MASTER

Inventory control in a multi-location rental system

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Inventory Control in a Multi-Location Rental System

Conducted at EQIN B.V.

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Abstract

This thesis presents the results of a master thesis project that has been conducted at EQIN B.V., a provider of industrial equipment in The Netherlands. The master thesis project aimed to improve the rental inventory control system. Since EQIN’s rental inventory is very divers, multiple mathematical models have been applied to the organization’s rental inventory. Firstly, a single-location model is applied that uses queueing theory to allocate a fixed number of items to each warehouse. Secondly, a lateral transshipment model is used to increase pooling advantages. Thirdly, a model is used that pools the entire demand; the resulting stock levels are divided over the warehouses using an allocation policy. Lastly, a simulation model has been applied as an extension of the first model (single-location). This model assumes that a warehouse orders additional rental items if the on hand inventory drops below a specified safety level. The models are evaluated on the investment costs of its outcomes and the resulting fill rate from a simulation based on actual historical data.

Management summary

This master thesis project has been conducted at EQIN B.V. EQIN provides industrial equipment to its customers in The Netherlands and Belgium through rental and sales. This project focuses on EQIN's rental inventory.

Problem statement

Currently, EQIN has calibrated its inventory based on location managers' desires. It is their responsibility to update these levels over time when they notice that demand changes. However, as demand has drastically changed for some warehouses, these calibrated levels have not been updated. Therefore, EQIN's management wants to apply a method for determining inventory levels that is based on historical demand data. Also, since EQIN has thousands of different product types and multiple warehouses, computing these levels manually is time consuming. Therefore, there is a need for an automated system that can determine the inventory levels.

Research methodology

In the project, the appropriateness of multiple rental inventory control models has been tested on EQIN's rental inventory. Since EQIN's product portfolio is divers, their products are classified in three fleets, based on their initial investment value:

- A-Fleet: Products above €2500
- B-Fleet: Products between €500 and €2500.
- C-Fleet: Products below €500

The outcomes of the rental inventory control models are evaluated for each of these fleets. Demand is assumed to arrive in batches and unmet orders result in (partial) lost sales. In this thesis, unplanned demand is distinguished from planned demand as planned orders may be fulfilled from another location.

C-Fleet inventory: Single-location approach

A single location model is tested and shown optimal for EQIN's C-Fleet inventory. The model assumes that a fixed number of items is allocated to each warehouse. A greedy algorithm is used to optimize this number of items for each product type, realizing a cost reduction of 25%.

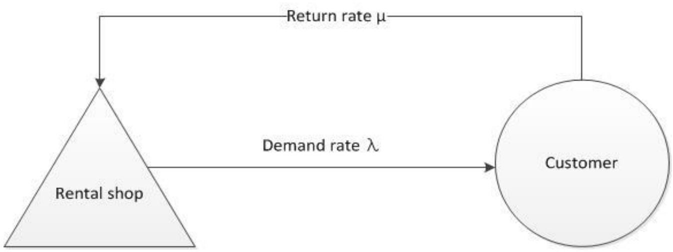

Figure 1: Single-location rental system

An extension has been proposed in which additional items are ordered if the on hand inventory drops below a certain level. This model leads to limited cost reductions compared to the original single-location system (7%) while its transportation increases significantly. This extension is not implemented in the delivered tool as its computations take a lot of time over an hour for single (main) warehouse.

B-Fleet inventory: Multi-location model

For the B-Fleet inventory, a multi-location model is used to model EQIN's inventory per region (Figure 2). The model assumes that main warehouses (hub) can send lateral transshipments to local warehouses to fulfill excess demand. Depending on the willingness of customers to wait for a lateral transshipment, this model can obtain a 20-40% reduction in inventory that is allocated to a local warehouse.

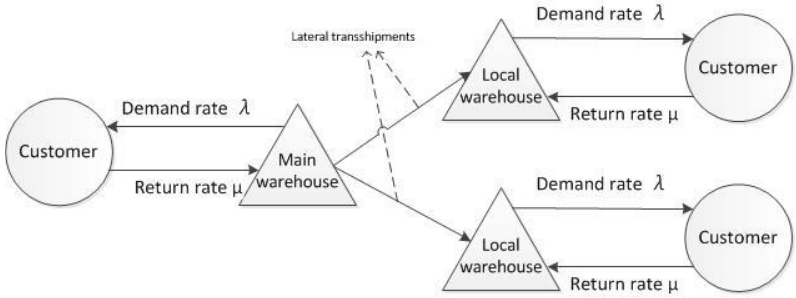

Figure 2: Multi-location rental system

A-Fleet inventory: Complete pooling approach

For the A-Fleet inventory, an approach is studied using a completely pooled demand. The pooled demand results in total inventory levels for all locations. An allocation heuristic is suggested to allocate these items to the individual warehouses. The optimization method resulted in a reduction in inventory value of approximately 10% (depending on the optimization target).

Inventory planning tool

An Excel VBA tool has been developed which computes optimal inventory levels for EQIN's rental fleet. In the tool, the first three models are used. The last model has not been implemented due to its computation time.

Recommendations

The inventory planning tool will provide EQIN a fast way to determine good information on their inventory. EQIN can improve its inventory control system even further without too much effort. The improvement can be realized if EQIN has more knowledge on its demand variables *arrival rate*, *rental duration* and *order size*. This can be obtained quite easily by registering separate orders instead of the state of rental items and by registering lost sales. This is a consideration that EQIN could take into account in the implementation of its new ERP system.

Preface

This thesis concludes not only this research project at EQIN B.V., but also my master program at Eindhoven University of Technology, where I spent a total of five wonderful years. In this preface I would like to take the opportunity to thank everyone who helped me during the project.

Firstly, I would like to thank my first supervisor, Ivo Adan, for his help and guidance during the project. Even during his sabbatical I could always reach out to him. I really appreciate the help and feedback that you offered to me.

Secondly, Simme Douwe Flapper, my second supervisor. I met with you during the mentor assignment phase, so I was happy to learn that you were my second supervisor. You always offer critical, honest and direct feedback, which has nothing but improved my thesis!

I would like to thank everyone from EQIN. Everyone was willing to help me during the project, I am very grateful for that. Also, I enjoyed my time with the other graduate students at EQIN: Iris, Renske and Veerle, thank you for enjoyable lunch breaks in which we could discuss both our projects and private matters.

From EQIN I would especially like to thank Ipe Oosterhof, my supervisor and Jacqueline de Putter. Ipe, you would always take your time to explain EQIN's processes and challenges. Thank you for giving me the opportunity to conduct my thesis project at EQIN. Jacqueline, as an OML graduate you perfectly helped me balance both EQIN's and my university's expectations. I would like to thank you for this but also for our talks on personal matters. I wish you nothing but the best with your baby.

As this thesis also marks the end of my student life, I would also like to thank all my friends and family for their support during these five years.

Vincent Bremer

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List of definitions

Term	**Definition**
Allocated inventory level	Fixed number of items for a product type that is allocated to a location. It is the sum of on hand inventory and the number of rented items.
Blocking probability	Probability that demand cannot be fulfilled from warehouse where initial demand takes place under the assumption of partial fulfillment.
Costs of ownership	The sum of all allocated rental items, multiplied with its corresponding purchasing value. The costs of ownership can be specified either for a single product type, or for an entire rental fleet.
Forwarded order	Planned order that is (in accordance with the maintained fulfillment policy) fulfilled by another warehouse than where initial demand takes place
Lateral transshipment	Excess demand that is fulfilled by a main warehouse instead of the local warehouse where initial demand takes place
Local warehouse	Warehouse that is set up with the main goal to satisfy unplanned demands from local customers
Lost sales ratio	The ratio of the total demand that is lost due to insufficient inventory: $\frac{Lost\ Sales}{Total\ demand}$
Main warehouse (hub)	Central location in each of EQIN's regions. Besides fulfilling customer demands, hubs also replenish local warehouses.
Minimum Stock Quantity (MSQ)	This term is maintained by EQIN. It serves as a threshold for warehouse managers to return stock to other warehouses. This is elaborated more clearly in Section 1.3.2.
Order arrival rate	The expected number of customers that arrive per time unit and demand an order of one or more items of the same product type.
Order size	The number of items of the same product type in a single order.
Partial fulfillment	If an order cannot be completely fulfilled, but on hand inventory is above 0, customers are assumed to accept their order to be fulfilled by the number of items that are on hand. The number of items that cannot be fulfilled is lost

Planned order	Order that has been reserved by a customer at least the day before it is demanded.
Pooling	Merging inventory points to reduce total inventory costs
Rental duration	The cycle that starts when an item is rented and ends when it is returned and ready to be rented again.
Rental items / Rental product	To avoid confusion, the term ‘Items’ is used over ‘Products’ if it refers to a number of rental products of the same product type.
Reorder level	If the on hand inventory drops below this level, managers are expected to order items from another location (unless they have explicit knowledge of returning items).
Single- and Batch-arrival process	A single-arrival process assumes that each customer demands 1 product. In a batch-arrival process, customers order a batch (order size) of the same product type.
Single/Batch-return process	Single-return systems assume a one-by-one return process in which one items is returned at the time. A batch-return assumes that a batch is returned at once
Unplanned order	Order that needs to be fulfilled immediately (no reservation)

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Symbol	Definition
Generic	
P_i	Purchasing price of a single item of product i
q_i	Mean order size of product i (in multi-location assumed location independent)
T_i	Mean rental duration of product i (in multi-location assumed location independent)
Single-location system	
S_i	Allocated number of items of product i to a given warehouse
λ_i	Mean order arrival rate of product i
$\lambda_{i,t}$	Mean order arrival rate of product i during season t
$\boldsymbol{\lambda}$	Total demand rate of all items
$\beta_i(S_i)$	Mean blocking probability of product i in a given warehouse
$\beta_{i,t}(S_i)$	Mean blocking probability of product i in a given warehouse during season t
$\boldsymbol{\beta}$	Aggregate blocking probability
$\beta^{Objective}$	Target aggregate blocking probability
Γ_i	Decrease in blocking probability per invested price
Extended single location system	
S_i	Optimal allocated number of items according to regular single-location system
$MiSQ_i$	If the on hand inventory drops below $MiSQ_i$, additional items are ordered up to $MiSQ_i$
$MaSQ_i$	If the on hand inventory exceeds $MaSQ_i$, overage inventory is sent to another warehouse
β_i	Blocking probability for product i
C_i^o	Costs of mean owned stock
Multi-location system	
S_i^j	Allocated number of items of product i to warehouse j
$\boldsymbol{S_i}$	Vector containing each S_i^j for a given product i
λ_i	Mean order arrival rate of product i in entire system
$\lambda_{i,t}$	Seasonal mean arrival rate of product i in entire system
$\lambda_{i,t}^j$	Mean order arrival rate of product i during season t at warehouse j
$\tilde{\lambda}_{i,t}^k$	Total demand intensity at main warehouse k during season t
$\boldsymbol{\lambda}$	Total demand rate of all items
$\beta_{i,t}^j(S_i^j)$	Blocking probability at warehouse j, during season t
$A_{i,t}(\boldsymbol{S_i})$	Seasonal transshipment ratio of product i
$A_i(\boldsymbol{S_i})$	Mean transshipment ratio of product i
$L_{i,t}(\boldsymbol{S_i})$	Seasonal ratio of lost sales of product i
$L_i(\boldsymbol{S_i})$	Mean ratio of lost sales of product i
$\boldsymbol{L}(\boldsymbol{S})$	Aggregate ratio of lost sales
$L^{Objective}$	Target aggregate ratio of lost sales
$f_i(\boldsymbol{S_i})$	Mean fill rate of product i
w	Fraction of customers that is willing to wait for a transshipment
c_i^a	Transshipment cost of item i

Γ_i^j	Decrease in lost sales ratio per invested price
e_i^j	Vector that denotes that S_i^j increases by 1
Complete pooling model	
$\lambda_{i,t}^J$	Pooled seasonal order arrival rate of product i of all locations j
$\lambda_{i,t}^j$	Seasonal order arrival rate of product i of location j
S_i^*	Total number of items of product i allocated over all locations j
S_i^j	Allocated number of items of product i to main warehouse j
$\boldsymbol{\beta}$	Aggregate blocking probability in system
$\beta^{Objective}$	Target aggregate blocking probability
$\beta_i^j(S_i^j)$	Mean blocking probability for product i at warehouse j

1. Introduction

This report describes a study on the optimization of a rental inventory system. The research has been performed in collaboration with the company EQIN BV, a company that is specialized in selling and renting out industrial material. This study is conducted as a master thesis project for the study Operations Management & Logistics at Eindhoven University of Technology.

This chapter serves as an introduction to the thesis. After reading this chapter, the reader will have an understanding of rental inventory systems and have an idea of EQIN's operations regarding its rental inventory control system. In Section 1.1, the context of rental inventory management is described and Section 1.2 introduces the background of EQIN. This is followed up by an analysis of EQIN's rental inventory system in Section 1.3. Hereafter, the thesis continues with the design of this study, identifying the inventory control problem and stating the research questions.

1.1 Inventory management

With increasing competition, companies strive for operational excellence. Currently still a lot of operational decisions are based on subjective employee opinions instead of scientific methods. An example of this is the decision process in inventory control. In the present day almost all companies make use of information systems to store all kinds of data. Enterprise resource planning systems are tracking business processes in real time. When gathering information for longer periods of time, it can serve as a foundation for decision making in business processes. For instance, historical data can be used to understand and predict demand patterns. Using this information, one can optimize its inventory policies using quantitative models.

Inventory control is studied widely over the years, which makes sense because of its financial importance. In March 2017, the value of the total manufacturing and trade inventories in the United States were estimated at 1,84 billion [1], indicating the interest in optimal inventory control policies. The goal of inventory control is to avoid stock outs while minimizing the costs of inventory. Steven Nahmias [2] describes classic inventory management with the two questions: How often should be ordered and how much should be ordered?

This study focusses on inventory control in rental systems. Rental businesses are very common in our society. One can immediately think of libraries, car rental business and the conventional movie-rental industry. The rental industry is however not limited to consumer products. For companies it may be more attractive to rent certain resources instead of purchasing them. Rental businesses are expected to grow even more in popularity, since customers become less interested in physically owning products [3]. Böckmann [4] classifies the drivers for a shared economy into three classes: **Societal drivers**, such as the customer's drive for sustainability [5]. **Economic drivers**, renting products is especially attractive when purchasing and/or maintenance costs are high and products are only used for a limited amount of time [3]. **Technological drivers**, emerging technological solutions help the shared economy by creating a real time market place [6].

Rental systems show many similarities to traditional inventory systems but do face additional challenges. In contrast to traditional inventory systems, rental systems deal with the reverse flow of goods. When businesses operate from multiple locations this may lead to problems with relocating products and the use of centralized warehouses or repair shops [3]. Although inventory systems have been studied extensively in the past, there has been performed little research on inventory control in rental systems [7]. This is remarkable considering the size of its industries and its emerging popularity. In the United States the annual revenue of 15 rental industries exceeded $1 billion in 2013 and the industrial equipment rental industry even surpassed $25 billion [8].

This thesis has been performed with the goal to analyze and optimize the inventory control system of EQIN BV. Studying this rental inventory system has contributed to the limited research on inventory control in rental systems.

1.2 Company background

This master thesis study has been conducted at EQIN B.V., operating in The Netherlands and Belgium. The organization's headquarter is located in Rotterdam. EQIN (Equipment Intelligence) provides industrial equipment for their customers through both rentals and sales. Typical products in its portfolio are generators, compressors, lighting solutions, welding equipment and tools. Additionally, EQIN provides education in professional practices such as welding. This makes EQIN a complete supplier in the industrial sector.

EQIN is part of the more familiar company Stork. Stork is a multinational that offers service on large assets of their customers, focusing on the oil, gas, chemical and power sectors. Stork is headquartered in Utrecht (The Netherlands). Since 2010, Stork was owned by Arle Capital Partners, a British investment company. Recently however, Stork has been acquired by Fluor Corporation. Fluor Corporation is a multinational service provider from the United States that employs around 40.000 people globally. The merger of Fluor Operations and Maintenance branch and Stork creates a company of 19.000 employees with a turnover of €2,1 billion. Although Stork operates in the same sector as EQIN, collaboration is limited. In some projects, Stork even chooses to work with EQIN's competitors. After the acquisition by Fluor, Stork's new board has stated that it wants to utilize the opportunities for collaboration between the organizations in a better way.

In 2014, EQIN was created by a merger of three companies owned by Stork: 2Rent, Interlas and WTT-lasopleidingen (WTT). WTT provided education and has formed EQIN's educational branch EQIN Opleidingen. 2Rent was a company specialized in rental of industrial equipment where Interlas is specialized in selling industrial equipment. These two companies were expected to merge into a single complete supplier of industrial equipment but the implementation has not been entirely successful. The two branches still operate separately from each other with limited collaboration. EQIN management is currently trying to improve the integration by implementing a new, shared ERP system. This should help to overcome the difficulties for integration between the two value streams.

EQIN tries to differentiate itself from competitors by being a complete supplier and by reducing total cost of ownership for its customers. For regular customers, EQIN does this by offering products that match best with their customer needs instead of offering cheaper solutions. If EQIN has to deliver equipment for an entire project, they may set up a temporary shop next to the project site.

All EQIN's sites are presented in Figure 3. In June 2017, EQIN merged three of its warehouses in the North-East region into a single warehouse (Delfzijl); the warehouses in Emmen and Groningen were closed. The main reason for the merger is the decreasing demand from gas extraction companies. The sites in Stein and Veldhoven are only for educational purposes, so EQIN owns 12 warehouses to distribute their equipment.

Figure 3: EQIN's warehouses and other sites

1.3 Current inventory control system

1.3.1 ABC-Fleet classification

EQIN currently operates across The Netherlands and Belgium but also used to operate in Germany and France. EQIN uses multiple warehouses (also functioning as rental stores), which are divided in five regions. Each region has a main warehouse (at EQIN referred to as hub) that is used to replenish products and perform complex repairs. The largest warehouse is located in Rotterdam, where central management is also located. Additionally, each region has a regional manager and each warehouse has its own local manager.

The company has classified its rental fleet into three classes; the A-, B- and C-Fleet. Table 1 gives the characteristics of the fleet classes. Products are classified based on their purchasing price.

Fleet	Purchasing price	Purchasing	Inventory
A	> €2500	Central	Mainly at main warehouses
B	€500 - €2500	Depends	Depends
C	< €500	Regional	Local warehouse

Table 1: Characteristics of fleet classes

- A-Fleet items are the most expensive products and have a purchasing price above €2500. Due to the high purchasing costs, A-Fleet items are purchased by central management. Because of its high purchasing price, it is preferred to transport items than to have high inventories Therefore, the A-Fleet inventory is generally stored in a main warehouse (hub) to increase the utilization $\left(\frac{Time\ used}{Total\ time}\right)$ of products and, as a result, decrease the inventory levels that are required to fulfill demand.

- An item is classified as a B-Fleet item if its purchasing value is between €500 and €2500. The inventory policy used for these items is less clear. Products are stored locally if considered necessary. However, a quick delivery from another location could be cheaper than buying additional items. Purchasing decisions can be either regional or central.

- The cheaper C-Fleet items are purchased for less than €500. C-Fleet items are bought by regional managers and are stored locally. Since these products are relatively simple, there are many competitors in this market. Customer satisfaction is EQIN’s main priority for these product types and therefore, the company strives for a high fill rate at local warehouses.

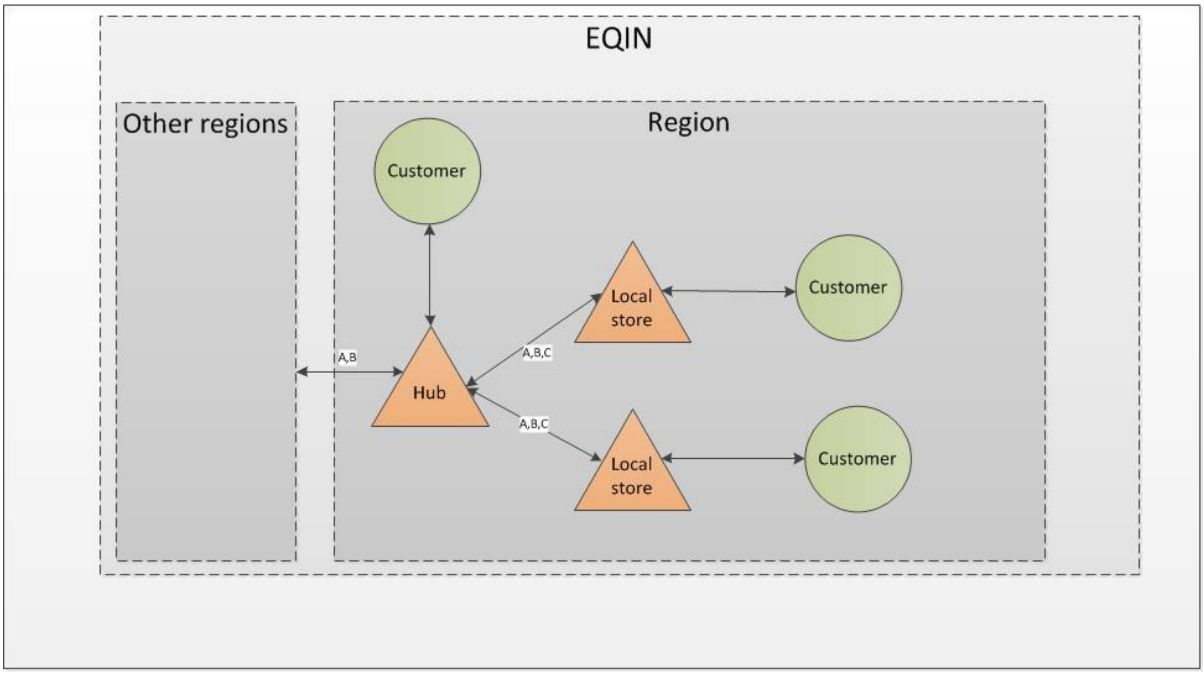

Figure 4: Flow of products

Figure 4 gives a graphical representation of the flow of goods. The figure shows that A- and B-Fleet items can be exchanged among regions if shortages arise at other regions. For C-Fleet items it is felt that these transportation costs outweigh the purchasing costs of new items. Hubs exchange items with their assigned local stores to deal with overages and underages. Purchasing for C items (and sometimes B items) is done by a regional employee, otherwise it is done by a central purchasing employee.

Figure 4 does not show how products leave the system. Products leave the system either by a purchase or a failure that cannot be repaired. However this is not shown as these events are not part of the scope of this research. The scope of the research is elaborated extensively in Section 2.3.1 The figure does also not show how lateral transshipments among locations are dealt with. This will be treated in Chapter 7, where multi-location models are introduced.

1.3.2 Current inventory control system

The current inventory control system is most similar to a (s, S) inventory control system. In a (s, S) system, warehouses order products up to S if the on hand inventory drops below s. EQIN has specified two thresholds for each product type. Firstly, the 'Minimum stock quantity' (MSQ) is calibrated for a location, which would correspond to the large S. Secondly, a reorder level is set, corresponding to the small s. The values for S and s are based on opinions of local warehouse managers (in accordance with the logistics manager).

The current information system is a very practical tool that requires little knowledge of IT systems from local managers. This is preferred by EQIN since these managers are not educated in operating complex IT-systems. In the system, employees can track the real-time status of products, such as the on hand inventories and orders that are rented out. The inventory levels are classified in four states, marked by a specific color. Figure 5 shows the states and their respective colors.

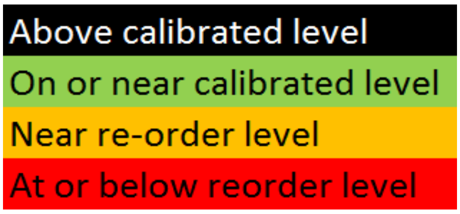

Figure 5: Color system for inventory

1. **Black**: The on hand inventory is above the calibrated MSQ level. In contrast to classic inventory models, where the on-hand inventory can never exceed the calibrated order-up-to level, EQIN's rental inventory can. This is possible due to the returning flow of material. If the system is in this state for a certain product type, items may be sent to another location, if that location has a shortage of the product type.
2. **Green**: This level means that the on-hand inventories are sufficient. There are no actions required. The calibrated MSQ level is above the green 'zone'.
3. **Orange**: Above but near the re-order level means that managers need be aware of the inventory status of these products. If the managers feel that additional items are required, they can request the additional items. At first sight this sounds strange, because of its subjective basis. However, it can be justified since location managers may have information on returning orders.
4. **Red**: If the stock is at or below the reorder level, managers need to take action and order new products. Material managers (located at each hub) will make the decision whether stock is moved from another location or purchased. This decision is based on the number of available items in other locations and purchasing price.

EQIN's location managers have indicated that they like working with this system. It is easy to understand and gives clear indications of what actions need to be followed. Therefore, it is preferred to keep using this tool in the future.

2. Project design

This chapter will present the design of the study. In the previous chapter, Section 1.3 introduced EQIN's inventory control system, this chapter will continue with the problem statement in Section 2.1 and the appropriate research questions in 2.2. Thereafter the scope of the research is described. The chapter will end with an outline of the rest of the thesis.

2.1 Problem statement

Customers expect EQIN to deliver its products on a very short notice. If EQIN is not able to deliver within a demanded lead time, customers are dissatisfied and will often go to a competitor. Customers are able to reserve items and therefore, EQIN distinguishes *planned* and *unplanned* demand. Demand is classified as "Planned" if a reservation has been made at least the day before the demand needs to be satisfied. The calibrated MSQs and reorder levels are based on unplanned demand since EQIN management feels that delivering planned orders is cheaper than keeping additional inventory. To satisfy unplanned demand, EQIN has calibrated the MSQs and reorder levels at their local warehouses across The Netherlands and Belgium. As with all inventory control problems, this results in a trade-off between service level and inventory costs.

The current MSQs and reorder levels are based on the desires of location managers. The logistics manager feels that the specified levels are suboptimal and wants to optimize the inventory. One of the major problems of the current policy is that local managers do not update their desired MSQs. This problem is illustrated in Figure 6. The figure shows the values of all items rented, the sum of all calibrated MSQs, and the on hand inventory levels over time for a single location. Note that the figure shows cumulative values, so for example in July 2015, the total value for the calibrated MSQ levels is approximately €2.000, the value of the on hand inventory is approximately €4.000 and the value of all items rented out is also €4.000.

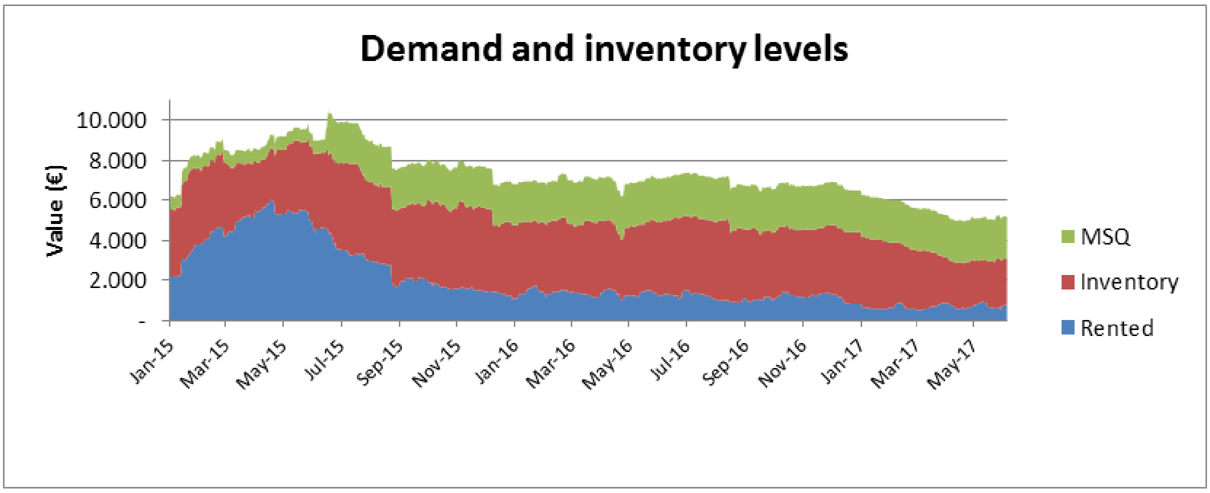

Figure 6: Rental demand, calibrated inventory level and actual inventory level

As Figure 6 shows, the demand (blue) has decreased drastically since 2015. The large decrease in 2015 was caused by a large customer that left the organization so the decrease is not considered as a future trend. In January 2017, the total value of the calibrated MSQs is according to the figure still approximately €2.000, however, the total value of items rented out is €500. This is an indication that the location manager has not responded to the change in demand by decreasing the MSQ's. This figure shows the value for a single location and its decrease in demand is an exceptional case, however, it illustrates EQIN's problem regarding the actuality of the MSQ's.

Additionally, EQIN management wants the optimal inventory policy to be determined for EQIN's rental fleets. Because EQIN has many locations and offers thousands of different product types, updating the inventory levels manually is too much work. EQIN needs a software tool that automatically calibrates the optimal inventory levels based on historical data and updates over time.

2.2 Research questions

To give direction to this research the following research question is defined.

What is the optimal inventory policy and what are the appropriate inventory levels for EQIN's rental system?

This implies that the research will investigate which inventory models are appropriate for EQIN's rental system. Furthermore, the project will deliver a tool that EQIN can use to determine stock levels. The following sub-questions are defined for this research:

- *What are the demand characteristics of EQIN's rental fleet?*
  A data analysis is performed to investigate EQIN's demand process. The variables that have been investigated are the order arrival rate, order size and rental duration. These variables vary among the rental fleets and influence which inventory policy is optimal. Seasonal trends are investigated since many products experience seasonal demand patterns (heaters, lighting, air conditioning, etc.) and projects are often conducted in spring. Chapter 3 will go more into detail on the data analysis.

- *What previous research has been conducted on optimizing rental inventory?*
  A literature review has been performed to get insight in previous research performed regarding rental systems. The main goal of the literature review was to find optimization techniques for rental inventory systems. Chapter 4 discusses this review extensively. This research question will serve as a foundation for the third research question.

- *How should the rental system of EQIN be modeled?*
  For this research questions, multiple inventory models have been studied. Although EQIN owns a multi-location rental network, also single-location models are studied to model the inventory control system since EQIN aims to keep transportation low for its

C-Fleet. The optimization models will be evaluated on the performance indicators presented in Section 2.4.

- *In what way can the optimal inventory control system be implemented?*
  The project will end with a phase that focusses on the implementation of the new inventory control system. In this phase the best inventory models will be implemented in a software tool that can be used by EQIN's logistics manager.

2.3 Scope and assumptions

2.3.1 Scope

Since this study had to be performed in a limited time span, certain choices had to be made regarding the scope of this research.

- As stated in Section 2.1, EQIN wants to calibrate its stock for the unplanned demand. However, EQIN's logistic manager admits that sometimes, planned demand is fulfilled from the inventory that is meant for unplanned demand. This occurs most when a customer demands an order of a small size since the delivery costs of these orders are relatively high. Therefore, this study will investigate the possibility to include planned demand in the optimization of the inventory control system. This is done by three *fulfillment policies*, which are described extensively in Chapter 6.
- When orders are returned by customers, the rental items are inspected whether they are in need of cleaning or repair. This study will not investigate this process since EQIN keeps no track of repair shop throughput times. Another reason that justifies this decision is that repair shop managers do not apply sequencing rules to determine the order in which items are repaired, which would influence the throughput time.
- For large products EQIN may open temporary warehouses, these are excluded in this project. Only EQIN's fixed warehouses are studied.
- Demand for all product types is included in the project.

2.3.2 Assumptions

Repair time

As stated in the scope, an analysis of the repair time is not included in the scope. EQIN's data gives the number of items are available for customers and the number of items rented out, which are used to calculate a total cycle time (elaborated extensively in Chapter 3). Therefore, the concept of rental duration that is used throughout this thesis is assumed to also contain the repair time.

Lost sales

The system does not allow backorders. EQIN operates in a competitive industry where customers can easily leave for competitors. Therefore, all unmet demand is assumed to result in lost sales. This assumption will lead to an overestimation of the ratio of lost sales as in real life EQIN can satisfy a part of the unmet demand through emergency shipments (from other warehouse's inventory) or emergency purchases.

Partial fulfillment
It is assumed that all customers partial fulfillment, i.e. if there is inventory on hand, but an order exceeds the on hand inventory, the customer will accept the number of items that are on hand. This assumption will be further elaborated in Section 5.1.1, which presents the applied lost sales model with its assumptions. As in real life not each customer will accept partial fulfillment, the assumption of partial lost sales will result in an underestimate.

Transshipments
Demand may be satisfied from another location, which is referred to as a transshipment. Transshipments are only allowed from a main warehouse (referred to as hub by EQIN) to a local warehouse in the same region. This is discussed in Chapter 8.

Transportation costs
Since EQIN has a portfolio of thousands of different product types that differ in size and weight, determining the exact transportation costs of each product is too time consuming. However, since the organization operates from 16 locations, transportation is an inevitable factor. It is assumed that transport costs within a rental fleet (A,B,C) are equal to one another. Although these products also differ in characteristics, this subdivision can somehow be justified. EQIN has agreements with PostNL for emergency shipments, which they will use for C-Fleet products. However, A- and B-Fleet items are really expensive, so these items are transported by special transport. The outcomes of the research will be evaluated on the number of times that items have to be transported instead of the actual costs.

Substitutes and correlation in demand
For some product types, there are clear substitutes that may further improve inventory levels (for instance: extension cables of different lengths). Additionally, there are correlations in demand between certain products. However, because of the high number of different product types, this is not researched, so the demand patterns of all product types are assumed to be independent from one another. A method to deal with substitutes will be mentioned under recommendations.

Constant number of items
The computations of the models are based a constant number of items. This implies the assumption that items do not break down or additionally bought (or replaced with the same lead time as the repair time when it breaks down).

2.4 Key performance indicators

The outcome of the inventory models are evaluated on the following indicators:

- *Service level:*
  This is the ratio of demand that is satisfied, either immediately or via transshipment. Demand that cannot be fulfilled result in lost sales. As stated in the assumptions, it is assumed that customers accept partial fulfillment. Therefore, the service level is expressed in number of items: $\left(\frac{Number\ of\ items\ satisfied}{Number\ of\ items\ demanded}\right)$

- *Costs of ownership*
  The goal of this research is to optimize the number of rental items that is allocated to each of EQIN's warehouses. Besides the service level, the optimization will be evaluated on the costs of the allocated rental items. This cost driver is defined as the sum of the purchasing price of each allocated rental item. If c_i and S_i denote the purchasing price and number of allocated items of product i, the costs of ownership are computed with: $\sum_{i \in I} c_i * S_i$.
- *Transportation*
  As stated in Section 2.3.2, transportation is an inevitable cost in EQIN's rental system. Although the actual transportation costs will not be computed, the evaluation of each model will discuss the number of times that transportation is required in the system.

2.5 Thesis outline

Chapter 1 introduced the context of the problem and the organization EQIN, where the research is conducted. In this chapter the problem has been formulated and research questions have been drafted. The following chapters are dedicated to answer the research questions. Chapter 3 presents an analysis of EQIN's data on demand arrivals and returns. This is followed by a review of the available literature on rental inventory systems in Chapter 4. In Chapter 5, a function that calculates the ratio of lost sales is introduced and tested. This function is of importance for the following chapters. Three analytical models are used to model the rental inventory system in the Chapter 6, 8 and 9. Chapter 7 will discuss an extension of the first model of Chapter 6. The outcomes of this extension are computed through simulation. Finally, Chapter 10 presents the software tool that is developed for EQIN to determine its inventory levels. Figure 7 gives an overview of how the chapters contribute to answering the research questions.

RQ1: What are the demand characteristics of EQIN's rental fleet?
- Chapter 3: Analysis of demand and returns

RQ2: What previous research has been conducted on rental inventory?
- Chapter 4 Literature on rental systems

RQ3: How should the rental process of EQIN be modeled?
- Chapter 5: Batch-arrival loss function
- Chapter 6: Single-location rental inventory system
- Chapter 7: Single-location system with safety stock
- Chapter 8: Multi-location rental inventory system with lateral transshipments

RQ4: In what way can the optimal inventory control system be implemented?
- Chapter 10: Inventory planning tool

Figure 7: Thesis outline

3. Analysis of demand and returns

The data analysis has been performed to gain knowledge on the *demand process* and *rental duration*. Firstly, Section 3.1 presents the available data. Thereafter, the methodology of the analysis is given in Section 3.2. Sections 3.3 through 3.6 present results of the actual data analysis.

3.1 Available data

This section describes the available data on EQIN's inventory system. EQIN's ERP system registers the number of items in a specific state, but it does not specify the number of orders. This would be helpful when determining order arrival rates, order sizes and rental durations. The available data dates back to 2013. Table 2 gives an impression of a data sheet when extracting data of two product types for a period of two days. This data is fictional and solely meant to give an impression of the available data. Firstly, the sheet states the location(s) and the extracted products with their corresponding purchasing value. Thereafter, the sheet states the state of rental items for each day (at end of day):

- ***Rented out***: This column states the number items that are rented out. For the first product, the value of this column increases. Customers can place an order that consists out of multiple items (of the same product type), so this increase indicates that one or more customers have arrived during that day.
- ***On location***: This column states the on hand inventory. As the number of rented items increases by 3, the on hand inventory decreases by 3 (assuming that the number of items owned by a location remains equal).
- ***Reserved***: This column states the number of reserved items. For the first product, the number of reserved items decreases at the same day as a customer has arrived, indicating that the demand had been reserved. As indicated in section 2.3, EQIN distinguishes planned and unplanned demand. A demand is classified as planned if the column "Reserved" decreases at the same day as the demand occurs.
- ***Owned***: This column states the number of items allocated to a location (its inventory position). This is the sum of the items rented out and the on hand inventory (on location). A change in this column implies that an item has been transported from or to another location.

Location 00		**Day 1**				**Day 2**			
Product ID	**Value €**	**Rented out**	**On location**	**Reserved**	**Owned**	**Rented out**	**On location**	**Reserved**	**Owned**
1234567	60.00	6	6	3	12	9	3	0	12
7654321	1000.00	1	1	0	2	0	1	2	1

Table 2: Impression of available data

Unfortunately, this data does not state the details on order arrivals and returns:

- The exact numbers of customers that have arrived to rent or return an item are unknown.
- Order arrivals and returns may cancel each other out.
- Lost sales are not registered

It is required to know these details to compute the demand and rental duration exactly. To do this, one could check each invoice manually. However, since this should be done for each day, thousands of products at multiple locations, this is too time consuming. To overcome these limitations, the following extra assumptions are made:

On a day, there are either demands or returns, but not both
On days with a positive change in rented items only demand has occurred. On days with a negative change it is assumed that only returns have occurred. On days with no change no events have happened. This assumption is reasonable due to the low order arrival rate of the products; the daily order arrival rate is for most products below 0.2. Therefore, the implication of this assumption will be limited.

Lost sales are not estimated
It is very hard to make assumptions about the number of lost sales lost sales. One method to estimate the lost sales is by using the demand intensity while on hand inventory is 0. However, in EQIN's case, there are cases where the on hand inventory increases the day before a demand occurs, indicating that this demand was known by EQIN employees (and thus placed when on hand inventory is 0). Also, since customers can order more than one item at once, lost sales can occur when on hand is present. Therefore, the total demand is purely estimated on the registered demands. The implication of this assumption is that the demand is underestimated.

All items demanded on a single day are demanded by a single customer
This assumption will be elaborated in Section 3.4.1 after it shows that the system follows a batch-arrival demand process.

3.2 Data analysis

3.2.1 Selected data

To analyze EQIN's demand, the daily number of items rented is analyzed (column 'Rented out'). Figure 8 gives an impression of how this column would change over time. Using this data, variables on the demand can be analyzed.

In the data analysis, no distinction has been made between planned and unplanned orders. The rental inventory models will not only be evaluated for unplanned, but also for planned orders and therefore, these orders are also of importance in this data analysis.

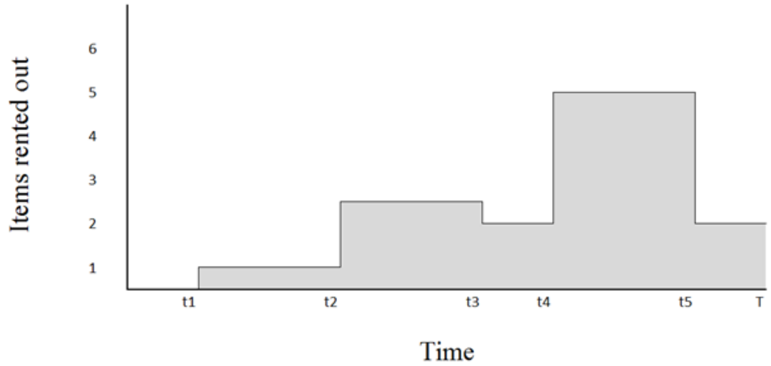

Figure 8: Figure of items rented out

EQIN's rental fleet is classified in the A, B and C-Fleets. Since the outcomes of the models will be evaluated separately for these fleets, the data analysis is performed separately as well. The available data dates back to 2013 but all tests are performed over a time span of two years (2015 and 2016) as to analyze actual data. For the A-,B- and C-Fleet, different samples are used in the analysis, this decision is based on the following considerations:

A requirement of the applied goodness-of-fit test is the sample size (Section 3.2.2). For many products, demand intensities are too low (below 0.2 arriving customers per day) to perform a goodness-of-fit test. For A- and B-Fleet items, this implies that the tests cannot be performed on data from a single-warehouse. Therefore, it is decided to perform the tests for the C-Fleet on a single location, the B-Fleet on a complete region and the A-Fleet is tested for EQIN's entire demand.

3.2.2 Goodness-of-fit test

The analysis makes use of a goodness-of-fit method in order to determine the required parameters. The applied goodness-of-fit method is the Chi-Square goodness-of-fit test, which is applicable to discrete functions [9]. The test measures discrepancies between the expected number of observations according to a stochastic distribution and the actual number of observations. The outcome of the test determines how well a stochastic distribution fits the set of observations. The Chi-Square test statistic is defined as:

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{3.1}$$

where O_i and E_i denote the number of observed and expected instances in class i respectively and k denotes the number of classes. The null hypothesis states that the set of observation follows a certain stochastic distribution. The null hypothesis h_0 is rejected if $\chi_0^2 > \chi_{\alpha,d}^2$, where α indicates the significance level of the test and is set at 0,05, which is a widely used value of α. This implies that the null-hypothesis is rejected with a certainty of 95%. d denotes the degrees of freedom. Additionally, the corresponding P-values can be computed. The P-values give the probability that, if h_0 is true, its resulting values would be as extreme as the observed values.

One of the requirements of the Chi-Square test, is the sample size. This study uses the method described in the book by Montgomery & Runger [9] to perform the test. The authors state that there is no general agreement regarding the minimal expected number of observations in a class but values of 3, 4 and 5 are widely used as minimal. Due to the low order arrival rates, this analysis will use a minimum of 3. If the class has an expected value below 3, the class should be merged with a neighboring class. The number of degrees of freedom (d) is $k - m - 1$ in which k is the number of classes that are specified and m is the number of parameters of the distribution function that are estimated from the data such as the mean. The number of degrees of freedom should be equal or larger than 1 to compute $\chi^2_{\alpha,d}$. Also, the Chi-Squared test requires independence of data. As the data meets these requirements, it is concluded that the Chi-Squared test can be applied.

3.3 Seasonal patterns

EQIN's products follow seasonal patterns but there is a large variety in seasonality. Many products have peak periods in Spring or Summer, when most industrial projects are conducted but some product types have other peak periods. For example, Figure 9 shows the demand pattern of a (1) distributor, (2) chassis and (3) extension cable. The figure shows the existence of seasonal patterns. For this reason, seasonal factors will be introduced when modeling the rental inventory system.

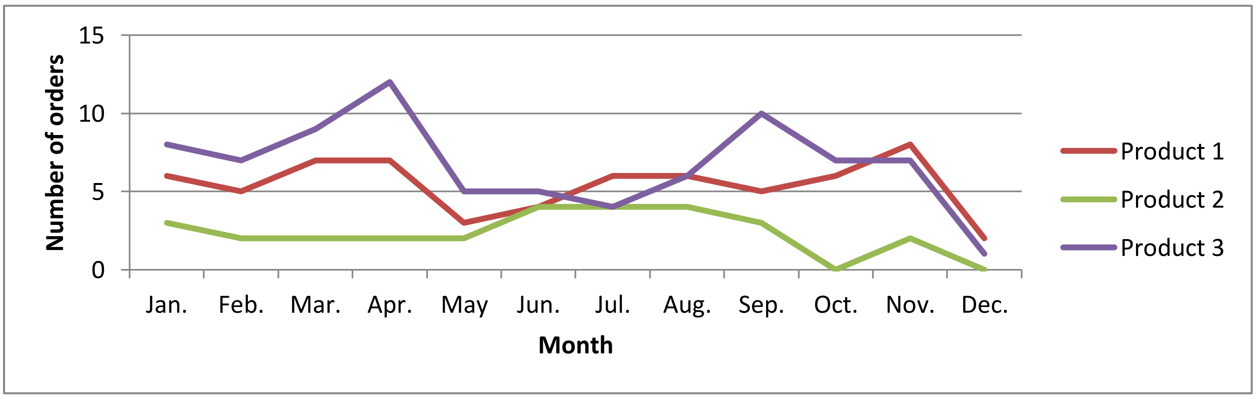

Figure 9: Seasonal factors per month

For the model, four seasons of three months will be distinguished. If a product has a mean order arrival rate of λ and a seasonal order arrival rate λ_t during season t, its season factor would be $c_t = \frac{\lambda_t}{\lambda}$. The seasons that are distinguished are January-March, April-June, July-September, and October-December.

3.4 Arrival process of demand

3.4.1 Batch-Arrival process

This section aims to determine the arrival process of the demand. Table 3 gives data on the demand of three C-Fleet products (same products as used in the example in 3.3).

Index	Product type	Mean demand (items per day)	Fraction of days with demand
1	Distributor	0,318	0,041
2	Chassis	0,289	0,134
3	Ext. cable	0,182	0,034

Table 3: Analysis of daily demand

The third column gives the mean demand in number of items per day $\left(\frac{Total\ demand}{Number\ of\ days}\right)$. The fourth column gives the fraction of days with demand $\left(\frac{Days\ with\ demand}{Number\ of\ days}\right)$. Using these numbers, one can calculate the mean per day, given that a demand has occurred $\left(\frac{mean\ demand}{fraction\ of\ days\ with\ demand}\right)$. This results in such high numbers, that it is concluded concluded that demand arrives according to a batch-arrival process. The importance of this lies in the fact that the appropriate loss-model that needs to be applied depends on the arrival process [10,11].

This finding leads to another assumption regarding the available data. It is assumed that if the number of rented items increases at a certain day, the entire increase is caused by a single order (batch). As this assumption may sometimes combine multiple orders, the estimated order size will be overestimated and the order arrival rate will be underestimated. Section 3.5 will investigate the distribution of the order size, but first, the following section will test whether batches arrive according to a Poisson process.

3.4.2 Inter-arrival process

This section investigates the inter-arrival times of EQIN's rental system. Figure 10 how inter-arrival times are computed. In the Figure, three orders arrive at $T1$, $T2$ and $T4$. The inter-arrival times of these orders are $(T2 - T1)$ and $(T4 - T2)$.

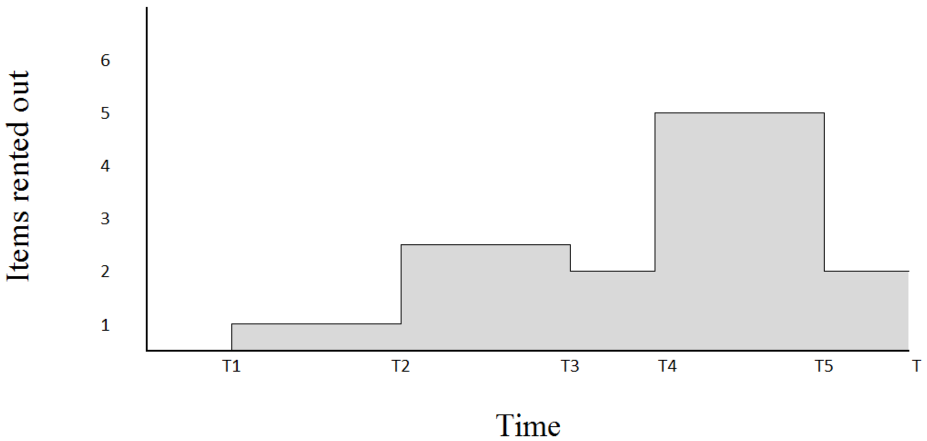

Figure 10: Illustration of inter-arrival times

As stated in Section 3.3, EQIN's demand is exposed to seasonal influences. To model this seasonality, four seasons are distinguished. When analyzing the inter-arrival times of EQIN's orders, demand is assumed to be stationary within these seasons. In Section 3.2.2 it has been explained that the Chi-Square test requires a minimum of three classes with an expected value of at least three items (within a single season). In the analysis, all four seasons of each product has been analyzed. Each season that met the requirements of the Chi-Square test has been tested.

Before conducting the Chi-Square test, for four products a histogram is made of their observed and expected inter-arrival times, according to the exponential distribution. Figure 11 gives the histograms of these products. The first two products seem to follow an exponential distribution. The other two products seem to deviate from the exponential distribution.

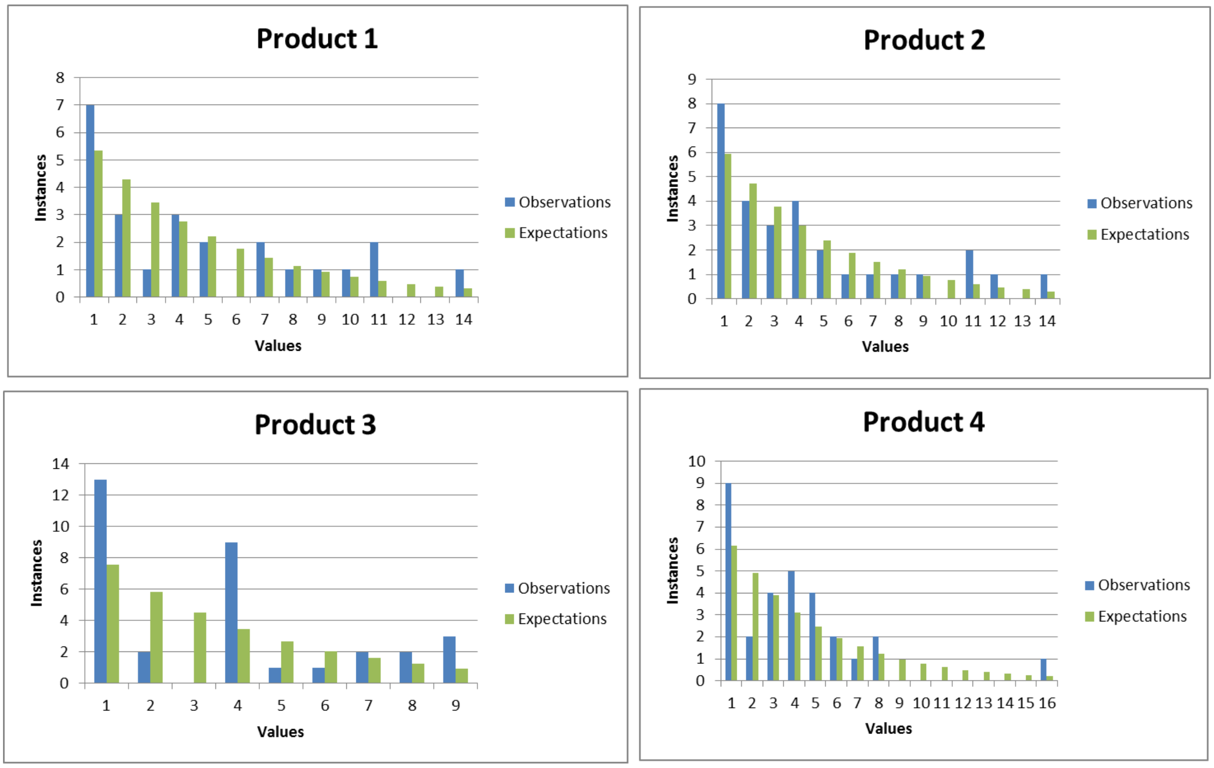

Figure 11: Observed and expected inter-arrival times

One of the characteristics of the exponential distribution is that the standard deviation (σ) is equal to the mean (μ). Table 4 gives the mean and standard deviation of the four products. For all products, the mean is larger than the standard deviation. In line with the expectations the standard deviation is closer to the mean for the first two products.

Product	μ	σ	$\frac{\mu}{\sigma}$
1	4,79	3,85	1,24
2	4,45	3,73	1,19
3	3,67	2,76	1,33
4	3,87	2,21	1,75

Table 4: μ and σ of inter-arrival time

3.4.3 Chi-Square test on inter-arrival times

Table 5 gives the results of the Chi-Squared test. The Chi-Squared test tests if the inter arrival times are exponentially (h_0). For the C-Fleet, the analysis has been performed for the warehouse in Moerdijk. For the B-Fleet, the warehouse in Botlek (Rotterdam) has been used since demand in the warehouse in Moerdijk are too low. For the A-Fleet, EQIN's entire demand has been used.

The table shows that for the larger part, the exponential distribution is rejected at the 0,05 level (approximately 58% of all analyzed seasons). At the 0,01 level, the exponential distribution is rejected for approximately 40% of all products.

Fleet	**Sample**	**Number of seasons analyzed**	$\boldsymbol{h_0}$ **rejected** $(\boldsymbol{\alpha = 0,05})$	$\boldsymbol{h_0}$ **rejected** $(\boldsymbol{\alpha = 0,01})$
A	Total	195	128	97
B	Botlek	172	94	61
C	Moerdijk	124	61	39

Table 5: Chi-Squared test on Poisson arrival process

Figure 12 shows the Chi-Square test of a single product (ProductID: 6612922, welding clamp). For the first season, only 2 classes could be formed, so this season has not been analyzed. The other seasons have 3 classes and are thus analyzed. For these seasons, h_0 is accepted on the 0,05 level.

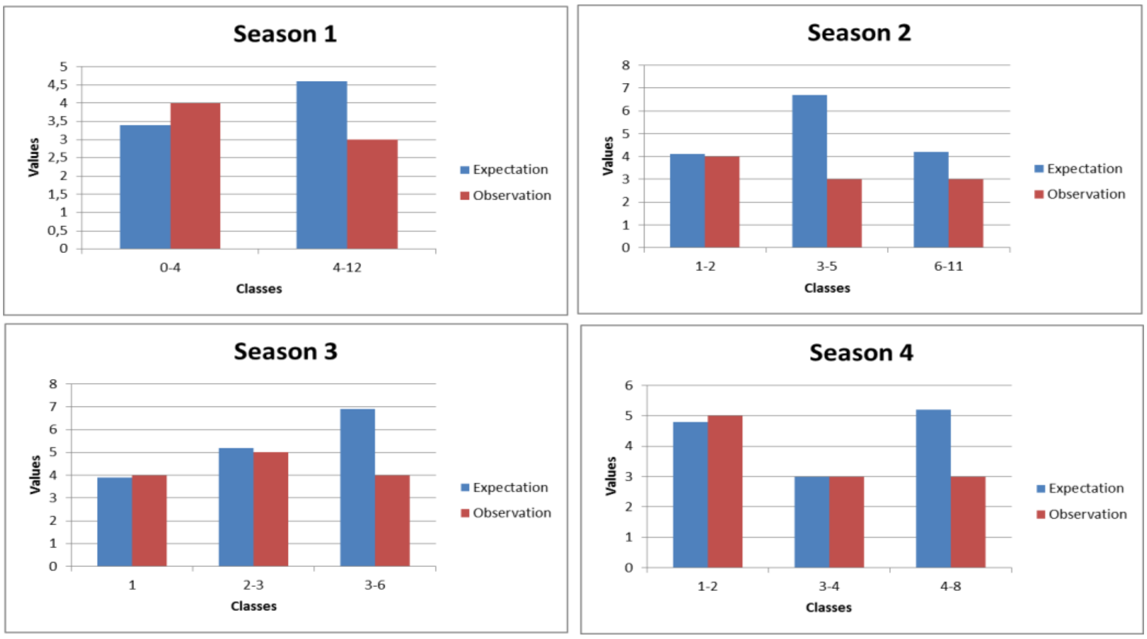

Figure 12: Chi-Square test (Poisson) of single product

A further analysis on the rejected seasons, revealed for how many products all seasons are rejected. For the C-Fleet, 39 products are tested over more than two seasons ($N \geq 2$). For only 7 of those 39 products, all seasons are rejected. This implies that for the other 32 products, for at least one season the Poisson process is accepted. Table 6 gives the results for the other fleets.

Fleet	**Number of products** $\boldsymbol{N \geq 2}$	$\boldsymbol{h_0}$ **rejected for all seasons** $(\boldsymbol{\alpha = 0,05})$
A	57	16
B	44	7
C	39	7

Table 6: Chi-Square test on multiple seasons

From Table 6 it is learned that for a vast majority of the products, a Poisson process is accepted for at least one of its seasons. It is expected that the high total number of rejected seasons (Table 5) is partially caused by the seasonality, explicitly by the following two reasons:

1. Seasons are defined equally for each product. However, the length and timing of seasons are different among products. This causes demand to not be stationary within some seasons for some products.
2. It is assumed that that demand is stationary during a season. However in real life, demand also fluctuates within a season. To model seasonality an assumption had to be made on the length of seasons. By reducing the lengths of seasons, seasonal factors could get really extreme due to low order arrival rates.

Considering the results this section it is chosen to model the order arrival process according to a batch-arrival Poisson distribution. We have seen that the demand arrival process of most products follow, for at least one or more periods of time, the Poisson distribution. Hence, it seems reasonable to assume this for modeling EQIN's rental system. This assumption might influence the performance of inventory models as for some products the distribution of the inter-arrival times is different from the stationary Poisson distribution.

3.5 Distribution of order size

In Section 3.4 it has been determined that orders will be modeled according to a batch-arrival Poisson process, so this section analyzes the order (or batch) size. As mentioned before, previous research has indicated that the lost sales function in a system with group-arrival processes depends on the mean service time, independently of service time distribution [10]. However, the authors state that this property only holds if the order sizes are geometrically distributed. Therefore, this section will investigate if the order size follows a geometric order size distribution (h_0). The probability distribution function of the geometric order size is given in Appendix 1. Table 7 shows a summary of the chi-squared test on h_0 for EQIN's rental fleet.

Fleet	Number or products	Products tested	h_0 accepted
A	187	39	35
B	351	65	56
C	601	100	68

Table 7: Chi-squared test for geometric order sizes on rental fleets

For the A- and B-Fleet, results are satisfactory. For the A-Fleet, 35 out of the 39 tested products are accepted to have a geometric distribution of order sizes ($\chi_0^2 > \chi_{\alpha,d}^2$). The test on the B-Fleet gives similar results, 56 out of 65 products are accepted for the geometric distribution. From these results is concluded that it is reasonable to model the order sizes of A- and B-Fleet items geometrically. Unfortunately, the results of test on the C-Fleet items are less convincing. Of the 100 tested products, 32 items are rejected.

To get a better insight in the C-Fleet products, six products are tested individually. Table 8 gives the results of this test. For product 1 to 3, the chi-squared test accepted the null-hypothesis for these products which implies that its order size is geometrically distributed. For the last three products the null-hypothesis is rejected.

Index	Product type	Number of orders	Mean order size	χ_0	$\chi_{0,95,d}$	P-value
1	Ext. cable	24	7,71	3,17	9,49	0,5300
2	Chassis	78	1,65	0,92	5,99	0,8938
3	Adapter	20	3,90	4,87	5,99	0,0875
4	Ext. cable	52	2,35	8,30	7,81	0,0402
5	Adapter	93	3,11	29,78	12,59	0,0000
6	Scaffold	45	18,60	74,68	18,31	0,0000

Table 8: Chi-Square test on distribution of order size

Figure 13 gives a better view of the order size distributions of the six products from Table 7. The figure consists out of histograms that show the number of observed products with a certain order size (blue) with the expected number of observations according to the geometric

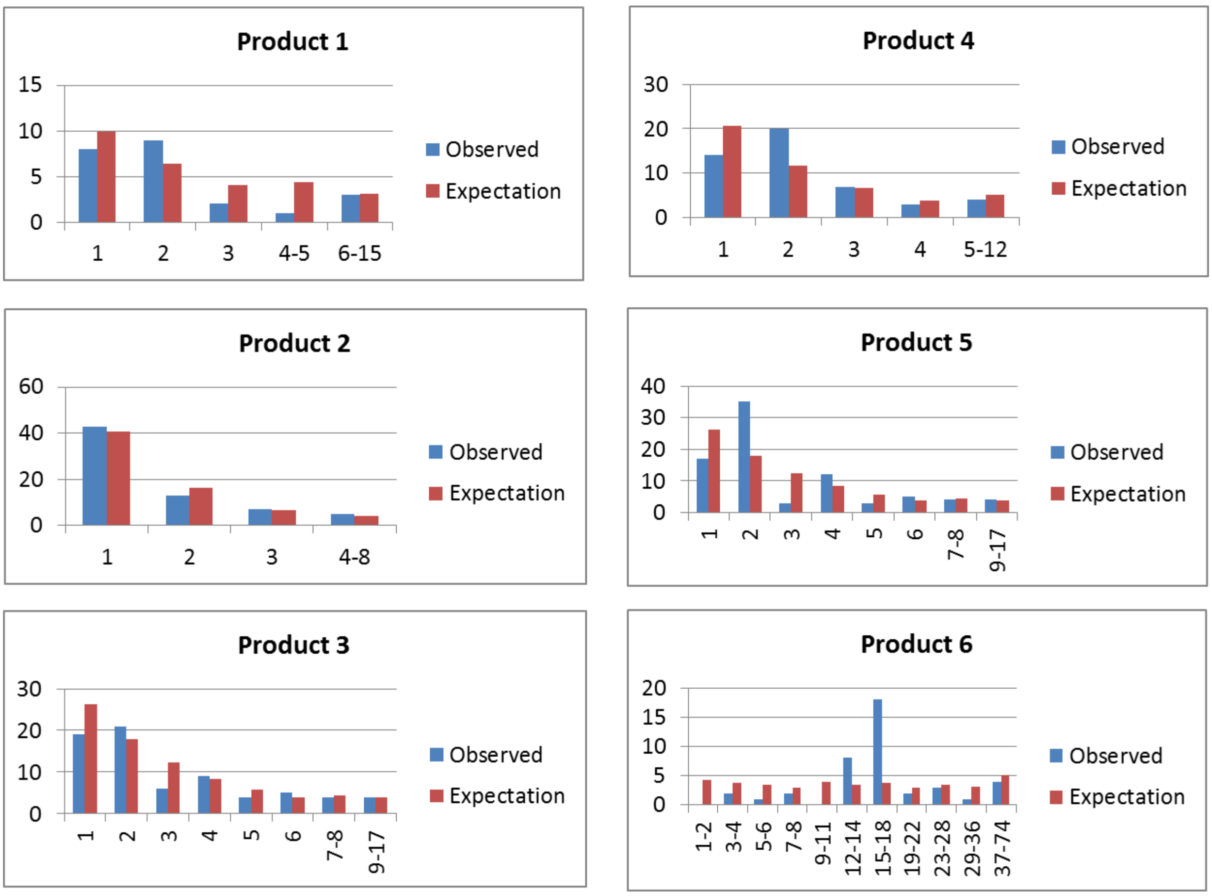

Figure 13: Distribution of order sizes of six products

distribution (red) using the same mean order size. The figure shows the observations and expectations for classes, not specific values. So for product 1, the bars at ‘6-12’ denote the sum of the observations and expectations for the values 6 to 12.

From the figure we conclude that product 2 matches the geometric order size distribution best, in line with the result from the table ($\chi_0 = 0{,}92$). Its observations are similar to the expected values. The observations for products 1 and 3 also show similarities with the expected values.

For products 4 through 6 the geometric order size distribution has been rejected. However, when analyzing the graphs for products 4 and 5 in Figure 13 one will notice that the order size distributions show similarities to the geometric distribution. The distribution of product 6 is nothing like the geometric order size distribution.

In the literature review, loss models usually assume geometric batch (order) size distributions [10]. Because not all products follow the geometric distribution, Chapter 4 will present a sensitivity analysis on the batch size distributions for these models.

3.6 Rental duration

The exact rental durations are not available in the extracted data, therefore it has to be estimated. In this section, we suggest two methods for doing so:

Little's law
In Chapter 4 it was learned that rental systems are usually modeled as queueing systems. Using this insight, Little's Law is used to estimate the mean rental duration ($Rental\ duration = \frac{Rented\ out}{Demand\ rate}$). As the rented inventory and arrival rates are known, the available data should be sufficient to use this method. Figure 14 gives a graphical representation of the known data. Note that the demand rate is in rental items, so the demand rate in the figure is $\frac{6}{T}$.

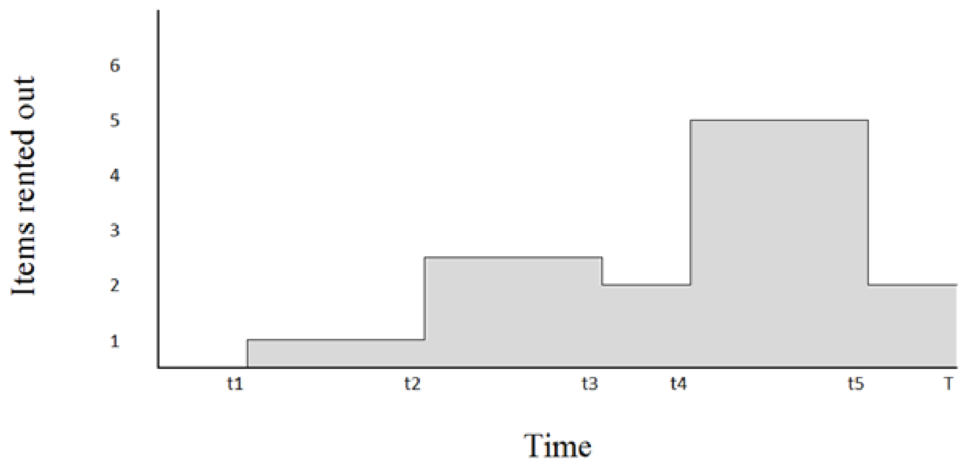

Figure 14: Estimation of rental duration

However, due to low order arrival rates for many products, this method may result in unrealistic values if there are rented items at the start of the considered horizon. Therefore, if this is the case, a *dummy order arrival* is created at the start of the period (red marked order at T_0). Doing so, the rental duration results in an underestimation of the actual duration.

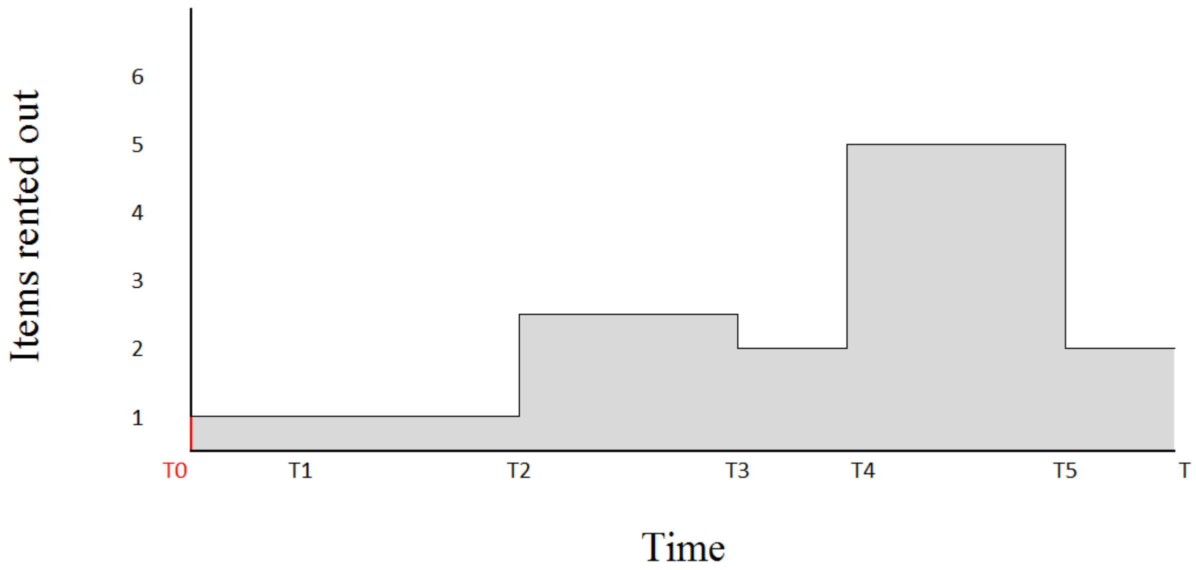

Figure 15: Exceptional case for computation of rental duration

4. Literature on rental systems

As stated in the introduction, there has been limited research conducted on rental businesses [7]. Fortunately, many of the stochastic processes in these systems are similar to other, well studied processes that are, from a modeling point of view, equal to queueing processes. Many researchers have used this property when studying rental systems. For instance, Tainiter [12] suggests that a rental process is equal to a company that deals with stochastic lead times when replenishing its sales. Also, rental systems have many similarities with spare parts inventory systems [3]. However, in most rental systems unmet demand is assumed to lead to lost sales where most spare parts problems assume backorders.

Because of the great variety in products, both single-location and multi-location models are studied. Also, EQIN experiences a batch-arrival demand process for a significant part of its products, so this is an important subject to study.

4.1 Single-location rental systems

To describe a single-location rental system one can think of a stand-alone movie-rental business or a library. Typical problems that are studied in single-location rental systems are total the number of items purchased by a company, with stationary or non-stationary demand or problems regarding prioritization of certain customers [7].

The first researchers to describe rental systems date back to the 1960s [13,14]. These papers use queueing processes to model rental systems. Thereafter a couple of extensions were introduced. For rental systems, decreasing demand is a well-studied subject [7,15,16] since many consumer rental industries are focused on fashion products (especially movies). Other extensions that have been studied are the option to buy an item [11] during the rental period (rent-to-own system) and the impact of usage based loss [8] which is modeled by a failure rate that increases over the time that an item is rented out. In the future, these papers may be used to model repairs, if EQIN collects more data on failure rates of its products.

Most researchers use an arrival process that assumes one-by-one (single) arrivals. However, Papier and Thonemann [10] consider a fleet sizing problem that assumes a batch-arrival and batch-return process with partial fulfillment. Calculating the number of lost sales is significantly more complex in a batch-arrival system. Papier and Thonemann use an exact loss function [17] that is independent from the rental duration distribution. Because the function is time consuming for large values, they also suggest an approximation formula.

4.2 Multi-location rental systems

Examples of multi-location rental systems are city bike rental systems or car rental systems. When multi-location systems allow transshipments among locations, it allows a warehouse to satisfy demand from another warehouse. This brings pooling advantages [18], which may lead to major cost reductions. The left part of Figure 16 gives a representation of such a system. In the figure circles denote regular warehouses, triangles denote central warhouses. Regarding the return process of rental systems one can distinguish two streams of rental systems: The fixed-return systems and the free-return rental systems [3]. In fixed-return system customers return items to the location where they rented the item, this is usual in movie rental or library systems. This is in contrast to, for example, many vehicle rental systems, which often allow customers to return items to a rental location of their own choice. Such a system is defined as a free-return system. These free-return systems bring the challenge of relocating items among the locations [19].

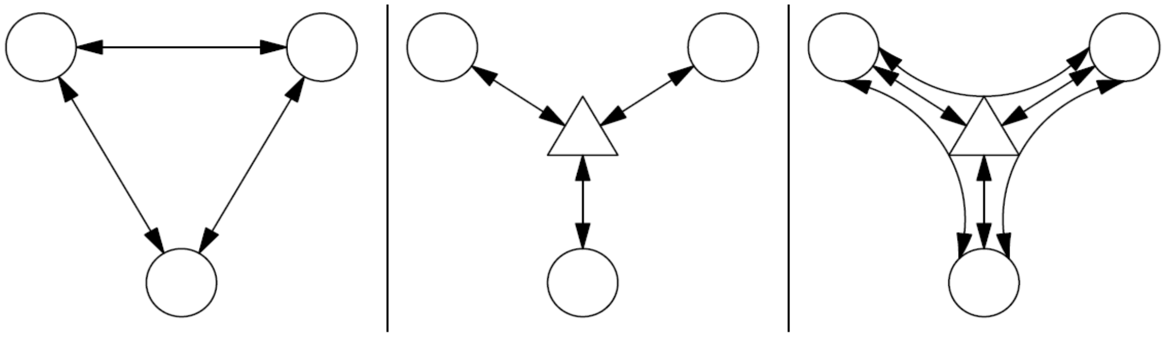

Figure 16: Multi-location rental situations. [3]

Some multi-location rental systems make use of a central warehouse or depot that supports local rent stores, like in the middle of Figure 16. These central points are used to store rental items and optionally to receive returned items. In some of these systems it is allowed to transfer items between local stores without going through the central warehouse [3], this is represented in the right part of Figure 16. Although very little research has been performed on these rental systems, models from spare parts inventory theory can be used to describe such systems [20,21]. In spare parts inventory models it is very common to make use of multiple echelons to increase performance. Van Houtum and Kranenburg [20] describe how lateral transshipments and the allowance of emergency shipments can help to optimize multi-location rental networks. Lateral transshipments are an important extension when modeling the multi-location system for EQIN.

4.3 Gap in literature

This master thesis distinguishes itself from most studies since it is performed in collaboration with a rental business in an industry where customers order more than one product at once. From a modeling point of view, this results in a *batch-arrival, batch-return* (queuing) system with lost sales. The single location application of this system has been investigated before, but this thesis will also study *multi-location* systems. Also, analyzing this system in a *case study* distinguishes this from previous studies.

5. Batch-arrival loss function

This section introduces a model that is used to compute the expected ratio of lost sales in a batch-arrival, batch-return rental system. An entire chapter is dedicated to the lost sales model since the existing literature did not provide information on the sensitivity of the ratio of lost sales towards batch size and rental duration distributions; this is treated in Section 5.2. Because of its importance in the following chapters, a simulation study is performed to test this sensitivity.

5.1 Model

In this section, a loss function is suggested for a batch-arrival, batch-return system. The single-arrival model has been studied widely in relation to inventory rental models, spare-part inventory models and also conventional inventory models with stochastic lead times. The batch-arrival model however, has been studied significantly less. Similarly to the single-arrival, the batch-arrival system is modeled as a queueing system, graphically represented in Figure 17. In some papers that present this system, the considered model is referred to as a $M/G/K$ -system [10,17], where M denotes a memoryless demand arrival process, G denotes a general distribution of the rental durations and K denotes that multiple servers are used. However, to avoid confusion about the batch-arrival and batch-return process, this thesis will refer to it as the $M^x/G^x/c$-system, in accordance with the Kendall's notation [22].

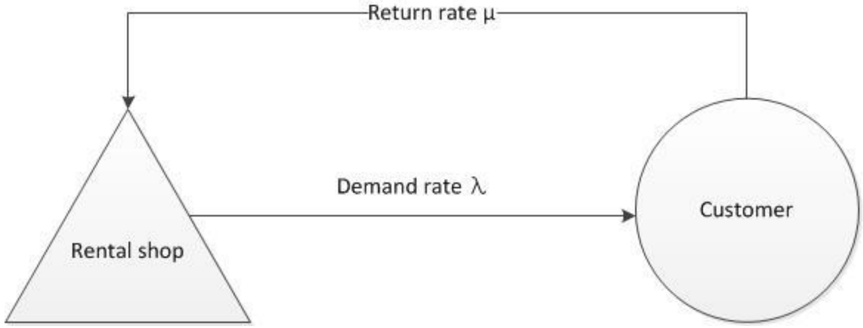

Figure 17: Single-location rental system

5.1.1 Parameters and assumptions

The following variables are of importance in the model:

Parameter	**Definition**
S_i:	*Allocated inventory level of product type i*
λ_i:	*Order arrival rate of product type i*
q_i:	*Mean order size of product type i*
T_i:	*Mean rental duration of product type i.*
$\beta_i(S_i)$:	*Blocking probability while using allocated inventory level S for product type i*

The $M^x/G^x/c$ -model considers a single location that owns a fixed number of S_i items of a certain product type i. The model assumes that customers arrive according to a stationary Poisson distribution with arrival rate λ_i. Each customer demands an order with a stochastic order size, distributed with mean q_i. The service time duration follows an arbitrary distribution with mean T_i. If an item cannot be fulfilled from the on hand inventory of the considered warehouse, it is *blocked*. In multi-location systems, blocked demand might be fulfilled by another warehouse. In a single-location system, all blocked demand is *lost*. The blocking probability $\beta_i(S_i)$ is defined as the probability that the demand for an item of product type i cannot be fulfilled by the warehouse that experiences the demand. In this single-location model, the blocking probability $\beta_i(S_i)$ is equal to the ratio of lost sales for product type i.

This study assumes that customers accept partial fulfillment of their order. This implies that when n items are available for rent, for an arriving customer that demands j items (of the same product type), the lost sales of this order are equal to $\max(0, j-n)$. For EQIN this assumption is reasonable since customers often order large orders therefore only a small part of their order is rejected. In real life however, not all customers will accept partial fulfillment. Therefore the ratio of lost sales will be underestimated.

The model results in a Markov chain with states $(X_0, X_1, \ldots, X_S)$, where state X_i denotes the state where i items are rented. As the model considers both batch arrival and batch return, computing state probabilities $(p_0, p_1, \ldots, p_S)$ is complex. This is because in a batch-return system transition probabilities of returns depend on the number of batches rented. Papier and Thonemann (2008) suggest an exact loss-formula for the $M^x/G^x/c$ –model that assumes partial fulfillment [10].

5.2 Blocking probability function

The formula for the blocking probability is valid for any distribution of the service time, but the order size needs to be geometrically distributed [17]. Details on the geometric distribution can be found in Appendix 1. They derive the following formulas:

$$\beta_i(S_i) = 1 - \frac{\sum_{k=1}^{S_i}\left(\left(kg(k)\right) + S_i(q_i-1)g(S_i)\right)}{\lambda_i q_i T_i\left(G(S_i) + (q_i-1)g(S_i)\right)} \qquad 5.1$$

$$g(k) = e^{-\lambda_i T_i}\sum_{a=1}^{k}\left(\frac{(\lambda_i T_i)^a}{a!}\right) f^a(k) \qquad 5.2$$

$$f^a(k) = \frac{(k-1)!}{(a-1)!\,(k-a)!} * \left(\frac{1}{q_i-1}\right)^a * \left(\frac{q_i-1}{q_i}\right)^k \qquad 5.3$$

Formula 5.3 gives the a-fold convolution of the geometric order size function. $f^a(k)$ can only be computed in the case that both k and a are larger than 0 and $a \leq k$. Since Formula 5.3 results in high factorials for larger values of S_i, the authors suggest an approximation formula. However, with recursive formulas it is possible to compute exact values for $g(k)$, even for large values of k. This is explained in Appendix 2. Additionally, an Excel VBA script to compute the blocking probability $\beta_i(S_i)$ is given in Appendix 3.

The loss function depends on the mean service time duration, but not on its distribution. For the exact method, Papier & Thonemann state that this property holds if and only if order sizes are geometrically distributed. However, in the data analysis we learned that not all order sizes are geometrically distributed. Therefore, a simulation study is performed to evaluate the sensitivity of this formula.

5.3 Simulation of blocking probability function

A simulation study is performed to investigate the sensitivity of the blocking probability towards order size distributions and rental duration distributions. This sensitivity analysis is performed because:

- The data-analysis in Chapter 3 on EQIN's rental inventory indicated that not all products are geometrically distributed. Therefore it is interesting to find the sensitivity of the blocking probability towards other order size distributions.
- Literature indicates that $\beta_i(S)$ is independent of the service time distribution if the order size distribution is geometric [10]. Since this is not the case and the rental duration distribution is unknown, the sensitivity of the actual blocking probability towards rental duration distributions is investigated.

5.3.1 Simulation values

The study has been performed for four cases, using different sets of parameters, given in Table 9. The parameter values that have been used are four sets that also have been computed by Papier and Thonemann [10]. These sets are chosen since it was preferred to simulate the system for different traffic intensities. The fifth row gives the traffic intensity $\rho_i = \frac{\lambda_i T_i q_i}{S_i}$, which can be described as the ratio between demand intensity and return rate. The last row gives the blocking probability according to the exact blocking probability function $\beta_i(S_i)$.

	Case			
Parameter	*1*	*2*	*3*	4
Arrival rate (λ_i)	4	4	10	4
Number of rental items (S_i)	500	200	2000	200
Mean service time (T_i)	5	10	10	3
Mean order size (q_i)	20	5	20	20
Traffic intensity (ρ_i)	0.8	1	1	1.2
Blocking prob. (β_i)	**0,10**	**0,15**	**0,10**	**0,35**

Table 9: Parameters used in simulation study

5.3.2 Probability distributions

The study is performed using two rental duration distributions, firstly an exponential distribution is applied, with parameter $\mu_i = \frac{1}{T_i}$. The exponential distribution is commonly used to model times between random events as the distribution is memoryless. Furthermore, it is easy to compute. Secondly, the service time is modeled to be deterministic. As the deterministic service time distribution has no variability, it is interesting to compare the outcomes of this simulation to the outcomes of the simulation using the exponential distribution.

To simulate the order sizes, three approaches have been used. Firstly, a Poisson distribution has been simulated with mean q_i Secondly, a geometric distribution has been used to simulate the order size, using geometric parameter $p_i = 1 - \frac{1}{q_i}$. The simulation has also been performed using a deterministic order size. For a Poisson distribution, its variance is equal to its mean. For a geometric distribution, the variance is equal to $\frac{p}{(1-p)^2}$ and therefore increases with p (and mean q_i). Therefore, is expected that simulations that use a geometric order size distribution result in higher values for the blocking probability.

5.3.3 Results

The simulation is performed for all combinations of order size and service time distributions using Excel VBA. The horizon of the simulation is set at 5000 days, this number of days showed to give repeatedly equal results. The first 100 days of the simulation are used to fill up the system, and therefore not used to compute the blocking probability.

			Order size distribution		
***Case*:**	β	**Service time distribution**	*Poisson*	*Geometric*	*Deterministic*
1	*0,10*	*Exponential*	0,06	0,11	0,07
		Deterministic	0,07	0,11	0,06
2	*0,15*	*Exponential*	0,13	0,15	0,13
		Deterministic	0,13	0,16	0,12
3	*0,10*	*Exponential*	0,09	0,11	0,09
		Deterministic	0,09	0,11	0,09
4	*0,35*	*Exponential*	0,33	0,35	0,33
		Deterministic	0,32	0,36	0,32

Table 10: Simulation of blocking in batch-arrival, batch-return system

Table 10 gives the results of the simulation study. The second column (β) gives the computed blocking probability, using the exact function. Column 4 through 6 give the simulated blocking probabilities, using all combinations of order size and service time distributions.

As stated in Section 5.2, β should give exact results for the case that the order size is geometrically distributed. Furthermore, if the order size is geometrically distributed, the blocking probability should be independent of the service time distribution. Regarding these statements, the results of the simulation are satisfactory. When computing the order sizes with a geometric distribution, the difference between β and the simulated blocking probability is never larger than 0,01. Also, between the simulations that use different service time distributions, the discrepancies are never larger than 0,01.

Between the order size distributions there are some discrepancies. The geometric distribution results in a higher ratio of lost sales. This is most likely caused by the larger variation geometric distribution. However, the discrepancies are never larger than 0,05, and therefore satisfactory.

5.3.4 Conclusion

This simulation study is performed to investigate the validity of the blocking probability formula $\beta(S)$ when using different order size and service time distributions. Since the order size is not geometrically distributed for all C-Fleet items, this was required to compute EQIN's ratio of lost sales. The conclusion of the study is that $\beta(S)$ gives satisfactory results, for both different rental size and order size distributions, and therefore can be used to calculate blocking probabilities.

6. Single location model: The $M^x/G^x/c$ -system

In this section, locations are decoupled from each other and approached separately using a single location model. This results in separate $M^x/G^x/c$ -systems, as introduced in Chapter 4. This chapter optimizes the base-stock levels for EQIN's rental inventory.

6.1 Model description

In Chapter 4 the $M^x/G^x/c$ –system has been introduced. The $M^x/G^x/c$ –system allocates a fixed number of items of product type i to the considered warehouse (S_i). which is not replenished if the on hand inventory drops below a reorder level. This is the main difference with EQIN's current system.

Figure 18 illustrates how EQIN's multi-location rental system is modeled as separate single-location systems. Each warehouse has its own customers demand and returns. The main advantage of the single-location system over EQIN's current system is that it reduces transportation between warehouses.

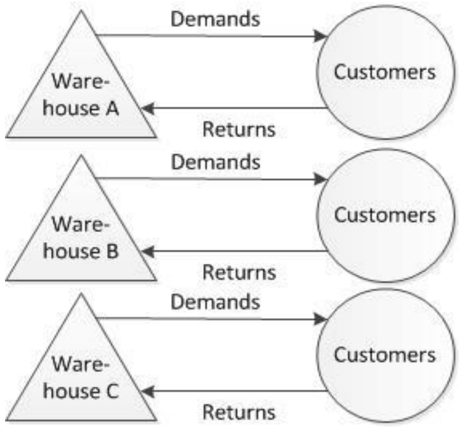

Figure 18: Three warehouses as separate single-location systems

6.1.1 Previous research on the $M^x/G^x/c$ -system

As presented in the literature review, modeling a rental system as a queueing system is a common approach. The first papers to describe rental inventory systems already used queueing theory to describe the stochastic processes [12,14]. Using these theories, researchers are able to analyze rental inventory systems using relatively simple but accurate computations. A rental inventory system has great similarities with classic inventory systems with stochastic lead times and spare part inventory systems [3]. The $M^x/G^x/c$ -model is a multi-item, single-location problem that has been studied in multiple applications. In the literature on rental systems, Papier and Thonemann [10] studied this problem. This paper presented the loss formula that has been described in Chapter 5. Literature on spare parts inventory offers a wide range of multi-item models. The research on multi-item, single location problems helps to develop an optimization algorithm [20].

6.2 Fulfillment policies

As stated in Chapter 2 of this thesis, the inventory in EQIN's local warehouses is introduced to satisfy *unplanned demand*, which is defined as the demand that has not been reserved by its customers. However, EQIN has not specified a hard rule to deal with planned demand. To model EQIN's rental system, rules have to be set to deal with planned orders. From a modeling point of view, the fulfillment policies have the following implications. Each order is classified as either (A) an order that should be satisfied by the considered warehouse or (B) an order that is (completely) forwarded to and satisfied from a main warehouse. It is assumed that these forwarded orders can always be fulfilled. Figure 19 illustrates the flow of these orders.

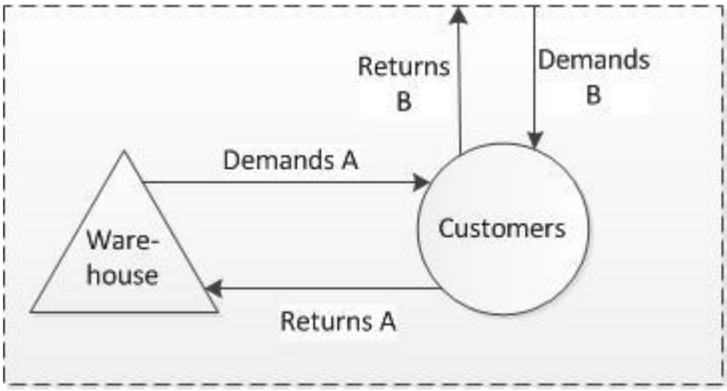

Figure 19: Fulfillment of planned orders

Forwarded orders (B) are not used in the calculation of the order arrival rate, order size and rental duration. To compute these parameters, only the orders that should be satisfied by the considered warehouse (A) are used. Therefore, these parameters will be different among the fulfillment policies and therefore, also, the computed inventory levels (S_i).

Three policies are introduced to specify whether a planned order is fulfilled by the local warehouse or forwarded to a main warehouse:

Policy 1: All orders are satisfied from the local inventory, both planned and unplanned. Since each order is meant to be satisfied from the local inventory, this policy is expected to result in higher inventory costs than the other policies. However, this implies that no orders are forwarded and thus transported from another warehouse. This policy is simple to understand and easy to implement in a local warehouse, since it makes no distinction between planned and unplanned orders.

Policy 2: Only the unplanned orders are satisfied from the local inventory and all planned orders are forwarded. This is the other extreme policy. This policy will reduce the local inventory significantly but the high number of forwarded orders is expected to result in much transport.

Policy 3: All unplanned orders are satisfied from the local inventory. Additionally, small planned orders are satisfied from the local inventory. Orders are defined as small if they are smaller than or equal to the mean order size (of all orders). The remaining *large* orders are forwarded to the main warehouse. This policy is expected to be somewhere in between policy 1 and 2 in terms of inventory and transportation costs. This policy is expected to be attractive, since one could argue that it is not cost attractive to forward a very small order, since transportation costs would be relatively high. On the other hand, fulfilling large, unplanned orders from a local warehouse could result in both unnecessary high inventories and lost sales. Therefore, this hybrid policy is suggested.

6.3 Mathematical formulation of the $M^x/G^x/c$ -Model

This section proposes a method to optimize the number of items that are allocated to a location. As explained in Section 6.1, the method approaches the inventory problem as a single location problem, meaning that the optimal number of items for a single product is independent of the items owned at other locations.

The following assumptions are made regarding this system:

- If a product type i has not been demanded in the last year, this product is removed from the analysis. This product is assumed to be either removed from the portfolio or so rarely demanded that it is transported from a hub.
- Since demand fluctuates over seasons, seasonal factors are introduced to model these fluctuations. However, constant levels of S_i will be assumed. This is reasonable since in a rental system, items are owned by the company. Using a flexible S_i would result in overages throughout the system
- Within a season, batches are assumed to arrive according to a stationary Poisson process.
- If customer demand is unmet, it is lost.

6.3.1 Seasonal demand

As stated in the data analysis in Chapter 3, four seasons are introduced to model the seasonal fluctuations. During these seasons, the order arrival rate for a product type i is assumed to be stationary ($\lambda_{i,t}$). Using multiple seasons has implications for the blocking probability $\beta_i(S_i)$ for a product i, since the blocking probability varies among seasons. If for season t the blocking probability is defined as $\beta_{i,t}(S_i)$, the blocking probability over an entire cycle (consisting out of T seasons) is defined as:

$$\beta_i(S_i) = \frac{1}{T} * \sum_{t=1}^{T} \beta_{i,t}(S_i) * \frac{\lambda_{i,t}}{\lambda_i} \qquad 6.1$$

where S_i is constant throughout the year, seasons t are equal in length, and λ_i denotes the mean order arrival rate over the entire cycle. To analyze the consequences of the nonstationary demand, the optimization algorithm below has been executed on EQIN's rental fleet using both stationary and non-stationary demand. Using non-stationary demand has an enormous effect, increasing the total value of inventory levels by 40-60%.

6.3.2 Optimization algorithm

For the optimization of the rental inventory a greedy algorithm has been applied. The greedy algorithm is easy to understand and leads to a set of efficient solutions [20]. For any given objective (in EQIN's case ratio of lost sales), the greedy algorithm will develop feasible, near optimal solutions. The greedy algorithm optimizes over all product types instead of optimizing each product type separately. The benefit of the greedy algorithm is that it takes the investment costs of items into account while optimizing all inventory levels using an aggregate optimization objective.

A disadvantage of the aggregate measure is that the optimization algorithm does not take the strategic importance of some products into account. The optimization may result in a solation in which an expensive, but strategic product has a low fill rate. To tackle this problem, functionality has been built in the software tool (Chapter 10) to optimize inventories for an individual product.

The aggregate blocking probability of the rental inventory can be computed as the weighted sum of all blocking probabilities [23], with $\frac{\lambda_i * q_i}{\boldsymbol{\lambda}}$ as weight for each product type. Resulting in an aggregate blocking probability of:

$$\boldsymbol{\beta} = \sum_{i}^{I} \beta_i(S_i) * \frac{\lambda_i * q_i}{\boldsymbol{\lambda}} \qquad 6.2$$

$$\boldsymbol{\lambda} = \sum_{i}^{I} \lambda_i * q_i \qquad 6.3$$

where $\boldsymbol{\lambda}$ is the total demand rate (order arrival rate multiplied with order size) for the warehouse and q_i is the mean order size. Note that the arrival rate is multiplied by the mean order size, because it is a batch-arrival system. The model results in the following optimization problem:

$$\text{Minimize} \quad \textstyle\sum_i^I S_i * P_i \qquad 6.4$$

$$\text{Subject to} \quad \boldsymbol{\beta} \leq \beta^{Objective}$$

$$S_i \geq 0 \; for \; all \; i \in I$$

The greedy algorithm increases S_i of the product type that will decrease the aggregate blocking probability the most per invested euro (Γ_i). This is calculated by:

$$\Gamma_i = \frac{(\beta_i(S_i + 1) - \beta_i(S_i)) * \frac{\lambda_i * q_i}{\boldsymbol{\lambda}}}{P_i} \qquad 6.5$$

where P_i is the product investment value. As proven by Papier and Thonemann [10], the function for the blocking probability is convex and decreasing in S_i. In this chapter, $\beta_i(S_i)$ is the sum of multiple of these convex function, and therefore, also convex and decreasing. For a product i, Γ_i is independent of other products. This allows the optimal inventory levels to be calculated by the greedy algorithm. The greedy algorithm increases S_i's with 1, until $\boldsymbol{\beta} \leq \beta^{Objective}$, described by the algorithm below [20]. In the algorithm, the function $arg\,max\{\Gamma_i : i \in I\}$ finds the product type i with the largest value of Γ_i.

Greedy Algorithm $M^x/G^x/c$ *-model (based on: Van Houtum & Kranenburg, 2015)*

1. $S_i = 0$ $for\ all\ items\ i \in I$
2. $Compute\ \Gamma_i\ for\ all\ i \in I$
3. $\boldsymbol{Do\ until}: (\boldsymbol{\beta} \leq \beta^{Objective})$
4. $k = arg\,max\{\Gamma_i : i \in I\}$
5. $S_k = S_k + 1$
6. $Compute\ \Gamma_k$
7. $\boldsymbol{Loop}$
8. $\boldsymbol{S}^* = \boldsymbol{S}$

6.4 Outcomes of model

To evaluate the performance of the model, a fair comparison has to be made with the current performance. Currently, EQIN has the calibrated Minimum Stock Quantities (MSQ), which are set by location managers. When inventory decreases below the reorder level, they request items from another warehouse to maintain a safety stock. The allowance to order additional products causes the total stock in a single location's system to exceed the calibrated level most of the time. Table 11 shows the costs of the current system. The cost of owning items is from now on referred to as ownership costs. The ownership costs of the calibrated levels in the table are calculated using the mean MSQ level of each product (using calibrated level at start of each month over 2016). In the middle part, the table shows the analysis of the average number of items that are in the system of a rental location. In the last columns, the table shows the number of times that the number of owned stock has increased. This is an indication of how often products have been transported to the location.

	Average calibrated MSQs (2016)		**Average level in system (2016)**		**Transportation (2016)**
	#Items	**Ownership costs (x1000€)**	**#Items**	**Ownership costs (x1000€)**	**Increases owned stock**
A-Fleet	14	98	111	516	199
B-Fleet	111	115	310	336	472
C-Fleet	1.878	140	4.041	352	1.527

Table 11: Ownership cost related to the current MSQ levels and average inventory

The table shows that EQIN chooses to minimize the on hand inventory levels of expensive items, as the MSQ levels for A-Fleet items are low compared to the average level in the system.

Based on the findings in Table 11, the outcomes of the $M^x/G^x/c$ -model can be evaluated. In Table 12 the inventory levels are optimized with the greedy algorithm using blocking probability targets of 0.05 and 0.1. After the optimization using a target of 0.05, EQIN indicated that it would also be interesting to them what the inventory costs of 0.1 would be.

The optimization is performed for all three policies introduced in in Section 6.2. The second column of Table 12 shows which policy is used in the analysis.

If, according to the policy, an order is not fulfilled by the local warehouse, it is forwarded to a main warehouse. The number of forwarded orders is given in the third column. The table gives the outcomes of the optimal number of items allocated to the local warehouse and the corresponding ownership costs. Additionally, the column 'ΔCosts (%)' shows how the current ownership costs would change compared to the average number of items owned in the current system $\left(\frac{Ownership\ costs\ optimization - Ownership\ costs\ 2016}{Ownership\ costs\ 2016}\right)$. The outcomes are given for the A, B and C-Fleet.

			$\boldsymbol{\beta^{\text{objective}} = 0,05}$			$\boldsymbol{\beta^{objective} = 0,1}$		
Fleet	**Policy**	**Forwarded orders (2015-2016)**	**# Allocated items**	**Ownership cost (x1000€)**	**ΔCosts (%)**	**# Allocated items**	**Ownership cost (x1000€)**	**ΔCosts (%)**
A-Fleet	**1**	0	205	1.173	*127%*	167	851	*65%*
	2	328	119	677	*31%*	91	504	*-2%*
	3	65	161	937	*82%*	127	715	*39%*
B-Fleet	**1**	0	591	611	*82%*	472	480	*43%*
	2	638	308	320	*-5%*	254	258	*-23%*
	3	171	424	441	*31%*	345	352	*5%*
C-Fleet	**1**	0	7.197	468	33%	6.058	358	*2%*
	2	2518	3.909	255	-27%	3.245	191	*-46%*
	3	971	4.459	305	-13%	3.764	232	*-34%*

Table 12:Performance of the three policies for the A, B and C-Fleet (Location: Moerdijk)

The outcomes for the A-Fleet make it clear that the ownership costs are really high. When using a target blocking probability of 0.1 and maintaining policy 2, which has the lowest ownership costs, the costs would decrease by only 5% (± €12k). Evaluating this result, it is likely that this single-location model is not optimal. Although a cost reduction could be obtained, using this model would result in losing a lot of customers. The results makes sense because EQIN management chooses to centralize the inventory for these expensive items and has MSQ inventory levels for just a few items. The $M^x/G^x/c$ -model allocates fixed numbers of items to a location, resulting in a higher average number of items in the system, with a low utilization rate. Also, the $M^x/G^x/c$ –model makes no use of the other warehouses, losing pooling advantages.

In comparison to the expensive A-Fleet, the outcomes of the C-Fleet seem better regarding the ownership costs. As stated in Section 1.3.1, EQIN strives for high fill rates for these products. Policy 2 and 3 both have significant lower ownership costs in comparison to the average ownership costs from 2016. Even when using $\beta^{objective} = 0.05$ the ownership costs are reduced by 41% and 25% respectively. Additionally, the analysis of 2016 showed that 1.527 times a stock was increased, where policy 2 and 3 only would require 2.518 and 971 transported orders respectively over a time span of two years.

The outcomes for the B-Fleet items show no significant improvement. The second policy has lower ownership costs when using $\beta^{objective} = 0.1$ and shows a slight improvement when optimizing with $\beta^{objective} = 0.05$. However, since this model assumes no other warehouses, it makes no use of the pooling advantages that a multi-location inventory system. Therefore, the model loses efficiency. It is expected that a multi-location system would perform better for the B-Fleet inventory.

Because the $M^x/G^x/c$ -model is best suited for the C-Fleet, the comparison of the fulfillment policies is based on the C-Fleet. Policy 2 has the lowest ownership costs but transports more orders from a central stock point. Policy 1 has no transporting costs because it fulfills all orders from its own stock, resulting in higher inventory levels. It is hard to determine the actual transportation costs, since orders differ in the quantity of products and product characteristics such as size and weight. However, it is felt that policy 3 would perform best. Policy 2 requires around 750 more transported orders per year, while decreasing ownership costs by €50.000. Policy 1 demands ownership costs that are €150.000 above than the ownership costs for policy 3, with only 500 more transported orders per year. Section 6.5 goes further into these policies.

6.4.1 Sensitivity analysis

As stated in the data analysis in Chapter 3, data on EQIN's demand is quite limited. There is no data on repairs and rental duration, so the cycle time of rentals had to be estimated. Also, there could be discrepancies between the assumed arrival rate and the actual arrival rate. For this reason, a sensitivity analysis is performed on these two parameters. The sensitivity analysis is only performed for the C-Fleet, since the A- and B-Fleet will be modeled later on with a multi-location model. Figure 20 shows the results of this analysis. In the analysis, the variables arrival rate and rental duration are increased by 10%, 20% and 30%. The figure shows the increase in total cost of ownership.

		Increase arrival rate (%)				
		0%	**10%**	**20%**	**30%**	
Increase rental duration (%)	**0%**	0	3%	8%	13%	Below current costs
	10%	14%	24%	27%	32%	
	20%	32%	41%	52%	64%	Above current costs
	30%	57%	70%	89%	97%	

Figure 20: Sensitivy analysis of policy 3

The sensitivity analysis is performed on all policies, shown in Appendix 4 but since policy 3 works best for the C-Fleet, this analysis is treated in this section. As Figure 20 shows, the resulting ownership costs remain below the current ownership costs with small discrepancies. Even though the data was limited, it is quite unlikely that the arrival rate would actually be much higher. For the rental durations, discrepancies between the estimated and actual values will be larger, however, it is not expected that they will exceed the current costs. These results are in line with the expectations as it makes sense that the inventory costs increases with the arrival rate and rental duration.

6.5 Simulation

Since the output of the $M^x/G^x/c$ –model (fixed allocated stock) is different from the current system that is used by EQIN (mean ownership costs), a simulation is performed to obtain a better understanding of its outcomes. The simulation uses the optimized inventory levels to compare the fill rate and the on-hand inventory value of the $M^x/G^x/c$ -model to the current situation, using the actual demand data of 2015 and 2016 as input for the simulation.

The simulation has only been performed for the C-Fleet, since Section 6.4 has indicated that the policy is not optimal for the A- and B-Fleet. In Table 13 the outcomes of the simulation are presented. For each of the policies, a simulation has been performed, considering all orders that should be fulfilled by the warehouse according to the policy (Figure 19, demands and returns A). For these orders, the fill rate is computed. The results are in line with the expectation as the fill rate is near 95% and the cost savings are in line with the findings in Table 12. Appendix 5 shows the results for other warehouses. These results are in line with the findings above.

Policy	Fill rate (%)	Δ Inventory value (%)
1	97%	11%
2	96%	-38%
3	96%	-27%

Table 13: Simulation outcomes of $M^x/G^x/c$ –model using optimal values of S_i

6.6 Conclusion

This chapter studied a solution to solve a single location rental inventory problem and applied it to a location of EQIN's rental system. The $M^x/G^x/c$ -model considers a fixed amount of rental items in the system that is optimized using a greedy algorithm. To make the necessary computations, the loss formula has been tested for multiple stochastic order size and rental duration distributions. Additionally, the analysis has been performed for three different fulfillment policies that can be used regarding planned orders.

The analysis of the optimization outcome showed that the $M^x/G^x/c$ -model is inferior to the current performance for the expensive A-Fleet. Optimization of the B-Fleet results in ownership costs that are similar to the current system. The C-Fleet optimization results in a significant cheaper solution.

When modeling all warehouses as separate $M^x/G^x/c$–systems in a multi-location rental system, its effectivity is proven to be limited. The effectivity is limited by the fact that rental items cannot be shared among locations, losing a lot of pooling advantages. This is less important for items with relatively high transportation costs and for products that customers do not accept to wait for.

The main goal of this chapter was to test the appropriateness of the $M^x/G^x/c$ -model for EQIN's rental system. Since C-Fleet items are relatively cheap and customers can easily go to competitors, fill rates should be high at local warehouses. Also, individual transportation of an order is expensive in comparison to its revenue. Therefore, it can be concluded that the $M^x/G^x/c$ -model is appropriate for EQIN's C-fleet. For the A- and B-Fleet, the analysis showed that this solution is not optimal. The following chapter will study multi-location models to find a better solution for these products

7. Single location system with safety stock

In an addition to the proposed analytical model, a simulation model has been developed to further optimize the single location inventory model treated in Chapter 6. The model aims to decrease the inventory costs even further while keeping the lost sales rate low with a safety stock. Additional to the reduced inventory costs, this system provides another benefit. Its outcomes will be most similar to the current system, making it easy to implement. The main goal of this chapter is to test if the extended model performs better than the $M^x/G^x/c$- model.

7.1 Model description

The model considers a variable number of items that is allocated to a local warehouse. The model is quite similar to the system that is currently used, explained in Section 1.3.2. The considered system has two thresholds:

- Minimum inventory level: If the on-hand inventory drops below the minimum inventory level due to demands, items are ordered up to the minimum inventory level.
- Maximum inventory level: If the on-hand inventory exceeds the maximum inventory level due to orders that are returned by customers, the overage inventory is returned to a main warehouse.

Figure 21 serves as a graphical representation of the system where the $MiSQ$ and $MaSQ$ denote the minimum inventory level and maximum inventory level respectively. The red line denotes an order of additional inventory; the green line denotes a return of overage inventory. Note that $MiSQ \geq 0$ and if $MiSQ = 0$ the system is equal to the $M^x/G^x/c$-model with $S = MaSQ$. Obviously, $MiSQ \leq MaSQ$. It is assumed that the decision to return or order inventory is made at the end of each day. Also, the assumptions are made that items are ordered from an infinite stock and that orders are replenished with zero lead time.

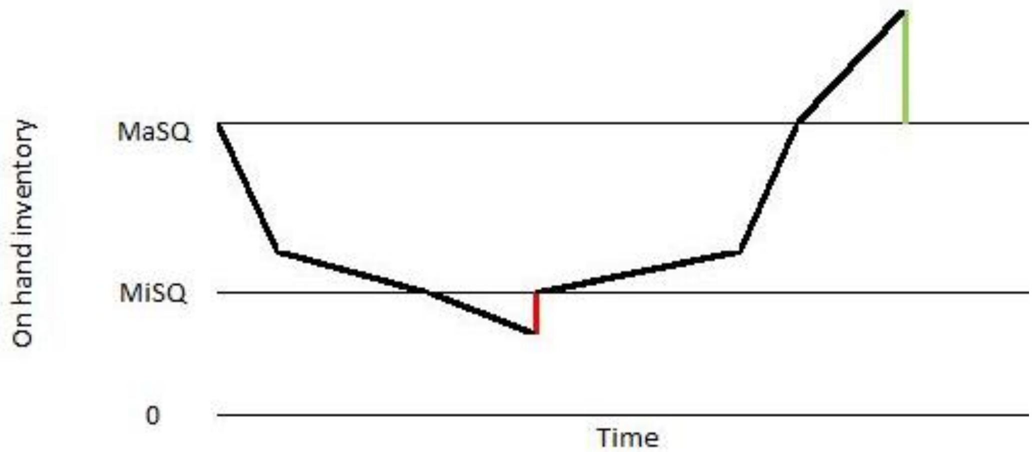

Figure 21: Extended single-location system

Compared to the current system, the maximum inventory level is the main addition. The current system uses a minimum inventory level but there is no hard threshold that determines when items are sent back to a hub (main warehouse). However, for modeling purposes this is required since it would lead to infinite inventories.

7.2 Algorithm and simulation

The model is only performed for the C-Fleet inventory, as the upcoming chapters will describe multi-location models for the A- and B-Fleet. Although the single-location model already proved an improvement for the C-Fleet inventory, it seems suboptimal to not use the multi-location rental system that EQIN has. Since EQIN's rental system experiences a batch-arrival demand process, this system is too complicated to optimize using an analytical model. Therefore, an algorithm is proposed using a simulation model to generate the optimal values for $MiSQ_i$ and $MaSQ_i$. As pointed out in Section 7.1 the system is equal to the $M^x/G^x/c$-model if $MiSQ_i = 0$. Therefore, the optimal solution of the $M^x/G^x/c$- model (S_i) is used as an upper bound for $MaSQ_i$ in this algorithm. The optimization is performed for each product independently:

$$\begin{aligned} &\text{Minimize} && C_i^o && (7.1)\\ &\text{Subject to} && MaSQ_i \leq S_i \\ & && 0 \leq MiSQ_i \leq MaSQ_i \end{aligned}$$

where C_i^o denotes the mean owned inventory cost of item i. The ownership cost for a single day is computed with $(on\ hand\ inventory + \ items\ rented) * purchasing\ costs$. $MaSQ_i$ is decreased one by one and the minimum value of $MiSQ$ is determined to satisfy the blocking probability objective $\beta \leq 0{,}05$. If C_i^o is below the previous 'best value' this becomes the best

<u>Algorithm for solution extended single location problem</u>

1. $Perform\ M^x/G^x/c\ \ model\ to\ compute\ optimal\ S_i$
2. $Set\ MaSQ_i = S_i$
3. $\boldsymbol{Do\ until: MaSQ_i = 0}$
4. $Set\ MaSQ_i = MaSQ_i - 1$
5. $Set\ minLevel_i = 0$
6. $\boldsymbol{Do\ until:\ beta_i \leq 0,05}$
7. $MiSQ_i = MiSQ_i + 1$
8. $Simulate\ (minLevel_i, maxLevel_i) \rightarrow \beta_i, C_i^o,$
9. $\boldsymbol{if\ \beta_i < 0,05\ and\ C_i^o < C_i^{o*}}$

   $\boldsymbol{set}\ optimal\ values$
10. $\boldsymbol{Loop}$
11. $\boldsymbol{Loop}$

solution. The algorithm stops either if $MaSQ = 0$ or no solutions can be obtained for a value of $MaSQ_i$ that satisfy $\beta \leq 0{,}05$.

In step 8 a simulation is performed to determine the lost sales and costs of ownership. The simulation is performed over four seasons of 250 days. Because the simulation has to be performed for so many days, the algorithm takes a lot of time to complete. For a single location (Moerdijk) it takes 20 minutes.

7.3 Outcomes

Since the goal of this chapter is to test the performance of this single-location model, the outcomes of this analysis are compared with the outcomes of Chapter 6. In line with the previous chapters the outcomes are compared based on the optimization results and a simulation on the actual historical data. Table 14 compares the outcomes of these single location systems.

	Optimization results				Simulation	
Model	**Stock increases**	**#Allocated items**	**Ownership cost (x1000€)**	**ΔCosts (%)**	**Fill rate (%)**	**Δ Inventory value (%)**
$M^x/G^x/c$	-	2.880	252	*-28%*	94%	-39%
Extension	566	2.835	235	-33%	91%	-53%

Table 14: Comparison of single location systems

As the $M^x/G^x/c$-model served as an upper bound for the best solution, the ownership costs of the simulation model can never exceed the ownership costs of the $M^x/G^x/c$-model. Therefore, the extension model obviously has less ownership costs than the $M^x/G^x/c$-model. However, the cost savings are quite limited; the extended solution reduces the mean ownership costs by less than 7% (€17.000), while decreasing the simulated fill rate. In a deeper analysis is found that the decreased fill rate is often caused by large orders that are also partially lost by the $M^x/G^x/c$-model.

The extension brings another side effect: Transportation costs. As the second column indicates, the $M^x/G^x/c$-model does not require any transportation. The extended model orders expectedly 566 times additional stock. Since some transport may be merged into a single truck if it is planned at the same moment, it is hard to say what the exact costs of the transportation will be. However, it is clear that this will have a significant impact on the total costs.

7.4 Conclusion

The main goal of this chapter was to compare the two single-location systems. The extended single location model aims to reduce the inventory costs by introducing a safety level. The safety level allows the system to reduce inventory costs while keeping the fill rate high.

It is fair to conclude that this extension works well for EQIN's C-Fleet inventory. It reduces the required number of items even further than the $M^x/G^x/c$-model while maintaining a high fill rate. Although the simulated fill rate of the extension is below the simulated fill rate of the $M^x/G^x/c$-model, it is hard to say whether the performance is actually worse. In real life, it is possible to negotiate with customers whether demand is fulfilled by a transport from another location, which is very reasonable for large orders. This is not possible in a simulation. It is expected that the model would be optimal for EQIN's C-Fleet if transportation is planned efficiently.

8. Multi location system: Lateral transshipments

This chapter models the problem as a multi-location system. The model has been applied since Chapter 6 indicated that the single location does not perform good for each rental fleet. The single location model performs badly for products with relatively high purchasing price and low demand rates. In this chapter, a model using lateral transshipments is introduced.

8.1 Introduction of model

When multi-location systems allow transshipments among locations it brings pooling advantages for the inventory levels [18], this may lead to major cost reductions. Lateral transshipments are transshipments between warehouses in the same echelon. The literature on lateral transshipments can be classified in the categories *proactive transshipments* and *reactive transshipments* [18]. Proactive transshipments occur to protect stocks against possible stock outs. A reactive transshipment occurs when one location is not able to fulfill an order but another location is. This study will focus on reactive lateral transshipments as it is expected that customers are willing to wait a lateral transshipment for A- and B-Fleet products.

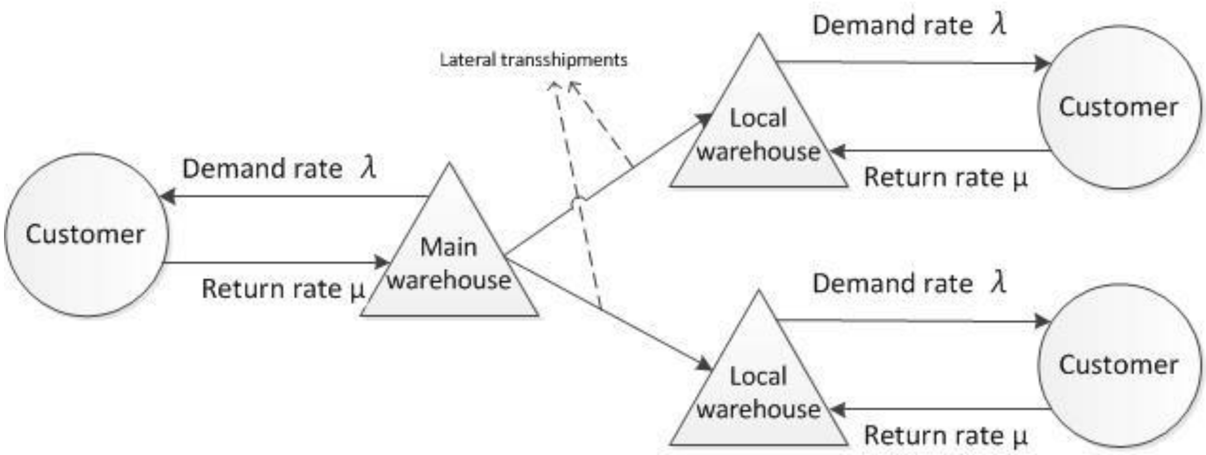

Figure 22: The multi-location system using lateral transshipments

The situation considered in this chapter is graphically represented in Figure 22. It is based on the the lateral transshipment model described by Van Houtum and Kranenburg [20]. The model consideres two types of warehouses: Main warehouses and local warehouses. Main and local warehouses are in the same echelon since they both experience demand from customers. All local warehouses are assigned to a main warehouse. When a local warehouse cannot fulfill a demand, the assigned main warehouse will fulfill the demand using a lateral transshipment (if the main warehouse's on hand inventory is sufficient, else lost). Altough Figure 22 does not show this, the lateral transshipment is returned to the main warehouse as soon as it is returned by the customer. The figure does not illustrate this to make clear that only main warehouses are allowed to send transshipments. The authors state that the model is very effective for situations with low demand rates. In a case study at ASML, inventory costs were decreased by a factor 2 while improving service rates by a factor 10 [20].

In the model by Van Houtum and Kranenburg, multiple main warehouses are considered. This implies that local warehouses can be assigned to more than one main warehouse and main warehouses are assigned to each other. In this study however, the model is used to compute inventory levels for EQIN's individual regions. In EQIN's rental system, each region has one main warehouse (hub). Decoupling the regions simplifies the model significantly, since computing the performance of multiple main warehouses is complex.

In line with the method suggested by Van Houtum and Kranenburg (2015), the following system parameters are computed regarding the fulfillment of the demand:

- Fillrate, as the fraction of demand that is fulfilled directly from stock (i.e. from the warehouse where the initial demand occurs).
- Transshipment ratio, as the fraction of demand that is fulfilled via transshipment (i.e. from the main warehouse instead of the local warehouse where the initial demand occurs).
- Lost sales ratio, as the fraction of demand that is lost.

The sum of these variables is equal to 1. As an extension to the model, parameter w is introduced, that represents the ratio of customers that is willing to wait for a transshipment. EQIN employees have contradicting opinions on the willingness of customers, so it is interesting to see how this ratio influences the system. The advantage of this model is that the computations are quite easy to understand using the $M^x/G^x/c$ –model from the previous chapter. Additionally, it allows pooling advantages for the more expensive products. Other extensions to the model are the batch-demand process and the seasonal fluctuations.

8.2 Mathematical formulation

In this section each symbol is explained, however, an overview of all symbols is given in the list of variables on page xii. The following assumptions are made regarding the system:

- Rental durations and order sizes are assumed to be location independent. To compute these variables, the weighted mean of each location j is used:

$$q_i = \sum_{j \in J} q_i^j * \frac{\lambda_i^j}{\lambda_i} \quad (mean\ order\ size\ in\ region\ j\ for\ product\ type\ i)$$

$$T_i = \sum_{j \in J} T_i^j * \frac{\lambda_i^j}{\lambda_i} \quad (mean\ rental\ duration\ in\ region\ j\ for\ product\ type\ i)$$

  This assumption is reasonable as product characteristics and its customer's purpose are location independent.
- Transshipment costs are equal for each item in a rental fleet. It is hard to determine exact transshipment costs, due to the large variety in products. This is a reasonable assumption since many A-Fleet items have to be transported individually. B-Fleet items are more easy to transport.

- At local warehouses, demand is assumed to be fulfilled according to the second fulfillment policy, i.e. all reserved orders are forwarded to the main warehouse. This is realistic since A- and B-Fleet items are expensive. EQIN would rather deliver planned orders wihin a region than buy additional items.
- At the main warehouse, fulfillment policy 1 is used.
- Arriving overflow demand at hubs is assumed to be Poisson, for modeling purposes this simplifies the problem siginifcantly.

8.2.1 Overflow demand

In the mathematical formulas we denote a warehouse with j. To distinguish main warehouses from local warehouses we use k. It is assumed that overflow demand from local warehouses arrives as a Poisson distributed batch-arrival process at the main warehouse, so that its total demand intensity ($\tilde{\lambda}_{i,t}^{k}$) also follows a Poisson distributed batch-arrival process. The equations in Appendix 1 show that the order size distribution of the overflow demand in a local warehouse is geometrically distributed with the same parameter as the original order size distribution. This property simplifies the computation siginicantly. When using the willingness ratio w, the total order arrival rate in a season for the main warehouse k ($\tilde{\lambda}_{i,t}^{k}$) can be computed with:

$$\tilde{\lambda}_{i,t}^{k} = \lambda_{i,t}^{k} + w * \sum_{j \in J, j \neq k} \lambda_{i,t}^{j} * \beta_{i,t}^{j}(S_i^j) \tag{8.1}$$

where $\beta_{i,t}^{j}(S_i^j)$ is the blocking probability at warehouse j during season t. Let $\beta_{i,t}^{k}(S_i^k)$ denote the seasonal blocking probability at warehouse k, which is calculated using the main warehouse's total demand intensity $\tilde{\lambda}_{i,t}^{k}$ (during season t).

8.2.2 Ratio of transshipments

If a demand is fulfilled through transshipment, a transshipment cost c_i^a is incurred. Therefore, the ratio of demand that is fulfilled through transshipment is calculated. For all local warehouses, the summed blocked demand in a season is computed with: $\sum_{j \in J, j \neq k} \lambda_{i,t}^{j} * \beta_{i,t}^{j}(S_i^j)$. The blocked demand can only be transshipped if customers are willing to wait for a transshipment, which is modeled by w. Furthermore, a part of this 'requested transshipments' is blocked when the main warehouse has insufficient on hand inventory (computed by $\beta_{i,t}^{k}$). The seasonal transshipments are computed as a ratio of the seasonal demand of product i ($\lambda_{i,t}$) in the entire region. The total transshipments are computed as a ratio of the mean demand rate of product i: $\lambda_i = \frac{1}{T}\sum_{t=1}^{T} \lambda_{i,t}$.

The seasonal transshipment ratio $A_{i,t}(\boldsymbol{S_i})$ and mean transshipment ratio $A_i(\boldsymbol{S_i})$ is obtained by:

$$A_{i,t}(\boldsymbol{S_i}) = \frac{w * \left(1 - \beta_{i,t}^{k}\right) * \sum_{j \in J, j \neq k} \lambda_{i,t}^{j} * \beta_{i,t}^{j}(S_i^j)}{\lambda_{i,t}} \tag{8.2}$$

$$A_i(\boldsymbol{S_i}) = \sum_{t=1}^{T} A_{i,t}(\boldsymbol{S_i}) * \frac{\lambda_{i,t}}{\lambda_i} \tag{8.3}$$

where $\boldsymbol{S_i}$ denotes the vector containing S_i^j for each location j.

8.2.3 Ratio of lost sales

The lost sales ratio of the entire system consists out of two streams:

1) Blocked customers at local warehouses are not willing to wait for the transshipment from another location. For a season, this stream is calculated by: $(1-w) * \sum_{j\in J, j\neq k} \lambda_{i,t}^j * \beta_{i,t}^j(S_i^j)$
2) The main warehouse has insufficient rental items when a direct or transshipment demand arrives: $\beta_{i,t}^k(S_i^k) * \tilde{\lambda}_{i,t}^k$

The ratio of lost sales is calculated with:

$$L_{i,t}(\boldsymbol{S_i}) = \frac{(1-w) * \sum_{j\in J, j\neq k} \lambda_{i,t}^j * \beta_{i,t}^j(S_i^j) + \beta_{i,t}^k(S_i^k) * \tilde{\lambda}_{i,t}^k}{\lambda_{i,t}} \tag{8.4}$$

$$L_i(\boldsymbol{S_i}) = \sum_{t=1}^{T} L_{i,t}(\boldsymbol{S_i}) * \frac{\lambda_{i,t}}{\lambda_i} \tag{8.5}$$

The fill rate is calculated by $f_i(\boldsymbol{S_i}) = 1 - L_i(\boldsymbol{S_i}) - A_i(\boldsymbol{S_i})$.

8.2.4 Optimization method

Similar to the optimization method for single-location models, the inventory levels are optimized using a greedy algorithm. In mathematical terms, the optimization problem is described as follows:

$$\begin{aligned} &\text{Minimize} && \textstyle\sum_i^I(\boldsymbol{S_i} * P_i + A_i(\boldsymbol{S_i}) * c_i^a * \lambda_i) && 8.6 \\ &\text{Subject to} && \boldsymbol{L}(\boldsymbol{S}) \leq L^{Objective} && \\ & && S_i \geq 0 \; for\ all\ i \in I && \end{aligned}$$

The optimization problem is very similar to the single location model, however the total costs also include transshipment costs. Moreover, the first restriction is expressed in the aggregate lost sales ratio $\boldsymbol{L}(\boldsymbol{S})$, which is calculated by $\sum_i^I L_i(\boldsymbol{S_i}) * \frac{\lambda_i}{\boldsymbol{\lambda}}$. $\boldsymbol{S}$ denotes the vector that contains the vector S_i for each product type i (that contains all S_i^j for each j). In Chapter 6, the problem was restricted by the blocking probability. In the single-location system, the blocking probability is equal to the ratio of lost sales. However, in this multi-location system, blocked demand can be fulfilled through lateral transshipment, so it is important to distinguish blocking probability from ratio of lost sales.

In the greedy algorithm, the inventory level is increased by the item that decreases $\boldsymbol{L}(\boldsymbol{S})$ most per additional cost:

$$\Gamma_i^j = \frac{\left(L_i(\boldsymbol{S_i}) - L_i(\boldsymbol{S_i} + e_i^j)\right) \ * \frac{\lambda_i}{\boldsymbol{\lambda}}}{P_i + \left(A_i(\boldsymbol{S_i}) - A_i(\boldsymbol{S_i} + e_i^j)\right) * c_i^a * \lambda_i} \tag{8.7}$$

where e_i^j is a vector of the same size as $\boldsymbol{S_i}$ which denotes the increase of S_i^j by 1. Note that increasing $S_{i,j}$ effects $\Gamma_{i,j}$ for other locations but only for product i. For the largest $\Gamma_{i,j}$, the corresponding S_i^j is increased by 1. After an increase, new values for Γ_i^j need to be calculated for each j (but only the single i). The change in transshipment rate can either be positive or negative, and therefore, Γ_i^j might be negative. If so, S_i^j should be increased. The allocated inventory levels are increased until $L(S) \leq L^{Objective}$. The following algorithm gives a better formulation of the method:

Greedy Algorithm for lateral transshipment model (based on: Van Houtum & Kranenburg, 2015)

1. $S_{i,j} = 0 \ for \ all \ items \ i \in I, locations \ j \in J$
2. a. $Compute \ \Gamma_{i,j} \ for \ all \ i \in I, j \in J$
   b. $\boldsymbol{If} \ (\Gamma_i^j < 0) \ \boldsymbol{Then} \ \{$
      $S_i^j = S_i^j + 1$
      $Compute \ \Gamma_{i,j} \ for \ all \ j \in J$
      $Return \ to \ 2b\}$
   c. $\boldsymbol{If} \ (\Gamma_i^j > \Gamma_i) \ \boldsymbol{Then} \ \{\Gamma_i = \Gamma_i^j\}$
3. $\boldsymbol{Do \ until}: \boldsymbol{L}(\boldsymbol{S}) \ \leq L^{Objective}$
4. $k = \arg\max\{\Gamma_i : i \in I\}$
5. $l = \arg\max\{\Gamma_k^j : j \in J\}$
6. $S_{i,j} = S_{i,j} + 1$
7. $l = \arg\max\{\Gamma_i : i \in I\}$
8. $S_l = S_l + 1$
9. $Compute \ \Gamma_l^j \ for \ all \ locations \ j \in J$
10. $\boldsymbol{Loop}$
11. $\boldsymbol{S}^* = \boldsymbol{S}$

8.3 Performance

In this section, the performance of the multi-location model is compared with the current performance. This section will present the outcomes of the model for the Dutch South-West region of EQIN, consisting out of two locations: The hub (main warehouse) Rotterdam and its local warehouse Moerdijk. Management intends to keep fill rates for C-Fleet items high; therefore the model is not applied to C-Fleet inventory. Due to the high competiveness in this market, it is assumed that unmet demand for C-Fleet items immediately results in lost sales. For A- and B-Fleet items, it is assumed that customers are willing to wait for their order (with probability w).

8.3.1 Current performance

Table 15 presents the inventory levels of the South-West region. The table shows the A- and B-Fleet inventory in the two locations in euros and number of items.

	Rotterdam		Moerdijk		Total	
	Average MSQs x€1000	**Average levels x€1000**	**Average MSQs x€1000**	**Average levels x€1000**	**Average MSQs x€1000**	**Average levels x€1000**
A-Fleet	1.270	7.506	106	487	1.376	7.993
B-Fleet	521	3.256	151	306	672	3.562

Table 15: Current inventory value of region South-West

An important consideration for this evaluation is that the inventory levels stored in hubs (main warehouses) are often high; this is for the following reason. When items return from a (large) project, the items are stored at a hub. Therefore, the total inventory that is registered in EQIN's information system consists out of inventory that is meant for both regular orders and projects. As this study determines inventory levels for regular rental orders, it is not fair to compare the outcomes with the inventory levels that are actually measured over 2016.

8.3.2 Performance of multi-location model

The optimization of the inventory levels for the South-West region has been performed using a target lost sales probability of 0.05. Table 16 gives the outcomes of the optimization. The table shows the sum of the ownership costs for the optimal inventory levels for Rotterdam and Moerdijk. The columns 'ΔCosts' compare the ownership costs of the optimal inventory levels with the costs of the current average owned stock.

			Rotterdam		Moerdijk		Total	
Fleet	**Willing-ness ratio**	**Trans-shipments**	**Ownership cost (x1000€)**	**ΔCosts (%)**	**Ownership cost (x1000€)**	**ΔCosts (%)**	**Ownership cost (x1000€)**	**ΔCosts (%)**
A-Fleet	**0**	0	9.366	*25%*	783	*61%*	10.149	*27%*
	0.25	14	9.522	*27%*	544	*12%*	10.066	*26%*
	0.5	36	9.553	*27%*	480	*-1%*	10.033	*26%*
	0.75	76	9.688	*29%*	294	*-40%*	9.982	*25%*
	1	230	9.932	*32%*	29	*-94%*	9.961	*25%*
B-Fleet	**0**	0	2.469	*-24%*	307	*0%*	2.776	*-22%*
	0.25	30	2.468	*-24%*	280	*-8%*	2.748	*-23%*
	0.5	76	2.476	*-24%*	241	*-21%*	2.717	*-24%*
	0.75	186	2.512	*-23%*	176	*-42%*	2.688	*-25%*
	1	518	2.597	*-20%*	16	*-95%*	2.613	*-27%*

Table 16: Ownership costs of the multi-location model outcomes

The optimization has been performed using multiple values for the willingness rate. The influence of the willingness rate on the total costs is clear. Increasing w decreases the required stock at local warehouse Moerdijk but increases the stock at Rotterdam. These outcomes are in line with the expectations.

Transshipments are calculated by multiplying the transshipment rate with the number of working days in a year. The transshipment costs have very little influence on the outcome of the model. If $w = 1$, the total number of items allocated to Moerdijk are very low.

For the A-Fleet, the model is more expensive than the current method. If most customers are willing to transship ($w > 0{,}5$), cost savings are obtained in local warehouse Moerdijk. However, the system causes high inventory levels in the hub in Rotterdam since its total demand increases with willingness ratio w (equation 8.1). It was expected that this model might be suboptimal for the A-Fleet, since a main warehouse cannot pool its inventory.

The model works well for the B-Fleet. In Rotterdam, the cost savings seem enormous, however as stated in Section 8.3.1, it is not fair to evaluate the model based on cost savings in a main warehouse. Even for a low willingness rate ($w = 0{,}25$) the model obtains cost savings in Moerdijk. If half of the customers would be willing to wait for a transshipment, ownership costs would be reduced by 21%.

The third column denotes the number of orders that are fulfilled by transshipments. These outcomes can be used to evaluate the transportation costs. In Chapter 6, Table 11 showed the number of times that the number of items that are allocated to the local warehouse in Moerdijk increased. For this study, the increases are interpreted as all transport between a hub and local warehouse. The allocated stock increased 199 and 472 times for the A- and B-Fleet respectively, both between the number of transshipments when assuming willingness rates of 0,75 and 1. This finding implies that the transportation costs will not significantly increase when applying the lateral transshipment model.

8.3.3 Sensitivity analysis

As stated in Chapter 6, it is interesting to test the sensitivity of the rental inventory models, with regards to the rental duration and demand rate. The analysis is performed on the B-Fleet in region south-west, on the total ownership costs, using a willingness rate of 0,75. In the analysis, when the arrival rate is increased, the arrival rate of both warehouses is increased. Table 17 shows the results of this sensitivity analysis; the increase of the total ownership costs when increasing the product variables.

		Increase arrival rate (%)			
		0%	**10%**	**20%**	**30%**
Increase rental duration (%)	**0%**	0%	8%	17%	25%
	10%	7%	16%	25%	36%
	20%	13%	23%	35%	43%
	30%	20%	32%	41%	52%

Table 17: Sensitivity analysis of lateral transshipment model

The optimal solution when $w = 0{,}75$ would lead to cost savings of 21% compared to the current performance. The table shows that for many discrepancies, the model would perform better. Only for large discrepancies, the ownership costs would exceed the current costs. The results are similar to the findings in Chapter 6 and in line with the expectations.

8.4 Simulation

In line with the previous chapter, a simulation is performed to gain more insight on the outcomes. The simulation is based on the actual demand over the years 2015 and 2016. The model is simulated for each region but the inventory changes are only calculated for the local warehouses. As explained in the previous sections, it is unfair to compare the outcome of the optimization model with actual inventory levels at main warehouses, since many items are stored here for large projects.

Location	Fullfilled	Lost sales ratio	Transshipment ratio	Inventory cost change
Moerdijk	0,94	0,06	0,19	-60%
Velsen-Noord	0,99	0,01	0,01	35%
Luik	0,92	0,08	0,08	-48%
Wachtebeke	0,86	0,14	0,14	-67%

Table 18: Simulation of multi-location transshipment model at local warehouses

Table 18 shows the results for EQIN's local warehouses. For most warehouses, the results are satisfactory, with the warehouse in Velsen-Noord as an exception. It seems strange that the inventory costs in this simulation are above the current costs for a single warehouse; however, this finding can be explained. The warehouse in Velsen-Noord is specifically set up for EQIN's customer Tata Steel. The warehouse is located at Tata Steel's terrain and is not accessible for other customers. The tight collaboration with Tata Steel helps EQIN's location manager to keep inventories very low as the customer specifies its rental duration.

8.5 Conclusion

In this chapter a lateral transshipment model has been studied to solve a multi-location rental inventory problem. The lateral transshipment model allows reactive transshipments from a main warehouse to a local warehouse but not the other way around. The chapter starts with an explanation of the mathematical formulas that are used to evaluate the transshipment model. The model assumes that customers accept a transshipment with probability w, so the inventory levels have been optimized for multiple values of w.

The optimization indicates that the model is not suitable for the A-Fleet inventory. At local warehouses cost savings can be obtained, however ownership costs increase at the main warehouse. This is caused by the restriction that main warehouses cannot pool their inventory with other locations. This indicates that another solution has to be studied for the A-Fleet inventory. The lateral transshipment model is appropriate for the B-Fleet inventory. For

EQIN's south-west region, the model obtains significant cost reductions, even at low values for w.

The main conclusion of this chapter is that the lateral transshipment model is appropriate for optimizing EQIN's B-Fleet inventory levels. Items in the A-Fleet inventory are too expensive, so these items need to be shared among more locations. The following chapter suggests a method to realize this.

9. Complete pooling approach

Chapter 7 showed that a multi-location model that uses lateral transshipments does not result into cost savings for the A-Fleet rental inventory. The model considers only a single region and therefore its pooling advantages are limited. To determine the A-Fleet rental inventory, this chapter suggests another approach. The method will compute the A-Fleet inventory levels, based on the completely pooled demand for these products. Doing so, the approach will maximize the utilization of these expensive products.

9.1 Model introduction

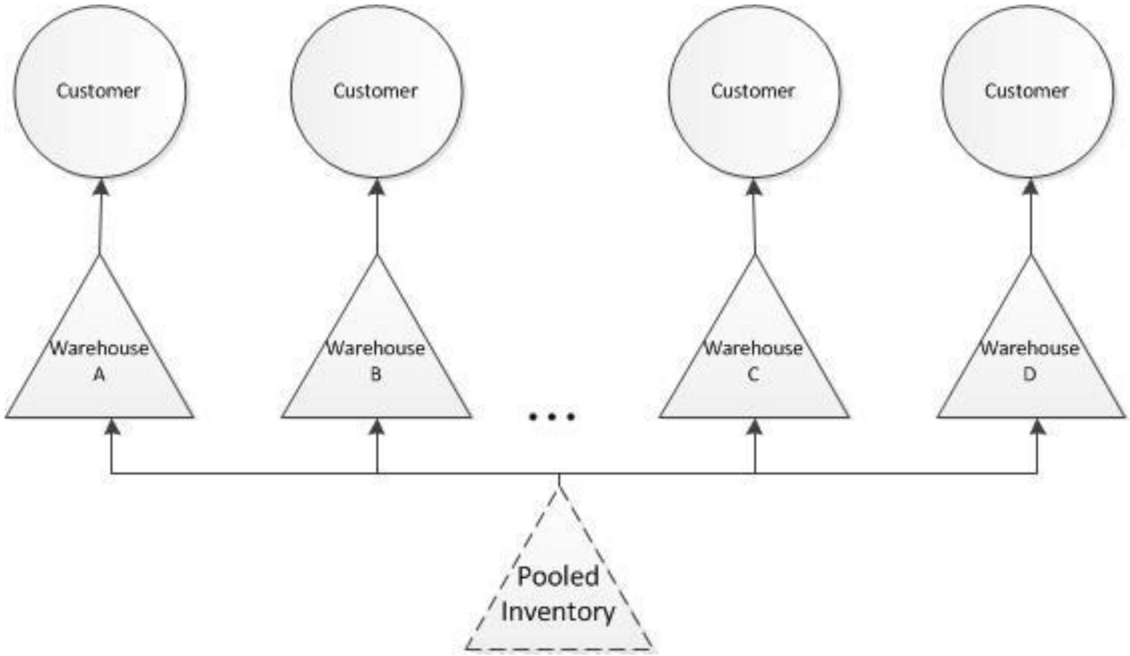

Figure 23: Complete pooling approach for EQIN's A-Fleet inventory

Figure 23 illustrates the completely pooled inventory system. Each warehouse in EQIN's system has its own customers that demand A-Fleet items. However, all warehouses share a pooled inventory. The pooled inventory is not stored a single central warehouse, but divided over EQIN's hubs (main warehouses).

The optimal pooled inventory of A-Fleet items is computed using the $M^x/G^x/c$ –model that has been introduced in Chapter 6. For each product i, the resulting seasonal order arrival rate is the pooled order arrival rate of all locations together:

$$\lambda_{i,t}^{J} = \sum_{j=1}^{J} \lambda_{i,t}^{j} \qquad 9.1$$

The greedy algorithm from Chapter 6 is applied to calculate the total required number of items S_i^* for each A-Fleet product in EQIN's entire system. The appropriate optimization function is defined as:

$$\text{Minimize} \quad \sum_i^I S_i^* * P_i \tag{9.2}$$

$$\text{Subject to} \quad \boldsymbol{\beta} \leq \beta^{Objective}$$

$$S_i^J \geq 0 \; for \; all \; i \in I$$

Where P_i denotes the purchasing price of product i and $\boldsymbol{\beta}$ denotes the aggregate blocking probability.

9.2 Performance

Table 19 shows the current ownership costs of EQIN's total A-Fleet inventory. Considering that the A-Fleet consists out of almost 200 product types, it is clear that the calibrated MSQ levels are low. This makes it obvious that EQIN prefers to transship its inventory instead of allocating a fixed number to each location.

Average MSQs		**Average levels**	
#Items	**Costs x€1000**	**#Items**	**Costs x€1000**
420	€3.286	2.694	€18.417

Table 19: Current ownership costs of total A-Fleet

Since the demand for the entire A-Fleet is pooled, fulfillment policy 1 is used in the optimization method. Policy 1 considers that a location fulfills all orders instead of (partially) forwarding planned orders. As this chapter considers the pooled demand of all warehouses, forwarding planned orders is not an option. Table 20 shows the outcomes for the optimization of all inventory levels. Since the entire A-Fleet's demand is used as input for the optimization, it is fair to compare the outcomes of the model to the stock that was owned on average in the current system. For the previous parts of the study, the value for $\beta^{objective}$ was set at 0,05. For A-Fleet items, the values lower values of $\beta^{objective}$ are also evaluated. This is interesting because EQIN's entire A-Fleet inventory is pooled and therefore, it is impossible to send additional items in emergency cases (for example if a large order depends on a single item). For B- and C-Fleet items, EQIN can always send an item from another warehouse.

$\boldsymbol{\beta^{obj}}$	**#Items**	**Costs x€1000**	**ΔCosts (%)**
0,05	1.972	14.220	-22%
0,04	2.048	15.415	*-15%*
0,03	2.099	15.968	*-12%*
0,02	2.147	16.623	*-8%*
0.01	2.287	18.244	*-1%*

Table 20: Optimization outcomes for complete pooling approach

Table 20 shows that even at a $\beta^{objective}$ value of 0,01, the optimal ownership costs are less than the current ownership costs. For $\beta^{objective} = 0{,}01$ the optimal costs are 1% below the current ownership costs. The cost reductions are higher than expected. However, this might be explained by the fact that, if required, EQIN might buy additional stock for large projects, even though these additional items would result in overage inventory for regular rentals.

Simulation

The resulting inventory levels from the optimization above have been simulated against the actual demand and returns in 2015 and 2016. The results from this simulation are shown in Table 21. The table shows the lost sales ratio and the change in on-hand inventory value, compared to the actual data. It is notable that the values for the on-hand inventory savings from Table 21 are so much higher than the ownership costs from Table 20. This can only be achieved if the utilization of rental items is very high.

$\beta^{objective}$	**Lost sales**	**Δ Inventory value (%)**
0,05	0,046	-60%
0,04	0,041	-56%
0,03	0,037	-52%
0,02	0,031	-46%
0,01	0,023	-35%

Table 21: Simulation of entire A-Fleet

9.3 Allocation procedure

The greedy algorithm above determines the number of items owned for each item in EQIN's entire A-Fleet, denoted by S_i. However, as stated in section 9.1, the inventory is not stored in a single location. This section describes a simple policy to allocate the A-Fleet items to main warehouses. For this policy the number of transportations are minimized, not the actual transportation distance or costs.

For the allocation procedure, it is assumed that A-Fleet inventory is stored at the main warehouses (hubs), using a level S_i^j. The entire demand of a region is satisfied through its hub and if demand is unmet, additional items are transported from another location. For the allocation of the total number items S_i^* to hubs, an optimization function is formulated that minimizes the total blocking probability of each hub:

$$\text{Minimize} \quad \sum_{i=1}^{I} \sum_{j=1}^{J} \beta_i^j\left(S_i^j\right) * \lambda_{i,j} \qquad 9.3$$

$$\text{Subject to} \quad \sum_{j=1}^{J} S_i^j = S_i^* \; for\ all\ i \in I$$

$$S_i^j \geq 0 \; for\ all\ i \in I$$

For the allocation policy, it is assumed that demand can always be satisfied from another location. This is reasonable because of the low lost sales rates. In emergency cases, EQIN may offer a substitute or even rent an additional A-Fleet item from a competitor to satisfy its customer. The expected rate of demand that requires transportation is computed by $\beta_i^j(S_i^j) * \lambda_{i,j}$. The blocking probabilities $\beta_i^j(S_i^j)$ for separate regions are independent from each other and convex in $S_{i,j}$.

Therefore, an algorithm can be performed for each product type i to allocate each item to a hub. In the allocation algorithm the inventory level of the location with the largest decrease of its transportation rate for product i ($\Gamma_{i,j}$) is increased, with S_i as the total number of items. Since the ownership costs are independent from the location, $\Gamma_{i,j}$ is calculated with:

$$\Gamma_i^j = \left(\beta_i^j(S_i^j) - \beta_i^j(S_i^j + 1)\right) * \lambda_{i,j} \qquad 9.4$$

<u>*Algorithm for allocation A-Fleet inventory (based on: Van Houtum & Kranenburg, 2015)*</u>

1. $S_i^j = 0 \; for \; all \; items \; i \in I, locations \; j \in J$
2. $\boldsymbol{Do \; for \; each \; product \; i \; \in I}$
3. $Compute \; \Gamma_i^j \; for \; all \; locations \; j \in$
4. $\boldsymbol{Do \; until}$: $\sum_{j=1}^{J} S_i^j = S_i^J$
5. $k = \arg\max\{\Gamma_i^j : j \in J\}$
6. $S_{i,j} = S_{i,j} + 1$
7. $Compute \; \Gamma_i^k$
8. $\boldsymbol{Loop}$
9. $\boldsymbol{Next \; product}$

9.4 Conclusion

This chapter proposed a method to optimize EQIN's A-Fleet inventory. To minimize the number of rental items the demand of all warehouses is pooled. Using the pooled demand, the $M^x/G^x/c$ –model is applied to calculate the total number of items that are required to fulfill the demand. Thereafter, an allocation procedure is proposed to set inventory levels at the individual warehouses.

Due to the complete pooling approach, transportation costs will be high, however due to the high ownership costs of A-Fleet items (> €2500) EQIN would rather focus on maximizing the utilization of these products. The model leads to significant cost reductions for the ownership costs and therefore it is concluded that this method is appropriate for EQIN's A-Fleet rental inventory.

10. Inventory planning tool

As agreed with EQIN, an inventory planning tool has been developed that calculates inventory levels for EQIN's warehouses. The tool bases its calculations on the same data sheets that are used throughout this thesis (elaborated in Table 2).

10.1 Introduction of the tool

The tool has been developed in Excel VBA, Figure 25 shows the home screen of the tool. The tool consists out of five tabs that can be used to compute and update inventory levels. The first tab allows the user to choose for which warehouse he wants to calculate inventory levels and for which warehouse. The outcomes of the optimization are displayed in excel sheets like displayed in Figure 25.

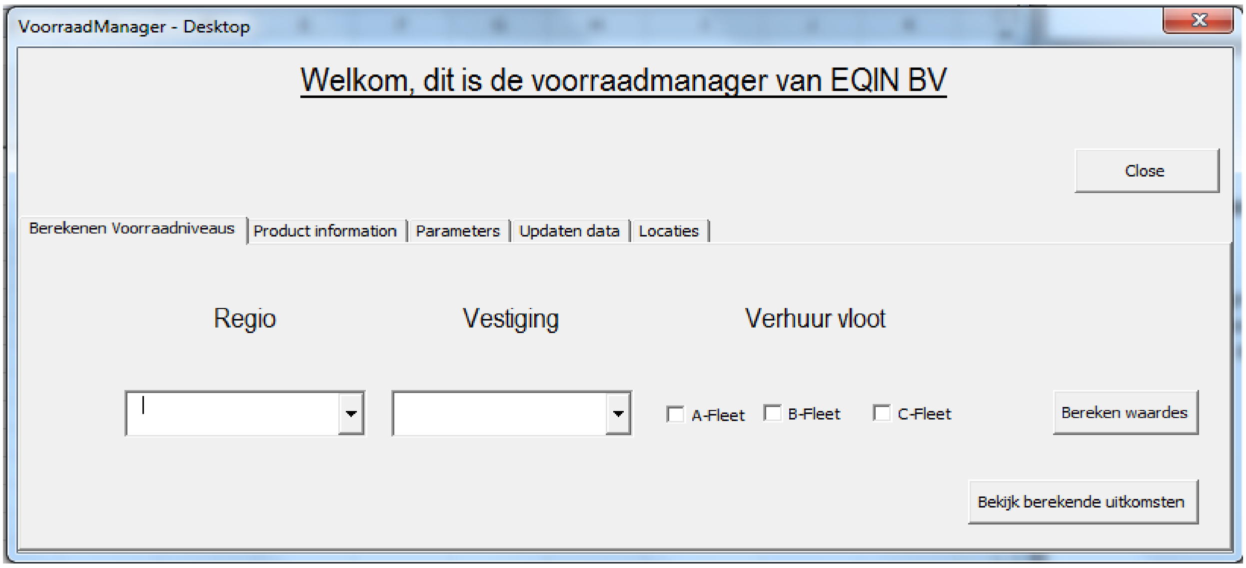

Figure 25: Inventory planning home screen

B-Vloot		C-Vloot	
Naam	Voorraad	Naam	Voorraad
[1113010] AGGREGAAT 5KVA 230V BENZ INCLISOBEW07	0	[1129101] ONDERSTEL TBV VERDEELKAST STAPELBAAR	11
[1124240] VERDEELKAST G 32A/1X32A/3X16A 6X230V CEE	3	[1131200] U VERLENGKAB.12/24V 16A2PCEE 25M(2,5MM2)	18
[1124260] VERDEELKAST H 32A/1X32A 6X230V CEE	11	[1133111] VERLENGKABEL 230V 16A 2P RA 10M	1
[1124340] VERDEELKAST E 63A/1X63A 3X32A 6X230V CEE	26	[1133141] VERLENGKABEL 230V 16A 2P RA 25M HASPEL	7
[1124360] VERDEELKAST F 63A/1X32A 4X16A 6X230V CEE	0	[1133226] U VERLENGKAB. 230V 16A 3P CEE 25M 2,5MM2	16
[1124440] VERDEELKAST D 125A/4X63A/4X32A 6X230V CE	0	[1133251] KABELHASPEL 230V 16A 3P CEE 25M 2,5MM2	3
[1134317] VERLENGKABEL 400V 125A 5P CEE 25M 5G35	4	[1133326] U VERLENGKABEL 230V 16A 2P+PA 25M 2,5MM2	16
[1134318] VERLENGKABEL 400V 125A 5P CEE 25M 5G50	0	[1134107] VERLENGKABEL 400V 16A 5P CEE 25M 2,5MM2	3
[1138250] VERLENGKABEL 240MM2 25M AARDE POWERSAFE	1	[1134207] VERLENGKABEL 400V 32A 5P CEE 10M 5G6	7
[1138251] VERLENGKABEL 240MM2 25M N POWERSAFE	1	[1134208] U VERLENGKABEL 400V 32A 5P CEE 25M 5G6	11
[1138252] VERLENGKABEL 240MM2 25M L1 POWERSAFE	1	[1134314] U VERLENGKABEL 400V 63A 5P CEE 10M 5G25	4
[1138253] VERLENGKABEL 240MM2 25M L2 POWERSAFE	1	[1134315] VERLENGKABEL 400V 63A 5P CEE 25M 5G25	2
[1138254] VERLENGKABEL 240MM2 25M L3 POWERSAFE	1	[1134341] VERLENGKABEL 400V 63A 5P CEE 25M 5G16	8
[1143200] BESCHERMINGSTRAFO 230V 16A 3P CEE 4000VA	7	[1136014] B AARDKABEL GROEN/GEEL 35MM2 10M MET OGE	2

Antwerpen | Wachtebeke | Elsloo

Figure 25: Inventory planning outcomes page

The outcomes are computed using the models that are presented throughout this thesis. To compute the C-Fleet inventory, the $M^x/G^x/c$ –model is used, for the B-Fleet, the multi-location system that uses lateral transshipment is used. To compute the A-Fleet inventory, the complete pooling approach is used.

On the second tab, the user can request product information on a single product, for a single location. After choosing the warehouse and entering the product number, the screen in Figure 26 will appear. The advised inventory level is based on the single location $M^x/G^x/c$-model. This feature can be used if the logistics manager would like to have a more in-depth insight in a product's characteristics.

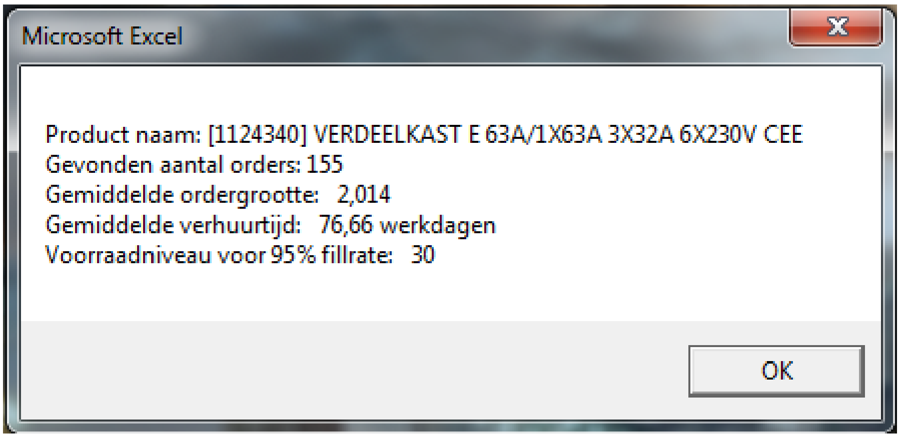

Figure 26: Product information screen

In the parameter page, the user can change the fulfillment policies that are applied in the optimization and the optimization target ($\beta^{Objective}, L^{Objective}$).

In the fourth tab, the user can open the data sheets. The data sheets need to be updated manually due to authorization rules by EQIN. The data sheets are linked to EQIN's servers. The last sheet tab be used to add, delete or rename warehouses.

10.2 Limitations

The main disadvantage of the tool is that its input data needs to be updated manually. The data sheets are linked to EQIN's servers and cannot be updated through an Excel VBA code. Therefore, the user must update the sheets one by one (input the appropriate dates). Furthermore, for the hubs, the amount of data for two years is too large to download from EQIN's servers. Therefore, the sheets of these warehouses only contain data of one year.

10.3 Company's satisfaction

EQIN's logistic manager, Ipe Oosterhof, has indicated that he liked working with the tool as he just needs a few clicks to compute inventory levels. As he has meetings with local managers once every quarter, he did not mind the updating time that much (although obviously it is experienced as inconvenient). He stated that he will definitely use the tool when he will meet the local managers.

11. Conclusions and recommendations

This master thesis project aimed to optimize EQIN’s rental inventory control system. EQIN management has difficulties to determine and update the right inventory levels due to its large number of different product types and multiple locations. Therefore they need a tool that is able to automatically compute appropriate inventory levels. EQIN distinguishes its A, B and C-Fleet, based on product investment value. In this research, the performance of multiple inventory models is evaluated on these fleets. The study has led to the following conclusions:

C-Fleet inventory
A single-location approach is recommended for the C-Fleet inventory. The proposed single-location system allocates a fixed number of items to each warehouse. The extended model that has been studied is promising as it leads to inventory cost reductions. However, due to the transportation costs, the cost reductions are limited.

B-Fleet inventory
For EQIN’s B-Fleet inventory, a lateral transshipment model has been used to model the inventory control system in an entire region. This model has been shown to be optimal for the B-Fleet inventory.

A-Fleet inventory
For the A-Fleet inventory, the other systems proved (cost wise) inferior to the current system, since main warehouses cannot use pooling advantages. Therefore, the A-Fleet inventory levels have been determined using a pooled demand of EQIN’s entire rental system.

11.1 Recommendations

This report provides EQIN with a methodology to control their inventories and additionally, the inventory planning tool can automatically determine the optimal inventory levels. To improve EQIN’s inventory system even further, some recommendations are proposed:

11.1.1 Use ERP system to determine product characteristics more accurately

EQIN is currently working on the selection of a renewed ERP system. This system must improve the integration of EQIN’s rental and sale departments. However, this also brings the opportunity to include some functionalities that help to improve their inventory control system. The rental inventory models calculate inventory levels based on three variables: *Order arrival rate*, *order size* and *rental duration*. The inventory planning tool that is developed in this project calculates these variables on a limited set of data that needs to be updated manually. Using an ERP system, these variables can be computed more accurately and updated automatically.

11.1.2 Research product repairs

Also, as stated in Section 2.3.2, product inspection, failures and repair time are not studied in this thesis. Instead, a total rental cycle time is assumed. If EQIN would have more knowledge about these processes in its repair shop, the total cycle time can be computed more accurately. Usage based loss has already been studied in rental systems [8]. The influence of the failures and repairs would also be an interesting subject of study as it presents the tradeoff between repairing products and buying new products.

11.1.3 Research unsatisfied customers

This study has assumed two critical assumptions regarding lost sales. Firstly, each unsatisfied demanded item is lost. This assumption leads to an overestimation of the lost sales. Secondly, each customer accepts partial fulfillment. This assumption leads to an underestimation of the lost sales. To improve the calculations on lost sales EQIN should research the behavior of customers regarding unsatisfied demand.

11.1.4 Deal with substitutes to decrease inventories

Secondly, the inventory control system described in this thesis does not consider substitute products. However, EQIN distributes thousands of different product types, for many of those products EQIN also distributes substitute products (extension cables of different lengths, aggregates with different capacities, etc.). Dealing with substitutes in a smart way, EQIN can optimize their inventory levels even further. The company can pool the demand of substitute products which will result in a decreased total number of allocated items (thus: $S_{123} \leq S_1 + S_2 + S_3$). However, when determining the individual inventory level from this total inventory level, EQIN should be aware of (at least partial) one-way substitution; Customers are willing to substitute a 10m extension cable for a 20m extension cable but not the other way around.

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Appendix 1 Geometric distribution

Consider geometric parameter p with $E[X] = \frac{1}{(1-p)}$

The variance of the geometric distribution: $V = \frac{p}{(1-p)^2}$

The probability distribution function is defined as:

$$P(X = n) = (1 - p)p^{n-1}, \qquad n = 1,2, \ldots$$

And:

$$P(\ X > n) = p^n$$

From the law of conditioning ($P(A|B) = \frac{P(A \cap B)}{P(B)}$) follows that:

$$P(X = n + k \mid X > n) = \frac{(1-p)p^{n+k-1}}{p^n} = (1 - p)p^{k-1}$$

For the order size distribution, this implies that if the order size is larger than n (overflow demand), the order size of this overflow demand follows a geometric with the same parameter p.

Appendix 2 Blocking probability function

Papier and Thonemann [10] suggest the following formulas:

$$\beta_i(S) = 1 - \frac{\sum_{k=1}^{S_i}\big(kg(k)\big) + S_i(q_i-1)g(S_i)}{\lambda_i q_i T_i\big(G(S_i) + (q_i-1)g(S_i)\big)}$$

$$g(k) = e^{-\lambda_i T_i} \sum_{a=1}^{k} \left(\tfrac{(\lambda_i T_i)^a}{a!}\right) f^a(k)$$

$$f^a(k) = \frac{(k-1)!}{(a-1)!\,(k-a)!} * \left(\frac{1}{q_i-1}\right)^a * \left(\frac{q_i-1}{q_i}\right)^k$$

$f^a(k)$ gives the a-fold convolution of the geometric order size function. $f^a(k)$ can only be computed in the case that both k and a are larger than 0 and $a \leq k$.

Example if $q_i = 3$:

a	k	$f^a(k)$
1	3	0,33
2	3	0,66
3	3	0,33

For $g(k)$ a recursive method is suggested:

$$\frac{(\mathrm{\lambda_i T_i})^{a+1}}{a+1} = \frac{(\lambda T)^a}{a!} * \frac{(\lambda T)}{a+1}$$

$$f^{a+1}(k) = f^a(k) * \left(\frac{1}{q-1}\right) * \left(\frac{k-a}{a}\right)$$

and when $a = 1$:

$$\frac{(\lambda T)^1}{1!} = \lambda T$$

$$f^1(k) = \frac{1}{q-1} * \left(\frac{q-1}{q}\right)^k$$

$$\Delta\beta(S) = \beta(S+1) - \beta(S)$$

Appendix 3 Excel VBA script blocking probability function

```
Function beta(lambda As Double, c As Integer, t As Double, q As Double) As Double
   'This function calculates the blocking probability (beta)
   'Suitable for batch-arrival, batch-return system
   'Input variables:
   'lambda = Order arrival rate
   'c = number of servers
   't = mean rental duration
   'q = mean order size
   Dim u As Double
   u = lambda * t
   If lambda = 0 Then
       beta = 0
   Else:
       'betaA denotes the numerator
       betaA = 0
       Dim k As Integer
       Dim gCum As Double
       Dim gC As Double
       gCum = 0
       gC = 0
       For k = 1 To c
           gValue = g(k, u, q)
           gCum = gCum + gValue
           betaA = betaA + k * gValue
           If k = c Then
               gC = gValue
           End If
       Next k
       betaA = betaA + (c * (q - 1) * gC)
       'betaB denotes denominator
       betaB = 0
       gCum = gCum + Exp(-u)
       betaB = (lambda * q * t) * (gCum + (q - 1) * gC)
       'final computation
       beta = 1 - (betaA / betaB)
   End If
End Function
Function g(k As Integer, u As Double, q As Double) As Double
    Dim gsum As Double
    Dim gsum1 As Double
    Dim gsum2 As Double
    gsum = 0
    gsum1 = 0
    gsum2 = 0
    Dim A As Integer
    For A = 1 To k
        If A = 1 Then
        gsum1 = u
        gsum2 = (1 / (q - 1)) * ((q - 1) / q) ^ k
        Else:
        gsum1 = gsum1 * (u / A)
        gsum2 = gsum2 * (1 / (q - 1)) * ((k - (A - 1)) / (A - 1))
        End If
        gsum = gsum + (gsum1 * gsum2)
    Next A
    g = Exp(-u) * gsum
End Function
```

Appendix 4 Sensitivity Analysis

			Increase arrival rate (%)			
			0%	**10%**	**20%**	**30%**
Increase rental duration (%)	***Policy1***	**0%**	0	5%	14%	20%
		10%	5%	15%	22%	27%
		20%	14%	22%	28%	33%
		30%	20%	27%	33%	40%
	Policy 2	**0%**	0	6%	8%	11%
		10%	8%	12%	19%	21%
		20%	20%	22%	27%	35%
		30%	26%	35%	40%	45%
	Policy 3	**0%**	0	3%	8%	13%
		10%	14%	24%	27%	32%
		20%	32%	41%	52%	64%
		30%	57%	70%	89%	97%

Appendix 5 Outcomes Single Location Model

				$\beta^{objective} = 0,05$			Simulation	
Location	**Minimum stock (x1000€)**	**Ownership cost (x1000€)**	**Policy**	**#Allocated items**	**Ownership cost (x1000€)**	**ΔCosts (%)**	**Fill rate**	**ΔCosts (%)**
Luik	86	192	**1**	2.206	117	-39%	88%	-22%
			2	1.298	74	-62%	89%	-50%
			3	1.581	92	-52%	92%	-41%
Velsen-Noord	244	423	**1**	5.669	474	12%	95%	-9%
			2	3.283	286	-32%	96%	-42%
			3	3.889	336	-20%	97%	-32%
Joure	7	21	**1**	147	15	-31%	98%	-22%
			2	68	7	-67%	98%	-47%
			3	121	12	-45%	98%	-39%
Geleen	76	454	**1**	6.542	428	-6%	98%	-24%
			2	3.862	275	-40%	98%	-50%
			3	4.630	323	-29%	99%	-42%