BACHELOR

Arrival processes at traffic intersections

Vonken, Hugo M.

Award date:
2017

Bachelor final project

2WH40

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Author:
Hugo Vonken, ID 0860541

Supervisor:
dr. i.r. M.A.A Boon

January 27, 2017

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Chapter 1

Introduction

The goal of this report is to better understand the arrival processes in urban traffic networks. Every day more and more people are using vehicles to drive to work, school, friends etcetera. As a result roads are getting crowded and congestions might arise at intersections. Therefore analysing intersections is interesting, because a better understanding might help to avoid congestions. There is already a lot of literature about queueing models concerning intersections, however that literature mainly focusses on the queue lengths, waiting times and the saturation rate of the intersection. A commonly made assumption is that the arrival process will be a Poisson process. However in real life this is often not the case. This report will focus on other suitable arrival processes for signalized and non-signalized intersections. In Figure 1.1 a simple example of a possible situation is shown.

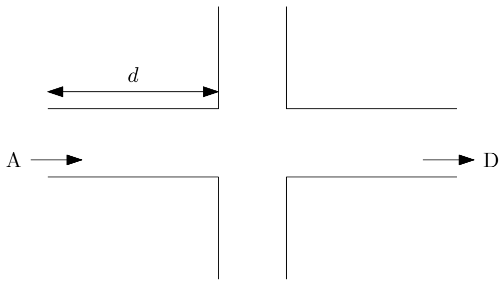

Figure 1.1: The basic situation.

Figure 1.1 shows an arrival process A, which is taking place at a distance d from an intersection. The arrival process will contain vehicles only. This means no pedestrians nor cyclists. Every vehicle will be driven with a random speed S. On the road of length d overtaking is forbidden. Therefore, drivers will have to adapt their speed to other slower drivers in a safe way if needed. The term headway will be defined as the time between a car and his predecessor. This means that a larger headway will reflect in a larger distance between two successive vehicles and thus a safer situation. An arriving vehicle belongs to an arrival process A with a random speed S. This vehicle will arrive at the intersection after travelling a distance d. As a consequence, the arrival process at the intersection will not be exactly the same as arrival process A. However it is interesting to know what will change. This report will consider the following main questions:

- Which model would give a good representation of arrival process A?
- What is the influence of the distance d on arrival process A. Remark that arrival process A is before travelling the distance d. What arrival process will take place at the intersection?
- What is a good representation of the departure process D from the intersection?
- What happens to the arrival pattern in a network of intersections?

Notice that the arrival process and the influence of distance d are independent of the type intersection. It does not matter whether a signalized or non-signalized intersection is concerned. However the departure process does depend on whether the intersection is signalized or not.

The report will be structured as follows:

- First, in Chapter 2 existing literature about arrival and departure processes will be analysed.
- Second, in Chapter 3 the influence of travelling a distance d on the arrival process A will be studied.
- Next, the arrival process after crossing a network of intersections will be looked into. This will be done in Chapter 4.
- Finally, there will be checked whether the main questions can be answered and conclusion will be drawn. This will be done in Chapter 5.

Chapter 2

Literature study

Traffic congestion at intersections is a well studied topic. In this chapter the existing literature will be discussed and the different types of arrival and departure processes will be mentioned.

2.1 Arrival process

Perhaps the best way to model an arrival process at a traffic intersection has been introduceed by Cowan [1]. Since cars need to keep a safe distance between each other while travelling Cowan considered the headway as important factor in the arrival process. He considered a headway X to have two different components, $X = V + U$ where U is the free component and V the tracking component. V and U may both be random variables. The free component U can be seen as the inter arrival time distribution. The tracking component V can be seen as the headway which is kept between two successive cars to keep a safe distance, so when the vehicles already caught up with each other. V is a restriction on U, it makes sure a minimum headway is kept. The following headway models were introduced by Cowan, were $F(x)$ represents the cumulative distribution:

Model 1. $F(x) = \begin{cases} 0 & \text{if } x < 0, \\ 1 - e^{-\lambda x} & \text{if } x \geq 0. \end{cases}$

$\mathbb{E}[X] = \frac{1}{\lambda}.$

In the first model the arrival process is a Poisson process, because the inter arrival times are simply exponentially distributed. The tracking component in this model is considered to be 0, while U, the free component, is exponentially distributed.

Model 2. $F(x) = \begin{cases} 0 & \text{if } x < \tau, \\ 1 - e^{-\gamma(x-\tau)} & \text{if } x \geq \tau. \end{cases}$

$\mathbb{E}[X] = \tau + \frac{1}{\gamma}, \qquad \lambda = \frac{\gamma}{\gamma\tau + 1}.$

Model 2 is considering a shifted exponential distribution. The tracking component V equals a constant τ and the free component U is again exponentially distributed. In other words the headway will always be at least τ. The flow rate of each model is defined as λ.

Model 3. $F(x) = \begin{cases} 0 & \text{if } x < \tau, \\ 1 - (1-\theta)e^{-\gamma(x-\tau)} & \text{if } x \geq \tau. \end{cases}$

$\mathbb{E}[X] = \tau + (1-\theta)\frac{1}{\gamma}, \qquad \lambda = \frac{\gamma}{\gamma\tau + (1-\theta)}.$

In Model 3 the headway is τ with a probability θ and with probability $(1 - \theta)$ larger than τ. Therefore, the traffic flow will contain bunches and gaps. A bunch is a group of vehicles with minimum headway τ. These vehicles will create platoons. A gap is created by vehicles with a headway larger than τ, these vehicles will end the previous platoon. The bunch size M is geometrically distributed. Thus holds $\mathbb{P}[M = m] = (1-\theta)\theta^{m-1}$ with $m \geq 1$. Furthermore the gap Y between two bunches is caused by the free component of the headway and is distributed as $\mathbb{P}[Y \leq y] = 1 - e^{-\gamma y}$.

Model 4. $F(x) = \begin{cases} 0 & \text{if } x < 0, \\ \theta B(X) + (1-\theta)\int_0^x B(x-u)\gamma e^{-\gamma u} du & \text{if } x \geq 0. \end{cases}$
In this case $B(x)$ is the distribution of random variable V.
$$\mathbb{E}[X] = \mathbb{E}[V] + (1-\theta)\frac{1}{\gamma}, \qquad \lambda = \frac{\gamma}{\gamma\mathbb{E}[V] + (1-\theta)}$$
Model 4 is an extension of Model 3, the tracking component no longer is a constant but a random variable with distribution $B(x)$.[1]

As can be reasoned, Model 3 and Model 4 are the most realistic models. They include the minimal headway which is kept in reality, but they also include possible group arrivals caused by slower moving vehicles. Hagring (1998) claimed that: "Recent research by Sullivan and Troutbeck (1994), Akçelik and Chung (1994) and Hagring (1996) have shown that the M3 distribution introduced by Cowan (1975), provides good fit to observed headway distributions.". [2] The M3 distribution introduced by Cowan which is mentioned by Hagring is also known as Model 3 in this report.

2.2 Arrival at an intersection

As depicted in Figure 1.1 a car arriving according to the arrival process A travels a distance d before reaching the intersection. Since every car has a random speed S, this might influence the arrival process when arriving at the intersection. In Cowan (1975) the following was introduced considering this situation on a road of length r. Assume every driver has a random speed S, for which holds $\mathbb{P}[S \leq s] = D(s)$. The driver will keep this speed until he reaches his predecessor, at that point he will directly adapt his speed to that of his predecessor. However, the driver will of course have to keep an appropriate tracking headway. Cowan denoted the cumulative distribution of the travelling time T as Ω(t). Before analysing Ω(t), Cowan first introduced the following concepts. The optimal travel time $P = r/S$, with distribution $\Phi(p) = 1 - D(\frac{r}{p})$. The optimal travel time can be seen as the situation when a car does not need to slow down for his predecessor. Furthermore if we consider the travel time of his predecessor to be T', then it holds that $T = \max(P, T' - U)$. Eventually Cowan showed that:

$$\Omega(t) = \frac{(1-\theta)\Phi(t)}{1-\theta\Phi(t)} \exp\left[-\gamma\int_t^\infty \frac{(1-\theta)\Phi(u)}{1-\theta\Phi(u)} \mathrm{d}u\right]. \tag{2.1}$$

for every $\Phi(t)$, θ en $0 \leq \lambda < \mathbb{E}[V]^{-1}$.[1]

2.3 Departure process

The departure process, as earlier said, depends on the type of intersection. There is a distinction between signalized and non-signalized intersections, both types will be discussed in the next subsections.

2.3.1 Signalized intersections

First, we will consider a signalized intersection. When cars arrive at a signalized intersection, they can arrive in 3 different types of situations. A car might arrive in a red period, that car needs to join the queue and wait for the green light. A car might arrive in a green period when there is nobody left in the queue and he can immediately leave the intersection. Finally, a car might arrive in a green period while there are still cars waiting and he also joins the queue in front of the traffic lights.

In Shao (2012) the departure process of the cars waiting in a queue during a red period is described. Shao introduced the term "queue discharge headway" as the time interval between two successive cars crossing an specified reference line. This leads to $h_s = t_r + \frac{L_{hj}}{v_s}$, with h_s the discharge headway; t_r the reaction speed of the driver; L_{hj} is the waiting space per vehicle including the length of the vehicle; v_s the speed of the vehicle. Next, he collected data from 11 different intersections in Beijing at the busiest period of the day. From the data he determined the distribution of the inter departure times. However, he disregards the outliers due to cyclists, distractions etcetera. Furthermore, he ignored the effect of the acceleration time and start up time by leaving out the first 5 recorded headways of every green period. Eventually he concluded that the mean headway is larger than the median headway. Also the skewness is positive which indicates that the distribution of the queue discharge headways is asymmetric. He also tested whether the queue discharge headway was normal or log-normal distributed. He showed that for 8 of the 11 intersections the queue discharge headway distribution is the log-normal distribution and 3 out of 11 the normal distribution. [5]

Another paper on this topic is by Jin (2009). In this paper the departure headways are defined in the same way as in Shao (2012); Thus as the time between two successive cars crossing an in advanced determined reference line. However, this paper considers only cars departing during a red period. He considers the place in the queue to be an important factor. He concluded in his paper that the departure process is place dependent. In other words, the place you have in the queue does influence the distribution of the departure process. When a vehicle is the second vehicle waiting in the queue or one behind that one, the headway distribution will follow a log-normal distribution. Every spot in queue is distributed by a log-normal distribution with different parameters. However the departure process of the 1^{st} car in queue is not log-normal distributed. Furthermore in this article a reason why cars follow a log-normal distribution is discussed. A driver will adjust its speed from time to time to track leading vehicles. This process contains increasing and decreasing speeds, but since these vehicles are considered to move in a queue and the leading vehicle's driving is unpredictable, the actual accelerations are typically smaller than the actual decelerations. This is due to the accelerations are constrained by the separation distance to the leading vehicle. Next, a comparison between other dynamic processes which are log-normal distributed is looked for. They claim that it is somewhat like particles falling down a log-normal type Galton board. [4]

In Cowan (1975) the number of departures until the queue is empty after a red period is studied. The traffic signals will be red during a time period of length $(0, \alpha)$. In this time a traffic queue will build up. Assume the length of the exit headway of a queued vehicle is τ. Furthermore let N be the number of departures until the queue is empty if we condition on the number of arrivals K within the time interval $(0,\alpha)$ the following results can be obtained. For Model 1 from Section 2.1 holds that N has a Borel-Tanner distribution,

$$\mathbb{P}[N = n|K = k] = \frac{ke^{-n\lambda\tau}(n\lambda\tau)^{n-k}}{n(n-k)!}, \text{ for } n = k, k+1, \ldots \tag{2.2}$$

However, since Model 3 is more realistic Cowan also shows that for Model 3 holds:

$$\mathbb{P}[N = n] = \frac{\lambda e^{-\gamma\alpha}}{\gamma} \sum_{i=0}^{n-1} \binom{n-1}{i} \theta^{n-1-i}(1-\theta)^{i+1} \left[\sum_{j=o}^{i} \frac{(\gamma\alpha)^j - (\gamma\alpha+\tau)^j e^{-\gamma\tau}}{j!} - \frac{\gamma(1-\lambda\tau)}{\lambda(i+1)!}(\gamma\alpha)^{i+1} \right]. \tag{2.3}$$

The density can be calculated explicitly for λ, γ, θ and τ for which holds $\lambda = \dfrac{\gamma}{\gamma\tau + 1 - \theta}$. [1]

2.3.2 Non-signalized intersections

We now consider two roads with one way traffic were overtaking is forbidden. One road will be called the minor road and one the major road, were the major road will have priority over the minor road. A minor road vehicle can only cross the road if there is a free period of at least β (the first gap acceptance time) in the major road stream, if no queue has formed at the minor road. Otherwise he starts waiting at the intersection until the first period of at least α (second gap acceptance time) between successive vehicles on the major road. Furthermore suppose the minor road vehicles arrive in bunches of size i in a Poison process with parameter λ. The arrival process at the major road is in a similar way. In Yeo and Weesakul 1964 the similarity between this situation and a $M/G/1$ queue was first noticed. The headway process does correspond exactly to the epochs of the $M/G/1$ customers entering service of duration V, with V a random variable which represents the tracking component of the headway as defined in Cowan

(1975). Furthermore the bunch size distribution will be the same as that of the number of customers in a busy period.[6] Cowan (1975) shows that when considering Model 3 from Section 2.1 at a non-signalized intersection the bunch size M will have a Borels distribution. Thus headways are not sequentially independent.

In Heidemann (1997) a short recap of frequently applied literature is given. The goal is to generalize the assumption within the frame of the $M/G2/1$-queuing system as far as possible. A lot of results are written about that in this article. However, the main focus now on the arrival process models. In Heidemann (1997) it stated that the most frequently used inter arrival time models are the expontional distribution, the shifted-exponential distribution and the bunched-exponential distribution. These are also known as Models 1,2 and 3 given within Section 2.1. Furthermore, the log-normal and Erlang distribution seem to be used too, but they are more common within simulation models. The rest of the article basically focuses on providing insight in the gap-acceptance and move up time models needed for crossing a non-signalized intersection. The corresponding gap-rejection and queue length distributions are explained after that.[3]

Chapter 3

The influence of travelling

In this chapter the influence of travelling a distance d will be discussed. At first, in Section 3.1 we will take a look at the possible ways to distribute the speed. So, for certain types of speed distribution the influence of travelling will be studied. We are doing this because the distribution of a vehicle's speed is yet unknown. Next, we will check for correlation within the arrival processes before and after travelling to get a better understanding in what way the arrival process is changing. It is important to know if we still can assume exponential inter arrival times or whether there is too much correlation to assume this. The platoon size distribution is also an important factor for better understanding the arrival process and therefore will be studied as well.

3.1 Speed distributions

In Section 2.1 the arrival processes have been introduced. In this chapter we will compare those processes using simulation. Models 1, 2 and 3 from Section 2.1 will be considered as arrival processes in this chapter. For each of these models, we will use simulation to give an impression of the headways generated with these models. The simulation describes the events in units of meter per second. In Figure 3.1 the inter arrival times of each model are plotted in a histogram. The data is generated based on a mean headway of 5 seconds, also known as 2 arrivals per 10 seconds. As a consequence, for the arrival parameter λ of model 1 it holds that $\lambda = 0.2$. Furthermore assume that $\theta = 0.3$, then 30% of the cars in model 3 will arrive within a platoon and which means keeping a minimum headway. The minimum headway is assumed to be 2 seconds, thus $\tau = 2.0$. With this information the γ for model 2 and 3 can be calculated. Note that the mean headway for models 2 and 3 needs to be 5 seconds. In Section 2.1 a relation between λ, γ, τ and θ is given. This can be rewritten to the following for model 2 and model 3. For model 2 holds that $\gamma_2 = \frac{\lambda}{1-\lambda\tau}$ and for model 3 holds that $\gamma_3 = \frac{\lambda(1-\theta)}{1-\lambda\tau}$. Thus $\gamma_2 = \frac{1}{3}$ and $\gamma_3 = \frac{7}{30}$.

Figure 3.1 gives an indication of the density of the inter arrival times of the 3 models. As can be seen in Figure 3.1 model 1 contains inter arrival times which are smaller than the minimum headway of 2 seconds. This is in practice not very common. Drivers tend to keep a safe distance between each other. Therefore models 2 and 3 might give a better representation of the arrival process. In model 2 the minimum headway is taken into account, this results in a higher density for headways of 2 seconds or higher. However the shape of the density remains the same. Models 2 could give a good representation of a real arrival process. Model 3 adds the feature of group arrivals to model 2, also known as platoons. This model can be used when the arrival process considers a road where overtaking is hard or forbidden, because in that case cars will arrive in groups. Notice the shape of the density of model 3 is different in comparison to model 1 and 2. There is a lot more mass at the minimum headway. This is caused by the group arrivals which are created with probability θ.

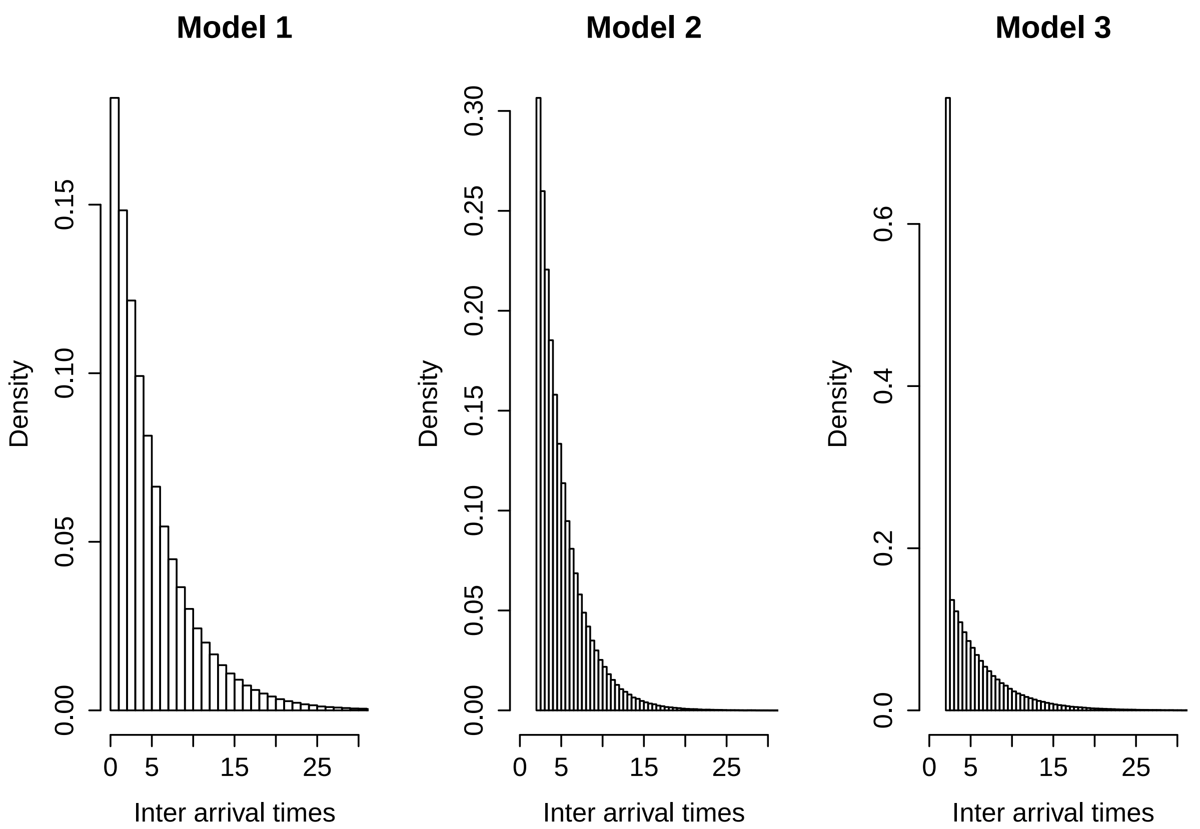

Figure 3.1: Histogram of the inter arrival times.

As was mentioned earlier a vehicle has to travel a distance d before arriving at the intersection. Since every car has a random speed S, this will influence the arrival process. We will consider the following distributions for the random speed S:

- S is constant,
- $S = \begin{cases} S_{min} & \text{with probability } 0.5, \\ S_{max} & \text{with probability } 0.5, \end{cases}$
- S is uniformly distributed, $S \sim \text{Unif}(S_{min}, S_{max})$,
- S is exponentially distributed, $S \sim \text{Exp}(1/S_{mean})$,
- S is normal distributed, thus $S \sim \text{N}(S_{mean}, S_{var})$.

The simulation will consider arrivals at a 80km/h road. We also assume $S_{min} = 75$km/h, $S_{max} = 85$km/h, $S_{mean} = 80$km/h, $S_{var} = 5\text{km}^2/\text{h}^2$, distance $d = 1.0$km and the simulation will contain 100 runs of 10,000 arrivals or 1 run of 1,000,000 arrivals in some other cases. The simulation output is in meters per second.

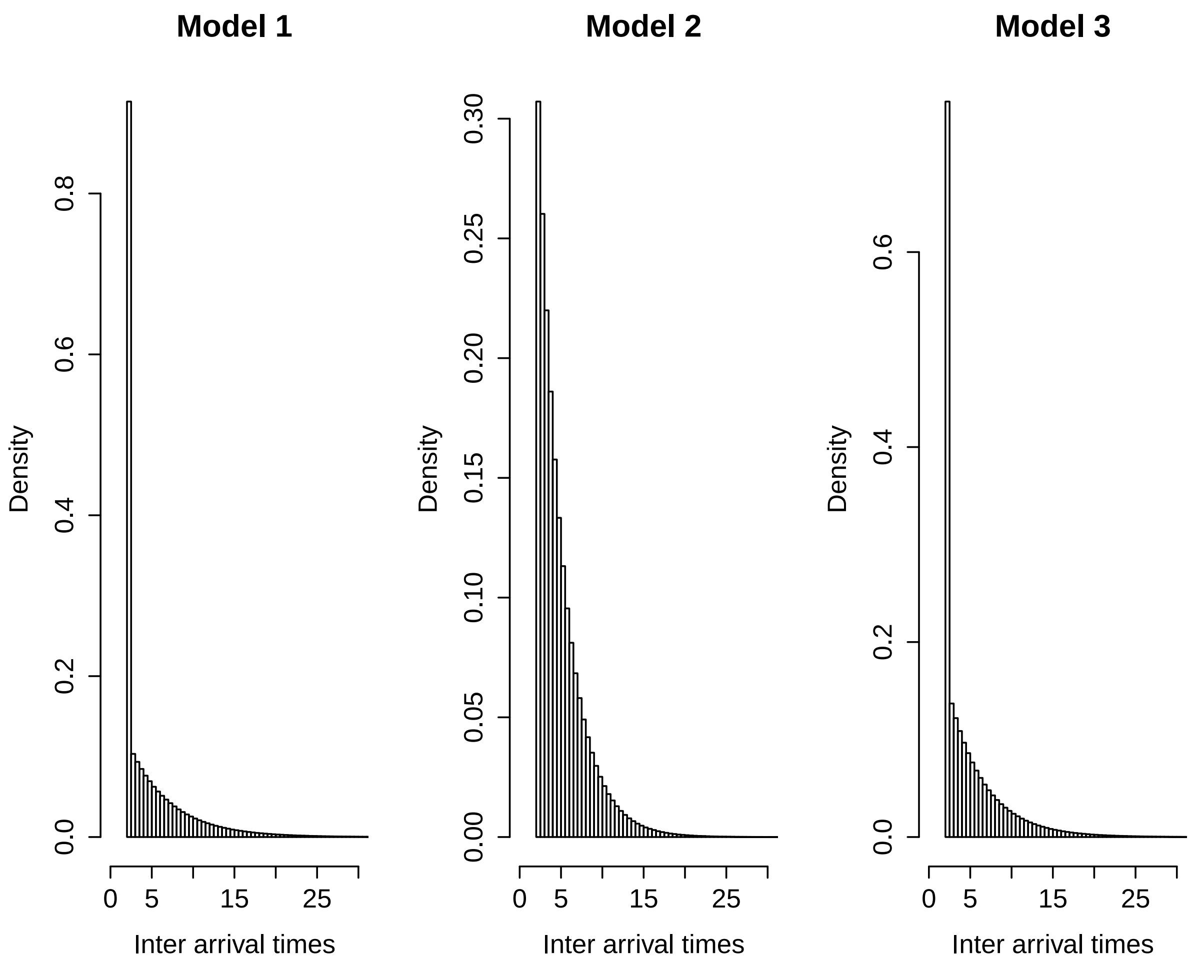

Figure 3.2: Histogram of the inter arrival times after distance d with constant speed.

S is constant implies that all vehicles will have the same speed. Therefore, the distance d will not influence the arrival process of model 2 and model 3. However, the arrival process of model 1 will change due the fact that a minimum headway will have to be kept. Figure 3.2 shows that the inter arrival times of model 2 and model 3 do indeed remain the same. Furthermore the distribution of model 1 did indeed change. The distribution of model 1 does contain a lot more mass at the minimum headway. This is caused by all the headways which were smaller than 2 seconds. Notice that the shape of model 1 now corresponds more with the shape of model 3 instead model 2 which was the case earlier. The probability of a headway smaller than 2 represents the same as the probability θ of arriving in a platoon, in both cases the platoon size increases. This causes the similarity, however the relationship between this probability and θ is not completely clear yet. This will be further investigated later on.

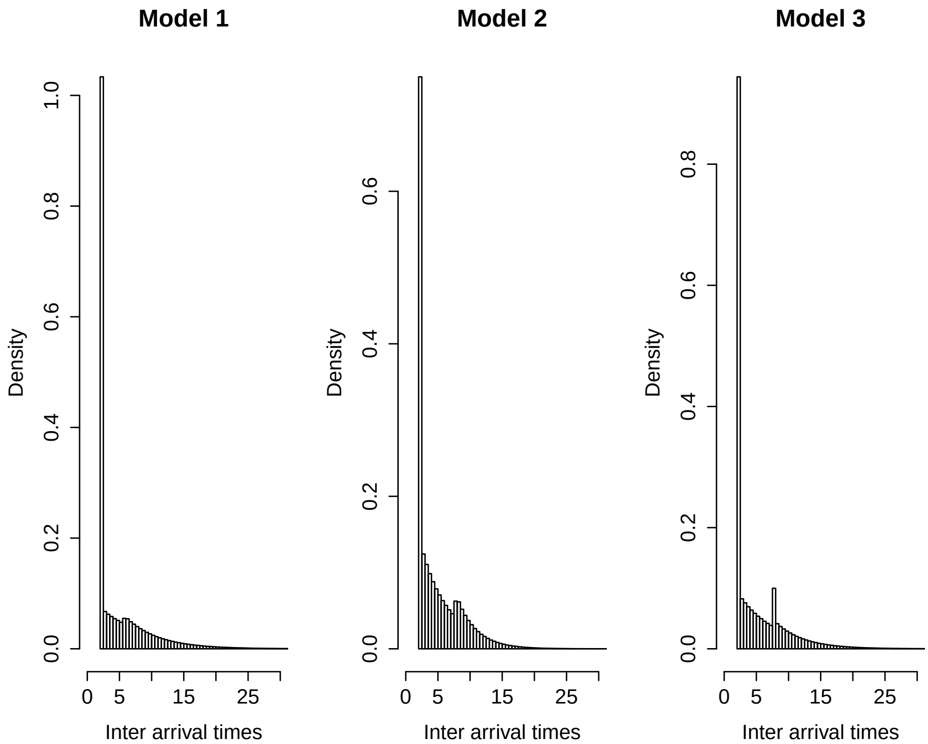

Figure 3.3: Histogram of the inter arrival times after distance d with a mininum and maximum speed.

A model considering a constant speed for each vehicle is not realistic, therefore we introduce a minimum and maximum speed. The vehicles will have a speed of 75km/h or 85km/h with probability 0.5, thus the mean speed will still be 80km/h. In Figure 3.3 the inter arrival times at the intersection of this situation is shown. Notice that because of this minimum and maximum speed the shape of the density has changed. The density distribution now contains two peak values, the first still coming from vehicles arriving within a platoon. But the second is caused by a vehicle driving at a minimum speed behind a vehicle driving with a maximum speed in which case the original inter arrival time was 2 seconds. In this case the original headway will increase, resulting in a peak near 7-10 seconds. A more detailed explanation can be given when considering model 3. Since $\theta = 0.3$ we can calculate the probability that inter arrival time $X_i = 2$.

$$\mathbb{P}\big[X_i = 2\big] = \theta = 0.30. \tag{3.1}$$

When considering a total number of Y inter arrival times, 30% of those inter arrival times will be 2 seconds and 70% will be larger than 2 seconds. As explained earlier, we are interested in the situation when arrival A_i has the maximum speed and A_{i+1} the minimum, when at the same time the inter arrival time between A_i and A_{i+1} is 2 seconds. Since the inter arrival time distribution is independent of the vehicle speed distribution, the following can be concluded. In 25% of the cases arrival A_i has the maximum speed and A_{i+1} the minimum. Because of the independence relationship we can conclude that in $0.25 \cdot 0.30 \cdot 100\% = 7.50\%$ of the cases this phenomenon takes place. In this case the inter arrival time would be the inter arrival time at the beginning of the road, plus the extra time the vehicle takes crossing the road which is together:

$$X_{Total} = X_{Begin} + \frac{d}{S_{min}/3.6} - \frac{d}{S_{max}/3.6} \approx 7.65 \text{ seconds.} \tag{3.2}$$

As we can see in Figure 3.3 this corresponds with the second peak for model 3, this peak does also have approximately the same mass. The mass in the second peak is even a little higher because a vehicle can be considered to be in the same bin within the histogram as those type of vehicles, while not meeting those requirements. The same explanation can be used for model 1 and 2. However, these models do not have the geometric distributed mass for inter arrivals times of 2 seconds.

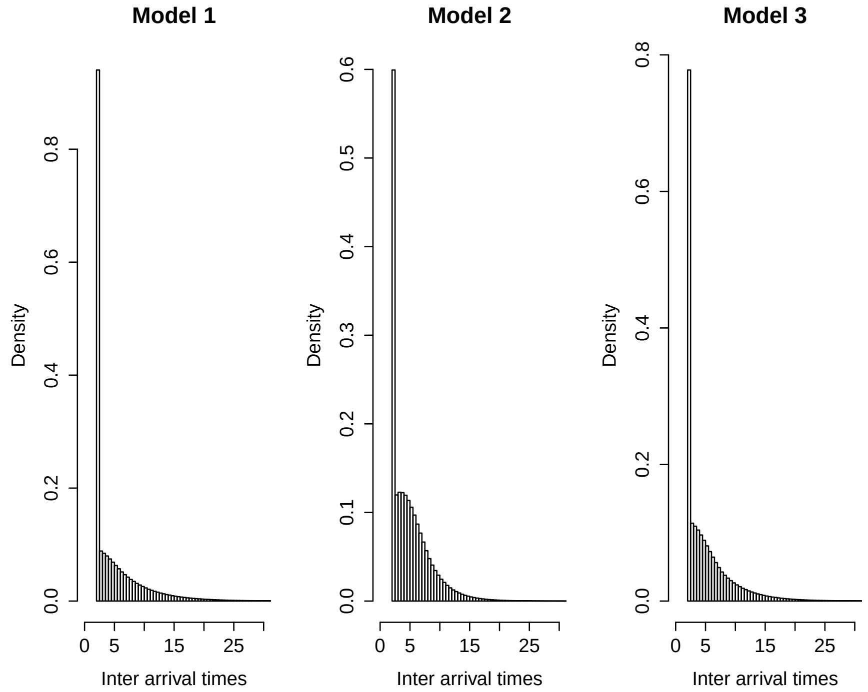

Figure 3.4: Histogram of the inter arrival times after distance d with an uniformly distributed speed.

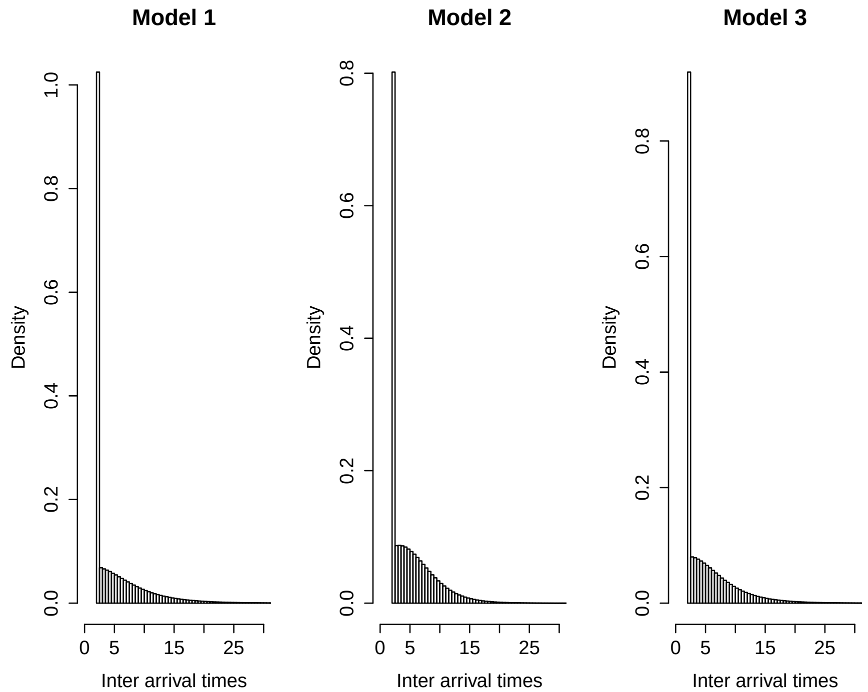

Figure 3.5: Histogram of the inter arrival times after distance d with a normal distributed speed.

	at arrival	constant	min/max	uniform	exponential	normaal
Mean model 1	4.999	4.999	4.999	4.999	390.362	4.999
Variance model 1	24.970	20.967	22.263	21.193	50646514295	22.493
Mean model 2	5.003	5.003	5.003	5.003	225.363	5.003
Variance model 2	9.046	9.046	13.407	10.941	3057759677	13.927
Mean model 3	5.008	5.008	5.008	5.008	234.193	5.008
Variance model 3	16.788	16.788	18.705	16.777	3738279291	18.668

Table 3.1: Mean and variance of the inter arrival times after distance d.

In Table 3.1 the mean and variance of the headways are given at the point of arrival, but also at the intersection. As can be seen the mean headways are constant for the parameters introduced before. Thus the number of cars entering the road is the same as the number of cars leaving the road within this time frame. However, this does not hold when the speed is exponentially distributed. As can be seen in Table 3.1 an exponential distribution for the speed is not realistic. It will cause an extremely large mean and variance headway because extremely low speeds will be reached. A speed of less than 10 km/h on a 80 km/h road might be reached with this model, this is however not realistic for a road considering basic type vehicles. Furthermore Table 3.1 shows that the variance of each model differs. Notice that the variance of the min-max model and the normal distribution are quite the same for all arrival models. Furthermore for model 2 and 3 the uniform distribution has almost the same variance as the constant speed distribution and the situation at arrival. Even though shape of the density distributions differ.

3.2 Correlation

In Section 2.1 the bunch size distribution of model 3 is mentioned. Bunches are geometrically distributed with probability θ. However the bunch size distribution of model 1 is not known. Furthermore the influence of distance d needs to be analysed. Since the bunch size of model 3 is geometrically distributed there can be noticed that a vehicle will be within the same bunch independent of the previous vehicles inter arrival time. Thus can we claim independence between inter arrival times. However through varying speeds over a distance d dependence might arise. We will test for this by using the auto correlation function. This function will check for correlation between the headway of car i and the headway of car $i + j$.

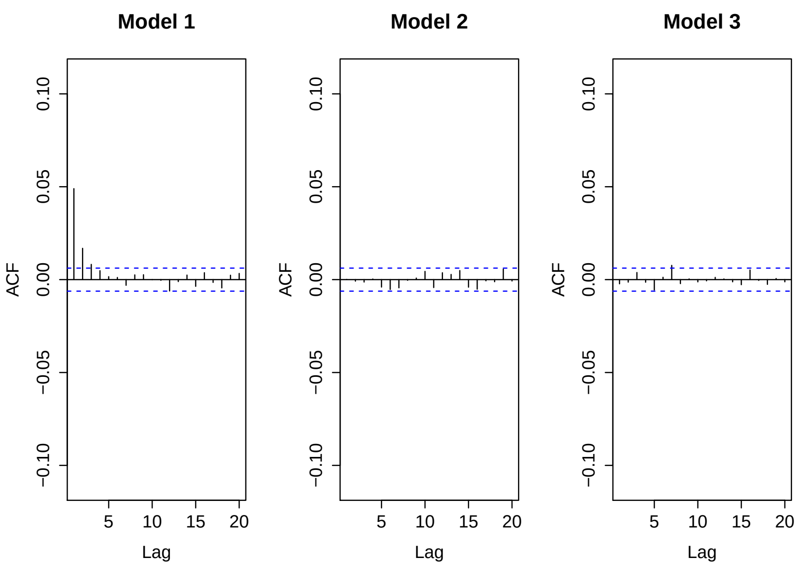

Figure 3.6: Constant speed.

In Figure 3.6 the auto correlation function of the inter arrival times from each model is shown considering a constant travel speed. Since a constant speed does not change the inter arrival times of model 2 and 3, the inter arrivals will remain independent of each other. However for model 1 it holds that the minimum headway needs to be met. Thus inter arrivals of less than 2 seconds will be set to 2 seconds. Therefore dependence arises. Apparently there is dependence between a vehicle and the next 2 vehicles arriving after that one. Since geometric bunch sizes assume independence we can already conclude that, however in Figure 3.2 the density distribution of the inter arrival times of model 1 and model 3 look quite similar they probably do not have the same distribution. Since model 1 contains dependence, while model 3 has independent inter arrivals. We can partly explain the correlation within model 1 by looking at the following probability:

$$\mathbb{P}\big[X_i \leq 2 \wedge X_i + X_{i+1} \leq 4 \wedge X_{i+1} > 2\big]. \tag{3.3}$$

The probability in Equation (3.3) can be estimated using simulation and equals 0.041321. This is the probability that a car will get a follow time of 2 seconds even though it was originally bigger than 2 seconds because of his predecessor. As an example take a look at the following situation. If inter arrival time $X_i = 1.7$ and $X_{i+1} = 2.1$ seconds, then after travelling inter arrival time X_i will be set to the minimum headway of 2 seconds, however this will influence inter arrival time X_{i+1}. X_{i+1} will now become 2.1 - 2 + 1.7 = 1.8 seconds which is smaller than 2 seconds. Of course this can hold for more than just 1 car and therefore it just partly explains the correlation.

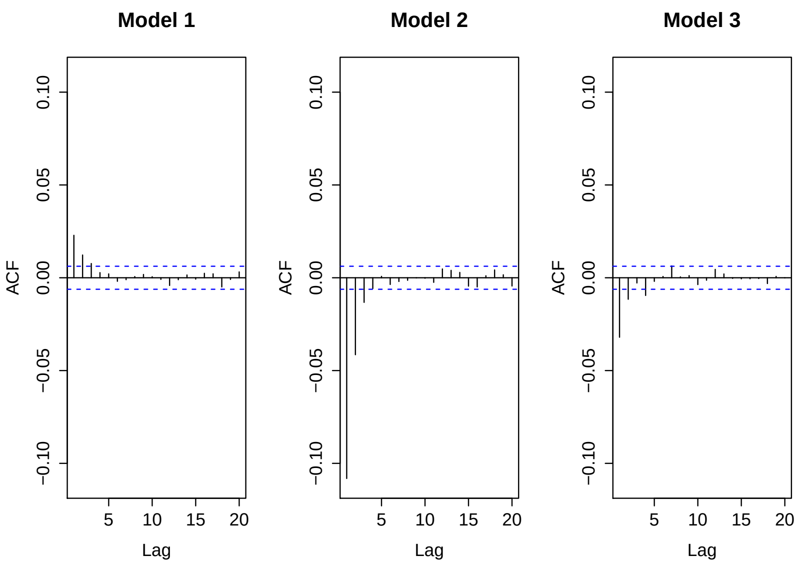

Figure 3.7: Min-max Speed.

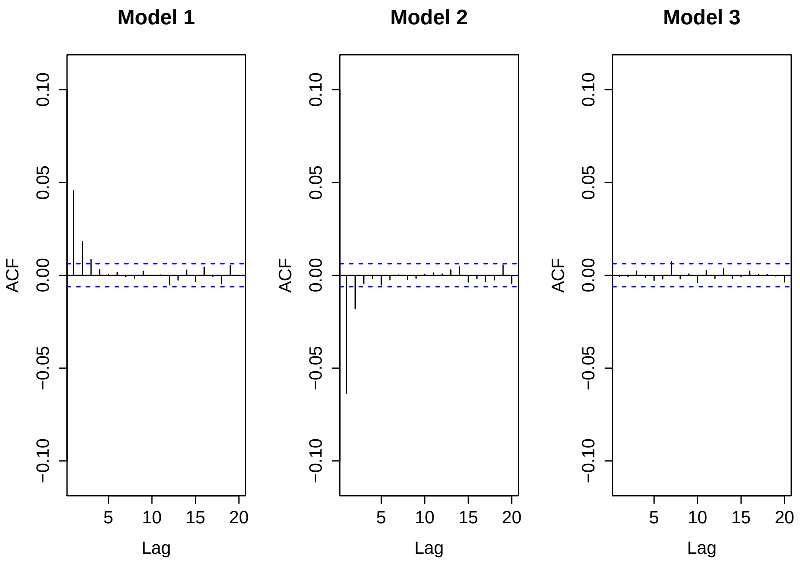

Figure 3.8: Uniformly distributed speed.

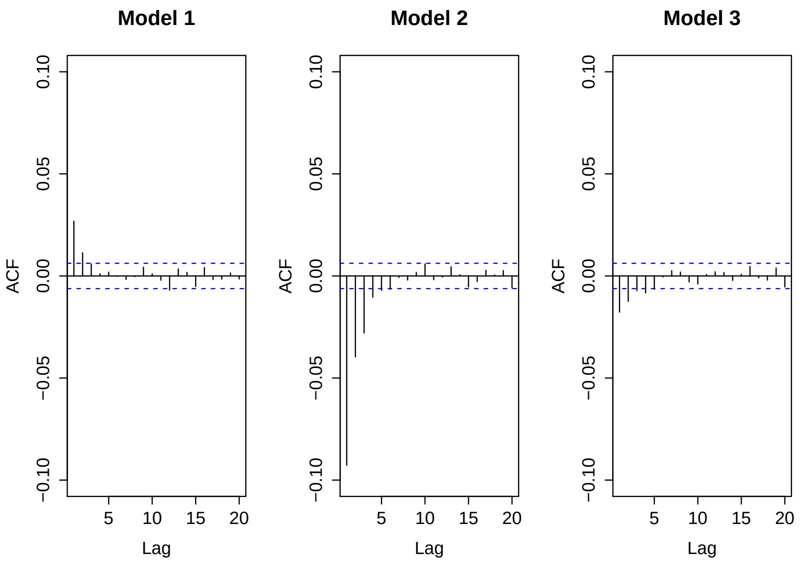

Figure 3.9: Normally distributed speed.

In Figure 3.7 the auto correlation function is plotted with the speed distributed according to the min-max speed distribution. For model 1 there is again correlation between headway i and headway $i + 1$ and $i+2$. For model 3 independence is questionable, there now is negative correlation between headway i and headway $i + 1$, however this correlation is significantly smaller than for model 2. For model 2 it is clear to see that there is dependence with at least the next 2 headways. The correlation is also negative, this means that a large headway will result in two smaller headways. In other words, a slow travelling car will result in a large headway and the next vehicles behind this car will catch up and even be slowed down by that car and thus have a smaller headway. The same results can be concluded from Figure 3.8 and 3.9 concerning model 1 and 2. However when considering model 3 with a uniformly distributed speed, the headways will remain independent. With normally distributed speed independence is questionable. The correlation between headway i and $i + 1$ is not that big, thus might be considered insignificant.

Using the auto correlation function we showed that a geometric distribution is probably not a good distribution for most of the bunch sizes because of the dependence between the inter arrival times. Nevertheless it is interesting to take a look at what the platoon size distribution itself looks like. This will be done in the upcoming section.

3.3 Platoons

In this section we will further investigate the distribution of the platoon sizes. We define a platoon to be of at least length 1. Whether the platoon size increases depends on the next arrivals. An arrival A_i will increase the platoon size if the inter arrival time $X_i \leq 2$. However, since Model 1 contains inter arrival times smaller than 2 seconds some extra conditions need to be satisfied. We can state the platoon size distribution of model 1 as follows:

Platoon size distribution model 1. $\mathbb{P}\big[Z = n\big] = \frac{(2\lambda n)^{n-1}}{(n)!} \cdot e^{-2\lambda n}$

We will now discuss why this platoon size distribution is correct. As mentioned earlier an arrival can have an inter arrival time X_i larger than 2 seconds and still be within the platoon. We considered $X_i = 2.1$ wherefore holds that as long as $X_{i-1} \leq 1.9$, than X_i will still increase the platoon size. This is due the fact that the headway has to be at least 2 seconds and therefore the inter arrival time will be set to 2 seconds. This results in a real inter arrival time of $X_{i-1} = 2$ and $X_i = 2$. Furthermore, since model 1 has exponentially distributed inter arrival times there holds that X_i is independent. Therefore we can state the platoon size distribution in terms of probabilities, these are stated in Table 3.2.

Platoonsize	Model 1
1	$\mathbb{P}(X_1 > 2)$
2	$\mathbb{P}(X_1 \leq 2 \wedge X_1 + X_2 > 4)$
3	$\mathbb{P}(X_1 \leq 2 \wedge X_1 + X_2 \leq 4 \wedge X_1 + X_2 + X_3 > 6)$
$\vdots$	$\vdots$
n	$\mathbb{P}(X_1 \leq 2 \wedge \cdots \wedge X_1 + \cdots + X_n > 2n)$

Table 3.2: Probability of the platoon size i for Model 1.

When calculating these probabilities a certain pattern occurs. We now denote the platoon size to be Z then the following holds:

$$\mathbb{P}\big[Z = n\big] = c \cdot \lambda^{n-1} e^{-2\lambda n}, \qquad \text{with } n \geq 1, \text{ c} \in \mathbb{R}. \tag{3.4}$$

However c has to be further examined. This is were recursion will be needed. State the term to be integrated as $f(k, i)$, with i the platoon size minus 1 then there holds that:

$$f(k, i) = \int_0^{2k - \sum_{j=0}^{k-1} X_j} f(k+1, i) dx_k, \qquad \text{with as restriction } f(i, i) = 1. \tag{3.5}$$

In this case it is easier to calculate the integrals, however a more explicit function of c is wanted. Using the recursion to find more terms can provide inside in the process. Eventually we can notice that the following integer sequence represents c:

$$c = \frac{2^i (i+1)^{i-1}}{i!}. \tag{3.6}$$

Thus we find that the $\mathbb{P}\big[Z = n\big]$ can be stated as an explicit closed formula. This formula is stated in Equation 3.7.

$$\mathbb{P}\big[Z = n\big] = \frac{2^{n-1}(n)^{n-2}}{(n-1)!} \cdot \lambda^{n-1} e^{-2\lambda n} = \frac{(2\lambda n)^{n-1}}{(n)!} \cdot e^{-2\lambda n} \tag{3.7}$$

Thus now we know the platoon size distribution of model 1, 2 and 3 at arrival instance. In Table 3.3 the probability of platoon size n is given for a certain model at arrival instance, notice $\theta = 0.5$ because in this case model 1 looks very similar to model 3. We can now verify using $\lambda = 0.2$ that this data is similar to the results from above.

Platoonsize	Model 1	Model 2	Model 3	Model 1 algebraic
1	0.670455796	1	0.4995576	0.67032
2	0.178972584	0	0.2463506	0.179732
3	0.072856967	0	0.1267744	0.0722866
4	0.034543921	0	0.0641814	0.034457
5	0.018185486	0	0.0310251	0.0180447
6	0.009983763	0	0.0160051	0.0100327
7	0.005799551	0	0.0073793	0.00581424
8	0.003365707	0	0.0042828	0.00347366
9	0.002195461	0	0.0023123	0.00212421
10	0.001300273	0	0.0010858	0.00132312
11	0.000831841	0	0.0005429	0.000836518
12	0.000585123	0	0.0001206	0.000535431
13	0.000305064	0	0.0002815	0.000346281
14	0.000215045	0	4.02e-05	0.000225937
15	0.000135028	0	2.01e-05	0.000148543

Table 3.3: Probability of the platoonsize at arrival.

Even though the model 1 looks very similar to model 3 in the case of $\theta = 0.5$, Table 3.3 shows that the platoon size distribution is very different. It also shows that the simulation results match with the results from Equation 3.7. Furthermore notice that we could generalize Equation 3.7 even more. Earlier we stated that the minimum headway was τ and thus the equation could be restated to:

$$\mathbb{P}\big[Z = n\big] = \frac{(\tau\lambda n)^{n-1}}{(n)!} \cdot e^{-\tau\lambda n}, \qquad n = 1, 2, 3, \ldots \tag{3.8}$$

Note that Equation 3.8 is very similar to the Borel distribution with $\mu = \tau\lambda$ and $n = n$. For a Borel distribution holds that $\mu \in [0, 1]$. This is always the case in a stable system, since $\tau \cdot \lambda \leq 1$. In that case the probability mass function of the Borel distribution is stated as:

$$\mathbb{P}\big[Z = n\big] = \frac{(\tau\lambda n)^{n-1}}{(n)!} \cdot e^{-\tau\lambda n}, \qquad n = 1, 2, 3, \ldots \tag{3.9}$$

Thus Equation (3.8) is equal to Equation (3.9). Therefore we can conclude that the platoon size distribution of model 1 equals a Borel distribution with probability mass function $f(\tau\lambda; n)$.

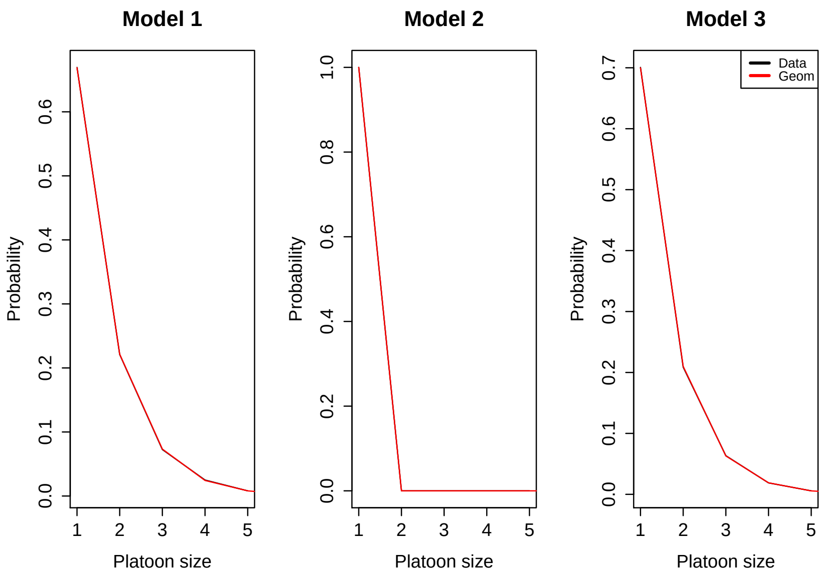

Figure 3.10: Platoon size distribution vs. geometric distribution at arrival.

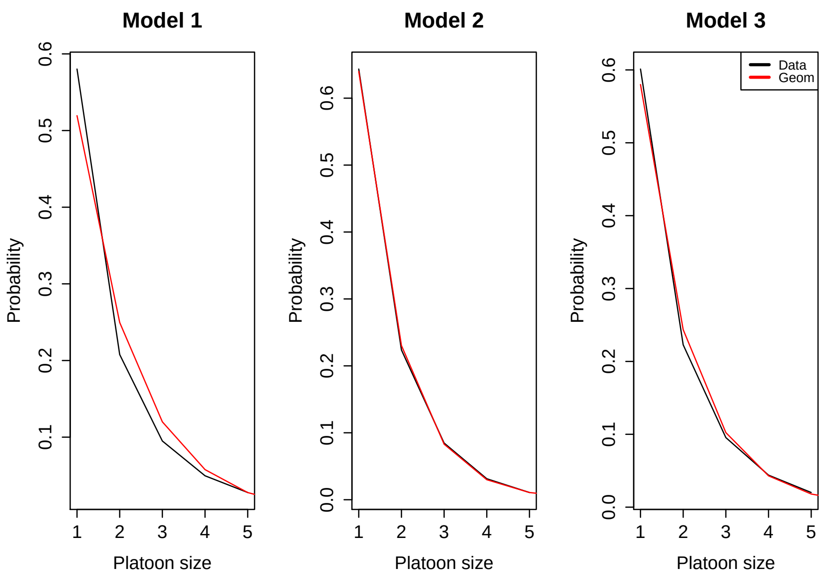

Figure 3.11: Platoon size distribution vs. geometric distribution at the intersection.

In Figure 3.10 the platoon size distribution of each model is plotted in comparison to a geometric distribution before travelling. The maximum-likelihood estimator is used to estimate the parameter of the geometric distribution. Notice that, however we just showed that model 1 does not follow a geometric distribution, the data of model 1 does match quite well with the geometric distribution at the moment before travelling. Furthermore the shape of the distribution of model 1 seems to be fit with the shape of model 3. However a distance d has to be travelled. In Figure 3.11 the effect of distance d is plotted. The speed is considered to be normally distributed. Notice that the platoon size distribution of model 2 still matches with a geometric distribution, but the platoon size distribution of model 1 and 3 differs now. As shown in Figure 3.9 the data of model 1 and 3 contained dependency at this point. At that instance a geometric distributed platoon size was getting less likely. This coincides with Figure 3.11.

3.4 Sensitivity analysis

The results contained within this chapter are dependent of the parameters chosen. A different distance d can for instance cause more or less correlation between the headways. Furthermore a lower on average speed causes the variance to have relatively more important. Therefore we will consider some fluctuation within the parameters. We will variate:

- The minimum headway τ;
- The arrival intensity λ, which influences γ;
- The variance of the vehicle's speed S_{var} and thus also S_{min} and S_{max};
- The travelling distance d;

3.4.1 Arrival intensity

The arrival intensity λ is related to the mean headway. The relation for each model has been shown in Section 2.1. In general it holds that when λ decreases that the mean headway X increases. The result of this is shown in Figure 3.12 with $\lambda = 0.1$. Notice that the shape is similar with Figure 3.1, however the mass shifted to the right. The same can be said about the inter arrival times after travelling. An example of this can be seen in Figure 3.13, again the mass is shifted to the right. Furthermore when we consider the correlation in Figure 3.14 there can be noticed that the correlation decreases when λ decreases.

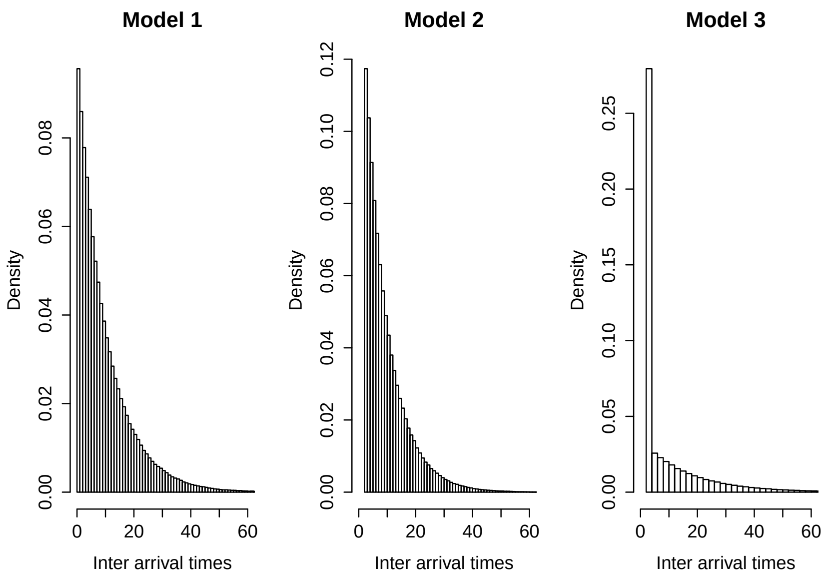

Figure 3.12: Histogram of the inter arrival times with $\lambda = 0.1$ at arrival.

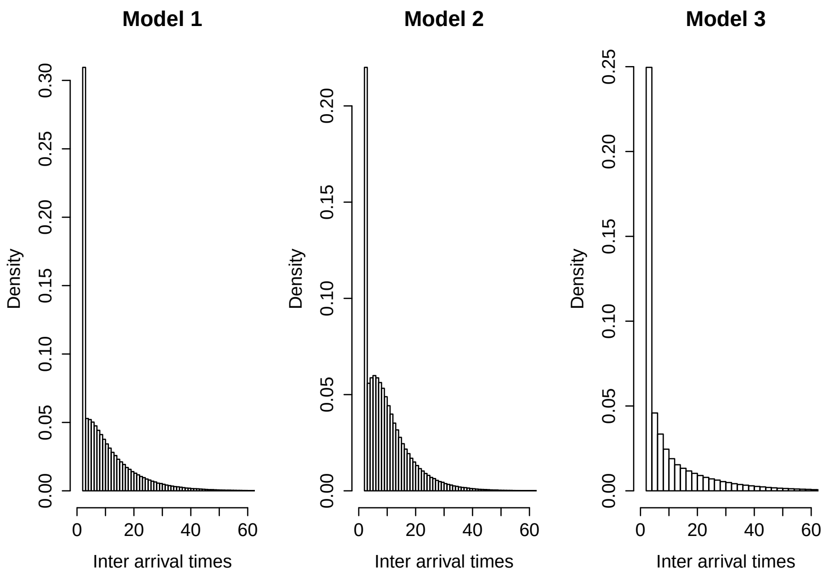

Figure 3.13: Histogram of the inter arrival times with $\lambda = 0.1$ and normally distributed speed.

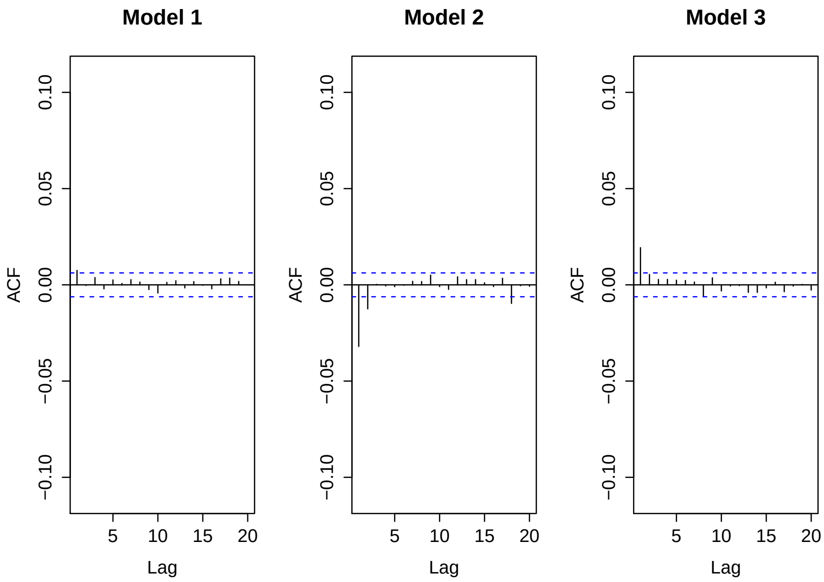

Figure 3.14: Auto correlation function with $\lambda = 0.1$ and normally distributed speed.

Next we consider $\lambda = 0.4$. In this case the arrival intensity is close to the maximum of $\lambda = 0.5$. This is the maximum arrival intensity because of the minimum headway which equals 2 seconds. In this case the mass for inter arrival times is shifted to the left and the shape of the density remains the same. The effect after travelling is shown in Figure 3.15. Furthermore there now is significant correlation between the headway of vehicle i and vehicle $i + j$ with $j = 1, 2, \dots, 15$. The small inter arrival times do apparently cause a lot of correlation, this can be seen in Figure 3.16.

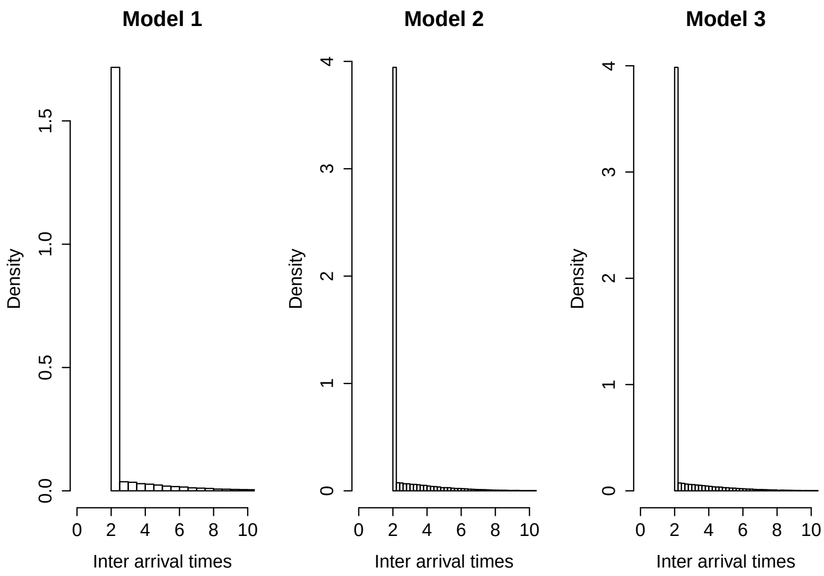

Figure 3.15: Histogram of the inter arrival times with $\lambda = 0.4$ and normally distributed speed.

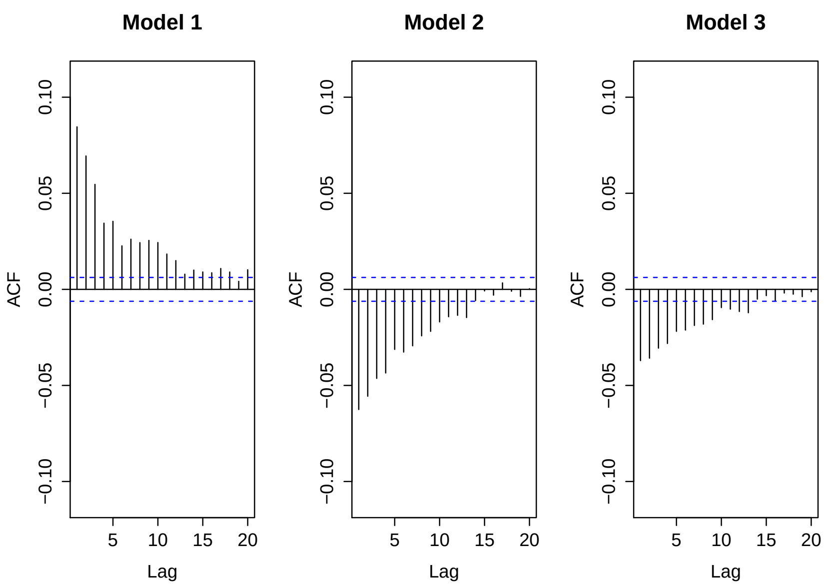

Figure 3.16: Auto correlation function with $\lambda = 0.4$ and normally distributed speed.

3.4.2 The minimum headway

The minimum headway is an important factor for the maximum capacity of the road. Since $\frac{1}{\tau}$ is the maximum number of arrivals per second. We derived the platoon size distribution of model 1 which depends on τ. Therefore we do already know in what way the platoon sizes will change. When considering a minimum headway $\tau = 1$, the headway distribution will remain quite the same. However the data gets less correlated, especially for model 1. Since the platoon size distributed depends on τ, we know that each arrival is correlated. When τ gets smaller the correlation decreases. This can be seen in Figure 3.17 and 3.18 in which the correlation after a uniformly and normally distributed speed is plotted. As we can see there is no longer a significant correlation between headway i and headway $i+1$ nor $i+2$ when looking at model 1.

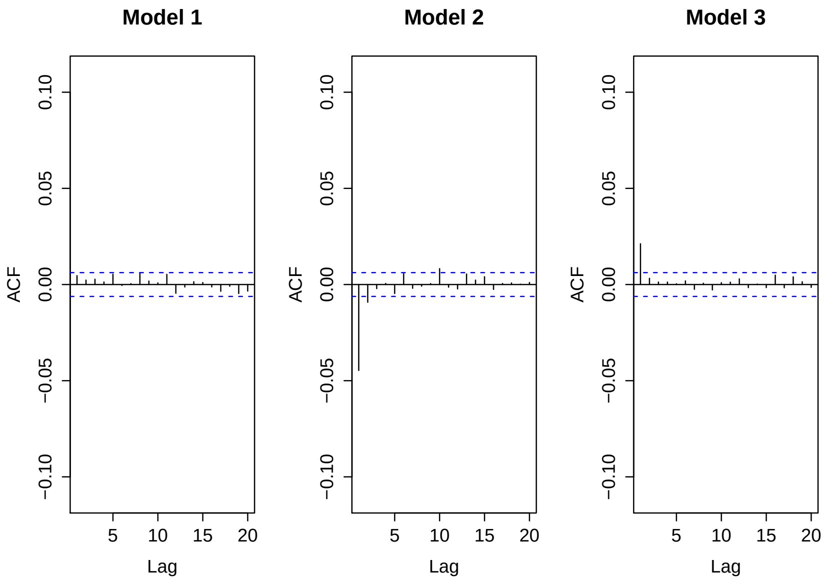

Figure 3.17: ACF with $\tau = 1$ and uniformly distributed speed.

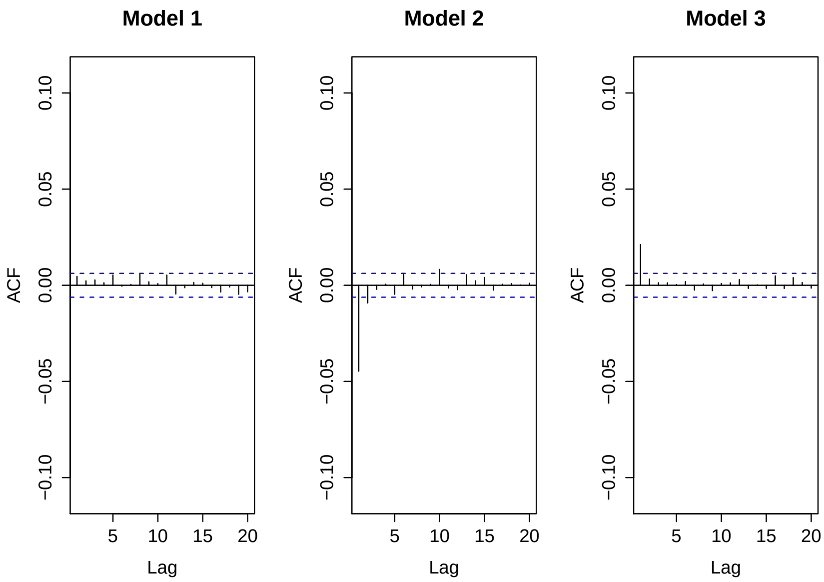

Figure 3.18: ACF with $\tau = 1$ and normally distributed speed.

When considering $\tau = 4$, the mass of the inter arrival time distribution model 2 and 3 gets more concentrated. Because only inter arrival times larger than 4 seconds will be allowed and thus a large part of the previous distribution is cut of. This can be seen in Figure 3.19. After travelling a lot of cars will catch up with each other, since the minimum headway is large. This will result in even more mass at the minimum headway. Furthermore a larger headway will cause more dependency because the probability of ending in a platoon increases. This can be seen in Figure 3.20. The correlation between headways for model 1 and 2 increased significantly. However model 3 already contains platoons at arrival instance and therefore is less sensitive to an increase of τ.

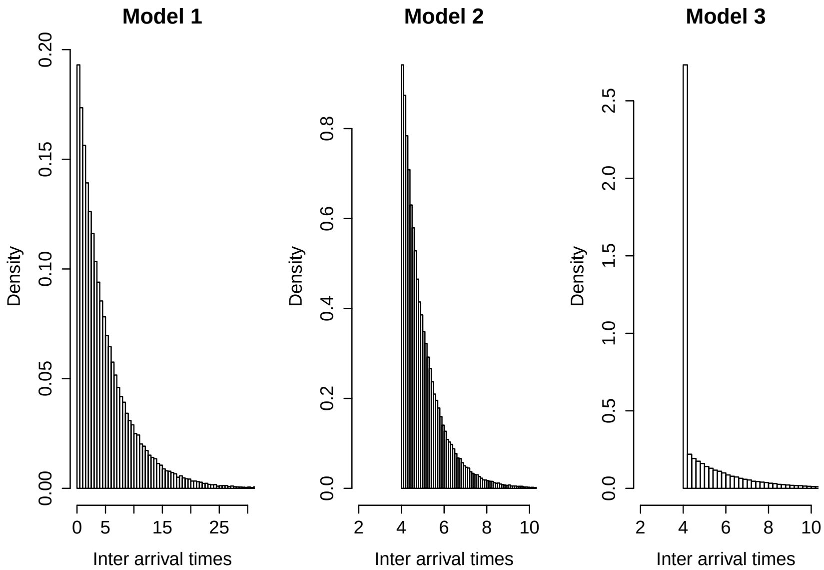

Figure 3.19: Histogram of the inter arrival times with $\tau = 4$.

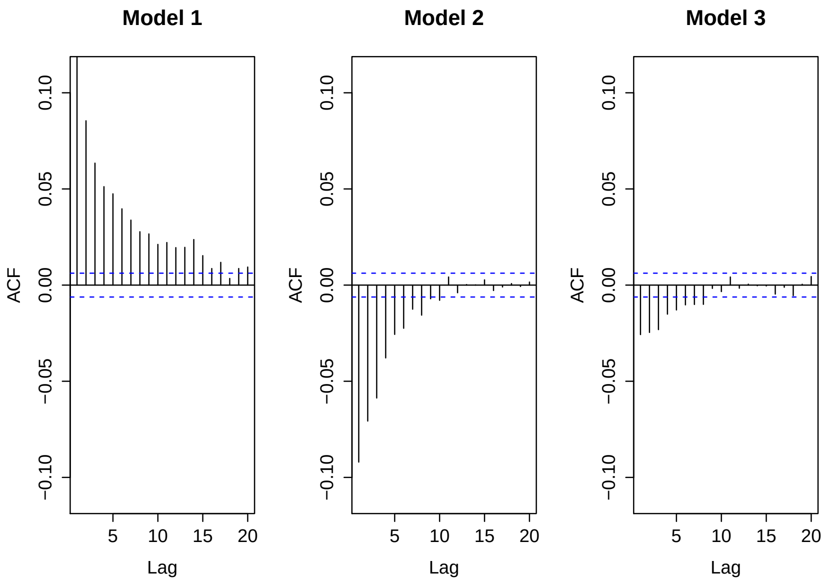

Figure 3.20: ACF with $\tau = 4$ and normally distributed speed.

3.4.3 Distance

The distance d which has to be travelled to reach the intersection is an important factor when cars have varying speeds. The longer the distance, the more the headway distribution will change. In general a longer distance will cause more correlation between the headways, since slower cars will be caught up with by faster travelling cars. The results can be seen in Figure 3.21. Almost every car will end up within a platoon caused by a slower moving vehicle. Furthermore the auto correlation function is plotted for a longer distance and notice that there is correlation between car i and at least $i + 10$, but also that the correlation became negative for model 1 and 3. This is as earlier explained the effect of a large headway followed by a sequence of smaller headways, caused by a slower moving vehicle.

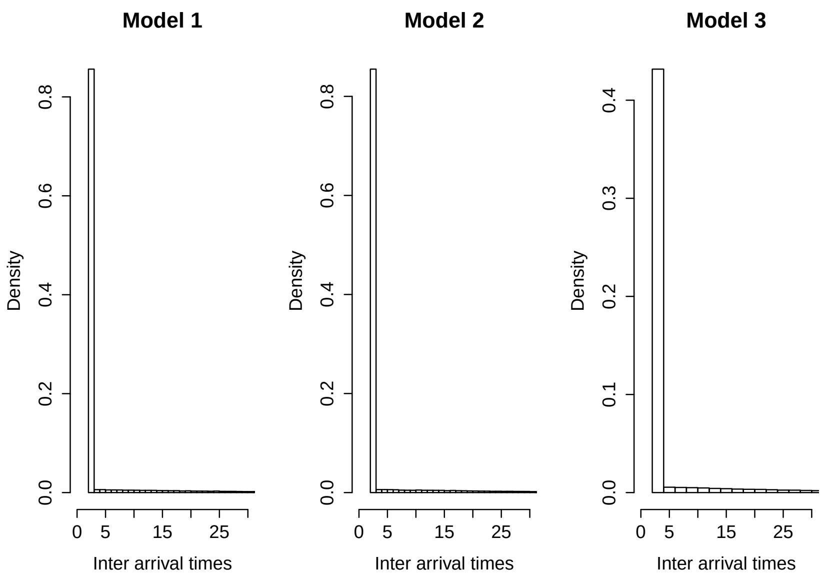

Figure 3.21: Histogram of the inter arrival times with $d = 10,000$ meter.

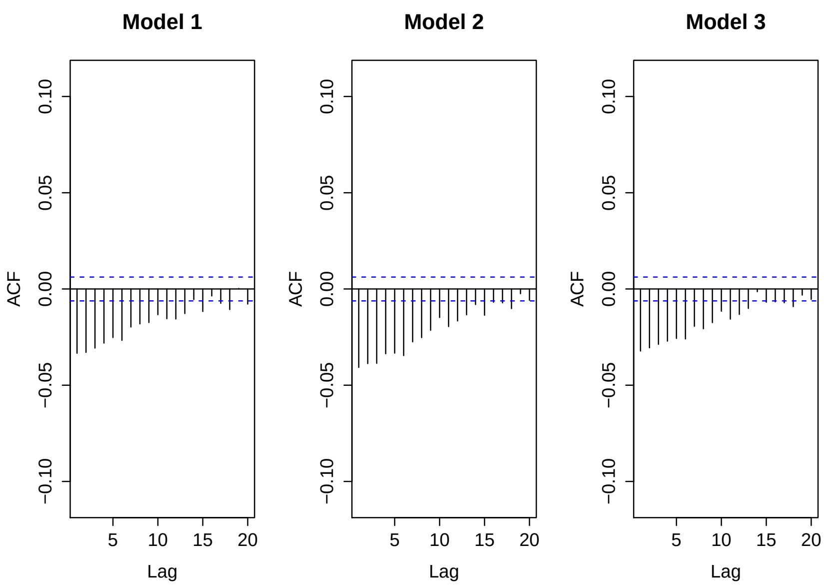

Figure 3.22: ACF of the headway with $d = 10,000$ meter.

3.4.4 Speed

The distribution of the speed is an important factor which influences the headway distribution. We could fluctuate the mean and variance of the speed. A lower mean speed would make the influence of the variance and the travelling distance larger. Therefore we will just briefly take a look at the variance. When we increase the variance to 10 km/h instant of the original 5 km/h this will result in bigger correlation, but also negative correlation. This can be seen in Figure 3.23. The correlation became negative, because an increase in variance creates slower and faster travelling vehicles. A slow driving vehicle will now slow down even more vehicles. The result of this is that a large headway between the slow driving vehicle and its predecessor results in a lot of small headways of the vehicles slowed catching up with this slow driving vehicle.

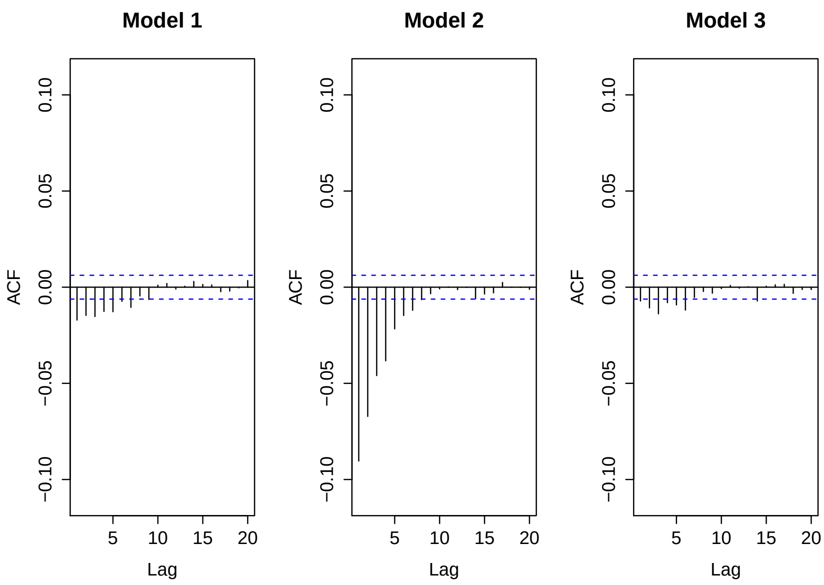

Figure 3.23: ACF of the headway with $S_{var} = 10$ km/h.

Chapter 4

Network of intersections

The basic situation has been introduced in Chapter 1 and the influence of travelling a distance d has been discussed in Chapter 3. In this chapter the departure process for a signalized intersection will be studied. In Figure 4.1 the situation is outlined. In a series of intersections a main way with arrival intensity λ will be considered. At each intersection a side road is connected with arrival intensity μ.

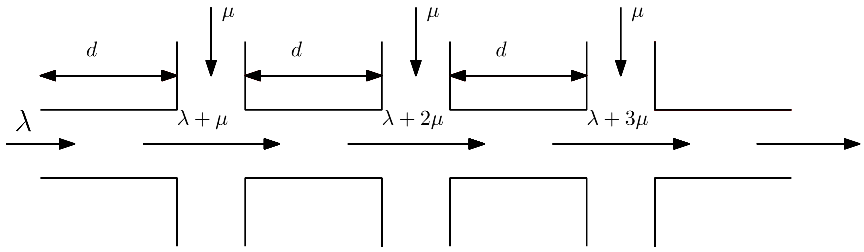

Figure 4.1: Situation 1.

Another variant of this situation is given in Figure 4.2. In this case at every intersection vehicles from the main way will departure with intensity μ. The side way arrival intensity will remain μ, thus the traffic stream will remain constant, assuming the intersection can process a traffic stream of λ. Furthermore notice that μ can be stated as $\mu = \lambda \cdot p$ with $p \leq 1$.

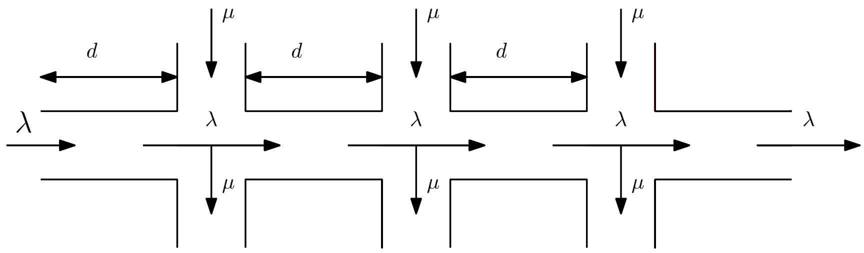

Figure 4.2: Situation 2.

4.1 Assumptions

Before situation 1 and 2 can be modelled some important features must be specified. However most of these features can vary, therefore assumptions need to be made. Consider the following assumptions:

- The arrival process A of the main way is Poisson distributed with parameter λ, and Poisson distributed with parameter μ when considering the side ways.

- The vehicle's speed V is normally distributed with parameters μ and σ^2 and will remain constant when travelling.
- A vehicle will immediately stop if needed and will immediately reach his original speed after crossing the intersection, thus no acceleration/deceleration is considered.
- A vehicle will never overtake another vehicle and thus adept its speed if needed. A vehicle driving a speed V will always keep a minimum headway. This is also the case if a vehicle does not have to stop in front of the intersection.
- The traffic lights will have a constant green-red cycle, which can be defined as follows:

$$C = G_{main} + t_{red} + G_{side} + t_{red}$$

  In other words, the cycle length equals the green time of the main way plus the green time of a side way plus two red periods.
- The departure headway D is log-normal distributed when cars arrive in a red period or when a queue is formed at the intersection.

To be able to compare these results with those of Chapter 3, similar speed distributions and distances have been chosen. The mean speed will be 80 km/h and the variance will be 5 km^2/h^2. Furthermore there has been chosen for a $C = 25$, $G_{main} = 15$, $G_{side} = 6$ and $t_{red} = 2$. Notice C_{length}, G_{main}, G_{side} and t_{red} are expressed in units of seconds. To create an in the Netherlands so called "green wave" the distance d will be set to $\frac{50 \cdot 80}{3.6}$ which equals circa 1111 metres. A "green wave" means that a car driving a mean speed $V = 80$ km/h will have green lights at every intersection, if no delays occur. At last the minimum headway will yet again be 2 seconds. The parameters of the arrival intensity will be varied and therefore stated explicitly for each situation.

4.1.1 Departure headway distribution

In Chapter 3 the influence of travelling a distance d has been discussed. However the influence of a signalized intersection was not taken into account yet. To take the traffic lights into account a departure headway distribution for the vehicles arriving at the intersection has to be introduced. A distinction will be made between two situations. Situation 1, a vehicle arriving when the traffic light is green and there is no queue. This vehicle can departure immediately but has to take into account the minimum headway. Situation 2, a vehicle arrives when there is queue and/or the traffic lights are turned on red. In this case the vehicle will slow down and a departure headway will be needed. In Section 2.3 there has been spoken about a log-normal departure headway in the report of Shao (2012) and Jin (2009). Thus the departure headway distribution will look like:

Departure headway distribution $$\text{D} \sim \begin{cases} \max(0, t_{predecessor} + \tau - t_{vehicle}) & \text{if situation 1,} \\ lnN(\mu, \sigma^2) & \text{if situation 2.} \end{cases}$$

Remark that lnN stands for log-normal. Thus in situation 1 the departure time will be 0 or the departure time of his predecessor plus the minimum headway minus his own arrival time at the intersection. However the parameters μ and σ are still unknown. To get an indication of how to determine these parameters the results of Shao (2012) and Jin (2009) can be used. In Figure 4.3 and Figure 4.4 some results of Shao (2012) respectively Jin (2009) are shown.

Code of surveyed site	Sample size	Average	Median	Stand error	Skewness	Kurtosis
1	96	1.95	1.88	0.52	0.66	0.88
2	95	2.36	2.28	0.55	0.61	1.22
3	75	2.30	2.23	0.49	0.43	−0.10
4	89	2.22	2.14	0.37	0.88	1.09
5	93	2.19	2.03	0.43	0.97	1.21
6	94	2.08	1.98	0.47	0.43	−0.34
7	93	2.21	2.09	0.44	0.35	−0.39
8	91	2.04	2.00	0.43	0.81	1.11
9	100	2.31	2.30	0.45	0.02	−0.75
10	99	2.18	2.16	0.46	0.07	−0.73
11	98	2.11	2.07	0.42	0.02	−0.20

Figure 4.3: Departure headway data Shao (2012)[5].

Position	Sample size	Maximum	Minimum	Range	Mean	Median	Variation
1	423	9.74	1.26	8.48	4.53	4.26	1.51
2	423	8.36	1.05	7.31	3.05	2.81	1.08
3	423	6.69	1.05	5.64	2.76	2.56	0.93
4	422	9.32	1.05	8.27	2.57	2.39	0.90
5	416	7.77	1.05	6.72	2.51	2.35	0.88
6	394	6.26	1.05	5.21	2.36	2.23	0.78
7	298	4.91	1.05	3.86	2.19	2.10	0.69
8	146	5.25	1.09	4.16	2.04	1.89	0.66
9	42	3.15	1.05	2.10	1.87	1.83	0.56

Figure 4.4: Departure headway data Jin (2009)[4].

As can be seen in Figure 4.3 and 4.4 the collected data does differ in mean and variance when considering the departure headway. Remark that Shao (2012) investigated the departure headway with leaving out the first x cars, but Jin (2009) included those. In Shao (2012) the average departure headway is circa. 2.18 seconds. When considering Jin (2009) this is quite a bit higher, namely 2.65 seconds if we merely add the mean values of each position and divide it by the total number of positions. Figure 4.4 shows that the first vehicles in queue have a higher departure headway than their predecessors. This can be the reason that Shao (2012) has a lower mean departure headway. In Shao (2012) the sample size is smaller than in Jin (2009). Remark that in Jin (2009) they considered the departure headway for position 1 to not follow a log-normal distribution. This let to the decision to not include this data when calculating the mean departure headway and variance departure headway. This resulted in a mean departure headway of 2.27 seconds and variation of 0.81 seconds^2. Knowing this $\mu_{headway}$ and $\sigma_{headway}$ can be calculated, because for a log-normal distributed random variable X holds that:

$$\begin{cases} \mathbb{E}[X] = e^{\mu_{headway}+\frac{1}{2}\sigma^2_{headway}} = 2.27, \\ \mathbb{V}ar[X] = (e^{2\mu_{headway}+\sigma^2_{headway}})(e^{\sigma^2_{headway}}+1) = 0.81. \end{cases} \tag{4.1}$$

Solving this system of equation leads to $\mu_{headway} = 0.746781$ and $\sigma_{headway} = 0.382096$. These are the parameters that will be used in the departure headway distribution in the simulation. Furthermore as can be seen in Figure 4.1 the focus will be on a situation with three intersections. It is interesting to investigate a situation in which the main road of the intersections can handle the traffic stream. It must hold that the workload $\rho_{main} < 1$. This is the case when:

$$\rho_{main} = (\lambda + \mu + \mu)\mathbb{E}[B]\frac{1}{f_{main}} < 1. \tag{4.2}$$

Remark that λ is the arrival intensity of the main road and μ the arrival intensity of each side road. Furthermore $\mathbb{E}[B]$ is the mean services time and $\frac{1}{f_{main}}$ is the fraction of time the traffic light is green on the main road. The service time is in this case the time it takes for a vehicle to be processed by the intersection. Knowing this, there can be concluded that: $(\lambda + \mu + \mu) < 0.264$. Thus the arrival intensity of the main road and first two side roads together needs to be smaller than 0.264. For the side road holds that:

$$\rho_{side} = (\mu)\mathbb{E}[B]\frac{1}{f_{side}} < 1. \tag{4.3}$$

Therefore we can conclude that $\mu < 0.105$ to create a stable situation.

4.2 Description of the simulation

In the previous section the assumptions which were needed before modelling the situation have been introduced. Therefore it is now time to describe the simulation which has been build. The program which has been written contains different classes. These will be explained in the next paragraphs. Next, the construction of the main class will be illustrated, using these classes.

Event This class keeps track of what type of event takes place, at a certain time unit, considering a specific vehicle.

FES This stands for Future Event Set, which is a list containing all events sorted in a chronological order.

Queue The class Queue contains the information whether a queue is empty, and which vehicle is within the queue.

QueueOvertaking The class QueueOvertaking is an ArrayList containing the time of the next event of the previous vehicles. It is used to check whether vehicles do not overtake each other.

Server A server represents a traffic light. This class contains the information whether a traffic light is processing a vehicle crossing the intersection and if this is the case which vehicle is being processed.

SimResults In this class the before travel and after travel arrival times are registered with the corresponding arrival side.

Vehicle This class keeps track of:

- The arrival time of each customer;
- The arrival side;
- The arrival type, main or side arrival;
- The vehicle speed;

IntersectionsUpdate In the main class the known values will be initialised. Furthermore the needed queues, servers, first arrivals etcetera are created. The simulation starts with planning the first arrivals at each arrival side. The corresponding ARRIVAL or SIDEARRIVAL events will be skewed and the vehicle will endure the following process.

- The ARRIVAL event. In this event the before travel arrival time will be registered. Next, the time needed to travel the distance d will be added and the after travel time will be registered. This time corresponds to the before travel time plus the maximum of either the distance divided by the speed of the vehicle or the after travel time of the previous vehicle plus the minimum headway. After this the TRAFFICLIGHT event will be planned. If the corresponding vehicle's arrival side is 0 then a new arrival will be created and a new ARRIVAL event will be planned.

- In the SIDEARRIVAL event the SIDETRAFFICLIGHT event and the next SIDEARRIVAL will be planned.
- In the TRAFFICLIGHT event the following will be checked for a vehicle arriving at the intersection:
  1. Whether the traffic light is green at that time instance;
  2. Whether $trafficLight$ is not occupied, $trafficLight$ is of type Server;
  3. Whether the *queue* in front of the traffic light is empty, *queue* is of type Queue;

  If the first requirement is not met, the vehicle will be added to the *queue* and a QUEUE event will be planned. If requirement 1 is met, but requirement 2 or 3 is not then the vehicle will be added to the *queue* without planning a QUEUE event. If requirement 1,2 and 3 are met then a DIRECTINTERSECTION event will be planned. In this case there will be checked whether a vehicle will not overtake another vehicle.
- The SIDETRAFFICLIGHT event is build up in the same way as the TRAFFICLIGHT event, but in this case the *sideQueue* needs to be empty. Furthermore a SIDEQUEUE event will be planned instant of a QUEUE event.
- The QUEUE event occurs when a vehicle arrives at the intersection at a time t which corresponds with a red traffic light. In this case the next green period of the main way will be calculated after time t and then the EMPTYINGQUEUE event will be planned.
- The SIDEQUEUE event has the same principle as the QUEUE event, however in this case the next green period for the side way will be calculated and the EMPTYINGSIDEQUEUE event will be planned.
- In the EMPTYINGQUEUE event the next INTERSECTION event will be planned for the first vehicle in the queue if the $trafficLight$ server is not occupied. This event will take place after adding a departure headway.
- The EMPTYINGSIDEQUEUE event does the same as the EMPTYINGQUEUE event, but now regarding to the *sideQueue*.
- In the DIRECTINTERSECTION event there will be checked whether side departures can take place, if so a random number will determine whether a vehicle will make a side departure or not. If a vehicle is going to make a side departure, the SIDEDEPARTURE event will be planned. If a vehicle is not going to make a side departure then the arrival side of the vehicle will be increased by 1. Next there will be checked whether a vehicle's arrival side is smaller than the number of servers. If so, the next ARRIVAL event will be planned for that vehicle, if not a DEPARTURE event will be skewed. This is also the case if we are not considering side departures.
- In the INTERSECTION event the same process as in the DIRECTINTERSECTION event will take place. However the next INTERSECTION event will be planned for the first person waiting in the *queue* or *sideQueue* of the corresponding intersection if there still is a green light at the corresponding arrival side.
- In the DEPARTURE event the last before travel arrival time will be registered and the number of departures will be increased.
- In the SIDEDEPARTURE event only the number of departures will increase, the departure times will not be registered, since we are only interested in those for the main way.

4.3 Simulation results

In this section the simulation results will be discussed. As Figure 4.1 shows there are two different situations which are interesting. Namely the before travel situation and the after travel situation. Notice that after crossing the intersection this is again a before travel situation and therefore we can simply look at just these two situations. As Figure 4.1 indicates the main focus will be on a road with 3 intersections. For every situation which is looked into the inter arrival times before and after travel are shown with the corresponding arrival intensities.

4.3.1 Without side departures

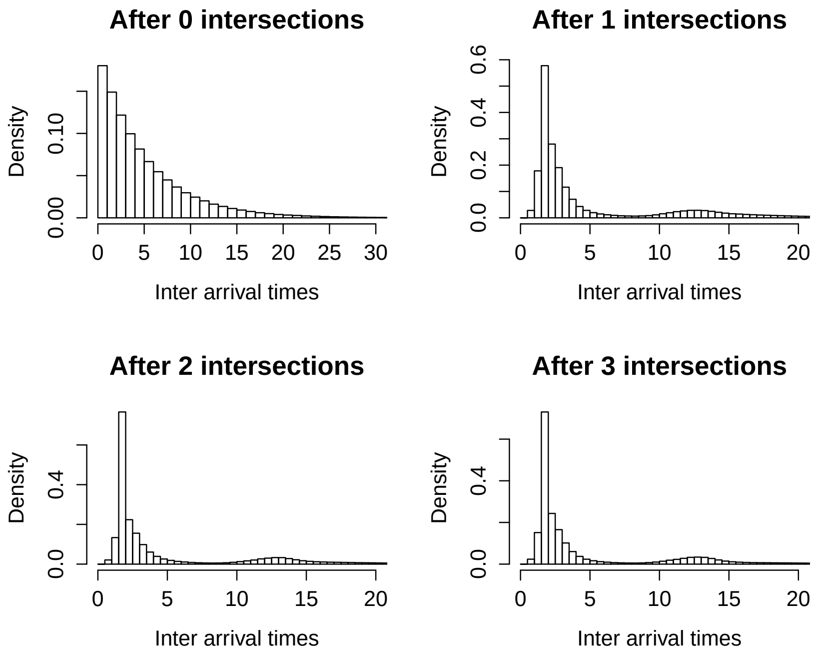

Figure 4.5: Inter arrival times after crossing i intersections, with $\lambda = 0.2$.

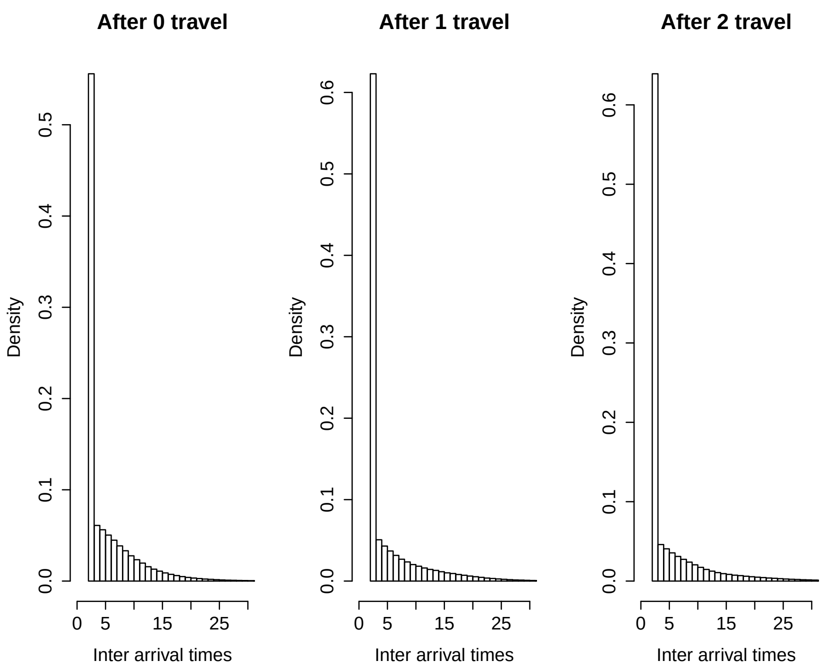

Figure 4.6: Inter arrival times after travelling, with $\lambda = 0.2$.

In Figure 4.5 the inter arrival times after crossing i intersections are shown, with $i = 0, 1, 2, 3$. In this figure only arrivals on the main road are considered. As you can see after 0 intersections the inter arrival times are exponentially distributed, nothing has happened at this point. Furthermore after i intersections, with $i = 1, 2, 3$ the shape of the inter arrival time distributions will remain the same. Thus the first intersection already determines the shape of the inter arrival time distribution when side arrivals are not allowed. Notice that there are two different peaks after i intersections with $i = 1, 2, 3$. The first peak is created by vehicles crossing the intersection within the same green period. The inter arrival time between these vehicles is log-normal distributed. This explains the shape of the first peak. However a vehicle can depart directly from the intersection when there is no queue and the traffic light is showing a green light. In this case a vehicle's inter arrival time is dependent on the inter arrival times created by travelling a distance d. This will mainly influence the size of the peak at the minimum headway time. This is because a vehicle departing when traffic lights are green and there is no queue get an instant departure or the departure time of the previous vehicle plus the minimum headway to maintain a safe driving distance. Since a lot of vehicles will approach the intersection with a minimum headway with respect to its predecessor, which can be seen in Figure 4.6, a lot of vehicles will remain this minimum headway when they direct departure from the intersection. The second peak in the picture is created partly by vehicles with an after travel time larger than 10 seconds, but the main contribution comes from vehicles departing in a green period after its predecessor. This peak does again have a log-normal shape because of the log-normal headway distribution, but the peak is now $C - G_{main} = 10$ seconds later. The peak is smaller because this situation occurs in fewer instances. Notice that a vehicle can still be departing a little after the green period as long as he starts departing within the green period, therefore the second peak starts a little before 10 seconds.

In Figure 4.6 the inter arrival times after travelling a distance d which is equal to circa 1111 metres is plotted. In the figure the title "after i travel" is used, this means that for the i time a distance d is travelled. As can be seen in Figure 4.6 most vehicles will have a headway which equals the minimum headway, like in Chapter 3. Furthermore, the two peaks shown in Figure 4.5 are spread out again or absorbed within the minimum headway peak because of the varying speed of each vehicle.

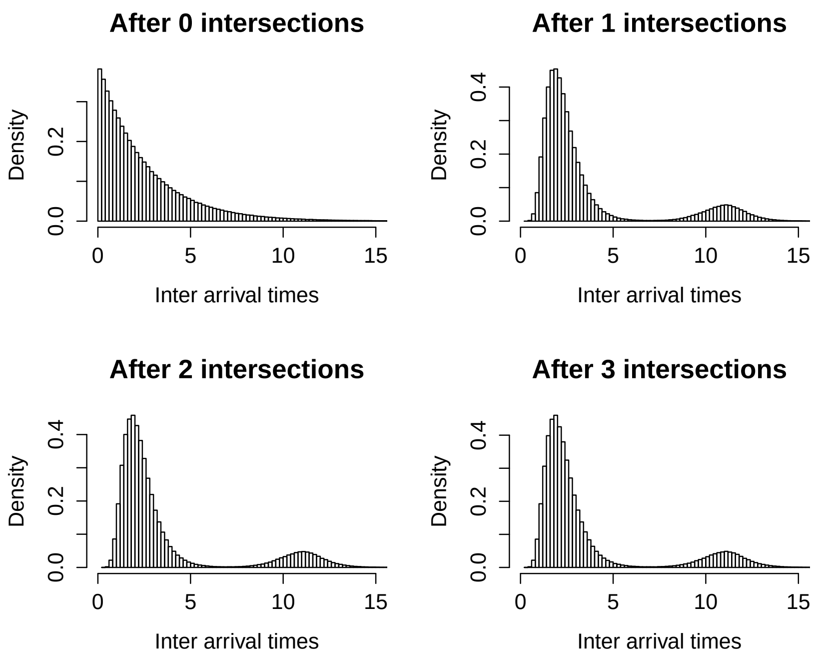

Figure 4.7: Inter arrival times after i intersections, with $\lambda = 0,4$.

The maximum capacity of the intersection itself is of course an important factor for the inter arrival times distribution. Recall that for the maximum capacity of the intersection needs to hold that: $\rho < 1$. This resulted in $\lambda < 0.264$ for the main road and $\mu < 0.105$ for the side roads. In Figure 4.7 the results for

$\lambda = 0.4$ are presented. There can be noticed that the inter arrival times after 0 intersections are of course smaller than in the situation before. But more interesting is that after i intersections, with $i = 1, 2, 3$ the shape of the inter arrival time distribution is still very similar to the first situation. The mass in the first peak has changed, the peak is now more closely to a log-normal distribution. This is because of the higher arrival intensity, which makes it less likely that direct departures take place and therefore the peak at the minimum headway of 2 seconds will not be significantly larger than the surrounding values. Furthermore a lower percentage of the times there will be a vehicle which will have a green period later then its predecessor. Therefore the second peak is slightly smaller than before. Since the green period is 15 seconds and the minimum headway 2.27 seconds, there can be reasoned that 6-7 vehicles will cross on average per green period. Thus the mass in the first peak will on average be 5 to 6 times larger than in the second peak.

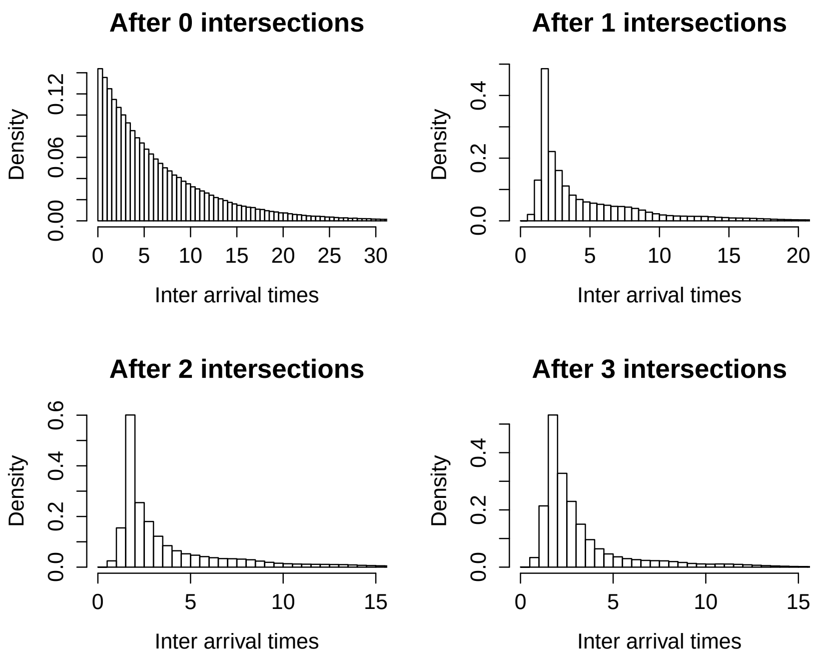

Figure 4.8: Inter arrival times i intersections, with $\lambda = 0.15$ and $\mu = 0.05$.

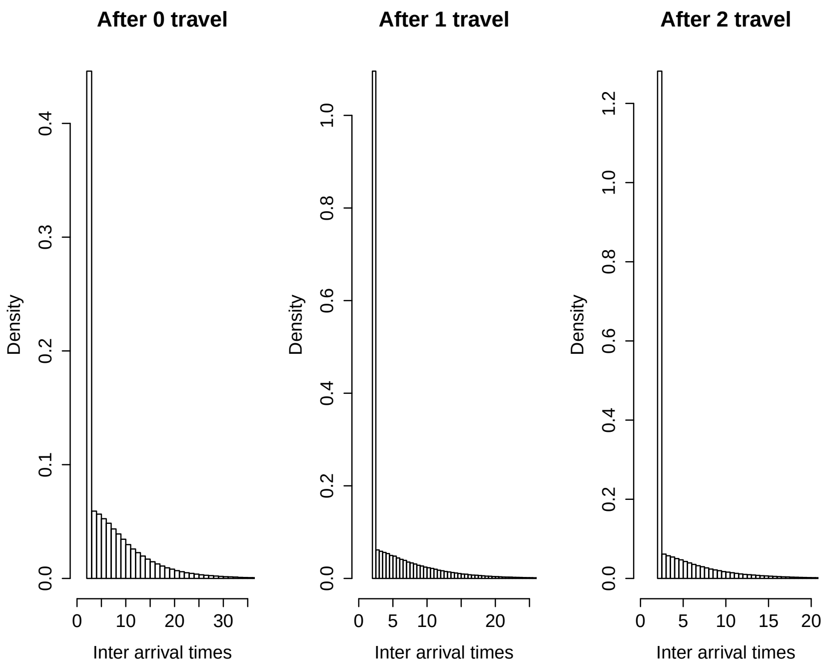

Figure 4.9: Inter arrival times after travelling, with $\lambda = 0.15$ and $\mu = 0.05$.

When including side arrivals, the inter arrival time distribution will change. This can be seen in Figure 4.8. Since we do not want to exceed the maximum arrival intensity, the following parameters are chosen: $\lambda = 0.15$ and $\mu = 0.05$. In Figure 4.8 the effect of the side arrivals can be noticed after i intersections, with $i = 1, 2, 3$. There is no longer negligible density for inter arrival times of the interval from (5,10) seconds. Instant of a 10 second red period which was the case in the previous situation when there were only main arrivals. There now is a 2 second red period before the side arrivals can take place. Notice that if the number of intersections which are crossed increases, the arrival intensity increases and thus less direct departures take place. Also the shape of the inter arrival time distribution shifts more closely to that of a log-normal distribution. Since μ is relatively low, in other words on average just 1 arrival per 20 seconds, there will always be a mass for inter arrival times until at least 10 seconds. This is the cycle length minus the green time of the main way. This peak is caused by the situation in which there are no side arrivals after the last main road arrival. Thus the next departure from the intersection will take place at the main road and is at most 10 seconds plus a departure headway later. This is because, there can be assumed that an other vehicle which wants to departure is waiting at the main road with high certainty, because of the high arrival intensity. However there is also mass before the 10 seconds, because the departure headway of the last vehicle can be outside the green period. Thus the departure headway might be less than 10 seconds if the departure headway of the next main way vehicle is less than the residual departure headway of its predecessor. Another part of the mass can be caused by the difference between vehicles arriving in a red period at the side way which departure at the next green period and the next departure at the main way. In this case a departure headway will occur at both the main way and side way and therefore they sort of cancel each other. Thus the inter arrival time is mainly determined by the red time and the green time of the side way which equals 6 plus 2 seconds and thus is around 8 seconds or less when a side way vehicle does arrive within a green period. This explains the more fluently descending shape of the inter arrival time distribution.

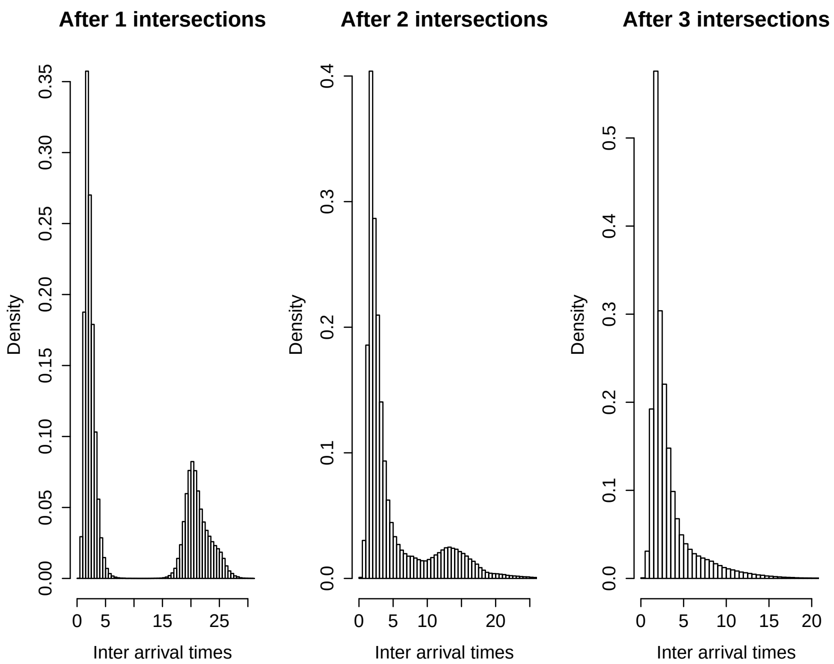

Figure 4.10: Inter arrival times after i intersections, with $\mu = 0.1$.

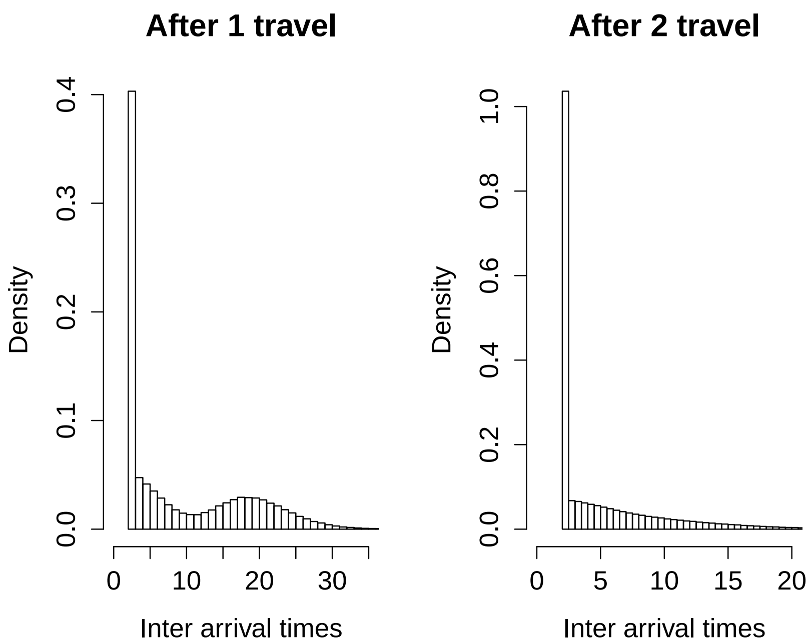

Figure 4.11: Inter arrival times after travelling, with $\mu = 0.1$.

When considering situation 1 which is presented in Figure 4.1 the assumption that $\lambda = 0$ can be made. In this case the arrival intensity will completely depend on μ. In Figure 4.10 the results of this situation is shown. As we expect after 1 intersection there is a peak for the inter arrival times after 15 seconds. This peak is caused by the red time which in this case can be seen as $G_{main} + 2 \cdot t_{red}$, because there are no vehicles at the main way. This peak shifts slightly to the left after 2 intersections. The mean departure headway equals circa 2 seconds and thus on average 3 vehicles will pass the traffic light on the first side road within a green period. Thus on average 3 vehicles will departure at the main way at the second intersection. Therefore the second peak shifts $t_{red} + 3 \cdot \mathbb{E}[departureHeadway]$ to the left which is circa 8-9 seconds. This depends on the moment at which these vehicles arrive at the intersection after travelling the distance d. These vehicles will travel on average with a speed of 80 km/h and thus end up 50 seconds later at the next intersection, at which there is a green period for the side way. Thus they will end up in the main road queue. However they can also arrive within a green period at the main way causing other possibilities for the inter arrival time distribution, creating smaller headways. For the third intersection holds that the main arrival intensity is even higher and therefore the second peak shifts more to the right causing it to behave more like a log-normal distribution. The fact that there is such a big gap between the inter arrival times after the first intersection does also cause a peak in the after travel inter arrival times shown in Figure 4.11. Notice this peak does disappear as soon as vehicles arrive at the main way, which causes the inter arrival times to get lower because there now is a more consistent stream of vehicles.

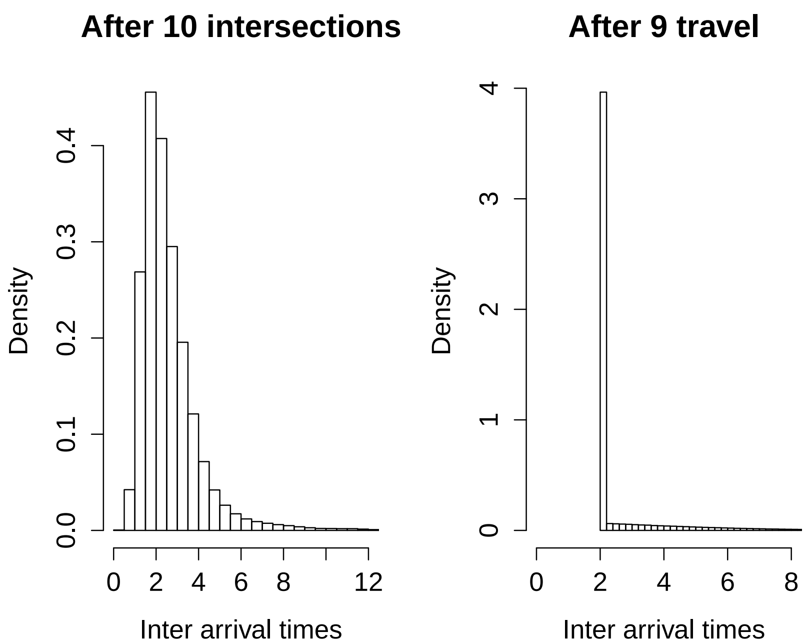

Figure 4.12: Inter arrival times in a limiting situation, with $\mu = 0.1$.

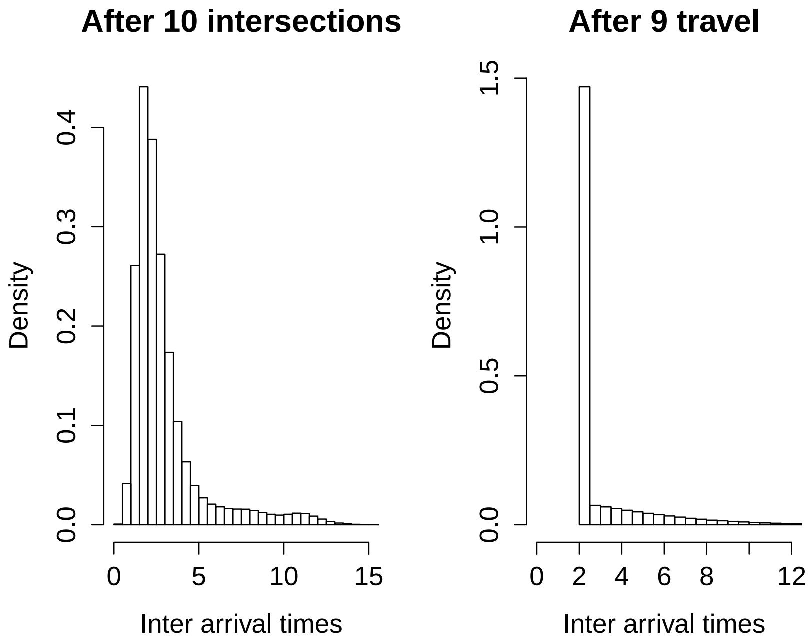

Figure 4.13: Inter arrival times in a limiting situation, with $\lambda = 0.1$ and $\mu = 0.05$.

In Figure 4.12 the situation after 10 intersections is plotted for $\mu = 0.1$. As we can see this situation does not differ a lot from the situation in Figure 4.10 for 3 intersections. This is because the intersection has a maximum capacity, since the arrival intensity of the side ways remain constant and the one of the main way is already near the maximum value the shape does not change a lot. The same reasoning holds for Figure 4.13 in comparison to Figure 4.8.

4.3.2 With side departures

The second situation represented in Figure 4.2 contains side departures. A vehicle driving on the main road has a probability p to cross the intersection and stay on the main road and $1 - p$ to departure and leave at a side road. As a consequence of this the arrival intensity remains constant over a certain time period. The results of this situation will be discussed now. Remark that there is chosen for $p = 0.5$ and remark that $\mu = p \cdot \lambda$.

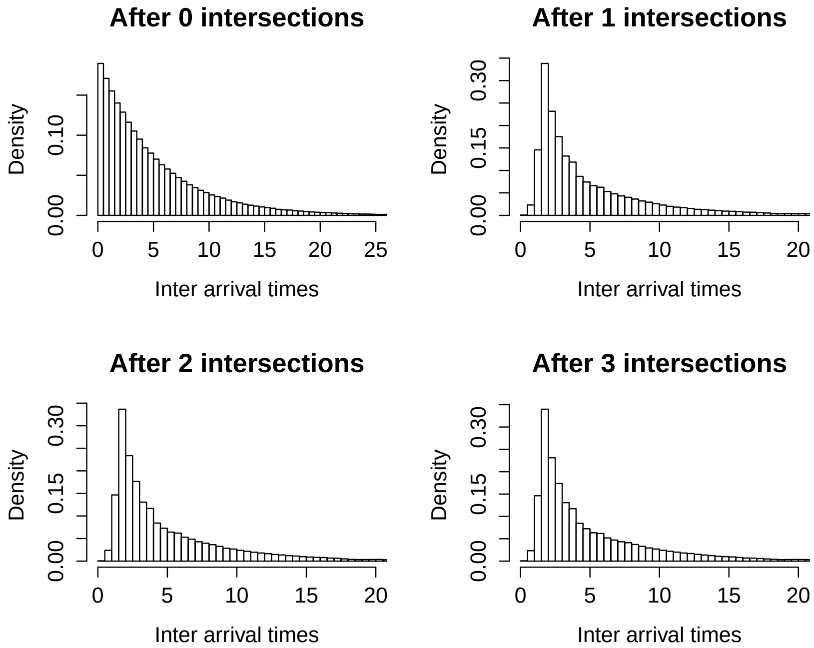

Figure 4.14: Inter arrival times with side departures, with $\lambda = 0.2$.

In Figure 4.14 the situation is shown with $\lambda = 0.2$. Notice that after crossing 1 intersection the inter arrival times distribution remains the same. This could be expected because the departure intensity is the same as the arrival intensity at every intersection. Furthermore notice that the log-normal distribution can still be recognised in the plots, however the peak is lower than a log-normal peak would be. A lot of the mass is spread out to the right. This is because of the red periods between the main way and side way. Another reason is that there is another peak peak within the large peak. This peak is around $t_{red} + \mathbb{E}(departureHeadway)$, which equals 4.27 seconds and causes the fact that the distribution flattens more slowly than a log-normal distribution. Furthermore there are also more larger headways because some vehicles departure on a side way and therefore leave a gap at the main way stream. This causes inter arrival times to increase and thus larger inter arrival times happen more frequently.

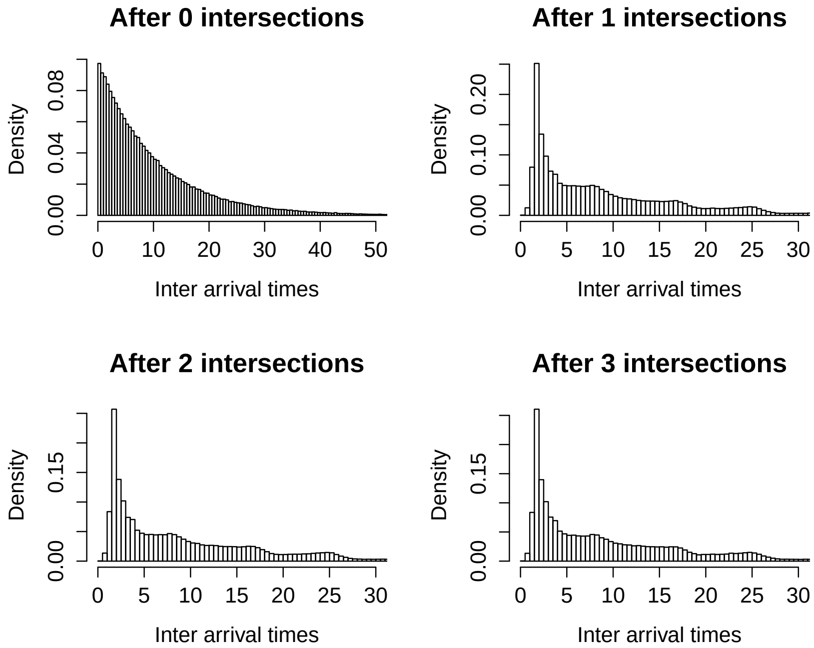

Figure 4.15: Inter arrival times with side departures, with $\lambda = 0.1$.

In Figure 4.15 the same situation is regarded with a lower arrival intensity. Because of the lower arrival intensity the features of the traffic lights show more clearly. The size of the green periods are in this case very determining because there is less traffic. The same reasoning as in the situation without side arrivals can be used to explain the peaks. However in this case a difference is made by random departing vehicles in the main stream causing longer inter arrival times to be more frequent.

4.4 Approximations

The results discussed in Section 4.3 do not give concrete distributions. In this section we will try to regenerate the data using standard distributions. This is the easiest for the situations with high arrival intensity and thus were the settings of the intersections are very determining.

At first the situation with $\lambda = 0.4$ and $\mu = 0.0$ will be reconsidered without side departures. As we can see in Figure 4.7 there are two peaks defining this situation. Both peaks are dependent of the departure headway distribution. In Figure 4.16 the simulation data got estimated with the following estimation method, where $P_1 = lnN(\mu, \sigma^2)$ which estimates the first peak and $P_2 = t_{red} + G_{side} + t_{red} + lnN(\mu, \sigma^2) - (lnN(\mu, \sigma^2) - G_{main} + \mu_{headway} \cdot \lfloor \frac{G_{main}}{\mu_{headway}} \rfloor)$ which estimates the second peak.

Estimation 1. $D_i \stackrel{d}{=} \begin{cases} P_2 & \text{if } i \text{ modulo } \lceil \frac{G_{main}}{\mu_{headway}} \rceil = 0, \\ P_1 & \text{Otherwise.} \end{cases}$

D_i stands for the i^{th} departure. Notice that $lnN(\mu, \sigma^2)$ corresponds with a random draw from the log-normal distribution and thus they won't cancel each other because both draws are different. As Figure 4.16 shows, the first peak does correspond with the basic log-normal distribution. However the estimation of the second peak is a little off. Due the fact that each departure headway is random it is not necessarily true that every seventh car will be the one with the large headway. For the log-normal distribution holds that the median is smaller than the mean. Thus in most cases the residual headway caused by: $lnN(\mu, \sigma^2) - G_{main} + \mu_{headway} \cdot \lfloor \frac{G_{main}}{\mu_{headway}} \rfloor$ is underestimated. Resulting in the fact that the second peak is a little too much to the right.

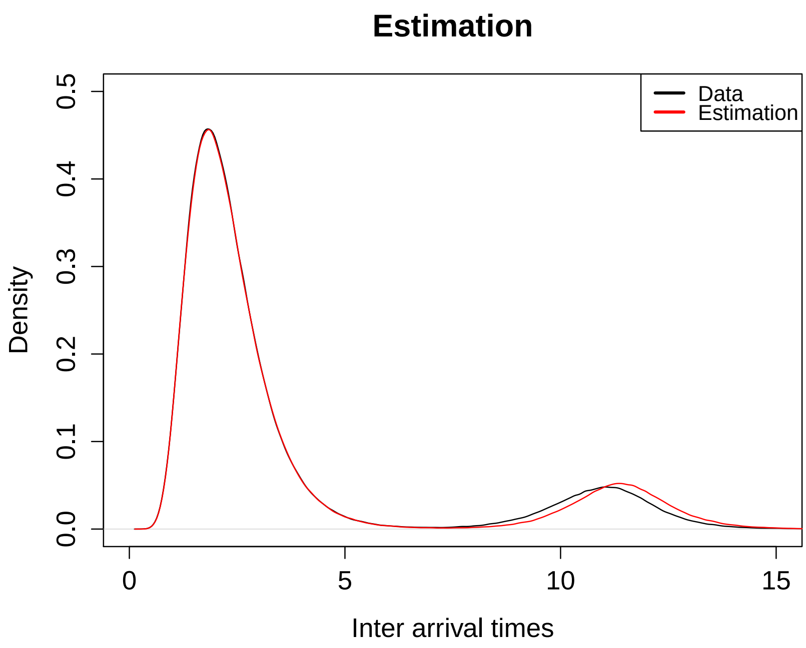

Figure 4.16: Comparison between the simulation data and the estimation.

Estimation 1 can be further simplified, instead of trying to estimate the residual headway we could assume that the residual headway is $0.5 \cdot \mu_{headway}$. This simplifies the situation, but in this case also helps with the problem that we overestimated the second peak. The result can be seen in Figure 4.17. This estimation already fits better with the data than in the previous situation due the fact that value of the residual headway is more close to estimation 2 than to estimation 1.

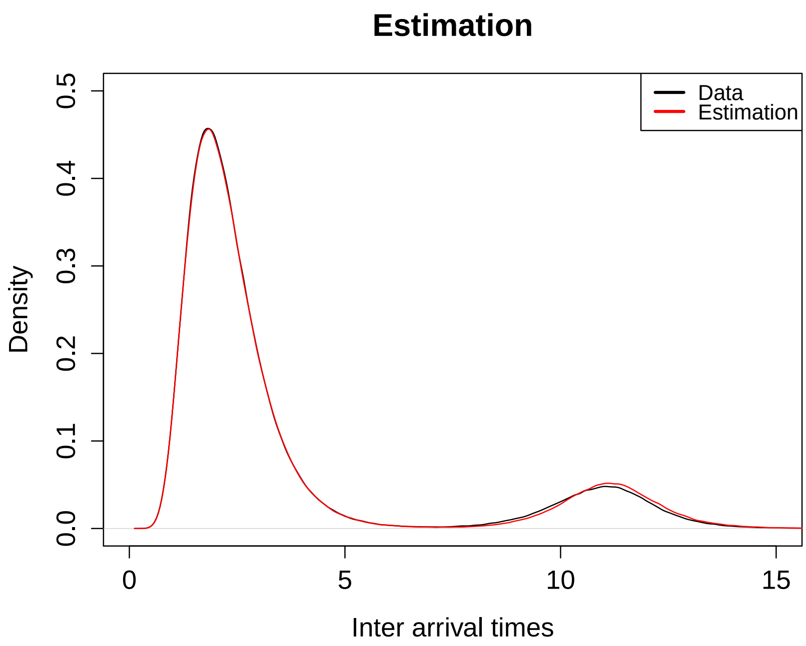

Figure 4.17: Comparison between the simulation data and the estimation.

If the situation gets expanded with side arrivals the estimation method has to be adapted too. Lets consider the the situation with $\lambda = 0.4$ and $\mu = 0.2$. At both ways the arrival intensity is more than the maximum capacity at the intersection. Therefore we can be sure that every green period starts and ends with vehicles departing, because there are always vehicles waiting. In this case the previous estimation can be easily adapted. Now $E_1 = t_{red} + lnN(\mu, \sigma^2) - (lnN(\mu, \sigma^2) - G_{side} + \mu_{headway} \cdot \lfloor \frac{G_{side}}{\mu_{headway}} \rfloor)$, $E_2 = t_{red} + lnN(\mu, \sigma^2) - (lnN(\mu, \sigma^2) - G_{main} + \mu_{headway} \cdot \lfloor \frac{G_{main}}{\mu_{headway}} \rfloor)$ and $E_3 = lnN(\mu, \sigma^2)$.

Estimation 2. $D_i \stackrel{d}{=} \begin{cases} E_1 & \text{if } i \text{ modulo } \lceil \frac{G_{main}+G_{side}}{\mu_{headway}} \rceil = 0, \\ E_2 & \text{if } i \text{ modulo } \lceil \frac{G_{main}+G_{side}}{\mu_{headway}} \rceil = 7, \\ E_3 & \text{Otherwise.} \end{cases}$

Notice that since side arrivals take place with on average 3 cars per green period the same process can be done as in the main period, which led to estimation 2. The results can be seen in Figure 4.18. As you can see in the estimation some mass has shifted to the right. In Figure 4.19 estimation 2 is changed with the residual headway now determined by $0.5 \cdot \mu_{headway}$. This causes some mass to shift to the left and provides a better fit. Again we can conclude that the residual headway is a little overestimated. However these estimations do seem to fit quite well.

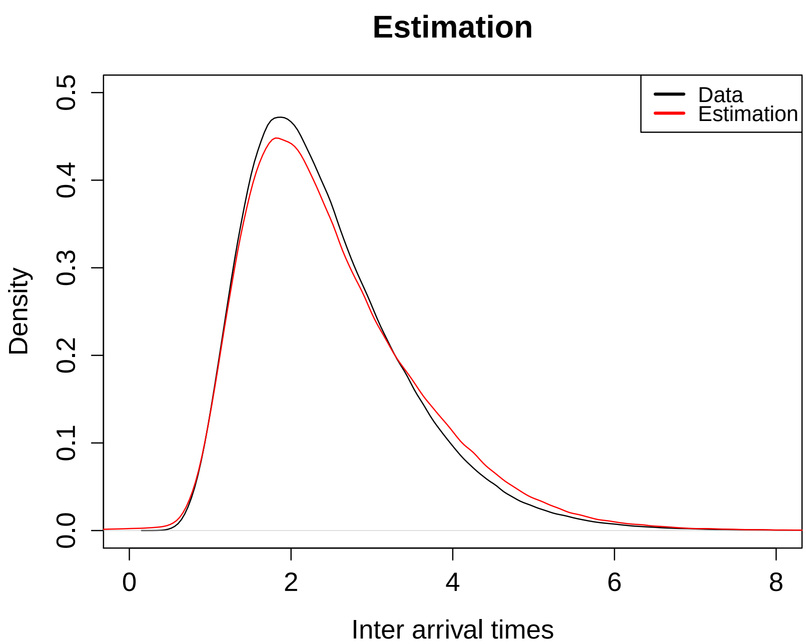

Figure 4.18: Comparison between the simulation data and the estimation.

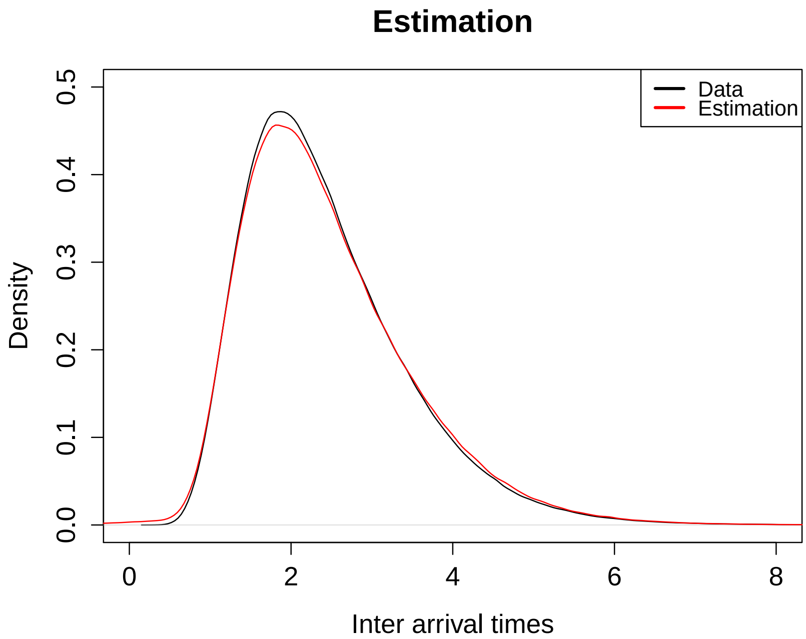

Figure 4.19: Comparison between the simulation data and the estimation.

Even though we can properly approximate the data, we should study the dependence between the vehicles more extensively to get a better understanding of the process which takes places. Only then the process can be well described.

Chapter 5

Conclusion

Throughout this report the arrival and departure processes at signalized intersections have been discussed using two different simulations. However, in Chapter 1 four main question were formulated, which have not been answered yet. The questions were stated as follows:

1. Which model would give a good representation of arrival process A?
2. What is the influence of the distance d on arrival process A. What arrival process will take place at the intersection?
3. What is a good representation of the departure process D from the intersection?

It is now time to answer these question, starting with the first question. In Chapter 2 a literature study is conducted. In most literature the exponential, shifted-exponential and bunched-exponential distribution are considered as the best headway models. These models are also known as model 1, 2 and 3 given in Section 2.1. Each model adds an extra property to the arrival process. The exponential model generates inter arrival times which are continuous and independent. However as said earlier this model does not consider a safe headway. Therefore model 2 is introduced. The principle of model 2 is the same as model 1, however model 2 does not ignore the minimum headway. This makes model 2 a more realistic model. At last model 3 added the aspect of group arrivals to model 2, when θ is larger than zero. If theta is zero than model 3 is the same as model 2. Now to answer the question, all of these models can be used to represent arrival process A. However model 3 is the most comprehensive and realistic. By varying θ, γ and τ this model can be used for roads where overtaking is hard or easy, crowded or not and the minimum headway can be large, small or ignored.

The influence of distance d has been investigated within Chapter 3. Remark that at the road of distance d overtaking is not allowed. As can be seen in Figure 3.1, if the minimum headway is not ignored, then the inter arrival times after travelling of model 1 seem very similar to those of model 3 when every vehicle has the same speed. But also when varying speed distributions are introduced. This gives an indication that model 3 seems to provide a good fit of the arrival process after travelling when overtaking is not allowed. However the goodness of fit depends on the speed distribution, the travelling distance and other factors. As can be seen in the sections of Chapter 3 a larger distance, higher arrival intensity, larger minimum headway result in a higher correlation between the inter arrival times and thus larger bunch sizes. Since model 3 is based on geometric bunch sizes which consists of independent inter arrival times increasing these factors are making model 3 less appealing. However as can be seen in Figure 3.11 the geometric bunch sizes distribution does still provide a good fit under certain circumstances. Therefore when choosing the right θ model 3 can still be assumed to give a realistic representation of the after travel arrival process.

In Chapter 4 the departure process D from the intersection is investigated. At first, existing literature is used to find a departure headway distribution for queued vehicles at the intersection. A log-normal distribution is often stated in literature and therefore considered within this report. As we can see in Chapter 4 this log-normal distribution can be recognised in the results, however the degree of recognition depends on the arrival intensity. In Section 4.4 estimations of the departure process D are given when high arrival intensities are considered. These estimations seem to give a good representation of the data and are easy to generate. As can be seen the departure process is very depending on the traffic light adjustments and thus the cycle-length, green periods and red periods, but also on the chosen queue departure distribution.

Bibliography

[1] R. J. Cowan. Useful headway models. *Transportation Research*, 9(6):371–375, 1975.

[2] O. Hagring. A further generalization of tanners formula. *Transportation Research Part B: Methodological*, 32(6):423–429, 1998.

[3] D. Heidemann and H. Wegmann. Queueing at unsignalized intersections. *Transportation research part B: methodological*, 31(3):239–263, 1997.

[4] X. Jin, Y. Zhang, F. Wang, L. Li, D. Yao, Y. Su, and Z. Wei. Departure headways at signalized intersections: A log-normal distribution model approach. *Transportation research part C: emerging technologies*, 17(3):318–327, 2009.

[5] C.-q. Shao and X.-m. Liu. Estimation of saturation flow rates at signalized intersections. *Discrete Dynamics in Nature and Society*, 2012, 2012.

[6] G. Yeo and B. Weesakul. Delays to road traffic at an intersection. *Journal of applied probability*, pages 297–310, 1964.