MASTER

Short term forecasting of 24 hour sales in an online retail environment

Platenburg, N.

Award date:
2021

Industrial Engineering & Innovation Sciences
Information Systems Group

Short Term Forecasting of 24 Hour Sales in an Online Retail Enviroment

Master's Thesis

In partial fulfillment of the requirements for the degree of Master of Science in Operations Management and Logistics

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Eindhoven
February 26, 2021

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Preface

This master thesis represents the end of three and a half years of studying at Eindhoven University of Technology. During these years, I was lucky to meet many great people and I am grateful for all the unforgettable memories I have made. I would like to take the opportunity to show my gratitude to several people who have always supported me during my study period.

First of all, I would like to thank Banu Aysolmaz, my first supervisor, for her constructive feedback, enthusiasm and guidance throughout the project. I appreciate the fast responses, extensive feedback and clear way of working. Moreover, I would like to express my gratitude to Yingqian Zhang for her useful input, suggestions and feedback during our meetings. Although there were only a few, our contact was extremely valuable to raise the research to a higher level.

Secondly, I would like to thank the most dynamic and inspiring company: bol.com. I am grateful for all members of the Capacity Management team who gave me a warm welcome and for all the laughter we had together at C4 and during the Teams meetings and drinks. In particular, I would like to thank Wouter Valk and Timo van Hooijdonk for providing me continuous support and all information needed to finish this master thesis.

Furthermore, I would like to thank my friends and family who provided endless support throughout my complete study period. First of all, a big thanks to my friends of 'Borreltje?' that I have met during my pre-master program with whom I have experienced a lot of study time, fun chats, coffees at Industria and other activities. You have made the university an enjoyable place where I could study and make fun. Additionally, I would like to thank my hockey team and my wonderful housemates for the great contribution to my student life in Eindhoven. Last but not least, I would like to thank my parents and brother who always provided me with support whenever I needed it. Thank you for your unconditional love and support.

I am proud of how I have grown the last years during my study period at the Eindhoven University of Technology and I am very satisfied with the result of my graduation research project that is in front of you. I hope you will enjoy reading it.

Naomi Platenburg, February 2021

Abstract

Due to the annual growth of e-commerce companies, it gets harder and harder to make the right operational decisions concerning demand planning. Delivering a highly accurate real-time forecast is of high importance to make quick and accurate decisions on warehouse operations. An accurate forecast in the beginning of the days for 24 steps ahead is used to form decisions on up- or downscaling within these warehouse operations. This master thesis conducts an extensive research to short-term forecasting techniques that can be applied to a multi product online retail environment. An extensive overview of the time series forecasting field is given that elaborates on several forecasting models, data preparation techniques, multi-product handling, multi-step ahead forecasting techniques, exogenous variables and accuracy measures. Firstly, a clustering based approach is used prior to forecasting to handle the multi-product environment. Next, four models have been selected for a comparative study on short-term forecasting: exponential smoothing (ES), autoregressive integrated moving average (ARIMA), support vector regression (SVR) and multilayer perceptron (MLP). For each model, an optimal configuration in terms of parameter settings and exogenous variables has been searched. Optimal parameters were used to create forecasts based on cluster and aggregate level for 24 steps ahead of which their performance was evaluated by the RMSE and MASE. It was found that within the clusters, the ARIMA model had the best ability to predict the sales, while on the total set, SVR outperformed on short time series. Finally, the performance of cluster selection is compared to the performance of aggregate selection. It was concluded that cluster selection was outperformed by SVR on aggregate selection. Overall, the SVR is able to outperform the existing forecasting process, especially on the far future. Therefore, it is recommended that the forecasting model is used to improve the current forecasting process.

Keywords: Time Series Forecasting, Short-term, E-commerce, Exponential Smoothing, ARIMA, Support Vector Regression, Neural Networks, MLP, Multi-step ahead Forecasting

Executive Summary

This report contains the study of increasing the accuracy of short-term forecasting within an online multi-product environment. Given the enormous market growth and high customer requirements in the e-commerce sector, accurate forecasting of daily sales is of high importance. This study aims on developing a highly accurate short-term forecasting model in combination with time series clustering to obtain better insights in the daily sales. These insights help to steer faster and more precisely on daily warehouse operations. The study is conducted at the largest online retailer of the Benelux, bol.com.

Problem Context

Due to the annual growth of e-commerce companies, it gets harder and harder to satisfy increasing customer demands and optimize logistic processes. Delivering a highly accurate real-time forecast is of high importance to make quick and accurate decisions on warehouse operations. Daily decision in balancing personnel and production to achieve the highest productivity. Therefore, the aim is to maximize customer satisfaction against minimal costs and maximum order proposition. An accurate forecast during the day is used to form decisions on up- or downscaling within warehouse operations. Actual patterns observed from customer behavior are compared with the forecasted ones to form these decisions. A short-term forecast is a new development of bol.com and includes analysis and updates every 10 minutes during the day reacting on the customer behavior. Effective changes in the way of short-term forecasting the upcoming intake of sales for the rest of the day might increase the accuracy and consequently, the quality of decision making. Making the right decisions, will potentially lead to an increase in productivity, increase in customer satisfaction and a positive impact on the financial costs. The need for a highly accurate forecast of bol.com sales intake leads to the following main research question:

How to increase accuracy in the short-term forecast of the intake of 24 hour sales in an online multi-product retail environment?

An extensive overview of the time series forecasting field is given that elaborates on several forecasting models, data preparation techniques, multi-product handling, multi-step ahead forecasting, exogenous variables and accuracy measures. The model selection aimed on short-term forecasting techniques that can be applied to a multi product online retail environment.

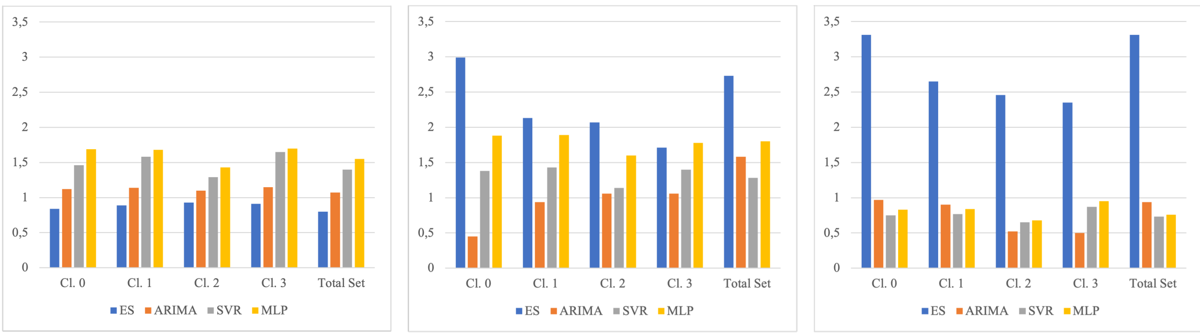
Data Description

Data on historic sales for 1 year were retrieved from bol.com, which shows every order and the product group it belongs to. In order to identify the orders that are relevant for short-term

forecasting the rest of the day, only 24 hour sales are included in the data. Therefore, orders that cause workload in the warehouse on the same day are included in the data. To decrease the number of data points in the sales data, sales are aggregated to 10 minutes time slots for each product group. To deal with the multi-product environment and reduce data points, product groups are clustered prior to forecasting based on shape and characteristics. Next, relevant exogenous variables such as steering effects, calendar effects and weather effects were identified and data on these variables were retrieved. An exploratory data analysis was performed on the sales data to reveal possible trend and seasonality patterns. It could be clearly identified that the data includes double seasonality, daily and weekly. Next, Pearson's correlation coefficients were determined to identify significant correlations between the exogenous variables and the actual sales. Three multivariate sets were created that each consists of a subset of relevant exogenous variables for each cluster and the total set.

Results

Four models have been selected for a comparative study on short-term forecasting: exponential smoothing (ES), autoregressive integrated moving average (ARIMA), support vector regression (SVR) and multilayer perceptron (MLP). Forecasts are obtained based on cluster and aggregate level by the MIMO strategy for the upcoming 24 hours. A time series cross validation is used in which the train set gets increased by 24 data points at each iteration. The models are evaluated on the near, middle and far future, in which the first 8 hours represent the near future, the next 8 hours the middle future and the last 8 hours the far future. For each model, an optimal configuration in terms of parameter settings was search by use of grid search. The forecast performance is evaluated by root mean squared error (RMSE) and mean absolute scaled error (MASE). An overview of the best MASE per model for each cluster and the total set is shown in Figure 0.1. The figures represents the scores on the near, middle and far future.



(a) Near Future (b) Middle Future (c) Far Future

Figure 0.1: Best MASE per Model for each Cluster and Total Set

It was observed that in the near future, statistical methods did outperform machine learning methods. In the middle and far future, ARIMA did outperform for the clusters, while SVR outperformed for the total aggregate set. To evaluate if cluster selection outperforms aggregate selection, the best forecast model is selected for each cluster. ARIMA is applied to all clusters, while SVR is applied to the total aggregate set. It was shown that aggregate selection outperforms cluster selections in terms of RMSE and MASE.

Additionally, aggregate selection is able to outperform the current forecast method on selected days. It is able to capture the behaviour of normal and special days like Saturdays and holidays. Therefore, aggregate selection has the potential to improve the current forecast method. To conclude, the SVR model was selected as the best performing forecast method in combination with aggregate selection.

Conclusion and Recommendations

It can be concluded that forecasting models have a great potential to provide accurate forecasts for the rest of the day in an online multi-product environment, even up to 24 hours ahead. Therefore, the results of the forecasting model can be used as guideline for the process of daily steering in which domain knowledge of business experts can be added to maintain the best decision making.

Short Time Series

This research has proved that forecasting models are able to provide accurate forecasts based on only one year of data. Machine learning models were able to capture double seasonality patterns and provide accurate forecasts on special days. Since it is common sense that longer time series provide even more accurate forecasts, it would be preferred to increase the data set to two or three years.

Exogenous Variables

Exogenous variables such as calendar effects and weather effects have a significant positive impact on the performance of the forecast models. It proves to outperform the current method which only includes basic effects such as weekday and current time. Therefore, it is recommended to include additional exogenous variables to bring the model into more detail.

Cluster Approach

This research concluded that aggregate selection outperforms cluster selection. Cluster selection did not contribute to the improvement of the forecasts since the forecast methods were not able to capture higher correlations with external variables in comparison to the correlations with external variables of the total aggregate set. It might me more viable to split the clusters into smaller clusters to find higher correlations with external variables.

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List of Abbreviations

ARIMA Autoregressive Integrated Moving Average.

BFC bol.com Fulfillment Center.

DTW Dynamic Time Warping.

ES Exponential Smoothing.

FNK Fulfillment Network System.

FOP Fulfilment Optimization.

LCM Local Cost Matrix.

LOR Logistics Order Realisation.

LvB Logistics via bol.com.

MLP Multi Layer Perceptron.

NPS Net Promotor Score.

PG Product Group.

SODA Sales & Operations Planning Dashboard.

SOR Shop Orders Service.

SVR Support Vector Regression.

Chapter 1

Introduction

This report contains the study of maximizing the accuracy of short-term forecasting within an online multi-product environment. Given the enormous market growth and high customer requirements in the e-commerce sector, accurate forecasting of daily sales is of high importance. This study aims on developing a highly accurate short-term forecasting model in combination with time series clustering to obtain better insights in the daily sales. These insights help to steer faster and more precisely on daily warehouse operations. The study is conducted at the largest online retailer of the Benelux, bol.com.

1.1 E-commerce

Over the last few years, e-commerce has become an essential part of the global retail framework. Like many other industries, the retail landscape has undergone a substantial transformation following the arrival of the internet and ongoing digitalization of modern life. As internet access is rapidly increasing around the world, the number of digital buyers worldwide keeps climbing every year (Clement, 2020). "Between 1999 and 2005 online retail sales grew with 720 percent in the UK, 1,060 percent in Germany, and 1,403 percent in The Netherlands". However, despite the enormous growth, in 2005 the proportion of online retail sales compared to total retail sales was only around 3% for each of the countries (Weltevreden, 2008). As the rapid expansion of online shopping continued in the future, sales values got really impressive. In 2019, 25.8 billion euros were spent online in The Netherlands, which was a growth of 7% compared to 2018 and represents 26% of total retail sales. This 23.7 billion euros arises from 13.5 million different customers that purchased over 258 million online products (Thuiswinkel.org, 2019). 2.8 billion of the total online sales in 2019 were created by bol.com, which was almost four times the revenue of 730 million euros in 2015. Hence, bol.com is the biggest online retailer of The Netherlands and is able to outperform general market growth by making effective use of its market knowledge, loyal customer base and platform. On average, the companies revenue and number of orders currently grow with a rate of 30% each year. Although, worldwide players like Alibaba and Amazon still overrule the size of the Dutch retail market. Alibaba's annual revenue for 2019 was 56.1 billion US dollars, an increase of 40,7% from 2018. In 2019, Amazon generated a worldwide revenue of 280.5 billion US Dollars, which increased by 20,5 % compared to 2018. Amazons market share in the entire US e-commerce amounts almost 45% in 2019 (Clement, 2020).

The development of the e-commerce sector has evolved in a complex challenge for both oldfashioned brick-and-mortar retailers who tend to go online and pure-play online retailers who enter new markets. The landscape of retailing has changed significantly due to the arrival of the internet. Competition has become larger, delivery windows smaller and customer expectations higher. Agatz et al. (2008) address the specific supply chain management issues of e-fulfillment in a multi-channel environment. They show that although online markets and multi-channel retailing provide many opportunities, "the design of the underlying distribution processes also confronts companies with novel complexities" (Agatz et al., 2008). From an operational perspective, these complexities are mainly applicable to the fulfillment process. This process is considered as the most expensive and critical part of the supply chain (Lummus & Vokurka, 2002). Fulfillment includes the unique chain of events that are activated by a single click on a purchase button. Due to the easiness of the order, the customer expects the delivery to be as fast and easy as the purchase (De Koster, 2003). Besides traditional aspects of delivery as speed, quality and service, customers expect high product availability and frequent order status updates. Therefore, order management, customer care, web presentation and reverse logistics have become of high importance as well (Lummus & Vokurka, 2002).

The accelerating growth of the e-commerce sector and high customer expectations have resulted in many complex challenges for e-commerce companies. One of these challenges is the problem of short-term forecasting of daily sales introduced in Section 1.3. A highly accurate short-term forecast of the daily intake of sales is of high importance to make quick and accurate warehouse operations decisions and deal with the existence of aggressive order services. The customer behavior is observed to form decisions on up- or downscaling within warehouse operations. An accurate forecast helps to steer in the right direction on time and increase productivity. How to optimize the short-term forecast to maximize productivity is the challenge tackled in the remainder of this paper.

1.2 Company Description

At 1999, bol.com opened the first online bookstore of the Netherlands. Nowadays, bol.com is the biggest e-commerce company in the Netherlands and Belgium with more than 2 million visits per day at their platform, more than ten million customers and more than 21 million products in store, that reach out from books to games, electronics, beauty, clothes, sports, pets and even more (Schaeffer, 2017). The company has experienced incredible growth in the last years and has currently more than four thousand pick-up points, around two thousand employees at their headquarters in Utrecht and thousands of external employees at their warehouses. The company's success led to the acquisition by Ahold in July 2012. The biggest warehouse of bol.com (BFC), which has fifty thousand squared meters, is located in Waalwijk which has already started expanding. Compared to previous years, the company still grows with a rate of on average 30% each year. By innovative projects such as special last mile services and platform initiatives, it is the company's goal to generate 7.2 billion euros of revenue in 2024.

Bol.com's motto is clear: the company wants to offer everything to everyone. The company changes retail to make everyday life easier and more enjoyable and shares enormous passion for continuous improvement. Realizing this objective in the dynamic world of e-commerce requires enormous efforts and an ever changing company environment. Due to high market competition, online customers have high expectations regarding delivery times, prices and services. To satisfy these expectations, bol.com must keep innovating and optimizing their logistic processes. This has already lead to same day delivery options, a 2-hour delivery hub in Amsterdam and extensive collaborations with partners such as PostNL and BPost. Furthermore, the company has a very active online social media presence, customer service and a virtual assistant Billie. This has resulted in a current Net Promoter Score (NPS) of 54, which measures the customer experience. Also, bol.com won several prices, as for example the Dutch Marketing awards 2019. In addition, bol.com focuses on increasing their market share because Amazon has approached the Netherlands and will be a big competitor.

The scale bol.com operates on is unique, just like its platform. In addition to the company's own assortment, products of over twenty thousand business partners are offered on bol.com. These partners are called Plaza partners, of which a selection of partners make use of Logistics via bol.com (LvB) which means outsourcing of the fulfillment of customers orders to bol.com. The non-LvB partners only make use of the platform. These elements have shaped a complex landscape of products, retailers and suppliers and require innovative and efficient solution approaches. The logistic department of bol.com, aims at optimizing these complex logistic processes and satisfy all parties involved, especially the customer.

1.3 Problem Formulation

The e-commerce sector has exploded over the past few years. More and more people have opted to shop online, while many new businesses have flooded the market. In the Netherlands, bol.com is the biggest online retailer and is still continuing to grow. Due to the unique size, the growth and the competitive market, it is a complex challenge for their logistic professionals and their fulfillment partner Ingram Micro to satisfy increasing customer demands and optimize logistic processes. Looking at the current situation, one worth mentioning challenge is that it gets harder and harder to make an accurate forecast concerning demand planning. Bol.com aims on maximizing customer satisfaction against minimal costs and maximum order proposition. To achieve this, many difficult operational decisions need to be made by team Control Tower.

The Control Tower team is the link between bol.com and the daily operations at the warehouses. Warehousing operations and fulfillment are handled by Ingram Micro. Ingram Micro bases its outbound production planning, personnel schedule and operating lines on forecasts delivered by the Control Tower team, which are reviewed every week during a joint production planning meeting. Delivering a highly accurate forecast is of high importance to make the right daily decisions and balance in personnel and production to achieve the highest productivity. Therefore, at regular days two times a day, in peak period four times a day, a call is conducted with the control rooms in both warehouses. Here, the actual patterns observed from customer behavior are compared with the forecasted ones to form decisions on up- or downscaling within warehouse operations. In other words, 'levers' can be activated to direct orders and production into the desired direction. There are several levers that can be activated to influence the intake of sales. For example, the levers can have impact on warehouse operations, specific product types on sale or special services for customers. The possible degree of precision on up- or downscaling depends on the time of the day since some 'lever' options will not be available later in the day. The levers will be described in more detail in Section 3.3. The impact of levers must not be understated because this has impact on thousands of sales (revenue) and on the other hand a high impact on the customer satisfaction.

Currently, lever decisions are mainly made manually, Excel based, which has definitely reached their scale limits, but also causes high costs, low speed and complexity. In addition, manual decision making is really sensitive to errors and the multi-warehouse environment of the company of interest increases complexity. Employees are allocated to a specific warehouse beforehand and there is no possibility to switch employees across warehouses during day.

Due to these developments and this complexity, team Fulfillment Optimization (FOP) is searching for a way to support team Control Tower in the daily utilization process in the full logistical chain through insights and decision automation. To provide these insights, Control Tower seeks for a highly accurate automated forecast dashboard that shows the estimated sales for the upcoming hours. Bol.com calls this dashboard the Sales & Operations Planning Dashboard (SODA). The short-term forecast is a new development of bol.com and includes analysis and updates every 10 minutes during the day reacting on the customer behavior. Effective changes in the way of short-term forecasting the upcoming intake of sales for the rest of the day might increase the accuracy and consequently, the quality of decision making. Making the right decisions, will potentially lead to an increase in productivity, increase in customer satisfaction and a positive impact on the financial costs.

1.4 Research Objectives

The goal of bol.com and team FOP and Control Tower in particular is to generate an optimization model that forecasts the intake of daily sales on short term in the most accurate way. Hence, it should take into account order information, capacity of warehouses and so on. This results in a complex problem which must be simplified in order to provide a well-structured base for a master thesis project. Optimizing the short-term forecast leads to better decision making within warehouse operations and to a big positive impact regarding to productivity and financial costs. A model that forecasts the intake of daily sales on short-term for BFC, the biggest and most important warehouse of bol.com, is considered as great added value by bol.com.

From an academic perspective, the main goal is to close the research gap defined in Section 1.8. Due to its ever increasing demands and changing behaviour, the e-commerce market demands new research continuously. Current research regarding short-term prediction is mainly focused on other industries and areas like environmental perspectives. Regarding modelling purposes, the academic value of the model should be sound. The model should have a clear goal and modelling assumptions must be stated that make practical sense. These assumptions have to be realistic and translatable to general situations. Hence, the model should be reproducible or extendable by practitioners and academia. Especially, it is the goal that practitioners from (e-commerce) companies are able to retrieve valuable information from the short-term forecasting model. It will not only be valuable for warehouse capacity management, it can also contribute to enhance planning efficiency upstream and downstream in the supply chain. Furthermore, it could help inbound and stock departments to understand demand more precisely, helping them to manage stock levels more optimally.

1.5 Research Questions

For conducting effective research, appropriate research questions that guide the researcher through the project are essential. This section transforms the previously defined research objectives into research questions and states the research questions that decompose the main problem into smaller sub-problems. Combining both practical and academic goals, the main research question can be defined as:

How to increase accuracy in the short-term forecast of the intake of 24 hour sales in an online multi-product retail environment?

To facilitate an efficient solution procedure for answering this central question, the following sub-questions are defined. The first question serves to provide an overview of possible influential factors that might effect online sales. The second question investigates possible techniques to handle a multi-product environment.

1. What are possible influential factors effecting online sales?

2. How should a multi-product environment be handled within time series forecasting?

The third question investigates possible statistical and machine learning models that can be applied to short term time series forecasting. Different models will be compared based on e.g. sensitivity, computation times and understandability. The models will be tested using different test sets and parameters.

3. What statistical- and machine learning models can be applied for short term time series sales forecasting and how can parameters be optimized?

The fourth question focuses on the techniques that exist for multi step ahead time series forecasting. It will be discovered which technique suits best to the problem on hand.

4. What techniques exist to create multi-step ahead time series forecasts and which techniques are appropriate for short term sales forecasting?

Subquestion five outlines several performance metrics that can be used in time series forecasting. It will be discovered which performance metrics exists, which are used most in literature and which could be applied best to the problem on hand.

5. What accuracy measures are available to evaluate forecasting performance and which accuracy measures are appropriate to evaluate short term sales forecasts?

The last subquestion serves to find the best combination of input variables, forecast model, data transformation, parameters and multi-step ahead technique to achieve the most accurate results. Therefore, several combination will be compared in the research.

6. What combination of input variables, forecasting model, data transformation(s), parameter configuration and multi-step ahead technique achieves the most accurate results?

1.6 Scope

As discussed in Section 1.3, bol.com faces the complex challenge to satisfy increasing customer demands and optimize logistic processes. This research should provide the company an accurate short-term forecast of the daily intake of sales to improve decision making regarding warehouse operations. Providing accurate insights in daily sales will have a positive impact on productivity, customer satisfaction and financial costs. It is essential to scope the project in line with assumptions and exclusion criteria to keep the problem manageable and scope the project to the desired size. Assumptions are formed in Section 1.6.1, followed by the description of exclusion criteria in Section 1.6.2.

1.6.1 Assumptions

Below a list of assumptions is defined, which help in scoping the project to the desired size. The assumptions have to be realistic and translatable to general situations. Additional, specific (modelling) assumptions are provided in later sections.

- The IT landscape that assigns customer orders to warehouse is considered as given. The fulfillment network systems (FNK) assigns orders to different warehouses based on several rules and shop order characteristics. This is only possible if stock is located at multiple warehouses which is only a small percentage. Therefore, it is assumed that the dynamic and biased choice of FNK can be neglected.

1.6.2 Exclusion Criteria

Given the problem formulation and scope defined in previous sections, the following warehouse, product and lever exclusion criteria are defined:

- The focus is only on one warehouse which will be BFC, the biggest warehouse of all. All other warehouses are excluded but the model could eventually be tested on performance for other warehouses in the end. With this exclusion, products that are sold by non-LvB Plaza partners are immediately excluded from the research since these products do not require logistic fulfillment by bol.com.
- Only 'STOCK' and 'DLY' lever impact will be included in the model. The STOCK lever shifts the delivery promise of a subset of products on sale to another moment. The DLY lever closes a warehouse so the delivery promise of all products on sale located at that specific warehouse are shifted. These levers are used in most of the cases and have the highest impact. Other levers like switching off the possibility to wrap your order or switching off commercial levers are only used very often during peak period, are badly registered and are therefore not included in the model.
- Since bol.com has grown and changed significantly over the past few years, only data collected in the year of 2019 is used. Data points of 2020 are not included in the model due to the Coronavirus, which causes unrealistic sales values and a lot of up- and downscaling. In addition, the reason for restricting the data to one year is to minimize the effects on the data set due to changes within the companies structure, such as warehouse or assortment expansions, while allowing to have all possible seasonal effects included.

1.7 Methodology

The methodology of this research is based on the popular and well-proven framework used for data mining, the CRISP-DM method, shown in Figure 1.1. Data mining will be used to uncover the reasoning behind certain results and why the current forecasting model is not achieving the desired accuracy. The CRISP-DM framework is an idealised sequence of events and requires many loops. This includes going back and forward between steps and revisiting and redefining the problem statement (Kotu & Deshpande, 2014). The CRISP-DM method is typically applied to continuously changing environments where continuous improvement is necessary.

Figure 1.1: CRISP-DM Model (Smart Vision, 2018)

This research project is divided into four main phases: literature review, data preparation, model development and evaluation. An overview of the methodology with the four main phases is shown in Figure 1.2. The four phases are distinguished by use of different colors.

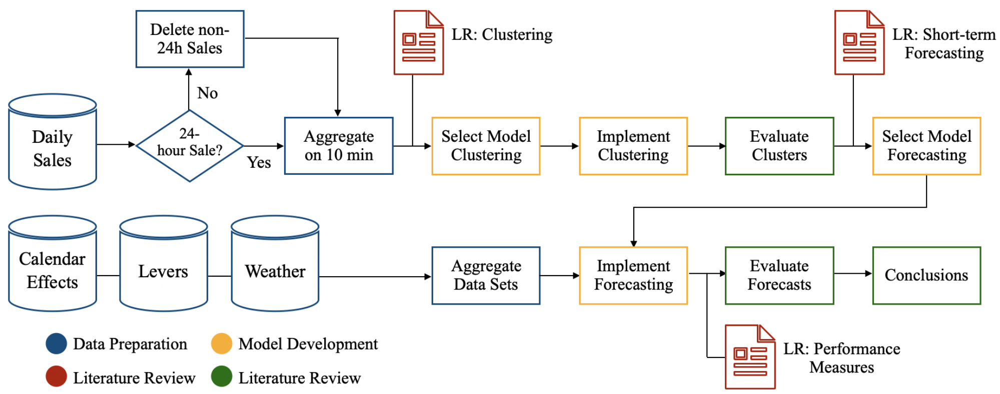


Figure 1.2: Overview of Methodology - 4 Phases

The literature review phase involves an extensive literature review to identify existing time series clustering techniques and short-term forecasting models and assess their applicability to the problem at hand. Additionally, the literature review outlines available multi-step ahead forecasting techniques, data transformation techniques and performance measure techniques. The data preparation phase involves collection and preparation of the data that is used as input for the models that are developed. This phase involves the exclusion and aggregation of data points to come up with a final data set. Model development will take place in phase three and is concerned with the development of the clustering- and short-term forecasting model. Selection criteria will be taken into account and a founded choice of models will be made. Lastly, the evaluation phase revolves around the evaluation of the implemented models. Performance metrics resulted from the literature review are used to measure accuracy of the models. Besides model evaluation, this phase will draw a final conclusion on the overall thesis project.

1.8 Scientific Contribution

This research involves a comparative study on short-term time series forecasting with low volume sales data. While studies comparing various statistical- and machine learning models already exist in recent research, most studies are applied to longer time series and other research areas. Within the environmental perspective, there is a long history of research on the modeling of hourly real-time electricity load, for example in the research of Karthika et al. (2017), Ryu et al. (2017) and Kim et al. (2019). The demand for electricity varies with time, with different hours of the day and different days of the week having an effect on the load. Daily variations in human activity due to working, leisure, and sleeping periods can introduce a cyclical pattern within a 24-hour period (Braun et al., 2014). This study contributes to the field by comparison of various statistical- and machine learning models applied to short-term forecasting in an online multi-product environment. The nature of e-commerce order arrivals has raised the need to perform demand prediction in a short interval including external variables (Leung et al., 2020). An extensive literature review on short-term time series forecasting is conducted to create an overview of the research field. The study contains several forecasting models, data preparation techniques, multi-step ahead forecasting techniques and accuracy measures. Both statistical and machine learning models were selected and applied to the case study of bol.com to discover best performing models in short-term online sales forecasting.

1.9 Report Outline

The remainder of this research is structured as follows. Chapter 2 outlines an extensive literature review on time series clustering and short-term forecasting. Chapter 3 defines the company case setting and depicts the company situation by showing current figures. Next, the data exploration and data preparation process is described in Chapter 4. The selection criteria for model selection and the final models and techniques that will be applied to the case study will be elaborated in Chapter 5. Chapter 6 explains the selected models for time series clustering and elaborates on their performance. This chapter is followed by Chapter 7 which explains the selected models for short-term time series forecasting together with their performance. Chapter 8 discusses the practical implementation phase of the final model at the company at hand. Lastly, Chapter 9 provides the main findings together with recommendations, limitations of the conducted research and directions for future research.

Chapter 2

Literature Review

In this chapter, a literature review on models and techniques within time series are presented, particularly on clustering and forecasting. Section 2.1 outlines the literature review protocol. Next, Section 2.2 points out various techniques on time series clustering provided by the literature. Third, Section 2.3 outlines available forecasting models resulted from the literature review. Section 2.4 provides various data preparation techniques required for modelling. This Section is followed by Section 2.5 which points out multi-step ahead forecasting techniques. Lastly, suitable performance measures to evaluate forecasts are discussed in Section 2.6.

2.1 Literature Review Protocol

A literature review with selective criteria is conducted on available methods for short-term times series demand forecasting. The main goal is to identify existing forecasting methods to create demand predictions on a short time interval based on time series data. Preliminary, time series clustering techniques are identified. A simple literature review is conducted and different techniques are described. For short-term forecasting, a systematic approach, provided by (Kitchenham & Charters, 2007), is used to effectively find relevant studies and separate the relevant from irrelevant studies.

In this literature review, four academic databases were used that are complimentary to each other in terms of topics and journals: IEEE[1], ACM Digital Library[2], Web of Science[3] and ScienceDirect[4]. The keywords and synonyms used to represent the research question are stated in Table 2.1. In addition, several inclusion criteria were used to restrict the initial lists to only available and relevant articles. Accessibility criteria ensured that documents were written in English and the full text was available. Content criteria were concerned with the number of citations and subject based on title and/or keywords. After application of the query and inclusion criteria, the total number of documents found in each database are stated in Table 2.1.

[1] http://ieeexplore.ieee.org.dianus.libr.tue.nl/Xplore/dynhome.jsp
[2] https://dl-acm-org.dianus.libr.tue.nl
[3] http://apps.webofknowledge.com.dianus.libr.tue.nl
[4] https://www-sciencedirect-com.dianus.libr.tue.nl/search

Table 2.1: Initial Search Strategy - Short-term Forecasting

Final Query				
forecast* OR predict* AND multi-item OR demand OR sales AND short-term OR daily OR hour*				
Database	IEEE	ACM Digital Library	Web of Science	Science Direct
Number of Results	23	6	64	66
Unique Results			116	

A total number of 159 documents were found from which 43 duplicates were removed resulting in a total of 116 documents. Search criteria were formulated that were applied after screening the abstract, method and conclusion of the remaining list:

- The focus of the document is on proposing a method to forecast demand;
- The proposed method is explained to a level on which it is possible to replicate the method;
- The document includes a measure to validate the proposed method;
- The proposed method is used to create short-term forecasts.

After effective judgement, concerning short-term forecasting, 25 articles remained.

2.2 Time Series Clustering

To deal with a multi-product environment, time series clustering can be very useful prior to forecasting. Firstly, homogeneous clusters can be identified within the data set. Next, the best performing forecasting methods in terms of accuracy should be selected for each cluster. Such effective clustering and then forecasting has the potential to outperform aggregate selection, which selects a single forecasting method for the entire data set (Vangumalli et al., 2019; Dantas & Oliveira, 2018). In addition, clustering prior to forecasting could save a lot of time and money (Vangumalli et al., 2019). In this section, results from the literature review regarding time series clustering methods are summarized. Various clustering methods are found, investigated and elaborated, followed by a first comparison on which model can be applied best.

2.2.1 Model Elaboration

Literature review identifies different clustering models to address a time series clustering problem. Clustering time series data has been used in diverse scientific areas to discover patterns which empower data analysts to extract valuable information from complex and massive datasets (Aghabozorgi et al., 2015). Clustering identifies structure in an unlabelled data set by objectively organizing data into similar groups. The process of time series clustering is complicated, because time series data are naturally noisy and include outliers and shifts. To deal with this complexity, various clustering methods are found in literature. A summary of potential clustering methods with main features, advantages and disadvantages are shown in Table 2.2. In addition, the models are explained more extensively below. To complete, various distance measurement and quality measurement methods are identified.

Hierarchical Clustering

Hierarchical clustering is an approach of cluster analysis which makes a hierarchy of clusters using agglomerative or divisive algorithms (Aghabozorgi et al., 2015). First, hierarchical clustering takes a distance matrix which contains distances between each pair of observations, in this case, series. The agglomerative algorithm considers each item as a separate cluster and then gradually merges the clusters into larger clusters, relying on the distance matrix. This is called the bottom-up approach. In contrast, divisive algorithm starts with all objects as a single cluster and then splits the cluster to reach the clusters with one object. This is called the top-down approach.

A disadvantage of hierarchical clustering is the fact that they cannot adjust the clusters after splitting or merging a cluster. To overcome this problem, hierarchical clustering algorithms must be combined with other clustering approaches. In addition, it is not able to work with effectively large time series due to its computational complexity. A big advantage of hierarchical clustering is that the algorithm creates a dendrogram, which visualizes the hierarchy of clusters to which each observation belongs and therefore, it has a great visualization power. Additionally, hierarchy clustering does not require the number of clusters as an initial parameter and it has the possibility to cluster time series with unequal length (Aghabozorgi et al., 2015). However, this is only possible if an appropriate elastic distance measure, such as Dynamic Time Warping (DTW) is used to compute the dissimilarity/similarity of the time series. DTW and more distance measurement methods are explained more extensively later on in this Section.

Table 2.2: Potential Methods Used for Time Series Clustering

Method	Feature	Advantages	Disadvantages
Hierarchical Clustering (Aghabozorgi et al., 2015)	Cluster approach which makes a hierarchy of clusters using agglomerative or divisive algorithms	Great visualization power; no need to pre-define k, the number of clusters	No possibility to adjust the clusters; computationally complex
Partitioning Clustering - k-Means (Swarndeep Saket & Pandya, 2016)	Identifies k centroids, allocates every data point to the nearest cluster, while keeping the centroids as small as possible; based on mean	Fast computation time; produces tight clusters	Need to pre-define k, the number of clusters; difficult to compare quality of cluster
Partitioning Clustering - k-Medoids (Swarndeep Saket & Pandya, 2016)	Identifies k centroids, allocates every data point to the nearest cluster, while keeping the centroids as small as possible; based on medoid	Simple to understand and easy to implement; fast algorithm and converges in a fixed number of steps; less sensitive to ouliers	More costly than k-Means due to time complexity; does not scale well of large data sets; results and total run time depends on intitial partitions

Partitioning Clustering

A partitioning clustering approach makes k groups from n unlabelled objects in a way that each group contains at least one object. There are two popular algorithms that belong to the partitioning clustering approaches: k-Means and k-Medoids.

K-means clustering is one of the simplest and popular partitioning clustering techniques. A pre-defined number of centroids, k, refer to the number of centroids needed in a data set. Every data point is allocated to the nearest cluster, while keeping the centroids as small as possible. The distance measure commonly used in this algorithm is the Euclidean distance. The k-Means algorithm starts with a first group of randomly selected centroids and than performs iterative calculations to optimize the positions of the centroids. This is repeated until there is no change in the mean value of the objects for each cluster (Swarndeep Saket & Pandya, 2016).

Another member of the partitioning clustering family is the k-Medoids algorithm, also known as Partition Around Mediods (PAM). K-Medoids algorithm uses an actual point in the cluster to represent the center of a cluster instead of using the mean point. K-Medoids is less sensitive to noise and outliers than k-Means since the mean of k-Means is easily influenced by extreme values. Therefore, k-Medoids represent a better cluster center (Swarndeep Saket & Pandya, 2016). An example of k- Mediods against k-Means clustering is presented in Figure 2.1.

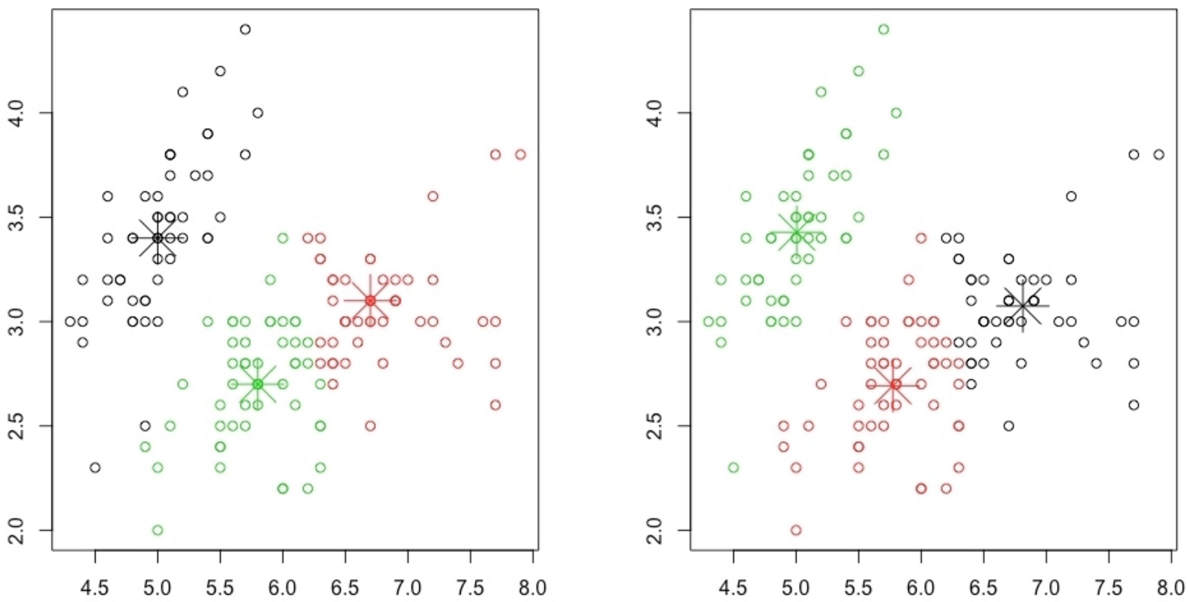


Figure 2.1: Example k-Medoids clustering (left) VS k-Means clustering (right)

Distance Measurement Methods

As mentioned before, time-series clustering relies on distance measure to a high extent. Distance measurement methods are used to measure the distance among time series. The two most popular distance measurement methods that are used for time series data are investigated: Euclidean Distance and Dynamic Time Warping (DTW).

Euclidean distance is the straight-line distance between two points that are on the same time. Therefore, time series needs to be equal in length. DTW can measure similarity between two temporal sequences that do not align exactly in time, speed or length (Aghabozorgi et al., 2015). There is no need to match indices one-to-one on the same time value, but indices can be matched with one or more other indices from the other time series. This rule applies for all indices, except of the first and last indices. DTW is highly recommended for distance measures in time series. An example of Euclidean distance against DTW is presented in Figure 2.2.

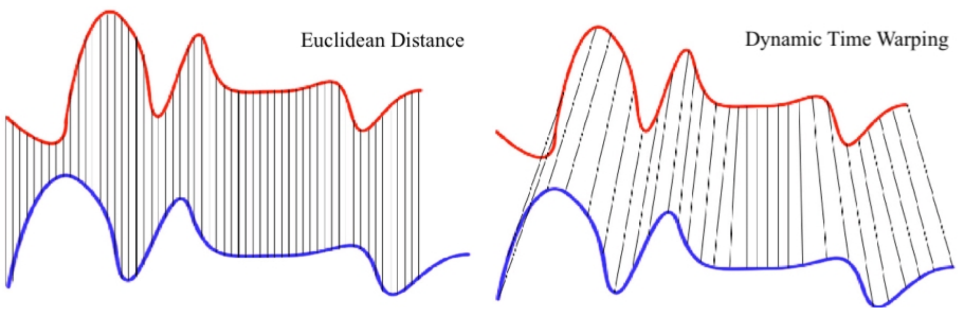


Figure 2.2: Example Euclidean Distance VS Dynamic Time Warping

Quality Measurement Methods
To measure the quality of the clusters generated by a method, different quality measurement methods can be used. In this case, the ground truth is not available and the ideal clustering is unknown, so an intrinsic method must be applied. Intrinsic methods, also known as unsupervised methods, evaluate the goodness of a clustering by considering how well the clusters are separated and how compact the clusters are (Han et al., 2011). The silhouette coefficient is an example of an intrinsic method that determines how well each object lies within its cluster. The silhouette coefficient can be calculated as follows for a particular data point, where $a(o)$ is the average distance between o and all other data points in the cluster to which o belongs. $b(o)$ represents the minimum average distance from o to all clusters to which o does not belong.

$$\text{Silhouette coefficient} \qquad s(o) = \frac{b(o)-a(o)}{maxa(o),b(o)}$$

The silhouette coefficient has a range of [-1,1] where a value near +1 indicates that the sample is far away from the neighboring cluster. A negative value indicates that samples might have been assigned to the wrong cluster. To determine the overall silhouette coefficient of all samples, the mean silhouette coefficient over all samples is obtained. The average silhouette has a range of [-1,1] where a value near +1 indicates that clusters are far away from each other. A negative value indicates that clusters are more similar to each other. Thus, a high average silhouette width indicates a good clustering. The silhouette coefficient can also be used to determine the optimal number of k in which the silhouette coefficient returns the highest value.

2.3 Short-term Forecast Methods

In this section, results from the literature review regarding short-term forecast methods are summarized. Various forecasting methods are found, investigated and elaborated, followed by a first comparison on which model can be applied best.

2.3.1 Model Elaboration

Literature review identifies different predictive models to address a forecasting problem. They range from standard regression and time series approaches to methods that use machine learning algorithms (Braun et al., 2014; Hyndman & Athanasopoulos, 2018). Autoregressive (AR), autoregressive integrated moving average (ARIMA) and regression-based are statistical models that make use of statistical tools to describe the behaviour of a time series and make predictions. Other statistical models that will be investigated are exponential smoothing (ES) and moving average (MA). Potential machine learning models that will be investigated are fuzzy time series (FTS), random forest (RF), support vector regression (SVR) and neural network (NN). These models include non-parametric techniques and have a higher complexity but can result in more powerful models. A summary of potential methods with main features, advantages and disadvantages are shown in Table 2.3. The predictive models can be further dis-aggregated to more specific model variations. Every model has different features, advantages and disadvantages and therefore, it gets easier to define the model that suits best for the problem on hand. All predictive models can be visualized in an extensive framework which visualizes all main predictive model categories and its model variations. In addition, articles that make use of the prediction model are mentioned next to the main categories. The extensive framework can be found in Figure 2.3. The main predictive models and its model variations are explained below after which Table 2.4 represents an overview of the models used in the investigated articles, along with their rank. In addition, the table shows the horizon on which the forecast is based, if there is trend and/or seasonality in the data and in which research area the research is conducted..

Autoregressive and Moving Average

Autoregressive (AR) and moving average (MA) are time series models that uses observations from previous time steps as input to predict the value at the next time step. In an AR model, the variable of interest is forecasted using a linear combination of past values of the variable. The term autoregression indicates a regression of the variable against itself (Hyndman & Athanasopoulos, 2018). AR models are flexible at handling a wide range of time series patterns.

In comparison to AR, which uses past values of the forecast variable in a regression, a MA model uses past forecast errors in a regression-like model. MA models are able to use different time frames in which the average is calculated. The shorter the time frame used to create the average, the more sensitive it will be to changing numbers. The longer the time frame, the less sensitive the average will be. In addition, MA models are able to cope with significant trend.

The autoregressive integrated moving average (ARIMA) model combines the AR and MA models. The traditional ARIMA model uses AR to include historic values and MA to include historic residual errors. It integrates both terms by use of differencing to stationarize time series with trend. Since the ARIMA model combines AR and MA, it is flexible in handling a wide range of different time series patterns and is able to cope with trend. Furthermore, the ARIMA model can be used for non-seasonal and seasonal predictions, where a seasonal parameter is included in the seasonal ARIMA (SARIMA) model (Hyndman & Athanasopoulos, 2018).

Regression-based

Regression-based models describe the relationship between variables by fitting a line to the observed data. Linear regression models use a straight line, while logistic and nonlinear regression models use a curved line. Regression allows you to estimate how a dependent variable changes as the independent variable(s) change (Hyndman & Athanasopoulos, 2018). Simple linear regression is used to estimate the relationship between two quantitative variables. The basic concept is to forecast the time series y, assuming that it has a linear relationship with time series x. For example, simple linear regression is used when you want to know the amount of sales (dependent variable) at a certain level of rainfall (independent variable). When there are two or more independent variables, the model is called a multiple regression model, which is the most common regression-based model. For example, multiple linear regression is used when you want to know the amount of sales (dependent) at certain levels of rainfall and temperature (independent).

Logistic regression is a statistical model that uses a logistic function to model a binary dependent variable, e.g. pass/fail, win/lose. Nonlinear regression models are very diverse and can fit an enormous variety of curves. The flexibility of this method results in a huge number of application possibilities. However, because there are many forms, extensive research is needed to determine which functional form provides the best fit for the data. While regression-based models are easy to calculate and good at analyzing multi-factor models, a disadvantage of using a regression-based method is that the results cannot reflect periodic wave (Lusis et al., 2017).

Exponential Smoothing

Exponential smoothing (ES) is a time series forecasting method that provide simple methods to derive forecasts based on historical observations and has motivated some of the most succesful forecasting methods. ES can handle univariate data that can be extended to support data with a systematic trend or seasonal component (Hyndman & Athanasopoulos, 2018). It is a powerful forecasting method that may be used as an alternative to the popular ARIMA family of methods. ES forecasting methods are similar in that a prediction is a weighted sum of past observations, but the model explicitly uses an exponentially decreasing weight for past observations. The most recent the observation, the higher the associated weight (Hyndman & Athanasopoulos, 2018).

ES is easy to learn and apply, can handle a wide range of time series and can generate reliable forecasts quickly. To cope with trend and seasonality, there are variations of ES. Holt's linear trend method can calculate strong trend patterns while Holt Winter's seasonal method can cover a strong trend and seasonal pattern variations.

Random Forest

Random forest is a machine learning algorithm that is based on the decision tree algorithm but has a different decision making technique. Random forests creates randomly multiple decision trees and each node of the decision tree uses a random subset of features to calculate the output. The final output is created by combining the output of the individual decision trees and is called ensemble learning. Predictive powers of multiple learners are combined and result in a single model which gives the aggregated output from several models. The ensemble method used is called bagging. Bagging is the part of the random forest where random samples are created from the training data set where it is allowed to use doubles (Hastie et al., 2009).

Random forests are not dependent on the feature importance given by a single decision tree and can therefore better generalize over the data. In addition, most of the times, random forests algorithm give more accurate output then simple decision trees (Hastie et al., 2009). In addition, the algorithm provide a reliable estimation of the features that are of importance (Svoboda et al., 2020). On the other hand, the algorithm is harder to interpret than a normal decision tree and the predictions are slower. This causes high computation times and therefore possibly have high computational costs. A possibility to solve this is to split the process to multiple machines to run and run the processes in parallel (Svoboda et al., 2020).

Support Vector Regression

Support Vector Regression (SVR) models are designed to fit known data based on performance criteria and then forecast unknown data based on the trained model (Lusis et al., 2017). The model tries to find a linear separating hyperplane by mapping data into the feature space with the higher-dimensional space using the so-called kernel trick by use of a weight factor, regularization parameter and a kernel function (Herrera et al., 2010). Because of model complexity, selection of the optimal parameters is complicated. The application of SVR was successful in several cases where it outperformed statistical models (Lusis et al., 2017).

Table 2.3: Potential Methods Used for Time Series Forecasting

Method	Feature	Advantages	Disadvantages
Autoregressive (Hyndman & Athanasopoulos, 2018)	Dependent variable is a linear combination of past values of the variable	Flexible at handling a wide range of different time series patterns	The method is restricted to stationary time series
Moving Average (Hyndman & Athanasopoulos, 2018)	Dependent variable is a linear combination of past forecast values	Able to cope with significant trend	It requires the model to be invertible
ARIMA (Hyndman & Athanasopoulos, 2018)	Dependent variable is a combination of its lagged values and the present value of the random error term	Flexible in handling a wide range of different time series patterns and is able to cope with trend	Require stationary and invertible time series; model cannot reflect non-linear relationships
Regression-based (Hyndman & Athanasopoulos, 2018)	Dependent variable is a function of independent variable(s)	Good at analyzing multivariate models; provide error checking of model estimation parameters; easy to calculate	Does not consider the untestability of certain influence factors; results cannot reflect periodic wave
Exponential Smoothing (Hyndman & Athanasopoulos, 2018)	Dependent variable is a function of historic values with weights	Can handle univariate data; supports data with a trend or seasonal component	Does not consider error terms
Random Forest (Svoboda et al., 2020)	Dependent variable is categorical and is the result of a combination of decision trees	Provides a reliable estimation of the features that are of importance	Hard to interpret; predictions are slow
Support Vector Regression (Lusis et al., 2017)	Dependent variable is the result of a complex model based on limited sample information and learning ability	Computationally efficient; can improve generalization performance; less parameters to solve	Sensitive to missing data; difficult to implement large-scale training samples
Fuzzy Time Series (Mamlook et al., 2009)	Dependent variable is forecasted by fuzzy judgement for systems with unknown models	Capability to handle vagueness more efficiently; good at uncertain situation prediction of input variables	Cannot reflect the relationship between predicted values and historical data
Neural Networks (Lusis et al., 2017)	Dependent variable is the result of a complex model that includes multiple transformations; abstracted from the human brain	Provide self-learning function and high-speed search for optimal solutions; approximate any arbitrarily complex nonlinear relationship	No ability to explain reasoning; cannot work when data is insufficient
k-Nearest Neighbor (Sudheer & Suseelatha, 2015)	Dependent variable is the result of a model that finds the k nearest neighbors and calculates the average of the numerical target	Simple and easy to implement; no need to tune several parameters or make additional assumptions; can handle fluctuations in a data set	Time consuming to find the optimal k; with large data sets the prediction might be slow

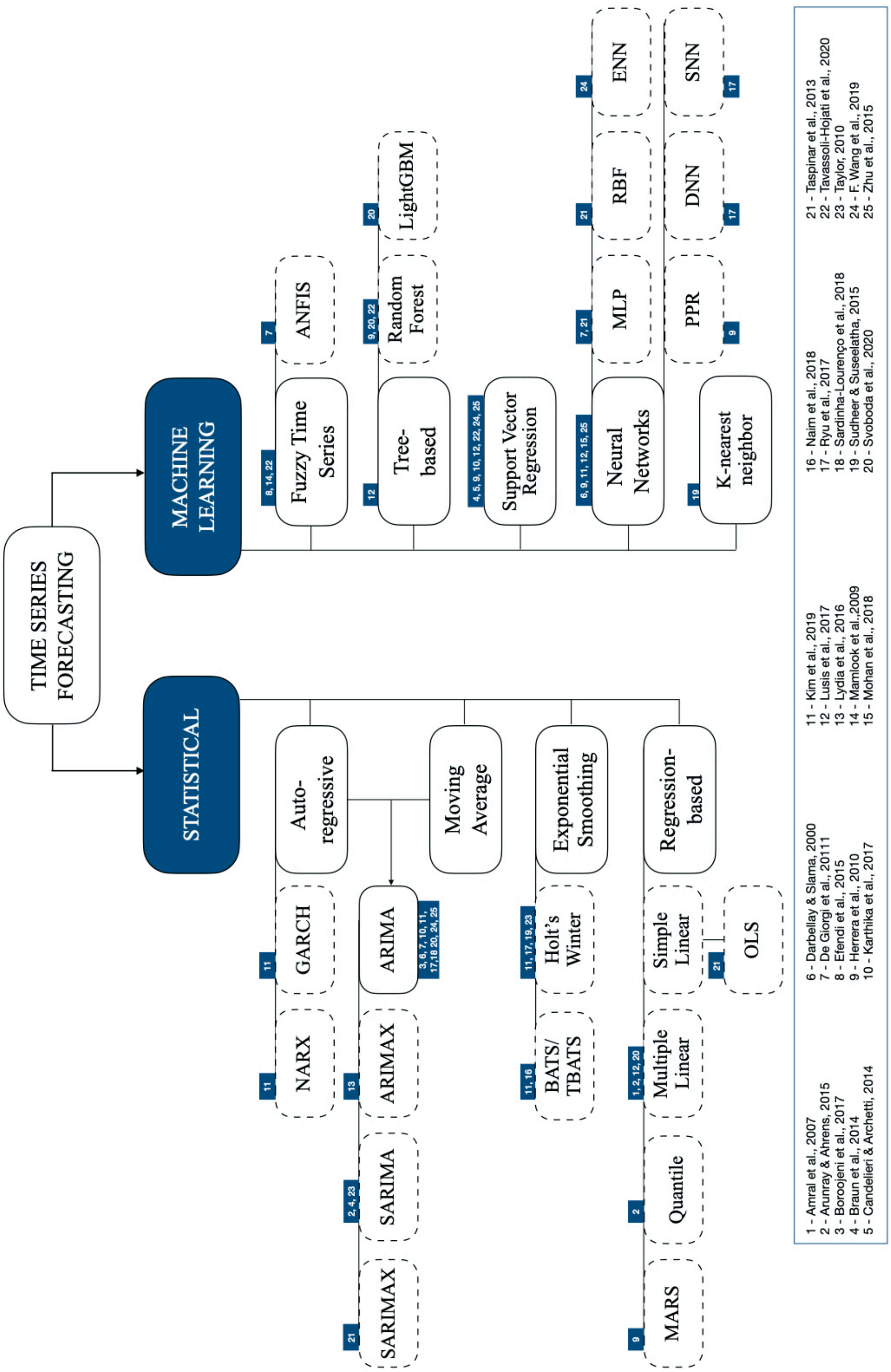


Figure 2.3: Framework Forecasting Models

Fuzzy Time Series

Fuzzy time series (FTS) is a forecasting technique that breaks down the entire range of numbers in a time series into many fuzzy sets. This causes losing some information but retaining the most important part of it. This information is enough to make a rough and quick forecast (Mamlook et al., 2009). Because of the fact of dealing with linguistic terms instead of real values of the time series, these methods have the capability to handle vagueness more efficiently and thus become a forefront technique in time series forecasting (Tavassoli-Hojati et al., 2020). FTS is good at uncertain situation prediction of input variables but cannot reflect the relationship between predicted values and historical data. However, FTS has been successfully used to deal with various problems such as temperature forecasting and load forecasting (Efendi et al., 2015).

Neural Networks

Artificial neural networks (ANN) are forecasting methods that are based on simple mathematical models of the brain. They allow complex nonlinear relationships between the response variable and its predictors (Hyndman & Athanasopoulos, 2018). A neural network is a network of simple elements, called "neurons" which are organised in layers. The predictors (or inputs) form the bottom layer, and the forecasts (or outputs) form the top layer. There may also be intermediate layers containing "hidden neurons" (Hyndman & Athanasopoulos, 2018). A shallow neural network has three layers of neurons: an input layer, a hidden layer, and an output layer and is equivalent to a logistic regression. A Deep Neural Network (DNN) has more than one hidden layers, which increases the complexity of the model and can significantly improve prediction power.

NN is a machine learning technique that can solve very complex problems, classify inputs or even make difficult predictions and provide high-speed accurate answers. However, NN cannot work when data is insufficient, are computationally intensive and turns all reasoning into numerical calculations which results in the loss of information (Lusis et al., 2017). In addition, when a regular regression model can be used, a neural network is overkill. A regression model is easier to understand and calculate.

k-Nearest Neighbor

The k-Nearest Neighbor (kNN) algorithm is a simple, easy to implement supervised machine learning algorithm that can be used for time series forecasting. The algorithm relies on labeled input data to learn a function that produces an appropriate output when given new unlabeled data. The algorithm assumes that similar things are near to each other. Therefore, it uses the distance between the query-instance and all the training samples. Euclidean distance is the default distance metric in kNN but dynamic time warping can also be used in the algorithm. The total distance and index of each training sample is added to an ordered collection and sorted in ascending order by the distances. Next, the mean of the k nearest neighbors is used as the prediction value of the query instance (Sudheer & Suseelatha, 2015). In the kNN algorithm, it is not easy to find the right value of k. A small value of k means that noise will have a higher influence on the result and a large k will make it computationally expensive. To find the optimal k with a minimum prediction error, the algorithm should be executed several times which is time consuming. On the other hand, there is no need to tune several parameters or make additional assumptions. In addition, the algorithm is useful in modeling the fluctuation component in a data set (Sudheer & Suseelatha, 2015).

Table 2.4: Best Performing Forecasting Method

Article	**1**	**2**	**3**	**Horizon**	**Trend**	**Seasonal**	**Research Area**
(Amral et al., 2007)	MLR			Hourly			Load Forecasting
(Arunraj & Ahrens, 2015)	SARIMA-QR	SARIMA-MLR		Daily	N	N	Food Retail Forecasting
(Boroojeni et al., 2017)	MA	AR		Daily	N	Y	Load Forecasting
(Braun et al., 2014)	SVR	SARIMA		Hourly		Y	Water Demand Forecasting
(Candelieri & Archetti, 2014)	SVMR			Hourly			Water Demand Forecasting
(Darbellay & Slama, 2000)	NN	ARIMA		Hourly		Y	Load Forecasting
(De Giorgi et al., 2011)	MLP NN	ELMAN NN	ANFIS	Hourly			Wind Power Forecasting
(Efendi et al., 2015)	Proposed	Yu's	Cheng's	Daily	Y	Y	Load Forecasting
(Herrera et al., 2010)	SVR	PPR	RF	Hourly	Y	Y	Water Demand Forecasting
(Karthika et al., 2017)	SVM-ARIMA	SVM	ARIMA	Daily			Load Forecasting
(Kim et al., 2019)	NARX	ARIMA-GARCH	TBATS	Hourly	Y	Y	Load Forecasting
(Lusis et al., 2017)	SVR	Regression Tree	NN	30 min		Y	Load Forecasting
(Lydia et al., 2016)	Linear ARMAX	SMOreg	M5R	10 min	Y	N	Wind Power Forecasting
(Mamlook et al., 2009)	FTS			Hourly		Y	Load Forecasting
(Mohan et al., 2018)	DMD	NN		Hourly	Y	Y	Load Forecasting
(Naim et al., 2018)	TBATS	BATS		Hourly	Y	Y	Load Forecasting
(Ryu et al., 2017)	DNN	SNN	DSHW	Hourly		Y	Load Forecasting
(Sardinha-Lourenço et al., 2018)	Heuristic			Daily		Y	Water Demand Forecasting
(Sudheer & Suseelatha, 2015)	Proposed	HWNN	TES	Hourly	Y	Y	Load Forecasting
(Svoboda et al., 2020)	WA Ensemble	RF	LGB	Daily	N		Load Forecasting
(Taşpınar et al., 2013)	SARIMAX	MLP	RBF	Daily	Y	Y	Load Forecasting
(Tavassoli-Hojati et al., 2020)	SPLNF	SVR	RF	Hourly			Load Forecasting
(Taylor, 2010)	TS ARMA-HWT	TS ARMA	TS HWT	Hourly	N	Y	Load Forecasting
(F. Wang et al., 2019)	SVR	ENN	ELM	Hourly	Y		Electricity Price Forecasting
(Zhu et al., 2015)	FNF-SVRLP	SVRLP	ANN	Daily			Load Forecasting

Table 2.4 represents an overview of the models used in the investigated articles, along with their rank. In addition, the table shows the horizon on which the forecast is based, if there is trend and/or seasonality in the data and in which research area the research is conducted. It can be seen that a statistical model was only able to outperform a machine learning model in (Taşpınar et al., 2013), while the opposite is found in (Braun et al., 2014; Darbellay & Slama, 2000; Ryu et al., 2017). Karthika et al. (2017) and Sudheer and Suseelatha (2015) use a combination of a machine learning model with a statistical model which outperforms the use of a single model. It was also found out that most of the models can perform a short-term forecast based on a hourly or even smaller time horizon. Lydia et al. (2016) is even able to forecast on a ten minutes horizon with a statistical model. To zoom in on the research areas of the short-term forecasts, it can be seen that 24 of a total of 25 articles did research in load forecasting or water demand forecasting. It describes the research gap very clearly and therefore, it is very interesting to explore the applicability of short-term forecasting techniques on online sales.

2.4 Data Preparation

Data preparation is the process of cleaning and transforming raw data before moving on to processing and analysis of the data. It is an important step which often involves transforming data sets and making corrections to data. Good data preparations makes processed data more accessible to users. The required data preparation depends on the data and the model that will be applied. This section explains the four most popular data transformation techniques including an overview on which techniques are applied in literature.

Power transformation is a technique used to remove a shift from a data distribution to make the distribution more normal (Gaussian). For example, on a time series data set, this can have the effect of removing a change in variance over time. Two specifications on power transformation are the log transform and a generalized versions such as the Box-Cox (BC) transform.

The Box-Cox transformation uses an optimal parameter λ to transform data into a normal distribution. In the Box-Cox, λ reaches from -5 to 5 until the best value is found. When $\lambda = 0$, a special case of the Box-Cox transformation is used: log transformation. It replaces each variable x with a $log(x)$ and reduces the skewness of the original data. Therefore, the data has to (approximately) follow a log-normal distribution (Hyndman & Athanasopoulos, 2018).

$$w_t = \begin{cases} (y_t^\lambda - 1)/\lambda, & \text{otherwise} \\ log(y_t) & \text{if } \lambda = 0 \end{cases}$$

Difference transformation is a simple way for removing a systematic structure from the time series, such as trend or seasonality. Trend can be removed by subtracting the previous value from each value in the series and is called first order differencing. The process can be repeated to remove second order trends, and so on. Higher order differencing can be applied to series that seem to follow a higher order function due to a nonlinear trend. Seasonality can be removed in a similar way by first order differencing a time series at a lag equal to the period. For each autoregressive (AR) model, trend and seasonality should be removed to stationarize the series and make it more reliable. A time series is stationary when it does not matter when it is observed and it look much the same at any point in time (Hyndman & Athanasopoulos, 2018).

First order difference	$y'_t = y_t - y_{t-1}$
Second order difference	$y''_t = y'_t - y'_{t-1}$
Seasonal difference	$y'_t = y_t - y_{t-m}$

Normalization is a rescaling transformation technique that scales data from the original range to a new range, without distorting differences in the range of values. Normalization is required when features have different ranges to make sure every feature has equal importance. Therefore, data is normalized to bring all the variables to the same range. Data can be normalized using the minimum and maximum of the data set or by z-score normalization. The min-max scalar guarantees the same scale but does not handle outliers well (Tian et al., 2012) while the z-score scalar does the other way around. Normalization is a good technique when the distribution of the data is now known. It is useful for models such as k-nearest neighbors and neural networks since they do not make assumptions about the distribution of data.

Min-Max scalar	$X_{norm} = \frac{X - X_{min}}{X_{max} - X_{min}}$
Z-score scalar	$X_{norm} = \frac{X - \mu}{\sigma}$

Fuzzification is a data transformation technique applied for fuzzy time series. It determines the degree to which an input data belongs to each of the appropriate fuzzy sets via the membership functions. Triangular or trapezoidal-shaped membership functions are the most common membership functions and should overlap in order to allow smooth mapping of the system. Any particular input is interpreted from the fuzzy set and a degree of membership is obtained.

An overview on the different data transformation techniques applied in literature is provided in Table 2.5. The articles that leave a blank line did not specify or did not use any of the data transformation techniques. Logarithmic transformation is used in articles where the goal was to reduce the variability of the time series. The forecasting methods that go along with this articles are ARIMA, SARIMA, SARIMAX and MLR. The general Box-Cox transformation is only used in three articles using ARMA, BATS and TBATS (Boroojeni et al., 2017; Naim et al., 2018; Kim et al., 2019). Difference transformation is applied in most of the AR models. When the data in the article faces clear trend or seasonality, difference transformation is used, otherwise no difference transformation needs to be applied (Hyndman & Athanasopoulos, 2018). Normalization is applied in most of the articles with neural networks, except for (De Giorgi et al., (2011); Kim et al., (2019); F. Wang et al., (2019); Zhu et al., (2015)) which did not specify any data transformation for their neural network. Fuzzification is applied to FTS forecast methods. No data transformation technique was found for the FTS of (Tavassoli-Hojati et al., 2020).

Table 2.5: Data Transformation Applied in Literature

Article	**Forecast Method**	**Log**	**BC**	**Diff.**	**Nor.**	**Fuz.**
(Amral et al., 2007)	MLR					
(Arunraj & Ahrens, 2015)	SARIMA-MLR SARIMA-QR					
(Boroojeni et al., 2017)	ARIMA		X	X		
(Braun et al., 2014)	SARIMA SVR	X				
(Candelieri & Archetti, 2014)	SVMR					
(Darbellay & Slama, 2000)	ARIMA			X		
	NN				X	
(De Giorgi et al., 2011)	ARIMA Weighted NN, ANFIS					
(Efendi et al., 2015)	FTS					X
(Herrera et al., 2010)	NN, PPR, MARS RF, SVR				X	
(Karthika et al., 2017)	ARIMA SVM			X		
(Kim et al., 2019)	ARIMA			X		
	HWT Exp. smoothing NN's					
	TBATS		X			
(Lusis et al., 2017)	MLR, SVR, NN's Regression Trees	X				
(Lydia et al., 2016)	ARIMAX					
(Mamlook et al., 2009)	FTS					X
(Mohan et al., 2018)	DMD				X	
(Naim et al., 2018)	BATS, TBATS		X			
(Ryu et al., 2017)	NN's DHWT ARIMA				X	
(Sardinha-Lourenço et al., 2018)	ARIMA Heuristic			X		
(Sudheer & Suseelatha, 2015)	HWT exp. smoothing Weighted neirest neighbor					
(Svoboda et al., 2020)	ARIMA, MLR RF, LightGBM WA			X		
(Taşpınar et al., 2013)	SARIMAX OLS, NN's	X		X		
(Tavassoli-Hojati et al., 2020)	FTS					
(Taylor, 2010)	SARIMA HWT exp. smoothing			X		
(F. Wang et al., 2019)	ARIMA NN's SVR			X		
(Zhu et al., 2015)	ARIMA NN, SVRLP, FNF-SVRLP			X		

2.5 Multi-step Ahead Forecasting

Time series forecasting is typically discussed when only a one-step ahead prediction is required. When multiple steps into the future need to be predicted, like in this master thesis project, a multi-step ahead forecasting technique is required. This chapter discusses the four most popular strategies discussed in literature that can be applied to multi-step ahead forecasting.

Direct multi-step ahead forecasting involves developing a separate model for each forecast time step. For example, when predicting the sales for the next to hours, two separate models are developed for hour 1 and hour 2. This strategy requires a high computation time and there is no opportunity to model dependencies between the two separate predictions (Taieb et al., 2010).

$$\begin{gathered} \textit{-Direct-} \\ prediction(t+1) = model1(obs(t-1), obs(t-2), ..., obs(t-n)) \\ prediction(t+2) = model2(obs(t-2), obs(t-3), ..., obs(t-n)) \end{gathered}$$

Recursive multi-step ahead forecasting involves using a one-step model multiple times where the prediction for the prior time step is used as an input for making a prediction on the following time step. For example, a one-step forecasting model is used to predict the next hour. Next, this prediction will be used as an observation input in order to predict the second hour, using a one-step forecasting model again. The recursive strategy allows prediction errors to accumulate, so the performance of the model can quickly degrade.

$$\begin{gathered} \textit{-Recursive-} \\ prediction(t+1) = model(obs(t-1), obs(t-2), ..., obs(t-n)) \\ prediction(t+2) = model(prediction(t+1), obs(t-1), ..., obs(t-n)) \end{gathered}$$

The direct-recursive hybrid strategy is a combination of direct and recursive multi-step ahead forecasting. A separate model can be constructed for each time step to be predicted, where each model may use the predictions made by models at prior time steps as input values (Taieb et al., 2012). This strategy requires the most computation time of all strategies.

$$\begin{gathered} \textit{-Direct Recursive-} \\ prediction(t+1) = model1(obs(t-1), obs(t-2), ..., obs(t-n)) \\ prediction(t+2) = model2(prediction(t+1), obs(t-1), ..., obs(t-n)) \end{gathered}$$

The multiple output strategy is a multi-step ahead forecasting technique that is able to develop one model that is capable of predicting the entire forecast sequence in once. An example of the multiple output strategy is the multi-input multi-output (MIMO) strategy (Taieb et al., 2010). The MIMO strategy returns a vector of future values in a single step. This strategy is complex and has a low flexibility since it constrains all the horizons to be predicted with the same model structure (Taieb et al., 2012) . In addition, the strategy requires a medium-high computation time and needs more data to avoid overfitting.

$$\begin{gathered} \textit{-Multiple Output-} \\ prediction(t+1), prediction(t+2) = model(obs(t-1), obs(t-2), ..., obs(t-n)) \end{gathered}$$

Research of Taieb et al. (2012) shows that the multiple-output strategy is invariably the best strategy. They beat single-output strategies such as direct, recursive and direct recursive. Among the single-output strategies, the recursive strategy outperforms the direct and direct recursive strategy. Direct recursive gives especially low accuracy when no deseasonalization is performed. Since the multiple-output strategy invariably outperforms the single-output strategies, the multiple-output strategy will be applied in the problem at hand.

2.6 Performance Measures

Validation of a forecasting method is of high importance to be able to compare different forecasting techniques and decide on which method works best for the problem on hand. The most important criterion for model selection is forecast accuracy, which can be measured in many ways (Hyndman & Athanasopoulos, 2018). To find out which performance measure are used most commonly, an overview on the different techniques applied in literature is provided in Table 2.6. Next, the performance measures are explained in detail to have a better understanding of their way of working.

Table 2.6: Performance Measure Applied in Literature

Article	**MSE**	**RMSE**	**MAE**	**MAPE**	**RE**
(Amral et al., 2007)				X	
(Arunraj & Ahrens, 2015)		X		X	
(Boroojeni et al., 2017)		X	X	X	
(Braun et al., 2014)					X
(Candelieri & Archetti, 2014)				X	
(Darbellay & Slama, 2000)	X			X	
(De Giorgi et al., 2011)	X			X	
(Efendi et al., 2015)				X	
(Herrera et al., 2010)		X	X		
(Karthika et al., 2017)		X		X	
(Kim et al., 2019)		X		X	
(Lusis et al., 2017)		X			
(Lydia et al., 2016)		X	X	X	
(Mamlook et al., 2009)			X		
(Mohan et al., 2018)		X	X	X	
(Naim et al., 2018)		X		X	
(Ryu et al., 2017)		X		X	
(Sardinha-Lourenço et al., 2018)				X	
(Sudheer & Suseelatha, 2015)				X	
(Svoboda et al., 2020)	X		X	X	
(Taşpınar et al., 2013)		X		X	
(Tavassoli-Hojati et al., 2020)		X	X	X	
(Taylor, 2010)				X	
(F. Wang et al., 2019)		X		X	
(Zhu et al., 2015)			X	X	

The first three performance measures, MSE, RMSE and MAE, are scale-dependent measures that are useful for comparing different models to data with the same scale. If the scale is different, these measures cannot be used. The two most commonly used scale-dependent measures are based on the squared errors (RMSE) or absolute errors (MAE), which is also confirmed in literature. The MSE shows the average of a set of errors and literature shows that it is only used in two out of 25 discovered articles. The RMSE takes a step further and computes the standard deviation of prediction errors and tells how concentrated the data is around the line of best fit. The RMSE is more difficult to interpret but widely used in forecasting. Both RMSE and MSE are more sensitive to outliers than MAE. The MAE is a popular measure for comparing single time series or several time series with the same scale since it is easy to compute and understand (Hyndman & Athanasopoulos, 2018).

Mean Squared Error $MSE = \frac{1}{n}\sum_{j=1}^{n}(y_j - \hat{y}_j)^2$

Root Mean Squared Error $RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(y_j - \hat{y}_j)^2}$

Mean Absolute Error $MAE = \frac{1}{n}\sum_{j=1}^{n}|y_j - \hat{y}_j|$

Another very popular performance measure is the MAPE, which is used most often in literature. Only 4 out of 25 articles did not specifically mention or use the MAPE performance metric. The MAPE is based on the percentage error and has the advantage of being scale-independent which enables comparison across data sets (Hyndman & Athanasopoulos, 2018). The MAPE works best if there are no extremes and no zeros in the data set. In addition, the MAPE is easy to interpret and can convey information when the volume is not known. The MAPE has the disadvantage of putting a heavier penalty on negative errors than on positive errors.

Mean Absolute Percentage Error $MAPE = \frac{1}{n}\sum_{j=1}^{n}\left|\frac{y_j - \hat{y}_j}{y_j}\right| * 100$

The relative error (RE) can be used as a measure of accuracy and is an error relative to the true value. Therefore, the relative error is used to put an error into perspective. The relative error is not a common technique and is, in the investigated literature, only used by Braun et al. (2014). Braun et al. (2014) uses a histogram to show the relative errors of the hourly demand prediction of all points from a one year data set. With this technique, it can be easily seen how many of the prediction points fall within a specified relative error range.

Relative Error $RE = \frac{|y_j - \hat{y_j}|}{y_j}$

In conclusion, as confirmed by the literature, the most widely used performance measures for short-term forecasting are the RMSE and the MAPE. Since more than half of the articles use these performance measures, the RMSE and MAPE will be used for further research.

Chapter 3

Company Descriptives

This chapter describes the company case environment and serves to answer the first two subquestions defined in Section 1.5. Firstly, relevant company figures are shown in Section 3.1 to display the current sales of the company and their specifications. Secondly, Section 3.2 outlines the current categorization structures applied at bol.com. Lastly, the current forecast method and its performance is explained in Section 3.3.

3.1 General Sales

To get insights in the high level statistics of bol.com and their current sales, this section displays an overview of relevant statistics and figures that help understand the problem situation.

Figure 3.1 shows the weekly total consumer sales for the main warehouse of bol.com, BFC, over the year of 2019. Hence, a clear pattern is observed at bol.com around peak season, November and December, sales are significantly higher in comparison to other months of the year. This is due to holidays and promotion days like Black Friday. During peak season, consumer sales are tripling in comparison with the sales of other months. Additionally, Figure 3.1 shows the share of 24 hour sales in the total consumer sales. The 24 hour sales are representing the sales that cause direct workload in the warehouse on a specific day, since the orders should be delivered the same or next day to the consumer. The 24 hour sales represent the sales on which this research is focused and cannot include sales on days for which delivery is not possible the next day. It can be concluded that approximately 75% of the total consumer sales present 24 hour sales. The pattern of 24 hour sales is very similar to the total consumer sales, facing a clear peak around November and December and an increasing trend during the year.

Figure 3.2 shows the sales per weekday during the year of 2019. The sales are summed by weekday and plotted in the graph on a hourly x-axis. Figure 3.2a displays the total sales and shows that all weekdays behave quite similar. Each day has a clear fall down in sales between 17h and 18h, which is probably caused by diner time. Additionally, each day sales increase significantly after 18h and decreases again around 21h. The lowest sales values are on Saturdays, followed by Fridays. The highest sales values arise on Mondays and Wednesdays.

In comparison, Figure 3.2b shows the 24 hour sales per weekday. The most remarkable insight is the enormous decrease of sales on Saturdays. This is due to 24 hour sales cannot

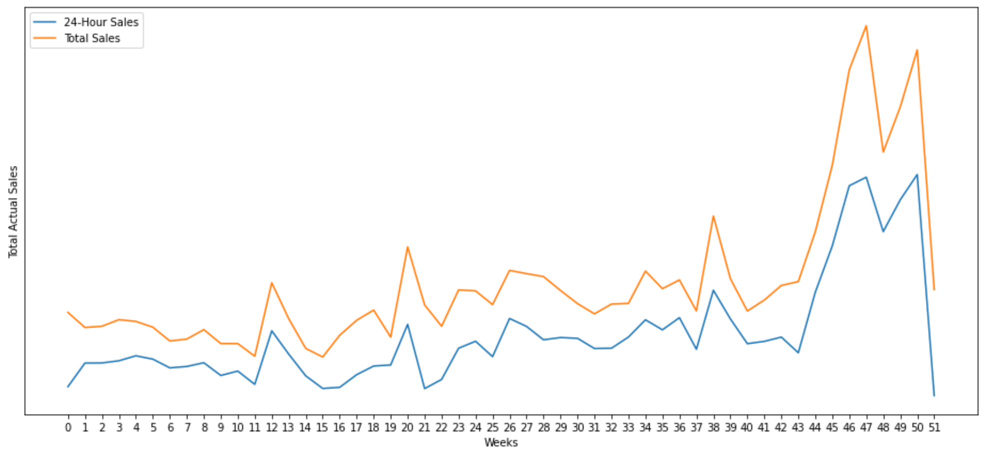
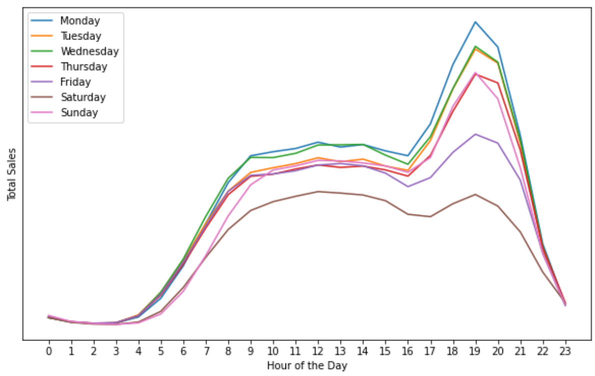
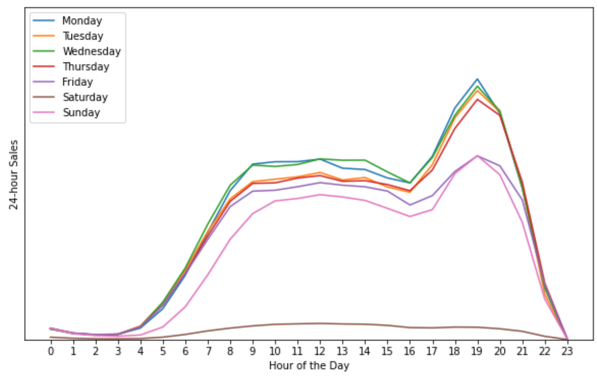


Figure 3.1: Weekly Consumer Sales 2019

include sales on days for which delivery is not possible the next day. Since most distribution partners do not operate on Sundays, the sales are decreased in this graph. However, 24 hour sales are not completely zero on Saturdays since bol.com offers some special delivery services, such as same day or Sunday delivery in return for extra payment. Therefore, Saturdays represent approximately one tenth of the workload in comparison to other weekdays. Additionally, sales on Sundays also decreased more than other weekdays compared to the total sales on Sundays. This is due to the warehouse that only operates on Sunday afternoon, so the capacity is lower compared to other weekdays. Other weekdays behave quite similar and cause quite an equal workload compared to each other. Within the 24 hour sales, the highest sales values arise on Mondays and Wednesdays just as in the total sales figure.



(a) Total Sales



(b) 24 hour Sales

Figure 3.2: Sales per Weekday

3.2 Categorization

Consumer sales can be divided into different categorization layers, defined by the finance department of bol.com. There are seven main clusters: 'Dagelijkse Verzorging & Dier', 'Elektronica', 'Huis & Tuin', 'Lezen & Leren', 'Speelgoed & Entertainment', 'Mode, Sport & Baby' and 'Zakelijk'. The partitioning of these clusters into subcategories is provided in Table 3.1. The subcategory 'Shops' is the highest level and stands for non-physical shops in the IT-landscape of bol.com. The main cluster 'Zakelijk' only contains gift cards and is therefore out of scope and not included in the upcoming figures and statistics. Given that bol.com's sales can be divided into different category levels, selecting an appropriate level will be essential for modelling purposes. The level should capture several characteristics of customer behaviour. For example, the level should exist of clear trend and seasonality factors which helps in making an accurate forecast. A final choice of level for further research and modelling will be made in Section 2.4.

Table 3.1: Partitioning of Clusters over Subcategories

	Shops	**Product Group**	**PG Sub**	**PG Subsub**
Dagelijkse Verzorging & Dier	8	8	51	90
Elektronica	9	18	51	82
Huis & Tuin	9	11	103	339
Lezen & Leren	4	8	147	146
Speelgoed & Entertainment	5	7	40	48
Mode, Sport & Baby	8	11	75	196
Zakelijk	1	1	1	1
Total	44	64	468	902

Figure 3.3 shows the six mains clusters and its share in sales for the year of 2019. It is shown that 'Dagelijkse Verzorging Dier', 'Elektronica' and 'Speelgoed Entertainment' significantly have the highest share in consumer sales. These clusters represent approximately 72.5% of all sales in 2019. 'Lezen en Leren' has the lowest share in sales with only 4.3% of all sales.

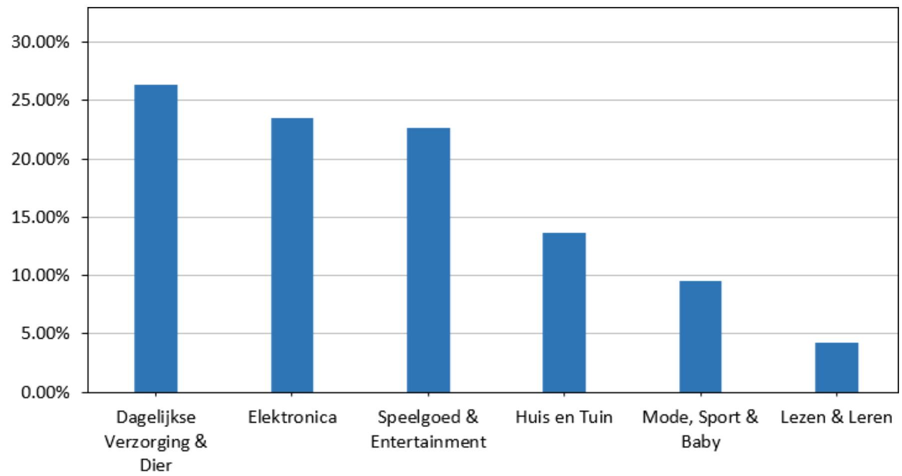


Figure 3.3: Share in Sales per Cluster - 2019

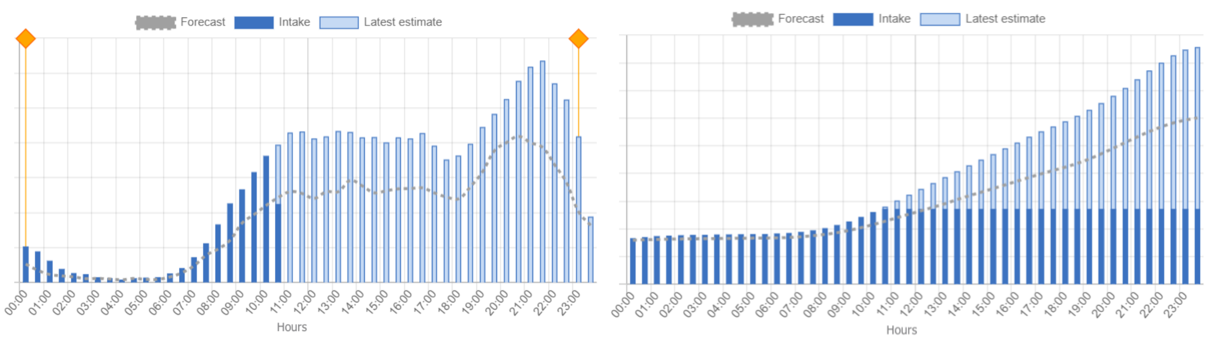
3.3 Current Forecast Method

The current forecast is displayed in the Sales Operations Planning Dashboard (SODA). The dashboard is provided with different insights into the item intake on a specific day. The sales that SODA shows are only the sales that cause direct workload on that specific day (today), also known as 24 hour sales. Thus, Control Tower only responds on the workload of today and >24 hour sales are pushed forward to the graphs and insights of the next days. The sales that are pushed forward are summed to the workload at the beginning of the day it has to be outbound. Therefore, the orders are included as workload on the right day and lever decisions are based on the total workload. Control Tower can activate several levers to steer in the daily sales intake:

- The delivery promise of a product on sale is shifted to another moment. Therefore, the delivery promise is adjusted from 'ordered before 23:59, delivered tomorrow' to a later moment in time, e.g. 'ordered today, delivered day after tomorrow'. This lever is applied on a subset of products and is called the 'STOCK'-lever.

- The warehouse capacity has exceeded and therefore, the warehouse is closed. The delivery promises of all products on sale are shifted to another moment. For products that are available at other warehouses, FNK switches the fulfillment to the other warehouse. This lever is applied on all products of a specific warehouse and is called the 'DLY'-lever.

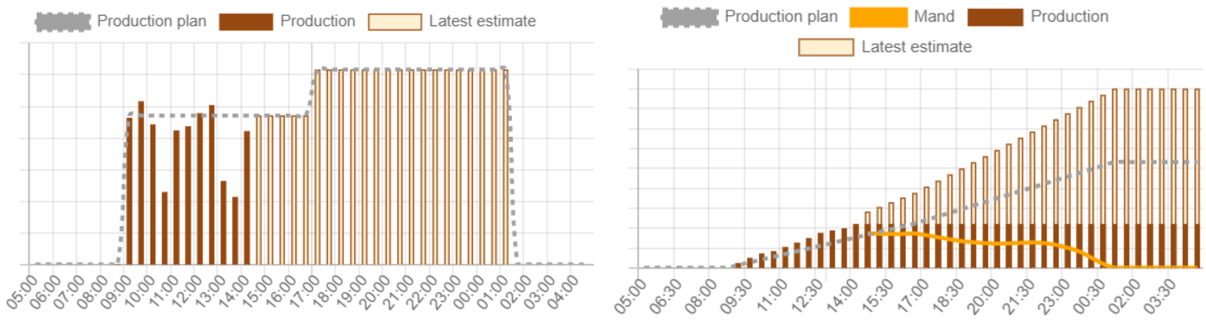
Figure 3.4 shows an example of the 24 hour item intake of the BFC warehouse at t=10:52AM. Figure 3.5a shows sales intake on each 30 minutes time slot, while Figure 3.5b shows the cumulative sales intake. The cumulative graph is most relevant since it shows the total workload of today. Workload that is elevated from the warehouse since the orders are processed are considered in separate production plans. Additionally, the grey dotted line represents the 24 hour forecast which is determined before the start of the day and will never change during the day. The dark blue bars represent the actual sales for each 30 minutes time slot that, in this case, happened until t=10:52. As the day progresses, the actual sales intake are overwriting the light blue bars and are visualised by dark blue bars. The forecast, also know as the latest estimate, that remains for the rest of the day, is visualised by the light blue bars. The actual and expected production is shown in Figure 3.5. The dark brown bars show the actual production and the light brown bars show the expected production for the rest of the day. The actual and expected production is taken into account when a lever decision needs to be made.



(a) Hourly Sales Intake

(b) Cumulative Sales Intake

Figure 3.4: Example Sales Intake BFC at t=10:52AM



(a) Hourly Production (b) Cumulative Production

Figure 3.5: Example Sales Intake BFC at t=10:52AM

Currently, the actual and latest estimate sales are updated every ten minutes. First, actual 24 hour sales are summed during the day for each 30 minutes time slot. Therefore, every 10 minutes, 24 hour sales that happened in the previous 10 minutes are added to the dark blue bars and overwrites the light blue bars. It could happen at the end of the 30 minutes time slot that the total of the actual sales is higher or lower then the latest estimate was expecting. This indicates that the latest estimate was not 100% accurate and could be improved.

In addition, the latest estimate for the rest of the day needs to be updated and fine-tuned every 10 minutes, simultaneously with adding the actual sales. The system compares the current day with reference days, which are the 54 same week days from the previous 54 weeks. Therefore, the system looks for the most similar reference day in terms of actual sales until that point in time. For example, if t=10:52AM, the system compares the sales of the current day until 10:52AM with all reference day sales until 10:52AM. The comparison gives a percentage of deviation with each reference day from which the most similar reference day is chosen.

Next, the most similar reference day is used to predict the sales intake of the rest of the current day. The cumulative percentage of sales are assumed to be the same for the current day and the reference day until point t in time. Given these percentages, the predicted sales intake can be determined for the rest of the day. This completes the update of the sales intake.

3.3.1 Performance

The performance describes the ability of the SODA system to predict the total end of the day sales on a specific moment during the day. This deviation is calculated as the mean relative error and describes the deviation between the expected total sales in the end of the current day, predicted at a specific time, versus the actual end of the day. These calculations are performed when the day has ended and the actual end of the day item intake is known. For example, if the system predicted at 10:00AM that sales at the end of the day would be 100.000 and the actual sales in the end were 110.000, the deviation at 10:00AM was (110.000 - 100.000)/100.000 = 10%. The final performance of the model is expressed as the average deviation of all previous days at a specific moment in time. For bol.com, the most important time during the day is 10:00AM. This is the time where bol.com aims on having a 95% performance, so an average deviation of 5%. This moment is important because at this time, it is still possible to make logistical decisions that have a quite low impact on the sales and costs, e.g. change capacity and employees for the rest of the day in the warehouse. After 10:00AM only levers can be activated to steer in the right direction which has direct impact on productivity and customer satisfaction. If the right logistical decision is made, it is more likely that no levers have to be activated later in the day. If no levers were required during the day, the forecast at 10:00AM was accurate enough and maximum capacity can be utilized. This also maximizes the productivity and customer satisfaction which is the goal of an highly accurate forecast.

Figure 3.6 shows the average performance of the current SODA system during a period of 5 months. It can be seen that the average deviation of the model at 10:00AM is currently 9.22%. To have a clear feeling of number of sales, this is about on average 17.500 items/day during 'normal' season and about on average 30.000 items/day during peak season.

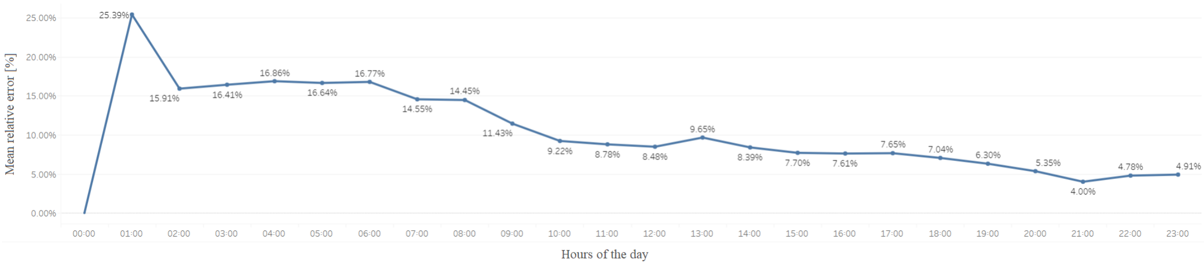


Figure 3.6: Average Deviation SODA prediction

The high deviation is caused by the limited amount of information used by the current forecast model, since it only uses actual sales in 30 minutes time intervals of the current day to find a proper reference day. At the start of the day, the progress is only a few hours so it is difficult to choose the right reference day only based on actual sales until point t in time.

Chapter 4

Data Collection and Exploration

This chapter explains the data exploration and data preparation process. For successful execution of the research project, it is important that there is sufficient quantity and quality of data available. The relevant data should be extracted, filtered from bol.com's data infrastructure and combined together for the successful analysis. Section 4.1 describes the information system structure at bol.com. Secondly, Section 4.2 outlines how the data is filtered and aggregated. Lastly, Section 4.3 gives an explanation of the actual sales data and other available internal and external variables.

4.1 Information System

Bol.com is supported by several information systems from which the three most important information systems are Shop Orders Service (SOR), Fulfillment Network (FNK) and Logistics Order Realisation (LOR). Each information system has its own core function. A schematic overview of the main information systems and their interaction is presented in Figure 4.1.

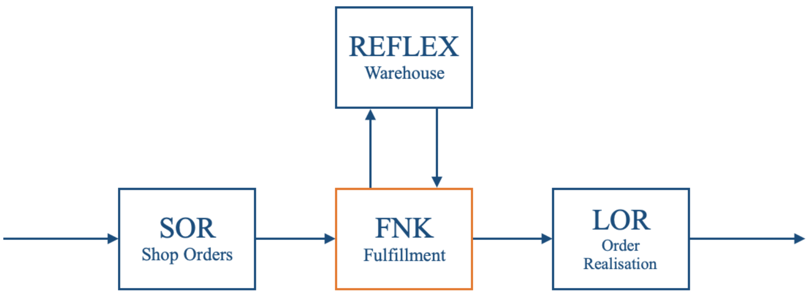


Figure 4.1: Information Systems at bol.com

SOR gives the raw data of the order of a customer in the webshop to FNK. After they received the order, FNK handles the order and decides to which warehouse the order should be allocated. To do this, FNK needs to check where the product is stored and assigns the order to the right warehouse for fulfillment. For logistics, FNK is the most important information system since they should communicate the order. If they miss this step, the order will never be completed. After the warehouse (REFLEX) handed over the package to the distribution partner, FNK gets informed again. They will give final confirmation and inform LOR about the completed order. LOR makes sure that the stock level goes down, revenue goes up and the customer gets informed about their completed order. This research will focus on the FNK information system to make sure that the forecasts are based on sales that are assigned to a specific warehouse, in this case BFC.

4.2 Data Exclusions and Aggregation

Since bol.com has grown and changed significantly over the past few years, only data collected in the year of 2019 at BFC is used. The reason for restricting the data to one year is to minimize the effects on the data set due to changes within the companies structure, while allowing to have all possible seasonal effects included in the data set.

The sales data shows every order in the year 2019 including the assigned warehouse and the product (sub(sub))group it belongs to. Since the research is focused on BFC, orders that belong to VW are excluded from the sales data. In addition, the research will include sales on product group level, so sales are divided over 57 product groups. This results in a total data set with 29,371,194 data points. In order to identify the orders that are relevant for short-term forecasting the rest of the day, only 24 hour sales should be included in the data. Therefore, orders that have the same order date and handover-to-distribution date are included in the data. This step also filters out the orders of products that still has to be released in the future, since their handover-to-distribution date is empty. After exclusion, 21,793,015 data points remain.

To decrease the number of data points in the sales data, sales are aggregated to 10 minutes time slots for each product group. The reason for aggregation on 10 minutes time slots is the strong desire to forecast on a really detailed time scale since it has a direct and specific impact on decision making during the day. The aggregation leads to a big reduction in data points and 2,995,977 remain for further data description. The total transition of number of data points after the exclusions and aggregation are shown in Table 4.1. At a later stage, Chapter 6 outlines the aggregation of product groups into clusters based on their yearly pattern to prepare for forecasting. This section bases further data descriptions on the previous explained data set.

Table 4.1: Data Points after Exclusions and Aggregation

	2019-BFC	**..-24h**	**..-10min**
Data points	29,371,194	21,793,015	2,995,977

4.3 Data Description

This section described and explores the gathered data sets for the analysis. First, the actual sales data set is described in Section 4.3.1. Next, available exogenous variables, internally and externally retrieved, are discussed in Section 4.3.2.

4.3.1 Actual Sales Data

To enable further data exploration, the actual sales data are plotted for the full time range. All product groups cover the full time range and are plotted in Figure 4.2. The sales are normalized, aggregated on a weekly level and separated into multiple graphs to maintain the readability of the graphs. It can be clearly seen that most of the product groups (PG) follow a trend during the year and have a clear peak period during the last months of the year. Chapter 6 deep dives into the similar behaviour of product groups to aggregate them further into clusters.

Figure 4.2: Actual Sales per Product Group

For further data exploration, the mean, standard deviation, minimum and maximum for all product groups per day are shown in Table 4.2. For this analysis, days on which the warehouse is closed are excluded, since no 24 hour sales are possible. The reason for this is to discover the behaviour of each product group during normal days. All values are normalized to 100 because of the sensitivity of the actual data. It can been seen that 'Beauty PG' achieved the highest sales value in 2019, followed by 'Hobby Spellen en Buitenspeelgoed PG' that has the highest mean and standard deviation. Ten product groups faced zero sales on at least one day in 2019.

Table 4.2: Summary Statistics of Product Groups

	Mean	St.Dev.	Min	Max
Algemene Internationale Boeken PG	0.022	0.020	0.000	0.163
Algemene Nederlandstalige Boeken PG	0.513	1.276	0.000	7.603
Auto	0.290	0.165	0.008	1.218
Baby Hardwaren PG	4.021	1.726	0.217	9.374
Baby Verzorging PG	4.502	3.532	0.097	21.416
Baby en Kindermode	1.119	0.918	0.015	8.456
Beauty PG	9.681	9.466	0.309	100.000
Beeld en Geluid Accessoires	3.252	1.480	0.174	8.403
Been en Ondermode	0.627	1.192	0.003	17.996
Camera	0.461	0.261	0.016	2.505
Dames en Herenmode	0.141	0.103	0.002	0.606
Desktop Monitor en Beamer	0.324	0.203	0.025	2.707
Dierbenodigdheden en Ruitersport PG	0.434	0.271	0.010	1.123
Drank PG	0.254	0.252	0.007	2.025
Educatief Nederlandstalig	0.069	0.203	0.000	1.371
Ereaders en Accessoires	0.445	0.249	0.016	1.545
Erotiek PG	2.892	1.540	0.131	9.266
Fiets	0.284	0.188	0.000	0.855
Film PG	0.034	0.023	0.000	0.151
Games Accessories	0.352	0.222	0.021	1.716
Games Consoles	0.021	0.020	0.000	0.150
Games Software Physical	0.052	0.076	0.000	0.393
Gereedschap en Verf PG	0.407	0.258	0.013	1.242
Gezondheid PG	1.705	0.820	0.071	5.003
Groot Huishoudelijk PG	0.047	0.024	0.000	0.107
Heat en Air	0.214	0.403	0.003	3.039
Hobby Spellen en Buitenspeelgoed PG	13.679	10.107	0.449	96.322
Home Entertainment	1.454	1.424	0.082	21.623
Household Appliances	0.721	0.323	0.016	2.886

	Mean	St.Dev.	Min	Max
Huishouden PG	2.598	1.235	0.110	5.775
Kamperen en Outdoor Hardwaren	0.538	0.387	0.013	2.556
Kern Speelgoed PG	10.642	9.341	0.319	78.733
Kitchen Machines	0.889	0.490	0.044	2.920
Koken en Tafelen PG	4.990	3.303	0.217	22.245
Laptop Computers	0.454	0.433	0.025	5.762
Meubelen PG	0.167	0.095	0.002	0.470
Motor	0.019	0.015	0.000	0.076
Opslag en Netwerk	2.599	1.197	0.163	9.562
PC Accessoires	3.527	1.836	0.138	10.318
Personal Audio	3.015	2.321	0.127	34.776
Personal Care	1.988	1.003	0.094	5.986
Persoonlijke Verzorging en Huishoudmiddelen PG	8.999	6.577	0.302	77.907
Printen en Inkt	1.495	0.666	0.056	3.254
Sanitair en Veilig Wonen PG	1.499	0.937	0.046	5.622
Schoenen PG	0.325	0.242	0.003	2.026
School en Kantoor PG	3.622	2.210	0.122	10.181
Sieraden en Horloges PG	1.102	0.628	0.044	5.460
Sport Hardwaren PG	0.847	0.546	0.012	2.235
Sport en Outdoor Kleding en Schoenen PG	0.494	0.580	0.000	4.132
Tassen Reisbagage en Modeaccessoires PG	1.027	0.772	0.020	4.197
Telefonie en Tablets	1.684	1.179	0.095	16.627
Telefoon en Tablet Accessoires PG	4.423	2.082	0.225	12.543
Textiel	1.171	0.553	0.039	2.819
Tuin en Kerst PG	0.523	0.318	0.008	1.635
Verlichting PG	2.025	1.397	0.076	12.747
Wearables	0.891	0.636	0.062	9.470
Woondecoratie	1.246	0.716	0.043	4.150

Figure 4.3 represents the total number of actual sales per product group. It can be seen that 'Hobby Spellen en Buitenspeelgoed PG' represents the biggest product group of all, followed by 'Kern Speelgoed PG', 'Beauty PG' and 'Persoonlijke Verzorging en Huishoudmiddelen PG'.

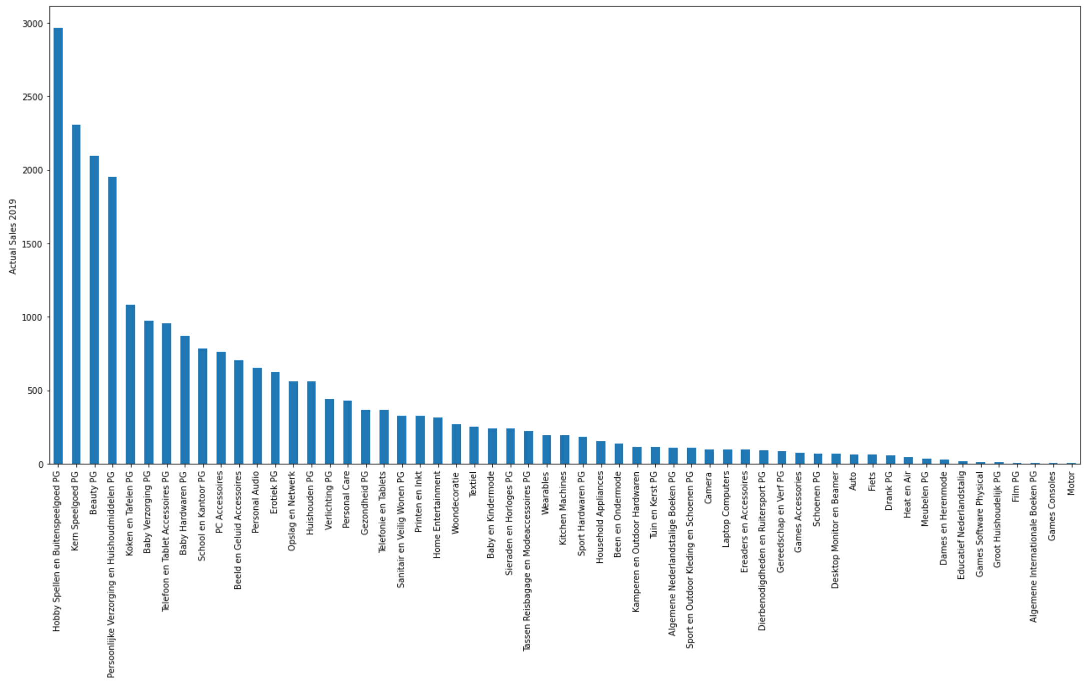


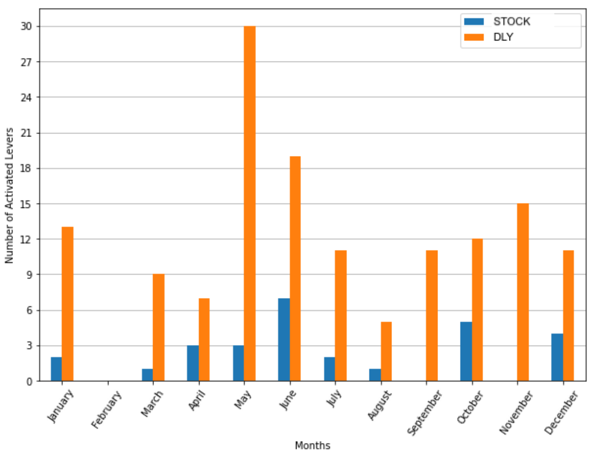
Figure 4.3: Total Actual Sales per Product Group

4.3.2 Exogenous Variables

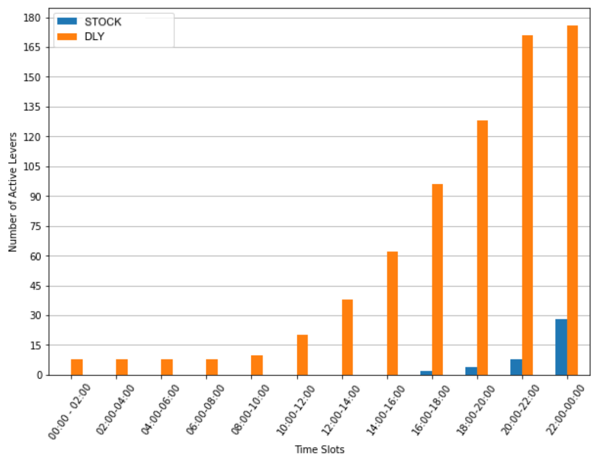
The data on exogenous variables can be split into variables that are internally retrieved and variables that are externally retrieved and are explained in more detail below.

The exogenous variable related to activated 'levers' is internally retrieved. In this data set, activated levers are registered including the type of lever and the time slot in which it was active. Figure 4.4a shows the frequency of activated levers per month in the year of 2019. It shows the total activated number of levers for the whole assortment since it is not registered for which product groups the lever was activated. Therefore, it is assumed that the activated lever has impact on each product group equally. It can be clearly seen that the months May and June required the most daily steering in sales. Additionally, the graph shows that the 'STOCK'-lever, which steers in a subset of products on sale, is activated much more in comparison to the 'DLY'-lever, which directly closes the complete warehouse and complete assortment.

Figure 4.4b shows how many times a specific lever was active during each time slot in 2019. It is shown that in the beginning of the day, the least levers are active since there is less need to steer in sales. Active levers increase cumulatively as the day progresses as in most cases an activated lever persist for the rest of the day. An activation of a lever denotes a wrong prediction of workload at the beginning of the day at 10:00AM. Again, it is clearly shown that the 'STOCK'-lever is much more active in comparison to the 'DLY'-lever. The 'DLY'-lever is only activated after 16:00h and was active on 17 days during 2019 at the end of the day. The 'STOCK'-lever was active on 87 days during 2019 at the end of the day, which represents almost 25% of all days. This means that in 25% of all days the prediction of workload at the beginning of the day was not accurate enough and lever activation was required.



(a) Activated Levers Per Month



(b) Active Levers Per Time Slot

Figure 4.4: Lever Information 2019

Another exogenous variable that will be included in this research is the effect caused by promotions, holidays and weekends. Literature proves that including these variables could significant improve the forecast (Kim et al., 2019; Ryu et al., 2017; Mohan et al., 2018). However, the number of variables should be considered properly to prevent the model from overfitting which causes memorizing rather than learning (Ryu et al., 2017).

Figure 4.5 shows the calendar of 2019 with promotion, holiday and weekend indications during the year. The yellow blocks indicate holidays on which the warehouse is completely closed. On these days, no 24 hour sales are possible since there is no possibility to fulfill the orders. Orders that are placed on these days, are automatically passed on to the fulfillment of the next day. The green blocks indicate holidays on which the warehouse operates normally. However, since these days are special, the workload is expected to be much higher on these days or the days in front. For example, Black Friday on the 29th of November, breaks a record of sold items every year on the day itself. In case of 'Sinterklaas' and Christmas, the workload especially gets increased on the days, or even weeks, in front. These also cause the high workload during the November and December months. The 'Bulk-weeks' at bol.com, which represent the big promotion weeks during the year, are indicated in blue. During these weeks, bol.com operates normally, but the workload is expected to be much higher due to the promotions.

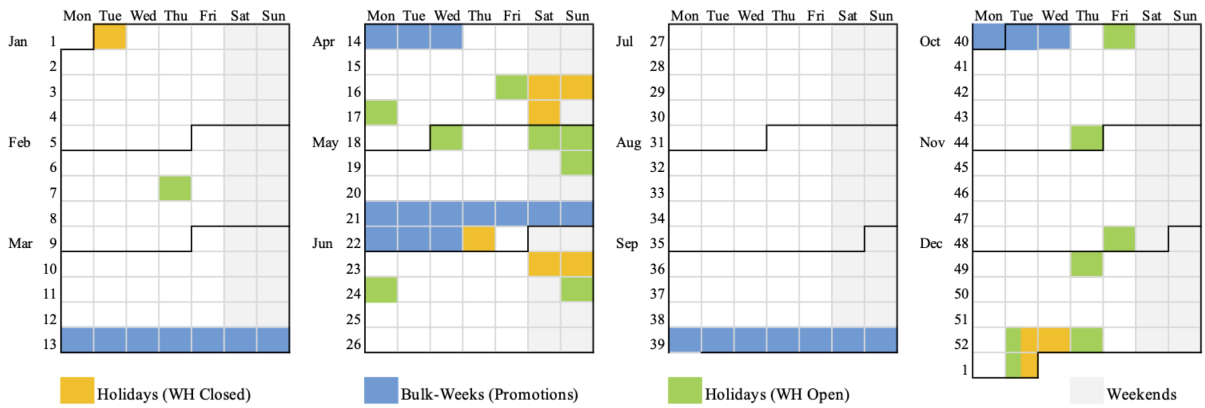


Figure 4.5: Calendar Effects 2019

Lastly, the light grey blocks indicate the weekends. To investigate the impact of weekends and investigate the behavior of 24 hour sales during weekdays, Figure 4.6 presents the average sales on each weekday. It can be seen that Saturday behaves very different from other weekdays.

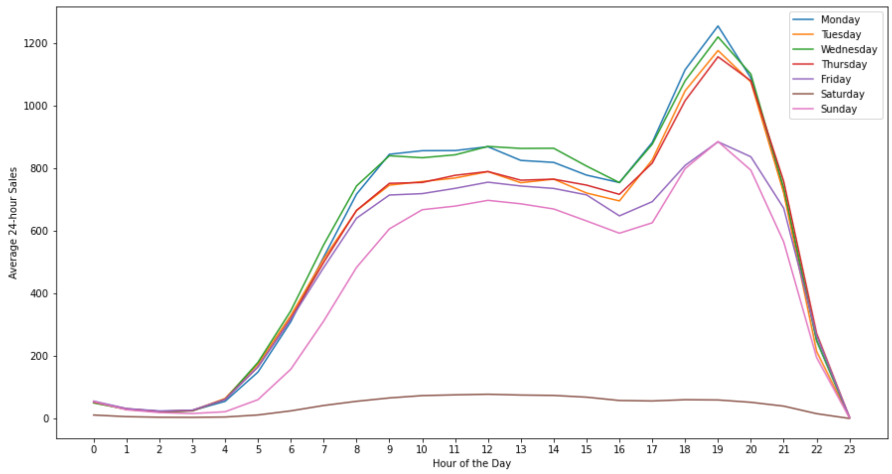
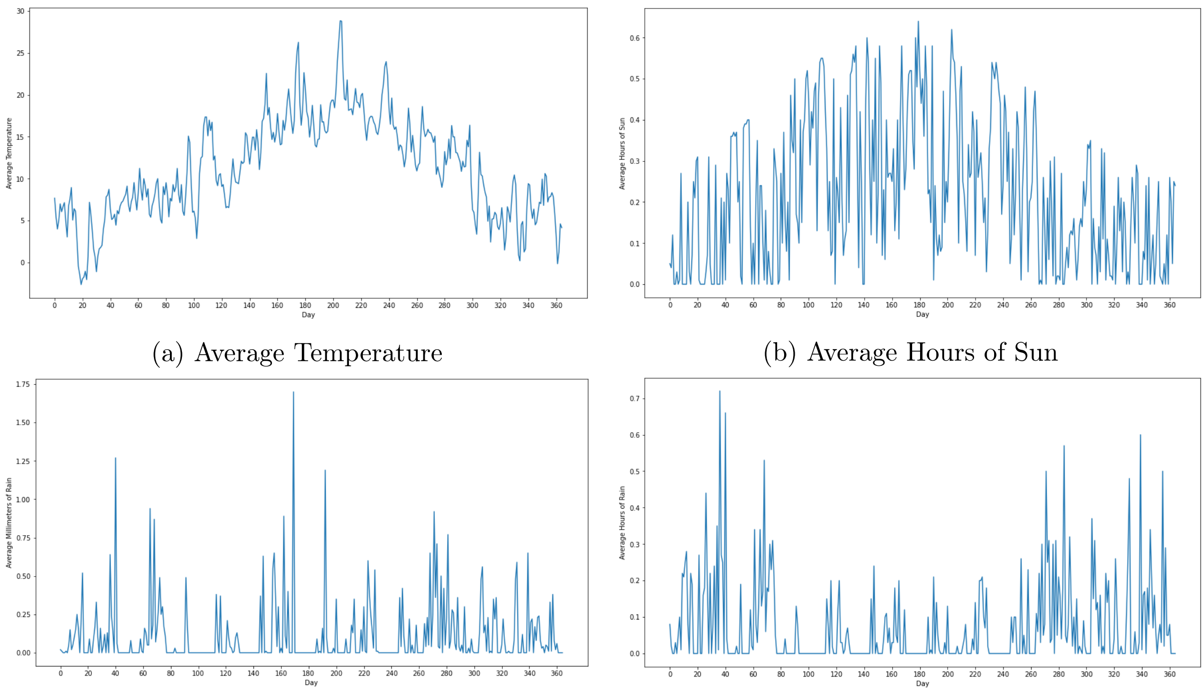


Figure 4.6: Average Sales per Weekday

This is caused by most of the distribution partners that do not operate on Sundays, so orders that are placed on Saturday are in most cases delivered on Monday. Therefore, this cannot be labeled as a 24 hour order. However, there are some exceptions since there are special services available at bol.com, such as same day delivery or Sunday delivery, in return for extra payment. It should be noted that only a few people make use of this service, which results in approximately one tenth of the workload on Saturday in comparison to other weekdays.

The last exogenous variable that will be included in the research is a variable that is externally retrieved: weather information. Steinker et al. (2017) proves in their research that including weather information in forecasting significantly improves forecast accuracy. They found that sales are generally lower on days with better weather, particularly during weekends. Figure 4.7 visualizes four weather information graphs during the year of 2019: average temperature, hours of sun, millimeters of rain and hours of rain. It is clearly shown that the average temperature and average hours of sun is higher during summer days. In terms of rain, the average millimeters of rain graph shows relatively similar peaks during the year. However, the average hours of rain are higher during autumn, winter and early spring days.



(a) Average Temperature

(b) Average Hours of Sun

(c) Average Millimeters of Rain

(d) Average Hours of Rain

Figure 4.7: Weather Information 2019

Chapter 5

Model Selection

This chapter describes the selection procedure of the models that are implemented in this research for clustering as well as for forecasting. First, the selection criteria are explained for both phases followed by which models found in the literature meet these selection criteria. Lastly, an overview is provided of the different phases of implementation.

5.1 Selection Criteria

The literature study on clustering and short-term forecasting conducted in Chapter 2 is used as a basis for model selection. To enable a proper comparison between clustering and forecasting models, different models will be compared so a comprehensive conclusion can be made. The following selection criteria are formulated for model selection:

1. The model is best performing in at least one article that is included in the conducted literature study;
2. The model is applicable to the available data;
3. The model is able to deal with a small amount of data;
4. The model is able to deal with seasonal behavior of a time series;
5. The model is able to deal with different exogenous variables.

5.2 Selection of Model

5.2.1 Clustering

Literature study identifies different clustering models to address a time series clustering problem. Since clustering is an unsupervised learning and performance is hard to measure, criterion 1 is excluded from the model selection. Additionally, criterion 5 is excluded since the clustering approach should identify structure in the internally retrieved data set of actual sales. Hierarchical and partitioning clustering are both clustering methods that are applicable to the available data and are able to deal with a small amount of data, so criteria 2 and 3 are met. Criterion 4 is met upfront since clustering is focused on identifying similar patterns and characteristics.

Partitioning clustering can be split into k-Means and k-Medoids clustering, from which k-Medoids clustering will be applied because of the lower sensitivity to noise and outliers. In terms of distance metric, euclidean distance and dynamic time warping will be applied and will be compared in both hierarchical and k-Medoids clustering. To measure quality, the silhouette score will be used. However, it should be noted that clustering is an unsupervised learning and performance is hard to measure. Therefore, human evaluation is applied additionally to the silhouette score to form a final conclusion.

5.2.2 Forecasting

In the literature study, exponential smoothing (ES) and auto regressive integrated moving average (ARIMA) are both found to be best performing statistical methods. Both models has proven to be best performing at least once in the conducted literature, so criterion 1 is met. Especially ARIMA has proven its success in several articles (Arunraj & Ahrens, 2015; Karthika et al., 2017; Lydia et al., 2016) and is also able to deal with the available, small amount of data. Therefore, ARIMA meets criteria 2 and 3. While ES has not proven to be the best performing method with a small amount of data, research has proven that the method is able to deal with a small amount of data (Ryu et al., 2017; Kim et al., 2019) and meets criteria 2 and 3. To meet criterion 4, the Holt Winters (HW) model of the ES model can capture seasonal behaviour of time series and ARIMA can be extended to SARIMA to capture seasonal behaviour. To deal with different exogenous variables and meet criterion 5, the ES model can be extended to multivariate methods by regression-based correction after univariate forecasting. ARIMA can be extended to ARIMAX or SARIMAX to include exogenous variables. Since ES and ARIMA satisfy all criteria, both statistical models will be applied in further research.

Additionally, literature study proves that machine learning models tend to perform very well in short-term forecasting. Therefore, neural networks (NN), support vector regression (SVR) and fuzzy time series (FTS) are assessed on the established criteria. NN and SVR models are both found to be best performing methods in recent literature and both meet criteria 1 (Karthika et al., 2017; Kim et al., 2019; Lusis et al., 2017; Mohan et al., 2018; Ryu et al., 2017; F. Wang et al., 2019). In contrast, FTS never outperforms other models so will be excluded from further research. Criterion 2 can be met by NN as well as SVR models, since these models do not require a very specific data type. Additionally, literature proves in both NN as SVR to be able to deal with a small amount of data, so criterion 3 is met for both models as well. NN models have the ability to deal with complex and nonlinear data and is able to deal with seasonal behaviour of a time series and exogenous variables. Research of (Karthika et al., 2017) proves that a SVR models are able to cope with seasonality and external variables. Therefore, criteria 4 and 5 are met for both NN and SVR models, so both will be applied in further research. Since there are several types of NN, the type of NN should be selected as well. The types of NN that are found in literature are Nonlinear Autoregressive Network with Exogenous Inputs (NARX) and Deep Neural Network (DNN) - Multilayer Perceptron (MLP). Since the MLP model is most widely used in investigated literature, applicable to the data and time horizon of the problem at hand and relatively easy to implement, MLP will be applied in further research.

5.3 Overview

To conclude, three models remain for further research: ES, ARIMA, SVR and MLP. The models will be applied on actual sales data and make use of additional internal and external variables as described in chapter 4. Prior to forecasting, clustering can be used as additional data preparation technique. Effective clustering and then forecasting has the potential to outperform aggregate forecasting, which forecasts the entire data set at once. Hierarchical and k-Medoids clustering will be used to identify similar product groups. It will be evaluated if cluster or aggregate selection performs best in forecasting. In addition, multiple feature cases will be assessed on performance and compared with each other. Case 1 represents an univariate model and uses only the actual sales data. Case 2 contains actual sales data, lever data and calendar input. Weather information is added to case 3, thus including actual sales data, lever data, calendar input and weather information. Models are divided into train and test sets using a nested time series cross-validation method, which creates many train/test splits and averages the errors over all the splits. This technique is called forward-chaining, also referred to in the literature as rolling-origin evaluation (Tashman, 2000). Using this method, each period is considered as the test set and assigns all previous periods into the training set.

The required and recommended data transformations depend on the forecast model as described in Chapter 2, Section 2.4. Required data transformations are applied to the models indicated in Figure 5.1, e.g. normalization is applied to neural networks and differencing is used for ARIMA models. Statistical, SVR and NN models are optimized by use of grid search. For multi-step ahead forecasting, the multiple-output strategy (MIMO) tend to outperform other techniques so will be applied in further research. As performance evaluation, firstly the RMSE will be used since these are confirmed to be most widely used in literature for short-term forecasting. Additionally, the MASE will replace the MAPE since the actual sales data contains many zero's for which the MAPE is not applicable. An overview of the models, connections and techniques that will be applied in further research is shown in Figure 5.1.

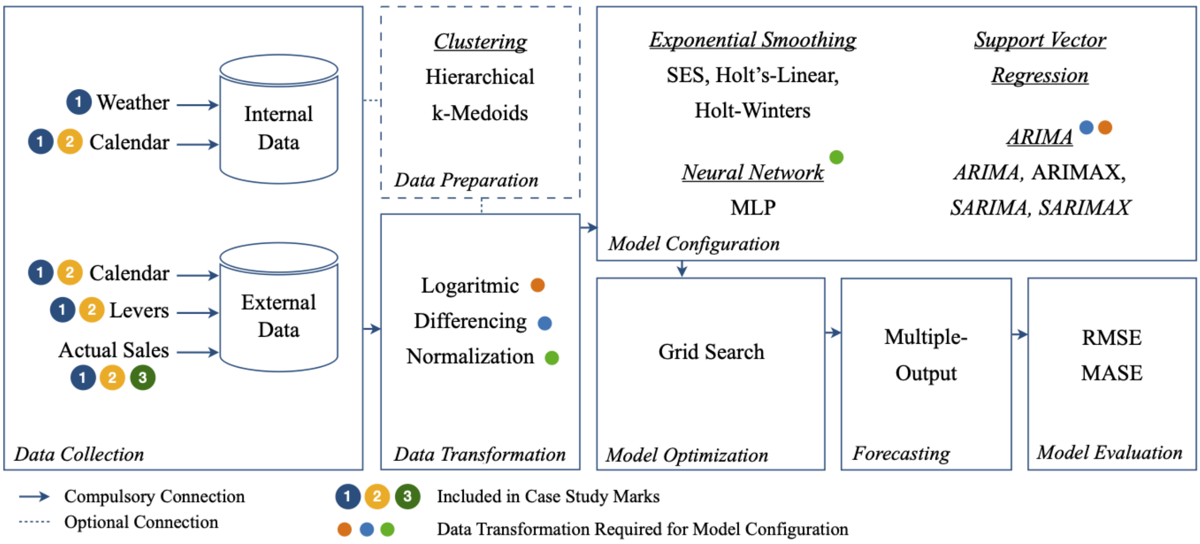


Figure 5.1: Overview of Models Included in Research

Chapter 6

Cluster Evaluation

To deal with a multi-product environment, time series clustering can be very useful prior to forecasting. Firstly, homogeneous clusters can be identified within the data set. Next, the best performing forecasting methods in terms of accuracy should be selected for each cluster. Such effective clustering and then forecasting has the potential to outperform aggregate selection, which selects a single forecasting method for the entire data set (Vangumalli et al., 2019; Dantas & Oliveira, 2018). This chapter outlines the selected distance metrics and shape-based clustering methods used for product group clustering. Selected time series clustering methods are further explained and implemented. All models are modelled in Python 3.8 to obtain the final clustering results. Next, the clusters will be discussed and validated and a final conclusion about the clusters for further modelling will be formed.

6.1 Distance Metrics

To determine whether time series are similar, one must first decide on a distance measure to quantify this similarity. Literature review compared Euclidean Distance and Dynamic Time Warping (DTW) and identifies both as suitable distance measures for time series. This section describes both distance measures followed by the implementation and results.

6.1.1 Distance Descriptions

Euclidean distance is the most popular straight-line distance between two points that are on the same point in time. Therefore, time series needs to be equal in length. The distance formula for the Euclidean Distance between point x and y is shown in equation 6.1.

$$d_{euc}(x, y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2} \tag{6.1}$$

DTW can measure similarity between two temporal sequences that do not align exactly in time, speed or length (Aghabozorgi et al., 2015). There is no need to match indices one-to-one on the same time value, but indices can be matched with one or more other indices from the other time series. This rule applies for all indices, except of the first and last indices. DTW is highly recommended for distance measures in time series.

To determine the DTW distance between two time series, first a $(n \times m)$ local cost matrix (LCM) should be calculated, where element (i, j) refers to the distance between x_i and y_j. This distance is defined as the quadratic difference $d(x_i, y_j) = (x_i - y_i)^2$. Next, a warping path $W = w_1, w_2, ..., w_K$ is determined, where $max(n, m) <= K <= m + n - 1$. The warping path crosses the LCM under three constraints:

- **Boundary:** The path must start and end in the diagonal corners of the LCM, such as: $w_1 = (1, 1)$ and $w_K = (n, m)$.
- **Continuity:** Only adjacent elements in the matrix are allowed for steps in the path. Diagonal adjacent elements are allowed as well.
- **Monotony:** Subsequent steps in the path must be monotonic spaced in time.

The total distance for path W is obtained by the sum of the individual distances of the LCM that the path crosses. To obtain the DTW distance, the path with the minimum total distance is selected. Equation 6.2 shows the recurrence that can be used to find the path with the minimum cumulative distance. The optimal total DTW distance is the sum of distances of the path with the minimum cumulative distance, calculated by equation 6.3.

$$d_{cum}(i, j) = d(x_i, y_j) + min\left\{d_{cum}(i-1, j-1), d_{cum}(i-1, j), .d_{cum}(i, j-1)\right\} \tag{6.2}$$

$$d_{DTW}(x, y) = min\sqrt{\sum_{k=1}^{K} w_k} \tag{6.3}$$

6.1.2 Implementation

Distance matrices are constructed using Python 3.8 and indicate the distances between the 57 different product groups. The actual sales of all product groups range from the 1st of January until the 31st of December. Due to time complexity and the fact that time series values around zero are not very meaningful, sales values are aggregated by day. Therefore, each time series contains 365 data points on which the distance matrices are constructed.

Before moving on to the construction of distance matrices, the data should be preprocessed due to the variety of ranges within the different time series. Depending on the goal of clustering, it may also be desired to detrend of deseasonalize the data in advance. It is decided not to detrend and deseasonalize since it is desired to capture this behavior within the clusters. However, normalization is applied to scale each of the 57 individual time series into a range of $[0, 1]$.

For each distance measure discussed before, the corresponding distance matrix of the time series data containing 57 product groups is calculated. Therefore, the distance matrix requires the calculation of 57(57-1)/2 $=$ 1596 distances.

6.1.3 Results

Final distance matrices of Euclidean Distance and DTW can be found in Appendix A. It can be clearly seen that the DTW matrix give significant smaller distances in comparison to the Euclidean Distance matrix. Euclidean Distance gives an average distance between time series of 59257,98 where DTW gives an average distance between time series of 39538,77. However, both distance metrics will be used in the subsequent clustering process to compare the results and performance of both distance metrics.

6.2 k-Medoids Clustering

This section describes the k-Medoids clustering approach that is used to form clusters of similar product groups. Next, the implementation and results of the model are discussed.

6.2.1 Model Description

K-Medoids clustering is a clustering approach in which the objective is to partition the data into k sets. In the algorithm, observations are selected as medoids (centers) and heuristics are used to obtain k-Medoids partitions. The most popular heuristic for k-Medoids is the Partitioning Around Medoids (PAM) algorithm (Swarndeep Saket & Pandya, 2016). Algorithm 1 shows the pseudo-code of the algorithm. In the algorithm, swaps are considered in an iterative manner and are executed if a swap decreases the total sum of distances between all product groups.

Algorithm 1 Partitioning Around Medoids (PAM) Algorithm

```
Predetermine the value of k
Randomly choose k observations as the initial medoids
while no change in the centroid assignment do
  for each medoid m do
    for each non-medoid observation o do
      if swapping m and o improves the solution then
        Swap m and o
      end if
    end for
  end for
end while
return  the k medoids and the observations assigned to each cluster.
```

6.2.2 Implementation

K-Medoids clustering is executed in Python 3.8 and aims on the assignment of product groups into clearly different clusters. The Euclidean and DTW distance matrices described in the previous section are used as input for modelling. Therefore, clusters are created by the comparison of 57 individual time series that all contain 365 data points. The algorithm is executed for every k between 2 and 10 and returns for every situation the k medoids and the observations assigned to each cluster. Additionally, the silhouette score is calculated for every situation and indicates the performance of the clusters. The silhouette score indicates the compactness and separation of clusters. However, since clustering is an unsupervised learning and performance is hard to measure solely on the silhouette score, expert evaluation is additionally applied to form a final conclusion on the best clusters. The results will be discussed in the next section.

6.2.3 Results

Figure 6.1 shows the average silhouette score for k between 2 and 10 and indicates the performance of the clusters. It can be clearly seen, that the average silhouette score for DTW is decreasing when k gets increased. For Euclidean Distance, the silhouette score remains relatively stable when $k > 3$. The silhouette score indicates for Euclidean Distance a best performance for $k = 2$, with a score around 0.4, and significantly decreases when $k > 2$. The silhouette score indicates for DTW distance a quite stable performance for k between 2 and 4 with a score between 0.4 and 0.5. The best value the silhouette score can achieve is 1, so according to the performance metric, the clusters are relatively well separated. Expert evaluation is applied for k between 2 and 4 for both distance metrics to form a final conclusion on the best clusters.

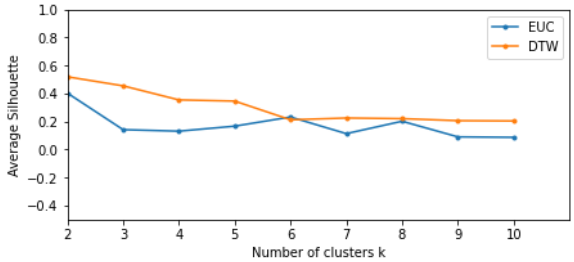


Figure 6.1: Average Silhouette Score k-Medoids

Looking at the behaviour of the time series, expert evaluation can clearly indicate that there are two 'main' behaviours within the data set: constant time series and slightly increasing time series. However, k-Medoids in combination with Euclidean Distance is not able to split the product groups into clusters of which these behaviours are clearly visible. Every number of k and every repeated iteration creates very similar clusters according to expert evaluation, so the highest silhouette score is assumed to form the best clusters. Therefore, Euclidean Distance gives the best result when $k = 2$. Figure 6.2 shows the normalized averaged sales of the product groups that are assigned to each cluster on a weekly x-axis. A small difference is notable in the figures, namely the moment in time that the graph starts increasing. Cluster plots for $k = 2$ with all time series assigned to each cluster can be found in Appendix B.1.

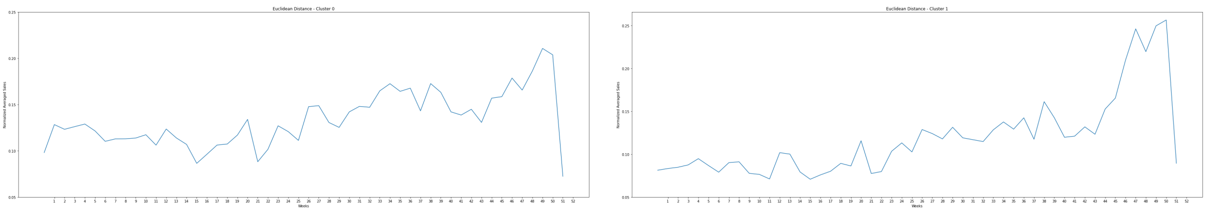


Figure 6.2: k-Medoids Clusters, Euclidean Distance - Normalized Averaged Sales

K-Medoids in combination with DTW Distance is just as Euclidean Distance not able to split the product groups into clusters of which a clear difference is visible. Every number of k and every repeated iteration creates very similar clusters according to expert evaluation, so again, the highest silhouette score is assumed to form the best clusters. Therefore, DTW Distance gives the best result when $k = 2$. Figure 6.3 shows the normalized averaged sales of the product groups that are assigned to each cluster on a weekly x-axis. The figures look very similar to the figures of k-Medoids in combination with Euclidean Distance. Cluster plots for $k = 2$ with all time series assigned to each cluster can be found in Appendix B.1.

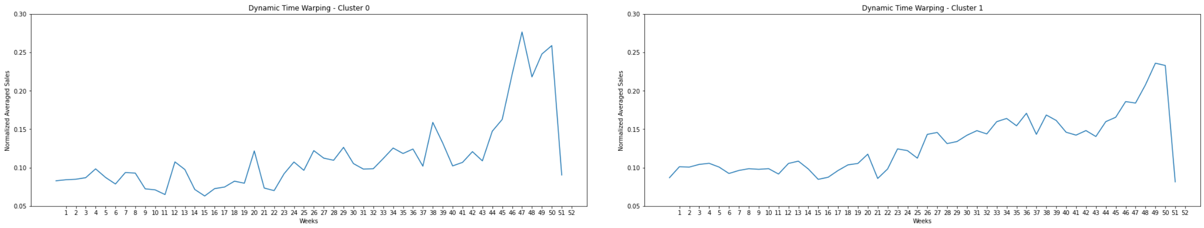


Figure 6.3: k-Medoids Clusters, DTW Distance - Normalized Averaged Sales

6.3 Hierarchical Clustering

This section describes the Hierarchical clustering approach that is used to form clusters of similar product groups. Next, the implementation and results of the model are discussed.

6.3.1 Model Description

Hierarchical clustering is a clustering approach in which the objective is to build a hierarchy of clusters. The algorithm organizes a set of nested clusters into a tree and can be visualizes by a dendrogram. The x-axis of a dedrogram represent the different observations and the y-axis indicates the distance between different clusters. A clustering is obtained by a cut in the dendrogram in which the number of crossed trees indicate the number of clusters. Algorithm 2 shows the pseudo-code of the agglomerative hierarchical clustering algorithm, in which at each step the closest pair of clusters are merged. Therefore, the algorithm starts with 57 clusters with only one product group and iteratively merges clusters until only one single clusters remain.

Algorithm 2 Agglomerative Hierarchical Clustering

```
Initialize all N observations as the initial N clusters
while number of clusters > 1 do
    Merge two closest clusters
end while
return set of nested clusters.
```

6.3.2 Implementation

Hierarchical clustering is executed in Python 3.8 and aims on the assignment of product groups into clearly different clusters. The Euclidean and DTW distance matrices described in the previous section are used as input for modelling. Therefore, clusters are created by the comparison of 57 individual time series that all contain 365 data points. The algorithm returns a dendrogram which can be split into clusters for k between 1 and 57, the number of product groups. The silhouette score is calculated for k between 2 and 10 and indicates the performance of the clusters. The silhouette score indicates the compactness and separation of clusters. However, since clustering is an unsupervised learning, performance is hard to measure solely on the silhouette score. Therefore, expert evaluation is additionally applied to form a final conclusion on the best clusters. The results will be discussed in the next section.

6.3.3 Results

Figure 6.4 shows the average silhouette score for k between 2 and 10 and indicates the performance of the clusters. It can be clearly seen, that the average silhouette score is decreasing when k gets increased. The silhouette scores indicate for both distance metrics a best performance for k between 2 and 4, with a score between 0.3 and 0.5. The best value the silhouette score can achieve is 1, so according to the performance metric, the clusters are relatively well separated. Additionally, the complete cluster separation can be found in Appendix B.2 which shows the dendograms of the algorithm in combination with both distance metrics. Expert evaluation is applied for k between 2 and 4 to form a final conclusion on the best clusters.

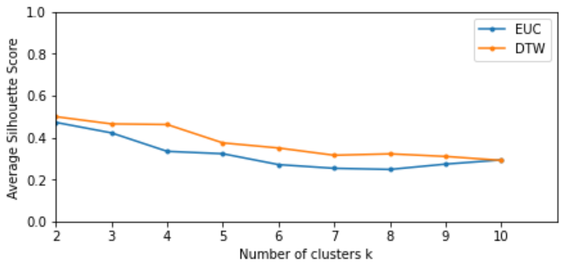


Figure 6.4: Average Silhouette Score Hierarchical

Looking at the behaviour of the time series, expert evaluation can clearly indicate that there are two 'main' behaviours within the data set: constant time series and slightly increasing time series. In combination with Euclidean Distance, the hierarchical clustering approach is not able to split these time series when $k = 2$, but creates two relatively similar clusters. However, when increasing k to $k = 3$, the algorithm is significantly improving its performance and indicates three clearly different clusters. Figure 6.5 shows the normalized averaged sales of the product groups that are assigned to each cluster on a weekly x-axis.

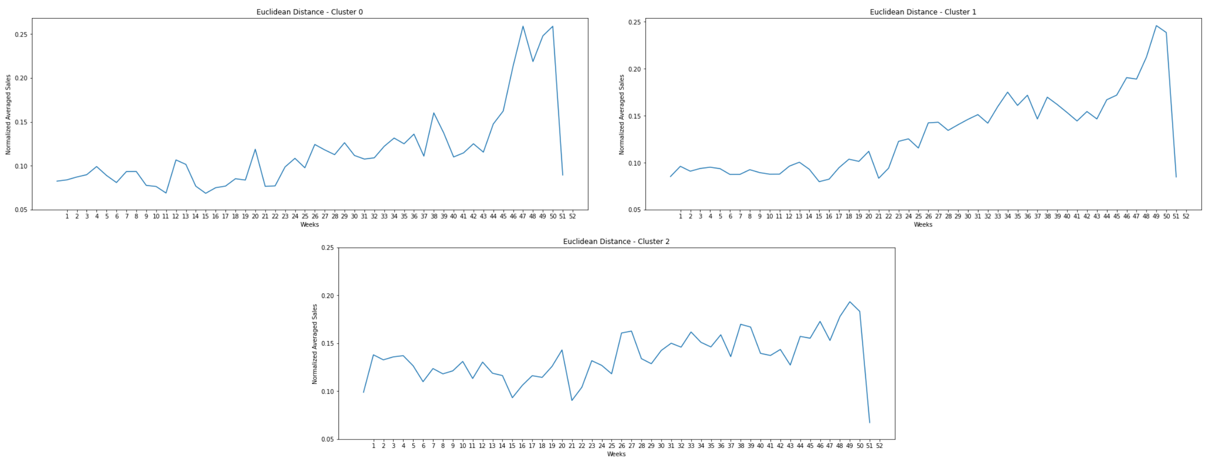


Figure 6.5: Hierarchical Clusters, Euclidean Distance - Normalized Averaged Sales

Despite the fact that cluster 0 and 1 still indicate a quite similar cluster, they are different in terms of the moment in which it starts increasing. Cluster 0 has a relatively constant behaviour until week 44 and then shows a clear peak. In cluster 1, the increase starts much earlier, around week 22, and also faces a clear peak in the end of the year. Cluster 2 shows a relatively constant cluster behaviour. When $k = 4$, cluster 0 is split into two similar clusters, so the performance is not increased. Therefore, Euclidean Distance gives the best results when $k = 3$. Cluster plots for $k = 3$ with all time series assigned to each cluster can be found in Appendix B.3.

With the use of DTW distance, the hierarchical clustering approach is not able split the product groups into clusters of which at least one cluster contains the constant time series product groups. Even if k gets increased to $k = 5$ or $k = 6$, the algorithm is not able to identify clearly different clusters. Although, the best clusters are created by the DTW distance when $k = 2$. Figure 6.6 shows the normalized averaged sales of the product groups that are assigned to each cluster on a weekly x-axis. The graphs are different in terms of the moment in which it starts increasing. Cluster 0 has a relatively constant behaviour until week 44 and then shows a clear peak. In cluster 1, the increase starts much earlier, around week 22, and also faces a clear peak in the end of the year. It can be seen that these clusters are quite similar to cluster 0 and 1 of the Euclidean Distance clusters. However, in contrast to Euclidean Distance, DTW is not able to identify the product groups with a constant time series when k gets increased. Therefore, DTW gives the best results when $k = 2$. Cluster plots for $k = 2$ with all time series assigned to each cluster can be found in Appendix B.3.

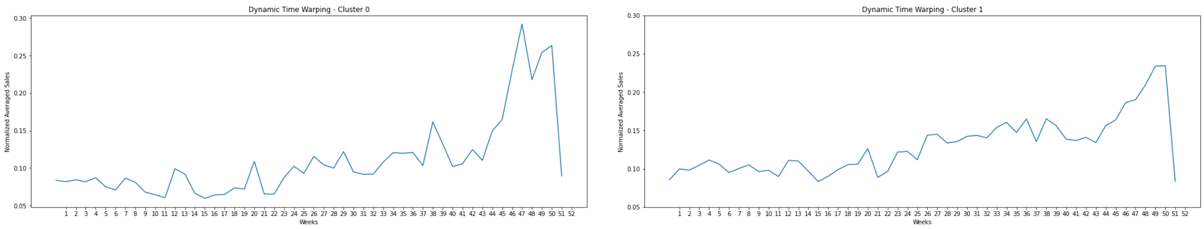


Figure 6.6: Hierarchical Clusters, DTW Distance - Normalized Averaged Sales

6.4 Characteristic-based Clustering

This section describes an additional clustering approach to further investigate similar products groups. A characteristic-based clustering approach is implemented to form clusters of similar product groups. This approach can serve as validation for the clusters formed in the other cluster approaches. This section outlines a model description, implementation and results of the model.

6.4.1 Model Description

Characteristic based clustering is a method for clustering of time series based on their structural characteristics. Unlike the other methods, the method does not cluster point values using a distance metric, rather it clusters based on global features extracted from the time series (X. Wang et al., 2006). Feature measures are obtained from each product group and can be fed into arbitrary clustering algorithms. Global measures describing the time series are obtained by applying statistical operations that best capture the underlying characteristics: trend, seasonality, serial correlation, skewness and kurtosis. Since the method clusters using extracted global measures, it reduces the dimensionality of the time series and is less sensitive to missing or noisy data (X. Wang et al., 2006). To perform a meaning full comparison with the results of the shape based clustering approaches, hierarchical clustering is applied. Algorithm 2 shows the pseudo-code of the agglomerative hierarchical clustering algorithm, in which at each step the closest pair of clusters are merged. Therefore, the algorithm starts with 57 clusters with only one product group and iteratively merges clusters until only one single clusters remain.

6.4.2 Implementation

Hierarchical clustering is executed in Python 3.8 and aims on the assignment of product groups into clearly different clusters. Clusters are created by the comparison of global features of 57 individual time series that all contain 365 data points. The global features that are extracted from the time series are trend, seasonality, serial correlation, skewness and kurtosis. Table 6.1 shows the global features of three example product groups: 'Heat & Air', 'Kern Speelgoed PG' and 'Printen & Inkt'. It clearly shows that 'Heat & Air' is a summer product group, while 'Kern Speelgoed PG' is sold mostly during the last quarter due to Sinterklaas and Christmas and thus faces a clear trend as well. 'Printen & Inkt' represents a relatively constant product group. Global features for all product groups can be found in Appendix B.4. The hierarchical clustering algorithm returns a dendrogram which can be split into clusters for k between 1 and 57, the number of product groups. Just as in other methods, the silhouette score is calculated for k between 2 and 10. However, expert evaluation is additionally applied to form a final conclusion on the best clusters. The results will be discussed in the next section.

Table 6.1: Global Feature Examples:
'Heat & Air', 'Kern Speelgoed PG' and 'Printen & Inkt'

	Season Q1	**Season Q2**	**Season Q3**	**Season Q4**	**Kurtosis**	**Trend**	**Skewness**	**Serial Corr**
Heat & Air	0.2034	0.5456	0.7167	0.3838	0.0122	0.0015	0.044	0.745
Kern Speelgoed PG	0.3322	0.3415	0.3784	0.7936	0.0098	0.1456	0.0266	0.659
Printen & Inkt	0.4829	0.4630	0.5078	0.5427	0.0146	0.0023	-0.0074	0.313

6.4.3 Results

Figure 6.7 shows the average silhouette score for k between 2 and 10 and indicates the performance of the clusters. It can be clearly seen that the average silhouette score keeps relatively stable when k gets increased. The silhouette score indicates a best score between 2 and 7, with a score around 0.35. The best value the silhouette score can achieve is 1, so according to the performance metric, the clusters achieve a medium score. To define the best clusters, expert evaluation is applied for k between 2 and 4. Since it is not preferred to increase k above 4 due to modelling computation times in further forecasting steps, k between 5 and 7 are not considered.

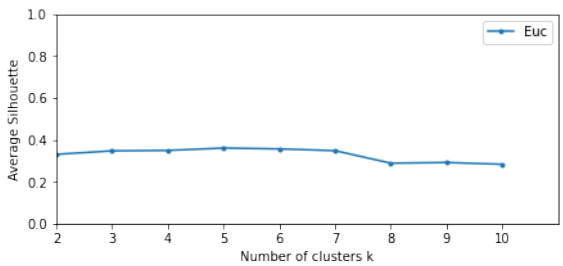


Figure 6.7: Average Silhouette Score Characteristic-based, Hierarchical

Looking at the behaviour of the time series, if $k = 2$, the characteristic-based clustering can identify two 'main' behaviours within the data set: constant time series and slightly increasing time series. When increasing to $k = 3$, the characteristic based approach is able to split time series with a clear summer peak from the others, where shape based was not able to. Therefore, the hierarchical clustering approach is able to split the time series in three clearly different clusters. Figure 6.8 shows the normalized averaged sales of the product groups that are assigned to each cluster on a weekly x-axis.

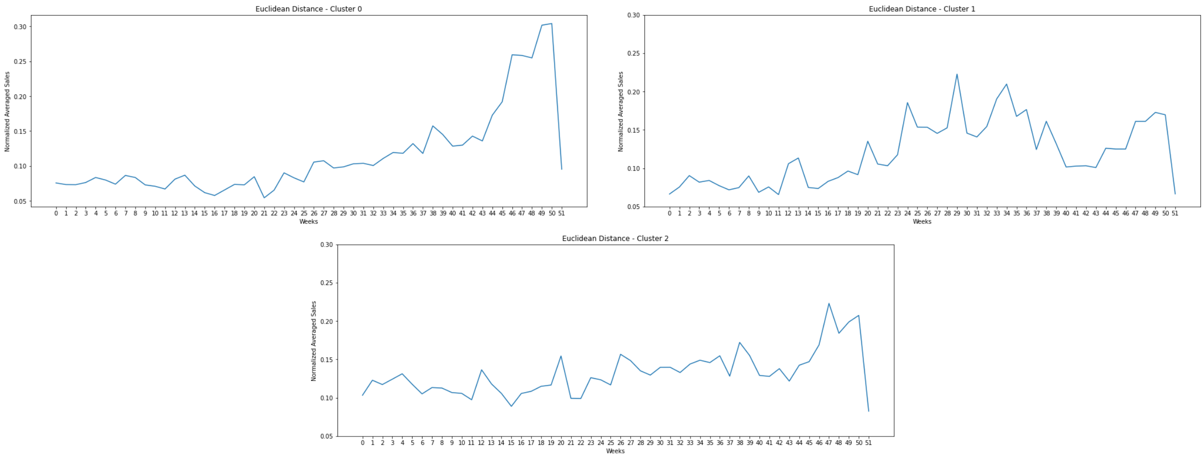


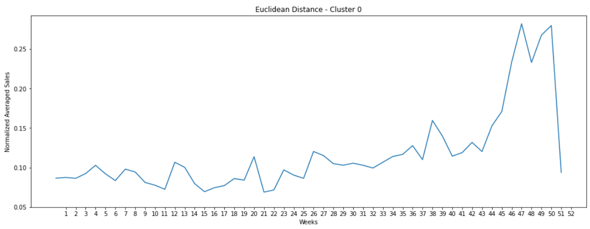
Figure 6.8: Characteristic-based Clusters
Euclidean Distance - Normalized Averaged Sales

It can be seen that the characteristic based approach identifies three clearly different clusters. Cluster 0 has a relatively constant behaviour until week 44 and then shows a clear peak. In cluster 1, summer peak product groups are identified, with a small peak in the last weeks of the year. Cluster 2 shows a relatively constant cluster behaviour but also starts increasing quite early and shows a small peak around the last weeks of the year. When $k = 4$, cluster 0 is split into two similar clusters, so the performance is not increased. Therefore, characteristic based clustering gives the best results when $k = 3$. Cluster plots for $k = 3$ with all time series assigned to each cluster can be found in Appendix B.5.

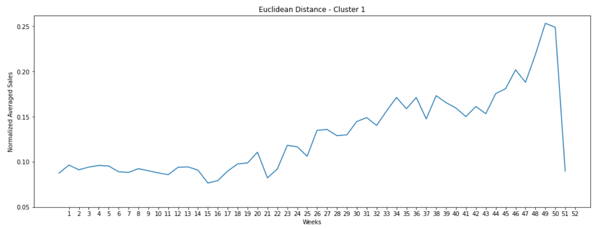
6.5 Conclusion

Within the clustering approaches, four main behaviours are identified: a late increasing with peak cluster, an early increasing with peak cluster, a constant cluster and a summer peak cluster. The only combination that was able to clearly split the constant time series product groups form the other time series, was the shape-based hierarchical clustering approach in combination with Euclidean Distance. For $k = 3$ the clusters show a late increasing peak, an early increasing peak and a constant cluster, which was shown in Figure 6.5. Therefore, the algorithm was able to identify three out of four main behaviours. The only combination that was able to identify the summer peak product groups, was the characteristic based hierarchical clustering approach in combination with Euclidean Distance. It identified for $k = 3$ a late increasing peak, an early increasing peak and a summer peak, shown in Figure 6.8. Therefore, the characteristic based approach was able to identify three out of four main behaviours as well. Clusters for $k = 2$ gave similar results, no matter which approach was used and increasing k did not improve as well.

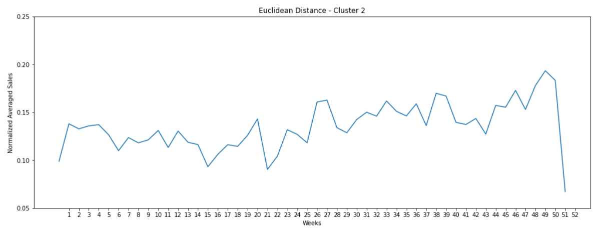
It can be concluded that the hierarchical clustering approach in combination with Euclidean Distance worked best for the identification of several well separated clusters. Both shape based and characteristic based approaches are able to identify the time series with an early increase with peak and the time series with an late increase with peak. However, where the shape based approach is able to extract the constant time series from the others, the characteristic based approach is able to extract the summer peak time series from the others. Since experts confirm the importance of both behaviours, the two approaches are combined to form a final conclusion about the clusters. Thus, the summer peak time series are extracted from the characteristic-based clustering and the constant time series are extracted from the shape-based clustering. The other products remain in the early increase with peak and late increase with peak clusters. This approach extracts the best of both methods and clusters are confirmed by expert evaluation. Additionally, the clusters are validated by the silhouette score, which outputs a score of 0.182. Despite the fact that the combination of approaches achieved a lower score than approaches on their own, experts confirmed the final clusters. Figure 6.9 shows the final normalized averaged sales of the product groups that are assigned to each cluster on a weekly x-axis. Cluster plots for $k = 4$ with all time series assigned to each cluster can be found in Appendix B.6.



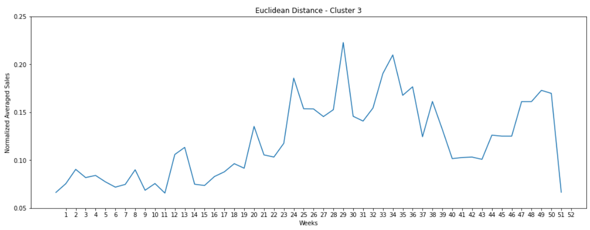
(a) Late increasing peak



(b) Early increasing peak



(c) Constant



(d) Summer Peak

Figure 6.9: Final Clusters, Euclidean Distance - Normalized Averaged Sales

Chapter 7

Forecast Evaluation

This chapter outlines the selected forecasting models applied to each cluster constructed in Chapter 6 and to the whole aggregate sales data set. Selected forecasting models, ES, ARIMA, SVR and NN, are further explained, implemented and optimized in Python 3.8. Next, the forecasting results are discussed and a final conclusion for the best performing forecasting method in term of accuracy is selected for each cluster. Finally, a comparison against aggregate selection, which selects a single forecasting method for the entire sales data set is conducted to compare if cluster forecasting improves the overall forecast accuracy.

7.1 Exponential Smoothing

The aim of exponential smoothing (ES) is to forecast based on historic values with weights. The simple ES model can be extended to ES models that can handle systematic trend and seasonality. This section describes the multiple model variations and identifies the optimal smoothing parameters for each data set. Finally, the results are presented and discussed for each cluster and the aggregate data set.

7.1.1 Model Descriptions

Simple Exponential Smoothing

Simple exponential smoothing (SES) is a time series forecasting method for univariate data that assigns exponentially decreasing weights over time to past observations. SES does not contain any trend or seasonal smoothing parameters and only requires a single smoothing parameter, alpha (α). The forecast and smoothing equation of SES are subsequently defined as,

$$\hat{y}_{t+h|t} = l_t \tag{7.1a}$$

$$l_t = \alpha y_t + (1-\alpha) l_{t-1} \tag{7.1b}$$

The smoothing parameter α can take a value between 0 and 1. If α is close to zero, more weight is assigned to observations further into the past. If α is close to one, more weight is assigned to the closer observations. The level parameter that presents the weighted average of previous observations at time t is denoted by l_t.

Holt

Simple exponential smoothing can be extended to allow the forecasting of data with systematic trend. This method involves two smoothing parameters and two smoothing equations. The first smoothing equation (l_t) is used for the level and the second smoothing equation (b_t) is used for the trend. The forecast and smoothing equations of Holt are defined as,

$$\hat{y}_{t+h|t} = l_t + hb_t \tag{7.2a}$$

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1}) \tag{7.2b}$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \tag{7.2c}$$

The level parameter at time t is denoted by l_t and b_t denotes an estimate of the trend of the series at time t. The smoothing parameter for the trend, β, can take a value between 0 and 1.

Holt Winters

The last extension is included in the Holt Winters model that is able to deal with trend and seasonality and includes three smoothing parameters. The seasonal component can be added through a additive or multiplicative method. The additive method is preferred when seasonal variations are constant over time and the multiplicative method is preferred when the seasonal variations are changing exponentially over time. The additive method is most widely used and is able to deal with zero-values in the data. Therefore, the additive method is applied in further research. The forecast and smoothing equations of the additive Holt Winters are defined as,

$$\hat{y}_{t+h|t} = l_t + hb_t + s_{t+h-m(k+1)} \tag{7.3a}$$

$$l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1}) \tag{7.3b}$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \tag{7.3c}$$

$$s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m} \tag{7.3d}$$

The frequency of the seasonality is denoted by m. In this case, $m = 24$ for daily seasonality on a hourly data set. The level equation (l_t) shows the weighted average between the seasonal adjusted observation ($y_t - s_{t-m}$) and the non-seasonal forecast ($l_{t-1} + b_{t-1}$) at time t. The trend equation (b_t) is unchanged in comparison to the Holt method. The seasonal equation (s_t) presents a weighted average between the current seasonal index ($y_t - l_{t-1} - b_{t-1}$) and the seasonal index of m time periods ago.

7.1.2 Configuration

Exponential smoothing models are constructed using the statsmodels.tsa.holtwinters package in Python. The univariate ES models are automatically optimized by the model by maximizing the log-likelihood. Therefore, no other optimization techniques are required.

The forecasts of the exponential smoothing models are obtained by the MIMO strategy for the upcoming 24 hours. This process is iterated for every step in the cross validation process. The training sets consists only of observations that occurred prior to the observations that form the test set. Since it is not possible to obtain a reliable forecast based on a small training set, the first train set consists out of 7 days, so $7 \times 24 = 168$ data points. The train set gets increased by 24 data points at each iteration. The cross validation strategy enables forecasting and performance evaluation on many points in time. To determine the model that performs best on a very short-term, mid-term and longer-term, all models are evaluated on each of the three horizons. Therefore, the forecasts for the first 8 hours are used to calculate the near future performance. The forecasts for the next 8 hours are used to calculate the middle future performance followed by the last 8 hours to calculate the far future performance.

The training data is used to determine the optimal parameters α, β and γ per group. Since the cross validation process contains many training sets, the optimal parameters are determined for each training set. Averaged optimal parameters for each group are shown in Table 7.1. It can be seen that α is 0.9950 for four out of five groups in the SES model. Therefore, more weight is assigned to the closer observations, which is also applied in Holt and Holt Winters. For Holt, cluster 0 and the total set show high trend smoothing parameters, so the slope significantly changes over time which could be explained by the peak period in November and December. Cluster 1, 2 and 3 slopes hardly changes over time. Holt Winters seasonal smoothing parameters show that seasonality is a little, but hardly changing over time in each group. The smoothing parameters for trend are near zero, so the slope hardly changes over time.

Table 7.1: Optimized Parameters - ES

	SES	**Holt**		**Holt Winters**		
	α	α	β	α	β	γ
Cluster 0	0.9950	0.9950	0.8529	0.8889	0.0001	0.0987
Cluster 1	0.9950	0.9950	0.0001	0.9243	0.0001	0.0541
Cluster 2	1.0000	0.9950	0.0001	0.9243	0.0001	0.0541
Cluster 3	0.9950	0.9950	0.0001	0.9243	0.0001	0.0649
Total Set	0.9950	0.9950	0.8055	0.8889	0.0001	0.1111

7.1.3 Results

Forecast results are evaluated by use of RMSE and MASE. The best RMSE and MASE scores for SES, Holt and Holt Winters for near, middle and far future are shown for all clusters and the total aggregate group in Table 7.2 and Table 7.3. Extensive k-step-ahead forecast performances for all ES models are shown in Appendix D. Each step shows the k-step-ahead performances in every hour for 24 steps ahead. In the results below, the best performing models for each group on each horizon are marked in bold.

Table 7.2: RMSE Exponential Smoothing

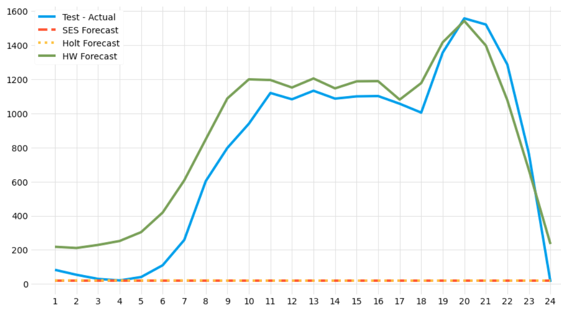
		Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
	SES	**552.37**	133.26	**99.70**	**93.94**	**789.53**
Near	Holt	2262.58	**133.09**	109.60	100.50	3524.41
	HW	934.07	188.94	120.72	96.74	1276.05
	SES	2033.76	502.57	347.50	325.21	3123.33
Middle	Holt	6844.42	502.85	380.86	338.36	10999.45
	HW	**1598.18**	**294.32**	**199.73**	**162.85**	**2225.55**
	SES	2209.54	537.24	384.08	351.29	3399.71
Far	Holt	11589.33	538.54	477.33	388.20	18569.75
	HW	**1877.01**	**352.30**	**225.70**	**211.77**	**2731.77**

Table 7.3: MASE Exponential Smoothing

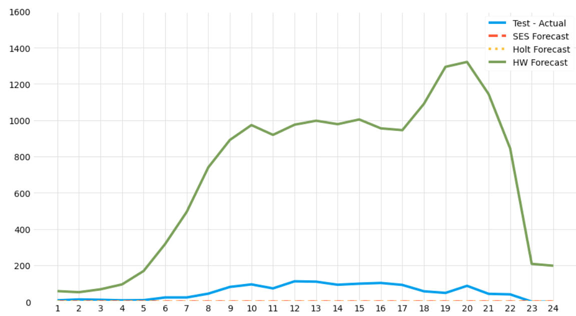
		Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	SES	**0.84**	0.90	**0.93**	**0.91**	**0.80**
	Holt	4.01	**0.89**	0.98	0.93	4.41
	HW	1.63	1.34	1.20	1.07	1.45
Middle	SES	4.12	4.02	4.00	3.97	4.19
	Holt	13.05	4.02	4.20	4.04	14.49
	HW	**2.99**	**2.13**	**2.07**	**1.71**	**2.73**
Far	SES	4.43	4.37	4.52	4.40	4.57
	Holt	21.77	4.38	4.91	4.52	24.32
	HW	**3.31**	**2.65**	**2.46**	**2.35**	**3.31**

It can be seen that RMSE and MASE give exactly the same best performing models. SES is selected as best performing method in the near future, except for cluster 1, where Holt tend to outperform SES. On the middle future horizon, SES and Holt seem to perform relatively similar for cluster 1, 2 and 3 but in the end, Holt Winters definitely outperforms both methods. On the far future, the best performing methods keep unchanged and still, Holt Winters outperforms SES and Holt. The fact that Holt outperforms other methods only once can be explained by a lack for trend in the data sets. Since the time series contain significant seasonality, it was expected that Holt Winters would outperform other ES models.

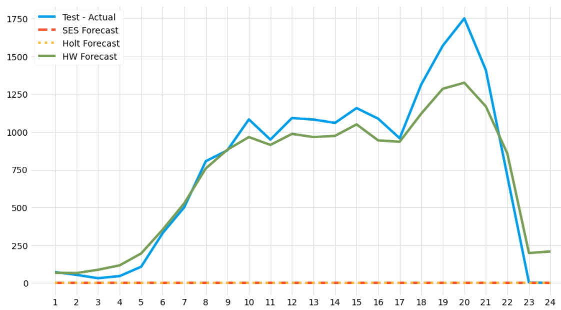
Figure 7.1 shows four examples of 24 hours ahead forecasts for cluster 1. Actual and predicted values on a randomly selected weekday, weekend, promotion day and holiday are compared from all ES models. The blue line shows the actual sales, where the red, yellow and green lines present the forecasts for SES, Holt and Holt Winters respectively.



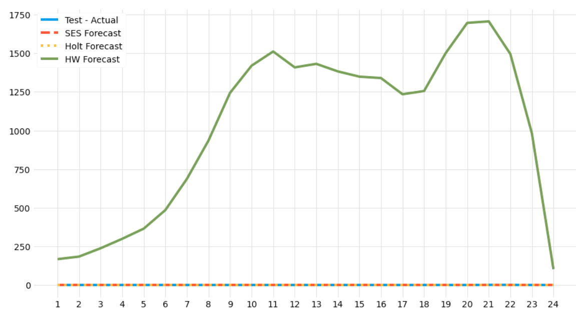
(a) Weekday (Nov 13, Wednesday)



(b) Weekend (Oct 19, Saturday)



(c) Promotion (Oct 2, Tuesday)



(d) Holiday (Dec 25, Wednesday)

Figure 7.1: Forecasts Exponential Smoothing, 24-step-ahead

The figures indicate that Holt Winters has the best results on weekday and promotion day, but is not able to react on the fall down in sales on Saturday and the holiday. It can be seen that SES and Holt predict a horizontal line since it is not able to include or it did not indicate any trend. Therefore, it performs accidentally well during Saturday and the holiday and also explains the good performance of the two methods in the near future. Holt Winters follows the clear seasonal pattern of 24 hours and did not show any trend, which was also explained by the parameter β.

7.1.4 Discussion

Since every aggregate group contains clear seasonality, it was expected that Holt Winters would outperform other ES models. None of the aggregate groups contain clear trend, so the similar behavior of SES and Holt was expected. However, SES performs best in the near future which could be explainable by the low amount of sales during night, which are close to zero. Cluster 0 and the total aggregate set yields worst performance which is caused by the big fluctuation of sales during the November and December months. Additionally, only one year of data is included in modelling, so a yearly seasonality could not be detected. Cluster 2 yields the best performance since it contains the product groups that behave constant during the year and is easiest to predict. None of the ES models were able to deal with holidays and Saturdays since it could only handle univariate data. To identify the fall down in sales on Saturdays, a double seasonality term or exogenous variable should be included in the models. ES models are not able to deal with these so it is expected that other models will outperform ES.

7.2 ARIMA

The aim of the ARIMA model is to describe autocorrelations in the data and to use these for forecasting purposes. The basic ARIMA model can be extended to a multivariate ARIMAX model and seasonality can be added into a SARIMA and SARIMAX model. This section describes the multiple model variations and identifies for each data set the optimal parameters to enable validation of the ARIMA models. Finally, the results are presented and extensively discussed for each cluster and the aggregate data set.

7.2.1 Model Descriptions

ARIMA

The basic ARIMA model consist of an autoregressive (AR), moving average (MA) and integration term. The first step in an ARIMA model is to make the time series stationary. The integration term d indicates the degree of differencing to make the time series stationary. The AR term in ARIMA, means it is a linear regression model that uses its own lags as predictors. The order of the AR term indicates how many lags are included in the model and is denoted by p. The MA term refers to the number of lagged forecast errors that should be included in the model and is denoted by q. ARIMA can be denoted as ARIMA(p, d, q) and is defined as,

$$y'_t = c + \phi_1 y'_{t-1} + ... + \phi_p y'_{t-p} + \theta_p \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{7.4}$$

where y'_t is the differenced time series, c is a constant, ϕ and θ are smoothing parameters, ε_{t-q} is the forecast error and ε_t is the white noise during t. ARIMA models are often expressed in 'backshift' notation in which B is the 'backshift' or 'lag' operator. Therefore, the function can be written in backshift notation as,

$$(1 - \phi_1 B - ... - \phi_p B^p) * (1 - B)^d y_t = c + (1 + \theta_1 B + ... + \theta_q B^q)\varepsilon_t \tag{7.5}$$

in which the first part before the multiplication denotes the AR(p) term, the second part denotes a dth-order difference and the part after the equal sign denotes the MA(q) term.

SARIMA

Another variant of ARIMA that is widely used and broadly known is the SARIMA model. This model constructs a seasonal ARIMA model including seasonal terms $(P, D, Q)_m$ in the traditional ARIMA model. The seasonal AR term is denoted by P, the seasonal difference order by D and the seasonal MA term by Q. The frequency of seasons is denoted by m. The SARIMA model can model a single seasonal effect. The SARIMA model can be denoted as ARIMA$(p, d, q)(P, D, Q)_m$ and is defined as,

$$\begin{aligned}(1 - \phi_1 B - ... - \phi_p B^p) * (1 - \Phi_1 B^m - ... - \Phi_P B^{Pm}) * (1 - B)^d * (1 - B^m)^D y_t = \\ c + (1 - \theta_1 B - ... - \theta_q B^q) * (1 - \Theta_1 B^m - ... - \Theta_Q B^{Qm})\varepsilon_t\end{aligned} \tag{7.6}$$

ARIMAX

Where ARIMA is an univariate model, different exogenous variables can be inserted in an ARIMAX model. ARIMAX is suitable for multivariate analysis where there are additional explanatory variables. The ARIMAX model can be defined as,

$$(1 - \phi_1 B - ... - \phi_p B^p) * (1 - B)^d y_t = c + (1 + \theta_1 B + ... + \theta_q B^q)\varepsilon_t + \sum_{i=0}^{J} \beta_{t-i}^{(1)} y_{t-i}^{(1)} \tag{7.7}$$

In this equation, $y_{t-i}^{(1)}$ represents the first independent variable at time $(t - i)$ and $\beta_{t-i}^{(1)}$ is the corresponding parameter. J denotes the number of exogenous variables used in the model.

SARIMAX

Similar to the extension of ARIMA to ARIMAX, the SARIMA model can be extended to the SARIMAX model, inserting different exogenous variables. SARIMAX is suitable for multivariate analysis and is able to model multiple seasonal patterns with Fourier terms. The standard SARIMAX model without Fourier terms can be defined as,

$$\begin{aligned}(1 - \phi_1 B - ... - \phi_p B^p) * (1 - \Phi_1 B^m - ... - \Phi_P B^{Pm}) * (1 - B)^d * (1 - B^m)^D y_t = \\ c + (1 - \theta_1 B - ... - \theta_q B^q) * (1 - \Theta_1 B^m - ... - \Theta_Q B^{Qm})\varepsilon_t + \sum_{i=0}^{J} \beta_{t-i}^{(1)} y_{t-i}^{(1)}\end{aligned} \tag{7.8}$$

7.2.2 Parameter Selection

To start, p, d and q parameters are determined manually to set initial parameters. Therefore, an ACF and PACF analysis is conducted to determine initial parameters p and q. As an example, Figure 7.2 shows the ACF and PACF plots for cluster 1 on the original time series. In addition to the ACF and PACF plots on the original time series, plots were created for logarithmic transformed and differenced time series to see if variability should and could be reduced. All ACF and PACF plots for original, logarithmic transformed and differenced time series can be found in Appendix C. The ACF and PACF plots were visually inspected to investigate if a time series is stationary or contains trend or seasonality.

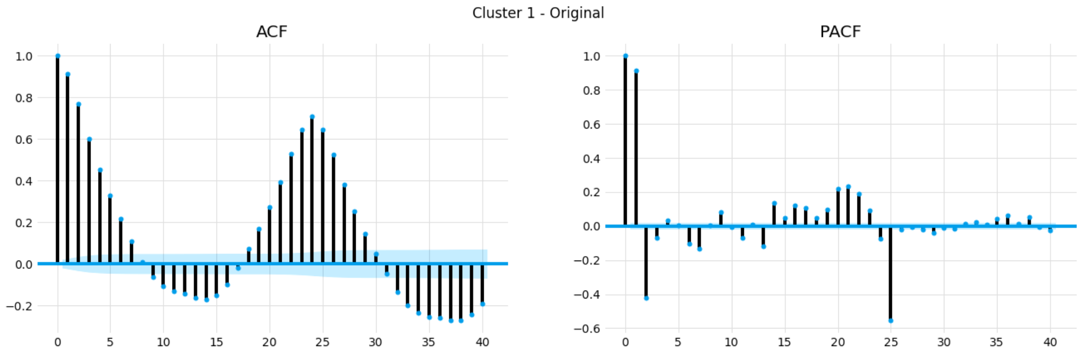


Figure 7.2: ACF and PACF Plot - Cluster 1

The ACF plots of the original time series show a positive autocorrelation to a number of lags for all clusters and the total set. Therefore, a difference term of d=1 is added for all groups. The ACF and PACF plot of cluster 1 after first order differencing is shown in Figure 7.3. A positive lag-1 autocorrelation remain in all groups and the overall auto correlations got smaller, so a difference term of $d = 1$ might be appropriate. Additionally, a p term of 2 and a q term of 1 are added to the initial parameters for all groups.

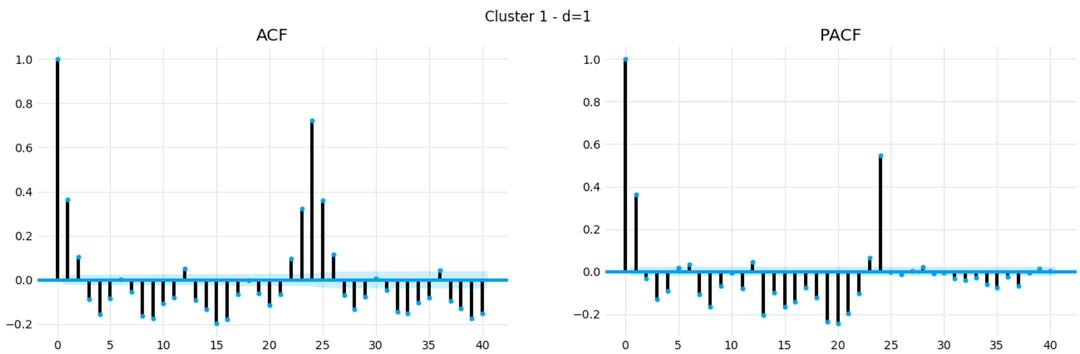


Figure 7.3: ACF and PACF Plot - Cluster 1, d=1

The ACF plots of all groups show a very clear seasonality at m=24 and a high correlation with data points near to the seasonal cycle. This denotes a clear seasonality and a seasonal differencing term should be included in the SARIMA and SARIMAX parameters. The seasonality follows a linear function so a difference term of D=1 is used to seasonal difference the time series. The

ACF and PACF plots for seasonal differenced time series of cluster 1 can be found in Figure 7.4. However, seasonal differencing with D=1 does not seem to remove the seasonality which could be caused by an additional seasonality of m=168. Therefore, a double seasonality term might be required in further modelling to improve forecast accuracy.

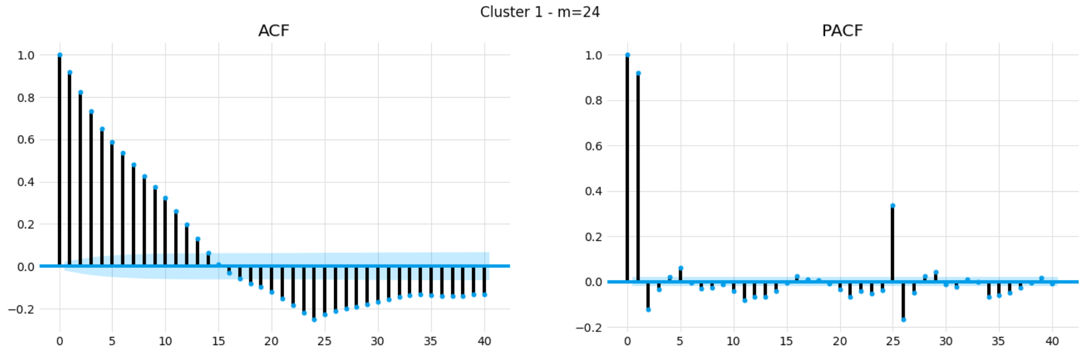


Figure 7.4: ACF and PACF Plot - Cluster 1, m=24

When inspecting the ACF and PACF plots of logarithmic transformed time series, for all groups, no significant reduction in variability could be identified. Therefore, no logarithmic transformation is required regarding visual inspection. Nevertheless, logarithmic transformation will be discovered in further modelling to see if it improves the forecast accuracy.

To obtain optimal parameters for all ARIMA models, parameters are optimized using grid search. Parameter values of q and Q were set between zero and three, where p and P varied from zero to four. Since the data showed a need for differencing to stationarize, d and D were set between zero and two. No higher order differencing was expected. The optimization objective for the combination of parameters was to minimize the Akaike Information Criterion (AIC) on four training data sets. The training sets are selected on four evenly spread moments in the cross validation process to decrease computation time. The AIC is defined as,

$$Tlog((\frac{\sum_{t=1}^{T} e_t^2}{T}) + 2(k+2)) \tag{7.9}$$

where T denotes the number of observations, k denotes the number of predictors and e_t represents the error at time t. Manual selected and final optimized parameters for ARIMA and SARIMA can be found in Table 7.4 for original and log transformed time series. It is assumed that ARIMAX uses the same parameters as ARIMA and SARIMAX uses the same parameters as SARIMA. The table shows in the optimized parameters that a d term of zero is included in all groups except for the total set with logarithmic transformation. A q term of 2 yields the best AIC score for all groups except cluster 0. The p term differs between 2, 3 and 4 which can be explained by the sharp cut offs that took place in the PACF plots of all groups around lag-2, 3 and 4. The P term equals 1 for all seasonal orders since one significant positive spike is shown in the PACF plots of all groups. A significant negative spike is shown in both original and logarithmic series so a Q term of 1 is added to all seasonal orders. Seasonal differencing is applied in each group in combination with the original data set. When log transformation is applied in advance, seasonal differencing is only required for cluster 0.

Table 7.4: Optimized Parameters - ARIMA and SARIMA

	Manual (p, d, q)	**ARIMA** (p, d, q)		**SARIMA** $(p, d, q)(P, D, Q)_m$	
	Original	*Original*	*Log*	*Original*	*Log*
Cluster 0	$(2, 1, 1)$	$(3, 0, 1)$	$(4, 0, 2)$	$(3, 0, 2)(1, 1, 1)_{24}$	$(2, 0, 2)(1, 1, 1)_{24}$
Cluster 1	$(2, 1, 1)$	$(4, 0, 2)$	$(4, 0, 2)$	$(4, 0, 2)(1, 1, 1)_{24}$	$(2, 0, 2)(1, 0, 1)_{24}$
Cluster 2	$(2, 1, 1)$	$(4, 0, 2)$	$(4, 0, 2)$	$(3, 0, 2)(1, 1, 1)_{24}$	$(2, 0, 2)(1, 0, 1)_{24}$
Cluster 3	$(2, 1, 1)$	$(3, 0, 2)$	$(4, 0, 2)$	$(3, 0, 2)(1, 1, 1)_{24}$	$(3, 0, 2)(1, 0, 1)_{24}$
Total Set	$(2, 1, 1)$	$(2, 0, 2)$	$(4, 1, 2)$	$(3, 0, 2)(1, 1, 1)_{24}$	$(3, 0, 2)(1, 0, 1)_{24}$

7.2.3 Selection of Exogenous Variables

To identify which exogenous variables are related to the actual sales and should be included in further modelling, Pearson's correlation coefficients are determined. Correlations were determined separately for each cluster and before data transformations. Table 7.5 shows the correlations between the actual sales and the most significant exogenous variables according to the Pearson's correlation coefficient. A distinction is made between four types of exogenous variables: lever impact (1), calendar effects (2), weather effects (3) and interaction effects (4). It can be seen that no higher correlations can be found than absolute 0.398, so only small correlations are derived. A correlation with November and December is only found by cluster 0 and the total set, which was expected due to the high peak in these clusters during these months. The highest negative correlation is derived with Saturday, which can be explained by a extreme fall down in sales on these days. In terms of correlation with weather variables, cluster 3 has the highest overall correlation which can be explained by the type of product groups the cluster contains; summer peak product groups.

Table 7.5: Pearson's Correlation Exogenous Variables

		Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
(1)	$WarehouseClosed$ $[0, 1]$	-0.161	-0.183	-0.192	-0.176	-0.175
	$NationalHoliday$ $[0, 1]$		-0.169	-0.177	-0.163	-0.161
	$WeekbeforeChrist.andSint.$ $[0, 1]$	0.287	0.153			0.230
	$PromotionWeek$ $[0, 1]$				0.211	
	$Autumn$ $[0, 1]$	0.299	0.219			0.258
(2)	$November$ $[0, 1]$	0.238				0.178
	$December$ $[0, 1]$	0.188				0.158
	$MonTueWed$ $[0, 1]$	0,236	0.272	0.272	0.282	0.258
	$Saturday$ $[0, 1]$	-0.340	-0.380	-0.398	-0.369	-0.366
	$Temperature_t$		0.172	0.186	0.228	
(3)	$Temperature_{t-1}$		0.184	0.193	0.235	
	$HoursSun_t$		0.169	0.220	0.209	0.153
	$HeatingDegree_t$		-0.163	-0.180	-0.210	
(4)	$Temp_t - Month9$					0.184
	$Temp_t - Month10$		0.159			0.182

'WarehouseClosed' and 'NationalHoliday' represent a relatively similar variable and give relatively similar correlation coefficients with the actual sales. However, the warehouse is not closed on each national holiday, so sales will not be zero on each national holiday. In contrast, sales will be always zero if the warehouse is closed. It should be kept in mind that the correlation would not be near one since zero sales also appear during nights while the warehouse is not closed. Since 'WarehouseClosed' has a higher correlation in comparison to 'NationalHoliday', the correlation between 'WarehouseClosed' and actual sales will be included in the input sets and 'NationalHoliday' will be removed from the input sets.

Additionally, a choice between temperature at time t and temperature at time $t-1$ will be made since these represent a similar variable. Both exogenous variables show correlation with cluster 1, 2 and 3. However, the temperature at time $t-1$ shows a higher correlation in all clusters in comparison to the temperature at time t. Therefore, temperature at time t will be removed from the input sets and temperature at time $t-1$ will be included.

Table 7.6 shows the final input sets included in further modelling specified for each cluster. Input set includes lever impact and calendar effects since these are expected to have the highest impact. Weather effects will be added to the second input set. The third input set will be extended with relevant interaction effects, thus includes all variables.

Table 7.6: Input Sets Exogenous Variables

Input Set	Exogenous Variable	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
(1, 2, 3)	$WarehouseClosed$ $[0,1]$	✓	✓	✓	✓	✓
	$WeekbeforeChrist.andSint.$ $[0,1]$	✓	✓			✓
	$PromotionWeek$ $[0,1]$				✓	
	$Autumn$ $[0,1]$	✓	✓			✓
	$November$ $[0,1]$	✓				✓
	$December$ $[0,1]$	✓				✓
	$MonTueWed$ $[0,1]$	✓	✓	✓	✓	✓
	$Saturday$ $[0,1]$	✓	✓	✓	✓	✓
(2, 3)	$Temperature_{t-1}$		✓	✓	✓	
	$HoursSun_t$		✓	✓	✓	✓
	$HeatingDegree_t$		✓	✓	✓	
(3)	$Temp_t - Month9$					✓
	$Temp_t - Month10$		✓			✓

7.2.4 Results

Forecast results are evaluated by use of RMSE and MASE. The best RMSE and MASE scores for ARIMA, ARIMAX, SARIMA and ARIMAX for near, middle and far future are shown for all clusters and the total aggregate group in Table 7.7 and Table 7.8. Extensive k-step-ahead forecast performances for all ARIMA models are shown in Appendix E. Each step shows the k-step-ahead performances in every hour for 24 steps ahead. In the results below, the best performing models for each group on each horizon are marked in bold. The time series transformation and multivariate set that yield the best results were added to the scores in subscript.

It can be seen that both RMSE and MASE give exactly the same best performing models including the same time series transformation and multivariate set. ARIMA dit not outperform in any stage, while SARIMA outperforms 9 out of 15 times which was expected due to clear seasonality. In the middle and far future, the preference goes to the ARIMAX or SARIMAX model in which exogenous set 1 is chosen most often. In terms of transformation, both the original data set and the logarithmic transformed set did good forecasting. In the near future, the original data set outperformed, while in the middle future the logarithmic transformed data set outperformed. The SARIMA model only showed a best performance with the original data set except for the near future in cluster 3. It was expected that the original data set would perform best, since for the time series that contain non-zero values, all zero sales were replaced with sales of one unit. However, when exogenous variables are added, the logarithmic transformed data set is preferred by the models. The little impact of the replacement of zero to one sales units can be explained by the relatively high number of sales that occur during the other hours of the day. Lastly, it was expected that there would be one multivariate set per group that performed best. The choice of multivariate sets looks quite stable but still deviates when the forecast horizon gets increased. For example, cluster 3 prefers set 2 in the near and middle future ARIMAX model but changes to set 1 in the far future.

Table 7.7: RMSE ARIMA Models

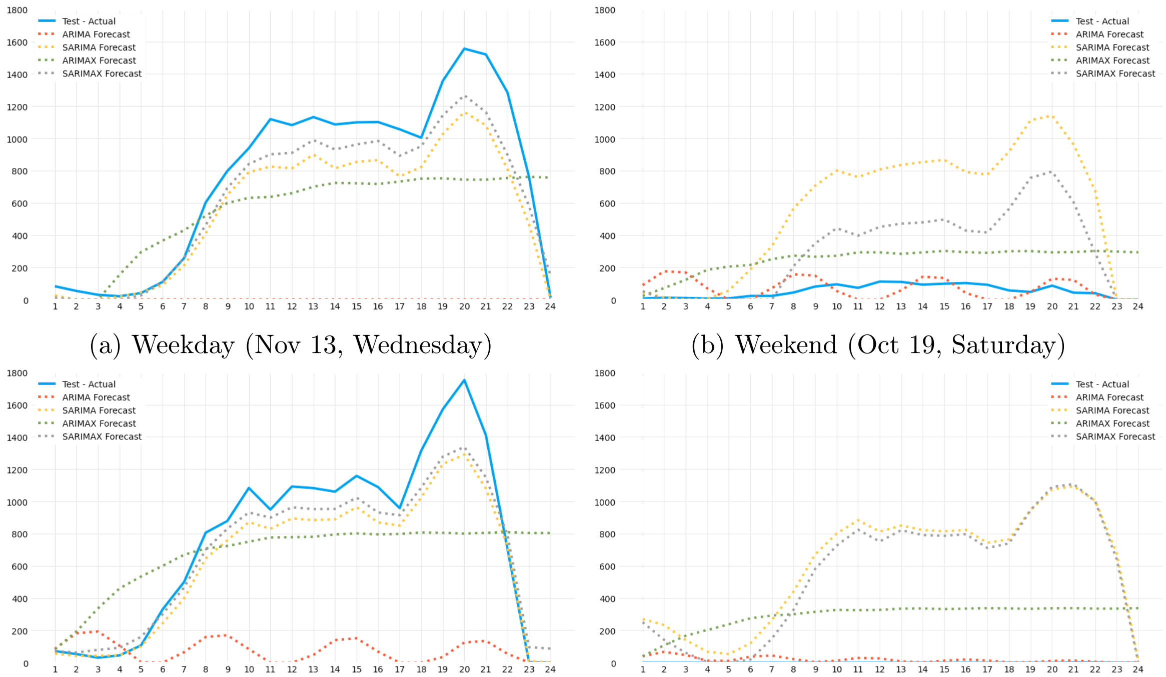
		Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	ARIMA	1059.89_{or}	223.45_{or}	157.83_{or}	153.04_{or}	1494.33_{or}
	ARIMAX	$970.06_{or,set3}$	$237.89_{or,set3}$	$178.36_{or,set1}$	$153.61_{or,set2}$	$1461.22_{or,set3}$
	SARIMA	$\mathbf{630.72}_{or}$	$\mathbf{158.92}_{or}$	$\mathbf{105.80}_{or}$	$\mathbf{106.68}_{log}$	$\mathbf{917.09}_{or}$
	SARIMAX	$1060.79_{or,set2}$	$237.88_{or,set1}$	$158.85_{or,set1}$	$108.09_{or,set1}$	$1459.00_{or,set2}$
Middle	ARIMA	2002.51_{or}	451.59_{or}	266.68_{or}	254.11_{log}	2707.68_{or}
	ARIMAX	$1062.86_{log,set3}$	$281.96_{log,set1}$	$\mathbf{87.67}_{log,set2}$	$\mathbf{85.58}_{log,set2}$	$2933.62_{or,set2}$
	SARIMA	949.42_{or}	216.57_{or}	137.75_{or}	138.20_{or}	$\mathbf{1348.05}_{or}$
	SARIMAX	$\mathbf{182.32}_{log,set1}$	$\mathbf{106.76}_{log,set1}$	$220.03_{or,set1}$	$148.84_{or,set2}$	$2226.88_{or,set2}$
Far	ARIMA	1295.15_{log}	295.06_{log}	202.94_{or}	199.17_{or}	1767.29_{or}
	ARIMAX	$773.36_{log,set3}$	$195.48_{log,set1}$	$\mathbf{41.74}_{log,set1}$	$\mathbf{38.82}_{log,set1}$	$1953.68_{or,set2}$
	SARIMA	$\mathbf{593.13}_{or}$	$\mathbf{130.61}_{or}$	92.79_{or}	98.28_{or}	$\mathbf{864.12}_{or}$
	SARIMAX	$750.89_{log,set1}$	$193.69_{or,set2}$	$157.80_{log,set1}$	$99.50_{or,set2}$	$1130.23_{or,set1}$

Table 7.8: MASE ARIMA Models

		Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	ARIMA	2.14_{or}	1.74_{or}	1.73_{or}	1.80_{or}	2.00_{or}
	ARIMAX	$1.98_{or,set3}$	$1.80_{or,set3}$	$1.71_{or,set2}$	$1.74_{or,set2}$	$1.93_{or,set1}$
	SARIMA	$\mathbf{1.12}_{or}$	$\mathbf{1.14}_{or}$	$\mathbf{1.10}_{or}$	$\mathbf{1.15}_{log}$	$\mathbf{1.07}_{or}$
	SARIMAX	$2.02_{or,set2}$	$1.83_{or,set2}$	$1.63_{or,set2}$	$1.28_{or,set1}$	$1.82_{or,set2}$
Middle	ARIMA	3.49_{log}	3.57_{or}	3.09_{or}	2.97_{log}	3.64_{or}
	ARIMAX	$2.11_{log,set3}$	$1.92_{log,set2}$	$\mathbf{1.06}_{log,set2}$	$\mathbf{1.06}_{log,set2}$	$3.93_{or,set3}$
	SARIMA	1.66_{or}	1.55_{or}	1.41_{or}	1.53_{or}	$\mathbf{1.58}_{or}$
	SARIMAX	$\mathbf{0.45}_{log,set1}$	$\mathbf{0.94}_{log,set1}$	$2.13_{or,set2}$	$1.74_{or,set2}$	$2.91_{or,set2}$
Far	ARIMA	2.49_{log}	2.31_{log}	2.39_{log}	2.44_{log}	2.47_{or}
	ARIMAX	$1.29_{log,set3}$	$1.33_{log,set1}$	$\mathbf{0.52}_{log,set1}$	$\mathbf{0.50}_{log,set1}$	$2.54_{or,set2}$
	SARIMA	$\mathbf{0.97}_{or}$	$\mathbf{0.90}_{or}$	0.92_{or}	1.02_{or}	$\mathbf{0.94}_{or}$
	SARIMAX	$1.68_{or,set2}$	$1.28_{or,set2}$	$0.97_{or,set2}$	$1.05_{or,set2}$	$1.33_{or,set1}$

Figure 7.5 shows four examples of 24 hours ahead forecasts for cluster 1. Actual and predicted values on a randomly selected weekday, weekend, promotion day and holiday are compared from all ARIMA models. The blue line shows the actual sales, where the red, yellow, green and grey dotted lines present the forecasts for ARIMA, ARIMAX, SARIMA and SARIMAX respectively. Multivariate set 1, only including lever and calendar effects, is used in the shown ARIMAX and SARIMAX models since it performs best in cluster 1. The figures indicate that SARIMA and SARIMAX performs best on weekday and promotion day, but are just as the other methods not able to react on the fall down in sales on Saturday and the holiday. None of the ARIMA models captures trend, which was expected since no significant trend is present in the data and

no differencing term was included. Both SARIMA and SARIMAX are able to capture the daily seasonality of $m = 24$ but are both not able to react on the weekly seasonality of $m = 168$. However, SARIMAX outperforms SARIMA on Saturday in which the SARIMAX model is able to keep the sales near zero in the first few hours of the day and keeps the sales relatively low during the rest of the day.



(a) Weekday (Nov 13, Wednesday)

(b) Weekend (Oct 19, Saturday)

(c) Promotion (Oct 2, Tuesday)

(d) Holiday (Dec 25, Wednesday)

Figure 7.5: Forecasts ARIMA, 24-step-ahead

7.2.5 Discussion

It was expected that a seasonal variant of the ARIMA model would outperform other ARIMA methods, since a clear seasonality is conducted in the data sets of each aggregate group. Therefore, a seasonal differencing term D is included for all groups. However, since the data set contains double seasonality, the model could only detect the daily seasonality of $m = 24$ but passes over the weekly seasonality of $m = 168$. Furthermore, it was expected that ARIMA models that include exogenous variables would outperform since it was expected that the sales highly depend on several variables. However, the Pearson's correlation test showed that only small correlations are found. This is due to the fact that finding meaningful correlations on a hourly base is relatively hard because there will always be other time periods with the same sales values. For example, it was expected that closing the warehouse would directly impact the sales values to go to zero, but sales values of zero also occur during night when the warehouse is not closed. Therefore, the correlation is low and the ARIMAX and SARIMAX model are not able to predict zero sales when the warehouse is closed. This happened with other exogenous variables as well which negatively influenced the performance of the models.

7.3 Support Vector Regression

Support vector regression (SVR) is a combination of support vector machines (SVM) and regression and tries to minimize the error within a certain nonlinear threshold. This section describes the model and identifies optimal parameters. The model description is derived from the research of Lusis et al. (2017) and Kavaklioglu (2011). Finally, results are presented and discussed for each cluster and the total aggregate data set.

7.3.1 Model Description

The main concept behind classical support vector regression is to find the decision function $\hat{y}_t$ that gives as flat as possible decision boundary. A linear model is proposed as,

$$\hat{y}(x) = f(x) = \langle w, \Phi(x) \rangle + b \tag{7.10}$$

where $\hat{y}$ is the estimated output of the model, w is a weight vector controlling the smoothness of the model and b is a bias term. $\Phi(x)$ denotes a features mapping that converts the features x into a high-dimensional feature space. The mapping is required to convert the problem from a nonlinear regression to a linear regression case, shown in Figure 7.6. Parameters w and b are found by minimizing the regularized risk function R which is defined as,

$$\begin{aligned} &\textit{minimize} && \frac{1}{2}\left\|w\right\|^2 + C\sum_{i=1}^{n}(\zeta_i + \zeta_i^*) && \\ &\text{subject to} && y_i - \langle w, \Phi(x_i) \rangle - b \leq \varepsilon + \zeta_i && i = 1, \ldots, n, \\ & && y_i - \langle w, \Phi(x_i) \rangle - b \leq \varepsilon + \zeta_i^* && i = 1, \ldots, n, \\ & && \zeta_i, \zeta_i^* \geq 0 && i = 1, \ldots, n \end{aligned} \tag{7.11}$$

The forecast values within the tube have an error of zero, whereas the values outside the tube have an error ζ equal to the distance between the data point and the tube wall as shown in Figure 7.6. The empirical risk term uses slack variables ζ and ζ^* to capture the residuals outside the tube. By minimizing the regularization term, the geometric margin between support vectors are maximized to reduce the complexity of the function and increase flatness. Constant C helps to prevent overfitting by giving the trade-off between the regularization term and empirical risk.

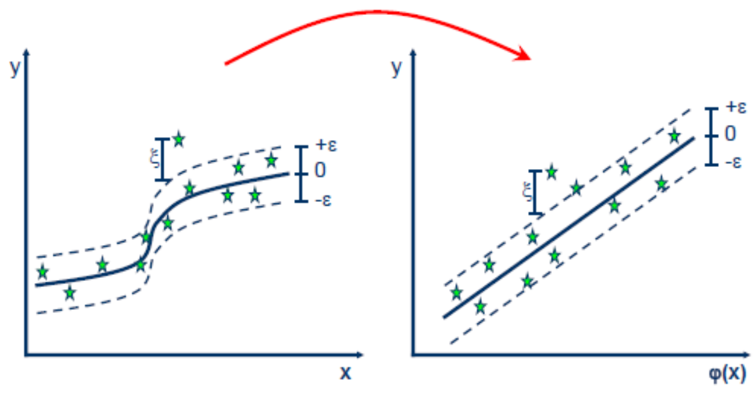


Figure 7.6: SVR Visualisation

The optimal regression function is shown in equation 7.12, where $K(x_i, x)$ is a Gaussian kernel function and n is the number of support vectors. For every training set data point there is a pair of α_i, α_i^*. The bias b is computed such that $\varepsilon - y_i + \hat{y}_i = 0$ condition is satisfied for all the support vectors.

$$\hat{y}(x) = f(x) = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) K(x_i, x_i) + b \tag{7.12}$$

7.3.2 Configuration

Support vector regression (SVR) models are constructed using the sklearn.svm package in Python. Multivariate models are constructed using one total exogenous variable set, including all exogenous variables explained in the (S)ARIMAX model in section 7.2.3.

The forecasts of the support vector regression are obtained by the MIMO strategy for the upcoming 24 hours. This process is iterated for every step in the cross validation process. The training sets consists only of observations that occurred prior to the observations that form the test set. Just as in the ARIMA models, the first train set consists out of 7 days and the train set gets increased by 24 data points at each iteration. The cross validation strategy enables forecasting and performance evaluation on many points in time. To determine the model that performs best on a short-term, mid-term and longer-term, all models are evaluated on each of the three horizons. Therefore, the forecasts for the first 8 hours are used to calculate the near future performance. The forecasts for the next 8 hours are used to calculate the middle future performance followed by the last 8 hours to calculate the far future performance.

The training data is used to obtain the optimal parameters C, ε and γ by brute force grid search. Parameter C varied from 5000 to 6000 for cluster 0 and the total set and from 500 to 2000 for cluster 1, 2 and 3. Parameter ε varied from 1 to 50 and γ varied from 0.1 to 0.5 for all groups. Therefore, a lot of different models were built and the one with the least test RMSE was chosen as the best model as it would have the best prediction ability. Since the cross validation process contains many training sets, all parameters were determined using four test sets that were followed by a test set on a promotion day, weekday, Saturday and holiday. Therefore, the parameters were tested on four very different and specific days to obtain optimal parameters that can handle several daily characteristics. Optimal parameters and the number of support vectors for each cluster and the total aggregate group are shown in Table 7.9.

Table 7.9: Optimized Parameters - SVR

	C	ε	γ	#SV
Cluster 0	5000	5	0.2	8156
Cluster 1	600	45	0.15	5004
Cluster 2	1300	1	0.15	8422
Cluster 3	1700	20	0.1	5856
Total Set	6000	5	0.4	8331

7.3.3 Results

Forecast results are evaluated by use of RMSE and MASE. The best RMSE and MASE scores for support vector regression for near, middle and far future are shown for all clusters and the total aggregate group in Table 7.10 and Table 7.11. The multivariate set that yield best results were added to the scores in subscript. Extensive k-step-ahead forecast performances for all SVR models are shown in Appendix F. Each step shows the k-step-ahead performances in every hour for 24 steps ahead.

In the results below, it can be seen that both RMSE and MASE give exactly the same best performing multivariate set. Cluster 0 indicates an equal performance for set 1 and 2 which is explained by the inclusion of the same external variables since cluster 0 did not find any correlation with weather variables. For the other aggregate groups, multivariate set 1 outperforms in most of the cases. Thus, only lever and calendar effects are included and weather effects seem to effect the forecasts in a negative way. For the total aggregate data set, including weather variables increases the performance, especially in the middle and far future. In terms of forecast horizon, SVR improves while increasing the forecast horizon. Therefore, the model performs best in the far future which could be explainable by the clear seasonality of 24 hours that makes it easier to forecast the last hours of the 24 hour cycle.

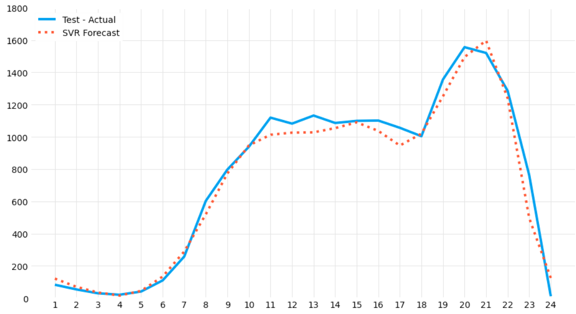
Table 7.10: RMSE SVR Model

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	$955.15_{set1=2}$	227.69_{set1}	133.64_{set1}	154.99_{set1}	1339.43_{set1}
Middle	$947.82_{set1=2}$	206.39_{set1}	121.66_{set1}	133.48_{set1}	1274.88_{set2}
Far	$492.23_{set1=2}$	107.35_{set1}	65.82_{set1}	86.15_{set1}	711.32_{set2}

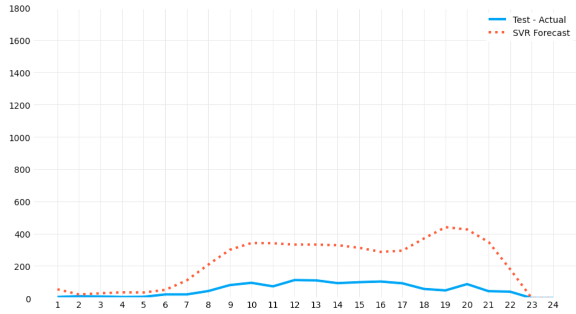
Table 7.11: MASE SVR Model

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	$1.46_{set1=2}$	1.58_{set1}	1.29_{set1}	1.65_{set1}	1.40_{set1}
Middle	$1.38_{set1=2}$	1.43_{set1}	1.14_{set1}	1.40_{set1}	1.28_{set2}
Far	$0.75_{set1=2}$	0.77_{set1}	0.65_{set1}	0.87_{set1}	0.73_{set2}

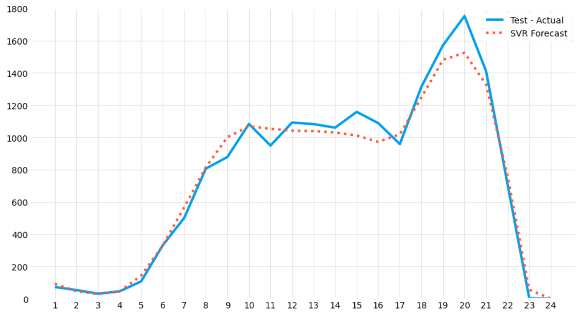
Figure 7.7 shows four examples of 24 hours ahead forecasts for cluster 1. Actual and predicted values on a randomly selected weekday, weekend, promotion day and holiday are compared from all ARIMA models. The blue line shows the actual sales, where the red dotted line presents the forecast for the SVR model. Multivariate set 1, only including lever and calendar effects, is used in the shown figures since it performs best. The figures indicate that the model performs best on weekday and promotion day and is able to react on the fall down in sales on Saturday and the holiday. Therefore, the model is able to deal with the double seasonality.



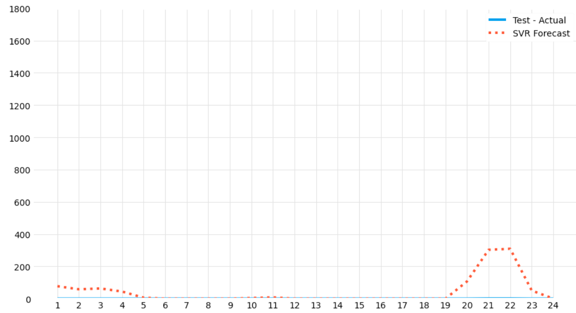
(a) Weekday (Nov 13, Wednesday)



(b) Weekend (Oct 19, Saturday)



(c) Promotion (Oct 2, Tuesday)



(d) Holiday (Dec 25, Wednesday)

Figure 7.7: Forecasts SVR, 24-step-ahead

7.3.4 Discussion

Since the data of every aggregate group contains clear daily and weekly seasonality, it was expected that support vector regression as machine learning model would perform very well. The model is able to detect the double seasonality and brings in very well performed forecasts on as well 'normal' days as on special days like holidays and Saturdays. However, especially on Saturday and the holiday, the model tends to predict higher sales values near the end of the day, which might be caused by the daily peak that occur during these hours on 'normal' days. Therefore, daily seasonality seem to have a really strong impact on the model and slightly overrules the external variables that are included in the model. In terms of external variables, weather variables do not improve the models except for the total aggregate set.

7.4 Neural Network

A multi layer perceptron (MLP) is a deep, feed forward Artificial Neural Network (ANN) with multiple layers between the input and output layers. Deep learning methods are able to deal with missing values, non-linear feature interactions and automatic feature extraction. This section describes the model and identifies optimal parameters. Finally, results are presented and discussed for each cluster and the total aggregate data set.

7.4.1 Model Description

A MLP model is based on the function of the human brain and is composed of an input layer to receive the signal and an output layer that makes a prediction about the input. The input layer consists of the same number of features as the data set. In between the input and output layer, an arbitrary number of hidden layers are located that are the true computational engine of the MLP. Each neuron in the hidden layer takes the weighted sum of the inputs and combines them using an activation function. Multi layer perceptrons are trained on a set of input-output pairs and learn to model the correlations and dependencies. In the training process of an MLP, back propagation is used to minimize the error, which can be measured in several ways, e.g. RMSE. Additionally, a number of epochs is used that defines the number of times the training data is used to update the weights of the inputs.

7.4.2 Configuration

For each cluster and the total aggregate set, MLP models were constructed using multivariate set 3, explained in section 7.2.3. Since deep learning methods are able to deal with automatic feature extraction, only multivariate set 3 is used, which includes all relevant exogenous variables. All MLP models use z-score normalization as data transformation to speed up the training process and reduce the chance of getting stuck in local optima.

The architecture of MLP models are critical to the performance of the model. Hyper parameter settings have a significant impact on the success of machine learning models. Since it is not computationally possible to test all possible combinations, a grid search is conducted for parameter tuning. Structure is determined by trial and error where the number of hidden layers varied from 1 to 6 and the number of neurons varied from 20 and 200, increasing by 20. Hidden layers have either a sigmoid unit for RBM pre-training or ReLU. ReLU is simpler than sigmoid and has a big advantage in training time. To prevent the model from overfitting, 100 epochs are used to train the model. A callback function is used to stop training if the model does not improve for more than three epochs in a row. As optimization algorithm for the weights of the input layer, Adam is used since it provides an algorithm that is computationally efficient. Optimized hyper parameters for each cluster and the total set are shown in Table 7.12.

Table 7.12: Optimized Parameters - MLP

	Activation	**Epochs**	**#Hidden Layers**	**#Neurons**	**Loss**
Cluster 0	ReLU	46	6	150	0.0005
Cluster 1	ReLU	68	5	80	0.0012
Cluster 2	ReLU	48	5	200	0.0012
Cluster 3	ReLU	41	6	150	0.0015
Total Set	ReLU	62	6	150	0.0007

It can be seen that ReLU is the most chosen activation function. The number of epochs used to construct the models for each cluster and the aggregate set varies from 41 to 68. It can be concluded that none of the models made use of the maximum of 100 epochs, so it presented a sufficient limit. All models use several hidden layers with all more then 80 neurons included in the hidden layers. The minimum loss in training is obtained by cluster 0 with a loss of 0.0005.

The forecasts of the MLP model are obtained by the MIMO strategy for the upcoming 24 hours. This process is iterated for every step in the cross validation process. The training sets consists only of observations that occurred prior to the observations that form the test set. Just as in the other models, the first train set consists out of 7 days and the train set gets increased by 24 data points at each iteration. The cross validation strategy enables forecasting and performance evaluation on many points in time. To determine the model that performs best on a short-term, mid-term and longer-term, all models are evaluated on each of the three horizons. Therefore, the forecasts for the first 8 hours are used to calculate the near future performance. The forecasts for the next 8 hours are used to calculate the middle future performance followed by the last 8 hours to calculate the far future performance.

7.4.3 Results

Forecast results are evaluated by use of RMSE and MASE. The best RMSE and MASE scores for the MLP model for near, middle and far future are shown for all clusters and the total aggregate group in Table 7.13 and Table 7.14. Extensive k-step-ahead forecast performances for all SVR models are shown in Appendix G. Each step shows the k-step-ahead performances in every hour for 24 steps ahead. In the results below, it can be seen that based on RMSE and MASE the model performs best in the far future and the middle future achieves the lowest performance. The best performance in the far future can be explained by the clear seasonality of 24 hours.

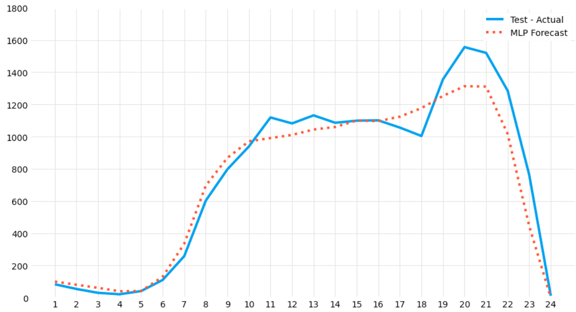
Table 7.13: RMSE MLP Model

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	1027.33	240.25	147.51	151.27	1454.79
Middle	1119.57	267.58	163.05	162.88	1610.34
Far	514.43	123.37	69.20	92.08	736.33

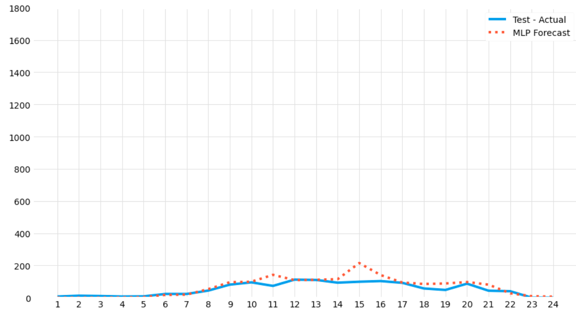
Table 7.14: MASE MLP Model

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	1.69	1.68	1.43	1.70	1.55
Middle	1.88	1.89	1.60	1.78	1.80
Far	0.83	0.84	0.68	0.95	0.76

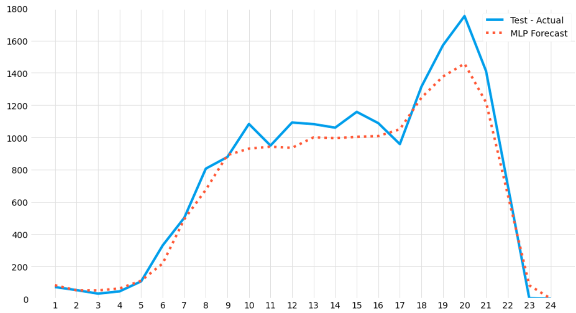
Figure 7.8 shows four examples of 24 hours ahead forecasts for cluster 1. Actual and predicted values on a randomly selected weekday, weekend, promotion day and holiday are compared from all ARIMA models. The blue line shows the actual sales, where the red dotted line presents the forecast for the MLP model. The figures indicate that the model performs well on weekday and promotion day. However, it predicts a smooth line according to the daily seasonality but is not able to predict the small peaky variations. Additionally, the model is able to react on the fall down in sales on Saturday and the holiday and performs very well. Therefore, it is proved that the model is able to deal with seasonality and automatic feature extraction.



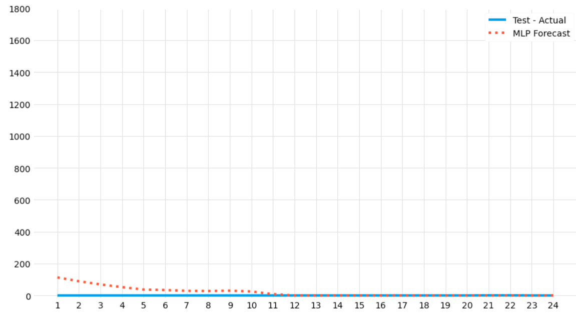
(a) Weekday (Nov 13, Wednesday)



(b) Weekend (Oct 19, Saturday)



(c) Promotion (Oct 2, Tuesday)



(d) Holiday (Dec 25, Wednesday)

Figure 7.8: Forecasts MLP, 24-step-ahead

7.4.4 Discussion

Since the data of every aggregate group contains clear daily and weekly seasonality, it was expected that MLP would perform very well. The MLP is able to deal with the double seasonality and is able to execute automatic feature extraction. The model was feed with a complete exogenous set, from which it should select the best features themselves. The model did a very good job in feature selection so it proves that there is no need to pre-define the most important features. In terms of predictions on 'normal' days, the MLP model is not able to deal with the small peaky variations but predicts a smoothed line. This results in a negative impact on the forecast accuracy in terms of RMSE and MASE. Since a simple grid search is conducted for hyper parameter tuning, the predictions might be influenced by non-optimized parameters. A more extensive optimization in hyper parameter tuning might lead to better results.

7.5 Overview

This section provides an overview and comparison of the results of the ES, ARIMA, SVR and MLP forecasting models. Additionally, the performance of selecting a single forecast method for each cluster (cluster selection) against aggregate selection will be evaluated. Lastly, a discussion of the results is provided.

7.5.1 Results

The best RMSE and MASE results per aggregate group and forecasting model are shown in Table 7.15 and Table 7.16. Only the best performing models are shown so the applied data transformation and multivariate set may differ per aggregate group and model type. Exact configurations can be found in the model sections in which all scores are discussed. It can be seen that both RMSE and MASE points to the same optimal models. In the near future, exponential smoothing outperforms other models, while in the middle and far future, ARIMA and SVR tend to outperform. In the middle future, SVR is closely followed by MLP, but MLP never outperforms. All clusters prefer ARIMA in the middle future and might change to SVR in the far future. However, ARIMA outperforms in most cases and closely follows SVR in cluster 0 and 1. For the total aggregate set, SVR outperforms on the middle and far future.

Table 7.15: RMSE Comparison all best Models

		Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	ES	**552.37**	**133.09**	**99.70**	**93.94**	**789.53**
	ARIMA	630.72	158.92	105.80	106.68	917.09
	SVR	955.15	227.69	133.64	154.99	1339.43
	MLP	1027.33	240.25	147.51	151.27	1454.79
Middle	ES	1598.18	294.32	199.73	162.85	2225.55
	ARIMA	**182.32**	**106.76**	**87.67**	**85.58**	1348.05
	SVR	947.82	206.39	121.66	133.48	**1274.88**
	MLP	1119.57	267.58	163.05	162.88	1610.34
Far	ES	1877.01	352.30	225.70	211.77	2731.77
	ARIMA	593.13	130.61	**41.74**	**38.82**	864.12
	SVR	**492.23**	**107.35**	65.82	86.15	**711.32**
	MLP	514.43	123.37	69.20	92.08	736.33

Table 7.16: MASE Comparison all best Models

		Cluster 0	Cluster 1	Cluster 2	Cluster 3	Total Set
Near	ES	**0.84**	**0.89**	**0.93**	**0.91**	**0.80**
	ARIMA	1.12	1.14	1.10	1.15	1.07
	SVR	1.46	1.58	1.29	1.65	1.40
	MLP	1.69	1.68	1.43	1.70	1.55
Middle	ES	2.99	2.13	2.07	1.71	2.73
	ARIMA	**0.45**	**0.94**	**1.06**	**1.06**	1.58
	SVR	1.38	1.43	1.14	1.40	**1.28**
	MLP	1.88	1.89	1.60	1.78	1.80
Far	ES	3.31	2.65	2.46	2.35	3.31
	ARIMA	0.97	0.90	**0.52**	**0.50**	0.94
	SVR	**0.75**	**0.77**	0.65	0.87	**0.73**
	MLP	0.83	0.84	0.68	0.95	0.76

To evaluate if cluster selection outperforms aggregate selection, the best forecast model is selected for each cluster. The choice will be made on the middle and far future performance since performances between models do not differ much in the near future. Cluster 2 and 3 are forecasted based on the log transformed ARIMAX model with multivariate set 1 since this method outperforms other ARIMA methods in these clusters. Since cluster 0 and 1 both prefer the ARIMA model on the middle future and the SVR model on the far future, a choice should be made between these two models. The negative impact on performance is relatively small when a choice is made for the ARIMA model instead of the SVR model in the far future, so the ARIMA model is applied. The forecasts are based on the SARIMA model without any data transformations since this method outperforms other ARIMA methods in both clusters.

Next, forecasts are made for every cluster with the best possible method, and the performance is compared to that of aggregate selection. The performance of the aggregate selection was already shown in the previous tables in the total set column, where it was shown that SVR with multivariate set 2 outperformed other methods. Table 7.17 and Table 7.18 show the results in terms of RMSE and MASE of the cluster and aggregate selection on the near, middle and far future. It can be seen that aggregate selection outperforms cluster selection in terms of RMSE and MASE. However, both methods achieve the best results in the far future.

Table 7.17: RMSE Comparison Cluster Selection - Aggregate Selection

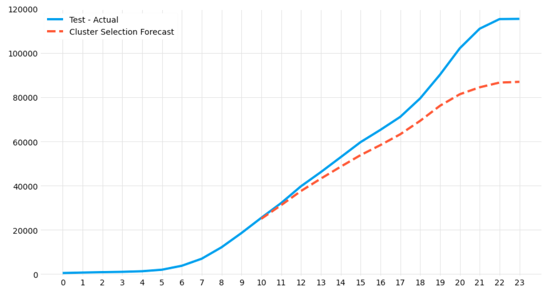
	Cluster Selection	Aggregate Selection
Near	1200.37	789.53
Middle	1710.44	1274.88
Far	1090.08	711.32

Table 7.18: MASE Comparison Cluster Selection - Aggregate Selection

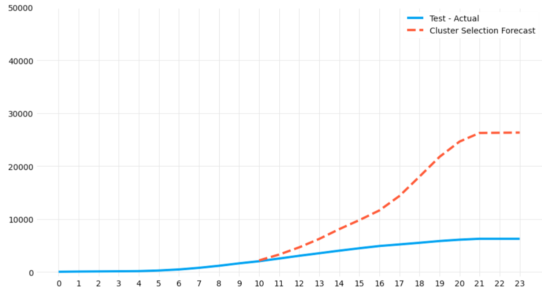
	Cluster Selection	Aggregate Selection
Near	1.62	0.80
Middle	2.17	1.28
Far	1.40	0.73

To compare the forecast results with the current forecast method of bol.com, performances in terms of MAPE are measured on 4 selected days. These days contain a normal weekday, Saturday, promotion day and holiday, thus include 'normal' and 'special' days. By this approach, the models are assessed on their prediction capability on several days with different characteristics. The current method is assessed on its prediction capability at 10:00AM in the morning to forecast the end of the day total. Therefore, the predictions of the models are assessed in the same way, so forecasts are made at 10:00AM for 14 steps ahead. The end of the day totals of the actual and forecasted sales are compared with each other in terms of MAPE to examine if the predictions are able to improved the MAPE of around 10% of the current method.

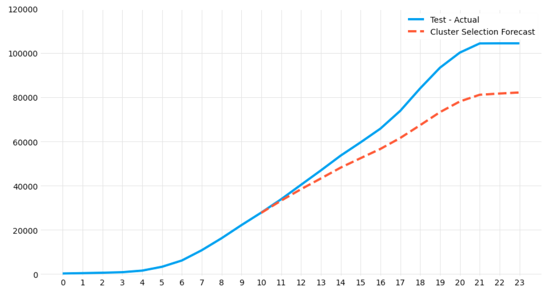
Figure 7.9 shows the four example days of 14 steps ahead cumulative forecasts for the cluster selection approach. It can be seen that a MAPE of 24.67% is achieved on a normal weekday and a MAPE of 12.29% is achieved on a promotion day. The Saturday and holiday have a very high MAPE of above 300% which could be explained by the disability of the ARIMA model to deal with double seasonality and abrupt fall downs in sales. Therefore, cluster selection is not able to outperform the current forecast method. However, on normal and promotion days it has the potential to outperform since it is not far from the desired output of 10%.



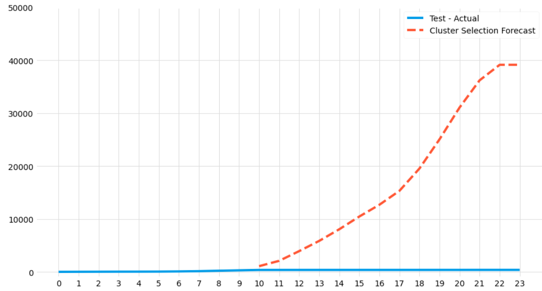
(a) Weekday (Nov 13, Wednesday)
MAPE 24.67%



(b) Weekend (Oct 19, Saturday)
MAPE 320.32%

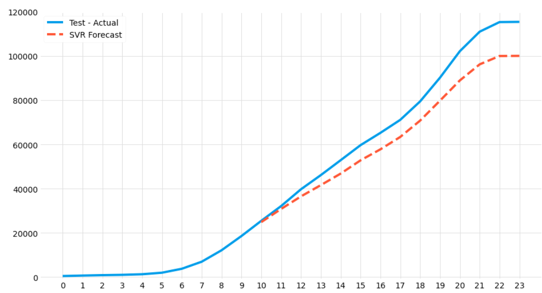


(c) Promotion (Oct 2, Tuesday)
MAPE 12.29%

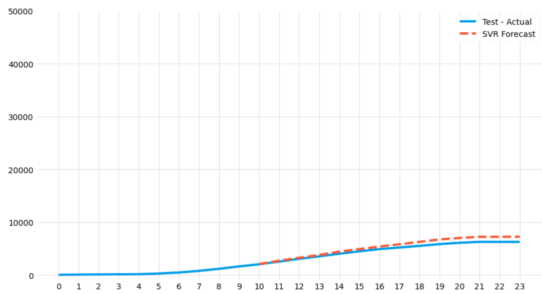


(d) Holiday (Dec 25, Wednesday)
MAPE >1000%

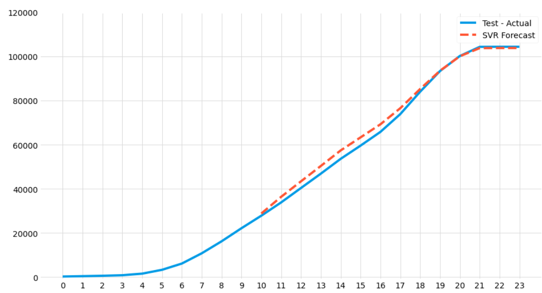
Figure 7.9: Cumulative Forecasts Cluster Selection SARIMA, ARIMAX



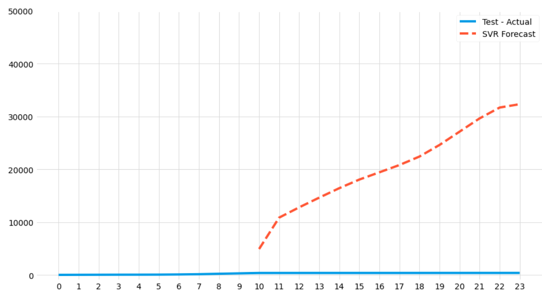
(a) Weekday (Nov 13, Wednesday)
MAPE 13.32%



(b) Weekend (Oct 19, Saturday)
MAPE 15.31%



(c) Promotion (Oct 2, Tuesday)
MAPE 0.54%



(d) Holiday (Dec 25, Wednesday)
MAPE >1000%

Figure 7.10: Cumulative Forecasts Aggregate Selection SVR

Figure 7.10 shows the four example days of 14 steps ahead cumulative forecasts of the aggregate selection approach. The figures show a very well performed forecast during the weekday, Saturday and promotion day with a MAPE of 13.32%, 15.31% and 0.54% respectively. The holiday shows a very high MAPE of above 1000%. Aggregate selection achieves good results but is still not able to outperform the current forecast method on each day. However, the promotion day achieved perfect results and the weekday and Saturday are very close by. Therefore, aggregate selection has the potential to improve the current forecast method.

7.5.2 Discussion

When inspecting the overall results, it can be concluded that statistical models outperform on clusters, while machine learning, especially SVR, outperformed on the total aggregate set. Statistical models are able to capture simple patterns by inclusion of a limited number of parameters, which was conducted in the clustered data sets. It could be seen that the statistical models were able to capture the daily seasonality very well, but failed when a special day occurred. Machine learning models include many parameters to search for complex patterns, which results in an SVR that achieved the best results on the total aggregate set. However, MLP only achieves good results in the far future, but fails in the prediction in the near and middle future. A more extensive optimization in hyper parameter tuning might lead to better results of the MLP.

A combination of ARIMA models were chosen for the cluster selection, while a SVR model was chosen for the aggregate selection. It showed that aggregate selection with an average MASE of 0.93 outperforms cluster selection, which achieved an average MASE of 1.73. This might be caused by the fact that the clusters do not contain higher correlations with external variables in comparison to the correlations with external variables of the total aggregate set. Therefore, cluster selection does not contribute to the improvement of the forecasts. It might be more viable to split the clusters into clusters that only contain a single or two product groups. However, this approach might be time consuming since a lot of models should be trained. Therefore, it can be concluded that the overall best performance is achieved by aggregate selection of SVR.

Chapter 8

Implementation

This chapter describes how bol.com can practically implement the forecasting model in their systems to derive forecasts. Next, it is explained how the forecasting model should be integrated in the daily steering process.

8.1 Practical Implementation

As discussed in the previous chapter, SVR derives the best forecast results by aggregate selection. The model is easy to implement, there is no need to transform data and it has a low computation time. Therefore, SVR is the most suitable for bol.com to use for short-term forecasting of 24 hour sales. The best performing multivariate set for the SVR model contains lever, calendar and weather effects. A Python file can be provided to bol.com that enables forecasting for the total aggregate group using the SVR model. Bol.com keeps responsible for feeding the model with external variables that might have impact on the sales in the future and are not fixed like holidays. This includes the collection and updating of for example the weather forecasts, scheduled promotions and activated levers.

8.2 Integration in Processes

The forecasts for the rest of the day should be derived at the beginning of the day and should be updated every 30 minutes just as the current method. It is most important that before 10:00AM, employees at S&OP have collected and updated the exogenous variables from different sources in an Excel file so the model can fit a good model for the rest of the day. In this way, forecast practitioners can use the SVR forecasts at 10:00AM to make adjustments to their warehouse operations plan. During the rest of the day, the model can be used as a guideline for the process in which domain knowledge of business experts should be added to maintain the best decision making in daily operations. With the high speed IT-services of bol.com, it is recommended to run the model every 10 minutes to maintain highly accurate forecasts and be able to steer in the right directions on time.

Chapter 9

Conclusions

In this comparitive study, extenstive research on time series forecasting methods, data transformations, forecasting techniques, multi-product approaches, accuracy measures and exogenous variables is conducted to develop a highly accurate forecasting model that creates short-term forecasts with short time series. The forecasts of the model are aimed to give guidance to practitioners in the daily steering process of bol.com logistics. An elaborate study was performed to form a good foundation for this study. Several model types and configurations have been tested and evaluated in this research. The aim was to answer the following main research question:

How to increase accuracy in the short-term forecast of the intake of 24 hour sales in an online multi-product retail environment?

Several sub questions were answered to structurally achieve the answer on the main research question. Upon these results, recommendations and conclusions are formed for general short-term time series forecasting and bol.com specific. This chapter outlines the answers to the research questions, the recommendations and discusses the limitations of the research.

9.1 Research Questions

1. *What are possible influential factors effecting online sales?*

Literature study point to several exogenous variables that have influence on short-term forecasting. Several researches that focus on load forecasting, points to calendar and weather variables as important exogenous variables (Kim et al., 2019; Lydia et al., 2016). Additionaly, Arunraj and Ahrens (2015) includes holiday effects, promotional effects and weather effects in their research of forecasting daily sales of a perishable food. Steinker et al. (2017) proves in their research that including weather information in forecasting significantly improves forecast accuracy. As a result of the literature research calendar, promotions and weather effects are further discovered. Additionally, lever effects that are internally retrieved are added to the exogenous variables. It was concluded that sufficient data was available on all exogenous variables so for each variable, Pearson's correlation coefficients are determined. As a result, the following relevant exogenous variables were identified: WarehouseClosed (lever), WeekbeforeChristmasandSinterklaas, PromotionWeek, Autumn, November, December, MonTueWed, Saturday, Temperature, HoursSun and HeatingDegree. Additionally, two interaction variables are included: Temperature/Month9 and Temperature/Month10. Three multivariate sets were created that were used for model construction for each aggregate group. The first multivariate set consist of the lever and calendar effects. The second multivariate set includes lever, calendar and weather effects and interaction variables are added to the third set.

2. *How should a multi-product environment be handled within time series forecasting?*

To deal with a multi-product environment, time series clustering can be very useful prior to forecasting. Firstly, homogeneous clusters can be identified within the data set. Next, the best performing forecasting methods in terms of accuracy should be selected for each cluster. Such effective clustering and then forecasting has the potential to outperform aggregate selection, which selects a single forecasting method for the entire data set (Vangumalli et al., 2019; Dantas & Oliveira, 2018). In addition, clustering prior to forecasting could save a lot of time and money (Vangumalli et al., 2019). Therefore, to deal with the multi-product environment, clusters are formed based on several clustering techniques combined with different distance metrics. A combination of shape-based and characteristic based clustering gave the best clusters. Four clusters were formed with four clearly different shapes: early increase of sales with peak period, late increase of sales with peak, summer peak sales, constant sales. The clusters are used to discover the forecast performance when selecting the best performing forecasting method for each cluster against selecting a single forecasting method for the entire data set.

3. *What statistical- and machine learning models can be applied for short term time series sales forecasting and how can parameters be optimized?*

Extensive literature research on short term forecasting techniques is conducted to answer this research question. Literature review identifies different predictive models to address a forecasting problem. They range from standard regression and time series approaches to methods that use machine learning algorithms. Main models including model variations of time series forecasting models are explained and reviewed. Four models are selected based on formulated selection criteria, including the model performance in previous studies, applicability to the available data and the ability to deal with a small amount of data, seasonality and exogenous variables. Exponential smoothing (ES) and autoregressive integrated moving average (ARIMA) were selected as appropriate statistical models and support vector regression (SVR) and a neural network - multilayer perceptron (MLP) are selected as machine learning models. For all models, grid search is applied to find the optimal parameters.

4. *What techniques exist to create multi-step ahead time series forecasts and which techniques are appropriate for short term sales forecasting?*

A literature research on multi-step ahead forecasting techniques was performed to answer this research question. A multi-step ahead forecasting technique is required to predict multiple steps into the future. The literature research provided four popular strategies that can be applied to multi-step ahead forecasting: Direct, Recursive, Direct Recursive and Multiple Output (MIMO). Research of Taieb et al. (2012) shows that the multiple-output strategy is invariably the best strategy. They beat single-output strategies such as direct, recursive and direct recursive. Since the multiple-output strategy invariably outperforms the single-output strategies, the multiple-output strategy was selected as most appropriate technique for short term sales forecasting.

5. *What accuracy measures are available to evaluate forecasting performance and which accuracy measures are appropriate to evaluate short term sales forecasts?*

A literature research on multi-step ahead forecasting techniques was performed to answer this research question and identify most widely used accuracy measures in short term forecasting. The literature research provided four popular accuracy measures: MSE, RMSE, MAE and MAPE. It was observed that most articles from the literature study used the mean average percentage error (MAPE) and root mean squared error (RMSE). However, since the data contains many zero's and MAPE is not able to deal with these, the MASE is proposed instead. Therefore, RMSE and MASE were selected as appropriate accuracy measures for short term sales forecasting.

6. *What combination of input variables, forecasting model, data transformation(s) and parameter configuration achieves the most accurate results?*

Main findings of previous sub questions are combined and applied to actual sales data of bol.com sales to derive an answer for this last sub question. Different data transformations and multivariate sets are applied to the several forecasting models to discover what combination yields the best results. In the near future, exponential smoothing outperforms other models, while in the middle and far future, ARIMA and SVR tend to outperform. All clusters prefer ARIMA in the middle future and might change to SVR in the far future. However, ARIMA outperforms in most cases and closely follows SVR. A combination of ARIMA models were chosen for the cluster selection, while a SVR model optimized by grid search was chosen for the aggregate selection. It showed that aggregate selection with an average MASE of 0.93 outperforms cluster selection, which achieved an average MASE of 1.7. In terms of exogenous variables, it is found that interaction variables do not increase the performance of the forecasting method. Therefore, multivariate set 1 and 2 perform best, depending on the aggregate group and the forecast horizon. For aggregate selection based on SVR, multivariate set 1 yields the best results so including weather variables do not contribute to the performance. Additionally, SVR does not require any data transformation in advance. Therefore, SVR is selected as the best performng model to forecast short term 24 hour sales of bol.com. However, it must be noted that the SVR model might be used as guideline for the process in which domain knowledge of business experts should be added to maintain the best decision making in daily operations.

9.2 Recommendations

The conducted research and developed forecasting models provide some general and bol.com specific recommendations which are discussed in this section.

Longer time series

Time series forecasting yields better results if longer time series are provided as the model input. It is common sense that this improves the forecasting, since more information can be retrieved by the model from the historical data. Therefore, it would be preferred to include more sales data, so increase the data set to two or three years of sales.

Use forecasts as guideline

The designed forecast method can be used as a guideline for the daily operations process. Since bol.com operates in a fast moving, complex environment domain knowledge of business experts is essential. Especially on the short term, business experts have more information on the current pipeline of incoming orders. It is strongly believed that the forecasting model can improve daily steering by providing the business experts with information on the 24 hour sales. Domain knowledge of business experts should be added to maintain the best decision making.

Update every 10 minutes

Since the model requires some computation time, high speed IT-services are required to update the forecast frequently. It is recommended to run the model every 10 minutes to maintain highly accurate forecasts and be able to steer in the right directions on time.

Include calendar and weather data

The current method only includes basic calendar effects such as weekday and current time. It is recommended to include additional calendar effects and weather effects in the forecast model since it proves to outperform models that do not include any additional variables. This will bring the model to more detail and more meaningfull insights.

9.3 Discussion

The gap between research and practice provides some boundaries which are discussed in this section by revisiting the model's assumptions and stating its limitations. Additionally, it suggests directions and interesting areas for future research.

9.3.1 Limitations and Future Research

Warehouse assumption

The IT landscape that assigns customer orders to warehouse is considered as given. The fulfillment network systems (FNK) assigns orders to different warehouses based on several rules and shop order characteristics. This is only possible if stock is located at multiple warehouses which is only a small percentage. Therefore, it is assumed that the dynamic and biased choice of FNK can be neglected. However, since bol.com keeps on increasing double stock and moving stock, it is important to take this into account. When a forecast is built based on product group, the model should be able to denote if the product is still available and located at the warehouse before it gives a prediction. This is not considered in modelling but might be required in future modelling to maintain accuracy.

Warehouse exclusion

The focus of this research was only on BFC, the biggest warehouse of bol.com. All other warehouses were excluded from the research which do not reflect the real situation of bol.com which keeps on increasing in warehouses. Therefore, in combination with the above mentioned suggestion, the model should be extended to be able to deal with the multi warehouse environment in which stock is allocated at multiple locations.

Data of the year 2019

This research contains a data set from the year of 2019. Since there are many changes in the way of living and the way of shopping due to the Corona virus, the data set might not be representative anymore to make predictions for the upcoming years. Other external variables might be needed to maintain a highly accurate forecast and be able to detect special events.

Exogenous variables

Exogenous variables are included that have a really high impact on sales. For example, the holidays are previous known 'special' days where 24 hour sales never occur. A high weight should be assigned to this variable to make sure the model will always take the variable into account. In this research, all external variables are equally weighted and the model tries to find the most important variables itself. Since the correlations with external variables were small, finding the most important variables might be hard. Therefore, it might be useful in future research to assign weights to external variables to help the model in finding the most important variables.

Lever exclusion

Only 'STOCK' and 'DLY' lever impact are included in the model. These levers are used in most of the cases and have the highest impact. However, it might be interesting and useful to include other levers as well to increase forecast performance. Especially in a year as 2020 in which many levers are activated, it might be required to include the other levers. To include the levers, experts have to provide the model with well registered lever activations from the past periods including all details, e.g. on which products it has impact.

Cluster approach

This research concluded that aggregate selection outperforms cluster selection. This might be caused by the fact that the clusters do not contain higher correlations with external variables in comparison to the correlations with external variables of the total aggregate set. Therefore, cluster selection does not contribute to the improvement of the forecasts. It might be more viable to split the clusters into smaller clusters that only contain a single or two product groups that have exact the same behavior. With this approach, the model can capture its behavior, find higher correlations and improve forecasts. However, this approach might be time consuming since a lot of models should be trained.

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Appendices

Appendix A

Distance Matrices

Table A.1: Euclidean Distance Matrix - Part 1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0.0	43.521	46.717	107.313	45.296	34.103	27.756	93.486	37.368	37.905	51.418	26.671	101.189	29.663	44.567	65.652	70.905	81.892	44.085
1	43.521	0.0	70.864	135.518	73.398	50.762	30.549	118.918	19.545	57.405	69.178	42.465	116.999	43.406	7.458	89.33	94.801	97.659	73.125
2	46.717	70.864	0.0	71.037	41.487	45.437	53.223	56.596	73.06	26.393	31.658	44.1	62.684	48.638	74.054	38.729	35.071	44.792	38.203
3	107.313	135.518	71.037	0.0	83.847	109.072	119.82	26.852	141.266	89.205	73.759	111.546	46.613	110.147	139.314	58.033	47.181	50.013	79.222
4	45.296	73.398	41.487	83.847	0.0	41.932	49.139	70.992	64.574	36.765	44.924	40.475	80.116	39.973	74.879	52.339	53.354	65.474	47.31
5	34.103	50.762	45.437	109.072	41.932	0.0	25.955	94.891	37.43	29.179	46.954	16.817	95.901	28.394	51.661	62.052	69.83	77.5	45.762
6	27.756	30.549	53.223	119.82	49.139	25.955	0.0	104.642	24.077	34.204	51.331	18.934	104.096	21.553	32.635	70.398	76.977	84.904	51.17
7	93.486	118.918	56.596	26.852	70.992	94.891	104.642	0.0	126.256	72.728	60.215	95.237	49.214	96.558	123.48	42.924	32.389	46.673	66.918
8	37.368	19.545	73.06	141.266	64.574	37.43	24.077	126.256	0.0	54.783	71.587	32.382	125.598	34.614	18.385	91.418	99.528	106.392	70.45
9	37.905	57.405	26.393	89.205	36.765	29.179	34.204	72.728	54.783	0.0	36.275	24.388	76.115	36.407	59.973	39.158	47.205	57.585	34.689
10	51.418	69.178	31.658	73.759	44.924	46.954	51.331	60.215	71.587	36.275	0.0	45.975	58.22	49.195	73.123	43.77	35.108	40.75	49.685
11	26.671	42.465	44.1	111.546	40.475	16.817	18.934	95.237	32.382	24.388	45.975	0.0	97.072	21.608	43.617	60.108	70.26	77.725	43.366
12	101.189	116.999	62.684	46.613	80.116	95.901	104.096	49.214	125.598	76.115	58.22	97.072	0.0	96.471	121.867	54.276	42.297	25.941	85.171
13	29.663	43.406	48.638	110.147	39.973	28.394	21.553	96.558	34.614	36.407	49.195	21.608	96.471	0.0	44.946	66.823	69.941	78.846	49.42
14	44.567	7.458	74.054	139.314	74.879	51.661	32.635	123.48	18.385	59.973	73.123	43.617	121.867	44.946	0.0	92.695	99.163	102.75	75.284
15	65.652	89.33	38.729	58.033	52.339	62.052	70.398	42.924	91.418	39.158	43.77	60.108	54.276	66.823	92.695	0.0	32.539	48.457	51.452
16	70.905	94.801	35.071	47.181	53.354	69.83	76.977	32.389	99.528	47.205	35.108	70.26	42.297	69.941	99.163	32.539	0.0	32.329	52.943
17	81.892	97.659	44.792	50.013	65.474	77.5	84.904	46.673	106.392	57.585	40.75	77.725	25.941	78.846	102.75	48.457	32.329	0.0	70.104
18	44.085	73.125	38.203	79.222	47.31	45.762	51.17	66.918	70.45	34.689	49.685	43.366	85.171	49.42	75.284	51.452	52.943	70.104	0.0
19	39.096	56.718	22.99	83.469	40.894	35.643	41.601	66.677	61.864	20.595	31.264	31.478	69.133	40.019	60.513	40.921	43.861	51.527	35.3
20	32.371	37.134	42.72	105.51	48.735	35.75	26.977	90.357	39.187	35.632	43.696	28.044	90.86	31.848	39.829	60.895	65.704	73.87	46.551
21	34.535	32.592	57.141	113.112	65.368	52.934	38.366	100.509	39.849	56.221	58.369	44.493	96.914	42.453	34.06	81.098	78.748	78.904	62.495
22	75.632	95.575	37.019	54.745	62.02	75.136	83.176	46.031	104.561	53.848	42.906	75.297	40.086	75.275	100.891	49.315	32.63	29.374	58.991
23	77.653	103.848	41.263	34.599	57.75	79.468	88.703	24.28	109.773	58.303	45.248	80.359	48.339	79.246	108.628	40.087	25.291	38.137	55.902
24	114.235	140.299	78.022	40.92	91.363	114.018	126.004	41.355	146.436	95.184	85.378	117.518	62.345	115.492	144.772	67.73	59.142	64.756	87.63
25	43.179	35.036	71.213	134.702	60.776	38.25	28.636	120.731	18.978	53.608	71.007	35.3	118.455	39.083	34.565	84.335	94.785	102.29	70.784
26	28.374	41.867	37.466	103.845	40.653	23.534	21.529	88.662	39.825	23.512	41.879	18.665	88.616	27.922	44.774	57.056	63.713	69.203	39.177
27	32.281	27.613	62.088	130.166	54.539	27.644	15.596	113.859	14.546	41.752	61.492	19.801	114.679	25.153	28.232	78.632	88.689	95.342	59.091
28	49.187	77.658	21.17	65.825	39.544	49.066	57.61	50.766	77.876	27.77	32.563	47.572	63.11	51.419	80.597	31.682	31.03	46.833	35.546
29	115.36	140.523	77.234	22.527	92.555	116.179	126.735	32.907	148.318	96.304	79.281	118.892	39.467	116.616	145.614	66.124	52.448	47.376	88.429
30	42.889	57.199	22.37	80.909	41.637	38.798	43.338	64.378	63.341	24.465	27.698	36.423	63.722	43.819	61.447	39.628	41.428	44.538	43.933
31	28.699	39.361	41.092	107.35	43.505	24.142	20.4	92.465	37.41	29.602	45.456	16.97	90.714	25.28	41.817	63.143	69.095	71.13	43.372
32	66.64	88.297	30.129	54.733	55.933	67.228	75.193	37.636	96.539	44.559	37.442	66.102	51.079	69.034	93.539	39.614	26.306	38.124	48.749
33	43.737	63.252	21.147	77.312	44.374	41.16	47.403	61.121	68.231	23.45	31.38	39.294	67.015	46.805	67.681	38.236	36.67	49.759	33.218
34	31.765	34.5	58.681	126.063	51.864	25.373	18.753	109.762	19.675	38.789	57.467	17.718	111.081	26.717	34.986	74.533	84.526	91.654	57.342
35	86.759	107.661	48.348	43.292	69.779	85.048	93.841	42.214	115.104	64.027	47.844	85.704	27.135	86.129	111.65	46.72	31.982	27.034	70.94
36	67.076	87.61	50.654	74.087	56.098	55.382	63.912	64.348	80.42	46.313	47.373	57.858	62.293	62.072	89.241	44.694	49.14	55.015	62.685
37	55.957	83.371	26.297	58.388	41.081	56.64	64.807	42.53	85.741	32.907	34.209	54.237	58.818	57.143	86.303	29.736	25.239	43.703	39.994
38	78.342	103.869	44.288	38.571	62.391	79.802	89.277	30.019	110.211	57.733	46.307	79.139	51.032	79.912	108.036	46.063	32.008	39.219	56.348
39	26.745	31.567	55.211	123.241	48.172	21.426	10.355	106.934	20.907	34.679	54.321	13.209	107.753	19.414	32.661	71.707	81.701	88.418	51.936
40	75.868	98.036	37.867	44.092	58.707	76.852	84.828	29.802	106.539	53.107	42.76	75.744	44.88	76.84	103.161	39.271	24.003	32.648	55.862
41	28.949	42.29	45.666	112.808	37.604	20.051	18.696	97.931	31.156	29.163	49.196	16.217	98.109	16.075	43.673	63.134	70.294	79.566	46.051
42	117.689	144.107	81.327	27.32	92.555	119.125	130.326	36.671	151.913	99.127	85.544	121.522	57.237	120.524	148.704	68.943	60.338	59.117	88.945
43	54.61	74.929	21.999	66.929	50.43	54.778	62.123	51.08	83.293	35.85	34.856	53.028	56.901	56.875	80.187	44.543	33.724	42.076	43.216
44	37.87	54.292	38.282	97.018	41.231	29.816	32.48	82.758	46.732	26.601	34.143	28.005	81.768	34.091	55.777	50.412	56.689	62.996	47.732
45	84.6	106.108	48.125	42.882	71.813	84.525	93.776	38.255	114.935	64.933	49.064	85.324	44.827	85.109	111.479	53.571	36.035	33.584	66.005
46	40.694	60.205	24.591	82.494	40.876	33.79	39.635	67.119	60.804	14.374	33.195	31.509	71.875	42.188	63.309	35.61	40.933	54.316	32.01
47	95.786	113.386	55.649	46.721	77.468	92.866	100.947	45.968	123.006	71.319	54.436	93.505	21.687	92.864	118.593	50.362	36.37	23.682	82.459
48	29.108	31.009	47.114	111.143	51.6	35.025	23.75	96.415	31.424	38.78	43.883	29.126	95.557	31.092	34.051	67.194	69.602	76.241	53.213
49	53.674	67.912	26.768	73.236	47.896	49.168	53.985	57.339	75.281	34.313	22.467	47.544	52.022	51.252	72.61	45.327	35.352	34.402	51.426
50	30.423	40.009	50.329	118.037	42.319	18.545	16.042	101.729	26.694	30.344	51.57	11.982	102.956	19.753	41.031	66.51	76.714	83.995	48.71
51	82.64	108.17	47.327	34.141	61.695	83.302	93.278	18.862	113.849	61.061	49.731	83.066	43.667	84.193	112.357	33.721	23.717	39.969	60.047
52	103.691	127.33	64.922	25.087	82.224	104.491	114.394	23.607	136.019	83.701	68.025	106.355	39.89	106.145	132.268	52.078	41.199	43.479	78.499
53	78.064	96.579	41.791	51.419	60.84	72.816	81.795	40.803	102.194	53.411	42.402	73.666	33.787	75.859	100.883	36.565	28.744	30.163	63.812
54	26.397	48.739	33.444	97.336	36.177	25.644	27.194	83.154	45.796	26.012	34.183	20.19	83.891	24.59	50.9	52.732	56.883	65.235	39.442
55	30.183	35.726	52.977	120.683	45.733	20.098	12.889	104.352	23.67	32.356	53.459	12.686	105.422	21.1	36.819	69.102	79.084	86.264	50.189
56	65.217	87.293	28.783	54.783	57.254	65.983	73.789	38.683	94.851	44.81	36.625	65.638	52.251	68.07	92.179	41.511	27.272	37.571	46.319

Table A.2: Euclidean Distance Matrix - Part 2

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
0	39.096	32.371	34.535	75.632	77.653	114.235	43.179	28.374	32.281	49.187	115.36	42.889	28.699	66.64	43.737	31.765	86.759	67.076	55.957
1	56.718	37.134	32.592	95.575	103.848	140.299	35.036	41.867	27.613	77.658	140.523	57.199	39.361	88.297	63.252	34.5	107.661	87.61	83.371
2	22.99	42.72	57.141	37.019	41.263	78.022	71.213	37.466	62.088	21.17	77.234	22.37	41.092	30.129	21.147	58.681	48.348	50.654	26.297
3	83.469	105.51	113.112	54.745	34.599	40.92	134.702	103.845	130.166	65.825	22.527	80.909	107.35	54.733	77.312	126.063	43.292	74.087	58.388
4	40.894	48.735	65.368	62.02	57.75	91.363	60.776	40.653	54.539	39.544	92.555	41.637	43.505	55.933	44.374	51.864	69.779	56.098	41.081
5	35.643	35.75	52.934	75.136	79.468	114.018	38.25	23.534	27.644	49.066	116.179	38.798	24.142	67.228	41.16	25.373	85.048	55.382	56.64
6	41.601	26.977	38.366	83.176	88.703	126.004	28.636	21.529	15.596	57.61	126.735	43.338	20.4	75.193	47.403	18.753	93.841	63.912	64.807
7	66.677	90.357	100.509	46.031	24.28	41.355	120.731	88.662	113.859	50.766	32.907	64.378	92.465	37.636	61.121	109.762	42.214	64.348	42.53
8	61.864	39.187	39.849	104.561	109.773	146.436	18.978	39.825	14.546	77.876	148.318	63.341	37.41	96.539	68.231	19.675	115.104	80.42	85.741
9	20.595	35.632	56.221	53.848	58.303	95.184	53.608	23.512	41.752	27.77	96.304	24.465	29.602	44.559	23.45	38.789	64.027	46.313	32.907
10	31.264	43.696	58.369	42.906	45.248	85.378	71.007	41.879	61.492	32.563	79.281	27.698	45.456	37.442	31.38	57.467	47.844	47.373	34.209
11	31.478	28.044	44.493	75.297	80.359	117.518	35.3	18.665	19.801	47.572	118.892	36.423	16.97	66.102	39.294	17.718	85.704	57.858	54.237
12	69.133	90.86	96.914	40.086	48.339	62.345	118.455	88.616	114.679	63.11	39.467	63.722	90.714	51.079	67.015	111.081	27.135	62.293	58.818
13	40.019	31.848	42.453	75.275	79.246	115.492	39.083	27.922	25.153	51.419	116.616	43.819	25.28	69.034	46.805	26.717	86.129	62.072	57.143
14	60.513	39.829	34.06	100.891	108.628	144.772	34.565	44.774	28.232	80.597	145.614	61.447	41.817	93.539	67.681	34.986	111.65	89.241	86.303
15	40.921	60.895	81.098	49.315	40.087	67.73	84.335	57.056	78.632	31.682	66.124	39.628	63.143	39.614	38.236	74.533	46.72	44.694	29.736
16	43.861	65.704	78.748	32.63	25.291	59.142	94.785	63.713	88.689	31.03	52.448	41.428	69.095	26.306	36.67	84.526	31.982	49.14	25.239
17	51.527	73.87	78.904	29.374	38.137	64.756	102.29	69.203	95.342	46.833	47.376	44.538	71.13	38.124	49.759	91.654	27.034	55.015	43.703
18	35.3	46.551	62.495	58.991	55.902	87.63	70.784	39.177	59.091	35.546	88.429	43.933	43.372	48.749	33.218	57.342	70.94	62.685	39.994
19	0.0	30.257	47.032	46.791	53.261	90.66	62.13	25.012	49.542	26.676	90.142	21.455	28.722	37.567	19.391	46.426	57.208	53.331	30.521
20	30.257	0.0	34.037	68.991	75.124	112.407	46.357	25.901	32.799	46.917	112.375	38.946	24.585	60.631	38.164	34.513	79.688	65.358	52.698
21	47.032	34.037	0.0	75.363	85.239	121.346	50.173	41.204	41.433	63.768	118.454	50.857	38.122	71.588	56.133	44.716	88.188	84.176	68.835
22	46.791	68.991	75.363	0.0	31.375	65.007	103.142	67.546	93.88	34.589	50.293	45.897	69.707	22.669	38.05	89.823	32.029	66.964	31.317
23	53.261	75.124	85.239	31.375	0.0	50.236	105.603	72.939	98.869	36.066	40.554	51.453	77.613	26.086	45.865	94.803	34.383	56.886	28.566
24	90.66	112.407	121.346	65.007	50.236	0.0	140.491	109.421	135.762	73.976	43.763	87.204	112.912	62.346	84.061	131.793	59.301	86.774	67.897
25	62.13	46.357	50.173	103.142	105.603	140.491	0.0	40.08	19.711	75.508	143.208	59.388	40.756	94.667	68.448	23.72	108.715	74.251	82.64
26	25.012	25.901	41.204	67.546	72.939	109.421	40.08	0.0	28.727	43.282	110.868	28.574	10.453	59.752	32.968	30.51	77.558	57.821	48.903
27	49.542	32.799	41.433	93.88	98.869	135.762	19.711	28.727	0.0	66.009	137.514	52.417	26.291	84.7	57.437	11.711	104.103	70.276	72.834
28	26.676	46.917	63.768	34.589	36.066	73.976	75.508	43.282	66.009	0.0	72.772	31.073	48.71	25.568	25.968	62.13	46.659	48.539	15.813
29	90.142	112.375	118.454	50.293	40.554	43.763	143.208	110.868	137.514	72.772	0.0	86.111	113.577	56.015	81.132	133.41	43.627	80.628	65.545
30	21.455	38.946	50.857	45.897	51.453	87.204	59.388	28.574	52.417	31.073	86.111	0.0	32.251	38.871	23.213	48.92	54.654	48.242	35.118
31	28.722	24.585	38.122	69.707	77.613	112.912	40.756	10.453	26.291	48.71	113.577	32.251	0.0	63.785	38.025	27.935	81.019	61.478	54.525
32	37.567	60.631	71.588	22.669	26.086	62.346	94.667	59.752	84.7	25.568	56.015	38.871	63.785	0.0	29.41	80.627	36.876	59.752	21.577
33	19.391	38.164	56.133	38.05	45.865	84.061	68.448	32.968	57.437	25.968	81.132	23.213	38.025	29.41	0.0	54.613	52.979	54.643	29.363
34	46.426	34.513	44.716	89.823	94.803	131.793	23.72	30.51	11.711	62.13	133.41	48.92	27.935	80.627	54.613	0.0	100.055	66.162	68.775
35	57.208	79.688	88.188	32.029	34.383	59.301	108.715	77.558	104.103	46.659	43.627	54.654	81.019	36.876	52.979	100.055	0.0	57.849	42.492
36	53.331	65.358	84.176	66.964	56.886	86.774	74.251	57.821	70.276	48.539	80.628	48.242	61.478	59.752	54.643	66.162	57.849	0.0	47.957
37	30.521	52.698	68.835	31.317	28.566	67.897	82.64	48.903	72.834	15.813	65.545	35.118	54.525	21.577	29.363	68.775	42.492	47.957	0.0
38	51.625	75.34	85.969	35.402	22.308	56.225	108.132	72.529	97.761	37.464	42.459	53.904	77.047	29.404	47.925	93.656	38.575	59.398	28.7
39	42.264	27.81	39.48	86.955	91.944	128.884	25.179	21.518	9.053	59.039	130.588	45.286	19.147	77.774	50.351	12.584	97.177	64.276	65.909
40	46.738	69.771	78.263	21.976	19.875	54.366	103.246	68.803	94.335	31.708	45.713	46.1	72.273	15.53	39.988	90.276	30.99	60.324	25.172
41	36.107	30.623	45.191	76.875	80.937	117.658	32.482	21.154	20.975	49.541	119.479	39.119	20.587	68.735	41.802	22.381	87.923	57.055	56.97
42	92.91	116.011	123.115	60.984	45.906	42.604	146.736	114.084	140.13	75.909	31.664	90.664	117.04	62.391	87.86	136.053	53.543	85.606	67.88
43	25.888	48.772	58.546	26.229	37.305	74.749	83.329	46.04	71.551	24.064	69.956	31.933	49.694	21.695	22.609	68.007	41.734	62.465	27.422
44	33.572	40.316	52.223	65.222	67.007	103.727	43.861	29.698	35.925	39.616	104.373	28.298	31.862	57.142	38.593	32.634	71.284	43.021	45.446
45	57.677	80.334	88.342	31.007	28.295	58.716	113.166	77.253	103.872	45.714	40.266	56.533	79.59	34.504	50.187	99.782	37.209	64.018	39.329
46	20.564	36.077	56.886	47.802	51.677	88.786	59.478	27.516	49.497	26.161	89.076	24.222	34.071	38.148	15.918	46.058	58.299	47.322	30.15
47	64.889	87.824	92.974	31.866	44.653	62.491	114.823	85.073	111.905	56.524	41.434	60.165	87.316	44.168	58.954	107.824	28.616	64.996	52.265
48	35.554	25.948	30.19	75.06	80.448	118.044	36.81	29.708	29.713	53.891	118.28	36.241	27.528	67.55	41.507	27.757	85.83	66.367	59.919
49	26.485	45.747	53.767	32.233	43.093	80.598	74.522	40.094	64.278	32.168	74.857	21.157	40.846	27.413	27.741	60.662	44.585	52.289	32.866
50	37.907	30.364	45.952	81.976	86.741	123.674	28.58	21.225	14.33	54.112	125.384	41.616	19.234	72.784	45.788	14.976	92.396	60.087	60.704
51	56.03	78.603	91.382	39.695	20.753	47.503	108.063	77.523	101.6	39.932	40.658	55.392	82.705	30.856	50.143	97.495	35.716	54.204	31.172
52	77.288	99.709	106.931	43.1	31.013	43.313	128.448	98.431	124.943	60.383	24.99	73.647	101.065	42.38	68.998	120.879	37.461	70.502	52.962
53	48.397	70.96	82.501	41.305	37.278	63.097	91.102	65.783	91.331	42.29	56.254	43.204	69.37	37.832	49.006	87.526	32.397	45.692	39.164
54	24.342	26.859	41.097	62.047	66.876	102.895	49.309	20.003	35.082	36.402	104.611	29.505	21.553	54.794	31.3	34.008	71.276	56.982	42.552
55	39.925	29.604	44.13	84.345	89.381	126.411	26.286	21.846	11.603	56.453	128.011	43.741	20.065	75.189	47.969	13.794	94.569	62.13	63.3
56	37.626	59.609	69.987	21.226	26.423	62.854	94.28	58.105	84.037	26.103	55.362	37.794	60.612	13.154	27.627	80.103	38.884	61.281	24.049

Table A.3: Euclidean Distance Matrix - Part 3

	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
0	78.342	26.745	75.868	28.949	117.689	54.61	37.87	84.6	40.694	95.786	29.108	53.674	30.423	82.64	103.691	78.064	26.397	30.183	65.217
1	103.869	31.567	98.036	42.29	144.107	74.929	54.292	106.108	60.205	113.386	31.009	67.912	40.009	108.17	127.33	96.579	48.739	35.726	87.293
2	44.288	55.211	37.867	45.666	81.327	21.999	38.282	48.125	24.591	55.649	47.114	26.768	50.329	47.327	64.922	41.791	33.444	52.977	28.783
3	38.571	123.241	44.092	112.808	27.32	66.929	97.018	42.882	82.494	46.721	111.143	73.236	118.037	34.141	25.087	51.419	97.336	120.683	54.783
4	62.391	48.172	58.707	37.604	92.555	50.43	41.231	71.813	40.876	77.468	51.6	47.896	42.319	61.695	82.224	60.84	36.177	45.733	57.254
5	79.802	21.426	76.852	20.051	119.125	54.778	29.816	84.525	33.79	92.866	35.025	49.168	18.545	83.302	104.491	72.816	25.644	20.098	65.983
6	89.277	10.355	84.828	18.696	130.326	62.123	32.48	93.776	39.635	100.947	23.75	53.985	16.042	93.278	114.394	81.795	27.194	12.889	73.789
7	30.019	106.934	29.802	97.931	36.671	51.08	82.758	38.255	67.119	45.968	96.415	57.339	101.729	18.862	23.607	40.803	83.154	104.352	38.683
8	110.211	20.907	106.539	31.156	151.913	83.293	46.732	114.935	60.804	123.006	31.424	75.281	26.694	113.849	136.019	102.194	45.796	23.67	94.851
9	57.733	34.679	53.107	29.163	99.127	35.85	26.601	64.933	14.374	71.319	38.78	34.313	30.344	61.061	83.701	53.411	26.012	32.356	44.81
10	46.307	54.321	42.76	49.196	85.544	34.856	34.143	49.064	33.195	54.436	43.883	22.467	51.57	49.731	68.025	42.402	34.183	53.459	36.625
11	79.139	13.209	75.744	16.217	121.522	53.028	28.005	85.324	31.509	93.505	29.126	47.544	11.982	83.066	106.355	73.666	20.19	12.686	65.638
12	51.032	107.753	44.88	98.109	57.237	56.901	81.768	44.827	71.875	21.687	95.557	52.022	102.956	43.667	39.89	33.787	83.891	105.422	52.251
13	79.912	19.414	76.84	16.075	120.524	56.875	34.091	85.109	42.188	92.864	31.092	51.252	19.753	84.193	106.145	75.859	24.59	21.1	68.07
14	108.036	32.661	103.161	43.673	148.704	80.187	55.777	111.479	63.309	118.593	34.051	72.61	41.031	112.357	132.268	100.883	50.9	36.819	92.179
15	46.063	71.707	39.271	63.134	68.943	44.543	50.412	53.571	35.61	50.362	67.194	45.327	66.51	33.721	52.078	36.565	52.732	69.102	41.511
16	32.008	81.701	24.003	70.294	60.338	33.724	56.689	36.035	40.933	36.37	69.602	35.352	76.714	23.717	41.199	28.744	56.883	79.084	27.272
17	39.219	88.418	32.648	79.566	59.117	42.076	62.996	33.584	54.316	23.682	76.241	34.402	83.995	39.969	43.479	30.163	65.235	86.264	37.571
18	56.348	51.936	55.862	46.051	88.945	43.216	47.732	66.005	32.01	82.459	53.213	51.426	48.71	60.047	78.499	63.812	39.442	50.189	46.319
19	51.625	42.264	46.738	36.107	92.91	25.888	33.572	57.677	20.564	64.889	35.554	26.485	37.907	56.03	77.288	48.397	24.342	39.925	37.626
20	75.34	27.81	69.771	30.623	116.011	48.772	40.316	80.334	36.077	87.824	25.948	45.747	30.364	78.603	99.709	70.96	26.859	29.604	59.609
21	85.969	39.48	78.263	45.191	123.115	58.546	52.223	88.342	56.886	92.974	30.19	53.767	45.952	91.382	106.931	82.501	41.097	44.13	69.987
22	35.402	86.955	21.976	76.875	60.984	26.229	65.222	31.007	47.802	31.866	75.06	32.233	81.976	39.695	43.1	41.305	62.047	84.345	21.226
23	22.308	91.944	19.875	80.937	45.906	37.305	67.007	28.295	51.677	44.653	80.448	43.093	86.741	20.753	31.013	37.278	66.876	89.381	26.423
24	56.225	128.884	54.366	117.658	42.604	74.749	103.727	58.716	88.786	62.491	118.044	80.598	123.674	47.503	43.313	63.097	102.895	126.411	62.854
25	108.132	25.179	103.246	32.482	146.736	83.329	43.861	113.166	59.478	114.823	36.81	74.522	28.58	108.063	128.448	91.102	49.309	26.286	94.28
26	72.529	21.518	68.803	21.154	114.084	46.04	29.698	77.253	27.516	85.073	29.708	40.094	21.225	77.523	98.431	65.783	20.003	21.846	58.105
27	97.761	9.053	94.335	20.975	140.13	71.551	35.925	103.872	49.497	111.905	29.713	64.278	14.33	101.6	124.943	91.331	35.082	11.603	84.037
28	37.464	59.039	31.708	49.541	75.909	24.064	39.616	45.714	26.161	56.524	53.891	32.168	54.112	39.932	60.383	42.29	36.402	56.453	26.103
29	42.459	130.588	45.713	119.479	31.664	69.956	104.373	40.266	89.076	41.434	118.28	74.857	125.384	40.658	24.99	56.254	104.611	128.011	55.362
30	53.904	45.286	46.1	39.119	90.664	31.933	28.298	56.533	24.222	60.165	36.241	21.157	41.616	55.392	73.647	43.204	29.505	43.741	37.794
31	77.047	19.147	72.273	20.587	117.04	49.694	31.862	79.59	34.071	87.316	27.528	40.846	19.234	82.705	101.065	69.37	21.553	20.065	60.612
32	29.404	77.774	15.53	68.735	62.391	21.695	57.142	34.504	38.148	44.168	67.55	27.413	72.784	30.856	42.38	37.832	54.794	75.189	13.154
33	47.925	50.351	39.988	41.802	87.86	22.609	38.593	50.187	15.918	58.954	41.507	27.741	45.788	50.143	68.998	49.006	31.3	47.969	27.627
34	93.656	12.584	90.276	22.381	136.053	68.007	32.634	99.782	46.058	107.824	27.757	60.662	14.976	97.495	120.879	87.526	34.008	13.794	80.103
35	38.575	97.177	30.99	87.923	53.543	41.734	71.284	37.209	58.299	28.616	85.83	44.585	92.396	35.716	37.461	32.397	71.276	94.569	38.884
36	59.398	64.276	60.324	57.055	85.606	62.465	43.021	64.018	47.322	64.996	66.367	52.289	60.087	54.204	70.502	45.692	56.982	62.13	61.281
37	28.7	65.909	25.172	56.97	67.88	27.422	45.446	39.329	30.15	52.265	59.919	32.866	60.704	31.172	52.962	39.164	42.552	63.3	24.049
38	0.0	90.836	25.039	81.869	47.278	38.405	67.621	18.839	52.86	47.703	81.423	45.569	85.644	25.013	38.619	43.67	66.671	88.214	28.991
39	90.836	0.0	87.41	15.116	133.204	64.564	30.576	96.946	42.284	104.983	27.142	57.265	9.64	94.675	118.017	84.434	27.248	7.132	77.098
40	25.039	87.41	0.0	77.861	51.515	29.066	64.308	29.739	47.512	39.362	76.703	34.384	82.329	25.092	32.814	35.935	64.405	84.802	18.508
41	81.869	15.116	77.861	0.0	123.601	56.206	27.087	86.964	35.587	94.997	30.382	50.485	13.56	85.785	107.873	74.864	25.528	14.636	67.355
42	47.278	133.204	51.515	123.601	0.0	74.37	108.822	52.084	93.012	59.402	122.12	82.285	128.001	44.516	36.972	65.454	107.54	130.586	63.474
43	38.405	64.564	29.066	56.206	74.37	0.0	49.722	40.9	32.108	49.271	54.55	24.781	59.744	43.818	57.707	42.583	42.005	62.046	21.228
44	67.621	30.576	64.308	27.087	108.822	49.722	0.0	72.373	30.703	79.285	32.609	36.998	29.57	71.117	92.065	59.176	31.3	30.015	55.714
45	18.839	96.946	29.739	86.964	52.084	40.9	72.373	0.0	57.705	40.071	85.34	45.539	91.85	33.316	38.908	45.942	72.093	94.396	30.025
46	52.86	42.284	47.512	35.587	93.012	32.108	30.703	57.705	0.0	66.354	39.724	33.084	38.081	55.342	76.728	51.0	29.19	39.949	37.246
47	47.703	104.983	39.362	94.997	59.402	49.271	79.285	40.071	66.354	0.0	92.595	47.867	99.927	41.504	38.674	33.463	79.923	102.499	44.633
48	81.423	27.142	76.703	30.382	122.12	54.55	32.609	85.34	39.724	92.595	0.0	45.698	31.03	85.396	106.149	73.981	27.049	30.027	65.819
49	45.569	57.265	34.384	50.485	82.285	24.781	36.998	45.539	33.084	47.867	45.698	0.0	53.204	49.386	62.096	36.261	37.128	55.4	24.303
50	85.644	9.64	82.329	13.56	128.001	59.744	29.57	91.85	38.081	99.927	31.03	53.204	0.0	89.5	112.813	79.705	25.955	7.685	72.226
51	25.013	94.675	25.092	85.785	44.516	43.818	71.117	33.316	55.342	41.504	85.396	49.386	89.5	0.0	30.1	33.513	70.364	92.05	32.671
52	38.619	118.017	32.814	107.873	36.972	57.707	92.065	38.908	76.728	38.674	106.149	62.096	112.813	30.1	0.0	42.217	93.428	115.402	42.843
53	43.67	84.434	35.935	74.864	65.454	42.583	59.176	45.942	51.0	33.463	73.981	36.261	79.705	33.513	42.217	0.0	62.451	82.23	39.657
54	66.671	27.248	64.405	25.528	107.54	42.005	31.3	72.093	29.19	79.923	27.049	37.128	25.955	70.364	93.428	62.451	0.0	27.002	54.048
55	88.214	7.132	84.802	14.636	130.586	62.046	30.015	94.396	39.949	102.499	30.027	55.4	7.685	92.05	115.402	82.23	27.002	0.0	74.574
56	28.991	77.098	18.508	67.355	63.474	21.228	55.714	30.025	37.246	44.633	65.819	24.303	72.226	32.671	42.843	39.657	54.048	74.574	0.0

Table A.4: DTW Distance Matrix - Part 1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0.0	28.095	29.448	60.776	29.309	24.186	21.82	55.177	26.323	25.268	30.981	21.509	54.145	22.929	25.669	35.939	40.476	48.737	30.023
1	28.095	0.0	32.208	46.639	35.877	26.288	17.852	45.036	12.057	29.489	28.83	24.464	40.388	23.606	5.43	37.128	38.404	39.443	38.225
2	29.448	32.208	0.0	54.111	35.662	35.699	33.271	48.695	37.959	22.284	26.118	35.107	42.69	34.932	27.778	30.572	29.495	34.785	27.507
3	60.776	46.639	54.111	0.0	54.658	67.451	66.206	25.292	58.419	69.711	57.148	83.018	42.741	59.489	43.613	50.908	42.05	42.801	57.582
4	29.309	35.877	35.662	54.658	0.0	31.623	36.803	53.688	42.704	32.305	37.486	35.13	53.203	29.473	32.836	37.204	42.262	47.313	35.534
5	24.186	26.288	35.699	67.451	31.623	0.0	20.46	64.419	21.865	23.872	34.996	14.138	52.073	20.997	22.752	45.539	54.07	47.238	37.409
6	21.82	17.852	33.271	66.206	36.803	20.46	0.0	59.375	14.398	27.713	30.9	15.326	51.078	16.056	17.422	42.838	44.911	46.036	39.568
7	55.177	45.036	48.695	25.292	53.688	64.419	59.375	0.0	62.217	62.61	47.372	68.556	45.952	56.556	41.49	38.909	31.14	39.307	54.371
8	26.323	12.057	37.959	58.419	42.704	21.865	14.398	62.217	0.0	32.006	39.388	19.505	49.728	19.798	11.305	49.813	50.502	47.53	41.958
9	25.268	29.489	22.284	69.711	32.305	23.872	27.713	62.61	32.006	0.0	28.545	20.827	50.92	28.325	26.548	32.446	39.404	40.514	30.405
10	30.981	28.83	26.118	57.148	37.486	34.996	30.9	47.372	39.388	28.545	0.0	33.26	37.71	31.842	29.309	27.616	26.605	31.955	39.16
11	21.509	24.464	35.107	83.018	35.13	14.138	15.326	68.556	19.505	20.827	33.26	0.0	65.656	18.151	23.423	46.185	49.702	52.699	38.249
12	54.145	40.388	42.69	42.741	53.203	52.073	51.078	45.952	49.728	50.92	37.71	65.656	0.0	47.318	36.767	45.781	38.384	22.86	61.54
13	22.929	23.606	34.932	59.489	29.473	20.997	16.056	56.556	19.798	28.325	31.842	18.151	47.318	0.0	21.389	42.001	43.683	47.227	35.361
14	25.669	5.43	27.778	43.613	32.836	22.752	17.422	41.49	11.305	26.548	29.309	23.423	36.767	21.389	0.0	32.966	32.67	36.281	32.751
15	35.939	37.128	30.572	50.908	37.204	45.539	42.838	38.909	49.813	32.446	27.616	46.185	45.781	42.001	32.966	0.0	24.274	39.759	35.665
16	40.476	38.404	29.495	42.05	42.262	54.07	44.911	31.14	50.502	39.404	26.605	49.702	38.384	43.683	32.67	24.274	0.0	29.67	42.034
17	48.737	39.443	34.785	42.801	47.313	47.238	46.036	39.307	47.53	40.514	31.955	52.699	22.86	47.227	36.281	39.759	29.67	0.0	51.908
18	30.023	38.225	27.507	57.582	35.534	37.409	39.568	54.371	41.958	30.405	39.16	38.249	61.54	35.361	32.751	35.665	42.034	51.908	0.0
19	26.512	27.763	19.087	63.572	35.911	27.785	27.791	55.478	34.916	18.045	25.877	25.43	36.971	31.185	25.169	35.291	32.264	32.148	30.748
20	20.72	22.015	26.525	55.071	34.105	26.819	19.695	49.671	25.67	23.936	23.369	20.651	42.288	23.79	22.223	34.264	32.596	37.479	31.16
21	26.908	17.006	28.308	45.788	34.588	30.562	24.265	42.195	23.546	29.656	25.583	28.61	34.208	25.783	18.037	33.437	31.441	33.759	35.252
22	46.899	38.727	29.565	48.144	48.749	50.769	46.79	41.5	42.401	39.449	35.17	54.673	30.681	46.799	34.95	38.736	27.517	25.863	44.986
23	47.531	43.943	35.879	32.61	48.182	60.819	53.676	23.012	55.016	49.461	38.924	60.661	44.827	53.169	39.542	34.563	24.533	35.663	43.946
24	71.51	61.001	62.538	36.144	66.151	78.349	74.517	38.821	75.516	79.443	66.589	89.295	54.935	68.744	56.076	56.845	53.015	54.1	64.843
25	28.011	12.475	33.771	56.623	40.506	24.241	19.646	54.51	11.234	30.285	35.547	23.117	47.344	23.962	12.326	43.76	45.913	46.576	40.908
26	22.023	22.059	28.389	70.686	34.971	20.084	17.9	59.996	25.384	19.506	26.169	13.842	44.669	22.046	24.522	38.148	42.343	41.496	35.259
27	23.882	16.401	39.292	84.756	42.378	20.96	12.264	69.842	8.25	30.936	38.025	16.152	75.984	18.43	15.919	49.638	52.296	63.607	42.185
28	27.881	34.036	18.851	58.111	34.373	40.267	34.759	46.451	36.031	25.21	28.842	38.762	52.629	35.978	28.257	28.513	27.367	40.286	30.281
29	63.502	50.265	60.054	21.271	60.374	72.224	72.519	30.557	66.235	73.59	57.461	90.878	35.712	62.063	46.869	54.427	46.951	40.562	67.321
30	28.616	26.566	20.359	57.832	34.88	28.318	25.213	51.021	30.871	21.003	22.197	27.35	39.449	30.305	26.859	29.156	36.805	33.001	34.556
31	24.361	20.554	30.658	67.383	36.456	19.435	15.371	62.538	21.536	25.68	28.605	13.644	43.302	20.97	21.964	41.16	46.347	38.825	40.029
32	41.566	35.843	26.913	47.943	45.21	50.999	42.354	35.485	41.609	37.051	32.059	50.051	39.946	45.891	34.057	34.114	25.044	33.073	39.119
33	30.643	33.923	19.012	61.525	36.698	30.846	35.064	54.04	34.732	20.153	26.837	31.004	40.74	34.925	26.359	32.818	31.381	33.784	30.541
34	23.836	17.934	34.816	77.516	40.883	19.585	13.658	65.686	10.38	26.322	34.526	15.047	70.444	19.144	17.331	44.927	49.099	59.936	39.008
35	51.761	41.657	35.591	38.991	48.897	54.226	51.311	36.568	47.868	42.538	37.443	59.803	24.699	50.806	36.541	38.142	29.31	24.257	51.046
36	39.64	37.14	39.238	49.228	35.578	42.174	39.956	43.409	45.535	37.821	33.473	40.95	43.914	37.48	35.197	32.007	38.364	37.807	39.783
37	31.412	35.962	22.333	51.057	34.843	46.21	38.708	40.192	40.024	29.785	29.088	43.071	49.382	40.156	31.272	26.506	22.497	38.281	33.439
38	47.228	42.101	35.73	36.549	45.568	56.69	53.163	26.449	52.045	46.939	39.08	60.855	46.036	52.52	39.153	33.632	29.112	36.97	41.473
39	22.414	19.083	36.296	84.346	41.587	18.118	8.662	68.558	10.579	29.396	35.76	12.37	73.137	17.115	17.028	48.609	53.926	58.174	42.716
40	45.412	37.396	31.915	41.569	48.864	57.051	47.885	28.688	46.519	45.292	36.374	57.832	37.393	49.232	34.166	36.064	23.276	30.166	43.556
41	22.594	26.819	34.986	74.869	34.074	16.233	15.008	66.365	17.16	24.632	35.568	13.093	69.176	14.729	23.021	48.422	53.015	60.997	33.786
42	69.746	52.498	63.387	25.591	65.106	75.804	74.577	34.43	63.543	82.377	65.06	93.71	49.924	70.12	49.56	57.708	51.369	51.577	69.257
43	35.889	33.546	19.877	52.415	41.949	42.33	39.826	42.882	41.431	28.187	27.216	41.703	35.391	40.874	29.416	34.83	25.424	28.851	33.959
44	23.373	24.613	29.765	64.183	32.97	23.33	21.788	56.318	22.359	21.315	27.882	20.284	50.653	22.589	22.376	33.537	40.597	43.267	30.422
45	54.491	45.812	38.725	39.09	53.007	61.586	57.845	32.7	56.34	49.022	39.791	64.026	40.343	56.7	43.893	37.681	31.555	30.833	47.777
46	26.486	32.368	21.677	62.597	33.731	28.996	31.48	53.687	36.171	13.437	26.606	26.5	43.836	31.612	26.061	30.325	33.98	36.944	28.363
47	51.655	43.278	40.708	44.97	53.968	55.098	53.842	42.464	50.156	47.484	38.623	65.262	20.001	51.189	38.293	44.146	32.967	21.506	59.255
48	24.187	16.506	26.404	53.66	34.348	24.497	17.831	50.242	21.062	22.332	24.561	19.291	39.483	23.352	14.974	35.762	35.07	36.1	33.547
49	34.72	30.247	24.871	57.333	39.392	34.256	32.354	47.321	36.724	28.892	18.755	32.137	29.787	36.347	29.558	32.617	29.373	25.025	42.61
50	23.229	23.159	37.876	71.65	35.291	15.054	13.052	62.224	13.897	26.379	36.02	10.648	63.752	14.997	19.704	48.47	51.945	56.116	36.158
51	48.302	41.394	39.828	32.464	47.856	62.207	56.157	17.31	56.706	50.499	38.058	61.068	39.68	51.909	38.679	31.065	22.431	35.124	48.14
52	57.341	45.903	56.013	23.361	59.035	68.218	64.299	22.177	59.599	68.662	51.484	79.746	37.607	56.346	41.95	45.057	37.943	37.203	61.298
53	44.652	34.902	33.99	46.892	41.177	47.223	41.647	34.238	49.413	40.994	27.529	46.015	31.904	42.941	34.071	28.702	26.803	26.889	49.459
54	20.435	21.129	25.99	70.292	32.942	21.098	19.224	61.733	27.027	20.016	25.108	16.343	48.227	21.328	23.51	36.071	41.958	42.915	34.506
55	22.455	20.763	37.351	88.87	39.532	16.093	10.265	73.481	12.363	28.801	36.102	11.241	77.826	16.876	18.447	50.453	54.815	61.965	39.648
56	40.552	36.134	26.674	48.749	45.852	49.883	44.743	36.326	42.813	37.082	31.434	48.94	38.578	45.07	34.361	34.747	24.621	31.487	36.57

Table A.5: DTW Distance Matrix - Part 2

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
0	26.512	20.72	26.908	46.899	47.531	71.51	28.011	22.023	23.882	27.881	63.502	28.616	24.361	41.566	30.643	23.836	51.761	39.64	31.412
1	27.763	22.015	17.006	38.727	43.943	61.001	12.475	22.059	16.401	34.036	50.265	26.566	20.554	35.843	33.923	17.934	41.657	37.14	35.962
2	19.087	26.525	28.308	29.565	35.879	62.538	33.771	28.389	39.292	18.851	60.054	20.359	30.658	26.913	19.012	34.816	35.591	39.238	22.333
3	63.572	55.071	45.788	48.144	32.61	36.144	56.623	70.686	84.756	58.111	21.271	57.832	67.383	47.943	61.525	77.516	38.991	49.228	51.057
4	35.911	34.105	34.588	48.749	48.182	66.151	40.506	34.971	42.378	34.373	60.374	34.88	36.456	45.21	36.698	40.883	48.897	35.578	34.843
5	27.785	26.819	30.562	50.769	60.819	78.349	24.241	20.084	20.96	40.267	72.224	28.318	19.435	50.999	30.846	19.585	54.226	42.174	46.21
6	27.791	19.695	24.265	46.79	53.676	74.517	19.646	17.9	12.264	34.759	72.519	25.213	15.371	42.354	35.064	13.658	51.311	39.956	38.708
7	55.478	49.671	42.195	41.5	23.012	38.821	54.51	59.996	69.842	46.451	30.557	51.021	62.538	35.485	54.04	65.686	36.568	43.409	40.192
8	34.916	25.67	23.546	42.401	55.016	75.516	11.234	25.384	8.25	36.031	66.235	30.871	21.536	41.609	34.732	10.38	47.868	45.535	40.024
9	18.045	23.936	29.656	39.449	49.461	79.443	30.285	19.506	30.936	25.21	73.59	21.003	25.68	37.051	20.153	26.322	42.538	37.821	29.785
10	25.877	23.369	25.583	35.17	38.924	66.589	35.547	26.169	38.025	28.842	57.461	22.197	28.605	32.059	26.837	34.526	37.443	33.473	29.088
11	25.43	20.651	28.61	54.673	60.661	89.295	23.117	13.842	16.152	38.762	90.878	27.35	13.644	50.051	31.004	15.047	59.803	40.95	43.071
12	36.971	42.288	34.208	30.681	44.827	54.935	47.344	44.669	75.984	52.629	35.712	39.449	43.302	39.946	40.74	70.444	24.699	43.914	49.382
13	31.185	23.79	25.783	46.799	53.169	68.744	23.962	22.046	18.43	35.978	62.063	30.305	20.97	45.891	34.925	19.144	50.806	37.48	40.156
14	25.169	22.223	18.037	34.95	39.542	56.076	12.326	24.522	15.919	28.257	46.869	26.859	21.964	34.057	26.359	17.331	36.541	35.197	31.272
15	35.291	34.264	33.437	38.736	34.563	56.845	43.76	38.148	49.638	28.513	54.427	29.156	41.16	34.114	32.818	44.927	38.142	32.007	26.506
16	32.264	32.596	31.441	27.517	24.533	53.015	45.913	42.343	52.296	27.367	46.951	36.805	46.347	25.044	31.381	49.099	29.31	38.364	22.497
17	32.148	37.479	33.759	25.863	35.663	54.1	46.576	41.496	63.607	40.286	40.562	33.001	38.825	33.073	33.784	59.936	24.257	37.807	38.281
18	30.748	31.16	35.252	44.986	43.946	64.843	40.908	35.259	42.185	30.281	67.321	34.556	40.029	39.119	30.541	39.008	51.046	39.783	33.439
19	0.0	20.241	24.131	31.968	42.861	74.316	29.771	20.931	32.492	23.97	65.233	19.39	23.396	31.709	17.163	30.433	36.379	37.979	28.445
20	20.241	0.0	19.947	35.208	42.736	68.484	26.433	19.985	23.209	29.015	57.631	24.784	19.972	32.703	23.978	23.611	40.807	36.684	31.439
21	24.131	19.947	0.0	34.5	39.23	54.831	25.491	24.18	26.433	31.468	46.707	25.252	23.282	32.105	29.736	27.141	37.837	34.358	30.995
22	31.968	35.208	34.5	0.0	28.872	59.636	43.426	47.515	53.478	31.953	43.076	34.619	48.053	19.963	26.484	53.336	26.669	43.743	27.973
23	42.861	42.736	39.23	28.872	0.0	46.646	49.013	56.176	57.926	31.886	38.494	43.369	58.53	24.901	40.082	57.216	31.59	42.569	26.155
24	74.316	68.484	54.831	59.636	46.646	0.0	70.473	80.298	92.18	66.123	38.885	67.103	78.341	57.331	72.063	86.965	50.565	55.237	60.621
25	29.771	26.433	25.491	43.426	49.013	70.473	0.0	25.993	13.918	36.431	59.572	31.402	22.357	40.864	33.518	14.706	49.072	42.858	38.312
26	20.931	19.985	24.18	47.515	56.176	80.298	25.993	0.0	22.085	34.134	70.988	23.946	9.421	45.143	28.748	20.292	48.739	36.534	36.743
27	32.492	23.209	26.433	53.478	57.926	92.18	13.918	22.085	0.0	36.285	88.493	33.394	21.232	45.907	37.438	8.768	67.734	45.273	40.086
28	23.97	29.015	31.468	31.953	31.886	66.123	36.431	34.134	36.285	0.0	63.393	28.481	39.796	23.598	24.056	33.153	37.551	39.619	14.076
29	65.233	57.631	46.707	43.076	38.494	38.885	59.572	70.988	88.493	63.393	0.0	58.548	71.39	46.554	59.279	81.742	39.731	53.741	57.398
30	19.39	24.784	25.252	34.619	43.369	67.103	31.402	23.946	33.394	28.481	58.548	0.0	21.912	34.536	21.598	31.898	37.978	36.517	29.885
31	23.396	19.972	23.282	48.053	58.53	78.341	22.357	9.421	21.232	39.796	71.39	21.912	0.0	46.79	30.97	22.13	49.499	37.297	42.395
32	31.709	32.703	32.105	19.963	24.901	57.331	40.864	45.143	45.907	23.598	46.554	34.536	46.79	0.0	25.748	45.384	30.871	44.082	19.78
33	17.163	23.978	29.736	26.484	40.082	72.063	33.518	28.748	37.438	24.056	59.279	21.598	30.97	25.748	0.0	33.124	37.543	42.091	26.949
34	30.433	23.611	27.141	53.336	57.216	86.965	14.706	20.292	8.768	33.153	81.742	31.898	22.13	45.384	33.124	0.0	63.193	41.67	40.547
35	36.379	40.807	37.837	26.669	31.59	50.565	49.072	48.739	67.734	37.551	39.731	37.978	49.499	30.871	37.543	63.193	0.0	42.564	35.75
36	37.979	36.684	34.358	43.743	42.569	55.237	42.858	36.534	45.273	39.619	53.741	36.517	37.297	44.082	42.091	41.67	42.564	0.0	38.262
37	28.445	31.439	30.995	27.973	26.155	60.621	38.312	36.743	40.086	14.076	57.398	29.885	42.395	19.78	26.949	40.547	35.75	38.262	0.0
38	41.433	41.485	37.187	31.051	21.015	48.052	49.219	51.854	60.728	33.116	38.476	40.903	54.759	26.345	39.025	60.547	35.629	43.16	25.689
39	31.932	21.294	26.183	63.207	63.649	88.591	17.164	17.868	8.269	41.185	91.753	29.548	15.915	53.309	41.168	11.351	66.753	44.083	46.62
40	38.183	37.37	34.642	19.759	18.887	52.355	43.983	51.2	52.074	28.092	39.431	36.866	52.478	14.439	33.381	52.551	28.877	45.552	23.874
41	29.83	23.711	29.935	54.868	61.405	87.66	23.507	18.487	15.453	35.603	83.077	31.2	18.823	48.707	33.956	15.904	65.961	42.543	42.552
42	77.727	68.259	53.052	58.371	43.657	35.966	62.655	78.666	96.695	70.163	28.852	68.379	76.594	56.177	76.708	89.551	48.992	54.623	62.029
43	21.636	26.722	29.692	20.907	32.765	62.475	38.674	34.646	48.863	22.764	52.184	28.344	37.046	20.283	18.606	42.502	29.268	40.366	24.667
44	27.726	23.757	25.392	47.031	49.533	75.662	25.758	21.141	22.299	29.941	68.946	24.595	20.245	42.099	31.644	18.57	48.17	31.614	34.193
45	41.564	45.177	41.32	27.403	25.819	48.037	53.274	56.187	70.161	39.935	36.902	42.991	57.114	31.322	40.078	68.922	32.572	46.925	34.297
46	17.622	22.917	29.106	31.278	43.634	72.715	32.831	24.223	35.537	23.558	63.456	20.289	28.16	29.715	12.642	29.589	39.607	39.866	26.886
47	36.92	40.712	36.691	25.078	41.245	57.057	48.618	48.313	74.865	48.405	39.845	39.179	47.18	34.803	35.069	67.759	24.854	42.549	44.677
48	20.299	18.908	20.298	38.292	42.209	69.095	18.786	19.546	20.981	27.673	57.782	21.572	18.708	35.074	26.093	19.589	40.689	35.213	32.076
49	21.46	24.781	25.842	28.637	39.531	67.034	34.119	27.171	39.56	30.305	52.991	19.33	24.559	25.503	24.724	37.435	32.321	32.013	30.828
50	30.395	22.646	27.873	55.496	58.518	83.014	18.957	17.389	12.377	39.873	78.688	31.133	16.454	48.876	36.871	12.792	60.736	42.219	45.023
51	43.944	43.281	37.995	32.409	19.594	44.95	50.285	53.261	61.665	35.939	37.11	45.517	55.224	27.869	43.243	58.787	30.623	39.919	29.433
52	61.961	52.329	43.499	38.272	29.122	39.54	52.434	66.186	78.843	54.386	23.442	55.053	67.021	37.887	58.64	72.181	34.848	49.118	48.232
53	31.822	36.497	30.922	34.952	34.53	53.063	42.343	38.843	55.562	37.575	49.846	31.952	37.062	35.06	38.242	52.125	29.092	35.719	34.427
54	21.371	19.126	26.02	49.034	52.671	78.395	25.509	15.608	24.233	30.896	73.914	20.876	15.789	43.77	27.734	22.095	48.485	36.787	33.751
55	31.293	21.857	28.055	58.29	63.848	94.243	18.858	17.664	9.939	37.226	96.591	30.261	16.695	51.168	37.571	11.51	68.812	46.049	43.015
56	31.22	31.697	33.199	18.811	24.536	59.638	42.754	43.938	48.464	24.219	45.289	35.19	44.661	11.85	24.045	46.982	31.635	41.731	22.232

Table A.6: DTW Distance Matrix - Part 3

	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
0	47.228	22.414	45.412	22.594	69.746	35.889	23.373	54.491	26.486	51.655	24.187	34.72	23.229	48.302	57.341	44.652	20.435	22.455	40.552
1	42.101	19.083	37.396	26.819	52.498	33.546	24.613	45.812	32.368	43.278	16.506	30.247	23.159	41.394	45.903	34.902	21.129	20.763	36.134
2	35.73	36.296	31.915	34.986	63.387	19.877	29.765	38.725	21.677	40.708	26.404	24.871	37.876	39.828	56.013	33.99	25.99	37.351	26.674
3	36.549	84.346	41.569	74.869	25.591	52.415	64.183	39.09	62.597	44.97	53.66	57.333	71.65	32.464	23.361	46.892	70.292	88.87	48.749
4	45.568	41.587	48.864	34.074	65.106	41.949	32.97	53.007	33.731	53.968	34.348	39.392	35.291	47.856	59.035	41.177	32.942	39.532	45.852
5	56.69	18.118	57.051	16.233	75.804	42.33	23.33	61.586	28.996	55.098	24.497	34.256	15.054	62.207	68.218	47.223	21.098	16.093	49.883
6	53.163	8.662	47.885	15.008	74.577	39.826	21.788	57.845	31.48	53.842	17.831	32.354	13.052	56.157	64.299	41.647	19.224	10.265	44.743
7	26.449	68.558	28.688	66.365	34.43	42.882	56.318	32.7	53.687	42.464	50.242	47.321	62.224	17.31	22.177	34.238	61.733	73.481	36.326
8	52.045	10.579	46.519	17.16	63.543	41.431	22.359	56.34	36.171	50.156	21.062	36.724	13.897	56.706	59.599	49.413	27.027	12.363	42.813
9	46.939	29.396	45.292	24.632	82.377	28.187	21.315	49.022	13.437	47.484	22.332	28.892	26.379	50.499	68.662	40.994	20.016	28.801	37.082
10	39.08	35.76	36.374	35.568	65.06	27.216	27.882	39.791	26.606	38.623	24.561	18.755	36.02	38.058	51.484	27.529	25.108	36.102	31.434
11	60.855	12.37	57.832	13.093	93.71	41.703	20.284	64.026	26.5	65.262	19.291	32.137	10.648	61.068	79.746	46.015	16.343	11.241	48.94
12	46.036	73.137	37.393	69.176	49.924	35.391	50.653	40.343	43.836	20.001	39.483	29.787	63.752	39.68	37.607	31.904	48.227	77.826	38.578
13	52.52	17.115	49.232	14.729	70.12	40.874	22.589	56.7	31.612	51.189	23.352	36.347	14.997	51.909	56.346	42.941	21.328	16.876	45.07
14	39.153	17.028	34.166	23.021	49.56	29.416	22.376	43.893	26.061	38.293	14.974	29.558	19.704	38.679	41.95	34.071	23.51	18.447	34.361
15	33.632	48.609	36.064	48.422	57.708	34.83	33.537	37.681	30.325	44.146	35.762	32.617	48.47	31.065	45.057	28.702	36.071	50.453	34.747
16	29.112	53.926	23.276	53.015	51.369	25.424	40.597	31.555	33.98	32.967	35.07	29.373	51.945	22.431	37.943	26.803	41.958	54.815	24.621
17	36.97	58.174	30.166	60.997	51.577	28.851	43.267	30.833	36.944	21.506	36.1	25.025	56.116	35.124	37.203	26.889	42.915	61.965	31.487
18	41.473	42.716	43.556	33.786	69.257	33.959	30.422	47.777	28.363	59.255	33.547	42.61	36.158	48.14	61.298	49.459	34.506	39.648	36.57
19	41.433	31.932	38.183	29.83	77.727	21.636	27.726	41.564	17.622	36.92	20.299	21.46	30.395	43.944	61.961	31.822	21.371	31.293	31.22
20	41.485	21.294	37.37	23.711	68.259	26.722	23.757	45.177	22.917	40.712	18.908	24.781	22.646	43.281	52.329	36.497	19.126	21.857	31.697
21	37.187	26.183	34.642	29.935	53.052	29.692	25.392	41.32	29.106	36.691	20.298	25.842	27.873	37.995	43.499	30.922	26.02	28.055	33.199
22	31.051	63.207	19.759	54.868	58.371	20.907	47.031	27.403	31.278	25.078	38.292	28.637	55.496	32.409	38.272	34.952	49.034	58.29	18.811
23	21.015	63.649	18.887	61.405	43.657	32.765	49.533	25.819	43.634	41.245	42.209	39.531	58.518	19.594	29.122	34.53	52.671	63.848	24.536
24	48.052	88.591	52.355	87.66	35.966	62.475	75.662	48.037	72.715	57.057	69.095	67.034	83.014	44.95	39.54	53.063	78.395	94.243	59.638
25	49.219	17.164	43.983	23.507	62.655	38.674	25.758	53.274	32.831	48.618	18.786	34.119	18.957	50.285	52.434	42.343	25.509	18.858	42.754
26	51.854	17.868	51.2	18.487	78.666	34.646	21.141	56.187	24.223	48.313	19.546	27.171	17.389	53.261	66.186	38.843	15.608	17.664	43.938
27	60.728	8.269	52.074	15.453	96.695	48.863	22.299	70.161	35.537	74.865	20.981	39.56	12.377	61.665	78.843	55.562	24.233	9.939	48.464
28	33.116	41.185	28.092	35.603	70.163	22.764	29.941	39.935	23.558	48.405	27.673	30.305	39.873	35.939	54.386	37.575	30.896	37.226	24.219
29	38.476	91.753	39.431	83.077	28.852	52.184	68.946	36.902	63.456	39.845	57.782	52.991	78.688	37.11	23.442	49.846	73.914	96.591	45.289
30	40.903	29.548	36.866	31.2	68.379	28.344	24.595	42.991	20.289	39.179	21.572	19.33	31.133	45.517	55.053	31.952	20.876	30.261	35.19
31	54.759	15.915	52.478	18.823	76.594	37.046	20.245	57.114	28.16	47.18	18.708	24.559	16.454	55.224	67.021	37.062	15.789	16.695	44.661
32	26.345	53.309	14.439	48.707	56.177	20.283	42.099	31.322	29.715	34.803	35.074	25.503	48.876	27.869	37.887	35.06	43.77	51.168	11.85
33	39.025	41.168	33.381	33.956	76.708	18.606	31.644	40.078	12.642	35.069	26.093	24.724	36.871	43.243	58.64	38.242	27.734	37.571	24.045
34	60.547	11.351	52.551	15.904	89.551	42.502	18.57	68.922	29.589	67.759	19.589	37.435	12.792	58.787	72.181	52.125	22.095	11.51	46.982
35	35.629	66.753	28.877	65.961	48.992	29.268	48.17	32.572	39.607	24.854	40.689	32.321	60.736	30.623	34.848	29.092	48.485	68.812	31.635
36	43.16	44.083	45.552	42.543	54.623	40.366	31.614	46.925	39.866	42.549	35.213	32.013	42.219	39.919	49.118	35.719	36.787	46.049	41.731
37	25.689	46.62	23.874	42.552	62.029	24.667	34.193	34.297	26.886	44.677	32.076	30.828	45.023	29.433	48.232	34.427	33.751	43.015	22.232
38	0.0	66.705	23.19	62.575	44.737	33.482	48.46	17.948	40.921	39.263	45.601	41.949	59.5	23.13	33.149	36.542	51.602	64.38	27.217
39	66.705	0.0	59.897	12.698	94.526	48.5	20.787	74.085	35.049	75.599	18.964	36.638	9.169	65.988	80.573	49.366	19.291	6.56	56.32
40	23.19	59.897	0.0	57.546	49.415	25.629	47.295	27.495	38.675	34.156	37.451	32.268	56.738	23.656	31.119	34.024	49.941	60.695	16.938
41	62.575	12.698	57.546	0.0	88.028	43.985	19.452	70.343	29.59	71.872	23.419	36.382	10.471	63.851	73.851	54.057	21.597	11.668	48.577
42	44.737	94.526	49.415	88.028	0.0	64.004	75.735	47.583	75.939	52.992	60.847	64.711	86.581	42.387	34.015	51.363	80.131	101.717	56.753
43	33.482	48.5	25.629	43.985	64.004	0.0	37.719	34.876	23.325	31.784	29.684	23.268	44.899	34.26	49.656	32.28	33.752	46.926	19.484
44	48.46	20.787	47.295	19.452	75.735	37.719	0.0	57.241	25.201	52.742	20.11	29.865	21.403	48.835	60.569	39.433	21.376	22.962	41.978
45	17.948	74.085	27.495	70.343	47.583	34.876	57.241	0.0	42.721	33.318	49.377	41.07	68.264	29.832	33.535	37.828	55.707	70.979	28.274
46	40.921	35.049	38.675	29.59	75.939	23.325	25.201	42.721	0.0	40.81	23.944	26.911	32.35	45.743	62.251	40.189	23.19	32.471	29.035
47	39.263	75.599	34.156	71.872	52.992	31.784	52.742	33.318	40.81	0.0	44.299	33.154	66.208	37.416	37.258	31.305	50.452	79.157	35.596
48	45.601	18.964	37.451	23.419	60.847	29.684	20.11	49.377	23.944	44.299	0.0	23.517	22.479	43.698	52.609	33.411	19.547	21.841	35.534
49	41.949	36.638	32.268	36.382	64.711	23.268	29.865	41.07	26.911	33.154	23.517	0.0	37.675	39.498	49.058	26.396	26.802	38.293	23.386
50	59.5	9.169	56.738	10.471	86.581	44.899	21.403	68.264	32.35	66.208	22.479	37.675	0.0	58.959	68.874	47.195	20.577	7.073	51.906
51	23.13	65.988	23.656	63.851	42.387	34.26	48.835	29.832	45.743	37.416	43.698	39.498	58.959	0.0	27.935	31.303	52.043	66.145	29.99
52	33.149	80.573	31.119	73.851	34.015	49.656	60.569	33.535	62.251	37.258	52.609	49.058	68.874	27.935	0.0	37.852	67.371	85.795	38.799
53	36.542	49.366	34.024	54.057	51.363	32.28	39.433	37.828	40.189	31.305	33.411	26.396	47.195	31.303	37.852	0.0	38.811	55.113	35.927
54	51.602	19.291	49.941	21.597	80.131	33.752	21.376	55.707	23.19	50.452	19.547	26.802	20.577	52.043	67.371	38.811	0.0	18.972	44.364
55	64.38	6.56	60.695	11.668	101.717	46.926	22.962	70.979	32.471	79.157	21.841	38.293	7.073	66.145	85.795	55.113	18.972	0.0	55.421
56	27.217	56.32	16.938	48.577	56.753	19.484	41.978	28.274	29.035	35.596	35.534	23.386	51.906	29.99	38.799	35.927	44.364	55.421	0.0

Appendix B

Cluster Results

B.1 k-Medoids Clustering - Clusters

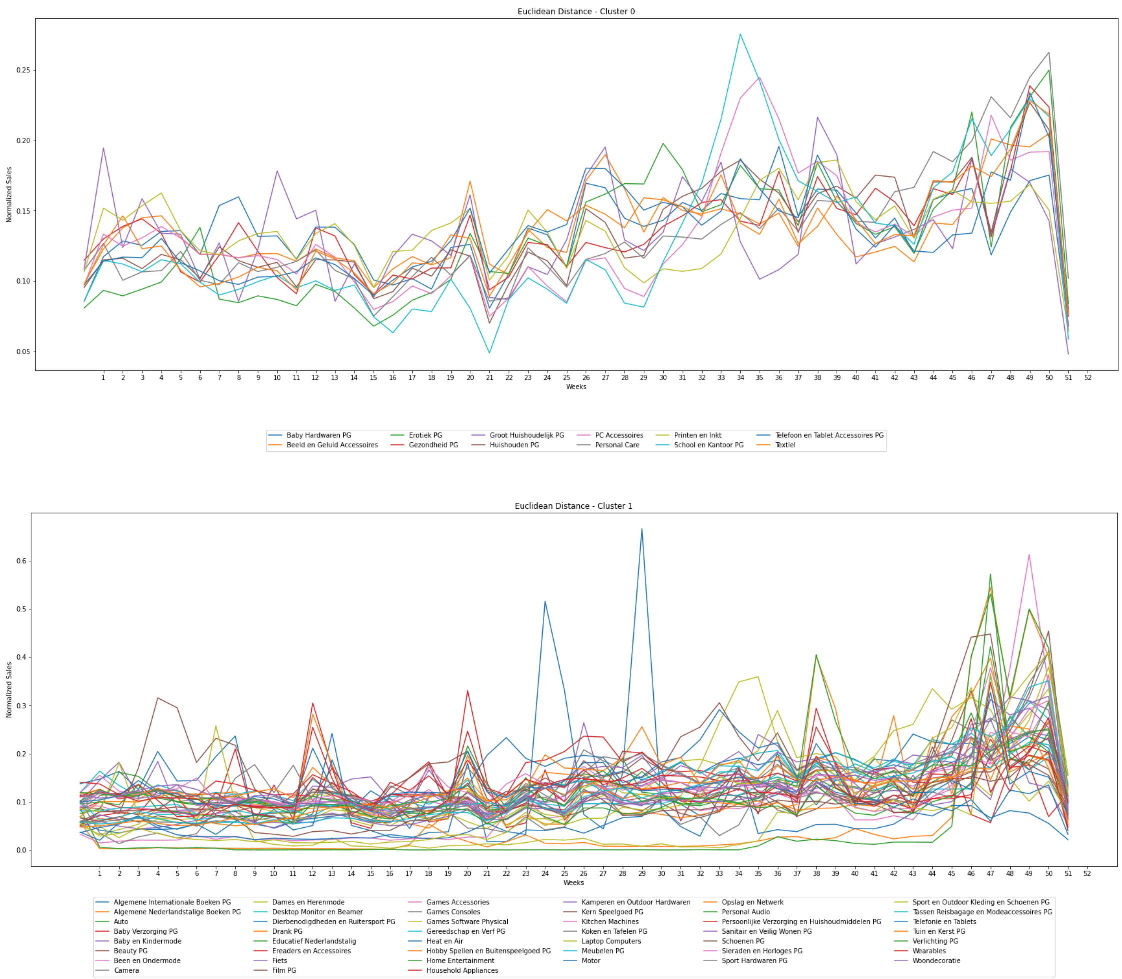


Figure B.1: k-Medoids, Euclidean Distance Clusters, k=2

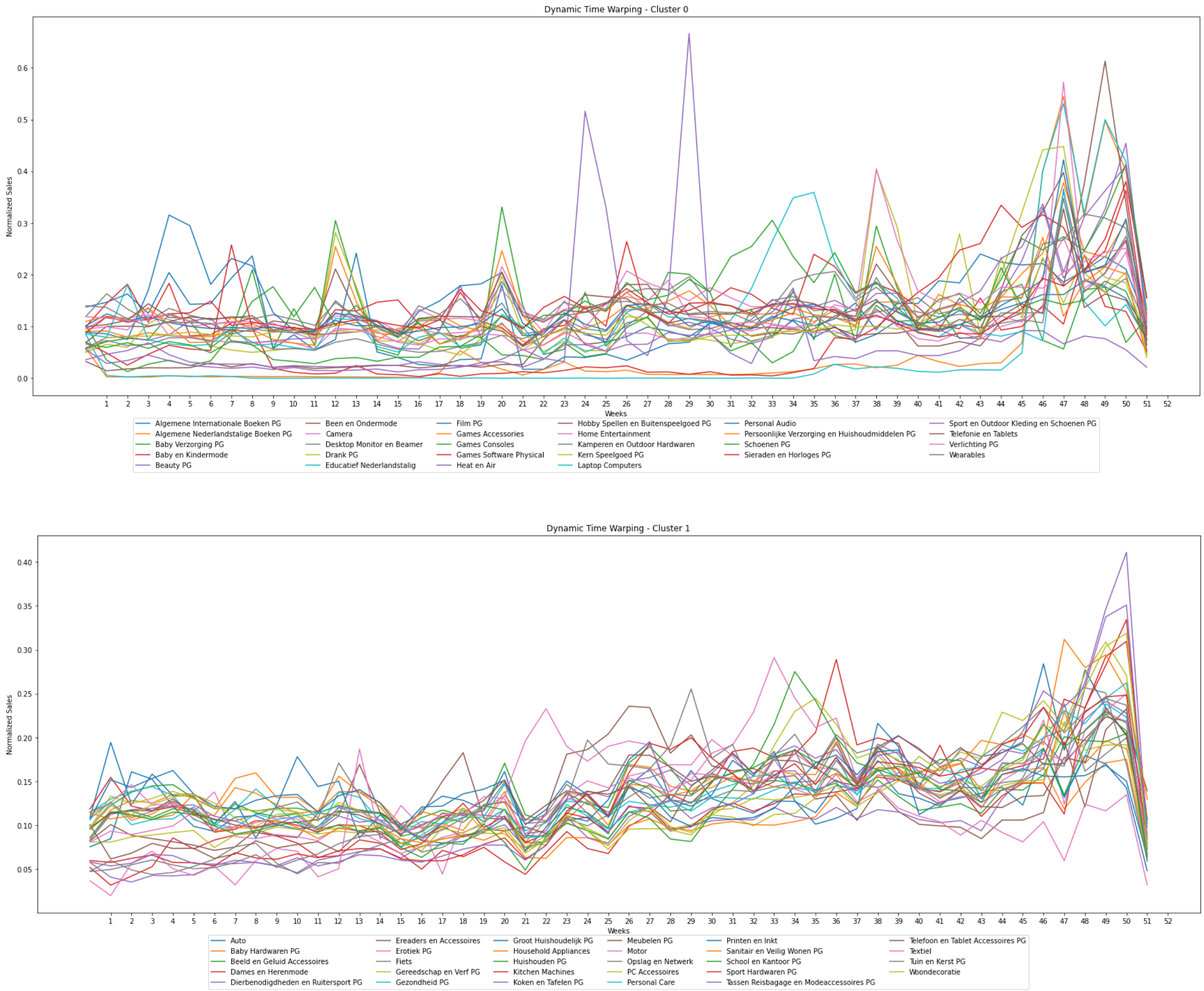


Figure B.2: k-Medoids, DTW Distance Clusters, k=2

B.2 Hierarchical Clustering - Dendrograms

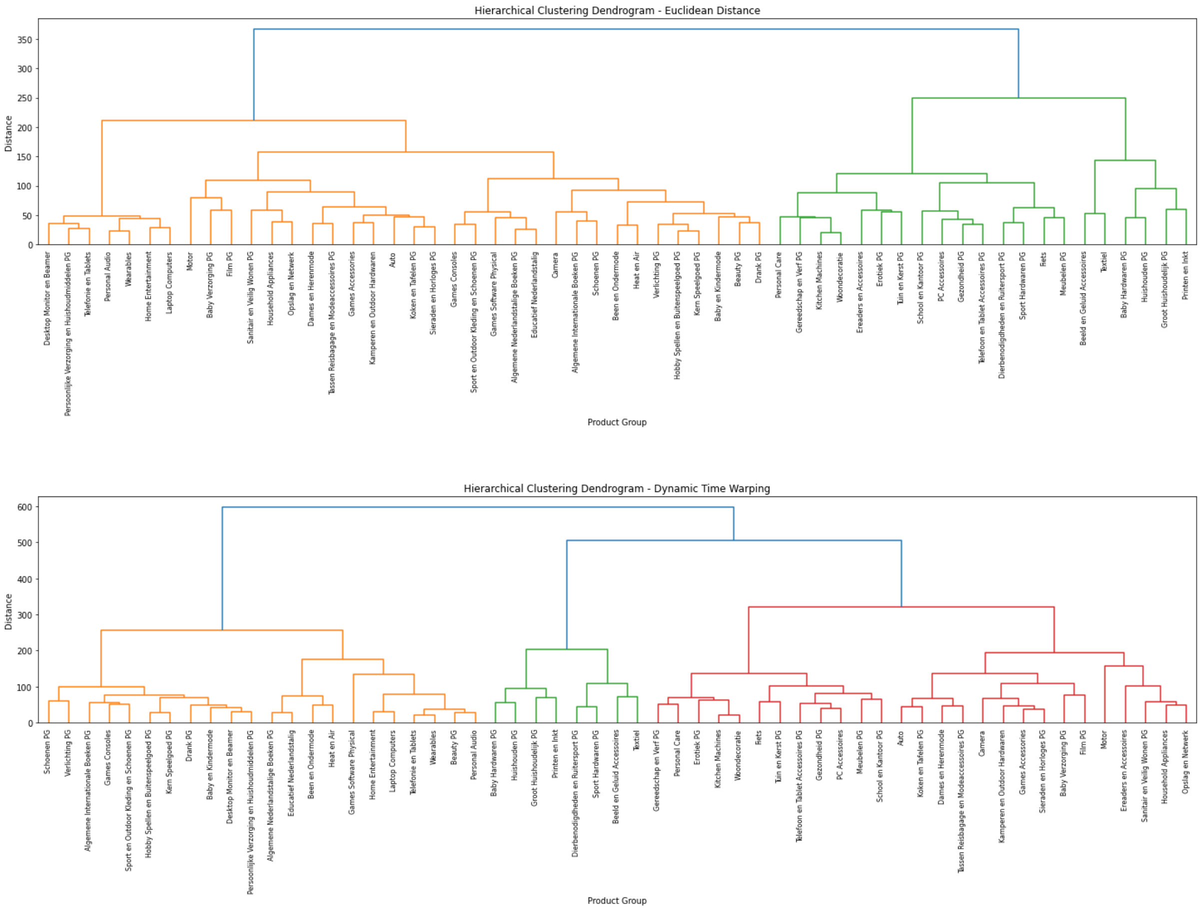
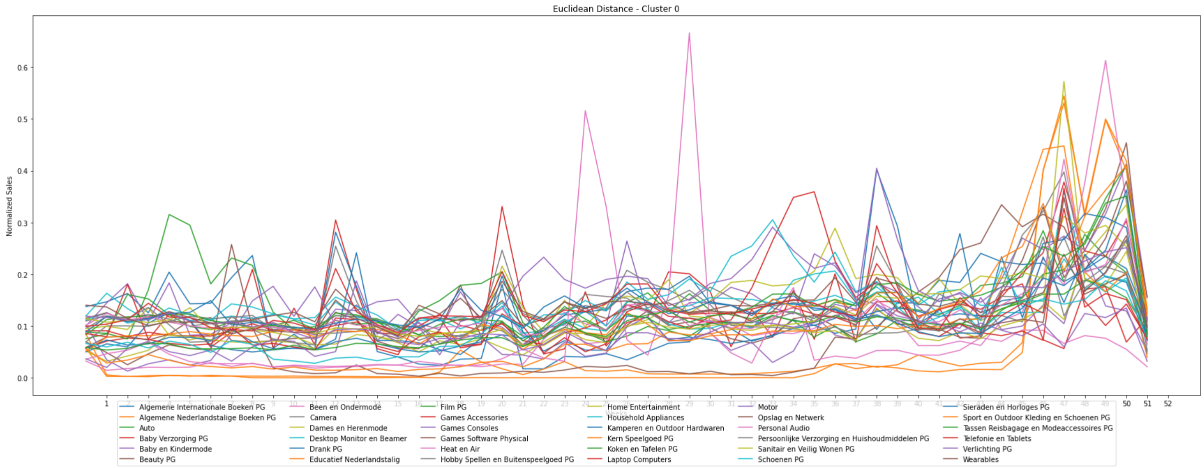
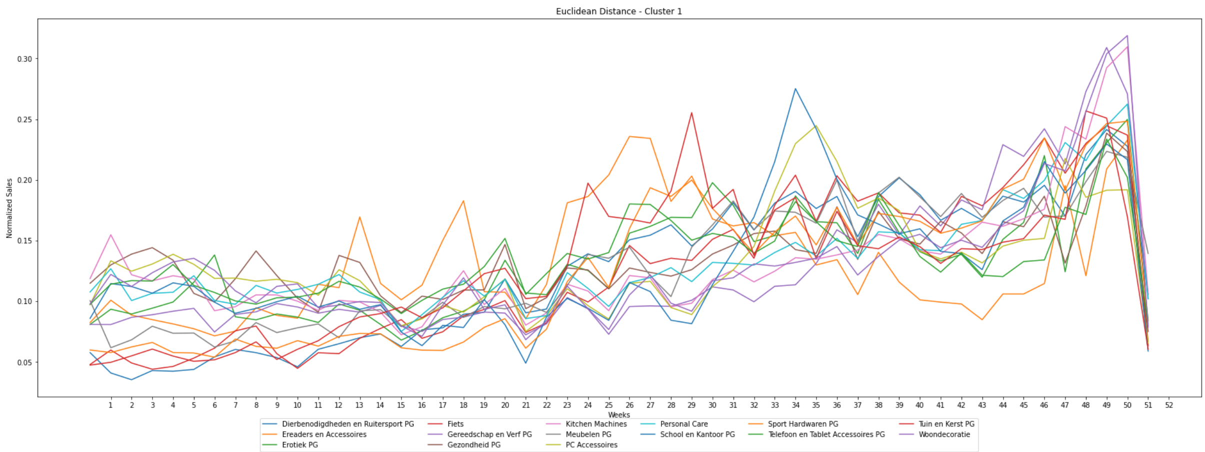


Figure B.3: Dendrograms Euclidean Distance (top) and DTW Distance (bottom)

B.3 Hierarchical Clustering - Clusters





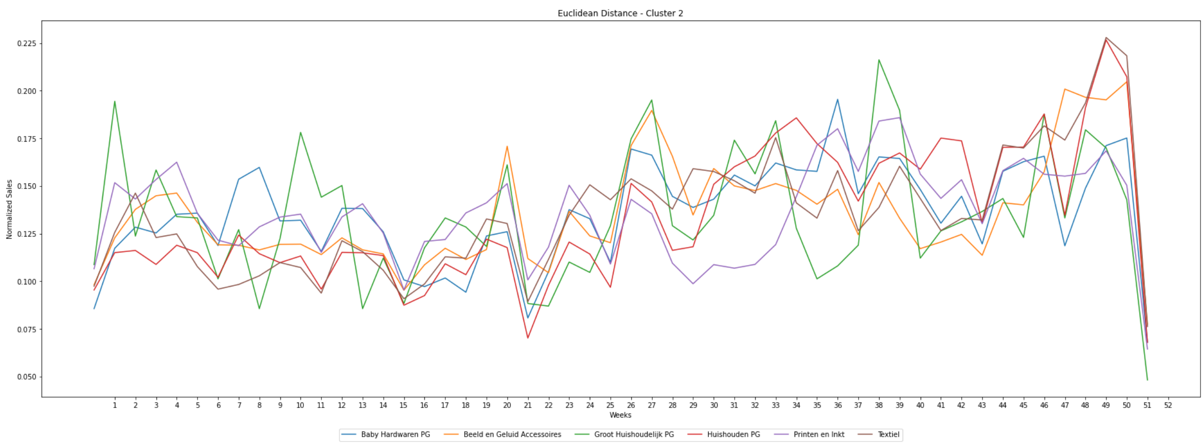


Figure B.4: Hierarchical, Euclidean Distance Clusters, k=3

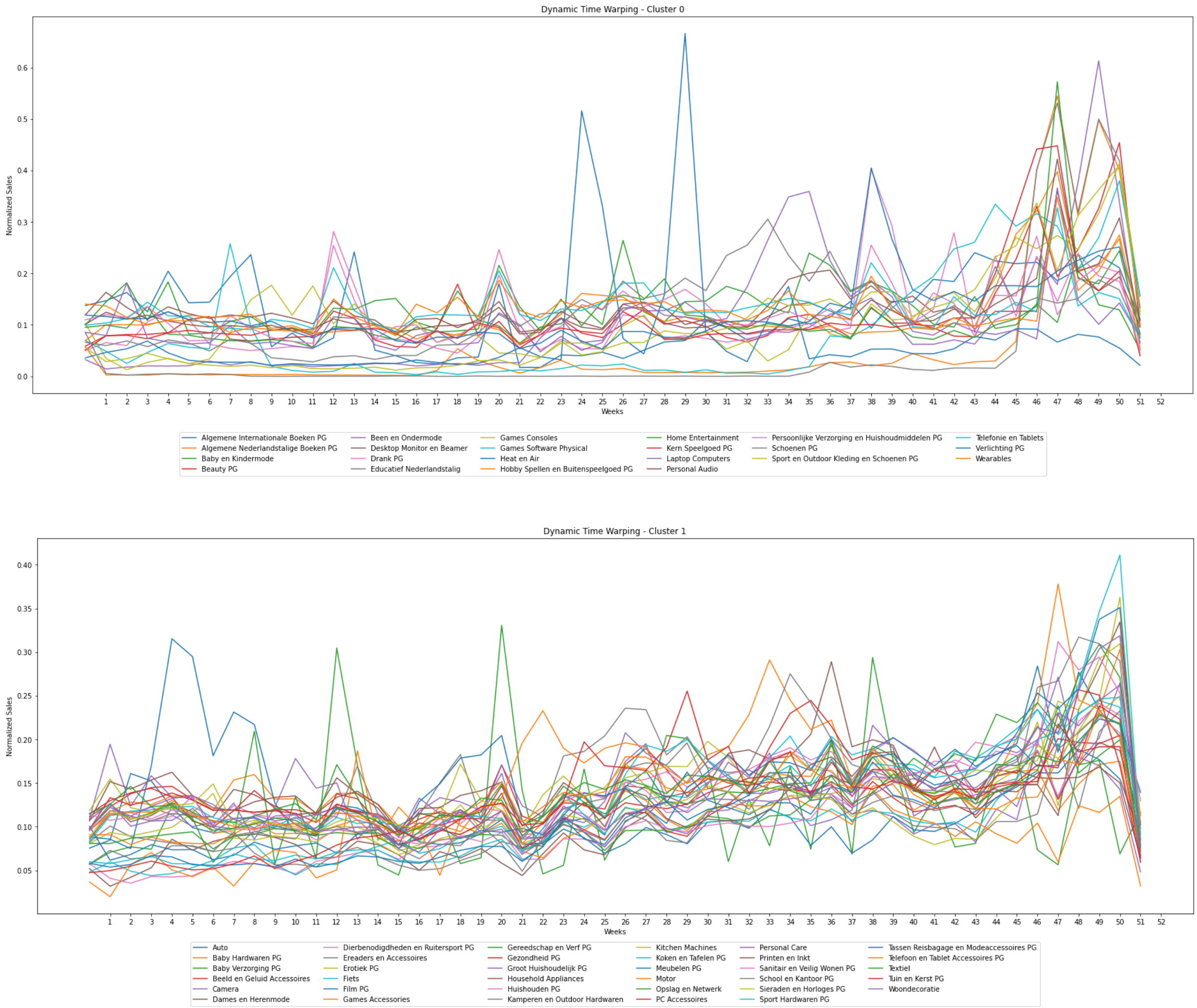


Figure B.5: Hierarchical, DTW Distance Clusters, k=2

B.4 Global Features Product Groups

Table B.1: Global Features - Characteristic-based Clustering

	Season Q1	Season Q2	Season Q3	Season Q4	Kurtosis	Trend	Skewness	Serial Corr
Algemene Internationale Boeken PG	0.5165	0.2352	0.3459	0.7471	0.0634	0.0001	0.0175	0.456
Algemene Nederlandstalige Boeken PG	0.0145	0.0781	0.0694	0.9944	0.0268	0.0254	0.0315	0.801
Auto	0.3975	0.3539	0.5187	0.669	0.0146	0.0025	0.0084	0.461
Baby Hardwaren PG	0.461	0.4114	0.5849	0.5254	0.0122	0.011	-0.0078	0.259
Baby Verzorging PG	0.4452	0.4501	0.6035	0.4848	0.0122	0.0128	0.0113	0.522
Baby en Kindermode	0.4288	0.4337	0.6358	0.4731	0.0122	0.0023	0.0366	0.082
Beauty PG	0.3319	0.3599	0.4067	0.7713	0.0122	0.1329	0.0402	0.385
Beeld en Geluid Accessoires	0.446	0.4361	0.5649	0.5402	0.0146	0.0105	-0.0042	0.327
Been en Ondermode	0.0895	0.1832	0.5251	0.8263	0.0146	0.0186	0.0908	0.297
Camera	0.3949	0.459	0.5725	0.5528	0.0122	0.0023	0.0152	0.282
Dames en Herenmode	0.2525	0.2616	0.661	0.6564	0.0146	0.0023	0.0105	0.609
Desktop Monitor en Beamer	0.4596	0.3904	0.5337	0.593	0.0146	0.0016	0.0387	0.287
Dierbenodigdheden en Ruitersport PG	0.178	0.3469	0.6208	0.6801	0.0146	0.0075	0.0016	0.543
Drank PG	0.3559	0.3416	0.4792	0.726	0.0146	0.0029	0.0263	0.448
Educatief Nederlandstalig	0.0104	0.0	0.0339	0.9994	0.0317	0.0039	0.0337	0.777
Ereaders en Accessoires	0.327	0.5222	0.6295	0.4734	0.0146	0.002	0.0046	0.427
Erotiek PG	0.3424	0.3714	0.6222	0.5981	0.0122	0.0267	0.0029	0.358
Fiets	0.201	0.3197	0.6075	0.6988	0.0171	0.0049	0.0043	0.572
Film PG	0.6117	0.4736	0.365	0.518	0.0366	-0.0001	0.0122	0.314
Games Accessories	0.3312	0.3549	0.4685	0.7382	0.0122	0.0045	0.0173	0.456
Games Consoles	0.2987	0.2811	0.325	0.8521	0.0439	0.0004	0.0177	0.542
Games Software Physical	0.2153	0.0471	0.1144	0.9687	0.0488	0.0016	0.017	0.741
Gereedschap en Verf PG	0.3205	0.3159	0.4506	0.7711	0.0146	0.0059	0.0084	0.581
Gezondheid PG	0.4349	0.4102	0.5338	0.598	0.0146	0.0087	-0.0019	0.344
Groot Huishoudelijk PG	0.4973	0.4177	0.562	0.5122	0.0341	0.0	-0.0023	0.232
Heat en Air	0.2034	0.5456	0.7167	0.3838	0.0122	0.0015	0.044	0.745
Hobby Spellen en Buitenspeelgoed PG	0.3931	0.3931	0.4171	0.7191	0.0146	0.1284	0.0227	0.565
Home Entertainment	0.342	0.4642	0.4886	0.6548	0.0122	0.0128	0.0753	0.276
Household Appliances	0.4345	0.3999	0.548	0.5924	0.0122	0.0037	0.0007	0.166
Huishouden PG	0.3966	0.3819	0.5729	0.6072	0.0122	0.018	-0.0035	0.366
Kamperen en Outdoor Hardwaren	0.2546	0.3101	0.5694	0.7175	0.0146	0.0085	0.0124	0.637
Kern Speelgoed PG	0.3322	0.3415	0.3784	0.7936	0.0098	0.1456	0.0266	0.659
Kitchen Machines	0.3996	0.3611	0.4718	0.6981	0.0122	0.0082	0.0082	0.419
Koken en Tafelen PG	0.4124	0.4027	0.4318	0.6938	0.0122	0.0431	0.0189	0.573
Laptop Computers	0.3216	0.3029	0.755	0.4847	0.0146	0.0043	0.0521	0.381
Meubelen PG	0.2591	0.3627	0.5841	0.6783	0.0146	0.0023	0.0018	0.4
Motor	0.1873	0.539	0.7354	0.3654	0.0537	0.0001	0.0078	0.515
Opslag en Netwerk	0.4307	0.4208	0.5495	0.5792	0.0122	0.0139	-0.0002	0.268
PC Accessoires	0.4412	0.3529	0.5932	0.5735	0.0122	0.0204	0.0009	0.489
Personal Audio	0.3869	0.3869	0.4787	0.6867	0.0146	0.0277	0.0644	0.251
Personal Care	0.392	0.3726	0.4985	0.6775	0.0146	0.0176	0.002	0.328
Pers. Verzorging en Huishoudm. PG	0.401	0.4356	0.5445	0.5941	0.0122	0.0541	0.0306	0.294
Printen en Inkt	0.4829	0.4630	0.5078	0.5427	0.0146	0.0023	-0.0074	0.313
Sanitair en Veilig Wonen PG	0.368	0.3075	0.4146	0.7734	0.0146	0.0186	0.0098	0.524
Schoenen PG	0.1934	0.3148	0.769	0.5217	0.0146	0.0046	0.0128	0.51
School en Kantoor PG	0.3758	0.3131	0.607	0.6263	0.0122	0.0354	0.0043	0.613
Sieraden en Horloges PG	0.412	0.4765	0.5212	0.5758	0.0098	0.0056	0.0122	0.458
Sport Hardwaren PG	0.2202	0.2982	0.6238	0.6881	0.0122	0.0138	0.0034	0.577
Sport en Outdoor Kl. en Schoenen PG	0.0923	0.1192	0.4421	0.8842	0.0171	0.016	0.02	0.656
Tassen Reisbagage en Modeacc. PG	0.2102	0.2772	0.5231	0.778	0.0146	0.0201	0.012	0.641
Telefonie en Tablets	0.4236	0.4684	0.5581	0.5382	0.0146	0.0062	0.0517	0.149
Telefoon en Tablet Accessoires PG	0.3903	0.4348	0.6027	0.5435	0.0122	0.0235	-0.0023	0.343
Textiel	0.4008	0.4304	0.5492	0.5937	0.0146	0.0068	-0.0026	0.243
Tuin en Kerst PG	0.2067	0.4323	0.6437	0.5967	0.0146	0.0066	0.0033	0.536
Verlichting PG	0.3939	0.2847	0.4935	0.7213	0.0122	0.0231	0.0193	0.519
Wearables	0.4008	0.4899	0.5196	0.574	0.0146	0.0046	0.0604	0.13
Woondecoratie	0.4135	0.3414	0.4615	0.7067	0.0122	0.0114	0.0081	0.49

B.5 Characteristic-based Clusters

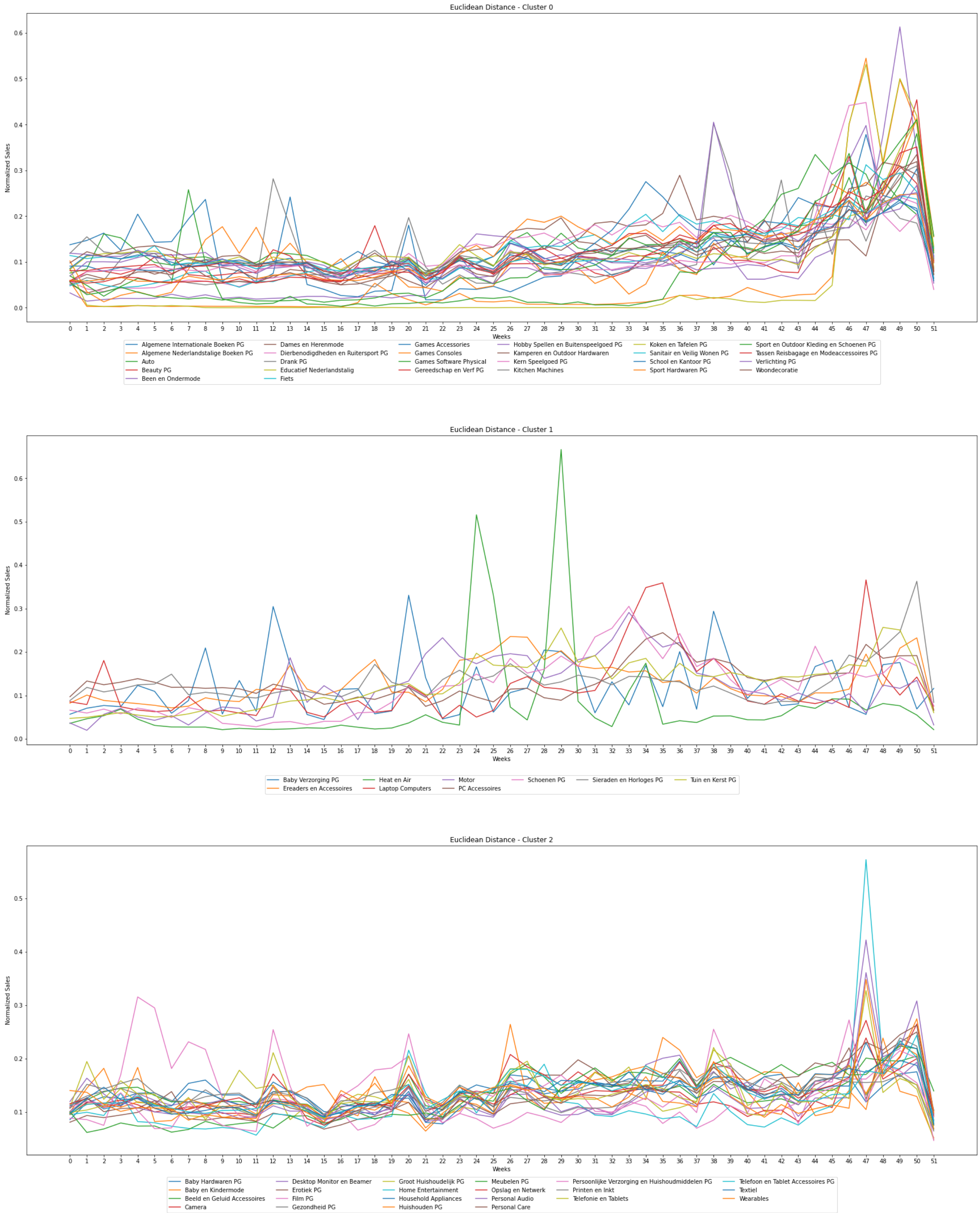


Figure B.6: Validation, Euclidean Distance Clusters, k=3

B.6 Final Clusters

Figure B.7: Final, Euclidean Distance Clusters, k=3

Appendix C

ACF and PACF Plots

ACF and PACF plots can be found from the next page on.

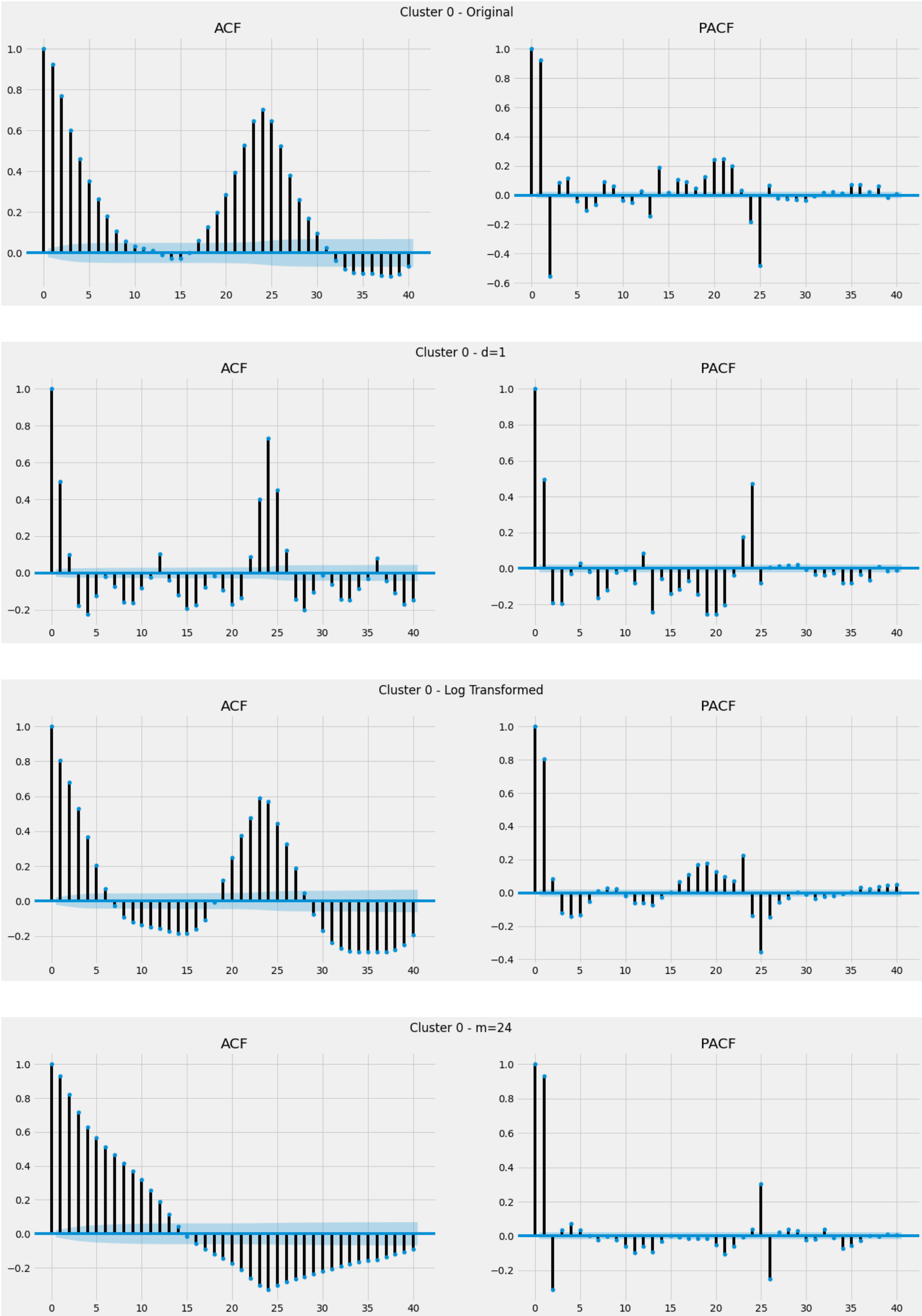


Figure C.1: ACF and PACF Plots - Cluster 0

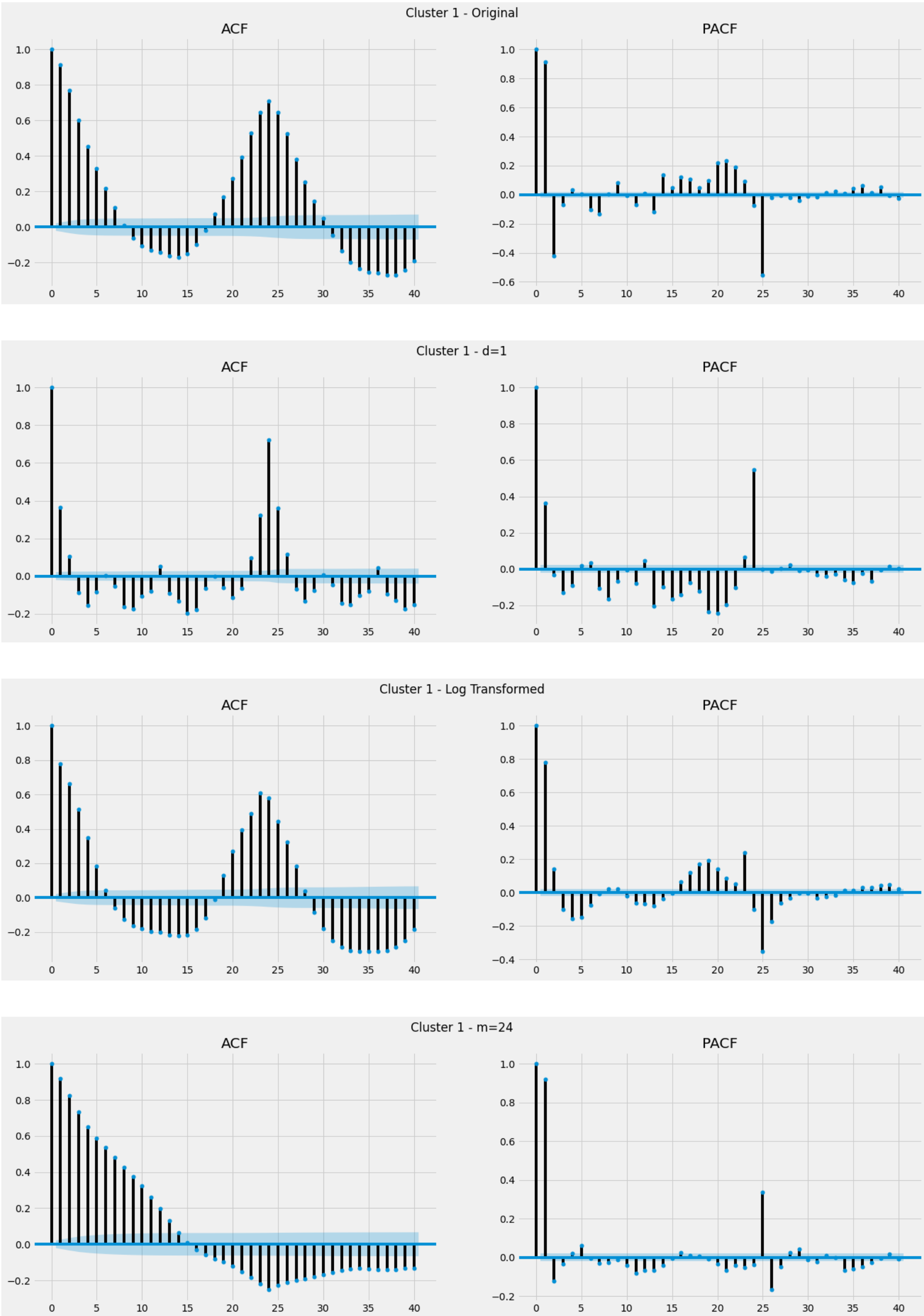


Figure C.2: ACF and PACF Plots - Cluster 1

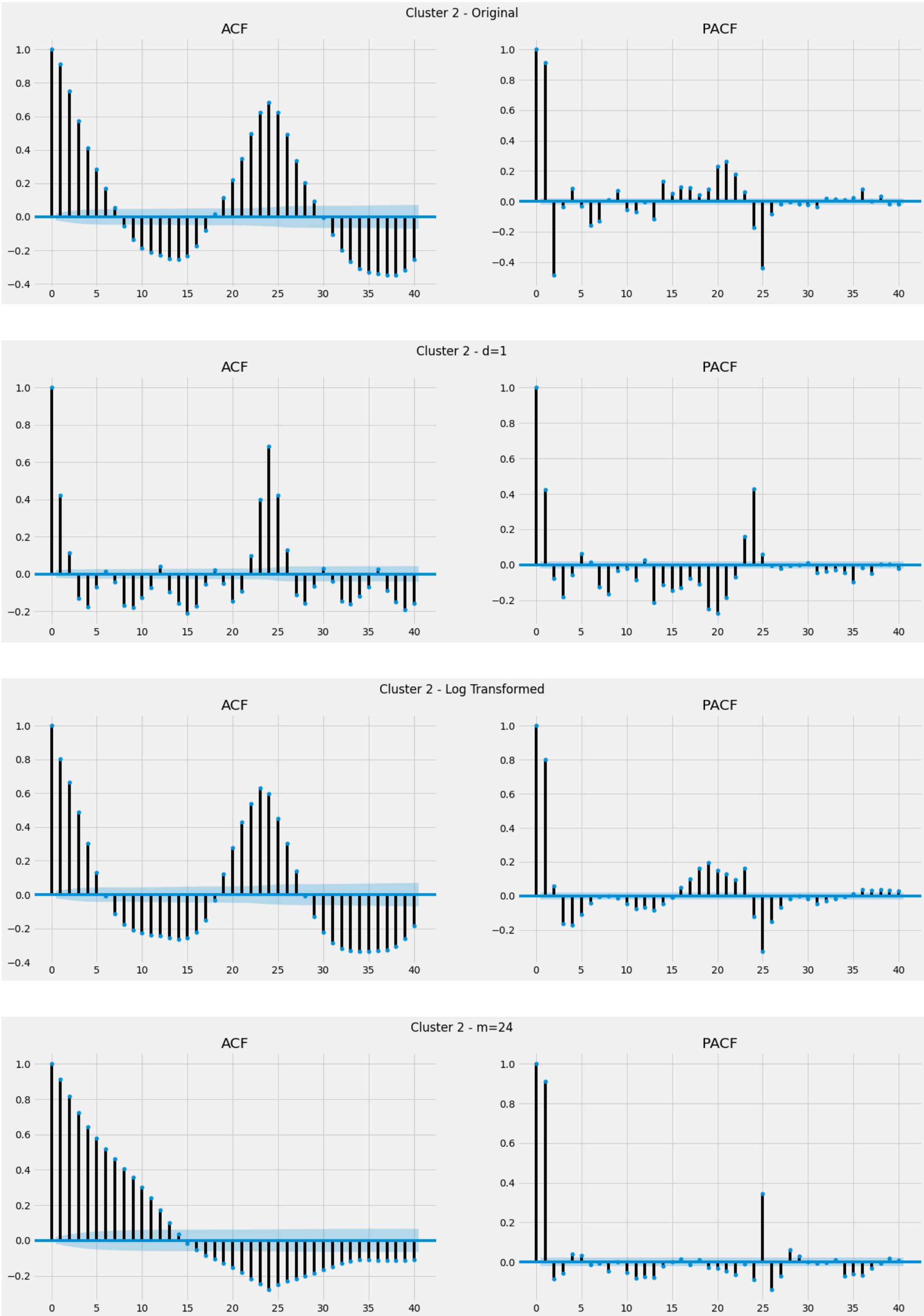


Figure C.3: ACF and PACF Plots - Cluster 2

Figure C.4: ACF and PACF Plots - Cluster 3

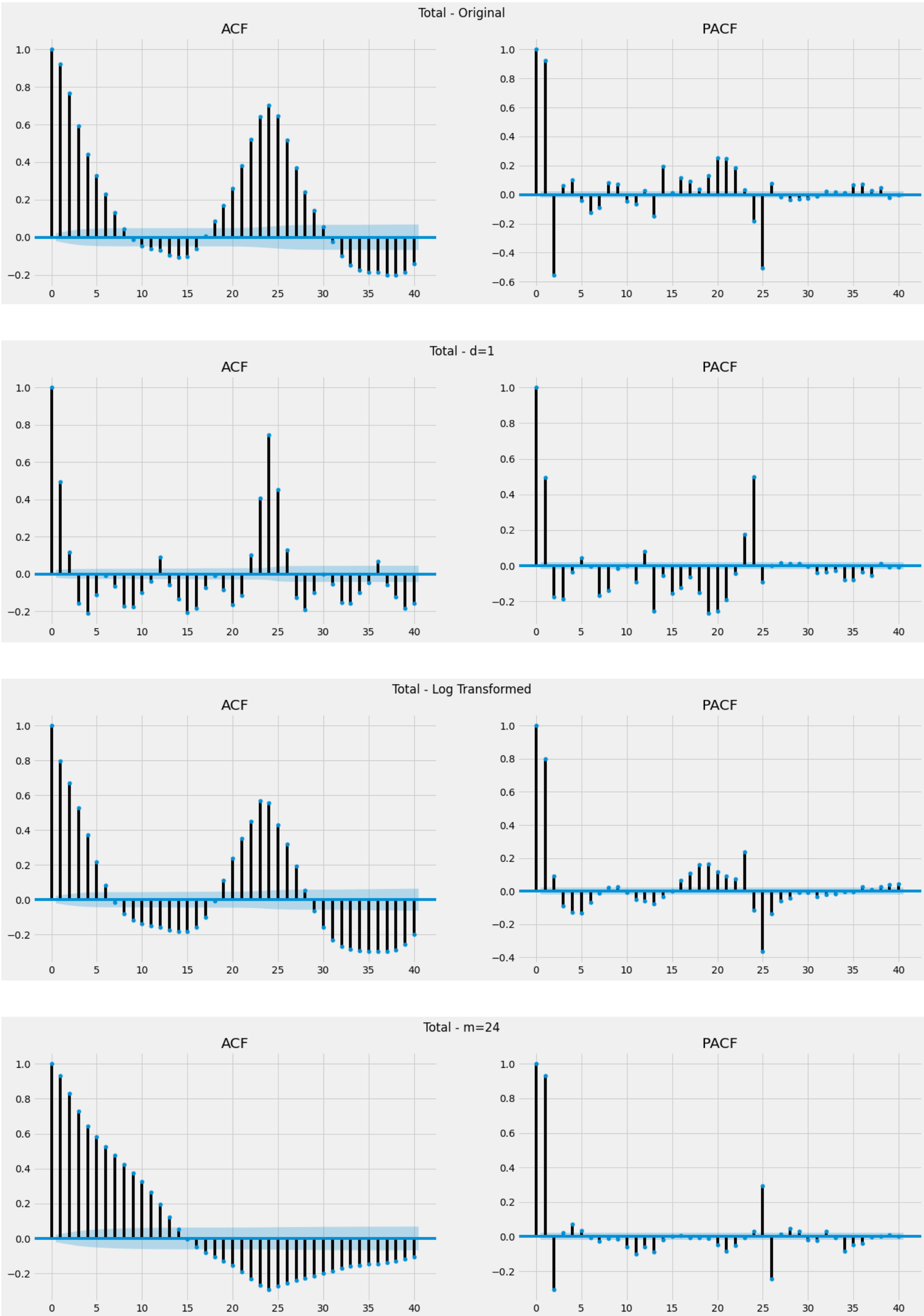


Figure C.5: ACF and PACF Plots - Total Set

Appendix D

Exponential Smoothing Results

Table D.1: RMSE - Exponential Smoothing - SES, Holt

	Simple Exponential Smoothing				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	355.26	94.72	69.48	65.86	487.03
2	389.2	90.59	72.04	77.09	528.68
3	484.36	124.65	96.25	80.66	710.14
4	526.06	146.89	100.40	90.09	782.24
5	600.41	149.61	102.90	97.28	861.99
6	640.94	155.71	122.74	107.32	942.85
7	699.75	138.01	127.59	112.94	984.01
8	723.01	165.93	106.21	120.28	1019.26
9	1151.04	291.35	168.00	197.44	1737.93
10	1760.83	399.3	231.64	253.29	2554.59
11	1740.9	368.19	200.53	186.77	2371.77
12	1239.88	276.61	220.13	193.46	1797.76
13	1999.2	580.37	436.58	397.69	3367.62
14	2869.4	730.29	519.97	469.49	4515.01
15	2715.47	674.28	496.84	449.02	4262.19
16	2793.37	700.18	506.28	454.54	4379.77
17	2818.97	709.75	508.49	459.16	4421.94
18	2815.23	708.15	505.33	457.05	4411.2
19	2721.82	678.96	485.44	437.89	4249.77
20	2430.8	600.1	430.64	381.11	3768.94
21	2018.38	492.21	352.66	310.63	3097.21
22	1679.08	389.11	276.28	259.92	2511.42
23	1551.65	349.51	248.36	246.09	2298.4
24	1640.42	370.15	265.43	258.49	2438.82

	Holt Exponential Smoothing				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	521.89	94.67	69.44	66.80	760.51
2	1003.32	90.52	73.07	78.60	1509.76
3	1373.00	124.49	96.89	84.16	2107.82
4	1961.87	146.72	104.29	95.42	3037.96
5	2524.69	149.37	112.37	104.56	3901.61
6	3056.59	155.49	132.93	116.34	4790.8
7	3691.93	137.90	147.08	124.94	5837.66
8	3967.34	165.57	140.75	133.21	6249.14
9	3846.03	290.73	191.12	204.84	6011.02
10	3844.04	398.56	247.92	259.90	6008.36
11	4453.29	367.49	224.56	197.45	7096.24
12	5773.35	276.47	255.3	209.23	9369.43
13	7688.26	581.16	467.42	410.01	12557.35
14	9318.05	731.45	560.76	484.89	15019.79
15	9634.21	675.49	542.78	466.61	15517.58
16	10198.16	701.45	557.04	473.92	16415.86
17	10704.4	711.09	564.32	480.47	17229.08
18	11181.91	709.58	566.87	480.58	17991.56
19	11584.53	680.48	554.72	464.56	18624.58
20	11798.18	601.69	512.36	413.59	18946.77
21	11837.65	493.84	451.76	351.93	18983.99
22	11841.23	390.6	399.67	308.48	18917.71
23	11781.75	350.45	377.72	297.44	18781.72
24	11984.98	370.57	391.23	308.55	19082.6

Table D.2: RMSE - Exponential Smoothing - Holt Winters

	Holt Winters Exponential Smoothing				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	544.64	105.15	73.72	57.96	735.06
2	699.59	128.24	88.83	85.16	998.22
3	821.38	163.22	111.82	80.93	1192.06
4	976.77	215.45	132.8	101.26	1385.49
5	1090.42	224.56	145.01	108.31	1499.98
6	1085.15	233.91	141.3	104.61	1449.23
7	1103.11	215.12	132.49	103.35	1428.21
8	1151.46	225.86	139.77	132.34	1520.16
9	1332.72	285.46	188.32	185.61	1849.18
10	1767.33	346.76	226.18	205.51	2385.65
11	2155.85	402.09	252.37	187.03	2862.86
12	1945.85	392.26	242.00	163.27	2780.43
13	1539.86	335.59	199.55	150.14	2236.56
14	1174.95	171.43	160.83	132.08	1635.56
15	1391.34	203.87	163.21	140.2	1939.28
16	1477.55	217.11	165.34	138.95	2114.9
17	1544.69	240.61	169.8	147.51	2279.15
18	1602.18	262.45	174.82	155.09	2393.36
19	1645.4	279.19	176.9	159.94	2450.21
20	1692.55	300.7	187.25	170.12	2493.15
21	1802.73	342.56	211.93	206.04	2626.06
22	1995.32	406.51	255.59	256.74	2899.5
23	2298.27	475.33	303.12	292.7	3262.37
24	2434.9	511.04	326.18	305.98	3450.35

Table D.3: MASE - Exponential Smoothing - SES, Holt

	Simple Exponential Smoothing				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	0.55	0.68	0.7	0.66	0.53
2	0.57	0.63	0.7	0.76	0.53
3	0.79	0.88	0.94	0.77	0.77
4	0.77	0.95	0.92	0.85	0.75
5	0.85	0.99	0.91	0.91	0.81
6	0.98	1.02	1.17	1.03	0.96
7	0.97	0.86	1.18	1.09	0.91
8	1.21	1.15	0.93	1.19	1.10
9	2.32	2.29	1.76	2.12	2.27
10	3.4	3.19	2.53	2.91	3.29
11	2.92	2.8	1.98	2.09	2.72
12	2.16	1.8	2.21	2.00	2.07
13	4.16	4.26	4.92	4.82	4.48
14	6.19	6.2	6.36	6.13	6.44
15	5.83	5.68	6.04	5.81	6.05
16	5.99	5.91	6.17	5.9	6.22
17	6.05	6.00	6.20	5.97	6.28
18	6.04	5.99	6.17	5.95	6.27
19	5.79	5.73	5.92	5.69	6.01
20	5.03	5.01	5.23	4.93	5.23
21	4.06	4.07	4.28	3.96	4.21
22	3.14	3.05	3.19	3.14	3.2
23	2.6	2.52	2.55	2.74	2.64
24	2.72	2.55	2.6	2.79	2.72

	Holt Exponential Smoothing				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	0.94	0.68	0.69	0.67	0.95
2	1.71	0.63	0.7	0.78	1.81
3	2.46	0.87	0.94	0.79	2.64
4	3.52	0.95	0.94	0.87	3.82
5	4.42	0.99	0.96	0.93	4.83
6	5.52	1.02	1.22	1.06	6.12
7	6.63	0.86	1.29	1.13	7.44
8	6.85	1.15	1.07	1.23	7.69
9	6.52	2.29	1.88	2.15	7.16
10	6.72	3.19	2.62	2.95	7.22
11	8.14	2.79	2.1	2.13	8.96
12	11.01	1.8	2.39	2.06	12.42
13	15.3	4.27	5.17	4.9	17.24
14	18.22	6.22	6.64	6.22	20.21
15	18.71	5.69	6.33	5.91	20.78
16	19.75	5.92	6.48	6.01	21.95
17	20.67	6.01	6.53	6.08	23.00
18	21.52	6.01	6.52	6.06	23.97
19	22.14	5.75	6.29	5.81	24.71
20	22.25	5.03	5.62	5.06	24.91
21	22.03	4.08	4.69	4.09	24.7
22	21.76	3.07	3.62	3.27	24.35
23	21.67	2.53	2.99	2.88	24.22
24	22.14	2.55	3.04	2.92	24.67

Table D.4: MASE - Exponential Smoothing - Holt Winters

	Holt Winters Exponential Smoothing				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	0.86	0.82	0.78	0.69	0.82
2	1.18	0.88	0.88	0.93	1.1
3	1.47	1.15	1.13	0.86	1.35
4	1.74	1.48	1.33	1.06	1.54
5	1.88	1.58	1.37	1.16	1.64
6	1.82	1.67	1.36	1.12	1.6
7	1.9	1.55	1.3	1.15	1.62
8	2.2	1.61	1.41	1.59	1.9
9	2.5	2.18	1.9	2.22	2.36
10	3.32	2.65	2.39	2.43	3.03
11	3.95	2.95	2.58	2.11	3.44
12	3.63	2.76	2.51	1.83	3.45
13	3.01	2.48	2.15	1.44	2.81
14	2.13	0.99	1.51	1.04	1.82
15	2.74	1.5	1.73	1.34	2.4
16	2.66	1.54	1.75	1.27	2.51
17	2.66	1.71	1.8	1.34	2.7
18	2.72	1.88	1.87	1.45	2.82
19	2.82	2.06	1.93	1.58	2.92
20	2.95	2.31	2.09	1.87	3.03
21	3.24	2.73	2.45	2.46	3.31
22	3.67	3.2	2.89	3.08	3.67
23	4.11	3.55	3.26	3.43	3.91
24	4.34	3.72	3.38	3.61	4.1

Appendix E

ARIMA Results

Table E.1: RMSE - ARIMA

	Original					Log Transformed				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	390.16	105.14	75.59	76.07	476.52	860.88	187.58	138.76	104.78	1258.29
2	537.55	122.32	104.84	121.61	706.4	1263.77	279.92	226.65	173.21	2420.71
3	877.34	177.7	148.97	149.3	1161.36	1688.41	362.24	283.14	232.8	2886.64
4	1039.53	224.27	169.18	155.95	1407.29	1719.73	422.99	307.69	249.56	3244.48
5	1234.95	249.3	182.21	164.9	1736.25	1690.11	456.5	325.08	267.55	3034.94
6	1343.85	261.32	173.93	153.73	1883.18	1699.93	472.66	309.51	265.61	3119.87
7	1312.56	263.26	161.9	149.8	1884.68	1584.34	444.35	283.15	258.31	2739.98
8	1743.17	384.3	246.01	252.92	2698.93	1942.76	544.81	350.2	344.68	3296.42
9	2548.47	564.33	395.45	391.72	4072.97	2713.7	710.42	483.81	460.8	4268.52
10	3258.75	682.29	477.54	468.65	5122.31	3407.28	836.9	558.6	516.28	5217.76
11	3175.69	647.51	414.52	386.03	4903.03	3379.04	788.58	491.67	433.01	4886.92
12	2248.68	483.35	260.59	201.38	3462.77	2483.96	594.33	330.11	245.8	3674.38
13	1296.01	353.51	165.19	155.61	1729.09	1292.09	332.98	161.82	121.34	2662.37
14	1249.69	305.73	163.86	171.97	1019.55	749.31	201.08	101.16	114.96	2438.85
15	1094.73	270.31	129.79	140.29	680.99	632.59	160.53	68.55	73.28	2527.22
16	1148.04	305.7	126.46	134.68	670.73	685.61	168.56	67.53	67.4	2802.29
17	1161.47	300.28	115.51	129.1	616.7	644.86	164.66	71.34	66.88	2648.94
18	1147.9	256.27	100.84	118.09	525.98	572.64	141.97	78.04	66.62	2342.46
19	1066.00	188.83	75.77	93.12	354.55	587.42	119.91	96.42	77.01	2141.65
20	892.88	133.27	63.27	71.93	631.58	796.9	124.22	133.66	124.74	2006.62
21	898.54	182.66	140.31	152.64	1488.51	1226.86	224.49	201.28	212.75	2143.12
22	1285.42	321.86	269.48	271.72	2593.69	1705.78	389.06	294.95	315.19	2532.22
23	1874.17	477.86	395.03	357.11	3659.5	2244.78	544.85	398.75	380.7	3119.1
24	2234.68	595.93	463.33	399.67	4267.84	2581.99	651.28	463.17	412.82	3578.83

Table E.2: RMSE - ARIMAX, Original

	Original											
	Cl. 0		Cl. 1			Cl. 2		Cl. 3		Total Set		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	365.22	371.32	95.16	106.01	106.45	75.58	80.55	77.58	80.46	487.06	501.21	533.80
2	518.35	544.49	136.07	139.09	141.17	108.73	109.46	124.21	117.88	777.89	776.99	828.45
3	805.89	761.91	218.83	215.00	207.54	162.96	153.29	154.15	147.41	1247.14	1234.38	1220.50
4	954.83	897.61	279.40	250.78	245.42	195.42	166.42	166.45	153.48	1504.42	1444.32	1473.67
5	1167.66	1119.29	319.39	288.24	276.06	214.46	184.9	179.56	162.05	1771.26	1714.25	1713.96
6	1276.76	1188.39	332.51	309.87	289.66	211.54	190.83	171.39	173.95	1872.91	1824.61	1778.68
7	1283.63	1225.36	320.76	287.37	273.35	195.74	174.21	169.37	163.61	1809.54	1784.55	1758.15
8	1698.24	1652.10	420.45	374.9	363.43	262.46	217.45	268.4	230.05	2414.94	2414.71	2382.58
9	2496.49	2440.51	584.78	554.27	539.56	395.18	353.86	401.39	362.40	3564.98	3627.86	3560.59
10	3183.65	3113.72	696.97	683.29	666.24	462.82	435.02	475.25	447.07	4458.16	4534.04	4431.58
11	3083.31	2997.71	645.09	639.4	621.03	393.14	369.24	392.18	369.20	4152.09	4243.52	4115.47
12	2134.28	2057.66	461.69	432.7	411.31	258.96	212.30	212.56	208.51	2797.66	2845.23	2726.86
13	1137.45	1104.00	371.53	346.07	340.56	234.81	211.17	164.52	226.16	1997.28	1887.82	1901.60
14	1393.62	1477.03	427.29	392.44	419.15	265.73	269.11	185.34	240.62	2552.88	2283.99	2466.26
15	1240.89	1339.91	377.85	337.8	366.22	249.32	248.36	160.27	225.35	2298.85	1969.07	2191.02
16	1319.21	1398.86	401.87	367.18	393.88	256.48	261.32	160.03	227.36	2410.29	2077.4	2287.45
17	1355.45	1431.32	408.07	371.00	397.78	254.51	260.98	158.44	229.93	2447.10	2100.98	2299.18
18	1347.84	1426.06	405.51	375.60	404.23	249.68	262.38	151.59	226.68	2434.43	2090.67	2306.20
19	1283.79	1368.02	380.41	347.92	377.22	232.44	247.48	129.68	213.16	2296.00	1933.07	2135.99
20	1069.2	1121.25	313.21	285.12	310.82	188.72	208.21	97.57	167.39	1938.26	1561.26	1703.51
21	916.37	1013.57	238.58	192.40	219.70	148.77	141.25	146.59	125.33	1586.93	1258.91	1378.79
22	1111.27	1217.89	236.80	144.38	163.59	182.22	106.16	252.35	142.8	1759.54	1589.86	1657.06
23	1579.11	1667.94	327.26	217.81	217.96	271.75	160.99	333.83	191.27	2354.95	2317.37	2365.68
24	1895.39	1973.68	409.48	287.15	278.57	330.16	198.72	373.53	215.71	2823.86	2777.35	2823.33

Table E.3: RMSE - ARIMAX, Log Transformed

	Log Transformed											
	Cl. 0		Cl. 1			Cl. 2		Cl. 3		Total Set		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	2430.95	2453.70	619	761.43	765.67	465.75	406.27	417.66	354.14	5249.03	5622.48	9634.95
2	2189.96	2236.20	569.45	1004.89	1010.36	387.44	301.09	348.13	255.83	6531.94	5809.91	34323.05
3	2086.36	2155.18	502.57	1347.53	1353.97	336.89	241.01	302.11	204.27	8920	6930.85	90525.92
4	1920.62	2000.77	458.82	1895.58	1903.14	290.47	194.13	260.68	168.03	12317.48	9404.81	170588
5	1780.46	1833.05	449.14	2412.49	2416.77	250.95	162	225.17	144.08	15788.7	12929.12	306321.81
6	1710.33	1724.5	403.57	2632.10	2632.57	217.85	140.75	195.65	127.21	17613.6	10946.67	131348.06
7	1604.74	1570.29	357.67	2585.03	2583.17	189.84	117.72	170.75	111.23	18599.41	12548.56	105481.16
8	1507.76	1443.14	357.59	1797.26	1795.17	165.78	102.08	149.19	96.61	18915.09	12916.72	24154.63
9	1448.17	1362.00	355.12	990.78	991.18	145.01	93.38	130.68	89.8	18845.06	13944.78	14943.96
10	1332.15	1229.16	306.8	377.11	380.94	127.24	88.62	114.97	85.56	18478.95	14542.42	31270.97
11	1241.19	1130.39	300.19	328.68	333.03	112.36	86.27	101.62	83.07	18251.01	15200.42	41792.67
12	1198.02	1082.09	307.7	440.32	443.43	98.34	84.42	89.88	82.31	17923.02	15479.5	42946.6
13	1111.19	995.56	264.65	380.54	384.42	89.25	86.13	82.11	84.72	17795.37	15901.93	14691.42
14	1048.92	930.29	238.23	298.21	303.15	81.48	87.72	75.45	85.74	17587.45	15994.5	11564.38
15	970.52	924.77	247.71	215.14	220.95	72.62	87.29	66.55	86.79	25553.65	23125.62	12273.15
16	900.33	848.63	235.28	186.27	192.96	66.82	87.55	60.19	86.62	25418.19	23197.83	12824.51
17	871.71	821.24	192.73	176.91	183.51	60.23	87.74	54.57	86.61	25436.05	23410.87	11185.32
18	896.90	872.02	201.84	177.79	186.42	54.04	87.65	49.31	86.10	25373.27	23409.88	22428.03
19	874.68	854.53	225.14	195.01	204.58	48.19	86.9	44.09	85.27	25428.69	23564.84	28471.60
20	847.01	820.74	185.33	241.96	252.08	42.68	87.31	39.08	84.66	25383.56	23521.34	19073.62
21	827.80	798.43	179.23	342.8	351.04	37.88	88.15	34.83	86.52	25446.27	23649.73	22601.23
22	748.26	709.41	214.47	505.23	511.17	33.74	90.83	31.17	89.01	25401.38	23628.76	53390.80
23	697.80	650.75	199.28	817.72	819.20	30.07	96.04	28.07	95.55	25461.45	24416.43	399486.92
24	695.80	659.75	165.82	1273.32	1272.33	27.06	100.99	25.44	99.74	25412.63	27768.31	1352484.88

Table E.4: RMSE - SARIMA

	Original					Log Transformed				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	275.48	82.6	62.68	75.63	384.29	488.52	96.27	84.09	56.63	613.4
2	436.2	103.43	82.74	94.27	605.57	659.84	152.7	111.37	91.05	1060.89
3	586.21	141.29	103.03	102.42	855.46	790.72	176.79	139.64	105.2	1279.88
4	621.17	166.33	107.23	99.92	939.66	869.61	217.71	150.04	116.43	1433.36
5	693.59	175.94	118.56	105.03	1029.88	953.21	231.36	159.21	110.1	1497.83
6	729.6	184.68	116.14	101.32	1051.63	1001.81	247.81	163.72	119.53	1575.03
7	750.45	181.89	111.31	113.7	1063.53	1029.95	229.45	152.61	113.29	1513.13
8	953.06	235.17	144.72	170.31	1406.7	1256.21	288.71	168.64	141.23	1657.98
9	1337.56	320.0	217.58	244.24	1997.8	1740.04	371.37	224.6	202.1	2294.81
10	1791.39	383.97	260.06	283.28	2561.77	2430.46	443.54	285.74	246.48	3178.06
11	1843.56	385.55	244.78	239.37	2545.55	2773.89	509.92	335.78	283.76	3774.95
12	1378.15	312.2	175.5	142.06	1876.18	2351.99	472.62	294.62	220.7	3289.12
13	695.5	189.36	99.84	74.33	968.08	1355.41	320.83	177.03	122.38	2028.75
14	175.82	42.54	34.55	43.18	271.17	31.56	3.99	3.64	2.24	41.12
15	214.93	57.31	37.65	42.54	323.51	217.64	57.46	28.57	25.89	365.18
16	158.42	41.65	32.01	36.62	240.36	97.39	29.19	14.84	20.26	169.59
17	138.89	37.78	29.09	32.63	209.01	65.09	18.97	12.0	9.49	108.09
18	130.33	32.16	27.01	27.81	190.65	62.39	17.46	10.58	9.56	95.27
19	144.62	37.57	29.7	28.46	213.47	130.41	34.76	20.05	16.96	174.12
20	323.01	69.04	51.91	56.43	468.16	392.12	87.67	46.92	44.77	522.76
21	622.88	126.61	85.62	102.15	885.14	823.12	165.98	90.78	90.31	1163.42
22	887.91	191.81	131.87	156.62	1291.05	1243.33	260.99	145.43	159.32	1786.36
23	1183.83	255.09	183.79	184.15	1726.12	1806.99	368.23	226.23	216.69	2693.64
24	1313.56	294.8	203.29	198.0	1929.39	2087.62	434.2	270.89	239.73	3170.71

Table E.5: RMSE - SARIMAX, Original

	Original											
	Cl. 0		**Cl. 1**			**Cl. 2**		**Cl. 3**		**Total Set**		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	546.83	536.66	113.61	109.22	176.54	84.32	81.33	61.26	61.41	781.86	684.85	870.19
2	732.86	692.32	154.89	143.22	210.79	111.5	101.53	85.88	89.87	1145.73	922.39	1311.73
3	1019.11	974.51	213.52	205.87	283.22	150.26	142.07	102.51	112.82	1796.37	1349.5	2112.8
4	1087.19	1053.16	257.49	261.02	295.3	165.65	161.41	110.95	118.63	1922.55	1476.72	2243.03
5	1225.02	1191.53	282.94	284.09	308.75	187.34	179.25	121.96	122.22	2098.43	1691.98	2273.09
6	1359.26	1322.56	302.35	313.58	321.64	197.93	187.71	130.09	133.34	2276.12	1867.14	2382.98
7	1293.24	1273.55	281.9	282.86	289.6	184.73	168.67	122.03	117.67	2074.52	1736.69	2223.96
8	1358.57	1442.01	296.32	318.15	307.81	189.08	189.94	130.02	145.2	2214.11	1942.73	2163.04
9	1975.95	2130.14	387.13	431.04	470.58	237.01	264.31	176.06	210.35	3038.55	2835.31	3131.53
10	2896.15	3049.25	482.04	530.94	603.76	301.43	334.83	229.9	271.08	4084.71	3928.39	4799.0
11	3290.36	3415.72	566.06	599.45	627.99	332.97	356.35	253.96	286.95	4455.51	4370.8	5307.71
12	2685.3	2733.77	521.56	526.9	530.25	298.55	297.55	198.14	206.53	3581.76	3616.51	4034.59
13	1488.84	1428.05	385.64	350.81	390.07	200.01	172.68	131.52	112.21	1934.59	1991.61	2000.31
14	620.02	241.15	193.01	114.74	226.39	124.99	22.67	100.52	29.92	445.26	286.04	2151.93
15	690.35	355.06	201.09	121.84	135.07	134.08	35.86	114.13	40.79	587.5	488.91	1358.86
16	641.48	243.05	191.0	107.02	192.99	131.17	22.65	112.08	32.86	409.72	297.44	1531.27
17	639.01	209.29	189.7	104.43	240.93	131.1	19.18	119.43	31.65	416.17	232.82	1752.53
18	630.98	193.52	185.9	101.05	237.14	130.2	17.69	122.76	28.77	420.48	206.37	1968.55
19	633.34	200.07	182.81	95.33	196.28	130.47	21.11	120.49	25.99	443.47	228.68	2115.07
20	710.39	405.76	185.19	107.64	147.73	139.13	43.39	121.29	38.14	605.12	531.55	1965.07
21	1076.73	914.46	227.96	165.17	139.02	160.4	88.82	147.65	86.81	1060.95	1211.8	1536.56
22	1383.72	1352.5	258.82	234.89	138.05	184.47	139.01	186.37	154.18	1443.49	1819.43	1347.71
23	1890.82	1976.47	317.82	339.2	244.9	220.49	212.3	211.54	206.03	2165.96	2673.71	2088.66
24	2139.04	2250.52	358.12	401.83	344.09	244.77	247.07	224.63	224.42	2486.16	3043.05	2572.97

Table E.6: RMSE - SARIMAX, Log Transformed

	Log Transformed											
	Cl. 0		Cl. 1			Cl. 2		Cl. 3		Total Set		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	4448.07	5316.07	978.7	1193.41	1045.25	575.09	586.02	475.83	492.15	6494.86	6676.57	6879.56
2	4269.02	6910.16	950.24	1332.57	1157.35	537.87	552.11	436.05	459.29	5879.14	6138.83	6571.30
3	3890.49	8672.47	840.26	1273.88	1206.95	558.46	574.98	443.49	470.82	6014.62	6350.79	6544.21
4	3448.72	10167.31	717.43	1152.0	1219.02	527.17	542.61	407.83	435.45	5579.11	5923.55	6189.45
5	2937.24	10443.85	624.79	1051.59	1165.99	496.62	509.9	407.08	433.82	5265.58	5609.4	5866.50
6	2348.93	9251.09	566.28	975.13	1056.02	480.24	490.57	391.03	416.13	5113.77	5426.06	5557.99
7	1718.28	6928.61	514.56	883.89	886.04	406.72	412.68	338.69	357.21	4372.18	4621.82	4722.39
8	1117.84	4394.69	441.3	741.97	669.38	418.43	420.53	352.44	368.93	4466.28	4675.7	4699.43
9	636.36	2350.9	337.76	548.08	447.18	507.16	504.11	430.29	443.85	5490.49	5653.09	5731.85
10	308.5	1061.15	218.15	340.86	258.33	554.23	544.62	482.99	490.81	6354.58	6389.39	6401.08
11	122.69	401.05	112.05	172.16	126.21	464.01	451.17	410.14	407.97	6000.27	5876.32	5948.12
12	36.33	124.59	41.90	66.90	49.96	341.71	325.55	285.56	275.37	4580.15	4342.62	4596.30
13	7.55	30.31	11.00	19.90	15.73	179.92	168.99	149.27	139.81	2515.5	2325.51	2412.87
14	2.24	6.27	3.23	5.71	5.04	2.62	2.49	2.53	2.33	28.2	25.86	29.44
15	168.16	500.53	68.95	101.74	124.51	35.29	33.23	32.45	30.83	469.75	470.5	475.12
16	176.73	556.27	61.06	93.79	114.74	17.47	17.55	18.8	19.27	229.35	241.02	265.96
17	203.31	670.31	61.59	96.97	117.48	12.31	12.4	12.23	12.56	153.6	163.09	173.10
18	251.56	862.09	70.09	111.26	128.82	14.76	14.95	10.56	11.11	135.64	146.39	147.48
19	329.71	1164.83	88.29	139.09	150.14	23.11	23.81	17.46	18.55	214.27	237.63	257.63
20	452.21	1627.45	120.88	185.32	186.36	60.88	61.89	49.28	52.78	545.43	611.29	524.77
21	641.18	2314.8	177.29	257.58	240.33	135.47	137.44	112.02	120.41	1260.4	1400.87	1456.02
22	928.11	3298.24	267.5	363.19	320.75	235.29	238.68	193.49	208.8	2193.5	2427.16	2579.74
23	1342.57	4632.89	393.17	506.55	430.68	356.69	363.23	292.42	316.36	3372.06	3727.38	3824.25
24	1858.46	6315.08	537.34	684.14	574.77	423.9	432.87	336.98	366.47	4011.42	4455.02	4587.41

Table E.7: MASE - ARIMA

	Original					Log Transformed				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	0.57	0.77	0.77	0.73	0.45	1.62	1.32	1.49	1.17	1.42
2	0.94	0.90	1.04	1.30	0.85	2.59	2.20	2.59	1.98	2.88
3	1.62	1.30	1.60	1.76	1.53	3.42	2.79	3.30	2.71	3.43
4	2.04	1.68	1.87	1.89	1.90	3.57	3.27	3.63	2.98	3.89
5	2.54	1.94	2.07	2.04	2.39	3.56	3.51	3.85	3.32	3.74
6	2.73	2.02	1.91	1.88	2.54	3.49	3.66	3.63	3.37	3.75
7	2.78	2.14	1.81	1.83	2.60	3.26	3.55	3.29	3.27	3.33
8	3.86	3.17	2.76	2.98	3.76	4.01	4.39	4.01	4.14	3.96
9	5.63	4.67	4.57	4.76	5.71	5.51	5.66	5.49	5.46	5.18
10	6.92	5.63	5.59	5.79	7.07	6.54	6.68	6.34	6.13	6.20
11	6.33	5.24	4.76	4.81	6.51	6.01	6.23	5.48	5.09	5.52
12	4.04	3.68	2.74	2.34	4.23	3.98	4.51	3.55	2.81	3.87
13	2.84	2.77	1.91	1.89	2.16	2.10	2.42	1.72	1.32	3.15
14	2.65	2.40	2.00	2.24	1.46	1.56	1.65	1.23	1.34	2.99
15	2.28	1.97	1.58	1.80	1.00	1.12	1.17	0.81	0.84	2.76
16	2.36	2.21	1.53	1.73	0.97	1.12	1.20	0.81	0.77	2.9
17	2.38	2.15	1.39	1.66	0.88	1.07	1.20	0.81	0.74	2.93
18	2.30	1.82	1.21	1.52	0.74	0.99	1.03	0.78	0.70	2.65
19	2.12	1.28	0.87	1.16	0.43	0.87	0.79	0.71	0.69	2.47
20	1.72	0.90	0.73	0.85	0.82	1.35	0.97	1.15	1.33	2.31
21	1.67	1.44	1.61	1.71	2.05	2.40	1.77	2.14	2.47	2.73
22	2.62	2.67	3.20	3.33	3.64	3.60	3.09	3.43	3.81	3.36
23	3.85	4.05	4.78	4.59	5.14	4.52	4.37	4.67	4.67	3.96
24	4.64	5.03	5.61	5.15	6.02	5.11	5.28	5.45	5.07	4.40

Table E.8: MASE - ARIMAX, Original

	Original											
	Cl. 0		**Cl. 1**			**Cl. 2**		**Cl. 3**		**Total Set**		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	0.54	0.55	0.68	0.77	0.76	0.77	0.83	0.75	0.82	0.50	0.52	0.53
2	0.96	0.95	1.01	1.03	1.02	1.13	1.16	1.35	1.34	0.96	1.00	1.03
3	1.60	1.49	1.66	1.58	1.54	1.75	1.63	1.84	1.66	1.58	1.67	1.64
4	1.95	1.81	2.11	1.87	1.83	2.11	1.81	2.03	1.77	1.94	1.97	1.96
5	2.49	2.35	2.48	2.18	2.11	2.31	1.99	2.24	1.91	2.34	2.38	2.36
6	2.63	2.45	2.55	2.25	2.18	2.17	2.02	2.11	2.01	2.41	2.46	2.40
7	2.74	2.59	2.47	2.14	2.09	2.01	1.85	2.05	1.87	2.34	2.45	2.41
8	3.75	3.63	3.35	2.89	2.84	2.88	2.35	3.20	2.53	3.33	3.43	3.38
9	5.54	5.41	4.85	4.39	4.31	4.53	4.02	4.92	4.19	5.06	5.23	5.15
10	6.76	6.64	5.82	5.51	5.44	5.38	5.01	5.92	5.29	6.24	6.38	6.27
11	6.15	6.07	5.32	5.10	5.03	4.43	4.16	4.9	4.31	5.55	5.73	5.62
12	3.88	3.81	3.53	3.20	3.12	2.56	2.09	2.51	2.20	3.42	3.47	3.38
13	2.25	2.18	2.74	2.75	2.72	2.35	2.25	1.88	2.72	2.49	2.5	2.49
14	2.66	2.66	3.28	3.34	3.48	2.81	3.26	2.22	3.10	3.22	2.96	3.05
15	2.37	2.30	2.89	2.90	3.04	2.62	3.04	1.87	2.92	2.85	2.55	2.66
16	2.49	2.43	3.11	3.16	3.30	2.74	3.21	1.86	2.95	3.02	2.68	2.78
17	2.54	2.46	3.14	3.20	3.34	2.71	3.2	1.83	3.00	3.04	2.69	2.77
18	2.50	2.44	3.12	3.24	3.39	2.65	3.23	1.71	2.96	3.00	2.66	2.76
19	2.33	2.28	2.95	3.00	3.15	2.52	3.04	1.4	2.81	2.78	2.43	2.52
20	1.95	1.95	2.47	2.42	2.56	2.16	2.52	1.06	2.17	2.39	1.87	1.93
21	1.73	1.91	1.83	1.50	1.64	1.65	1.56	1.63	1.48	2.05	1.54	1.62
22	2.21	2.48	1.76	1.07	1.17	1.79	1.13	3.07	1.56	2.25	2.06	2.10
23	3.32	3.59	2.34	1.61	1.60	2.96	1.73	4.28	2.21	3.03	3.18	3.22
24	4.07	4.33	3.17	2.15	2.10	3.74	2.21	4.83	2.43	3.78	3.91	3.95

Table E.9: MASE - ARIMAX, Log Transformed

	Log Transformed											
	Cl. 0		**Cl. 1**			**Cl. 2**		**Cl. 3**		**Total Set**		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	5.27	5.29	5.26	6.2	6.24	5.74	4.98	5.59	4.73	7.00	7.08	7.73
2	4.73	4.78	4.76	6.51	6.57	4.78	3.65	4.66	3.39	6.30	6.07	10.37
3	4.49	4.56	4.24	6.97	7.04	4.15	2.85	4.03	2.65	6.35	6.07	18.48
4	4.08	4.16	3.83	7.46	7.54	3.58	2.28	3.48	2.15	6.41	6.29	30.29
5	3.74	3.79	3.61	7.69	7.77	3.10	1.91	3.00	1.81	6.78	6.85	50.41
6	3.56	3.57	3.29	7.34	7.39	2.69	1.64	2.61	1.56	6.76	5.66	23.88
7	3.27	3.23	2.96	6.35	6.38	2.35	1.40	2.28	1.36	6.89	5.94	20.2
8	3.00	2.93	2.86	4.79	4.82	2.05	1.23	1.99	1.2	6.73	5.49	7.60
9	2.88	2.79	2.78	3.25	3.29	1.80	1.13	1.74	1.11	6.77	5.71	6.20
10	2.63	2.52	2.53	2.12	2.15	1.58	1.08	1.53	1.06	6.57	5.66	8.77
11	2.41	2.3	2.42	1.88	1.92	1.39	1.05	1.35	1.04	6.64	5.91	10.49
12	2.27	2.16	2.33	1.82	1.86	1.21	1.02	1.18	1.02	6.44	5.77	10.59
13	2.07	1.96	2.09	1.69	1.73	1.10	1.04	1.08	1.04	6.54	5.98	6.25
14	1.91	1.80	1.88	1.59	1.62	1.01	1.06	0.99	1.06	6.39	5.84	5.65
15	1.81	1.76	1.82	1.52	1.55	0.90	1.06	0.88	1.08	7.70	7.00	5.79
16	1.65	1.60	1.67	1.48	1.51	0.83	1.07	0.79	1.08	7.53	6.82	5.80
17	1.55	1.50	1.47	1.47	1.49	0.75	1.07	0.72	1.08	7.66	7.00	5.57
18	1.54	1.50	1.44	1.49	1.52	0.67	1.06	0.65	1.08	7.55	6.85	7.43
19	1.44	1.40	1.45	1.61	1.65	0.60	1.06	0.58	1.07	7.69	7.02	8.43
20	1.36	1.32	1.30	1.81	1.85	0.53	1.06	0.51	1.05	7.57	6.9	6.94
21	1.34	1.29	1.26	2.13	2.17	0.47	1.07	0.45	1.07	7.69	7.14	7.64
22	1.22	1.17	1.32	2.60	2.65	0.42	1.10	0.40	1.11	7.57	7.15	12.46
23	1.14	1.09	1.26	3.33	3.37	0.37	1.17	0.35	1.19	7.69	8.2	64.92
24	1,12	1,07	1,13	4,34	4,38	0,33	1,23	0,32	1,25	7,57	8,67	206,18

Table E.10: MASE - SARIMA

	Original					Log Transformed				
k	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**	**Cl. 0**	**Cl. 1**	**Cl. 2**	**Cl. 3**	**Total Set**
1	0.5	0.61	0.66	0.73	0.44	0.69	0.7	0.81	0.62	0.66
2	0.73	0.74	0.89	0.99	0.68	1.0	0.89	1.0	0.9	1.04
3	1.0	0.98	1.07	1.11	0.96	1.35	1.09	1.33	1.05	1.32
4	1.1	1.2	1.12	1.12	1.09	1.54	1.41	1.44	1.22	1.55
5	1.24	1.28	1.22	1.19	1.23	1.77	1.5	1.59	1.17	1.66
6	1.29	1.33	1.21	1.14	1.24	1.87	1.68	1.53	1.28	1.79
7	1.32	1.32	1.15	1.27	1.24	1.93	1.69	1.6	1.27	1.8
8	1.78	1.67	1.5	1.81	1.68	2.57	2.26	1.85	1.67	2.12
9	2.55	2.32	2.27	2.62	2.48	3.64	3.04	2.52	2.47	3.08
10	3.31	2.83	2.78	3.16	3.16	4.86	3.62	3.23	3.06	4.23
11	3.22	2.85	2.62	2.8	3.01	5.26	4.03	3.7	3.41	4.92
12	2.35	2.27	1.79	1.69	2.14	4.39	3.71	3.2	2.68	4.31
13	1.02	1.13	0.87	0.79	0.95	2.18	1.9	1.64	1.28	2.37
14	0.25	0.26	0.3	0.35	0.28	0.03	0.03	0.04	0.03	0.03
15	0.35	0.42	0.37	0.44	0.37	0.33	0.37	0.27	0.26	0.37
16	0.26	0.3	0.31	0.37	0.28	0.17	0.2	0.15	0.21	0.2
17	0.22	0.27	0.27	0.33	0.24	0.12	0.14	0.13	0.11	0.13
18	0.2	0.23	0.25	0.28	0.22	0.12	0.14	0.12	0.11	0.12
19	0.23	0.28	0.29	0.3	0.24	0.27	0.27	0.21	0.19	0.22
20	0.52	0.47	0.5	0.54	0.5	0.75	0.68	0.48	0.49	0.62
21	0.98	0.83	0.83	1.0	0.9	1.5	1.25	0.97	1.02	1.34
22	1.46	1.3	1.29	1.59	1.39	2.39	2.09	1.62	1.84	2.23
23	1.94	1.75	1.87	1.98	1.89	3.53	2.99	2.59	2.62	3.48
24	2.19	2.05	2.06	2.13	2.16	4.16	3.46	3.11	2.91	4.18

Table E.11: MASE - SARIMAX, Original

	Original											
	Cl. 0		**Cl. 1**			**Cl. 2**		**Cl. 3**		**Total Set**		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	0.97	0.88	0.93	0.88	1.34	0.87	0.86	0.72	0.74	0.9	0.76	1.05
2	1.35	1.19	1.16	1.06	1.58	1.15	1.07	1.02	1.07	1.36	1.05	1.55
3	2.03	1.86	1.68	1.54	2.21	1.59	1.53	1.23	1.34	2.22	1.7	2.66
4	2.16	2.06	1.97	1.97	2.29	1.71	1.76	1.29	1.39	2.43	1.88	2.88
5	2.42	2.3	2.24	2.15	2.18	1.92	1.89	1.41	1.42	2.66	2.16	2.84
6	2.72	2.61	2.4	2.43	2.31	2.04	2.01	1.56	1.53	2.92	2.42	3.06
7	2.46	2.45	2.2	2.15	1.99	1.91	1.83	1.44	1.4	2.57	2.17	2.64
8	2.65	2.83	2.3	2.45	2.39	1.93	2.1	1.56	1.67	2.76	2.45	2.58
9	3.92	4.28	2.96	3.31	3.92	2.47	2.9	2.18	2.42	3.83	3.66	4.38
10	5.74	6.23	3.7	4.21	5.01	3.24	3.71	2.84	3.16	5.26	5.08	6.5
11	6.67	7.05	4.62	4.98	5.02	3.68	4.17	3.11	3.39	5.85	5.76	6.78
12	5.59	5.77	4.28	4.49	4.09	3.23	3.5	2.4	2.57	4.82	4.99	4.99
13	2.92	2.91	2.9	2.77	3.08	1.95	1.91	1.42	1.29	2.47	2.62	2.76
14	1.14	0.38	1.22	0.48	1.77	1.15	0.21	1.04	0.27	0.56	0.3	2.8
15	1.48	0.64	1.63	0.75	1.06	1.46	0.35	1.38	0.48	0.82	0.53	1.88
16	1.31	0.44	1.43	0.56	1.5	1.38	0.25	1.33	0.37	0.56	0.35	2.1
17	1.27	0.37	1.38	0.51	1.97	1.35	0.21	1.36	0.35	0.57	0.27	2.44
18	1.24	0.33	1.35	0.48	1.93	1.34	0.19	1.39	0.32	0.58	0.24	2.79
19	1.27	0.35	1.35	0.5	1.57	1.38	0.23	1.36	0.3	0.62	0.27	3.03
20	1.41	0.69	1.47	0.67	1.18	1.55	0.39	1.5	0.38	0.74	0.53	2.79
21	1.89	1.52	1.65	1.08	0.94	1.78	0.85	1.88	0.85	1.07	1.23	2.02
22	2.55	2.36	1.75	1.64	0.89	1.94	1.31	2.36	1.58	1.55	1.95	1.49
23	3.6	3.6	2.25	2.42	1.86	2.17	2.06	2.54	2.18	2.52	3.05	2.44
24	4.13	4.25	2.62	2.95	2.85	2.49	2.51	2.61	2.46	3.0	3.6	3.22

Table E.12: MASE - SARIMAX, Log Transformed

	Log Transformed											
	Cl. 0		**Cl. 1**			**Cl. 2**		**Cl. 3**		**Total Set**		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	9.85	11.59	8.57	10.3	9.08	7.0	7.14	6.37	6.58	9.22	9.49	9.74
2	9.76	14.63	8.28	11.41	10.02	6.56	6.76	5.87	6.2	8.37	8.77	9.01
3	9.11	17.35	7.24	10.86	10.32	6.77	7.03	5.96	6.35	8.53	9.07	9.41
4	8.23	19.26	6.25	9.89	10.4	6.41	6.66	5.49	5.9	7.93	8.49	8.99
5	7.11	19.0	5.51	9.05	9.95	6.05	6.28	5.49	5.9	7.55	8.13	8.25
6	5.74	16.44	5.01	8.36	9.01	5.83	6.03	5.25	5.64	7.29	7.84	8.05
7	4.23	12.19	4.54	7.53	7.58	4.94	5.08	4.57	4.88	6.27	6.74	7.54
8	2.76	7.73	3.89	6.29	5.73	5.08	5.19	4.75	5.05	6.45	6.87	7.23
9	1.58	4.16	2.98	4.64	3.82	6.2	6.27	5.84	6.13	7.94	8.34	9.07
10	0.76	1.89	1.92	2.88	2.21	6.76	6.77	6.5	6.74	9.08	9.34	9.55
11	0.3	0.72	0.98	1.45	1.08	5.55	5.51	5.37	5.45	8.19	8.25	8.71
12	0.09	0.22	0.36	0.55	0.41	3.77	3.67	3.47	3.42	5.86	5.72	5.87
13	0.02	0.05	0.09	0.16	0.13	1.8	1.73	1.63	1.56	2.97	2.81	3.10
14	0.0	0.01	0.03	0.05	0.04	0.03	0.03	0.03	0.03	0.04	0.03	0.05
15	0.4	0.87	0.61	0.87	1.07	0.41	0.4	0.41	0.4	0.66	0.68	0.71
16	0.43	0.97	0.54	0.8	0.98	0.21	0.22	0.24	0.25	0.32	0.35	0.37
17	0.5	1.16	0.55	0.83	1.01	0.15	0.16	0.16	0.17	0.23	0.25	0.29
18	0.62	1.49	0.62	0.96	1.1	0.19	0.19	0.15	0.16	0.21	0.23	0.31
19	0.81	2.01	0.79	1.2	1.29	0.28	0.29	0.23	0.25	0.33	0.37	0.40
20	1.13	2.8	1.09	1.61	1.6	0.73	0.76	0.67	0.73	0.83	0.94	1.04
21	1.61	3.98	1.61	2.24	2.06	1.67	1.73	1.56	1.7	1.94	2.17	2.32
22	2.32	5.67	2.41	3.17	2.75	2.95	3.05	2.72	2.99	3.36	3.78	3.89
23	3.32	7.96	3.51	4.42	3.71	4.5	4.69	4.09	4.53	5.1	5.75	6.42
24	4.55	10.85	4.8	5.96	4.96	5.35	5.59	4.71	5.25	6.05	6.87	6.97

Appendix F

Support Vector Regression Results

Table F.1: RMSE - SVR

	Cl. 0		**Cl. 1**			**Cl. 2**		**Cl. 3**		**Total Set**		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	941.93	941.93	196.93	209.88	219.27	122.43	147.45	155.26	155.27	1236.48	1246.55	1428.96
2	963.17	963.17	223.4	249.81	262.92	131.02	172.75	165.93	184.22	1408.14	1435.23	1718.39
3	1065.14	1065.14	231.36	278.56	296.64	139.89	191.45	162.41	197.38	1475.92	1510.97	1933.11
4	966.2	966.2	238.15	286.67	305.28	139.24	199.27	153.59	189.92	1348.38	1372.64	1827.17
5	941.58	941.58	238.84	290.18	311.15	138.36	197.14	147.52	179.78	1319.24	1325.02	1730.42
6	958.02	958.02	230.72	285.86	307.12	138.05	189.87	142.51	174.22	1316.52	1318.33	1708.93
7	855.7	855.7	210.79	269.17	288.91	120.31	165.47	134.96	168.41	1205.43	1210.14	1535.53
8	949.45	949.45	251.29	303.5	326.76	139.8	186.97	177.77	204.04	1405.3	1391.32	1749.55
9	1208.57	1208.57	293.86	356.32	387.2	175.86	239.39	226.52	263.63	1729.53	1731.72	2269.97
10	1653.4	1653.4	329.91	397.62	434.46	195.63	258.53	253.2	293.41	2184.78	2183.9	2820.69
11	1819.37	1819.37	328.32	399.59	434.51	197.07	254.87	228.61	263.76	2319.94	2325.3	2881.14
12	1499.56	1499.56	293.89	363.58	382.67	187.14	214.96	170.32	192.37	1922.06	1927.59	2342.64
13	1035.28	1035.28	286.23	267.34	266.53	180.8	151.78	143.66	126.41	1500.98	1505.26	1590.79
14	154.03	154.03	53.0	89.24	138.84	7.2	61.57	14.04	54.8	178.91	165.96	816.55
15	126.5	126.5	37.93	69.3	88.59	18.78	51.11	17.6	46.82	207.01	203.46	636.6
16	85.83	85.83	27.99	65.48	87.85	10.78	46.64	13.9	42.74	163.14	155.86	670.1
17	80.38	80.38	32.54	66.21	90.65	9.38	44.56	12.63	40.34	161.63	153.12	704.52
18	83.82	83.82	32.26	68.05	90.01	9.48	42.8	11.22	39.85	169.09	160.47	707.45
19	143.74	143.74	37.02	66.95	87.1	22.6	47.55	20.91	50.04	247.19	231.06	697.47
20	340.88	340.88	71.23	77.79	91.72	55.6	61.09	59.93	72.51	539.27	517.68	748.32
21	554.98	554.98	112.41	105.82	115.72	80.17	80.65	109.96	117.0	829.32	793.0	873.88
22	766.08	766.08	164.54	150.59	159.33	104.76	111.27	146.8	151.3	1121.2	1105.12	1182.16
23	959.39	959.39	196.09	199.48	215.21	117.15	143.24	158.09	181.29	1328.07	1340.28	1574.29
24	1008.53	1008.53	212.71	239.38	263.55	127.41	168.11	169.62	195.02	1388.7	1389.8	1767.56

Table F.2: MASE - SVR

	Cl. 0		Cl. 1			Cl. 2		Cl. 3		Total Set		
k	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**	**set 1**	**set 2**	**set 1**	**set 2**	**set 1**	**set 2**	**set 3**
1	1.32	1.32	1.37	1.48	1.51	1.18	1.48	1.61	1.67	1.26	1.26	1.42
2	1.5	1.5	1.57	1.84	1.9	1.28	1.8	1.78	2.07	1.49	1.53	1.78
3	1.59	1.59	1.63	2.03	2.1	1.36	1.98	1.75	2.24	1.51	1.53	1.86
4	1.52	1.52	1.67	2.12	2.2	1.34	2.04	1.67	2.16	1.43	1.47	1.83
5	1.48	1.48	1.66	2.12	2.22	1.33	2.05	1.63	2.08	1.41	1.43	1.79
6	1.43	1.43	1.58	2.03	2.12	1.32	1.89	1.54	2.01	1.33	1.35	1.67
7	1.26	1.26	1.44	1.92	1.99	1.13	1.68	1.42	1.92	1.2	1.21	1.5
8	1.56	1.56	1.73	2.22	2.31	1.37	1.97	1.83	2.34	1.55	1.5	1.84
9	1.95	1.95	2.05	2.62	2.73	1.69	2.47	2.34	2.95	1.92	1.89	2.36
10	2.43	2.43	2.28	2.87	3.01	1.91	2.7	2.63	3.31	2.29	2.28	2.81
11	2.4	2.4	2.18	2.84	2.99	1.78	2.6	2.41	3.03	2.19	2.19	2.67
12	2.12	2.12	2.04	2.66	2.74	1.75	2.16	1.83	2.17	1.9	1.9	2.26
13	1.71	1.71	2.09	1.77	1.78	1.67	1.34	1.55	1.29	1.58	1.58	1.66
14	0.13	0.13	0.3	0.48	0.67	0.04	0.37	0.14	0.37	0.11	0.1	0.5
15	0.17	0.17	0.28	0.46	0.54	0.15	0.36	0.19	0.39	0.18	0.18	0.45
16	0.11	0.11	0.2	0.39	0.48	0.09	0.27	0.14	0.31	0.13	0.12	0.44
17	0.09	0.09	0.26	0.42	0.52	0.07	0.24	0.15	0.28	0.11	0.11	0.45
18	0.1	0.1	0.26	0.45	0.53	0.08	0.25	0.13	0.29	0.13	0.12	0.46
19	0.23	0.23	0.29	0.45	0.54	0.22	0.36	0.22	0.42	0.26	0.24	0.56
20	0.55	0.55	0.53	0.55	0.61	0.54	0.56	0.6	0.71	0.58	0.55	0.79
21	0.86	0.86	0.8	0.73	0.79	0.81	0.82	1.05	1.21	0.87	0.8	0.93
22	1.21	1.21	1.16	1.06	1.11	1.07	1.16	1.46	1.62	1.21	1.15	1.28
23	1.46	1.46	1.38	1.44	1.5	1.17	1.48	1.62	2.06	1.42	1.41	1.65
24	1.5	1.5	1.49	1.77	1.86	1.25	1.78	1.75	2.15	1.45	1.43	1.77

Appendix G

Multi Layer Perceptron Results

Table G.1: RMSE - Neural Network - MLP

k	Cl. 0	Cl. 1	Cl. 2	Cl. 3	Total Set
1	910.53	184.05	116.71	126.57	1176.16
2	978.69	217.52	135.79	150.96	1308.02
3	1080.53	233.91	147.32	154.92	1499.8
4	1093.81	256.15	157.71	157.95	1547.83
5	1063.29	256.58	153.45	153.0	1569.08
6	1046.16	249.26	153.93	145.97	1545.56
7	1045.81	250.38	151.93	144.57	1505.35
8	999.84	274.13	163.21	176.2	1486.48
9	1241.32	322.6	212.68	229.67	1847.58
10	1624.81	375.33	248.12	273.67	2357.97
11	1769.76	386.23	244.07	252.58	2526.73
12	1544.17	341.22	206.68	201.62	2280.23
13	1358.97	339.43	207.28	186.0	1981.17
14	1142.59	256.13	140.89	117.66	1458.57
15	161.6	63.93	24.33	22.15	237.12
16	113.35	55.73	20.32	19.72	193.36
17	122.18	50.28	20.22	26.06	247.5
18	138.68	56.07	22.17	33.14	282.99
19	190.33	73.97	30.02	35.69	294.41
20	326.13	89.42	46.24	58.86	472.52
21	497.12	114.68	65.83	106.62	710.98
22	711.92	156.46	95.57	142.31	977.6
23	1007.12	204.41	126.65	160.99	1363.09
24	1121.97	241.7	146.93	172.93	1541.52

Table G.2: MASE - Neural Network - MLP

k	Cl. 0	Cl. 1	Cl. 2	Cl. 3	Total Set
1	1.41	1.36	1.11	1.43	1.24
2	1.57	1.58	1.33	1.68	1.43
3	1.71	1.65	1.45	1.73	1.6
4	1.75	1.78	1.51	1.76	1.63
5	1.74	1.78	1.51	1.72	1.65
6	1.73	1.69	1.49	1.63	1.58
7	1.77	1.70	1.44	1.68	1.59
8	1.8	1.88	1.59	1.96	1.71
9	2.37	2.31	2.12	2.55	2.22
10	2.87	2.76	2.58	3.03	2.76
11	2.93	2.84	2.45	2.89	2.78
12	2.61	2.52	2.12	2.28	2.54
13	2.33	2.48	2.08	1.94	2.36
14	1.45	1.46	1.09	1.06	1.31
15	0.27	0.42	0.22	0.24	0.25
16	0.21	0.31	0.17	0.21	0.19
17	0.20	0.28	0.17	0.19	0.18
18	0.22	0.30	0.18	0.22	0.20
19	0.31	0.42	0.27	0.34	0.29
20	0.55	0.60	0.46	0.63	0.49
21	0.83	0.77	0.64	1.09	0.73
22	1.17	1.11	0.96	1.5	1.05
23	1.56	1.46	1.28	1.78	1.44
24	1.81	1.75	1.47	1.84	1.66