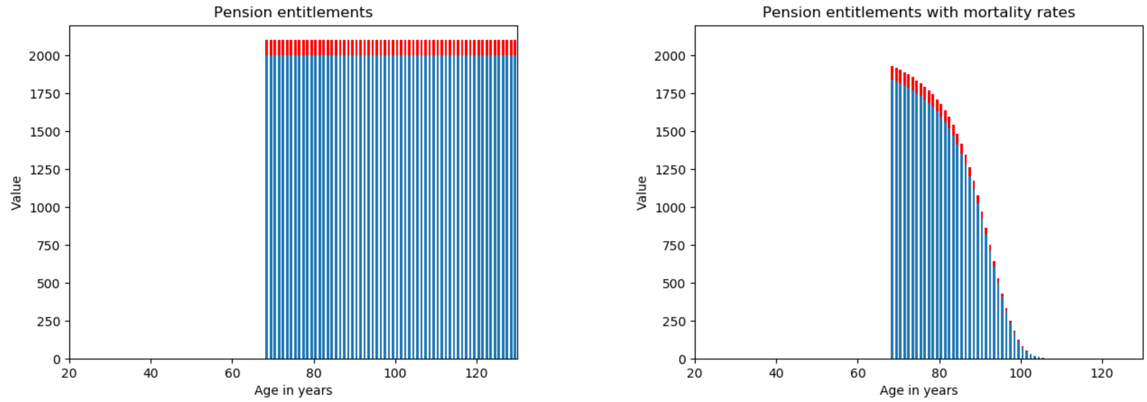


Figure 2.1: Basic pension fund.

We will now expand the basic idea of what happens in a pension fund by describing the steps needed to turn a premium payment into a pension payment. We assume that premium payments are constant. The pension fund buys pension entitlements for the participant based on these premium payments. The amount of pension entitlements which can be bought depend on the purchase rate. The purchase rate helps to convert the premium payments into an amount of life long pension, which is indicated by the pension entitlements. So, the pension fund has capital at this moment in time, which needs to be used to pay pension payments in the future. These future pension payments can be seen as liabilities. In Figure 2.2 we can see an example of how this works. In Figure 2.2a we can see the pension entitlements of a participant of age a. This participant is entitlement to €2000 of life long pension, this is indicated in blue. As the participant ages, he will keep paying premium payments to the pension fund. These premium payments will increase his pension entitlements. So, at an age of $a+1$ this participant might be entitled to €2100 of life long pension. The added value of €100 can be seen in red.

Let us assume that participants pay premium payments, from an age of 20-67, to the pension fund and receive pension payments from an age of 68-130. The latter is expressed in Figure 2.2. However, there is a distinction between the left and the right figure. In Figure 2.2a we look at the pension entitlements, while in Figure 2.2b we look at the pension entitlements adjusted with a persons mortality rate in a given year. We make this distinction to highlight the difference between the liability cash flow and the pension entitlement cash flow. A pension fund should have sufficient money to payout the liability cash flow, not the pension entitlement cash flow. The liability cash flow of the pension fund contains the expected value of the upcoming payments of all participants over a time horizon of T years. In Figure 2.2 we showed the impact of mortality rates, a participant does only receive pension payments when he is actually alive. However, there are many other factors which help to convert the pension entitlement to the pension liabilities. An example of such factor is the interest rate, which helps with pricing the liabilities. We will explain this more thoroughly in Chapter 5 and 6 based on the pension contract at state.



(a) Standard. (b) Adjusted with mortality rates.

Figure 2.2: Pension entitlements.

2.2 Step by step explanation

Up to now, we have sketched a picture of how premium payments of a participant are converted to their pension payments. However, from now on we will start looking from the viewpoint of a pension fund. To do so, we will first recognise three different moments in time which occur every year for the pension fund. These moments correspond to three different values of the pension entitlements of the participants. Namely, at the start of the year, in which we indicate the pension entitlements by PE_{start}. Next, after the participants have paid their premium payments and the fund bought the corresponding pension entitlements based on these premium payments. We call this moment in time, the moment before indexation, in which the pension entitlements are indicated by PE_{before}. At last, the moment after indexation, in which we call the pension entitlements PE_{after}. The yearly step by step explanation of the pension fund then becomes:

1. Calculate the value of the pension liabilities corresponding to PE_{start}, we call these $PL_{entitlements}$.
2. Next, calculate the value of the pension liabilities corresponding to the premium paid this year, we call these $PL_{premium}$.
3. Adding $PL_{premium}$ and $PL_{entitlements}$ we get the total liability cash flow before indexation, also known as PL.
4. Update the value of the pension assets, also known as capital, which we will describe as PA.

5. Calculate the funding ratio FR, based on PL and PA.
6. Index based on the funding ratio, this will lead to a new value of the pension liabilities, which we call $PL_{indexed}$.
7. Calculate the value of the pension entitlements before and after indexation based on PL and $PL_{indexed}$.

In the above step by step explanation, we explained the steps which a pension fund has to take every year to convert premium payments into pension entitlements. However, in this process it is important that the fund keeps a sufficient asset to liability ratio, which is described by the funding ratio. To maintain an appropriate funding ratio indexations are applied on the pension liabilities when needed. Since, the pension liabilities are correlated to the pension entitlements the value of the pension entitlements are also changed by the indexations. The value of the pension entitlements after indexation will be used as the starting value of the pension entitlements for the upcoming year. This process will be repeated.

As the step by step explanation indicates, there is a division within a pension fund into two sides. Namely the liabilities and the assets. Earlier, we described the liabilities of a pension fund by a cash flow of future payments and the assets of the pension fund by the amount of capital the pension fund has. We will now describe this split in more detail. We will look at the factors which impact the change in liabilities and in assets. We start by looking at the pension liabilities in the next section.

2.3 Pension liabilities

The liabilities a pension fund has to their participants change throughout the year. Pension liabilities are influenced by premium payments and pension indexations or benefit cuts. They also change on a yearly basis through the change of mortality rates, interest rate curves and ageing of the participants. The liabilities from a pension fund are based on the liabilities they have to each participant. The liabilities per participant are shown in the following equation:

$$PL_n = \sum_{t=0}^{T} L_{n,t}, \tag{2.1}$$

in which PL_n is the value of the pension liabilities of participant n. This value consists of the sum of liabilities $L_{n,t}$ which the pension fund will have to pay to participant n at time t. So, in case participant n is 68 year old, $L_{n,t}$ will represent the value of the liability which has to be paid in t years from now to this participant. An example of what $L_{n,t}$ might look like can be seen in Figure 2.2b, however note this only contains the mortality rates, to calculate the value of the liabilities we should take into account other factors, which will be explained later on.

2.4 Pension assets

A pension fund also has pension assets, aka capital. This capital should be sufficient to pay out the pension liabilities. The value of pension assets changes due to premium payments by participants and pension payments to participants. But a pension fund can also do several things to increase its capital and protect its capital-liability ratio themselves. The capital-liability ratio is described by the funding ratio. A pension fund can invest its capital in bonds and assets in order to increase the capital and ensure that pension liabilities can be met. They can also use interest rate hedges to protect the capital to liability ratio. This will be discussed in more detail later on.

2.5 Basic pension fund

In the previous sections of this chapter we explained the concept of the liabilities and assets of a pension fund. We also gave a step by step explanation of how a pension fund works on a yearly basis. We will now summarize all this information and introduce the basic formulas to describe this process. We will again, briefly indicated what influences the pension fund throughout the year. We will use both the liabilities and assets to sketch a picture of how a pension fund works.

The yearly change within a basic pension fund can be described as follows. Every participant starts with an amount of pension entitlements, PE_{start}, at the start of the year. These entitlements will be discounted such that we get the individual cash flows of the liabilities. The sum of these individual cash flows will give the liability cash flow of the pension fund, which is equal to:

$$PL_{entitlements} = \sum_{t=0}^{T} \sum_{n=1}^{N} L_{n,t} \tag{2.2}$$

Next, each participant will then pay premium to the pension fund. The premium will be converted to new pension entitlements, which we named PE_{before}. PE_{before} indicated the value of the pension entitlements before indexation. However, to calculate PE_{before} we have to know how much pension entitlements can be bought for €1 of premium. Therefore, we used the purchase rate. The purchase rate indicates the price of €1 of life long pension. So, it can be used to convert premium payments into pension entitlements.

The new pension entitlements will be discounted and will increase the liability cash flow by $C_{n,t}$. The value and calculation of $C_{n,t}$ will be explained per pension contract in the upcoming chapters. Note, that this value is strongly dependent on the purchase rate. The value of the pension liabilities of the fund then changes to:

$$PL = PL_{fund} = \sum_{t=0}^{T} \sum_{n=1}^{N} L_{n,t} + C_{n,t} \tag{2.3}$$

To get in line with the notation used later on we will define $PL^n_{before,t,y}$ as the value of the pension liabilities before indexation. This will be equal to:

$$PL^n_{before,t,y} = L_{n,t} + C_{n,t} \tag{2.4}$$

In (2.3), we see step 3 of the process, which adds the liability cash flow of the entitlements and the premium payments. So, we have now seen the change due in liabilities due to the premium payments. Thus, we can start to look at the pension assets. We look at the pension assets, because a pension fund will always steer based on a relation between pension assets and liabilities. Currently, they use the funding ratio for this. So, we will do the same in this basic model. The funding ratio (FR) is equal to:

$$FR = \frac{\text{Pension Assets}}{\text{Pension Liabilities}} \tag{2.5}$$

To calculate the funding ratio, we will calculate the value of the pension assets. The value of the pension assets of the previous year is known. To get the value of the current year, we will have to calculate the change due to pension payments, premium payments, Return on Investment(RoI) and a possible Interest Rate Hedge(IRH). The premium payments and pension payments are self-explanatory, however the return on investment and the interest rate hedge are not. The calculation of these two, will be explained per contract in the upcoming chapters. However, we can already introduce a basic formula to update the pension assets, based on an ultimo year payments. This gives:

$$PA_{\text{current year}} = PA_{\text{previous year}} \cdot (1 + RoI) + \text{premium} - \text{expected payments } + IRH \tag{2.6}$$

Now, we know the value of the pension assets and the value of the pension liabilities before indexation, we can calculate the funding ratio. Based on the value of the funding ratio and the features of the pension contract used an appropriate indexation or cut will be applied to the pension liabilities. The possible contracts will be introduced in Chapter 3. After indexation, we can recalculate the pension entitlements based on the changed pension liabilities, giving PE_{after} which represents the pension entitlements after indexation. The pension entitlements after indexation will then be used to calculate the pension liabilities of next year. To do so, we have to account for the change in age and mortality rates.

This process will be repeated yearly and depends highly on how pension entitlements are bought, pension liabilities and assets are calculated and updated. But also on how the steering mechanism for indexations and benefit cuts are designed. In the upcoming chapters we will specify the different contracts and techniques needed in each contract to perform these yearly calculations. In addition to that, we will also introduce the mathematics needed for the three comparison methods, namely the certainty equivalent, adjustment factor and market value.

Chapter 3

Contracts

In this chapter the different contract will be compared. To do so, we will first introduce each pension contract. We will explain their characteristics and emphasize the parts which are different. Next, we will make the assumptions needed to make a fair comparison between contracts.

3.1 Current FTK-contract

The FTK-contract described below is a simplified version of the real FTK-contract. We will set the indexation boundaries such that they correspond to a pension fund which represents the average of the Netherlands. These boundaries are also used by De Nederlandsche Bank in [5]. Financial shocks are spread over a period of 10 years, which is conventional in the Netherlands. The FTK-contract used is then as follows:

Rules for positive indexation:

- At a funding ratio between 110% and 125% pension entitlements increase linearly with price inflation, in which 110% corresponds to 0% of the price inflation and 125% with 100% of the price inflation. Note, this only holds whenever the price inflation is positive.
- At funding ratios above 125% pension entitlements increase with 100% of the price inflation and $1/5^{\text{th}}$ of the funding ratio above 125%.

Rules for benefit cuts:

- The funding ratio is not allowed to be under 104.2% for longer than 5 years, otherwise pension entitlements will be unconditionally reduced over a period of 10 years to bring the funding ratio back to 104.2% immediately.
- Whenever the funding ratio drops below 95% pension entitlements will be conditionally reduced over a period of 10 years, to increase the funding ratio to 95%.

In both indexation cases the excess/deficit amount will be distributed uniformly over all participants independent of their age and accumulated pension.

Rules regarding the premium policy:

- Basic assumption: purchase of pension entitlements at corresponding market interest.
- Constant premium of 22% of the pension basis.

Remark that unconditionally reduced over a period of 10 years implies that the negative indexation can be directly traced back to the cash flows corresponding to the 10 upcoming years. An example will be given later on in Table 5.1. Conditionally reduced over a period of 10 years, means that you implement the first negative indexation and then revise the situation next year to see what has changed. The market interest is based on the DNB-UFR curve[1], in which UFR stands for 'Ultimate Forward Rate'. Meaning, an adjustment is made to the DNB-curve for the longer maturities.

3.2 2019-Ambition-contract

Rules for positive indexation:

- At a funding ratio between 100% and 120% the pension entitlements are increased with $1/10^{\text{th}}$ of the funding ratio above 100%.
- At funding ratios above 120% pension entitlements are increased with $1/10^{\text{th}}$ of the funding ratio between 100% and 120% and $1/5^{\text{th}}$ of the funding ratio above 120%.

Rules for benefit cuts:

- At a funding ratio below 100% the pension entitlements decrease by $1/10^{\text{th}}$ of the difference between the funding ratio and a funding ratio of 100%
- The funding ratio is not allowed to be under 100% for longer than 5 years, otherwise pension entitlements will be unconditionally reduced over a period of 10 years to bring the funding ratio back to 100% immediately.
- Whenever the funding ratio drops below 90% pension entitlements will be unconditionally reduced over a period of 10 years, to increase the funding ratio to at least 90%.

In both indexation cases the excess/deficit amount will be distributed uniformly over all participants independent of their age and accumulated pension.

Rules regarding the premium policy:

- Basic assumption: purchase of pension entitlements at corresponding market interest.
- Constant premium of 22% of the pension basis.

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3.3 IRR-ambition-contract

Within the IRR-ambition-contract there are several steering mechanisms possible. Namely:

- A fixed internal rate of return
- A fixed accrual percentage corresponding to the pension base
- A fixed difference between Expected Return and Internal Rate of Return
- A fixed level of certainty concerning being able to pay their pension liabilities.
- A fixed ambition to compensate for example price inflation or economic growth through indexation.

In this report, we will only consider the options for which we can assume a constant premium of 22% of the pension basis. So the IRR-ambition-contract becomes:

Rules for indexation:

- Funding ratio no longer key.
- Depending on the pension funds goals pension entitlements are either increased or decreased. A pension fund can steer towards:
  - A fixed internal rate of return

  - A fixed difference between Expected Return and Internal Rate of Return
  - A fixed ambition to compensate for price inflation

Financial shocks are no longer spread over a period of at most 10 years, but directly absorbed by the current pension liabilities.

Pension indexations will be distributed over all participants according to their age and pension. The specific way of distributing the pension indexations will be determined with the IRR-algorithm[17] which will be explained in Section 6.7. Note that pension indexations are no longer uniformly distributed among participants.

Rules regarding the premium policy:

- Basic assumption: purchase of pension entitlements at corresponding internal rate of return.
- Constant premium of 22% of the pension basis.

3.4 Main differences between contracts

As the name of this section indicates we will briefly highlight the main differences between each of the contracts.

FTK-contract vs. 2019-ambition-contract:

As we can see in Section 3.1 and 3.2 the FTK-contract and 2019 Ambition-contract are very similar. They both buy pension entitlements against the corresponding market interest. Furthermore, they have a similar system for indexation and cuts. The difference within these systems, are primarily the funding ratio for which to start indexing/cutting. The FTK-contract is more risk averse, and will start indexing at a funding ratio of 110%, while the 2019-ambition-contract already starts at 100%. The same holds for cutting benefits, the FTK-contract will start reducing benefits at higher funding ratios. Another small difference is between the proportion of the indexation values. In both contract the pension entitlements change based on the funding ratio, but in the FTK-contract there is a relation with price inflation. In the 2019-ambition-contract this is no longer the case, for this contract it will only depend on the funding ratio.

FTK-contract and 2019-ambition-contract vs IRR-ambition-contract:

The IRR-ambition-contract varies from the FTK/2019-ambition-contract several ways. Both the FTK-contract and 2019-ambition-contract are driven by the funding ratio. These contracts are designed to keep a buffer in case of funding ratios above 100% to decrease the probability of benefit cuts. While also spreading financial shocks such that whenever benefit cuts occur they tend to be small and take place more gradually. This helps to protect the benefits of the retirees. In case of the IRR-ambition-contract the funding ratio is no longer the driving factor. The IRR-ambition-contract has the internal rate of return as main driver. This contract will keep no buffers, and will always directly absorb indexations and benefit cuts. This will make indexations take place more frequently and perhaps pension entitlements more volatile.

Within the FTK-contract and the 2019-ambition-contract indexations and benefit cuts are always uniformly distributed among participants. In case of benefit cuts, this brings along a risk for the retirees. However, this is not the case in the IRR-ambition-contract. In this contract the indexations and benefit cuts are no longer uniformly distributed among participants. Younger people will take a larger share of the indexations and cuts to protect the retirees, but also benefit from this due their long horizon until retirement age.

The last main difference lies in how pension entitlements are bought. In the FTK-contract and 2019-Ambition-contract the pension entitlements are bought against the interest curve, while in the IRR-ambition-contract pension entitlements are bought against the internal rate of return. A problem of buying pension entitlements against the interest rate curve is that conversion of pension premiums into new pension entitlements will be very expensive when the interest rate is low, while they may be very cheap when the interest rate is high. This can lead to good luck and bad luck situations[2]. Buying pension entitlements against the internal rate of return protects against this unfairness.

Chapter 4

Simulation

In this chapter the assumptions which are needed to make a comparison between contracts are introduced. We consider three types of assumptions, namely assumptions regarding the data sets, assumptions simplifying the simulation and assumptions related to specific calculations based on the input data. Every contract will start with the same input variables to make sure no contract starts with an advantage.

4.1 Input data

To goal is to compare the three contracts. To do so, we have to make sure every contract starts with the same input variables. The following data sets have been used:

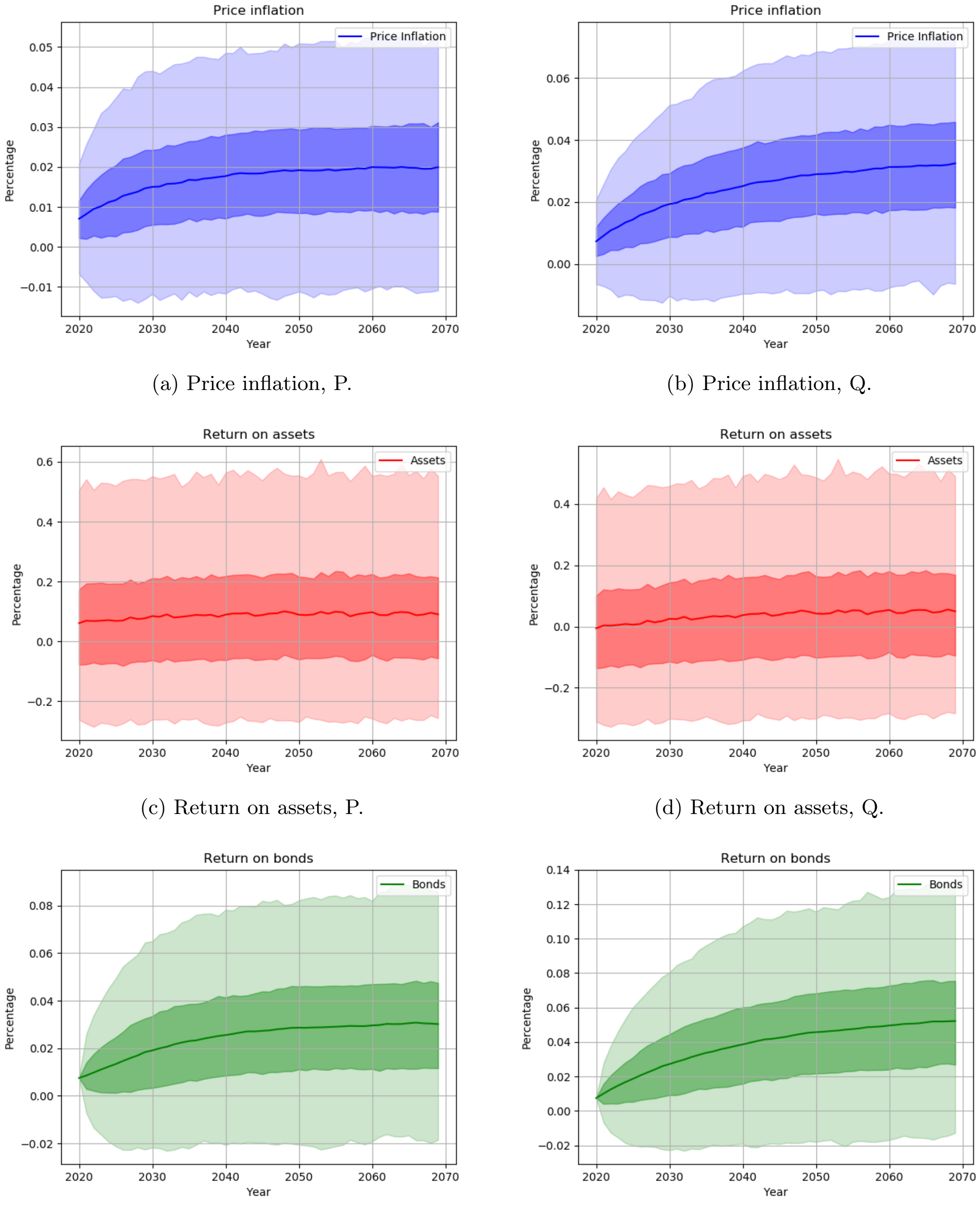
- The participant data is based on the CBS data set [3]. This data describes the number of participants according to a certain salary and age. The data is shown in the first four columns of Table 4.3. For simplicity we assume that every participant is a male, and born in January. Furthermore, we will set the age of each group to the average age of that group, meaning the age of a person from the 25-35 year old age group will be set on 30.
- The economic data is based on the KNW-capital model from [7]. The use of the KNW-capital model is based on the advise of the commission of parameters [10]. The economic data concerns price inflation, return on investment, state variables and nominal-interest parameters.
- The mortality data is based on the CBS prognosis table of 2018. An example of such a table is given in [4]. This table will contain forecasted mortality rates based on data up to and including 2018.

The KNW-capital market model is used to generate a real world uniform scenario set and a risk-neutral uniform scenario set which enables comparable feasibility tests of pension funds. In the world of pension, it is very common to work with scenario sets. As mentioned in the introduction, pension funds have to meet certain standards which can be found in the Dutch Pension Law and the FTK-contract. To check whether a fund meets these requirements, they need to use the scenario set provided by the pension regulator. Thus, it is mandatory for pension funds to work with these scenario sets, but it is also convenient. Since, everybody works with the same scenario sets, results can be easily compared. Therefore, we also use these scenario sets. The real world scenario set will be used to calculate the certainty equivalent and adjustment factor, while the risk-neutral scenario set is used to calculate the market value. The risk-neutral uniform scenario set is used to calculate the market values, because this scenario set excludes arbitrage. Therefore, arbitrage cannot be used to give a favourable market value to one contract over another. The model describes the stock and bond market. Net benefits of pensions can be considered as a derivative of bonds and equity, because both benefits and premiums depend on investment results. This capital model can appropriately evaluate derivative products. This model is very stylized and models four uncertain variables (two concerning interest, one for inflation and one for stocks) and two state variables. These variables are commonly used in the Netherlands, and we will use these estimations [7]. The estimated variables and their format is shown in Table 4.1. As the table indicates there are 2500 scenarios generated. Each scenario represents a different state of the economy, some states will represent possible economies in which the economy is thriving, while in other scenarios the economy is doing very poorly, this is also called a bad weather scenario. The table indicates that there are estimations of the stock returns and price inflations for 100 upcoming years. The nominal interest parameters are estimated for 101 possible times.

Description	Size	Point of time
Stock returns	2500x100	all
Price inflation	2000x100	all
State variable 1 (X1)	2500x101	all
State variable 2 (X2)	2500x101	all
Nominal interest-parameter "a"	101x1	-
Nominal interest-parameter "b"	101x 2	-

Table 4.1: Scenario set variables used to calculate the interest curve.

In Figure 4.1 and Figure 4.2, we plotted several aspects of the different scenario sets. We refer to the real world scenario set as P, and the risk-neutral uniform scenario set as Q. We can see several differences between the data sets. First, looking at the price inflation, we can see that the real world set and risk-neutral start with the same values, however in the risk-neutral set the price inflation increases faster than the real world set. When we look at the return on assets, the real world set starts at a higher value. This value changes slightly over time. Looking at the risk-neutral set, we can see a clear increase in the return on assets, this was not the case in the real world set. For bonds, we can see the same happening as with the price inflation. Both sets start at a similar value, however the return on bonds in the risk-neutral set increases faster than in the real world set. This can be explained by looking at Figure 4.2 in which we plotted the 1-year, 10-year and 30-year interest rate for both scenario sets. As the figure indicates, both scenario sets start at the same interest rate, but the interest rate of the risk-neutral scenario set increases faster than that of the real world scenario set. This holds for the 1-year, 10-year and 30-year interest rates.



(a) Price inflation, P.

(b) Price inflation, Q.

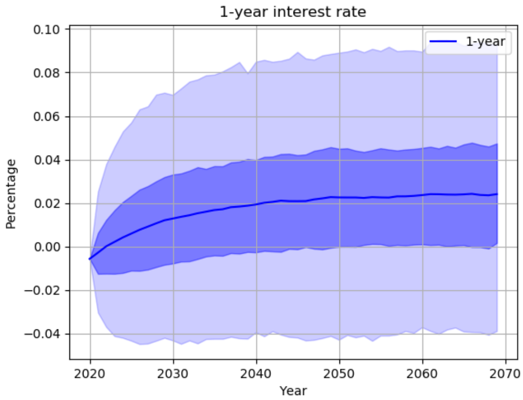
(c) Return on assets, P.

(d) Return on assets, Q.

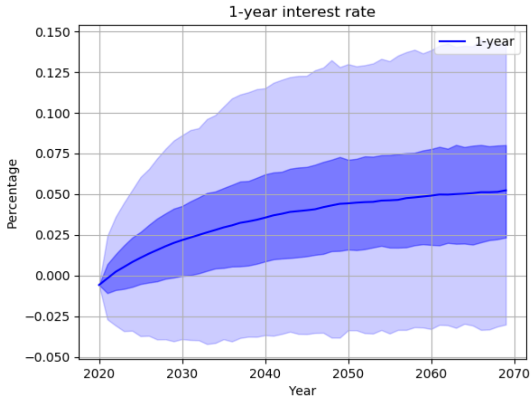
(e) Return on bonds, P.

(f) Return on bonds, Q.

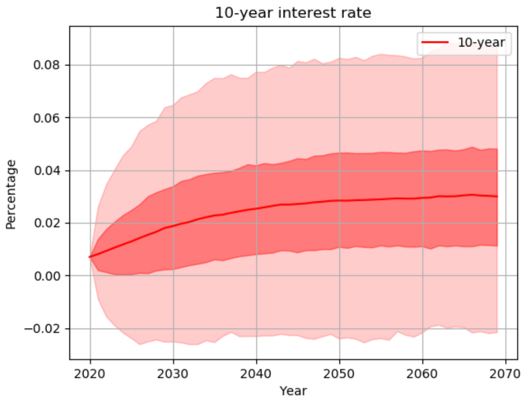
Figure 4.1: Scenario set plots, which show the mean, 25%-75% and 2.5%-97.5% confidence bounds.



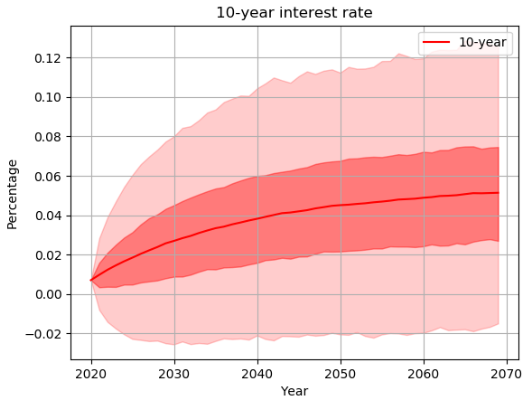
(a) 1-year interest rate, P.



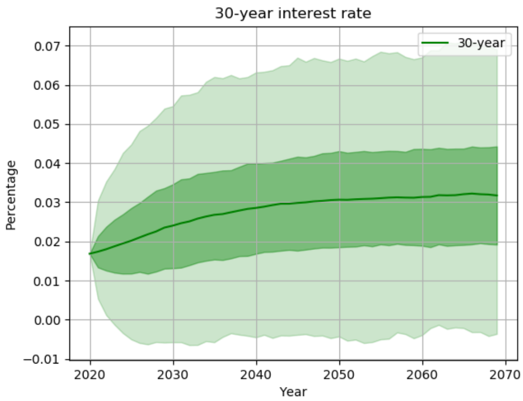
(b) 1-year interest rate, Q.



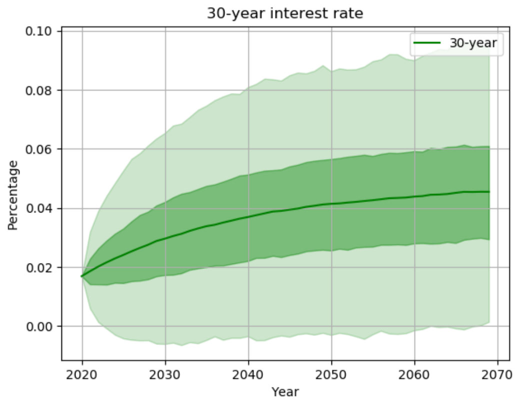
(c) 10-year interest rate, P.



(d) 10-year interest rate, Q.



(e) 30-year interest rate, P.



(f) 30-year interest rate, Q.

Figure 4.2: Scenario set plots, which show the mean, 25%-75% and 2.5%-97.5% confidence bounds.

4.2 Simplifications

To compare the contracts, we used some simplifications in the simulation. These simplifications are as follows:

- Every participant in the pension fund is assumed to be working full time from an age of 20 until 68. The pension entitlements accrued are corresponding to this.
- We only consider "ouderdomspensioen", also known as Old age Pension(OP). This corresponds to premium payments of the participants equal to 22% of the pension base.
- The portfolio mix will be static throughout time, and will consist of 40% assets and 60% bonds. This is in line with the optimal portfolios used in [5].
- We assume that premium payments paid by the participants to the pension fund take place once a year. This will be ultimo year. In reality the fund receives premium and pays pension payments monthly.
- The pension payments will also take place ultimo year, once a year. However, these will take place after indexation. So, this can also be seen as paying out the pension payments at the start of every new year, before anything has happened.
- We will simulate 50 years into the future. We do this because this ensures a generational shift from all working people to retirees.
- The cash flow of future payments of each participant will have the same length as the maximum possible duration based on the KNW data.

The above assumptions are all made to simplify the simulation. We will elaborate on these assumptions. First, we assumed that every participant works full time from an age of 20-68. This assumptions is made because there is no difference within the accumulation of pension entitlements for someone working part-time compared to someone working full time with the same salary. However, this assumption also covered that a participant do not become unemployed. We leave out unemployment, because this simplifies the simulation, without having a great impact on the results of the simulation. The same goes for the fact that we only consider Old Age pension, and thus ignore partner pensions.

4.3 Calculated input variables

Not every variable used as input for the simulation, can be directly traced to a data set. For some variables, we first need to make some additional assumptions and do some calculations based on the input data. These assumptions and the calculations will be discussed in this section. We will start by listing the variables calculated based on the input data. These are:

- The starting value of the pension assets
- The value of the interest rate curve, during the entire simulation.
- The starting value of the pension entitlements of each participant.

The assumptions and calculations needed for each of these variables will now be discussed in the upcoming subsections.

4.3.1 Pension assets

To compare all three contracts we need to make sure each contract starts with the same amount of pension assets. However, we are considering different scenarios, in which the value of the liabilities is determined by the interest rate curve of that scenario. Therefore, we cannot simply use the same starting value for every scenario, because that would result in different funding ratios. For every scenario, we want to start with a healthy fund. Hence, we calculate the value of the pension assets based on a funding ratio of 1.10. We do this with the following equation:

$$PA = FR \cdot PL = 1.10 \cdot PL \tag{4.1}$$

The value of the pension liabilities will be calculated based on the interest rate curve in the starting year. We will come back on how to calculate the value of the pension liabilities of the pension fund in Chapter 5. The interest rate curve is based on the KNW-capital model data set and is calculated using (4.2). This equation will be explained in the next subsection.

4.3.2 Interest rate curve

To calculate the value of the liabilities throughout the simulations we need to know the interest rate curve for every simulation year. We can calculate the interest rate curve using the KNW-capital model data. The interest rate depends on the state variables and the nominal interest-parameters. The notation used to describe the state variables and nominal interest-parameters was given in Table 4.1. Combining these variables we can calculated the interest rate term structure. In (4.2) we show how to do this for scenario i ($i = 1, \ldots, 2500$) in projection year y ($y = 0, \ldots, 100$) and for duration t ($t = 1, \ldots, 101$).

$$R^i_{t,y} = e^{a^t + b^t(1)\cdot X_{i,1}(y) + b^t(2)\cdot X_{i,2}(y)} - 1 \tag{4.2}$$

4.3.3 Pension entitlements

The pension entitlements in Table 4.3 are calculated based on data. Note that the goal of an Old age Pension is to replace 70% of average pension base during retirement. Therefore, we can get a good estimation of the pension entitlements of a person based on their income, franchise and yearly increase in income. We assume the following for the franchise and increase in income:

- The franchise will be equal to €15.178, based on [14]. We consider the franchise to increase with the same factor as income increases.
- The increase of income is shown in Table 4.2.

So, we can now estimate the pension entitlements at the start of the simulation. We do this by using the following equation:

$$PE_n = 0.7 \cdot \min\left(\frac{a-20}{68-20}, 1\right) \cdot \sum_{i=0}^{68-20} (I_{n,i} - F_{n,i}) \tag{4.3}$$

In which PE_n represent the pension entitlements of person n, at age a. $F_{n,i}$ is the franchise of person n at working year i, and $I_{n,i}$ the income of person n at working year i. Note, PE_n is independent of age a because every participant has a fixed starting age a at the start of the simulation. Therefore, PE_n suffices to describe the pension entitlements of every participant at the start of the simulation. In case we look at a participant of type 14, $I_{14,20} = 35200$ and $F_{14,20} = 15178$. To get the other values we can simply increase and decrease $I_{14,20}$ and $F_{14,20}$ corresponding to the yearly increase in income given in Table 4.2. The resulting pension entitlements are given in the last column of Table 4.3.

Age	Fiscal standard
20-35	3.00%
36-45	2.00%
46-55	1.00%
56-68	0.00%

Table 4.2: Yearly increase of income, based on their age.

Participant type	Age	Income(per year)	Number of participants ($\times$1000)	Pension entitlement
1	20	24700	116.3	0
2	20	34300	63. 6	0
3	20	43900	20.2	0
4	20	61000	9.2	0
5	20	128200	0.7	0
6	20	495500	0.2	0
7	30	25500	189.3	1932
8	30	35000	250.6	3710
9	30	44500	183.9	5489
10	30	63200	193.3	8989
11	30	126200	15.4	20782
12	30	347700	1.5	62243
13	40	25400	116.4	2983
14	40	35200	172.8	5842
15	40	44800	173.3	8644
16	40	66900	337.9	15093
17	40	129400	58.8	33330
18	40	308700	8.3	85650
19	50	25300	134.6	3801
20	50	35300	192.3	7556
21	50	44800	208.6	11124
22	50	67900	439.6	19798
23	50	130400	92.4	43269
24	50	344300	19.1	123594
25	60	25300	145.2	4789
26	60	35200	188.1	9473
27	60	44700	186.3	13968
28	60	67100	351.0	24566
29	60	130200	64.1	54420
30	60	359400	12.8	162861
31	70	24700	253.0	5406
32	70	34600	173.5	11027
33	70	44500	95.5	16648
34	70	65300	117.3	28457
35	70	128900	18.1	64566
36	70	337300	2.7	182886
37	80	24400	113.2	5236
38	80	34400	58.8	10913
39	80	44400	28.9	16591
40	80	64400	28.7	27946
41	80	127800	3.1	63941
42	80	345600	0.4	187598
43	90	24400	26.4	5236
44	90	34400	13.6	10913
45	90	44400	6.6	16591
46	90	64600	6.9	28059
47	90	127600	0.7	63828

Table 4.3: Participant information.

Chapter 5

FTK- & 2019-ambition-contract

As discussed in Section 4.1 we are looking at an economic scenario set, for which we can calculate 50 years into the future. We will determine the certainty equivalent, adjustment factor and market value based on this data. In this chapter we will explain how to do this for the FTK- and 2019-ambition-contract. We will do this per scenario, so we will ignore the subscript indicating the scenario number, which is chosen to be i in (4.2). We start this chapter by an explanation of the most important used variables of the simulation, we will then explain the mathematical techniques needed in order to calculate these variables such that we can describe the simulation. At the end of the chapter, we will combine these techniques and explain how they work together in order to simulate the process.

5.1 List of variables

In the simulation we use several different variables. These variables are dependent of factors such as the prognosis year y, the participant number n, and the age of the participant a. The simulation will be based on a year-to-year process which represents the activities of a pension fund. This process will be an elaborated version of the step by step explanation of how a pension fund works, which is given in Section 2.2. In the notation we make a distinction between participants, the prognosis year and the time and ignore other dependencies such as age and the time span of the liabilities. We do this because starting ages as well as the starting year of the simulation are predetermined. Therefore, we can deduce the age of a participant based on the prognosis year and the starting year. Furthermore, we will ignore the time span of the liabilities which is indicated by T, because T is also predetermined. We assume $T = 101$ and the starting year to be 2020. This implies that the prognosis year indicated by y is equal to 0, whenever we are in the year 2020. However, we did like to keep formulas as general as possible and therefore we will use T within the formulas. We will now introduce the notation belonging to the variables within the program:

$DP^n_{y,t}$ is the purchase price of €1 of pension for participant n at year $y + t$ given that we are currently in prognosis year y. The purchase price indicates the costs for a pension fund to pay out €1 of pension t years from prognosis year y to the participant.

P^n_y is the purchase rate of €1 of life long pension for participant n at prognosis year y. The purchase rate is the sum of all purchase prices.

$PE^n_{start,t,y}$ is the value of the pension entitlements of participant n in year $y + t$ at the start of the simulation year y.

$PE^n_{before,t,y}$ is the value of the pension entitlements of participant n in year $y + t$ before indexation at simulation year y.

$PE^n_{after,t,y}$ is the value of the pension entitlements of participant n in year $y + t$ after indexation at simulation year y.

$PL^n_{premium,t,y}$ is the value of the pension liabilities based on the premium payments in prognosis year y of participant n in year $y + t$.

$PL^n_{entitlements,t,y}$ is the value of the pension liabilities based on the pension entitlements at the start of prognosis year y of participant n in year $y+t$.

$PL^n_{before,t,y}$ is the value of the pension liabilities of participant n in year $y+t$ before indexation at simulation year y. $PL^n_{before,t,y}$ is the sum of $PL^n_{premium,t,y}$ and $PL^n_{entitlements,t,y}$.

$PL^n_{after,t,y}$ is the value of the pension liabilities of participant n in year $y+t$ after indexation at simulation year y.

$PL_{premium,y}$ is the value of the pension entitlements based on the premium payments in prognosis year y of the pension fund.

$PL_{entitlements,y}$ is the value of the pension entitlements based on the the pension entitlements at the start of prognosis year y of the pension fund.

PA_y is the value of the pension assets in year y of the pension fund.

RoI_y is the value of the return on investment in year y.

IRH_y is the value of the interest rate hedge in year y.

We only gave a short description of each of the variables. In the upcoming sections we will elaborate more on how to calculate these variables, and why the pension fund uses these variables.

5.2 Purchase rate

In Chapter 2 we discussed the very basic principle of how a pension fund works. We will start filling in the factors needed to simulate the entire pension fund, starting with the purchase rate. The purchase rate is the price for €1 of life long pension. Ergo, it will help convert premium payment to pension entitlements and corresponding liability cash flows. Remark that this only refers to Old age Pension and no survivor's pension.

We can calculate the purchase rate based on four steps. To do so, we note that the price of €1 of life long pension equals the discounted expected costs of life long pension. So, we look at the cost a pension fund makes to pay out €1 of life long pension. These costs depend on the survival probabilities of the participant, the interest rate curves and pension age of the participant.

Before we can start with step 1, we will note that life long equals the time span of $T = 101$ years for the pension fund. So, we have to calculate the expected costs of the cash flow of €1 for the upcoming 101 years. In step 1, we note that the expected costs depend on the survival probabilities of the participant. A participant will only receive the €1 every year if he is alive. Note that the survival probability of a participant depends on the participants age a and the prognosis year y. By looking at the cumulative survival probabilities, we can calculate the probability that participant n with age a_n, starting in prognosis year y remains alive for at least t years from now, which we will indicate by $SP^n_{y,t}$. We calculate $SP^n_{y,t}$ using the following equation:

$$SP^n_{y,t} = \prod_{i=0}^{t}(1 - q_{y+i,a_n+i})$$

where q_{y,a_n} stands for the probability of death in year y of participant n at age a_n. These death probabilities are based on the data from [4]. The vector elements $SP^n_{y,t}$ however are representing the costs of the future cash flows, so they have to be transformed to present cash flows. This will be done in step 2. Step 2, converts the future cash flows back to the present cash flows. We do this by dividing them by the interest rate curve, examples of this are given in [12]. The interest rate curve is calculated using (4.2). The t^{th} element of the present value cash flow vector in year y of participant n, indicated by $PV^n_{y,t}$ will be:

$$PV^n_{y,t} = \frac{\prod_{i=0}^{t}(1 - q_{y+i,a_n+i})}{(1 + R_{t,y})^t}$$

So, now we have a vector which contains the discounted expected costs of €1 of pension for the upcoming 101 years. To calculate these, we took into account the survival probabilities and interest rate curves. However, we should note that Old age Pension only has to be paid when the participant is at least the retirement age (RA). We assumed RA to be equal to 68. This leads to step 3. In step 3 we will define the purchase price as $DP^n_{y,t}$, which takes into account the pension age. We note that participants will only receive pension whenever they are at least an age of 68. The purchase price will indicate the price of €1 of pension at t years from prognosis year y. We will define $DP^n_{y,t}$ as:

$$DP^n_{y,t} = \frac{\prod_{i=0}^{t}(1 - q_{y+i,a_n+i})\mathbb{1}_{a_n+i\geq RA}}{(1 + R_{t,y})^t} \tag{5.1}$$

$DP^n_{y,t}$ takes into account the mortality rates, the discount factor needed to discount future cash flows to present cash flows and the pension age. All what is left to do now is sum the purchase prices to calculate the purchase rate. This will be step 4. In (5.2) the equation to calculate the purchase rate is given.

$$P^n_y = \sum_{t=0}^{T} \frac{\prod_{i=0}^{t}(1 - q_{y+i,a_n+i})\mathbb{1}_{a_n+i\geq RA}}{(1 + R_{t,y})^t} \tag{5.2}$$

a_n	age of participant n	RA	retirement age
y	prognosis year	q_{y,a_n}	probability of death at prognosis year y and age a
t	time		
T	time span to take into account	$R_{t,y}$	interest rate curve given maturity t and prognosis year y

By looking at (5.2) we can summarize that the purchase rate can be interpreted as the sum of the cumulative survival probabilities discounted against the interest rate curve for ages older than the pension age. As a result we can now define the conversion from the premium payments per participant to the total amount of pension liabilities bought from these premium payments. A premium payment with value PP^n_y can purchase PP^n_y/P^n_y amount of pension entitlements, in which n indicates the participant number and y the prognosis year. To convert these pension entitlements into liabilities, we merely have to take into account the purchase price $DP^n_{y,t}$. So, we get the following equation to calculate the pension liabilities based on premium payments given by $PL_{premium,y}$:

$$PL_{premium,y} = \sum_{n=1}^{N}\left(\sum_{t=0}^{T}\frac{DP^n_{y,t}}{P^n_y}\cdot PP^n_y\right) \tag{5.3}$$

y	prognosis year	PP^n_y	premium payments of participant n at prognosis year y
t	time		
T	time span to take into account	$DP^n_{y,t}$	the purchase price in year $y+t$ of participant n at prognosis year y
N	number of participants		
		P^n_y	the purchase rate of participant n at prognosis year y

We can also look at the liabilities corresponding to the premium payments on individual level. An element of the individual cash flow corresponding to the premium payments of participant n in prognosis year y is equal to:

$$PL^n_{premium,t,y} = \frac{DP^n_{y,t}}{P^n_y}\cdot PP^n_y \tag{5.4}$$

5.3 Indexation

Now we introduced the purchase rate, we can start discussing the first step in the simulation. This is the initialization phase, we indicate the initialisation year by 0^*. In year 0^* we will only initialize the starting values, meaning participants will not pay premium payments and the pension fund will not take any actions. This means no investing, hedging, indexing or any other actions for the pension fund. We start the simulation by calculating the value of the pension assets. We calculate these based on the funding ratio and the pension entitlements. At the start of each scenario we assume the pension fund to have a funding ratio equal to 1.10. Based on the starting value of the pension entitlements of each participant we then calculate the value of the pension assets of the fund. To do so, we first note that:

$$FR_y = \frac{\text{Pension Assets in year } y}{\text{Pension Liabilities in year } y} = \frac{PA_y}{PL_y} \Rightarrow PA_y = FR_y \cdot PL_y \tag{5.5}$$

The starting value of the funds pension liabilities is equal to the sum of the participants pension liabilities. We can split the participants pension liabilities into two parts. Namely, the liabilities based on their pension entitlements and the liabilities caused by premium payments. Thus, in general it holds that:

$$PL_y = PL_{premium,y} + PL_{entitlements,y}, \tag{5.6}$$

in which PL_y represents the total amount of pension liabilities in year y. However, at the starting situation we want FR_{0*} to be equal to 1.10 based on the pension entitlements, no premium payments are paid in the initialisation phase. So, $PL_{premium,0^*}$ is equal to 0 in the initialisation phase. Thus, we need to calculate $PL_{entitlements,y}$ in case $y = 0*$. These can be calculated in a similar way as we calculated the purchase rate. Note that pension entitlements in year $y + t$ based on prognosis year y indicate the amount of life long pension a participant receives in year $y + t$. While the purchase price in year y, indicates the price of €1 of pension in year $y + t$ years based on prognosis year y. In both cases $t \in [0, T]$. Therefore, we can simply multiply the purchase price with the value of the pension entitlements and sum these to get the total value of the liabilities of the pension entitlements. In the equation below this concept is mathematically presented in general form.

$$PL_{entitlements,y} = PL_{N,y,a,T,R_{t,y}} = \sum_{n=1}^{N} \left(\sum_{t=0}^{T} \frac{\prod_{i=0}^{t}(1 - q_{y+i,a_n+i})\mathbb{1}_{a_n+i>RA}}{(1 + R_{t,y})^t} \cdot PE^n_{start,t,y} \right) \tag{5.7}$$

The notation $PL_{\sum N,y,a,T,R_{t,y}}$ is introduced because it helps calculating the interest rate hedge. Nevertheless, we can simplify (5.7) to:

$$PL_{entitlements,y} = \sum_{n=1}^{N} \left(\sum_{t=0}^{T} DP^n_{y,t} \cdot PE^n_{start,t,y} \right) \tag{5.8}$$

y	prognosis year	$DP^n_{y,t}$	the purchase price in year $y + t$ of participant n at prognosis year y
t	time		
T	time span to take into account	$PE^n_{start,t,y}$	pension entitlements of participant n at year $y + t$ in prognosis year y
N	number of participants		

We can also look at the liabilities on individual level. An element of the individual cash flow corresponding to the pension entitlements of participant n at the start of prognosis year y is equal to:

$$PL^n_{entitlements,t,y} = DP^n_{y,t} \cdot PE^n_{start,t,y} \tag{5.9}$$

Remark that in the initialization phase, the value of the pension entitlements, which are indicated by $PE^n_{start,t,0^*}$, are calculated using (4.3). Thus, it holds that $PE_{start,t,0^*} = PE_n$ in which PE_n is based on (4.3). Since, no premium is paid in the initialisation phase, it also holds that $PE_{start,t,0^*} = PE_{start,t,0}$ in which $PE_{start,t,0}$ indicates the value of the pension entitlements in prognosis year 0. So, at this point we can calculate the value of the liabilities of the pension entitlements in the initialisation phase. Therefore, we can calculate the value of the pension assets. To do so, we recall that PL_{0^*} at the start is entirely based on $PL_{entitlements,0^*}$. Thus, we can use (5.8) to calculate the starting value of the pension liabilities of the fund indicated by PL_{0^*}. Hence, PL_{0^*} and FR_{0^*} are known for the starting situation, so we can now calculate PA_{0^*} in the starting situation.

Next, we need to consider what influences PA_y and PL_y. The pension assets increase by yearly premium payments of the participant (PP^n_y) and decrease by the pension payments to the participant ($PE^n_{after,0,y-1}$). We look at the pension entitlements at year $y - 1$ with $t = 0$ because the indexations take place after the ultimo year premium payments. So, to determine the value of the pension entitlements which have to be paid in year $y-1$, we need to be at the end of year $y-1$, because we want to take into account the indexations of year $y-1$. At this moment in time the pension assets are already updated, therefore we pay out the pension entitlements of year $y-1$ in year y. The value of the pension assets in year y are also influenced by the Return on Investment (RoI_y) and an Interest Rate Hedge (IRH_y) in year y. The pension liabilities change due to indexation, pension payments and new mortality rates. We will first consider the change in pension assets. Note that we assumed that the pension fund receives premium payments ultimo year. Thus we get:

$$PA_y = PA_{y-1} \cdot (1 + RoI_y) + IRH_y + \sum_{n=1}^{N} \left(PP_y^n - PE_{after,0,y-1}^n\right) \qquad \text{ultimo year} \tag{5.10}$$

We will explain how to calculate the Return on Investment and the value of the Interest Rate Hedge in the upcoming subsections.

5.3.1 Return on Investment

In Section 4.1 we assumed a static portfolio mix. This portfolio mix invests A percentage in stocks and $1-A$ percentage in obligations every year. Adding up both returns will result into the Return on Investment. The stock returns are stated directly in the scenario set, however the return on obligations have to be calculated. Obligations can be represented by coupon bonds. The pricing of coupon bonds is discussed in [12]. A coupon is the annual interest rate paid on a bond, expressed as a percentage of the face value, also referred to as the coupon rate. We introduce to following variables to calculate the return on obligations, aka the return on a coupon bond:

D	Bond duration	$R_{t,y}$	interest rate curve given maturity t and prognosis year y
y	prognosis year	VB_y	value of the coupon bond at the begin of year y.
t	duration	VE_y	value of the coupon bond at the end of year y, after receiving 1 coupon payment
F	face value of the bond		
C	coupon rate		

To calculate the value of a coupon of duration D, we will note that the value of a coupon depends on D yearly coupon payments and the face value of the coupon bond after D years. So, VB_y will be split into $VB_{face,y}$ and $VB_{coupon,y}$. The same holds for VE_y. However, we are not interested in keeping the coupon bond up to duration D. We are interested in buying a coupon primo year, and selling it ultimo year. Therefore, we will calculate VB_y and VE_y for both the coupon rate and the coupon face value.

$$VB_{coupon,y} = \sum_{t=1}^{D} \frac{C}{(1+R_{t,y})^t} = \sum_{t=1}^{D} \frac{F \cdot R_{D,y}}{(1+R_{t,y})^t} \tag{5.11}$$

$$VE_{coupon,y} = \sum_{t=1}^{D} \frac{C}{(1+R_{t,y})^{t-1}} = \sum_{t=1}^{D} \frac{F \cdot R_{D,y}}{(1+R_{t,y})^{t-1}} \tag{5.12}$$

$$VB_{face,y} = \frac{F}{(1+R_{D,y})^D} \tag{5.13}$$

$$VE_{face,y} = \frac{F}{(1+R_{D,y})^{D-1}} \tag{5.14}$$

As we can see the value gained by buying a coupon of duration D primo year, and selling it ultimo year is equal to the value of the end of the year minus the value of the begin of the year. Thus, the return on the obligation becomes:

$$R_{Obligations,y} = \frac{(VE_{coupon,y} + VE_{face,y}) - (VB_{coupon,y} + VB_{face,y})}{VB_{coupon,y} + VB_{face,y}} \tag{5.15}$$

To calculate the return on investment it now suffices to combine the return on stocks in year y, indicated by $R_{stocks,y}$, with the return on obligations in year y. Within the simulation we assume that $D = 10$. This will give:

$$RoI_y = A \cdot R_{Stocks,y} + (1 - A) \cdot R_{Obligations,y} \tag{5.16}$$

5.3.2 Interest Rate Hedge

The current FTK-contract and 2019-ambition-contract are hedged partly against the interest rate. A decrease of the interest rate leads to an increase of the liabilities which results into a decrease in the funding ratio. Interest rate swaps are used to maintain a stable funding ratio whenever interest rates change. The value of an interest rate swap depends on the interest rate mutation. Whenever the mutation is positive, and thus the interest rate increases, the liabilities decrease. Therefore, we will have a negative value of the interest rate swap, because the assets are allowed to decrease as the liabilities decrease. But, whenever the mutation is negative, and thus interest rate decreases, the liabilities will increase. To limit the decrease in the funding ratio, the interest rate swap will be positive, and thus increase the value of the pension assets. This limits the drop in the funding ratio. We assumed that 50% of the interest rate risk is hedged on a yearly basis. We will mimic an interest rate swap as follows. At the end of every year after indexing, a participant has pension entitlements corresponding to the interest rate curve at year y. Next year, this person will have the same entitlements, however the value of the liabilities of these entitlements have changed, due to the change in the interest rate curve and mortality rates. We only want to protect against the change due to the interest rate curve. We do so by using the following equation:

$$IRH_y = 0.50 \cdot \sum_{n=1}^{N} \left(\sum_{t=0}^{T} \left(\frac{\prod_{i=0}^{t} (1 - q_{y+i,a_n+1}) \mathbb{1}_{a_n+1>RA}}{(1 + R_{t,y})^t} - \frac{\prod_{i=0}^{t} (1 - q_{y+i,a_n+1}) \mathbb{1}_{a_n+1>RA}}{(1 + R_{t,y-1})^t} \right) \cdot PE^n_{start,t,y} \right)$$

which we simplify to:

$$IRH_y = 0.5 \cdot \left(PL_{N,y,a,T,R_{t,y}} - PL_{N,y,a,T,R_{t,y-1}} \right) \tag{5.17}$$

5.3.3 Pension entitlements

Thus far we have shown how to calculate the value of the liabilities of the pension entitlements and the value of the liabilities of the premium payments. However, the premium payments and pension entitlements will add up to new pension entitlements ultimo year before indexation. We calculate those on participant level by adding up the pension liabilities on participant level and dividing them by the purchase price which is given in (5.1). This will give:

$$PE^n_{before,t,y} = \frac{PL^n_{entitlements,t,y} + PL^n_{premium,t,y}}{DP^n_{y,t}}$$

which simplifies to the following equation:

$$PE^n_{before,t,y} = PE^n_{start,t,y} + \frac{PP^n_y}{P^n_y} \tag{5.18}$$

The same also holds for the pension entitlements after indexation. These can be expressed in the value of the liabilities at time t divided by the purchase price. This is done in (5.19), in which $PL^n_{after,t,y}$ indicates the value of the pension liability of participant n at time t after indexation. $PL^n_{after,t,y}$ will be calculated using (5.26), (5.27) or (5.29) depending on the type of indexation and the pension contract used.

$$PE^n_{after,t,y} = \frac{PL^n_{after,t,y}}{DP^n_{y,t}} \tag{5.19}$$

However, to calculate $PL^n_{after,t,y}$ we first need to calculate $PL^n_{before,t,y}$. This can be done in a similar way as $PE^n_{before,t,y}$, but in this case we do not have to take into account the purchase price $DP^n_{y,t}$. This gives:

$$PL^n_{before,t,y} = PL^n_{entitlements,t,y} + PL^n_{premium,t,y} \tag{5.20}$$

5.3.4 Benefit cuts

In Section 3.2 we described that the value of the pension liabilities depends on the funding ratio through indexations. However, indexations are not always directly absorbed but sometimes spread over a period of time length T_D, we call this the buffer period. In Dutch we call the spread of the indexation over a time period "Dakpannen". Each "dakpan" represent an indexation. These indexations have the same value, meaning the cash flow will decrease with a constant value x every year. However, this x cannot be calculated directly based on the total indexation value and the buffer period. To illustrate this let us take a look at Table 5.1 in which a simple example of a 10 year cash flow with 5 year buffer period is illustrated. We assume the present value of the cash flow equals the future value. We also ignore survival probabilities. In this example we want to go from a FR = 0.81 to 0.90 by reducing pension entitlements in a 5 year period. The pension assets equal 810 and the pension liabilities 1000. Thus the pension liabilities need to go from 1000 to 900 to reach a funding ratio of 0.90. As the second column of Table 5.1 indicates we cannot simply divide the 10% decrease in pension liabilities by the period of 5 to get the appropriate indexation within cash flows. We need to calculate adjusted indexation levels for the cash flows. To do so we introduce the following notation:

Cash flow before indexation	Cash flow after indexation using: total indexation/time period	Cash flow after indexation using: adjusted indexations
100	98	97.5
100	96	95
100	94	92.5
100	92	90
100	90	87.5
100	90	87.5
100	90	87.5
100	90	87.5
100	90	87.5
100	90	87.5
Total = 1000	Total = 920	Total = 900

Table 5.1: Indexation example.

t	indicator for time	PL_y	pension liabilities before indexation in year y
n	indicator for a participant	PL_y^*	pension liabilities after indexation in year y
N	number of participants	PL_y^+	pension liabilities corresponding to distribution of indexation in year y
T_D	length buffer period	PA_y	value pension assets in year y
δ_t	factor needed for buffer period	Δ_y	size indexation in year y
FR_y	the funding ratio, in year y		

Note, the funding ratio FR refers to the boundaries for indexation in Section 3.1 and 3.2. So, FR could be 1.042 in case of the FTK-contract. T_D refers to the 10 year buffer period mentioned in both contracts in case of negative indexation.

Pension-law article 63a section 6, which can be found at [19], states that "During a buffer period exclusively payments of participants which are in the allocation group at start of the buffer period will be adjusted in equal steps". Thus we assume the following function for δ_t:

$$\delta_t = \begin{cases} t/T_D & t < T_D \\ 1 & t \geq T_D \end{cases}$$

The current liabilities of the fund can be determined by the sum of the individuals liabilities. For individual liabilities it holds that the payment per participant is fluctuating. It depends on the purchase price and the payment before distribution of indexation. The payment before distribution is equal to the pension entitlements before indexation. As earlier indicated the purchase price is basically an element of the purchase rate before summation. This means that:

$$PL_y = \sum_{n=1}^{N} \sum_{t=1}^{T} PE^n_{before,t,y} \cdot DP^n_{y,t} \tag{5.21}$$

For the liabilities after indexation the same holds as before but with different payment values. So we get:

$$PL^*_y = \sum_{n=1}^{N} \sum_{t=1}^{T} PE^n_{after,t,y} \cdot DP^n_{y,t} \tag{5.22}$$

The allocation process is designed such that after distribution of indexation the funding ratio equals K. This means that:

$$\frac{PA_y}{PL^*_y} = FR_y \Leftrightarrow PA_y - FR_y \cdot PL^*_y = 0 \Leftrightarrow PA_y - FR_y \cdot PL_y = FR_y \cdot PL^+_y \tag{5.23}$$

Looking at the pension payments, knowing that the indexation equals Δ, we get that the new payments for participant i at time t are equal to:

$$PE^n_{after,t,y} := PE^n_{before,t,y} \cdot (1 + \Delta_y \cdot \delta_t) = \begin{cases} PE^n_{before,t,y} \cdot (1 + \Delta_y \cdot \delta_t), & t < T_D \\ PE^n_{before,T_D,y} \cdot (1 + \Delta_y), & t \geq T_D \end{cases} \tag{5.24}$$

If we fill in Equations (5.21), (5.22) and (5.24) into (5.23) we get that:

$$\begin{aligned} PA_y - FR_y \cdot PL_y &= FR_y \cdot PL^+_y \\ &= FR_y \cdot \sum_{n=1}^{N} \sum_{t=1}^{T} \left(PE^n_{after,t,y} - PE^n_{before,t,y}\right) \cdot DP^n_{y,t} \\ &= FR_y \cdot \sum_{n=1}^{N} \sum_{t=1}^{T} PE^n_{before,t,y} \cdot \Delta_y \cdot \delta_t \cdot DP^n_{y,t} \\ &= FR_y \cdot \Delta_y \cdot \sum_{n=1}^{N} \sum_{t=1}^{T} PE^n_{before,t,y} \cdot \delta_t \cdot DP^n_{y,t} \end{aligned}$$

Now we solve for Δ_y, this gives:

$$\Delta_y = \frac{\frac{1}{FR_y} \cdot PA_y - PL_y}{\sum_{n=1}^{N} \sum_{t=1}^{T} PE^n_{before,t,y} \cdot \delta_t \cdot DP^n_{y,t}} \tag{5.25}$$

Since, the size of the indexation is the same for the pension entitlements as for the pension liabilities, we now get the following equation for the pension liabilities after indexation:

$$PL^n_{after,t,y} = PL^n_{before,t,y} \cdot (1 + \Delta_y \cdot \delta_t) = \begin{cases} PL^n_{before,t,y} \cdot (1 + \Delta_y \cdot \delta_t), & t < T_D \\ PL^n_{before,t,y} \cdot (1 + \Delta_y), & t \geq T_D \end{cases} \tag{5.26}$$

If we now go back to our example given in Table 5.1 we have $K = 0.90$, $PA = 810$, $PL = 1000$, and $E^i_t = 1$ for all t and we only considered 1 participant. So, in this case Δ equals:

$$\Delta = \frac{\frac{10}{9} \cdot 810 - 1000}{100 \cdot \frac{1}{5} + 100 \cdot \frac{2}{5} + 100 \cdot \frac{3}{5} + 100 \cdot \frac{4}{5} + 100 \cdot \frac{5}{5} \cdot 6} = -0.125$$

Now, one can simply verify using (5.24) that the cash flow after indexation using adjusted indexations in Table 5.1 is correct.

5.3.5 Indexation

From the previous subsection we learned how to do benefit cuts in the FTK- and 2019-ambition-contract. We will now take a look at how to do positive indexations in both contracts. We will describe the procedure based on the pension liabilities, instead of the pension entitlements. However, the indexation ratio remains intact when going from liabilities to pension entitlements.

FTK-contract In case of the FTK-contract we get the following two possibilities based on Section 3.1 and the notation from the basic model:

$$PL^n_{after,t,y} = \begin{cases} PL^n_{before,t,y} \cdot (1 + CPI_y) + PL^n_{before,t,y} \cdot \frac{1}{5} \cdot (FR_y - 1.25) & \text{,if } FR_y \geq 1.25. \\ PL^n_{before,t,y} \cdot (1 + CPI_y \cdot \frac{FR_y - 1.10}{1.25 - 1.10}) & \text{,if } 1.25 > FR_y \geq 1.10. \end{cases} \tag{5.27}$$

CPI_y stands for the cumulative price inflation in year y, this is the sum of not yet indexed price inflations. The FTK-contract will always try to compensate for missed indexations. We can express the size of the indexation Δ_y as:

$$\Delta_y = \begin{cases} (1 + CPI_y) + \frac{1}{5} \cdot (FR_y - 1.25) & \text{,if } FR_y \geq 1.25. \\ (1 + CPI_y \cdot \frac{FR_y - 1.10}{1.25 - 1.10}) & \text{,if } 1.25 > FR_y \geq 1.10. \end{cases} \tag{5.28}$$

2019-ambition-contract The 2019-ambition-contract also has two possibilities which look very similar to that of the FTK-contract and are based on the information given in Section 3.2:

$$PL^n_{after,t,y} = \begin{cases} PL^n_{before,t,y} \cdot (1 + (FR_y - 1.20) \cdot \frac{1}{5}) + PL^n_{before,t,y} \cdot \frac{1}{10} \cdot 0.20 & \text{,if } FR_y \geq 1.20. \\ PL^n_{before,t,y} \cdot (1 + (FR_y - 1.00) \cdot \frac{1}{10}) & \text{,if } 1.20 > FR_y \geq 1.00. \end{cases} \tag{5.29}$$

In this case the size of the indexation will be:

$$\Delta_y = \begin{cases} (1 + (FR_y - 1.20) \cdot \frac{1}{5}) + \frac{1}{10} \cdot 0.20 & \text{,if } FR_y \geq 1.20. \\ (1 + (FR_y - 1.00) \cdot \frac{1}{10}) & \text{,if } 1.20 > FR_y \geq 1.00. \end{cases} \tag{5.30}$$

5.4 Ageing process

Within the simulation each participant becomes older. Every time a participant gets a year older, he has a probability to die. We have taken this into account by calculating the purchase rate and pension liabilities. Therefore, we should also take this into account by updating the pension payments. Otherwise the assets, will not be enough for the liabilities. We do this by looking at the expected pension payments, instead of the real pension payments. A simple way to take into account the probability of death during the simulation is the following:

$$PE^n_{start,t,y+1} = PE^n_{after,t,y} \cdot (1 - q_{y,a_n}) \tag{5.31}$$

So, the pension entitlements at the start of the year of a participant n with age $a_n + 1$ at year $y + 1$ is equal to their pension entitlements after indexation of the year y compensated for the probability that participant n dies at age a_n. Thus, instead of removing a person, who dies, from the simulation we consider the probability that this person remains alive. We do this by decreasing their pension entitlements by a fraction equal to their probability of death.

An example: Participant n is in the simulation with age a_n and has a probability of 0.01 to die at age a_n. For this person holds: $PE^n_{start,t,y+1} = PE^n_{after,t,y} \cdot (1 - 0.01)$. So, his pension entitlements will decrease by a fraction of 0.01.

5.5 Certainty Equivalent

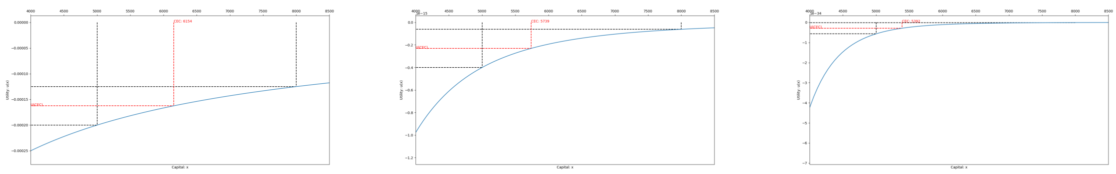
One of the criteria we will use to compare the contracts is the certainty equivalent. The certainty equivalent is the guaranteed return that someone would accept now, rather than taking a chance on a higher, but uncertain return. A more mathematical description of the certainty equivalent can be found in [12]. From [12] it is directly clear that the certainty equivalent is based on utility functions. Therefore, we cannot simply look at the discounted total amount of pension payments over all scenarios, and pick the contract which has the highest discounted value, as the best contract. We have to take into account risk aversion, participants tend to give very low utilities to bad scenarios. So, the contract which performs better in bad scenarios and slightly worse in normal scenarios can be rewarded for this. An extreme example would be, a person with a pension of €2.000 per year in scenario 1 appreciates an increase of €1.000 in pension more than the same person who already has €50.000 per year in scenario 2. We want to take into account this effect. Therefore, we will use a $CRRA$ utility function, this type of utility function is also used in [11], [5] and [13]. More general information about utility functions can be found in [8]. The utility function is given by:

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}. \tag{5.32}$$

Note that γ represents the level of risk aversion, in general we assume $\gamma = 5$. In [5] they also consider $\gamma = 3$ and $\gamma = 7$ to visualise the effect of the risk-aversion parameter. We want to amplify this effect, for that reason we will look at $\gamma = 2$ and $\gamma = 10$. In Figure 5.1 we give an example in which we calculate the certainty equivalent of two pension payments. The participant has a probability of 0.5 to receive a pension payment equal to 5000 and a probability of 0.5 to receive 8000. As the figure indicates the lower the risk aversion parameter, the higher your certainty equivalent is. In this simple example we calculate the certainty equivalent using (5.33).

$$0.5 \cdot \frac{8000^{1-\gamma}}{1-\gamma} + 0.5 \cdot \frac{5000^{1-\gamma}}{1-\gamma} = 1 \cdot \frac{CEC^{1-\gamma}}{1-\gamma} \tag{5.33}$$

The certainty equivalent varies from 6154/5739/5392 for $\gamma = 2/5/10$. As we can see, the risk aversion parameter clearly impacts the weight a person gives to the height of a pension payment. A zoomed in version of Figure 5.1 can be seen in Figure 3 which is placed in Appendix B.



(a) $\gamma = 2$. (b) $\gamma = 5$. (c) $\gamma = 10$.

Figure 5.1: CRRA utility function for different values of γ.

In an ideal situation, we would record every pension payment made to each participant. We assumed that a participant dies at age 130 thus $130 - RA$ pension payments will be made. The total utility of person j in scenario i for these pension payments would then be

$$UT_{j,i} = \sum_{t=0}^{130-RA} \rho^t u\Big(\frac{U_{t,j}}{\Pi_t}\Big), \tag{5.34}$$

in which $U_{t,j}$ indicates the pension payments, Π_t the price index and ρ the discount factor. ρ discounts the value of each individual their pension payments. The average utility of person j over Q scenarios is equal to UT_j.

$$UT_j = \frac{\sum_{i=1}^{Q} UT_{j,i}}{Q} \tag{5.35}$$

Next, we express the total welfare. The total welfare is equal to the discounted sum over all participants their average utility. We indicate the number of participants by L.

$$TW_{overall} = \sum_{j=1}^{L} \delta^{RA-a_j} \cdot UT_j \tag{5.36}$$

Here, δ is also a discount factor, but it discounts the value of the pension payments with respect to the start of the simulation. This implies that the eventual pension payments of a 25 year old would be less significant than the pension payments of a 65 year old whenever $\delta < 1$. However, we want the pension payments of all participants to be independent of their starting age, thus we assume $\delta = 1$.

We will now calculate the overall certainty equivalent. Based on the simulation we get the value of the total welfare. This value is calculated based on L participants, for which we have calculated their average utility over the scenarios. In each scenario they received $130 - RA$ pension payments, each payment belongs to a different year in their life. The certainty equivalent will be the utility which replaces all different utilities of each participant at every state of their life. This results in (5.37).

$$TW_{overall} = \sum_{j=1}^{L} \sum_{t=0}^{130-RA} \rho^t u(CEC) = \frac{\rho^{130-RA+1} - 1}{\rho - 1} \cdot L \cdot u(CEC) \tag{5.37}$$

Equation (5.37) indicates that the total welfare equals the discounted value of the certainty equivalent ($CEC_{overall}$) summed over each year in which the participant receives pension payments and summed over all participants. We will rewrite this, such that we can calculate the certainty equivalent.

$$CEC_{overall} = \left[\frac{TW_{overall}(1-\rho)(1-\gamma)}{L(1-\rho^{130-RA+1})} \right]^{\frac{1}{1-\gamma}} \tag{5.38}$$

We can also look at the certainty equivalent per scenario. To do so, we slightly adept the total welfare formula. We will no longer look at the average utility of person j over Q scenarios which was given by UT_j, but instead we will look at the utility of person j in scenario i given by $UT_{j,i}$.

$$TW_{scenario,i} = \sum_{j=1}^{L} UT_{j,i} = \sum_{j=1}^{L} \sum_{t=0}^{130-RA} \rho^t u(CEC_i) = \frac{\rho^{130-RA+1} - 1}{\rho - 1} \cdot L \cdot u(CEC_i) \tag{5.39}$$

Again, we can rewrite this such that we can calculate the certainty equivalent.

$$CEC_i = \left[\frac{TW_{scenario,i}(1-\rho)(1-\gamma)}{L(1-\rho^{130-RA+1})} \right]^{\frac{1}{1-\gamma}} \tag{5.40}$$

Next, we look at the overall certainty equivalent of a person at age 68. In this case the total welfare will be

$$TW_{68} = \frac{\sum_{i=1}^{Q} u\left(\frac{U_{0,j}}{\Pi_0}\right)}{Q} = u(CEC). \tag{5.41}$$

So, we get that

$$CEC_{68} = \left[TW_{68}(1-\gamma) \right]^{\frac{1}{1-\gamma}}. \tag{5.42}$$

Nevertheless, we are not in an ideal situation. Participants might already be retired, therefore several previous pension payments will be unknown. The missing pension payments of a participant older than the retirement age will be estimated by their first pension payment in the simulation also using the first price inflation. Also, we will only simulate up to 50 years forward. Thus, at the end of the simulation there might still be participants alive, with entitlement to pension payments. These missing pension payments will be estimated using the expected upcoming cash flows. Doing so, every participant will have $130 - RA$ pension payments. This ensures the usage of the formulas presented above and provides a more adequate approximation of the real certainty equivalent.

5.6 Adjustment factor

Another criteria on which we will compare contracts is the adjustment factor. The adjustment factor is defined as the factor of change within the pension entitlements due to indexation. Remark that the adjustment factor does not incorporate change in pension entitlements due to different purchase rates, premium payments etcetera. The adjustment factor is calculated on a yearly base for every scenario and every person. If indexations would always be directly absorbed, meaning there would be no buffer period in case of benefit cuts, we could do this by simply dividing the pension entitlement(PE) after indexation by those before indexation. However, this is not the case for the FTK- and 2019-ambition-contract. Therefore, we look at the size of the indexation which is given by Δ. So, we get that the adjustment factor in year y is equal to:

$$\text{Adj. Factor}_y = 1 + \Delta_y \tag{5.43}$$

From this we can calculate several interesting factors, like how many times a negative indexation appears on average, but also the average adjustment factor over all scenarios. We come back to this issue when we discuss the results of the simulation. Note that Δ_y is calculated with either (5.25), (5.28) or (5.30).

5.7 Market value

In this section we want to calculate the market value. Reference [9] gives a good indication of how we can do this. However, in this case we are interested in the market value per generation. The market value approach is also known as the generational effect model. The approach is used to indicate the financial consequences of a change between pension contracts per age cohort. To do so, we first need to calculate the Generational Account per age cohort at time 0. The generational account measures the added market value of a specific generation over a period of time. The time period we consider is 50 years. To calculate the generational account we take into account the pension fund their income and the expenses based on the participants over these 50 years. So, this means we take into account the premium payments participants paid to the pension fund in these 50 years. The premium payments will have a negative impact on the market value of their generation, since we look at the value based on the participant point of view. Furthermore, we take into account pension payments to their generation, these have a positive value. At last, we need to take into account the change in pension assets of the pension fund assigned to their generation. The pension assets are assigned to the participants based on the liabilities. A generation, with a large liability cash flow is entitled to a large part of the assets, because these assets are there to pay out future pension payments. So, we can now properly formulate the generational account. We indicate an age cohort by x, such cohort will represent a generation. The pension payments of cohort x at year y are indicated by U_y^x. The premium payments to the fund from cohort x at year y are equal to P_y^x and the value of the pension assets of cohort x at time t are A_t^x. This gives:

$$GA_0^x = V_0(A_T^x) - A_0^x + \sum_{y=0}^{50} V_0(U_y^x) - V_0(P_y^x) \tag{5.44}$$

$V_0(.)$ is a function which discounts the value of the variable back to the market value at time $t = 0$. To do so, it uses the yearly one year interest rate. Thus if we calculate the market value of U_2^x we get:

$$V_0(U_2^x) = \frac{U_2^x}{(1 + R_{0,0}) \cdot (1 + R_{0,1}) \cdot (1 + R_{0,2})}$$

The value of the pension assets per cohort will be determined by ratio of the liabilities. Thus we get:

$$A_y^x = \frac{A_y \cdot L_y^x}{L_y} \tag{5.45}$$

To get a good picture of what changed per cohort if we swap from the FTK-contract to the 2019-ambition contract we will calculate the net-profit-pictures. These pictures give the percentage change in the Generational Accounts per cohort. To indicate the contract used to calculate the generational accounts we added a subscript indicating the used contract.

$$\Delta\% GA^x_{0,FTK\to 2019} = \frac{GA^x_{0,2019} - GA^x_{0,FTK}}{\mid GA^x_{0,FTK} \mid} \tag{5.46}$$

5.8 Difference between contracts

In the previous sections of this chapter we described all tools needed to simulate both contracts. The mathematics behind both contracts is very similar, both contracts use the same way to calculate the purchase rate, and therefore buy pension entitlements in the same manner. The only difference between contracts is within the indexation rules, as explained in Subsections 5.3.4 and 5.3.5. In case of positive indexation, the FTK-contract uses (5.27) while the 2019-ambition-contract uses (5.29).

In case of negative indexations the difference is even more subtle. We calculate negative indexations by using (5.25). However, the FTK- and 2019-ambition-contract use a different funding ratio FR in (5.25). We recall that FR represents the funding ratio to which the fund wishes to steer, and note based on Sections 3.1 and 3.2 that these are different in the contracts. Another difference is whether indexations are unconditionally or conditionally reduced. In both cases we calculate Δ with (5.25). However, when we unconditionally reduce the pension liabilities, we will use each of the 10 indexations calculated, while in case of conditionally reduced indexations we only use the first indexation. This would correspond to changing the cash flow to 97.5 in all the years, when we return to the example given in Table 5.1, and revisiting the situation again a year later.

5.9 Simulation

We introduced all mathematics needed to simulate the entire situation for the FTK-contract and the 2019-ambition-contract. Within the simulation we keep track of the following variables, namely:

Fund level	Participant level
Pension assets	Pension entitlements before indexation
Price index	Pension entitlements after indexation
Pension liabilities	Pension liabilities
Market value discount rate	Premium payments

Table 5.2: Important variables within the simulation.

These variables help to calculate the certainty equivalent, adjustment factor and generational account. So, they help with a good comparison between contracts.

5.9.1 Simulation description

As mentioned before the simulation takes as input the pension age, starting year, participant data and premium percentage. Using this input we will first use the pension entitlements of each participant and calculate, by using the purchase price, as described in Section 5.3 what the pension liabilities are for the pension fund. Based on these pension liabilities and the starting value of the funding ratio, we will calculate the starting value of the pension assets. Next, we will start a iterative procedure for which each step can be repeated for each year simulated. The iterative procedure does the following. It first starts calculating the amount of premium paid by the participants based on their pension base, these will be used in the generational account. It will then adjust the value of the pension assets of the fund based on (5.10), in which the interest rate hedge in the starting year is equal to zero. This is because the first change in interest rates is when we go from the starting year to the next simulation year. It will also calculate the net value of this premium payment and the net value of the pension entitlements using the purchase rate and purchase price, this will change the value of the pension liabilities. Based on these we calculate the value of the pension entitlements before indexation. This leaves us with calculating the new funding ratio. This new funding ratio will change the down counter, which counts the years below the benefit cut threshold which is 1.042 in the FTK-contract and 1.0 in the 2019-Ambition-contract. Depending on the new funding ratio and the down counter an appropriate indexation will be applied. The indexation options were mentioned in Subsection 5.3.4 and 5.3.5. The indexation will then be applied on the cash flows. This will give the pension entitlements after indexation. We use those to calculate the adjustment factor and utility corresponding to that pension payment if the retirement age is reached. We also use the pension payments to calculate the generational account, however note that the pension payments needed for the generational account are based on a different input data set. The pension entitlements after indexation are then used to calculate the pension entitlements at the start of next year. This process repeats until we have reached the number of years which we wanted to simulate. This was a very compact description of the simulation process. The algorithm below will also provide a compact description, but will use references to the introduced equations.

Algorithm 1 FTK-contract

```
Result: Certainty equivalent, adjustment factor
Initialize: pension age, starting year, participant data, funding ratio;
 Set: down counter = 0, prognosis year y = 0;
 Calculate: starting value of pension liabilities using (5.7);
 Calculate: starting value of pension assets using (5.5);
 while prognosis year y < 50 do
    Update PL_y using (5.6), which uses (5.7) and (5.3)
     Update PA_y ultimo year using (5.10), which uses (5.16), (5.17) and (5.19);
     Update PE^n_{before,t,y}, using (5.18);
     Calculate FR_y using (5.5);
     if FR_y ≥ 1.10 then
        Update PL^n_{after,t,y} using (5.27);
         Calculate Δ_y using (5.28);
         Down counter = 0;
    end
    if 1.042 ≤ FR_y < 1.10 then
        Δ_y = 0;
         Down counter = 0;
    end
    if FR_y ≤ 1.042 then
        if down counter == 5 then
            Unconditional benefit cuts using (5.25) with FR_y = 1.042;
             Calculate Δ_y using (5.25);
        else
            if FR_y < 0.95 then
                Conditional benefit cuts using (5.25) with FR_y = 0.95;
                 Calculate Δ_y using (5.25);
            end
        end
    end
    Update PE^n_{after,t,y}, using (5.19);
     Calculate adj. Factor, using (5.43);
     if Age participant ≥ RA then
        Calculate utility of pension payment, using (5.32);
    end
    Calculate PE^n_{start,t,y} by using (5.31);
     Age participants = age participants + 1;
     Prognosis year y = prognosis year y + 1;
end
Calculate CEC, using (5.38);
```

The same works for the calculation of the generational account. Note, that the generational account calculations are based on a risk-neutral data set. To avoid confusion these steps are not taken into account within the algorithm. Furthermore, a similar algorithm works for the 2019-ambition-contract, one could determine the modifications of the algorithm based on the differences between the contracts explained in Section 5.8.

Chapter 6

IRR-ambition-contract

In this chapter we will discuss the simulation for the IRR-ambition-contract. Several tools from the FTK- and 2019-Ambition-contract can be used here as well. As in Chapter 5 we will first start by listing the import variables. Next, we will adept the step-by-step explanation of a pension fund to the IRR-ambition-contract. After that we introduce the concepts and mathematical formulas needed. At last, we will describe how these formulas fit together to simulate the contract.

6.1 List of variables

As in Section 5.1 we will give a list of important variables. We will use the same notation as in Chapter 5. Some of these variables have already been explained, however several of these variables will be new. These new variables will be explained throughout the rest of the chapter.

$FDP^n_{y,t}$ is the future purchase price of €1 of pension for participant n at year $y+t$ given that we currently are in prognosis year y. Note, we did not discount from future value to present value.

P^n_y is the purchase rate of €1 of life long pension for participant n at prognosis year y. The purchase rate is the sum of all purchase prices.

$PE^n_{start,t,y}$ is the value of the pension entitlements of participant n in year $y+t$ at the start of the simulation year y.

$PE^n_{before,t,y}$ is the value of the pension entitlements of participant n in year $y+t$ before indexation at simulation year y.

$PE^n_{after,t,y}$ is the value of the pension entitlements of participant n in year $y+t$ after indexation at simulation year y.

$L^n_{entitlements,t,y}$ is the expected future cost in year $y+t$ for participant n based on prognosis year y. These costs correspond to a value of $U^n_{start,t,y}$ pension entitlements.

$C^n_{premium,t,y}$ is the expected future cost in year $y+t$ for participant n based on prognosis year y corresponding to the premium payments of participant n in year y.

PA_y is the value of the pension assets in year y of the pension fund.

PA^n_y is the value of the pension assets in year y for participant n.

$U^n_{t,y}$ is the total expected future cost in year $y+t$ for participant n based on prognosis year y. Thus, we get that: $U^n_{t,y} = L^n_{entitlements,t,y} + C^n_{premium,t,y}$.

$U^n_{scaled,t,y}$ is the total expected future cost in year $y+t$ for participant n based on prognosis year y scaled based such that it correspond to $IRR_{fund_{new}}$ and PA^n_y.

RoI_y is the value of the return on investment in year y.

γ_y^n is the scale factor of participant n in year y.

$IRR_{start,y}$ is the internal rate of return at the start of the year y.

$IRR_{fund,y}$ is the internal rate of return before indexation in year y. This is different from $IRR_{start,y}$ due to the change in value of the pension assets and pension entitlements.

$IRR_{fund_{new},y}$ is the internal rate of return during the indexation algorithm. We assume that $IRR_{fund_{new},y-1} = IRR_{start,y}$.

We will use these variables to explain how the fund works on a yearly basis. We will do this by changing the step-by-step explanation of a pension fund used to describe the FTK- and 2019-ambition-contract.

1. First, we calculate the value of the pension liabilities corresponding to $PE^n_{start,t,y}$. These liabilities will be called $L^n_{entitlements,t,y}$, and are no longer discounted from future cash flows to present cash flows.
2. Next, we calculate the value of the pension liabilities in year y based on the premium payments of the participants. An element of the vector of these liabilities is indicated by $C^n_{premium,t,y}$.
3. Adding $L^n_{entitlements,t,y}$ and $C^n_{premium,t,y}$ we get the total future cash flow before indexation per participant. An element of this vector is described by $U^n_{t,y}$. We can also sum over the participants to get $U_{t,y}$, which represents an element of the total future cash flow before indexation of the pension fund.
4. Then, we update the value of the pension assets, which we will describe as PA_y.
5. Based on the value of the pension assets and the liability cash flow of the pension fund in year y we calculate the internal rate of return of the pension fund, known as $IRR_{fund,y}$.
6. We use the internal rate of return $IRR_{fund,y}$ and the liability cash flow of the participants $U^n_{t,y}$ to calculate the value of the pension assets of participant n, indicated by PA^n_y.
7. Next, we steer based on a given steering mechanism. This will result in a new internal rate of return for the pension fund $IRR_{fund_{new},y}$. In Section 3.3 we introduced several steering mechanisms, which we will explain in Subsection 6.7.1.
8. At last, we will scale the total future cash flow per participant such that the new internal rate of return of the fund matches the value of the pension assets per participant according to their scaled pension liabilities.

From the step-by-step explanation we can see that we no longer use the funding ratio to determine the indexation size. This will be determined by the steering mechanism and the scaling of the liability cash flows after steering. Furthermore, when we introduce the formulas for the liabilities, we can see that we no longer discount the liabilities to present cash flows, but instead look at future cash flows. Based on these future cash flows, we will determine the internal rate of return corresponding to this cash flow and the value of the pension assets. What this means, will become clear in Section 6.3 in which we explain the internal rate of return and how to calculate the internal rate of return.

6.2 Starting value of the Assets and Liabilities

In the IRR-ambition-contract the funding ratio is no longer a key argument. The total valuation of the liabilities of the fund is no longer needed, within this contract we are interested in the liability cash flows. We will say more over this in the upcoming sections. We will now focus on using the same starting value for each contract, therefore we will use (5.7). We determine the starting value of the pension liabilities with the interest rate curve. Next, we use that we assumed a funding ratio of 1.10 to calculate the value of the pension assets using (5.5). In fact, we apply the exact same procedure as we used for the FTK- and 2019-ambition-contract to calculate the starting value of the pension assets. Doing so, we ensure that all contract starts with the same value of pension assets.

Since, we determined the value of the pension assets at the start of the simulation we can also look at updating the value of the pension assets. We will do this in a similar way as we did in (5.10). Nonetheless, we do no longer hedge against interest rate risk. Pension entitlements are bought against the internal rate of return, hence this is not longer needed. This results in,

$$PA_y = PA_{y-1} \cdot (1 + RoI_y) + \sum_{n=1}^{N} \left(PP_y^n - PE_{after,0,y-1}^n \right) \qquad \text{ultimo year} \tag{6.1}$$

in which PP_y^n are the premium payments of participant n in year y and $PE_{after,0,y-1}^n$ the value of the pension entitlements of participant n at year $y-1$. Furthermore, we should note that the Return on Investment is calculated based on stock returns and bond returns. The stock returns are given in the economic data set based on the KNW-capital model [7] and therefore will remain the same. The bond returns will remain dependent on the interest rate curve, so these will also stay the same. Thus, we can again use (5.16) to calculate the RoI_y.

6.3 Internal rate of return

The IRR-Ambition-contract is based on the internal rate of return. Hence, we need to introduce how to calculate the internal rate of return to explain the contract. The internal rate of return can be calculated based on the value of the pension assets and the pension payments, aka cash flows which have to be paid. This can be done on participant level and on fund level. We will discuss how to do it on fund level, but the mathematics is exactly the same on individual level. First, we recall the definition of internal rate of return. We use the definition which was first introduced in [6]. The internal rate of return is defined as the minimal yearly rate of return that a pension fund needs on their assets to pay out all their upcoming pension liabilities. So this means that after T years the value of the pension assets needs to be 0, based on receiving a return of IRR and pension payments U_t. On a yearly basis this implies that the value of pension assets changes as follows in case of ultimo year pension payments:

$$PA_{t+1} = (1 + IRR) \cdot PA_t - U_t \tag{6.2}$$

We can use this recursion to express PA_{t+1} only in terms of U_t, IRR and the starting value of the pension assets PA_0. This leads to:

$$PA_{t+1} = PA_0 \cdot (1 + IRR)^{t+1} - \sum_{k=0}^{t} U_k (1 + IRR)^{t-k} \tag{6.3}$$

Considering that $PA_T = 0$, PA_t is a function of IRR and U_t is known for $t = 0, \ldots, T-1$ we can solve this for IRR. We will do this using the Newton-Raphson method. To do so, we first define PA_T as function of IRR and estimate its derivative.

$$PA_T(IRR) = PA_0 \cdot (1 + IRR)^T - \sum_{k=0}^{T-1} U_k (1 + IRR)^{T-1-k} \tag{6.4}$$

We will estimate the derivative using the symmetric difference quotient, with ϵ very small. Thus we get that:

$$PA_T'(IRR) = \frac{PA_T(IRR + \epsilon) - PA_T(IRR - \epsilon)}{2 \cdot \epsilon} \tag{6.5}$$

So now we can use the above equations in the Newton-Raphson method, which is stated below. We do this to calculate for which value of IRR we get that $PA_T = 0$.

$$IRR_{n+1} = IRR_n - \frac{PA_T(IRR_n)}{PA_T'(IRR_n)} \tag{6.6}$$

We stop iterating when $\mid IRR_{n+1} - IRR_n \mid < \epsilon_2$, in the simulation we have chosen for $\epsilon_2 = 10^{-8}$. We used the interest rate curve to determine the value starting value of the pension assets, therefore we can use the geometric mean of the interest rate curve as a educated guess of IRR_0 in the starting year.

6.4 Purchase rate

The purchase rate of this contract is very similar to the purchase rate mentioned in (5.2). However instead of purchasing pension entitlements against the interest rate curve, pension entitlements are now purchased against the internal rate of return. The interest rate curve, can be very volatile, and therefore buying pension entitlements against the interest rate curve can lead to good and bad luck generations. This is no longer the case when the fund buys pension entitlements against the internal rate of return. The internal rate of return tends to be stable. As with the FTK- and 2019-ambition-contract pension entitlements are bought ultimo year. The internal rate of return used to buy the pension entitlements is the internal rate of return calculated previous year after indexation, which we assumed to be equal to $IRR_{start,y}$. We have seen that the internal rate of return is independent of time t, unlike the interest rate curve. The new formula for the purchase rate can be seen in (6.7).

$$P^n_{y,T} = \sum_{t=0}^{T} \frac{\prod_{i=0}^{t}(1 - q_{y+i,a_n+1})\mathbb{1}_{a_n+1>RA}}{(1 + IRR_{start,y})^t} \tag{6.7}$$

a_n	age of participant n	q_{y,a_n}	probability of death at prognosis year y at age a
y	prognosis year	PP^n_y	premium payments of participant n at prognosis year y
t	time	RA	retirement age
T	time span to take into account		
n	indicating participant n		

We will now calculate the future purchase price. The future purchase price is a part of (6.7). The difference between the purchase price used in Chapter 5 and the future pension price is that it does not discount the future cash flows to the present cash flows using the interest rate curve. Therefore, we can state the future purchase price as:

$$FDP^n_{y,t} = \prod_{i=0}^{t}(1 - q_{y+i,a_n+i})\mathbb{1}_{a_n+i>RA} \tag{6.8}$$

We can now express the cash flow of the premium payments based on the purchase rate. However, we do not discount from future to present cash flows using the internal rate of return, because we need to calculate a new internal rate of return. The change in mortality rates, age and accumulation of new pension entitlements all influence the internal rate of return. So, previous year internal rate of return is no longer accurate. Since, we do not account for discounting from future to present cash flow, the cash flow of the premium payments and pension entitlements will be:

$$C^n_{premium,t,y} = \prod_{i=0}^{t}(1 - q_{y+i,a_n+i})\mathbb{1}_{a_n+i>RA} \cdot \frac{PP^n_y}{P^n_y} = FDP^n_{y,t} \cdot \frac{PP^n_y}{P^n_y} \tag{6.9}$$

$$L^n_{entitlements,t,y} = \prod_{i=0}^{t}(1 - q_{y+i,a_n+i})\mathbb{1}_{a_n+i>RA} \cdot PE^n_{start,t,y} = FDP^n_{y,t} \cdot PE^n_{start,t,y} \tag{6.10}$$

We will add these together to get the expected future pension payments according to their mortality rates. These will represent the cash flow of the participants. We call these $U^n_{t,y}$.

$$U^n_{t,y} = FDP^n_{y,t} \cdot \left(\frac{PP^n_y}{P^n_y} + PE^n_{start,t,y}\right) \tag{6.11}$$

Next, we sum the cash flows of the participants to get the cash flow of the fund. We call these elements $U_{t,y}$. These will be used in next section when we consider the internal rate of return of the pension fund.

$$U_{t,y} = \sum_{n=1}^{N}\left(FDP^n_{y,t} \cdot \left(\frac{PP^n_y}{P^n_y} + PE^n_{start,t,y}\right)\right) \tag{6.12}$$

From this we can also determine the value of the pension entitlements before indexation. These are calculated in a similar way as (5.18) indicates. We add the liability cash flow based on the premium payments and the liability cash flow based on the pension entitlements together. This is already done in (6.11). Next, we divided the total future expected cost by the future purchase price per euro to get the pension entitlements. In formulas this is represented by:

$$PE^n_{before,t,y} = \frac{U^n_{t,y}}{FDP^n_{y,t}} = PE^n_{start,t,y} + \frac{PP^n_y}{P^n_y} \tag{6.13}$$

We can do the same for the pension entitlements after indexation. These are calculated based on the scaled total expected future costs of participant n in year $y+t$ given prognosis year y, which are indicated by $U^n_{scaled,t,y}$. So, we get:

$$PE^n_{after,t,y} = \frac{U^n_{scaled,t,y}}{FDP^n_{y,t}} \tag{6.14}$$

How we determine $U^n_{scaled,t,y}$, will be explained in the upcoming sections of this chapter.

6.5 Pension assets participants

Until now, we only considered the value of the pension assets of the entire fund. But we can also assign the pension assets to their participants using the internal rate of return. Based on the individual cash flows and the internal rate of return, we can calculate for each participant their pension assets PA^n_y at year y. We do this by using that PA^n_{y+T} should be equal to 0, after paying all pension payments $U^n_{y,t}$ for $t = 0, \ldots, T-1$. The following recursion occurs:

$$PA^n_{y+t} = \frac{PA^n_{y+t+1} + U^n_{y,t}}{1 + IRR_{fund,y}} \tag{6.15}$$

We can simplify this recursion, such that it is only dependent on $U^n_{y,t}$ and the internal rate of return. The equation then becomes:

$$PA^n_y = \sum_{t=0}^{T-1} \frac{U^n_{y,t}}{(1 + IRR_{fund,y})^{t+1}} \tag{6.16}$$

So, the part of the pension assets of the pension fund which will belong to the participant n depend on which fraction of the future liabilities discounted to the present value belong to this participant, in comparison to the fund.

6.6 Scaling factor

As the internal rate of return, the scaling factor is an important part of the IRR-ambition-contract which we did not yet explain. We will do this in this section. In the IRR-ambition-contract we change the internal rate of return and index the cash flows based on a steering mechanism. This leads to a new internal rate of return and indexed cash flows, for the fund and for the individuals. However, these cash flows should still match the new internal rate of return based on their pension assets. The value of the pension assets remained unchanged due the indexation phase, therefore we need to scale the indexed cash flows. We scale the cash flows, such that they match the new internal rate of return based on the pension assets, on individual level as well as for the entire fund. Calculating the scaling factor of the cash flow will go very similar to calculating the internal rate of return. It will make use of the same type of recursion, in which the elements of the pension payments cash flow given by U are now replaced by U times a scaling factor. The known factors at this point are the value of the pension assets at the individual level, the cash flows at individual level, the internal rate of return on fund level and the fact that the pension assets at individual level need to be 0 at time T. Thus, we no longer have a function depending on the internal rate of return. Remark, the cash flows are future values, not present values and that the internal rate of return on fund level also needs to apply on individual level. We will define the value of the pension assets at individual level as PA^n_{y+T} and the scaling factor as γ^n_y where n indicates the participant number and y the prognosis year and T the time span. So we need to solve:

$$PA^n_{y+T}(\gamma^n_y) = PA^n_y \cdot (1 + IRR_y)^T - \sum_{t=0}^{T-1} U^n_{t,y} \cdot \gamma^n_y \cdot (1 + IRR_y)^{T-1-t} = 0 \tag{6.17}$$

As we can see this equation is very similar to (6.4), however we now look at the participant level. Also the value of the pension assets are now a function of γ^n_y instead of a function of the internal rate of return. The internal rate of return used in (6.17) is $IRR_{fund_{new},y}$. This is a known number. We use $IRR_{fund_{new},y}$, since the scaling takes place after the pension fund steered towards a certain goal. As with the calculations of the internal rate of return we will now estimate the derivative of (6.17) using the symmetric difference quotient with very small ϵ. Finally, we will use the Newton-Raphson method to solve (6.17) for γ^n_y. This will give the scaling factor, we use the same precision as we used for the internal rate of return when applying the Newton-Raphson method.

6.7 Indexation algorithm

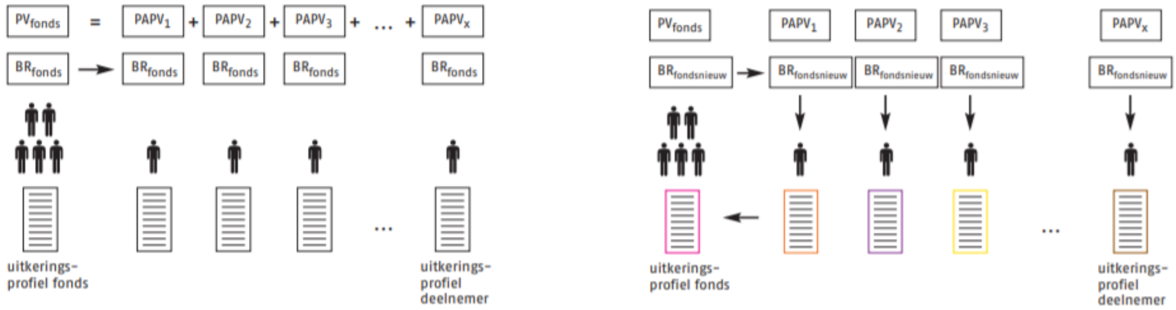
In Section 3.3 we introduced three steering mechanisms for a pension fund. Namely, steering based on a fixed internal rate of return, a fixed difference between the expected return and the internal rate of return and steering to compensate price inflation. The implication of all three steering principles can be described by an indexation algorithm. The indexation algorithm is described in the article of van Mullekom [17]. The algorithm does the following. First, it combines the cash flows of each participant to get the cash flow on fund level. Based on the value of the pension assets and the fund its cash flow it will then calculate the internal rate of return of the fund. As Figure 6.1a shows the internal rate of return of the fund will become the internal rate of return of the participants. We then calculate the value of the pension assets on individual level, belonging to this internal rate of return. Next, we will change the individual cash flows based on what the fund chooses to steer for. Since the cash flows on individual level changed, the cash flow for the fund will also be changed. Thus, we need to recalculate the internal rate of return on fund level. Note that there is no extra capital generated, so value of the pension assets individually and for the fund remain unchanged. The new internal rate of return of the fund together with the individual indexed cash flows does not match the value of the pension assets on individual level, because individual cash flows do not necessarily share the characteristics of the fund. Therefore we scale the individual cash flows, such that the new internal rate of return corresponding to the fund also matches each individual participant. This process can be seen in Figure 6.1b. Note, participants might require different scaling factors. The scaling factors will represent the indexation level, if a scaling factor is larger than 1, you receive a positive indexation on your pension entitlements. But whenever your scaling factor is smaller than 1 your pension entitlements are reduced. In short the algorithm does:

1. Use PA_y and $U_{t,y}$ to calculate $IRR_{fund,y}$.
2. Calculate PA^n_y for participant n based on $U^n_{t,y}$ and $IRR_{fund,y}$.
3. Steering mechanism.
4. Scale $U^n_{t,y}$ such that $IRR_{fund_{new},y}$ is the new internal rate of return of PA^n_y and $U^n_{scaled,t,y}$.

We calculate $U^n_{scaled,t,y}$ by simply multiplying the cash flow before scaling with the scaling factor. The cash flow before scaling depends on the steering mechanism. We will abbreviate the expected return by ER. Thus, now we can express $U^n_{scaled,t,y}$ in terms of the before scaling cash flow using (6.18).

$$U^n_{scaled,t,y} = \begin{cases} (1+\gamma^n_y) \cdot U^n_{y,t} & \text{,if steering based on a fixed } IRR. \\ (1+\gamma^n_y) \cdot U^n_{y,t} & \text{,if steering based on fixed difference ER and IRR.} \\ (1+\gamma^n_y) \cdot U^n_{indexed,y,t} & \text{,if steering based on price inflation.} \end{cases} \tag{6.18}$$

As we can see only the price inflation based steering mechanism makes use of indexed cash flows. The other steering mechanisms leave the cash flow before scaling unchanged, and steer by using the internal rate of return. We will determine the value of $U^n_{indexed,y,t}$ for the price inflation based steering method in Subsection 6.7.1.



(a) Fund to participant. (b) Participant to fund.

Figure 6.1: Visual representation of the IRR-algorithm.
Figures from [17].

6.7.1 Steering mechanism

As the algorithm above states in step 3, we have a steering mechanism which depends on whether we steer based on a fixed internal rate of return, a fixed difference between expected return and internal rate of return or to compensate price inflation. We will discuss what to do for each of these three methods.

A fixed internal rate of return We steer to have a fixed internal rate of return equal to C over the entire time period of 50 years. This means, whenever we get an $IRR_{fund,y} \neq C$ in step 2, we set $IRR_{fund_{new},y} = C$ in step 3. To ensure this internal rate of return is within the capabilities of our portfolio, we link them. We do this by using the geometric mean of the return on investment over the entire time period of 50 years.

$$GMP = \sqrt[50]{(1 + RoI_0) \cdot \ldots \cdot (1 + RoI_{50})} \tag{6.19}$$

We calculate the return on investment of all 50 years, using (5.16). Since, we are looking at the geometric mean portfolio return, we know that some returns will be lower than the geometric mean. Thus, we will set the $IRR_{fund_{new},y}$ slightly lower than the geometric mean of the portfolio return.

$$IRR_{fund_{new},y} = GMP - \alpha \tag{6.20}$$

We choose $\alpha = 0.005$. So, we have a small buffer of 0.5% in comparison to the expected portfolio return.

A fixed difference between Expected Return and Internal Rate of Return Since, we linked the fixed internal rate of return to the expected portfolio return this steering mechanism will be very similar. Previously, we approximated the expected portfolio return for every year by using the geometric mean of the return on investment over a 50 year period. However, we will now look at a variable expected portfolio return. The geometric mean of the return on investment will no longer take into account the entire period of 50 years. We reduce the period in which we are interested to 10 years, which will make the average portfolio return change yearly.

$$GMP_y = \sqrt[10]{(1 + RoI_y) \cdot \ldots \cdot (1 + RoI_{y+10})} \tag{6.21}$$

The return on investment in year y is indicated by RoI_y. This will result in the following equation to calculate $IRR_{fund_{new}}$:

$$IRR_{fund_{new},y} = GMP_y - \alpha \tag{6.22}$$

in which we assume $\alpha = 0.005$ as we did before.

A fixed ambition to compensate for price inflation In case of price inflation based steering, we will compensate the cash flows based on price inflation. We will only compensate whenever the price inflation is positive and there is sufficient capital available. This means that we cannot keep compensating for price inflation when the internal rate of return increases too much. The return needed to compensate for price inflation has to be possible to achieve on the market. Missed price inflations, will be caught up with whenever possible. This gives the following Indexation policy (I_y) in year y:

$$I_y = \min(CPI_y, CMP_y - IRR_y) \tag{6.23}$$

CPI_y indicates the cumulative price inflation in year y, it will add missed price inflations whenever they are positive. $CMP_y - IRR_y$ represents the room for indexation left based on the market, in which CMP_y is calculated using (6.21). Looking back to the algorithm in step 3, this steering mechanism will replace the old individual cash flows by the new cash flows which are equal to $U^n_{indexed,y,t} = (1 + I_y) \cdot U^n_{y,t}$. Next, we will calculate the $IRR_{fund_{new},y}$ based on $U^n_{indexed,y,t}$ and PA_y. After that we execute step 4.

6.8 Adjustment Factor

We will compare the contracts based on the adjustment factor, for the FTK- and 2019-ambition-contract we could use (5.43) to do so. We needed to use the size of the indexations to calculate the adjustment factor in year y, because benefit cuts had a 10 year buffer period. This is in the IRR-ambition-contract no longer the case. Both positive and negative indexations are directly absorbed. Therefore, we can define the adjustment factor based on the change in pension entitlements before and after indexation. Since, indexations in the IRR-ambition-contract are no longer uniformly distributed to their participants, we will make a distinction between participants. A participant will only have pension entitlements when he is alive, therefore we will always consider $t = 0$ when we calculate the adjustment factor of participant n in year y. This gives us the following equation for the adjustment factor:

$$\text{Adj. Factor}^n_y = \frac{PE^n_{after,0,y}}{PE^n_{before,0,y}} \tag{6.24}$$

The other criteria used to compare the contracts will be calculated in the exact same way as in the FTK- and 2019-ambition-contract. So, for these calculations we refer back to Section 5.5 and Section 5.7.

6.9 Simulation

In this section we will combine the previous sections of this chapter to explain how the simulation works. Within the simulation we keep track of the following variables, namely:

Fund level	Participant level
Pension assets	Pension entitlements before indexation
Price index	Pension entitlements after indexation
Pension liabilities	Pension liabilities
Market value discount rate	Premium payments

Table 6.1: Important variables within the simulation.

The simulation requires the same input as the 2019 Ambition-contract. Those input variables were the pension age, starting year, participant data and premium percentage. Using this input we will first use the pension entitlements of each participant and calculate, by using the purchase price based on the interest rate curve, what the pension liabilities are for the pension fund, this is described in Section 5.3. Next, based on the funding ratio of 1.10 we calculate the starting value of the pension assets of the fund. Then we start the iterative procedure, within the loop we first calculate cash flow of the expected future pension payments. We do this by summing the cash flow corresponding to the premium payments and the pension entitlements. Next, we update the value of the pension assets of the fund and participant based on the premium paid, pension payments received by the participants and the return on investment. Next, we will calculate the internal rate of return of the fund based on the pension assets of the fund and the liability cash flow. We use this internal rate of return to calculate the corresponding the pension assets of the participant. Now, we can start the indexation procedure based on the chosen steering method. The steering method is either based on the fixed internal rate of return or fixed difference between expected return and internal rate of return or based on price inflation. The steering mechanism will result in a new internal rate of return of the fund and indexed cash flows. Thus, we scale the individual cash flows such that the internal rate of the fund matches that of the indexed cash flows. Based on the scaled cash flows we calculate the pension entitlements after indexation. The pension entitlements after indexation are then used to calculate the pension entitlements at the start of next year. The start of the year pension entitlements together with the pension which is build next year will be used to calculate the new internal rate of return in the next loop. This process is repeated until the number of years selected has past. We can also see this process in a very compact form in the algorithm below. As before we indicate the used equations to run the simulation within the algorithm.

Algorithm 2 IRR-ambition-contract

Result: Entitlements OP

Initialize: pension age, starting year, participant data, funding ratio;
Set: prognosis year $y = 0$;
Calculate: starting value of pension liabilities using (5.7);
Calculate: starting value of pension assets using (5.5);
while *prognosis year* $y < 50$ **do**
    Update $U_{t,y}$ and $U^n_{t,y}$ using (6.12) and (6.11), which uses (6.9) and (6.10);
    Update PA_y ultimo year using (6.1), which uses (5.16) and (6.14);
    Calculate $IRR_{fund,y}$ using (6.6), which uses (6.4) and (6.5);
    Calculate PA^n_y using (6.16);
    Update $PE^n_{before,t,y}$ using (6.13);
    if *Steering mechanism == fixed IRR* **then**
        set $IRR_{fund_{new},y}$ based on (6.20);
    end
    if *Steering mechanism == fixed difference between* $\mathbb{E}[P_y]$ *and IRR* **then**
        Set $IRR_{fund_{new},y}$ based on (6.21);
    end
    if *Steering mechanism == price inflation* **then**
        Calculate indexation I_y based on (6.23);
        Calculate $U^n_{indexed,y,t} = (1 + I_y) \cdot U^n_{t,y}$;
        Calculate $IRR_{fund_{new},y}$ based on PA_y and $U^n_{indexed,y,t}$ using (6.6);
    end
    Calculate γ^n_y based on $IRR_{fund_{new},y}$ and PA^n_y using (6.17);
    Calculate $U^n_{scaled,t,y}$ using (6.18) and scale factor γ^n_y;
    Update $PE^n_{after,t,y}$, using (6.14);
    Calculate Adj. Factor, using (6.24);
    if *Age participant* $\geq$ *RA* **then**
        Calculate utility of pension payment, using (5.32);
    end
    Calculate $PE^n_{start,t,y}$ by using (5.31);
    Age participants = age participants + 1;
    Prognosis year y = prognosis year y + 1;
end
Calculate CEC, using (5.38);

The same works for the calculation of the generational account. Again, the generational account calculations are based on a risk-neutral set. Therefore, to avoid confusion these steps are not taken into account within the algorithm.

Chapter 7

Results

In this chapter, we will discuss the results of the simulation. For each contract, we will look at their certainty equivalent and adjustment factor. We will compare based on these criteria. Next, we will also compare based on the market value. Note, in Section 5.7 we showed that the market value is a tool to calculate the net-profit-pictures, which use the current FTK-contract as basis.

7.1 FTK-contract

In this section we will discuss the results of the FTK-contract. We will start by looking at the certainty equivalent. We will look at several different interpretations of the certainty equivalent. First, we look at the overall certainty equivalent, introduced in (5.38) and the certainty equivalent per scenario, which is introduced in (5.40). The results are shown in Table 7.1. We calculated the statistic in Table 7.1 based on the certainty equivalent per scenario and the overall certainty equivalent.

	Overall	Mean	Median	Max	Min	Std
$CEC_{\gamma=2}$	17900	20320	19460	53500	4720	7160
$CEC_{\gamma=5}$	7070	9510	9790	14330	1770	2100
$CEC_{\gamma=10}$	2320	6650	7100	8500	990	1380

Table 7.1: Certainty equivalent FTK-contract.

The overall certainty equivalent is lower than the mean certainty equivalent for every γ. Furthermore, we note that the difference between the mean certainty equivalent and the overall certainty equivalent increases as γ increases. This is because lower values are weighed heavier whenever γ increases, since a more risk-averse person considers lower pension payments less desirable.

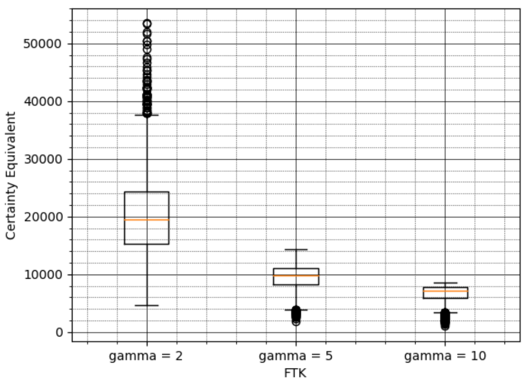


Figure 7.1: Boxplot of the certainty equivalent.

In Figure 7.1 we plotted the certainty equivalent per scenario. The figure shows the clear decrease in the certainty equivalent whenever γ increases. We also see that the spread decreases whenever γ increases. This coincides with the results from Table 7.1.

In Table 7.2 we show overall, mean, median and standard deviation of the certainty equivalent per scenario per age group. The table clearly shows that retirees which started with 70% of their pension basis as pension entitlements are worse of then working people which had their pension entitlements bought according to the FTK-contract. For $\gamma = 2$, we see an increasing certainty equivalent from age 20-30, which then starts to decrease from age 30-90. However, for $\gamma = 5, 10$, the overall, mean and median certainty equivalent decrease from age 20-70 and then start to increase after. So, participants going through the system are better off, this explains the decrease from 20-70. The increase from 70-90 can be partly explained by looking at the participant data. Based on the pension entitlements per age group, which can be found in Table 4.3, we can calculate an average pension entitlement per age group. One can verify that the average pension entitlement decreases as the age increases from 70-90. This partly explains the decrease in overall, mean and median certainty equivalent for $\gamma = 2$. However, when we start to look at the more risk averse values of γ, we should note that the percentage of participants with high pension entitlements is lower when the age increases from 70-90. We refer to participant type 35-36 for age 70, 41-42 for age 80 and 47 for age 90. These participants do have a lot of impact, concerning the average value of the pension entitlements per age group. However, as γ increases the weight corresponding to these participants decreases when calculating the certainty equivalent. Thus, because of the more risk averse nature when $\gamma = 5$ or 10 the overall, mean and median certainty equivalent increases from 70-90.

Age participants	20	30	40	50	60	70	80	90
Overall$_{\gamma=2}$	27460	31010	28550	22440	16770	9690	8150	7990
Overall$_{\gamma=5}$	14150	13050	10380	8060	6730	5290	5530	5890
Overall$_{\gamma=10}$	6190	5320	4100	3020	2320	1910	2890	3710
Mean$_{\gamma=2}$	37980	43110	37950	27920	19320	10370	8390	8080
Mean$_{\gamma=5}$	32510	30060	21790	15060	10820	7050	6260	6190
Mean$_{\gamma=10}$	30190	23870	15760	10690	7880	5790	5400	5430
Median$_{\gamma=2}$	32150	36440	33060	25110	18240	10140	8360	8070
Median$_{\gamma=5}$	27510	25830	19470	13940	10470	7230	6470	6350
Median$_{\gamma=10}$	25550	20700	14200	10010	7830	6150	5760	5720
Std$_{\gamma=2}$	24150	27330	22010	13840	7470	2670	1410	860
Std$_{\gamma=5}$	20690	18400	11890	6890	3710	1560	970	660
Std$_{\gamma=10}$	19230	14100	8250	4710	2560	1270	930	720

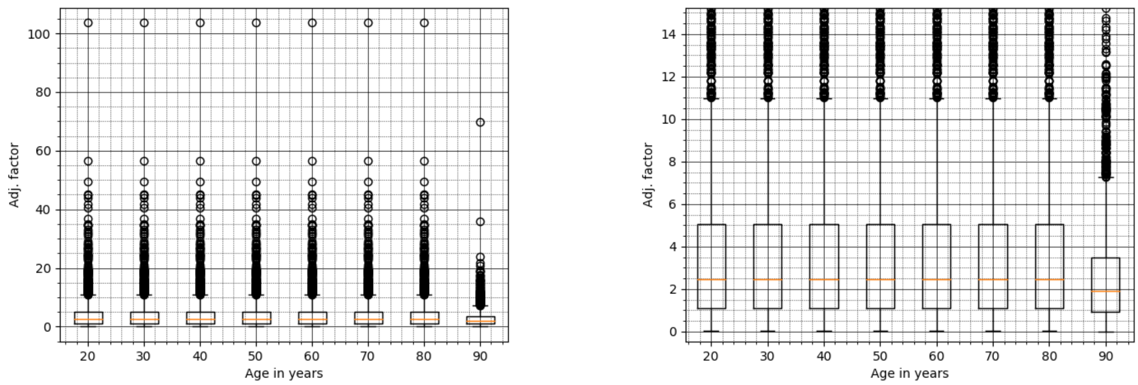
Table 7.2: Info about the certainty equivalent per age group FTK-contract.

In Table 7.3 we give several statistics corresponding to the adjustment factor per age group. The average adjustment factor is approximately 4. So, on average the pension entitlements of every age group tend to improve with a factor 4. Furthermore, we note that all statistics are the same for every age group within the simulation, besides the 90 year group. This is because the FTK-contract spreads out their indexations and benefit cuts uniformly. So, the pension of every participant is adjusted in the same manner. The 90-year old age group has different values, because the simulation length is 50 years, but a participant dies at an age of 130. So, they do not participate the last 10 years. In Table 7.3 we also give the probability of a negative indexation and a negative scenario. A negative indexation is defined as a single adjustment factor which decreases a participant its benefits, so the adjustment factor is smaller than 1. A negative scenario is defined as a scenario for which the product of adjustment factors is smaller than 1, this means that the adjustment factor over the entire period of 50 years has to be smaller than 1. As the table shows negative scenarios happen in 22.7% of the cases, which is relatively high. The amount of negative indexations, on the other hand, is approximately 11.6%, which is low.

Age	20	30	40	50	60	70	80	90
Mean	4.150	4.150	4.150	4.150	4.150	4.150	4.150	2.736
Median	2.484	2.484	2.484	2.484	2.484	2.484	2.484	1.913
Max	103.595	103.595	103.595	103.595	103.595	103.595	103.595	69.785
Min	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.017
Std	5.535	5.535	5.535	5.535	5.535	5.535	5.535	3.082
Prob. neg. sce.	0.227	0.227	0.227	0.227	0.227	0.227	0.227	0.273
Prob. neg. index.	0.116	0.116	0.116	0.116	0.116	0.116	0.116	0.098

Table 7.3: Adjustment Factor FTK-contract.

As we can see in Figure 7.2 there are several outliers corresponding to the adjustment factor. These outliers are extreme scenarios with high adjustment factors, so these are not a problem. Furthermore, based on the plot and Table 7.3 we know that only 11.6% percent have an adjustment factor below 1. So, the contract tends to perform very well.



(a) Normal. (b) Zoomed in.

Figure 7.2: Boxplot of the adjustment factor per age group.

Participant related data can be found in Appendix B. This data coincides with the per age group data.

7.2 2019-ambition-contract

In this section we discuss the same statistics as in the FTK-contract for the 2019-ambition-contract. As we can see in Table 7.4 the overall, mean and median certainty equivalent are slightly higher for the 2019-ambition-contract than for the FTK-contract. The maximum, minimum and standard deviation are very similar. So, based on this the 2019-ambition-contract tends to perform better.

	Overall	Mean	Median	Max	Min	Std
$CEC_{\gamma=2}$	18080	20490	19580	53310	4830	7210
$CEC_{\gamma=5}$	7340	9650	9910	14640	1870	2150
$CEC_{\gamma=10}$	2460	6750	7170	8680	1040	1400

Table 7.4: Certainty equivalent 2019-ambition-contract.

In Figure 7.3 we can see a more visual representation of the data from Table 7.4. Due to the nature of the utility function in (5.32), we will keep seeing the decreasing certainty equivalent as γ increases.

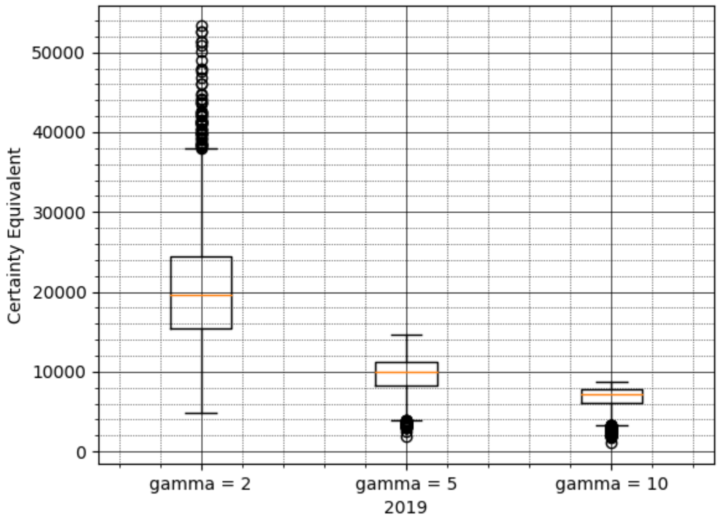


Figure 7.3: Boxplot of the certainty equivalent.

In Table 7.5 we can see an increasing overall, mean and median certainty equivalent from ages 20-40, after that the certainty equivalent starts to decrease from age 40-90. This was also the case in the FTK-contract. The standard deviation, however does only decrease from age 20-90. This is because the group of 20-year old participants will endure the longest period for which they both have deviations caused by purchasing pension and pension indexations. Other age groups will lose the deviation coming from pension purchases in an earlier moment in time within the 50 years. Therefore, their pension payments will be less volatile and thus their certainty equivalent will be less volatile. When we compare Table 7.5 with Table 7.2 we see that the 2019-ambition-contract performs better for every age group based on the mean, median and overall certainty equivalent.

Age participants	20	30	40	50	60	70	80	90
Overall$_{\gamma=2}$	27840	31330	28740	22590	16940	9810	8240	8070
Overall$_{\gamma=5}$	15210	13930	11030	8530	7050	5470	5590	5930
Overall$_{\gamma=10}$	7750	6550	4780	3330	2440	2020	3050	3690
Mean$_{\gamma=2}$	38140	43170	37920	27950	19500	10520	8500	8170
Mean$_{\gamma=5}$	32650	30130	21800	15110	10960	7170	6340	6250
Mean$_{\gamma=10}$	30320	23920	15750	10730	8000	5890	5460	5480
Median$_{\gamma=2}$	32170	36490	33010	25220	18360	10280	8450	8180
Median$_{\gamma=5}$	27550	25880	19510	13970	10620	7300	6540	6430
Median$_{\gamma=10}$	25580	20830	14260	10030	7910	6210	5810	5780
Std$_{\gamma=2}$	24260	27300	21880	13710	7520	2750	1460	900
Std$_{\gamma=5}$	20780	18410	11730	6790	3750	1610	1010	690
Std$_{\gamma=10}$	19310	14110	8070	4630	2590	1300	960	740

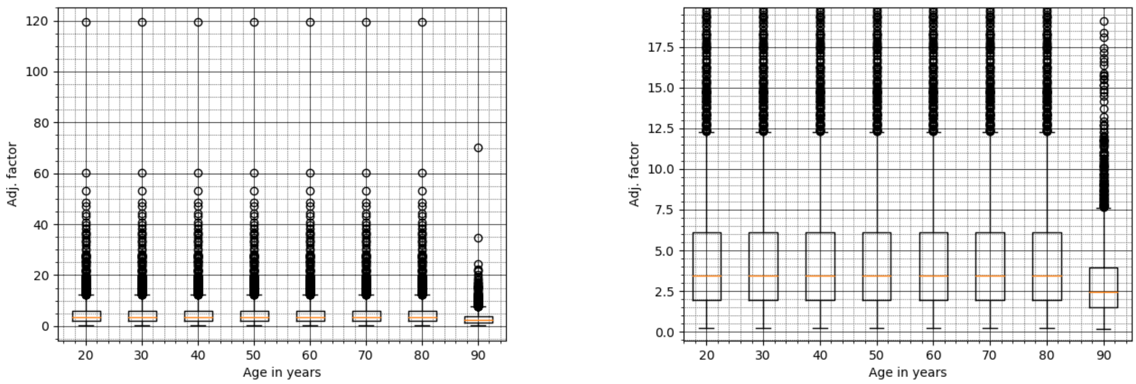
Table 7.5: Info about the certainty equivalent per age group 2019-ambition-contract.

In Table 7.6 and Figure 7.4 we can see the adjustment factor of the 2019-ambition contract. Again, we see the uniform distribution of pension indexations. The mean adjustment factor of the 2019-ambition-contract is almost 1 higher than the FTK-contract, the same holds for the median. Furthermore, we can see that the minimum is a factor 10 higher, which is very desirable, since benefit cuts are unwanted. We can also see that the amount of negative scenarios decreases from almost 23% to 8%, even though the amount of negative indexation increases from almost 11.5% to 18.0%. Thus, while the number of benefit cuts increases, the amount of scenarios which end up reducing the pension entitlement decreases. So, we can conclude that benefit cuts take place more frequently, than in the FTK-contract, but the size of them is smaller.

Age	20	30	40	50	60	70	80	90
Mean	5.049	5.049	5.049	5.049	5.049	5.049	5.049	3.188
Median	3.445	3.445	3.445	3.445	3.445	3.445	3.445	2.429
Max	119.335	119.335	119.335	119.335	119.335	119.335	119.335	70.118
Min	0.236	0.236	0.236	0.236	0.236	0.236	0.236	0.169
Std	5.829	5.829	5.829	5.829	5.829	5.829	5.829	3.021
Prob. neg. sce.	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.112
Prob. neg. index.	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.154

Table 7.6: Adjustment Factor 2019-ambition-contract.

The adjustment factor boxplot, given below, is very similar to the adjustment factor boxplot in Figure 7.2. Individual based data for the 2019-ambition-contract can also be found in Appendix B, the corresponding tables are Table 4 and 5.



(a) Normal. (b) Zoomed in.

Figure 7.4: Boxplot of the adjustment factor per age group.

7.3 IRR-ambition-contract

In this section, we will show the results based on the three different steering mechanisms. The results correspond to a fixed internal rate of return, a fixed difference between the expected portfolio return and the internal rate of return and the ambition to compensate for price inflation. We will start with the latter.

7.3.1 Compensation of price inflation

When we introduced the price inflation based steering in (6.23), we did not mention the risk averse nature of this steering principle. The steering mechanism only compensates the minimum of the cumulative price inflation and the difference between the expected portfolio return and internal rate of return. As a consequence, we will have a low internal rate of return during the simulation. This is positive, whenever it is hard to get a good return on our portfolio. However, this is in general not the case. It also has the negative effect that, a low internal rate of return leads to a high purchase rate, and thus expensive pension purchases. So, because we look at the minimum of the cumulative price inflation and the difference between the expected portfolio return and internal rate of return we cannot simply steer for a higher internal rate of return to avoid this problem. The effect of the expensive purchase rate can be seen in Table 7.7. As Table 7.7 shows, this leads to a low overall certainty equivalent, and low standard deviation. This can also be seen in Figure 7.5. This also leads to a lot of excess capital, which will not be used. We can see this more clearly when we look at the market value.

	overall	mean	median	max	min	std
$CEC_{\gamma=2}$	10850	11090	11110	16320	5620	1580
$CEC_{\gamma=5}$	5330	6040	6080	9830	2240	1180
$CEC_{\gamma=10}$	2670	4240	4200	7420	1320	1080

Table 7.7: Certainty Equivalent IRR-ambition-contract, price inflation based steering.

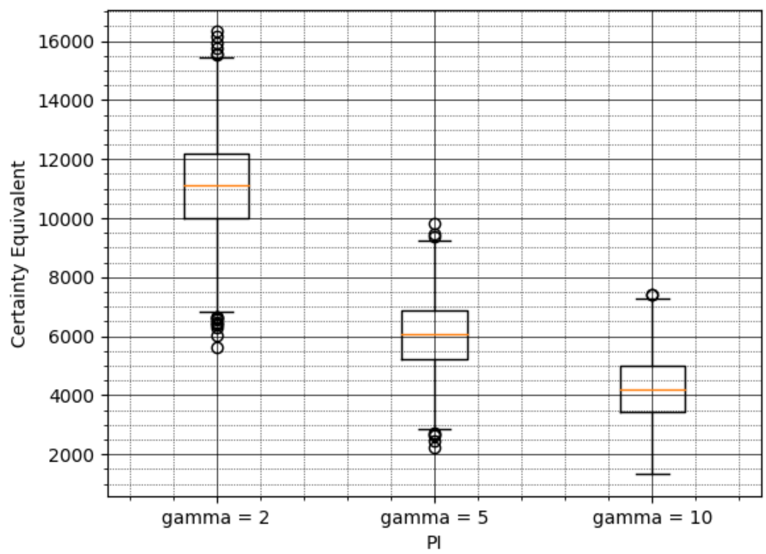


Figure 7.5: Boxplot of the certainty equivalent.

Based on Table 7.8 we can now see that the difference between age groups decreased a lot when looking at the overall, mean and median certainty equivalent. Especially, the younger participants suffer from this expensive purchase price. We can also see a low standard deviation in comparison to the previous contracts.

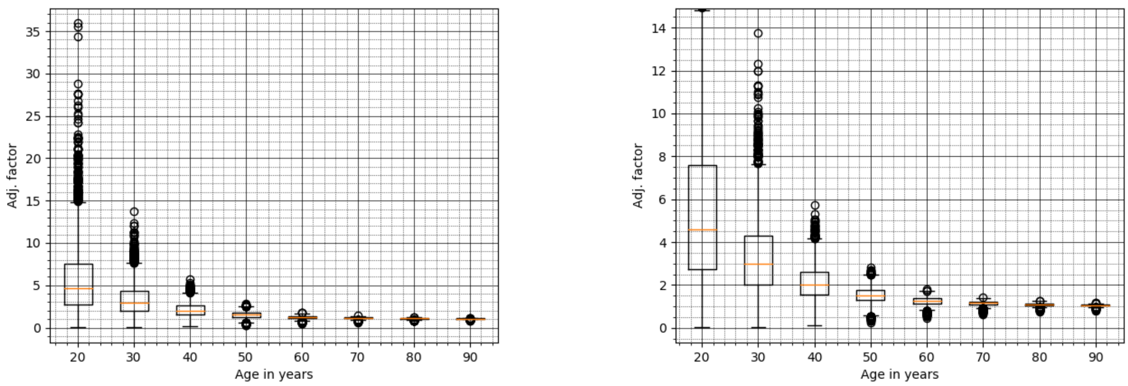
Age participants	20	30	40	50	60	70	80	90
$\text{Overall}_{\gamma=2}$	10280	16010	15720	12950	10310	6500	6160	6780
$\text{Overall}_{\gamma=5}$	5230	10010	9040	6990	5430	3870	3950	4760
$\text{Overall}_{\gamma=10}$	2830	6770	5900	4330	3090	2230	2250	3120
$\text{Mean}_{\gamma=2}$	15250	17720	16350	13310	10590	6660	6250	6820
$\text{Mean}_{\gamma=5}$	13060	12810	10100	7710	6200	4540	4500	5060
$\text{Mean}_{\gamma=10}$	12140	10690	7790	5750	4610	3660	3650	4190
$\text{Median}_{\gamma=2}$	12850	16870	16160	13290	10540	6660	6270	6850
$\text{Median}_{\gamma=5}$	11010	12180	9980	7710	6180	4560	4560	5140
$\text{Median}_{\gamma=10}$	10230	10170	7700	5730	4560	3620	3630	4240
$\text{Std}_{\gamma=2}$	9960	5770	3160	2140	1730	1020	750	530
$\text{Std}_{\gamma=5}$	8530	4170	1980	1390	1280	1000	880	700
$\text{Std}_{\gamma=10}$	7930	3480	1560	1130	1100	980	950	850

Table 7.8: Info about the certainty equivalent per age group IRR-ambition-contract, price inflation based steering.

The price inflation based steering, as it is now, gets outperformed by both the FTK-contract and 2019-ambition-contract because it keeps too much capital during the high purchase rate. However, looking at the adjustment factor in Table 7.9 we can see a desirable factor. The adjustment factor decreases as the age of the participant increases. This provides stable pension to the retirees. Looking at Figure 7.6 we can note that the adjustment factor in general is lower and less volatile. The spread within the boxplot is smaller, especially at higher ages. We can also see that the number of negative indexations is very low, namely 10.3%. Just as the number of negative scenarios is low. This is also explained by the safe nature of the steering mechanism.

Age	20	30	40	50	60	70	80	90
Mean	5.769	3.314	2.096	1.514	1.253	1.144	1.096	1.056
Median	4.602	2.968	2.020	1.512	1.264	1.154	1.100	1.060
Max	35.930	13.750	5.718	2.820	1.787	1.422	1.257	1.164
Min	0.012	0.039	0.115	0.264	0.467	0.641	0.755	0.812
Std	4.505	1.908	0.833	0.381	0.184	0.101	0.062	0.040
Prob. neg. sce.	0.064	0.075	0.085	0.094	0.094	0.083	0.064	0.083
Prob. neg. index.	0.103	0.103	0.103	0.103	0.103	0.103	0.103	0.095

Table 7.9: Adjustment Factor IRR-ambition-contract, price inflation based steering.



(a) Normal. (b) Zoomed in.

Figure 7.6: Boxplot of the adjustment factor per age group.

7.3.2 Fixed internal rate of return

We will now consider a fixed internal rate of return. Thus pension entitlements are always bought against the same price as previous years. We did this by linking the internal rate of return to the expected portfolio return over the entire simulation horizon. This was described in (6.19). Table 7.10 shows that this method also has a low standard deviation. This can be explained through the fact that the internal rate of return used to purchase entitlements is constant. We also see a great improvement in comparison to the price inflation based steering, however the FTK- and 2019-ambition-contract still perform better when looking at the overall certainty equivalent.

	overall	mean	median	max	min	std
$CEC_{\gamma=2}$	15690	16500	16230	32620	6370	3680
$CEC_{\gamma=5}$	6660	7380	7330	12440	3290	1420
$CEC_{\gamma=10}$	3510	4910	4860	8380	1860	1160

Table 7.10: Certainty Equivalent IRR-ambition-contract, steering for a fixed IRR.

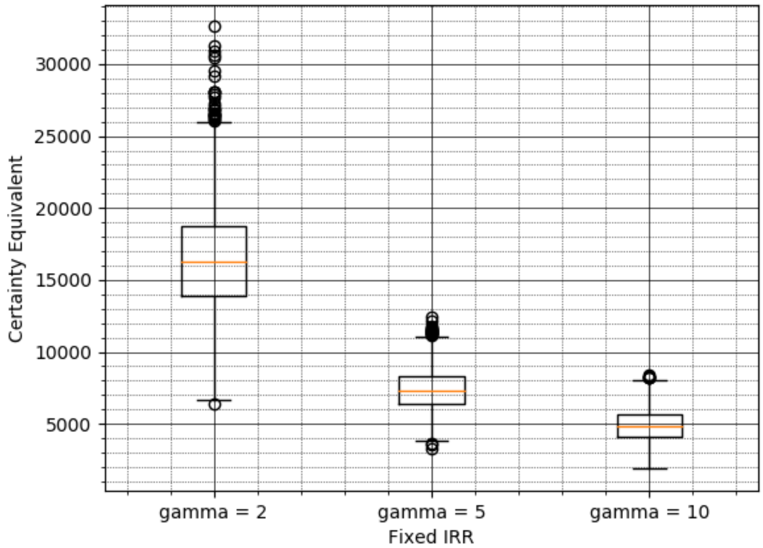


Figure 7.7: Boxplot of the certainty equivalent.

In Table 7.11 we can see that the 20-60 year old participants benefit from the fixed internal rate of return through the purchase rate. Pension entitlements are less expensive than in the price inflation based steering method. We can also see that the retirees benefit from this steering method. In addition, we note that the standard deviation for the 20 year old age group exceeds all other contracts. This coincides with an overall, mean and median certainty equivalent which is higher than in previous contracts. This type of steering tends to favour the younger participants age 20-30 over the other participants age 40-90 when we compare with the FTK- and 2019-ambition-contract.

Age participants	20	30	40	50	60	70	80	90
$\text{Overall}_{\gamma=2}$	30530	32440	26960	19960	14230	7730	6980	7330
$\text{Overall}_{\gamma=5}$	13240	15180	12670	9760	7480	4680	4530	5160
$\text{Overall}_{\gamma=10}$	7020	7430	6690	5530	4670	2970	2790	3490
$\text{Mean}_{\gamma=2}$	57530	44450	32540	22250	14980	7910	7060	7370
$\text{Mean}_{\gamma=5}$	49240	31650	19730	12650	8580	5270	4980	5400
$\text{Mean}_{\gamma=10}$	45690	25960	15000	9320	6310	4210	4000	4420
$\text{Median}_{\gamma=2}$	40020	37180	29330	20910	14540	7840	7030	7370
$\text{Median}_{\gamma=5}$	34270	26520	17890	12010	8320	5240	5010	5460
$\text{Median}_{\gamma=10}$	31790	21690	13590	8820	6130	4160	4000	4470
$\text{Std}_{\gamma=2}$	63830	29070	15360	7570	3420	1180	750	510
$\text{Std}_{\gamma=5}$	54630	20650	9070	4160	2020	1070	870	670
$\text{Std}_{\gamma=10}$	50720	17030	6810	3010	1550	1030	940	830

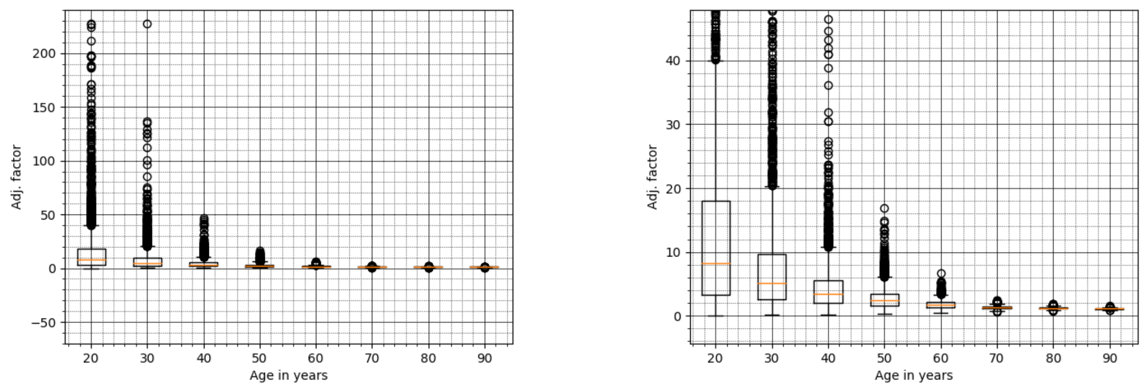
Table 7.11: Info about the certainty equivalent per age group IRR-ambition-contract, steering for a fixed IRR.

Table 7.12 shows what we just concluded. The 20-30 year old have a high adjustment factor of 18.6 respectively 8.6. Their pension entitlements are very volatile through indexation, while the pension entitlements of the retirees are not. The retirees have a very low standard deviation and even in the worst case they tend to keep $66.8 - 81.7$ percent of their pension. This is not the case in both the FTK- and 2019-ambition-contract. Moreover, we see that the probability of a negative scenarios is also very low, at most 9.2% for the 90-year group. The probability of negative indexations lies around 49%, this can be explained by the fact that we steer based on the average portfolio return. Thus, around 50% should be above the average and around 50% should be below. So, this variable lost most of its use.

Age	20	30	40	50	60	70	80	90
Mean	18.617	8.596	4.556	2.742	1.825	1.340	1.213	1.116
Median	8.254	5.111	3.384	2.389	1.745	1.335	1.208	1.111
Max	1407.297	227.486	46.430	16.880	6.650	2.406	1.849	1.584
Min	0.070	0.106	0.175	0.276	0.441	0.668	0.791	0.817
Std	44.730	12.519	4.380	1.712	0.678	0.239	0.144	0.089
Prob. neg. sce.	0.049	0.058	0.068	0.071	0.073	0.066	0.059	0.092
Prob. neg. index.	0.486	0.486	0.486	0.486	0.486	0.486	0.486	0.392

Table 7.12: Adjustment Factor IRR-ambition-contract, steering for a fixed IRR.

In Figure 7.8 we can clearly see that the adjustments are mainly absorbed by the age groups form 20-30 and partly by the age groups from 40-60. We can also see that the retirees, so the age group from 70-90, have a very stable pension.



(a) Normal. (b) Zoomed in.

Figure 7.8: Boxplot of the adjustment factor per age group.

All the results for the fixed IRR are based on a difference of $\alpha = 0.005$, between the IRR and the expected portfolio return. We will briefly look into the case where $\alpha = -0.005$ and $\alpha = 0.0$.

	overall	mean	median	max	min	std
$CEC_{\gamma=2}$	15360	16160	15860	31760	6410	3600
$CEC_{\gamma=5}$	6550	7320	7260	12250	3190	1470
$CEC_{\gamma=10}$	3380	4830	4760	8640	1800	1190

Table 7.13: Certainty Equivalent IRR-ambition-contract, steering for a fixed IRR, $\alpha = -0.005$.

Looking at Table 7.13 we see that steering based on $\alpha = -0.005$ gives a slightly lower mean, median and overall certainty equivalent than for $\alpha = 0.005$. As Table 7.14 shows, the reason for this is the decreased value in for the 20-50 old groups. However, the 60-90 year old seem to benefit from this. We also see more moderate adjustment factors when looking at Table 7.15. The probability of negative indexations increased, due to the fact that we now aim for an internal rate of return higher than the expected portfolio return. So, cash flows need to be scaled down more often to meet this requirement and keep sufficient capital.

Age participants	20	30	40	50	60	70	80	90
Overall$_{\gamma=2}$	23280	27000	24840	19880	14630	7830	7130	7470
Overall$_{\gamma=5}$	10480	13040	11810	9640	7520	4580	4490	5180
Overall$_{\gamma=10}$	5400	6670	6450	5490	4640	2850	2710	3440

Table 7.14: Info about the certainty equivalent per age group IRR-ambition-contract, steering for a fixed IRR with $\alpha = -0.005$.

Age	20	30	40	50	60	70	80	90
Mean	10.413	6.233	3.918	2.613	1.795	1.293	1.177	1.098
Median	4.593	3.696	2.912	2.282	1.721	1.288	1.172	1.093
Max	799.336	161.236	39.807	15.719	6.473	2.284	1.776	1.557
Min	0.0427	0.081	0.159	0.275	0.451	0.667	0.785	0.809
Std	25.057	9.086	3.747	1.610	0.653	0.221	0.134	0.087
Prob. neg. sce.	0.114	0.105	0.092	0.080	0.073	0.085	0.083	0.125
Prob. neg. index.	0.532	0.532	0.532	0.532	0.532	0.532	0.532	0.429

Table 7.15: Adjustment Factor IRR-ambition-contract, steering for a fixed IRR with $\alpha = -0.005$.

In Table 7.16 we see the data based on $\alpha = 0.0$. Steering based on $\alpha = 0.0$ does better than $\alpha = -0.005$, but worse than $\alpha = 0.005$ when we compare based on the overall, mean and median certainty equivalent. Based on Table 7.17 we can see that the difference is caused by a shift in value from the 20-50 year old to the 60-90 when we compare with $\alpha = 0.005$. The shift, is however smaller than in the case of $\alpha = -0.005$. From this we can conclude that the value of α can determine whether pension is more allocated to young or old participants.

	overall	mean	median	max	min	std
$CEC_{\gamma=2}$	15560	16370	16100	32250	6400	3650
$CEC_{\gamma=5}$	6620	7360	7310	12330	3250	1450
$CEC_{\gamma=10}$	3450	4880	4820	8520	1830	1180

Table 7.16: Certainty Equivalent IRR-ambition-contract, steering for a fixed IRR, $\alpha = 0.0$.

Age participants	20	30	40	50	60	70	80	90
Overall$_{\gamma=2}$	26750	29710	25970	19960	14450	7790	7060	7400
Overall$_{\gamma=5}$	11830	14140	12290	9730	7520	4640	4510	5170
Overall$_{\gamma=10}$	6210	7080	6610	5530	4670	2910	2760	3470

Table 7.17: Info about the certainty equivalent per age group IRR-ambition-contract, steering for a fixed IRR with $\alpha = 0.0$.

7.3.3 Fixed difference between ER-IRR

In this subsection, we will discuss the results based on steering for a fixed difference between the expected portfolio return and internal rate of return. As (6.21) indicates, we look at a variable portfolio return. Therefore, we will abbreviate this steering method to ER-IRR(variable).

Based on Table 7.18 we can see that the ER-IRR(variable) steering method performs similar to the fixed internal rate of return method, but slightly worse.

	overall	mean	median	max	min	std
$CEC_{\gamma=2}$	15290	16070	15720	30970	6020	3580
$CEC_{\gamma=5}$	6420	7150	7100	11730	3130	1410
$CEC_{\gamma=10}$	3320	4750	4700	8100	1720	1130

Table 7.18: Certainty Equivalent IRR-ambition-contract, steering for a fixed difference between ER-IRR(variable).

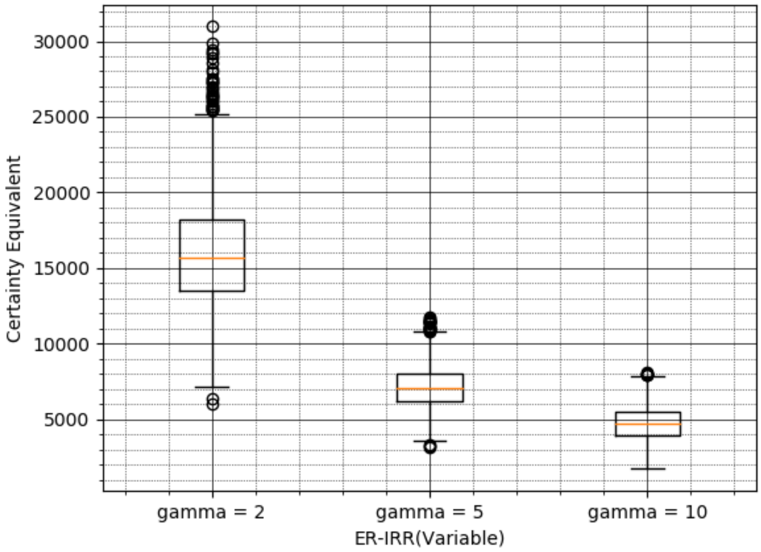


Figure 7.9: Boxplot of the certainty equivalent.

When we compare Table 7.19 with Table 7.8 we can see that steering based on a fixed internal rate of return gives a higher mean, median and overall certainty equivalent for every age group. However, the standard deviation is significantly smaller for the age groups from 20-40 years in case we steer based on the ER-IRR(variable). So, this contract tends to give more stable pension payments, but in general lower.

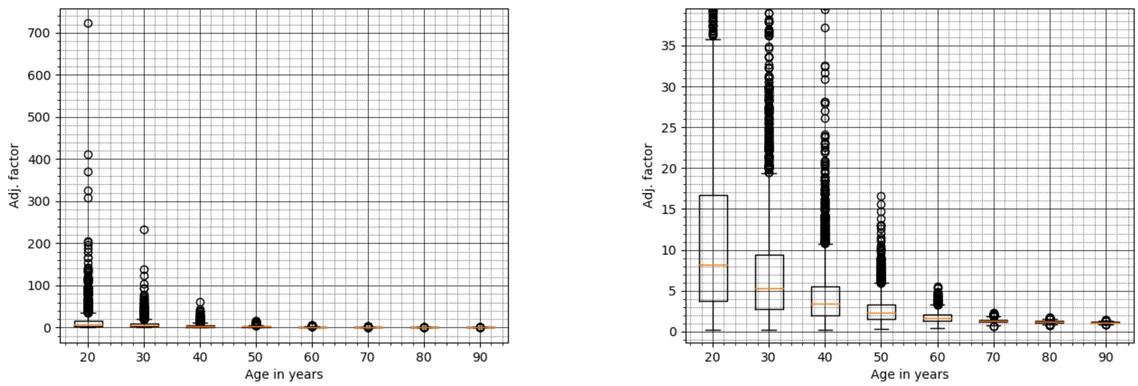
Age participants	20	30	40	50	60	70	80	90
Overall$_{\gamma=2}$	29340	31960	27160	19680	13590	7510	6760	7160
Overall$_{\gamma=5}$	15030	16230	12690	9320	6890	4530	4400	5050
Overall$_{\gamma=10}$	8310	8410	4910	4800	4110	2830	2650	3380
Mean$_{\gamma=2}$	41530	40540	32120	22050	14470	7700	6850	7200
Mean$_{\gamma=5}$	35520	28880	19320	12350	8190	5110	4850	5300
Mean$_{\gamma=10}$	32910	23700	14570	9020	5990	4080	3900	4350
Median$_{\gamma=2}$	35180	35790	29240	20600	13980	7630	6830	7190
Median$_{\gamma=5}$	30120	25500	17640	11560	7880	5090	4880	5370
Median$_{\gamma=10}$	27880	20920	13310	8430	5720	4040	3900	4410
Std$_{\gamma=2}$	26770	21800	14200	7800	3680	1200	790	540
Std$_{\gamma=5}$	22900	15510	8480	4350	2190	1050	860	660
Std$_{\gamma=10}$	21230	12790	6420	3210	1680	990	930	810

Table 7.19: Info about the certainty equivalent per age group IRR-ambition-contract, steering for a fixed difference between ER-IRR.

Based on Figure 7.10 and Table 7.20 we again see that the indexations are absorbed mostly by the 20-30 year groups. We see that the minimum adjustment factor for the retirees are very similar in comparison to the fixed internal rate of return steering. In addition, we note that the mean and median adjustment factor is slightly lower than with the fixed IRR method. This coincides with what we saw in Table 7.19. In Table 7.20 we can also see that the probability of a negative scenario is really low for retirees and young participants. This probability is at its highest for the 60 year old group, and even in this case it is just 9.2% which is lower than for the FTK-contract and slightly higher than for the 2019-ambition-contract.

Age	20	30	40	50	60	70	80	90
Mean	15.581	8.169	4.535	2.690	1.741	1.317	1.191	1.110
Median	8.224	5.250	3.419	2.302	1.649	1.310	1.191	1.109
Max	722.655	233.598	61.919	16.601	5.540	2.307	1.749	1.380
Min	0.182	0.172	0.205	0.293	0.447	0.660	0.790	0.884
Std	28.592	10.877	4.242	1.710	0.650	0.232	0.135	0.077
Prob. neg. sce.	0.038	0.044	0.058	0.080	0.092	0.084	0.072	0.076
Prob. neg. index.	0.476	0.476	0.476	0.476	0.476	0.476	0.476	0.379

Table 7.20: Adjustment Factor IRR-ambition-contract, steering for a fixed difference between ER-IRR.



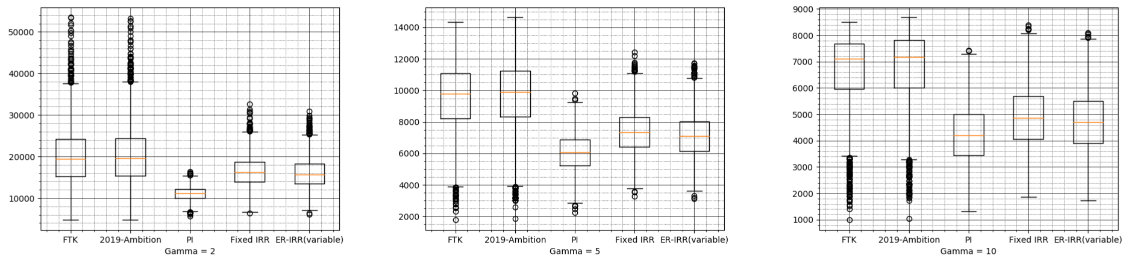
(a) Normal. (b) Zoomed in.

Figure 7.10: Boxplot of the adjustment factor per age group.

7.4 Comparing contracts

In previous sections we discussed the results of each contract and briefly started with comparing the contracts. In this section we will continue comparing the contracts. We start by revisiting the certainty equivalent per scenario.
In Figure 7.11a we see that the 2019-ambition-contract performs similar but slightly better than the FTK-contract. Next, we see that the fixed IRR and ER-IRR(variable) contract perform very similar. We can also see that the price inflation has the lowest certainty equivalent, however its spread is significantly smaller than in case of the other contracts. As γ increases the order of the contracts based on the certainty equivalent will remain unchanged. However, the spread of the price inflation based IRR-ambition-contract will be more similar to those of the fixed IRR and ER-IRR(variable) contract. We can also see that the ratio of the distance between the minimum and Q1 and the distance between the maximum and Q3 increase in the FTK- and 2019-ambition-contract when looking at Figure 7.11c, in which Q1 and Q3 refer to the 25% and 75% quantiles within the boxplot.

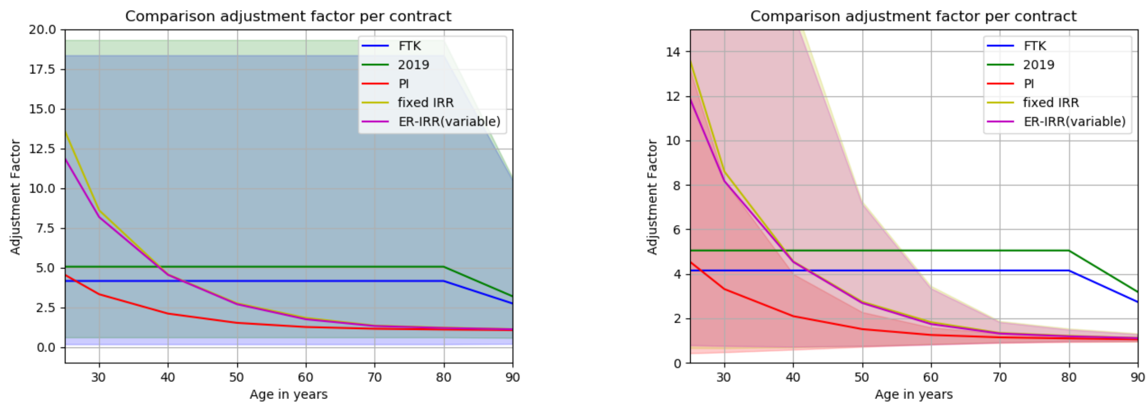
In Figure 7.12 we revisit the adjustment factor per contract. In Figure 7.12a we plotted the 2.5-97.5 confidence bounds of the FTK- and 2019-ambition-contract, and in Figure 7.12b we did this for the IRR-



(a) $\gamma = 2$. (b) $\gamma = 5$. (c) $\gamma = 10$.

Figure 7.11: Box-plots used to compare contracts for different γ values.

ambition-contracts. Based on Figure 7.12b we note that the price inflation IRR-ambition-contract hardly indexes their pension entitlements. We also see that the IRR-ambition-contracts protects the elderly against indexations by inducing them on the younger participants. This property is very desirable, the figure clearly indicates that even at the 2.5-97.5 bounds, the retirees pension hardly fluctuates. While Figure 7.12a shows that this is not the case for the FTK- and 2019-ambition-contract. In those contracts the pension of the retirees can still be changed by a factor between 0 and 19 in 95% of the situations. Remark that the distinction between the fixed-IRR and ER-IRR(variable) steering methods in the adjustment factor is very small, even at the confidence bounds. One needs to look really good to see the transition in colours in the upper and lower bound belonging to the difference between the fixed-IRR and ER-IRR(variable) steering methods. Therefore, Figure 4 shows the confidence bounds of all three IRR based contracts separately. Since, Figure 4 shows the confidence bounds separately, we do not get a mixture of colours which occurs in Figure 7.12b. Therefore, we get a more clear image of what the corresponding confidence bounds are for each contract.



(a) FTK-contract and 2019-ambition-contract. (b) IRR-ambition-contract.

Figure 7.12: Plots with 95% confidence bounds for the contracts.

In Figure 7.13 we plotted the median net-profit per age group calculated based on (5.46). The blue line indicates a change in contracts from the FTK-contract to the FTK-contract. Therefore, no groups will benefit from this and thus we get a flat line at zero. When we look at the green line, corresponding to the 2019-ambition-contract we see that the market value for the 20-30 year group increased in comparison to the FTK-contract. This is also the case for the 70-90 year old. However, the working classes from 40-60 year are worse off. We can see this more clear in Figure 1a, in which we plotted the 90% confidence bounds around the median value.

In case of the price inflation based steering for the IRR-ambition-contract, we noticed that the certainty equivalent of the pension payments were low compared to the other contracts. This resulted in a high value of the pension assets at the end of the simulation. Since, we assign these based on liabilities at the end of the simulation, these will be assigned mostly to the younger groups, aka 20-40 year old. These groups just retired and thus have a higher horizon of payments left, but they also did not get influenced as much by the probability of death as the older participants. Remember, that liabilities are calculated based on survival probabilities and we considered that the pension entitlements decreased based on the ageing process in (5.31). Together this explains why the market value is so much higher for the 20-40 year groups, in comparison to the FTK-contract. However, as we can see, this is at the cost of the 50-90 year groups. These groups received lower pension payments, and do not have a lot of the remaining assets assigned to them. This is more clearly presented in Figure 1b.

In case of steering based on a fixed internal rate of return, we see again that the younger age groups benefit largely, from the non-uniform distribution of the indexations. However, for this contract we also see that the 60-90 year group has a positive factor of change in comparison to the FTK-contract.
The last option is steering based on a fixed difference between the expected portfolio return and internal rate of return. As we can see the market value of the 20-50 year group increased in comparison to the FTK-contract. However the retirees, 70-90 year, are disadvantaged at their cost. We see this very clearly in Figure 2b. Even though in several situations the 70-90 year group is better of, in most situations they are not.

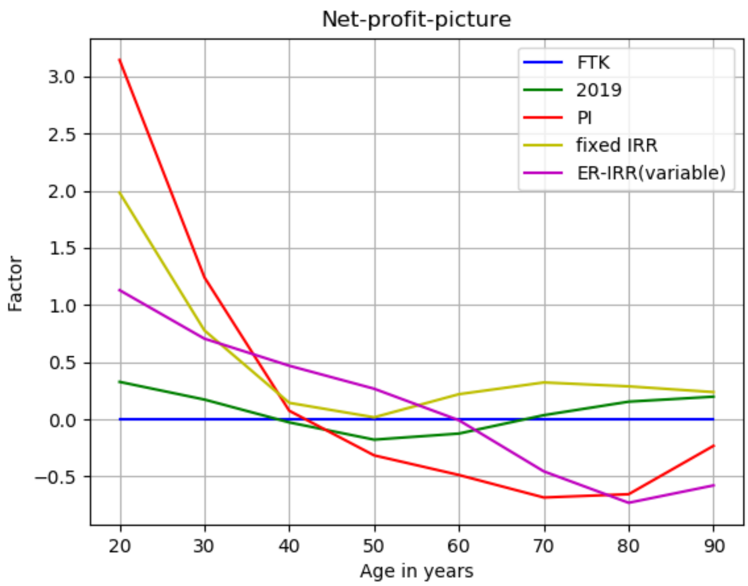
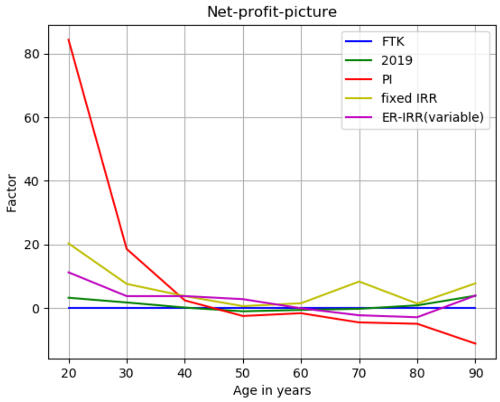


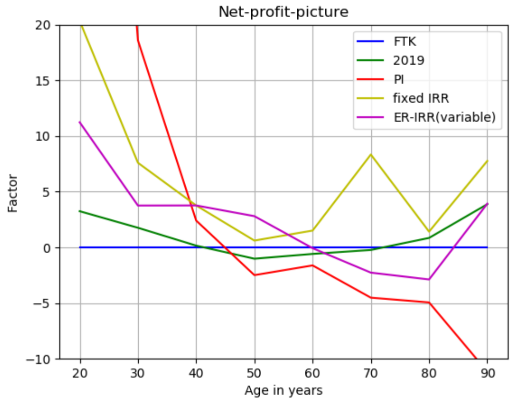
Figure 7.13: Plot of the median change in the market value.

The above conclusions were based on the median and confidence bounds, nevertheless it is also interesting to look at the mean of the situations. We can see the mean in Figure 7.14. As we compare the net-profit-pictures of the mean with the median we notice that the shape of the lines is very similar, but there are some small differences. First, we can notice that the factors of change are much higher. This implies that there are large outliers, upwards as well as downwards.

Next, we can see that the mean market value in the 2019-ambition-contract is higher for the 20-40 and 80-90 groups, and lower for the 50-70 groups. This is a slight change in comparison to the situation of the median value, in which it performed better for the 20-30 and 70-90 groups. In case of the price inflation based IRR-contract we can make the same conclusion as before. The 20-40 group is better off, at the cost of the 50-90 group. However, the increase going for ages 80 to 90 changed to an even stronger decrease. For this contract, we again see the large factors of change in comparison to the other contracts. When looking at the fixed IRR contract we can see that it outperforms the FTK- and 2019-ambition-contract for every age group. Furthermore, we can see that steering based on a fixed difference between the expected portfolio return and internal rate of return does again better for the 20-50 groups. However, the 90 year group is also better of in the mean of the situations. This was not the case in the median. Nevertheless, the 70-80 group remains worse off in comparison to the FTK-contract.



(a) Normal.



(b) Zoomed-in.

Figure 7.14: Plot of the mean change in the market value.

Chapter 8

Discussion

In the previous chapter we showed the results of the simulation, and compared the results of the contracts. There is no contract which outperforms the other contracts based on all three factors, namely the certainty equivalent, adjustment factor and market value. So, we cannot define one contract as the absolute best contract. However, we can make several observations looking at the distinct factors.

8.1 Certainty Equivalent

When we compare purely based on the overall certainty equivalent and the certainty equivalent per scenario, which are calculated by (5.38) and (5.40), then we conclude that the 2019-ambition-contract outperforms all other contracts. This contract has the highest certainty equivalent when we look at the entire population together. If we distinguish between age groups it will start depending on which goal you have. If we want to put the weight on giving younger people a higher pension, then we recommend using the IRR-ambition-contract with fixed internal rate of return. This one dominates the other internal rate of return based contracts. However, if we want to distribute the amount of pension more equal between every group the 2019-ambition-contract is preferred. The 2019-ambition-contract dominates the FTK-contract based on this criteria.

8.2 Adjustment Factor

Based on the adjustment factor we can clearly state that the 2019-ambition-contract does better than the FTK-contract. The 2019-ambition-contract has a higher mean, median, max and minimum adjustment factor, while the standard deviation is lower. Furthermore, based on the probability of a negative indexation, we can conclude that the number of negative indexations is lower. We can also clearly see that the amount of negative scenarios is lower for the 2019-ambition-contract.
When we compare the three different steering methods within the IRR-ambition-contract, we can conclude that steering for a fixed IRR gives the highest mean and median adjustment factors for all age groups. However the standard deviation is also higher for this contract. The probability of a negative scenario is for all three steering methods very similar. Therefore, our opinion is that the fixed IRR based steering method does better, based on the higher adjustment factors. This can also be seen in Figure 7.12.

However, when we compare the 2019-ambition-contract with the IRR-ambition-contract with a fixed IRR the comparison is less straightforward. In this case we should consider what is more important, a more stable pension when retired or a higher in general adjustment factor when retired. We prefer a more stable pension when retired and more volatile when young. Therefore, we would recommend the IRR-ambition-contract with fixed IRR over the 2019-ambition-contract when we look at the adjustment factor. In this case people going through the entire contract from young age will benefit from the high adjustment factor when young, but also benefit from the stability provided to them when they are older.

8.3 Market value

At last, we compare based on the market value. We used the market value to create the net-profit-pictures. Note that to calculate the net-profit we used the FTK-contract as basis. When we compare based on the market value we can see that the 2019-ambition-contract performs better than the FTK-contract for the 20-30 and 70-90 year group, however it loses for ages 40-60 based on the median. In case of the mean, it does better for the 20-40 and 80-90 group, and loses for ages 50-70. So, none of these dominates the other. However, the fixed IRR contract seems to outperform the FTK-contract for every age at the median and mean value. We saw this in Figure 7.13 and Figure 7.14. When looking at Figure 2a, we see that in most of the scenarios this is still true, however there are scenarios in which the FTK-contract is better off. This will always be the case since both contracts have their pros and cons. Therefore, we would still recommend the IRR-ambition-contract with fixed IRR over the FTK-contract based on the market value. Since, it does better in most situations. We can also note, that the fixed IRR contract outperforms the 2019-ambition-contract for every age when we look at the mean and median net-profit-pictures. Next, we can see that steering based on a fixed IRR performs better for the 20-30 and 60-90 year group than the ER-IRR(variable) based steering when we look at the median. The ER-IRR(variable) contract does slightly better for the 40-50 year group. Looking at the mean, it does better for every age group besides the 50 year old group. Since, this advantage of the ER-IRR(variable) steering method does not outweigh the disadvantages for the other groups, we recommend the fixed IRR based steering over the ER-IRR(variable). The same can be said about steering based on price inflation, this method does better for the 20-30 year groups, but a lot worse after. We do not think the benefits received for the 20-30 year groups outweighs the disadvantages for the 40-90 year group. Thus, based on the market value we would recommend the IRR-ambition-contract with steering based on a fixed internal rate of return.

8.4 Conclusion

As we mentioned at the start of this chapter, we can conclude there is not one best contract, which outperforms all others based on the certainty equivalent, adjustment factor and market value. We did however note that the 2019-ambition-contract is more desirable than the FTK-contract, based on the certainty equivalent and adjustment factor. When looking at the market value there is a slight downside to the 2019-ambition-contract, but also multiple upsides. The same can be said about the IRR-ambition-contracts, steering based on a fixed IRR outperforms the other steering methods based on the certainty equivalent and adjustment factor. In case of the market value this is also true, for most of the age groups, but not for every age group. Looking at the performance measures separately we can give more clear preferences. For the certainty equivalent we noted that the 2019-ambition-contract does better than the other contracts. Based on the adjustment factor, we prefer the IRR-ambition-contract with steering based on a fixed IRR. The stability for the retirees, is a decisive factor for choosing this contract. At last, we also choose the IRR-ambition-contract with fixed IRR when we compare based on the market value. So, the 2019-ambition-contract and the IRR-ambition-contract seem to come out as the best options.

8.5 Improvements

The simulation we ran is based on simplifications. Therefore, there is room for improvement, for future research we can improve several things. We will take a look at non data related improvements. The reason for this is because pension data is often not publicly available.

We started the simulation using data based on the average income of males in the Netherlands. From this data we calculated their expected pension entitlements by using (4.3). However, this influences the comparison between contracts. The entitlements based on (4.3) are in general lower than the entitlements a participant would receive from building up pension based on the contracts discussed. To decrease this effect, one could start with a burn-in period. This period can be used to calculate the starting value of the pension entitlements based on a specific contract. By doing this, 70 year old persons in the IRR-ambition-contract will have received both benefits from the high adjustment factor earlier on, and the stability when retired. Based on the calculated pension entitlements, we could now start simulating the real situation with the participants as specified in Table 4.3. This would improve the comparison between contracts. This will also help filling in the gaps in pension payments. To calculate the certainty equivalent we now used the first pension payment a person receives whenever that person belong to the 70-90 group to fill in the missing pension payments. Therefore, this first payment will have a big influence, especially for the 90 year group. For those participants we use this one payment to fill up 22 missed pension payments. Therefore, it greatly impacts the calculations for the certainty equivalent.

Next, within the IRR-ambition-contract we steer based on three mechanisms. Two of these mechanisms are linked to the difference between the expected portfolio return over a time horizon T and the internal rate of return. We steer on a difference of $\alpha = 0.005$, but as we showed for the fixed IRR based steering method, the value of α does influence the distribution. An improvement would be to search for the optimal value of α based on the goal one wants to reach.

In addition, to make the simulation more realistic we could also look at partner pensions. In the event of death, the employee's partner is entitled to partner pension. This will protect against the loss of income, due to the death of your partner. The activation of a partner pension is independent of the retirement age and therefore adds an interesting addition to the simulation.

Moreover, we could look at optimal investment policies. Currently, we are looking at a $40 - 60$ percent split in assets and bonds. However, this might not be optimal for each contract, especially for the IRR-ambition-contract. Investing in bonds works like an interest rate hedge, it helps to protect against fluctuating interest rates. This is important when we consider the FTK-contract and 2019-ambition-contract. However, an interest rate hedge was no longer needed for the internal rate of return based contracts. Therefore, results will change when we consider optimal investment policies based on the pension contract.

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Appendix A

In the actuarial and financial science there is a lot of terminology used, which is not commonly used in general mathematics. To help out we provide a list of several actuarial and/or financial terms with brief indication of their meaning.

Funding Ratio the funding ratio (FR) expresses the financial state of a pension fund. The funding ratio can be calculated by: dividing the value of the pension assets by the value of pension liabilities.

$$\textbf{FR} = \frac{\text{pension assets}}{\text{pension liabilities}} \cdot 100\%$$

Future value is the value of a product or cash flow which takes place in the future.

Present value is the value of a product or payment in the future discounted to the present according to a certain belief of the discount rate.

Cash flow the total amount of money being transferred into and out of a business, especially as affecting liquidity. In this case we will often look at cash flows into and out a pension fund. Later on there will be a division between cash flows on fund level and on participant level.

Purchase rate is the amount of money paid for €1 of life-long pension after retirement. The purchase rate depends on survival probabilities of the participant and the interest rate curve. For example €1 of life-long pension is cheaper for a 30 year old than for a 60 year old because a 60 year old has a higher probability of living until retirement age and actually receiving pension benefits. A more in depth description of the purchase rate will be given later on.

Rate of Return (RR) is the net gain or loss on an investment over a specified time period, expressed as a percentage of the investment's initial cost.

$$\textbf{RR} = \frac{\text{Current value - Initial value}}{\text{Initial value}} \cdot 100\%$$

Internal Rate of Return is the minimal yearly rate of return that a pension fund needs on their pension assets to pay out all their upcoming pension liabilities. Thus for a pension fund it is sufficient to have a zero pension assets after paying all its participants their promised pension benefits.

Pension benefit another word for pension payment, but in this case you look at it from participant perspective. The participant receives something and therefore it is called a benefit.

FTK is a Dutch abbreviation which stands for "Financieel Toetsingskader", aka Financial assessment framework. A pension contract will be a financial assessment framework for pension providers. Pension providers are obligated to follow such financial assessment framework.

Pension base is the part of your salary on which pension is accumulated.

Primo year term used to indicate the start of the year.

Ultimo year term used to indicate the end of the year.

Pension entitlement indicates the amount of yearly life long pension you already accumulated. A pension entitlement of €2000 means that you will receive €2000 yearly at retirement.

Certainty equivalent is the guaranteed return that someone would accept now, rather than taking a chance on a higher, but uncertain return.

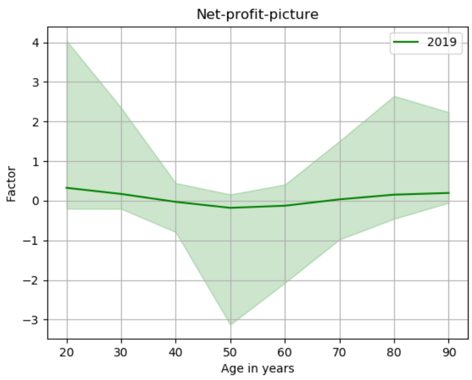
Market value the value for which something can be sold on a given market at a given time point.

Interest rate hedge the usage of financial products to protect against future changes in interest rate. Interest rate hedging helps protect your borrowing from the risk of fluctuations in interest rates.

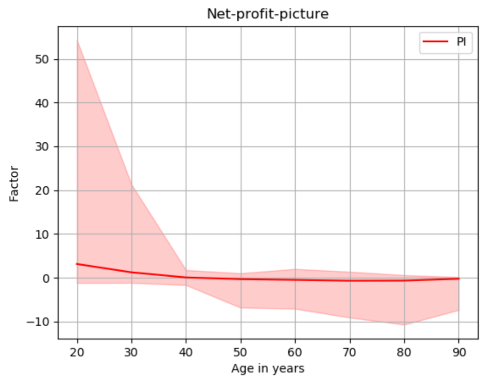
English term	Dutch term
Funding Ratio	Dekkingsgraad
Cash flow profile	Uitkeringen profiel
Purchase rate	Inkooptarief
Internal Rate of Return	Benodigd Rendement
Pension benefit	Pensioenuitkering
Pension base	Pensioengrondslag
Pension entitlement	Pensioenaanspraak
Future value	Toekomstige waarde
Present value	Huidge waarde
Pension assets	Pensioenvermogen
Pension liabilities	Pensioenverplichtingen

Table 1: Dutch translation of the common used terminology.

Appendix B

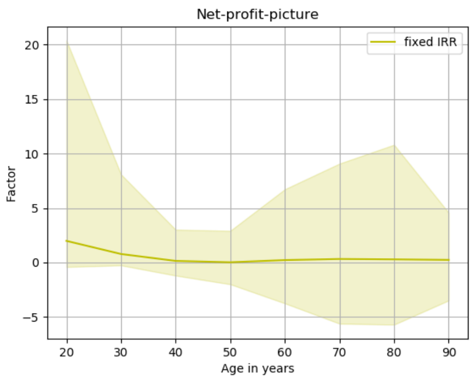


(a) 2019-ambition-contract.



(b) IRR-ambition-contract, price inflation based steering.

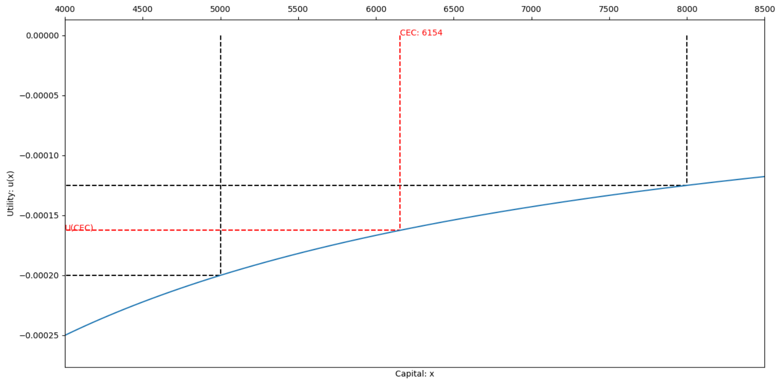
Figure 1: Plot of the median market value with 90% confidence bounds.



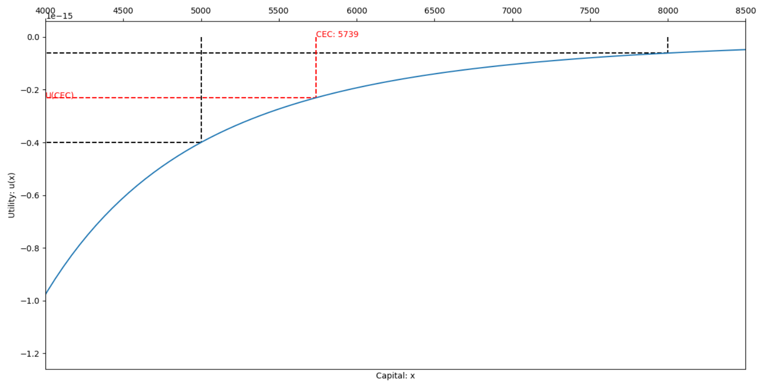
(a) IRR-ambition-contract, fixed IRR.

(b) IRR-ambition-contract, ER-IRR(variable).

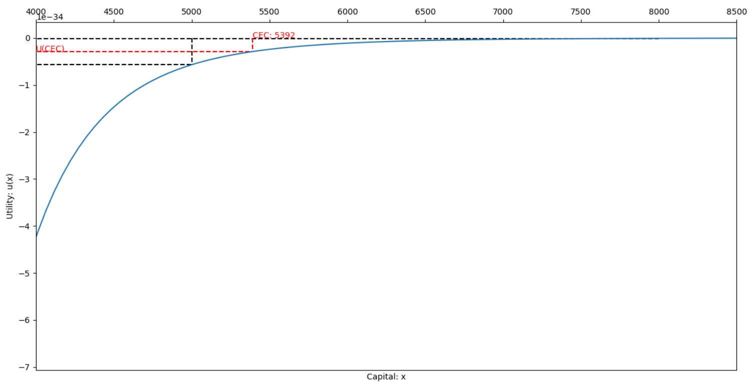
Figure 2: Plot of the median market value with 90% confidence bounds.



(a) CRRA utility function, with $\gamma = 2$.

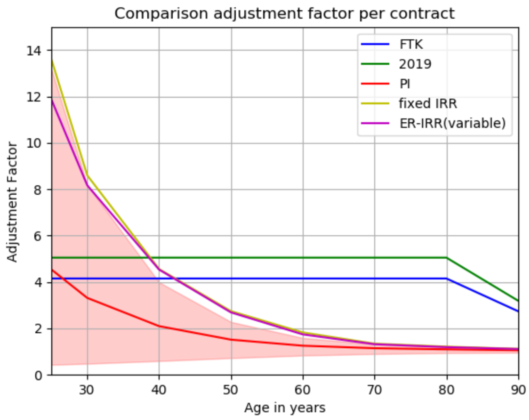


(b) CRRA utility function, with $\gamma = 5$.

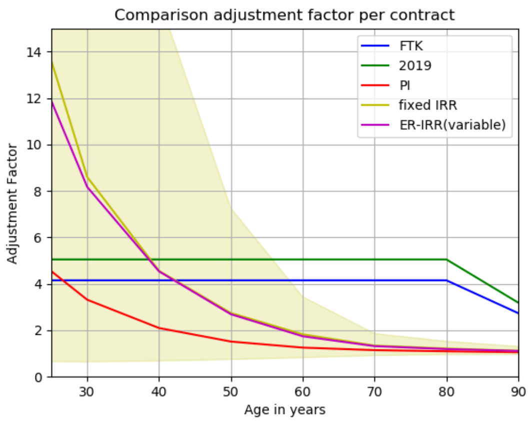


(c) CRRA utility function, with $\gamma = 10$.

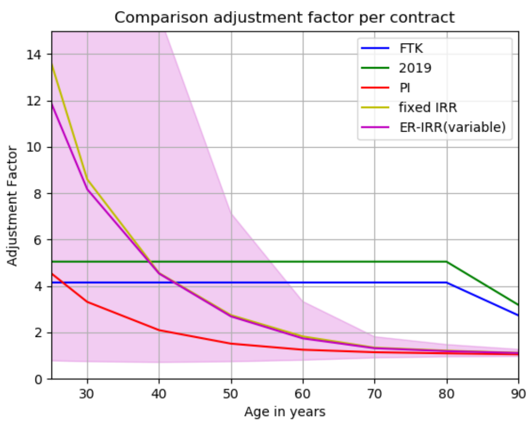
Figure 3: Zoomed in version of the CRRA plots.



(a) Price inflation based steering.



(b) Fixed IRR.



(c) ER-IRR(variable).

Figure 4: Confidence bounds IRR based contracts.

Participant type	$\text{Overall}_{\gamma=2}$	$\text{Overall}_{\gamma=5}$	$\text{Overall}_{\gamma=10}$
1	20460	12310	5800
2	41100	24730	11650
3	61730	37150	17500
4	98480	59260	27920
5	242900	146170	68860
6	1032260	621180	292640
7	15900	9250	4520
8	30530	17770	8670
9	45150	26290	12830
10	73950	43050	21010
11	170970	99520	48570
12	512070	298080	145480
13	11080	6470	3280
14	21710	12680	6420
15	32110	18750	9500
16	56070	32740	16590
17	123830	72310	36640
18	318210	185820	94160
19	8170	4920	2400
20	16250	9770	4760
21	23920	14390	7010
22	42570	25610	12480
23	93040	55970	27280
24	265760	159860	77910
25	6700	4320	1880
26	13260	8540	3720
27	19550	12590	5490
28	34380	22140	9650
29	76160	49050	21380
30	227920	146790	63970
31	5770	4210	1720
32	11760	8590	3510
33	17760	12960	5290
34	30350	22160	9040
35	68870	50280	20520
36	195070	142420	58130
37	5460	4660	2670
38	11380	9700	5570
39	17300	14750	8460
40	29140	24840	14250
41	66660	56840	32610
42	195580	166770	95670
43	5360	4950	3420
44	11160	10330	7140
45	16970	15700	10850
46	28700	26550	18350
47	65290	60400	41740

Table 2: Overall certainty equivalent per participant FTK-contract.

Participant type	$\text{CEC}_{\gamma=2}$	$\text{CEC}_{\gamma=5}$	$\text{CEC}_{\gamma=10}$
1	21260	15080	10110
2	42690	30280	20310
3	64120	45480	30510
4	102300	72560	48670
5	252320	178960	120050
6	1072300	760550	510190
7	15390	11880	8590
8	29550	22810	16500
9	43720	33740	24410
10	71590	55260	39980
11	165520	127770	92430
12	495740	382670	276840
13	10490	8680	6700
14	20550	16990	13130
15	30410	25140	19430
16	53090	43890	33920
17	117250	96940	74910
18	301300	249100	192500
19	7680	6850	5900
20	15280	13610	11730
21	22490	20040	17270
22	40020	35670	30730
23	87470	77950	67170
24	249850	222650	191860
25	6340	6160	5940
26	12550	12190	11750
27	18500	17970	17330
28	32530	31610	30470
29	72070	70020	67510
30	215680	209550	202030
31	5380	5380	5380
32	10980	10980	10980
33	16580	16580	16580
34	28340	28340	28330
35	64300	64300	64290
36	182140	182120	182100
37	5210	5210	5210
38	10870	10870	10870
39	16520	16520	16520
40	27830	27830	27830
41	63680	63680	63670
42	186830	186820	186790
43	5210	5210	5210
44	10870	10870	10870
45	16520	16520	16520
46	27950	27940	27940
47	63570	63560	63550

Table 3: Certainty equivalent of pension payment at age 68 FTK-contract.

Participant type	Overall$_{\gamma=2}$	Overall$_{\gamma=5}$	Overall$_{\gamma=10}$
1	20750	13230	7260
2	41660	26580	14570
3	62570	39920	21890
4	99830	63680	34920
5	246230	157080	86120
6	1046430	667540	366000
7	16060	9880	5560
8	30850	18970	10670
9	45630	28070	15780
10	74730	45960	25850
11	172770	106260	59760
12	517460	318270	178980
13	11160	6880	3820
14	21850	13470	7490
15	32330	19930	11080
16	56450	34800	19350
17	124670	76840	42740
18	320380	197460	109820
19	8230	5210	2640
20	16350	10350	5260
21	24070	15230	7740
22	42840	27110	13770
23	93640	59250	30090
24	267460	169240	85960
25	6770	4530	1980
26	13390	8950	3910
27	19750	13200	5770
28	34730	23220	10150
29	76930	51440	22490
30	230220	153930	67290
31	5840	4350	1820
32	11910	8880	3710
33	17980	13410	5600
34	30740	22910	9580
35	69750	51990	21730
36	197560	147260	61540
37	5520	4700	2810
38	11500	9800	5860
39	17490	14900	8910
40	29460	25100	15000
41	67400	57440	34320
42	197740	168510	100700
43	5420	4990	3410
44	11290	10400	7110
45	17160	15820	10800
46	29020	26750	18270
47	66010	60840	41560

Table 4: Overall certainty equivalent per participant 2019-ambition-contract.

Participant type	$CEC_{\gamma=2}$	$CEC_{\gamma=5}$	$CEC_{\gamma=10}$
1	21260	15220	10700
2	42690	30570	21490
3	64110	45920	32270
4	102290	73260	51490
5	252290	180700	126990
6	1072190	767940	539700
7	15260	11880	8770
8	29300	22810	16840
9	43350	33730	24920
10	70990	55250	40810
11	164120	127730	94340
12	491560	382560	282550
13	10420	8670	6720
14	20420	16980	13150
15	30200	25120	19460
16	52740	43860	33980
17	116460	96850	75030
18	299280	248890	192810
19	7720	6920	6000
20	15350	13750	11920
21	22600	20240	17550
22	40230	36030	31240
23	87920	78730	68270
24	251140	224890	195020
25	6470	6270	6010
26	12790	12390	11890
27	18860	18270	17530
28	33170	32140	30830
29	73470	71190	68300
30	219870	213060	204400
31	5440	5440	5440
32	11100	11100	11090
33	16750	16750	16740
34	28640	28630	28620
35	64980	64960	64940
36	184060	184010	183950
37	5270	5270	5270
38	10980	10980	10980
39	16700	16690	16690
40	28130	28120	28110
41	64350	64340	64310
42	188800	188760	188690
43	5270	5270	5270
44	10980	10980	10980
45	16700	16690	16690
46	28240	28230	28220
47	64240	64220	64200

Table 5: Certainty equivalent of pension payment at age 68 2019-ambition-contract.

Participant type	$\text{Overall}_{\gamma=2}$	$\text{Overall}_{\gamma=5}$	$\text{Overall}_{\gamma=10}$
1	7660	4550	2650
2	15380	9130	5320
3	23100	13720	7990
4	36860	21890	12740
5	90910	53990	31430
6	386360	229450	133580
7	8210	7100	5740
8	15770	13630	11030
9	23320	20160	16310
10	38200	33020	26710
11	88300	76330	61750
12	264470	228630	184940
13	6100	5640	4720
14	11960	11040	9240
15	17690	16330	13670
16	30880	28520	23870
17	68200	62990	52720
18	175270	161860	135470
19	4720	4270	3440
20	9370	8480	6830
21	13800	12480	10050
22	24560	22220	17890
23	53680	48550	39100
24	153320	138680	111680
25	4120	3480	2510
26	8150	6890	4960
27	12010	10160	7310
28	21130	17870	12860
29	46800	39590	28480
30	140060	118470	85220
31	3870	3080	2010
32	7890	6280	4090
33	11920	9480	6180
34	20370	16200	10560
35	46220	36760	23960
36	130930	104110	67860
37	4120	3320	2070
38	8600	6920	4320
39	13070	10520	6570
40	22010	17720	11060
41	50370	40540	25310
42	147770	118930	74270
43	4540	4010	2890
44	9470	8360	6010
45	14400	12700	9140
46	24350	21480	15460
47	55400	48860	35170

Table 6: Overall certainty equivalent per participant IRR-ambition-contract with price inflation based steering.

Participant type	$CEC_{\gamma=2}$	$CEC_{\gamma=5}$	$CEC_{\gamma=10}$
1	7621	4527	2635
2	15304	9091	5291
3	22987	13655	7947
4	36672	21785	12678
5	90454	53733	31271
6	384412	228356	132896
7	8394	7315	6033
8	16119	14047	11586
9	23845	20779	17139
10	39052	34031	28069
11	90284	78675	64893
12	270411	235640	194361
13	6931	6566	5655
14	13575	12861	11076
15	20085	19027	16386
16	35069	33223	28611
17	77446	73368	63184
18	199016	188538	162368
19	5989	5871	5580
20	11905	11672	11093
21	17526	17182	16330
22	31193	30581	29064
23	68170	66834	63519
24	194723	190907	181437
25	5672	5644	5588
26	11219	11165	11054
27	16542	16462	16299
28	29093	28953	28666
29	64450	64139	63503
30	192877	191946	190043
31	5370	5370	5369
32	10954	10953	10951
33	16537	16536	16533
34	28268	28266	28262
35	64137	64132	64123
36	181671	181656	181632
37	5200	5200	5199
38	10840	10839	10838
39	16479	16478	16476
40	27757	27755	27752
41	63509	63505	63498
42	186329	186317	186296
43	5200	5200	5199
44	10839	10838	10837
45	16477	16476	16474
46	27867	27865	27862
47	63390	63386	63379

Table 7: Certainty equivalent of pension payment at age 68 IRR-ambition-contract with price inflation based steering.

Participant type	$\text{Overall}_{\gamma=2}$	$\text{Overall}_{\gamma=5}$	$\text{Overall}_{\gamma=10}$
1	22750	11520	6570
2	45690	23130	13200
3	68630	34740	19820
4	109480	55420	31620
5	270040	136700	78000
6	1147630	580950	331470
7	16630	10770	6300
8	31940	20670	12110
9	47250	30580	17910
10	77380	50080	29330
11	178880	115790	67800
12	535770	346790	203070
13	10470	7900	5350
14	20500	15480	10490
15	30330	22900	15510
16	52950	39990	27090
17	116940	88310	59820
18	300500	226930	153730
19	7270	5950	4390
20	14450	11830	8720
21	21270	17420	12840
22	37860	31010	22850
23	82740	67760	49930
24	236320	193560	142630
25	5690	4800	3790
26	11250	9500	7500
27	16590	14000	11060
28	29170	24630	19450
29	64630	54560	43090
30	193400	163280	128960
31	4600	3720	2670
32	9390	7590	5440
33	14180	11460	8220
34	24230	19590	14040
35	54980	44450	31860
36	155730	125910	90250
37	4680	3810	2580
38	9750	7940	5380
39	14820	12070	8170
40	24960	20330	13770
41	57120	46510	31500
42	167570	136460	92420
43	4920	4340	3220
44	10250	9050	6720
45	15590	13750	10220
46	26360	23260	17280
47	59970	52910	39300

Table 8: Overall certainty equivalent per participant IRR-ambition-contract with steering based on a fixed IRR.

Participant type	$CEC_{\gamma=2}$	$CEC_{\gamma=5}$	$CEC_{\gamma=10}$
1	22680	11330	6490
2	45540	22760	13030
3	68410	34190	19570
4	109130	54540	31220
5	269180	134530	77000
6	1143950	571740	327240
7	17010	9870	5700
8	32660	18960	10940
9	48310	28050	16190
10	79110	45940	26510
11	182900	106200	61290
12	547810	318070	183580
13	12070	8160	5250
14	23630	15970	10290
15	34970	23630	15220
16	61050	41270	26570
17	134830	91130	58690
18	346480	234180	150810
19	9490	7170	4780
20	18860	14260	9500
21	27770	20990	13990
22	49420	37360	24900
23	108000	81650	54420
24	308500	233240	155430
25	8170	6960	5300
26	16160	13770	10480
27	23820	20300	15450
28	41900	35700	27170
29	92820	79080	60200
30	277780	236660	180150
31	6750	6390	5800
32	13760	13040	11830
33	20770	19690	17860
34	35500	33660	30520
35	80550	76360	69250
36	228160	216300	196150
37	6050	5920	5700
38	12610	12330	11880
39	19160	18750	18060
40	32280	31580	30410
41	73850	72270	69590
42	216680	212020	204170
43	5690	5650	5580
44	11870	11780	11630
45	18040	17910	17680
46	30520	30290	29900
47	69420	68890	68020

Table 9: Certainty equivalent of pension payment at age 68 IRR-ambition-contract with steering based on a fixed IRR.

Participant type	$\text{Overall}_{\gamma=2}$	$\text{Overall}_{\gamma=5}$	$\text{Overall}_{\gamma=10}$
1	21860	13080	7790
2	43910	26260	15630
3	65950	39440	23480
4	105220	62920	37460
5	259520	155200	92400
6	1102920	659570	392690
7	16390	11510	7140
8	31470	22100	13710
9	46550	32690	20280
10	76230	53540	33210
11	176230	123770	76770
12	527840	370700	229940
13	10540	7910	3930
14	20650	15500	7700
15	30550	22930	11390
16	53340	40040	19880
17	117790	88430	43910
18	302690	227240	112840
19	7170	5690	3800
20	14250	11300	7560
21	20970	16640	11130
22	37330	29620	19810
23	81570	64730	43300
24	233000	184900	123670
25	5430	4420	3340
26	10740	8750	6610
27	15840	12900	9740
28	27850	22680	17130
29	61700	50250	37950
30	184650	150370	113560
31	4470	3600	2540
32	9120	7350	5190
33	13770	11100	7830
34	23540	18970	13390
35	53400	43050	30370
36	151260	121930	86020
37	4530	3700	2440
38	9440	7710	5090
39	14350	11720	7740
40	24170	19730	13030
41	55300	45150	29820
42	162240	132470	87480
43	4800	4250	3120
44	10010	8860	6500
45	15210	13470	9870
46	25730	22780	16700
47	58520	51830	37990

Table 10: Overall certainty equivalent per participant IRR-ambition-contract with steering based on a fixed difference between the ER and IRR.

Participant type	$CEC_{\gamma=2}$	$CEC_{\gamma=5}$	$CEC_{\gamma=10}$
1	22010	12980	7620
2	44190	26070	15310
3	66380	39160	23000
4	105900	62470	36690
5	261200	154080	90490
6	1110040	654820	384550
7	17560	10240	5720
8	33730	19660	10980
9	49890	29080	16250
10	81710	47620	26610
11	188910	110100	61510
12	565810	329770	184240
13	12540	6730	2800
14	24570	13190	5480
15	36350	19510	8110
16	63470	34070	14170
17	140170	75240	31290
18	360200	193350	80400
19	9190	5780	3130
20	18270	11480	6220
21	26900	16900	9150
22	47870	30090	16290
23	104620	65750	35610
24	298830	187820	101710
25	7240	5210	3420
26	14310	10300	6770
27	21100	15190	9980
28	37120	26710	17550
29	82220	59170	38890
30	246060	177070	116380
31	5860	5180	4150
32	11960	10570	8470
33	18060	15960	12790
34	30870	27280	21860
35	70030	61890	49600
36	198370	175300	140480
37	5570	5310	4900
38	11600	11080	10210
39	17640	16840	15520
40	29710	28360	26140
41	67970	64890	59810
42	199430	190390	175490
43	5440	5360	5220
44	11340	11160	10880
45	17230	16970	16540
46	29150	28700	27970
47	66300	65290	63620

Table 11: Certainty equivalent of pension payment at age 68 IRR-ambition-contract with steering based on a fixed difference between the ER and IRR.