

MASTER

Identifying leading indicators for tactical truck parts sales predictions using LASSO

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Identifying Leading Indicators for Tactical Truck Parts Sales Predictions using LASSO

Final Report for Master Thesis Project

D.M. Gerritsen 0809239

In partial fulfillment of the requirements for the degree of Master of Science in Operations Management & Logistics

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> > January 24, 2021

Abstract

Leading indicators are defined as variables that contain predictive information and ideally can predict a certain movement for a target variable in advance. This paper aimed to identify leading indicators for a case company that supplies truck parts to the European truck aftersales market. We used LASSO to extract relevant information from a collected pool of business, economic, and market indicators. We propose the efficient one-standard error rule, as an alternative to the default one-standard error rule, to reduce the influence of sampling variation on the LASSO tuning parameter value. We found that applying the efficient one-standard error rule over the default one, improved forecasting performance with an average of 0.73%. Next to that, we found that, for our case study, applying forecast combination yielded the best forecasting performance, outperforming all other considered models, with an average improvement of 2.38%. Thus, including leading context information did lead to more accurate parts sales predictions for the case company. Also, due to the transparency of LASSO, using LASSO provided business intelligence about relevant predictors and lead effects. Finally, from a pool of 34 indicators, 7 indicators appeared to have clear lead effects for the case company.

Management Summary

Introduction

PACCAR Parts Europe (PPE) is a leading supplier of high-quality truck parts to the European truck aftersales market. Generally, it is well known that truck aftersales results are highly related to the ongoing economic activities. According to Currie and Rowley (2010) using additional information can enhance forecasting performance in volatile environments. For example, leading indicators, such as macroeconomics, can contain leading context information in terms of changing economic conditions (Sagaert, Aghezzaf, Kourentzes, & Desmet, 2018). Currently, PPE's Business Development & Intelligence department tracks a number of macroeconomic indicators to qualitatively analyze and assess the truck aftersales market conditions. However, it is unclear if there are certain indicators that possibly contain leading context information for PPE's parts sales. As a result, they would like to gain insight into whether there are indicators that contain leading information for their business in a quantitative manner. Next to that, they want to know if using information extracted from these indicators actually leads to more accurate parts sales predictions. Accordingly, the business objective was defined as:

Identify and gain insight into leading indicators for PACCAR Parts Europe's part sales. Next to that, explore and analyze whether including information extracted from leading indicators actually leads to more accurate parts sales predictions.

Modeling

In order to identify leading indicators for PPE's parts sales, it was necessary to specify a certain target variable. For this project, the target variable was specified as the total monthly truck parts sales reported by all DAF dealers, located in the EU27+2 area. With regard to the indicators, we collected a pool of 34 business, economic and market indicators which had the potential of being a leading indicator for PPE's part sales. The

business indicators covered PPE's business activities by means of indicators that were related to observed parts sales in the past and the number of DAF truck deliveries. The economic indicators covered Europe's overall economic climate by means of indicators that were mainly related to the industrial manufacturing and construction sectors, as these sectors are the major drivers of commercial vehicle transport. The market indicators covered the ongoing activities in Europe's road transport sector by means of indicators that were related to the number of placed truck orders, diesel consumption, tire consumption, carrier sentiment and road activity. Thereafter, in order to model and identify any lead effects, each considered indicator was lagged in time multiple times, increasing the number of predictors significantly. For this project, we assumed a maximum lead effect of 12 months and thus the number of predictors increased to a total of 408. Given a large number of predictors together with the frequent occurrence of small sample sizes in sales forecasting, the identification of leading indicators in a monthly sales forecasting environment resulted in a high-dimensional (p > n) problem. Moreover, since each indicator is lagged in time multiple times, there exists correlation among the predictors and thus the problem of multicollinearity is present. Hence, the identification of leading indicators resulted in a high-dimensional problem with the presence of multicollinearity among the predictors. Therefore, LASSO was chosen as modeling technique as LASSO performs both variable selection and regularization that involves penalizing the absolute size of the regression coefficients. Due to these shrinkage properties, LASSO is capable of effectively dealing with multicollinearity among the predictors. Next to that, the use of LASSO contributed to the business objective as "the LASSO forecast is transparent, and provides insights into the selected leading indicators. Experts can benefit by gaining a better understanding of their market and can thus improve their understanding of market dynamics and interactions" (Sagaert, Aghezzaf, Kourentzes, & Desmet, 2017, p. 127)

Results

We found that 7 indicators appeared to have clear lead effects for PPE: observed parts sales, DAF truck deliveries, construction spending, short-term diesel consumption growth, automotive diesel deliveries, short-term OEM truck orders and carrier demand expectations. In order to assess whether the inclusion of leading context information actually led to more accurate predictions, we benchmarked the performance of LASSO to SARIMA and Holt-Winters, which are two univariate time series forecasting methods, often used in businesses. Initially, it turned out that Holt-Winters predicted most accurate on the shorter horizons (1-5 months), whereas SARIMA mainly predicted most accurate on the longer horizons (7-12 months). Hence, despite the fact that LASSO used external information, it actually predicted less accurate than the traditional forecasting methods. As a result, two experiments were conducted in order to explore whether forecasting performance could be improved by applying efficient tuning parameter selection and forecast combination. With regard to the efficient tuning parameter selection experiment, we introduced the efficient one-standard error rule which, instead of choosing the parameter value corresponding to the most regularized model within one-standard error of the minimum CV error estimate, chooses all parameter values within one-standard error of the minimum, and subsequently calculates a weighted average. The purpose of applying the efficient one-standard error rule over the default one-standard error rule, when using CV, is to reduce the influence of sampling variation on the tuning parameter value. Accordingly, it was found that applying the efficient one-standard error rule improved forecasting performance in 10 out of 12 models, with an average improvement of 0.73%. Next to that, we analyzed model diversities and explored whether applying forecast combination could lead to more accurate predictions. Accordingly, it was found that combining the predictions of LASSO and Holt-Winters yielded the most accurate predictions, outperforming the individual LASSO, Holt-Winters and SARIMA models for almost all horizons. Hence, for PPE, the inclusion of leading indicators led to more accurate parts sales predictions, with an average improvement of 2.38% over all horizons.

Recommendations

The results obtained during this field project led to a number of recommendations towards PACCAR Parts Europe. First of all, it is recommended to implement the additive Holt-Winters model with smoothing parameters $\alpha = 0.2$, $\beta = 0.1$, $\gamma = 0.2$. Next to that, it is recommended to implement the LASSO model with the indicators: observed parts sales, DAF truck deliveries, construction spending, short-term diesel consumption growth, automotive diesel deliveries, short-term OEM truck orders, and carrier demand expectations. With regard to LASSO's tuning parameter λ , it is recommended to implement and use 10-fold CV with the efficient one-standard error rule, proposed in this report, as the tuning parameter selection method. In order to obtain the final prediction, the predictions of both the LASSO and Holt-Winters models should be averaged.

Preface

This report marks the end of my master thesis project and, at the same time, my student life at the University of Technology Eindhoven. During my time at university, I was fortunate enough to meet many lovely people with whom I have experienced unforgettable moments and have made great memories with. Moreover, during this time I have gained a lot of knowledge and experiences in various areas that will definitely contribute to the further development of myself and my career. I am very grateful for all the opportunities that have presented themselves and would like to express my gratitude.

First of all, I would like to thank my former first supervisor Anna Wilbik for helping me with the problem and project definition. Unfortunately, we were not able to finish this project together as you have found another challenging job. Furthermore, I would like to thank Vahideh Reshadat, who took over the role of first supervisor, for helping me improve the overall quality of the thesis. Additionally, I would like to thank my second supervisor Yingqian Zhang for spending her time on this project and providing valuable feedback.

Secondly, I would like to thank DAF Trucks NV for giving me the opportunity to carry out a graduation project at such an enormous and state-of-the-art company. In particular, I would like to thank Jing Li and Max Jacobs for helping me throughout the project. Unfortunately, due to the coronavirus situation, I had to work from home for the vast majority of the time and therefore I was not really able to fully experience working at DAF.

Finally, I would like to thank all my student friends with whom I had good times during the Thursday afternoon drinks in the Villa and all other activities. Last but certainly not least, I would like to thank my family for their continuous support and positivity. For now, I am ready for the next challenge that awaits.

List of Abbreviations

ACF	Autocorrelation Function
\mathbf{ADF}	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
ARIMA	Autoregressive Integrated Moving Average
CRISP-DM	Cross-Industry Standard Process for Data Mining
\mathbf{CV}	Cross-Validation
DAF	DAF Trucks NV
\mathbf{EU}	European Union
FRED	Federal Reserve Economic Data
KPSS	Kwiatkowski-Philips-Schmidt-Shin
LASSO	Least Absolute Shrinkage and Selection Operator
MAE	Mean Absolute Error
OECD	Organisation for Economic Cooperation and Development
OEM	Original Equipment Manufacturer
OLS	Ordinary Least Squares
PACF	Partial Autocorrelation Function
PPE	PACCAR Parts Europe
RMSE	Root Mean Square Error

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Chapter 1

Introduction

This chapter aims at defining and describing the research environment by introducing DAF Trucks NV and PACCAR Parts Europe. Furthermore, the European truck aftersales and road freight markets are briefly described in order to improve market understanding and provide relevant market insights. Finally, this chapter is concluded by introducing the research problem and the corresponding research questions for this field project.

1.1 DAF Trucks NV

DAF Trucks NV, further denoted as DAF, is a technology company and a leading commercial vehicle builder in Europe. DAF is a full subsidiary of PACCAR Inc, a worldwide leader in designing and manufacturing high-quality light, medium and heavy-duty commercial vehicles under the Peterbilt, Kenworth and DAF nameplates. The core activities of DAF consist of the development, production, marketing and sales of medium and heavy-duty commercial vehicles. The upcoming sections will elaborate on DAF's product portfolio, market performance and organizational structure.

1.1.1 Product Portfolio

DAF produces vehicles according to the build-to-order principle. Hence, vehicles are specified to the customer's needs and are only built after a confirmed customer order is received. At the time of writing, DAF has a number of production facilities in Eindhoven (The Netherlands), Westerlo (Belgium), Leyland (United Kingdom) and Ponta Grossa (Brasil). DAF offers high-quality solutions for long-distance, distribution, and construction transport needs by offering trucks on three distinct vehicle platforms. First, the small LF series is designed for city and regional transport, and is characterized by its maneuverability, accessibility, and an exceptional low weight. Secondly, the CF series is a multifunctional truck designed for regional, national or international transport, both on flat roads and rough terrains. Lastly, the XF series is designed for maximum transport efficiency and is mainly used for long-distance transport due to its spacious and luxurious interior.

1.1.2 Market Performance

DAF operates in a highly competitive market in which it competes with several other original equipment manufacturers (OEMs) offering comparable solutions. In 2019, DAF obtained a market share of 16.2% in the heavy-duty (>16 tons) segment. As a result, they were market leaders in the Netherlands, the United Kingdom, Poland, Hungary, Belgium, Luxembourg, and Bulgaria. With regard to the medium-duty (6-16 tons) segment, DAF were market leaders in the United Kingdom and Ireland as they increased their market share in this segment up to 9.7%.

1.1.3 Organizational Structure

DAF's headquarters is located in Europe's leading innovative technology region, Eindhoven (the Netherlands). At the time of writing, DAF employs approximately 9,400 employees within their organization. Figure 1.1 shows the organizational structure of DAF in 2020. As can be seen, the company structure consists of eight main divisions: Marketing & Sales, Operations, Product Development, Finance, PACCAR Financial Europe, PACCAR IT Europe, PACCAR Parts Europe, and PACCAR Purchasing Europe.

The project described in this report was executed at the Business Development & Intelligence department of the PACCAR Parts Europe division in Eindhoven. The core activities of PACCAR Parts Europe, and especially the Business Development & Intelligence department are discussed in Section 1.2.

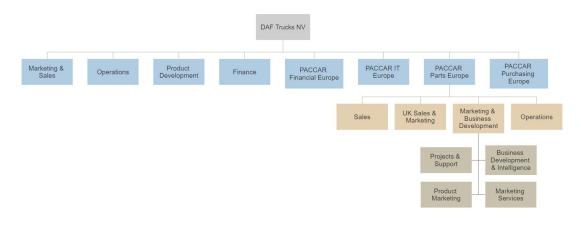


Figure 1.1: Organizational structure at DAF Trucks NV (DAF Trucks NV, 2020).

1.2 PACCAR Parts Europe

PACCAR Parts Europe, further denoted as PPE, is a leading provider of high-quality truck parts, integral service concepts and support solutions in the European truck aftersales market. The core activities of PPE consist of optimizing parts availability and logistics operations across the DAF dealer network in order to maximize customer vehicle uptime. It is generally known that the profitability and total cost of ownership of a commercial vehicle is highly dependent on its availability (i.e. vehicle uptime). Hence, the availability of truck parts for repairs and maintenance play a crucial role in this. With a product portfolio consisting of PACCAR Genuine Parts, DAF Genuine Parts, TRP universal parts and other supplier brands; PPE is able to offer a complete range of truck spare parts suitable for mixed fleets of all truck ages. Moreover, together with four state-of-the-art distribution centers in Eindhoven, Leyland, Madrid and Budapest and a strong DAF dealer network; PPE achieves the highest standards in distribution efficiency and commercial support in the increasingly demanding truck aftersales market.

The Business Development & Intelligence department is responsible for identifying new market developments, conducting market studies, determining vehicle parks, calculating aftersales potential, measuring dealer performance and mapping competitors and the transport market. Accordingly, the upcoming sections will analyze and elaborate on the current state of both the European truck aftersales and road freight market.

1.2.1 Aftersales Market Developments

In the past decade, the relevance of the truck aftersales business has grown significantly due to the fact that margins in new truck sales are shrinking considerably. Therefore, aftersales has become the most profitable business and a major profit driver for OEMs, despite the fact that it only accounts for a small portion of revenues (Roland Berger, 2015). For clarity, Figure 1.2 shows the contrast between margins and revenues in the truck sales and aftersales business. As can be seen, margins in aftersales are significantly higher than margins in new truck sales. As a result, competition in the aftersales market has increased noticeably over the years due to the fact that new participants (e.g. original equipment suppliers and independent wholesalers) have entered the market. Next to that, the truck aftersales market is facing a number of key influences (Roland Berger, 2014).

First of all, road freight transportation across Europe is expected to keep on increasing in the future. In the past decade, the European road freight market size grew from 276.2 billion Euros in 2010 to 355.1 billion Euros in 2019, with continuous year-on-year increases except for 2012. Hence, the road freight market increased a significant 22% by the end of 2019. Moreover, estimates suggest that the freight transport market will increase with 60% by 2050 (European Commission, 2019).

Secondly, truck vehicle parcs across Europe are expanding and getting older year-on year. In 2014, a total amount of 6.1 million trucks were driving the roads in the European Union (EU). Ever since, this number has increased year-on-year, resulting into 6.6 million trucks driving the European roads by the end of 2018. With a fleet size of over 1.1 million trucks, Poland has the largest truck fleet, followed by Germany and Italy with fleet sizes of 946,541 and 904,308 trucks respectively. Moreover, the average age of trucks has been increasing likewise. In 2014, trucks in the EU were 11.9 years old on average, whereas the average truck age had increased up to 12.4 years by the end of 2018. With an average fleet age of 14.4 years, Spain has the oldest truck fleet, followed by Italy with an average fleet age of 14.0 years. Conversely, with an average fleet age of 7.2 years, France has the youngest truck fleet among the EU's major markets, followed by Germany with an average fleet age of 9.5 years (European Automobile Manufacturers' Association, 2019).

Lastly, requirements for vehicle uptime and efficiency are rising. In order to satisfy these requirements, OEMs tend to design and develop components in a more sustainable manner. As a result, maintenance intervals are increasing as wear components become more durable, which positively affects vehicle uptime. For example, DAF used to apply main-

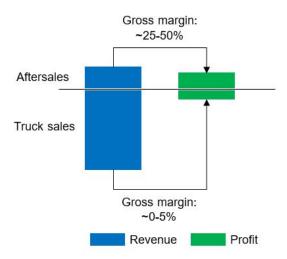


Figure 1.2: Aftersales is a major profit driver for OEMs (Roland Berger, 2015).

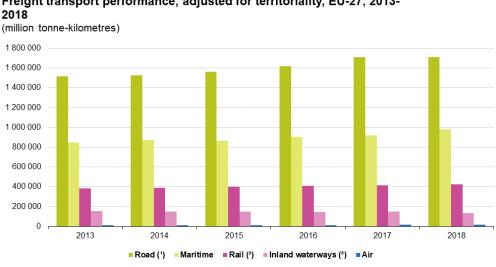
tenance intervals of 100,000 kilometers in the past; whereas at the time of writing, DAF is applying maintenance intervals up to 200,000 kilometers. Logically, larger maintenance intervals have a negative impact on the aftersales market. On the other hand, higher expectations in terms of vehicle uptime and efficiency offer opportunities for introducing new business models. An example of one of these business models is predictive maintenance. Predictive maintenance is a data-driven approach based on machine learning which uses data from thousands of vehicles in terms of mileage, speed, temperature, and other performance data to calculate the service life of certain vehicle components and ultimately predict when certain vehicle components are likely to break down. Accordingly, predictive maintenance enables components which are likely to break down in the near future to be replaced in advance and thus minimizing and preventing unplanned vehicle downtime. Another example of new business models are fleet management systems. Fleet management systems are data-driven mobility services developed in order to provide valuable insights and assistance to operators that manage a large number of trucks. The purpose of fleet management is to improve operational efficiency and minimize costs by optimizing vehicle maintenance, fuel consumption and fuel costs, driver management, asset utilization and route planning.

1.2.2 Importance of Road Transport

Road transport is a vital part of, and a major contributor to the European economy. It can be grouped into the transportation of goods and materials (i.e. road freight transport)

and the transportation of passengers (i.e. passenger transport) on roads. Undoubtedly, the truck aftersales market highly depends on the activity of road freight transport.

According to Boylaud and Nicoletti (2001), road freight transport is a key sector for all economies as it plays a major role in market integration and it directly impacts transaction costs in corresponding economies. With a total number of 3.2 million employees spread over approximately 590,000 businesses, road freight transport dominates the European freight market in terms of employment (European Commission, 2018). Figure 1.3 shows the modal split of freight transport in the EU for the period 2013-2018. As can be seen, freight is transferred in the EU by road, maritime, rail, inland waterways and air transport. Moreover, it can be concluded that road freight transport dominated the freight market between 2013-2018 in terms of tonne-kilometres, followed at a significant distance by maritime transport. For clarity, a tonne-kilometer represents the transport of one ton of goods, including packaging and tare weights of containers and pallets, over a distance of one kilometer.



Freight transport performance, adjusted for territoriality, EU-27, 2013-

(³) Includes estimates for Finland in 2017-2018 and does not include Sweden in 2013-2015 (negligible).

Figure 1.3: Modal split of freight transport in the EU (Eurostat, 2020).

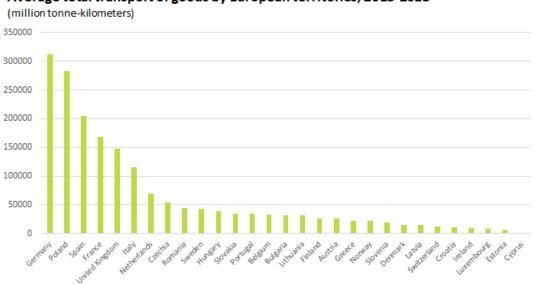
The popularity of road freight transport can easily be explained by the fact that it is the most flexible, responsive and economic mode of goods transportation (European Automobile Manufacturers' Association, 2017). Also, it is often the case that all the other freight transport modes depend on road transport in order to transfer freight from

Note: Maritime cover only intra-EU transport (transport to/from countries of the EU) and exclude extra-EU transport (1) Includes estimates for Malta Includes estimates for Belgium

and to marine terminals, rail terminals and airports. Hence, road freight transport is an indispensable link in facilitating door-to-door logistic supply chains across Europe.

1.2.3 Market Understanding

Given the fact that road freight is of great importance for the European logistics network, it is of importance to analyze and understand how road freight is distributed across the European territories. Figure 1.4 shows the average total transport of goods across European territories in terms of million tonne-kilometers over the period 2013-2018. As can be seen, Germany and Poland were leading countries in the European road freight market with a market share of 17.0% and 15.4% in terms of million tonne-kilometers respectively. Thereafter, Spain, France and the United Kingdom follow with market shares of 11.1%, 9.1%, and 8.0% respectively. Together, these countries accounted for 60.8% of the total goods transported. Moreover, if Italy is also included, the total share increases to 67.0% and thus six out of twenty-nine countries accounted for about twothirds of the total European market.



Average total transport of goods by European territories, 2013-2018

Figure 1.4: Transport of goods on country level (Eurostat, 2020).

1.3 Problem Formulation

Sales forecasting plays a significant role in business strategies nowadays. In particular, tactical (i.e. up to 12 months) forecasting often supports short-term decision-making in supply chain management as it serves as a basis for raw material purchase, inventory planning and production scheduling. Alternatively, strategic forecasting is often referred to as long-term forecasting and principally supports decision-making in the development of overall strategies and capacity planning. In fact, both forecasting strategies commonly use observed values of the past and available knowledge to predict the future as accurately as possible (Sagaert et al., 2018). However, including external information could enhance the performance of a sales forecasting model (Currie & Rowley, 2010).

According to Currie and Rowley (2010) using additional information can enhance forecasting performance in volatile environments. The main focus of previous research has been enhancing operational forecasts (i.e. up to 48 hours ahead). For example, Williams, Waller, Ahire, and Ferrier (2014) successfully integrated supply chain information into a forecasting model, whereas Ma, Fildes, and Huang (2016) used additional price and promotional data to improve forecast accuracy. Conversely, the dynamics of tactical forecasts can be different due to the relevant horizons and business models. Moreover, leading indicators, such as macroeconomics, can contain leading context information in terms of changing economic conditions (Sagaert et al., 2018). These indicators are mainly published on a monthly or quarterly basis and are therefore useless for operational forecasting purposes. However, for medium to long-term horizons (i.e. 3 to 12 months ahead) macroeconomic information is relevant and could enhance forecast performance (Sagaert et al., 2018). In many cases, tactical forecasts are based on autoregressive information (i.e. observed values of the past) and therefore unable to respond to changing conditions in the market. As a result, expert adjustments are often used to enhance forecasting accuracy. Fildes, Goodwin, Lawrence, and Nikolopoulos (2009) concluded that these adjustments generally suffer from biases towards optimism. Moreover, using human input increases forecasting complexity and limits automation capabilities. Although including leading indicators is not entirely new, Verstraete, Aghezzaf, and Desmet (2020) assumed that, based on their experiences, macroeconomic indicators are primarily used as a qualitative aid in forecasting.

Generally, it is well known that truck aftersales results are highly related to the ongoing economic activities. Currently, PPE's Business Development & Intelligence department reports several indicators in their market intelligence report, to qualitatively analyze and assess the truck aftersales market conditions. However, up to now, it is unclear if there are any indicators that possibly contain leading context information for predicting PPE's parts sales. As a result, they would like to gain insight into whether there are indicators that contain relevant leading information for their business in a quantitative manner. Next to that, they want to know if using information extracted from these indicators could lead to more accurate parts sales predictions. Accordingly, resolving this problem could help extend and improve the quality of PPE's market intelligence report.

1.4 Research Questions

This section presents the research questions that have been defined with regard to this field project. The research questions are based on the problem described in Section 1.3 and the literature review written in preparation for this project. In order to create a clear structure within this research, three research questions have been defined. The research questions are defined as:

- Research Question 1: How to identify leading indicators for PACCAR Parts Europe's parts sales?
- Research Question 2: Are there any leading indicators for PACCAR Parts Europe's business, and if so, which indicators exactly are relevant for predicting PACCAR Parts Europe's parts sales?
- Research Question 3: Does the inclusion of leading context information actually lead to more accurate predictions of PACCAR Parts Europe's parts sales?

1.5 Methodology

For this project, the problem-solving cycle defined by van Aken and Berends (2018) will be used. This cycle is a problem-solving methodology designed for field projects in business and management. The problem-solving cycle consists of five process steps: problem identification, analysis & diagnosis, solution design, intervention, and evaluation. In addition, the process diagram is shown in Figure 1.5. Typically, student projects are performed up to and including the solution design step (van Aken & Berends, 2018). Therefore, this field project is scoped to the first three steps: problem identification, analysis & diagnosis and solution design. The upcoming sections will elaborate more on these first three process steps related to this field project.

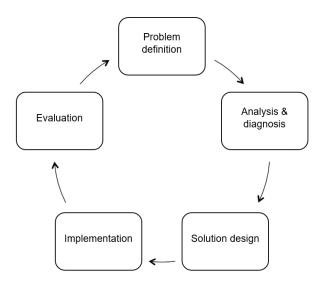


Figure 1.5: Problem-solving cycle for field projects (van Aken & Berends, 2018).

1.5.1 Problem Definition

The problem definition step forms the base for the whole problem-solving project. It is based on an agreement between the initiator of the project and the project executor. It is often the case that the initial problem statement is part of an underlying problem. Therefore, the initial problem statement should be placed in the context of the problem mess, a complex system of interacting problems (Ackoff, 1981). Thereafter, the problem should be scoped thoroughly. Moreover, the problem identification includes designing the project plan and the approaches for the subsequent steps.

1.5.2 Analysis & Diagnosis

The analysis & diagnosis step produces specific knowledge on the problem context. With regard to this field project, knowledge on the problem context is produced by conducting a literature review on which methods are useful to identify leading indicators in a sales forecasting environment and how to apply regression techniques in time series forecasting. The literature review will provide insight into useful and suitable modeling approaches and will clarify how to apply these modeling approaches on a time series forecasting problem. In this case, the analysis & diagnosis step will serve as a basis to explore and answer the first research question.

1.5.3 Solution Design

The solution design step proposes a solution concept for the business problem. Consequently, solution concepts are often designed based on literature review results or on interviews with members of the client organization. With regard to this field project, acquired knowledge in the analysis & diagnosis step will contribute to the development of the solution concept. In this case, the solution design step will serve as a basis to explore and answer the second and third research questions. For the solution design step, the cross-industry standard process for data mining (CRISP-DM) framework will be used. Accordingly, CRISP-DM is an open standard process model that describes a common approach for data mining. The process model consists of six major phases: business understanding, data understanding, data preparation, modeling, evaluation, and deployment (Chapman et al., 2000). For clarity, the process diagram is shown in Figure 1.6. As can be seen, there is no strict sequence between the six separate phases as switching between the different phases is often required. This section will elaborate on the six major phases with respect to this field project.

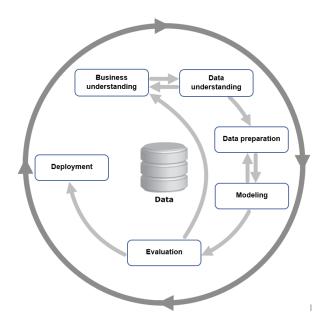


Figure 1.6: Cross-industry standard process for data mining.

Business Understanding

The first phase in CRISP-DM is business understanding. This phase requires the definition of the business objective and the corresponding success criteria. The business objective describes the main goal of the project, whereas the success criteria will determine whether the project has been successful from a business perspective. Additionally, this phase requires the definition of the data mining objectives. The difference between these objectives are that the business objective is defined in terms of business terminology, whereas data mining objectives are defined in more technical terms.

Data Understanding

The next phase in CRISP-DM is data understanding. This phase involves collecting, describing, exploring and verifying useful data.

Data Preparation

The third phase in CRISP-DM is data preparation. This phase involves cleaning and constructing data sets which can be directly used as input in the modeling phase. As a result, it is often the case that the data preparation phase is most time-consuming.

Modeling

The fourth phase in CRISP-DM is modeling. This phase involves selecting the right modeling techniques, building the actual model and interpreting the generated data mining results.

Evaluation

The fifth phase in CRISP-DM is evaluation. This phase involves assessing the degree to which extent the model meets the predetermined business objective. Moreover, this phase offers the possibility to explore business reasons why the model could be considered as deficient. Accordingly, the evaluation phase is concluded with recommendations regarding the next steps which could be proceeding to the deployment phase or opting for additional model improvements.

Deployment

The final phase in CRISP-DM is deployment. This phase involves determining and summarizing the deployment strategy including the necessary steps to be taken. Moreover, it is often the case that during this phase a report is written to summarize the data mining results. With regard to this field project, deployment includes solely writing this final report which summarizes the project and the corresponding data mining results.

Chapter 2

Related Work

This chapter aims at summarizing useful findings and insights gained from available literature on identifying leading indicators in a sales forecasting environment and using regression techniques on a time series forecasting problem. It should be noted that the available literature on leading indicators in a sales forecasting environment focuses exclusively on including macroeconomics as leading indicators. As a result, the upcoming sections will refer to macroeconomic indicators as leading indicators.

2.1 Sales Forecasting using Macroeconomic Indicators

According to Verstraete et al. (2020) using macroeconomic indicators as input variables for tactical sales forecasting introduces two major challenges. The first challenge is the limited sample size of available sales data, which is considered as a typical challenge in sales forecasting in general. It is often the case that companies lack having effective data management practices and therefore cannot access historical data. On the other hand, even if companies do have effective data management practices to a certain extent, it is often the case that historical data is not representative anymore due to changing product portfolios and customer behaviors. Moreover, as macroeconomic data is mainly reported on monthly or higher aggregation levels, it could be that the amount of usable data becomes even more limited. The second challenge is the large number of available macroeconomic indicators across multiple publicly available data sources. For example, the best known economic database sources such as Organisation for Economic Cooperation and Development (OECD), Federal Reserve Economic Data (FRED) and Eurostat provide access to thousands of macroeconomic time-series. As a result, selecting potential macroeconomic indicators could become very time consuming and quite complex. According to Sagaert et al. (2018), these two challenges together create the distinct tactical sales forecasting problem. Hence, the tactical sales forecasting problem consists of a considerably large set of predictors (p) with limited sales data sample sizes (n).

If the number of predictors exceeds the number of observations (p > n), traditional regression methods such as ordinary least squares (OLS) regression cannot handle these problems because of non-unique coefficient estimates. As a result, it becomes necessary to select and simultaneously reduce the number of predictors up to a maximum of nvariables. According to the existing literature, there are several approaches to select useful predictors amongst a larger group of potential indicators.

For example, stepwise regression is a step-by-step model building procedure that uses statistical significance in order to automatically select a subset of predictors. The procedure adds or removes one predictor at each step and in the end produces one final regression model. Despite the fact that after the emergence of stepwise regression it was proposed as an efficient way to perform variable selection, it has been critized for having bias in parameter estimations, inconsistencies in model selection and the problem of multiple hypothesis testing (Flom & Cassell, 2007; Whittingham, Stephens, Bradbury, & Freckleton, 2006). Moreover, Smith (2018) demonstrated that stepwise regression may prefer to choose nuisance predictors over predictors that do have causal effects on the dependent variable and therefore the out-of-sample accuracy may be significantly worse than the in-sample fit. Next to that, Smith (2018) highlighted that the larger the number of predictors, stepwise regression becomes less effective and tends to be more misleading.

Besides stepwise regression, another approach to select useful predictors is best subset selection. Best subset selection is a model selection approach that fits and compares all possible models based on the number of predictors. The end result is a number of best-fitting models and their summary statistics. Consequently, if there are p predictors, best subset selection fits and compares a total of $2^{(p)} - 1$ models. As a result, best subset selection is generally considered infeasible when p > 30 (Tibshirani, 1996).

Given the fact that both stepwise regression and best subset selection are not applicable to larger datasets and simultaneously the emergence of big data over the past decade, least absolute shrinkage and selection operator (LASSO) regression has attracted more and more interest and accordingly has been applied to various large data problems (Tibshirani, 2011). LASSO is a linear regression technique that uses regularization in order to enhance forecast accuracy and prevent overfitting of high-dimensional data (Tibshirani, 1996). To do so, regularization methods often introduce penalties in cost functions. For instance, the LASSO cost function has similarities with OLS; however, it is expanded by adding a penalty equal to the absolute magnitude of the coefficients. As a result, LASSO is capable of shrinking some coefficients to exactly zero, resulting into uninformative variables being eliminated from the model and thus performing variable selection. It should be noted that unlike stepwise regression and best subset selection, LASSO fits a model starting with all p predictors. Accordingly, LASSO has been found useful in enhancing forecast accuracy and simultaneously selecting relevant predictors. Li and Chen (2014) used LASSO regression to forecast 20 macroeconomic time series using a large set of 107 publicly available macroeconomic variables. The study demonstrated that LASSO outperformed dynamic factor models for almost all 20 target variables, while additionally identifying and providing insights in relevant predictors. Moreover, Zhang, Ma, and Wang (2019) used LASSO regression to forecast oil prices with a large set of predictors. Again, the study demonstrated that the LASSO model yielded substantially better predictive ability than several other competing models.

It is obvious that these studies mainly focused on predicting macroeconomic variables, whereas at the same time LASSO has been found useful in enhancing forecast accuracy and identifying leading indicators in tactical sales forecasting. Sagaert et al. (2018) proposed a forecasting framework that automatically identifies leading indicators from an enormous set of macroeconomics, tracks changes in the business environment and generates more accurate forecasts. Next to that, Verstraete et al. (2020) proposed a comparable framework that automatically generates tactical sales forecasts taking into account relevant predictors from a large set of macroeconomics and sales data decomposition. Accordingly, the frameworks of Sagaert et al. (2018) and Verstraete et al. (2020) will be analyzed more in-depth in the upcoming sections.

2.1.1 Framework by Sagaert

Sagaert et al. (2018) proposed a framework to improve tactical sales forecasting using macroeconomic leading indicators. The proposed framework automatically identified key leading indicators from an enormous set of macroeconomic indicators for a supplier to the tire industry. In many studies the gross domestic product (GDP) is used to represent the ongoing economic activity at some point in time (Muhammad, 2012; Vahabi, Seyyedi, & Alborzi, 2016; Gao, Xie, Cui, Yu, & Gu, 2018; Fantazzini & Toktamysova, 2015). For clarity, GDP represents the value of all finished goods and services produced within a country in a specific time period. However, Sagaert et al. (2018) indicated that GDP is

principally an aggregate variable and therefore does not provide detailed changes in the various sectors and economic activities. Moreover, they mention that using more detailed macroeconomic indicators could possibly provide relevant information and therefore the number of potentially relevant indicators will increase extensively, especially for supply chains across multiple markets and countries.

The case company of Sagaert et al. (2018) has a global supply chain and supplies numerous tire manufacturers across multiple markets and as a result Sagaert et al. (2018) initially selected 67,851 monthly macroeconomic variables from several sections of the FRED database. To model any indicator leading effects to the sales variable, each input variable is lagged in time up to a maximum considered time lag. Sagaert et al. (2018) assumed a maximum leading effect of 12 months and therefore the number of input variables increased to a total of 67,851 * 12 = 814,212 predictors. Due to the extensive number of predictors causal-regression modeling becomes highly complex and truly impossible. For this reason, Sagaert et al. (2017) proposed to use LASSO regression. Moreover, Sagaert et al. (2017) opted for the use of LASSO regression as "the LASSO forecast is transparent, and provides insights into the selected leading indicators. Experts can benefit by gaining a better understanding of their market and can thus improve their understanding of market dynamics and interactions" (Sagaert et al., 2017, p. 127). Additionally, since each input variable is lagged in time multiple times, Sagaert et al. (2017) highlights that multicollinearity may be present among the input variables. Due to the shrinkage properties of LASSO, Sagaert et al. (2017) mentions that LASSO is capable of effectively dealing with multicollinearity.

Next to a large set of macroeconomic indicators, seasonal dummy variables and autoregressive information is included as this might contain potential sales dynamics at no additional data cost (Sagaert et al., 2017). It follows that by applying and implementing the proposed framework Sagaert et al. (2017) were able to improve sales forecasts by 16.1% in terms of the mean absolute percentage error (MAPE). Sagaert et al. (2018) extended the framework by integrating judgemental preselection of leading indicators. In fact, Sagaert et al. (2018) incorporated judgemental preselection by interviewing the company's supply chain manager. They based their interview on the latest quarterly industry report and asked the manager to explicitly state the assumptions why certain factors could impact the company sales. Based on these assumptions, they constructed a list of keywords and selected indicators related to these keywords (Sagaert et al., 2018). As a result, the number of indicators reduced significantly from 67,851 to a filtered set of 1,082 indicators. Ultimately, Sagaert et al. (2018) were able to improve sales forecasts by 18.8% in terms of MAPE. Hence, they proved that incorporating expert knowledge is an appropriate method to effectively decrease the number of potential indicators without loss of forecasting accuracy.

Finally, Sagaert et al. (2018) benchmarked the performance of LASSO regression model against stepwise regression, OLS, Holt-Winters and ETS. Accordingly, LASSO outperformed al the aforementioned methods on a horizon of 2 up to 9 months, whereas on further horizons, LASSO is outperformed by ETS. Sagaert et al. (2018) indicated that the loss of performance on further horizons is due to absence of predictive information in longer leads compared with shorter leads. In other words, it is easier to find an indicator that is leading a couple of months ahead than an indicator that is leading one year ahead.

2.1.2 Framework by Verstraete

Verstraete et al. (2020) proposed a comparable methodology that automatically generates tactical sales forecasts by using a large group of macroeconomic indicators. A noticeable difference is Verstraete et al. (2020) assumed that macroeconomic conditions determine the trend of sales. Therefore, they opted to use the LASSO regression technique of Sagaert et al. (2018) to forecast the trend component.

The sales data is decomposed into a trend, a seasonal and a remainder component using the STL decomposition proposed by Cleveland, Cleveland, McRae, and Terpenning (1990). Additionally, they motivate their choice for STL decomposition because it can be robust to outliers, the seasonal component may alter across different time periods and smoothness of the trend component is controllable. Furthermore, as the data is divided into three independent components, each individual component is forecasted separately. Verstraete et al. (2020) used the seasonal naive method as proposed by Xiong, Li, and Bao (2018) to forecast the seasonal component. For clarity, the seasonal naive method uses the latest seasonal observation as a forecast for the consecutive seasonal period. Verstraete et al. (2020) assumed that the remainder component is determined by other factors than macroeconomic indicators. For instance, they mention social media, promotions, weather and random noise as factors that will mainly determine the remainder component. However, they considered predicting the remainder component out-of-scope as they assumed that the predictive power of these factors are out of the tactical time window. Moreover, Verstraete et al. (2020) used the framework to examine the influence of the number of variables included in the indicator set. They composed three different indicator data sets in order to examine the influence of the number of variables in the indicator set. The data sets were composed in such a way that whenever the set becomes smaller, it is more related to the operating sectors of the case companies. Consequently, the first data set included 2,500 indicators, whereas the second data set consisted of 300 indicators based on keywords. The third and last data set included approximately 10 manually selected indicators based on expert judgement. As a result, Verstraete et al. (2020) observed that the forecasting accuracy consistently increased as the data set size decreased. Moreover, they highlighted that the increase in accuracy is due to a higher density of causal variables in the data set, which will eventually enhance predictive ability.

Finally, Verstraete et al. (2020) benchmarked the performance of the LASSO regression model against the naive method, seasonal naive method, OLS, Theta, ARIMA and ETS. Accordingly, LASSO outperformed all the aforementioned techniques on a horizon of 1 up to 6 months. However, on a horizon of 7 up to 12 months, Theta, ARIMA and ETS outperformed LASSO. Verstraete et al. (2020) concluded that incorporating non causally related indicators will enlarge prediction errors for further horizons due to the fact that "as indicators are selected by optimizing fit and the near future pattern will most likely not differ too much, the short term prediction of the variables will often lead to accurate predictions even if they are not causally related. However, on the further horizons, selecting non causally related indicators will lead to large prediction errors" (Verstraete et al., 2020, p.6).

2.2 Time Series Forecasting using Regression Techniques

According to Bontempi, Ben Taieb, and Le Borgne (2013) a time series is a sequence of observations at discrete points in time. It should be noted that time series data differ from other types of data due to the fact that the temporal aspect contains additional information that can be used. Time series analysis originated in the late 1920s and has served as the foundation for time series forecasting (Tsay, 2000).

Within time series forecasting, a distinction is made between two different modeling approaches: univariate models and multivariate models (Nahmias & Olsen, 2015). Univariate models use only historical data of the series being forecasted and are based on the assumption that existing patterns in past observations can be used to forecast future values of the series. Multivariate models use data from additional sources other than the time series itself, as patterns in other variables might be linked to the series being forecasted. Time series analysis recognizes several patterns that occur frequently (Nahmias & Olsen, 2015):

- 1. Trend: a stable pattern of growth or decline.
- 2. Seasonality: a pattern that repeats at fixed time intervals.
- 3. Cycles: patterns that are to some extent similar to seasonality, except that the length and magnitude of cycles are not fixed and vary.
- 4. Randomness: data without any recognizable patterns or structures.

There are many methods available for addressing time series problems. For a long time, time series forecasting has been dominated by linear statistical methods such as ARIMA and exponential smoothing. These methods are often limited to univariate analysis and therefore future estimates are based on the patterns present in the time series itself. For example, in case we want to estimate the total sales for September, we assume that the patterns found in the sales data will continue into the future. Then, we could use these patterns to predict the expected total sales in September. A drawback of this assumption is that univariate methods are unable to respond to changing conditions in the market. Alternatively, including external information (i.e. leading indicators) could enhance the performance of a sales forecasting model (Currie & Rowley, 2010).

Over the years, some of these statistical methods have been expanded to include multivariate analysis into their method. However, with the increasing popularity and success of machine learning in many different fields, a lot of research has been conducted in solving time series problems as a regression problem. Before regression techniques can be used, time series forecasting problems must be restructured as supervised learning problems. Next to that, when moving from a one-step to a multi-step forecasting problem it becomes necessary to select a forecasting strategy in order to forecast multi-steps-ahead. Hence, the upcoming sections will elaborate on restructuring one-step-ahead forecasting problems as supervised learning tasks and present an overview of existing forecasting strategies when dealing with multi-step-ahead forecasting problems.

2.2.1 Supervised Learning Setting

Supervised learning is the task of learning a function that maps an input, often a vector, to an output based on a set of example input-output pairs, also known as the training set. Given a sequence of numbers for a time series, the data can be restructured as a supervised learning problem by using previous time steps as input variables and the next time step as the output variable. For clarity, Figure 2.1 shows an example of how a time series sequence is restructured as a supervised learning problem in case of a univariate model. As can be seen, the training set consists of three examples that can be used to learn a machine learning function that maps the given input to the output.

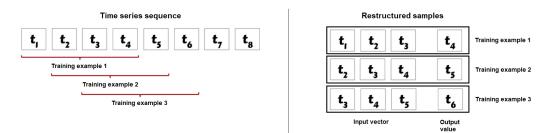


Figure 2.1: Time series sequence as a supervised learning task.

2.2.2 Multi-Step-Ahead Forecasting Strategies

A one-step-ahead time series forecasting problem predicts a single time step into the future (e.g. the next month). In many scenarios a time series forecasting problem consists of a multi-step-ahead problem where forecasts are required for longer horizons (e.g. the next 12 months). In other words, multi-step-ahead forecasting involves predicting the next H observations $(y_{t+1}, \ldots, y_{t+H})$ given a time series sequence (y_t, \ldots, y_{t-n+1}) of n observations at time t. In general, there are three common strategies for multi-step-ahead forecasting: recursive, direct and multiple output (Bontempi et al., 2013; Ben Taieb, Sorjamaa, & Bontempi, 2010).

Recursive Strategy

The recursive strategy first involves training a one-step-ahead function f:

$$y_{t+1} = f(y_t, \dots, y_{t-d+1}) + w_{t+1}$$
(2.1)

Where w_{t+1} represents missing information and d the number of past observations taken into account for predicting future observations. Then, the trained model is used iteratively to generate multi-step forecasts, using forecasted values in prior steps as input for the next time steps. For example, if we want to forecast the total sales of September and October, the forecasted sales of September are used as input for October. Hence, after the training process, the next H observations are estimated by:

$$\hat{y}_{t+h} = \begin{cases}
f(y_t, \dots, y_{t-d+1}) & \text{if } h = 1 \\
f(\hat{y}_{t+h-1}, \dots, \hat{y}_{t+1}, y_t, \dots, y_{t-d+h}) & \text{if } h \in \{2, \dots, d\} \\
f(\hat{y}_{t+h-1}, \dots, \hat{y}_{t+h-d}) & \text{if } h \in \{d+1, \dots, H\}
\end{cases}$$
(2.2)

A drawback of the recursive strategy is that the use of predictions instead of observations, allows errors to accumulate, especially on longer horizons.

Direct Strategy

The direct strategy involves training h separate functions f_h , one for each time step:

$$y_{t+h} = f_h(y_t, \dots, y_{t-d+1}) + w_{t+h}$$
(2.3)

Where w_{t+h} represents missing information and d the number of past observations taken into account for predicting future observations. After the training process, the next Hobservations are estimated by:

$$\hat{y}_{t+h} = f_h(y_t, \dots, y_{t-d+1}) \qquad h \in \{1, \dots, H\}$$
(2.4)

Since the direct strategy does not make use of predicted values, it is not sensitive to the accumulation of errors. Nevertheless, a drawback of the direct strategy is that statistical dependencies among forecasts are ignored as each function is trained independently.

Multiple Output Strategy

The multi-input multi-output strategy involves training a function that is capable of predicting all time steps in once:

$$\{y_{t+H}, \dots, y_{t+1}\} = F(y_t, \dots, y_{t-d+1}) + w$$
(2.5)

Where w represents missing information and d the number of past observations taken into account for predicting future observations. After the training process, the next Hobservations are estimated by:

$$\{\hat{y}_{t+H}, \dots, \hat{y}_{t+1}\} = F(y_t, \dots, y_{t-d+1}) \tag{2.6}$$

The multi-input multi-output strategy avoids the ignorance of statistical dependencies and the accumulation of errors of both the direct and recursive strategies. Nevertheless, a drawback of the multi-input multi-output strategy is that the strategy constrains all horizons to be forecasted with the same model structure, set of inputs and learning procedure.

2.3 Summary and Evaluation

The previous sections have shown that identification of leading indicators in a sales forecasting environment results in a high-dimensional problem with the presence of multicollinearity among the predictors. Accordingly, LASSO has already been found useful in the identification of leading indicators, due to its shrinkage properties and transparency. As a result, LASSO is chosen as modeling technique for the case study described in Chapter 3. As described in the previous sections, LASSO is a linear regression analysis method that performs both variable selection and regularization in order to prevent overfitting of high-dimensional data, and in order to enhance both prediction accuracy and model interpretability (Tibshirani, 1996). The LASSO solution minimizes a penalized residual sum of squares, yielding coefficients that are shrunken to/towards zero (Kirkland & Millard, 2015):

$$\hat{\beta}^{lasso} = \operatorname{argmin}_{\beta} \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{p=1}^{p} \beta_p x_{ip} \right) + \lambda \sum_{p=1}^{p} |\beta_p|$$
(2.7)

The solution and thus the $\hat{\beta}^{lasso}$ estimator highly depends on the magnitude of regularization, which is a value between 0 and 1, and is represented by tuning parameter λ . The solution produced by LASSO is a linear solution path, starting with $\hat{\beta}^{lasso}$ containing all zero coefficients and ending with $\hat{\beta}^{lasso}$ containing all non-zero coefficients as the tuning parameter is reduced to 0.

With regard to the multi-step-ahead forecasting strategies, as will become clear in Section 3.1.2, we are dealing with a 12-step-ahead forecasting problem, and therefore we have to decide which forecasting strategy to apply in order to predict multi-steps-ahead. Since we have chosen to use LASSO as modeling technique, the multiple output strategy is excluded, as LASSO can only handle a single output. Consequently, we have to choose between the remaining recursive and direct forecast strategies. A large number of studies have compared the performances of both strategies in linear models, and accordingly this has led to the overall conclusion that "if the model is correctly specified, the recursive strategy tends to benefit from more efficient parameter estimates, while the direct strategy tends to be more robust to model misspecifications" (Ben Taieb, 2014, p. 37). Accordingly, as the LASSO solution highly depends on λ , there is a likelihood of any model misspecification if λ is chosen incorrectly. Given the fact that the direct strategy tends to be more robust to this, applying the direct strategy is preferred over the recursive strategy in order to predict on further horizons. Hence, this means that, for the case study described in Chapter 3, a total number of 12 separate models must be built and trained independently, in order to predict the next 12 observations.

Chapter 3

Case Study

3.1 Business Understanding

This section aims at covering the business understanding phase of the CRISP-DM framework. First, the business objective is described in order to understand the primary goal of this project from a business perspective. Thereafter, the corresponding data mining objectives are described in order to understand the desired project goals from a more technical perspective.

3.1.1 Business Objective

As indicated earlier in Section 1.3, macroeconomics can contain leading context information in terms of changing economic conditions. Generally, leading indicators are defined as variables that contain predictive information and ideally can predict a certain movement for a target variable in advance. Additionally, Figure 3.1 shows the dynamics between a leading indicator and a specified target variable. Sagaert et al. (2017) illustrated the potential of leading indicators by showing an example from the tire industry. For instance, an economic upturn causes an increase in road transport, which results in an increase in truck tire wear and hence tire consumption will probably rise likewise. It should be noted that truck tires are replaced whenever old tires are worn out and therefore their replacement will likely lag behind the economic upturn. Hence, variables that represent ongoing economic activities have the potential to be a leading indicator of replacement tire sales.

As a major part of PPE's business consists of supplying truck wear and maintenance parts to the European aftersales market, the same reasoning can be applied to PPE's

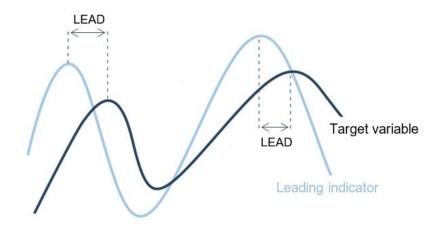


Figure 3.1: Dynamics between a leading indicator and a specified target variable.

business. In other words, variables that represent ongoing economic activities, have the potential to be a leading indicator for PPE's business. As a result, the primary objective from a business perspective is defined as:

Identify and gain insight into leading indicators for PACCAR Parts Europe's part sales. Next to that, explore and analyze whether including information extracted from leading indicators actually leads to more accurate parts sales predictions.

3.1.2 Data Mining Objectives

In order to achieve the business objective, the business objective must be translated into specific data mining objectives. The data mining objectives state the desired project goals in more technical terms. As a result, the data mining objectives from a more technical perspective are defined as:

- 1. Provide insight into which potential leading indicators are actually relevant for predicting PACCAR Parts Europe's parts sales at $\{(t + 1), (t + 2), \dots, (t + 11), (t + 12)\}$.
- 2. Build, validate and benchmark a forecasting model that is capable of predicting PACCAR Parts Europe's parts sales at $\{(t+1), (t+2), \ldots, (t+11), (t+12)\}$.

3.2 Data Understanding

This section aims at covering the data understanding phase of the CRISP-DM framework. At first, three different types of potential leading indicators are introduced. Thereafter, a brief description is given of the data sources and the collected data itself. Finally, the data availability of both the target variable and the included indicators are discussed.

3.2.1 Data Collection

The studies of Sagaert et al. (2018) and Verstraete et al. (2020) only included publicly available macroeconomic indicators as potential leading indicators. With regard to this field project, it was possible to access non-publicly available databases with data specifically related to DAF and the European road freight market. Therefore, a distinction is made between three different types of potential leading indicators: business, economic and market indicators.

In order to collect data with regard to the target variable and the potential leading indicators, different data sources were used. First, the internal departments PPE Control and PPE Business Development & Intelligence served as data sources for PPE's sales sales data and the business indicators described in Section 3.2.2 and 3.2.2. Secondly, the Eurostat and OECD publicly available economic databases provided access to thousands of macroeconomic time series related to European territories and served as data sources for the economic indicators described in Section 3.2.2. Thirdly, Rementum Research & Management, further denoted as Rementum, is a market research and advisery firm specialized in both the European road freight market and commercial vehicles with a gross vehicle weight over 6 tonnes. Rementum collects data from a broad array of sources relevant for heavy commercial road transport such as road carriers, transport equipment OEMs and OE & aftermarket component suppliers in order to analyze the European road transport market conditions. The expertise of Rementum was used to collect the market indicators described in Section 3.2.2.

3.2.2 Data Description

Together with PPE's sales data, a total number of 33 potential leading indicators have been collected. All potential leading indicators are directly or indirectly related to the European road freight market. For example, the vast majority of the economic indicators are related to the industrial manufacturing and construction sectors and are therefore not directly related to the road freight market. However, the industrial manufacturing and construction sectors are considered as the major drivers of commercial vehicle road transport and are therefore indirectly related to transport. Additionally, it should be noted that the exact European areas which each indicator covers, are not revealed in this report due to confidentiality. The upcoming sections will provide more information on the collected data in terms of time series properties.

Parts Sales

As explained in Section 3.1.1, in order to identify which potential leading indicators are relevant for predicting PPE's parts sales, it is necessary to specify a certain target variable. In this case, the target variable is specified as the total monthly truck parts sales reported by all DAF dealers located in the $EU27+2^1$ area. Figure 3.2 shows the specified target variable, which will be further denoted as PPE's parts sales. It should be noted that the y-axis is normalized due to data confidentiality. As can be seen, observed sales data is available from January 2002 to February 2020 which is equivalent to 218 monthly observations. Moreover, it becomes clear that the sales data is following a certain trend and that there may be seasonality present in the data. The partial autocorrelation function and a seasonal subseries plot of the PPE truck parts sales data in Appendix A show that the sales data indeed has a seasonal pattern that repeats every 12 months.



Figure 3.2: PPE's parts sales from January 2002 to February 2020 in EU27+2 area.

¹Included countries: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden and Switzerland.

Business Indicators

The business indicators represent the most important internally reported variables that are likely to affect PPE's parts sales. A total number of two business indicators have been selected which are related to observed sales in the past (i.e. autoregressive information) and the total number of DAF truck deliveries. Sagaert et al. (2017) mentioned that autoregressive information contains potential sales dynamics at no additional data cost and should therefore always be included as an indicator. Moreover, PPE's parts sales consists of both parts sales related to wear/maintenance (e.g. brake pads, oil filters) and parts sales related to new truck sales (e.g. side skirts, mudguards). As a result, it is likely that the total number of DAF truck deliveries affects PPE's parts sales and it is therefore included as an indicator. Accordingly, Table 3.1 presents an overview of all included business indicators and their time series properties.

Table 3.1: All business indicators and their corresponding time series properties.

Indicator description	\mathbf{Unit}	Source
Y Observed parts sales: Current month, NSA X_1 DAF truck deliveries: Current month, NSA	€ Trucks	PPE Control PPE Business Development & Intelligence

NSA = Not seasonally adjusted;

Economic Indicators

The economic indicators represent ongoing developments in Europe's overall economic climate and relevant sectors. A total number of 12 economic indicators have been selected which are likely to affect PPE's parts sales. The vast majority of these indicators are related to the industrial manufacturing and construction sectors as these sectors are the major drivers of commercial vehicle transport. Additionally, macroeconomic variables such as passenger car registrations, gross domestic product and economic sentiment are commonly used to assess the current state of the overall economy and are therefore included as indicators likewise. Accordingly, Table 3.2 presents an overview of all included economic indicators and their time series properties.

Indic	cator description	Unit	Source
X_2	Business climate: Current month, SA	Index	Rementum
X_3	Economic sentiment: Current month, SA	Index	Eurostat
X_4	Passenger car registrations: Current month, NSA	Cars	Rementum
X_5	Retail confidence: Current month, SA	Balance	Eurostat
X_6	Industrial confidence: Current month, SA	Balance	Eurostat
X_7	Industrial production: Current month, SA	Index	Eurostat
X_8	Gross domestic product: Current month, SA	Index	OECD
X_9	Producer price index: Current month, SA	Index	Eurostat
X_{10}	Construction confidence: Current month, SA	Balance	Eurostat
X_{11}	Construction spending: Current month, NSA	Index	Rementum
X_{12}	Construction activity: Current month, SA	Index	Eurostat
X_{13}	Construction and mining equipment sales: MA	Index	Rementum
	(3mos), NSA		

Table 3.2: All economic indicators and their corresponding time series properties.

SA = Seasonally adjusted; NSA = Not seasonally adjusted; MA = Moving average; mos = months;

Market Indicators

The market indicators represent the ongoing activities in Europe's transport sector. A total number of 19 market indicators have been selected which are likely to affect PPE's parts sales. These indicators cover a wide range of variables that provide valuable information about the current and expected market conditions, such as the number of truck orders, diesel consumption, tire consumption, carrier sentiment and road activity. Accordingly, Table 3.3 presents an overview of all included market indicators and their time series properties.

Indie	cator description	Unit	Source
X_{14}	Replacement truck tire sales (ST) : MA $(2mos)$, NSA	Index	Rementum
X_{15}	Replacement truck tire sales (LT): MA (12mos), NSA	Index	Rementum
X_{16}	OE truck tire sales (ST): MA (2mos), NSA	Index	Rementum
X_{17}	OE truck tire sales (LT): MA (12mos), NSA	Index	Rementum
X_{18}	Aftermarket truck tire deliveries: Current month, NSA	Index	Rementum
X_{19}	Diesel consumption growth (LT): MA (12mos), NSA	%	Rementum
X_{20}	Diesel consumption growth (ST): MA (2mos), NSA	%	Rementum
X_{21}	Retail diesel price: Current month, NSA	€	Rementum
X_{22}	Automotive diesel deliveries: Current month, NSA	m^3	Rementum
$X_{23}^{}$	OEM truck orders growth (LT): MA (12mos), NSA	%	Rementum
X_{24}	OEM truck orders (ST): MA (3mos), NSA	Trucks	Rementum
X_{25}	OEM order intake expectations: MA (6mos), NSA	Balance	Rementum
X_{26}	OEM production expectations: MA (6mos), NSA	Balance	Rementum
X_{27}	Road transport activity: Current month, NSA	Index	Rementum
X_{28}	Road transport capacity: Current month, SA	Index	Rementum
X_{29}	Freight volume index: MA (3mos), NSA	Index	Rementum
X_{30}	Carrier confidence: Current month, NSA	Balance	Rementum
X_{31}	Carrier demand expectations: Current month, NSA	Balance	Rementum
X_{32}	Carrier hiring expectations: Current month, NSA	Balance	Rementum
X_{33}	Carrier pricing expectations: Current month, NSA	Balance	Rementum

Table 3.3: All market indicators and their corresponding time series properties.

ST = Short-term; LT = Long-term; SA = Seasonally adjusted; NSA = Not seasonally adjusted; MA = Moving average; mos = months;

3.2.3 Data Availability

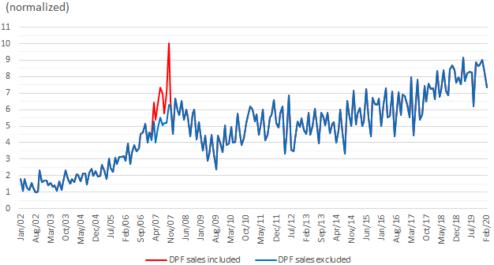
The previous sections have shown that a total number of 34 variables (33 potential leading indicators and PPE's historical parts sales), coming from various data sources such as PPE, Eurostat, OECD and Rementum, were collected. As a result, the availability of historical data differs for each data source. For example, the historical records of the indicators coming from the Eurostat and OECD databases generally start in the early 90's and in some cases even the early 80's. On the other hand, the historical record of PPE's parts sales data is considerably shorter and Figure 3.2 shows that the historical record of Rementum are decisive as the data from Rementum is only available from January 2005. Hence, the number of observations available for modeling is limited to the period from January 2020, which is equivalent to 182 monthly observations.

3.3 Data Preparation

This section aims at covering the data preparation phase of the CRISP-DM process model. At first, the data cleaning proceedings that have been applied are discussed. Then, all variables are analyzed and if necessary transformed to achieve stationarity. Finally, a brief description is given of how the data is restructured to a supervised learning setting and what consequences this has on the data availability.

3.3.1 Data Cleaning

Data cleaning consists of assessing and ideally improving the data quality by removing or modifying data that is considered as incomplete, incorrect or irrelevant. Therefore, all variables were checked for the presence of missing values in the period from January 2005 to February 2020. Since all used data sources apply their own data management practices, no missing values were found. Then, a visual inspection was carried out on each variable in order to possibly detect any incorrect or doubtful values. As a result, during the inspection of PPE's parts sales data (see Figure 3.2) it was noticed that in October 2007 there were significantly higher sales than usual. After this finding, the PPE control department was approached with the question whether there was a clear explanation for this extremely high value. It turned out that for the year 2007 the Dutch government had introduced a subsidy scheme for the installation of diesel particulate filters (DPF) in order to encourage and promote the reduction of air pollution. Since PPE does a lot of business in the Netherlands, this subsidy scheme led to a significant increase in DPF sales throughout the year 2007. As the increase in sales is clearly visible in the data, it was decided to exclude the results of these additional DPF sales. Figure 3.3 shows the result of excluding the additional DPF sales from the data.



PPE truck parts sales, EU27+2, Jan 2002 - Feb 2020 (normalized)

Figure 3.3: Result of removing additional DPF sales.

3.3.2 Data Stationarity

Many time series forecasting techniques require stationary data as input in order to be able to forecast future values. Generally, a stationary time series is defined as a time series which statistical properties, such as mean and variance do not change over time. In consequence, it is assumed that a stationary time series is easier to predict as the statistical properties in the future must correspond to the current statistical properties. Hence, all variables must be checked for stationarity. There are several statistical tests that can determine whether a series is likely to be stationary or alternatively whether the series contains a unit root. If a series contains a unit root, it shows a systematic pattern that is hard to predict and control. In practice, two statistical tests are often used interchangeably: the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Philips-Schmidt-Shin (KPSS) test. The ADF and KPSS tests differ from each other due to the different hypotheses that are being tested. In particular, the null (H_0) and alternative (H_1) hypothesis for the ADF test are defined as follows:

- H_0 : the series contains a unit root.
- H_1 : the series does not contain a unit root.

Alternatively, the null and alternative hypothesis for the KPSS test are defined as follows:

• H_0 : the series is stationary around a deterministic trend.

• H_1 : the series contains a unit root.

As can be seen, the null hypothesis of the ADF test states that the series contains a unit root, which is equal to the alternative hypothesis of the KPSS test. Practically, a series is often classified as a stationary series if the null hypothesis of the ADF test is rejected and at the same time the null hypothesis of the KPSS is accepted. Accordingly, Appendix B presents an overview of the results of both the ADF and KPSS test applied on all included variables.

The results show that for many variables the ADF and KPSS test contradict each other. For example, applying both tests on variable X_4 led to the acceptance of both the ADF null hypothesis (indicating non-stationarity) and the KPSS null hypothesis (indicating stationarity). Since this occurs a considerable number of times, it becomes necessary to choose if we rely solely on the results of the ADF or the KPSS test. As the KPSS test was specifically developed for economic series, the decision was made to rely on the results of the KPSS test (Kwiatkowski, Phillips, Schmidt, & Shin, 1992). According to these results, the variables Y, X_9 , X_{11} , X_{22} are non-stationary and must be transformed in order to achieve stationarity.

A well-known method to make a non-stationary series stationary is differencing. Differencing computes the differences between consecutive observations and helps eliminating/reducing trend. When dealing with a non-stationary series Y_t , the differenced series Y_t^d is obtained through:

$$Y_t^d = Y_t - Y_{t-1} (3.1)$$

If the series contains seasonality, seasonal differencing computes the differences between consecutive observations of the same season and helps eliminating/reducing trend and seasonality. When dealing with a non-stationary series Y_t , the seasonal differenced series Y_t^{sd} is obtained through:

$$Y_t^{sd} = Y_t - Y_{t-s} (3.2)$$

Where s represents the number of seasonal periods in one cycle (e.g. 12 for monthly seasonality, 4 for quarterly seasonality). A consequence of applying seasonal differencing is that s observations are lost as it is not possible to calculate the seasonal differenced values for the first s observations. Consequently, Table 3.4 shows the applied data transformations in order to achieve stationarity for the variables Y, X_9, X_{11}, X_{22} . As can be seen, both differencing and seasonal differencing have been applied to achieve stationarity for all variables. As a consequence of applying seasonal differencing, a total number of

12 observations are lost as in this case s equals 12 and thus the number of observations available for modeling reduces from 182 to a total of 170 observations.

	Variable	Data transformation
Y	Observed parts sales	Seasonal differencing
X_9	Producer price index	Differencing
X_{11}	Construction spending	Seasonal differencing
X_{22}	Automotive diesel deliveries	Seasonal differencing

Table 3.4: Data transformations applied to achieve stationarity.

3.3.3 Supervised Learning Setting

In order to use regression techniques on a time series forecasting problem, the data must be restructured to a supervised learning task. As demonstrated in Section 2.2.1, given a sequence of numbers for a time series, the data can be restructured as a supervised learning task by using previous time steps as input variables and the next time step as the output variable. As Sagaert et al. (2018) indicated that macroeconomic indicators can contain leading context information up to a maximum horizon of 12 months, the decision was made to include all 12 previous time steps as input variables. This means that every potential leading indicator is lagged in time 12 times and therefore the number of indicators/predictors increases significantly from 34 (business, economic and market indicators + autoregressive information) to a total number of 34 * 12 = 408input variables/predictors. Hence, we are dealing with a high-dimensional problem since the number of predictors exceeds the number of observations (p > n). Figure 3.4 shows how the data is restructured to a supervised learning task for the first three observations when predicting (t + 1) ahead.

Input Output

M	MODEL t+1																						
J	F	М	А	М	J	J	Α	S	0	Ν	D	L.	F	Μ	Α	Μ	J	J	Α	S	0	Ζ	D
2006								2007															
J	F	М	Α	М	J	J	Α	S	0	Ν	D	L.	F	Μ	Α	Μ	J	J	Α	S	0	Z	D
200	06											2007											
J	F	М	А	М	J	J	Α	S	0	Ν	D	J	F	М	Α	М	J	J	Α	S	0	Ν	D
2006								2007															

Figure 3.4: Time series data to supervised learning setting.

As can be seen, as a consequence of lagging each indicator in time 12 times, the first 12 observations are lost and thus a total number of 158 observations are left over for modeling purposes. Next to that, as a consequence of restructuring the data to a supervised learning task when predicting multi-steps ahead, additional observations are lost.

Input Output Lost observations												
MODEL t+3												
JFMAMJJ	A S O N D	JFMAMJJASOND										
2006		2007										
MODEL t+12	MODEL t+12											
JFMAMJJ	A S O N D	J F M A M J J A S O N D										
2006 2007												

Figure 3.5: Restructuring the data causes additional observation losses.

For clarity, Figure 3.5 shows how many observations are lost due to restructuring the data when predicting (t+3) and (t+12) ahead. As can be seen, the number of observations that are lost depends on the forecasting horizon and thus the number of observations available for modeling will differ for each forecasting horizon. Therefore, Table 3.5 presents an overview of how many observations are available for modeling per model M_{t+h} , where hrepresents the forecast horizon in months.

Model	Period available for modeling	Number of observations
M_{t+1}	Jan 2007 to Feb 2020	158
M_{t+2}	Feb 2007 to Feb 2020	157
M_{t+3}	Mar 2007 to Feb 2020	156
M_{t+4}	Apr 2007 to Feb 2020	155
M_{t+5}	May 2007 to Feb 2020	154
M_{t+6}	Jun 2007 to Feb 2020	153
M_{t+7}	Jul 2007 to Feb 2020	152
M_{t+8}	Aug 2007 to Feb 2020	151
M_{t+9}	Sep 2007 to Feb 2020	150
M_{t+10}	Oct 2007 to Feb 2020	149
M_{t+11}	Nov 2007 to Feb 2020	148
M_{t+12}	Dec 2007 to Feb 2020	147

Table 3.5: Data available for modeling after data preparation activities.

3.4 Modeling

This section aims at covering the modeling phase of the CRISP-DM process model. At first, the proposed modeling approach is explained and discussed in detail. Then, a

brief description is given of the tuning parameter selection procedure and the benchmark models that are considered.

3.4.1 Modeling Approach

As explained in Section 2.3, the LASSO solution highly depends on the amount of regularization, which is controlled by the tuning parameter λ . Hence, choosing an appropriate value for λ is crucial and therefore paying serious attention to the design of the modeling approach is necessary.

Hastie, Tibshirani, and Friedman (2009) proposes to determine λ based on the crossvalidation estimate of the prediction error. Typically, K-fold cross-validation (KCV) randomly splits the data into K-folds and subsequently fits a model using K - 1 folds and uses the K^{th} fold for testing. This process is repeated until every K^{th} fold is used for testing once. The estimated prediction error is determined by averaging the errors over all K^{th} folds used for testing. For clarity, Figure 3.6 shows the principle of KCV when using 5-fold cross-validation. As the data is partitioned randomly, using KCV in a

Iteration 1	Test	Train	Train	Train	Train
Iteration 2	Train	Test	Train	Train	Train
Iteration 3	Train	Train	Test	Train	Train
Iteration 4	Train	Train	Train	Test	Train
Iteration 5	Train	Train	Train	Train	Test

Figure 3.6: K-fold cross-validation.

time series environment does not seem applicable as temporal dependencies are ignored. Nevertheless, using CV in a time series environment was extensively studied by Bergmeir and Benítez (2012) and they did not find any practical problems with standard crossvalidation. Moreover, they suggest to use standard KCV or blocked CV together with stationary data as this uses all available information for training and testing. In this case, applying standard KCV is preferred over applying blocked CV as applying blocked CV will lead to additional observation losses. With regard to KCV, it is well known that there exists a bias-variance trade-off when choosing K. When K is small, more observations are removed from the training set and therefore error estimates are biased upwards while suffering less variance. Otherwise, when K is large, error estimates are less biased as the training set becomes larger; however, due to the increasing similarities between the training sets, the variance of the error estimates increases. The choice of K was extensively studied by Breiman and Spector (1992) and Kohavi (1995) and accordingly the use of K = 5 or K = 10 is recommended as a good compromise (Hastie et al., 2009). Consequently, in order to evaluate whether to apply K = 5 or K = 10, the observations must be split into two subsets: in-sample observations (i.e. training data) and out-of-sample observations (i.e. test data). Next to that, it is necessary to determine which error measure will be used for the cross-validation estimate of the prediction error.

A common approach to split training and test sets when dealing with time series data, is time series cross-validation (Hyndman & Athanasopoulos, 2018). This approach uses a series of test sets, with each test set consisting of a single observation. The corresponding training set consists only of observations prior to the observation in the test set. Figure 3.7 shows an example of how the training and test sets are defined for models M_{t+1} and M_{t+12} . Since it is not possible to obtain reliable predictions when using a small training

	Training set Test set																									
M	DDE	L t	-1	_		_	_			_	_		_	 	_											
J.,	F	Μ	Α	М	J.	J.	Α	S	0	Ν	D				J.	F	М	Α	М	J.	J.	Α	S	0	N	D
200	07														2015											
J	F	М	Α	М	J	J	Α	S	0	Ν	D				J	F	М	Α	М	J	J	А	S	0	Ν	D
200	07														201	15										
J	F	М	Α	М	J	J	Α	S	0	Ν	D				J	F	М	А	М	J	J	Α	S	0	Ν	D
2007														201	15											

MODEL t+12	
J F M A M J J A S O N D	J F M A M J J A S O N D J F M A M J J A S O N D
2007	2015 2016
J F M A M J J A S O N D	J F M A M J J A S O N D J F M A M J J A S O N D
2007	2015 2016
JFMAMJJASOND	J F M A M J J A S O N D J F M A M J J A S O N D
2007	2015 2016
	·
	(b)

(a)

Figure 3.7: Time series cross-validation data split when predicting (t + 1) ahead (a) and (t + 12) ahead (b).

set, the initial size of the training set is set to 70% of the smallest available sample size. The smallest sample size equals 147 observations for model M_{t+12} and thus the initial size of the training set is set to 102 observations for all 12 models. Accordingly, Table 3.6 presents an overview of the remaining observations that are used as test sets for model

Model	Period used as test sets	Number of test sets
M_{t+1}	Jul 2015 to Feb 2020	56
M_{t+2}	Aug 2015 to Feb 2020 $$	55
M_{t+3}	Sep 2015 to Feb 2020	54
M_{t+4}	Oct 2015 to Feb 2020	53
M_{t+5}	Nov 2015 to Feb 2020	52
M_{t+6}	Dec 2015 to Feb 2020	51
M_{t+7}	Jan 2016 to Feb 2020	50
M_{t+8}	Feb 2016 to Feb 2020	49
M_{t+9}	Mar 2016 to Feb 2020 $$	48
M_{t+10}	Apr 2016 to Feb 2020 $$	47
M_{t+11}	May 2016 to Feb 2020	46
M_{t+12}	Jun 2016 to Feb 2020	45

Table 3.6: Data available for model performance evaluation.

performance evaluation. As can be seen in Table 3.6, model M_{t+1} has a total number of 56 observations available for performance evaluation, whereas model M_{t+12} has a total number of 45 observations available for performance evaluation. When aggregating all observations across all horizons, a total number of 606 observations are available for model performance evaluation.

With regard to the prediction error estimates, two commonly used metrics, when dealing with continuous variables, are the root mean square error (RMSE) and the mean absolute error (MAE) (Chai & Draxler, 2014). The RMSE is defined as the square root of the quadratic mean of the prediction errors and is mathematically represented as:

RMSE =
$$\sqrt{\frac{1}{n} \sum_{j=1}^{n} (y_j - \hat{y}_j)^2}$$
 (3.3)

On the other hand, the MAE is defined as the sum of the absolute values of the prediction errors and is mathematically represented as:

$$MAE = \frac{1}{n} \sum_{j=1}^{n} |y_j - \hat{y}_j|$$
(3.4)

A major difference between the RMSE and MAE lies in the contribution of individual prediction errors to the overall error. In the case of MAE, individual prediction errors contribute proportionally to the total amount of error and thus larger errors will contribute linearly to the overall error. In the case of RMSE, the contribution of individual prediction errors to the total amount of error grows quadratically and thus larger errors contribute more to the overall error. While both metrics have been widely used in many applications, there is no consensus on which is the most appropriate metric for model evaluation (Chai & Draxler, 2014). As a result, the choice between RMSE and MAE should depend on the problem domain. Accordingly, as it is assumed that penalizing larger errors disproportionately is unnecessary in a sales forecasting environment, the use of MAE is preferred over RMSE in the modeling approach.

On the whole, we used the first 102 observations as training set and the MAE to analyze the KCV error curves for K = 5 and K = 10. Since it is not practically feasible to consider all possible λ values, λ is chosen on a fixed logarithmic grid of $\{-2.5, \ldots, -0.5\}$ with equally spaced intervals of 0.1. Figure 3.8 shows an example of the CV error curves for model M_{t+1} . As can be seen, the error curve when K = 5 is biased upwards as expected, while the error curve when K = 10 is still quite stable, despite the increased variance. As this was the case in all 12 models, the decision was made to apply CV with K = 10.

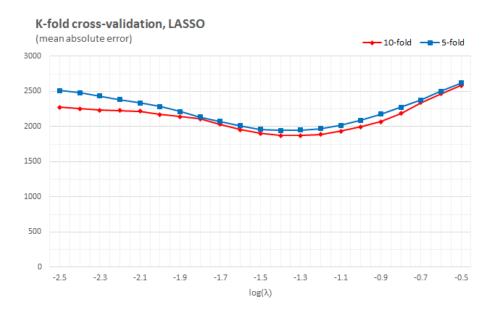
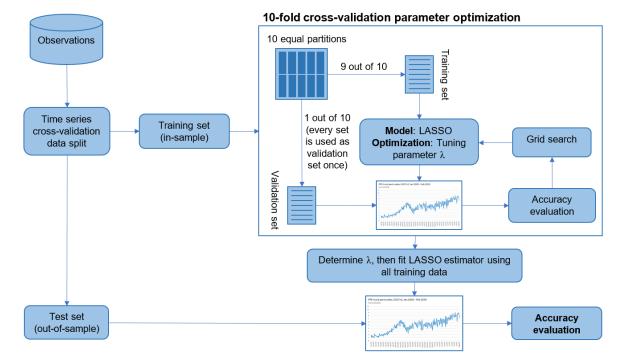


Figure 3.8: Comparison of CV error curves when K = 5 and K = 10.

On the whole, Figure 3.9 shows the overall modeling approach that will be used for all models. At first, we determine the value of tuning parameter λ using a 10-fold CV grid search on the in-sample data. Then, we fit a $\hat{\beta}^{lasso}$ estimator using the entire training set and the determined tuning parameter λ , after which the $\hat{\beta}^{lasso}$ estimator is used to



predict the observation in the final test set.

Figure 3.9: Design of the modeling approach.

3.4.2 Tuning Parameter λ

The existing literature describes two options to choose tuning parameter λ when using CV (Hastie et al., 2009):

1. Choose λ corresponding to the smallest CV error estimate, mathematically expressed as:

$$\lambda_{min} = \operatorname{argmin} \operatorname{CV}(\lambda) \tag{3.5}$$

2. Choose λ corresponding to the most regularized model within one-standard error of the minimum CV error estimate (one-standard error rule), mathematically expressed as:

$$\lambda_{1se} = \operatorname{argmax}_{\lambda > \lambda_{min}} \operatorname{CV}(\lambda) \text{ with } \operatorname{CV}(\lambda_{1se}) \le \operatorname{CV}(\lambda_{min}) + \operatorname{SE}(\lambda_{min})$$
(3.6)

The one-standard error rule assumes equal performance for all models within one-standard error of the minimum and as a result opts to choose the most regularized model. In other words, the one-standard error rule chooses the most simple model as more coefficients

will be shrunken to 0. The one-standard error rule additionally requires to calculate the standard errors of each $CV(\lambda)$. Accordingly, for $k = \{1, \ldots, K\}$ the standard errors are obtained by:

$$CV_k(\lambda) = \frac{1}{n_k} \sum_{i=1}^{n_k} |y_i - \hat{\beta}_{\lambda}^{lasso, -k}(x_i)|$$
(3.7)

$$SD(\lambda) = \sqrt{\operatorname{var}(CV_1(\lambda), \dots CV_K(\lambda))}$$
(3.8)

$$SE(\lambda) = \frac{SD(\lambda)}{\sqrt{K}}$$
(3.9)

Figure 3.10 shows an example of how tuning parameter values λ_{min} and λ_{1se} are determined based on the CV error curve with vertical standard error bars. As can be seen in Figure 3.10, the CV error estimate is minimized when $\log(\lambda) = -1.3$ and thus -1.3is chosen as value for $\log(\lambda_{min})$. Moreover, the most regularized model that does not exceed $CV(\lambda_{min}) + SE(\lambda_{min})$, which is represented by the green horizontal line, is found at $\log(\lambda) = -1.0$ and thus -1.0 is chosen as value for $\log(\lambda_{1se})$. In practice, choosing λ_{1se} is often considered as standard (Hastie et al., 2009).

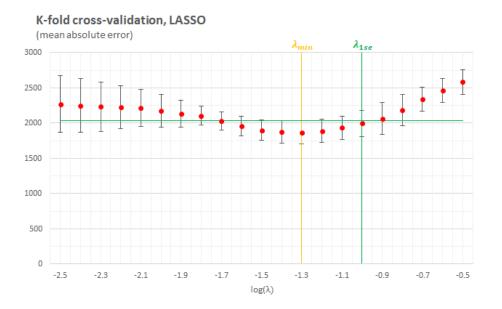


Figure 3.10: Tuning parameter values λ_{min} and λ_{1se} .

3.4.3 Benchmark Models

The purpose of extracting information from leading indicators is to ideally enhance forecasting performance by including information in terms of changing economic and market conditions. In order to assess whether including leading indicators enhances forecasting performance, the performance of LASSO is benchmarked to commonly used univariate methods that are unable to respond to these changing conditions. Since we are dealing with a very small sample size, complex machine learning techniques, such as the recurrent neural network, are considered out of scope as these techniques require a large sample size for training purposes. The methods used as benchmark are listed below.

Holt-Winters

The Holt-Winters method, also known as triple exponential smoothing, is a statistical method that can be used to model and predict a series containing both trend and seasonal variations (Hyndman & Athanasopoulos, 2018). It distinguishes three components within the time series: the level component l_t , the trend component b_t and the seasonal component s_t . The three components are expressed as three distinct types of exponential smoothing, with corresponding smoothing parameters α , β , γ . Additionally, the frequency of the seasonality, i.e. the number of observations within a seasonal period, is represented by m, which equals 12 in this case. Within Holt-Winters there are two variations that distinguish between additive and multiplicative seasonality. The additive method is often used when the seasonal variations in the series can be considered as practically constant, while the multiplicative method is often used when the seasonal variations are changing proportional to the level of the series. As the seasonal variations in PPE's parts sales data seems to fit the description of both the additive and multiplicative seasonal variations to some extent, both variations will be used as benchmark. Accordingly, the component expressions and the forecast equation for the additive method are:

$$\hat{y}_{t+h|t} = l_t + hb_t + s_{t+h-m} \tag{3.10}$$

$$l_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(l_{t-1} + b_{t-1})$$
(3.11)

$$b_t = \beta (l_t - l_{t-1}) + (1 - \beta) b_{t-1}$$
(3.12)

$$s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1 - \gamma)s_{t-m}$$
(3.13)

In addition, the component expressions and the forecast equation for the multiplicative method are:

$$\hat{y}_{t+h|t} = (l_t + hb_t)s_{t+h-m} \tag{3.14}$$

$$l_t = \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1})$$
(3.15)

$$b_t = \beta(l_t - l_{t-1}) + (1 - \beta)b_{t-1}$$
(3.16)

$$s_t = \gamma \frac{y_t}{(l_{t-1} - b_{t-1})} + (1 - \gamma)s_{t-m}$$
(3.17)

Where y_t equals the observed sales at time t and $\hat{y}_{t+h|t}$ equals the prediction for hmonth(s) ahead made at time t. The smoothing parameters α , β , γ were automatically chosen in the range of 0.1-0.2 by minimizing the prediction error on a validation set, as this often ensures stable forecasts (Nahmias & Olsen, 2015).

SARIMA

ARIMA is, next to exponential smoothing, one of the most widely used approaches to model and predict a series. ARIMA, an acronym for Autoregressive Integrated Moving Average, aims to describe the autocorrelations in the data by using a linear combination of past observed values, known as the autoregressive part, and past forecast errors, known as the moving average part (Hyndman & Athanasopoulos, 2018). Accordingly, an autoregressive model of order p, referred to as AR(p), is written as:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \epsilon_t$$
(3.18)

Where ϵ_t equals white noise. Note that varying the parameters ϕ_1, \ldots, ϕ_p will result in different time series patterns. Moreover, a moving average model of order q, referred to as MA(q), is written as:

$$y_t = c + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \dots + \theta_q \epsilon_{t-q} + \epsilon_t$$
(3.19)

Where, again, ϵ_t equals white noise and varying the parameters $\theta_1, \ldots, \theta_p$ results in different time series patterns. Consequently, if autoregressive and moving average models are combined, we obtain an ARMA(p,q) model, which is logically written as:

$$y_t = c + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \theta_1 \epsilon_{t-1} + \dots + \theta_q \epsilon_{t-q} + \epsilon_t \tag{3.20}$$

In order to use ARMA(p,q), AR(p) and MA(q) models, the time series must be stationary. Otherwise, if autoregressive and moving average models are combined together

with differencing, we obtain an ARIMA(p, d, q) model, where d refers to the order of differencing. Accordingly, an an ARIMA(p, d, q) model which is written as:

$$y'_t = c + \phi_1 y'_{t-1} + \dots + \phi_p y'_{t-p} + \theta_1 \epsilon_{t-1} + \dots + \theta_q \epsilon_{t-q} + \epsilon_t \tag{3.21}$$

Where y'_t equals the differenced series of order d. Furthermore, when dealing with seasonal variations, additional seasonal terms are included and as a result we obtain a SARIMA $(p, d, q)(P, D, Q)_m$ model, where m represents the number of observations in a seasonal period. Note that the seasonal part of the model consists of terms that are similar to the non-seasonal components of the model, but involve backshifts of the seasonal period. Since we are dealing with trend and seasonal variations, a SARIMA $(p, d, q)(P, D, Q)_m$ model with m = 12 will be used as benchmark. Accordingly, plots of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) were used to determine appropriate values for (p, d, q)(P, D, Q).

Chapter 4

Results

This chapter aims at presenting and describing the results related to the previously described case study. Additionally, this chapter will elaborate on two experiments that have been conducted in order to explore, investigate and analyze whether forecasting performance can be enhanced by applying efficient tuning parameter selection and forecast combination.

4.1 Case Study

This section presents the forecasting performance results of the LASSO, Holt-Winters and SARIMA models. With regard to LASSO, the one-standard error rule was used to select the value of tuning parameter λ . Furthermore, to account for the randomness of CV, all models were run five times and thus the total number of test sets available for model evaluation, presented in Table 3.6, are multiplied by five. Hence, model M_{t+1} will have 280 test sets available for evaluation, model M_{t+2} will have 275 test sets available for evaluation and so on. On the other hand, with regard to the benchmark models, both the additive (AHW) and multiplicative Holt-Winters (MHW) methods were implemented with optimal smoothing parameters $\alpha = 0.2$, $\beta = 0.1$, $\gamma = 0.2$, whereas regarding SARIMA, a model with $(p, d, q)(P, D, Q)_m$ equal to $(2, 1, 0)(1, 1, 1)_{12}$ was selected using the Akaike information criterion (AIC). Overall, Table 4.1 shows the mean absolute prediction error and the mean absolute percentage error across all considered models and forecast horizons.

Horizon	LASSO	SARIMA	AHW	MHW
t+1	1,865.93 (4.75)	1,876.55 (4.76)	1,732.56 (4.43)	1,994.59(5.03)
t+2	1,883.23 (4.79)	1,778.85(4.51)	1,636.93 (4.17)	1,935.19(4.85)
t+3	1,895.85 (4.79)	1,955.72 (4.95)	1,669.06 (4.23)	1,999.18(5.03)
t+4	1,808.90 (4.57)	1,778.09 (4.48)	$1,722.89\ (4.34)$	2,043.26 (5.10)
t+5	1,803.76 (4.56)	1,820.68 (4.57)	$1,726.69 \ (4.33)$	2,087.26(5.20)
t+6	$1,785.07 \ (4.51)$	1,872.53 (4.73)	1,887.68(4.74)	2,140.95(5.35)
t+7	2,067.32(5.17)	$1,968.04 \ (4.93)$	2,125.34 (5.31)	2,313.06(5.74)
t+8	2,092.85 (5.19)	$1,939.43 \ (4.83)$	2,038.74 (5.04)	2,248.73 (5.56)
t+9	2,242.76(5.58)	$2,021.24 \ (4.99)$	2,120.01 (5.24)	2,291.26(5.64)
t + 10	$2,\!138.72\ (5.34)$	2,232.01 (5.57)	2,342.59(5.84)	2,547.03(6.32)
t + 11	2,377.19(5.91)	2,363.90 (5.84)	2,405.85(5.91)	2,680.01 (6.57)
t + 12	2,349.52 (5.84)	$2,\!316.47\ (5.78)$	2.378.72(5.88)	$2,\!614.66\ (6.42)$

Table 4.1: Mean absolute prediction error and the mean absolute percentage error (in parentheses) across all models and forecast horizons.

As can be seen in Table 4.1, when comparing AHW with MHW, AHW consistently outperforms MHW and thus the seasonal variations could be considered additive. Moreover, when comparing LASSO, SARIMA and AHW, their model performances seem more competitive as no model consistently outperforms the other. In particular, AHW seems to predict more accurate on the shorter horizons, whereas on the longer horizons, SARIMA seems to predict more accurate. Thus, despite the fact that LASSO uses information from external indicators, forecasting performance has not improved compared to traditional time series forecasting methods. A possible reason why the traditional methods perform reasonably well in this case, may be due to the limited volatility in the out-of-sample parts sales. According to Currie and Rowley (2010) using additional information can enhance forecasting performance, especially in volatile environments. The period used for out-of-sample evaluation, July 2015 to February 2020, was a reasonably stable period from an economic point of view, and as a result, the volatility in the parts sales was limited during this period. Hence, it could be that in more volatile times, the performance of LASSO might be more competitive.

4.2 Efficient Tuning Parameter Selection

With regard to the case study, the commonly used one-standard error rule was used for choosing the value of λ . Jung (2016) stated that tuning parameter selection is often one of the crucial parts in high-dimensional modeling and hence using CV to select a single value as optimal value for the tuning parameter can be unstable due to the sampling variation. A possible solution to account for these sampling variations is to apply repeated CV.

Nevertheless, applying repeated CV exponentially increases computational costs when predicting multi-steps ahead and as a result Jung (2016) proposed the use of efficient CV. Efficient CV selects multiple candidates of parameter values and calculates an average based on different weights depending on their performance without significant additional computational costs. As a criterion to select C candidates, Jung (2016) opts to select the top C best performing parameter values. This follows that efficient CV proposed in Jung (2016) cannot be implemented when the one-standard error rule is used for choosing the optimal value of λ . Therefore, this experiment will explore and analyze an extension that combines efficient CV with the one-standard error rule. Thus, instead of choosing the top C best performing parameter values as candidates, all parameter values which are considered by the one-standard error rule $\{\lambda_{min}, \ldots, \lambda_{1se}\}$ are selected as candidates. The combination of efficient CV with the one-standard error rule will be further denoted as the efficient one-standard error rule. The efficient one-standard error rule will calculate a weighted average of all candidates with different weights depending on the CV error estimates as proposed by Jung (2016). The estimates of the weights are designed in such a way that candidate values with lower CV errors are assigned a greater weight. Additionally, the weights are normalized and thus the weights of all candidate models add up to 1. The tuning parameter corresponding to the efficient one-standard error rule is obtained by:

$$\hat{\lambda}_{eff1se} = \sum_{c=1}^{C} w_c \lambda_c \quad \text{with} \quad w_c = \frac{\left(\frac{1}{\text{CV}(\lambda_c)}\right)}{\sum_{c=1}^{C} \left(\frac{1}{\text{CV}(\lambda_c)}\right)}$$
(4.1)

Accordingly, Figure 4.1 shows an example of which models are selected as candidate models by the efficient one-standard error rule. As can be seen in Figure 4.1, a total number of 4 candidate models $\log(\lambda) = -1.3$, $\log(\lambda) = -1.2$, $\log(\lambda) = -1.1$, $\log(\lambda) = -1.0$ were selected. After determining the weights w_c s, the efficient one-standard error tuning parameter is calculated at $\log(\lambda_{eff1se}) = -1.14$. It should be noted that the value of λ_{eff1se} is not a value on the grid used for the parameter search. Hence, the efficient one-standard error rule is capable of finding parameter values on a finer grid without any additional computational costs. Overall, Table 4.2 presents an overview of the model performances when using both the default and the efficient one-standard error rule.

Model	LASSO, one-standard error rule (λ_{1se})	LASSO, efficient one-standard error rule (λ_{eff1se})
M_{t+1}	1,865.93 (4.75)	1,855.95 (4.72)
M_{t+2}	1,883.23 (4.79)	1,855.71 (4.71)
M_{t+3}	1,895.85 (4.79)	1,891.97 (4.78)
M_{t+4}	1,808.90 (4.57)	1,788.00 (4.51)
M_{t+5}	1,803.76 (4.56)	1,786.83 (4.51)
M_{t+6}	1,785.07 (4.51)	1,818.96 (4.59)
M_{t+7}	2,067.32 (5.17)	2,063.71 (5.16)
M_{t+8}	2,092.85 (5.19)	2,082.58 (5.16)
M_{t+9}	2,242.76 (5.58)	$2,\!190.58$ (5.45)
M_{t+10}	2,138.72 (5.34)	2,150.93 (5.36)
M_{t+11}	2,377.19 (5.91)	2,335.49 (5.82)
M_{t+12}	2,349.52 (5.84)	2,297.18 (5.72)

Table 4.2: Mean absolute prediction error and the m	ean absolute percentage error (in
parentheses), when using the default and efficient	ent one-standard error rule.

As can be seen in Table 4.2, the proposed efficient one-standard error rule outperforms the default one-standard error rule for 10 out of 12 models, with an average improvement of 0.73%. Hence, it seems that in our case study sampling variation did affect the value of the tuning parameter. Accordingly, choosing multiple candidate values, instead of choosing one optimal value, and calculating a weighted average, to reduce the influence

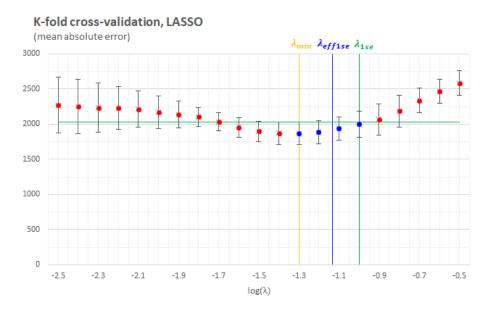


Figure 4.1: Tuning parameter values λ_{min} , λ_{1se} and λ_{eff1se} .

of sampling variation on the tuning parameter value, did seem to cause improvements in both tuning parameter selection and forecasting accuracy.

4.3 Forecast Combination

The case study has shown that LASSO did not outperform traditional time series forecasting methods, whereas the studies of Sagaert et al. (2018) and Verstraete et al. (2020) reported forecasting performance losses on the longer horizons compared to traditional methods. It should be noted that these studies solely compared the forecasting performance of individual models. Bates and Granger (1969) noted that combining sets of forecasts can lead to improvements if each set contains independent information. Moreover, Bates and Granger (1969) indicated that this independent information could be of two types: (1) forecasts are based on variables or information that other forecasts have not considered and (2) forecasts make different assumptions about the form of relationships between variables. For clarity, Figure 4.2 illustrates whenever forecast combinations are superior to individual forecasts.

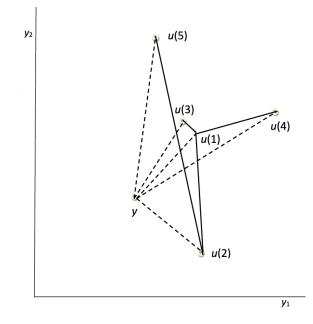


Figure 4.2: Forecast combinations considering five forecast vectors u(1), u(2), u(3), u(4) and u(5) and two steps ahead y_1 and y_2 . The solid lines represent the forecast combination in pairs of two, whereas the dotted lines to y represent the corresponding error of the forecast vectors.

As can be seen in Figure 4.2, forecasts u(1) and u(3) are highly correlated and there-

fore combining these forecasts will not improve the forecasting performance significantly. Moreover, forecast u(5) is a considerably poor forecast as the distance to y is large. However, combining forecasts u(5) with u(2) will improve the forecasting performance significantly as the distance between y and the solid line between u(5) with u(2) is reduced. Clearly, the performance improvement is due to the diversity of both models (Atiya, 2020). Accordingly, this experiment will explore and analyze whether using a combination of individual forecasts potentially enhances forecasting performance.

With regard to LASSO, SARIMA and AHW, each individual model creates forecasts based on independent information. For example, LASSO extracts information from leading indicators, whereas SARIMA extracts information from autocorrelations and AHW extracts information from level, trend and seasonal variations. Hence, in order to quantitatively assess their model diversities, correlation between the individual forecast errors are presented in Table 4.3.

 Table 4.3: Correlation of individual forecast errors.

	LASSO	AHW	SARIMA
LASSO	1	-	-
AHW	0.758	1	-
SARIMA	0.825	0.944	1

As can be seen in Table 4.3, the least correlation exists between the forecast errors of LASSO and AHW. Thus, combining the individual forecasts of LASSO and AHW will have the highest potential for enhanced forecasting performance. In order to obtain combined forecasts, weights must be allocated to the individual forecasts. Accordingly, Bates and Granger (1969) introduced numerous methods for determining the weights of each individual forecast as it is preferred to assign a greater weight to an individual forecast with a higher accuracy. However, Armstrong (2001) mentioned that applying weights is only beneficial if there is strong evidence that particular forecasting models are likely to predict better than others. Otherwise, the use of equal weights is likely to perform better under almost all other circumstances (Clemen, 1989). In our case study, there is no strong evidence that LASSO outperforms AHW or vice versa and thus the decision was made to allocate equal weights to the individual forecasts, i.e. the individual forecasts of LASSO and AHW are simply averaged. Overall, Table 4.4 shows the forecasting performance of the combined LASSO and AHW forecasts (LASSO-AHW) in comparison to all other individual models.

Horizon	LASSO	SARIMA	AHW	LASSO-AHW
t+1	1,855.95 (4.72)	1,876.55 (4.76)	1,732.56(4.43)	1,618.09 (4.13)
t+2	1,855.71 (4.71)	1,778.85(4.51)	1,636.93 (4.17)	1,581.07 (4.03)
t+3	1,891.97 (4.78)	1,955.72 (4.95)	1,669.06 (4.23)	$1,\!650.04\ (4.19)$
t+4	1,788.00 (4.51)	1,778.09(4.48)	1,722.89(4.34)	$1,\!648.04\ (4.17)$
t+5	1,786.83 (4.51)	1,820.68(4.57)	1,726.69(4.33)	1,642.26 (4.15)
t+6	1,818.96 (4.59)	1,872.53 (4.73)	1,887.68(4.74)	1,736.84 (4.39)
t+7	2,063.71 (5.16)	1,968.04 (4.93)	2,125.34(5.31)	$1,928.84 \ (4.84)$
t+8	2,082.58(5.16)	1,939.43 (4.83)	2,038.74(5.04)	1,939.34 (4.81)
t+9	2,190.58(5.45)	2,021.24 (4.99)	2,120.01(5.24)	2,047.29(5.08)
t + 10	2,150.93(5.36)	2,232.01(5.57)	2,342.59(5.84)	2,120.66 (5.28)
t + 11	2,335.49(5.82)	2,363.90(5.84)	2,405.85(5.91)	2,279.48(5.64)
t + 12	2,297.18 (5.72)	2,316.47 (5.78)	2.378.72 (5.88)	2,316.44 (5.75)

Table 4.4: Mean absolute prediction error and the mean absolute percentage error (in parentheses) across all models and forecast horizons.

As can be seen in Table 4.4, after combining the individual forecasts of LASSO and AHW, LASSO-AHW outperforms all other individual models for almost all forecast horizons. Thus, it seems that both the LASSO and AHW models are so diverse, that combining the predictions of these models results into enhanced forecasting performance, with an average improvement of 2.38%. Apparently, LASSO extracted valuable information from leading indicators, whereas Holt-Winters extracted valuable information from level, trend and seasonal variations, and ultimately, combining all of this information resulted in forecasting performance improvements. Hence, with regard to this case study, the inclusion of information extracted from leading indicators actually did lead to more accurate parts sales predictions.

4.4 Summary

The results have shown that the individual LASSO model does not outperform the traditional forecasting methods SARIMA, and Holt-Winters. Moreover, we proposed the efficient one-standard error rule, as an alternative to the default one-standard error rule, to reduce the influence of sampling variation on the value of tuning parameter λ . As a result, forecasting performance improved with an average of 0.73% over all horizons. Lastly, we found that combining LASSO with the additive Holt-Winters model, outperformed all other individual models, with an average improvement of 2.38% over all horizons.

Chapter 5

Leading Indicators

This chapter will provide insight into which indicators contain leading context information by analyzing which predictors were consistently included in the $\hat{\beta}^{lasso}$ estimators.

5.1 Inclusion Ratio

A major advantage of LASSO is that the $\hat{\beta}^{lasso}$ estimator is transparent and could provide business intelligence about relevant predictors. The $\hat{\beta}^{lasso}$ estimator is a vector of size p with zero and non-zero regression coefficients, representing uninformative and relevant predictors respectively. Accordingly, Table 5.1 presents an overview of how many relevant predictors and unique indicators were included in all LASSO models.

Model	Number	r of relevant predictors	Numbe	er of unique indicators
	Mean	Standard deviation	Mean	Standard deviation
M_{t+1}	16.04	9.75	9.81	3.07
M_{t+2}	11.53	3.42	7.57	1.95
M_{t+3}	9.28	3.01	6.44	1.30
M_{t+4}	13.55	5.69	10.57	1.50
M_{t+5}	13.66	3.70	10.71	1.48
M_{t+6}	12.96	4.55	9.85	1.65
M_{t+7}	19.67	2.59	12.02	1.00
M_{t+8}	21.94	9.89	13.11	2.39
M_{t+9}	22.02	6.73	13.69	1.74
M_{t+10}	21.29	7.60	12.65	2.26
M_{t+11}	23.33	9.89	12.13	2.85
M_{t+12}	37.10	12.22	15.54	2.71

Table 5.1: The number of included relevant predictors and unique indicators.

As can be seen in Table 5.1, when predicting (t + 1) ahead a total number of 16.04 predictors spread over 9.81 unique indicators were included on average, whereas, when predicting (t + 12) ahead, a total number of 37.10 predictors spread over 15.54 unique indicators were included on average. Hence, when predicting on further horizons, more predictors and more unique indicators are included in the $\hat{\beta}^{lasso}$ estimators. This insight is at a considerably high level and does not provide business intelligence about any lead effects. In order to explore and identify any lead effects, we will use the inclusion ratio to assess the inclusion consistency of each predictor. The inclusion ratio represents the percentage of how many times a predictor turned out to be relevant in all fitted $\hat{\beta}^{lasso}$ estimators. For example, if the inclusion ratio for predictor p equals 0.30, then in 30% of the fitted $\hat{\beta}^{lasso}$ estimators the regression coefficient of the corresponding predictor turned out non-zero, whereas in the other 70% the regression coefficient was shrunken to zero. Accordingly, the inclusion ratio is mathematically expressed as:

Inclusion ratio predictor
$$p = \frac{\sum \left[\hat{\beta}^{lasso} : \hat{\beta}_p \neq 0\right]}{\sum \left[\hat{\beta}^{lasso} : \hat{\beta}_p \neq 0\right] + \sum \left[\hat{\beta}^{lasso} : \hat{\beta}_p = 0\right]}$$
 (5.1)

A comprehensive overview of the calculated inclusion ratios is shown in Appendix C.

5.2 Lead Effects

It is likely that if a certain predictor actually contains relevant information, it should be consistently included in the $\hat{\beta}^{lasso}$ estimator as a non-zero coefficient. Therefore, we assume that a predictor could be considered as consistently relevant if the inclusion ratio has a value of at least 0.50 or greater. Accordingly, the inclusion ratios across all forecast horizons were analyzed to explore the existence of any consistent patterns that might indicate a lead effect. As a result, consistent patterns were found in the inclusion ratios of the following indicators:

- Y Observed parts sales (i.e. autoregressive information)
- X_1 DAF truck deliveries
- X_{11} Construction spending
- X_{20} Short-term diesel consumption growth
- X_{22} Automotive diesel deliveries
- X_{24} Short-term OEM truck orders
- X_{31} Carrier demand expectations

The upcoming sections will elaborate more on the exact patterns found in all these indicators.

5.2.1 Observed Parts Sales

With regard to the parts sales observed in the past, Table 5.2 presents the inclusion ratios for each lagged predictor. As can be seen, there is a consistent diagonal pattern running from $Y_{(t-2)}$ in M_{t+1} to $Y_{(t)}$ in M_{t+3} , indicating an autoregressive lead effect of 3 months. Next to that, there is a consistent diagonal pattern running from $Y_{(t-11)}$ in M_{t+1} to $Y_{(t)}$ in M_{t+12} , indicating an additional autoregressive lead effect of 12 months. The latter can be explained by the existence of a seasonal pattern within the sales data.

Table 5.2: Inclusion ratios of observed parts sales.

	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$Y_{(t)}$	0.05	0.00	1.00	0.00	0.00	0.07	0.00	0.20	0.00	0.00	0.24	1.00
$Y_{(t-1)}$	0.00	1.00	0.00	0.00	0.02	0.00	0.05	0.00	0.00	0.19	1.00	0.23
$Y_{(t-2)}$	1.00	0.00	0.00	0.11	0.00	0.01	0.00	0.01	0.23	1.00	0.00	0.00
$Y_{(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	1.00	0.00	0.00	0.00
$Y_{(t-4)}$	0.00	0.01	0.00	0.00	0.00	0.00	0.61	1.00	0.00	0.00	0.06	0.01
$Y_{(t-5)}$	0.00	0.00	0.00	0.02	0.00	0.45	1.00	0.00	0.00	0.00	0.00	0.48
$Y_{(t-6)}$	0.00	0.00	0.00	0.00	0.45	1.00	0.00	0.00	0.02	0.00	0.00	0.25
$Y_{(t-7)}$	0.07	0.02	0.00	0.14	1.00	0.00	0.00	0.02	0.00	0.01	0.18	0.01
$Y_{(t-8)}$	0.02	0.00	0.04	1.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.00
$Y_{(t-9)}$	0.00	0.11	1.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.00	0.29
$Y_{(t-10)}$	0.19	1.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.47	0.00
$Y_{(t-11)}$	1.00	0.00	0.00	0.05	0.00	0.00	0.19	0.00	0.00	0.45	0.01	0.68

5.2.2 DAF Truck Deliveries

With regard to the number of DAF truck deliveries, Table 5.3 presents the inclusion ratios for each lagged predictor. As can be seen, there is a consistent diagonal pattern running from $X_{1,(t-8)}$ in M_{t+4} to $X_{1,(t)}$ in M_{t+12} , indicating a lead effect of 12 months.

	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$X_{1,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.89	0.00	0.00	0.15	0.01	1.00
$X_{1,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.66	0.00	0.00	0.12	0.00	1.00	0.04
$X_{1,(t-2)}$	0.00	0.00	0.00	0.00	0.47	0.00	0.00	0.43	0.00	1.00	0.00	0.00
$X_{1,(t-3)}$	0.00	0.00	0.00	0.12	0.00	0.00	0.53	0.01	1.00	0.00	0.00	0.00
$X_{1,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.01	0.00
$X_{1,(t-5)}$	0.00	0.04	0.00	0.00	0.24	0.00	1.00	0.00	0.00	0.00	0.00	0.00
$X_{1,(t-6)}$	0.04	0.00	0.00	0.50	0.00	0.51	0.00	0.00	0.00	0.00	0.00	0.00
$X_{1,(t-7)}$	0.00	0.00	0.00	0.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{1,(t-8)}$	0.00	0.00	0.00	0.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08
$X_{1,(t-9)}$	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
$X_{1,(t-10)}$	0.00	0.06	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00
$X_{1,(t-11)}$	0.24	0.00	0.00	0.18	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.37

Table 5.3: Inclusion ratios of DAF truck deliveries.

As a result, it seems that the seasonal pattern existing in the DAF truck deliveries data contributes to the seasonal pattern in PPE's parts sales. Noticeably, this finding seems plausible as a part of PPE's parts sales are related to new truck sales. Moreover, a possible reason for the fact that in M_{t+1} to M_{t+3} no lead effect is identified, is that these models receive additional autoregressive information as shown in Section 5.2.1.

5.2.3 Construction Spending

With regard to the construction spending indicator, Table 5.4 presents the inclusion ratios for each lagged predictor. As can be seen, there is a consistent diagonal pattern running from $X_{11,(t-6)}$ in M_{t+1} to $X_{11,(t)}$ in M_{t+7} , indicating a lead effect of 7 months. Hence, it seems that the construction spending indicator contains relevant predictive information up to 7 months in advance.

Table 5.4: Inclusion ratios of construction spending.

	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$X_{11,(t)}$	0.01	0.00	0.00	0.00	0.00	0.00	1.00	0.24	0.20	0.84	0.40	0.72
$X_{11,(t-1)}$	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.55	0.23	0.16	0.95
$X_{11,(t-2)}$	0.00	0.00	0.00	0.00	0.99	0.00	0.00	0.34	0.21	0.14	0.72	0.00
$X_{11,(t-3)}$	0.05	0.00	0.00	0.88	0.00	0.00	0.00	0.00	0.17	0.23	0.00	0.77
$X_{11,(t-4)}$	0.05	0.00	0.59	0.00	0.00	0.00	0.00	0.09	0.27	0.00	0.03	1.00
$X_{11,(t-5)}$	0.00	0.62	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.02	0.98	0.65
$X_{11,(t-6)}$	0.63	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.89	0.01	0.00
$X_{11,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.98	0.01	0.00	0.23
$X_{11,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.00	0.00	0.00	0.55
$X_{11,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.79	0.00	0.00	0.00	0.33	0.91
$X_{11,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.11	0.48	0.00
$X_{11,(t-11)}$	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.09	0.20	0.22	0.00	0.51

5.2.4 Short-term Diesel Consumption Growth

With regard to the short-term diesel consumption growth indicator, Table 5.5 presents the inclusion ratios for each lagged predictor. As can be seen, there is a consistent diagonal pattern running from $X_{20,(t-7)}$ in M_{t+1} to $X_{20,(t)}$ in M_{t+8} , indicating a lead effect of 8 months. Hence, it seems that the short-term diesel consumption growth indicator contains relevant predictive information up to 8 months in advance.

Table 5.5: Inclusion ratios of short-term diesel consumption growth.

	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$X_{20,(t)}$	0.00	0.01	0.00	0.00	0.00	0.00	0.00	1.00	0.21	0.00	0.00	0.00
$X_{20,(t-1)}$	0.10	0.00	0.00	0.00	0.00	0.00	0.98	0.00	0.00	0.03	0.00	0.00
$X_{20,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.55	0.00	0.00	0.21	0.00	0.00	0.45
$X_{20,(t-3)}$	0.00	0.00	0.00	0.00	0.57	0.00	0.00	0.27	0.00	0.00	0.01	0.00
$X_{20,(t-4)}$	0.04	0.00	0.00	0.74	0.00	0.02	0.21	0.00	0.00	0.07	0.00	0.00
$X_{20,(t-5)}$	0.00	0.00	0.64	0.00	0.00	0.10	0.00	0.00	0.13	0.00	0.00	0.00
$X_{20,(t-6)}$	0.00	0.81	0.00	0.00	0.02	0.00	0.00	0.16	0.00	0.00	0.00	0.00

$X_{20,(t-7)}$	0.99	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{20,(t-8)}$	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49
$X_{20,(t-9)}$	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
$X_{20,(t-10)}$	0.20	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.23	0.00	0.00
$X_{20,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00	0.00	0.00

5.2.5 Automotive Diesel Deliveries

With regard to the automotive diesel deliveries indicator, Table 5.6 presents the inclusion ratios for each lagged predictor. As can be seen, there is a consistent diagonal pattern running from $X_{22,(t-11)}$ in M_{t+1} to $X_{22,(t)}$ in M_{t+12} , indicating a lead effect of 12 months. As a result, it seems that, just as the number of DAF truck deliveries, the seasonal pattern in the automotive diesel deliveries contributes to the seasonal pattern in PPE's parts sales. Again, this finding seems plausible since in addition to parts related to new truck sales, PPE's sales consists of parts related to wear and maintenance. Hence, the number of DAF truck deliveries provides predictive information related to new truck sales, whereas the amount of automotive diesel deliveries seems to be a good approximation of the kilometers driven/road activity and therefore provides predictive information related to wear and maintenance parts.

Table 5.6: Inclusion ratios of automotive diesel deliveries.

	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$X_{22,(t)}$	0.31	0.03	0.00	0.13	0.00	0.00	0.01	0.12	0.00	0.01	0.43	1.00
$X_{22,(t-1)}$	0.22	0.00	0.00	0.02	0.00	0.02	0.34	0.00	0.00	0.30	0.92	0.05
$X_{22,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.19	0.00	0.13	0.32	0.94	0.03	0.04
$X_{22,(t-3)}$	0.00	0.00	0.00	0.00	0.44	0.00	0.00	0.23	1.00	0.00	0.00	0.34
$X_{22,(t-4)}$	0.00	0.00	0.00	0.12	0.00	0.00	0.13	0.99	0.00	0.00	0.73	0.00
$X_{22,(t-5)}$	0.00	0.01	0.02	0.00	0.00	0.07	1.00	0.00	0.00	0.43	0.00	0.04
$X_{22,(t-6)}$	0.13	0.02	0.00	0.03	0.00	1.00	0.00	0.00	0.95	0.00	0.00	0.22
$X_{22,(t-7)}$	0.04	0.00	0.00	0.00	1.00	0.00	0.00	0.88	0.00	0.01	0.10	0.10
$X_{22,(t-8)}$	0.04	0.01	0.00	1.00	0.00	0.00	1.00	0.00	0.00	0.04	0.03	0.04
$X_{22,(t-9)}$	0.04	0.00	0.88	0.00	0.00	0.24	0.00	0.00	0.05	0.00	0.00	0.80
$X_{22,(t-10)}$	0.05	0.89	0.00	0.00	0.13	0.00	0.00	0.29	0.00	0.00	0.68	0.85
$X_{22,(t-11)}$	0.98	0.00	0.00	0.12	0.00	0.00	0.15	0.00	0.00	0.82	0.49	0.70

5.2.6 Short-term OEM Truck Orders

With regard to the number of OEM truck orders on the short-term, Table 5.7 presents the inclusion ratios for each lagged predictor. As can be seen, instead of a consistent diagonal pattern indicating a lead effect, there is a consistent horizontal pattern running from $X_{24,(t)}$ in M_{t+1} to $X_{24,(t)}$ in M_{t+11} . It seems like that the number of OEM truck orders on the short-term was used to assess the market position of road freight at time t. As a result, the number of short-term OEM truck orders at time t seems to be a good indication of the current market conditions.

	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$X_{24,(t)}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	0.00
$X_{24,(t-1)}$	0.04	0.53	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-2)}$	0.33	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-3)}$	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-4)}$	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
$X_{24,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30
$X_{24,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
$X_{24,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00

Table 5.7: Inclusion ratios of short-term OEM truck orders.

5.2.7 Carrier Demand Expectations

With regard to the carrier demand expectations, Table 5.8 presents the inclusion ratios for each lagged predictor. As can be seen, there is a consistent diagonal pattern running from $X_{31,(t-5)}$ in M_{t+1} to $X_{31,(t)}$ in M_{t+6} , indicating a lead effect of 6 months. Hence, it seems that the carrier demand expectations indicator contains relevant predictive information up to 6 months in advance.

Table 5.8: Inclusion ratios of carrier demand expectations.

	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$X_{31,(t)}$	0.00	0.00	0.72	0.00	0.00	1.00	0.34	1.00	0.01	0.98	1.00	0.63
$X_{31,(t-1)}$	0.00	0.76	0.00	0.00	1.00	0.00	0.99	0.00	0.58	0.88	0.37	0.00
$X_{31,(t-2)}$	0.66	0.00	0.00	1.00	0.00	0.58	0.00	0.24	0.69	0.05	0.00	0.00
$X_{31,(t-3)}$	0.00	0.00	1.00	0.00	0.55	0.00	0.00	0.41	0.28	0.00	0.00	0.00
$X_{31,(t-4)}$	0.00	1.00	0.00	0.26	0.00	0.02	0.16	0.00	0.00	0.00	0.00	0.00
$X_{31,(t-5)}$	1.00	0.00	0.20	0.00	0.00	0.19	0.00	0.00	0.00	0.00	0.00	0.12
$X_{31,(t-6)}$	0.00	0.28	0.00	0.01	0.16	0.00	0.00	0.00	0.00	0.00	0.01	0.00
$X_{31,(t-7)}$	0.24	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.39
$X_{31,(t-8)}$	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.92
$X_{31,(t-9)}$	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.61	0.00
$X_{31,(t-10)}$	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.86	0.02	0.73
$X_{31,(t-11)}$	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99	0.17	0.99	0.70

5.3 Summary

We used the inclusion ratio to assess the inclusion consistency of each predictor over all horizons. Accordingly, we assumed that a predictor is considered as consistently relevant if the inclusion ratio equals 0.50 or greater, after which, we analyzed the inclusion ratios to find any consistent patterns that might indicate a lead effect. Ultimately, 2 business, 1 economic, and 4 market indicators appeared to have clear lead effects for PPE's parts sales.

Chapter 6

Conclusion

This chapter will elaborate on the overall conclusion and recommendations of this field project.

6.1 Evaluation

In the business understanding phase of the CRISP-DM framework, the business objective of this field project was defined as:

Identify and gain insight into leading indicators for PACCAR Parts Europe's part sales. Next to that, explore and analyze whether including information extracted from leading indicators actually leads to more accurate parts sales predictions.

In line with the business objective, a total number of three research questions were defined, which are addressed and answered below.

• Research Question 1: How to identify leading indicators for PACCAR Parts Europe's parts sales?

Leading indicators are defined as variables that contain predictive information and ideally can predict a certain movement for a target variable in advance. Thus, in order to identify leading indicators for PPE's parts sales, it was necessary to specify a certain target variable. For this project, the target variable was specified as the total monthly truck parts sales reported by all DAF dealers, located in the EU27+2 area. With regard to the indicators, we collected a pool of 34 business, economic and market indicators which had the potential of being a leading indicator for PPE's parts sales. In order to model and identify any lead effects, each considered indicator was lagged in time multiple times, increasing the number of predictors significantly. For this project, we assumed a maximum lead effect of 12 months and thus the number of predictors increased to a total of 408. Given a large number of predictors together with the frequent occurrence of small sample sizes in sales forecasting, the identification of leading indicators in a monthly sales forecasting environment resulted in a high-dimensional (p > n) problem. Moreover, since each indicator is lagged in time multiple times, there exists correlation among the predictors and thus the problem of multicollinearity is present. Hence, the identification of leading indicators resulted in a high-dimensional problem with the presence of multicollinearity among the predictors. Therefore, LASSO was chosen as modeling technique as LASSO performs both variable selection and regularization that involves penalizing the absolute size of the regression coefficients. Due to these shrinkage properties, LASSO is capable of effectively dealing with multicollinearity among the predictors. Next to that, the use of LASSO contributed to the business objective as "the LASSO forecast is transparent, and provides insights into the selected leading indicators. Experts can benefit by gaining a better understanding of their market and can thus improve their understanding of market dynamics and interactions" (Sagaert et al., 2017, p. 127).

• Research Question 2: Are there any leading indicators for PACCAR Parts Europe's business, and if so, which indicators exactly are relevant for predicting PACCAR Parts Europe's parts sales?

We collected a pool of 34 business, economic and market indicators which had the potential of being a leading indicator for PPE's part sales. The business indicators covered PPE's business activities by means of indicators that were related to observed parts sales in the past and the number of DAF truck deliveries. The economic indicators covered Europe's overall economic climate by means of indicators that were mainly related to the industrial manufacturing and construction sectors, as these sectors are the major drivers of commercial vehicle transport. The market indicators covered the ongoing activities in Europe's road transport sector by means of indicators that were related to the number of placed truck orders, diesel consumption, tire consumption, carrier sentiment and road activity. Eventually, the transparency of LASSO provided business intelligence about leading indicators that are relevant for predicting PPE's part sales. It turned out that a total number of 2 business indicators, 1 economic indicator and 4 market indicators have clear lead effects for PPE's business. The identified leading indicators and their corresponding lead effects are shown in Table 6.1.

Ind	licator name	Indicator type	Lead effect
1.	Observed parts sales	Business	3 and 12 months
2.	DAF truck deliveries	Business	12 months
3.	Construction spending	Economic	$7 \mathrm{months}$
4.	Short-term diesel consumption growth	Market	8 months
5.	Automotive diesel deliveries	Market	12 months
6.	Short-term OEM truck orders	Market	1 up to 12 months
7.	Carrier demand expectations	Market	6 months

Table 6.1: Indicators with clear lead effects.

• **Research Question 3**: Does the inclusion of leading context information actually lead to more accurate predictions of PACCAR Parts Europe's parts sales?

In order to assess whether the inclusion of leading context information actually led to more accurate predictions, we benchmarked the performance of LASSO to SARIMA and Holt-Winters, which are two univariate time series forecasting methods, often used in businesses. Initially, it turned out that Holt-Winters predicted most accurate on the shorter horizons (1-5 months), whereas SARIMA mainly predicted most accurate on the longer horizons (7-12 months). Hence, despite the fact that LASSO used external information, it actually predicted less accurate than the traditional forecasting methods. As a result, two experiments were conducted in order to explore whether forecasting performance could be improved by applying efficient tuning parameter selection and forecast combination. With regard to the efficient tuning parameter selection experiment, we introduced the efficient one-standard error rule which, instead of choosing the parameter value corresponding to the most regularized model within one-standard error of the minimum CV error estimate, chooses all parameter values within one-standard error of the minimum and subsequently calculates a weighted average. The purpose of applying the efficient one-standard error rule over the default one-standard error rule, when using CV, is to reduce the influence of sampling variation on the tuning parameter value. Accordingly, it was found that applying the efficient one-standard error rule improved forecasting performance in 10 out of 12 models, with an average improvement of -0.73%. Next to that, we analyzed model diversities and explored whether applying forecast combination could lead to more accurate predictions. Accordingly, it was found that combining the predictions of LASSO and Holt-Winters yielded the most accurate predictions, outperforming the individual LASSO, Holt-Winters and SARIMA models for almost all horizons. Hence, for PPE, the inclusion of leading indicators led to more accurate parts sales predictions, with an average improvement of -2.38% over all horizons.

6.2 Recommendations

The results obtained during this field project led to a number of recommendations towards PACCAR Parts Europe. First of all, it is recommended to implement the additive Holt-Winters model with smoothing parameters $\alpha = 0.2$, $\beta = 0.1$, $\gamma = 0.2$. Next to that, it is recommended to implement the LASSO model with the indicators: observed parts sales, DAF truck deliveries, construction spending, short-term diesel consumption growth, automotive diesel deliveries, short-term OEM truck orders and carrier demand expectations. With regard to LASSO's tuning parameter λ , it is recommended to implement and use 10-fold CV with the efficient one-standard error rule, proposed in this report, as the tuning parameter selection method. In order to obtain the final prediction, the predictions of both the LASSO and Holt-Winters models should be averaged.

6.3 Limitations

This research has several limitations which are addressed and discussed. First of all, no distinction was made between new truck parts sales and wear/maintenance parts sales. In the end, we found leading indicators that are specifically related to new truck parts sales (DAF truck deliveries, short-term OEM truck orders) and wear/maintenance parts (short-term diesel consumption growth, automotive diesel deliveries). Hence, it might be the case that if new truck parts sales and wear/maintenance parts are distinguished, and predicted separately, forecasting performance could be enhanced. Secondly, with regard to the determination of LASSO's tuning parameter λ , we used K-fold CV and thus ignored the temporal dependencies between parts sales. As an alternative to K-fold CV, Bergmeir and Benítez (2012) recommended to use blocked CV together with stationary data, however, using blocked CV will lead to additional observation losses. It might be the case that when using blocked CV, forecasting performance could be improved, but due to the additional observation losses, it was considered as not applicable in our case study. Additionally, since we applied the direct forecast strategy and each model was retrained at every time step, applying repeated CV was considered out of scope, as this would result into exponentially increased computational costs. Nevertheless, applying repeated CV should reduce the influence of sampling variation on the tuning parameter value and perhaps could improve forecasting performance. Lastly, since we were dealing with limited sample sizes, complex machine learning techniques, such as the recurrent neural network, were considered out of scope as these techniques require larger data sets for training purposes.

6.4 Contribution and Future Research

This research has several contributions to the existing academic literature. First of all, we proposed the efficient one-standard error rule, as an alternative to the default one-standard error rule, by combining efficient CV, proposed in Jung (2016), with the commonly used one-standard error rule, described in Hastie et al. (2009). The purpose of the efficient one-standard error rule is to reduce the influence of sampling variation on the actual tuning parameter value. As stated earlier, we found that applying the efficient onestandard error rule over the default one, improved forecasting performance in 10 out of 12 models. With regard to future research purposes, there is a need to explore and analyze whether the efficient one-standard error rule improves performance, compared to the default one-standard error rule, when applied on multiple and larger data sets. Secondly, the studies of Sagaert et al. (2018) and Verstraete et al. (2020) reported forecasting performance losses of LASSO on the longer horizons, compared to traditional methods. These studies solely considered and compared the forecasting performance of individual models, whereas we found that, for our case study, applying forecast combination resulted into improved forecasting performance over almost all horizons.

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Appendix A

Truck Parts Sales

A.1 Partial Autocorrelation Function

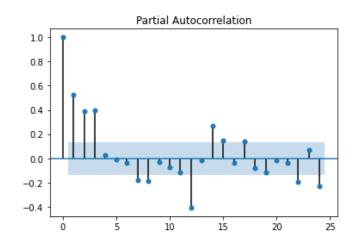


Figure A.1: Partial autocorrelation function of PPE truck parts sales.

A.2 Seasonal Subseries Plot

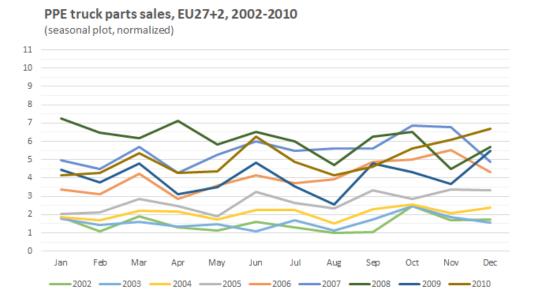


Figure A.2: Seasonal subseries plot of PPE truck parts sales (2002-2010).

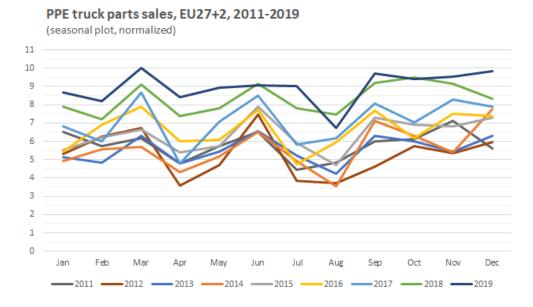


Figure A.3: Seasonal subseries plot of PPE truck parts sales (2011-2019).

Appendix B

Data Stationarity

B.1 Augmented Dickey-Fuller Test

Variable	ADF test statistic	Critical value at 95% confidence interval	\mathbf{Result}
Y	-1.291	-2.878	H_0 not rejected
X_1	-3.211	-2.878	H_0 rejected
X_2	-4.103	-2.878	H_0 rejected
X_3	-2.876	-2.878	H_0 rejected
X_4	-1.723	-2.878	H_0 not rejected
X_5	-2.752	-2.878	H_0 not rejected
X_6	-4.220	-2.878	H_0 rejected
X_7	-2.643	-2.878	H_0 not rejected
X_8	-3.292	-2.878	H_0 rejected
X_9	-1.950	-2.878	H_0 not rejected
X_{10}	-1.342	-2.878	H_0 not rejected
X_{11}	-1.832	-2.878	H_0 not rejected
X_{12}	-1.282	-2.878	H_0 not rejected
X_{13}	-5.270	-2.878	H_0 rejected
X_{14}	-3.263	-2.878	H_0 not rejected
X_{15}	-3.232	-2.878	H_0 not rejected
X_{16}	-3.206	-2.878	H_0 rejected
X_{17}	-2.476	-2.878	H_0 not rejected
X_{18}	-3.244	-2.878	H_0 rejected
X_{19}	-2.075	-2.878	H_0 not rejected
X_{20}	-2.289	-2.878	H_0 not rejected
X_{21}	-2.915	-2.878	H_0 rejected

Table B.1: Results of the Augmented Dickey-Fuller test.

X_{22}	-1.405	-2.878	H_0 not rejected
X_{23}	-3.519	-2.878	H_0 rejected
X_{24}	-3.671	-2.878	H_0 rejected
X_{25}	-4.480	-2.878	H_0 rejected
X_{26}	-3.304	-2.878	H_0 rejected
X_{27}	-2.426	-2.878	H_0 not rejected
X_{28}	-2.093	-2.878	H_0 not rejected
X_{29}	-2.338	-2.878	H_0 not rejected
X_{30}	-2.838	-2.878	H_0 not rejected
X_{31}	-3.113	-2.878	H_0 rejected
X_{32}	-2.655	-2.878	H_0 not rejected
X_{33}	-2.525	-2.878	H_0 not rejected

B.2 Kwiatkowski-Philips-Schmidt-Shin Test

Variable	KPSS test statistic	Critical value at 95%	\mathbf{Result}
		confidence interval	
Y	1.244	0.463	H_0 rejected
X_1	0.190	0.463	H_0 not rejected
X_2	0.180	0.463	H_0 not rejected
X_3	0.247	0.463	H_0 not rejected
X_4	0.273	0.463	H_0 not rejected
X_5	0.369	0.463	H_0 not rejected
X_6	0.160	0.463	H_0 not rejected
X_7	0.267	0.463	H_0 not rejected
X_8	0.101	0.463	H_0 not rejected
X_9	1.024	0.463	H_0 rejected
X_{10}	0.294	0.463	H_0 not rejected
X_{11}	0.578	0.463	H_0 rejected
X_{12}	0.425	0.463	H_0 not rejected
X_{13}	0.305	0.463	H_0 not rejected
X_{14}	0.251	0.463	H_0 not rejected
X_{15}	0.351	0.463	H_0 not rejected
X_{16}	0.151	0.463	H_0 not rejected
X_{17}	0.172	0.463	H_0 not rejected
X_{18}	0.246	0.463	H_0 not rejected
X_{19}	0.244	0.463	H_0 not rejected
X_{20}	0.207	0.463	H_0 not rejected
X_{21}	0.204	0.463	H_0 not rejected
X_{22}	1.074	0.463	H_0 rejected
X_{23}	0.061	0.463	H_0 not rejected

Table B.2: Results of the Kwiatkowski-Philips-Schmidt-Shin (KPSS) test.

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X_{24}	0.136	0.463	H_0 not rejected
X_{25}	0.306	0.463	H_0 not rejected
X_{26}	0.382	0.463	H_0 not rejected
X_{27}	0.349	0.463	H_0 not rejected
X_{28}	0.316	0.463	H_0 not rejected
X_{29}	0.396	0.463	H_0 not rejected
X_{30}	0.111	0.463	H_0 not rejected
X_{31}	0.106	0.463	H_0 not rejected
X_{32}	0.222	0.463	H_0 not rejected
X_{33}	0.154	0.463	H_0 not rejected

Appendix C

Inclusion Ratios

Table C.	1: Indicator	inclusion	ratios.
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	M_{t+1}	M_{t+2}	M_{t+3}	M_{t+4}	M_{t+5}	M_{t+6}	M_{t+7}	M_{t+8}	M_{t+9}	M_{t+10}	M_{t+11}	M_{t+12}
$Y_{(t)}$	0.05	0.00	1.00	0.00	0.00	0.07	0.00	0.20	0.00	0.00	0.24	1.00
$Y_{(t-1)}$	0.00	1.00	0.00	0.00	0.02	0.00	0.05	0.00	0.00	0.19	1.00	0.23
$Y_{(t-2)}$	1.00	0.00	0.00	0.11	0.00	0.01	0.00	0.01	0.23	1.00	0.00	0.00
$Y_{(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	1.00	0.00	0.00	0.00
$Y_{(t-4)}$	0.00	0.01	0.00	0.00	0.00	0.00	0.61	1.00	0.00	0.00	0.06	0.01
$Y_{(t-5)}$	0.00	0.00	0.00	0.02	0.00	0.45	1.00	0.00	0.00	0.00	0.00	0.48
$Y_{(t-6)}$	0.00	0.00	0.00	0.00	0.45	1.00	0.00	0.00	0.02	0.00	0.00	0.25
$Y_{(t-7)}$	0.07	0.02	0.00	0.14	1.00	0.00	0.00	0.02	0.00	0.01	0.18	0.01
$Y_{(t-8)}$	0.02	0.00	0.04	1.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.00
$Y_{(t-9)}$	0.00	0.11	1.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.00	0.29
$Y_{(t-10)}$	0.19	1.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.47	0.00
$Y_{(t-11)}$	1.00	0.00	0.00	0.05	0.00	0.00	0.19	0.00	0.00	0.45	0.01	0.68
$X_{1,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.89	0.00	0.00	0.15	0.01	1.00
$X_{1,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.66	0.00	0.00	0.12	0.00	1.00	0.04
$X_{1,(t-2)}$	0.00	0.00	0.00	0.00	0.47	0.00	0.00	0.43	0.00	1.00	0.00	0.00
$X_{1,(t-3)}$	0.00	0.00	0.00	0.12	0.00	0.00	0.53	0.01	1.00	0.00	0.00	0.00
$X_{1,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.01	0.00
$X_{1,(t-5)}$	0.00	0.04	0.00	0.00	0.24	0.00	1.00	0.00	0.00	0.00	0.00	0.00
$X_{1,(t-6)}$	0.04	0.00	0.00	0.50	0.00	0.51	0.00	0.00	0.00	0.00	0.00	0.00
$X_{1,(t-7)}$	0.00	0.00	0.00	0.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{1,(t-8)}$	0.00	0.00	0.00	0.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08
$X_{1,(t-9)}$	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
$X_{1,(t-10)}$	0.00	0.06	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00
$X_{1,(t-11)}$	0.24	0.00	0.00	0.18	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.37
$X_{2,(t)}$	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-2)}^{2,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-10)}^{2,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{2,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t)}^{2,(t-11)}$	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-1)}^{3,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

$X_{3,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{3,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
			0.00	0.00	0.00	0.00	0.00	0.00		0.00	0.00	
$X_{3,(t-10)}$	0.00	0.00							0.00			0.00
$X_{3,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{4,(t)}$	0.06	0.00	0.00	0.03	0.00	0.00	0.01	0.00	0.00	0.26	0.02	0.00
$X_{4,(t-1)}$	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.02	0.00	0.00
$X_{4,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.03	0.00	0.00	0.01
$X_{4,(t-3)}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.02
$X_{4,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
$X_{4,(t-5)}$	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{4,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.51
$X_{4,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
$X_{4,(t-8)}^{4,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.03	0.00	0.00
$X_{4,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.01
$X_{4,(t-9)}$		0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	
$X_{4,(t-10)}$	0.00											0.07
$X_{4,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
$X_{5,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00
$X_{5,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-6)}$	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-8)}$	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-9)}$	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{5,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.03	0.00	0.33
$X_{5,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	
$X_{6,(t)}$												0.00
$X_{6,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{6,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t)}$	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-5)}$												
$X_{7,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-10)}$	0.00	0.00	0.00	0.00	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{7,(t-11)}$	0.00	0.00	0.00	0.28	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
$X_{8,(t)}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{8,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
$X_{8,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.23
$X_{8,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.26	0.10
$X_{8,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.15	0.27	0.23	0.12
$X_{8,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.25	0.42	0.57	0.09
$X_{8,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.10	0.62	0.32	0.14	0.00
$X_{8,(t-6)}$ $X_{8,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.52	0.03	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.02	0.00	0.00	0.00	0.00
$X_{8,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.08	0.00	0.00	0.00	0.00
$X_{8,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00

$X_{8,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{8,(t-11)}$	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{9,(t)}$	0.02	0.00	0.05	0.00	0.12	0.00	0.00	0.00	0.00	0.01	0.00	0.00
$X_{9,(t-1)}$	0.00	0.09	0.00	0.10	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.48
$X_{9,(t-2)}$	0.19	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.16
$X_{9,(t-3)}^{5,(t-2)}$	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.25	0.82
$X_{9,(t-3)}$ $X_{9,(t-4)}$	0.26	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.03	0.17	0.98	1.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.11	0.98	0.14
$X_{9,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.95	0.23	0.20
$X_{9,(t-6)}$												
$X_{9,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.89	0.01	0.14	0.79
$X_{9,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.09	0.04	0.21
$X_{9,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.01	0.21	0.01	0.00	0.24
$X_{9,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.01	0.00	0.22	0.34
$X_{9,(t-11)}$	0.04	0.00	0.00	0.00	0.00	0.05	0.02	0.07	0.05	0.21	0.23	0.00
$X_{10,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.03	0.00	0.06
$X_{10,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00
$X_{10,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
$X_{10,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
$X_{10,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{10,(t-11)}$ $X_{11,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.24	0.20	0.84	0.40	0.72
	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.24	0.20	0.23	0.40	0.95
$X_{11,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33 0.21	0.23	0.10	0.95
$X_{11,(t-2)}$												
$X_{11,(t-3)}$	0.05	0.00	0.00	0.88	0.00	0.00	0.00	0.00	0.17	0.23	0.00	0.77
$X_{11,(t-4)}$	0.05	0.00	0.59	0.00	0.00	0.00	0.00	0.09	0.27	0.00	0.03	1.00
$X_{11,(t-5)}$	0.00	0.62	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.02	0.98	0.65
$X_{11,(t-6)}$	0.63	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.89	0.01	0.00
$X_{11,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.98	0.01	0.00	0.23
$X_{11,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.00	0.00	0.00	0.55
$X_{11,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.79	0.00	0.00	0.00	0.33	0.91
$X_{11,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.11	0.48	0.00
$X_{11,(t-11)}$	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.09	0.20	0.22	0.00	0.51
$X_{12,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{12,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{12,(t-2)}$	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{12,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
$X_{12,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{12,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{12,(t-6)}$	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16
$X_{12,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{12,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{12,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00
$X_{12,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.41
$X_{12,(t-11)}$	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.23	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{13,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{13,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{13,(t-2)}$												
$X_{13,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
$X_{13,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{13,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
$X_{13,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00
$X_{13,(t-7)}$	0.00	0.00	0.00	0.00	0.37	0.00	0.00	0.20	0.00	0.03	0.00	0.00
$X_{13,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00	0.15	0.00	0.00	0.00
$X_{13,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.19	0.00	0.00	0.00	0.00
$X_{13,(t-10)}$	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00
$X_{13,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{14,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32
$X_{14,(t-1)}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
$X_{14,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
$X_{14,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38
14,(1-3)												

$X_{14,(t-4)}$	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.02	0.00	0.00	0.05	0.00
$X_{14,(t-5)}$	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
$X_{14,(t-6)}$	0.00	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
$X_{14,(t-7)}$	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00
$X_{14,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{14,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.02
$X_{14,(t-9)}$ $X_{14,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{14,(t-10)}$ $X_{14,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.85
$X_{14,(t-11)}$ $X_{15,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-7)}$												
$X_{15,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{15,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{16,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{16,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
$X_{16,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.00	0.00
$X_{16,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{16,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
$X_{16,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{16,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{16,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
$X_{16,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00
$X_{16,(t-9)}$	0.00	0.00	0.00	0.00	$0.00 \\ 0.00$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{16,(t-10)}$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	0.00	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$
$X_{16,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-3)}$ $X_{17,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-4)}$ $X_{17,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-9)}^{17,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{17,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{18,(t)}$	0.05	0.15	0.00	0.00	0.00	0.00	0.00	0.71	0.00	0.00	0.00	0.68
$X_{18,(t-1)}^{13,(t)}$	0.11	0.00	0.00	0.00	0.00	0.01	0.93	0.00	0.00	0.02	0.40	0.01
$X_{18,(t-2)}$	0.00	0.00	0.00	0.00	0.03	0.31	0.00	0.00	0.00	0.55	0.00	0.36
$X_{18,(t-3)}$	0.00	0.00	0.00	0.01	0.49	0.00	0.00	0.03	0.29	0.00	0.09	0.65
$X_{18,(t-4)}$	0.00	0.00	0.00	0.51	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00
$X_{18,(t-5)}$	0.00	0.00	0.20	0.00	0.00	0.00	0.00	0.01	0.17	0.00	0.00	0.00
$X_{18,(t-6)}$	0.00	0.39	0.00	0.00	0.01	0.00	0.00	0.31	0.00	0.00	0.00	0.08
$X_{18,(t-7)}$	0.62	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00
$X_{18,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.15	0.00	0.58
$X_{18,(t-9)}$	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.12	0.15
$X_{18,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00	0.02	0.33	0.00
$X_{18,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.52	0.00	0.07	0.33	0.00	0.00
$X_{19,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00
$X_{19,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00
$X_{19,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

$X_{19,(t-10)}$	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{19,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{20,(t)}$	0.00	0.01	0.00	0.00	0.00	0.00	0.00	1.00	0.21	0.00	0.00	0.00
$X_{20,(t-1)}$	0.10	0.00	0.00	0.00	0.00	0.00	0.98	0.00	0.00	0.03	0.00	0.00
$X_{20,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.55	0.00	0.00	0.21	0.00	0.00	0.45
$X_{20,(t-3)}$	0.00	0.00	0.00	0.00	0.57	0.00	0.00	0.27	0.00	0.00	0.01	0.00
$X_{20,(t-4)}$	0.04	0.00	0.00	0.74	0.00	0.02	0.21	0.00	0.00	0.07	0.00	0.00
$X_{20,(t-4)}$ $X_{20,(t-5)}$	0.00	0.00	0.64	0.00	0.00	0.10	0.00	0.00	0.13	0.00	0.00	0.00
$X_{20,(t-5)}$	0.00	0.81	0.00	0.00	0.02	0.00	0.00	0.16	0.00	0.00	0.00	0.00
$X_{20,(t-6)}$	0.99	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{20,(t-7)}$	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49
$X_{20,(t-8)}$												
$X_{20,(t-9)}$	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
$X_{20,(t-10)}$	0.20	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.23	0.00	0.00
$X_{20,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00	0.00	0.00
$X_{21,(t)}$	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.00	0.55	0.99	1.00	0.27
$X_{21,(t-1)}$	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.80	0.93	0.03	0.00	0.58
$X_{21,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.22	0.00	0.00	0.28	0.00
$X_{21,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.04	0.02	0.00	0.00	0.00	0.00	0.00
$X_{21,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{21,(t-5)}$	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{21,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{21,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{21,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{21,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
$X_{21,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{21,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{22,(t)}$	0.31	0.03	0.00	0.13	0.00	0.00	0.01	0.12	0.00	0.01	0.43	1.00
$X_{22,(t-1)}$	0.22	0.00	0.00	0.02	0.00	0.02	0.34	0.00	0.00	0.30	0.92	0.05
$X_{22,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.19	0.00	0.13	0.32	0.94	0.03	0.04
$X_{22,(t-3)}$	0.00	0.00	0.00	0.00	0.44	0.00	0.00	0.23	1.00	0.00	0.00	0.34
$X_{22,(t-3)}$ $X_{22,(t-4)}$	0.00	0.00	0.00	0.12	0.00	0.00	0.13	0.99	0.00	0.00	0.73	0.00
$X_{22,(t-4)}$ $X_{22,(t-5)}$	0.00	0.01	0.02	0.00	0.00	0.07	1.00	0.00	0.00	0.43	0.00	0.04
$X_{22,(t-5)}$ $X_{22,(t-6)}$	0.13	0.02	0.00	0.03	0.00	1.00	0.00	0.00	0.95	0.00	0.00	0.22
	0.04	0.00	0.00	0.00	1.00	0.00	0.00	0.88	0.00	0.00	0.10	0.10
$X_{22,(t-7)}$	0.04	0.01	0.00	1.00	0.00	0.00	1.00	0.00	0.00	0.01	0.03	0.10
$X_{22,(t-8)}$			0.88	0.00		0.00	0.00	0.00		0.04		0.04
$X_{22,(t-9)}$	$0.04 \\ 0.05$	$0.00 \\ 0.89$	0.88	0.00	$0.00 \\ 0.13$	0.24	0.00	0.00	$0.05 \\ 0.00$	0.00	0.00 0.68	0.80
$X_{22,(t-10)}$												
$X_{22,(t-11)}$	0.98	0.00	0.00	0.12	0.00	0.00	0.15	0.00	0.00	0.82	0.49	0.70
$X_{23,(t)}$	0.23	0.03	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{23,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{23,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{23,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{23,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{23,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{23,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32
$X_{23,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.72	0.22
$X_{23,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.09	0.70	0.15	0.00
$X_{23,(t-9)}$	0.01	0.00	0.00	0.00	0.00	0.00	0.05	0.12	0.82	0.02	0.00	0.00
$X_{23,(t-10)}$	0.01	0.00	0.00	0.00	0.00	0.49	0.43	0.27	0.07	0.00	0.00	0.00
$X_{23,(t-11)}$	0.00	0.03	0.04	0.95	1.00	0.84	0.87	0.00	0.00	0.00	0.00	0.00
$X_{24,(t)}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	0.00
$X_{24,(t-1)}$	0.04	0.53	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-2)}$	0.33	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-3)}$	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-4)}$	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
$X_{24,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30
$X_{24,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
$X_{24,(t-8)}$ $X_{24,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-9)}$ $X_{24,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{24,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
$X_{25,(t)}$		0.00	0.00	0.22			0.00					
$X_{25,(t-1)}$	0.00				0.00	0.00		0.00	0.00	0.00	0.00	0.00
$X_{25,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

$X_{25,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{25,(t-11)}$	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-1)}$ $X_{26,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-1)}$ $X_{26,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-2)}$ $X_{26,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-7)}$												
$X_{26,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{26,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18
$X_{26,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.32	0.13	0.41
$X_{27,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00
$X_{27,(t-1)}$	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00
$X_{27,(t-2)}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00
$X_{27,(t-3)}$	0.00	0.05	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{27,(t-4)}$	0.11	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{27,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{27,(t-6)}$	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{27,(t-7)}$	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{27,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12
$X_{27,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
$X_{27,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{27,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00
$X_{28,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00
$X_{28,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.58
$X_{28,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
$X_{28,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
$X_{28,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{28,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{28,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{28,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{28,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{28,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{28,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{28,(t-11)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-2)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-5)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-6)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-10)}$				0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00								
$X_{29(t-11)}$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	$0.00 \\ 0.00$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$X_{29,(t-11)}$ $X_{30,(t)}$	0.00					$0.00 \\ 1.00$	$0.00 \\ 0.76$	$0.00 \\ 0.40$	$0.00 \\ 1.00$	$0.00 \\ 0.05$	0.00 0.00	$0.00 \\ 1.00$
$X_{30,(t)}$	$0.00 \\ 0.00$	0.00	0.00	0.00	0.00							
$X_{30,(t)} X_{30,(t-1)}$	$0.00 \\ 0.00 \\ 0.12 \\ 0.00$	$0.00 \\ 0.02 \\ 1.00$	$0.00 \\ 1.00 \\ 0.00$	$0.00 \\ 0.00 \\ 0.00$	$0.00 \\ 0.00 \\ 1.00$	$1.00 \\ 0.00$	$\begin{array}{c} 0.76 \\ 0.00 \end{array}$	$\begin{array}{c} 0.40 \\ 0.96 \end{array}$	$\begin{array}{c} 1.00 \\ 0.00 \end{array}$	$0.05 \\ 0.00$	$\begin{array}{c} 0.00\\ 0.94 \end{array}$	$1.00 \\ 0.00$
$\begin{array}{c} X_{30,(t)} \\ X_{30,(t-1)} \\ X_{30,(t-2)} \end{array}$	0.00 0.00 0.12 0.00 0.93	$0.00 \\ 0.02 \\ 1.00 \\ 0.00$	$0.00 \\ 1.00 \\ 0.00 \\ 0.00$	$0.00 \\ 0.00 \\ 0.00 \\ 1.00$	$0.00 \\ 0.00 \\ 1.00 \\ 0.00$	$1.00 \\ 0.00 \\ 0.00$	$0.76 \\ 0.00 \\ 0.98$	$0.40 \\ 0.96 \\ 0.00$	$1.00 \\ 0.00 \\ 0.00$	$0.05 \\ 0.00 \\ 0.84$	$0.00 \\ 0.94 \\ 0.00$	$1.00 \\ 0.00 \\ 0.64$
$\begin{array}{c} X_{30,(t)} \\ X_{30,(t-1)} \\ X_{30,(t-2)} \\ X_{30,(t-3)} \end{array}$	0.00 0.00 0.12 0.00 0.93 0.00	0.00 0.02 1.00 0.00 0.00	0.00 1.00 0.00 0.00 0.13	0.00 0.00 1.00 0.00	0.00 0.00 1.00 0.00 0.00	$1.00 \\ 0.00 \\ 0.00 \\ 0.11$	$0.76 \\ 0.00 \\ 0.98 \\ 0.00$	$0.40 \\ 0.96 \\ 0.00 \\ 0.02$	$1.00 \\ 0.00 \\ 0.00 \\ 0.73$	$0.05 \\ 0.00 \\ 0.84 \\ 0.00$	$0.00 \\ 0.94 \\ 0.00 \\ 0.25$	$1.00 \\ 0.00 \\ 0.64 \\ 0.00$
$\begin{array}{c} X_{30,(t)} \\ X_{30,(t-1)} \\ X_{30,(t-2)} \\ X_{30,(t-3)} \\ X_{30,(t-4)} \end{array}$	0.00 0.00 0.12 0.00 0.93 0.00 0.00	0.00 0.02 1.00 0.00 0.00 0.19	0.00 1.00 0.00 0.00 0.13 0.00	0.00 0.00 1.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00	$1.00 \\ 0.00 \\ 0.00 \\ 0.11 \\ 0.00$	0.76 0.00 0.98 0.00 0.00	0.40 0.96 0.00 0.02 0.61	1.00 0.00 0.73 0.00	$0.05 \\ 0.00 \\ 0.84 \\ 0.00 \\ 0.05$	$0.00 \\ 0.94 \\ 0.00 \\ 0.25 \\ 0.00$	$1.00 \\ 0.00 \\ 0.64 \\ 0.00 \\ 0.00$
$\begin{array}{c} X_{30,(t)} \\ X_{30,(t-1)} \\ X_{30,(t-2)} \\ X_{30,(t-3)} \\ X_{30,(t-4)} \\ X_{30,(t-5)} \end{array}$	0.00 0.00 0.12 0.00 0.93 0.00 0.00 0.32	0.00 0.02 1.00 0.00 0.00 0.19 0.00	0.00 1.00 0.00 0.13 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00	1.00 0.00 0.11 0.00 0.00	0.76 0.00 0.98 0.00 0.00 0.14	0.40 0.96 0.00 0.02 0.61 0.00	1.00 0.00 0.73 0.00 0.02	0.05 0.00 0.84 0.00 0.05 0.00	0.00 0.94 0.00 0.25 0.00 0.00	1.00 0.00 0.64 0.00 0.00 0.00
$\begin{array}{c} X_{30,(t)} \\ X_{30,(t-1)} \\ X_{30,(t-2)} \\ X_{30,(t-3)} \\ X_{30,(t-4)} \\ X_{30,(t-5)} \\ X_{30,(t-6)} \end{array}$	0.00 0.00 0.12 0.00 0.93 0.00 0.00 0.32 0.03	0.00 0.02 1.00 0.00 0.19 0.00 0.00	0.00 1.00 0.00 0.13 0.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00	1.00 0.00 0.11 0.00 0.00 0.00 0.07	0.76 0.00 0.98 0.00 0.00 0.14 0.00	0.40 0.96 0.00 0.02 0.61 0.00 0.10	$ 1.00 \\ 0.00 \\ 0.73 \\ 0.00 \\ 0.02 \\ 0.00 $	0.05 0.00 0.84 0.00 0.05 0.00 0.00	0.00 0.94 0.00 0.25 0.00 0.00 0.00	$1.00 \\ 0.00 \\ 0.64 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00$
$\begin{array}{c} X_{30,(t)} \\ X_{30,(t-1)} \\ X_{30,(t-2)} \\ X_{30,(t-3)} \\ X_{30,(t-4)} \\ X_{30,(t-5)} \\ X_{30,(t-6)} \\ X_{30,(t-7)} \end{array}$	0.00 0.00 0.12 0.00 0.93 0.00 0.32 0.03 0.00	0.00 0.02 1.00 0.00 0.19 0.00 0.00 0.00	0.00 1.00 0.00 0.13 0.00 0.00 0.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00	1.00 0.00 0.11 0.00 0.00 0.00 0.07 0.00	0.76 0.00 0.98 0.00 0.00 0.14 0.00 0.00	0.40 0.96 0.00 0.02 0.61 0.00 0.10 0.00	$ \begin{array}{c} 1.00\\ 0.00\\ 0.73\\ 0.00\\ 0.02\\ 0.00\\ 0.00\\ 0.00\\ \end{array} $	0.05 0.00 0.84 0.00 0.05 0.00 0.00 0.00	0.00 0.94 0.00 0.25 0.00 0.00 0.00 0.00	1.00 0.00 0.64 0.00 0.00 0.00 0.00 0.00
$\begin{array}{c} X_{30,(t)} \\ X_{30,(t-1)} \\ X_{30,(t-2)} \\ X_{30,(t-3)} \\ X_{30,(t-4)} \\ X_{30,(t-5)} \\ X_{30,(t-6)} \end{array}$	0.00 0.00 0.12 0.00 0.93 0.00 0.00 0.32 0.03	0.00 0.02 1.00 0.00 0.19 0.00 0.00	0.00 1.00 0.00 0.13 0.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00	0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00	1.00 0.00 0.11 0.00 0.00 0.00 0.07	0.76 0.00 0.98 0.00 0.00 0.14 0.00	0.40 0.96 0.00 0.02 0.61 0.00 0.10	$ 1.00 \\ 0.00 \\ 0.73 \\ 0.00 \\ 0.02 \\ 0.00 $	0.05 0.00 0.84 0.00 0.05 0.00 0.00	0.00 0.94 0.00 0.25 0.00 0.00 0.00	$1.00 \\ 0.00 \\ 0.64 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00$

APPENDIX C. INCLUSION RATIOS

	$X_{30,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$X_{30,(t-11)}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	$X_{31,(t)}$	0.00	0.00	0.72	0.00	0.00	1.00	0.34	1.00	0.01	0.98	1.00	0.63
	$X_{31,(t-1)}$	0.00	0.76	0.00	0.00	1.00	0.00	0.99	0.00	0.58	0.88	0.37	0.00
	$X_{31,(t-2)}$	0.66	0.00	0.00	1.00	0.00	0.58	0.00	0.24	0.69	0.05	0.00	0.00
	$X_{31,(t-3)}$	0.00	0.00	1.00	0.00	0.55	0.00	0.00	0.41	0.28	0.00	0.00	0.00
	$X_{31,(t-4)}$	0.00	1.00	0.00	0.26	0.00	0.02	0.16	0.00	0.00	0.00	0.00	0.00
	$X_{31,(t-5)}$	1.00	0.00	0.20	0.00	0.00	0.19	0.00	0.00	0.00	0.00	0.00	0.12
	$X_{31,(t-6)}$	0.00	0.28	0.00	0.01	0.16	0.00	0.00	0.00	0.00	0.00	0.01	0.00
	$X_{31,(t-7)}$	0.24	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.39
	$X_{31,(t-8)}$	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.92
	$X_{31,(t-9)}$	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.61	0.00
	$X_{31,(t-10)}$	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.86	0.02	0.73
	$X_{31,(t-11)}$	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99	0.17	0.99	0.70
	$X_{32,(t)}$	0.98	0.39	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.08	0.28
	$X_{32,(t-1)}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
	$X_{32,(t-2)}$	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$X_{32,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$X_{32,(t-4)}$	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27
	$X_{32,(t-5)}$	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.18	0.00
	$X_{32,(t-6)}$	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00
	$X_{32,(t-7)}$	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.10	0.00	0.00	0.04
	$X_{32,(t-8)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00
	$X_{32,(t-9)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$X_{32,(t-10)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$X_{32,(t-11)}$	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.00	0.00	1.00
	$X_{33,(t)}$	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.52	0.01	0.00
	$X_{33,(t-1)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.93	0.00	0.00	0.53
	$X_{33,(t-2)}$	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.82	0.06	0.00	0.03	0.00
	$X_{33,(t-3)}$	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.68	0.00	0.50	0.00	0.04
	$X_{33,(t-4)}$	0.00	0.00	0.00	0.00	0.00	0.69	0.00	0.00	0.21	0.00	0.00	0.00
	$X_{33,(t-5)}$	0.01	0.00	0.00	0.00	0.71	0.00	0.00	0.10	0.00	0.00	0.00	0.00
	$X_{33,(t-6)}$	0.00	0.00	0.00	0.69	0.00	0.00	0.21	0.00	0.00	0.00	0.00	0.00
	$X_{33,(t-7)}$	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07
	$X_{33,(t-8)}$	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$X_{33,(t-9)}$	0.05	0.00	0.00	0.24	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.00
	$X_{33,(t-10)}$	0.04	0.00	0.00	0.00	0.26	0.00	0.00	0.00	0.40	0.00	0.00	0.00
	$X_{33,(t-11)}$	0.04	0.00	0.00	0.43	0.00	0.00	0.14	0.29	0.00	0.00	0.00	0.00
-													