MASTER

Inventory control for a lost-sales system with joint-replenisment in an e-commerce environment

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Inventory Control for a Lost-sales system with Joint-replenishment in an e-commerce environment

A Master Thesis Project at Optiply: Supply Management Software

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Abstract

Companies with webshops in an e-commerce environment face many different uncertainties regarding inventory control. Uncertainties such as lost-sales resulting from excess demand, stochastic joint replenishment, highly variable but slow demand and stochastic lead-times. This report describes the development of a model and associated decision support tool that address these uncertainties. The model includes replenishment policies that were developed throughout the design process. Functions from the model were verified and validated based on a scenario analysis and a model comparison with the use of a self-developed simulation tool. The findings and results of this analysis are combined in the development of a decision support that assists in making inventory control decisions by estimating fill rates, inventory cost and other relevant parameters for as many products as desired.

Management summary

In this master thesis report we present our research on inventory control in a single-echelon lost-sales system that takes into account joint replenishment under stochastic highly variable demand and stochastic lead-times. As a case study the supply chain of Company B Webshops B.V. is chosen from the customer network of the supervising company Optiply B.V.

Problem statement

Optiply is an innovative young company operating in an business-to-consumer e-commerce environment. Their customer network mainly consists of companies that own one or more webshops. These so called e-tailers sell their products online and rarely own a physical shop but do store their products. The core business model of Optiply is designed to automate the replenishment process of their customers such that inventory control is of a high standard and does not need much expertise nor manual labor. In this way the Optiply attempts to 'replace the logistic experts of this world' with their model. The case study company we call 'Company B' is a company that manages numerous webshops and has an assortment of thousands of different products. Companies such as Company B order their products to multiple suppliers from all over the world. They face challenges such as high uncertainty in demand of their products, uncertainty in the lead-times of their suppliers and a rapidly growing e-commerce market with many competitors. Within this uncertain environment inventory levels have to be maintained correctly and the replenishment process for all products has to be cost efficiently coordinated.

Based on the problem analysis and the gained business insight on the problem similarities of the companies in the customer network of Optiply, we defined a research assignment that has the goal to solve the different problems that these companies face in their daily inventory control. This main assignment is defined as:

> *Develop a decision support tool that assists in minimizing total inventory cost in a single-echelon lost-sales system taking into account joint-replenishment under lead-time, order moment and demand uncertainty for a given target service level.*

Analysis of inventory control in the e-commerce environment

Inventory control problems in the as-is situation of the case study company were found to be a combination of business environmental factors and the fact that inventory control is often overlooked because companies are rapidly growing and putting all business effort in this growth only. By environmental factors we refer to uncertainties in demand making it difficult to forecast, variable lead-times from suppliers and online customers that are price and delivery time sensitive. Because customers can compare prices or check delivery times online with great ease, switching to competitors occurs often and products are rarely backordered. As a result, excess demand is typically lost and the related inventory system is transformed from a backorder system into a lost-sales system. Lost-sales result in the fact that demand in periods without stock on hand is not known. This unobserved demand is difficult to uncover and replenishment decisions are therefore often based on historical sales, if it is based on anything at all. Taking into account the holding cost of holding stock on hand, the ordering cost of ordering to the suppliers and aiming at satisfying customer demand (i.e. target service level), the replenishment process results in a joint replenishment problem in a lost-sales system where inventory and replenishment has to be coordinated tightly. Therefore, the four main components of the inventory control problem are:

1. *Taking into account lost-sales (i.e. unobserved demand)*
2. *Which replenishment model(s) to use for the n replenishment problems*
3. *Coordinating replenishments by joint replenishment*
4. *Target service level and s-level setting*

New model development

During the design phase of our research we considered different solution concepts which we described extensively in this report. In solving the inventory control problems we combined or adapted some of these concepts and developed a new inventory control model. This model includes two approaches that estimate the unobserved demand in periods without stock on hand, a method to determine the supplier review period and product order quantities to coordinate product replenishment and two periodic replenishment policies. These two policies are largely based on the (R, S) and (R, s, S) replenishment policies but differ in their parameters. We therefore defined R^δ as the review period that is determined for a supplier δ. This review period is based on the demand and cost from all products i ordered to that supplier and coordinates the replenishment of all these products to the supplier. Furthermore, we defined reorder level s_i and order-up-to-level S_i which are both based on a specific product i. Once, every review period the inventory position (i.e. stock on hand plus inventory on order in a lost-sales system; stock on hand plus inventory on order minus backorders in a backorder system) is brought back up to this order-up-to level. However, in the (R^δ, s_i, S_i) replenishment policy the inventory position is only is brought back up to the order-up-to level if the inventory position is below the reorder level s_i. The calculations of both policies in the new model were found to be accurate under highly variable stochastic demand and stochastic lead-times. By using the new model policies, reorder level calculations, cost calculations and other output parameters, calculations become more tractable and rational within the decision making process. Furthermore, we described multiple s-level correction methods to achieve target service levels when demand is forecasted or simulated. However, these correction methods are not one-on-one applicable to the inventory system with characteristics such as lost-sales, compound renewal demand and more complex forecasting methods than simple exponential smoothing.

Case study

Additional to a scenario analysis where we verified the calculations of the new model, we compared the performance of the new model compared with the model currently utilized by Optiply. In almost all simulated scenarios, the new model policies outperformed the policies from the Optiply model with respect to inventory cost efficiency and achieved fill rate. Moreover, we showed that the new model policies perform acceptable under highly variable and unpredictable demand taking demand forecasts as input parameter. Comparing the two replenishment policies within the new model only, the (R^{δ}, S_i) replenishment policy outperformed the (R^{δ}, s_i, S_i) replenishment policy with respect to total inventory cost and achieved fill rate. The reason for this is that the order-up-to level (i.e. reorder level plus economic ordering quantity) is calculated in such a way that there should be enough inventory to last for the whole replenishment cycle. In the (R^{δ}, s_i, S_i) replenishment policy the inventory level is sometimes not brought back to the order-up-to level due to variability in demand during the review period. Therefore, the inventory level is lower in the coming replenishment cycle and the probability of a stock-out is higher resulting in a lower fill rate. Only focusing on holding cost and ordering cost, the (R^{δ}, S_i) policy obviously performed worse because the inventory position is brought back to the order-up-to level in almost every review period which results in ordering more frequently and higher inventory levels. An interesting finding is that the 95% confidence intervals of the simulated output parameters of the (R^{δ}, S_i) replenishment policy were tighter in almost every scenario, which means that in the long term the different inventory cost and the achieved fill rate are less variable under the (R^{δ}, S_i) replenishment policy than under the (R^{δ}, s_i, S_i) replenishment policy. Additionality we research target service level setting for the fill rate service level and the ready rate service level and how both service level relate to each other. We showed that if we increase the variability of demand in a compound renewal process, the deviation between the fill rate and the ready rate becomes larger (i.e. $P_2 < P_3$).

Decision support tool

We developed a decision support tool that encompasses the calculations and decision rules of the new model and its components. This tool can be integrated into the model of Optiply or be used independently. The tool enables the calculation of reorder levels, order-up-to levels, supplier review periods, order quantities and other relevant inventory parameters for as many products as desired. In the basis, the tool only requires position of sales data (POS data), stock changes data and cost and target fill rate setting. Based on demand input parameter calculations, a forecast method and an approach to take into account unobserved demand, the tool determines the continuous demand parameters. Thereafter the tool calculates all relevant output parameters and provides the user with suggestions on the setting of supplier review periods, economic order quantities, reorder-levels and order-up-to levels for every product. Additionally, the tool provides estimations for the holding cost, ordering cost, average inventory levels and service levels.

Preface

Six months, and therefore approximately 15% of my time at the Eindhoven University of Technology, were spend on conducting my Master Thesis Project. One could say that it is the largest project that I have undertaken in my whole life up until this moment, and I think this is indeed the case. This report describes the results of that Master Thesis Project, which I conducted at Optiply Supply Management Software in Eindhoven. It concludes my Master in Operations Management and Logistics and brings my time at the Eindhoven University of Technology to an end.

I would like to thank the people that supervised me during my Master Thesis Project. First of all, I would like to thank Henny van Ooijen, for being my first supervisor. I am grateful for having him as my mentor during the whole process of conducting my Master Thesis Project. I could always come to you if I had questions or wanted to discuss certain design choices related to the project. Your feedback throughout my project was very helpful and constructive.

Secondly, I would like to thank my second supervisor Ton de Kok for his feedback and suggestions throughout my project. During the end of my project I got very familiar with the ladies of the eSCF secretary due to the fact that they helped me with the intensive process of planning my meetings with you.

I would also like to thank my third supervisor Karel van Donselaar for taking the time to review my report and showing interest in my work.

Lastly, I would like to show appreciation to both my supervisors of Optiply, Sander van den Broek and Wiebe Konter. Thanks to you both I was able to conduct my project at a young, innovative and interesting company where I learned much about inventory control in an e-commerce environment. It has been a good time discussing how we could overcome problem situations by looking at it from different perspectives and attempting to find the best practical solution. I am grateful to have learned so much about programming in *R* and applying it in the field of inventory control and replenishment planning.

For me personally, this is an end of an era. My struggle against the Ragnarøk has ended (Lindholm, 1991). It is time to earn back my student loan.

Roel Buying

Eindhoven, March 2017

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CV:	Coefficient of Variation
DGS:	Direct Grouping Strategy
EM:	Expectation Maximization
EOQ:	Economic Order Quantity
ETS:	Exponential Smoothing State Space Model
IGS:	Indirect Grouping Strategy
iid:	Independent and Identically Distributed
JRP:	Joint Replenishment Problem
KPI:	Key Performance Indicator
LSP:	Lost-Sales Problem
MAD:	Mean Absolute Deviation
MASE:	Mean Absolute Scaled Error
ME:	Mean Error
MSE:	Mean Squared Error
POS:	Point Of Sale
RMSE:	Root Mean Squared Error
NBD:	Negative Binomial Distribution
NN:	Neural Network
nOO:	Number of Outstanding Orders
OPAC:	Operations Planning Accounting & Control
R^{δ}:	Review period on a supplier level based on all product ordered to supplier δ
SJRP:	Stochastic Joint Replenishment Problem
SKU:	Stock Keeping Unit
TPOP:	Time Phased Order Point

Introduction

In this master thesis report we present our research on inventory control in a single-echelon lost-sales system taking into account joint replenishment under stochastic highly variable demand and stochastic lead-times. The research is conducted at Optiply Supply Management Software in Eindhoven. Optiply operates in an innovative and agile e-commerce environment. Customers in the network of Optiply are mostly webshops that vary in assortment from hundreds to thousands of different SKUs. These companies operate in an environment that is rapidly growing, where products have short lifecycles and demand is difficult to predict.

Optiply's core business is based on an innovative developed inventory control model that includes several approximation algorithms to estimate relevant parameters. The algorithm is separated in a tactical and an operational model. The tactical model includes replenishment models with sampled demand which are used to determine relevant parameters required for inventory control. In the operational model these replenishment models are combined with demand forecasting models to estimate output parameters including safety stocks and order quantities. Optiply would like to improve their algorithms and its fit on replenishment in an e-commerce environment. With the current model it seems that target service levels are not met and inventory levels suggested by the model are not accurate enough. Moreover, the model is based on sales only and the company would like to take into account the demand in periods without stock on hand.

Scientific research on the application of existing replenishment models and demand forecasting methods in an e-commerce environment is scarce and in reality, inventory control is often outdated or even overlooked by e-commerce companies. This Master Thesis therefore addresses the problem of companies in an e-commerce environment that face many uncertainties in their replenishment process but do not have the knowledge to improve their inventory control and grip on the replenishment process. In this project, a mathematical model is developed that attempts to minimize the total inventory cost of an inventory control system under certain assumptions and a target fill rate. Thereafter, a decision support tool is developed for Optiply that assists in the decision process regarding inventory control. The tool takes sales/demand data and stock changes data as input parameters and provides the user with relevant output parameters such as reorder levels, holding cost, ordering cost and expected inventory levels.

Chapter One

Optiply & Company B

This first chapter introduces two companies. First, an introduction is made to Optiply, the supervising company of the Master Thesis Project. Thereafter, an introduction is made to Company B, a company that inspired us to perform a case study on. Company B therefore, will be referred to as the case study company in this Master Thesis Project. In section 1.1 the supervising company Optiply is briefly introduced. Section 1.2 introduces the case study company 'Company B' and the environment the company is working in. In section 1.3 the problem statement and the as-is situation of Company B is described briefly.

1.1 Supervising company introduction - Optiply

Optiply specializes in inventory optimization and focuses their business on the e-commerce industry. The e-commerce industry has several large market segments such as B2B e-commerce and B2C e-commerce. Customers in the customer network of Optiply include companies such as webshops and retailers with a focus on online sales (e-tailers). The company developed a model that helps in decision making with regard to inventory control. The improved decision making may reduce inventory levels and increases revenue by helping webshops to determine when and how much to order of which product. Certain parameters in the model can be adjusted to make it useful in different inventory control situations that are experienced by customers. The model is sold as a service package in combination with potential replenishment advice, implementation and training.

The company was founded in 2015 by two freshly graduated students and has grown into a current team of seven. These seven persons come from different technology backgrounds such as Operations Management & Logistics, Software Science, Data Science and Data Engineering. The company is growing and increasingly hiring employees and graduation interns.

Optiply has always been looking to improve and strengthen their inventory control model. The most relevant problems that occur in this process are focused on process control, replenishment models, cost setting and demand forecasting. One of the next steps for Optiply is to improve their inventory control model by using big data techniques to implement external data such as the weather, Google positions and web analytics in the algorithm. This external data could help enhance the forecasts by explaining a larger part of the variance in the demand data. Optiply is a company that is located on the intersection between replenishment models, data science and e-commerce.

1.1.1 Office locations and processes of Optiply

The main office is based in Eindhoven and is located on the campus of the Eindhoven University of Technology. This location focusses on development processes. Located in the Multi Media Pavilion of the university, the company is close to information and knowledge that flows from the

Operations Planning Accounting & Control (OPAC) department of the university. Furthermore, appropriate new members for their team can be attracted from the university. Marketing processes are being outsourced for approximately 90% and are supervised by the main Office in Eindhoven.

The second office is based in Amsterdam and is especially focused on data science. Some of the development is performed here by integrating the achieved external data into the inventory control model. The location of this office is chosen because studies such as Data Science and Econometrics are located in Amsterdam. It is possible for employees to work on both locations.

1.1.2 Inventory control model of Optiply

Optiply developed an inventory control model that is based on a statistical approximation algorithm, which is separated in a tactical model and an operational model. The tactical model contains replenishment models and is used to determine important parameters such as the safety time and the order quantity time which are the safety stock and order quantity expressed in time. The operational model involves a combination of the tactical model with certain forecasting methods. These forecasting methods are based on historical sales or demand data and are able to cope with trends and seasons in demand.

1.2 Case study company introduction – Company B

Company B is a company from within the customer network of Optiply. The company's main office is located in Groningen and manages approximately 400 webshops in The Netherlands and Germany. Their product assortment throughout the year consists of more than 3000 SKUs ranging from computer accessories to household appliances and from cleaning products to food supplements. Approximately 40% of their assortment is non-active. Products are offered for sale on their own websites and on larger webshop platforms such as Bol.com, Coolblue.nl and Markplaats.nl. Company B has been rapidly growing throughout the last years and introduced some fast selling products since the fall of 2016. The average stock on hand of Company B has a value ranging from 180,000 to 190,000 euros and Company B orders on average 120 different SKUs per day. Figure 1 shows the value of the stock on hand from *2015-10-21* to *2016-09-01*. The *warehouse management system* (*WMS*) that is used by Company B is called *Picqer*. Picqer is an online warehouse tool that helps in managing the warehouse and sales channels and is focused on webshops. *Picqer* was implemented on *2015-10-21* and for this reason the graph starts at this date. Stock data for analysis in the Master Thesis Project is therefore available since this date.

The black line in figure 1 represents the total stock value in Euros. The large peak around the middle of May 2016 is due to a large purchase of four products that were expected to sell in the coming periods. We see that the stock value has an increasing trend, which is due to the fact that the company is growing.

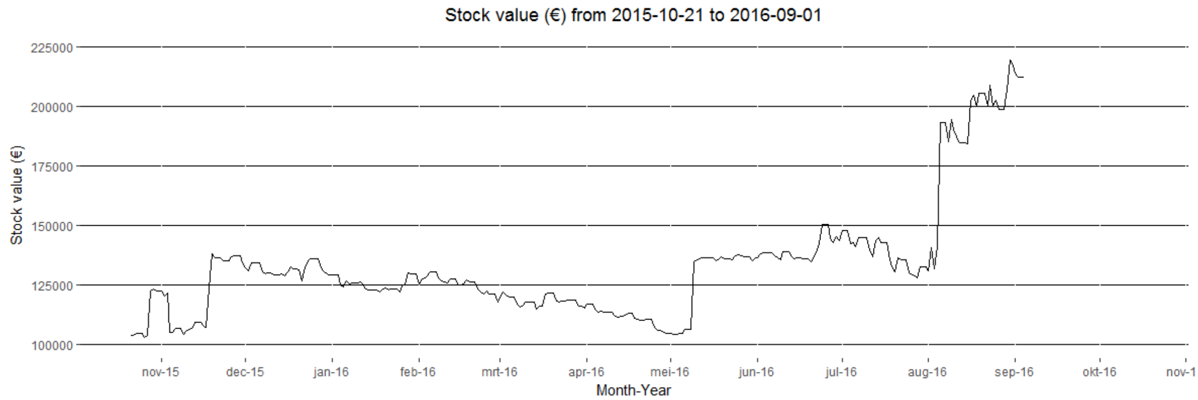


Figure 1: The stock value of the stock on hand in Euros from 2015-10-21 to 2016-09-01

1.2.1 Environment of Company B

Company B is a company that manages a set of webshops that sell a large range of different products. Their core business processes are therefore performed in an e-commerce environment. This environment may be described by a set of key characteristics. An overview of the most important characteristics is presented in table 1.

Table 1: Characteristics of the current environment of Optiply and Company B

Characteristic	Brief description
Large assortment	It is common for webshops to have a larger assortment of different products compared to their brick-and-mortar counterparts.
Improved information availability	Information can be retrieved rather rapidly and easily because almost everything is digitalized. Via the internet, suppliers and webshops can be compared based on prices, lead-times and more.
Complete lost-sales	Backorders are not rational and often result in a lost sale in an e-commerce environment because the potential customer can probably order the same product at another webshop more easily without having to backorder the product and having to wait for it a longer period of time.
Complex nature of demand	Demand is proving more difficult to forecast because it is not always stable and constant. Due to a large assortment of products and the fact that some products are very customer specific, product demand is intermittent or lumpy and does not seem to follow a known demand distribution.
Fast growing companies	Many companies in e-commerce are growing rapidly due to performing business via the internet. Companies are focused on this growth and often forget to improve their other business processes such as inventory control along the way.
Just-in-time processes	Developments such as 'one-day-delivery' and 'same-day-delivery' change the traditional business processes and service level setting.

These characteristics describe what differentiates inventory control in e-commerce from that in traditional commerce. A more extensive elaboration of each characteristic can be found in appendix A.

Intermittent (or irregular) demand. *Random demand with a large proportion of values equal to zero (Willemain, Smart, & Schwarz, 2004). Intermittent demand series are characterized by zero demand occurrences interspersed by positive demands* (Aris A. Syntetos, Zied Babai, & Gardner, 2015).

Lumpy demand. *This is demand that appears randomly with some time periods having zero demand. Demand, when it occurs, is (highly) variable* (A.A. Syntetos & Boylan, 2005).

For more on information on demand patterns, please refer to the literature study (Buying, 2016).

1.3 Problem statement

Company B manages a very large variety of products and sells these products through approximately 400 webshops. Some of these webshops offer many different products and some only offer one product (i.e. one product webshops). A large part of the assortment consists of slow moving products that only sell a couple times per year. These products often follow demand patterns that are hard to forecast, resulting in inappropriate stock levels. Some products have an unnecessary high stock level, while many other products have zero stock because these products are simply forgotten about.

The large number of different products is ordered to a relatively small number of suppliers, which means that for in some situations hundreds of products are ordered to the same supplier. The ordering of these products has to be coordinated due to fixed ordering cost that are made every time an order is placed to the supplier. Therefore, using a replenishment policy that takes into account this coordination of placing orders for the different products is important. However, the process of deciding which policy to use and how to implement it may prove difficult. Standard replenishment policies may not be applicable and an adapted or extended replenishment policy is required. The inventory control problem we have here can be described as a *joint replenishment problem* (JRP). The standard form of the JRP is described in appendix F. More case specific elaboration on the JRP can be found in section 4.3 and appendix B.2.

Demand in the case situation is not forecasted. Replenishment of products is performed based on managerial judgment, what in the case situation boils down to ordering what is deemed to sell in the coming period based on sales in the past. Factors such as historical demand of the product, historical demand of other products and supplier information may be essential in the demand forecasting process, but is left out in the case situation. Because backorders are not rational in the environment Company B is working in, a non-sale typically results in a lost-sale. Whenever a product is out of stock, the product is typically removed from the website of the webshop. In this way potential customers cannot find the product and essential information about the demand for the specific product is lost. An inventory control system where excess demand results in lost-sales can be seen as a *lost-sales system* and is described extensively in section 4.1 and appendix B.1 (K. Van Donselaar, De Kok, & Rutten, 1996).

Currently, the fixed ordering cost is not clearly specified. It is not specified per supplier or per product group or individual product. Fixed ordering cost is an important parameter in the inventory control system. This parameter is included in the trade-off between ordering products and holding products in storage. Moreover, the decision of which replenishment policy to use is, among other parameters, also based on the fixed ordering cost.

Chapter Two

Research assignment

This chapter describes the research assignment that is foundation of the Master Thesis Project and the gaps in the literature that led to this assignment. Section 2.1 elaborates on the scientific papers that were analyzed in the literature study and in the preparation of the modeling phase of the project. Section 2.2 defines the main assignment of the project and defines the underlying research objectives. The deliverables of the project are described in section 2.3. In section 2.4 the scope of the project is described. Section 2.5 elaborates on the used methodologies in the project. The last section, section 2.6, provides the outline of the thesis. The notations and expressions in this chapter are largely based on the definitions and expressions by de Kok et al. (2012) and are presented in appendix E.1.

2.1 Literature review & first considerations

This section briefly describes the relevant literature from the literature study that was performed in preparation of the Master Thesis Project. Note that numerous relevant additional scientific papers were analyzed during the project. A review on this literature can be found in appendix B. We would also like to refer to the literature study itself (Buying, 2016). This section will only elaborate on the main findings from reviewing the literature. Note that much of the reviewed literature will function as an input for the model development in chapter four.

2.1.1 Literature study conclusions

This section describes what relevant gaps could be found in the scientific literature. The performed analysis combines the scientific literature reviewed in the literature study and the scientific papers reviewed throughout the first part of the Master Thesis Project.

In the stochastic joint replenishment literature demand if often assumed to be (compound) Poisson and excess demand is completely backordered in the inventory systems, which makes them backorder systems. Lead-times are often assumed to be fixed, deterministic or a multiple integer of the review period, which itself is also assumed deterministic or fixed. In almost every paper a major fixed ordering cost is incurred per order in combination with minor ordering cost per product. In practice, this is not always the case. Furthermore, many scientific papers focus on finding the optimal solution to the s-levels while according to Khouja and Goyal (2008) researchers should focus more on developing applicable models for the real life inventory problems (Khouja & Goyal, 2008).

While lost-sales systems are relevant in real world practices, most scientific papers on stochastic inventory models assume excess demand being backordered. The reason for this limited attention for lost-sales system in the scientific literature is the fact that discrete-time inventory models in combination with stochastic demand are very difficult and (optimal) solutions always include dynamic programming (Zipkin, 2008).

In line with this complexity, scientific papers on periodic lost-sales systems often assume the lead-time to be a fixed or a random integral multiple of the review period. Only a few papers address a periodic inventory system with fractional lead-times; i.e., the lead time is smaller than the length of a review period. Models with fractional lead-times may prove useful in an e-commerce environment for the products that are slow moving (i.e. these products have a long review period because they are only sold once or twice per year; the lead-time is likely to be smaller than the review period for such products). Sezen (2006) studied he impact of the review period length on the average stock on hand and the fill rate through a simulation approach in case of fractional lead-times. The results show that the variability in the demand process is the most important factor to set the duration of a review period. No analytical procedure is proposed to determine the length of a review period or on how to set the order-up-to level (Bijvank & Vis, 2011). Furthermore, most developed models focus on inventory systems in which no fixed order cost is charged. Although many papers proposed properties and bounds on the optimal order quantities, they still require much computational effort to find the optimal order quantities, especially for large inventory systems. The combination of a stochastic joint replenishment problem in a lost-sales system is rarely studied in the scientific literature. All lost-sales literature that was reviewed, led us to believe that providing an optimal solution for the joint replenishment lost-sales problem is very computational intensive and an heuristic or self-developed approach would be useful in the conceptual model of our research.

SKU classification may be performed on two different levels. Firstly, SKU classification considering demand forecast methods. Fast selling products experience a different demand pattern than slow selling products. Therefore, products with certain characteristics may be forecasted differently than other products. Secondly, SKU classification considering replenishment policies and service level setting. Products differ in characteristics such as supplier, fixed ordering cost and holding cost. Therefore, it seems intuitive to classify products based on different relevant characteristics rather than only on demand value and demand volume or no classification at all.

What is missing in the demand forecasting literature is a trade-off between error performance of a demand forecasting method and other performance measures such as service levels and inventory costs. Demand forecasting methods are often evaluated considering forecasting errors such as MAD, MSE and RMSE. However, the output of these errors is rarely combined with inventory control parameters to represent the impact of the forecasting method on the performance of the inventory system. Related to combining demand forecasting performance and inventory control performance, is the earlier described statistical process control (SPC). An interesting future research direction would be to develop a method that can trigger changing the safety stock levels or reorder levels when experienced demand deviates from demand forecasts so greatly that the difference between experienced demand and forecasted demand reaches a certain threshold. When this threshold value is reached, safety stock or reorder levels have to be adjusted. In this way it can be decided when and how much the safety stocks, and possible other safety measures, have to be adjusted to guard against future demand uncertainties. The same holds for deviations in target service levels.

2.2 Assignment

The characteristics of the environment, the as-is situation at Company B and the described issues in inventory control, led us to define the following main assignment:

Develop a decision support tool that assists in minimizing total inventory cost in a single-echelon lost-sales system taking into account joint-replenishment under lead-time, order moment and demand uncertainty for a given target service level.

2.2.1 Underlying research objectives

In this section the underlying research objectives are described that will help in accomplishing the main assignment of our Master Thesis Project. Every underlying research objective is divided into a set of research questions or tasks to clarify what actions should be performed to complete the underlying research objective. The underlying research objectives form the structure of our project.

1. *Describe the as-is situation of inventory control at Company B.*
   (a) What are the they characteristics of the environment Company B is working in and what are the characteristics of Company B itself?
   (b) What are the processes that are carried out by Company B for inventory control in the case situation? (e.g. replenishment, inventory management)
   (c) How does Company B measure the performance of their inventory control and which KPIs are defined?
   (d) What are the problems that occur in the case situation of inventory control and in which areas is there a scope for improvement?
   (e) What requirements does Company B have considering improving their inventory control?

2. *Provide conceptual solutions aimed at the joint replenishment problem, complete lost-sales and the other problem areas by combining theoretical and practical knowledge and model these solutions into a mathematical model*
   (a) Which model can be used as input in the process of developing a solution for the overall joint replenishment problem and what adaptions or extensions have to be made?
   (b) Which model can be used as input in the process of developing a solution for the complete lost-sales component and what adaptions or extensions have to be made?
   (c) What are relevant parameters in the conceptual model and how should these parameters be measured?
   (d) How are the key components of the conceptual model connected to each other?
   (e) Develop a mathematical model where the conceptual solutions for the different components and parameters of the inventory control system are modeled.

3. *Analyze the inventory control model developed by Optiply and relate it to the conceptual solutions in the process of developing an inventory control model that can perform in an e-commerce characteristic environment.*
   (a) In what way can the existing inventory control model of Optiply be used as input for the development of decision support tool that takes into account the characteristics of an e-commerce environment?
   (b) What relevant (new) inventory control components can be adapted or developed to integrate into the inventory control model?

4. *How can the current inventory control model and the suggested improvements of the conceptual model be combined into a decision support tool that takes into account the components of an e-commerce characteristic environment?*
   (a) Combine the different components of the conceptual model and integrate the components into a decision support tool with different parameters that can be altered.
   (b) Define the relevant KPI of the model and the decision support tool.
   (c) Test the decision support tool and simulate different inventory control situations by measuring their KPI output under an service level or cost efficiency constraint.
   (d) Compare the output of the new inventory control situations with the as-is situation of Company B and test what settings of parameters achieve the best results.
   (e) Perform a sensitivity analysis to test the importance of the different parameters in the model and determine measures of uncertainty for the model.
   (f) Develop and generalize the decision support tool such that it can be utilized to analyze (demand) data from other companies within and outside of the customer network of Optiply.

5. *Write an implementation plan on the recommended use of the decision support tool for inventory control.*
   (a) Describe the features of the tool and how Optiply or customers in the network from Optiply can use these features.
   (b) What actions need to be performed by Optiply to implement the usage of the decision support tool?

2.3 Deliverables

The deliverables of this Master Thesis Project are based on the five underlying questions and have the objective of answering them for the case situation in chapter four. The description of the deliverables and our research setup can be found in appendix C.

2.4 Scope

In this section of the report, the scope of our research is described. The project will be conducted in a time period of approximately six months. Due to this time limit, the project should be converged accordingly. In this section of the scope and the level of detail of the project are described. The project is carried out within the Development team of Optiply, therefore Data

Science implementations such as external data are excluded from the scope of this research. Company B will be analyzed as case study company and the developed model and decision support tool that result from this analysis will thereafter be generalized into a tool that can be used by Optiply and potential other companies. Figure 2 represents the scope of the our research and is divided in the case study and the generalization. Moreover, Optiply and its customer network is displayed and Company B, as one of the customer companies, is shown with its customers and suppliers.

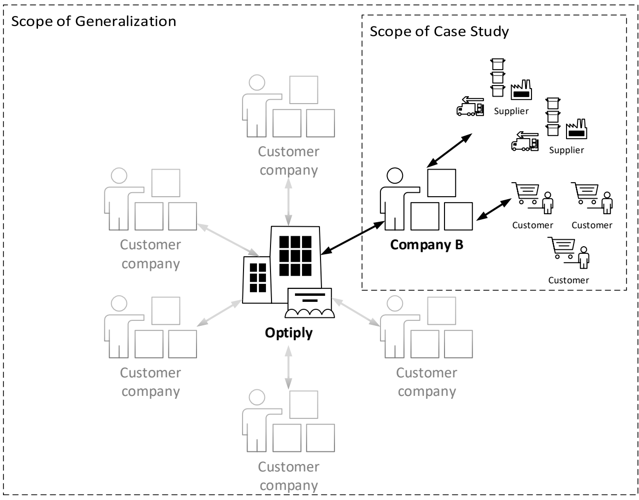


Figure 2: Scope of the Master Thesis Project

Within the replenishment process of company B and other companies in the customer network of Optiply, we see that the only incurred ordering cost are fixed ordering cost per replenishment order. No additional fixed ordering cost are incurred per product. Lead-times are assumed to be stochastic and vary per supplier. Lastly, backorders are out of scope in this project due the reason described in section 1.2 and section 1.3.

2.5 Methodologies

The methodologies that will be used in carrying out the project are the reflective cycle including the regulative cycle as by Heusinkveld & Reijers (2009) and the operations research model by Mitroff et al. (1974). More information on these methodologies and how they are utilized in our project can be found in appendix D.

2.6 Thesis outline

Chapter one of this report gave a brief introduction on the supervising company Optiply and the case study company. It elaborates on the environment the case study company is working in and defines the problem statement. In chapter two relevant literature from the performed literature study was combined with literature that was studied throughout our research. This studied scientific literature led to considerations on how to solve the inventory control problem in the case situation. The chapter also defined the main assignment of the project and its deliverables, scope and methodology. Chapter three describes the analysis of the case study company. The analysis is focused on the current inventory control in the case situation. Chapter four part one proposes different solution concepts for the inventory control problem described in chapter two and chapter three. The literature in this chapter functions as an important input for the solution concepts proposed in chapter four. Chapter four part two defines a new inventory model including two newly modified replenishment policies that address the components of the inventory control problem. In developing a mathematical model, expressions were derived for all relevant parameters of the inventory system under these replenishment policies including the fill rate calculation and cost calculations. The chapter concludes with defining the cost functions for both replenishment policies. In the case study of chapter five the derived expressions and functions of the mathematical model are verified by simulating multiple scenarios. These simulations also show the performance of the newly modified replenishment policies with data from the case situation. Chapter six describes the functions and components of the developed decision support tool in the form of an implementation plan. In the last chapter, which is chapter seven, our research is concluded and recommendations are given to the supervising company and for potential future research.

Chapter Three

Analysis of the case situation

In this chapter the case situation of Company B's inventory control will be described in more detail. In section 3.1 the supply chain of Company B is described. Section 3.2 elaborates the demand parameter of the inventory control system. In Section 3.3 the current replenishment process is analyzed and describes the problems that occur during this process in the current situation. Section 3.4 describes the current KPIs utilized by Company B to measure the performance of their inventory control system. Section 3.5 defines the requirements of the case study company considering a new automated inventory control system. Section 3.6 concludes on the challenges the case study company faces in their inventory control. The notations and expressions in this chapter are based on the definitions presented in appendix E.1.

3.1 Supply chain

Company B is a retailer that sells its products through the internet (e-tailer). Within the supply chain this makes Company B the chain-link between the customers that actually buy and use the products downstream and the distributors and suppliers upstream in the supply chain.

3.1.1 Sales channels and suppliers

Products are sold with 3 different sales systems via approximately 400 webshops (website domains). Company B can have a website online in 30 minutes and can get it offline whenever they see fit, which results in a large flexibility in sales channels where Company B can offer their products.

The company has multiple suppliers which are based in China, The Netherlands and Germany. Orders for replenishment are placed to suppliers once a week on average. Figure 3 represents the supply chain of Company B in a simplified and structured way. However, a large part of the supply chain is not structured and replenishment is not performed following a set of decision rules.

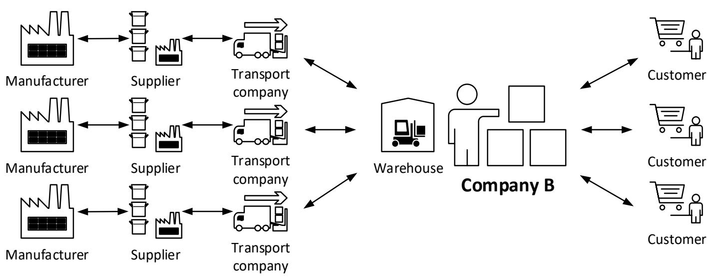


Figure 3: A structured representation of the Company B's supply chain

3.1.2 Assortment

Company B has an extensive and broad assortment of products. In e-commerce, having a large range of different products in the assortment is feasible because the products do not have to be placed in physical stores where there are storage and shelf restrictions. Instead, products can be put on a website of a webshop and the product itself can be put in storage in a warehouse. Company B manages approximately 3000 products that vary in characteristics such as sales price, cost price, supplier origin and demand volume. In the case situation approximately 1600 of these products were active in the case situation. Because of this large assortment many of their products exist in the *long tail*, which implicates that such products have a non-voluminous and often non-stable demand. These long tail products typically make up for 80% of the total assortment in product quantity but only add 20% to the total revenue from product sales. Products in the long tail often have long periods of zero demand interspersed with some periods of non-zero demand. Demand patterns such as intermittent demand and lumpy demand are more difficult to forecast and result in poor estimations and higher forecasting errors. In the case situation this does not fully hold because one fastmoving product takes up almost 80% of the total revenue as we can see in figure 4. On the left we see a graph inclusive the fastmover and on the right a graph is shown without the fastmover. Although companies focus the most attentions to the 20% of the products that make up for 80% of the total revenue, our research will focus on all products. If the fastmover is not taken into account, there are still 500 products that would be needed to manage to make up for 80% of the total revenue without the fastmover. Therefore, these products are still relevant.

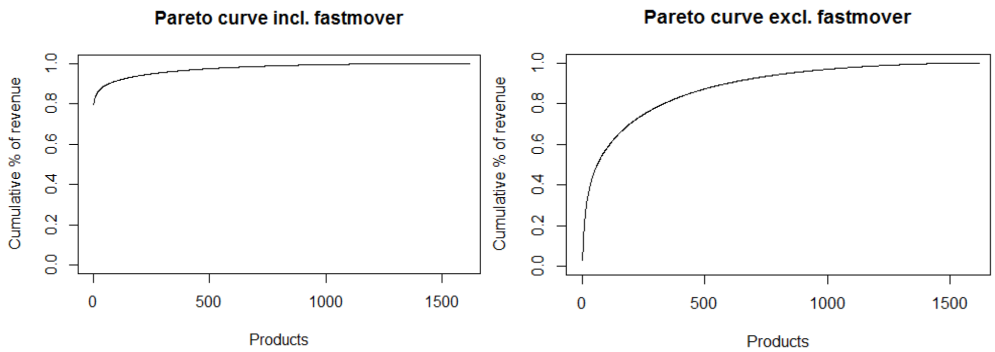


Figure 4: Pareto curves of percentage of total revenue in case situation

Because of high forecasting errors, safety stocks and other inventory levels deviate from the level that is required in operations. Inventory control becomes less cost efficient because inventory levels are higher or lower than they have to be and orders are placed to suppliers to frequently. In the case of shortage occurrences, service levels such as the fill rate or the ready rate may become lower because stock is insufficient.

Fill rate. The fill rate is a trivial expression for the P_2 service level, which is the long run fraction of total demand which is being delivered from stock on hand immediately. This specific service is also often referred to as the customer service level.

Ready rate. The ready rate is a trivial expression for P_3 service level, which is the probability of no stock-out in a replenishment cycle.

Offering a broad assortment of products often results in holding more stock. If the stock is large and contains products that are not sold often, this may result in excess stock. In the case situation excess stock amounts a little more than 16,000 euros. This excess stock includes products that will probably not sell for the original selling price or worse, never sell and have to be depreciated.

3.2 Demand

The demand for the approximately 3000 products is different for each product. A part of Company B's assortment contains fast moving products that are sold every day and a part contains products that are sold only once or twice per year. To give an example, in the past year one of the products was sold more than 42,000 times while other products exist that only sold once in the same year. In between these two extremes there are products that range from selling a few times per year to a couple hundreds or thousands per year. These products or product groups follow different demand patterns which may cause difficulties in making decisions with respect to forecasting and replenishment. Different decision rules may be followed for products with different demand characteristics. For products that are demanded very rarely the decision could be made to hold zero to very low stock and replenish only when demand is expected in the coming period. A graphical representation of the difference in sales between different products is shown in figure 5.

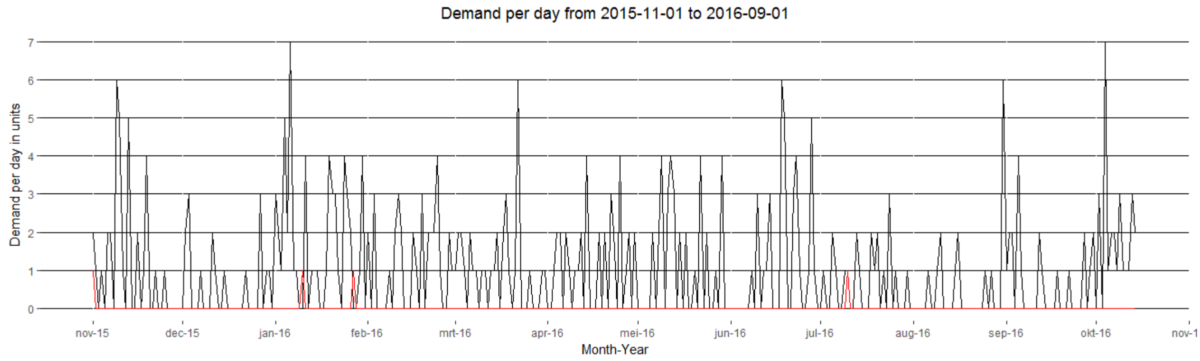


Figure 5: Sales per day of product "269452" and product "1272290" from 2015-09-01 to 2016-09-01

The red line in figure 5 represents the sales per day of a steam cleaning product. The sales pattern may be described as intermittent (i.e. long periods of zero sales interspersed with positive sales). The black line represents the sales per day of a skincare product. The sales pattern of this product is rather variable and more voluminous than that of the product represented by the red line. Online customers are very sensitive for pricing of products and low prices at the competition. For example, a weekly discount at a competitor webshop can result in downward peaks in the historical demand that may cannot be explained at a later moment in time. These peaks in demand are hard to explain and therefore difficult to forecast

3.2.1 Complete lost-sales vs. complete backordering

Complete lost-sales are common in an e-commerce environment. However, it results in the problem of having intractable or unobserved demand, meaning that there is no information available about the demand for that specific product in the period that it was out of stock and offline. This information could be essential in forecasting demand of the product for future time periods and assuring that demand can be filled from stock. The information on how many lost-sales occurred in this period without stock is probably more important than the backordering of products itself.

3.2.1.1 Sales data

The available data for analysis is sales data. In a situation where excess demand is backordered completely, the number of sales is approximately equal to the demand and a part of this demand may be backordered.

In the case situation backorders are not accepted and we have a lost-sales system (K. Van Donselaar et al., 1996). The paper states that a target service level should be set in each period in a lost-sales system. Because excess demand is not backordered, demand is equal to a sale if and only if the stock on hand positive. Moreover, there is no information available about demand in situations without stock on hand because products are often removed from the website of the webshop and a customer cannot place an order for that product. It could be possible to have experienced demand, and therefore sales, in the period without stock on hand if there had been stock on hand to satisfy the demand. Therefore, the sales of product i can be expressed as:

$$W_i(t) = \begin{cases} D_i(t) & if\ X_i(t) > 0 \quad for\ t = 1,2,\ldots \\ 0 & if X_i(t) = 0 \quad for\ t = 1,2,\ldots \end{cases} \tag{3.1}$$

An intuitive assumption would be that in a period without stock on hand, the demand for that period is the same as the demand in periods with positive stock on hand or follows the same probability distribution. This assumption is further elaborated in chapter four.

3.2.1.2 Demand through time

Demand for some products follows a trend, which means that demand may be increasing or decreasing over time. Moreover, for some product demand follows a seasonal trend, meaning that demand may increase or decrease in the same time periods and with the same seasonal pattern every year, every month or even every day of the week. Demand shows high peaks and lows over time. Some demand is highly variable when we look at demand on a daily basis. An example of this can be seen in figure 6, where the line represents the daily sales of a food supplement product. If we analyze the first two moments of the demand in periods with positive stock we see that a large part of the assortment experiences demand with a high *coefficient of variation* (CV). The CV of demand is defined as:

$$\frac{\sigma_D}{\mu_D} \tag{3.2}$$

Only two products have a CV below 1. Approximately 40 products have a CV between 1 and 2 and the rest of the approximately 1600 products have a CV ranging from 2 to 17. Therefore, the demand for products in the assortment can be seen as rather (highly) variable.

3.2.1.3 Demand distribution

Demand is stochastic and not fixed. The exact demand per period is not known and expected future demand needs to be forecasted. Furthermore, demand is not stationary. The average demand in period $t + 1$ may be higher or lower than the average demand in period t. Moreover, the variance of the demand in period may be less or more than in period t. Figure 6 shows the variable daily demand pattern of a food supplement product.

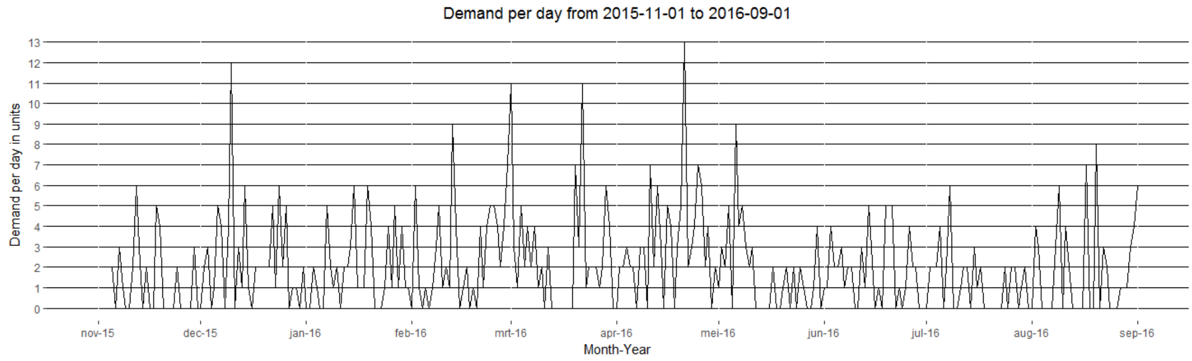


Figure 6: Sales per day of product "270498" from 2015-11-01 to 2016-09-01

Due to different demand patterns, of which some may be highly variable or intermittent, controlling processes such as determining the safety stock and replenishment may prove to be more troublesome than in situations where demand is voluminous and rather stable. Calculating safety stocks to guard for the variability of demand in the coming period, is not performed by Company B. The number of products held in inventory is based on common sense and often products are ordered if the stock on hand equals zero or when it is expected that a product will sell in the comping period because it sold reasonably in past periods. This setup can be enhanced by setting the right set of parameters at the appropriate moment in time.

3.2.1.4 New product introductions

It is common that Company B introduces new products into the webshop assortment throughout the year. As described before, the assortment is large and new products are added to this assortment throughout the year. New products lack direct information about historical demand, stock and service levels. To forecast such parameters certain assumptions are required. Assumptions including correlations of the newly introduced product with current available products in the assortment and the underlying demand distribution of the newly introduced product. Other relevant input information could be external data such as Google positions and Google test advertisements.

3.3 The replenishment process

3.3.1 Ordering

The current replenishment process does not include fixed ordering moments. However, it is preferred by the company to review the inventory of the products approximately once a week and orders are placed to the supplier if deemed necessary. Replenishment is performed considering individual products; there is not much attention in checking which product comes from which supplier. Situations occur that only fastmoving products or products that have a net stock of zero are ordered for replenishment. This is due to the fact that orders are placed on common sense and not of fixed decision rules such as: 'order if the inventory level is below a certain point'. Net stock in this situation is equivalent to the stock on hand because backorders are not accepted. Note that not all products are individually reviewed for their stock on hand and ordered if needed (not specified by a certain decision rule). The ordering process and the number of products ordered is based on product sales from the past periods instead. The ordered amount is based on what the purchaser deems to be reasonable to order. An order-up-to level S or an economic order quantity Q^* does not exist in the ordering process. The order quantity is not based on historical demand data, current review period and the lead-time of a products. It is based on gut feel taking into account the past sales of the product. Often it occurs that the stock on hand of such a product is zero (sometimes it is zero for a longer time period). This ordering process is performed by hand which means that nothing is automated by a system that is based on a replenishment policy with decision rules. No attention is paid to characteristics of products such as demand volume, holding costs, shortage costs and the average order quantity. Orders should be interconnected and decisions have to be made how many products should be ordered when and to which supplier.

3.3.2 Deliveries

Products that are ordered at a certain moment in time are delivered in one batch at the same moment. For example, an order for 20 products is not delivered in two sets of 10 products on two subsequent days but rather in one batch on the same day. There is an uncertainty present in the number of products actually delivered. It may be possible that a supplier does not deliver the fully ordered number of products due to miscommunication, machine failures or delivery problems. Note that this 'yield' is not in the scope of our project. For some suppliers the restriction holds that the transportation and delivery of products is bound to fixed moments in time. Suppliers in China send their orders via container shipping or air mail. Especially container shipping takes time and is bound to fixed shipments. In the replenishment process, this sort of transportation restrictions should be taken into account.

Every supplier has a certain lead-time for production, procurement or delivery. This lead-time varies per supplier and is available to us through historical ordering data. Lead-times are non-deterministic and may vary due to uncertainties such as availability of products and transport. The lead-time is not dependent on the ordered number of products if the number is small. When large numbers of products need to be ordered, the lead-time may increase because a full pallet of products has to be send in a different way than just one box of products. The lead-time parameters of 16 relevant suppliers are shown in table 2. As can be seen in the table, the mean lead-time of the suppliers varies from 1 to 17 days and some suppliers have a rather low delivery reliability and result a higher standard deviation and CV. Supplier lead-times with a CV higher than 1,0 are displayed in bold. We performed Kolmogorov-Smirnov tests on a sample of the lead-times and most of the lead-times were not found to come from a Gamma distribution nor from a Normal distribution. However, in our research we will assume that lead-times are Gamma distributed which is described in chapter four. Note that the lead-time standard deviations are sometimes assumed to be 0,25 times the mean lead-time due to lack of data.

Table 2: Mean and standard deviation of supplier's lead-times

	Supplier	Mean lead-time (days)	Std. dev. lead-time (days)	CV lead-time (σ/μ)
1	858	4,00	10,00	2,50
2	862	6,33	11,55	1,82
3	863	17,00	5,66	0,33
4	865	7,00	1,75	0,25
5	868	4,00	1,00	0,25
6	869	3,00	0,75	0,25
7	872	1,67	1,15	0,69
8	873	4,00	1,00	0,25
9	874	6,50	0,71	0,11
10	879	7,00	1,75	0,25
11	880	2,50	2,12	0,85
12	881	2,66	1,15	0,43
13	882	1,00	0,25	0,25
14	884	1,75	0,50	0,29
15	889	1,00	0,25	0,25
16	897	10,33	10,00	0,97

3.4 Current key performance indicators

Considering inventory control, the only parameter that was measured and monitored by Company B was the total stock value of the products in storage. The cost price and the amount of products in stock on hand were known and in that way the stock value could be calculated. If we consider KPIs outside the context of pure inventory control, three other KPIs arise that were measured by Company B: (1) sales of products: the number of products that were sold in a certain time period, (2) revenue: the amount of money that was earned by selling products during normal operations and (3) profit: the surplus (or loss) in a certain period that is equivalent to the revenue deducted by total costs and after taxes. These KPIs were not further used in their inventory control system. Only sales KPI was in a way used to decide on how much to order a product: if the number of sales of a product were deemed good, the product was weekly reviewed and an order was placed to the supplier such that the product's inventory level was deemed enough for the coming period

3.5 Conclusion

Summarizing chapter three, where the relevant components of the current inventory control system were analyzed, the case study company faces the following inventory control challenges:

- Stochastic demand. In some situations demand is highly variable and unpredictable, making it even more difficult to forecast and to control for.
- Stochastic lead-times. Lead-times vary per supplier and are not fixed.
- Review periods are not fixed and may be dependent on exogenous factors such as delivery moments and day of the week (weekends). Review periods for all products ordered to the same supplier are not coordinated.
- Excess demand is lost due to the fact that backorders are not accepted. Due to these lost-sales demand in periods without stock on hand is unobserved.
- Abundant different products are ordered from a relative small set of suppliers. Due to ordering cost of placing an order to a supplier, the ordering process should be coordinated.
- Lack of an automated inventory control system that suggest which product to order at which moment.

Chapter Four

Model development

This chapter defines the model that incorporates the different components of our inventory control problem. The main issue is:

Determining appropriate inventory levels for many different products which are ordered to a small set of suppliers, taking into account unobserved stochastic demand and stochastic lead-times in a lost-sales system.

This main issue can be split into four problem components:

1. *Taking into account lost-sales (i.e. unobserved demand)*
2. *Which replenishment model(s) to use for the n replenishment problems*
3. *Coordinating replenishments by joint replenishment*
4. *Service level and s-level setting*

Part one of this chapter elaborates what the possibilities are for the different components of the inventory control problem and describes the decision process of choosing appropriate solutions concepts. In part two of this chapter we describe the process of combining the chosen solution concepts into two newly modified replenishment policies. For these two replenishment policies all derivations of relevant parameters and cost functions will be defined and formed into a mathematical model. The notation in this chapter is largely based on the notation from de Kok et al. (2012) and can be found in appendix E.1.

Part one – Conceptual solutions

Section 4.1 on lost-sales describes different solution approaches to the lost-sales problem. Relevant related information can be found in appendix B.1 on the lost-sales problem. Section 4.2 and 4.3 on replenishment policies and joint replenishment describe the considerations for appropriate replenishment policies for the case situation. Relevant related information can be found appendix B.2 on the joint replenishment problem. Section 4.4 elaborates on service levels and s-level setting in an inventory system were stochastic demand is forecasted or simulated. Section 4.6 concludes part one of chapter 4.

4.1 Lost-sales problem

The characteristics of a lost-sales system are described in appendix B.1 of this report. Different approaches for the lost-sales problem are briefly discussed in this section. More information on these approaches and our considerations related to these approaches are described in appendix G.1. The different approaches are the following and the chosen option is displayed bold:

1. *Assume that sales equal demand and do not take into account demand in period without stock on hand.*
2. *Correct the fill rate due to the fact that backorder system equations and decision rules are used in a lost-sales system.*
3. *Uncensoring of unobserved demand in the periods without stock such that the model can be based on demand data instead of sales data.*
4. ***Taking into account the unobserved demand by using a method that makes an assumption about the demand in periods without stock on hand.***

Our preference is focused on uncensoring or untruncating the unobserved demand in periods where stock-outs occurred. The main reason for this preference is because we would like to take into account the demand in periods without stock. This demand should be satisfied in the best way possible. Only basing the inventory system on sales and correcting the fill rate such is not the solution concept that will be used in this project. However, the process of uncensoring unobserved demand data is rather complex and researched iterative algorithms result in *NP-hard* problems if the number of products becomes large. Furthermore, the available customer preferences data is not sufficient to say something relevant about product substitution.

Therefore, we decided to work with option 3 and make the assumption in our project that the demand in periods without stock can be estimated by using the sales in the periods close to the stock-out period without taking into account substitution effects. By making this assumption, we assume that the demand in periods without stock resembles the same characteristics as the demand in periods with positive stock on hand (i.e. sales). The approach to impute sales data in the periods without stock based on sales in the periods with positive stock on hand is elaborated on in the section 4.6 of this chapter. More background information on the method can be found in appendix G. A simplified version of the code of the method can be found in appendix I.

Using the impute demand method in attempting to estimate the demand in a time period instead of using sales only, makes for a more realistic demand parameter that can be utilized in setting parameters including the reorder level, the order quantity Q the order-up-to level S and the review period R. Additionally, it aims at setting more realistic P_2 service levels per product or product group and should result in more genuine holding, ordering and shortage cost. Note that by taking into account this demand in periods without stock, no correction is needed on the fill rate to take into account the fact that we operate in a lost-sales system. In chapter 5 we elaborate on a method that resembles the same steps as this method but where the demand in periods without stock is imputed based on the same distribution (Gamma distribution) as the demand in periods with positive stock on hand.

4.2 Replenishment policy

The decision on which replenishment policy to use and to incorporate joint replenishment are closely related. Deciding on the nature of reviewing is dependent on the environment in which the replenishment policy is to be implemented. Additionally, may be restrictions and requirements that influence the decision between certain replenishment policies. The different possibilities for a replenishment policy are based on two main components and the chosen option is displayed bold:

1. ***Order-up-to policy*** *or Fixed order quantity policy*
2. *Continuous policy or* ***Periodic policy***

Fixed order quantities Q are not often seen in the e-commerce. Replenishment policies with a fixed order quantity Q such as the (s, Q) and (R, s, Q) replenishment polices are out of scope in our research. This is due to the fact that the *WMS* of the case study company does not allow for fixed order quantities but rather sets a minimum and maximum for every product in the assortment. Therefore, we will focus our research on order-up-to policies only.

If products are ordered to the same supplier or shipped in the same transportation mode, coordination of replenishments may be appealing. In such cases periodic review is particularly appealing because all items in the group can be given the same ordering interval. Because of the periodic property of the (R, S) and (R, s, S) replenishment policies, they are more preferred to continuous systems such as the (s, S) policy in terms of coordinating the replenishments of related products. In addition, the periodic policies offers a regular opportunity (every R units of time) to adjust the s-level(s), which is a wanted property if the demand pattern is changing over time.

4.3 Joint replenishment

The joint replenishment problem results from the fact that many products have to be ordered to a handful of suppliers and that with every order fixed ordering cost are incurred. Furthermore, every supplier has its own lead-time with a certain lead-time uncertainty. To coordinate the replenishment of every product in the assortment in a cost efficient manner we have the following options and the chosen option is displayed bold:

1. *Replenishment on a company level; the same review period is set for all products in the assortment and economic order quantities per product are calculated accordingly.*
2. ***Replenishment on a supplier level; the same review period is set for all products ordered to the same supplier and economic order quantities are calculated per product based on this review period.***
3. *Replenishment on the product level; a review period is calculated for every product individually and economic order quantities are calculated accordingly.*

Products are ordered to different suppliers and fixed ordering cost are incurred every time an order is placed. The products have different demand parameters and differ in holding cost. Because we do not want to order for every product individually an want to replenish cost efficient and, the third option is eliminated from our spectrum. Another reason for not using the first option is that in all reviewed JRP literature, the replenishment problem is based on a major and minor fixed ordering cost. The minor fixed ordering cost are often used to determine the optimal review period for every individual product. One of the most relevant differences with the JRP literature is that in the case situation no additional costs are made when adding one or more products to the order that is placed to the supplier (i.e. no minor fixed ordering cost). The first option is also eliminated because we want to be more flexible in setting the review period per supplier. In setting a companywide period, the different characteristics of the supplier and its products are not taken into account. We therefore will determine a method to find the appropriate review period on a supplier level. The parameter setting of the different products ordered to the same supplier can be seen as n single-item replenishment problems that each follow an (R, S) or (R, s, s) replenishment policy (Federgruen, Groenevelt, & Tijms, 1984). In our research for finding an appropriate joint replenishment method we found several differences with our problem and what was described in the JRP literature. The most important differences between the case situation and the paper of Atkins and Iyogun (1988) and other relevant literature on joint-replenishment are shown in table 3.

Table 3: Differences between SJRP in the case situation and the SJRP literature

	Atkins & Iyogun (1988) and other relevant JRP literature	Case situation
Demand	In the JRP literature demand is often assumed to be (compound) Poisson. Atkins and Iyogun assume that demand follows a Poisson distribution.	In the case situation demand follows a compound renewal process with Gamma inter-arrivals and Gamma order sizes (as we will elaborate later on in this report).
Inventory system	Backorder system: excess demand is completely backordered.	Lost-sales system: excess demand is completely lost.
Lead-times	Lead-times are assumed to be deterministic or Normally distributed and often an integer multiple integer of the review period.	Lead-times are stochastic and follow a Gamma distribution (as we will elaborate later on in this report).
Review periods	The review periods R are assumed to be deterministic and fixed.	Review periods are in some situations bounded by exogenous factors (e.g. delivery moments, weekends without deliveries). Review periods may depend on the demand of products coming from the same supplier. More information on this topic is given in section 4.6.
Ordering cost	The ordering cost include a fixed major and minor cost part. This minor ordering cost is product specific but is fixed in the context of that it does not matter how many products of that specific product are ordered; the minor ordering cost remains the same.	In the case situation this minor ordering cost does not exist. No additional ordering cost are made when ordering more products to the same supplier. Furthermore, the extra time of placing an order of $2,3, \dots, n$ and putting the order in the warehouse is negligible.
s-levels	Order-up-to level M_i	Order-up-to-level S_i and if suitable for a certain supplier or product a reorder level s_i.

More information on the decision process of choosing which replenishment policy to use for the individual replenishment problems and how to incorporate joint replenishment can be found in appendix B.2.

A method that is currently used by Optiply is elaborated on in appendix G. The method calculates the total *economic order quantity* (EOQ) and review period on a supplier level and then translates this back to an EOQ on product level for the products that are ordered to the same supplier. The problem with this method is that it calculates the EOQ on a supplier level by simply adding all the demand and costs from different products that are ordered to the same supplier. Note that the EOQ calculation uses costs and average demand from an individual product as input; simply summing up these parameters is meaningless. A new method to determine the review period per supplier is elaborated on in section 4.6 of this chapter.

4.4 Service level and s-level setting

In our research, the expressions for certain inventory control parameters such as the demand during lead-time or during the review period are derived based on the assumption that demand is stationary. In practice however, demand is often non-stationary and future demand is based on forecasts. Strijbosch et al. (1997) conclude that even in the simplest possible setting that the standard procedures from the literature do not guarantee the desired service levels. In a situation with non-stationary demand this may only exacerbate (Strijbosch, Moors, & de Kok, 1997). This section briefly elaborates on service level setting under both stationary and non-stationary demand and the difference between the P_2 and P_3 service level.

4.4.1 Service levels

The P_2 service level or fill rate can be described as the long-run fraction of total demand which is being delivered from stock on hand immediately. The fill rate is most commonly used as the target service level an inventory system. Based on this target service level the inventory levels such as the reorder level s or order-up-to level S are set. The fill rate requires the knowledge of the demand in a replenishment cycle as we will see in section 4.6 where we derive the expressions for the fill rate. The P_3 service level, also denoted as the ready rate, is defined as the fraction of time during which the system has positive net stock, which is the same as the probability of no stock-out at the end of an arbitrary period. Net stock in our situation, is equal to the stock on hand. The ready rate as by Køhler-Gudum and de Kok (2002) can be expressed as:

$$P_3 = P\{X(t) \geq 0\} = 1 - \frac{N_B}{N_P} \tag{4.1}$$

where N_B is the number of periods with additional backordered demand and N_P is the total number of time periods considered. Therefore, the ready rate represents a time dimension of demand satisfied without backorders (or in our case, no lost-sales). In case of continuous Poisson distributed demand the P_2 and P_3 service levels are equal. Hence, for $CV_D = 1$, it holds that $P_2 = P_3$. This can also be shown by deriving the functions for the P_3 service level under the (R, S) and (R, s, S) replenishment policy as we will show in section 5.4 where we relate the P_3 service level to the P_2 service level.

In practice, having a CV_D equal to 1 is often not the case and customer demands vary in size. If the stock on hand contains a small number of units on hand most of the time, the ready rate can be high. Still, the fill rate may be low if there are some large customer demand sizes (i.e. demand sizes are (highly) variable). For Normally distributed demand the fill rate is also equivalent to the ready rate according to Axsäter (2006). It is easy to reason that the more variable daily demand, the larger the difference $P_2 - P_3$ becomes. We will verify this by simulation in section 5.4. as well.

Let us define:

$b = \text{shortage cost per unit per unit time}$
$p = \text{shortage cost per unit}$

A shortage cost of type b, for example, is relevant for a spare part when a shortage implies that a machine has to stop working until the spare part is available again. The costs are proportional to the customer waiting time. The shortage cost of type p can be interpreted as a cost per average number of backorders. It is very important to mention here that the shortage cost of type b can only be used in a backorder system because the time that the backorders are present needs to be tracked to calculate the shortage cost per replenishment cycle or per unit time. Backorders at time t are equal to the net stock when it is below zero (i.e. $-X(t)$). In a lost-sales system this is not possible because the stock on hand is at least zero. The whole idea of a customer 'waiting' for his backordered product does not exist.

In the optimal solution, the optimal reorder level s can be characterized as the largest reorder level providing a ready rate not higher than $\frac{b_i}{b_i+h_i}$. This fractile is also called the Newsvendor fractile. An advantage of the newsvendor fractile is that it does not need information on the demand in a replenishment cycle like with the other service levels. Note that the shortage cost b_i is a shortage cost per unit per time unit just like the holding cost h_i is a holding cost per unit per time unit. Assuming compound Poisson demand and if s_i^* is the optimal reorder level for product i, we have:

$$P_3(s_i^*) \leq \frac{b_i}{b_i + h_i} < P_3(s_i^* + 1) \tag{4.2}$$

This relation also holds for the fill rate if demand is purely Poisson. For Normally distributed demand, it is shown that the optimal solution entails:

$$P_2 = P_3 = \frac{b_i}{b_i + h_i} \tag{4.3}$$

Given a certain shortage cost b_i for product i, we can determine from (4.3) the service level providing exactly the same reorder level:

$$b_i = \frac{h_i P_2}{1 - P_2} = \frac{h_i P_3}{1 - P_3} \tag{4.4}$$

This also works the other way around, i.e., given a target service level, we can determine the implicit shortage cost. However, as described before, the shortage cost of type b can only be used in a backorder system because the time that the backorders are present needs to be tracked. The number of backorders is equal to the negative stock on hand. In a lost-sales system this is not possible because the stock on hand is at least zero. In our situation, we have shortage cost of type p, which are incurred over the number of shortages per replenishment cycle. For this kind of shortage cost, the scientific literature focusses especially on solving for the P_1 service level instead of the P_2 or P_3 service level (Axsäter, 2006; Silver, Pyke, & Peterson, 1998).

De Kok (1991) also suggest another form of the P_3 service level in one of his 6 research reports from 1991 on the basics of inventory management. He describes the P_3 service level as the long-run average shortage at an arbitrary moment in time. This is equivalent to the average backorders during a replenishment cycle. In the compound renewal demand case under an (R, S) replenishment policy, the P_3 based on a certain order-up-to-level S can be expressed as:

$$P_3(S) = E[X] - \left(S - \left(E[L] + \frac{R}{2}\right)\frac{E[D]}{E[A]}\right) \tag{4.5}$$

In the inventory system under an (R, s, S) or (R, S) replenishment policy and a target service level constraint, one particularly considers linear holding cost and fixed ordering cost. The holding cost are derived from the average stock on hand and the ordering cost depend on the review period R. Shortage cost are incurred implicitly by achieving a target service level. Because shortage cost are typically hard to obtain, often a service level approach is utilized. However, once the inventory cost associated with the replenishment policy under a target service level are known, the expressions for the P_2 and P_3 service levels can be used to obtain the implicit shortage cost that is assumed. This can be done by taking the shortage cost per unit or per unit time as a variable and determine the value of this variable for which the replenishment policy is cost-optimal (de Kok, 1991).

If we set a certain target fill rate and set the reorder level s or order-up-to level S accordingly, we can use expression (4.5) to calculate the expected shortage per replenishment cycle and the related shortage cost per replenishment cycle. Taking both cost into account, we should then be able determine the cost-efficient target fill rate. We will perform a verification analysis on expression (4.5) in section 5.4.

4.4.2 s-levels

If we want to calculate a dynamic reorder level or order-up-to level under forecasted demand, de Kok (1991) rewrites the random variables $D(0, R]$ and $D(0, L]$ as follows:

$$D(0, R] = D^F(0, R] + \varepsilon(0, R] \tag{4.6}$$

$$D(0, L] = D^F(0, L] + \varepsilon(0, L] \tag{4.7}$$

Here $D^F(0, R]$ and $D^F(0, L]$ are forecasts and therefore known constants. The forecast error or deviation from the forecast is given by $\varepsilon(0, R]$ and $\varepsilon(0, L]$, which are random variables. Often, the forecast error is assumed to be Normally distributed. A more robust approach is to assume that (4.3) and (4.4) are Gamma distributed. Then, the method to determine the appropriate s-level is the following:

1. Determine $D^F(0, R]$ and $D^F(0, L]$.
2. Determine $\sigma(\varepsilon(0, R])$ and $\sigma(\varepsilon(0, L])$.
3. Calculate the reorder level s and/or order-up-to level S using the PDF-method, assuming the Gamma distribution.

The expected average stock on hand during a replenishment cycle is then given by (de Kok, 1991):

$$E[X] \cong S - D^F(0, L] - \frac{D^F(0, R]}{2} \tag{4.8}$$

Silver et al. (1998) and Axsäter (2006) suggest multiple methods to incorporate the forecast error in the process of setting appropriate safety stocks. One of these methods is described in appendix B.3.1. The cost effect of using estimated standard deviation instead of the true forecast error value and suggestions for making adjustments to the s-levels is research by Strijbosch, Moors and de Kok (1997).

Strijbosch et al. (1997) suggest a heuristic to improve the setting of the order-up-to level under an (R, S) replenishment policy if demand is forecasted and under a P_1 service level. They state the standard procedure of setting the reorder level s (i.e. quantile of the distribution of demand during lead-time or review period plus lead-time)does not guarantee stock-out probabilities smaller than $1 - P_1$ and that they can deviate substantially. The procedure of the heuristic involves correcting the order-up-to level S under the assumption of Normally distributed demand and a Simple Exponential Smoothing (SES) forecast method (Strijbosch et al., 1997). The relevant expressions can be found in appendix B.3.2.

Køhler-Gudum and de Kok (2002) propose a technique, called the safety stock adjustment procedure (SSAP), which enables the determination of safety stocks to ensure target service levels in simulation studies of inventory systems where demand is forecasted (i.e. non-stationary). The procedure can be used for the P_1, P_2 and P_3 service level. An essential constraint in the usage of the technique is that excess demand is backordered. If not, the important property of the safety stock being independent cannot be maintained. A more extensive description of this work can be found in appendix B.3.3.

We can conclude that reorder levels and order-up-to levels need to be updated throughout the replenishment process due to non-stationary demand. Note that this is even required if demand is stationary, as Strijbosch et al. (1997) showed. Demand forecasts may be run daily and s-levels should be updated in-between review periods such that the replenishment policy becomes a cyclical policy where every replenishment cycle performs under a target service level constraint.

4.5 Conclusion

We found that the standard (R, S) and (R, s, S) replenishment policies can be used in solving the n single-item replenishment problems in the aggregate joint replenishment problem. Within these single-item replenishment problems, excess demand is completely lost, making the inventory system a lost-sales system. Therefore, the sales data needs to be corrected such that lost-sales in periods without stock on hand are taken into account to the best of our abilities. Updating of s-levels based on demand forecasts may be performed in-between review periods. However, updating the s-levels too frequent, could lead to the fact that the inventory system reacts to 'noise' too much.

Part two – The modified replenishment policies

This part focusses on the mathematical model concerning the newly modified replenishment policies and the derivations of the expressions for the relevant parameters of the policies. Section 4.6 introduces the (R^{δ}, S_i) replenishment policy and the (R^{δ}, s_i, S_i) replenishment policy that take into account stochastic demand sizes D_i and stochastic demand inter-arrivals A_i for every product i, joint replenishment, complete lost-sales, stochastic lead-times L and supplier based review periods R^{δ}.

4.6 The $(\boldsymbol{R^{\delta}}, \boldsymbol{S_i})$ and $(\boldsymbol{R^{\delta}}, \boldsymbol{s_i}, \boldsymbol{S_i})$ replenishment policy

In this section we introduce our model with two modified replenishment policies. The subscript i relates to the fact that for every product i a reorder level s_i and order-up-to level S_i is determined. The R^{δ} represents the aggregated review period of all products i that are ordered to the same supplier. We start with a brief introduction to the two replenishment policies and the assumptions that hold in the analysis of both policies. In section 4.6.1 the relevant parameters of the replenishment policies are introduced including the demand during lead-time, demand during the review period, the undershoot, the expected average stock on hand and the service levels. This section also describes the cost functions of the relevant costs of the inventory system under each of the replenishment policies. All derivations of the expressions and formulas can be found in appendix J and appendix K. The notation used in the derivations of the expressions of the relevant parameters of both replenishment policies are for a large part based on the notation of de Kok et al. (2012). The notation can be found in appendix E.1 of this report.

The following assumptions were made in deriving the expressions of the relevant parameters of both replenishment policies:

- (i) Demand is stochastic but stationary. Demand over time intervals of fixed length does not depend on time itself. Although this assumption may appear somewhat unrealistic, note that the decision rules can be adapted (i.e. they can be updated through time to learn from more recent historical demand data). Demand is continuously distributed with demand inter-arrivals and demand order sizes. Therefore, the demand for a product follows a compound renewal process.
- (ii) Replenishment orders do not cross in time (i.e. an order placed at a later moment in time cannot arrive earlier).
- (iii) Lead-times are stochastic and independent .
- (iv) Time between review moments (i.e. the review period) is independent. Review periods are based on the review period determination method described in section 4.6.1.2.
- (v) All excess demand is lost (i.e. excess demand is not backordered but results in a lost sale).
- (vi) The entire ordered quantity of a replenishment order is delivered at the same time (i.e. there is no variance in the delivery time within the order).
- (vii) When placing an order, only major fixed ordering cost are incurred; there are no minor fixed ordering cost based on the number of products ordered.
- (viii) The unit variable cost of any of the products does not depend on the quantity; there are no discounts in either the unit purchase cost or the unit transportation cost.

4.6.1 Mathematical model

4.6.1.1 Lost-sales component (more details in appendix G.1.3)

Let us define:

D_i^d = daily demand of product i (i.e. NOT demand order size as in continuous demand)
j = number of days back in the historical data
$X_i(t-j)$ = stock on hand from every historical date that is used in the calculation
$I_{x_i(t-j)}$ = an indicator function that indicates if the stock on hand of product i was positive on day j;

$$\left(I_{x_i(t-j)} \coloneqq \begin{cases} 1 & if\ X_i(t-j) > 0 \\ 0 & if\ X_i(t-j) = 0 \end{cases} \right)$$

N = total number of days that is looked back in the historical data

Imputed demand on days without stock can be expressed as:

$$D_i^d(t) = \frac{\sum_{j=1}^{N} D_i^d(t-j)}{\sum_{j=1}^{N} I_{x_i(t-j)}} \qquad \begin{matrix} for\ t = 1,2,\dots,n \\ for\ j = 1,2,\dots,N \end{matrix} \tag{4.9}$$

subject to:

$$10 < \sum_{j=1}^{N} I_{x_i(t-j)} \leq N$$

4.6.1.2 Review period component (more details in appendix G.2.1)

Let us define the subset $V^\delta = \{1,2,\dots,i,\dots,n\}$ which contains all the product i that are ordered to the same supplier δ and D_i^d as the daily demand of product i (i.e. NOT demand order size as in continuous demand). Then, the review period for all products i in V^δ is given by:

$$R^\delta = \sqrt{\frac{2K}{\sum_{i \in V^\delta} h_i D_i^d}} \tag{4.10}$$

The economic order quantity for every product i in V^δ becomes:

$$Q_i^* = R^\delta * \frac{E[D_i]}{E[A_i]} \tag{4.11}$$

4.6.1.3 Derivations of relevant parameters (more details in appendix J)

1. The first two moments of the demand during lead-time: $E[D(0,L)]$ and $\sigma^2(D(0,L))$,
2. The first two moments of the demand during the review period: $E[D(0,R)]$ and $\sigma^2(D(0,R))$.
3. The first two moments of the demand: $E[U]$ and $\sigma^2(U)$.
4. The expected stock on hand: $E[X]$.
5. The service levels: P_1 and P_2.

4.6.1.4 Holding cost (more details in appendix K.1)

The total expected daily holding cost for the (R^δ, s_i, S_i) replenishment policy can be expressed as:

$$TEDHC_{RsS} = \sum_{i=1}^{n} h_i * c_i * \left(s_i - E[D(0,L)] - \frac{E[U_i]}{2} + \frac{E\left[D(0,R^\delta)\right]}{2} \right) \tag{4.12}$$

The total expected daily holding cost for the (R^{δ}, S_i) replenishment policy can be expressed as:

$$TEDHC_{RS} \quad = \sum_{i=1}^{n} h_i * c_i * \left(S_i - E\big[D(0,L)\big] - \frac{E\left[D\left(0, R^{\delta}\right]\right]}{2} \right) \tag{4.13}$$

with:

$n =$ number of products
$h_i =$ holding cost rate for one unit of product i for one time unit
$c_i =$ cost price of one unit of product i

An alternative for the term between brackets (i.e. $E[X_i]$) is a term based on de Kok (2002):

$$E[X] = s_i + \frac{\left(\frac{(S_i - s_i)^2}{2} - \frac{E^2[U_i] + \sigma^2(U_i)}{2} + \frac{E^2\left[D\left(0, R^{\delta}\right]\right] + \sigma^2\left(D\left(0, R^{\delta}\right]\right)}{2E\left[D(0, R^{\delta}]\right]\left((S_i - s_i) + E[U_i]\right)} \right)}{\left(\left((S_i - s_i) + E[U_i]\right) - \frac{E[D_i]}{E[A_i]} E[L] - P_2 \frac{E\left[D(0, R^{\delta}]\right]}{2} \right)}$$

In chapter five we show that this approximation is more accurate than the approximation by Silver et al. (1998).

4.6.1.5 Ordering cost (more details in appendix K.2)

The total expected daily ordering cost for ordering products to a supplier δ under a (R^{δ}, s_i, S_i) replenishment policy can be expressed as:

$$TEDOC_{RsS} = \sum_{\delta=1}^{N} \frac{OC_2(\rho_i)}{E[R^{\delta}]} \tag{4.14}$$

with

$$OC_2(\rho_i) = K \left(1 - \prod_{i \in V^{\delta}} (1 - \rho_i) \right) \tag{4.15}$$

$\rho_i =$ ordering probability $= \frac{E[D_i]E[R^{\delta}]}{(S_i - s_i + E[U_i])E[A_i]}$.

$N =$ number of suppliers

The total expected daily ordering cost under a (R^{δ}, S_i) replenishment policy can be expressed as:

$$TEDOC_{RS} = \sum_{\delta=1}^{N} EDOC_{RS}^{\delta} = \sum_{\delta=1}^{N} \frac{OC_1}{E[R^{\delta}]} \tag{4.16}$$

with:

$OC_1 = K$
$N =$ number of suppliers

4.6.1.6 Shortage cost (more details in appendix K.3)

The total daily expected shortage cost for the (R^{δ}, s_i, S_i) replenishment policy can be expressed as:

$$TEDSC_{RsS} = \sum_{\delta=1}^{N} \sum_{i \in V^{\delta}} \frac{(E[D(0,L] + U_i - s_i)^+] - E[D(0,L] - S_i)^+])}{R^{\delta}} \tag{4.17}$$

with:

$N =$ number of suppliers

The total daily expected shortage cost for the (R^{δ}, S_i) replenishment policy can be expressed as:

$$TEDSC_{RS} = P_i \sum_{\delta=1}^{N} \sum_{i \in V^{\delta}} \frac{\left(E\left[\left(D(0,R^{\delta} + L] - S_i\right)^+\right] - E[(D(0,L] - S_i)^+]\right)}{R^{\delta}} \tag{4.18}$$

with:

$N =$ number of suppliers

4.6.2 Cost minimization

The total cost (TC) of the inventory system consists of the holding, ordering and shortage cost. Hence, we want to:

$$\min(\mathrm{TC}) \Leftrightarrow \min(TEDHC + TEDOC + TEDSC) \tag{4.19}$$

The three cost components are minimized by determining the appropriate cost efficient review period (i.e. review determination method) and calculating the appropriate reorder level s_i and/or order-up-to-level for every product i under a target fill rate P_2, as we can see in table 5.

Table 4: Replenishment policy parameters

	$(R^{\delta^*}, s_i^*, S_i^*)$	(R^{δ^*}, S_i^*)
Review period; EOQ:	$R^{\delta^*} = \sqrt{\frac{2K}{\sum_{i \in V^{\delta}} h_i D_i^d}}$	$R^{\delta^*} = \sqrt{\frac{2K}{\sum_{i \in V^{\delta}} h_i D_i^d}}$
Reorder level:	$s_i^* = \hat{x}_{L,i} + k\hat{\sigma}_{R+L,i}$	-
Order-up-to level:	$S_i^* = \hat{x}_{R+L,i} + k\hat{\sigma}_{R+L,i}$ $= s_i + (S_i - s_i)$ $= s_i + Q_i - U_i$	$S_i^* = \hat{x}_{R+L,i} + k\hat{\sigma}_{R+L,i}$

with:

$\hat{x}_{L,i}$ = forecasted or expected demand during leadtime for product i
$\hat{x}_{R+L,i}$ = forecasted or expected demand during the review period for product i
$\hat{\sigma}_{L,i}$ = forecasted or expected std. deviation of demand during lead time for product
$\hat{\sigma}_{R+L,i}$ = forecasted or expected std. deviation of demand during the review period for product i
k = safety factor specified based on the target fill rate; depends on L, μ_D, σ_D and Q_i

If shortage cost are taken into account with the service level setting, this results in the P_3 service level (i.e. $P\{X_i > 0\}$) which can be defined as the Newsvendor fractile: $\frac{b_i}{b_i + h_i}$. Note that this can only be implemented if the shortage cost b_i have the dimension cost per unit per unit time and that the inventory system is a backorder system. Expression (4.5) could be used to determine the shortage cost per replenishment cycle under a certain target service level. Based on the shortage cost and holding cost per replenishment cycle, a cost-efficient target service level could be determined.

Chapter Five

Case study

Chapter four described the chosen solution concepts and derived expressions for the relevant parameters of our inventory control model. This chapter describes the case study where we implement the inventory control solutions in the problem situation of the case study company. The case study will involve simulating day by day behavior of the inventory control system that results from utilizing the newly modified replenishment policies. This simulating is performed by a simulation tool that we developed for our research. Section 5.1 of this chapter starts with the description of the different components of the simulation model and the assumptions used in the case study. Simulation is performed because: (1) some of the derived expressions for the different relevant parameters have to be verified, which is done is section 5.2; (2) the performance of the new policies should be compared with the performance of the Optiply model during the same time period, which is done in section 5.3; (3) useful or interesting scenarios can be simulated that may assist in the inventory control decision making process, which are simulated in section 5.3 as well.

The initial goal was to additionally compare the new model with the old situation at the case study company before they became a customer at Optiply. However, no target fill rates were set by the company in that period, which means that although we can calculate the inventory cost over a certain period before *2016-09-01*, we cannot compare the performance of the new model over this period. If target fill rates (and actual experiences fill rates) are unknown, we cannot set the parameters of the new model to simulate over the same period. However, by knowing that Optiply improved inventory control at the case study company since *2016-09-01*, we can assume that the new model improves inventory control compared to the old situation if we show that our model outperforms the Optiply model. Therefore, this chapter will show that the new model outperforms the current Optiply model.

The simulation tool is developed in the software program *R* and simulates daily demand and the response of the two newly modified policies on this demand. Parameters are tracked and simulated per product. These parameters include the daily stock on hand, daily inventory position, stochastic lead-times, replenishment orders, replenishment deliveries, the undershoot and daily lost-sales. At the end of every simulation run the simulation tool creates 3 tables with relevant input and output parameters. This information is then automatically written to *Excel* files.

5.1 Components of the model

This section provides an overview of the different components of the simulation tool and describes how these components were integrated into the model. The replenishment process is simulated day by day and the sequence of events is the following:

1. Determine starting stock on hand and starting inventory position
2. Demand occurs
3. Place replenishment orders
4. Determine ending inventory position
5. Receive replenishment orders
6. Determine ending stock on hand
7. Determine lost-sales and cost calculation

Demand only decreases the inventory position and the stock on hand only and only if both the stock on hand and the inventory position are positive. Lost-sales occur if demand occurs when the stock on hand is zero. A replenishment order is placed to the supplier based on the inventory position and placing an order increases the inventory position. Hence, the ending inventory point of every day can be determined after ordering. Receiving a replenishment order (i.e. replenishment delivery) increases the stock on hand. Hence, the ending stock on hand of every day can be determined after receiving the replenishment order.

5.1.1 Historical data

Historical data from the period *2015-10-21* to *2016-09-01* is used as input for setting the s-levels (reorder level and order-up-to-level) of the different products. Data from this period will typically be used unless stated otherwise. Since *2015-10-21* the case study company started using a new *warehouse management system* (WMS) called "Picqer" and historical data is extracted from this WMS. The historical data includes point of sale (POS) collection data such that the inter-arrival time of demand and the size of the demand order could be determined in periods with positive stock. For a large part of the assortment stock data is available including order and delivery moments such that the average supplier lead-times and uncertainty of supplier lead-times could be determined.

The following assumptions are made when using the historical data:

1. Sales (demand) data and stock data over de period *2015-10-21* to *2016-09-01* was used to determine the input parameters and perform forecasts. Note that not all 1800 products have data over the full period because some products were introduced into the assortment during this period (see next assumption).
2. Only products that have enough data to forecast with are used in the analysis and simulation. Some of the newly introduced products are therefore excluded. Note that this is not too relevant since we will only simulate for several products due to limited time and the fact that if the verification holds for multiple products with varying characteristics, it holds for all products.

5.1.2 Demand generation

For the tool to work, demand has to be given as an input parameter. Relevant parameters such as the order quantity, reorder level s and the order-up-to-level S are determined by using demand or sales data as an input. This data is used for initial parameter setting. The dataset to test the model on should be new historical data which has not been used for parameter setting. We will simulate demand data for the period *2016-09-01* to *2017-01-15*. For this period we generate demand which is based on a products specific distribution. Historical sales/demand data is split up in demand inter-arrivals and demand order sizes (i.e. continuous demand parameters). Therefore, the demand follows a compound renewal process. The following section describes which demand distribution will be used throughout the model and the simulations.

5.1.2.1 Demand distribution

Heinecke et al. (2013) performed a Kolmogorov-Smirnov test on underlying demand distributions for 13.000 products. They concluded that the Poisson distribution gives a good estimation for all average demand intervals and a relatively low CV. For a CV in the range of 0 to 1 the Normal distribution showed a good fit, but fits poorly for erratic demand data and is therefore only appropriate for more smooth demand patterns. Additionally, the Normal distribution would generate negative demand due to its characteristics and therefore an additional assumption should be made that removes the generated negative values. The Gamma distribution showed a good fit for demand data with high inter-arrival times and high CV values and therefore the Gamma distribution can be used for generation of variable demand (Heinecke, Syntetos, & Wang, 2013). According to Burgin (1975) Gamma distribution is appropriate to represent demand in multiple situations. The distribution covers a wide range of distribution shapes, is defined for non-negative values only and is mathematically tractable in inventory control applications. An important note to make is that the Gamma distribution requires estimation of the mean and variance only and that the use of the distribution in practical applications is supported by ample empirical evidence (Burgin, 1975). The following assumptions are made for demand generation:

1. Demand inter-arrival times follow a Poisson distribution. By assuming that the demand inter-arrivals are Poisson distributed we assume that the inter-arrivals of demand are independent and identically distributed. Because testing for independence of all 1800 products, we therefore implicitly assume iid demand inter-arrivals in case we generate demand data. Demand order sizes follow a Gamma distribution.
2. Seasonality and trends are not taken into account because there is not enough historical data to make relevant assumptions about them. However, in the practical simulation scenarios in section 5.3 we do perform a forecast and therefore take into account trends and seasonality.
3. The time scale is 7 days per week.

In we assume that demand is Gamma distributed with mean $\alpha\beta$ and variance $\alpha\beta^2$, then the pdf is equal to:

$$f(x|\alpha,\lambda) = \frac{e^{-\beta x}x^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}} \tag{5.1}$$

With the 'Gamma function':

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1}e^{-y}dy \tag{5.2}$$

Furthermore, we will use the fact that $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$ and the cdf of the Gamma distribution $F(x|\alpha,\beta)$. The Gamma distribution is defined with a so called shape and a scale parameter which can be described as follows:

$$scale = \lambda = \frac{\sigma_D^2}{\mu_D} \tag{5.3}$$

$$shape = \alpha = \frac{\mu_D^2}{\sigma_D^2} \tag{5.4}$$

These shape and scale parameters can be used when there is data available to estimate μ_D and σ_D or when μ_D and σ_D are given. In our case the μ_D and σ_D from the historical data can be used to generate Gamma distributed demand data for simulation. Note that the scale parameter can also be described by the rate parameter $\beta = \frac{1}{\lambda} = \frac{\mu_D}{\sigma_D^2}$.

5.1.3 Ordering process

The simulation tool simulates the whole replenishment process: inventory levels are reviewed, replenishment orders are placed and received and inventory levels are updated accordingly. The starting state on *2016-09-01* is the moment of the first review moment $R_0 = 0$. The starting stock on hand and starting inventory position are assumed to be equal to the actual stock on hand on that day. Furthermore, it is assumed that there are no outstanding orders at that moment in time. Hence, the first replenishment order that is received is the order placed at R_0. In case of the (R^δ, s_i, S_i) replenishment policy, the inventory of every product i that is ordered to the same supplier is reviewed at the review moment R_t. If the inventory position of a product i is below s_i, a replenishment order of size $S_i - Y_i(R_t)$ is placed to the supplier. In case of the (R^δ, S_i) replenishment policy, a replenishment order is expected to be placed to the supplier every review period. The size of the replenishment order of a product i is $S_i - Y_i(R_t)$. Every replenishment order is given a lead-time for delivery that follows a Gamma distribution. The replenishment order is received 'a lead-time time units' later.

The following assumptions were made for the receiving and placing of replenishment orders:

1. The replenishment process is performed according to the newly modified replenishment policies.
2. For every replenishment order the ordered quantity is equal to the delivered quantity (i.e. there is a yield of 100 percent).
3. Lead-times of suppliers are assumed to follow Gamma distribution with the first two moments based on historical data or based on test data.

5.1.4 Cost calculation

With respect to the ordering process described in section 5.1.3, three different inventory cost are made: (1) holding cost, which are equal to the cost price of the product multiplied with the holding cost rate (2) ordering cost, which are equal to the fixed ordering cost incurred per order to a supplier and (3) shortage cost, which are incurred when demand cannot be met from stock on hand immediately and which is equal to the margin of a product multiplied with a certain goodwill factor α.

The following assumptions were made in the cost calculation:

1. Ordering cost are incurred per replenishment order to a supplier (i.e. major fixed ordering cost). No ordering cost are incurred based on the number of products in the replenishment order (i.e. minor fixed ordering cost).
2. Holding cost are based on invested capital. Therefore, the holding cost incurred are determined on the average inventory position level of each day.
3. Shortage cost are incurred when excess demand cannot be satisfied from stock on hand immediately; shortage cost are not taken into account in minimizing the inventory cost because we minimize cost under a target fill rate. Hence, we accept that a part of the demand is lost.
4. No form of trade credit is allowed (i.e. replenishment orders are paid when ordered and sold products are paid by customers when bought).
5. The goodwill factor α is set to 1 for all simulations since it is not relevant for cost minimization under a target fill rate.

For a brief description of the Company B specific cost specifications, please refer to appendix L.

5.1.5 Key Performance Indicators

In the analysis of the inventory system we will consider the following relevant KPIs:

1. P_2 service level (fill rate);
2. Expected (daily) holding cost, based on expected stock on hand;
3. Expected (daily) ordering cost;
4. Expected (daily) total cost.

This section will describe some of the KPIs in more detail because they need further explanation on their content and on their underlying distribution.

5.1.5.1 $\boldsymbol{P_2}$ service level

The P_2 service level is calculated by using expression $(J.24)$ from appendix J.5.2. For a given reorder level s and order quantity Q, Silver et al. (1998) fit a Gamma distribution to the expression for the P_2 service level.

($\boldsymbol{R, s, S}$) replenishment policy

If we fit a Gamma distribution to the first two moments of the demand during lead-time and the demand during lead-time plus undershoot, the expression becomes:

$$P_2 = 1 - \frac{\left(E\left[\left(D(0,L] + U_{R^\delta} - s\right)^+\right] - E[(D(0,L] - S)^+]\right)}{(S - s + E[U_{R^\delta}])} \tag{5.5}$$

with:

$$E\left[\left(D(0,L]+U_{R^\delta}-s\right)^+\right] \quad = \alpha\beta\left(1-\Gamma_{\alpha+1,\beta}(s)\right)-s\left(1-\Gamma_{\alpha,\beta}(s)\right) \tag{5.6}$$

$$E[(D(0,L]-S)^+] \quad = \alpha\beta\left(1-\Gamma_{\alpha+1,\beta}(S)\right)-S\left(1-\Gamma_{\alpha,\beta}(s)\right)$$

$(\boldsymbol{R},\boldsymbol{S})$ replenishment policy

If we fit a Gamma distribution to the first two moments of the demand during lead-time and the demand during the review period plus lead-time, the expression becomes:

$$P_2 = 1-\frac{(E[(D(0,R+L]-S)^+]-E[(D(0,L]-S)^+])}{E\big[D(0,R]\big]} \tag{5.7}$$

with:

$$E[(D(0,R+L]-S)^+] \quad = \alpha\beta\left(1-\Gamma_{\alpha+1,\beta}(S)\right)-S\left(1-\Gamma_{\alpha,\beta}(S)\right) \tag{5.8}$$

$$E[(D(0,L]-S)^+] \quad = \alpha\beta\left(1-\Gamma_{\alpha+1,\beta}(S)\right)-S\left(1-\Gamma_{\alpha,\beta}(S)\right) \tag{5.9}$$

5.1.5.2 Expected stock on hand

Expected stock on hand $E[X]$ is the stock on hand at the beginning of an arbitrary replenishment cycle just after a potential replenishment (i.e. at time $t=L$) subtracted by the stock on hand at the end of an arbitrary replenishment cycle just before a potential replenishment (i.e. at time $t=(R+L)^-$). It is common in scientific literature on periodic review systems to take the expected stock on hand at time $t=(R+L)^-$ as KPI for the stock on hand. However, if the review period is very large compared to the average order quantity (i.e. $S-s$ under an (R,s,S) replenishment policy), it is desired to take into account the expected stock on hand at the beginning of the replenishment cycle. This is accomplished by taking the average of $E[X(L)]$ and $E[X(R+L)^-]$. Heijden and de Kok (1998) developed a trapezoidal rule that linearly approximates the average stock on hand in a replenishment cycle (Heijden & Kok, 1998). Appendix J.4 elaborates on the approximation of the expected stock on hand in our research.

5.2 Verification

This section describes the verification of the expressions which we derived in part two of chapter four. Although we will verify both our mathematical model and our simulation tool in section 5.2.1 and section 5.2.2, we feel the need to verify how accurate the output from our functions of the mathematical model is compared to the simulated output. Therefore the content of this verification section is the following: in section 5.2.1 we describe the verification of the model in general. Section 5.2.2 describes the verification of the simulation tool. Section 5.2.3 elaborates on the fill rate calculation verification. In section 5.2.4 the cost functions verification is described. Both verifications are performed by comparing the simulation results with the analytical results.

In the process of verifying the analytical fill rate and cost functions, different scenarios were simulated. These scenarios vary on the values of the input parameters: lead-time, review period, inter-arrivals of demand orders and the demand order size. The cost functions are verified in all of the scenarios. The scenarios that are simulated for verification are the following:

1. Constant lead-time with Poisson distributed demand inter-arrivals and Gamma distributed demand sizes under an (R, s, S) replenishment policy (section 5.2.3 and 5.2.4).
2. Non-constant lead-time with Poisson distributed demand inter-arrivals and Gamma distributed demand sizes under an (R, s, S) replenishment policy (section 5.2.3. and 5.2.4).
3. Constant and non-constant lead-time with Poisson distributed demand inter-arrivals and Gamma distributed demand sizes under an (R, S) replenishment policy (section 5.2.3 and 5.2.4).

In section 5.3 we simulate joint-replenishment with n products under non-constant lead-times with Poisson distributed inter-arrivals of demand and Gamma distributed demand sizes. This scenario is performed for both the (R^{δ}, S_i) and (R^{δ}, s_i, S_i) replenishment policy. We also compare the Optiply model with our New model in this section where we simulate with both generated demand and real demand data.

We have chosen these scenarios because it is relevant to show how the tool performs under varying lead-times and using different decision rules based on the replenishment policy. Additionally, it is important to show that the tool functions for more than one product because it has to be used for all products i ordered to the same supplier δ. The closer the scenarios come to real-life inventory control, the more relevant the verification of our tool is for practical applications.

5.2.1 Model verification

We verified that the different functions of our model are correctly programmed into our decision support tool in *R* by comparing the output of calculating the functions manually with the output of the decision support tool. Furthermore, we verified the complex functions such as the fill rate function with the spreadsheet of de Kok (2002). The holding cost functions is largely based on the expected average stock on hand $E[X]$. The expected average stock on hand approximation by Silver et al. (1998) as well as the expected average stock on hand approximation by de Kok (2002) could be verified by comparing the output of our tool with the spreadsheet from de Kok (2002) and manually calculating it. The ordering probability ρ_i and the related ordering cost were verified manually by calculating them for one product and comparing with the output of the decision support tool.

5.2.2 Simulation tool verification

The verification of the simulation tool is shown by the simulation output of table 13 in appendix M. The table shows the simulation output of the relevant parameters of scenario 1.1. All parameters of demand in certain periods are calculated accurately with differences of about 0,15% from the simulated parameters, except the undershoot parameters. This difference is probably caused by the fact that the inventory system we have simulated is a lost-sales system: both the inventory position and the stock on hand cannot be negative. Therefore, the distance $-min\{0, Y(t) - s\}$ cannot be larger than s and the average undershoot will have a value between 0 and s instead of between $-min\{0, Y_i(t)\}$ and s. Hence, the expected undershoot $E[U]$ is lower in our simulation model.

Two other relevant parameters are the expected average stock on hand $E[X]$ and the ordering probability ρ_i. The calculated $E[X]$ amounted $24{,}96$ and the simulated $E[X]$ amounted $25{,}49$ with a 95% confidence interval of $\pm 0{,}0587$. The calculated ρ_i amounted $0{,}48$ and the simulated ρ_i amounted $0{,}47$ with a 95% confidence interval of $\pm 0{,}0036$. Showing that all these output parameters are accurate, verifies that our simulation tool simulates accurately. All output of the simulation runs will be handed in as proof for our verification.

5.2.3 Cost functions verification

In the last sections we verified our model functions, the simulation tool and showed that the fill rate calculations are accurate in the larger part of the scenarios. To show the accuracy of the cost functions we performed verifications within the same simulation scenarios. The verification of the cost functions and relevant conclusions on this verification can be found in appendix N. All input parameters used for the different scenarios can be found in appendix O.

5.2.4 Conclusion

This section and the related appendices described the verification of the mathematical model, the simulation tool, fill rate calculation functions, the inventory cost functions and other relevant parameter calculations. All calculations were found accurate by comparing them with the simulation output. Expected fill rate calculations often deviated less than 1% from the simulated values. Only in scenarios with high variable lead-time the deviation increased to approximately 2%. The initial expected holding cost function based on Silver et al. (1998) deviated too much from the simulated values to our liking. Therefore, the equations from de Kok (2002) were integrated which resulted in accurate expected holding cost calculation in all scenarios. Lastly, the expected ordering cost were verified by showing that both the ordering probability ρ_i and ordering cost itself were accurately calculated with differences often less than 1% compared to the simulated values. The verification in this section provides us with the opportunity to use the new model and its modified replenishment policies for other simulations and comparisons. Comparison with the performance of the new model with the performance of the Optiply model and the case situation.

5.3 Comparison with the Optiply model

In this section the new model will be tuned in such a way that it resembles the model from Optiply. First a simulation will be performed for both the (R^{δ}, s_i, S_i) and (R^{δ}, S_i) replenishment policy with generated discrete demand data (Gamma distributed). Thereafter a simulation with actual experienced demand data is performed. All input parameters such as the aggregated review period, the demand parameters and reorders level are based on actual historical data. The target fill rate in every scenario is 95% for all products. Because this section compares the new model with the Optiply model, we will first briefly described certain relevant aspects of the Optiply model:

- Undershoot is not taken into account in the process of calculating the reorder level s_i or the order-up-to level S_i under the target fill rate.
- In situations with highly variable demand the iteration method does not find the appropriate reorder level s_i because the iteration process is not large enough. The method of calculating the reorder level is based on an iteration method of de Kok (2002). In our new model we extended this method to search in an interval of such that it is somewhat more reliable in finding the appropriate reorder level.
- The expected inter-arrival times of demand are estimated by correcting the inter-arrival times of sales based on the historical service levels. Unobserved and actual demand is probably underestimated.
- The review period for all products i ordered to the same supplier is determined by calculating an EOQ on supplier level without weighted averages (see appendix H).
- The expected stock on hand is calculated based on Silver et al. (1998), which underestimates the expected stock on hand.

Review period determination

Comparing the supplier review periods that result from the Optiply method described in appendix H with the supplier review periods that result from method, we concluded that the newly calculated review periods are often shorter. Three advantages of shorter review periods are that: (1) the holding cost are lower because less stock on hand has to be held to overcome the review period, (2) the model becomes more flexible to changes in demand or delivery uncertainties and (3) the horizon where we need to forecast demand over becomes shorter which results in lower uncertainty in the forecast. A disadvantage of shorter review periods is that because replenishment orders are placed more often, the total ordering cost increase.

Simulation

The products being analyzed are the 5 products coming from a Chinese supplier. The relevant information of the 5 products is given in table 4. Note that the reorder level s_i is not relevant for the simulation of the (R^{δ}, S_i) replenishment policy.

As we can see in table 5 the products are very different with respect to their demand parameters. Some products are slow movers and some products are fast movers. Moreover, some of the products such as product 4 is highly variable with respect to its inter-arrivals of demand. Therefore, the 5 products from this supplier represent a good sample for the overall products population in the assortment.

Table 5: Relevant parameters of products used for comparison between Optiply model and the New model

Optiply model										
Product	$E[D_i]$ $(days)$	$\sigma(D_i)$ $(days)$	$E[A_i]$ $(days)$	$\sigma(A_i)$ $(days)$	R $(days)$	$E[L]$ $(days)$	$\sigma(L)$ $(days)$	s_i	S_i	Target P_2
1	1,7143	3,8076	2,6411	3,0809	9	7	1,75	16	22	95%
2	1,0588	0,2365	2,6831	3,6927	9	7	1,75	6	10	95%
3	1,9496	1,9034	0,0046	0,0713	9	7	1,75	4131	8587	95%
4	1,1364	0,4087	4,4158	8,2256	9	7	1,75	3	16	95%
5	1,3544	0,8479	0,9232	1,9259	9	7	1,75	17	44	95%
New model										
Product	$E[D_i]$ $(days)$	$\sigma(D_i)$ $(days)$	$E[A_i]$ $(days)$	$\sigma(A_i)$ $(days)$	R^δ $(days)$	$E[L]$ $(days)$	$\sigma(L)$ $(days)$	s_i	S_i	Target P_2
1	1,7143	3,8076	2,4804	3,2994	1	7	1,75	34	35	95%
2	1,0588	0,2365	2,3473	4,2634	1	7	1,75	12	13	95%
3	1,9496	1,9034	0,0046	0,0732	1	7	1,75	5854	6278	95%
4	1,1364	0,4087	4,4158	8,2256	1	7	1,75	9	10	95%
5	1,3544	0,8479	0,9232	1,9259	1	7	1,75	32	34	95%

5.3.1 Generated demand – Theoretical model comparison

Demand is generated for a simulation horizon of $m = 1000$ days, a warm-up period of $l = 20$ days and $n = 30$ replicas. The input parameter are set based on historical data from the period *2015-10-21* until *2016-09-01*. A practical simulation horizon of 1000 days is chosen, being a simulation horizon of about 3 years. The starting state for all 5 products in the simulation is at time R_0 with a stock on hand of zero (i.e. $X(0) = 0$). The confidence interval for the different simulation output for one specific model is equal to (Law, 2007):

$$CI_{95\%} = \bar{X}(n) \pm t_{n-1,1-\frac{\alpha}{2}}\sqrt{\frac{\sigma^2(n)}{n}} = \bar{X}(30) \pm t_{29,0.975}\sqrt{\frac{\sigma^2(30)}{30}} \tag{5.11}$$

Comparisons of the different simulation output from the two models can be compared based on the following 95% confidence interval:

$$CI_{95\%} = \bar{X}_1(n_1) - \bar{X}_2(n_2) \pm t_{\hat{f},1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2(n_1)}{n_1} + \frac{\sigma_2^2(n_2)}{n_2}} \tag{5.12}$$

with $\hat{f}$ being the estimated degrees of freedom: $\hat{f} = \left[\frac{\frac{\sigma_1^2(n_1)}{n_1} + \frac{\sigma_2^2(n_2)}{n_2}}{\frac{\left[\frac{\sigma_1^2(n_1)}{n_1}\right]^2}{(n_1-1)} + \frac{\left[\frac{\sigma_2^2(n_2)}{n_2}\right]^2}{(n_2-1)}}\right]$

Table 6 shows the simulation results from the Optiply model and the New model under an (R^δ, s_i, S_i) and (R^δ, S_i) replenishment for the 5 described products with generated demand under a target fill rate of 95%. With the results in table 7 we should keep in mind that the demand inter-arrival parameters of product 1 and 2 are somewhat different between the two models. The reason for this difference is the fact that both models estimate these parameters differently based on the unobserved demand estimation. However, we can still compare the different costs and the fill rate taking into account that the total demand over the simulation horizon is a little higher for product 1 and 2 in the New model.

The different columns from left to right are the total holding cost, total ordering cost, total shortage cost and the fill rate. If we use the results from **table 6** and compare the new (R^{δ}, s_i, S_i) replenishment policy with the policy in the Optiply model, the holding cost are somewhat higher for the majority of the products within the new model. Also the ordering cost are higher because the review period is shorter in the new model. However, where the new model really comes to play is the actual experienced fill rate and the resulting shortage cost. If we assume that the shortage cost are equal to every lost-sales multiplied by its margin, the cost savings are very large. Note that the extent of the cost savings is dependent on the fixed ordering cost, the cost price of the products, the margin of the products and the demand volume. If we would only look at products 3, 4 and 5, where the demand inter-arrival parameters are equal in both models, we also see a large difference in fill rate and shortage cost. For the (R^{δ}, S_i) replenishment policy we can draw the same conclusion although the cost differences are somewhat smaller. Additionally, almost all the 95% confidence intervals of the fill rate are much tighter in the new model than in the Optiply model meaning that the new model is more reliable.

Table 6: Simulation results Optiply model vs. New model under generated demand and target fill rate 95%
(2016-09-01 $+\ m = 1000\ days\ , l = 0\ days, n = 30$)

		Optiply model				New model			
Product		h_i	$K\ cost$	$b_i\ cost$	P_2	$h_i\ cost$	$K\ cost$	$b_i\ cost$	P_2 target: 95%
1		€1,00	-	€640,13	83,06%	€2,09	-	€172,84	96,83%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,01)		(±€87,00)	(±2,21%)	(±€0,02)		(±€49,14)	(±1,16%)
1		€1,09	-	€596,67	84,55%	€2,12	-	€141,85	96,56%
(R^{δ}, S_i)	$CI_{95\%}$	(±€0,02)		(±€119,90)	(±2,91%)	(±€0,90)		(±€51,48)	(±1,15%)
2		€9,95	-	€2682,75	85,07%	€19,43	-	€866,77	95,93%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,11)		(±€221,12)	(±1,21%)	(±€0,76)		(±€206,28)	(±0,93%)
2		€11,58	-	€1810,26	90,01%	€22,67	-	€804,65	96,23%
(R^{δ}, S_i)	$CI_{95\%}$	(±€0,13)		(±€196,35)	(±1,02%)	(±€4,63)		(±€193,14)	(±0,88%)
3		6925,08	-	€2.451.327,75	78,64%	€6472,95	-	€611.598,00	94,52%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€66,63)		(±€140.307,73)	(±1,21%)	(±€87,00)		(±€65.757,24)	(±0,57%)
3		€8245,74	-	893.157,81€	92,16%	€6662,51	-	€497.793,60	95,50%
(R^{δ}, S_i)	$CI_{95\%}$	(±€122,71)		(±€92.203,61)	(±0,79%)	(±€85,64)		(±€55.411,34)	(±0,50%)
4		€9,03	-	€503,54	87,00%	€352,38	-	€112,37	97,19%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,15)		(±€51,27)	(±1,23%)	(±€262,91)		(±€43,97)	(±1,06%)
4		€14,24	-	€50,52	98,69%	€475,69	-	€45,09	98,79%
(R^{δ}, S_i)	$CI_{95\%}$	(±€0,13)		(±€28,59)	(±0,74%)	(±€223,75)		(±€31,07)	(±0,84%)
5		€21,65	-	€2708,30	85,83%	€22,90	-	€736,30	96,20%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,30)		(±€274,18)	(±1,40%)	(±€0,37)		(±€148,37)	(±0,74%)
5		€29,66	-	€812,51	95,85%	€23,58	-	€633,88	96,67%
(R^{δ}, S_i)	$CI_{95\%}$	(±€28,91)		(±€177,33)	(±0,84%)	(±€0,38)		(±€123,71)	(±0,65%)
Total (R^{δ}, s_i, S_i)		**€6966,71**	**€2003,33**	**€2.457.862,48**		**€6869,75**	**€14.488,00**	**€613.486,27**	
Total (R^{δ}, S_i)		**€8301,56**	**€2220,00**	**€896.427,77**		**€7186,58**	**€20.000,00**	**€499.419,05**	

If focus our attention purely to the new model in **table 6**, we can conclude that the (R^{δ}, S_i) replenishment policy performs better than the (R^{δ}, s_i, S_i) replenishment policy with respect to the fill rate and cost efficiency. Furthermore, the (R^{δ}, S_i) replenishment policy results in much tighter

95% confidence intervals for the different inventory cost and the fill rate. One of the reasons for this is that the probability of ordering in a replenishment cycle is always close to one. This lets us to conclude that in the new model the (R^{δ}, S_i) replenishment policy is more reliable than the (R^{δ}, s_i, S_i) replenishment policy.

We calculated the ordering probability ρ_i for the 5 products in the new model under the (R^{δ}, s_i, S_i) and (R^{δ}, S_i) replenishment policy and compared it with the simulated ρ_i for the 5 products. For the (R^{δ}, s_i, S_i) replenishment policy the calculated ordering probability amounted $\rho_i = 0{,}7800$ and the simulated ordering probability amounted $\rho_i = 0{,}7244$. For the (R^{δ}, S_i) replenishment policy the calculated ordering probability amounted $\rho_i = 0{,}9756$ and the simulated amounted $\rho_i = 1{,}0000$. Taking into account the somewhat short simulation horizon, this lets us to conclude that the calculated ordering probability ρ_i is good approximation for the probability that an order is placed in a replenishment cycle taking into account joint replenishment of multiple products.

5.3.2 Real sales data – Practical model comparison

This section compares the Optiply model and the new model under actual experienced demand in the period of *2016-10-23* until *2017-01-23* (i.e. 3 months). The input parameter are set based on historical data from the period *2015-10-21* until *2016-10-23*. We ran $n = 30$ replicas with a warm-up period of $l = 0$ days. The same 5 products from the Chinese supplier are used in the simulation. Note that the parameters of the 5 products are different from what we saw in section 5.3.1 because more data is used as input for this simulation. The simulation starts at time R_0 and the starting stock on hand of every product equals the actual stock on hand on *2015-10-23*. In section 5.3.2.1 the that is used for simulation is the sales data that is based on the inventory control by the Optiply model. We mean by that, that due to inventory control from Optiply the stock of the products was zero on some days. Hence, no sales could be made on these days and therefore there is no demand data available on these days. The input data is based on the sales where demand is imputed in the periods without stock based on the *ImputeDemanData* function described in our research. In section 5.3.2.2 the same simulation is performed but here demand is imputed on the days with zero stock. By doing this, the dataset includes demand on the days without stock which follows the same distribution as the demand from the days with positive stock. The same holds for the input parameters for the new model.

5.3.2.1 Model comparison under sales data

Table 7 shows the output of the simulation for both models. The fill rate of product 1 is equal to 1 because there were no sales in the period since *2016-10-23*. Table 8 shows us that the new model replenishment policies provide higher fill rates and lower overall inventory cost than the policies in the Optiply model. For both the (R^{δ}, S_i) replenishment policy and the (R^{δ}, s_i, S_i) replenishment policy, the results are much better. Note that in some situations the actual experienced fill rate is higher or lower than the 95% target fill rate. This is caused by two things: (1) the demand for the products is very unpredictable (which is something different than variable) and does not necessarily follow a statistical distribution exactly (which is often the case for constant and voluminous demand) and (2) in reality, forecasts are ran every night such that new data can be processed into the forecast of demand and the replenishment process can be adapted accordingly. For this simulation we assumed that the order quantity, reorder level s_i and order-up-to level S_i were fixed for the period of 3 months. Hence, peaks or trend patterns in demand during this time period were not taken into account in the forecasting process in between days. However, the conclusion that the new model performs better than the Optiply model with respect to the fill rate and cost efficiency still holds. Also in this practical scenario, we see that the 95% confidence intervals of the fill rate under both replenishment policies are tighter in the new model than in the Optiply model. Therefore, we can conclude that the replenishment policies in the new model are more reliable.

Table 7: Simulation results Optiply model vs. New model under real sales data and target fill rate 95%
(2016-10-23 + $m = 3\ months$, $l = 0\ days, n = 30\ replicas$)

		Optiply model				New model			
Product		h_i	$K\ cost$	$b_i\ cost$	P_2	$h_i\ cost$	$K\ cost$	$b_i\ cost$	P_2 target: 95%
1 (R^δ, s_i, S_i)	$CI_{95\%}$	€0,10 (±€0,00)	-	€0,00 (±€0,00)	100% (±0,00%)	€0,26 (±€0,00)	-	€0,00 (±€0,00)	100% (±0,00%)
1 (R^δ, S_i)	$CI_{95\%}$	€0,14 (±€0,00)	-	€0,00 (±€0,00)	100% (±0,00%)	€0,26 (±€0,00)	-	€0,00 (±€0,00)	100% (±0,00%)
2 (R^δ, s_i, S_i)	$CI_{95\%}$	€0,69 (±€0,02)	-	€437,10 (±€47,35)	82,01% (±2,38%)	€28,32 (±€0,00)	-	€0,00 (±€0,00)	100% (±0,00%)
2 (R^δ, S_i)	$CI_{95\%}$	€0,84 (±€0,02)	-	€290,38 (±€27,37)	88,05% (±1,28%)	€28,81 (±€0,00)	-	€0,00 (±€0,00)	100% (±0,00%)
3 (R^δ, s_i, S_i)	$CI_{95\%}$	€389,02 (±€9,41)	-	€822.896,53 (±€14.067,42)	53,03% (±1,51%)	€263,29 (±€4,28)	-	€410.250,53 (±€10.927,41)	76,59% (±0,81%)
3 (R^δ, S_i)	$CI_{95\%}$	€345,36 (±€10,09)	-	€669.245,20 (±€19.279,88)	61,80% (±1,78%)	€263,68 (±€3,11)	-	€398.652,80 (±€11.486,38)	77,25% (±0,85%)
4 (R^δ, s_i, S_i)	$CI_{95\%}$	€0,44 (±€0,02)	-	€434,93 (±€10,09)	37,64% (±3,85%)	€78,51 (±€17,35)	-	€0,00 (±€0,00)	100% (±0,00%)
4 (R^δ, S_i)	$CI_{95\%}$	€0,52 (±€0,01)	-	€426,21 (±€7,46)	38,89% (±2,75%)	€75,99 (±€15,96)	-	€0,00 (±€0,00)	100% (±0,00%)
5 (R^δ, s_i, S_i)	$CI_{95\%}$	€3,93 (±€0,07)	-	€7095,31 (±€157,89)	38,09% (±3,62%)	€3,34 (±€0,03)	-	€6492,70 (±€125,58)	43,35% (±2,53%)
5 (R^δ, S_i)	$CI_{95\%}$	€4,09 (±€0,10)	-	€7005,09 (±€112,95)	38,88% (±2,54%)	€3,40 (±€0,03)	-	€6449,10 (±€96,81)	43,73% (±1,93%)
Total (R^δ, s_i, S_i)		**€394,22**	**€194,67**	**€830.863,87**		**€373,72**	**€1482,00**	**€416.743,23**	
Total (R^δ, S_i)		**€350,95**	**€228,67**	**€676.966,88**		**€371,64**	**€1682,67**	**€405.101,90**	

5.3.2.2 Model comparison under sales data with imputed demand

The same simulation is performed as in section 5.3.2.2 but here the assumption is made that the demand in the periods without stock follows the same distribution as the demand on the days with positive stock. Therefore demand is imputed on the days without stock based on the demand of the days with positive stock. This results in both higher demand parameters in the historical data where the model is set on and in the 3 months where is simulated over.

Table 8 shows the simulation output of both the Optiply model and the new model. Like in section 5.3.2.1 both models do not reach their target fill rate of 95% for some of the products (in particular the Optiply model). This mainly has to do with the fact that all parameters are set based on historical data of only less than one year and because parameters are set on demand forecasts. For almost all products the replenishment policies in the new model are more cost efficient and have a higher achieved fill rate. The only exception is product 5, where the two models perform roughly equal. Moreover, the 95% confidence intervals of the inventory cost and fill rates are tighter under both replenishment policies in the new model, meaning that the replenishment policies of the new model are more reliable compared to the replenishment policies of the Optiply model. Focusing on the new model only, we see that for almost all products the (R^{δ}, S_i) replenishment policy outperforms the (R^{δ}, s_i, S_i) replenishment policy with respect to cost and achieved fill rate. Again, product 5 forms a slight exception to this fact.

Table 8: Simulation results Optiply model vs. New model under actual experienced sales with imputed demand on the days without stock and target fill rate 95% (2016-10-23 + $m = 3\ months\ , l = 0\ days, n = 30\ replicas$)

		Optiply model				New model			
Product		h_i	$K\ cost$	$b_i\ cost$	P_2	$h_i\ cost$	$K\ cost$	$b_i\ cost$	P_2 target: 95%
1		€0,10	-	€77,86	79,22%	€0,09	-	€53,49	87,06%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,00)		(±€22,73)	(±6,00%)	(±€0,00)		(±€30,41)	(±6,33%)
1		€0,10	-	€55,55	85,33%	€0,09	-	€50,90	88,89%
(R^{δ}, S_i)	$CI_{95\%}$	(±€0,00)		(±€22,28)	(±5,65%)	(±€0,00)		(±€22,66)	(±5,22%)
2		€0,74	-	€408,07	83,21%	€28,41	-	€0,00	100%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,03)		(±€50,08)	(±2,48%)	(±€0,12)		(±€0,00)	(±0,00%)
2		€0,84	-	€235,36	90,31%	€28,42	-	€0,00	100%
(R^{δ}, S_i)	$CI_{95\%}$	(±€0,02)		(±€28,28)	(±1,22%)	(±€0,10)		(±€0,00)	(±0,00%)
3		€393,05	-	€812.429,80	53,63%	€244,37	-	€416.831,13	76,21%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€9,73)		(±€21.850,02)	(±2,33%)	(±€4,61)		(±€10.975,03)	(±0,82%)
3		€345,67	-	€653.666,00	62,29%	€247,66	-	€409.896,07	76,61%
(R^{δ}, S_i)	$CI_{95\%}$	(±€7,84)		(±€12.540,39)	(±1,14%)	(±€4,49)		(±€10.438,59)	(±0,78%)
4		€0,45	-	€431,54	38,12%	€86,66	-	€0,00	100%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,01)		(±€7,69)	(±2,89%)	(±€18,82)		(±€0,00)	(±0,00%)
4		€0,53	-	€417,50	40,14%	€81,47	-	€0,00	100%
(R^{δ}, S_i)	$CI_{95\%}$	(±€0,01)		(±€6,71)	(±2,40%)	(±€19,77)		(±€0,00)	(±0,00%)
5		€1,87	-	€8580,20	50,72%	€1,33	-	€8273,54	51,01%
(R^{δ}, s_i, S_i)	$CI_{95\%}$	(±€0,07)		(±€565,04)	(±3,59%)	(±€0,08)		(±€473,39)	(±2,87%)
5		€1,97	-	€8172,55	51,76%	€1,31	-	€8580,89	50,59%
(R^{δ}, S_i)	$CI_{95\%}$	(±€0,13)		(±€500,42)	(±2,91%)	(±€0,08)		(±€713,43)	(±3,99%)
Total (R^{δ}, s_i, S_i)		**€396,20**	**€193,33**	**€821.927,47**		**€360,87**	**€1568,67**	**€425.158,16**	
Total (R^{δ}, S_i)		**€349,87**	**€210,00**	**€662.546,96**		**€358,94**	**€1793,93**	**€418.527,85**	

If we compare the holding cost, ordering cost and shortage cost made by the Optiply model in table 7 and table 8 we see that the differences are rather small. This has to do with the fact that the stock on hand of product 2,3,4 and 5 almost included no days without stock. Therefore, not that much demand was imputed extra in the scenario of table 8 compared to the scenario of table 7. For product 1 however, we see that the achieved fill rate is much lower under both policies of the Optiply model in table 8 compared to table 7. The reason that cost are not always lower in table 8 for the Optiply model has to do with the variability of the lead-time and the variability of the imputed Gamma distributed demand in both scenarios.

5.4 Service level setting

In this section we will describe the simulation setup and simulation output of multiple scenarios that we simulated to compare the fill rate (i.e. P_2) and the ready rate (i.e. P_3) under different levels of demand variability. Scenario 4.1, 4.2 and 4.3 only differ based on their demand input parameters.

The parameters used for scenario 4.1, 4.2 and 4.3 are presented in **table 9** in **appendix O** and we assume that the demand parameters follow a compound renewal process with Poisson distributed demand inter-arrivals and Gamma distributed demand sizes just as in the verification sections of **chapter five**. Moreover, the simulation setup is the same as in the fill rate calculation verification scenarios (i.e. $m = 20.000 \textbf{ days}, l = 2000 \textbf{ days}, n = 10 \textbf{ replicas}$).

Based on a meeting with prof. de Kok about service levels, the P_2 and P_3 under an (s, Q) replenishment policy were given by:

$$P_2 = 1 - \frac{E[(D(0,L] + U - s)^+] - E[D(0,L] + U - (s+Q)]}{Q} \tag{5.13}$$

$$P_3 = 1 - \frac{E\left[\left(D(0,L] + \frac{E[D]}{E[A]} - s\right)^+\right] - E\left[D(0,L] + \frac{E[D]}{E[A]} - (s+Q)\right]}{Q} \tag{5.14}$$

This would indicate that if $U > \frac{E[D]}{E[A]}$, then $P_2 < P_3$ and hence, when the CV of daily demand is larger than 1 (i.e. $CV_{D^d} > 1$). For the periodic replenishment policies the following should hold: if $U > E\big[D(0,R]\big]$ then $P_2 < P_3$ and hence, when $\frac{\sigma(D(0,R))}{E[D(0,R]]} > 1$ (i.e. $CV_R > 1$). Note that for the (R, S) policy the U parameter is the undershoot from the perspective of the order-up-to-level S and not the reorder level s. In scenario 4.1 it holds that $U = E\big[D(0,R]\big] \iff CV_R = 1$ and the input demand parameters should result in an undershoot that is equal to the expected demand during the review period according to the spreadsheet of de Kok (2002). We increased the variability of demand in every subsequent scenario as can be seen in **table 9**. Our expectation prior to the simulation was that the difference between the fill rate and the ready rate would increase if the variability of demand increased.

The relevant simulation results are shown in **table 9**. The table presents the target fill rate, the adjusted target fill rate due to rounding up the order-up-to level to the nearest integer, the calculated order-up-to level, the achieved fill rate, the achieved ready rate and the simulated difference between the fill rate and the ready rate under the (R, S) replenishment policy.

Table 9: Simulation results scenario 4 ($m = 20.000, l = 2000, n = 10$)

Scenario		P_2 $(target)$	P_2 $(adj.\ target)$	S	P_2 $(achieved)$	P_3 $(achieved)$	$Diff.$
4.1 (R,S) $CV_R = 1$	$CI_{95\%}$	95,00%	95,23%	48	95,95% (±0,1158%)	96,00% (±0,1277%)	0,05%
4.2 (R,S) $CV_R = 1{,}65$	$CI_{95\%}$	95,00%	95,29%	44	96,24% (±0,1089%)	98,16% (±0,1050%)	1,92%
4.3 (R,S) $CV_R = 2$	$CI_{95\%}$	95,00%	95,24%	58	96,48% (±0,2377%)	98,74% (±0,0638%)	2,26%
4.3 (R,S) $CV_R = 3{,}16$	$CI_{95\%}$	95,00%	95,01%	121	96,37% (±0,3984%)	99,99% (±0,0059%)	3,62%

This table shows us that our expectations hold for our compound renewal process with Poisson demand inter-arrivals and Gamma distributed demand sizes under an (R, S) replenishment policy. In scenario 4.1 we see that the achieved fill rate and the achieved ready rate are close to being equal. In scenario 4.2 we see that the achieved fill rate is lower than the achieved ready rate, which was expected based on the fact that the CV_R is around $1{,}65$. In scenario 4.3 and 4.4 this difference is even larger because the CV_R is around 2 and 3 in those scenarios respectively. Due to the variability of demand, it is especially the achieved ready rate that starts to deviate from the target service level. This lets us to conclude what we already expected: the deviation between the achieved ready rate and fill rate increases with increasing the variability of demand. Moreover, we see in table 9 that the deviation of the achieved fill rate compared to the target fill rate also increases with increasing the variability of demand. Lastly, we can conclude that the 95% confidence interval of the achieved fill rate also increases with increasing the variability of demand.

We now briefly return to the expression of the average shortage during a replenishment cycle by de Kok (1991):

$$P_3(S) = E[X] - \left(S - \left(E[L] + \frac{R}{2}\right)\frac{E[D]}{E[A]}\right) \tag{4.5}$$

In the attempt to verify the calculation of the average shortage in a replenishment cycle with the spreadsheet of de Kok (2002) for varying demand parameters, expression (4.5) gave irrational or even negative values. This made us wondering if the expression worked for all demand input parameters. We found that the expression provides realistic approximations if the demand is rather constant (i.e. $CV_R \leq 1$) and that the expression provides negative and hence, unrealistic approximations when demand is (highly) variable (i.e. CV_R). Therefore, the expression cannot be applied in our problem situation. However, we would like to describe the fact that it would be possible to determine a cost-optimal service level in lost-sales system with (highly) variable demand and stochastic lead-times. The procedure would be a procedure that is performed in between review periods (i.e. a cyclical ordering policy) and the steps would be the following:

1. Set a target fill rate of $x_j\% = 99\%$ and determine the resulting holding cost and shortage cost based on the calculated reorder level s, order-up-to level S and other parameters.
2. Set a target fill rate of $x_{j+1}\% = (x_j - 1)\%$ and determine the resulting holding cost and shortage cost based on the calculated reorder level s, order-up-to level S and other parameters.
3. Redo step 2 until reaching a practical and acceptable lowest target fill rate (depending on context and managerial judgment; e.g. until 80%).
4. Determine which target fill rate results in the lowest total inventory cost and use this as target fill rate for the next review period.
5. After the next review period, redo all 5 steps to update the cost-optimal service level and s-levels accordingly (i.e. in-between review periods).

5.5 Conclusion

With the simulation results and comparisons made throughout section 5.3, we can conclude that the two newly modified replenishment policies of the new model outperform the replenishment policies from the Optiply model. Additionally, we showed that both newly modified policies performed reliable and adequate in the scenarios with generated demand and acceptable in the scenarios with actual experienced sales. We can conclude from our simulations that it is very difficult to promise the customer to achieve a target fill rate (this is probably only possible for fastmoving products with a rather constant demand pattern). The new model is able to perform the same calculations for as many products as desired. Therefore, the calculations (and simulations) made for 5 products also work for hundreds or thousands of products.

Throughout all calculations and simulations in section 5.3 the (R^{δ}, S_i) replenishment policy outperformed the (R^{δ}, s_i, S_i) replenishment policy with respect to total inventory cost and achieved fill rate. Note that these results would change in a context with different setting of holding, ordering and shortage costs. As described at the start of this section, comparing the new model with the inventory control situation before Optiply was not possible due to lack of 'inventory control' with target service levels or any logistic decision rules. However, by knowing that the Optiply model improved the old situation and showing that the new model outperforms the Optiply model we can assume that the new model improves inventory control compared to the old situation as well. Taking into account that in practice forecasts are performed daily and s-levels can be adapted accordingly, we conclude that the new model is useful for joint replenishment inventory control in a situation with characteristics such as stochastic highly variable demand, lost-sales and stochastic lead-times.

In section 5.4 we showed that the equalities between the P_2 and P_3 service level hold in a situation with compound renewal demand with Poisson distributed demand inter-arrivals and Gamma distributed order sizes. In the scientific literature this equalities were only shown for Poisson, Compound Poisson and Normally distributed demand. Moreover, we attempted to show that setting a target P_3 service level based on shortage cost per unit per time unit and holding cost per unit per time unit (i.e. Newsvendor fractile) leads to cost-optimal inventory control. However, this is not possible in a lost-sales system because in order to calculate the shortage cost per unit per time unit, one must know the number of backorders that are outstanding every time unit. However, it is possible to calculate the average shortage cost and average holding cost per replenishment cycle and determine the cost-optimal target service level based on these parameters.

Chapter Six

Implementation of the Decision support tool

This chapter describes the decision support tool that was developed for Optiply and potential customer webshops in the customer network of Optiply. The tool is designed in such a way that it can be used by non-experts (i.e. no extensive knowledge on inventory control is required to use the tool). The tool calculates required inventory control parameters based on the inserted input parameters and provides suggestions on how to set the supplier review period, ordering quantities, reorder levels, order-up-to levels and approximates the expected long-run holding and ordering cost. The tool is built in *R* which makes it a fast and reliable tool that enables the user to determine relevant inventory control parameters for as many products as desired. Determining the just described parameters for 1000 products takes less than 10 seconds on an average computer with 4gb RAM and an Intel i5 processor. Moreover, in contrast to often used programs such as *Excel* and *Arena*, *R* is an open source software program that can be used by any company or individual. All add-ons and required software packages are available on the internet and are free.

All relevant information on the parameters of the tool, the features of the tool and its user interface can be found in appendix P.

Chapter Seven

Conclusions & Recommendations

The last chapter of the Master Thesis Project describes the main findings of our research and the conclusions and recommendations based on these findings. This research was performed to accomplish the following main assignment:

Develop a decision support tool that assists in minimizing total inventory cost in a single-echelon lost-sales system taking into account joint-replenishment under lead-time, order moment and demand uncertainty for a given target service level.

In section 7.1 the general and case specific conclusions of the research are described. In section 7.2 we provide our recommendations for Optiply. The last section, section 7.3, provides our recommendations for future research.

7.1 Conclusions

This section briefly describes the main conclusion of our research. More detailed elaboration on the underlying research objectives of our research can be found in appendix Q. While we focused on a specific case study company with respect to historical data, the same conclusions hold for more general inventory control situations.

- Causes of the inventory control problem were found to be a combination of the environment for webshops to work in and the fact that inventory control is often overlooked when companies are rapidly increasing. In an e-commerce environment, backorders are often not accepted resulting in lost-sales when demand exceeds the stock on hand. The complex nature of demand and the fact that backorders are often not accepted, results in high inventory levels or more lost-sales than desired.

- We described how the aggregate joint replenishment problem can be approached by splitting it up in n single-item replenishment problems with a supplier aggregate review period to coordinate the ordering process. We concluded that the single-item problem solution concept should be of a periodic nature because the aggregate review period is the controlling variable in the joint replenishment process.

- With respect to the lost-sales problem we developed a method to take into account the demand in periods without stock based on the assumption that demand in periods without stock on hand follows the same demand pattern as demand in periods without stock on hand. Within this method two options can be chosen: (1) impute Gamma distributed demand on the days without stock on hand based on the days with positive stock on hand or (2) impute demand based on the last 10 to 30 days with positive stock on hand on the days without stock on hand.

- Relevant KPIs were found to be the (target) fill rate (implicitly taking shortage cost into account), demand during lead-time, demand during the review period, expected average stock on hand, expected daily holding cost and the expected daily ordering cost.

- Product demand is often highly variable and in some cases probably dependent on seasonality, trends or competitor pricing. Therefore, demand for all products has to be partly based on historical data and partly on demand forecasts.

- The expected average stock on hand approximation by Silver et al. (1998) often underestimated the actual average stock on hand.

- The fill rate calculations and other relevant parameter calculations of both the (R^{δ}, S_i) and $(R^{\delta}, \mathrm{s_i}, S_i)$ replenishment policy were found accurate under (highly) variable stochastic demand and stochastic lead-times; under highly variable stochastic lead-times the fill rate calculations somewhat underestimated the actual experienced fill rate.

- The inventory cost calculations of both replenishment policies in the new model were found accurate under (highly) variable stochastic demand and stochastic lead-times. Both the holding cost and the ordering cost function provides good approximations for the expected inventory cost. Therefore, we can conclude that both policies minimize total inventory cost under a target fill rate.

- In situations with highly variable demand the iteration method by de Kok (2002) does not find the appropriate reorder level s_i because the iteration process is not large enough. In our new model we extended this method to search in a larger interval such that is it more reliable in finding the appropriate reorder level. It searches in the interval $s_i = \left[0;\ 60 * E\left[D(0, L]\right]\right]$ and performs 80 iterations in calculating a reorder level that achieves the set target service level instead of 20 iterations.

- We showed that the achieved ready rate and fill rate are equal under an (R, S) replenishment policy where $CV_R = 1$ if demand follows a compound renewal process with Poisson distributed demand inter-arrivals and Gamma distributed demand sizes. In the literature this was only showed for Poisson, Compound Poisson or Normally distributed demand. Furthermore, we concluded that that the more variable demand in a replenishment cycle (i.e. $CV_R > 1\ to\ CV_R \gg 1$), the larger the difference between the achieved ready rate and the achieved fill rate (i.e. $P_2 < P_3$).

- If shortage cost have the dimension cost per unit per time unit, the P_3 target service level can be specified as: $P_3 = P\{X_i > 0\} = \frac{b_i}{b_i+h_i}$. Under this target P_3 service level, the inventory cost can be minimized in a backorder system.

- The approximation by de Kok (1991) for the expected shortage in a replenishment cycle (i.e. $P_3(S)$) does not seem to provide realistic estimates if demand is (highly) variable.

Conclusions that may be drawn on the performance of the new model compared to the Optiply model and the replenishment policies of the new model in general are:

- In both theoretical and practical situations the undershoot variable should be taken into account, not only for the (R^δ, s_i, S_i) replenishment policy but also for the (R^δ, S_i) replenishment to achieve the target fill rate (this was not done by Optiply before).

- In almost all simulation situations, both replenishment policies in the new model outperformed those in the Optiply model with respect to achieved fill rates, total inventory cost and reliability (i.e. tighter 95% confidence intervals); note that the cost efficiency is highly dependent on how the different cost parameters are set.

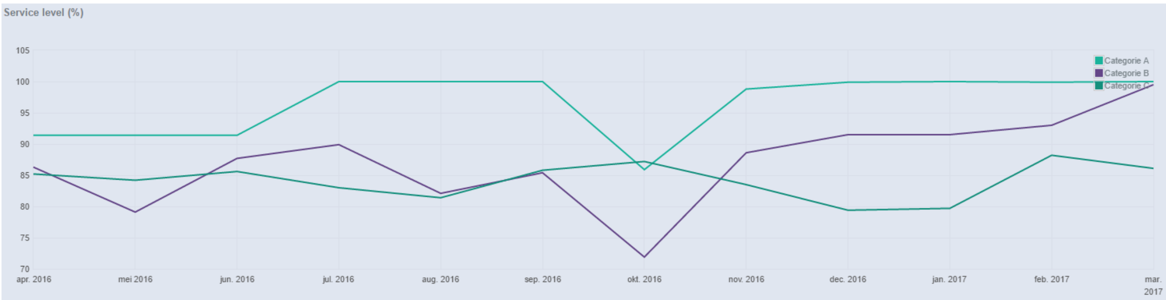
- In almost every simulation situation, the (R^δ, S_i) replenishment policy outperformed the (R^δ, s_i, S_i) replenishment policy regarding achieved fill rates, total inventory cost and reliability (i.e. tighter 95% confidence intervals); note that the cost efficiency is highly dependent on how the different cost parameters are set.

- Both policies in the new model performed reasonably under 'real demand data' in the simulation analysis with respect to achieved fill rate and cost efficiency, especially taken into account that replenishment was performed without new data information in between and was simulated for a 3 month horizon.

- The developed decision support tool uses historical sales/demand data and potential demand forecasts as input to provide appropriate inventory control parameters and inventory cost such as the reorder level, holding cost and ordering cost. The tool is able to perform this calculations for as many products and supplier as desired.

Summarizing, we approached a difficult joint replenishment problem in a lost-sales system under stochastic lead-times and uncertain demand, and split it up in smaller easier to solve problems. We then solved these problems individually and combined them back into the aggregate joint replenishment problem. The main (business) contributions we have made during this research are: the development of a method to determine the supplier review period and product order quantities for coordinated joint replenishment, two methods to estimate unobserved demand under certain assumptions, the development of a model which describes all relevant KPIs and parameters of the inventory control situation, a decision support tool associated to the model that may assist in making inventory control decisions and is able to calculate relevant output parameters for as many products as desired at the same time, the application of existing theoretical replenishment rules and new practical solutions in an e-commerce environment, a simulation tool that allows for test product and real product simulations under the (R, S) and (R, s, S) replenishment policies (which may be utilized by Optiply or other future graduation students).

Lastly, we would like to share a graphical representation of the service levels of Company B throughout the last 12 months. In October 2016 Optiply implemented our Impute Demand function and in January 2017 Optiply implemented our Review period determination method and the Determine output parameter function which, among other things, takes into account the undershoot and suggest appropriate s-level setting. 'A picture says more than a thousand words'.



7.2 Recommendations

Based on our performed research and gained insights, we would like to make the following recommendations to Optiply:

1. It is recommended to use the decision support tool to assist in the decision making process regarding the supplier review period and the product economic order quantities. Using the tool will lead to rational setting of the order moments to every supplier and calculate the appropriate economic order quantity of every product ordered to that supplier. Moreover, in the case study, the tool led to shorter review period which makes the inventory system more flexible to changes in demand or lead-times.

2. The tool is recommended to be used for all products in the assortment. The tool suggests reorder levels, order-up-to levels and when and how much to order in every replenishment cycle based on the used forecast method and historical data. Therefore, less manual calculations or judgmental decisions are required, making the inventory control decision process and related calculations more tractable. Although many products have non-stationary demand in the e-commerce environment, the decision support tool can give a good estimate of the adjusted fill rate, the holding cost, ordering cost and the expected stock on hand. Especially when the tool is used in combination with a demand forecast method that takes into account seasonality and other trends. Note that the output parameters are long-run approximations.

3. We recommend using the (R^{δ}, S_i) replenishment policy above the (R^{δ}, s_i, S_i) replenishment policy because it results in a higher achieved fill rate which is closer to the target fill rate. Furthermore, it results in lower overall inventory cost and tighter confidence intervals of the parameter estimates (i.e. more reliable). Note that this depends on the cost setting of the company (e.g. high ordering cost may results in a higher total inventory cost, especially under the (R^{δ}, S_i) replenishment policy).

4. The tool is built in such a way that it can easily be generalized such that I can be used in other inventory control situations. The tool is and its replenishment policies is applicable in a wide range of inventory control problems because it takes into account stochastic (highly variable) demand, stochastic lead-times and may function in both a lost-sales system and backorder system. Therefore the tool is advised to be used by Optiply in other company problem situations.

5. We recommend to make use of the two impute demand functions that are integrated into the tool. Using one of these functions results in a smaller probability of underestimating future demand. Furthermore, it is recommended to perform more research on including external data into the process of uncovering unobserved demand. Although this uncovering is a difficult process, this unobserved demand could shed more light on the number of lost-sales. Only then can the real achieved fill rate be accurately determined. Essential information about clicks per webpage or per product (i.e. online visitors looking for a certain product) may be used as an input. If clicks are monitored over a certain time period in combination with the sales in that period (i.e. demand if stock on hand is positive), a conversion rate could be calculated that entails the relation between clicks and sales. In that way demand in periods without stock on hand can be estimated even better resulting in more accurate demand forecasts and improved inventory control.

6. By recommending to use the decision support tool we implicitly recommend to take into account the undershoot in the process of calculating the reorder levels and other output parameters under a target fill rate. Furthermore, it implicitly recommends to increase the iteration method to find the 'optimal' reorder level for every product. This is integrated in the function $DetermineOutputParameters(x)$ and sub function $CalculateReorderlevel(x)$.

7. With the use of a demand forecasting method, comes the problem of having a certain forecast error because forecasted demand deviates from actual experienced demand. We recommend Optiply to conduct further research on integrating the forecast error into their demand forecasting process and relating the forecast error to inventory control. Suggestions based on SES, Normally distributed demand or backorder systems were given in section 4.4. By taking the accuracy of the demand forecast into account, the safety stocks can be adjusted in such a way that target service levels can be achieved with more success. Note that none of the described procedures in our research can be used one-on-one in the situation of

Optiply due to different distributed demand, lost-sales and a much more complex demand forecasting method than SES.

8. Regarding (cost) parameter setting we would like to recommend that Optiply communicates closely with its customers. Holding cost, ordering cost, shortage cost and the target fill rate have a large impact on inventory control because input and output parameters are based on these cost and target fill rate. If customers prioritize low inventory levels for example, more emphasis should be put on the holding cost rate by increasing it. If customers find it important that demand is satisfied, fill rates should be higher which typically leads to higher holding and ordering cost but less shortage.

9. The shortage cost is an important component of the total inventory cost, just as the holding cost and ordering cost. We recommend that Optiply discusses the setting of shortage cost per product closely with their customers. In a lost-sales system without backorders the shortage cost could be set as a cost per unit short and by calculating the holding cost and shortage under specific target service levels, a cost-optimal target service level could be determined. In case a client of Optiply works with a backorder system, the shortage cost could be set as a cost per unit per time unit. Then, the shortage cost can be integrated in the review period and order quantity calculation. The SKU classification by Teunter et al. (2010) which we described in this report would then be a good option to use instead of the standard ABC classification. When the shortage cost is defined as a cost per unit per time unit, the shortage cost could be used in setting a Newsvendor fractile: $P_3 = P\{X_i > 0\} = \frac{b_i}{b_i + h_i}$, instead of the common used fill rate P_2. By taking shortage cost into account the Newsvendor fractile can be determined per product and the reorder level can be set such that it is cost-optimal. Note that setting such a target service level is only enabled in a backorder system.

10. Lastly, we would like to make the recommendation for Optiply to research statistical process control (SPC) for their parameter setting. A method that alerts the user of the model when demand forecasts deviate 'too much' from actual experienced demand such that the user may update the forecast. Based on the updated forecast the user could then adapt certain inventory control parameters such as the reorder levels or order quantities of products to meet the updated expected demand.

7.3 Recommendations for future research

By reviewing the literature on relevant subjects including joint replenishment, lost-sales systems, replenishment models and demand forecasting we gained much insight on what research was performed in these fields and what would be interesting for our research. According to our knowledge, no research was performed before on minimizing the total inventory cost in a single-echelon taken into account joint replenishment with only major fixed ordering cost under highly variable stochastic demand and stochastic lead-times. Adding the fact that the inventory control system in our research entailed a lost-sales system with unobserved demand (due to an e-commerce environment), made our SJRP more than ordinary. We proposed a newly developed inventory control model including the (R^{δ}, S_i) and (R^{δ}, s_i, S_i) replenishment policy. The model performs well under highly variable demand, stochastic Gamma distributed lead-times and in combination with an *ETS* forecasting method. However, the model is not without flaws and in reality, demand is often not stationary, forecasting is subdue to errors and lead-times are not Gamma distributed. Based on our Master Thesis Project, we would like to make the following recommendations for future research in the context of our Operations Management and Logistics field:

1. *Analysis of the relation between demand forecasting and inventory control for the P_2 or P_3 service level in combination with current forecasting methods.* The two replenishment policies in our new model were verified for stationary demand with respect to fill rate calculation, cost calculations and other relevant inventory control parameters. We also performed a scenario analysis where we tested our model under real data and based our input parameters on a demand forecast method (i.e. exponential smoothing state space model). In this process, no extra inventory level precaution was taken with respect to the forecast error. Forecasts can deviate from actual experienced demand and this error should be taken into account in the parameter setting for inventory control. This probably results in setting higher reorder levels to achieve the same target service levels. Among others, much relevant research was already performed by de Kok (de Kok, 1991; Kohler-Gudum & de Kok, 2002; Strijbosch et al., 1997) for periodic replenishment policies. We would like to suggest more research on more practical exponential smoothing methods than the SES method and their effect on the P_2 or P_3 service level and s-level setting. Moreover, the applicability of these methods in a lost-sales system instead of a backorder system needs more attention.

2. *Analysis of different demand forecasting methods.* Due to limited data and limited time to perform our research, we let empirical and practical research on the performance of different forecasting methods out of scope in our project. However, much theoretical research was performed on forecasting in our literature study (Buying, 2016). Other forecasting methods than the *ETS* forecasting method could be analyzed and compared to find answers to the questions: which forecasting method performs best for which demand pattern and results in the smallest forecasting errors? What underlying demand distribution can be used apart from the Gamma, Normal and Poisson distribution in the demand forecasting process (we found in our literature study that the Negative Binomial distribution would be a good option for intermittent demand)? Two new TU/e graduation students at Optiply will perform research in the following directions: (1) New product introductions, where new product demand may be forecasted based on historical demand data from other products and (2) Different demand forecasting methods for replenishment, where different forecasting method will be tested regarding their forecast error and relation to inventory control.

3. *Analysis of shortage cost integration.* Unfortunately, shortage cost per product were not specified clearly by the supervising company nor by the case study company during our

research. We did make an assumption for the shortage cost which states that the shortage cost is equal to the margin of a product (i.e. penalty in the form of an opportunity cost equal to the margin of a product). If the shortage cost are specified as a cost per unit per time unit or a cost per time unit (de Kok, 1991), SKU classifications based on shortage cost, holding cost, ordering quantity and demand could be used instead of an SKU classification based on just demand value and demand volume (Teunter, Babai, & Syntetos, 2010). More research on shortage cost per unit (i.e. type p) in combination with the P_2 and P_3 service level would be interesting.

Appendix A

E-commerce characteristic environment

The characteristics of the environment Company B is working in was briefly described in section 1.3 of this report. The following section will elaborate on these characteristics more extensively.

A.1 Large assortment of products

Webshops tend to have a very large assortment of products in e-commerce. This large assortment often includes all sorts of products with different characteristics with respect to demand volume, demand value and other characteristics. The webshop tries to enlarge its market share by satisfying as many customers as possible with their large assortment. However, having such a wide variety of products results in having products with very slow demand or even intermittent demand. More about demand patterns can be found in appendix A.4.

A.2 Extensive information availability

Information and knowledge within the company (and the supply chain) about products is largely digitalized. Communication and data collection constraints are reduced due to web-based production and procurement of products and services. Data such as demand data, sales data, supplier and product data is easily accessible. The key however, is to retrieve the right data and process this into useful information that can help adjusting the relevant parameters in the inventory control system.

The online customer has an increased amount of digital information available as well. Relevant information about products or services and its price are retrieved with great ease. Prices of products of different e-tailers for example, can be compared very easily. This fact can have both a positive and a negative impact on the webshop that tries to sell its products or services.

Moreover, the relationship between suppliers, retailers and customers may be closer with respect to information sharing. Websites enable companies to keep suppliers and customers informed about developments that concern them in their practices (Turban, King, Lee, Liang, & Turban, 2015).

A.3 Backorders are not rational

In e-commerce, a non-sale (nee verkoop) is equivalent to a lost sale. If a product is out of stock at a certain webshop, the online customer can often easily buy the product at another webshop. Although there may be a slight price increase involved in this purchase, it is often more desired than waiting a longer time period for a backordered product. Therefore, the likelihood that a product is backordered is close to zero in this context.

A.4 Complex nature of demand

Where in traditional logistics demand is often stable and constant, demand in e-commerce may be seasonal, erratic, slow moving, intermittent and/ or lumpy. Erratic demand for example, is difficult to forecast because it is highly variable in demand size, while intermittent demand is difficult to forecast due to long periods of zero demand occurrences (A.A. Syntetos, 2001), (A.A. Syntetos & Boylan, 2005). Intermittent demand is a demand pattern that is commonly experienced in the service part industry where there is a demand for service parts that are needed for repairing broken down vehicles, machines or products (Willemain et al., 2004). Demand patterns such as the ones described above are more challenging to forecast because it proves to be more difficult to fit an underlying lying demand distribution to the demand data.

A.5 Fast growing companies

Many companies in the e-commerce market were fast growing throughout the last decade and the business processes were focused on growth. Some key business processes that grow more important when a company is maturing were somewhat forgotten throughout this time period. Resulting are underdeveloped business processes such as inventory control and replenishment. Companies often estimate their demand based on 'gut-feel' and do not make use of an efficient inventory control system to control their stock levels.

A.6 Just-in-time processes

In e-commerce, customers expect their products delivered very fast. Recent developments are 'one-day delivery' and even 'same-day delivery'. To satisfy these customer demands, webshops must adapt their business processes. These forms of delivery have an impact on the service levels that a webshop can guarantee to their customers and furthermore influence service levels and processes upstream in the e-commerce supply chain. Webshops want their stock minimized even more than traditional companies with examples where the webshop does not have stock at all When a retailer applies drop shipping to satisfy demand, the retailer forwards customers' order to the manufacturer who fills the orders directly to the customers and is paid a predetermined price by the retailer (Khouja, 2001).

Transport is often outsourced and products are produced or procured with a Just-in-time mentality. The production, procurement and purchasing cycles are reduced significantly due to e-procurement, which is the online purchase of supplies, materials, energy, labor and services (Turban et al., 2015). This makes that the total cycle time of a product is shorter than it would be in traditional commerce.

Appendix B

Literature review and solution concepts

B.1 Lost-sales systems

Lost-sales were briefly described in the problem statement of chapter one. Complete lost-sales in is not new to the world but is researched extensively throughout the last decades in a *retail environment*. The irrationality of backordering in some parts of the retail business can best be explained by the example of going to the bakery buying a bread. If the bread is not available at that moment, it seems rather senseless to backorder the bread. One would just buy another bread or go to another bakery.

Corsten and Gruen (2003) show that, in a retail environment, only 15% of all customers delay the purchase of a product if there is a stock-out for their preferred product (i.e. backorder the product). The other 85% of the customers decide to buy another product, buy the product at another shop or not buy the product at all. In all cases of this 85% the excess demand is lost (Corsten & Gruen, 2003). If excess demand is lost instead of being backordered, this is called *complete lost-sales* (Silver et al., 1998) and the resulting inventory control system can be described as a *lost-sales system* (K. Van Donselaar et al., 1996).

Generally the net stock can be described as (Silver et al., 1998):

$$\text{Net stock} = (on\ hand) - (backorders)$$

Backorders are not accepted and therefore:

$$\text{Net stock} = (on\ hand)$$

In a backorder model, the inventory position is used as the main indicator of the inventory status and is given by:

$$\text{Inventory position} = (on\ hand) - (on\ order) - (backorders)$$

In a lost-sales system this becomes:

$$\text{Inventory postion} = (on\ hand) + (on\ order)$$

The inventory position increases when an order is placed to the supplier and is decreases when a demand occurs. Backorders are typically included in the definition for the inventory position. When the demand is lost instead of being backordered, the inventory position does not decrease when the system is out of stock. It no longer holds that the amount of inventory after the lead time equals the inventory position after the order placement minus the demand during lead-time. For an (R, S) replenishment policy this means that:

$X((R_1 + L_1)^-) \neq S - D(0, R_1 + L_1]$

And for an (R, s, S) replenishment policy:

$X((R_1 + L_1)^-) \neq s - U_{1,R} - D(R_1, R_1 + L_1]$

For an (R, S) inventory system this is shown in figure 7. An order is placed to the supplier to bring the inventory position to the order-up-to level S since the inventory position is less than the reorder level s at review moment R. In the backorder model, the inventory level (solid line) is based solely on the inventory position (dashed line) and the demand during lead-time D_L. While in the lost-sales model the inventory position depends on the individual outstanding orders. For general information on the differences and similarities of the standard replenishment policies, please refer to (Silver et al., 1998) and (De Kok, Fortuin, & Donselaar, 2012)

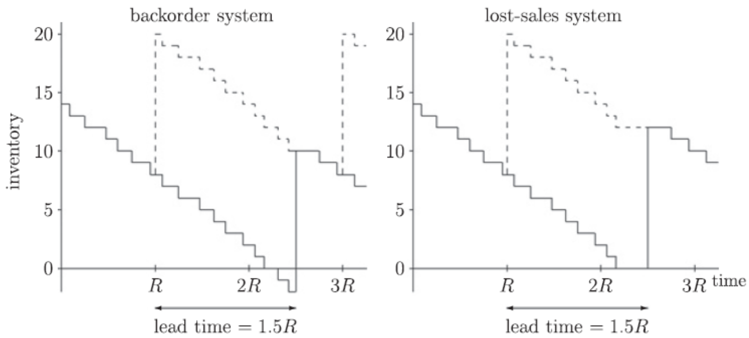


Figure 7: Inventory level under an (R, s, S) replenishment policy in a backorder system and lost-sales system

Contrary to the backorder model, it is not possible to track the changes in the inventory position independently of the on hand inventory level when excess demand is lost. Therefore, a lost-sales model has to keep track of the available inventory on hand and the quantities of the individual outstanding orders that were placed in the past and have not yet arrived. The information vector for a lost-sales model has a length equal to the lead-time and the state space to describe the inventory system increases exponentially fast with the length of the lead-time. This makes inventory models with a lost-sales assumption on excess demand more difficult to analyze compared to models where excess demand is backordered. To keep the analysis tractable, the exact approaches often assume that at most one (or two) order(s) can be outstanding at the same time. In practice, these assumptions cannot always be met.

The backorder model that is described in the paper by Tijms & Groenevelt (1984) is often used in practice. However, the backorder assumption for excess demand is not realistic in a retail environment, especially not in an e-commerce environment where the chance that a potential customer waits for the product to be on stock again is close to zero (Bijvank & Vis, 2011). According to Bijvank & Vis (2011), adding a minimal service level restriction to an inventory model with lost-sales makes the model more realistic to represent a retail environment, but the analysis and computations become more difficult. Hardly any scientific papers are available that studies this problem. Aardal et al. (1989) examine an (s, Q) replenishment policy with a service level constraint.

They show by using Lagrange multipliers that any service level restriction implies a penalty cost for lost-sales and relate the lost-sales model to the backorder model (Aardal, Jonsson, & Jönsson, 1989).

Tijms & Groenevelt (1984) propose a procedure to determine the s-levels of an inventory system under a service level restriction in a backorder model. In their paper both a continuous as well as a periodic review replenishment policy is proposed. They consider a stochastic inventory system in which the sequence of arrivals of demand can be described by a renewal process (i.e. the successive arrivals of demand form a sequence of positive, independent and identically distributed random variables). They define $N(t)$ as the number of arrivals of demand in the interval $(0, t]$ with the counting process $\{N(t), t \geq 0\}$ as a renewal process. The successive demand arrivals for a single product are nonnegative, independent random variables with common probability distribution function F with given mean μ_L and standard deviation σ_L. Successive demand arrivals are assumed to be independent of the process $N(t)$ which generates the demand arrivals. Their periodic review inventory replenishment policy follows equivalent decision rules as an (R, s, S) policy where excess demand is completely backordered and demand follows a renewal process. In the paper $S - s$ is set equal to the EOQ to determine the order-up-to level S and the authors show that their approximation procedure performs well for backordering systems (Tijms & Groenevelt, 1984). Bijvank and Vis (2012) proposed an approximation procedure to determine the order-up-to-level S for an (R, s, S) replenishment policy in a lost-sales system (i.e. where excess demand is lost). They partly follow the procedure of Tijms and Groenevelt (1984) to set the value of the reorder level s, but use the order-up-to level S resulting from an approximation procedure instead of the EOQ (Bijvank & Vis, 2012).

B.2 Joint replenishment

In standard inventory models, the total cost is composed of two parts: (1) The (fixed) ordering cost; the cost of preparing and receiving the order and the transportation cost and (2) the holding cost; the cost of holding inventory which includes the cost of capital tied up in inventory, taxes and insurance.

In the case situation, large numbers of products have to be ordered to a relatively small number of suppliers. Attempting to optimize the replenishment process of every individual product and ordering the products following this optimization may not be the best solution if cost efficiency is considered. A multi-product problem such as this can be described as a *Joint Replenishment Problem (JRP)* and abundant scientific papers are written on the subject (Khouja & Goyal, 2008). Joint replenishment is focused on minimizing cost while satisfying demand. Joint replenishment considering cost efficiency may be helpful in making replenishment decisions such as when and how much to order of which product to which supplier. Within a classic JRP, the cost of placing an order for a number of different products to the supplier has two components: (1) The major fixed ordering cost independent of the number of different products in the order and (2) the minor fixed ordering cost which depends on the number of products in the order. The assumptions of the classic JRP are similar to that of the standard EOQ assumptions. These assumptions include deterministic and uniform demand, no shortages allowed, no quantity discounts and linear holding cost. The classic JRP and its assumptions are defined in **appendix F** and is largely based on Khouja and Goyal (2008).

B.2.1 Stochastic joint replenishment

The *JRP under stochastic demand (SJRP)* involves demand that is stochastic but stationary in the mean and has the objective to minimize the expected total cost per unit time. Two main policies have been most common in the scientific literature on solving such SJRP: (1) *Can-order joint replenishment policies* and (2) *Periodic joint replenishment policies* (Khouja & Goyal, 2008).

Can-order policies are replenishment policies with a must-order level s_i, can-order level c_i and an up-to inventory level S_i. When the inventory position of any item drops to or below its s_j an order is placed to bring its inventory level to S_j and for all items $i \neq j$ with inventory level below c_i, inventory levels are replenished to S_i. This policy is known as the (s_i, c_i, S_i) policy (Johansen & Melchiors, 2003). Note that the earlier described can-order policies can be of a periodic nature as well. Can-order systems such as the (s_i, c_i, S_i) policy are especially focused on the situation where savings in the ordering costs are of primary concern as opposed to achieving a specified total replenishment size, which may be required for a quantity discount purpose. Our attention will be converged to periodic replenishment policies because they are more relevant to our research.

Federgruen et al. (1984) stated that the joint replenishment problem can be seen as n single-item problems which may be combined as one JRP where the joint replenishment cost is part of the total cost functions that should be minimized (Federgruen et al., 1984; Silver et al., 1998). Atkins and Iyogun (1988) developed periodic replenishment policies under unit Poisson demand. Their procedure outperforms the can-order policies and is easier to compute. One of their policies is called the periodic (R, T) policy. In this policy items of a base set are brought up to R at every review interval T, while other items are brought up to their R_i level every $n_i T$ time (as in the classic JRP where it is defined as $k_i T$). Joint cost K are allocated to items so that item i receives $\alpha_i K$ where $\alpha_i \geq 0$ and $\sum \alpha_i = 1$. The problem can be seen as n single-item inventory problem for which every item i incurs fixed cost of $\alpha_i K + k_i$. The minimum common cycle time is termed the *common base period* T (review period) and all items with $\alpha_i > 0$ is called the *base set*. All items in the base set have a common cycle time of length T, all other items is given a period of $n_i T$ closest to its cycle time (with n_i being an integer). For all items, R_i is chosen to minimize expected holding cost and shortage cost during the period $T_i + L_i$.

B.3 s-level adjustments

This section describes two s-level adjustment procedures by de Kok and other scientific authors. Although the adjustments are not fully applicable in our research, we will elaborate on them because they are relevant for the general problem of the inaccuracy of forecasted or simulated demand.

B.3.1 Safety stock correction (Axsäter, 2006; Silver et al., 1998)

A measure of variability of forecasts that is often used in fitting of squared errors of a straight line to the historical data is the mean square error (MSE). The MSE is given by:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(x_t - \hat{x}_{t-1,t}\right)^2 \tag{B.1}$$

where x_t is the actual observed demand and x_{t-1} the period ahead forecast of demand.

The standard deviation of the forecast errors, for the purpose of setting safety stocks, is given by the relation between the true value of σ_1 and the true MSE:

$$\sigma_1 = \sqrt{true\ MSE} \tag{B.2}$$

An estimate of σ_1 (i.e. $\hat{\sigma}_1$) is the square root of the MSE: RMSE. In theory, $(B.2)$ could be used to update the estimate of MSE each time an additional period's information becomes available. However, Silver et al. (1998) propose a simple exponential smoothing updating method:

$$MSE_t = \omega\left(x_t - \hat{x}_{t-1,t}\right)^2 + (1-\omega)MSE_t \tag{B.3}$$

where MSE_t is the estimate of MSE at the end of period t and ω is a smoothing constant. Thereafter, the authors suggest to derive a relation between the standard deviation of the forecast error for the next L periods (in continue replenishment policies) and R+L periods (in periodic replenishment policies) and the demand forecast for the next L or R+L periods. For situations with enough historical data this relationship is given by:

$$\hat{\sigma}_L = L^c \hat{\sigma}_1 \tag{B.4}$$

where:

$\hat{\sigma}_L =$ estimate of std. dev. of forecast errors over a leadtime of L periods
$\hat{\sigma}_1 =$ estimate of std. dev. of forecast errors over one period
$c =$ coefficient that must be estimated empirically

Assuming that the forecast errors in consecutive periods are independent and each has standard deviation $\hat{\sigma}_1$, the estimate $\hat{\sigma}_L$ can be approximated by:

$$\hat{\sigma}_L = \sqrt{L}\hat{\sigma}_1 \tag{B.5}$$

According to Axsäter (2006), the markup on top of the safety stock due to forecasted demand with SES is equal to:

$$M_\sigma, SES = \sqrt{1 + \frac{L\alpha}{(2-\alpha)}} \tag{B.6}$$

with α being the smoothing constant. This markup is increasing in L and σ .

B.3.1 Safety Stock Adjustment Procedure SSAP (Kohler-Gudum & de Kok, 2002)

The technique is based on a netting procedure constructed so that the net requirement process and the replenishment process are independent of the safety stock and that the inventory process satisfies an invariance relation. The procedure does not require knowledge about the demand distribution compared to other traditional inventory models. Inventory levels are controlled according to a Time Phased Order Point (TPOP) policy which means that ordering decisions are made periodically based on the information on so-called net requirements. Prior to the simulation, initial safety stock levels and net stock levels are specified. Within the simulation, at the beginning of each period demand is forecasted over the forecast horizon. Based on the outstanding orders, the current net stock and forecasted demand, net requirements are calculated. The expressions for the different parameters only hold in a system where demand is completely backordered.

For stationary Normally distributed demand, the safety stock $\boldsymbol{\Psi_0}$ is determined as:

$$\Psi_0 = k * \sqrt{L} * \sigma(D) \qquad (B.7)$$

where $\boldsymbol{k}$ is the safety factor depending of the target service level.

In the safety stock adjustment procedure the safety stock $\boldsymbol{\Psi^*}$ level required to achieve the target service level is determined by adding the adjustment quantity to the initial calculated safety stock $\boldsymbol{\Psi_0}$. (Silver et al., 1998). The safety stock is determined 'a posteori' meaning that the safety stock is adjusted retrospectively based on a sample path analysis of the historical data assuming that the demand process and lot sizing decisions will be similar in the future. The only assumption made in the paper is that the historical demand pattern, (i.e. the forecast error pattern), in a stochastic sense represents the pattern to be expected in the future, for which a decision is made on which rules to use on the safety stocks and lot sizing. Therefore, the data does not need be derived from some statistical model. The main constraint of using the SSAP procedure is that it requires using the Time Phased Order Point (TPOP) netting procedure to calculate net requirements and that excess demand needs to be backordered.

B.3.3 Forecasting correction for inventory control (Strijbosch et al., 1997)

The authors show that the actual stock-out probability may greatly exceed the target stock-out probability even in a simple case with stationary Normally distributed demand. In their heuristic for determining the reorder level and/or order-up-to level, they express the order-up-to level as:

$$Z^* = S_t + \left(u_{1-\gamma} - c(\alpha,\omega,\gamma)\right) k(\alpha,\omega,\gamma)\sqrt{V_t} \qquad (B.8)$$

with:

$S_t = \alpha E_t + S_{t-1} = \text{forecasted order} - \text{up} - \text{to level}$
$E_t = X_t - S_{t-1} = \text{last known forecast error}$
$V_t = \omega E_t^2 + (1-\omega)V_{t-1} = \text{variance of the forecast error}$
$\gamma = 1 - P1 = \text{probability of stock} - \text{out}; 1-\gamma = service\ level$
$u_{1-\gamma} \Leftrightarrow \Phi(u_{1-\gamma}) = 1-\gamma; F(\theta_{1-\gamma}) \text{ is known, e.g. } X_i \sim N(\mu,\sigma^2)$
$\alpha = \text{smoothing constant}$
$\omega = \text{smoothing constant}$

By multiple regression the following relations were established:

$$c(\alpha, \omega, \gamma) = 0{,}63\alpha - 0{,}39\omega - 0{,}08\gamma - 2{,}26\alpha\gamma + 1{,}66\omega\gamma \qquad (B.9)$$

$$k(\alpha, \omega, \gamma) = 1 + 0{,}29\alpha + 0{,}75\omega - 4{,}07\omega\gamma \qquad (B.10)$$

B.4 SKU classification

A number of authors have considered the usage of multiple criteria and developed multi-criteria SKU classifications. In a paper by Teunter, Babai and Syntetos, the authors perform an inventory cost analysis and show that for achieving a cost optimal solution, a single criterion is sufficient and simpler. This single criterion takes into account four parameters: demand volume, holding cost, shortage cost and the average order quantity. Across three large datasets, the cost criterion outperformed all other methods in minimizing the safety stock cost, considering target service levels (95% and 99%), and types of demand distribution (Normal and Gamma distribution). Both the demand value and the demand volume criteria more than doubled the safety stock cost in all considered situations. The most common criterion, the demand value criterion, resulted in the worst performance (Teunter et al., 2010).

Different approaches for inventory control may be required for different products in the assortment. A product that is only sold once or twice per year may require a different set of decision rules regarding replenishment than a product that is sold several times per day or per week. Often important decision rules and performance measures guided by a replenishment policy or a demand forecasting method are set for the assortment of products as a whole or for large groups (e.g. ABC classification). Service levels are probably the most important service measure in inventory control (Silver et al., 1998). Target service levels such as the fill rate drive the determination of the safety stocks and therefore the capital invested in inventory and the responsiveness of the inventory system to changes. Setting an appropriate fill rate is difficult and if often based on 'expert opinion'. The historical sales data used for analysis resulted from an inventory system without product classification and without specified fill rates per product group or per product. In the current situation Optiply implemented a classification of products based on demand value and demand volume only, not taking into account costs such as inventory cost and shortage cost. A product that has a high selling price and sells daily may seem attractive to focus on and is classified as an A product. However, if the cost price is relatively high compared to this selling price the margin on the product is low and furthermore, keeping stock of the product is expensive.

Teunter et al. (2016) suggest setting the target fill rate related to the individual SKU level. Safety stocks and ordering calculations are in theory performed at the individual item level, which makes it an intuitive decision to also set a target fill rate on the individual SKU level. In practice, organizations often deal with large numbers of SKUs in their assortment and targets are being set at an aggregate (system) level (Teunter, Syntetos, & Babai, 2016a). For more background information on the papers on the classification criterion and setting target fill rates on an individual SKU level, please refer to (Buying, 2016).

The SKU classification method that was considered to be used in our project is based on the cost criterion proposed by Teunter, Babai and Syntetos (2010). The classification was slightly modified to make it useful in the our situation by using shortage cost for a lost sale instead of using a penalty cost for backordering the product. The criterion then became the following:

$$\frac{b_i D_i}{h_i Q_i} \qquad (B.11)$$

with:

$b_i =$ shortage cost of on item of product i

$D_i =$ demand rate for product i

$h_i =$ inventory holding cost of product i

$Q_i =$ order quantity for product i

The cost criterion can be applied through the following these steps:

(1) *Rank all SKUs in descending order of* $\frac{b_i D_i}{h_i Q_i}$
(2) *Divide the SKUs into classes A,B and so on. The best results are given by using increasing class sizes of 20%, 30% and 50% for three classes and 4%, 7%, 10%,16%, 25% and 38% for six classes. Six classes always lead to better results regarding safety stock cost.*
(3) *Fix the target service level for each class, where A should have the highest service level, followed by B, and so on.*

(4) *An extension based on* (Teunter et al., 2016a) *could be: Fix the target service level for each SKU, where A should have the highest service level, followed by B, and so on. This process is time consuming and therefore products are often classified in groups.*

Teunter et al. (2010) show that using their SKU classification method classifying the products in the assortment and specifying fill rates to every product class accordingly leads to significant cost reductions compared to using the standard ABC classification (Teunter et al., 2010). Not only the fill rate could be specified differently for the various product classes. The type of replenishment policy utilized per product class may be specified differently as well. Silver (1998) for example suggests, in case of periodic review, using an (R, s, S) replenishment policy for higher classified products and an (R, S) replenishment policy for lower classified products. In the case situation shortage cost are not specified in such a way that the cost are paid as a percentage over a Euro per time unit. Moreover, showing that the Teunter classification performs better than the standard ABC classification is difficult and subjective. Fill rate setting remains a judgmental aspect of inventory control.

Ranking SKUs from inventory is typically based on demand value and demand volume (ABC classification). These classes are then receiving a certain service level based on their ranking. Service level targets determine safety stocks and hence, inventory investments. In practice, these service level targets are often not related to individual SKUs, while safety stock and ordering calculations are performed at the individual level. Based on this, it should be intuitively interesting to give distinctive items different treatments. In a yet unpublished study by Teunter, Syntetos and Babai that was requested for analysis, this problem is addressed (Teunter, Syntetos, & Babai, 2016b).

B.5 Process control

The term statistical process control (SPC) is often used in a quality and reliability engineering environment (e.g. (Jiang, 2015) or in a supply chain environment where the bullwhip effect is the primary subject (Costantino, Di Gravio, Shaban, & Tronci, 2015, 2016). SPC can be applied to inventory monitoring purposes according to Watts et al. (1994) and Pfohl et al. (1999). The key objective of SPC inventory management is to use historical inventory and demand data to optimize replenishment ordering and inventory levels in the future. When calculating replenishment orders, SPC takes into account the deviations of expected demand in the future periods and the variations in lead-time (Pfohl, Cullmann, & Stolzle, 1999). Lee & Wu (2006) developed an SPC-based inventory system based on the research of Pfohl et al. (1999). In their system, the replenishment quantities are adjusted dynamically according to a set decision rules. These rules trigger the changing of replenishment quantities and communicate high or low alert levels for reviewing inventory and demand.

B.6 Demand forecasting & underlying demand distribution

Comparing different demand forecasting methods and their performance under different demand patterns is done in research by Syntetos (2001) and Syntetos & Boylan (2005). In this research different demand patterns are categorized and the best fitting demand forecasting method is assigned accordingly. Most recent work is by Babai et al. (2014) where several demand forecasting methods are tested. These methods are all modifications of Croston's method and apply exponential smoothing separately to the inter-arrival intervals of demand and the size of demand when it occurs (Babai, Syntetos, & Teunter, 2014). Automatic forecasting for large time series that takes into account trend, seasonality and other characteristics of the demand data without the need for human intervention is studied by Hyndman et al. (2002). Moreover, relevant research on state space models exponential smoothing is also carried out by R.J. Hyndman. In collaboration with other researchers, he provides a combination of all material related to innovations state space models and exponential smoothing and other forecasting methods in forecasting time series data (R.J. Hyndman, Koehler, Ord, & Snyder, 2008).

Relevant work on fitting underlying demand distributions to non-normal demand patterns, especially intermittent demand, is performed by Syntetos (2001), Syntetos (2012), Heinecke et al. (2013) and Syntetos et al. (2015). These studies result in a Negative Binomial Distribution (NBD) and a Gamma distribution to be appropriate fitting underlying compound distributions to the demand data in both theoretical and real world testing (Heinecke et al., 2013; A.A. Syntetos, 2001; Aris A. Syntetos, Babai, & Altay, 2012; Aris A. Syntetos et al., 2015).

Appendix C

Detailed deliverables

The inventory control system in the case situation is a lost-sales system. Some issues have to be taken into account about demand, service levels and other relevant inventory control components. An assumption or model has to be made that takes into account the demand during the periods without stock on hand. Historical demand data is not available because demand in periods without stock is unobserved. Therefore, only sales data is available and sales data equals demand data in the periods with positive stock on hand. The main issue is can be described as:

Determining appropriate inventory levels for many different products which are ordered to a small set of suppliers, taking into account unobserved stochastic demand and stochastic lead-times in a lost-sales system.

This main issue can be split into three problem components:

1. *Taking into account lost-sales (i.e. unobserved demand)*
2. *Which replenishment model(s) to use for the $\boldsymbol{n}$ replenishment problems*
3. *Coordinating replenishments by joint replenishment*

The first action is to:

(1) *develop a solution for the fact that excess demand is completely lost and there is only sales data available for analysis. Note that this solution has to be combined with certain demand distributions for the inter-arrival times and the size of demand when it occurs.*

Service levels are often set for two or three SKU groups based on demand value and demand volume. Using the $P3$ service level, a service level could be set per SKU based on the holding cost and shortage cost of that product. Therefore a second action is to:

(2) *Set target service levels on an SKU level. Research the utilization of the $P3$ service level (i.e. ready rate) compared to the $P2$ service level (i.e. fill rate).*

If the required demand data is made available by a correction method, the following step is deciding on an appropriate replenishment policy and type of joint replenishment:

(3) *determine what type of replenishment policies may be considered and develop a policy or heuristic that can be used for a lost-sales system in a joint replenishment situation. Extensions of the standard assumptions such as single-item, fixed lead-time, fixed review period and deterministic demand are:*
   (i) *Joint replenishment with only major fixed ordering cost.*
   (ii) *Stochastic demand, inter-arrivals that follow a compound renewal process with Gamma distributed demand size (i.e. non-negative).*

(iii) Stochastic lead-times that vary per supplier.
(iv) Review periods may be based on demand of products ordered to the same supplier.

For the company to use the solutions to the inventory control problems, the conceptual model should be used as input for:

(1) development of a decision support tool that includes a modified replenishment policy with one or more of the above described components. The decision support tool should first be adapted and improved for the case situation.
(2) the decision support tool has to be generalized such that it can be used other inventory control situations in environments with similar characteristics.

Summarizing this section, the deliverables that will be completed throughout the Master Thesis Project are the following:

(1) A decision support tool that assist in making inventory control decisions in an environment that is characterized by a broad assortment of products, uncertain long tail demand, variable lead-times and where excess demand often results in lost-sales.
(2) A simulation report including the verification, validation and performance of the decision support tool.
(3) An implementation plan that describes how to implement and use the decision support tool.
(4) A Master Thesis that elaborates on the complete process of developing the decision support tool.

Requirements from Company B

Table 10 shows the requirements from the case study company for their inventory control.

Table 10: Requirements of Company B considering an inventory control system

Requirement	Brief description
Service level specification	Define total P_2 service levels (fill rate) and possibly define a fill rate specified on SKU group level or individual SKU level.
SKU classification	Implement product classification such as ABC classification or another self-developed classification framework.
Reduce inventory levels	Reduce inventory levels and thereby lower excess stock while keeping customer satisfaction the same or increasing it and keep costs the same or lower them.
Set fixed reviewing period	Reviewing product inventories once a week if ordering processes have to be performed manually.
Process automation	Automate the replenishment process to cut (personnel) costs and increase control of the exponential growth of the company.

Appendix D

Methodologies

According to van Aken (2005), knowledge produced by academic management can be of a descriptive or a prescriptive nature. The development of descriptive knowledge is generally theory-driven focusing on existing situations. The development of prescriptive knowledge however, is more field-problem driven and solution-oriented. It involves research in the called *design sciences* and it analyzes alternative courses of action in dealing with certain organizational problems. The typical research product in a design science is the *technological rule*, a set of general knowledge linking an intervention with an expected outcome or performance in a certain field of application. In other words, this technological rule is the solution concept for a certain organizational problem or even a set of problems (Van Aken, 2005). This research is based on this design sciences and has the objective to develop prescriptive knowledge in the form of a set of solution concepts. To accumulate the required design knowledge, the *reflective cycle* as proposed by van Aken (2004) and extracted from van Aken et al. (2012) will be used in this research (Van Aken, Berends, & Van Der Bij, 2012). Figure 8 represents the reflective cycle as proposed by Heusinkveld and Reijers (2009). As can be seen in the figure 12, selected problem cases will be addressed based on the *regulative cycle* by van Strien (1997), which involves a structured organizational problem solving process (Van Strien, 1997). The output of the regulative cycle involves a theory of practice, a so called *mini theory*. This theory is applicable to the individual case $N = 1$.

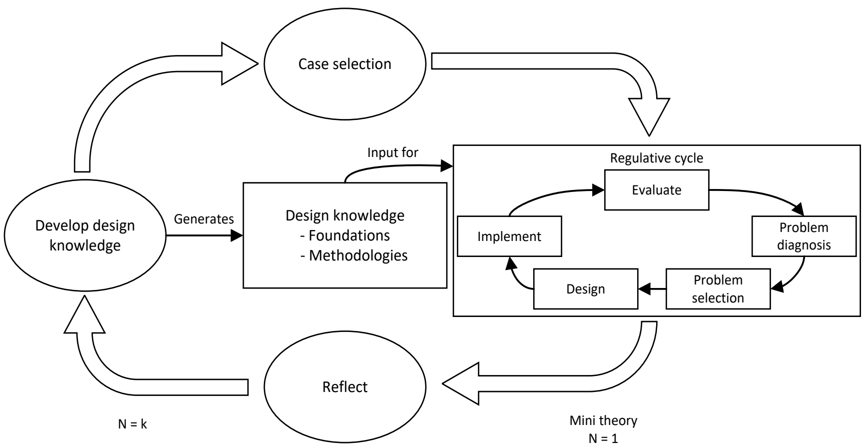


Figure 8: The reflective cycle including the regulative cycle (Heusinkveld & Reijers, 2009)

These organizational designs and interventions are studied by evaluating and classifying a set of selected and successful $N = 1$ theories. In the process of reflecting, these $N = 1$ theories may be generalized to $N = k$ theories, such that they can be applied in a wider context (Heusinkveld & Reijers, 2009). The methodology that will be used for the quantitative part of this research is the updated operations research model by Mitroff et al. (1974) as suggested by Bertand and Fransoo (2002). The model is represented in figure 9. The classic operations research model was first proposed by Sagasti and Mitroff (1973). The model can be used for quantitative research where a mathematical model is developed. In this model, the operational research approach consists of four structured phases (Mitroff, Betz, Pondy, & Sagasti, 1974):

(1) Conceptualization. In the conceptualization phase, a conceptual model of the problem and system is made. Decisions are made about the variables that need to be included in the model and about the scope of the problem and the model.
(2) Modeling. In the modeling phase, the quantitative model is built and causal relationships between the variables are established. The controllable and uncontrollable variables are defined in exact operational terms.
(3) Model solving. In de model solving phase, the quantitative model is solved for one or more situations defined in the scope of the research.
(4) Implementation. In de implementation phase, the quantitative model can be implemented and conclusions can be formed. Based on the conclusion, recommendations can be made to the company or for potential future research.

Other important steps are validation, where the scientific model is compared with reality and its degree of fit (accuracy) is established and feedback, where the relevance of the solutions can be tested by comparing them with the initial conceptualization of the problem situation.

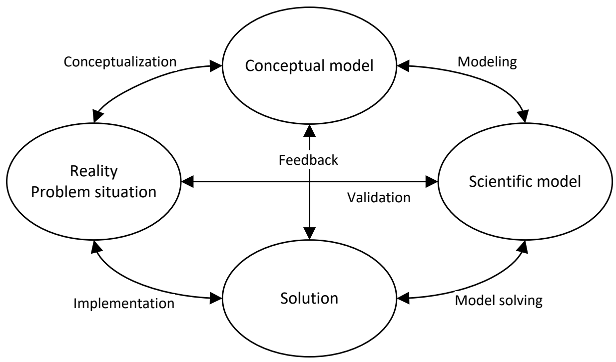


Figure 9: Operations research model (Mitroff et al., 1974)

The research methods that will be conducted in practice are:

- Desk research. A literature study will be performed on relevant scientific papers to the project. Furthermore, data from within the supervising and case study company will be analyzed. This research will be performed partly in phase 1 and partly in phase 2.
- Field research. Multiple meetings and interviews will take place to gain more insight in the issues of current inventory control. This research will be performed throughout the first 3 phases of the project.
- Modeling. A conceptual (theoretical) model will be developed during phase 2 of the project.
- Simulation. A decision support tool will be developed that is tested by simulation. By simulation a sensitivity analysis can be performed such that the impact of the tool and its parameters can be measured.

Appendix E

Single-echelon inventory model

E.1 Definitions

Variables and expressions largely extracted from de Kok et al. (2012). The subscript i is only used in situations with more than one product ordered to the same supplier, making the problem a joint replenishment problem

i	$:=$	i^{th} product
$X_i(t)$	$:=$	stock on hand of product i at time t
$Y_i(t)$	$:=$	inventory position of product i at time t
W_i	$:=$	sales per customer or sales per period of product i
D_i	$:=$	demand size per order or per period for product i
$D_i(t_1, t_2]$	$:=$	demand for product i during the interval $(t_1, t_2]$; $(t_1, t_2] = \{x \mid t_1 < x \leq t_2\}$
s_i	$:=$	reorder level of product i
S_i	$:=$	order up to level of product i; equals $s_i + Q_i$
Q_i	$:=$	order quantity of product i
τ_i	$:=$	$\mathrm{i^{th}}$replenishment order moment after time $\mathrm{t} = 0$; $(\mathrm{i} = 1,2, \dots)$
L_t	$:=$	moment of delivery of order placed to a supplier at time t; L_0 = first delivery moment of order placed at time 0
L	$:=$	lead time of order in days
$B(t_1, t_2]$	$:=$	demand for product i backordered during the interval $(t_1, t_2]$
R_t	$:=$	review moment with a potential order to a supplier at time j; R_0 = first review moment at time 0
R^δ	$:=$	time between successive review moments with a potential order to supplier δ; review period
A_i	$:=$	inter arrival time of demand orders for product i in days
ss_i	$:=$	expected stock on hand of product i immediately before the arrival of an order; safety stock
$U_{i,t}$	$:=$	undershoot of product i at time t

c_i	$\coloneqq$	cost price of one product i
h_i	$\coloneqq$	holding cost rate for holding inventory of one product i ($\%/€/\ time\ unit$)
K	$\coloneqq$	major fixed ordering cost of placing an order to a supplier
b_i	$\coloneqq$	penalty cost for a one unit shortage of product i
$P\{\dots\}$	$\coloneqq$	Probability $\{\dots\}$
$E[\dots]$	$\coloneqq$	Expectation$[\dots]$ (first moment)
$\sigma^2(\dots)$	$\coloneqq$	Variance$(\dots)$ (second moment)
x^+	$\coloneqq$	$max(0, x)$
$x(t^-)$	$\coloneqq$	x just before time t ➔ $x(t^-) = \lim_{t \uparrow t^-} x(t)$
$\bar{x}$	$\coloneqq$	the average of variable x
$x \mid y$	$\coloneqq$	x holds, given that y; x is dependent on y
$f(x)$	$\coloneqq$	probability density function of random variable X
$F(x)$	$\coloneqq$	cumulative distribution function of random variable X
P_1	$\coloneqq$	probability of not being out of stock just before a order arrives
P_2	$\coloneqq$	long run fraction of total demand, which is being satisfied directly from stock
μ_{D_i}	$\coloneqq$	average demand per period of product i
μ_{L_j}	$\coloneqq$	average lead time in time units
$D_i^d(t) = \frac{\sum_{j=1}^{N} D_i^d(t-j)}{\sum_{j=1}^{N} I_{x_i(t-j)}}$	$\coloneqq$	average demand of the last N days before time t with positive stock on hand with a given N
σ_D^2	$\coloneqq$	standard deviation of demand per period
i	$\coloneqq$	$1,2,\dots,n$; product index
j	$\coloneqq$	$1,2,\dots,\infty$; days

E.2 Generic expressions for single-item single-echelon replenishment policies

P_1 service level	$\coloneqq$	$P_1 = \{X((\tau_i + L_1)^-) \ \geq 0\}$
P_2 service level	$\coloneqq$	$P_2 = 1 - \dfrac{E\big[B(L_0, \tau_1 + L_1]\big]}{E\big[D(L_0, \tau_1 + L_1]\big]}$
Expected shortage in a replenishment cycle	$\coloneqq$	$B(L_0, \tau_1 + L_1] = \big(-X((\tau_1 + L_1)^-)\big)^+ - \big(-X(L_0)\big)^+$
Expected demand in a replenishment cycle	$\coloneqq$	$D(L_0, \tau_1 + L_1] = X(L_0) - X((\tau_1 + L_1)^-)$
Expected stock on hand in a replenishment cycle	$\coloneqq$	$E[X] = \dfrac{1}{2}(E[X(L_0)] + E[X((\tau_1 + L_1)^-)])$
safety stock	$\coloneqq$	$v = E[X((\tau_1 + L_1)^-)]$

Appendix F

Classic joint replenishment problem (JRP)

Joint replenishment is focused on inventory decision making and aims at minimizing cost while satisfying demand. The joint replenishment problem involves the coordination of when and how much to order at which supplier and is a problem that exists in every logistics environment where more than one different product is ordered to the same supplier. Additionally, fixed ordering cost are incurred with every order placed to the supplier. Joint replenishment models and method have successfully been utilized in different sectors and types of industry (e.g. spare parts, retail), which could make it a useful method for our joint replenishment problem in an e-tailing environment (Khouja & Goyal, 2008).

F.1 Definitions

Variables and expressions are largely extracted from Khouja and Goyal (2008) and are partly redefined for clarity.

T	$:=$	time between successive replenishments (years)
O	$:=$	major fixed ordering cost associated with each replenisment (€/year)
TC	$:=$	total annual holding and ordering costs for all the products (€/year)
i	$:=$	$1,2,\dots,n;$ a product index
n	$:=$	number of products
k_i	$:=$	k_i^{th} review moment with a potential order for product i
K	$:=$	set containing all k_i; $K = \{k_1, k_2, \dots k_n\} \in \mathbb{N}^n$
D_i	$:=$	annual demand for product i (units/year)
h_i	$:=$	annual holding cost of product i (€/unit/year)
o_i	$:=$	minor fixed ordering cost incurred if product i is ordered in a replenishment (€/order)
Q_i	$:=$	order quantity of product i
T_i	$:=$	time interval between successive replenishments of product i (years)

Strategies for solving the JRP can be classified into two types: (1) a *direct grouping strategy* (DGS) and (2) an *indirect grouping strategy* (IGS). Under DGS, products are partitioned into a predetermined number of sets and the products within each set are jointly replenished with the same cycle time. Under IGS, a replenishment is performed at regular time intervals and every product has a replenishment quantity sufficient to be enough for exactly an integer multiple of the regular time interval.

Under IGS the cycle time for every product is an integer multiple k_i of the replenishment cycle time T. Hence, the cycle time for product i is:

$$T_i = k_i T \tag{E.1}$$

The order quantity for product i is:

$$Q_i = T_i D_i = T k_i D_i \tag{E.2}$$

The total annual holding costs are:

$$C_H = \sum_{i=1}^{n} \frac{Q_i h_i}{2} = \frac{T}{2} \sum_{i=1}^{n} k_i D_i h_i \tag{E.3}$$

The total annual fixed ordering costs are:

$$C_O = \frac{O}{T} + \sum_{i=1}^{n} \frac{o_i}{k_i T} = \frac{\left(O + \sum_{i=1}^{n} \frac{o_i}{k_i}\right)}{T} \tag{E.4}$$

In equation $(E.4)$, cycles without replenishments (i.e. $k_i \geq 2, i = 1,2, \dots, n$) still incur a major fixed ordering cost of O.

The total annual cost are:

$$TC(T, K) = C_H + C_O = \frac{T}{2} \sum_{i=1}^{n} k_i D_i h_i + \frac{\left(O + \sum_{i=1}^{n} \frac{o_i}{k_i}\right)}{T} \tag{E.5}$$

where K is a set of integer multipliers. The policies defined by the basic cycle time and a set of multipliers are can be described as cyclic policies. There are two classes of cyclic policies for the JRP. Let ρ_c be the set of cyclic policies which can be expressed as:

$$\rho_c \coloneqq (T, K) : T > 0 \qquad K = (k_1, k_2, \dots, k_n) \in \mathbb{N}^n \tag{E.6}$$

A cyclic policy is called a strict cyclic policy ρ_{sc} if at least one product i has an integer multiplier $k_i = 1$, which mean it is included in every order. Therefore, $\rho_{sc} \subseteq \rho_c$ and ρ_{sc} can be expressed as:

$$\rho_{sc} \coloneqq (T, K) : T > 0 \qquad \begin{array}{l} K = (k_1, k_2, \dots, k_n) \in \mathbb{N}^n \\ k_i = 1 \; for \; some \; 1 \leq i \leq n \end{array} \tag{E.7}$$

This boils down to two optimization problems:

$$\min_{(T,K) \in \rho_{sc}} TC(T, K) \tag{JRPSC}$$

$$\min_{(T,K) \in \rho_c} TC(T, K) \tag{JRPC}$$

Arkin et al. (1989) proved that the JRP is an NP-hard problem and there probably is no polynomial time algorithm to solve the JRP. For a fixed $K = (k_1, k_2, \dots, k_n) \in \mathbb{N}^n$, the optimization problem becomes:

$$\min_{T>0} TC(T, K) \qquad (JRPT)$$

For a given $T > 0$, the optimization problem can be expressed as:

$$\min_{T>0} TC(T, K) \qquad (JRPK)$$

Solutions to both optimization problems are provided in Goyal (1974). Goyal also developed an algorithm to find the optimal solution to minimize $(B.5)$. In other words, Goyal's approach results in an optimal solution to the JRPSC. However, it may be computationally prohibitive for large problems.

F.2 Assumptions

The assumptions of the classic JRP as by Silver et al. (1998):

1. The demand rate of each item is constant and deterministic.
2. The replenishment quantity of an item need not be an integral number of units.
3. The unit variable cost of any of the items does not depend on the quantity; there are no discounts in either the unit purchase cost or the unit transportation cost.
4. The replenishment lead-time is zero; The extension with a fixed, known, nonzero lead-time that is independent of the magnitude of the replenishment, is easily made.
5. The entire order quantity is delivered at the same time.

Appendix G

Solution concept information

G.1 Lost-sales solution concept information

In this section we elaborate on the solution concepts for the lost-sales problem briefly described in chapter four.

G.1.1 $\boldsymbol{P_2}$ service level in a lost-sales system

The excess demand that is lost due to the fact that backorders are not accepted is important for the total cost function in the form of shortage cost and to the service levels; the P_2 service level in particular. The P_2 service level is the long run fraction of demand that is satisfied directly from stock; i.e. excess demand in our lost-sales system is the fraction of demand that is not satisfied directly from stock and therefore lost. Van Donselaar and Broekmeulen (2013) reason that in a lost-sales system the average sales will be lower than in a model in which excess demand is backordered. As a result the average stock on hand and the service level in a lost-sales model will be higher than in a similar backordering system. The different methods to correct the P_2 service level in a lost-sales system based on a backorder system are described hereunder.

To approximate the P_2 service level in a lost-sales system the simplest assumption would be that the P_2 service level of a lost-sales system is equal to that of a backorder system and is called the *P2BO-approximation* (K. H. Van Donselaar & Broekmeulen, 2013):

$$\hat{P}_2^{LS} = P_2^{BO} \tag{G.1}$$

Silver and Peterson (1985) suggest that this is a reasonable approximation if the customer service level is high. An improvement on this first approximation was proposed by Silver and Peterson (1985) for continuous review lost-sales systems with an (s, Q) replenishment policy and by Tijms and Groenevelt (1984) for the (s, S) replenishment policy. The approximation of the fill rate in a lost-sales system is called the *P2SPTG-approximation* (P_2^{LS1}) and is suggested to be (Tijms & Groenevelt, 1984):

$$\frac{1 - P_2^{LS1}}{P_2^{LS1}} = 1 - P_2^{BO} \Leftrightarrow P_2^{LS1} = \frac{1}{2 - P_2^{BO}} \tag{G.2}$$

For the first approximation the fill rate in the lost-sales system is always at least as high as in the backorder system. However, in some situations the difference between the fill rate in the lost-sales system and the fill rate in the backorder system becomes rather large (e.g. P_2^{LS1} is 90% while P_2^{BO} is only 60%). According to van Donselaar and Broekmeulen (2013) the service level in the lost-sales system for a given safety stock is always at least as high as the service level in the backordering system, since the actual sales per period in the lost-sales system (equal to $P_2^{LS1}\mu$) are less than the actual sales in a backordering system (equal to μ). Hence, with the same amount of inventory in the system, due to lower sales the same or a higher service level can be achieved in the lost-sales system.

Two important factors to take into account for determining the fill rate of a periodic lost-sales system are: (1) the uncertainty of demand during lead-time and the review period and (2) the number of outstanding orders. The first can be measured via the CV of the demand during lead-time plus the review period. If demand is identically and independently distributed, the CV is equal to:

$$CV_{R+L} = \frac{\sigma}{\mu\sqrt{R+L}} \qquad (G.3)$$

We described in **section 2.1.1** and **appendix B.1**, that for lost-sales systems it is necessary to take into account the number of outstanding orders and the times at which the orders are placed to the supplier. A simple and intuitive measure to take into account the outstanding orders is the variable nOO, which stands for number of outstanding orders. In case of a periodic fixed quantity replenishment policy this variable is defined as the ratio between the expected demand during lead-time $L\mu$ and an expression for the expected order size ($\max(Q, R\mu)$) (K. H. Van Donselaar & Broekmeulen, 2013):

$$nOO = \frac{L\mu}{\max(Q, R\mu)} \qquad (G.4)$$

This approximation is exact if $Q = 1$, excess demand is backordered and discrete demand in every review period is strictly larger than zero. In case of a periodic order-up-to replenishment policy, the average order quantity is equal to $R\mu$ and therefore:

$$nOO = \frac{L\mu}{\max(R\mu, R\mu)} = \frac{L\mu}{R\mu} = \frac{L}{R} \qquad (G.5)$$

The relationship between P_2^{LS} and P_2^{BO} for a given value of nOO and CV_{R+L} is given by the following relations:

$$P_2^{LS2} = \alpha(nOO) + \beta(nOO) * P_2^{LS3} \qquad (G.6)$$

and

$$P_2^{BO} = \alpha'(CV_{R+L}) + \beta'(CV_{R+L}) * P_2^{LS3} \qquad (G.7)$$

Where P_2^{LS3} is the third and last approximation for the P_2 service level. The parameters α, α', β and β' are the linear regression coefficients to be estimated. P_2^{LS2} is also called P_2^{DoBr} and is found by an iteration process (K. H. Van Donselaar & Broekmeulen, 2013). Important findings based on this relationship are that if $nOO < 5$, the relationship between P_2^{LS} and P_2^{BO} takes an almost linear relationship and the estimates are given by:

$$\beta(nOO) = 0{,}062nOO + 0{,}87$$

and

$\alpha(nOO) = 0,9980 - \beta(nOO)$

This leads to the following approximation for the fill rate in a lost-sales system with $nOO \geq 5$:

$$P_2^{LS3} = P_2^{LS2} - \frac{\alpha(nOO)}{\beta(nOO)} \qquad if\ nOO < 5 \qquad (G.8)$$

$$= P_2^{LS2} - \frac{(0,128 - 0,062nOO)}{0,062nOO + 0,87}$$

However, if $nOO \geq 5$, the relationship is not linear and the estimates are found to be:

$\beta'(CV_{R+L}) = CV_{R+L}^{-0,552} e^{0,279}$

and

$\alpha'(CV_{R+L}) = 1,0172 - \beta'(CV_{R+L})$

This leads to the following approximation for the fill rate in a lost-sales system with $nOO \geq 5$:

$$P_2^{LS3} = P2^{BO} - \frac{\alpha'(CV_{R+L})}{\beta'(CV_{R+L})} \qquad if\ nOO \geq 5 \qquad (G.9)$$

$$= P2^{BO} - \frac{(1.0172 - CV_{R+L}^{-0,552} e^{0,279})}{CV_{R+L}^{-0,552} e^{0,279}}$$

Assume that the target fill rate is at least 95% and the safety stocks are determined using formulas which assume excess demand being backordered. Van Donselaar and Broekmeulen (2013) take all the $P_2^{BO} \geq 95\%$ from their simulation and compare them with the P_2^{LS3} (i.e. the true service level as measured by the service in the simulated lost-sales system). As shown earlier in this section, the difference between P_2^{BO} and P_2^{LS3} depends on CV_{R+L} and nOO. They show that for very high values of CV_{L+R}, both the new formula as the old *P2PSTG-approximation* can be used (i.e. $P_2^{LS1} = \frac{1}{2-P_2^{BO}}$). For lower values of CV_{L+R} (i.e. $CV_{R+L} < 1$), the new formula adds the most value. Furthermore, the higher the value of nOO, the less accurate P_2^{BO}.

Van Donselaar and Broekmeulen (2013) found that if $CV_{R+L} < 0.5$ and $nOO \geq 1$, the service level which results from using the *P2BO-approximation* with a target service level of 95% led to a an actual service level in a lost-sales system that deviated at least 1% from the target service level, meaning it would generate a service level between 96% and 100%. If products come from far (e.g. long distance sea transportation from China) the lead-time is often longer than the review period and in almost all cases with order-up-to levels (i.e. inventory position is brought back to a constant order-up-to level S), $nOO = \frac{L}{R}$ is larger than 1. The CV_{R+L} is smaller than 0,5 is all cases simulated by van Donselaar and Broekmeulen (2013), except for the cases where the lead-time is rather small and the demand uncertainty is large.

G.1.2 Demand uncensoring

According to Vulcano et al. (2012) two important problems in retail demand forecasting are: (1) estimating the turned-away demand when products are out of stock (lost-sales) and (2) properly accounting for substitution effects among related products. If excess demand is lost and forecasts are based on sales data only, the resulting demand forecasts may be negatively biased. This underestimation exacerbates if products are out of stock for a longer period of time. Additionally, stock-out based substitution will increase sales of substitute products that are available (called *recaptured demand*). Not taking into account this recaptured demand may lead to overestimation bias among the available products. The recapture rate in retail can be rather high (e.g. supermarket where the customer chooses another type of potato chips because the one he prefers is out of stock) (Corsten & Gruen, 2003). Spilled and substituted demand is not directly observable from sales transactions. Numerous statistical techniques have been proposed to estimate this demand. These techniques are known as demand untruncation or demand uncensoring methods. One of the most popular methods is the *expectation-maximization* (EM) *algorithm*. EM uses iterative methods to estimate the underlying relevant parameters (product demand across a historical data set in our situation). The EM methods works by using alternating steps of computing conditional expected values of the parameter estimates to obtain an expected log-likelihood function (the E-step) and then maximizes this function to obtain improved demand estimates (the M-step). However, retail forecasts that utilize the EM approach have been limited to uncensoring historical sales data for individual products. The EM method and other newly developed methods are of a high complexity. Furthermore, knowledge is required on the set products customers are considering when making the decision for a certain product (Conlon & Mortimer, 2013). This knowledge is based on the aggregate estimate of the market share of those products (Vulcano, van Ryzin, & Ratliff, 2012).

Additionally, substitution in an e-commerce environment is less likely because the customer performs an online search for a specific product instead of going to a physical store. If the product is not available at a certain webshop the product is often removed from the website(s) of the webshop. Therefore, the product does not show on the internet at all and the customer will not notice that the preferred product is out of stock. Another situation may be that the customer actually sees that the preferred product is out of stock at a certain webshop. In this situation, the customer can go to other webshops with the ease of one mouse click and order the product there. Products that are bought online are often more specific than buying something in the supermarket to put on your bread or buying a pair of pants in a clothing shop.

G.1.3 The impute demand method

In this section we describe an approach developed for estimating the unobserved demand for products in periods without stock. The approach assumes realistic data: observed sales data and stock data such that it is known when products were available and when they were out of stock. There are two important aspects about the demand input parameter to describe in this section.

The first aspect is that the model is based on forecasting the demand for the coming period. This period is currently set to 90 days. There are several reasons for forecasting: (1) because we want to take into account potential trends and seasonal factors of demand per product because demand does not often behave stationary, (2) the more data is fed to the model as an input, the better the demand forecast (i.e. the model is learning from the data). The second aspect is that the demand input for forecasting in the model of Optiply should be a time series with a value at every time unit of the time series. If we would only use the demands (sales) of the periods with positive stock on hand (i.e. point of sales POS data), the model would not work. This POS data only contains the times when sales were made and how many sales were made at that time. We make the POS data a time series by manipulating the data by adding zero sales to every day without sales. For example, if we have had 4 days with sales in the past week, we should impute a value of zero sales to the 3 days without sales to make sure that every day of the input data has a value.

The reasons that we want to say something about the demand in periods without stock are that: (1) we want to take this demand into account to the best of our abilities and (2) because the forecast tends to go to zero if the input demand parameter includes many days with zero demand in the recent past. Demand for such products is variable which results in high safety times set by Optiply (i.e. safety stock expressed in time). Once there is demand for such products after a long period of time, the model picks up this demand and advises to replenish many items of the product because of the high safety times. This results in too much stock on hand and extra costs. Therefore, we want to create a more stable input demand parameter by imputing demand on the days without stock on hand.

The procedure of the method

Let us define:

$D_i^d = \text{daily demand of product } i \text{ (i.e. NOT demand order size as in continuous demand)}$
$j = \text{number of days back in the historical data}$
$X_i(t-j) = \text{stock on hand from every historical date that is used in the calculation}$
$I_{x_i(t-j)} = \text{an indicator function that indicates if the stock on hand was positive on day } j;$

$$\left(I_{x_i(t-j)} := \begin{cases} 1 & if\ X_i(t-j) > 0 \\ 0 & if\ X_i(t-j) = 0 \end{cases} \right)$$

$N = \text{total number of days that is looked back in the historical data}$

Average expected demand data is imputed in the sales dataset on the days without stock on hand (i.e. days where $X_i(t) = 0$). The demand that is imputed is equal to the average of the sales of the last j days with a total value of 30 days where the stock on hand is larger than zero. A value of maximal 30 days is chosen such that any annual seasonal trend can be taken into account. With this seasonal trend we aim for example at the annual returning increasing or decreasing of sales during the summer months or Christmas. Moreover, we only include products in our analysis that have more than 10 days with positive stock on hand. We make the assumption that the demand information of products with less than 10 days with positive stock on hand is insufficient for analyzing. Therefore, the average may be calculated with less than 30 values because sales were not made on every day and sales could only be made on days with stock on hand (this is elaborated later on in this paragraph). The imputed demand for product i at time t can be expressed as

Imputed demand on days without stock can be expressed as:

$$D_i^d(t) = \frac{\sum_{j=1}^{N} D_i^d(t-j)}{\sum_{j=1}^{N} I_{x_i(t-j)}} \qquad \begin{array}{l} for\ t = 1,2,\dots,n \\ for\ j = 1,2,\dots,N \end{array} \qquad (G.10)$$

subject to:

$$10 < \sum_{j=1}^{N} I_{x_i(t-j)} \leq N$$

If the availability is not appropriate $(j < 10)$, the imputed demand is set equal to zero. Furthermore, forecasting for products with too little data gives inaccurate forecasts and special methods for new products introductions should be implemented.

G.2 JRP solution concept information

In chapter four we briefly described the possibilities in solving the joint replenishment problem. This section elaborates more on these possibilities.

Based on the paper of Atkins and Iyogun (1988), an option that was considered is to set a base review period for every supplier δ by using the smallest integer that is divisible by all the optimal review periods of the products i that are in the same order to supplier δ (i.e. least common multiple). Using this procedure the base review period is R and every product i ordered to the same supplier has a review period k_iR with k_i being the k_i^{th} review moment with a potential order for product i.

A first problem is the fact that many of these problem solutions are based on the fact that there exists a major fixed ordering cost and a minor fixed ordering cost per product each time that an order is placed to the supplier. In the case situation there are no minor fixed ordering cost: no extra fixed ordering cost are incurred by adding one extra product to the order to the supplier. Van Eijs et al. (1992) compared solutions of the direct (DGS) and indirect (IGS) grouping strategies Under DGS, products are partitioned into a predetermined number of sets and the products within each set are jointly replenished with the same cycle time. Under IGS, a replenishment is performed at regular time intervals and every product has a replenishment quantity sufficient to be enough for exactly an integer multiple of the regular time interval(Khouja & Goyal, 2008). They measured the solution quality of each strategy by percentage savings in total cost over the total cost of the independent EOQ strategy. The authors found that two factors are important in determining the relative performance of using DGS or IGS: (1) the ratio of the major ordering cost K to the average minor ordering cost $\bar{k}_i$: $\frac{K}{\bar{k}_i}$ and (2) the number of products n. They found that for values of $\frac{K}{\bar{k}_i}$ above 75, DGS and IGS became the same because only a single replenishment group was created. This lets us to believe that if there are no minor fixed ordering cost k_i or if they are very small, the term $\frac{K}{\bar{k}_i}$ becomes very large and the solution will result in placing the products in one replenishment group.

A second problem is that finding the s-levels for every product i is difficult process and in the scientific literature often bounds or at best approximations for the order-up-to level S are realized. The computational complexity increases exponentially when the size of the set of $k_i's$ is increased. In our case situation there are suppliers where hundreds of different products are ordered to. Algorithms that give an optimal solution in a joint replenishment problem with up to 50 items, have a hard time with problems where the number of items increase to 100 or more (Bijvank & Vis, 2011; Wang, Dun, Bi, & Zeng, 2012). Linear programming has infinitely feasible solutions. A Greedy algorithm is efficient if it searches in a continuous spectrum but does not consider most of the feasible solutions. Finding optimal s-levels for 300 products that are ordered to the same supplier with an algorithm such as the Greedy algorithm results in $\mathbf{10^{300}}$ combinations and may not even result in a global optimum.

Review period determination method

Taking into account the fact that there are only major fixed ordering cost (i.e. no minor fixed ordering cost per product) in our JRP and knowing that the current method for coordinated ordering is not fully correct, we propose a new method for determining the review period per supplier. This section describes the determination of this review period which can be used in determining the relevant parameters per product i ordered to the same supplier. The difference with the current method utilized by Optiply is that our method takes into account the costs and demand from the different products i ordered to the same supplier δ by taking the weighted average.

Let us define the subset $V^{\delta} = \{1,2,\dots,i,\dots,n\}$ which contains all the product i that are ordered to the same supplier δ and D_i^d as the daily demand of product i (i.e. NOT demand order size as in continuous demand).

Then the total inventory costs for these products can be expressed as:

$$C(Q) \quad = \sum\nolimits_{i \in V^{\delta}} \frac{h_i Q_i}{2} + \frac{D_i^d K_i}{Q_i} \qquad (G.11)$$

We then substitute the order quantity with the order quantity interval $T_i = \frac{Q_i}{D_i}$:

$$C(T) \quad = \sum\nolimits_{i \in V^{\delta}} \frac{h_i D_i^d T_i}{2} + \frac{K_i}{T_i} \qquad (G.12)$$

We want to place all the different products in one predetermined group because we only have one major fixed ordering costs (Khouja & Goyal, 2008). Therefore, we can write expression $(G.12)$ as:

$$C(T) \quad = \sum\nolimits_{i \in V^{\delta}} \frac{h_i D_i^d T}{2} + \frac{K}{T} \qquad (G.13)$$

If we then rewrite this expression and minimize the total cost by taking the first derivative with respect to T and setting it equal to zero:

$$C(T) = T\left(\frac{1}{2}\sum_{i\in V^{\delta}} h_i D_i^d\right) + \frac{K}{T} \qquad (G.14)$$

$$C'(T) = 0 \quad \Leftrightarrow \frac{1}{2}\sum_{i\in V^{\delta}} h_i D_i^d - \frac{K}{T^2} \qquad (G.15)$$

$$\Leftrightarrow \frac{1}{2}\sum_{i\in V^{\delta}} h_i D_i^d = \frac{K}{T^2}$$

$$T^* = \sqrt{\frac{2K}{\sum_{i\in V^{\delta}} h_i D_i^d}} \qquad (G.16)$$

The T^* can be used as the review period for all the products i ordered to the same supplier. Therefore we state for every supplier δ with product subset $V^{\delta} = \{1,2,\ldots,i,\ldots,n\}$:

$$R^{\delta} = T^* \qquad (G.17)$$

The order quantity for every product i ordered to the same supplier therefore becomes:

$$Q_i^* = R^{\delta} * \frac{E[D_i]}{E[A_i]} \qquad (G.18)$$

Appendix H

Optiply review period on a supplier level

The following method is a description of a method that was implemented in the model used by Optiply. Every product i is ordered to a specific supplier and multiple products may be ordered to the same supplier. The review period for a certain supplier δ is based on historical demand data and is determined by the following method:

1. Calculate the total average annual demand of every individual product i ($TAAD_i$) ordered to the same supplier based on the average inter-arrival time of demand and average demand size in an order of product i:

$$TAAD_i = 365 * \frac{\mathrm{E[D_i]}}{\mathrm{E[A_i]}} \tag{H.1}$$

2. Calculate the total average demand on a supplier basis by summing the annual demand of all products i that are ordered to the same supplier δ ($TAAD^{\delta}$) :

$$TAAD^{\delta} = \sum_{i \in V^{\delta}} TAAD_i \tag{H.2}$$

3. Calculate the average selling price cost price average cost price of the products i that are ordered to the same supplier and calculate the average annual holding cost by multiplying the annual interest rate with the average cost price and the average fixed ordering cost by setting it equal to the fixed ordering cost.

$$\text{avg. selling price supplier } \delta = \frac{\sum_{i \in V^{\delta}} TAAD_i * \text{selling price } i}{TAAD^{\delta}} \tag{H.3}$$
$$\text{avg. cost price supplier } \delta = \frac{\sum_{i \in V^{\delta}} TAAD_i * \text{cost price } i}{TAAD^{\delta}}$$
$$\bar{C}_h = \text{avg. holding cost supplier } \delta = \text{holding cost rate} * \text{avg. cost price supplier } \delta$$
$$\bar{C}_o = \text{avg. fixed ordering cost supplier } \delta = \text{fixed ordering cost}$$

4. Calculate the *economic order quantity* (Q^{δ}) on supplier level based on demand, fixed ordering cost and holding cost and determine the review period on a supplier level:

$$Q^{\delta} = \sqrt{\frac{2 * TAAD^{\delta} * \bar{C}_o}{\bar{C}_h}} \tag{H.4}$$

$$R^{\delta} = \text{review period of supplier } \delta = \frac{Q^{\delta}}{TAAD^{\delta}} * 365 \tag{H.5}$$

5. Calculate the Q_i on product level based on the review period of supplier δ and the inter-arrival time of demand and the demand size for product i:

$$Q_i = R^{\delta} \frac{\mathrm{E[D_i]}}{\mathrm{E[A_i]}}$$

Appendix I

Impute demand data heuristic

Input: Sales_data (POS data), Stock_data

Output: Sales_data_imputed_demand

Function: $ImputeDemanddata(x)$

Note: In case of using the $ImputeGammaDemand$ function instead of the Impute, a Gamma distribution is fit to the product demand on the days with positive stock on hand. Then, demand is imputed on the days without stock on hand based on this Gamma distribution.

1. Create a function available (indicator function) that states the availability of data with positive stock on hand (stock_on_hand).

   Simplified R code:
   ```
   available = ifelse(test = stock_on_hand == 0,
                      yes = 0,
                      no  = 1))
   ```

   Output: $Binary()$

2. Create a function nonzero_lags that counts how many days have had positive stock on hand in the last $N+1$ days (with $N = 30$ and $+1$ for the day itself. $N = 30$ is chosen not larger than 30 days because of the seasonal aspect and not smaller than 10 days so it is not too sensitive for sudden changes or has too little data. This variable cumulatively sums the availability variable over the last $N+1$ days. To do this, it checks back into the last $N+1$ days of historical data; if less than $N+1$ days available: check back into the historical data until the boundary of the data set is reached.

   Simplified R code:
   ```
   nonzero_lags = roll_sum(available, N = min(31, length(!is.na(sales))), fill = 0))
   ```

   Output: $Integer(0,1, \dots N, N+1)$

3. Create a variable demand that represents demand on a specific day.
   - If stock on hand is zero on that specific day and there are more than $j = 10$ historical data points with positive stock on hand in the last 30 days, take the average of the number of sales of those data points and return this average as the demand output of the specific day. If we look at the dataset as a whole, this means that the function that calculates the average is a summation of the moving average (SMA) that rolls over the days of the dataset. The number 10 is to account for noise and makes the imputed demand not too dependent on recent historical data points (i.e. if the last one or two historical data points are outliers in the sense of being very high or very low compared to the average and the value of j is chosen small, the value for the imputed demand is too sensitive to these outliers).

- Moreover, ff stock on hand is zero on that specific day and there are less than 10 historical data points with positive stock on hand in the last 30 days, there is too little data to base a forecast on. Forecasting demand of a product based on a few days results in inaccurate forecasts. If this is the case, the demand on the days without stock on hand is set equal to the sales (i.e. zero).

Simplified R code:

```
demand = ifelse(test = stock_on_hand == 0,
                yes = ifelse(test = nonzero_lags > 10,
                            yes = SMA(x = sales, N = min(30, length(!is.na(sales)))),
                            no  = sales)
```

Output: $numeric()$

4. Lastly, as set of additional checks are performed on the dataset after imputing the artificial expected demand.

- If the stock on hand is not known (NA; i.e. not available), set the demand of that day equal to the number of sales. If the stock on hand is known, set demand equal to demand that was imputed.
- If demand is not known (NA), set the demand of that day equal to zero. If demand is known, set demand equal to demand that was imputed.
- Some sales are entered manually by Company B. Some of these sales are made on days where, according to the system, there was no stock on hand. However, these sales have to be accounted for in the expected demand. Therefore: if the number of sales on a day is larger than zero and the stock on hand is zero as well, we know that this is such a situation and the impute demand can be set equal to the number of sales. In all other situations the demand can be set equal to the imputed demand (which obviously can be equal to the number of sales of that day).

Simplified R code:

```
demand = ifelse(test = is.na(stock_on_hand),
                yes = sales,
                no = demand),
demand = ifelse(test = is.na(demand),
                yes = 0,
                no = demand),
demand = ifelse(test = sales > 0 & (stock_on_hand == 0 | is.na(on_hand_correct)),
                yes = sales,
                no = demand)
```

Output: $numeric()$

Appendix J

Derivations of relevant parameters

For the derivations of the relevant components of the model it is assumed that the inventory system analyzed is a single-item single-echelon inventory system. Therefore, the subscript i that depicts the concerning product i is removed throughout this section as it becomes irrelevant here. The demand during lead-time, demand during the review period, the undershoot, the expected stock on hand and the service levels are therefore derived for the (R, s, S) and (R, S) replenishment policy.

The generic functions for relevant parameters in an inventory system are given in appendix E.2 and hold for every single-item single-echelon inventory system. Let us define the potential ordering cycle (or review period) as the time interval between two subsequent review moments $(\tau, R]$ with τ being the review moment at the beginning of the ordering cycle (because of the assumption that we have identically and independent review periods we can also write $(0, R]$ here). Then let us define the (potential) replenishment cycle as the time interval $(L, R + L]$. Ordering and replenishment is potential under an (R, s, S) replenishment policy because not every cycle a replenishment order will be placed to the supplier. If the inventory position is above the reorder level at a review moment, no replenishment order will be placed.

Furthermore, it is assumed that the inventory system is analyzed from the moment in time that a potential order is placed. This moment in time is the start time $t = R_0 = 0$. This moment can be any point in time. The (potential) order that is placed at time $t = 0$ arrives at time $t = L_0$. The next (potential) order after $t = R_0$ is placed at time $t = R_1$ and this order will arrive at time $t = R_1 + L_1$. Therefore, the analysis will focus on one full ordering cycle and one full replenishment cycle; the time at which the first potential order is placed until the time that the second placed order is delivered.

J.1 Demand during lead-time

The first parameter to analyze is the demand during lead-time. In this section the first two moments of the demand during lead-time are derived. An order placed at time R_0 is delivered at time L_0. Assuming that $D(R_0, L_0]$ and $D(R_1, L_1]$ follow the same Gamma distribution, the problem of finding the first two moments of the demand during lead-time is reduced to finding an expression for $E[D(0, L_0]]$ (mean) and one for $\sigma^2(D(0, L_0])$ (variance). These expressions lead to correct parameters of the Gamma distribution and are elaborated on in chapter five. If we take a step back for now and only assume that both the demand and the lead-times are stochastic variables, we require estimates of the mean and variance of the demand during lead-time.

Let us assume that the lead-time L equals an constant and integer number of days and define the inter-arrivals of demand in the time period $(0, L]$ as K and the demand sizes of an inter-arrival as

D_k with k being the k^{th} inter-arrival of demand. The demand during this period is given by the sum of the demand sizes of each of the inter-arrivals and can be expressed as $\sum_{k=1}^{K} D_k$. If we assume that D_k is an independent and identically distributed stochastic variable the mean and variance can be expressed as:

$$E[D(0,L)] = L * E[D] \tag{J.1}$$

$$\sigma^2(D(0,L]) = L * \sigma^2(D) \tag{J.2}$$

If we then relax the assumption that L is constant and only assume it equals an integer number of days, the mean of the demand during lead-time can be derived as follows:

$$\begin{aligned} E[D(0,L]] &= E\left[\sum_{k=1}^{K} D_j\right] \\ &= \sum_{n=0}^{\infty} E\left[\sum_{k=1}^{n} D_k\right] P\{K=n\} \\ &= \sum_{n=0}^{\infty} nE[D]P\{K=n\} \\ &= E[D]\sum_{n=0}^{\infty} nP\{K=n\} \\ &= E[D]E[K] \end{aligned} \tag{J.3}$$

The following expression holds for a random variable: $\sigma^2[X] = E[X^2] - E[X]^2$. This expression can be rewritten as $E[X^2] = \sigma^2[X] + E[X]^2$ and may be used in deriving the expression for the variance of the demand during lead-time. First the expressions for $E[D^2(0,L]]$ is derived:

$$\begin{aligned} E[D^2(0,L] &= E\left[\left(\sum_{k=1}^{K} D_k\right)^2\right] \\ &= \sum_{n=0}^{\infty} E\left[\left(\sum_{k=1}^{n} D_k\right)^2\right] P\{K=n\} \\ &= \sum_{n=0}^{\infty}\left(\sigma^2\left(\sum_{k=1}^{n} D_k\right) + E^2\left[\sum_{k=1}^{n} D_k\right]\right) P\{K=n\} \\ &= \sum_{n=0}^{\infty} (n\sigma^2(D) + n^2E^2[D])\, P\{K=n\} \\ &= \sigma^2(D)\sum_{n=0}^{\infty} nP\{K=n\} + E^2[D]\sum_{n=0}^{\infty} n^2P\{K=n\} \\ &= \sigma^2(D)E[K] + E^2[D]E[K^2] \end{aligned} \tag{J.4}$$

With the use of the expression $\sigma^2(D(0,L)) = E\left[D^2(0,L)\right] - E^2\left[D(0,L)\right]$, the variance can be derived thereafter:

$$\begin{aligned}\sigma^2(D(0,L)) &= E[K]\sigma^2(D) + E[K^2]E^2[D] - E^2[K]E^2[D] \\ &= E[K]\sigma^2(D) + \sigma^2(K)E^2[D]\end{aligned} \tag{J.5}$$

Taking one step back and using the expressions from de Kok (1991a) for $E[K]$ and $E[K^2]$ based on a renewal process, the expressions for the mean and the variance of demand during lead-time can be derived. The mean of the demand during lead-time is expressed as:

$$E\left[D(0,L)\right] = \frac{E[L]}{E[A]}E[D] \tag{J.6}$$

Then, with the use of the expression for $E\left[D^2(0,L)\right]$:

$$\begin{aligned}E\left[D^2(0,L)\right] &= E^2[D]\left[\frac{E[L^2]}{E^2[A]} + \frac{E[L]}{E[A]} * (CV_A^2 + CV_D^2) + \frac{(1 - CV_A^4)}{6}\right] \\ &= E^2[D]\left[\frac{E[L^2]}{E^2[A]} + \frac{E[L]}{E[A]} * \left(\left(\frac{\sigma(A)}{E[A]}\right)^2 + \left(\frac{\sigma(D)}{E[D]}\right)^2\right) + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6}\right]\end{aligned} \tag{J.7}$$

and the expression $\sigma^2(D(0,L)) = E\left[D^2(0,L)\right] - E^2\left[D(0,L)\right]$, the variance of the demand during lead-time can be derived as follows:

$$\begin{aligned}\sigma^2(D(0,L)) &= E^2[D]\left[\frac{E[L^2]}{E^2[A]} + \frac{E[L]}{E[A]} * \left(\left(\frac{\sigma(A)}{E[A]}\right)^2 + \left(\frac{\sigma(D)}{E[D]}\right)^2\right) + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6}\right] \\ &\qquad - \left(\frac{E[L]}{E[A]}E[D]\right)^2 \\ &= E^2[D]\left[\frac{E[L^2]}{E^2[A]} + \frac{E[L]}{E[A]} * \left(\left(\frac{\sigma(A)}{E[A]}\right)^2 + \left(\frac{\sigma(D)}{E[D]}\right)^2\right) + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6} - \frac{E^2[L]}{E^2[A]}\right] \\ &= E^2[D]\left[\frac{E[L^2]}{E^2[A]} + \frac{E[L]}{E[A]} * \frac{\sigma^2(A)}{E^2[A]} + \frac{E[L]}{E[A]} * \frac{\sigma^2(D)}{E^2[D]} - \frac{E^2[L]}{E^2[A]} + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6}\right] \\ &= \frac{E[L]}{E[A]}\sigma^2(D) + E^2[D]\left[\frac{E[L^2]}{E^2[A]} + \frac{E[L]}{E[A]} * \frac{\sigma^2(A)}{E^2[A]} - \frac{E^2[L]}{E^2[A]} + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6}\right] \\ &= \frac{E[L]}{E[A]}\sigma^2(D) + E^2[D]\left[\frac{E[L^2] - E^2[L]}{E^2[A]} + \frac{E[L]}{E[A]} * \frac{\sigma^2(A)}{E^2[A]} + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6}\right]\end{aligned} \tag{J.8}$$

$$= \frac{E[L]}{E[A]}\sigma^2(D) + E^2[D]\left[\frac{\sigma^2(L)}{E^2[A]} + \frac{E[L]}{E[A]} * \frac{\sigma^2(A)}{E^2[A]} + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6}\right]$$

J.2 Demand during the review period

In an inventory system under an continuous replenishment policy, only demand during lead-time is relevant for analysis. However, when under an periodic replenishment policy the demand during the review period R is relevant as well because parameters such as the safety stock have to take into account the demand during the whole replenishment cycle $R + L$. In case of an (R, s, S) replenishment policy, an order is placed at time R_0 and the next order can be placed at time R_1. As for the demand during lead-time, we need the first two moment of the demand during the review period for analysis. Based on de Kok (1991a), the expressions for the mean and variance of the demand during the review period can be expresses as (de Kok, 1991):

$$E[D(0,R)] \quad = \frac{E[R]}{E[A]}E[D] \tag{J.9}$$

and

$$\sigma^2(D(0,R)) \quad = \frac{E[R]}{E[A]}\sigma^2(D) + E^2[D]\left[\frac{\sigma^2(R)}{E^2[A]} + \frac{E[R]}{E[A]} * \frac{\sigma^2(A)}{E^2[A]} + \frac{\left(1 - \left(\frac{\sigma(A)}{E[A]}\right)^4\right)}{6}\right] \tag{J.10}$$

J.3 Undershoot

Under a continuous can-order replenishment policy, the inventory position at the moment of ordering is not necessarily equal to s but rather equal to $s - U$, where U is a non-negative stochastic variable called *undershoot*. Under a periodic can-order replenishment policy such as the (R, s, S) replenishment policy, this fact is enhanced because, due to periodic reviewing, an order may be placed several time units after the moment that the inventory position went below s. The undershoot at the moment of placing an order at time t is equal to $(s - Y(t))^+$. The first two moments of the undershoot involve approximation due to complexity of finding the mean and variance of the undershoot. The approximations are based on de Kok (2002), which at their turn are based on Tijms (1994). The approximations can be expressed as (De Kok, 2002):

$$E[U] \quad \approx \frac{\sigma^2(E[D(0,R]]) + E[D(0,R]]^2}{2E[D(0,R]]} \tag{J.11}$$

and

$$\sigma^2(U) \quad \approx \left(1 + \left(\frac{\sigma(E[D(0,R]])}{E[D(0,R]]}\right)^2\right) * \left(1 + 2\left(\frac{\sigma(E[D(0,R]])}{E[D(0,R]]}\right)^2\right) * \frac{E[D(0,R]]^2}{3} - E[U]^2 \tag{J.12}$$

J.4 Expected stock on hand

The general expression for the expected stock on hand in a replenishment cycle is given by:

$$E[X] = \frac{1}{2}(E[X(L_0)] + E[X((\tau_1 + L_1)^-)]) \qquad (J.13)$$

And for a periodic replenishment policy:

$$E[X] = \frac{1}{2}(E[X(L_0)] + E[X((R_1 + L_1)^-)]) \qquad (J.14)$$

In order to determine the expected stock on hand we need to derive the expressions for the expected stock at the beginning of a replenishment cycle just after the arrival of a replenishment order $E[X(L_0)]$ and the expected stock at the end of the replenishment cycle just before the arrival of a replenishment order $E[X((\tau_1 + L_1)^-)]$. We assume that every replenishment cycle is stochastically identical and independent.

Furthermore, we assumed that replenishment orders do not cross in time and therefore we know that an order placed at time $R_0 = 0$ will arrive at time L_0. The subsequent potential replenishment order arrives at time $R_1 + L_1$. Between these two order arrivals no other orders arrive and only demand takes place in the interval $(L_0, R_1 + L_1]$ which reduces the stock on hand. The stock on hand at time L_0 is equal to the inventory position at time $R_0 = 0$ minus the demand during $D(0, L_0]$ and the expression for the stock on hand at time L_0 for both the (R, s, S) and the (R, s) replenishment policy is given by:

$$\begin{aligned} X(L_0) &= Y(0) - D(0, L_0] \\ &= S - D(0, L] \end{aligned} \qquad (J.15)$$

$(\boldsymbol{R}, \boldsymbol{s}, \boldsymbol{S})$ replenishment policy

At the moment of ordering at time R_1 the inventory position was equal to $Y(R_1) = s - U_1$. Because of the assumptions that orders do not cross in time, no orders will arrive before time $(R_1 + L_1)^-$ and only demand has to be subtracted from $Y(R_1)$ during the interval $(R_1, R_1 + L_1]$. Furthermore, we assume that every cycle is stochastically identical and independent such that $D(R_1, R_1 + L_1] = D(0, L]$. Therefore, the expression for $X((R_1 + L_1)^-)$ becomes:

$$\begin{aligned} X((R_1 + L_1)^-) &= s - U_1 - D(R_1, R_1 + L_1] \\ &= s - U - D(0, L] \end{aligned} \qquad (J.16)$$

The same expression can be found by rewriting $X((R_1 + L_1)^-) = X(L_0) - D(L_0, R_1 + L_1]$.

We find the expected stock on hand in a replenishment cycle by substituting the expressions $(J.15)$ and $(J.16)$ into $(J.14)$ and rewriting:

$$E[X] = \frac{1}{2}\left(E\left[S - D(0, L]\right] + E\left[s - U - D(0, L]\right]\right) \qquad (J.17)$$

$(\boldsymbol{R}, \boldsymbol{S})$ replenishment policy

At the moment of ordering at time R_1 the inventory position was equal to $Y(R_1) = S - D(0, R_1]$. Because of the assumptions that orders do not cross in time, no orders will arrive before time $(R_1 + L_1)^-$ and only demand has to be subtracted from $Y(R_1)$ during the interval $(R_1, R_1 + L_1]$. Furthermore, we assume that every cycle is stochastically identical and independent such that $D(R_1, R_1 + L_1] = D(0, L]$. Therefore, the expression for $X((R_1 + L_1)^-)$ becomes:

$$\begin{aligned} X((R_1 + L_1)^-) &= S - D(0, R_1] - D(R_1, R_1 + L_1] \\ &= S - D(0, R_1 + L_1] \end{aligned} \tag{J.18}$$

We find the expected stock on hand in a replenishment cycle by substituting the expressions $(J.15)$ and $(J.18)$ into $(J.14)$ and rewriting:

$$\begin{aligned} E[X] &= \frac{1}{2}(E[S - D(0, L]] + E[S - D(0, R_1 + L_1]]) \\ &= \frac{1}{2}\left(2S - 2E[D(0, L]] - E[\, D(L_0, R_1 + L_1]]\right) \\ &= S - E[D(0, L]] - \frac{E[D(0, R]]}{2} \end{aligned} \tag{J.19}$$

J.5 Service levels

Service levels are required for evaluating the performance of an inventory system under a certain replenishment policy. Therefore, this section describes the derivations of the service levels required to evaluate the modified (R^δ, S_i) and (R^δ, s_i, S_i) replenishment policy.

J.5.1 The $\boldsymbol{P_1}$ service level

The general expression for P_1 is given by:

$$P_1 = P\{X((R_1 + L_1)^-) \geq 0\} \tag{J.20}$$

$(\boldsymbol{R}, \boldsymbol{s}, \boldsymbol{S})$ replenishment policy

The expression for the expected stock at the end of the replenishment cycle just before the arrival of a replenishment order $E[X((\tau_1 + L_1)^-)]$ was already derived in **appendix I.4** and is given by:

$$E[X((\tau_1 + L_1)^-)] = s - U - D(0, L] \tag{J.21}$$

Substituting the expression of $(J.21)$ into expression $(J.20)$ we obtain:

$$\begin{aligned} P_1 &= P\{s - U - D(0, L] \geq 0\} \\ &= P\{D(0, L] + U \leq s\} \end{aligned} \tag{J.22}$$

In the previous sections we derived expressions for the first two moments of the demand during lead-time and the undershoot. Because we assumed that demand is mutually independent and identically distributed we define the variable Z which is the sum of the first two moments of demand during lead-time and the undershoot (De Kok, 2002):

$$E[Z] = E\big[D(0,L]\big] + E[U] = E[D(0,L] + U] \quad (J.23)$$

and

$$\sigma^2(Z) = \sigma^2(D(0,L]) + \sigma^2(U) = \sigma^2(D(0,L] + U) \quad (J.24)$$

$(\boldsymbol{R}, \boldsymbol{S})$ replenishment policy

The expression for the expected stock at the end of the replenishment cycle just before the arrival of a replenishment order $E[X((\tau_1 + L_1)^-)]$ was already derived in **appendix I.4** and is given by:

$$E[X((\tau_1 + L_1)^-)] = S - D(0, R_1 + L_1] \quad (J.25)$$

Substituting the expression of $(J.25)$ into expression $(J.20)$ we obtain:

$$\begin{aligned} P_1 &= P\{S - D(0, R_1 + L_1] \geq 0\} \quad (J.26) \\ &= P\{D(0, R + L] \leq S\} \\ &= P\{D(0,L] + D(0,R] \leq S\} \end{aligned}$$

J.5.2 The $\boldsymbol{P_2}$ service level

The P_2 service level is described as the long run fraction of total demand which is being delivered from stock on hand immediately. This service level is often referred to as the *customer service level* or the *fill rate*. The long run fraction refers to the behavior of demand and the inventory position in every replenishment cycle. By assuming that every replenishment cycle is stochastically identical and independent, we can use the following the expressions of de Kok (2012) for the P_2 service level:

$$P_2 = 1 - \frac{E\big[B(L, R + L]\big]}{E\big[D(L, R + L]\big]} \quad (J.27)$$

which means that:

$$P_2 = 1 - \frac{\text{the expected shortage in a replenishment cycle}}{\text{total demand in a replenishment cycle}} \quad (J.28)$$

$(\boldsymbol{R}, \boldsymbol{s}, \boldsymbol{S})$ replenishment policy

The expression for the expected shortage in a replenishment cycle $E\big[B[L, R + L]\big]$ can be derived by taking the expected number of backorders at the end of the replenishment cycle just before an order arrives (i.e. at time $(R + L)^-$) and subtracting the expected number of backorders that were present at the beginning of the replenishment cycle (i.e. at time L). The expression for $E\big[B(L, R + L]\big]$ is then given by:

$$\begin{aligned} E\big[B(L, R + L]\big] &= E\left[(-X((R_1 + L_1)^-))^+ - \big(-X(L_0)\big)^+\right] \quad (J.29) \\ &= E\left[(-(s - U - D(0,L]))^+ - \big(-(S - D(0,L])\big)^+\right] \\ &= E[(D(0,L] + U - s)^+ - (D(0,L] - S)^+] \\ &= E[(D(0,L] + U - s)^+] - E[(D(0,L] - S)^+] \end{aligned}$$

Thereafter, the expression for the total demand in a replenishment cycle $E\big[D(L, R+L]\big]$ can be derived by taking the stock on hand at hand just after delivery of an order (i.e. at time L) and subtracting the stock on hand just before the subsequent delivery of an order (i.e. at time $(R+L)^-$). The expression for $E\big[D(L, R+L]\big]$ is then given by:

$$
\begin{aligned}
E\big[D(L, R+L]\big] &= E[X(L_0) - X((R_1 + L_1)^-)] && (J.30)\\
&= E[(S - D(0, L]) - (s - U^R - D(0, L])] \\
&= S - s + E[U]
\end{aligned}
$$

If we combine the two expressions we get the expression for the P_2 service level:

$$
P_2 = 1 - \frac{(E[(D(0, L] + U - s)^+] - E[(D(0, L] - S)^+])}{(S - s + E[U])} \qquad (J.31)
$$

$(\boldsymbol{R}, \boldsymbol{S})$ replenishment policy

The expression for the expected shortage in a replenishment cycle $E\big[B[L, R+L]\big]$ can be derived by taking the expected number of backorders at the end of the replenishment cycle just before an order arrives (i.e. at time $(R+L)^-$) and subtracting the expected number of backorders that were present at the beginning of the replenishment cycle (i.e. at time L). The expression for $E\big[B(L, R+L]\big]$ is then given by:

$$
\begin{aligned}
E\big[B(L, R+L]\big] &= E\left[(-X((R_1 + L_1)^-))^+ - \big(-X(L_0)\big)^+\right] && (J.32)\\
&= E\left[(-(S - D(0, R+L]))^+ - \big(-(S - D(0, L])\big)^+\right] \\
&= E[(D(0, R+L] - S)^+ - (D(0, L] - S)^+] \\
&= E[(D(0, R+L] - S)^+] - E[(D(0, L] - S)^+]
\end{aligned}
$$

Thereafter, the expression for the total demand in a replenishment cycle $E\big[D(L, R+L]\big]$ can be derived by taking the stock on hand at hand just after delivery of an order (i.e. at time L) and subtracting the stock on hand just before the subsequent delivery of an order (i.e. at time $(R+L)^-$). The expression for $E\big[D(L, R+L]\big]$ is then given by:

$$
\begin{aligned}
E\big[D(L, R+L]\big] &= E[X(L_0) - X((R_1 + L_1)^-)] && (J.33)\\
&= E\big[(S - D(0, L]) - (S - D\big((0, R+L]\big)\big] \\
&= E\big[D(0, R+L] - D(0, L]\big] \\
&= E\big[D(0, R]\big]
\end{aligned}
$$

If we combine the two expressions we get the expression for the P_2 service level:

$$
P_2 = 1 - \frac{(E[(D(0, R+L] - S)^+] - E[(D(0, L] - S)^+])}{E\big[D(0, R]\big]} \qquad (J.34)
$$

Appendix K

Derivations of cost functions

In this section the relevant cost involved in performing operational inventory control at the case study company, that is within the scope of the project, are described. The objective of the newly defined replenishment policies is to minimize the total inventory cost under a target service level constraint. The cost described in this section are holding cost, ordering cost and shortage cost. For each of these cost, a cost function will be derived is used in evaluating the current situation and the newly defined policy. This section is based on the aggregated stochastic joint replenishment problem and therefore the subscript $\boldsymbol{i}$ comes into play again.

K.1 Holding cost function

The cost of holding inventory are based on the amount of inventory held, the cost price of that inventory and the holding cost rate. Because of the assumption that every replenishment cycle is stochastically identical and independent, the expected stock on hand derived in **appendix I.4** is a good approximation of the average stock on hand in a replenishment cycle and therefore in any arbitrary time period.

$(\boldsymbol{R^{\delta}}, \boldsymbol{s_i}, \boldsymbol{S_i})$ replenishment policy

The expected stock on hand for the (R, s, S) replenishment policy was described as:

$$E[X] = \frac{1}{2}\left(E\left[S - D(0, L)\right] + E\left[s - U - D(0, L)\right]\right) \tag{K.1}$$

The (R, s, S) replenishment policy assumes that there is a *minimum order quantity* (MOQ) equal to $S - s + 1$ for discrete demand and $S - s$ for continuous demand (K. H. Van Van Donselaar & Broekmeulen, 2014). If we use this to rewrite the expression for the expected stock on hand we get:

$$\begin{aligned} E[X] &= \frac{1}{2}\left(S + s - E[U] - 2E\left[D(0, L)\right]\right) \\ &= \frac{1}{2}\left(S - s + 2s - E[U] - 2E\left[D(0, L)\right]\right) \\ &= \frac{1}{2}\left(MOQ + 2s - E[U] - 2E\left[D(0, L)\right]\right) \\ &= s - E\left[D(0, L)\right] - \frac{E[U]}{2} + \frac{MOQ}{2} \end{aligned} \tag{K.2}$$

Using the expected stock on hand $E[X]$, the holding cost rate h and c_i, the cost price of one product i the expected daily holding cost of one product i under the (R^{δ}, s_i, S_i) replenishment policy ($EDHC_{RsS}$) can be expressed as:

$$EDHC_{RsS} = h_i * c_i * E[X_i] \tag{K.3}$$

$$= h_i * c_i * \left(s_i - E\left[D(0,L)\right] - \frac{E[U_i]}{2} + \frac{E\left[D\left(0,R^{\delta}\right]\right]}{2} \right)$$

The total expected daily holding cost ($TEDHC_{RsS}$) can be expressed as:

$$TEDHC_{RsS} \quad = \sum_{i=1}^{n} h_i * c_i * \left(s_i - E\left[D(0,L)\right] - \frac{E[U_i]}{2} + \frac{E\left[D\left(0,R^{\delta}\right]\right]}{2} \right) \tag{K.4}$$

With n being the total number of products.

(R^{δ}, S_i) replenishment policy

The expected stock on hand for the (R, S) replenishment policy was described as:

$$E[X] = S - E\left[D(0,L)\right] - \frac{E\left[D(0,R)\right]}{2} \tag{K.5}$$

Using the expected stock on hand $E[X]$, the holding cost rate h and c_i, the cost price of one product i the expected daily holding cost of one product i under an (R^{δ}, S_i) replenishment policy ($EDHC_{RS}$) can be expressed as:

$$\begin{aligned} EDHC_{RS} \quad &= h_i * c_i * E[X_i] \\ &= h_i * c_i * \left(S_i - E\left[D(0,L)\right] - \frac{E\left[D\left(0,R^{\delta}\right]\right]}{2} \right) \end{aligned} \tag{K.6}$$

The total expected daily holding cost ($TEDHC_{RS}$) can be expressed as:

$$TEDHC_{RS} \quad = \sum_{i=1}^{n} h_i * c_i * \left(S_i - E\left[D(0,L)\right] - \frac{E\left[D\left(0,R^{\delta}\right]\right]}{2} \right) \tag{K.7}$$

With n being the total number of products.

Note that with the simulation results from chapter five, we changed the term between brackets for both policies into (De Kok, 2002):

$$E[X] = s_i + \frac{\left(\frac{(S_i - s_i)^2}{2} - \frac{E^2[U_i] + \sigma^2(U_i)}{2} + \frac{E^2\left[D\left(0,R^{\delta}\right]\right] + \sigma^2\left(D\left(0,R^{\delta}\right]\right)}{2E\left[D(0,R^{\delta}\right]\right]\left((S_i - s_i) + E[U_i]\right)} \right)}{\left(\left((S_i - s_i) + E[U_i]\right) - \frac{E[D_i]}{E[A_i]} E[L] - P_2 \frac{E\left[D(0,R^{\delta}\right]\right]}{2} \right)}$$

K.2 Ordering cost function

The fixed ordering cost is incurred every time that an order is placed to the supplier. In the case situation there are no minor fixed ordering cost and therefore the size of the ordered quantity has no effects on the ordering cost. If this was the case, the ordering cost functions should be adapted accordingly. However, for generalization, expressions for the first two moments of the stochastic random variable *order quantity* (Q_i) are derived in this section. These expressions are useful if there are indeed minor fixed ordering cost or price discounts related to the ordered quantity.

Based on review period on a supplier level elaborated in **section 4.6** we can describe the ordering process of product $\boldsymbol{i}$ ordered to supplier $\boldsymbol{\delta}$ by deriving expressions for the frequency of the review moments and the ordering frequency.

($\boldsymbol{R^{\delta}}, \boldsymbol{s_i}, \boldsymbol{S_i}$) replenishment policy

For the (R^{δ}, s_i, S_i) replenishment policy we introduce a new variable ρ_i. Note that this variable is only relevant for the for the (R, s, S) replenishment policy. The variable ρ_i is introduced to take into account the fact that if there is a review moment R_t^{δ} and an order is allowed to be placed, the inventory position could be equal or higher than the reorder level s_i which results in not placing an order (i.e. $Y_i(t) \geq s_i$). In other words, we would like to know the probability that an order for product $\boldsymbol{i}$ is placed in a review period at the review moment. This is equal to the probability that in period $(0, R_1^{\delta}]$ the inventory position just before time R_1^{δ} is less than the reorder level s_i. The inventory position at time $\left(R_1^{\delta}\right)^{-}$ is equal to the inventory position at time $R_0^{\delta} = 0$ (just after the review moment) minus the demand during the review period. The inventory position at time $R_0^{\delta} = 0$ is given by $Y_i(0) = S_i$. Assuming that subsequent review moments are independent and demand is stochastically independent and identical, the expressions for this probability is given by:

$$\begin{aligned} \rho_i &= P\{S_i - D_i(0, R^{\delta}] < s_i\} \qquad (K.8) \\ &= P\{D_i(0, R^{\delta}] > S_i - s_i\} \end{aligned}$$

The review frequency for product $\boldsymbol{i}$ can be expressed as:

$$\text{Review frequency} = \frac{1}{E[R^{\delta}]}$$

The ordering frequency is dependent on the expected demand size of an order, the expected inter-arrival times between orders and the expected order quantity (i.e. the expected demand size of an order combined with the expected inter-arrival times between orders results in the expected demand in a time period).

$$\begin{aligned} \text{Ordering frequency} &= \frac{E[D_i]}{E[Q_i]E[A_i]} \qquad (K.9) \\ &= \frac{E[D_i]}{(S_i - s_i + E[U_i])E[A_i]} \end{aligned}$$

Therefore, the variable ρ_i can be expressed as:

$$\begin{aligned} \rho_i &= \frac{\frac{E[D_i]}{(S_i - s_i + E[U_i])E[A_i]}}{\frac{1}{E[R^{\delta}]}} \qquad (K.10) \\ &= \frac{E[D_i]E\left[R^{\delta}\right]}{(S_i - s_i + E[U_i])E[A_i]} \end{aligned}$$

The expected order quantity Q_i is a stochastic random variable. In the case situation this variable Q_i is only relevant because we want to know if an order is placed in the review period. The fixed ordering cost are taken into account with the derivation of the supplier based review period. We can cope with the ordering cost in two ways: (1) derive an expression for the probability that one of the products i of the set of products ordered to the same supplier has an inventory position lower than the reorder level s_i or (2) assume that there is always one product i in the set of products ordered to the same supplier that has an inventory position lower than the reorder level s_i.

Let us define the subset $V^\delta = \{1,2,\dots,i,i+1,n\}$ which contains all the product i that are ordered to the same supplier δ. The probability that one of the products i in V^δ has an inventory position lower than the reorder level s_i at the review moment and hence, an order is placed to the supplier, is equal to one minus the probability that none of the product i in V^δ has an inventory position lower than the reorder level s_i at the review moment. This probability can be expressed as:

$$P\{one\ product\ i\ \in V^\delta where\ D_i(0,R^\delta] > S_i - s_i\} = 1 - \prod_{i \in V^\delta}(1-\rho_i) \qquad (K.11)$$

If there is only one product i in the set of products ordered to the same supplier, the ordering cost function would be with fixed ordering cost K becomes:

$$OC_{1,i}(\rho_i) = \rho_i K \qquad (K.12)$$

If we indeed assume that there is always one product i in the set of products ordered to the same supplier that has an inventory position lower than the reorder level s_i, then the ordering cost function per supplier becomes:

$$OC_1 = K \qquad (K.13)$$

If we do not make this assumption, the ordering cost function per supplier becomes:

$$OC_2(\rho_i) = \left(1 - \prod_{i \in V^\delta}(1-\rho_i)\right)K \qquad (K.14)$$

with $\rho_i = \frac{E[D_i]E[R^\delta]}{(S_i - s_i + E[U_i])E[A_i]}$.

For product i, the probability that an order of size $E[Q_i]$ is placed at a review moment is equal to ρ_i. Hence, the expected order quantity of product i ordered to the supplier at a review moment is given by:

$$\begin{aligned} E[Q_{RsS,i}] &= \rho_i E[Q_i] \qquad (K.15) \\ &= \rho_i E[S_i - s_i + U_i] \end{aligned}$$

The expression for the variance of order quantity can be derived by using $\sigma^2(Q_i) = E[Q_i^2] - E[Q_i^2]$ in our derivation:

$$
\begin{aligned}
\sigma^2(Q_{RsS,i}) &= \rho_i E[(S_i - s_i + U_i)^2] - E[\rho_i(S_i - s_i + U_i)]^2 \qquad (K.16)\\
&= \rho_i(\sigma^2(S_i - s_i + U_i) + E[(S_i - s_i + U_i)]^2) - E[\rho_i(S_i - s_i + U_i)]^2\\
&= \rho_i(\sigma^2(S_i - s_i + U_i) + E[S_i - s_i + U_i]^2) - \rho_i^2 E[S_i - s_i + U_i]^2\\
&= \rho_i \sigma^2(U_i) + \rho_i E[S_i - s_i + U_i]^2 - \rho_i^2 E[S_i - s_i + U_i]^2\\
&= \rho_i(1 - \rho_i)E[S_i - s_i + U_i]^2 + \rho_i \sigma^2(U_i)
\end{aligned}
$$

We now derived the first two moments of Q_i. If a distribution is fit to these two moments, the resulting pdf can be used to determine the expected daily ordering cost for the (R^δ, s_i, S_i) replenishment policy.

The expected daily ordering cost function for ordering products to a supplier δ can be expressed as:

$$
EDOC_{RsS}^{\delta} = \frac{OC_2(\rho_i)}{E[R^\delta]} \qquad (K.17)
$$

The total expected daily ordering cost are given by:

$$
TEDOC_{RsS} = \sum_{\delta=1}^{N} EDOC_{RsS}^{\delta} = \sum_{\delta=1}^{N} \frac{OC_2(\rho_i)}{E[R^\delta]} \qquad (K.18)
$$

With N being the set of suppliers placing an order to.

$(\boldsymbol{R^\delta}, \boldsymbol{S_i})$ replenishment policy

The review frequency for product i under the (R^δ, S_i) replenishment policy can be expressed as:

$$
\text{Review frequency} \quad = \frac{1}{E[R^\delta]} \qquad (K.19)
$$

Let us assume that there is at least one demand order for a product i in a review period. Then the ordering frequency for product i under an (R^δ, S_i) replenishment policy is the same as the review frequency:

$$
\text{Ordering frequency} \quad = \frac{1}{E[R^\delta]} \qquad (K.20)
$$

For he (R^δ, S_i) replenishment policy it holds that if we assume that we have at least one demand in a review period of one of the products i that is in the set of products i that is ordered to the same supplier δ, the ordering frequency is equal to the review frequency. Furthermore, the order quantity is always equal to the demand in the most recent review period, assuming that demand is stationary (K. H. Van Van Donselaar & Broekmeulen, 2014). Therefore the mean and variance of the order quantity can be expressed as:

$$E[Q_{RS,i}] \quad = E[Q_i] \tag{K.21}$$

$$= D(0, R^{\delta}]$$

$$= \frac{E[R^{\delta}]}{E[A_i]} E[D_i]$$

and

$$\sigma^2(Q_{RS,i}) \quad = \sigma^2(D(0, R^{\delta}]) \tag{K.22}$$

$$= \frac{E[R^{\delta}]}{E[A_i]} \sigma^2(D_i) + E^2[D_i] \left[\frac{\sigma^2(R^{\delta})}{E^2[A_i]} + \frac{E[R^{\delta}]}{E[A_i]} * \frac{\sigma^2(A_i)}{E^2[A_i]} + \frac{\left(1 - \left(\frac{\sigma(A_i)}{E[A_i]}\right)^4\right)}{6} \right]$$

Let us use the subset V^{δ} with all product i ordered to the same supplier δ which we defined in this section for an (R^{δ}, s_i, S_i) replenishment policy. Because of the assumption that there is at least one product i in V^{δ} that has at least one demand in the duration of a review period, the ordering cost function can be expressed as:

$$OC_1 = K \tag{K.23}$$

The expected daily ordering cost function for ordering products to a supplier δ can be expressed as:

$$EDOC_{RS}^{\delta} = \frac{OC_1}{E[R^{\delta}]} \tag{K.24}$$

The total expected daily ordering cost are given by:

$$TEDOC_{RS} = \sum_{\delta=1}^{N} EDOC_{RS}^{\delta} = \sum_{\delta=1}^{N} \frac{OC_1}{E[R^{\delta}]} \tag{K.25}$$

With N being the set of suppliers placing an order to.

K.3 Shortage cost function

Excess demand results in a lost-sales because backorders are not accepted. We set a penalty cost p_i for not being able to deliver the product from stock. This penalty cost is set equal to the margin of the product. Additionally, it could be multiplied by a so called goodwill factor α. This α is to take into account the fact that a lost-sale does not only result in a penalty cost (lost margin) but may also result in a customer that will not return for the next potential purchase, or worse, never returns for any potential purchase. Let us assume that the goodwill factor can be set by the company based on the importance of the product and can vary between 1 and 2. For example, if losing a customer of a certain product is rather important, α could be set somewhere between 1.5 and 2, while if it is not, it could be set close to 1. Hence, not satisfying a demand for a product results in a shortage cost equal to α multiplied by the sales price minus the cost price of the product. In the derivations of the cost function this α is not taken into account.

We aim at minimizing inventory cost under a target fill rate which implicitly mean that we accept that a percentage of demand is not satisfied from stock (immediately). This accepted percentage of lost demand is equal to $1 - P_2$ and shortage cost become irrelevant in minimizing the inventory cost. However, if we want to minimize the inventory cost taking into account the shortage cost per product and aim at finding the 'optimal' fill rate per product, the shortage cost functions could be useful.

$(\boldsymbol{R^\delta}, \boldsymbol{s_i}, \boldsymbol{S_i})$ replenishment policy

The expected excess demand in a replenishment cycle can be described by the expression for the expected backorders in a replenishment cycle or by multiplying the expected demand in a replenishment cycle with $1 - P_2$. The expression for the expected backorders in a replenishment cycle were derived in appendix J5. The daily expected shortage cost of a product i in a replenishment can then be expressed as:

$$EDSC_{RsS,i} = b_i \frac{(E[(D(0,L] + U_i - s_i)^+] - E[(D(0,L] - S_i)^+])}{R^\delta} \tag{K.26}$$

If we define the stochastic random variables $Z = D(0,L] + U$ and $Y = D(0,L]$, we get the following expression:

$$EDSC_{RsS,i} = \frac{1}{R^\delta} b_i \int_{s_i}^{\infty} (z - s_i) f(x) dx - \int_{S_i}^{\infty} (y - S_i) f(y) dy \tag{K.27}$$

with $f(x)$ and $f(y)$ being the probability density functions (pdf) of Z and Y respectively.

The total expected daily shortage cost can be expressed as:

$$TEDSC_{RsS} = b_i \sum_{\delta=1}^{N} \sum_{i \in V^\delta} \frac{(E[(D(0,L] + U_i - s_i)^+] - E[(D(0,L] - S_i)^+])}{R^\delta} \tag{K.28}$$

$(\boldsymbol{R^\delta}, \boldsymbol{S_i})$ replenishment policy

The expression for the expected backorders in a replenishment cycle are derived in appendix J.5. The expected daily shortage cost ($EDSC_i$) of a product i in a replenishment cycle can then be expressed as:

$$EDSC_{RS,i} = b_i \frac{\left(E\left[\left(D\left(0, R^\delta + L\right] - S_i\right)^+\right] - E[(D(0,L] - S_i)^+]\right)}{R^\delta} \tag{K.29}$$

Because of the assumption that replenishment cycles are independent and identical, we can define the stochastic random variables $V = D(0, R + L] = D(0,R] + D(0,L]$ and $Y = D(0,L]$ and get the following expression:

$$EDSC_{RS,i} = \frac{1}{R^\delta} b_i \int_{S_i}^{\infty} (V - S_i) f(v) dv - \int_{S_i}^{\infty} (y - S_i) f(y) dy \tag{K.30}$$

The expected daily shortage cost for products ordered to the same supplier can be expressed as:

$$EDSC_{RS}^{\delta} = \frac{1}{R^{\delta}} b_i \sum\nolimits_{i \in V^{\delta}} \left(E\left[\left(D\left(0, R^{\delta} + L\right] - S_i \right)^{+} \right] - E[(D(0, L] - S_i)^{+}] \right) \tag{K.31}$$

$$= \frac{1}{R^{\delta}} b_i \sum\nolimits_{i \in V^{\delta}} \int_{S_i}^{\infty} (V - S_i) f(v) dv - \int_{S_i}^{\infty} (y - S_i) f(y) dy$$

The total expected shortage cost ($TEDSC$) can be expressed as:

$$TEDSC_{RS} = b_i \sum\nolimits_{\delta=1}^{N} \sum\nolimits_{i \in V^{\delta}} \frac{\left(E\left[\left(D\left(0, R^{\delta} + L\right] - S_i \right)^{+} \right] - E[(D(0, L] - S_i)^{+}] \right)}{R^{\delta}} \tag{K.32}$$

$$= b_i \sum\nolimits_{\delta=1}^{N} \frac{1}{R^{\delta}} \sum\nolimits_{i \in V^{\delta}} \int_{S_i}^{\infty} (V - S_i) f(v) dv - \int_{S_i}^{\infty} (y - S_i) f(y) dy$$

Appendix L

Company specific cost specifications

Holding cost

The holding cost can be determined by the cost of capital of the products in the assortment. In the case situation, the company works with a holding cost rate of 20% per Euro per year (%/€/year) to calculate the cost of invested capital. The first reason for using such a high percentage is the fact that the company wants to keep inventory as low as possible while keeping service levels at a target level. A second reason is that the same amount of money could have been invested in marketing for products in the form of Google ads or increasing Google ranking for a certain time period. Hence, the holding cost rate is not just the cost of not having the amount of money on the bank with a certain interest rate. The holding cost can be calculated by multiplying the *cost of goods sold* (COGS) with holding cost rate. Hence, The total annual holding cost can be calculated by summing all the annual holding cost of the different products in the assortment. To illustrate the holding cost, table 11 shows the cost price and holding cost per year of 10 randomly chosen SKUs.

Table 11: Holding cost representation for 10 randomly chosen SKUs

SKU name	Cost price (€/unit)	Holding cost (€/year)
803	46,75	9,35
1410	5,70	1,14
2546	340,22	68,04
3862	138,05	27,61
4201	4,88	0,98
1621	14,85	2,97
1084	48,29	9,66
3474	2,52	0,50
387	66,00	13,2
3829	78,41	15,68

Ordering cost

For every order placed to a supplier, a fixed ordering cost is incurred. However, fixed ordering cost are not precisely specified per supplier or per product. Every supplier has its own fixed ordering cost that is based on processes including order picking, administration and transportation. Knowing, or at least estimating this fixed ordering cost is required for decisions about which replenishment policy to utilize and how to set parameters such as the size and the timing of the order. For the case situation, the company sets the fixed ordering cost to equal to an amount of 20 Euros (€20) per order placed to the supplier. This fixed ordering cost was therefore used in the calculations.

Shortage cost

Excess demand results in a lost-sales because backorders are not accepted. We set the penalty cost for not being able to deliver the product from stock to account for the lost-sales in the lost-sales system. This penalty cost is set equal to the margin of the product multiplied by a so called goodwill factor α. This α is to take into account the fact that a lost-sale does not only result in a penalty cost (lost margin) but may also result in a customer that will not return for the next potential purchase, or worse, never returns for any potential purchase. Let us assume that the goodwill factor can be set by the company based on the importance of the product and can vary between 1 and 2. For example, losing a customer of a certain product is rather important, α could be set somewhere from 1,5 to 2, while if it is not, it could be set close to zero. Hence, not satisfying a demand for a product results in a shortage cost equal to α multiplied by the sales price minus the cost price of the product. The shortage cost of the same randomly chosen SKUs is presented in table 12.

Table 12: Shortage cost representation for 10 randomly chosen SKUs

SKU name	Cost price (€/unit)	Sales price (€/unit)	Margin (€/unit)	Shortage cost (€/unit)
803	46,75	99,13	52,38	52,38α
1410	5,7	15,66	9,96	9,96 α
2546	340,22	616,49	276,27	276,27 α
3862	138,05	138,05	139,59	139,59 α
4201	4,88	14,83	9,95	9,95 α
1621	14,85	33,02	18,17	18,17 α
1084	48,29	80,12	31,83	31,83 α
3474	2,52	10,7	8,18	8,18 α
387	66	107,4	41,4	41,4 α

Appendix M

Fill rate calculation verification

This section describes the verification and the simulation results of the fill rate calculation functions and all other relevant parameters such as demand during lead-time, demand during the review period and the reorder level.

Note that the simulation tool is built in such a way that it simulates daily demand. *R* functions that are used for generating demand therefore simulate a discrete demand process. Nevertheless, we use an *R* function called *momentCompound* that provides us with the first two moments of a compound distribution. The functions takes as input a parent distribution and its parameter(s), a compound distribution and its parameters and the choice of which moment to calculate. This function takes for example the Poisson distribution as the compound distribution and the Gamma distribution as the parent distribution and provides us with the first two moments of the compounding distribution (Nadarajah, Popović, & Ristić, 2013). The accuracy of the *R* functions is checked with help of the extended spreadsheet by de Kok (2002) where both discrete and continuous demand can be used as input for the (R, s, S) replenishment policy.

For the verification of the fill rate calculation and cost functions, multiple scenarios were simulated. For each scenarios we ran $n = 10$ replications with a length of $m = 20.000$ days. Every replication has a warm-up period of $l = 2000$ days. The warm-up period is determined by using the Welch graphical method by Welch (1983) on the fill rate. Described briefly, we ran 5 replicates with a moving average of $w = 1$ day and simulation horizon of $m = 10.000$ days to be able to see when the fill rate converges to a steady state. The moving average should be smaller or equal to $\frac{m}{4}$ and m should be chosen as large as practical regarding to the problem and the simulation time. If we simulate to test for the warm-up period with $m = 10.000$ days with a moving average of $w = 1$ day, the period is long enough to allow for infrequent events (Law, 2007). Figure 10 shows the graphical representation of the Welch graphical method. The line that represents the average fill rate per day over in the 5 replicas evens out after approximately 2000 days. For the determination of simulation results this warm-up period of 2000 days is removed from the simulation output data.

The simulation horizon should be as large as practical and should be significantly larger than the warm-up period (Law, 2007). A simulation horizon of 20.000 days is long enough to minimize potential outliers and provides simulations results that are close to identical. Taking this into account combined with the fact that time is limited, the number of simulation replications is set to $n = 10$. This results in a 95% confidence interval (i.e. $\alpha = 0{,}05$) for the fill rate and other simulation output equal to:

$$CI_{95\%} = \bar{X}(n) \pm t_{n-1,1-\frac{\alpha}{2}} \sqrt{\frac{\sigma^2(n)}{n}} = \bar{X}(10) \pm t_{10,0.975} \sqrt{\frac{\sigma^2(10)}{10}} \qquad (M.1)$$

Where $\bar{X}$ is the average simulation output, σ^2 the variance of the simulation outpur, n is the number of replicas and $t_{n-1,1-\frac{\alpha}{2}}$ is the t-statistic for an $100(1-\alpha)$ confidence interval with n replicas.

The fill rate calculation verification is performed for one product. If the fill rate calculation holds for one product, it also holds for joint replenishment situations because the replenishment problem of the different products can be seen as n single-item problems. Only ordering cost are dependent on the joint replenishment process which will be described in section 5.2.2. All verifications, simulation scenarios and relevant conclusions are presented in appendix M. All input parameters used for the different scenarios can be found in appendix O.

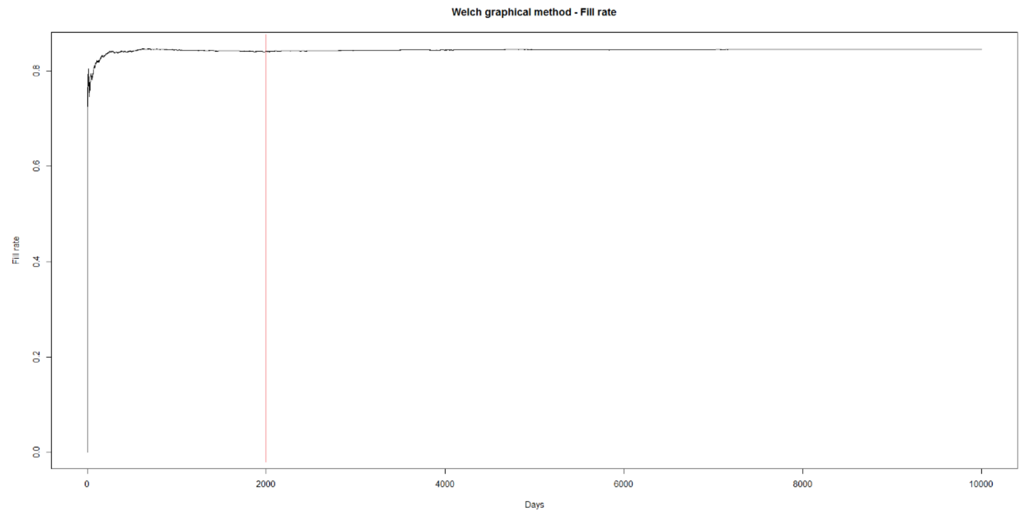


Figure 10: Welch graphical method

The first three scenarios are simulated under compound Poisson demand, with Poisson demand inter-arrivals and Gamma distributed order sizes. Hence, the CV of the inter-arrival times of demand orders is equal to 1 (i.e. $\frac{\sigma(A)}{E[A]} = 1$). Scenario 4 focusses on joint replenishment and simulation is done for multiple products from the same supplier. For each scenario a table with verification input and output is presented divided over two rows:

- *The first* row shows the fill rate according to the spreadsheet of de Kok (2002) based on the non-simulated input parameters which were used in every simulation run.
- *The second row* shows the average *post simulation* input parameters that result from the 10 simulations runs. The value for the analytical reorder level s on the second row is calculated beforehand based on the input parameters. The value for the fill rate on the second row is calculated with the average of all the *post simulation* input parameters. Note that this fill rate can therefore deviate from the target fill rate.
- *The third row* shows the average *post simulation* input parameters that result from the 10 simulations runs. The value for the reorder level s is calculated with the average of all *post simulation* input parameters under the initial target fill rate. The value for the fill rate on the lower row is the *post simulation* average fill rate that resulted from the 10 simulation runs.

- *The fourth row* shows the deviation of the simulated fill rate based on the 95% confidence interval.
- *The fifth* row shows the difference between the *post simulation* fill rate (i.e. from row 3) and the calculated fill rate with average *post simulation* input parameters (i.e. from row 2).

Table 13 shows the results from the first scenario where a target fill rate was used of 90% and 98% for scenario 1.1, 1.2 respectively. For a constant lead-time we can calculate the first two moments of the discrete daily demand with continuous input demand parameters. With help of the spreadsheet from de Kok (2002) and taking the input parameters from the upper row of table 13, we get a standard deviation of demand during lead-time of $\sqrt{58}$. Therefore the input parameters for demand generation are $\mu_D = \frac{E[D]}{E[A]} = \frac{10}{2} = 5$ and $\sigma_D^2 = 58$. According to the spreadsheet from de Kok (2002) these input parameters provide us with a fill rate of 90,34% for both the discrete distribution (R, s, S) and the continuous distribution (R, s, S). This fill rate results from a reorder level of 27 which is calculated based on a target fill rate of 90%. Interesting to mention here is that the *R momentCompound* function provides us with an $\mu_D = 7{,}994$ and $\sigma_D^2 = 75{,}144$ with the same continuous input parameters, which results in a fill rate of 90,39% in the spreadsheet of de Kok (2002) (Nadarajah et al., 2013).

Table 13: Simulation results scenario 1 ($m = 20.000, l = 2000, n = 10$)

Scenario		$E[D]$ $(days)$	$\sigma(D)$ $(days)$	$E[L]$ $(days)$	$\sigma(L)$ $(days)$	R $(days)$	$S - s$	s	P_2
1.1		5,0000	7,6158	1,00	0,00	2	10,00	27	90,34%
		4,9848	7,6212	1,00	0,00	2	10,00	27	90,32%
(R, s, S)		4,9848	7,6212	1,00	0,00	2	10,00	27	91,12%
	$CI_{95\%}$								(±0,3339%)
	Diff.								-0,80%
1.2		5,0000	7,6158	1,00	0,00	2	10,00	47	98,06%
		4,9929	7,6201	1,00	0,00	2	10,00	47	98,05%
(R, s, S)		4,9929	7,6201	1,00	0,00	2	10,00	47	98,09%
	$CI_{95\%}$								(± 0,1872%)
	Diff.								-0,04%

We can conclude from table 13 that the calculation of the analytical fill rate gives a good approximation of the simulated fill rate (i.e. actual experienced fill rate). We see that the simulated fill rate is slightly higher in both scenario 1.1 and 1.2. This is probably explained by the fact that the reorder level s is rounded up to the nearest integer and the fact that the simulated demand parameters are slightly lower than the non-simulated input parameters. However, the expected demand and standard deviation of demand calculations are accurately calculated.

Table 14 shows the simulation results for relevant parameters including the demand in certain periods and the undershoot. All parameters of demand in certain periods are calculated accurately with differences of about 0,15% from the simulated parameters, except the undershoot parameters. Especially the expected undershoot calculation differs greatly with the simulated expected undershoot. This difference is probably caused by the fact that the inventory system we have simulated is a lost-sales system: both the inventory position and the stock on hand cannot be negative. Therefore, the distance $-min\{0, Y(t) - s\}$ cannot be larger than s and the average undershoot will have a value between $\mathbf{0}$ and s instead of between $-min\{0, Y_i(t)\}$ and s. Hence, the expected undershoot $E[U]$ is lower in our simulation model. The calculated fill rates are partly based on the expected undershoot and are therefore important to incorporate. Although the simulated undershoot parameters are not very accurate, the calculated fill rate and other relevant parameter are.

Table 14: Simulation results of relevant parameters scenario 1 ($m = 20.000, l = 2000, n = 10$)

Scenario	Variable	$E[D(0,L)]$ $(days)$	$\sigma(D(0,L))$ $(days)$	$E[D(0,R)]$ $(days)$	$\sigma(D(0,R))$ $(days)$	$E[U]$	$\sigma(U)$
1.1	Analytical	5,0000	7,6158	10,0000	10,7703	10,8000	11,0635
	Simulated	4,9848	7,6212	9,9696	10,7780	6,2240	9,9151
(R, s, S)	$CI_{95\%}$	(±0,0480)	(±0,1038)	(±0,0960)	(±0,1468)	(±0,0647)	(±0,0701)
	Diff.	0,30%	-0,07%	0,30%	-0,07%	42,37%	10,38%
1.2	Analytical	5,0000	7,6158	10,0000	10,7703	10,8000	11,0635
	Simulated	4,9929	7,6201	9,9857	10,7764	7,2874	12,3258
(R, s, S)	$CI_{95\%}$	(±0,0782)	(±0,1374)	(±0,1564)	(±0,1943)	(±0,1647)	(±0,1831)
	Diff.	0,14%	-0,06%	0,14%	-0,06%	32,52%	-11,41%

Because the accuracy of the calculated input parameters is verified by the simulations performed for scenario 1.1 and 1.2, the simulation results of the input parameters expected daily demand and the standard deviation of daily demand are not included in the simulations results that follow in the remainder of this section. The simulated undershoot parameters are also excluded from the remainder of this section because they are not relevant for the simulated fill rate and inventory costs. Only the beforehand calculated parameters are dependent on the beforehand calculated undershoot parameters and these are verified by the spreadsheet from de Kok (2002).

In the case situation the lead-time is stochastic. Therefore stochastic lead-times have to be verified as well. Table 15 shows the simulation results of scenario 2.1 and 2.2. Both scenarios are run based on a target fill rate of 98% due to the fact that a target fill rate of 98% seems realistic for practical situations. Both scenarios include variable lead-time, a review period of $R = 4$ days and the demand input parameters are equal to those of scenario 1.1 and 1.2 ($E[D] = 5, \sigma(D) = \sqrt{58}$). The difference between scenario 2.2 and 2.1 is that in scenario 2.2 the lead-time variability is much higher. The reason for the post simulation expected lead-time to be higher than 12.00 is that the Gamma generated lead-times are rounded up to the nearest integer number of days.

Table 15: Simulation results scenario 2 ($m = 20.000, l = 2000, n = 10$)

Scenario		$E[L]$ $(days)$	$\sigma(L)$ $(days)$	$E[D(0,L)]$ $(days)$	$\sigma(D(0,L))$ $(days)$	$E[D(0,R)]$ $(days)$	$\sigma(D(0,R))$ $(days)$	R $(days)$	s	P_2
2.1		12,0000	2,0000	60,0000	28,2135	20,0000	15,2315	4	140	98,00%
		12,5011	2,0177	62,3507	28,6703	19,9505	15,1856	4	140	97,69%
(R, s, S)		12,5011	2,0177	62,3507	28,6703	19,9505	15,1856	4	144	98,30%
	$CI_{95\%}$									(±0,1546%)
	Diff.									-0,61%
2.2		12,0000	10,0000	60,0000	56,5332	20,0000	15,2315	4	212	98,00%
		12,5365	10,0478	62,5758	56,8479	19,9660	15,1156	4	212	97,87%
(R, s, S)		12,5365	10,0478	62,5758	56,8479	19,9660	15,1156	4	215	99,81%
	$CI_{95\%}$									(±0,0703%)
	Diff.									-1,94%

In both scenarios we can see that the fill rate is underestimated with the calculations. The post simulation fill rate is on average 0,61% and 1,94% higher in scenario 2.1 and 2.2 respectively. However, all the input parameters are very accurate if we compare the calculated input parameters with the average post simulation input parameters. Only the expected lead-time and expected demand during lead-time are slightly higher but this is due to rounding up the lead-time to the nearest integer in the lead-time generation process. Furthermore, if we use the *post simulation* input demand parameters $E[D]$, $\sigma(D)$, $E[L]$ and $\sigma(L)$ from table 15 in the spreadsheet of de Kok (2002), the resulting parameters such as $E\big[D(0,L)\big]$, $\sigma(D(0,L))$, $E\big[D(0,R)\big]$ and $\sigma(D(0,R))$ are almost exactly the same as those resulting from simulation. Hence, all parameters are correctly calculated beforehand and are very close to the simulated parameters, which lets us to believe that the beforehand set reorder level s is probably set too high and results in a fill rate that is a little bit higher than expected. Additionally, it could be due to the large variability of the lead-time.

Table 16 shows the simulation results of scenario 3.1 and 3.2. Scenario 3.1 simulates the (R,S) replenishment policy with constant lead-times while scenario 3.2 simulates the (R,S) replenishment policy with variable lead-times. Both scenarios have a target fill rate of 98%. An interesting finding with analyzing the (R,S) replenishment policy was that Optiply was not using the undershoot in the calculation of the order-up-to level S. Using the undershoot under an (R,S) replenishment policy sounds irrational because there is no reorder level s. However, we can use the (R,s,S) replenishment policy calculations for the (R,S) replenishment policy by setting $(S-s)=0$ and the order-up-to-level S equal to the expected demand and standard deviation of demand during lead-time and during the review period under a target fill rate. If the undershoot is not taken into account in calculating the reorder level s or order-up-to-level S under a certain target fill rate, one does not take into account the variance of the demand during the review period. This results in a lower actual experienced fill rate than was expected beforehand. Furthermore, both the spreadsheet from de Kok (2002) as the Optiply model were not performing enough iterations to find the optimal reorder level or order-up-to-level. A simple addition of performing more iterations in a larger search interval solved this problem (see chapter five for more details). Also for the (R,S) replenishment policy we can conclude that the fill rate calculations and calculations of the other relevant parameters are accurate. Only the fill rate calculation for scenario 3.2 is somewhat off. This probably has to do with the set reorder level which is set to high or the high lead-time variability.

Table 16: Simulation results scenario 3 ($m=20.000, l=2000, n=10$)

Scenario		$E[L]$ $(days)$	$\sigma(L)$ $(days)$	$E\big[D(0,L)\big]$ $(days)$	$\sigma(D(0,L))$ $(days)$	$E\big[D(0,R)\big]$ $(days)$	$\sigma(D(0,R))$ $(days)$	R $(days)$	S	P_2
3.1		1,00	0,00	5,0000	7,6168	10,0000	10,7703	2	55	98,15%
		1,00	0,00	5,0102	7,6221	10,0204	10,7792	2	55	98,15%
(R,S)		1,00	0,00	5,0102	7,6221	10,0204	10,7792	2	55	98,48%
	$CI_{95\%}$									(±0,1313%)
	Diff.									-0.33%
3.2		12,0000	10,0000	60,0000	56,5332	10,0000	10,7703	2	203	98,02%
		12,4506	9,9641	62,3009	56,6316	10,0078	10,7631	2	203	97,63%
(R,S)		12,4506	9,9641	62,3009	56,6316	10,0078	10,7631	2	207	99,91%
	$CI_{95\%}$									(±0,0275%)
	Diff.									-2,1%

Appendix N

Cost functions calculation verification

The following section describes the verification and simulation results of the developed cost functions of the new model. Section N.1 is on the verification of the holding cost function and section N.2 is on the verification of the ordering cost function.

N.1 Holding cost function

Table 17 shows the simulation results of the holding cost for all different scenarios. We see that the holding cost calculations in the 6th column (SPP) by Silver et al. (1998) are not very accurate compared to the simulated holding cost, especially the holding cost calculation from scenario 1.1. Such a difference in cost made us wondering if it would be better to use the expected stock on hand approximation by de Kok (2002) instead of the expected stock on hand approximation by Silver et al. (1998). Using the expected stock on hand approximations by de Kok (2002) takes the stock on hand at the beginning (i.e. $X(L_0)$) and the end of the replenishment cycle just before a replenishment order arrives (i.e. $X(R_1 + L_1)^-$). As can be seen in table 16 in the 3rd column this approximation resulted in much better estimations of the holding cost. The best holding cost approximation in ever scenario is displayed in bold. All holding cost simulation results have a 95% confidence interval smaller than $\pm 1\%$ with a minimum of $\pm 0.1473\%$ and a maximum of $\pm 0{,}7287\%$.

Table 17: Holding cost simulation results ($m = 20.000, l = 2000, n = 10$)

Scenario		Analytical (de Kok, 2002)	Simulated	Diff. %	Analytical (SPP, 1998)	Simulated	Diff. %
1.1	Total	**€984,63**	€1005,49	**-2,12%**	€852,16	€1005,49	-17,99%
(R,s,S)	$CI_{95\%}$		(±€1,6565)			(±€1,6565)	
	Daily	**€0,0547**	€0,0559		€0,0473	€0,0559	
1.2	Total	**€1757,89**	€1764,83	**-0,39%**	€1641.21	€1764,83	-7,53%
(R,s,S)	$CI_{95\%}$		(±€3,4285)			(±€3,4285)	
	Daily	**€0,0977**	€0,0980		€0.0912	€0,0980	
2.1	Total	**€3362,55**	€3303,09	**-1,98%**	€3239.01	€3303,09	-4,96%
(R,s,S)	$CI_{95\%}$		(±€14,6428)			(±€14,6428)	
	Daily	**€0,1868**	€0,1835		€0.1799	€0,1835	
2.2	Total	€6203,10	€6094,00	1,76%	**€6079,56**	€6094,00	**-0,24%**
(R,s,S)	$CI_{95\%}$		(±€44,41)			(±€44,41)	
	Daily	€0,3446	€0,3386		**€0,3378**	€0,3386	
3.1	Total	**€1768,77**	€1819,99	**-2,90%**	€1956.82	€1819,99	6,99%
(R,S)	$CI_{95\%}$		(±€2,6805)			(±€2,6805)	
	Daily	**€0,0983**	€0,1011		€0.1087	€0,1011	
3.2	Total	**€5437,81**	€5386,81	**0,94%**	€5625,86	€5386,81	4,25%
(R,S)	$CI_{95\%}$		(±€14,21)			(±€14,21)	
	Daily	**€0,3021**	€0,2993		€0,3125	€0,2993	

The holding cost approximation by de Kok (2002) is the most accurate in estimating the expected stock on hand. Therefore the $E[X]$ term in the holding cost functions becomes:

$$E[X] = s_i + \frac{\left(\frac{(S_i - s_i)^2}{2} - \frac{E^2[U_i] + \sigma^2(U_i)}{2} + \frac{E^2\left[D\left(0, R^\delta\right]\right] + \sigma^2\left(D\left(0, R^\delta\right]\right)}{2E\left[D(0, R^\delta]\right]\left((S_i - s_i) + E[U_i]\right)}\right)}{\left(\left((S_i - s_i) + E[U_i]\right) - \frac{E[D_i]}{E[A_i]}E[L] - P_2 \frac{E\left[D(0, R^\delta]\right]}{2}\right)} \quad (N.1)$$

where P_2 is the target fill rate and $(S - s)$ is the expected order quantity in every replenishment cycle. From the simulation results we can conclude that the newly defined holding cost functions give an accurate estimate of the holding cost.

N.2 Ordering cost function

Table 18 shows the simulation results for the ordering cost functions. It also shows the calculated ρ_i and the simulated ρ_i, which is the probability that an actual replenishment order is placed in a replenishment cycle. We also calculated and simulated this ρ_i for the (R, S) replenishment policy where we assumed that it is equal to 1. However, in scenario 3.2 the ρ_i was smaller than 1 meaning that an order is not necessarily placed in every replenishment cycle . All ordering cost simulation results have a 95% confidence interval which is smaller than $\pm 1\%$ with a minimum of $\pm 0\%$ and a maximum of $\pm 0.7717\%$. We can conclude that the ordering cost functions provide accurate ordering cost estimations according to the simulation of these cost.

Table 18: Ordering cost simulation results ($m = 20.000, l = 2000, n = 10$)

Scenario		Analytical	Simulated	Diff. %
1.1	Total	€85.538,46	€84.516,00	2,34%
(R, s, S)	$CI_{95\%}$		(±€462,33)	
	Daily	€4,81	€4,70	
	p_i	0,4808	0,4695	2,34%
1.2	Total	€86,538.46	€86216,00	0,37%
(R, s, S)	$CI_{95\%}$		(±€826,55)	
	Daily	€4,81	€4,79	
	p_i	0,4808	0,4790	0,37%
2.1	Total	€50279,33	€49598,00	1,36%
(R, s, S)	$CI_{95\%}$		(±€369.,91)	
	Daily	€2,79	€2,76	
	p_i	0,5587	0,5512	1,36%
2.2	Total	€50,279,33	€50.078,00	0,40%
(R, s, S)	$CI_{95\%}$		(±€386,45)	
	Daily	€2,79	€2,78	
	p_i	0,5587	0,5564	0,40%
3.1	Total	€180.000	€180.000	0,00%
(R, S)	$CI_{95\%}$		(±€0,00)	
	Daily	€10,00	€10,.00	
	p_i	1	1	0,00%
3.2	Total	€180.000	€179.972	0,02%
(R, S)	$CI_{95\%}$		(±€44.90)	
	Daily	€10,00	€10,00	
	p_i	1	0,9998	0,02%

In case of joint replenishment, the review period R^δ is set for all products in the subset $V^\delta = \{1,2, \ldots, i, n\}$. The probability of placing an order for product i in a replenishment cycle ρ_i should be calculated for every product i in subset V^δ. Because the calculations of ρ_i and the other relevant parameters are accurate, we can use the ordering cost function from section 4.6.2.2 for a joint replenishment situation under a (R^{δ}, s_i, S_i) replenishment policy:

$$OC_2(\rho_i) = \left(1 - \prod_{i \in V^{\delta}} (1 - \rho_i)\right) K \tag{N.2}$$

Let us give a short example: if we have 3 products: $V^{\delta} = \{1,2,3\}$ with $\rho_1 = 0.5$, $\rho_2 = 0.6$ and $\rho_3 = 0.8$. The probability of placing an order to the supplier in a replenishment cycle is equal to: $1 - \prod_{i \in V^{\delta}}(1 - \rho_i) = 1 - \big((1 - 0.5)(1 - 0.6)(1 - 0.8)\big) = 1 - 0.04 = 0.96$. The expected ordering cost per replenishment cycle is then equal to $0.96K$. For the (R^{δ}, S_i) replenishment policy we have shown that $\rho_i \approx 1$ and therefore the ordering cost function for a replenishment cycle is:

$$OC_1 = K \tag{N.3}$$

A practical case of the probability in a joint replenishment situation is given in section 5.3.

Appendix O

Simulation model input parameters

The input parameters of the different scenarios are shown in **table 19** and **table 20**.

Table 19: Simulation model input parameters

Scenario	$E[D_i]$ $(days)$	$\sigma(D_i)$ $(days)$	$E[A_i]$ $(days)$	$\sigma(A_i)$ $(days)$	$E[L]$ $(days)$	$\sigma(L)$ $(days)$	R^{δ} $(days)$	s_i	S_i	P_2 $(target)$
1.1	10	4	2	2	1	0	2	27	-	90%
1.2	10	4	2	2	1	0	2	47	-	98%
2.1	10	4	2	2	12	2	4	140	-	98%
2.2	10	4	2	2	12	10	4	212	-	98%
3.1	10	4	2	2	1	0	2	-	55	98%
3.2	10	4	2	2	12	10	2	-	203	98%

Table 20: Simulation model input parameters

Scenario	$E[D_i]$ $(days)$	$\sigma(D_i)$ $(days)$	$E[A_i]$ $(days)$	$\sigma(A_i)$ $(days)$	$E[L]$ $(days)$	$\sigma(L)$ $(days)$	R^{δ} $(days)$	CV_R	s_i	S_i	P_3 $(target)$
4.1	10	0	1	1	1	0	1	1	-	48	95%
4.2	10	6	2	2	1	0	1	1,65	-	44	95%
4.3	10	10	2	2	1	0	1	2	-	58	95%
4.4	10	20	2	2	1	0	1	3,16	-	121	95%

Appendix P

Implementation Decision support tool

Section P.1 describes the input parameters that are required for the tool such that it can provide the user the output parameters. In section P.2 the different features of the tool are elaborated on. Section P.3 provides a user manual on how to use the tool correctly, including explanation on how to read, write and update the input and output data. Section P.4 briefly describes the user interface of the tool.

Simulation tool

As described in chapter five, additional to the decision support tool we developed a simulation tool that is able to simulate daily replenishment operations for the (R^{δ}, S_i) and $(R^{\delta}, \mathrm{s_i}, S_i)$ replenishment policy. This simulation tool can be used for test products by inserting test data into the tool or for real products by inserting historical data or forecasted data. The simulation tool is not relevant for the decision support tool and will therefore not be explained in too much detail. However, all versions of the simulation tool will be made available for Optiply and its current and future graduation students. The simulation tool simulates stochastic demand (or real demand), stochastic lead-times, the daily stock on hand, the daily inventory position, daily undershoot, daily lost-sales, review moments, replenishment orders and replenishment deliveries. Simulation can be done for one or more products from the same supplier such that inventory cost such as holding cost, ordering cost and shortage cost can be simulated for a time period chosen by the user (e.g. 1000 days, 20.000 days etc.).

P.1 Input & output parameters

P.1.1 Input parameters

The decision support tool requires several input parameters to provide the output that is needed for the desired inventory control. The most important input parameters are the demand input parameters. Demand parameters are split up in demand inter-arrivals and demand order sizes (i.e. continuous demand). Therefore, demand is assumed to follow a compound renewal process.

P.1.1.1 Demand forecasting

Because in practice demand does not behave stationary, the demand parameters from the historical data are combined with an *ETS* forecast software package which combines 16 different forecasting methods that take into account any seasonal and trend factors (R.J. Hyndman et al., 2008; Rob J. Hyndman, Akram, & Archibald, 2008; Rob J. Hyndman, Koehler, Snyder, & Grose, 2002). Forecasting is performed over a horizon of 90 days. The forecast method in combination with the developed impute demand methods provide an estimation of the demand parameters and only requires the POS data and stock changes data. The demand input parameters are then balanced based on the historical data and the forecast. Hence, we could state that we only require

POS data and stock changes data to determine the input demand parameters. Note that in practice, the forecasted variance of demand during lead-time in one period may be substituted into the model as the variance of demand during lead-time (i.e. direct substitution of the forecast error). Other methods include corrections of demand during lead-time based on the forecast error and assumes the demand during lead-time to be normally distributed. This leads to corrected safety stocks based on forecast errors such as the RMSE or the MASE (Axsäter, 2006; Prak, Teunter, & Syntetos, 2017; Silver et al., 1998). From the stock data we acquire the lead-time information per supplier and in combination with the demand parameters this provides us with the mean and standard deviation of demand during lead-time. The decision support requires the following input parameters to function:

1. Product name or Product ID
2. Supplier name or Supplier ID
3. POS data, wherefrom the following parameters are calculated:
    a. Mean and standard deviation of the inter-arrival time of sales orders in days
    b. Mean and standard deviation of the sales order sizes in units
    c. Supplier aggregate review period for all products ordered to the same supplier
4. Demand forecast based on a 3-month forecast horizon, wherefrom the following parameters are calculated:
    a. Mean and standard deviation of the inter-arrival time of demand order in days
    b. Mean and standard deviation of the demand order sizes in units
5. Stock (changes) data, wherefrom the following parameters are calculated:
    a. Mean and standard deviation of the time between replenishment order and replenishment delivery per supplier in days (i.e. lead-time)
6. Current stock on hand of a product in units
7. Current purchase price of a product in preferred currency
8. Current selling price of a product in preferred currency
9. Annual holding cost rate in % per year; the daily holding cost rate can be calculated by taking annual holding cost rate divided by number of days in a year
10. Fixed ordering cost per replenishment order to a supplier in preferred currency
11. Shortage cost per product in preferred currency
12. Target fill rate (i.e. P_2 service level)

P.2.2 Output parameters

The output parameters of the tool include the reorder levels, order-up-to-levels and inventory cost of the different products. All output is saved in *R* data frames and can easily be written to Excel files if required. The decision support tool provides the following output parameters:

1. Suggested supplier review period per supplier
2. Suggested order quantity per product
3. Reorder level per product (in case of can order policy only)
4. Order-up-to level per product
5. Expected average stock on hand
6. Expected stock on hand just after replenishment delivery
7. Expected daily holding cost

8. Expected daily ordering cost
9. Probability of stock-out just before replenishment delivery (i.e. P_1 service level)
10. Adjusted fill rate (i.e. P_2 service level)

P.2 Features

This section describes the features that the decision support tool has to offer. The tool works by providing it with the required input parameters and running it for as many suppliers and products as needed. Input parameters such as the economic order quantity, holding cost rate and the fixed ordering cost can be altered manually if desired. All cost calculations are based on the expressions that we elaborated on throughout the mathematical model of chapter four and the relevant changes to the holding cost expressions in chapter five.

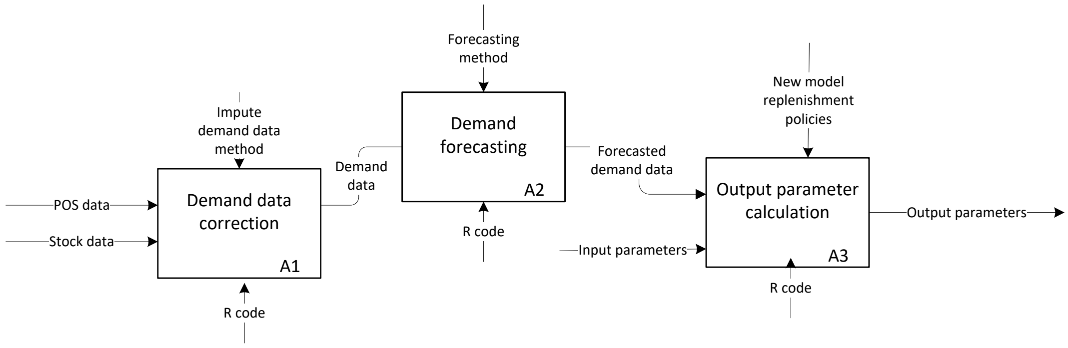


Figure 11: Decision support tool features in IDEF0 notation

Figure 11 shows a graphical representation of the features of the tool in IDEF0 notation. The rectangles represent the aggregate functions of the tool. Input parameters are shown to the left of every rectangle, output parameters to the right, resources/mechanisms on the bottom and control parameters on top. Figure 12 zooms in into process A3: Output parameter calculation.

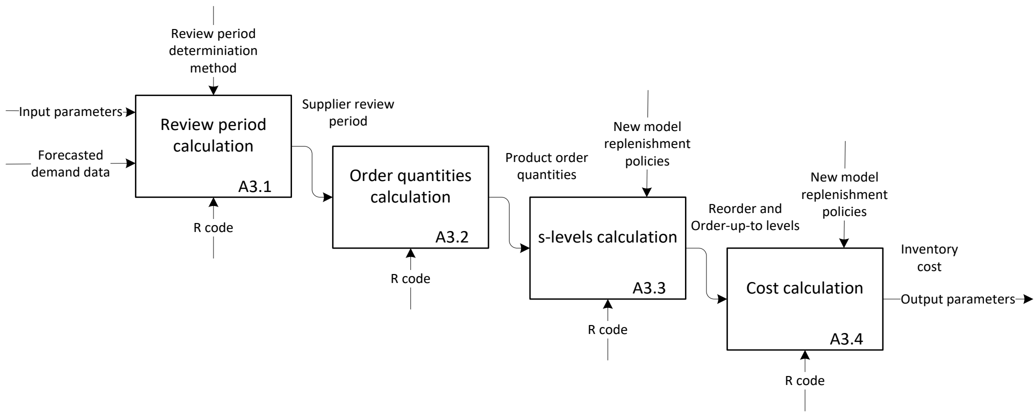


Figure 12: A3. Output parameter calculation in IDEF0 notation

P.2.1 Output parameter calculation

P.2.1.1 Supplier aggregate review period calculation and product ordering quantities

- The supplier aggregate review period is calculated per supplier based on the average daily demand of all products ordered to the same supplier, their daily holding cost and the major fixed ordering cost of placing an order to the supplier (see section 4.6). All products ordered to the same supplier follow this review period for their replenishment process.
- Based on the supplier aggregate review period, the economic order quantity of every product is calculated (see section 4.6).

P.2.1.2 Reorder level and order-up-to level calculation

- The following parameters are calculated with the $DetermineOutputParameters(x)$ function based on lead-times, review period and demand input parameters (see section P.3.2 for an extensive explanation of the functions):
  - mean and standard deviation of demand during lead-time; $E[D(0,L)]$ and $\sigma(D(0,L))$
  - mean and standard deviation of demand during the review period; $E\left[D\left(0,R^{\delta}\right)\right]$ and $\sigma\left(D\left(0,R^{\delta}\right]\right)$
  - mean and standard deviation of the undershoot; $E[U_i]$ and $\sigma(U_i)$
  - mean and standard deviation of demand during lead-time plus the undershoot; $E[(0,L]+U]$ and $\sigma(D(0,L]+U)$
- Based on the input parameters the tool calculates the reorder level s_i required to achieve the target fill rate P_2. The order-up-to level S_i equals the reorder level plus the economic order quantity EOQ.

P.2.1.3 Other relevant output parameters

- Additionally, the tool calculates the safety stock, the expected average stock on hand, the expected ordering probability of a product in an ordering cycle, the expected order quantity, the adjusted target fill rate, the expected daily shortage and the P_1 service level.
  - The safety stock ss_i is defined as the stock on hand just before the arrival of a replenishment order (i.e. the stock that is kept to guard for the demand variability during the replenishment cycle and the variability of the lead-time).
  - The expected average stock on hand $E[X_i]$ is defined as the stock on hand just after the arrival of a replenishment order subtracted by the stock on hand just before the arrival of a replenishment order.
  - The expected ordering probability ρ_i is defined as the probability that an order for product i is placed to the supplier in an ordering cycle.
  - The expected average ordering quantity $E[Q_i]$ is defined as the expected difference between the inventory position and the order-up-to-level just before placing a replenishment order to the supplier (i.e. $\rho_i E[S_i - s_i + U_i]$ or $\rho_i E\left[D\left(0,R^{\delta}\right] + U_i\right]$).
  - The adjusted target fill rate P_2 is the target fill rate that results from calculating the reorder level s_i that satisfies the target fill rate. Because the reorder level is rounded up to the nearest integer, the adjusted fill rate is often slightly higher than the initial target fill rate.

- o The expected daily shortage is defined as the fraction of daily demand that cannot be satisfied from stock on hand immediately. (i.e. adjusted $P_2 * \frac{E[D_i]}{E[A_i]}$).
- o The P_1 service level is defined as the probability of no stock-out just before the arrival of a replenishment order.

P.2.1.4 Cost calculations

- With respect to expected cost, the tool calculates the expected daily holding cost per product and the expected daily ordering cost per supplier.
  - o The expected daily holding cost per product is defined as the expected average stock on hand multiplied by the purchase price of a product and the daily holding cost rate.
  - o The expected daily ordering cost are defined as the product of the expected ordering probabilities of the products ordered to the same supplier multiplied by the major fixed ordering cost divided by the review period. Note that the ordering cost are on the aggregate supplier level and not on the individual product level.

P.2.2 Demand input parameters

- Demand forecasting, EOQ calculations, reorder level and cost calculations can be based on 3 types of demand input which are all based on historical sales data.
  - o Sales data: all calculations are based on historical sales data. Note that sales data only includes demand from the days where stock on hand was positive (there were no sales on days without stock and hence, demand is not known on such days);
  - o Sales data with Gamma imputed demand: a Gamma distribution is fit to the historical sales data from the days with positive stock on hand. Thereafter demand is imputed on the historical days without stock based on this Gamma demand distribution.
  - o Sales data with imputed demand from last month: demand is imputed on the historical days without stock on hand. This imputed demand is based on the historical sales from the last 10 to 30 days where stock on hand was positive (i.e. demand from the last 10 to 30 days). This procedure was elaborated on in section 4.6 and a simplification of the *R* code can be found in appendix I.

P.3 User manual

The different calculations should be made with the appropriate input parameters and it is important that the user knows which functionalities the different functions have. This section describes the different functions of the tool and how to use them. Note that all data manipulation processes in between the described functions are important but are performed automatically if all the code is run as a whole. Data manipulation includes steps such as filtering of relevant data, combining data and calculating parameters.

P.3.1 Supplier aggregate review period and product ordering quantities calculation

The supplier aggregate review period is based on the average daily demand of all products ordered to the same supplier, their daily holding cost and the major fixed ordering cost of placing an order to the supplier. Taking into account the fact that demand for products is changing through time it may be valuable to adapt the supplier review period throughout the year. In the current situation, this is done every 90 days. Calculating the supplier review period and the corresponding product ordering quantities is performed by two developed functions that should be called subsequently:

- The function $DetermineSupplierReviewPeriod(x)$ takes as input a data frame with the major fixed ordering cost per supplier, product information such as the purchase price, the holding cost rate, product demand data and provides as output a data frame with the review period per supplier.
- The function $DetermineProductEconomicOrderQuantities(x)$ takes as input the data frame with the review period per supplier and product demand data and provides as output a data frame with the economic order quantities per product.

P.3.2 Reorder level and order-up-to-level calculation

The decision support tool may be used on a daily basis if desired by the user. In theory, adapting the reorder levels and order-up-to levels of products is also possible every day. In practice however, it is recommended to perform the calculations with the tool every 30, 60 or 90 days, set the inventory control parameters accordingly and register the performance of the tool by gathering data on inventory control resulting from the tool. It is important to mention here that all calculations, approximations and suggestions made by the tool are based on long-run inventory control. Therefore, the tool will especially show positive results over a longer period of time. The reorder level and order-up-to level calculation is based on two developed functions with two sub functions:

- The function $DetermineOutputParameters(x)$ takes as input parameter the data frame with input demand parameters, the calculated supplier aggregate review period and economic order quantities per product and provides the user with all output parameters and cost calculations described in section P.2.1. Furthermore, the functions requires specification of: which replenishment policy to use, if backorders are accepted and if undershoot should be taken into account. Calling on the function $DetermineOutputParameters(x)$ for the (R^{δ}, s_i, S_i) replenishment policy with undershoot and no backorders would look like:
  $DetermineOutputParameters(\text{Input\_data} = \text{data}, \text{Policy} = \text{"RsS"},$
  $\text{Backorders} = \text{FALSE}, \text{Type\_of\_service} = \text{"P2"}, \text{Undershoot} = \text{TRUE})$
  Note that the reorder level and order-up-to level are calculated differently if undershoot is taken into account compared to when undershoot is not taken into account.
- Within the $DetermineOutputParameters(x)$ function two sub functions are automatically called upon, which together provide the 'optimal' reorder level based on the input parameters and the set target fill rate. For very slow moving products or products with too little data, the reorder level is set to 1 and not 0. If stock on hand is zero there is no demand information at all. The two sub functions of the $DetermineOutputParameters(x)$ are:
  - $CalculateReorderlevel(x)$, which performs an iteration process to find the 'optimal' reorder level based on the input parameters and the target fill rate. This iteration process sets a minimal reorder level ($s_i = 0$) and a maximal reorder level ($s_i = 12 * E[D(0, L]]$) and calculates different reorder levels until a reorder level is found that achieves the

target fill rate (De Kok, 2002). The fill rate is calculated using the other sub function: the $CalculateFillrate(x)$ function.
  - $CalculateFillrate(x)$, which calculates the expected achieved target fill rate based on the different reorder levels calculated by the $CalculateReorderlevel(x)$ function.
- Relevant to mention for some of the output parameters described in section P.2.1.3 is the following:
  - Expected average stock on hand $E[X_i]$: If undershoot is taken into account the expected average stock on hand is calculated with the formula by de Kok (2002). If undershoot is not taken into account the expected average stock on hand is calculated with the formula by Silver et al. (1998). The calculation of the expected average stock on hand has a direct effect on the expected daily holding cost $EDHC$.
  - The P_1 service level is calculated differently if undershoot is taken into account compared to when undershoot is not take into account.
- Output is saved in an *R* data frame that can easily be written to an *Excel* fil if needed. Moreover, after saving this *Excel* file, the user may change parameters such as the period of historical data, the order quantity per product or the holding and ordering cost. Thereafter, the function $DetermineOutputParameters(x)$ can be called again and will provide a new data frame with adapted output parameters. In this way, the user may compare the output and decide on making potential changes in the input parameters due to future promotions or increasing/decreasing holding and ordering cost.

P.3.3 Cost calculations

- The (long-term) expected daily holding cost $EDHC$ calculations per product are also integrated into the $DetermineOutputParameters(x)$ function. Once the functions is called it will include the holding cost calculations in the output data frame.
- To calculate the (long-term) expected daily ordering cost $EDOC$ per supplier the user has to call the function $CalculateDailyOrderingCost(x)$ because all the before described calculation are on a product bases while the calculation of the ordering cost is on a supplier basis. The function $CalculateDailyOrderingCost(x)$ takes as input the data frame resulting from the $DeterminineOutputParameters(x)$ function and provides the user with a data frame with al inventory control parameters including the just calculated expected daily ordering cost per supplier.

P.3.4 Demand input parameters

- The 3 forms of demand input can be created as follows:
  - Demand input parameters based on only sales data do not need any further function call. POS data contains the sales that were made on specific moments in time. The only step taken is impute zero demand on the days without sales to make the sales data a time series.
  - Demand input parameters based on sales data with Gamma imputed demand can be created by calling the function $ImputeGammaDemanddata(x)$. The function takes the sales data and stock data as input and provides the user with a data frame with demand

data on the days with positive stock (i.e. sales) and Gamma distributed demand on the days without stock (i.e. imputed demand).

  - Demand input parameters based on sales data with imputed demand from last month can be created by calling the function $ImputeDemanddata(x)$. The function takes the sales data and stock data as input and provides the user with a data frame with demand data on the days with positive stock (i.e. sales) and demand on the days without stock based on the demand of the last 10 to 30 days with positive stock (i.e. imputed demand).
- After creating the appropriate type of demand input parameters the demand input can be used in the demand forecasting method and thereafter for the supplier review period determination, EOQ calculation, s-level calculation, fill rate calculation and cost calculations.

P.3.4.1 Demand forecasting

As described in section P.1.1.1, demand forecasting is used for all products. By relaxing the assumption that demand for products is stationary, all developed calculations are not more than approximations. However, all calculations were compared with simulated values and under stochastic lead-times and unpredictable variable demand the new model replenishment policies performed reasonably. Due to limited historical forecast data and information on the used method, it is advised to use the forecast method in combination with the *ImputeDemand* functions and adapting the s-levels not more than once per review period because this has no effect. In short, this would mean that forecasts are performed daily and that s-levels of the different products could be adapted between subsequent review periods or once in a larger time interval (2,3,… review periods). Adapting the aggregate supplier review periods is recommended to be done with a frequency larger than 90 days due to the demand forecasting horizon of 90 days.

P.4 Tool user interface

This section will briefly describe the user interface of the decision support tool. The tool includes two scripts: (1) Decision support tool (DST): a script with all function calls and automatic data manipulation steps and (2) Functions: a script with all the code of the functions used in the decision support tool script. It is not recommended to make changes to the functions script because this script functions as a source for the decision support tool script. The different parts of code can be run in the decision support tool script to create data frames with the output parameters. Therefore, the tool is not an *Excel* of *VBA* tool where product and supplier data has to be imported manually or displayed on one tab. Product data of as many products required can be imported into *R* and the tool calculates the output parameters for every product that has enough historical data. We describe which output parameters are presented in which data frame. If required, these data frames can easily be written to an *Excel* file.

Sales_data_imputed_demand_table

The Sales_data_imputed_demand_table data frame is created after the functions $ImputeDemandData(x)$ or $ImputeGammaDemandData(x)$ are called upon. The data frame includes the products and their historical daily stock on hand and daily demand. Based on this data, the historical sales data is corrected and we have the mean and standard deviation of demand inter-arrivals and the mean and standard deviation of demand sizes which will be used in the forecast and the calculations that follow.

Review_period_supplier_table

The Review_period_supplier_table data frame is created after calling on the function $DetermineSupplierReviewperiod(x)$ and shows the suppliers and their calculated supplier aggregate review period. This review period is then linked to the different products coming from the corresponding supplier.

EOQ_product_table

The EOQ_product_table data frame is created by calling on the function $DetermineProductEconomicOrderQuantities(x)$ and shows the products and their economic order quantities and review period.

Calculated_output_parameters_table

The data frame Calculated_output_parameters_table shows all output parameters except the expected daily ordering cost per supplier and is created by calling on the function $DetermineOutputParameters(x)$.

Total_calculated_output_parameters_table

The data frame Total_calculated_output_parameters_table presents all the output parameters including the expected daily ordering cost per supplier and is created by calling on the function $CalculateDailyOrderingCost(x)$ with the Calculated_output_parameters_table as input.

The tool can easily be linked to other *R* code and to the WMS databases in the backend and an application or website in the frontend. With respect to updating the data, new POS and stock data can automatically be imported into the tool and therefore calculations can be made on a daily basis if desired. However, changing parameters such as the reorder level and target fill rate is not recommended on a daily basis because the performance of the tool under certain set parameters takes some time to present itself.

Figure 13 shows a flowchart of the different steps of the tool. The different functions in the tool can be run as a whole to calculate the output parameters in one run. The functions can also be run step by step such that changes to input parameters may be made manually in between the different steps

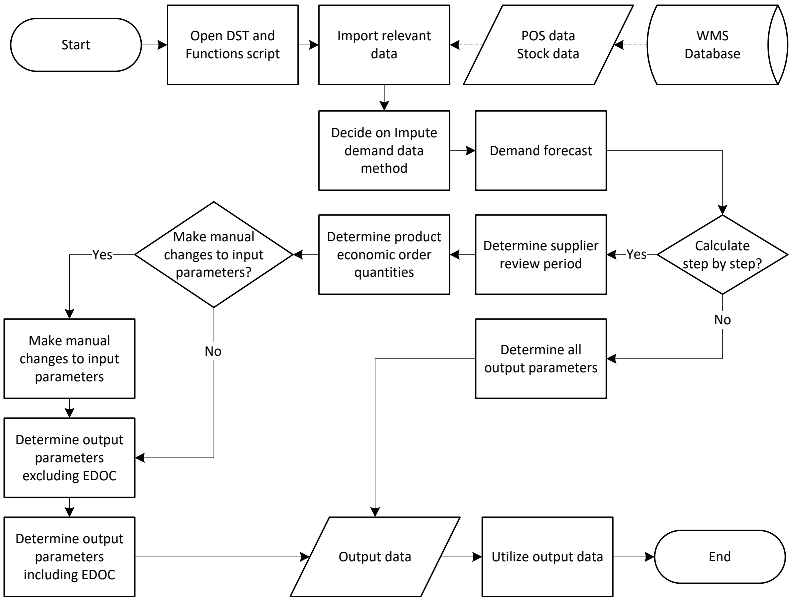


Figure 13: Basic flowchart of Decision Support Tool

Appendix Q

Detailed elaboration of underlying research objectives

In this section we give a more detailed elaboration of accomplishing the underlying research objectives of our research. A large part of the conclusions in this section can also be found in chapter seven.

1. *Describe the as-is situation of inventory control at Company B.*

Company B functioned as a good example for an e-tailer where almost all inventory control and purchasing process were performed manually and without strict decision rules. The as-is situation was analyzed in chapter one and chapter three. We thereby focused on the inventory control processes and identified the problems that occurred in these processes. Problems were found to be a combination of the environment for webshops to work in and the fact that inventory control is often overlooked when companies are rapidly increasing. In an e-commerce environment, backorders are often not accepted resulting in lost-sales when demand exceeds the stock on hand. The complex nature of demand and the fact that backorders are often not accepted, results in high inventory levels or more lost-sales than desired. The main components that were deemed relevant in analyzing the inventory control situation and the first literature review were periodic joint replenishment, lost-sales systems and stochastic demand and lead-times.

2. *Provide conceptual solutions aimed at the joint replenishment problem, complete lost-sales and the other problem areas by combining theoretical and practical knowledge and model these solutions into a mathematical model.*

Reviewed literature on replenishment policies, joint replenishment and lost-sales systems in combination with the problem analysis of Company B and Optiply resulted in the development of two newly modified replenishment policies. The (R^{δ}, S_i) and (R^{δ}, s_i, S_i) replenishment policies were introduced in chapter four after careful consideration of different solution concepts that had the purpose of solving the different components of the inventory control problem. We described how the aggregate joint replenishment problem can be approached by splitting it up in n single-item replenishment problems with a supplier aggregate review period to coordinate the ordering process. We concluded that the single-item problem solution concept should be of a periodic nature because the aggregate review period is the controlling variable in the joint replenishment process. With respect to the lost-sales problem, we pointed out multiple solution concepts that had to do with correcting the fill rate in a lost-sales system or with uncovering unobserved demand that occurs when the stock on hand of a product equals zero.

3. *Analyze the inventory control model developed by Optiply and relate it to the conceptual solutions in the process of developing an inventory control model that can perform in an e-commerce characteristic environment.*

4. *How can the current inventory control model and the suggested improvements of the conceptual model be combined into a decision support tool that takes into account the components of an e-commerce characteristic environment? (i.e. define KPIs, test the tool, compare output, generalize tool)*

The Optiply model was analyzed throughout the project and for different components of the model we provided improvements to existing components or came up with new components. In chapter four we introduced a method to take unobserved demand into account such that future demand is less underestimated. Based on the solution concepts we described in our research, we developed a method to take into account the demand in periods without stock based on the assumption that demand in periods without stock on hand follows the same demand pattern as demand in periods without stock on hand. Within this method two options can be chosen: (1) impute Gamma distributed demand on the days without stock on hand based on the days with positive stock on hand or (2) impute demand based on the last 10 to 30 days with positive stock on hand on the days without stock on hand. Furthermore, we suggested a different method to determine the review period that all products ordered to the same supplier should follow. This supplier aggregate review period is calculated based on the fixed ordering cost, holding cost per product and the average demand per product. We admit of course, that this method could also overestimate actual demand. However, because the method is based on a historical data set that is growing every day, the method will improve with every day because it 'learns' from the data.

Relevant KPIs were found to be the (target) fill rate (implicitly taking shortage cost into account), demand during lead-time, demand during the review period, expected average stock on hand, expected daily holding cost and the expected daily ordering cost.

Product demand is often highly variable and in some cases probably dependent on seasonality, trends or competitor pricing. Therefore, demand for all products has to be partly based on historical data and partly on demand forecasts.

The expected average stock on hand approximation by Silver et al. (1998) often underestimated the actual average stock on hand.

The fill rate calculations and other relevant parameter calculations of both the (R^{δ}, S_i) and $(R^{\delta}, \mathrm{s_i}, S_i)$ replenishment policy were found accurate under (highly) variable stochastic demand and stochastic lead-times; under highly variable stochastic lead-times the fill rate calculations somewhat underestimated the actual experienced fill rate.

The inventory cost calculations of both replenishment policies in the new model were found accurate under (highly) variable stochastic demand and stochastic lead-times. Both the holding cost and the ordering cost function provides good approximations for the expected inventory cost. Therefore, we can conclude that both policies minimize total inventory cost under a target fill rate.

In both theoretical and practical situations the undershoot variable should be taken into account, not only for the (R^{δ}, s_i, S_i) replenishment policy but also for the (R^{δ}, S_i) replenishment to achieve the target fill rate (this was not done by Optiply before).

In almost all simulation situations, both replenishment policies in the new model outperformed those in the Optiply model with respect to achieved fill rates, total inventory cost and reliability (i.e. tighter 95% confidence intervals); note that the cost efficiency is highly dependent on how the different cost parameters are set.

In almost every simulation situation, the (R^{δ}, S_i) replenishment policy outperformed the (R^{δ}, s_i, S_i) replenishment policy regarding achieved fill rates, total inventory cost and reliability (i.e. tighter 95% confidence intervals); note that the cost efficiency is highly dependent on how the different cost parameters are set.

Both policies in the new model performed reasonably under 'real demand data' in the simulation analysis with respect to achieved fill rate and cost efficiency (see section 5.3.2), especially taken into account that replenishment was performed without new data information in between and was simulated for a 3 month horizon.

The developed decision support tool uses historical sales/demand data and potential demand forecasts as input to provide appropriate inventory control parameters and inventory cost such as the reorder level, holding cost and ordering cost. The tool is able to perform this calculations for as many products and supplier as desired.

5. *Write an implementation plan on the recommended use of the decision support tool for inventory control.*

Chapter six provided an extensive description of the decision support tool and discussed the features of the tool and recommended use. The tool is built up in such a way that it is easily generalized and can be used in other inventory control situations. Its only true input parameters are POS data and Stock changes data. Therefore, the only aspects that need changing are the different variable names and the input for certain specifics such as the holding cost rate or the fixed ordering cost. Note that the ordering cost should be calculated differently once minor fixed ordering cost play a role in the replenishment process as well. For Optiply, the *R* code of the tool can be integrated into the Optiply model with little effort. Small additions such as variable names should be changed for smooth integration and some explanation on the different components may be given.

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