Eindhoven University of Technology

MASTER

Inventory management of perishable goods with an order constraint

Vermeulen, S.

Award date:
2020

Department of Industrial Engineering and Innovation Sciences
Operations Planning Accounting & Control Group

Inventory management of perishable goods with an order constraint.

By

S. Vermeulen

BSc Industrial Engineering – TU/e

Student identity number 0892890

In partial fulfilment of the requirements for the degree of

Master Of Science

in Operations Management and Logistics

Supervisors:

Ir. Dr. S.D.P Flapper, TU/e, OPAC

Dr. Z. Atan, TU/e, OPAC

H. van Herpen. Bonduelle

Eindhoven, January 2020

Note: All numbers used in this report are fictitious and serve for illustrative purposes only

Abstract

This thesis covers a research into a inventory policy that is able to deal with perishable inventory and has a minimum order constraint of always ordering Full Truckloads for the replenishment of the warehouse. The current supply chain operations are analysed in order to create a policy that could outperform the current policy. The current supply chain operations are classified as an (R,s,nQ) policy. The proposed policy differs from this policy only slightly since it outcounts for outdating. An algorithm is developed that makes sure the replenishment orders are actually Full Truckloads. A case study is performed where by use of a simulation the results of the policy developed in this thesis with the inventory policy that is currently used.

Keywords: Periodic (R,s,nQ) inventory policy, Perishable inventory, Minimal order constraint, Knapsack problem, Bin packing problem

Management Summary

In this thesis an inventory control policy is presented that is able to determine the quantities that should be ordered at a review moment. The inventory policy is a periodic inventory policy and is able to manage perishable inventory with an order constraint on the quantities that are ordered.

The current inventory control policy is not designed for dealing with the perishability of the inventory. Application of the current inventory policy often results in overstock which in turn leads to outdating cost for the products with a short remaining lifetime of the products. This results in the following research objective:

What kind of inventory control policy is able to handle perishability of inventory, joint replenishment and always orders full truckloads for replenishment?

In order to find an answer to the research objective the thesis is divided in to three parts.

Part 1 – Current setup

In this first part the current supply chain operations are examined. It starts with identifying the current inventory policy as an (R,s,nQ) policy. To be able to deal with the order constraint a factor is introduced. The sum of the demand over period R+L is than multiplied with this demand factor until one or multiple full truckloads can be ordered for the replenishment of the warehouse.

Part 2 – new inventory control policy

In the second part a new inventory control policy is developed. A literature review to identify the existing policies in the literature and solutions for the problems is performed. The heuristic of Broekmeulen & van Donselaar (2009) was deemed the best suited to model a perishable inventory in an (R,s,nQ) policy and the heuristic of Garey et al. (1976) was used to solve the bin packing problem when multiple trucks need to be ordered. The heuristic policy of Broekmeulen & van Donselaar (2009) is extended to include backorders. In order to be able to satisfy the order constraint an algorithm is developed. In the algorithm the heuristic of Garey et al. (1976) is used to solve the bin packing problem when more than 1 truck needs to be ordered to replenish the warehouse.

A tool is created that includes the complete inventory policy and the algorithm to satisfy the order constraint.

Part 3 – Performance of the thesis policy

In the third part the performance of the policy that is developed in the thesis is compared with the performance of the policy that is currently used. The performance of the policies is compared by means of a simulation of both policies. A couple of sensitivity analysis are executed to show the performance of the two policies for different scenarios. The two policies are compared on the following three scenarios:

- Changing the Safety stock levels

  In this scenario the safety stock levels are changed to see the performance of the policies when the reorder point is either higher or lower. From this scenario it can be concluded that the thesis policy is able to achieve a reduction of the total costs. However, the Weighted Service Level is decreased meaning more customers are not delivered with products in the time period as when the demand is issued.
- Changing the review period

  In this scenario analysis the performance of the current and the thesis policy is compared with values for the review period from a 1 day review period till a10 days review period (2 working weeks). From this sensitivity analysis it can been seen that the thesis policy is able to achieve a cost reduction compared to the current policy while having a lower Weighted Service Level when the review period is shorter or equal to 6. When the review period is actually bigger than 6 days the current policy is able to achieve a lower total costs than the thesis policy while also having a higher Weighted Service Level. Meaning that the current policy actually outperforms the policy that is presented in this thesis when the review period is bigger than 6 days.
- Sensitivity analysis of the outdating costs

  The biggest costs that are made in the policies are the outdating costs. In order to see the dependence of the total costs on the outdating costs a scenario analysis is done where the outdating costs are decreased and increased. The analysis of the outdating costs is done based on a periodic review moment of 5 working days (1 week). The thesis policy is able to achieve a reduction on the total costs while having a lower Weighted Service Level.

Recommendations

The main recommendations for Bonduelle is to evaluate the target weighted service level. The current target is rather high. This high target weighted service level means a lot of costs are made for the outdating. Second is to evaluate the products offered to the customers. It could be that the benefit of keeping the product on stock is outweighed by the total costs that have to be made in order to be able to keep the product on stock. If the current weighted service level is maintained with the same product composition, the periodic review period could be changed in order to satisfy the weighted service level while minimizing costs. If the periodic review period is not changed the safety stock level could be adjusted to satisfy the target weighted service level.

Preface

This report is the result of a research that has been conducted at Bonduelle Northern Europe in order to finalize the master Operations Management & Logistics at Eindhoven University of Technology (TU/e). This master thesis has been supervised by dr. ir. S.D.P. Flapper and dr. Z. Atan from Eindhoven University of Technology and H. van Herpen from Bonduelle Northern Europe.

I would like to thank a couple of people that helped me during the master thesis project. First of all, I would like to thank my first supervisor from TU/e, dr. ir. S.D.P. Flapper. All the effort in reviewing my work and discussing all the work that has been done really helped me during the master thesis and kept me motivated throughout the whole master thesis. The effort is really highly appreciated. Secondly, I would like to thank my second supervisor from TU/e, dr. Z. Atan. The critical comments and questions helped me to further improve my thesis.

At Bonduelle I would like to thank H. van Herpen for the opportunity to let me execute the project within her department. Furthermore, I would thank her for the discussions on the project to make sure a relevant result could be achieved. I would also like to thank all the people at Bonduelle for helping me during the project. Special thanks to W. Veneberg for the discussions that helped me with certain aspects during the project.

Finally, I would like to thank my family and friends who supported me during my studies at the TU/e, especially during the thesis project.

Bas Vermeulen

Contents

List of symbols used

Notation	Description
a	Variable that is used to determine what probability distribution is appropriate
$B_{i, t, r}$	Batch size of SKU i, at time t with r days remaining lifetime
$BO_{i, t}$	Number of boxes backordered for SKU i, at time t
β_i	Service level of SKU i
$C_{i, BO}$	Costs of backorders per box of SKU i
$C_{i, IO}$	Inventory holding costs per box of SKU i per day
C_{NO}	Costs of ordering 1 FTL
$C_{i, Z}$	Outdating costs per box of SKU i
$D_{i, t}$	The number of boxes demand for SKU i at time t
I	Total number of SKUs that are kept on stock
$IO_{i,t}$	Inventory on hand of SKU i at time t in number of boxes
$IP_{i,t}$	Inventory position of SKU i at time t in number of boxes
$IT_{i,t}$	Inventory in transit of SKU i, at time t in number of boxes
K_t	Number of trucks ordered at time t
L	Lead time in days
m_i	The fixed remaining lifetime in days of a batch of SKU i
$n_{i,t}^{k}$	The number of pallets ordered according to extended EWA policy of SKU i allocated to truck k at time t
$n_i^{k'}$	The number of pallets added of SKU i added to truck k
NO_t	Number of FTL that need to be ordered at time t
$O_{i,t}$	Removal of boxes of SKU i with 1 day remaining lifetime
$\hat{o}_{i, t}$	Estimated outdating of SKU i over time t
Q_i	Case Pack Size (number of boxes on a pallet) of SKU i

R	Review period in days
SS_i	Number of boxes Safety stock of SKU i
$s_{i,t}$	Reorder Level of SKU i at time t
T_i	The periods of mean demand that can be satisfied from one pallet of SKU i
τ_i	Periods of mean demand that the current on-hand inventory, inventory in transit and the products that need to be ordered would satisfy of SKU i
w_i	The weight of a pallet stocked with SKU i in kilos
$W_{i, t, r}$	Withdrawal of number of boxes for SKU i over time t with r days remaining lifetime
$Z_{i, t}$	Number of boxes outdated of SKU i over time t
$\mu_{i, t}$	Mean demand in boxes at period t for SKU i
$\sigma_{i, t}$	Standard deviation in boxes at period t for SKU i

List of definitions

AMBIENT:	Product that can be stored with a long shelf life ranging from 2 till 4 years
EWA POLICY:	The inventory control policy described by Broekmeulen and van Donselaar (2009) that deals with the outdating of products.
FROZEN:	Products that are kept at a temperature of at least – 18 degrees until used.
NORDICS:	Countries in Scandinavia: Denmark, Finland, Norway, and Sweden; the countries that are taken into consideration for this thesis
PLANT-BASED PRODUCTS:	Not all the products of Bonduelle are classified as vegetables. Bonduelle introduced new products that are seeds.
SERVICE LEVEL:	The number of products delivered divided by the number of products demanded

List of Abbreviations

AWSL: The average weighted service level. The average weighted service level of the simulation runs.

BELL: Bonduelle Europe Long Life; The business unit of Bonduelle where this project is performed.

DDP: Delivered Duty Paid; Selling company arranges transport of goods to an agreed-upon customer destination

FCA: Free Carrier; Customer is responsible for picking up products from the factory

FEFO: First Expired First Out

FFD: First Fit Decreasing algorithm

FIFO: First In First Out

FTL: Full Truck Load

KPI: Key Performance Index

LIFO: Last In First Out

SKU: Stock Keeping Unit

TU/e: Eindhoven University of Technology

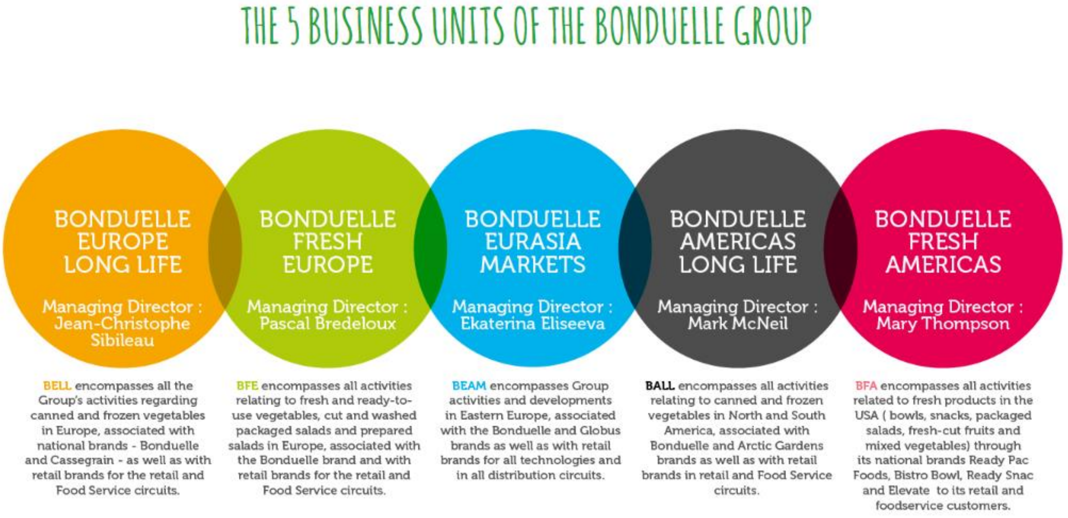
WSL: The weighted service level. The sum of the service level of each SKU divided by total demand.

List of Figures

List of Tables

1. Introduction

This master thesis study has been conducted at Bonduelle. Bonduelle is a French family-run company and is the current world leader in ready-to-use plant-based products, in all their forms. The products are divided into 4 different product classes: Ambient, Frozen, Refrigerated fresh and Prepared. Currently, the Bonduelle group provides over 100 countries with products that are preserved using natural processes. To offer the best service to its customers Bonduelle is split up into 5 major Business Units. The 5 major Business units are shown in Figure 1.



BELL encompasses all the Group's activities regarding canned and frozen vegetables in Europe, associated with national brands - Bonduelle and Cassegrain - as well as with retail brands for the retail and Food Service circuits.

BFE encompasses all activities relating to fresh and ready-to-use vegetables, cut and washed packaged salads and prepared salads in Europe, associated with the Bonduelle brand and with retail brands for the retail and Food Service circuits.

BEAM encompasses Group activities and developments in Eastern Europe, associated with the Bonduelle and Globus brands as well as with retail brands for all technologies and in all distribution circuits.

BALL encompasses all activities relating to canned and frozen vegetables in North and South America, associated with Bonduelle and Arctic Gardens brands as well as with retail brands in retail and Food Service circuits.

BFA encompasses all activities related to fresh products in the USA (bowls, snacks, packaged salads, fresh-cut fruits and mixed vegetables) through its national brands Ready Pac Foods, Bistro Bowl, Ready Snac and Elevate to its retail and foodservice customers.

Figure 1 - The 5 Business Units of Bonduelle Retrieved from Bonduelle Intranet

The Business unit where the thesis is performed is the Bonduelle Europe Long Life (BELL) unit. BELL is split up in different zones. The 5 zones that are defined in the BELL Business unit are France, Southern Europe, German-speaking countries, Northern Europe and Central Europe. The thesis is conducted at the headquarters for Northern Europe in Eindhoven. The Northern Europe zone consists of Netherlands, Belgium, Luxemburg, England, Scotland, Wales, Ireland, Denmark, Sweden, Norway, and Finland. An overview of the total division of departments is shown in Figure 2.

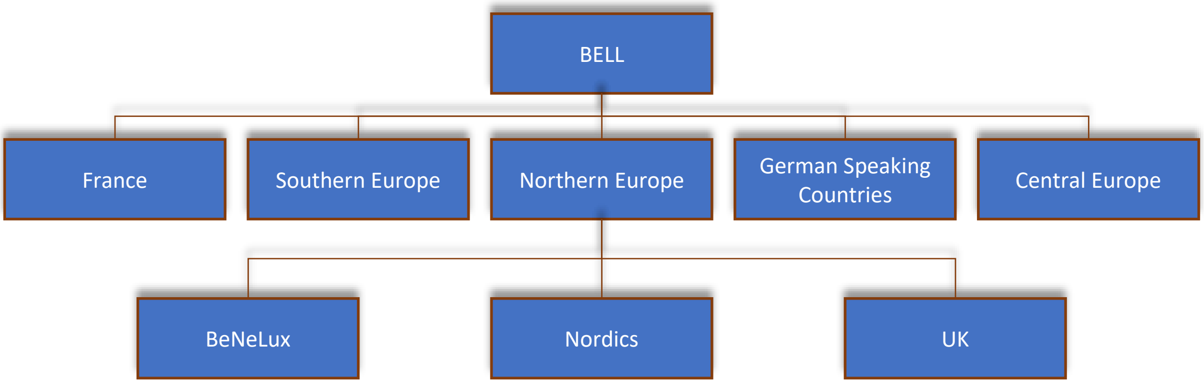


Figure 2 - Division of the zones within BELL

The thesis focuses on the supply chain for the Nordic countries (Denmark, Sweden, Norway, and Finland) also called the Nordics. The aim of this thesis is to design a general policy that is applicable in any situation given the same restrictions on ordering Full Truckloads (FTL).

2. Study design

This chapter will present the design of the study. This chapter will continue with the problem statement, model requirements and will give the research questions based on the problem statement and model requirements. Thereafter the scope of the research is described. The chapter will end with an outline of the rest of the thesis.

2.1 Problem Statement

Customers of Bonduelle expect Bonduelle to deliver products on a very short notice (1 day lead time). Bonduelle wants to be able to offer products with a shorter product lifetime to fully satisfy the customer needs. Additionally, to the shorter lifetime of the products, the customers only accept products that have a remaining lifetime of 2/3 of the original lifetime. Since the products are not produced at the warehouse, the actual time the products are kept on stock varies but per definition always has to be < 1/3 of the original lifetime. Because the current inventory control policy is not designed for products with short lifetimes, a lot of products expire and cannot be sold to the customers. This results in an increase of the waste that is produced in the supply chain and therefore increases costs. Furthermore, as with all inventory control problems, a trade-off between service level and inventory costs occurs.

The order that is made for the replenishment of a warehouse location of Bonduelle always includes multiple Stock Keeping Units (SKUs). This is done to make sure the transportation costs are minimized. The transportation costs are fixed per truck. So, including multiple SKUs in one truck is beneficial. Therefore, Bonduelle always wants to order FTL for the replenishment of the warehouse.

Bonduelle is interested in an inventory control model that is able to minimize costs while still being able to serve the customers with a certain service level. The way Bonduelle determines the service level is described in the next section.

2.2 Model requirements

In this section, the model requirements that are identified are explained. First, the supply chain is visualized in a figure. Following this, the different requirements are explained.

2.2.1 Supply chain design

In this section, the supply chain design is visualized.

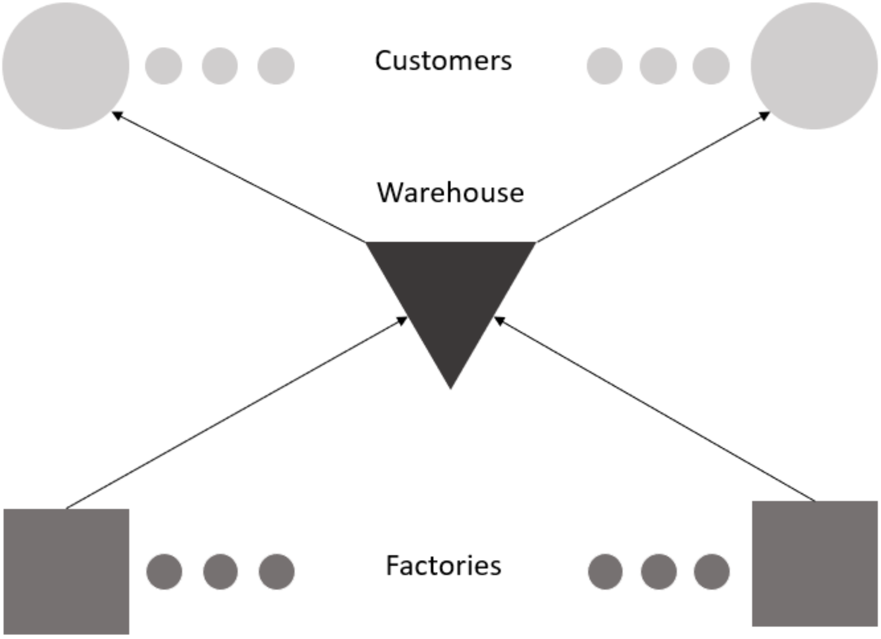


Figure 3 - Supply Chain Design Bonduelle Nordics

As can be seen in Figure 3, there is an intermediary location where inventories are kept in order to serve customers on time. The supply chain of Bonduelle is set up so that it is also possible for factories to directly serve the customers without first having to store products in the warehouse. The decision on when a product is delivered from the factories to the customers or via the warehouse is made based on lead times requested by the customer, order quantities and contracts with customers.

2.2.2 Service Level

The main performance indicator that is identified by Bonduelle is the service level. The service level is defined by Bonduelle as the number of boxes per SKU that is delivered divided by the total number of boxes that need to be delivered to the customers. The total number of boxes that need to be delivered consists of the demand in a period plus the backorders of the previous period for an SKU.

$$Service\ Level\ per\ SKU = \frac{\#Boxes\ of\ SKU\ delivered\ during\ a\ period}{\#total\ demand\ of\ Boxes\ of\ SKU\ during\ a\ period + \#Back\ orders\ from\ foregoing\ period} \tag{2.1}$$

2.2.3 Weighted service level

In order to evaluate the performance of the complete policy, the weighted service level (WSL) is introduced. First, the service level for all the SKUs individually is determined using formula 2.1. This service level is multiplied with the demand for the SKU. The sum off all the demand multiplied with the service level divided by the total demand give the WSL. This is given in formula 2.2 where the service level of SKU i is denoted by β_i , ranging from 0 to 1, and the demand of SKU i is denoted by $D_i \in \mathbb{N} \cup \{0\}$

$$WSL = \frac{\sum_{i=1}^{I} D_i * \beta_i}{\sum_{i=1}^{I} D_i} \qquad (2.2)$$

Using the WSL, the performance of two policies can be compared with each other. Where a lower WSL results in more products back ordered.

2.3 Research questions

For the thesis, the following research question has been defined.

What kind of inventory control policy is able to handle perishability of inventory, joint replenishment and always orders full truckloads for replenishment?

The following sub-questions are defined to answer the research question:

1. What is the current set up of the supply chain of Bonduelle?
2. What previous research has been conducted on joint replenishment, perishable inventory?
3. How should the outdating of products be modelled?
4. How should the joint replenishment be included in the inventory control policy?

2.4 Deliverables

Together with Bonduelle, some deliverables are defined:

1. A tool that makes decisions for replenishment based on inventory control policies found in the literature.
2. An algorithm that is used to assure always FTL is ordered for the replenishment of the warehouse.
3. The tool implemented in a software package so it can be used by an employee of Bonduelle.

2.5 Scope

2.5.1 The Nordics

As mentioned in the introduction to Bonduelle, the thesis is performed at Bonduelle Northern Europe and focuses on the Nordics specifically. Bonduelle has two types of customers in the Nordics. These two types of customers are Free Carrier customers (FCA) and Delivery Duty Paid customers (DDP). There is a difference in how Bonduelle delivers to these different types of customers. For instance, FCA customers have made agreements with Bonduelle to pick up the products at the factory while DDP customers want Bonduelle products to be delivered to them. The customers that are delivered from the warehouse are labelled as DDP customers. This thesis focusses on the DDP customers. Currently, Bonduelle has a single warehouse in the Nordics that supplies DDP customers. The single warehouse is located in Denmark.

2.5.2 Production and Inventory control at the factories

The products that are requested by the customers in the Nordics are produced in 4 different factories. The production of the Bonduelle products is not taken into consideration in this thesis since the production and inventory control at these factories is centrally controlled by the headquarters in France.

2.6 Assumptions

Backorders

Customers allow backorders. Meaning that when demand exceeds the inventory on hand the customer accepts the SKU that actually are currently available. Furthermore, Bonduelle operates in a market where a lot of suppliers are competing to supply customers. When a product is not available for a small period of time the product can be delivered with the next order. Since the demand of the customers is not immediately lost, the system cannot be classified as a lost sales system. For a lost sales model the demand that is not satisfied from inventory is completely lost. However, if a product is out of stock over a longer period of time, the customer will look for another supplier.

Full truckloads and Joint replenishment

A constraint to the problem is the FTL that have to be sent to the warehouse from the factories. To make sure FTL is ordered, sometimes a replenishment order for the warehouse is combined with direct delivery to an FCA customer. This however almost never occurs and therefore it is assumed that all orders that are placed for the replenishment of the warehouse are FTL. Combining products in order to better coordinate the replenishment is called Joint Replenishment.

Infinite stock at Factories

It is assumed that the factory can always deliver the products when ordered. However, the production of products is linked to the quality of harvest from farmers. If there is a bad harvest, not enough products can be produced. Furthermore, since Bonduelle sells plant-based food that needs to be farmed, the production of products is linked to seasons when the harvest is collected by the farmers. For instance, corn has a specific season in the year when it is harvested while black beans could have another season in which it might get harvested. This means that the factories of Bonduelle deal with cyclic production. Because of the cyclic production and the variable quality of the harvest, it could be that some SKU's are not always available in the factories. The percentage of SKU's that are unavailable when needed currently is not very high. To better integrate the supplier availability of the factories in France a supplier reliability study needs to be performed for the factories in France.

Lead time

In the thesis it is assumed that every truck that is ordered at time t has the same delivery lead-time L. The lead-time is assumed to be 10 working days (2 weeks).

Pallet loading

A pallet cannot contain multiple SKUs. This means a pallet is always loaded with only one SKU.

Single Location

The replenishment policy deals with SKU's from one factory. Because Bonduelle currently does not allow for transhipments, the replenishment policy is only able to deal with the ordering products for replenishment from a single location.

First Expired First Out

Bonduelle currently handles the First Expired First Out (FEFO) policy meaning that the products that shipped to the customers are the products with the smallest remaining lifetime. In the thesis, it is assumed the FEFO policy is the same as the First In First Out (FIFO) policy. The products that arrive at the warehouse for replenishment typically have a longer lifetime than the products that are already kept on stock at the warehouse. In that case, the FIFO is exactly the same as the FEFO policy. It is not possible for products to arrive at the warehouse with a shorter lifetime. It is possible that products arrive with a similar remaining lifetime.

2.6 Project outline

Chapter 1 introduced Bonduelle. In Chapter 2 the problem statement, model requirements, research questions, scope, and assumptions have been discussed. In Chapter 3 the current supply chain operations are discussed. This is followed by a literature study in Chapter 4. In the literature study perishable inventory, joint replenishment and the knapsack problem are examined. Chapter 5 describes the choices that are made to design an inventory policy suitable for Bonduelle and the heuristic perishable inventory that is used in this thesis. In Chapter 6 the algorithm that is used to allocate products to trucks is introduced. Chapter 7 discusses how the proposed model should be implemented at Bonduelle. In Chapter 8 the verification and validation of the simulation model are performed. In Chapter 9 a case study is performed on at Bonduelle and the results are presented in this chapter. In Chapter 10 some conclusions are given. Finally, in Chapter 11 the recommendations are given.

3. Current supply chain operations

In this chapter, the current supply chain operations that are performed by Bonduelle are discussed. Bonduelle does not have an Enterprise Resource Planning system (ERP) that is able to manage the inventory levels that should be kept at a warehouse in order to satisfy customer demand. Inventory control is done manually by an employee. Once a week, the employee checks the inventory that is currently on stock in the warehouse and places an order to the factories to resupply the warehouse. The products that are stored at the warehouse are classified into three groups. This classification is based on the historic inventory turnover. Bonduelle distinguishes between A, B or C classification. The reorder point of the SKUs is based on the classification. In order to resupply the warehouse, a replenishment order is placed based on the reorder level, current stock level, products in transit and the expected sales in the next week. For products to be classified as an A product the average sales for that product need to be more than 50 boxes per week. For B products the average demand per week is between 25 and 50. Finally, for C products the demand is below 25 boxes per week. For A products the safety stock level is 3 weeks of average demand. B products have safety stock levels for 4 weeks of average demand and C products have safety stock levels with 5 weeks of average demand. Currently, the Bonduelle employee responsible for replenishment of the warehouse uses a moving average of the demand from last year's sales data to predict the average demand per week. Bonduelle is transitioning into a situation where the demand forecast made by the production/demand planners is directly used for the replenishment for the warehouse. As of now, Bonduelle is still in progress of making this transition. Currently, the inventory policy results in a lot of products being outdated because the inventory control policy is not yet designed to handle SKU's with a shorter lifetime. As mentioned before, in order to be more cost-efficiently, the factories only send products to the Nordics if an FTL is ordered. Since the pallets cannot be stacked due to weight issues the maximum number of pallets that a truck is able to transport is 33. Figure 4 shows the setup of pallet places of 1 truck. The maximum weight one truck is able to transport is 22000 kilograms.

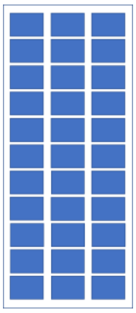


Figure 4 - Truck layout with 33 pallet

Currently, if a replenishment order is not sufficient to fill a truck, the person who is responsible for placing the replenishment orders manually increases the demand factor for all products evenly until a multiple of an FTL can be ordered. For instance, if an initial order contains 45 pallets of different kinds of products the demand factor of all these products is increased evenly until the model says 66 (2*33 = 2 FTL) pallets should be ordered. If the total weight exceeds the weight capacity of the FTL the demand factor is manually decreased until the weight capacity constraint is met.

The described inventory control policy is similar to an (R, s, nQ) policy with some added constraints on capacity, accounting for perishable products with backorders. The (R, s, nQ) policy is also called the base policy (Silver et al., 2001). The (R, s, nQ) policy is shown in Figure 5.

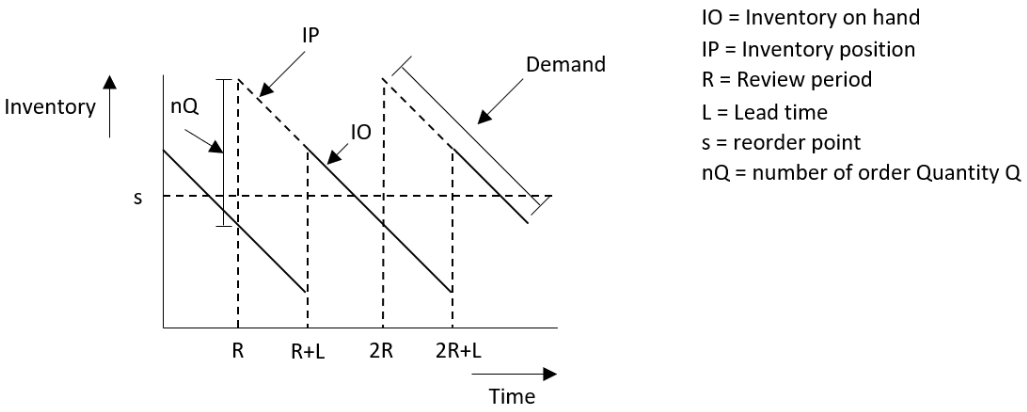


Figure 5 - The Base Stock policy

In the base policy, a replenishment order is created only when the inventory position at a periodic review moment is strictly below the reorder level s. When that's the case, Q or a number (n) of Q is ordered.

4. Literature Study

In this chapter, the literature study for the master thesis is discussed. First, the specific search terms are given. These search terms are found based on the analysis of the current situation and the model requirements that are specified by Bonduelle. In this literature study inventory control models are discussed.

4.1 Search Terms

Following the analysis of the current inventory policy and the model requirements the following search terms are identified:

- Periodic Review
- Joint Replenishment
- Perishable Goods
- Knapsack problem

The search engine that is used is LibrarySearch. LibraySearch is the discovery tool of the TU/e.

4.2 Inventory management

There already is a lot of research done in inventory management. In this thesis, plant-based food products are considered. As mentioned before Bonduelle is changing from products in a can, to pouches. The pouches have a smaller lifetime than products that are served in a can. In order to take the lifetimes into account, some literature is needed that deals with products that could perish over time. In their paper, Broekmeulen and Van Donselaar (2009) introduce a so-called EWA policy that can be used by retailers that have to manage a large assortment of perishable products assuming lost sales. It is mentioned that the cost reductions obtained with the EWA are especially large for products with a short remaining shelf life, the lead time is large, the review period is large, outdating is expensive and/or when the retailer aims for high product availability. Raafat (1991) provides an overview of different mathematical models for deteriorating products. Raafat (1991) mentions that for most of the models that are reviewed, except for the basic model, no simple decision rules are given. And mentions simpler heuristics or approximations are needed. Fries (1975) started describing order policies with fixed lifetimes for perishable products. Fries states that for specifying the target inventory the number of units in all age groups needs to be known and just the total inventory will not suffice. Van Donselaar and Broekmeulen (2012) derive approximations for the expected amount of waste in an inventory system with perishable products, periodic review, positive lead time, fixed case pack size and First-in-First-out withdrawal policy. The approximations enable the retailers to make trade-offs between the relative outdating (waste) and the customer service level when making strategic or tactical decisions on the (re)design of the perishable inventory system. The approximations are made under the assumption that demand is stationary.

Besides the perishability of the products, there is a restriction on the optimal order quantity for the replenishment policy of the warehouse. Bonduelle wants to make sure it utilizes the max capacity of the trucks that supply the warehouse. Therefore, Bonduelle is interested in combining the replenishment of multiple products into one order so the truck is filled to its maximum capacity. In the literature, the problem of combining orders of different products into one order is called the Joint Replenishment policy. The Joint Replenishment problem is a frequently studied subject in the literature. Atkins and Iyogun (1988) compare different models that deal with joint replenishment. In their paper Atkins and Iyogun (1988) compare periodic policies to a continuous "can order" policy. Federgruen et al. (1984) describe a can order policy (s, c, S) that at every demand epoch evaluates what it should order to bring the all the inventory levels back to an *order-up-to level* S. In the model introduced in the paper, an order is initiated when of product i the inventory reaches its *reorder level* s_i. Any item j for which the inventory level is at or below its *can-order level* c_j is included in the order. For every item k that is included in the order, the items are ordered up to level S_k. Note that the policy described is a continuous review policy. Atkins and Iyogrun (1988) find in their results that the can order policy proposed by Federgruen et al. (1984) performs better than the periodic policies when the major setup costs K are low but when K increases the periodic policies dominate the can order policy in most parameters settings. Viswanathan (1997) introduces a periodic-review $P(s, S)$ policy that is marginally better than the models that are introduced by Atkins and Iyogun (1988) however the policy that Viswanathan (1997) introduces is able to dominate the other policies in a wider range of parameters, making it a better policy. Both the papers of Atkins and Iyogun (1988) and Viswanathan (1997) extend the Joint Replenishment formulas for periodic review introduced by Hadley and Whitin (1963, Chapter 5). However, the can order policy still seems to perform the best when the major setup costs *K* are low. Furthermore, Viswanathan (1997) also introduces a QS policy named Q*(s, S)* policy. *However, computing the optimal Q(s, S) would be hard since the cost of the particular Q(s, S) cannot be evaluated analytically (to date) (Viswanathan, 1997).* Nielsen and Larsen (2005) show that it is possible to find an analytical solution procedure. The Q(s, S) policy works as follows: the total inventory is reviewed continuously while items are only reviewed when the total consumption since the last ordering reaches level Q. Then all items *i* with inventory position less than or equal to s_i are ordered up to level S_i. When $s_i = S_i - 1$ the Q(s, S) policy is equal to the QS policy. Nielsen and Larsen (2005) show that the optimal Q(s, S) outperform P(s, S) policies in all the examples tested. However, Nielsen and Larsen (2005) assume that the policy applies to a single-product-multiple-location setting. Most of the literature already done is based on the multiple-product-single-location setting. Nielsen and Larsen (2005) argue that the single-product-multiple-location setting is more intuitive and basically describe the same situation. However, in the case of Bonduelle, it is very clear a multiple-product-single-location setting should be used since the lead times of the products of one location are the same. Johansen and Melchiors (2003) introduce a can-order policy that outperforms other periodic policies for the periodic review joint replenishment problem. They contradict the claim of the textbook of Silver et al. (2001) that periodic replenishment policies outperform the can-order policy.

Coelho and Laporte (2014) introduced a policy that incorporates the joint replenishment problem and inventory control of perishable products. The type of perishability that is assumed by Coelho and Laporte (2014) is non-instantaneous deteriorating products. Furthermore, Coelho and Laporte (2014) solve the joint replenishment problem in combination with a routing problem. They modelled the problem under general assumptions as a MILP. They have shown that, for their test cases, the profit changes drastically depending on the revenue of the product. The MILP is solved exactly using a branch-and-cut algorithm. Xue-Yi et al. (2017) also extend the joint replenishment problem with deteriorating items. The results of their paper show that the costs can be significantly reduced when incorporating the perishability of the items in the joint replenishment problem. In their paper, Xue-Yi el al. (2017) use an approximation method and a heuristic branch and bound algorithm is used in order to obtain an optimal solution.

Finally, Chun-Wei el al. (2010) extends the classical joint replenishment problem to incorporate the transport capacity constraint. Due to the extension involves binary decisions of delivering of not delivering the problem needs to be solved with Binary Integer Programming. Furthermore, Chun-Wei et al. (2010) mention that the extension is an NP-Hard problem. In order to solve the problem anyway, two algorithms are introduced in the paper of Chun-Wei et al. (2010).

4.3 Knapsack Problem

Besides the inventory control management, Bonduelle also faces a hard restriction on ordering full truckloads from the factory. After the initial inventory control policy determines the safety stock and the ordering quantities, the total quantities that should be ordered should all be placed inside a truck. It never happens that the exact number of products that should be ordered, according to the inventory control policy, is the same as the maximum capacity of a truck. Therefore, a new type of problem arises what products should be ordered additionally to make sure a full truckload is ordered and how should these products be placed on the trucks. For instance, the initial inventory control model determines 40 pallets of products should be ordered. These 40 pallets exceed the capacity of 1 truck since a full truck has 32 pallet places or a maximum weight constraint of 22000 kilos. First, a decision has to be made about what products could be ordered extra to fill the second truck, and second, a decision has to be made about how the products are placed in the trucks. The problem of determining what products should be placed on what truck is called a knapsack problem. The goal of the classic knapsack problems is to maximize the value one can carry in its knapsack while satisfying the capacity constraint of the knapsack. The knapsack problem is a problem that is already widely studied in the literature. In their book, Kellerer et al. (2004) discuss a lot of different types of knapsack problems and discuss different types of algorithms to solve these knapsack problems since the knapsack problem is known to be NP-Hard. This means the problem cannot be solved in polynomial time but can only be approximated in polynomial time. Kellere et al. (2004) discuss basic algorithms to approximate the solution of the knapsack problem such as branch and bound algorithms, dynamic programming, linear programming approximation algorithms, and greedy algorithms. Kellere et al. (2004) also discuss advanced algorithmic concepts. Kellere et al. (2004) mention that for knapsack problems with a lot of products and weights of the products relatively small compared to the knapsack capacity the Greedy algorithm yields a solution not far from the optimal solution.

5. Thesis policy

In this chapter, the choices with respect to the inventory control policy are explained.

Periodic review

The policy that is proposed in this thesis has a periodic review period. This choice is done because Bonduelle currently has no ERP system to continuously track the inventory levels at the warehouse.

5.1 Inventory policy.

In this thesis, the heuristic called the EWA policy proposed by Broekmeulen and Van Donselaar (2009) is used to analyse the inventory level of a single product. The EWA policy is based on a relatively simple principle. The policy corrects the reorder level for outdating by estimating the amount of product that outdates in a period. Broekmeulen and Van Donselaar (2009) discuss two different withdrawal policies; First In First Out (FIFO) and Last In First Out (LIFO). Bonduelle uses a First Expired First Out (FEFO) policy. As mentioned in the assumptions it is assumed that the FIFO is the same policy as the FEFO policy.

The EWA policy of a single product is an extension of the (R, s, nQ) base policy of Silver (2001). In the EWA policy, the (R, s, nQ) base policy is extended to include the outdating of products. This outdating function influences the level of s and the number of quantities Q ordered.

5.1.1 Full Truckloads

As mentioned earlier Bonduelle wants to order Full Truck Loads of products for the replenishment of the warehouse. This problem can be defined as a multiple knapsack problem or a bin packing problem. In order to allocate the products to the trucks, the best fit decreasing heuristic of Garey et al. (1976) is used. The best fit decreasing heuristic first sorts all the products that need to be fitted into the trucks from high to low. Then it adds the product to the truck that has the least amount of leftover space available after adding the product. The algorithm is further explained in chapter 6.

5.2 Fitting Demand

To find a demand distribution function the fitting procedure of Adan et al. (1995) is used. In their paper Adan et al. (1995) consider the variable a to choose what probability distribution is appropriate for the data. a is defined by

$$a = \frac{\sigma^2/\mu - 1}{\mu} \tag{5.1}$$

Adan et al. (1995) choose the binominal distribution when $-1 < a < 0$, the Poisson distribution when $a = 0$, the negative binomial distribution when $0 < a < 1$, and the geometric distribution when $a \geq 1$. The fitting procedure of Adan et al. (1995) is chosen since the procedure only needs a mean and standard deviation to fully specify the probability distribution function.

5.3 The EWA policy

In the EWA policy the following model notations are made:

- A perishable product has a fixed lifetime m. The lifetime is defined by Broekmeulen and Van Donselaar (2009) as the remaining lifetime of the goods when they arrive at the location.
- The inventory is controlled with a periodic review system equal to R days.
- The inventory in the warehouse at the start of period t can consist of one or more batches. A batch is defined as a set of products that are available and all have the same remaining shelf life. The number of products available in the store at the start of day t having r days remaining shelf life is denoted by B_{tr}.
- Replenishment orders have a fixed lead-time of L days.
- Replenishment quantities are limited to multiples of an exogenously determined case pack size Q.
- Only products with a remaining shelf life of 2/3 of the total lifetime can be sold, W_{tr} is the amount products sold with remaining lifetime r at period t. Outdating O_t is removal of products with 1-day remaining product lifetime at the end of period t, these products cannot be sold anymore.

- The same safety stock SS is applied every period.

In the base policy at every periodic review moment if the inventory is below the reorder level s_t an order of n times Q is placed to get the inventory level back up to or just about the reorder level s_t. The reorder level is determined depending on the service level. In the base policy the reorder level is set as follows:

$$s_t = SS + \sum_{i=t+1}^{t+L+R} E[D_i] \tag{5.2}$$

where SS represents the safety stock and $\sum_{i=t+1}^{t+L+R} E[D_i]$ the expected demand during the lead-time plus review period. The value of n_t is chosen such that the inventory position just after replenishment decision is at or above s_t, but strictly less than $s_t + Q$. IP_t is defined as the inventory position at day t just before an order is placed.

$$\text{If } IP_t < s_t \text{ then } n_t = \left\lceil \frac{s_t - IP_t}{Q} \right\rceil \tag{5.3}$$

Note that $\lceil x \rceil$ rounds up x to the nearest integer. For instance, 2.1 is rounded up to 3. Inventory Position means the inventory on hand (the inventory available in the warehouse) plus the products that are already ordered but are not yet delivered, also called the pipeline inventory.

In the EWA policy, the inventory position is first corrected for the estimated amount of outdating and an order is placed if this revised inventory position drops below the reorder level s_t. The value of n_t is now determined as follows:

$$\text{If } IP_t - \sum_{i=t+1}^{t+L+R-1} \hat{o}_i < s_t \text{ then } n_t = \left\lceil \frac{s_t - IP_t + \sum_{i=t+1}^{t+L+R-1} \hat{o}_i}{Q} \right\rceil \tag{5.4}$$

To distinguish the estimated amount of outdating from the actual amount of outdating, the variable $\hat{o}_t$ is used in (6.3) rather than O_t. The outdating is estimated over L + R – 1 period. The outdating on the (L+R)th period does not affect the ability to meet demand during that day, outdating takes place at the end of the period after demand has taken place. Via recursive equations, using the age distribution of the inventory and the withdrawal behaviour, the estimated amount of outdating is determined. As mentioned before Bonduelle delivers according to the FIFO policy in the warehouse, so only the recursive function for the FIFO withdrawal policy is given below. The withdrawal is the minimum of the remaining batch size and the unsatisfied demand from older batches available.

$$W_{tr} = Min\left\{B_{tr}, D_t - \sum_{i=1}^{r-1} W_{ti}\right\} \tag{5.5}$$

Where B_{tr} denotes the number of items available in the warehouse at the start of period t having r periods remaining shelf life, D_t represents the expected demand that occurs at time period t.

At the end of each review period, an inventory replenishment decision is made which determines $B_{t+L+1,\, m}$. After each period, the batches are updated to account for aging, withdraw and outdating.

$$B_{t+1,r-1} = B_{tr} - W_{tr} \tag{5.6}$$

And for the batch with 1-period remaining life time :

$$O_t = B_{t,1} - W_{t,1} \tag{5.7}$$

The recursive equations above are the basis for calculating the estimated outdating quantities, which are needed in the EWA policy, in formula (6.3). The outdating quantities are estimated by calculating the withdrawal, the remaining batches and the outdating for consecutive weeks i (ranging from $i = t+1$ to $i = t+L+R-1$) under the assumption that in period i demand is equal to expected demand. This implies the following procedure, starting with $i = t + 1$:

1. Determine the estimated withdrawal at day i using (6.4) and by assuming demand at day i was equal to the expected demand.
2. Determine the estimated remaining batches available for the next period and the estimated outdating at day i using (6.5) and (6.6) with the withdrawal at day i equal to the estimated withdrawal as determined in step 1.
3. While $i < t + L + R - 1$ do $i := i + 1$ and continue with step 1, otherwise stop.

So, for example, to determine the expected outdating for a product, assume mean demand is 5 and the batches that are kept on stock are 10, 8, 8 with remaining lifetimes 1,2,3 periods respectively. Assume the lead time is 1 and the review period is 1. The expected outdating in period 1 is 10 – 5 = 5 and the expected outdating over L+R periods is (10 – 5) + (8 – 5) = 8.

5.4 Lost sales to Backorders

As mentioned in the literature study, Broekmeulen and van Donselaar (2009) based on the EWA policy on an (R,s,nQ) policy with lost sales. In the inventory policy of Bonduelle backorders are allowed.

Therefore, a couple of changes have to be made to make the EWA policy suitable for a policy that allows backorders.

In a lost sales policy, the demand only consists of random demand over time.

In a backorder policy, the demand in a period is also dependent on the unsatisfied demand of the previous period.

$$D_t = D_t + \ [D_{t-1} - IO_{t-1}, 0]^+ \tag{5.8}$$

where D_{t-1} denotes the demand in the previous period and IO_{t-1} denotes the inventory on hand in the previous period.

5.5 The cost function

After explaining the inventory policy, the costs function is defined as follows:

$$Total\ cost = \ \sum_{t=1}^{T}\sum_{i=1}^{I}\left(Z_{i,t} * C_{i,Z}\right) + \sum_{t=1}^{T}\sum_{i=1}^{I}\left(IO_{i,t} * C_{i,IO}\right) + \sum_{t=1}^{T}\sum_{i=1}^{I}\left(BO_{i,t} * C_{i,BO}\right) + \sum_{t=1}^{T}\left(NO_t * C_{NO}\right) \tag{5.9}$$

The total cost function depends on the safety stock level. The safety stock level influences the number of products that are kept on stock at every time period. Therefore, it directly influences the number of products that are outdated, the number of products that are back-ordered and the number of orders that are placed to replenish the warehouse.

6. Adding pallets and truck allocation algorithm

In this chapter, an algorithm to add pallets to satisfy the requirement of ordering full truckloads for a replenishment order is introduced. The volume of a truck is defined as the number of available pallet places in a truck. First, the decision tree related to the algorithm is given.

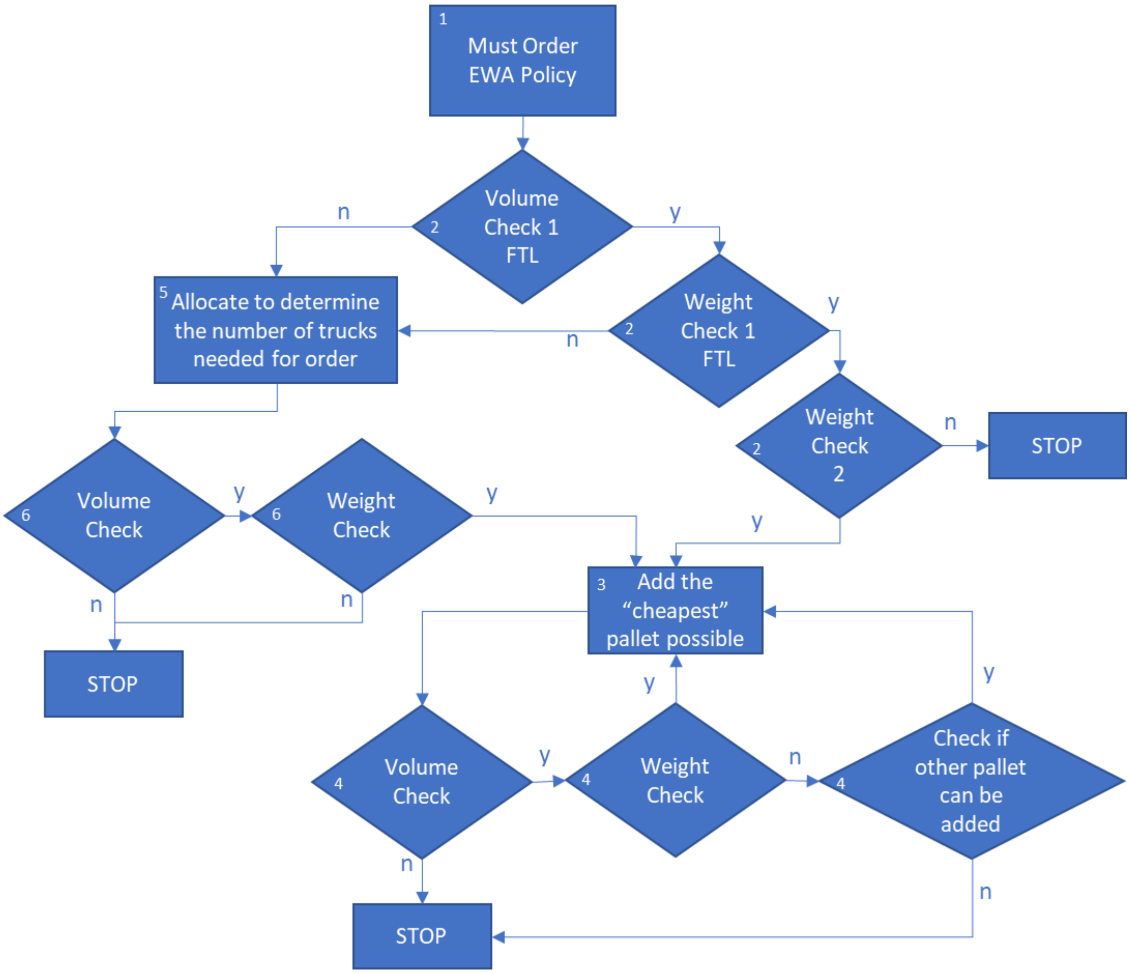


Figure 6 - Decision tree of the algorithm to add pallets

The explanation of the actions is given below. The numbers in the decision tree corresponding to the number in the explanation below.

1. Must order Extended EWA Policy:

   At a periodic review moment, the inventory position is checked, and an order is placed according to the inventory policy described in Chapter 5. Let I denote the total number of SKU’s that are kept on stock where I can be any natural number, $I \in \mathbb{N}$. The number of pallets that need to be ordered of SKU i is denoted by n_i whit $n_i \in \mathbb{N} \cup \{0\}$. Then the total number of pallets ordered is $\sum_{i=1}^{I} n_i$.

2. Volume and Weight check:

   First the Volume check is executed:

   If $\sum_{i=1}^{I} n_i <$ FTL$_{plts}$ only 1 truck is needed according to the volume check. If the sum of pallets exceeds the maximum volume capacity of 1 truck multiple trucks are needed. Go To Action 5.

   If the volume check is passed, a weight check is executed. The weight of 1 pallet with boxes of SKU i is denoted as w_i.

   If $\sum_{i=1}^{I} n_i w_i <$ FTL$_{weight}$ then the total weight of the order is less than the maximum capacity of 1 truck and a check to see for what SKU i a pallet can be added weight-wise is performed. If this check returns false, multiple trucks are needed. Go To action 5. Else a check is performed to see if a pallet actually can be added to the order. If FTL$_{weight} - \sum_{i=1}^{I} n_i w_i < w_j \; with \; j \in$ Ithe "cheapest" pallet is added. Else, the algorithm stops since the truck already reached the maximum number of pallets weight-wise for an FTL.

3. Add the "cheapest" pallet possible:

   The "cheapest" pallet is defined as the pallet for an SKU i that adds the least amount of costs to the order. To identify this pallet, the outdating and inventory holding costs are considered. To be able to determine the expected inventory holding and outdating costs, a time period needs to be defined. The time period consists of two parts.

   - First the time that the current inventory on hand plus the inventory in transit and the already ordered pallets can satisfy the average demand.

     $$\tau_i = \frac{inventory\ on\ hand + inventory\ in\ transit +\ n_i * Q_i + n_i' * Q_i}{\mu_i} = \frac{IO_i + IT_i + n_i * Q_i + n_i' * Q_i}{\mu_i}$$

     where $n_i' \in \mathbb{N}$ denotes the pallets of SKU i that already have been added extra to an order. μ_i represents the average demand for SKU i in boxes per week.

   - Secondly, the time one pallet can satisfy the average demand per period is included. $T_i = \frac{Q_i}{\mu_i}$.

   Over the period $T_i + \tau_i$ the inventory holding costs and outdating costs are determined. The inventory holding cost is the cost of keeping a pallet on storage overtime period $T_i + \tau_i$. For

1 pallet of SKU i the cost of keeping 1 pallet of SKU i on stock for 1 day, $C_{i,H}$, multiplied with the number of pallets of SKU i that is kept on stock during time $T_i + \tau_i$, $IO_{i,T_i+\tau_i}$ gives the total inventory holding costs of SKU i over time $T_i + \tau_i$. Cost is: $C_{i,H} * IO_{i,T_i+\tau_i}$.

The outdating costs are calculated by multiplying the expected number of boxes of SKU i that will outdate, assuming average demand, with the costs of outdating of 1 box of SKU i. The outdating costs are : $C_{i,Z} * Z_{i,T_i+\tau_i}$.

The pallet of SKU i that is the "cheapest', is added to the order. If there are multiple SKU's that are the cheapest, SKU i with the heaviest pallet filled with boxes of SKU i is chosen to be added to the order. This is done because adding a pallet to an order actually moves away from the optimal order quantity. Adding a pallet with more weight should result in adding fewer pallets.

4. Volume and Weight check:

   After adding a pallet, again a Volume and Weight check is performed. First, the Volume check is performed. In order to distinguish to what truck a pallet is allocated a superscript to the number of pallets is introduced, n_i^k represents the number of pallets of SKU i that are allocated to truck k, $n_i^k \epsilon \mathbb{N} \cup \{0\}$.Similarly, $n_i^{k\,\prime}$ represents the number of pallets of SKU i that are already added extra to truck k, $n_i^{k\,\prime} \epsilon \mathbb{N} \cup \{0\}$. If $\sum_{i=1}^{I} n_i^k + \sum_{i=1}^{I} n_i^{k\,\prime} < \mathrm{FTL}_{plts}^{k}$ there is still a free pallet place in truck k. If for all the trucks the check is not passed, no more volume is available, and the algorithm is stopped.

   Next, the Weight check is performed. The weight checks consist of two parts, first to identify which truck still has weight capacity left, the second weight check is to determine if there is enough capacity in truck k to add a pallet of SKU i.

   Because in the add "cheapest" pallet process the pallets that would fit weight wise are considered, the complete check of all trucks must always be true. If $\sum_{i=1}^{x} n_i^k w_i + \sum_{i=1}^{x} n_i^{k\,\prime} w_i < \mathrm{FTL}_{Weight}^{k}$ the "cheapest" pallet can be added to truck k. Next, a check is performed to see if there is still enough weight capacity left in one of the trucks to add another pallet. If $\mathrm{FTL}_{weight}^{k} - \sum_{i=1}^{I} n_i^k w_i - \sum_{i=1}^{I} n_i^{k\,\prime} w_i < w_i$ a pallet still can be added to truck k. If the test is not passed there is not enough remaining capacity left in any of the trucks to add a pallet, then the algorithm is stopped.

5. Allocate the initial must order pallets to determine the number of trucks that are needed: For the allocation of the pallets to the trucks the heuristic of Garey et al. (1976) is used. The heuristic of Garey et al. (1976) uses the principle of the first fit decreasing algorithm (FFD). The FFD sorts the pallets that need to be allocated to a truck from biggest to smallest on one single dimension. It then allocates the first pallet in the sorted list to a truck. The heuristic of Garey et al. (1976) extends this idea of the FFD algorithm to include multiple dimensions. In the case of allocating pallets to trucks, there are two dimensions. The first dimension is the volume of a truck also noted as the number of available pallet places. The second dimension is the weight. In the allocation heuristic of Garey et al. (1976) the pallets are sorted on both volume and weight. Then it takes the heaviest pallet and allocates it into a truck and so on. In this specific case, the weight is the main dimension each pallet occupies the same volume in a pallet place. If the capacity of a truck is exceeded and more trucks are needed, the allocation is restarted with a new number of trucks.
6. Volume and Weight check:

   After the allocation of the pallets to the trucks, a volume and weight check is performed on each truck. First, the volume check is performed. If $\sum_{i=1}^{I} n_i^k < \mathrm{FTL}_{plts}^{\mathrm{k}}$ then the truck can still fit another pallet volume-wise. If all trucks do not pass this check, the algorithm is stopped since there are no more pallet places available in the trucks. Second, the weight check is executed. If $\mathrm{FTL}_{Weight}^{\mathrm{k}} - \sum_{i=1}^{I} n_i^k w_i < w_i$ there is still room in the truck for a pallet of SKU i and step 3 is initiated. If the check is not passed, there is no more room in the truck for a pallet to be added and the algorithm is stopped.

7. Implementation at Bonduelle

The policy described in chapter 5 and algorithm chapter 6 is implemented in Bonduelle. The inventory policy and the algorithm should be used every review period in order to determine the quantities that need to be ordered. The tool that is designed for Bonduelle is an Excel file that can be used to determine the number of pallets of SKU i that need to be ordered. In order for the tool to function appropriately, some input is needed.

7.1 Input variables for tool

The following information is needed for the inventory control policy.

7.1.1 Demand forecast

To determine the number of pallets that should be ordered for each SKU, the demand forecast is needed. The reorder point is determined using formula 5.4. In this formula, the expected demand over the period L+R needs to be known.

7.1.2 Inventory position

When determining the number of pallets that need to be ordered for one single SKU, the current inventory level for each SKU and the number of pallets that are currently in-transit to the warehouse need to be known. The inventory position is used in formula 5.5 to determine the number of pallets that should be ordered according to the policy. Furthermore, for the inventory currently on stock, the number of days it has been kept on stock needs to be known. This data is needed to determine the expected outdating during a period. This will influence the number of pallets that need to be ordered in the inventory policy.

7.1.3 General product data

To make sure the inventory policy is able to determine the number of pallets that should be ordered for an SKU there are a couple of basic product-specific parameters needed. The data that is needed is summed up below:

- Number of boxes on one pallet
- Weight of one pallet of one SKU

- The factory where the SKU is produced
- The number of days a box of SKU can be held on stock before the box cannot be sold anymore due to outdating of the box

7.2 Safety stocks

The safety stock levels are based on the ABC classification given by Bonduelle. Where for SKUs classified as A products 3 weeks of average demand is kept as safety stock. For SKUs classified as B products 4 weeks of average demand is kept as safety stock and for SKUs classified as a C products 5 weeks of average demand is kept as safety stock.

8. Verification and Validation

This chapter describes how the simulation model is verified and validated. The simulation is used to compare the policy described in this thesis with the current inventory policy that is used by Bonduelle. To make sure the results that are obtained from the simulations are correct, the simulation of the policies needs to be tested. In this chapter, the simulation verification and simulation validation are discussed. According to Sargent (2013) simulation verification is defined as ensuring that the program and its implementation are correct. Sargent (2013) defines simulation validation as the “substantiation that a model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model”.

8.1 Simulation verification

Sargent (2013) defines two different methods to verify the simulation; static testing and dynamic testing. In static testing, the simulation is analysed to determine whether it is correct by using techniques such as structured walkthroughs, correctness proofs and examining the structural properties of the simulation. In dynamic testing, the computer program is executed with different conditions and the values obtained are used to determine whether the computer program and its implementation are correct. The techniques used in dynamic testing are traces, investigations of input-output relationships using different validation techniques, data relationship correctness and reprogramming critical components to determine whether the same results are obtained.

In this thesis, the verification techniques that are used are a combination of static and dynamic testing. A structured walkthrough of the program and a correctness test of the different functions in the simulation is performed during the programming of these functions (Static testing).

For the dynamic testing part, the input-output relationships are analysed using different validation techniques. This dynamic testing in touched upon in the next section where the different validation techniques are further discussed.

Furthermore, since the simulation is programmed in Python, Python also runs a check if the code that is programmed from a semantic point of view is correct. If the code is not correct, Python returns an error message.

8.2 Simulation Validation

The simulation validation consists of many different techniques. The validation techniques as described by Sargent (2013) that are found to be applicable for the simulation validation are extreme condition tests, data relationship correctness, and animation. The validation steps are discussed below.

8.2.1 Animation

Animation means the operational behaviour of the simulation over time. In order to validate the simulation that is created, a couple of variables are tracked over time:

- Inventory on hand of each SKU
- The quantities that are ordered at each review period for each SKU
- The backorders for each SKU
- The service level for each SKU
- Number of products outdated

These variables show the behaviour of the system over time. These variables are tracked and reported during the runs. An example of the inventory on hand that is tracked for one SKU in 1 simulation run is given in Figure 7.

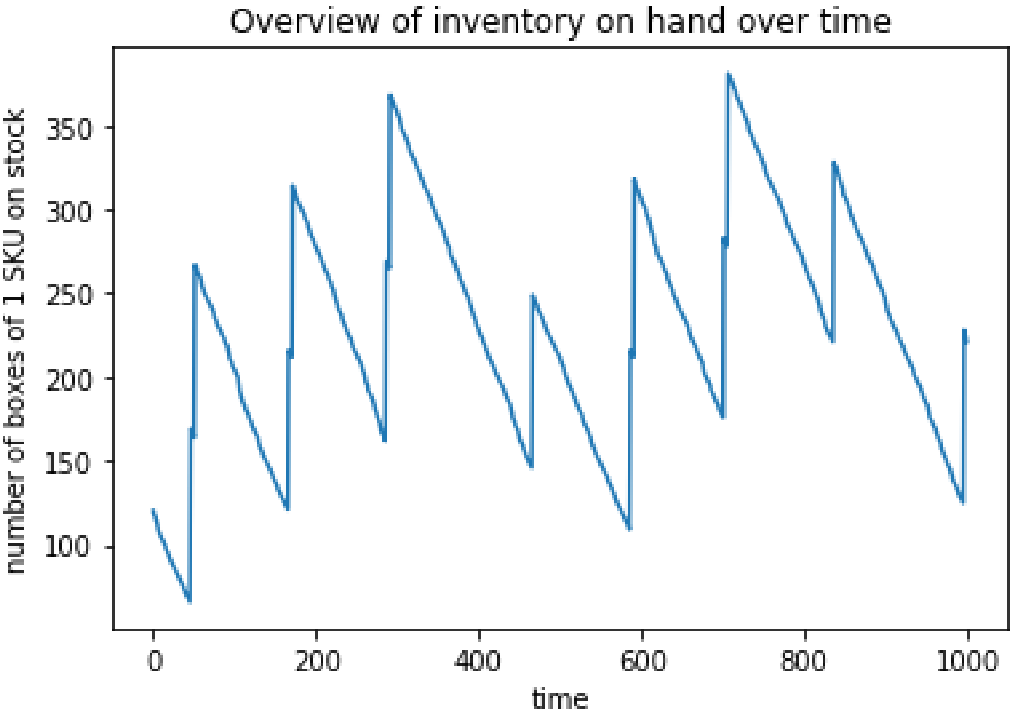


Figure 7 - Inventory on stock for 1 SKU over time for 1 simulation run

8.2.2 Extreme condition tests

According to Sargent (2013), the extreme condition means that the simulation model structure and outputs should be applicable for any extreme and unlikely combinations in the system. The extreme conditions that are tested in the simulation model presented in this thesis are: Very short remaining lifetime of products, very high inventory levels with infinite lifetime and extremely high demand.

Short remaining lifetime of products

If the remaining lifetime of the products is very short, there should be a lot off products lost due to the outdating of the products. In this specific test, the remaining lifetime of the products is set to 3 weeks. This would mean the products are only allowed to be kept on stock for three weeks. This, in turn, would lead to a significant increase in the outdating costs and therefore the overall costs. Furthermore, the low values for the remaining lifetime of products also affect the availability of the products meaning the percentage of backorders is expected to increase. Table 1 on page 32 shows the results of a simulation run with short remaining lifetimes of products and the results of a simulation run with normal values for the remaining lifetime of products. As can been seen in Table 1 the outdating costs indeed have indeed risen significantly compared to the results under normal input values. This is results are in line with the expected results.

Very high inventory levels with an infinite lifetime

When the starting inventory for each individual SKU is very high, while also assuming an infinite lifetime of the SKU’s. The simulation should never order new products to satisfy the demand in each period. This means the only costs that are encountered in this extreme test should be the holding costs of a pallet for an SKU. The results of this extreme test are also in line with the expected outcome. The inventory holding costs are very high and the service level for each SKU is equal to 100% meaning no backorders occur. The results of this extreme test are shown in Table 1 on page 32. As can been seen in Table 1 for this extreme test the total costs only consist of the inventory holding costs that are made. These results are in line with the expected outcome of the extreme test.

Extreme high demand

If the demand for each SKU is multiplied with a factor 1000 the simulation should order way more products each review period. Due to the high demand, the number of products that are outdated is likely to decrease since the inventory should not stay very long in the warehouse. Furthermore, the ordering costs will be increased significantly since way more FTL need to be ordered for the replenishment of the warehouse. Table 1 shows the results from the extreme high demand test and normal demand. Table 1 is shown on the next page.

As can been seen in Table 1 the outdating costs are extremely reduced when compared to the results under normal demand circumstances. From this test, it can be concluded the policy works very well when there is very high demand. Furthermore, as can been seen in Table 1 the percentage of products that are outdated is almost equal to zero. This is a very interesting result.

9.2.3 Data relationship correctness

Data relationship correctness is a check to see if the relationship between data actually occurs in the simulation model. In this specific case, such a relation does exist. There is a relation with the service level and the costs that are encountered for the Backorders and the inventory kept on stock in the system. The relation should be that when the service level is low the costs of Backorders should be high while the costs of inventory on hand should be lower. If the service level is high the costs of Backorders should be low and also the costs for keeping SKU’s on stock should be high. After performing the extreme tests, it can be concluded that the expected data relationship exists.

Table 1 - Extreme test extreme high demand and results from simulation based on normal demand values

	Percentage of products outdated	**Total costs**	**Inventory holding costs**	**Outdating costs**	**Ordering costs**	**Percentage of Backorders**	**Average WSL**
Extreme test 1: short remaining product lifetimes	97.3%	1,213,573,672	1,995	1,213,534,876	36,800	0.02%	0.9999
Extreme test 2: high inventory levels with infinite lifetime	0%	43,164,085	43,164,085	0	0	0%	1
Extreme test 3: extreme high demand	0%	78,119	7,069	13,550	57500	0.08%	0.9998
Results simulation run normal values	14.27%	205,068,248	9,763	205,035,025	23,460	12.27%	0.9202

9. Case study results

In this chapter, the results that were found by the simulation model for the case study performed at Bonduelle are presented. For the case study, a simulation length of 1000 is determined with 10 simulation runs and a warmup period of 200 time periods. The simulation length is based on the time it takes the simulated mean to convert to the before known mean of an SKU. This is demonstrated in Figure 8.

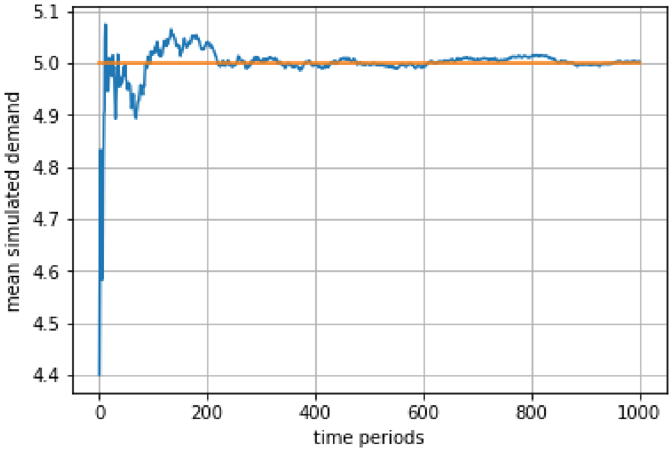


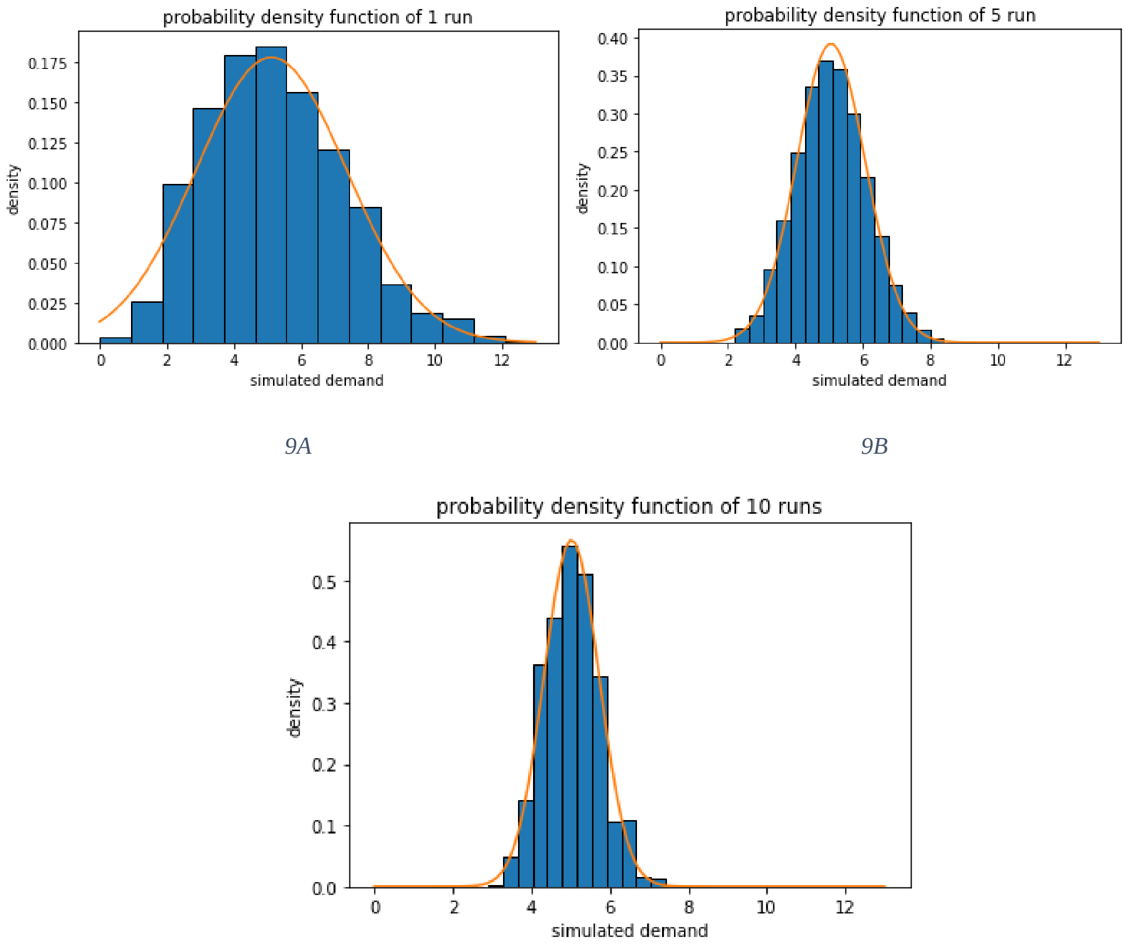
Figure 8 - Simulated mean of an SKU with a simulation run length of 1000 time periods

Figure 8 shows the simulated value of an SKU with an average of 5 demand per day with a standard deviation of 0.36. Following the fitting procedure of Adan et al. (1995) the distribution is a Binominal distribution. As can been in Figure 8 the simulated mean does not significantly change after 200 time periods meaning the simulation needs 200 time periods to be completely independent of the start-up effect. After examining the 95% confidence interval of a run length of 1000 and a warm-up period it can be concluded that a simulation run of 1000 time periods with a warmup period of 200 periods is a good enough predictor for the real simulation system. The 95% confidence interval is given in Table 2.

Table 2 - 95 % confidence interval of the simulated mean

Simulated Mean	Lower Bound 95%	Upper Bound 95%	Width	Accuracy
5.023	4.980	5.068	Narrow	Good

The central limit theorem (CLT) provides a means for determining the number of runs of the simulation model. The CLT relates the sample mean of a sufficiently large number of random values to the normal distribution. Figure 9 shows the probability density function of the demand for 1 simulation run, 5 simulation runs, and 10 simulation runs of the same SKU used in Figure 9.



9A *9B*

9C

Figure 9 - Three plots of the pdf on the normal distribution

Following this, it can be concluded that simulated values converge strongly to a normal distribution. Therefore, a choice is made to execute 10 simulation runs.

9.1 Simulation inputs

After defining the simulation length and the number of simulation runs, the simulation is used to compare the results of the current inventory policy without accounting for perishability of inventory and without the algorithm to always order full truckloads for the replenishment of the warehouse to the inventory policy and algorithm presented in this paper. The simulation also needs some input information. The input the simulation model needs is:

- Number of boxes on one pallet
- Weight of one pallet of one SKU
- The factory where the SKU is produced
- The number of days a box of SKU can be held on stock before the box cannot be sold anymore due to outdating of the box
- Safety stock level. For the simulation, the safety stock levels that are currently used with the ABC classification are used.

9.2 Cost factors

In order to compare the results from different situations, it is opted to compare the results based on the total costs and the weighted customer service level KPI. To be able to find total costs different cost factors are identified.

The outdating cost factor

The outdating cost factor is denoted as $C_{i,Z}$. The outdating costs are determined to be the money lost due to outdating. So the outdating costs for 1 box are the same as the price at which the box is sold to the customer. To give an example the price at which 1 box is sold for 1 particular SKU is 33.5 euros.

Ordering costs

The cost of ordering is defined as the cost paid of ordering 1 FTL denoted as C_{NO}. The costs are the transportation costs that are paid to transport an FTL from the factory to the warehouse. The transportation costs that are paid for ordering 1 FTL are 230 euros.

Inventory holding costs

The inventory holding costs are the costs that are kept of holding 1 pallet on inventory for 1 day denoted as $C_{i, IO}$. The inventory holding costs for keeping 1 pallet on inventory is 0.17 euro.

Backordering costs

In the situation of Bonduelle, the backorder costs are set equal to zero. The reason for setting the backorder costs to zero is because the backorder cost would be the transportation costs from the warehouse to the customers of the number of boxes that are back-ordered. However, the costs that are paid for transporting products from the warehouse to the customers is determined for sending complete client orders. So, the costs of the backorders are included in the costs Bonduelle has for transporting products to the customers.

9.3 Comparing the current policy to the thesis policy

In this section, the results of the current inventory policy are compared with the results of the inventory policy presented in this thesis. Table 3 shows the results of the thesis policy compared with the current policy. As can been seen in Table 3, the percentage of products which are outdated increases but the total outdating cost decreases. This means fewer products were kept on stock using the thesis policy. This is supported by the inventory holding costs and the ordering costs that are less than the inventory holding costs and ordering costs that are made in using the current policy. As also can been seen in Table 3 the reduction in inventory that is kept at the warehouse has a direct relation with the weighted service level and the percentage of backorders.

In order to further analyse the performance of the policy and algorithm that is developed in this thesis the results of the simulation runs when the current inventory policy is used in combination with the algorithm in chapter 6 and the policy in chapter 5 together with the current way of increasing the demand factor is also given. As can been seen in Table 3 the policy and algorithm are able to achieve a significant cost reduction compared to the current policy used with a factor to multiply with the demand to achieve FTL. As can been seen in Table 3 the thesis policy based on the EWA policy actually outperforms the current policy on both total costs and AWSL.

Table 3 - Results of simulation runs comparing thesis policy with the current policy

	Percentage of products outdated	Total costs	Inventory holding costs	Outdating costs	Ordering costs	Percentage of Backorders	Average WSL
Thesis policy + algorithm	14.27%	205,068,248	9,763	205,035,025	23,460	12.27%	0.9202
Current Policy+ Current way to order FTL	10,60%	277,161,559	17,380	277,161,559	34,040	5.42%	0.9519
Thesis policy + Current way to order FTL	8.23%	214,573,918	16,787	214,524,011	33,120	0.00%	0.9999
Current inventory policy + algorithm	18.20%	209,607,916	7,965	209,577,180	22,770	17.31%	0.9126

9.4 Changing the safety stock levels

As can been seen in Table 3 the main costs that are made are outdating costs. In this section, the performance of the simulation is examined when the safety stock levels are changed. As mentioned before the initial safety stock level that is used in the simulation runs is based on the current ABC classification. Table 4 shows the performance of the current policy when the safety stock levels are changed. As can been seen in Table 4, when the safety stock is decreased the total costs also decrease. However, the average weighted service level does not change a lot. What can be concluded from Table 4 is that for the current policy the current safety stock levels are too high. Since with lower safety stock levels, a similar average weighted service level can be achieved. Table 5 shows the results when the safety stock levels are decreased for the thesis policy. Similar to the results for the current thesis policy, the total costs decrease when the safety stock levels are decreased. Unlike the results in Table 4, the Average WSL over the simulations runs (AWSL) does drop a lot when the safety stock levels are reduced in the thesis policy. This behaviour is expected since lower safety stock levels mean less inventories and more backorders. This means there is a trade-off between the outdating costs that are paid and the AWSL that is guaranteed.

Table 4 - Changing safety stock levels current policy

	Percentage of products outdated	Total costs	Inventory holding costs	Outdating costs	Ordering costs	Percentage of Backorders	Average WSL
SS set to zero	9.21%	229,428,128	15,220	229,380,477	32,340	9.98%	0.9513
SS decreased by 3	9.32%	259,466,103	18,033	259,414,489	33,580	7.41%	0.9516
SS decreased by 2	11.24%	273,376,346	15,663	273,327,333	33,350	11.37%	0.9517
SS decreased by 1	10.65%	265,960,376	15,673	265,911,812	32,890	6.14%	0.9501
Current Policy	10.60%	277,161,559	17,380	277,161,559	34,040	5.42%	0.9519
SS increased by 1	10.43%	285,005,430	16,276	284,955,805	33,350	9.93%	0.9516
SS increased by 2	9.94%	281,351,858	17,213	281,301,066	33,580	7.24%	0.9499
SS increased by 3	11.78%	302,766,233	16,639	302,717,164	32,430	11.40%	0.9519

Table 5 - Changing safety stock levels thesis policy

	Percentage of products outdated	Total costs	Inventory holding costs	Outdating costs	Ordering costs	Percentage of Backorders	Average WSL
SS set to zero	13.67%	133,880,250	7,381	133,850,559	22,310	47.18%	0.6373
SS decreased by 3	10.89%	123,165,018	8,333	123,134,835	21,850	33.22%	0.7294
SS decreased by 2	7.92%	185,037,297	8,134	184,993,083	22,080	39.56%	0.6704
SS decreased by 1	18.08%	253,237,727	9,716	253,205,011	23,000	28.80%	0.8540
Thesis Policy	14.27%	205,068,248	9,763	205,035,025	23,460	12.27%	0.9202
SS increased by 1	13.43%	203,254,156	9,880	203,220,127	24,150	5.01%	0.9637
SS increased by 2	14.29%	208,802,857	8,981	208,769,955	23,920	0.86%	0.9929
SS increased by 3	15.61%	198,645,121	8,864	198,613,486	22,770	0.18%	0.9989

9.5 Joint Replenishment

In joint replenishment multiple, SKU's are ordered at the same time. All these SKU's do not necessarily share the same optimal reorder period. In the current situation at Bonduelle, the inventory check to see what products need to be replenished is done once every week. In order to check if the review period of 1 week Bonduelle currently uses is a good method of dealing with joint replenishment, the paper of Viswanathan (1997) is used. Viswanathan (1997) simulates a periodic inventory control policy a number of times while in each simulation the value of the review period is slightly increased with steps of 0.003 until an optimal value for the review period is found. The same principle applied here. The review period is increased with steps of 1 day. This value was chosen since the current review period is 5 working days (1 week) and 1 day means checking the inventory once a day. Table 6 shows the results of the simulation runs with the different values for the periodic review period for the current inventory policy. Again, the outdating costs are the biggest costs that are made in each simulation. It seems that for a periodic review period of 10 working days or 2 weeks the total costs are the lowest for the current policy. As also can been seen in Table 6 the AWSL does not change very much when the review period is extended meaning that doing period reviews once every two weeks would be the preferred option for the current inventory policy.

Table 7 shows the results of the simulation runs when the periodic review period is changed for the thesis policy. As can been seen in Table 7 the total costs are the lowest when a review period of 9 working days is used. Note that the ASWL decreases for the thesis policy when the review period is increased. When comparing the results from both Table 6 and Table 7 it can be concluded that the current policy outperforms the policy that is presented in this thesis when the review period is greater than 6 working days.

Table 6 - Periodic review period changed current policy

R	Percentage of products outdated	Total costs	Inventory holding costs	Outdating costs	Ordering costs	Percentage of Backorders	Average WSL
1 day	10.86%	435,345,186	116,889	435,187,358	40,940	3.14%	0.9791
2 days	9.92%	363,351,384	54,912	363,257,602	38,870	2.11%	0.9686
3 days	10.55%	338,111,646	33,930	338,040,456	37,260	1.15%	0.9543
4 days	10.90%	343,131,000	23,782	343,071,108	36,110	3.74%	0.9577
5 days	10.60%	277,161,559	17,380	277,161,559	34,040	5.42%	0.9519
6 days	10.04%	220,345,286	12,289	220,302,176	30,820	8.82%	0.9691
7 days	10.08%	188,084,136	8,619	188,047,227	28,290	9.56%	0.9284
8 days	9.51%	152,288,880	6,829	152,255,142	26,910	12.11%	0.9622
9 days	9.07%	97,274,571	4,845	97,244,886	24,840	15.72%	0.9212
10 days	8.94%	79,891,954	3,511	79,865,903	22,540	25.85%	0.9531

Table 7 - Periodic review value changed thesis policy

R	Percentage of products outdated	Total costs	Inventory holding costs	Outdating costs	Ordering costs	Percentage of Backorders	Average WSL
1 day	13.25%	253,532,503	58,940	253,448,954	24,610	10.09%	0.9989
2 days	14.05%	228,090,458	26,355	228,039,953	24,150	10.65%	0.9875
3 days	16.58%	383,900,650	22,898	383,849,692	28,060	11.32%	0.9532
4 days	17.90%	220,112,196	10,097	220,079,099	23,000	11.78%	0.9344
5 days	14.27%	205,068,248	9,763	205,035,025	23,460	12.27%	0.9202
6 days	17.81%	202,305,636	9,483	202,270,623	25,530	12.88%	0.9198
7 days	18.27%	200,121,118	8,527	200,086,371	26,220	14.36%	0.9080
8 days	17.32%	206,773,672	5,161	206,745,511	23,000	16.78%	0.8975
9 days	17.67%	124,305,567	2,972	124,282,125	20,470	13.45%	0.9035
10 days	19.37%	166,526,844	3,294	166,503,310	20,240	15.12%	0.8865

9.6 Sensitivity analysis on the outdating costs

In this section, a sensitivity analysis is conducted on what would happen if the outdating costs are changed. This sensitivity analysis will show how much the total costs are dependent on the outdating costs. Table 8 shows the sensitivity analysis over the current policy and Table 9 shows the sensitivity analysis over the thesis policy.

As can been seen in Table 8 the outdating costs decrease when the outdating costs per SKU is reduced. This is as expected. The same holds that the total costs increase when the outdating costs are increased. As can been seen in Table 9 the results for the thesis policy show the same effect. As can been seen in both Table 8 and Table 9 the thesis policy seems to be performing better than the current policy when the outdating costs are changed slightly.

Table 8 - Changing the outdating costs current policy

	Percentage of products outdated	Total costs	Inventory holding costs	Outdating costs	Ordering costs	Percentage of Backorders	Average WSL
- 20% outdating costs	10.36%	179,233,585	13,923	179,192,062	27,600	5.54%	0.9518
- 10% outdating costs	9.18%	247,421,863	17,036	247,371,477	33,350	9.60%	0.9545
Current policy	10.60%	277,161,559	17,380	277,161,559	34,040	5.42%	0.9519
+ 10% outdating costs	9.08%	283,948,217	16,353	283,899,204	32,660	10.99%	0.9534
+ 20% outdating costs	9.36%	298,517,371	13,463	298,477,688	28,220	5.57%	0.9514

Table 9 - Changing the outdating costs thesis policy

	Percentage of products outdated	Total costs	Inventory holding costs	Outdating costs	Ordering costs	Percentage of Backorders	Average WSL
- 20% outdating costs	15.34%	131,937,910	10,376	131,907,974	19,560	11.89%	0.9219
- 10% outdating costs	15.33%	166,876,211	10,357	166,847,754	18,100	11.41%	0.9256
Thesis policy	14.27%	205,068,248	9,763	205,035,025	23,460	12.27%	0.9202
+ 10% outdating costs	15.55%	22,257,925	11,151	22,225,215	21,560	12.98%	0.9188
+ 20% outdating costs	14.37%	234,973,973	10,310	34,946,332	22,330	12.43%	0.9205

10. Conclusions

This chapter gives an overview of the conclusions and recommendations that can be made based on the research done in this thesis project. The conclusions are based on the answers to the research questions. The main research question is formulated as:

What kind of inventory control policy is able to handle perishability of inventory, joint replenishment and always orders full truckloads for replenishment?

To help and find an answer to the main research questions four sub-questions are answered.

SQ1: What is the current set up of the supply chain of Bonduelle?

In chapter 3 the current supply chain operations are discussed. The current inventory policy is classified as a basic (R,s,nQ) policy. To deal with the full truckload constraint a factor is introduced. The sum of the demand over period R+L is than multiplied with this demand factor until one or multiple full truckloads can be ordered for the replenishment of the warehouse.

SQ2: What previous research has been conducted on joint replenishment, perishable inventory?

A literature study was performed, and the main findings of the literature study are presented in Chapter 4. The heuristic of Broekmeulen & van Donselaar (2009) was deemed the best suited to model a perishable inventory in an (R,s,nQ) policy and the heuristic of Garey et al. (1976) was used to solve the bin packing problem when multiple trucks need to be ordered.

SQ3: How should the outdating of products be modelled?

In chapter 5 the policies that are actually used are explained and the mathematical equations that are used are given. In this chapter, the heuristic policy proposed by Broekmeulen & van Donselaar (2009) that is able to incorporate the perishability of inventory in ordering decisions is explained. The policy is extended with an algorithm in chapter 6 to satisfy the constraint that always an FTL needs to be ordered.

SQ4: How should the joint replenishment be included in the inventory control policy?

In chapter 9 a sensitivity analysis is performed of the current policy and the thesis policy over the periodic review period. This sensitivity analysis is done based on the paper of Viswanathan (1997) that proposes to incrementally increase the value for the periodic review moment until an optimal value for the periodic review moment is found. As mentioned, before it is found in chapter 9 that the current policy actually outperforms the policy proposed in this thesis when the periodic review period is greater than 6 working days.

MQ: What kind of inventory control policy is able to handle perishability of inventory, joint replenishment and always orders full truckloads for replenishment?

All taken together a policy is created based on the heuristic of Broekmeulen & van Donselaar (2009) which is extended with an algorithm that is specially developed to ensure replenishment orders always are FTL where the heuristic of Garey et al. (1976) is used to solve the bin packing problem to allocate pallets to the trucks that are ordered. The performance of the described policy is compared to the current policy by use of a simulation. The policy is generic enough so that it could be used in other studies that also face problems with the perishability of inventory while satisfying an order constraint.

11. Recommendations

In this chapter some recommendations are made. First recommendations for Bonduelle are given. Secondly recommendations for the academic literature for further studies are given.

11.1 Recommendations for Bonduelle

The recommendations for Bonduelle are listed in this section.

Evaluate the target service level

Bonduelle has set a target weighted service level of 98.5%. This target for the weighted service level is rather high. With this high WSL the costs in the warehouse are also very high. If a small reduction is the WSL is considered a significant cost reduction could be achieved.

Implement the new inventory policy and algorithm

The policy that is presented in this thesis is able to achieve a reduction in total cost. However the WSL for the warehouse is also reduced. Therefore if the goal is to reduce the costs the policy presented in the thesis would be the best option. However, if the goal is to reduce the costs while keeping a target WSL of 98.5 % the extended EWA policy that is presented in the thesis in combination with the current way of adding pallets to an order to assure FTL would be a better policy.

Evaluate the periodic review period

Based on the results of the case study the periodic review period could be revised. If the target WSL of 98.5 % has to be achieved the thesis policy with a periodic review period of 2 days would result in the minimal costs while satisfying the WSL.

Evaluate the SKU composition

As can been seen in the results the outdating costs are very high. Concluding from the results of the extreme tests, this could be because the low demand for certain SKUs. The main issue with these kind of products and the outdating are that when a pallet of the specific SKU is ordered, a lot of demand can be satisfied with just 1 pallet. The issue of overstock is directly related to this problem. For instance for 1 particular SKU 1 pallet could satisfy 139 weeks of demand. Therefore focussing on the SKUs with higher demand could solve some of the issues Bonduelle faces with overstock.

11.2 Recommendations for Academic literature

Some subject is identified that are well suited for further research.

Allocation Heuristic

In this master thesis, the heuristic of Garey et al (1976) has been used. The allocation heuristic is used in combination with a method to add pallets to a truck on an order. In this algorithm of adding pallets, an allocation is made based on the quantities that are ordered according to the EWA policy. This initial allocation determines the number of trucks that are used to fulfil the demand. After finding the number of trucks, the remaining space in the trucks is analysed to see if a pallet can be added to a truck. For instance, if three trucks need to be ordered according to the first allocation of pallets, and truck 1 and 3 still have a pallet place available and enough remaining capacity on weight. A pallet is added to either truck 1 or 3 based on the weight of the trucks respectively. This process of adding pallets to the trucks is repeated till no pallet places are available or no pallet can be added anymore due to weight issues.

Solution to the knapsack problem

In this an algorithm is made to ensure FTLs are ordered for the replenishment of the warehouse. The algorithm that has been presented resembles a Greedy algorithm where the pallet that would add the least estimated cost is added to an replenishment order until a FTL can be ordered. Different solution approaches to solve this knapsack problem could be further developed and could lead to a better performance in future research.

References

Atkins, D., & Iyogun, P. (1988). Periodic versus 'can-order' policies for coordinated multi-item inventory systems. Management Science, 34(6), 791-796.

Broekmeulen, R., & Van Donselaar, K. (2009). A heuristic to manage perishable inventory with batch ordering, positive lead-times, and time-varying demand. Computers and Operations Research, 36(11), 3013-3018. doi:10.1016/j.cor.2009.01.017

Chun-Wei R. Lin, Hsian-Jong Hsiau & Neng-Shu Yan (2010) An approximate scheduling policy for joint replenishment problem with transport capacity constraint, Journal of the Chinese Institute of Industrial Engineers, 27:4, 316-330, DOI: 10.1080/10170669.2010.488025

Coelho, L., & Laporte, G. (2014). Optimal joint replenishment, delivery, and inventory management policies for perishable products. Computers and Operations Research, 47, 42-52. doi:10.1016/j.cor.2014.01.013

Federgruen, A., Groenevelt, H., & Tijms, H. (1984). Coordinated replenishments in a multi-item inventory system with compound Poisson demands. Management Science, 30(3), 344-357.

Fries, B. (1975). Optimal ordering policy for a perishable commodity with a fixed lifetime. Operations Research, 23(1), 46-61.

Garey, M., Graham, R., Johnson, D., & Yao, A. (1976). Resource-constrained scheduling as generalized bin packing. Journal of Combinatorial Theory, Series A, 21(3), 257-298. doi:10.1016/0097-3165(76)90001-7

Hadley, G., & Whitin, T. (1963). Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs, N.J.

Johansen, S., & Melchiors, P. (2003). Can-order policy for the periodic-review joint replenishment problem. Journal of the Operational Research Society, 54(3), 283-290. doi:10.1057/palgrave.jors.2601499

Sargent, R. (2013). Verification and validation of simulation models. Journal of Simulation, 7(1), 12-24. doi:10.1057/jos.2012.20

Kellerer, H., Pferschy, U., & Pisinger, D. (2004). Knapsack problems (Springer nature book archives millennium (2000-2004)). Berlin: Springer

Nahmias, S. (2011). Perishable inventory systems (International series in operations research & management science, v. 160). New York: Springer. doi:10.1007/978-1-4419-7999-5

Nielsen, C., & Larsen, C. (2005). An analytical study of the q(s, S) policy applied to the joint replenishment problem. European Journal of Operational Research, 163(3), 721-732. doi:10.1016/j.ejor.2004.02.003

Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models. The Journal of the Operational Research Society, 42(1), 27-37.

Silver, E., & Peterson, R. (1985). Decision systems for inventory management and production planning (2nd ed., Wiley series in production - operations management). New York: Wiley.

Silver, E., Pyke, D., & Peterson, R. (2001). Inventory management and production planning and scheduling (third edition). Journal- Operational Research Society, 52, 845-845.

Tekin, E., Gürler, U., & Berk, E. (2001). Age-based vs. stock level control policies for a perishable inventory system. European Journal of Operational Research, 134(2), 309-329. doi:10.1016/S0377-2217(00)00250-2

van Donselaar, K., & Broekmeulen, R. (2012). Approximations for the relative outdating of perishable products by combining stochastic modelling, simulation, and regression modelling. International Journal of Production Economics, 140(2), 660-669. doi:10.1016/j.ijpe.2012.02.023

van Donselaar, K., & Broekmeulen, R. (2013). Determination of safety stocks in a lost sales inventory system with periodic review, positive lead-time, lot-sizing, and a target fill rate. International Journal of Production Economics, 143(2), 440-448. doi:10.1016/j.ijpe.2011.05.020

Viswanathan, S. (1997). Periodic review (s, S) policies for joint replenishment inventory systems. Management Science, 43(10), 1447-1454.

Xue-Yi Ai, Jin-Long Zhang & Lin Wang (2017) Optimal joint replenishment policy for multiple non-instantaneous deteriorating items, International Journal of Production Research, 55:16, 4625-4642, DOI: 10.1080/00207543.2016.1276306