MASTER

Risk management practices in electricity trading

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Risk Management Practices in Electricity Trading

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Abstract

This thesis describes the risk management practice of an electricity trader in the Dutch electricity market. The thesis focuses on the imbalance and price risk that an electricity trader is exposed in the daily operation of its business. An analytical solution is derived for which the price and imbalance risk are minimized. Next, a pricing model was developed which was the basis for the investigation of the analytical formulas. Finally, a model was developed to assess the risk management of price risk and imbalance risk of the existing portfolio of customers of Anode. Based on those results recommendations are made for the management of risk of Anode's portfolio.

Management Summary

This thesis focuses on the commodities risk management at an electricity trading company. The company is actively buying and selling electricity for its customers. The liberalization of electricity markets introduced a market price setting process which enabled extreme spikes, high volatility and uncertainty of the cash flows generated by electricity flows. This has made it necessary to actively manage all risks associated with managing an electricity portfolio through buying or selling of financial contracts consisting of futures, forwards, options and swaps.

An electricity trader's business model consists of operating as a gateway between the customer and the wholesale markets. The trader exchanges a physical electricity flow for a cash flow in either direction. This direction depends on the type of customer, who can be a consumer of electricity or a producer or both. Specifically, the customer transfers electricity from or to the trader in exchange for a cash flow. Then the trader needs to transfer this electricity from or to the wholesale market, again in exchange for a cash flow. This is summarized in the picture below. Note that the trader's customers can have either positive or negative electricity flows (supplier or consumer) which are each other's opposite.

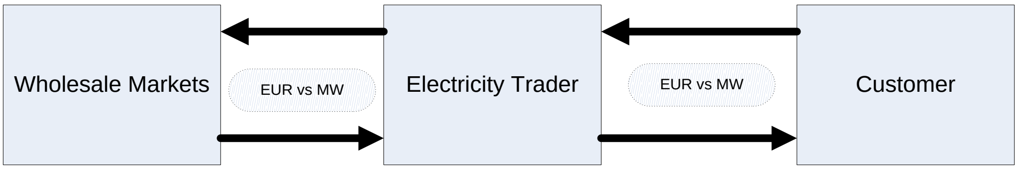


Figure 1: Conceptual overview of Cashflows versus Electrical Flows

By offering this gateway function the trader has multiple ways to earn money. First, the trader earns a spread on the cash flow between the customer and the corresponding cash flow with the wholesale markets. The margin however is quite low because of the highly competitive nature of the electricity market. Maintaining this margin is therefore an important point to consider in the risk management policy. In addition, the trader earns money by managing a customer's imbalance. Imbalance is defined as the difference between a projected electricity flow and the realized electricity flow. Normally, the customer himself would need to pay an uncertain price for this stochastic difference that occurs. At an adequate fee, a trader might be able to take on this risk which is highly unpredictable and uncertain.

Risk management should be the main concern for any electricity trader. The fact that this trader has no direct control on the production or consumption of the customers makes traders especially vulnerable to both price and imbalance risk. An operational hedge was designed by pooling different customers with price or imbalance risk together. Overall this results in a reduction of the total risks as both producers' and consumers' demand is netted. Ideally, a balanced pool of consumers and producers would result in zero price and imbalance risk, while earning a margin of both sides of the transaction. Because correlation between the customers is not equal to one, an imperfect match will still result in an overall reduction of risk.

In reality a perfectly netted portfolio is not available and the netted demand exposure curve is always different from zero. Ideally, a perfect hedge is bought in the wholesale market, which offsets the net exposure. For example, a long position from customers will be offset by shorting an identical quantity through futures. By buying a futures contract in the wholesale markets a fixed quantity needs to be bought. The result is that for most hours of the day, exposures still remain. To hedge these exposures a trade-off is identified, where having a long position in one set of hours is offset with a short position in another set of hours. It was these net exposures and the optimal hedging of price risk on these exposures that was mainly investigated. This is shown in figure 2 below, where the dark solid line is the demand per hour of this portfolio and the dashed line represents the forward position taken to hedge this demand. The green (long) and red (short) indicate where there is a mismatch (net exposure) between the two. It is impossible to remove these exposures completely.

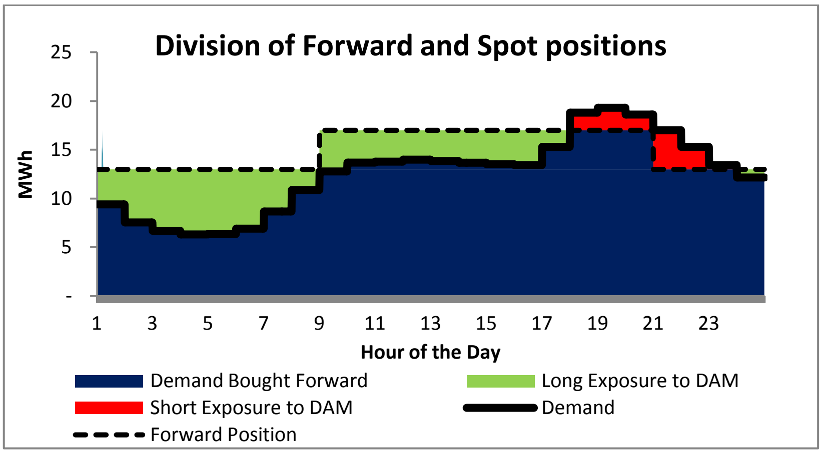


Figure 2: A schematic overview of the unhedged exposures to the DAM market

1.1 Results

From the analysis in this thesis an important finding is that the composition of the customer portfolio is important. The composition's effects are affected by the relationships between the customers themselves and between customers and electricity prices. Correlation between customers can increase or decrease the total risk. Therefore, it is important to include the customer characteristics that are used to construct the net exposure curve.

A model is developed that takes into account the characteristics of electricity prices of the Dutch electricity market. In addition, different demand scenarios are constructed and for each of those the risk minimizing hedging quantities are analytically determined. Using Monte Carlo simulations, the histograms of total cashflows are constructed for different scenarios. It can be seen that hedging, with either baseload-only or base- and peakload results in significant reductions of variance of cashflows and hence in risk. In short, price risk was minimized using the pooling procedure, followed by an optimal purchase of futures contracts in wholesale markets.

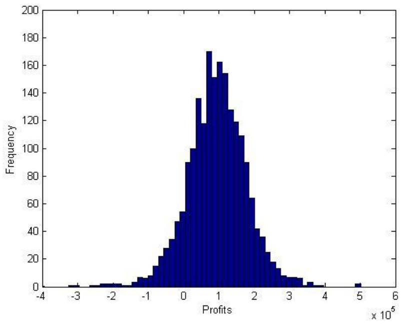


Figure 3: The distribution of cash flows from profiled customers with no hedging

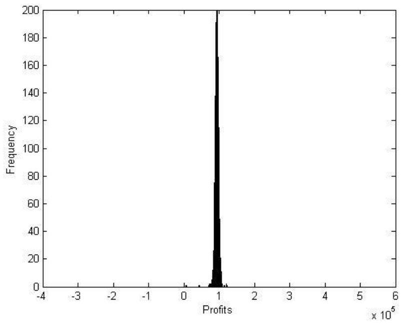


Figure 4: The distribution of cash flows for base and peakload

1.2 Recommendations

It is shown that for profiled customers the risk minimizing quantities are substantially higher than the average demand. This is caused by the fact that most of the demand occurs in peak hours that have a high price risk. Therefore, the risk-minimizing quantity is a trade-off between a reduction of peak-hour exposures as well as an increase of exposure in the off-peak hours. Using the risk minimizing portfolio of hedging instruments the variance of cashflows could be reduced by 93%, which leads of a guaranteed profit for this group of customers, versus an uncertain profit or loss in the other cases.

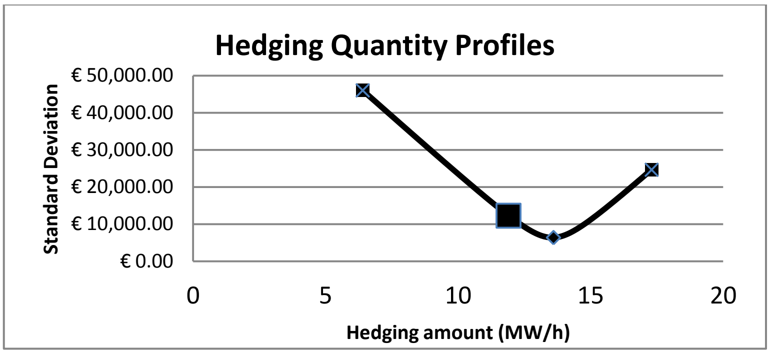


Figure 5: The standard deviation of cashflows as a function of the hedging quantity of baseload, the hedging quantities from minimum of demand to maximum, big square is average demand, while the little dot is the optimum.

The same is calculated for the wind customers, where the forecasted mean demand curve is flat. However, the variability of the demand and the correlation with electricity prices is different in this case. It is shown that because of the flat forecasted demand curve, the optimal hedge is equal to the average demand in the deterministic case. However, also in the stochastic demand case the peakload contract does not have any added value as there is no pattern in the mean forecasted demand curve. In both cases variance was reduced by 84%. The reduction leads to a guaranteed profit instead of an uncertain profit.

Finally, the procedure was repeated for the net exposure curve which was constructed of the five customer groups. Each customer's demand was individually simulated and the result shows that a significant reduction of risk occurs. The risk reduction is equal to 90% which is less than what is possible with only profiled customers, but more than only with wind. Again the difference in base- and peakload and baseload-only is results in an additional 20% reduction. For the overall exposure such a reduction means that a cashflows' confidence interval can be reduced from -1M EUR to 350k EUR (unhedged) to -400k to -300k EUR. Note that this is the cashflow from the exposures and is not equal to the total profit to the firm.

In general, we have shown that from a risk management perspective it is very important to reduce the short exposures during peak hours. It is not necessarily so that is achieved by both peak- and baseload contracts. Hedging electricity price risk can be done with baseload contracts only, to capture 80% of the gains. Hence, this finding explains the interesting observation that peakload contracts are not heavily traded and considered illiquid. In addition, the risk-minimizing quantity is not very sensitive to deviations from the optimal baseload-only or any interchange between peak- and baseload quantities. This shows that rounding quantities to the nearest integer is a requirement that can easily be met at a relatively low cost.

A software tool is developed that is able to calculate these risk minimizing quantities for the real portfolio of Anode. A screenshot of this software tool is shown in Appendix N.

Contents

2 Commodity Risk Management at an Electricity Trading Company

This thesis focuses on the commodities risk management at an electricity trading company. The company is actively buying and selling electricity for its customers. The liberalization of electricity markets introduced a market price process which enables extreme spikes, high volatility and uncertainty of the cash flows generated by electricity flows. This in turn has made it necessary to actively manage all risks attached to managing an electricity portfolio through buying or selling of financial contracts consisting of futures, forwards, options and swaps.

An electricity trader's business model consists of operating as a gateway between the customer and the wholesale markets. The trader exchanges a physical electricity flow for a cash flow in either direction. This direction depends on the type of customer, who can be a consumer of electricity or a producer or both. Specifically, the customer transfers electricity from or to the trader in exchange for a cash flow. Then the trader needs to transfer this electricity from or to the wholesale market, again in exchange for a cash flow. This is summarized in the picture below. Note that the trader's customers can have either positive or negative electricity flows (supplier or consumer) which are each other's opposite.

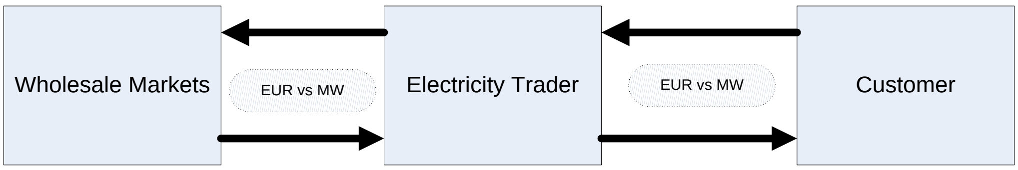


Figure 6: Conceptual overview of Cashflows versus Electrical Flows

By offering this gateway function the trader has multiple ways to earn money. First, the trader earns a spread on the cash flow between the customer and the corresponding cash flow with the wholesale markets. The margin however is quite low because of the highly competitive nature of the electricity market. Maintaining this margin is therefore an important point to consider in the risk management policy. In addition, the trader earns money by managing a customer's imbalance. Imbalance is defined as the difference between a projected electricity flow and the realized electricity flow per fifteen minute interval. Normally, a customer would need to pay an uncertain price for this unknown difference that occurs. At an adequate fee, a trader might be able to take on this risk which is highly unpredictable and uncertain.

Risk management is the main topic of the thesis, and it will focus on the two risks (price and imbalance) described above. Of course, these are not the only risks to which an electricity trader is exposed. Counterparty risk, margin risk and basis risks are also present, however for this thesis these risks are assumed to be out of scope.

First, it is important to understand the underlying motives for hedging. As Modigliani & Miller(1958) showed in their seminal paper, firm value is created on the left-had side of the balance sheet when companies make good investments that ultimately increase operating cash flows. The way in which companies structure their financing of those investments through debt, equity or retained earnings does not have any influence on the firm value. It only affects the way income is dispersed to the stakeholders. A corollary of the Modigliani and Miller theorem is that risk management strategies also have no effect on the total firm value. Risk management consists of purely financial transactions that do not influence operating cash flows and hence firm value. In addition, there are substantial transaction costs involved with hedging instruments and therefore, it could even be argued that risk management is detrimental to firm value.

In the postmodern paradigm companies need risk management in an *enabling* role for large investments (Stulz, 1996). Companies can basically choose between two alternatives; equity or debt. Equity can be very expensive because of information asymmetry. Therefore, managers are reluctant to issue equity. Debt does not include this information asymmetry problem since it can be valued easily and (with sufficient collateral) it can be quite available. However, small trading companies do not have sufficient assets that can serve as collateral. Therefore, these firms are severely capital constrained and need to manage their capital usage efficiently to optimize growth. In addition, high debt levels relative to equity could introduce significant limits to the access to lend new funds if needed, as well as cause distress, default and even bankruptcy. Therefore, companies prefer to finance investments with internal cash funds as much as possible, before turning to the external capital markets. This is called the pecking order theory and it is especially relevant in this case.

In the most extreme case these capital constrained companies could experience a large negative cashflow and then be forced to file for bankruptcy. However, even in less extreme cases significant costs can arise; credit lines might be withdrawn, interest rates might go up and good investment opportunities might be foregone. (Stulz, 1996). To the extent that these threats are credible, they are reflected in the company's current market value and associated cost of capital. A risk management program that costlessly eliminates the risk of bankruptcy effectively reduces these expected costs to zero and thereby increases firm value. Note that this only is valid if the company is healthy and a reduction in operating cashflow would put the company in bankruptcy.

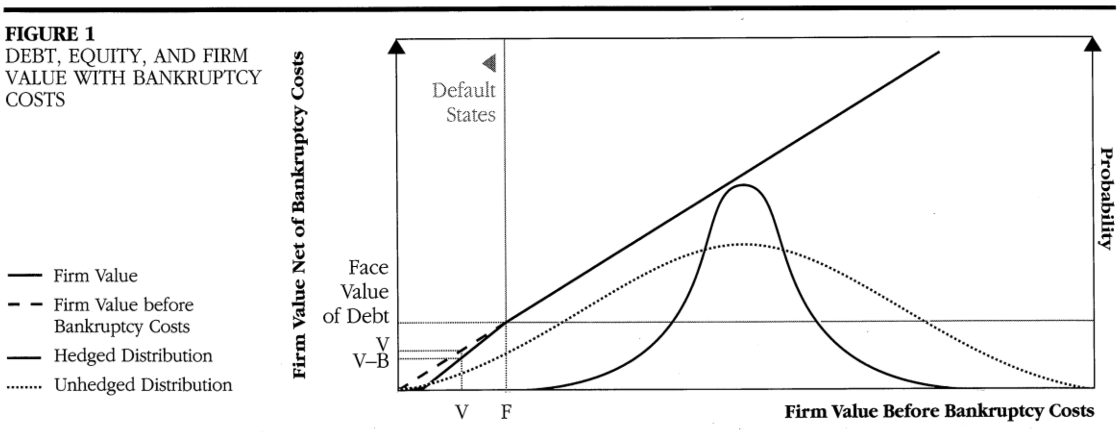


Figure 7 Debt Equity and Firm Value with Bankruptcy Costs (Stulz, 1996)

The figure above focuses on firm value. In the first situation the unhedged distribution (dotted line) has very fat tails. These fat tails enable the possibility of entering a bankruptcy on the left-hand side. By hedging the distribution's fat tails are trimmed and hence the bankruptcy states are no longer possible. The value itself is created because a good hedging policy transforms highly uncertain volatile cashflows (spreads) to more stable ones, enabling it to survive negative price movements and ensuring a certain profit. In addition, hedging eliminates the costly lower-tail outcomes. (Stulz, 1996). In fact, a good hedging policy reduces the expected costs of financial trouble while preserving the opportunity for the company to manage risks on which it has a comparative advantage.

Sometimes companies see external financing as so costly that investment is cut based on the available financing from internal funds. Internally generated cash is a very important competitive weapon that is able to reduce the cost of financing and enables investments. The role of risk management can therefore be described as ensuring that companies have the cash available to make sufficient investments that are value-enhancing. (Froot et al., 1994) Other reasons for hedging are taxes, where a convex tax curve results in a lower expected tax bill. (Stulz, 1996) However, in the Netherlands the tax curve is linear and hence the profit from hedging does not emerge.

3 Literature Review of Risk Management in Electricity Markets

Markets are liberalized in most countries and this introduces risks that were not present previously. Many risk factors are identified in the process of risk management; varying electricity prices, fuel prices, varying demand, equipment malfunction and defaulted payments (Bjorgan, 1999). The primary difference between electric power and other traded commodities is that it cannot be stored. Therefore, although the discipline of financial engineering has developed many methods for dealing with financial markets including commodity markets, the non-storability attribute makes most of these techniques not easily applied to electric power markets. Real-time delivery coupled with the lack of real-time metering and the need to constantly balance supply and demand are the most important differences for why these techniques must be adapted. (Dahlgren, 2003)

The majority of risk management papers utilize a mean-variance structure where the analogy between the profit of an electricity producer and return of investor leads to a consideration of the Portfolio Theory (Markowitz, 1952). The opportunity to diversify the sales of an energy commodity in a financial Portfolio Theory setting has already been studied in multiple papers (Huisman 2009, Falbo 2010). In addition, Bessembinder and Lemmon (2002) use the standard deviation as a measure of risk to determine the equilibrium price of electricity forward contracts. The early approach was to assume a utility function linear in the mean and the standard deviation of their profits. (Bjorgan, 1999) However, in more recent settings the variance itself is interpreted as a constraint, and choices are optimized to be efficient. (Falbo, 2010, Huisman, 2009) A choice is considered efficient if, for a chosen level of risk, it maximizes the expected profit. Hence, for different risk appetites resulting in different constraints, an efficient frontier of portfolio compositions can be build. Using this frontier, efficient choices can be distinguished from inefficient ones. Another optimization in such environment leads to the Sharpe Ratio (SR, i.e. the ratio between the increment of expected profit and the increment of risk) of the alternatives. (Sharpe, 1964) Factors that influence the efficient portfolio compositions are risk assessment of the day-ahead market, an expectation regarding the expected price in the day-ahead market in the delivery period, the amount of risk premium that needs to be paid and the personal appetite for risk (Huisman, 2009).

Falbo (2010) considers the risk management problem of an electricity producer, where an additional decision variable, namely the production quantity to be sold in the spot market, results in a spread option. Huisman considers the composition of an efficient portfolio for both a peak and off-peak futures position. However, both approaches result in a static portfolio composition. Hence it fails to capture an important concept of portfolio management: the trade-off between short-term and long-term consequences of an investment strategy based on the evolution of the stochastic variables. In response to this problem, a multistage programming approach where the hedging problem is solved for multiple periods was developed. Examples from Kuhn(2000), Carrión(2007) and Conejo(2007) who use a stochastic optimization programming approach to solve the 12 month rolling horizon problem of hedging. The objective is to maximize the expected value of profit from selling energy in the day-ahead market, subject to a certain risk level. They formulate it as a mixed-integer quadratic programming problem. Carrión proposes a risk-constrained stochastic programming framework to decide which forward contracts the retailer should sign and at which price it must sell electricity that is expected profit is maximized at a given risk level.

A new development is the Integrated Risk Management (IRM) setting, which investigates the dependencies between the risk factors affecting the profit function explicitly. “Upstream” and “downstream” prices (i.e. production costs and products prices) must be considered together because they are usually correlated. (Falbo, 2010). This approach was first investigated by Tanlapco(2010) who investigated the benefits of hedging electricity price risk directly or indirectly. Using historical data, Tanlapco investigates direct hedging strategies where the futures contract is directly based on the spot market under evaluation. It could be beneficial to hedge price movements using derivative products of demand and supply factors, because some parameters between the spot and forward do not match (i.e. time to maturity). Anderson(1981) indicates that this is possible as long as a significant correlation between the spot prices and other futures prices exist. Therefore, Tanlapco focuses on a test of futures contracts such as natural gas and crude oil to capture the supply factors and index futures such as the Dow Jones and SP500 to capture the demand factors. He concludes that direct hedging does outperform cross-hedging, even though the relationship between spot and futures was weak.

Tanlapco focuses on the reduction of risk as measured by the variance of the value of retailer’s position. The higher the variance the more difficult it is for the retailer to determine how much revenues it will receive for selling electricity at time t. Therefore, he uses a risk minimization framework to determine the optimal hedge. The assumption is that the optimal hedge ratio is derived after the number of MWs to be sold has already been determined.

In the same spirit, Bjorgan(1999) considers hedging price risk using futures contracts. These are not physically delivered but financially settled. One strong assumption is that he assumes that the futures price is equal to the expected spot price. In this setting, the expected value of the profit is independent of the amount of financial contracts bought. However, the variance of the total profit does change as a result of the procurement of futures contracts. Bjorgan shows how the variance of profit is minimized, and then he fine-tunes the formulas for different settings, including for a producer with limited fuel and unlimited fuel and, a consumer. For each of these scenarios he derives the analytical, variance minimizing, procurement quantities. He also notes that advanced modeling of the covariance terms is essential for a calculation of the total quantities.

Collins (2002) argues that hedging electricity through financially settled futures contracts represents significant difficulties. The amounts needed during the delivery period can be highly variable. Then it is impossible to procure the specific expected demand curve in the futures market and perfect hedging becomes impossible. Therefore, it is clearly not practical for power retailers to buy all their power by accepting delivery on futures contracts. (Collins, 2002) On the other hand, if the futures position is closed out while the open interest is still reasonably large, the manager would have to close the futures position one or two weeks before delivery. Because prices behave differently due to the weather and other short-term supply factors, closing out the position early does not hedge against these price risks. In addition, buying other futures contracts with other maturities is not practical since electricity consumption or production cannot be hedged with contracts that are not specifically made for the same delivery period. Collins calls this problem basis risk and defines it as the difference between the price at which a futures contract is closed out and the subsequent actual price of electricity in the spot market. (Collins, 2002) There are several possible ways for a power retailer to manage this risk. One way, is to enter into a long-term contract with a local generating company. This eliminates price uncertainty, but the negotiation of the price will be affected by the relative strength of the parties. (Collins, 2002). A more competitive price could be achieved if the power retailer could manage this risk by simply taking long futures positions to offset any increases in the spot price that occur. It is essential to take delivery of the electricity because else the problem of basis risk during the delivery period occurs. The careful hedging of this basis risk is far more important with respect to other hedging environments.

This thesis focuses on the electricity price risk management problem in the line as given by Bjorgan (1999) where a rolling horizon approach is chosen to minimize the price risk present in the electricity portfolio. In this thesis and in Bjorgans' paper, financial contracts are used solely for hedging purposes whose goal is to reduce the profit uncertainty. Multiple settings are explored by Bjorgan, where both input and output costs are taken into account. In short, given a certain expected profit, Bjorgan calculates the variance-minimization strategy.

In addition, in this thesis the definition of price risk also incorporates the risk that Collins(2002) identified as basis risk. In the Dutch futures market each contract involves physical delivery or a financial swap. However, there still is significant over- or under hedging which introduces a significant risk to an electricity trader. Collins proposes changes to the futures market, but that is unfortunately not an option in the current setting. This thesis gives an answer to the question, how to optimally hedge this basis risk in addition to the general price risk.

This paper also has some similarities to Huisman(2009), where the allocation problem for delivery of a base- and peakload contract in a one-period setting is solved. The trader buys a baseload and peakload contract, and all unfilled or overfilled demand is settled in the day-ahead market. They focus on the minimization of the total costs, subject to a variance-constraint. Using this approach two important issues arise, one is the determination of the risk appetite of the company. Second, the risk premium associated with forwards can have a significant impact. Huisman finds that marginal hedging costs can have an influence and that it causes that it is not always optimal to hedge a baseload profile with a baseload forward contract. Also, they find that expected risk premiums are positive, which would reward higher risk appetites. This thesis minimizes the variance where the expected profit is fixed. Using this approach the risk premium and the risk appetite of the firm are not relevant in order to determine on the optimal hedging quantity. In addition, this thesis focuses on a multiperiod setting where covariance is significant across multiple days, as well as within the day.

4 The Electricity Wholesale Markets

Commodities have been trading on forward markets and spot markets for over 200 years. The main reason for the establishment of forward and spot markets was that farmers with produce could not be certain whether the price received was a 'fair' price. Therefore, spot markets were developed in an attempt to discover the real price for produce. These markets gradually developed and increased the different products, different contracts and liquidity.

Futures trading started in the UK in 1990, in Scandinavia in 1995 and in 1996 NYMEX followed in the US. (Amundsen & Bergman, 2006) This thesis focuses on electricity markets which are a relatively new phenomenon in the Netherlands. The development of the Dutch power markets (APX and ENDEX) started with the introduction of the Electricity Act of 1998. The liberalization of electricity trading, and the unbundling of ownership of distribution activities from production activities in 2008, enabled new entrants to start competing in the electricity market. (De Nooij & Baarsma, 2009). In addition, European markets were integrated during the 2000s and price converged, while volatility was reduced significantly. (De Jonghe et al., 2008)

4.1 Futures and Forwards markets

On the Dutch exchanges two sets of products are traded, futures contracts and spot contracts. A futures contract enables a market participant to trade electricity at a certain price for future delivery. This is called physical settlement and requires that the market parties have the physical infrastructure to take delivery. Sometimes financially futures also trade which are settled in cash only. However, these are not traded on the Dutch exchanges but only OTC or in the German exchange. In addition, spot trading takes place which is usually defined as "*any transaction where delivery either takes place immediately (which is rarely the case in practice) or if there is a minimum lag, due to technical constraints, between the trade and delivery."*(Geman, 2000). In electricity markets the Day-Ahead market is considered a spot market, even though delivery takes place after 12-36 hours. As this lag increases beyond that period all trading contracts are called futures. In addition, Anode has access to the OTC market and is able to take on forward positions. Forwards are different from futures because the deals are no longer standardized. In addition, futures require the payment of margin deposits that are marked-to-market on a daily basis, while a forward requires a one-off payment. Only physical producers, consumers or retailers can trade forwards. Anode is a company that has such a PRP license and is able to trade forward agreements. Currently, Anode is not trading forwards because of the unsatisfactory rules about margining. High margins are required from Anode, while the big companies are not sufficiently willing to supply their own margins, or return margins as contracts increase in prices.

Currently, the Netherlands and Belgium have their own futures exchanges called ENDEX (NL/BE) which offer futures to be traded on their respective market place. The daily prices are published on the exchange's website[1]. The prices are often used for pricing contracts and billing customers because it provides a suitable reference point. However, the actually traded volume on the futures markets is quite low and the futures markets are therefore quite illiquid. It is difficult to find counterparties for nonstandard volumes and sometimes no trade happens for a specific contract, which implies that prices can be not available. Hence, the exchange developed a bidding procedure that provides two prices for each tradable contract at midday and close[2]. Because of this procedure the prices that are quoted do not necessarily represent physical trades, which indicates that pricing might not be competitive.

On the ENDEX different types of futures are traded. The first and most liquid contract is the baseload type. In a baseload contract a fixed amount of electricity is delivered for a specific time period at the contract's price. This contract is available with a weekly, monthly, quarterly and yearly delivery period and is can be traded in multiples of .1MW. In addition, the ENDEX offers peakload contracts. The peakload contract[3] delivers electricity for a

[1] APX ENDEX: http://www.apxendex.com

[2] Intraday: 11:45-12:00 CET and Close: 15:45-16:00 CET, whenever a trade happened in the pricing window then that transaction price is used as the price point for that period. In all other cases the bidding procedure is initiated. This procedure consists of contacting market players and asking for their bid and ask price of that moment. These prices are combined and an average is computed, excluding the highest and lowest value of each bid. These procedures apply to both countries, however the Belgium ENDEX only quotes close prices.

[3] The peakload contract mentioned is traded, another peakload contract only tradable OTC because it was eliminated by ENDEX. It had the peakhours defined as 7:00-23:00 CET. Its termination date is January 2014.

specified period during the peak hours. These peak hours are defined as all hours between 8:00 and 20:00, and only on weekdays excluding holidays. In Belgium all these contracts are available for the next three months, the next four quarters and the next three years. In the Netherlands more contracts are available, up to four weeks, six months, six quarters and five years.

Interval	Type of Market	Variants
Year	Forward / Futures / Swaps	Base, Peak
Quarter	Forward / Futures / Swaps	Base, Peak
Month	Forward / Futures / Swaps	Base, Peak
Week	Forward / Futures / Swaps	Base, Peak
Day	Spot (Strip) / OTC	Base, Peak, Off-peak
Hour	Spot (Day-Ahead) / Intraday	Hourly
PTU	Imbalance	PTU

Table 1: the different time intervals and the associated (financial) products

At the expiration date a monthly futures contracts will go into delivery and the holder of the contract will be required to consume or produce the amount traded. This amount will be delivered anyhow, but the position can physically be offset by buying or selling in the spot markets. At the moment of delivery, the electricity is settled using the imbalance calculation. The quarterly and yearly contracts will cascade into smaller contracts at the moment of expiration. A quarterly contract cascades into three monthly contracts while a yearly contract cascades into three monthly contracts and three quarterly contracts. The front month contract will expire the day after, giving holders the opportunity to close the contract and prevent delivery.

Finally forwards or futures can be procured in other markets, especially the larger and more liquid German market that settles its trades on the EEX exchange. Trading is possible because the CWE (Central Western Europe) block was formed in 2010. The EEX always trades multiple contracts and these can be used to import electricity into the Netherlands and/or into Belgium. Because of market coupling it is also possible to trade electricity from the French energy markets (EPEX) and the Austrian electricity markets (EXAA). It is important to note that cross-border transmission capacity is limited. To allocate the capacity efficiently, an auction structure is developed which is called CASC-EU. It is possible to buy capacity on a yearly, monthly or daily basis. Currently, the capacity for the Netherlands is more than sufficient and auction prices for capacity are very low. Again all auction results are published on the auctioneer's website.[4]

At the EEX another interesting financial product is traded. This is called a swap, and exchanges one cashflow for another one, and in the case of electricity swaps the EEX price is swapped for a fixed price. Hence, the swaps can be seen as a futures contract that is settled financially instead of physically. This removes the requirement that the market party has the physical capability to take physical delivery.

Anode does not have German customers, and therefore it has three options for buying electricity on the more liquid German market. First, it can buy a swap contract that settles financially. Second, it can take a position in a physical delivery future and close its position before the expiration date. Third, it can take the futures position and simultaneously buy transmission capacity on the German-Dutch border. Because of these (physical) constraints, a spread exists between the German and Dutch electricity prices, even though the grids are

[4] http://www.casc-eu.com

interconnected. Anode needs to evaluate the trade-off between the spread risk with the German electricity market and the less liquid position in the Dutch futures market.

4.2 Spot Energy Markets

The APX Day-Ahead Market (DAM) is the spot market in which most trades take place. Each country has its own DAM exchange and the Dutch is called APX, while the Belgian DAM is called the BELPEX. The DAM is not a market in which there is continuously buying and selling of electricity, but it a double-blind two-sided auction system in which all participants place hourly bids in which they want to buy or sell electricity. Each bid consists of a volume and a corresponding price. All bids are collected by the exchange and at 12h00 the DAM market closes. Next, a complex algorithm[5] calculates the market prices that settle the market and these are then published on the website. Often, transmission constraints are not binding and the prices are identical in all connected markets[6]. However, sometimes a cross-border transmission line is at capacity and then different market prices are calculated for different regions (usually countries).

The DAM market is an hourly market and hence a constant load will be bought or sold for the whole hour. In addition contracts have a minimum size of 1MW and can be changed with increments of 0.1 MW. The DAM market can be quite volatile and price peaks have occurred in the past. The market however is capped at the price range of -3000 EUR/MW to 3000 EUR/MW. Usually, the daily average price hovers around 35-50 EUR during 2011.

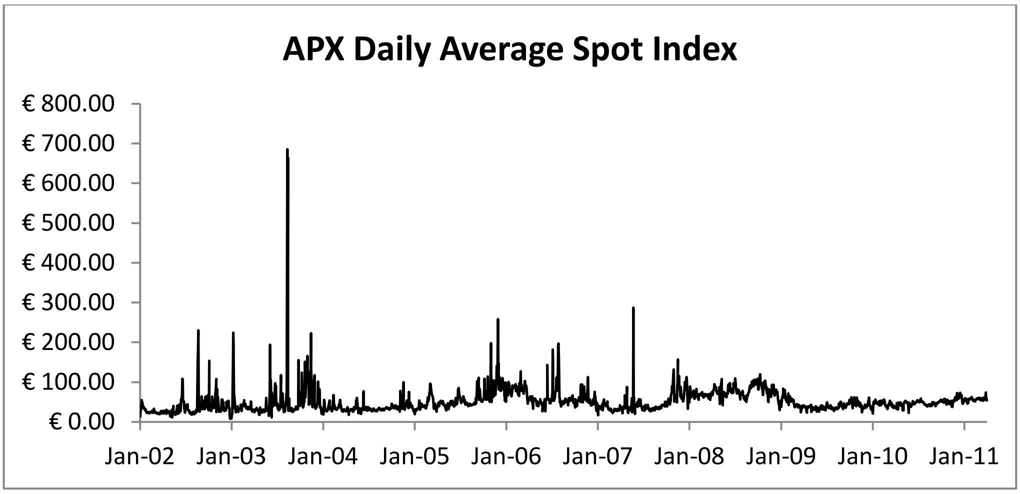


Figure 8: The Daily Average APX DAM market prices for 2002-2011

In addition to the DAM markets, intraday markets exist. These enable companies to buy or sell additional electricity for up to two hours before the physical delivery period. These markets are very new and the volume that is traded over these markets is very small. It is however, the market that most resembles a traditional spot market.

Because it is an intraday market, the prices can move even more volatile than the DAM markets. Again the prices are capped within a range of -99.999 EUR/MW and 99.999 EUR/MW. Unfortunately the relative newness of these markets make trading very thin, and it often happens that even no bid or ask quotes are available to market players. In order to increase liquidity APX ENDEX is trying to integrate the Scandinavian intraday markets with

[5] This algorithm is called COSMOS and is quadratic optimization algorithm that uses the bids, constraints put in by the TSOs to optimize social welfare

[6] The CWE market coupling includes: the Netherlands, Belgium, France, Luxembourg and Germany

the Central European intraday market. This was finalized in 2011 and will probably result in more trading, since the volume in the Scandinavian markets is significantly higher.

4.3 Imbalance Markets

Once all electricity is procured or sold the delivery period occurs. There, the electricity that was procured is delivered to the grid, and the electricity that was sold will be consumed from the grid. In order to explain the properties of this market, it is first necessary to briefly discuss the general structure.

The electricity market consists of connections to the high voltage grid. This high voltage grid is managed by one entity that is called the Transmission System Operator (TSO). The TSO is responsible for the efficiency and reliability of the grid and has a monopoly in each country. In the Netherlands the TSO is Tennet and in Belgium the TSO is called Elia.

The TSOs allocate the management of high voltage grid connections to program responsible parties (PRPs)[7]. These entities manage the connections by sending E-programs and T-programs to Tennet, and are responsible for the physical flows that are measured from these connections. For our discussion the E-program is most important since that includes the physical trades that happen with other market participants. The E-programs are checked for consistency by the TSO and form the basis on which imbalance is calculated.

In the E-program the consumption and production of electricity is listed with the specific counterparties with whom this electricity is settled. The net sum of all these positions is always zero, since every seller needs to have a buyer. However, some flows will be settled internally and are therefore not included in the E-program. For example, Anode expects to produce 100MW for a specific time period, then in the E-program it will have to state which counterparty has bought a contract from Anode to take delivery of this 100MW. Because of the different customers and flows, this can become a long list of positions.

During delivery of electricity every market participant has a deviation of the traded electricity and the realized demand or production. The difference between the estimated net production or consumption (listed in the e-program) and the realized production or consumption is called the imbalance. The TSO measures this imbalance real-time per connection and reports it back to the PRP within a specific period, usually one week.

In order to maintain a reliable electricity grid, it is necessary to continuously balance supply and demand. If supply is too low then the frequency of the electricity current drops below 50Hz and if it drops too far machines will no longer work. On the other hand, if the frequency increases too much then machines will break because of overloading of their systems. Hence, the TSO acts as the central counterparty to all players that continuously buys and sells the surplus or shortage from the market participants to other market participants. The price at which the transactions between the TSO and the market participants are settled is called the settlement price.

Hence, Tennet needs backup generators that are capable of producing electricity to counter the market shortage, and it needs a list of market parties that are willing to increase consumption if the market has surplus. These were the general goals when the market system was developed. And its workings can be seen in the figure in the appendix A.

The TSO maintains a bid-ask ladder that contains bids from market participants for

[7] This is the Dutch definition. In Belgium all concepts have other names but the market structure is identical unless stated otherwise.

electricity production or consumption. All market participants can put in bids, however some technical constraints apply. As the market moves in one direction, the bids are activated and the marginal settlement price increases or decrease. This curve changes exponential in both directions. A small increase does not influence the settlement price much, but as the distance from equilibrium increases, the price increases exponentially.

For each PTU the TSO determines the settlement price which is equal to the highest production bid price activated or the lowest consumption price and potentially includes a fee. In addition Tennet determines a scenario that applies to each specific PTU. In total there are four possible scenarios. A scenario can be a -1 or +1 a shortage or overage of electricity for the market was present and this price is used to settle all imbalance trades with Tennet. In this situation the whole imbalance market is zero sum game in which wealth is transferred from buyers to sellers or the reverse. Second the scenario can be ‘0’ in which case the settlement price is equal to average of the lowest bid and highest ask price. Finally, Tennet can also choose the scenario “2”. In this case there was both a shortage and overage of electricity during a PTU and both marginal prices are used. In this case, the zero-sum property is lost and Tennet incurs a spread (the difference between the buy and sell settlement price).

During the year Tennet thus earns money and this is then reallocated to the market at the end of the year. In this way good behavior (i.e. accurately forecasting the production/consumption in the e-program) is rewarded, since these players will receive more money from Tennet than they paid during the year.

The same procedure applies to Belgium, but the Elia does not have a 1 or -1 scenario. Elia always incurs a profit from the imbalance because in Belgium always two imbalance prices apply.. In the Dutch system it is possible to profit from imbalance, while in Belgium imbalance is always a cost.

5 Price and Imbalance Risk for Anode

In the first chapter the risks and model of a standard electricity trading company were discussed. Now, two risks will be elaborated upon specifically tailored to the situation as experienced from Anode’s perspective. This serves as the input of the next chapter in which the risks will be solved quantitatively. First price risk will be discussed as well as the customers that introduce it. Next, imbalance risk is discussed.

5.1 Price Risk

As stated in the first chapter an electricity trader acts as a gateway between the customer and the wholesale market. Anode has this function as well and in this section the price risk that Anode has will be discussed. Each transaction where electricity is exchanged for money occurs with two counterparties. The important aspect is that the price at which Anode trades with the customer is tried to be matched with the price that Anode pays to the wholesale market.

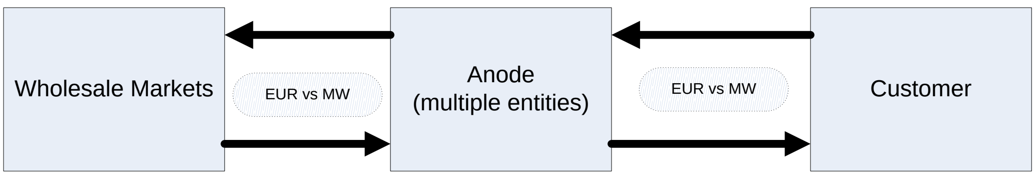


Figure 9: The business model of Anode

Anode has multiple customer groups, some of which do not allocate price risk to Anode. The specific contracts for which this applies are called the OTC access contracts. These customers only need access to the wholesale markets through Anode and want to maintain their exposure to the market. Examples of these companies are mainly larger companies that are capable of maintaining an own risk management department. In addition, some companies are present in markets in which it is common to pass the price of energy on to customers. Anyhow, these customers usually have a contract in which a fixed fee or fixed margin is paid to Anode for their logistic and administrative services, while keeping the exposure to energy prices. This situation looks in formulas like:

$$\pi_t = \mathrm{D}((1+m)S_t - S_t) \tag{4.1}$$

In these cases both cash flows are dependent upon S_t and each price movement cancels itself out. Ensuring that $m > 0$ if demand is positive (a consumer) than a certain profit will be made. In addition, if demand is negative (a producer) then $m < 0$ to ensure a certain profit from this customer. From a risk management perspective it is essential that Anode mimics the price movements in the wholesale markets. If the electricity would be procured forward, the opposite exposure would exist, where the sourcing of electricity has a fixed price P and the sales price would be variable. In fact the price risk would change from the price risk of a consumer to the price risk of producer. Not hedging these customers is therefore the best risk management strategy.

At Anode, the **Spot + OTC contract** is the type of contract that is the representation of the above situation. The contract enables customers to buy or sell electricity at the realized spot prices. Hence, there is no price exposure for Anode given that they did not buy or sell the electricity in advance. However, these customers also have the option to buy electricity forward from Anode. As soon as such a position is taken by the customer, this means that for that portion of demand the price risk has been transferred to Anode.

The challenge for Anode is that the transaction with the customer usually involves a fixed transaction price P that is determined at contract origination. The price for transactions with the wholesale markets is not known in advance. The hourly APX spot price (S_t) that will be used to settle the energy transfers with Anode is unknown and highly volatile. Therefore the margin that is earned by Anode is also highly volatile and unknown. In a formula the total cashflow to Anode that results from the transaction is shown in the formula.

$$\pi_t = \mathrm{D}(P - S_t) \tag{4.2}$$

The size and sign of this cashflow is uncertain and stochastic as it is dependent upon the APX spot price (S_t). First, the sign can depend on the type of customer; a producer has a negative demand, while a consumer will have a positive demand. This influences the sign that the premium $(P - S_t)$ needs to have. A positive spread is associated with a positive demand, while a negative spread should account for negative demand. The fact that the resulting cashflow π_t can be negative is called price risk. This risk will be the main focus of this section. Note that because the spread is quite thin in most situations, the probability that a negative cashflow will occur is significant. Electricity prices can have a big influence, a 1 EUR movement corresponds to an increase or loss of €87,600 for a yearly contract of 1MW. Hence, analyzing price movements is extremely important.

One major group of the customers of Anode are the **small-use profiled** customers, which mainly consist of small business, retail companies, households and small production plants. These customers are too small to monitor continuously, and therefore a method was developed to approximate the use of these customers. The method consists of multiple profiles of which each profile is an approximation of a specific type of customer. A profile splits the year into PTUs and then assigns a fraction of total demand that is consumed in each PTU. These fractions are updated yearly and remain constant throughout the year, as well as identical for the Netherlands as a whole. Hence, it does not matter whether a customer with an identical profile lives in Amsterdam or Nijmegen. The fractions are multiplied with expected total consumption of the individual customer and result in a quantity of electricity that is assigned to the trader. Below the average hourly profile, as well as the yearly profile are shown.

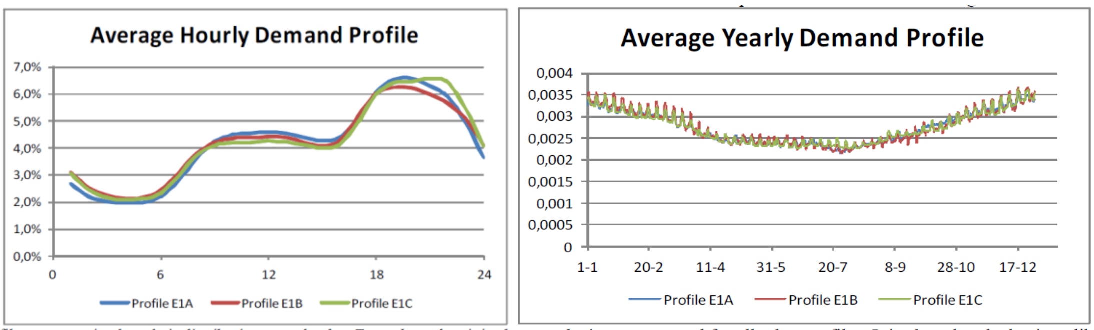


Figure 10: The total demand of profiled household customers on a hourly basis (left) and yearly basis (right)

Other contracts all correspond to large users that have some form of more frequent monitoring. Some are monitored on a continuous (PTU) basis, while others send in weekly or monthly figures. The **Fixed price contracts** and **click contract** enable customers to set a fixed price at which they consume electricity. The fixed price is the default contract, while the click contract enables customers to use ‘click’ prices in multiple time periods. However, in both contracts the prices are fixed before the first delivery takes place, after which these will not change. Customers that have these contracts therefore allocate the price risk to Anode, and need to be included when managing the price risk. In addition, the **reseller contracts**, need to be included in this group since they resell electricity to other electricity retailers. For Anode, these retailers are seen as a single entity, even though they consist of thousands of individual contracts between customers and retailers.

The **wind and solar contracts** also fix the price that customers will receive for generating electricity with their installations. Their volume can only be approximated by advanced climatologic models that are generated by a third party. Wind and solar are highly unpredictable and therefore the premium that is applied to the fixed contract price is substantial. Again because of the fixed price these contracts need to be considered when evaluating price risk.

Finally, there is a very special contract that is called the **fixed or better contract**, where the customer has the right but not the obligation to reset the contract at a lower price during the delivery period if a beneficial price movement occurs. In this case the problem of course is that by fixing the price at a very high level, this can mean that the profit evaporates if the prices drop and the customer is able to settle for a lower price. Hedging these contracts would involve some sort of options, since essentially a set of puts is sold to the customer.

5.1.1 Factors influencing the price risk

In the determination of the sales price P the baseload price is used as a starting point. This price can be observed in the market and provides a basis on which to price electricity in the future. If a customer has historic demand data available, this is used to assess a factor that conforms to a weighted average of the APX price. This weighted average is compared to the simple average of the APX to find the ratio that corresponds to the customer. This ratio is the first factor and provides the pricing of the customer with respect to the baseload price. Next, a second factor is introduced that corresponds to the correlation of the customers demand with prices, volatility and counterparty risk. Therefore it is essential to investigate a customer's correlation with market prices to determine the real price risk.

When hedging price risk, Anode essentially has two options available. It can use a form of operational hedges or financial hedges. Financial hedges refer to financial contracts that Anode can buy to offset demand or production from customers. Financial hedges include forwards, futures or swaps in the exchanges that can be physically or financially settled. The operational hedges refer to the fact that real production of electricity is used to offset a demand for electricity. Anode itself does not have production sources or large consumption options, but it does have a portfolio that consists of both parties. Hence, offsetting customers can be seen as operational hedges. The result is shown in figure 7, where the offsetting of a producer and consumer results in a far smaller net exposure that needs to be hedged. In addition, the correlation between the producer and consumer could further reduce the volatility in the net exposure.

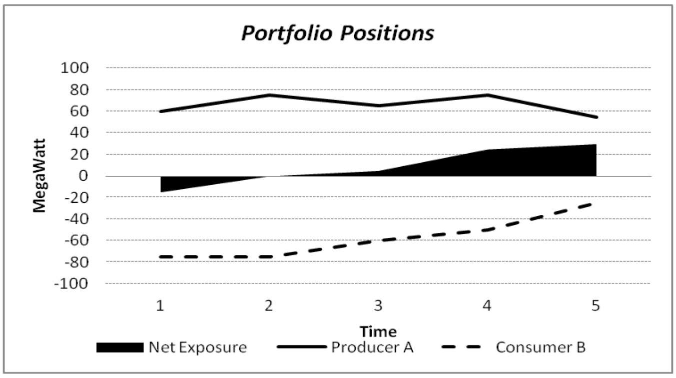


Figure 11: The net exposure from Producer A and Consumer B (operational hedging)

After the net exposures have been determined, these need to be hedged in the wholesale markets. The price dependencies between customer and market would not be important from a risk management perspective if a perfect hedge was available. Note that a perfect hedge would cover precisely each short and long positions for each hour. This would result in zero risk, as all hourly electricity quantities would be delivered at a predetermined price independent of the current market prices. However, there are two restrictions. First, a customer's demand or supply can change which means that a portfolio that was fully hedged, becomes partially unhedged. In contracts of Anode a 20+% deviation from projected demand is penalized, but a less than 20% deviation at the wrong moment could result in significant cash flows. Second, the market only offers a baseload and peakload contract to hedge the demand in the medium- and long-term, which deliver a fixed amount of electricity. Therefore, as can be seen in the figure 8 below hourly exposures remain because of over- or underhedging for specific hours. However, it is possible to shift these exposures somewhat by increasing or decreasing the overall amount bought. The result is trade-off whereby substantial long positions in the (off)peakhours need to be offset by short positions in peak-hours. Looking at the historical information on prices and volumes a risk minimizing quantity can be determined.

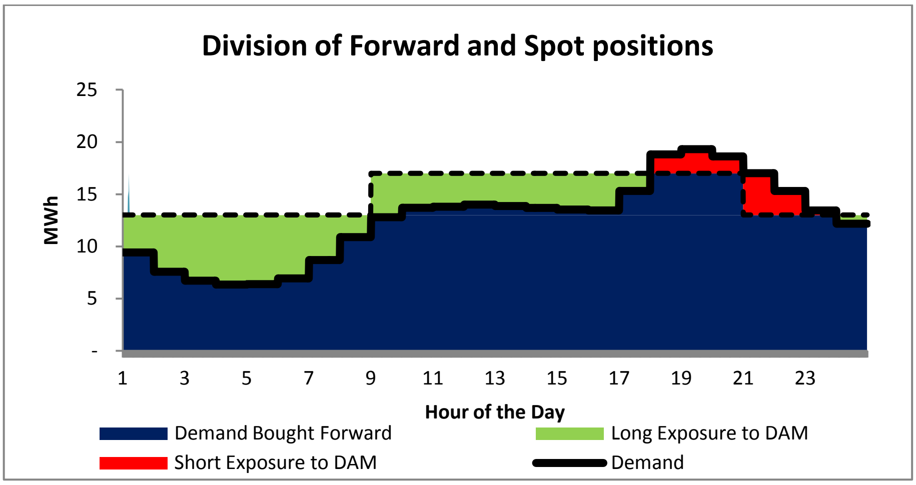


Figure 12: A schematic overview of the unhedged exposures to the DAM market

In the figure above, it is shown, that for a fixed demand curve, and only a peak- and baseload position both long (green) and short(red) exposures to the DAM market remain. The retailer has multiple options; would you buy the minimum amount possible during the day? Or would you buy the average usage, which would indicate a short position during peak hours and a long position during off-peak hours. Or would you buy the maximum amount, which would indicate substantial short positions during the day. In reality the customer does not have a constant profile, but has a clear weekly and monthly and yearly trend in is his demand.

5.2 Imbalance Risk

Another important risk that was already briefly discussed in the previous chapter is imbalance risk. This risk is managed on a day-ahead basis as it concerns the deviation from the hourly quantities that are procured in the day-ahead market. As the day-ahead auction is cleared and Anode's bids are filled, the forecasts will be updated. Also, the realization of demand will always be different from the final forecast. The deviations between the forecasted flow (on which the bids in the DAM are based) and the realized flow results in a physical imbalance. The price of at which the imbalance is settled is even more volatile than the APX DAM prices. Anode earns money on these transactions, as well as fixed fees that cover the administrative expenses.

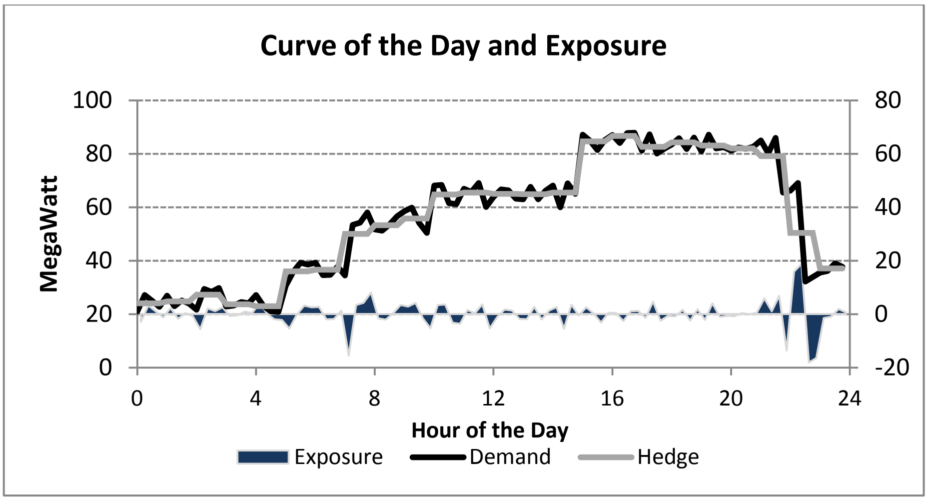


Figure 13: A schematic overview of the exposure to imbalance risk

Just before the DAM auction Anode has a forecast of the demand for the next day on a

PTU basis. A PTU is a quarter of an hour, and hence there are 96 different demand volumes to describe the day. Unfortunately, it is only possible to buy 24 hourly quantities to hedge the demand curve of 96 estimates. Each hourly hedge quantity therefore covers four PTUs that can have different quantities. The result is that a perfect hedge is unavailable and that every procurement of 24 hourly quantities will result in unhedged exposures in the imbalance market. Some trading is possible up to two hours in advance to cover major changes in demand, but this market is very new and highly illiquid. Therefore, Anode does not use that market frequently. Thus, after the DAM auction is settled, the quantities are fixed and all over- or underhedged quantities will be bought in the imbalance market at the imbalance price.

$$IR_{i,t} = \left(\sum_{p=1}^{4} [\frac{1}{4} Q_{i,t} - (D_{p,t} + \varepsilon_{p,t})] S_{p,t} \right) - Q_{i,t} F_{i,t} \tag{4.3}$$

Hence, the cashflow from imbalance ($IR_{i,t}$) is equal to the sum for the four PTUs of the demand and the forecast error minus the procured quantity. The result is a quantity that was sold or bought in the imbalance market at the imbalance price $S_{p,t}$ where p represents the PTU and t the day. In addition, whether a profit or loss was made depends on the cost of the procured quantity Q which is equal to the DAM price for that specific hour.

From the formula for imbalance risk we can see that multiple effects are important. First, the Q cannot be set in such a way that it covers all demand in the PTUs. In addition, the demand cannot be forecasted perfectly and therefore a forecast error exists. Finally the imbalance price is charged on a PTU-level and is highly volatile and uncertain. In addition, this price often turns negative indicating that a shortage can result in increased profit instead of a cost. Finally, whether imbalance is profitable also depends on the forward price from the DAM market. A high price could make it more profitable to buy more electricity in the imbalance market paying $S_{p,t}$ instead of $F_{i,t}$.

To analyze this situation a cost series was calculated that assumed that an overage would occur, as well as the situation where it was assumed that a shortage would occur. By comparing the costs of these series it is possible to determine whether it can be beneficial to influence the sign of the error. If anode buys a little more than the most reliable estimate indicates, the probability of a shortage is smaller than the probability of an overage. Hence, on average the cost of the overage would occur more often than the cost of shortage. In the opposite case, Anode would procure a small amount less than expected, resulting in a shortage, more often than an overage.

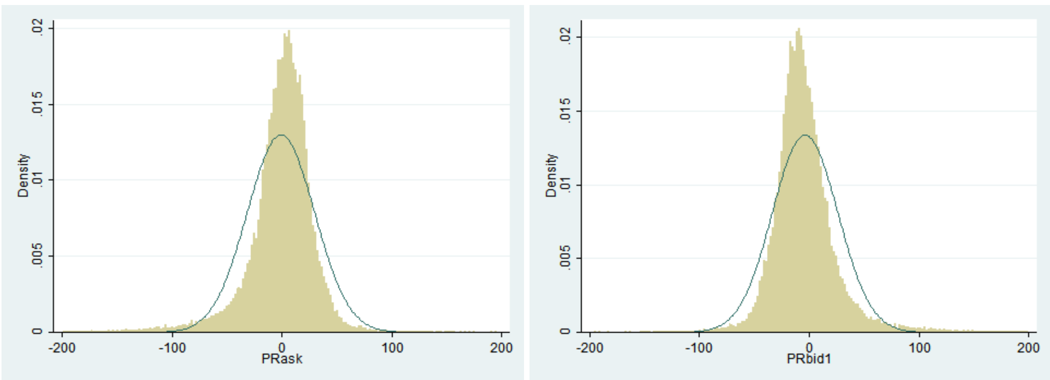


Figure 14: Histogram of profit from Buy Imbalance sell DAM (left) and Buy Forwards, Sell Imbalance (right) these histograms are capped at [-200,200]

First, we can minimize the forecasting error thereby reducing the unknown imbalance that will occur. In general the forecast has a certain error that has an expected value of zero, but is stochastic and unknown. The question that is then interesting is what amount should be bought to cover this unknown error? If imbalance prices were symmetric this would be equal to zero since the expected value of the error is equal to zero. However, the distributions of the imbalance prices are **not** symmetric and therefore an opportunity for improvement exists.

Secondly, the imbalance cost is charged on PTU basis, resulting in four different quantities and prices for an hour. Unfortunately the DAM and intraday market only trade hourly quantities, indicating that a small imbalance will always be present, even with deterministic demand. The algorithm that minimizes the risk of these exposures will be presented in the next chapter. The solution will be based on a long-term price risk scenario, but the same applies to the short-term day-ahead scenario.

The imbalance risk is allocated to Anode in almost all contracts. The only exception on this case is the CTG contract. This arranges for all risks to remain with the customer, which will receive a bill for the total imbalance that is generated by this customer. Anode only has a few customers that have applied for this type of contract, and each of them are experienced players that continuously monitor the market themselves and are trying to profit from the imbalance by exploiting extreme price movements.

In addition to the risk minimization procedures described above, another type of hedge is available. The operational hedge is again that by pooling the total imbalance of all customers together, numerous imbalances are opposite and offset each other. Hence the net imbalance will be lower than the imbalances of all customers individually. However, the problem is that the correlation between customers is not always zero, leading to less than optimal offsetting behavior. However, assuming that the fee for this service is set at the correct level on average the result will be a value-adding activity for Anode.

6 A Quantitative Analysis for the forward procurement of Electricity

In the previous section the risks of the electricity trader have been described. First, the procurement of a specific demand profile in the wholesale markets is not possible at competitive pricing. Hence, the challenge of the trader is to procure electricity in fixed quantities in such a way that it covers as much as possible of the ‘continuous’ demand profile. In this way the procurement of electricity covers demand, but leads to exposures in each period that are equal to the difference between the procured quantity and the forecasted demand. This quantity will be bought in the APX spot market at a highly volatile and uncertain price. This introduces a risk, where the cash required to procure this electricity leads to a loss that offsets the profit. Hence, this price risk is what will be attempted to be minimized.

The goal is to minimize the risk that Anode is exposed to managing its overall portfolio, including the financial hedges. It is assumed that a set of contracts has been acquired and that these contracts provide a positive expected return to Anode. The pricing and acquisition of the contracts is out of scope in this thesis. Given the existing contracts with customers and positions on the wholesale market, our goal is to minimize the price risk caused by any unhedged positions either with customers and/or on the wholesale markets. Since it is impossible to completely offset these (random) exposures, the goal is to minimize the price risk caused by the exposures. The minimization of price risk is considered as minimization of the variance of the cash flow caused by the settlement of these exposures in the APX spot market.

6.1 Risk-minimizing Hedge with Baseload Contract and Deterministic Demand

In this section a formula is derived to calculate the electricity order quantity (Q) that minimizes the variance of cash flows. The trader has a portfolio, consisting of customer's which each consist of a forecasted demand curve. These demand curves are netted and a single net demand curve remains. The problem will be solved in the simplest case in which the portfolio demand is deterministic and an analytical solution can be found. Next, a series of real-life complexities are added to make the problem more realistic. These complexities consist of stochastic demand, multiple product and multiple periods.

This demand will occur in a future time period and will differ for a number of intervals (I) in which this period can be divided, usually the interval will represent a single hour and therefore a monthly period will consist of 30×24 intervals, while for imbalance risk the problem consists of 24×4 intervals. Each interval is of equal size and can be identified by the integer i; $1 \leq i \leq I$. The trader wants to hedge the portfolio's net demand in the wholesale market by buying a fixed quantity Q (MW/i) forward at the forward price $F_{0,T}$. The challenge however, is that this contract delivers the fixed quantity Q, for each interval (i) of the next period. The problem then is to determine the optimal risk-minimizing quantity Q that the trader needs to buy.

Note that as Fama & French (1987) showed under the expectations hypothesis, the forward price is equal to the risk-adjusted expected spot price. This has two important consequences; first it is necessary to show that hedging has no influence on the expected profit if the risk premium on average is equal to zero. Second, it explains the difference between the real expected spot price and the current futures price as a risk premium. A positive risk premium would indicate a cost of going long, while a negative risk premium would indicate a discount. In the rest of this chapter, the assumption is that the risk premium is equal to zero.

The expected spot price for each interval is stated as $E\left[\tilde{S}_i\right]$ (EUR/MW). Using the expectations hypothesis (Fama&French, 1987) we hypothesize that $E\left[\tilde{S}_i\right]$ is equal to F_i which is the unobserved hourly forward price. Then, the $F_{0,T}$ is equal to the simple average of the unobserved F_i, so $F_{0,T} = \frac{\sum_{i=1}^{I} E[\tilde{S}_i]}{I} = \frac{\sum_{i=1}^{I} F_i}{I}$. In each of the formulas a stochastic variable will be denoted by the ~ on top of the stochastic variable.

The customer has a deterministic demand per interval equal to D_i $(\frac{MW}{i})$. If demand is positive this is defined as a short position by the customer and demand is equal to a load that Anode will need to deliver to the customer. If demand is negative, this indicates a long position for the customer, and hence Anode will need to supply this electricity to the wholesale market. Since the amount bought forward is Q (MW/i), (where a positive number indicates a long position and a negative number indicates a short position) the net exposure per interval is equal to $(Q - D_i)$. This quantity can be either positive indicating a long position in the DAM market, or negative indicating a short position. Note that it is not possible to sum these exposures per interval, since each interval is unique and has its own (imbalance) price movements.

Using all of the above the total expected cash flow formula can be written down. Denote the total cash flow per interval as X_i and the total cashflow for the whole period as $X = \sum_{i=1}^{I} X_i$. The total cashflow per interval X_i is equal to revenues $(D_i P)$ note that revenues can have a positive or negative sign depending on whether the customer is a producer or consumer. A consumer will have a positive demand and hence result in positive revenues. Next, the sum of the forward procured quantity in the wholesale market ($QF_{0,T}$) is deducted and the net exposure for that period multiplied with the realized spot price. Note that this can result in a discount or excess cost based on both the sign of the net exposure $(Q - D_i)\tilde{S}_i$ and the realized spot price. A positive net exposure indicates that excess electricity was bought in the forward market and hence this will result in a positive revenue stream. A negative quantity indicates that insufficient electricity was bought and hence the shortage will bought in the spot market. Finally the electricity has been sold to a specific customer for price P which results in the total cashflow. Note that the cashflow is equal to profit if the formula is used for one customer. However, when we look at the net exposure of a portfolio the formula excludes the margin that is made while buying and selling from internal customers.

The total cashflow X for the next period is hence equal to:

$$\tilde{X} = \sum_{i=1}^{I} D_i P + (Q - D_i)\tilde{S}_i - QF_{0,T} \quad (5.1)$$

Since $\tilde{S}_i$ is a stochastic variable and unknown at the moment of the procurement decision, it is possible to calculate the expectation of the total cashflow. $E[\tilde{X}]$ which then results in:

$$E[\tilde{X}] = \sum_{i=1}^{I} (Q - D_i)E[\tilde{S}_i] - QF_{0,T} + D_i P \quad (5.2)$$

Further derivation of this formula (Appendix B), and using the forward price as defined in the previous section, we derive the end result:

$$E[\tilde{X}] = \sum_{i=1}^{H} D_i(P - E[\tilde{S}_i]) \quad (5.3)$$

Hence under the expectations hypothesis, the total cashflow is independent of Q, i.e. it does not matter what forward position is taken since the total expected cashflows are always simply the summation of the product between demand times the forward price or expected spot price per hour. Hence, hedging does not create value.

Note that in case where the customer is a consumer of electricity, the demand will be positive and the spread $P - E[\tilde{S}_i]$should be positive, i.e. the sales price P should be higher than the expected spot price or futures price. If the customer is a producer however, the demand is negative and the spread should be negative as well; the expected spot price should be higher than the sales price.

However for this problem, both the expected spot price / futures price are a given, and the sales price P has already been set in the contracts. As Q does not influence the cashflow, hedging for profit motives is not relevant. However, there is a significant risk introduced through the expected spot price and management of this price risk is crucial to maintain the business. Hence, we would like to minimize the risk as the return is already fixed. Therefore, the expectation of profit is not our main interest but the variance is.

In this section, the variance of the total cashflow $Var(X)$ is derived. First, the variance per interval is determined, then the total variance for the period. Start with $X_i = (Q - D_i)\tilde{S}_i - QF_{0,T} + D_i P$ then the $Var(X_i)$ is equal to $(Q - D_i)^2 Var(\tilde{S}_i)$. However, we are interested in the total variance over all periods.

$$\mathrm{Var}\left(\sum \tilde{X}_i\right) = \sum_{i,j}^{I} (Q - D_i)(Q - D_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) \tag{5.4}$$

In this report, the covariances are calculated and put in a matrix. The (i,j)th element of this matrix is equal to the $Cov(\widetilde{S_i}, \widetilde{S_j})$. To determine the variance minimizing quantity Q, the derivate of the $Var(\tilde{X})$ with respect to Q is taken and set equal to zero. $\frac{dVar(\tilde{X})}{dQ} = 0$.

PROPOSITION 5.1. *With deterministic demand and only baseload contracts, the forward procurement quantity Q that minimizes the variance of total cashflows is equal to*

$$Q = \frac{1}{2}\frac{\sum_{i=1}^{I}\sum_{j=1}^{I}(D_i + D_j)Cov(\tilde{S}_i, \tilde{S}_j)}{\sum_{i=1}^{I}\sum_{j=1}^{I}Cov(\tilde{S}_i, \tilde{S}_j)} \tag{5.5}$$

Hence the variance-minimizing hedging quantity Q is equal to a sum of weighted covariance terms with respect to demand, divided by the sum of the covariance terms. Therefore increases in demand or increases in covariances in specific hours, will lead to higher order quantity.

6.2 Risk-minimizing Hedge Quantity with Base- and Peakload Contract & Deterministic Demand

In addition to baseload contracts, it is also possible to procure peakload contracts that deliver a fixed quantity (Q_P) during peak hours. Peak hours are defined as the hours between 08h00 and 21h00. In addition, these contracts only deliver the peak hours during weekdays. This contract gives an important second decision variable for our problem since this gives us an option to reduce the peakload that is apparent in most demand profiles.

First, we identify two sets of periods in the overall specification of the model. Set B, which contains all intervals i for which only the baseload contract is traded. Secondly, the set P, which contains all intervals I for which both the baseload and the peakload contract are traded. In mathematical terms:

$$i \in P \rightarrow Q = Q_b + Q_p \tag{5.6a}$$

$$i \in B \rightarrow Q = Q_b \tag{5.6b}$$

$$P \cup B = I \tag{5.6c}$$

The forward price at which these contracts can be bought differs as well, since the forward peak-price $F_{0,T}^{P}$ is defined as, where the subset P constitutes the set of intervals that conform to the two conditions, weekday and peakhour-interval.

$$F_{0,T}^{P} = \frac{\sum_{i \in P}^{I} E(\tilde{S}_i)}{\sum_{i \in P}^{I} 1_{i \in P}} \tag{5.7}$$

The inclusion of a peak contract complicates our model since it implies that it is always necessary to look at hourly data or quarterly hour data. Of course the whole period can be extended for as long as possible. The total cashflow function is then equal to:

$$\tilde{X} = \sum_{i \in P}^{I}(Q_B + Q_P - D_i)\tilde{S}_i - Q_B F_{0,T}^{B} - Q_P F_{0,T}^{P} + D_i P + \sum_{i \in B}^{I}(Q_B - D_i)\tilde{S}_i - Q_B F_{0,T}^{B} + D_i P \tag{5.8}$$

Two quantities for baseload and peakload need to be determined in order to minimize price risk. Fortunately, it is possible to derive the formulation of the optimal baseload and peakload quantities in two equations. By filling in the one equation in the other, it is possible to derive the analytical results.

In order to find the optimal procurement quantities Q_B and Q_P under deterministic demand, the following notations are used:

$$\sum_{i=1}^{I} c_i \tilde{S}_i - Q_B F_{0,T,B} - Q_P F_{0,T,P} + D_i P \quad \text{where } c_i = \begin{cases} Q_B + Q_P - D_i & i \in P \\ Q_B - D_i & i \in B \end{cases} \tag{5.9}$$

$$\mathrm{Var}(\tilde{X}) = \sum_{i=1}^{I} c_i c_j \mathrm{Cov}(\widetilde{S_i}, \widetilde{S_j}) \tag{5.10}$$

To find the risk-minimizing Q, we take the two partial derivatives with respect to Q_B and Q_P, set them equal to zero, and then combine the two formulas the find the solution for Q_B. To solve, we calculate the two partial derivatives, $\frac{\partial Var(X)}{\partial Q_B}$, and $\frac{\partial Var(X)}{\partial Q_P}$. The resulting partial derivatives solved for setting them equal to zero are shown below (The derivations can be found in Appendix D):

PROPOSITION 5.2. *With deterministic demand and both base- and peakload contracts, the forward procurement quantities Q_b and Q_p that minimize the variance of total cashflows is equal to*

$$Q_B{}^* = \frac{1}{2}\frac{\left(\sum_{i,j\in P}^{I}(-2Q_P + D_i + D_j)Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in B}^{I}(D_i + D_j)Cov(\tilde{S}_i,\tilde{S}_j) + 2\sum_{i\in P,j\in B}^{I}(-Q_P + D_i + D_j)Cov(\tilde{S}_i,\tilde{S}_j)\right)}{\sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j)} \tag{5.11}$$

$$Q_P{}^* = \frac{1}{2}\cdot\frac{\sum_{i,j\in P}^{I}(-2Q_B + D_i + D_j)Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in P,j\in B}^{I} 2(-Q_B + D_j)Cov(\tilde{S}_i,\tilde{S}_j)}{\left(\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right)} \tag{5.12}$$

Since there are two equations with two unknowns it is possible to solve these equations. The solution can be found in the Appendix D.

6.3 Risk-minimizing Hedge Quantity with Baseload Contract and Stochastic Demand

In the previous part it was assumed that demand was deterministic. However, this is almost never the case since unpredictable deviations occur frequently. Hence, we introduce a stochastic demand. The expected demand $(\widetilde{D_i})$ for each interval in the next period and a standard deviation (σ_{D_i}). Using the time series of realized demands for historic intervals, it is possible to estimate the correlation that this customer's demand deviation has with the main factor influencing prices. Conceptually, this correlation estimates the situation where most customers have a strong correlation with the overall market. For most customers the market deviation of expected demand is highly indicative of the customer's individual deviation. This is because both are influenced by common factors, such as the weather conditions. Hence, when the market consumes a more electricity than expected, there is a high probability that the customer does as well. In addition, windy periods will result in significant amounts of electricity produced by wind mills. Therefore, there is a correlation between wind production and electricity prices. Of course, some customers are not influenced by these circumstances and for those a very low correlation will be found.

Note that the overall period of interest, is between the forward procurement decision and the APX day-ahead procurement decision. Therefore, only effects that can be forecasted or estimated one-day ahead are included in this analysis. All other disturbances are dealt with in the imbalance market. Again we start with the total cashflow formula as defined in the previous section:

$$\tilde{X} = \sum_{i=1}^{I} (Q - \tilde{D}_i)\tilde{S}_i - QF_{0,T} + \tilde{D}_{i_i}P \qquad (5.13)$$

Remember that both $\tilde{S}_i$ and $\tilde{D}_i$ are stochastic now and therefore, when the formula is written out, the result is a summation of three parts, the first is stochastic in $\tilde{S}_i$, the second is a joint product of both the $\tilde{S}_i$ and $\tilde{D}_i$ stochastic variables and the last part is constant.We first want to know the expectation of the total cost. Therefore we take the expectation of the function below.

$$E[\tilde{X}] = \sum_{i=1}^{I} QE(\tilde{S}_i) - E(\tilde{D}_i\tilde{S}_i) + E(\tilde{D}_i)P - QF_{0,T} \qquad (5.14)$$

Because there is covariance between $\tilde{D}_i$ and $\tilde{S}_i$, the expectation of the product is not straightforward. The rule is that the $E[XY] = Cov[X,Y] + E[X]E[Y]$ Using this formula and incorporating the previous part, see appendix C leads to:

$$\text{Total Cashflow } (\tilde{X}) = \sum_{i=1}^{I} -Cov(\tilde{D}_i, \tilde{S}_i) - E(\tilde{D}_i)\,E(\tilde{S}_i) + E(\tilde{D}_i)P \qquad (5.15)$$

Hence the total profit is equal to the expected demand times the expected price but now in this case the covariance has an effect as well. Intuitively this shows that when there is a positive covariance between the two variables, then unexpected increases in price result in a higher demand as well. Note that the real impact on cashflow is dependent upon the sign of the $E(\tilde{D}_i)$. A customer that is a consumer has a positive $\tilde{D}_i$.Therefore, a positive covariance between prices and demand will add an additional cost to Anode. On the other hand in the case of a producer, the same covariance will result in an additional profit. Unfortunately, wind energy has a negative covariance, i.e. high wind production results in lower prices and hence it is also an additional cost. In addition, most consumers have a positive covariance with market prices.

One interesting insight, is that trying to tune the portfolio in such a way that it correlates negatively with the market demand, results in a discount on the total costs for consumers, while the opposite is important for producers. Note again, that the total expected cashflows are independent of the level of Q, indicating that hedging does not create value.

Next, a number of terms are reduced in form to show how these can be calculated more easily, see the appendix C. Next, for the optimization of the problem we take the derivative of the formula with respect to the optimal hedge level Q. Hence, using the derivative of those formulas we find the optimal hedging Q.

PROPOSITION 5.3. *With stochastic demand and only baseload contracts, the forward procurement quantity Q that minimizes the variance of total cashflows is equal to*

$$Q = \frac{1}{2} \cdot \frac{\sum_{i=1}^{I}\sum_{j=1}^{I} Cov(\tilde{S}_i, \tilde{D}_j\tilde{S}_j) + Cov(\tilde{D}_i\tilde{S}_i, \tilde{S}_j) - \mathrm{P}Cov(\tilde{S}_i, \tilde{D}_j) - \mathrm{P}Cov(\tilde{D}_i, \tilde{S}_j)}{\sum_{i=1}^{I}\sum_{j=1}^{I} Cov(\tilde{S}_i, \tilde{S}_j)} \qquad (5.16)$$

6.4 Risk-minimizing Hedge Quantity with Base- and Peakload Contract and Stochastic Demand

In the previous section the model was expanded with the inclusion of a stochastic demand variable in addition to the stochastic prices. Now, the inclusion of the peakload contract will be discussed. Note that the same approach is chosen as in section 4.2 for the deterministic demand case and that the peakload contract is identical to the one discussed there. In mathematical terms:

$$\mathrm{i} \in \mathrm{P} \rightarrow \mathrm{Q} = \mathrm{Q_b} + \mathrm{Q_p} \tag{5.17a}$$

$$\mathrm{i} \in \mathrm{B} \rightarrow \mathrm{Q} = \mathrm{Q_b} \tag{5.17b}$$

$$\mathrm{P} \cup \mathrm{B} = \mathrm{I} \tag{5.18c}$$

The inclusion of a peak contract however complicates our model since it implies that it is always necessary to look at hourly data, of course the whole period can be extended for as long as possible. The new total profit function is then equal to:

$$\widetilde{\mathrm{X}} = \sum_{\mathrm{i}\in\mathrm{P}}^{\mathrm{I}} (\mathrm{Q_B} + \mathrm{Q_P} - \widetilde{\mathrm{D}}_\mathrm{i})\tilde{\mathrm{S}}_\mathrm{i} - \mathrm{Q_B F^B_{0,T}} - \mathrm{Q_P F^P_{0,T}} + \widetilde{\mathrm{D}}_\mathrm{i}\mathrm{P} + \sum_{\mathrm{i}\in\mathrm{B}}^{\mathrm{I}} (\mathrm{Q_B} - \widetilde{\mathrm{D}}_\mathrm{i})\tilde{\mathrm{S}}_\mathrm{i} - \mathrm{Q_B F^B_{0,T}} + \widetilde{\mathrm{D}}_\mathrm{i}\mathrm{P} \tag{5.19}$$

Because only constants were added to this equation, the same expectation results as the previous section, however the additional terms are included:

$$\mathrm{E}(\widetilde{\mathrm{X}}) = \sum_{\mathrm{i}=1}^{\mathrm{I}} \left[-\mathrm{Cov}(\widetilde{\mathrm{D}}_\mathrm{i}, \tilde{\mathrm{S}}_\mathrm{i}) - \mathrm{E}(\tilde{\mathrm{S}}_\mathrm{i})\mathrm{E}(\widetilde{\mathrm{D}}_\mathrm{i}) + \mathrm{E}(\widetilde{\mathrm{D}}_\mathrm{i})\mathrm{P}\right] \tag{5.20}$$

Again the total costs are independent of Q_B or Q_P. And assuming the expectations hypothesis holds, it does not matter what quantity of electricity is bought forward. In order to find the optimal procurement quantities Q_B and Q_P under stochastic demand, the following notations are used:

$$\sum_{\mathrm{i}=1}^{\mathrm{I}} \mathrm{c_i}\tilde{\mathrm{S}}_\mathrm{i} - \widetilde{\mathrm{D}}_\mathrm{i}\tilde{\mathrm{S}}_\mathrm{i} - \mathrm{Q_B F_{0,T,B}} - \mathrm{Q_P F_{0,T,P}} + \widetilde{\mathrm{D}}_\mathrm{i}\mathrm{P} \quad \text{where } \mathrm{c_i} = \begin{cases} \mathrm{Q_B} + \mathrm{Q_P} & \mathrm{i} \in \mathrm{P} \\ \mathrm{Q_B} & \mathrm{i} \in \mathrm{B} \end{cases} \tag{5.21}$$

To find the variance-minimizing Qs, we take the two partial derivatives with respect to Q_B and Q_P, set them equal to zero, and then combine the two formulas the find the solution for Q_B. The resulting partial derivatives are solved by setting them equal to zero are shown below (The derivations can be found in Appendix E):

PROPOSITION 5.4. *With stochastic demand and both base- and peakload contracts, the forward procurement quantities Q_b and Q_p that minimize the variance of total cashflows are equal to*

$$Q_B{}^* = \frac{1}{2}\frac{\left(\sum_{i,j}^{I} Cov(\tilde{S}_i, \tilde{S}_j\widetilde{D}_j) + Cov(\tilde{S}_j, \tilde{S}_i\widetilde{D}_i) - \mathrm{P}\mathrm{Cov}(\widetilde{D}_j, \tilde{S}_i) - \mathrm{P}\mathrm{Cov}(\widetilde{D}_i, \tilde{S}_j) - 2Q_P\left[\sum_{i\in B, j\in P}^{I} Cov(\tilde{S}_i, \tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i, \tilde{S}_j)\right]\right)}{\sum_{i,j}^{I} Cov(\tilde{S}_i, \tilde{S}_j)} \tag{5.22}$$

$$Q_P{}^* = \frac{1}{2}\frac{\sum_{i,j\in P}^{I}\left[Cov(\tilde{S}_i, \tilde{S}_j\widetilde{D}_j) + Cov(\tilde{S}_j, \tilde{S}_i\widetilde{D}_i) - \mathrm{P}\mathrm{Cov}(\widetilde{D}_i, \tilde{S}_j) - \mathrm{P}\mathrm{Cov}(\widetilde{D}_j, \tilde{S}_i)\right] - 2Q_B\left(\sum_{i,j\in P}^{I} Cov(\tilde{S}_i, \tilde{\mathrm{S}}_j) + \sum_{i\in B, j\in P}^{I} Cov(\tilde{S}_i, \tilde{S}_j)\right)}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i, \tilde{S}_j)} + \frac{1}{2}\frac{\sum_{i\in B, j\in P}^{I} 2Cov(\tilde{S}_j, \tilde{S}_i\widetilde{D}_i) - 2\mathrm{P}\mathrm{Cov}(\widetilde{D}_i, \tilde{S}_j)}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i, \tilde{S}_j)} \tag{5.23}$$

Since there are two equations with two unknowns it is possible to solve these differentials. The formulas can be seen in Appendix E. Note that a higher P value reduces the risk of a loss and hence already reduces the amount needed to minimize variance. A high P value will result in lower hedge values, while a low P value will result in more hedging. It is however, also dependent on the type of exposure, which needs to be positive or negative based on the customer. Whether it should be positive or negative.

6.5 An Algorithm to Solve the Optimal Order Quantity for Multiple Periods

In the previous sections multiple models were developed that estimate the variance minimizing procurement quantity for a specific period. Next, we need to develop an algorithm to sequentially solve the base- and peakload quantities for a longer time horizon. Usually, customers sign a contract and negotiate a minimum duration of one year. At the moment such a contract is signed, the transacted volume needs to be hedged in the wholesale markets.

Since most demand curves inhibit a yearly trend it is important to try to maximize the number of periods in which the contract period can be split. However, a number of practical difficulties arise. First, not all contracts are tradable; i.e. the APX NL only trades six monthly contracts, six quarterly contracts and five yearly contracts. Hence, for the first year, the last six months cannot be bought yet, and will need to be hedged using a quarterly contract.

In addition, liquidity is major concern on the APX exchange. Trading in the spot markets is obligatory for all market participants and hence the spot market is quite liquid. However, since this does not apply to the forward markets liquidity is an issue there. For Anode this limits the acceptable products to the three front months, three front quarters and two front years. In addition, this puts important constraints on the size of the contracts. Because of these considerations a contract should have a size of 5MW or a multiple thereof, even though smaller and other quantities can be traded.

In the table in the Appendix F all tradable contracts under these considerations are listed. Note that products can overlap, which increases the number of periods that can be solved individually. The resulting number of periods are shown in the table in the fourth column ‘periods for the algorithm’. Hence, for each month that the algorithm is run, different periods are used. Fortunately the cascading of large contracts makes it quite efficient to switch from one month to the next.

The algorithm assumes that Anode has a certain portfolio and it wants to hedge it for the next twelve months. For this period a certain number of contracts have been signed and only eligible contracts (see the discussion in the previous chapter) are included in the algorithm. Again because of liquidity considerations, it will not be used on a day-to-day basis unless of course a large customer changes the whole exposure curve of Anode. The threshold for such considerations lies with the 5MW barrier, and the trading department of Anode wants to have.

For all of the periods that have been indentified both the base- and peakload variants of contracts are available. However, the peakload contracts are seen as a special case of the baseload contract and hence the liquidity of these peakloads is even lower than those of the baseloads. Taking positions in these products can result in difficulties when offsetting a current position.

Finally, transaction costs are not used, but have a significant role. The transaction costs consist mainly of fees to the exchange, but also include a significant spread. Often, it is quite difficult to find multiple bids and hence, it can be assumed that the futures are not always competitively priced. Again, this makes it more important to consider market developments when taking a decision to take a certain hedge.

The algorithm entails the following steps in hedging the overall portfolio of customers for Anode.

I. Generate forecasts of the demand to be expected for all customers with a contract in which the price risk has been transferred to Anode. Forecast the $D_{i,c}$ where i is an interval and c is the specific customer.
II. Net these forecast to find the net exposure curve for the next year. $\left(D_i = \sum_c D_{i,c}\right)$
III. Determine the front month, and pick the right set of periods as shown in table 2.
IV. Find the optimal baseload (and peakload) quantities using the analytical derivations of 5.2-5.6 to find the optimal hedge quantities.
V. Translate these optimal hedge quantities to the actual positions in the different products.
    a. The second and third months can be netted with the first quarterly whenever overlap is present
    b. The final month can be a contract that is not tradable yet. Anode hedges these amounts by buying an identical volume in another contract that overlaps with the period. Usually, this means taking a volume-wise position in a quarterly (whenever available) or a yearly contract. This introduces a new risk, but is the practical best solution.
VI. Round the final quantities to the nearest five-fold integer to ensure that liquid contracts are bought or sold

The results of this algorithm will be detailed in chapter 7. In these results different variants will be discussed.

7 The estimation of a pricing model for the APX DAM NL

In the previous chapter the analytical solutions for the management of price risk were determined. One important aspect is a pricing model that can be used to estimate the price trajectories that simulate electricity prices. These will be derived in this section.

7.1 Electricity Price Characteristics

As noted extensively in the literature, electricity is a commodity with unique characteristics. It is characterized by a high volatility that can mainly be attributed by the fact that it cannot be stored efficiently, and that the supply and demand need to be continuously balanced. In addition, electricity has substantial transportation constraints throughout different countries as the cross-border connections are utilized to capacity. Hence, local factors like transmission, plant outages and maintenance costs can influence prices as well. Finally, demand for electricity is highly inelastic, i.e. it is an essential commodity whose use is not easily reduced. (Geman, 2000)

These factors result in the behavior of electricity prices that can be characterized by the fact that small changes in demand or supply can cause large price movements, especially when reserve capacity is low (Huisman, 2011) In June 1998 the Midwest of the US suffered from a superheated market. In desperate search for replacement power prices soared, and topped out at around $7,500 per MWh compared to the normal price of $25 per MWh. This caused a gulf of defaults of power traders. (Weron, 2001) Such spikes have occurred in the Netherlands as well, especially during the 2003 summer. In Holland legislation sets limits on the temperature of the water that power plants may discharge in the rivers. Hence, power plants needed to reduce production in order to comply with the legislation, which resulted in lower electricity production and shortages for the market. (Boogert and Dupont, 2005).

The demand for electricity has a strong seasonal component. These seasonalities can be explained from a microeconomic perspective by interpreting the electricity spot price as an equilibrium between supply and demand curves. Since demand is relatively inelastic the marginal costs of the supply side determine the price. (Geman,2000) With a low load, the cheapest energy sources are utilized, while with high load the more expensive sources are used as well. Hence, the periodicity of the total load is responsible for the periodicity of the electricity prices. Total load consists of a random component, depending on short term weather conditions and other uncertainties, but is also predictable part based on the seasons and week of day seasonalities. (Burger et al. 2004). Looking at the autocorrelations it was found that both the -1 and -7 lag are highly significant. This indicates that there is both a week component as well as the previous day component. The coefficients are negative indicating a mean-reverting behavior. (Weron, 2001)

To test whether these seasonalities are also present in the Dutch market, an empirical analysis was setup. The analysis starts from the highest aggregation level and then disaggregates to the smallest interval of one hour. In the chart below the yearly average base- and peakload price are shown. Looking at the chart it is clear that strong mean reverting behavior is present. A high average price is followed by years with lower than average prices. In addition, it can be seen that the spread between base- and peakload is slowly declining as the market matures. Finally, it seems that the extreme volatility is reduced in the last years since 2009, especially considering that the winters were extremely cold in comparison to the previous years. This can partially be explained by structural changes to the Dutch electricity market, where significant capacity has been added to the national grid, as well as increased integration with the German electricity market.

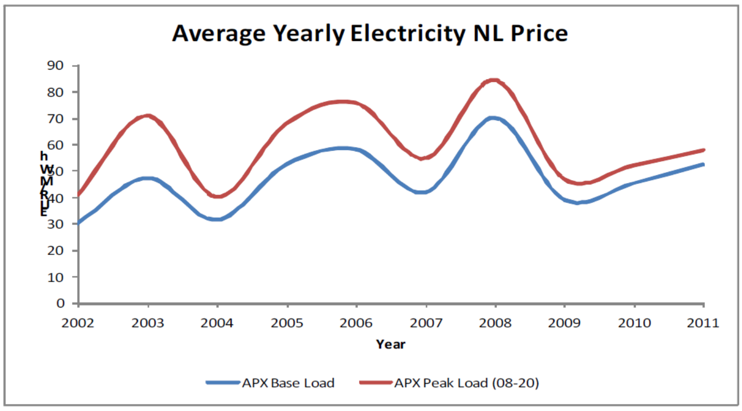
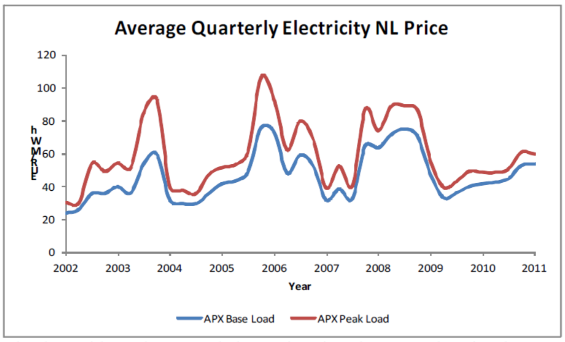


Figure 15 Time series of the average APX price calculated as the simple average of the realized hourly APX prices 2002-2011

Next, the interval was disaggregated to the quarterly level. It is clear that the smooth mean-reverting time series now incorporates a seasonal trend. The seasonal trend indicates lower prices during the spring/summer period and high prices in autumn and winter. Spikes occur more frequently in the winter periods but not always. In 2004 and 2006 price spikes occurred during the winter, however in 2005 this spike was not present. Prices change significantly from an average 30 EUR baseload in the summer seasons to up to 80 EUR average baseload in the winter. In addition, a chart was made where the quarterly prices were indexed on the yearly prices. It is clear that there is a seasonal movement that corresponds to the four seasons of the year. Again as expected peaks are present in the final months of the year, when seasonal weather is at its coldest and most energy is consumed for heating purposes.



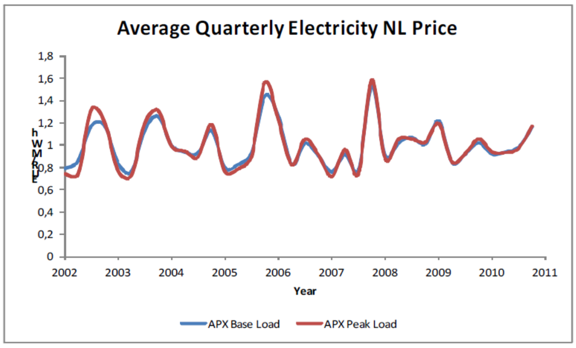


Figure 16: Chart of the quarterly average realized APX DAM NL market prices (left) & indexed prices (right)

The monthly interval in figure 10 shows a similar picture to the quarterly chart. A clearer pattern becomes visible where months clearly follow the seasons and again a strong mean reversion appears present. Months higher than the average are followed by months with lower values than the average. On the left hand side the realized average prices are shown, while on the right hand side each month is represented by an index on the yearly average. A clear pattern becomes visible where the November and December months show the highest peaks. However, the January and February charts, surprisingly, show a less than average price. Again volatility seems to be decreasing in the winter period. Note that these deviations are based on ten or eleven observations and therefore, can still be quite sensitive to extreme movements in trend. This is probably what happened during 2001-2004, 2005-2006 and 2007-2008. This introduces a linear trend in the time series and therefore this would overestimate the decrease in the first months of the year, and underestimate the increase in the final months of the year.

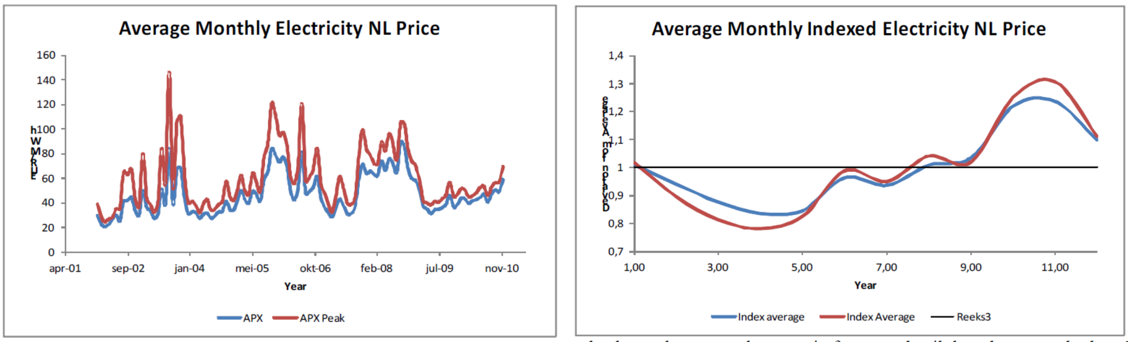


Figure 17 Chart of the monthly average realized APX DAM NL market prices (left) and indexed (right)

In order to investigate whether a weekly trend is present in the data as well, the same indexation procedure was repeated for the daily averages scaled on the weekly average. The result can be seen in figure 8, where the deviation is plotted. It is clear that prices on average are higher during weekdays and significantly decrease during the weekend. On average this decrease amounts to 30%! It is therefore very important to incorporate the effect of this on the portfolio in order to increase the accuracy.

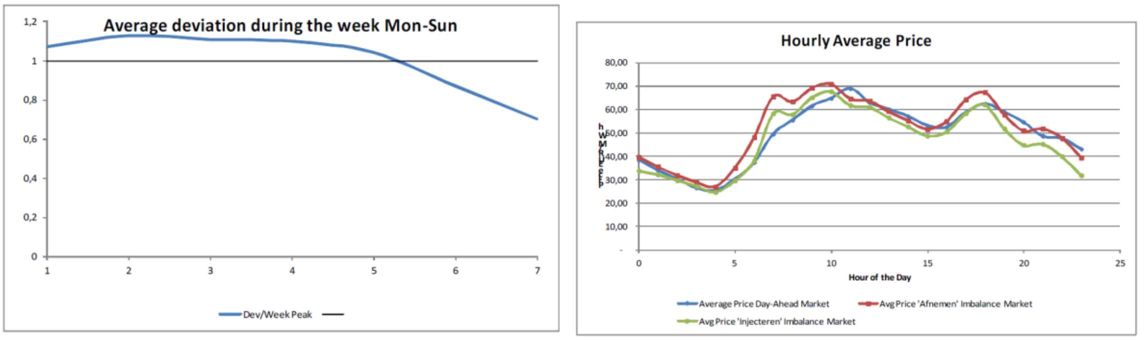


Figure 18: The average deviation from the weekly average as a ratio (left). (APX DAM NL 2002-2011) and hourly prices (right)

Above the hourly realized average spot price is show in figure 9. In addition to the Day-Ahead spot price, the two imbalance prices are shown. It is clear that on average these prices are quite close to the realized average spot price. However, the volatility of these series is multiples higher than the volatility of the Day-Ahead market. See the summary statistics in the appendix H. The average shown below in figure 10 is based on data from 2002 until 2011. It is clear that there is a peak and off-peak period. However, there are two peaks present in the data. The first peak occurs during the typical working hours from 09.00 to 17.00. The second peak starts at 17.00 and moves to about 22.00. It is very difficult to estimate these two peaks, as they are both also dependent upon the seasonal trends.

7.2 The Pricing Model

On the exchange's website the electricity spot prices are reported as 24 individual prices, each used to settle all transactions in that specific hour. This spot price will be denoted as $S_{i,t}$ where the i corresponds to the specific hour of the day ($0 < i \leq 24$), and t is an index that indicates the day ($t \geq 0$). Because the prices are published simultaneously the $S_{i,t}$ will be interpreted as 24 different dependent time-series in a panel-data setting. (Huisman, 2007) The dependencies of the prices $S_{i,t}$ are both cross-sectional, i.e. prices are correlated within the same day i as well as serial, i.e. prices are correlated from one period t to the next period $t + 1$. In order to filter out the smallest seasonality part, which is equal to 24-hourly time series of spot electricity prices is transformed into a 24 x 1 vector-timeseries $S_t =$

$\left(S_{1,t} \dots S_{24,t}\right),\ \ t = 1, \dots, T$. Because hourly data is available, the time series regression for the daily vector S_t has a periodic model for the hourly series.

The model estimates prices based on three different components. The first component captures the seasonality through the estimation of a conditional average. The second component captures cross sectional structures and will be modeled using a dynamic factor model. Finally serial correlation will be estimated using ARIMA models. This structure is variation on the model that Garcia-Martos et al. (2011) used to forecast electricity prices.

$$S_{i,t} = \mu_{i,t} + \sum_{j=1}^{J} \beta_{i,j} F_{j,t} + \varepsilon_{i,t} \tag{6.1}$$

The seasonal component is estimated using a multiple regression with multiple dummies as the independent variables. The goal is to estimate a conditional average $E_{t_0}\left[\mu_{i,t+n} \middle| X_{i,t+n}\right] = f\left(X_{i,t+n}\right)$ where X represents all available information and $f(X)$ is a function that is based on all available information on time period t_0. Using the literature and the price analysis the following effects were indentified. First, a yearly effect was estimated, since the market has undergone several structural changes that can be captures using the yearly dummy. Next, monthly effects are present that attempt to capture the seasonal influences from the weather. Finally, seasonality is captured on the daily level because of working days and the weekend or holidays. The conditional average is denoted by $\mu_{i,t}$, and the equation is shown below:

$$\mu_{i,t} = \beta_0 + \beta_i + \beta_{year} d_{year} + \beta_{month,i} d_{month} + \beta_{day,d,i} d_{day} + \beta_{holiday,d,i} d_{holiday} + \eta_{i,t} \tag{6.2}$$

The regression models were run in the statistical software package Stata. In the appendix H, the summary statistics and the final regression results are shown. The model was simplified by assuming that certain clusters of hours responded identically to an effect. First of all, it was shown that weekdays are identical for Monday through Thursday, hence the number of days was reduced to 4. In addition, the yearly effects were only estimated for blocks of hours, because such long-term effects cannot be estimated with hourly indices. The monthly effect was still estimated on an hourly basis, because the impact differs substantially based on the hours. The regression model has an R^2 of 42.5%. For both models the residuals were calculated and these were tested for normality, independence and homoscedasticity, all of these tests showed that there was dependence, non-normality and heteroskedasticity.

Next, a multivariate dynamic factor model was fitted to capture the structure in the correlated residuals from the previous regression equation. The multivariate factor model approach was first developed by Watson&Engle (1981). The approach is based on state space modeling of multivariate unobserved components models (the dynamic factors). The factors are estimated using principal components analysis (PCA). The results from the PCA are shown in Appendix I. The four factors that were extracted – based on the eigenvalue > 1 – could explain approximately 80% of the correlation structure. Note that as the data represents the residuals from the regression, the dynamic factors need to be explained in terms of deviations from the conditional mean. The factors can be interpreted as the different ‘forces’ that influence the prices during the day. The first factor corresponds with a general level of volatility that is based on each of the hourly prices. The second factor can be interpreted as the general peak / off-peak factor that explains differences between the peak and off-peak hours. The third factor accounts for the evening peak that is caused mainly by households. And the fourth factor represents some effect in the transition from peak to off-peak and vice versa.

After the factors are identified from the residual data of the regression, the unobserved common factors $F_{k,t}$ are assumed to follow a seasonal multiplicative ARIMA model $(p,d,q)\times(P,D,Q)_s$ with GARCH(1,1) disturbances and a seasonality of seven days. This simulation method was first used by Garcia-Martos et al. (2011) who assume that such a process for the factors is modeled using a $\text{SARIMA}(1,0,0)\times(1,1,0)_7$ combined with GARCH(1,1) disturbances. The ARIMA accounts for the autocorrelation in the factors with respect to the previous days, the seasonal ARIMA accounts for the autocorrelation with respect to the same day a week(s) before. The GARCH model captures the volatility clustering of the factor values. Even though the long-term seasonality has been captured by the regression, short-term seasonalities still exist. Finally, the random innovations of these factors should be normal and IID. Using the ARIMA model it was possible to remove autocorrelation for a large extent, however not completely. In addition, it was shown that a student-t_3 distribution better fitted the random noise data. Finally, the PCA that was used to estimate the common factors, assures us that the factors themselves are uncorrelated. The SARIMA specification was evaluated for multiple values; after running multiple models, an AR process proved optimal. For all the factors an AR(1 2 3), $AR(3)_7$. Each of the factors (k) can be written as:

$$F_{k,t} = f_{k,t} \tag{6.3}$$

Where $f_{k,t}$ follows the above VARIMA-type model with a seasonal component of 7 and no differencing resulting in equation (6.4) excluding the moving average terms:

$$\left(1-\sum_{j=1}^{p}\alpha_j B^j\right)\left(1-\sum_{j=1}^{p}\alpha_j B^{7j}\right)f_{k,t} = z_{k,t} \tag{6.4}$$

In addition, the $z_{k,t}$ are random drawings from a student-t_3 distribution with a variance of $h_{k,t}$ or $\sim T(0,h_{k,t})$. The variance follows a GARCH(1,1) process which is described by the following equation:

$$h_{i,t} = \omega + \alpha_1 z_{t-1}^2 + \beta_1 h_{t-1} \tag{6.5}$$

Finally the factors were regressed on the individual hourly residuals from the regression to find the β −coefficients of equation (6.1). The residuals for each hour from the regression of the factors are saved again and the $\varepsilon_{i,t}$ (see equation 6.1) will be modeled with an ARMA(p,q) process as shown below. The residual for each hour i is the sum of a moving average of the previous resulting residuals, as well as an autoregressive from the previous impulses $z_{i,t}$ which are all normally distributed $\sim N(0,s)IID$. The B^j is the backward operator that is used to indicate the steps back in time. $B\varepsilon_{i,t} = \varepsilon_{i,t-1}$ and $B^j\varepsilon_{i,t} = \varepsilon_{i,t-j}$.

$$\left(1-\sum_{j=1}^{p}\alpha_j B^j\right)\varepsilon_{i,t} = \left(1+\sum_{j=1}^{q}\theta_j B^j\right)z_{i,t} \tag{6.6}$$

After the model was estimated it was used as the basis for a Monte Carlo simulation in Matlab. An important choice there is the specific distribution to be chosen for the random shocks of the factors, and the individual hourly residuals. The standard approach is to assume a Gaussian distribution on the residuals. However, this is often not the case for price series, since these show a high kurtosis or skew. In order to counter this problem a number of different distributions are fitted to describe the data.

A student-t distribution, as well as the student t_3-distribution are fitted to the residual data. The performance of these distributions was compared to the original dataset. The student t_3-

distribution was fitted to the data using the build-in functions from the statistics toolbox from Matlab. The student-t distribution was fitted using the method of moments. This method uses a series of moments (as much as parameters need to be estimated) in order to find the set of parameters that matches all moments of the distribution.

The student t-distribution has multiple moments that can be used to fit the model. The probability density function is dependent upon the parameter ν which needs to be estimated.The student distribution has as the first moment equal to zero. However, the second moment, which is equal to the variance is the equal to $\frac{\nu}{\nu-2}$ for $\nu > 2$. Hence we can fit the distribution by calculating the second moment of the time series and setting it equal to the above formula..

$$\frac{\nu}{\nu-2} = \sigma^2 \qquad \text{rewrite to } \nu = \frac{2\sigma^2}{1-\sigma^2} \tag{6.7}$$

First the standard Gaussian model was used to generate a new pricing series that covered the whole original estimation period. This data was then subjected to an identical analysis as the previous section. The regression results show that the data has already changed in a significant way. This is caused by the fact that more of the deviations are explained by the regression 73% instead of the original 50%. However, when we look at the PCA analysis, it is clear that no factors are found by the PCA. The reason for this find is unknown.

In order to choose the best model the three specifications were used to simulate an identical period as the original data. The charts are shown in the Appendix M. It is clear that the prices show large spikes during peak hours. The model that is best able to capture these effects is the student-t3 model. The student-t3 model will therefore be used throughout the report. To confirm whether the distribution influences the optimal Q, multiple specifications were run with all the four models. It was shown that the choice of distribution does not significantly influence the Q. (Appendix K)

7.3 Demand Model

In the scenarios where demand is stochastic, a model for the demand needs to be assumed. Demand is assumed to be normally distributed with a specific mean and standard deviation. The mean is equal to the best forecast that Anode can provide for their future demand. This forecast is made for each separate customer group and varies significantly from one customer group to the other. In addition, using historical data for 2011, each customer group got assigned a coefficient of variation (CV). This CV was used with the forecasted mean demand to calculate the standard deviation, hence the standard deviation can vary hour by hour, but is linearly related to the forecast mean. For profiles, a forecast is based on an hourly quantity which results in different hourly means and standard deviation. On the other hand, wind is forecasted with a level monthly mean, hence the hourly demand mean and standard deviation are identical for all hours.

In addition, a correlation coefficient was estimated for each customer group with the first pricing factor from the dynamic factor analysis. This correlation coefficient was saved and used to generate the demand scenarios after the pricing scenarios have been generated. All random residuals the first dynamic factor were saved. This set of residuals was combined with the new residuals for the demand with the normal distribution and the parameters as shown in the previous paragraph. Using formula (Schumacher, 2010) below, the two sets of residuals $(\varepsilon_{1,t}\varepsilon_{2,t})$, were combined in such a way that they were correlated with (ρ).

$$z_{i,t} = \rho\varepsilon_{1,i,t} + \sqrt{(1-\rho^2)}\varepsilon_{2,t} \quad (6.8)$$

New correlated residuals were calculated, and those were used to generate the demand paths. Each hourly demand can be written as the mean expected forecast plus a disturbance $z_{i,t}$ which has an expected value of zero and is correlated with the residual that was used to generate the price path of the first factor. This means that each scenario of prices needs to be combined pair-wise with a scenario of demand. Note that the factor residuals are a daily residual, while the individual demand residuals are hourly. Therefore there is significant correlation between the random residuals.

8 Results for Anode's Risk Management and Discussion

In this section the formulations as found in chapter five, are applied to real data from Anode. First, two separate cases are investigated; the profiled customers of Anode, and the wind producers. Finally, the whole portfolio is analyzed to find the risk-minimizing hedging quantities.

8.1 Case 1: Profiled Customers

To illustrate how a portfolio consisting of profiled customers should be hedged, an analysis is made where the risk-minimizing hedging quantities are calculated using the models derived in the previous chapters. This timeseries focuses on the month of October 2011 and represents the sum of forecasts for the different profiled customers. Below, in chart 1, the time series is shown. The forecast is based on the customer portfolio of Anode.

To calculate the cashflow from the net exposure from profiled customers, it is necessary to assume a sales price. This price was set at the futures price plus a premium of 15 EUR. This illustrates the profit from this customer group. In addition, a probability distribution has to be assumed for the stochastic demand. Based on historical data a normal distribution was chosen with two parameters that needed to be estimated, the coefficient of variation (0.05) and the correlation with the first pricing factor (0.1). These values were put in the model and based on these characteristics random demand data was generated. The random data from profiled customers reflects a set of different factors; the change in number of customers, the change in type of contract of customers, and changes in expected demand updated by the incumbent DSOs. Using the price process, 2000 price paths for the month of October were simulated.

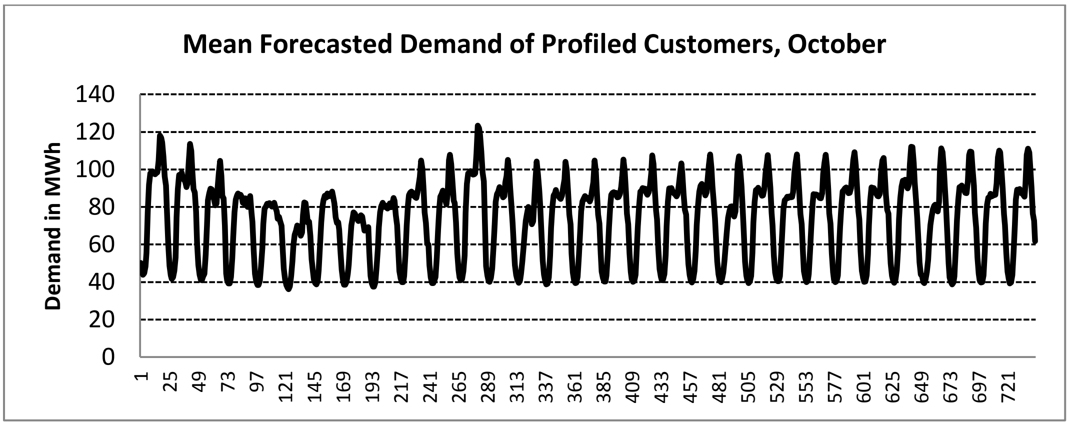


Figure 19: Demand Curve of the Profiled Customers October 2012 (positive values indicates consumption)[8]

Using the formulas from chapter 4 the optimal Qs were calculated from the Monte Carlo simulation. The four scenarios are based on 'base-only' contract or 'base&peak' contracts, and the demand is considered deterministic or stochastic.

	Scenario			
	Base-only Deterministic	Base + Peak - Deterministic	Base-only Stochastic	Base + Peak Stochastic
Baseload (Qb)	85.9	69.1	85.9	70.3
Peakload (Qp)	-	29.9	-	28.0

Table 2: Variance-minimizing order quantities for the different scenarios for profiled customers[8]

[8] Both the forecast data, as well as the calculations in table 2 have been randomized because of confidentiality.

In the table it is clear that on for both stochastic and deterministic demand the risk-minimizing hedge quantity is equal to an 85.9 MW long position in a baseload contract. The average demand during the same period is equal to 69.1MWh and hence, the risk minimizing quantity indicates that an overall long position reduces risk substantially. Demand is a natural short position which implies that upward price spikes are detrimental to the cash position of the firm. Because these price spikes mainly occur during peak hours when demand is at its highest, increasing the long position causes a reduction in exposure for peak hours, and an increased exposure in off-peak hours. Because both the correlation between prices and demand, and the volatility of demand are quite low, there is not a significant difference between stochastic demand and deterministic demand.

Comparing the baseload-only case with the base & peak case, it is clear that introducing a peakload contract reduces the amount of baseload procured. This makes sense, as the reduction in baseload reduces the excess position in the off-peak hours, while the peakload contract still reduces the short position in peak hours. When taking into consideration that peakload is only delivered half the time (ignoring weekends) then in the base case 85.9 MW on average for the day, compares to 84MW on average for the day in the B&P case. Again the deterministic and the stochastic case are not that different from each other.

In figure 2 below, the variance curve is shown for deviations from the optimal Q-base and Q-peak. The curve is fairly smooth and insensitive to redistributions of MW between peakload and baseload quantities. However, adding additional MWs by buying both peak- and baseload simultaneously then the variance increases very rapidly.

Below, the cashflow histograms for the three scenarios in the stochastic case are shown. From these charts the trade-off can be seen. The two hedging policies reduce the risk enormously by reducing the range of possible cashflow scenarios. In the no-hedge case the cashflows occur in the interval of -400k EUR to 500k EUR with one spike to -1 Mln EUR. While in the hedged scenarios the range is reduced to approximately 0 and 125k EUR. Note that in all three scenarios the average cashflow is identical and that hedging has no effect on the expected cashflow. While it is clear that avoiding scenario cashflows of -400k EUR, it is not clear that preventing profits from increasing to 400k EUR is profitable. This is the tradeoff introduced by hedging, where minimization of risk implies losing bad scenarios, but also reducing good one.

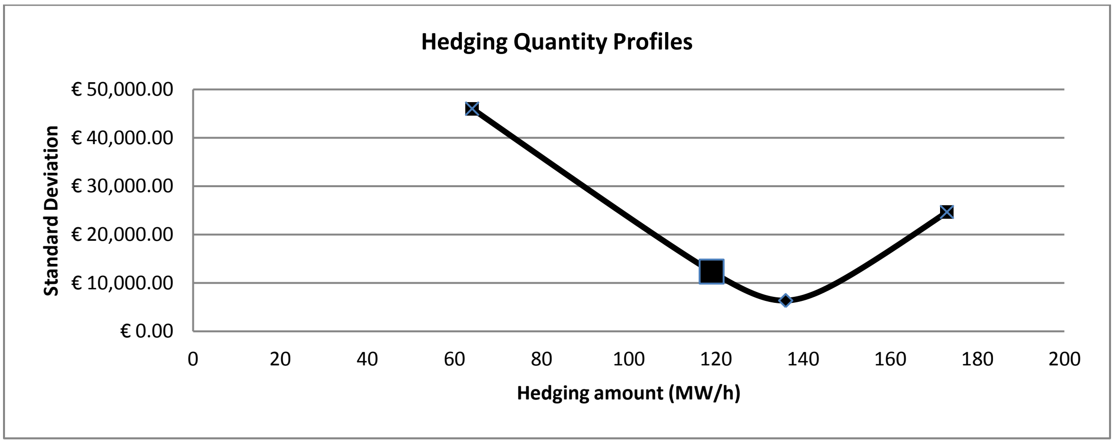


Figure 20: The standard deviation of cash flows from profiled customers as function of hedging quantity

It can be seen from figure 20, that the hedging amount is not that sensitive. The minimum however, is significantly different from the average demand of the profiled customers. The line start with the minimum demand, and ends with maximum demand. Again buying too much is not as bad as buying too little.

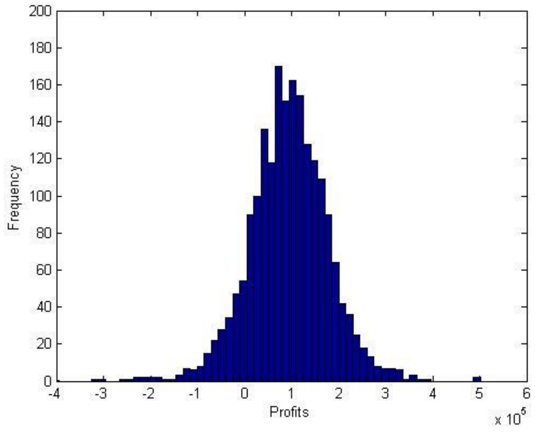


Figure 21: The distribution of cash flows from profiled customers

Summary Statistics of Cashflows from Net Exposure Profiles

		Base and Peak hedging		Base-only hedging		No hedging
Mean	€	93,730.00	€	93,730.00	€	93,730.00
Standard Deviation	€	5,527.00	€	6,974.00	€	87,659.00
Reduction SD		93.7%		92.0%		
Minimum	€	6,100.00	€	11,900.00	€	-1,003,000.00
Maximum	€	122,870.00	€	123,840.00	€	501,300.00
95% Confidence Interval						
Low	€	84,610.45	€	82,222.90	€	-50,907.35
High	€	102,849.55	€	105,237.10	€	238,367.35

Table 3: Summary Statistics

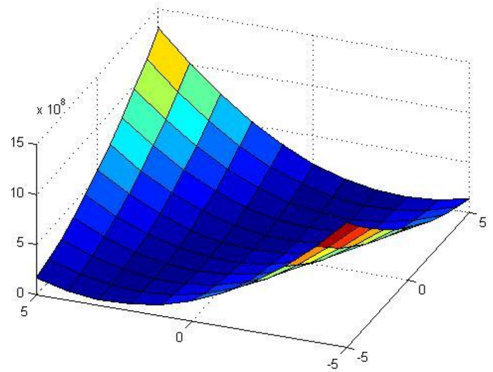


Figure 22: The variance surface

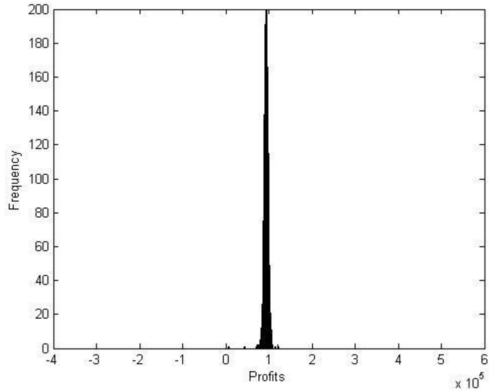


Figure 23: The distribution of cash flows for base and peakload

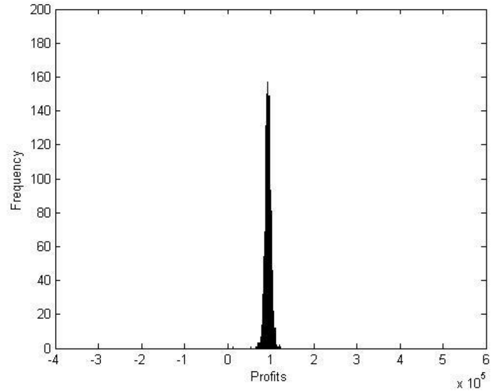


Figure 24: The distribution of cash flows for baseload

8.2 Case 2: Wind energy

Anode manages a significant amount of electricity production from wind farms in its portfolio. Wind energy is one of the most unpredictable and volatile sources of electricity and is therefore extremely difficult to manage the risks correctly. The production from wind turbines cannot be accurately predicted for more than a few days in advance, if it all. In addition, the market prices react quite strongly to wind energy because it is becoming a significant component of the overall energy production, and because it has a marginal cost that is equal to zero. (some argue it is negative as wind energy is highly subsidized by the government) Anode always receives electricity from wind turbines as the weather permits and cannot reject electricity. These factors combined make wind energy the most risky customer group of Anode.

Because of these characteristics the analysis of a wind-only portfolio is very interesting. Wind is highly unpredictable and therefore the long-term forecasts are merely based on long term averages. Hence, wind is considered a constant source of electricity. This makes it very easy to hedge in the deterministic case, since a constant production quantity can easily be bought in the wholesale market, reducing the net exposure to zero. However, in the stochastic case with a coefficient of variation of 0.8 and a correlation of -0.4, interesting patterns should emerge. Looking at historic data, it was found that wind production could be anywhere from 35MW to 0MW. Again prices were simulated in the Monte Carlo simulation with 2000 price and demand paths.

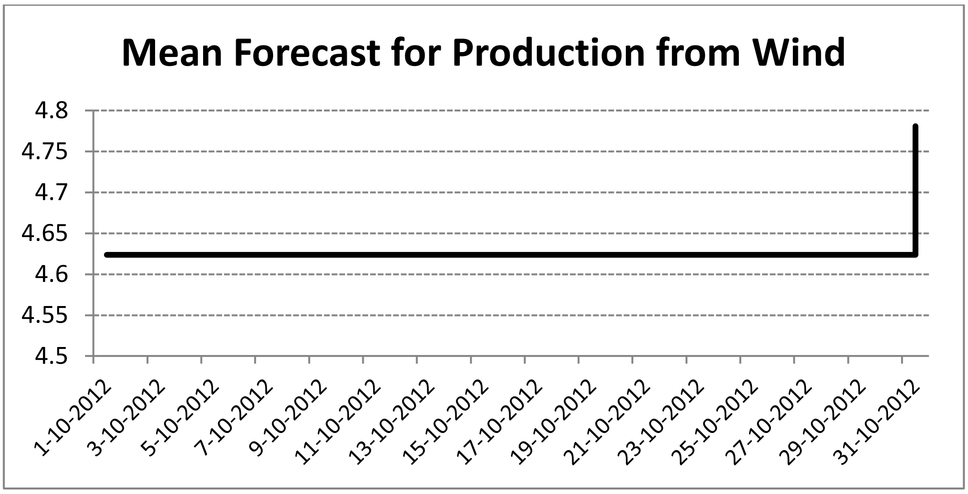


Figure 25: Electricity Production from Wind Energy Anode 2011 (Daily scenarios through the day (Shown as the negative of demand, randomized)

Below in table the results can be found for the four different models. The scenarios again differ in the type of contracts available (base-only and base and peakload) and the demand (stochastic or deterministic).

	Scenarios			
	Base-Deterministic	Base + Peak - Deterministic	Base -Stochastic	Base + Peak Stochastic
Baseload	-4.6	-4.6	-4.8	-4.7
Peakload	-	0	-	-0.2

Table 4: Variance-minimizing order quantities for the different models for wind producers (randomized)

From the risk-minimizing order quantities it can be seen that the addition of wind energy again results in a hedging quantity that is higher than average. Since wind is a natural long position of electricity, the hedging quantity should be negative, indicating that an offsetting short position should be taken. What is surprising, is that a small peakload position is taken, as well as the baseload position. This indicates that some volatility reduction during peakhours is beneficial as well. However, given that the variance surface is relatively flat, this could also be due to randomness. Should the peakload short position of 0.2 MW be decreased to zero, then the variance would not change significantly. Therefore, it is possible to state that wind energy, should be hedged using only baseload contracts, and peakload contracts do not add any additional value over the baseload hedge. Possibly, a more complex model of wind forecasts would result in different solutions.

The difference between the deterministic case and stochastic is surprisingly small. Where in the deterministic case variance can be removed completely due to the expected value being constant, a highly correlated wind production does not result in significant deviations from that basic value. This could be caused by the fact that electricity prices are very uncertain, and influenced by multiple factors, of which wind production is only one.

Looking at the cash flow distribution that results from the three scenarios, it can be seen that the expected profit does not change. Note that a premium was estimated, to generate a profit. The unhedged cash flows are quite volatile, with a minimum of -250k EUR and a maximum of 380k EUR. Using the hedging policies as indicated by the formulas of chapter 4, the interval is reduced to 15kEUR to 106k EUR. The trade-off occurs that while bad scenarios are removed, this is also happens to some beneficial scenarios. Overall, the total risk in terms of standard deviation is decreased with 84.9%.

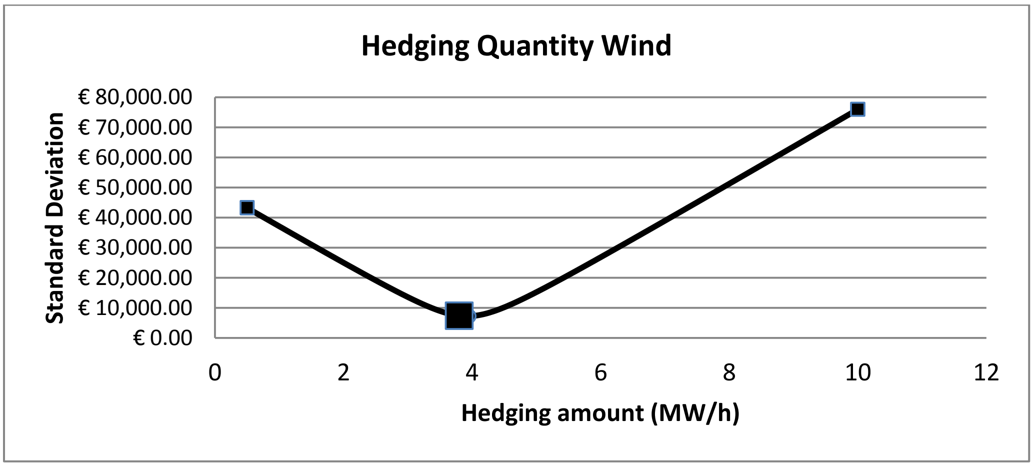


Figure 26: The standard deviation of cash flows from wind energy as function of hedging quantity

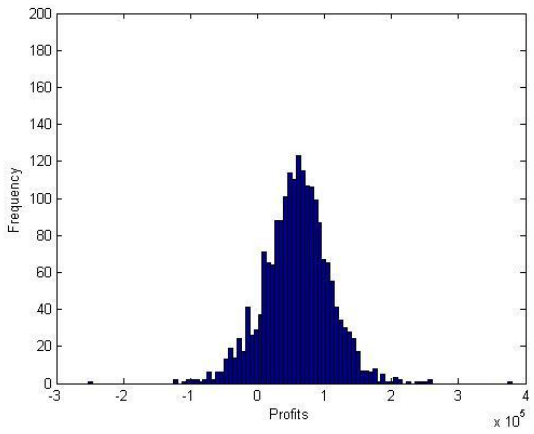


Figure 27: The unhedged casfhlows from Wind

Summary Statistics of Cashflows from Net Exposure Wind

	Base and Peak hedging		Base-only hedging		No hedging	
Mean	€	59,100.00	€	59,100.00	€	59,100.00
Standard Deviation	€	7,546.00	€	7,550.00	€	49,853.00
Reduction SD		84.9%		84.9%		
Minimum	€	14,660.00	€	14,480.00	€	-252,000.00
Maximum	€	106,800.00	€	106,690.00	€	380,320.00
95% Confidence Interval						
Low	€	46,649.10	€	46,642.50	€	-23,157.45
High	€	71,550.90	€	71,557.50	€	141,357.45

Table 5: Summary Statistics of the cashflows

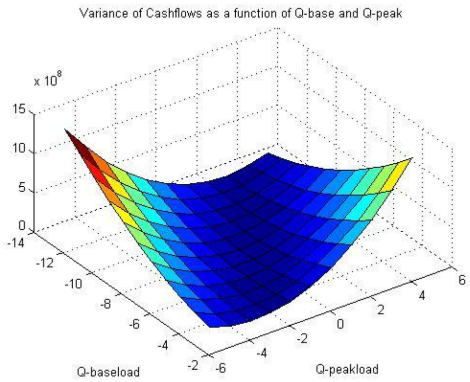


Figure 28: Variance of Cashflows from simulation based on Qbase and Qpeak

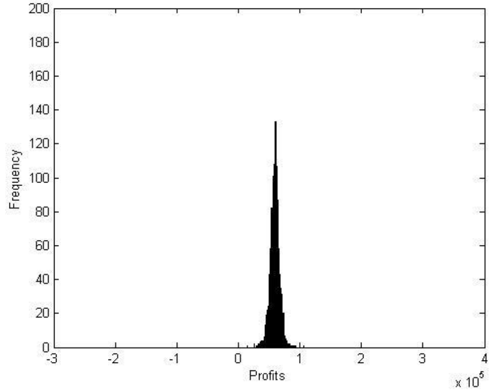


Figure 29: Histogram of cashflows for hedge policies the cashflows from peak and baseload contracts

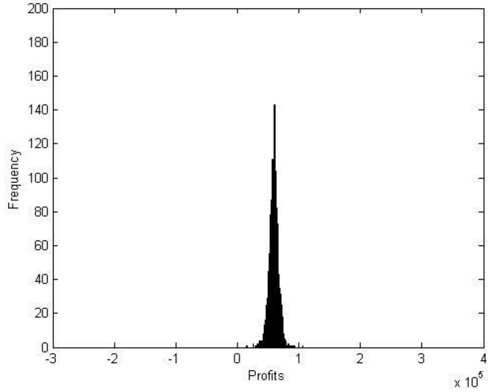


Figure 30: Histogram of cashflows for hedge policies the cashflows from baseload contracts

8.3 The overall portfolio of Anode

Now that two specific parts of the portfolio are evaluated, we now focus on the complete portfolio and on the whole timeline. Anode basically has five different types of customers. The demand forecast as available on the 1st of September 2011 was chosen as the expected demand curves for each customer and the portfolio as a whole. The model was started at the 1st of September as well, (excluding a warm-up period) and was run to estimate the distribution of prices for each hour of the next year. In addition, the demand was determined stochastically and correlated with the prices with parameters that were estimated for data of realized demand and factor prices of 2011.

The optimal order quantities for each period were determined. More specifically, there were seven periods, four monthly periods (Sep, Oct, Nov, Dec) two quarterly periods (Jan-Mrt, Apr-Jun) and one two-month period (Jul-Aug). These periods do not directly refer to tradable products, but show the net result that a portfolio of products should result in. The combination has been chosen in such a way that the periods themselves are as small as possible to increase the ‘fit’ to the data. After the optimal Qs were calculated the cashflow distributions were shown.

Below the average daily consumption and production patterns are shown. It is clear that there is a significant deviation, and that good management of the exposures is necessary. Especially for the two customers that were already discussed; wind and profiled customers. Production and Consumption from large customers is actually quite constant and should not result in great difficulties in order to hedge it away. On the other hand special attention is warranted for the profiled customers, that have quite some variation in expected consumption throughout the day. Wind is quite constant as well, but care should be taken since wind energy incorporates large and highly correlated production and prices.

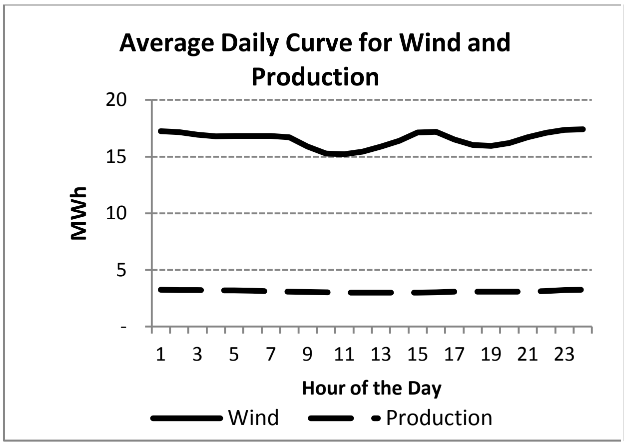


Figure 31: The average daily wind and other power production

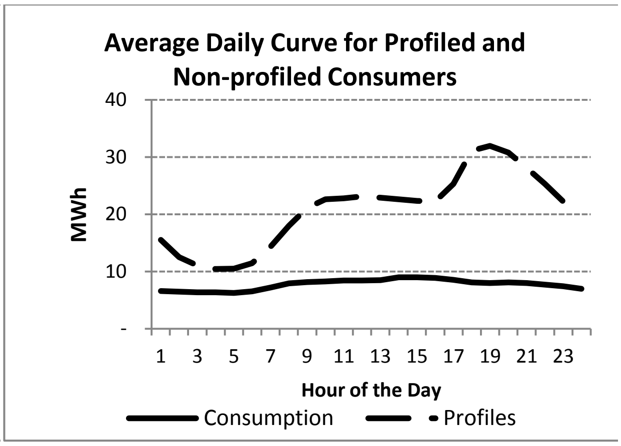


Figure 32: The average daily consumption from profiled and other customers

Below is shown the chart of the overall portfolio exposures on the APX Day-Ahead market in 2011. These quantities reflect the amount that Anode bought in the DAM in order to settle its expected demand. This does not represent realized flows, as these would incorporate forecast errors and other information sets. The exposures are the result from the sum of all production facilities (solar, wind and large customers) as well as consumption (profiles and large customers). It can be seen that the amount bought or sold is highly volatile and is large driven by the wind energy production. It is this net exposure that we will simulate using stochastic distributions for all the customer groups, as well as correlations with the main price factor (and hence with each other).

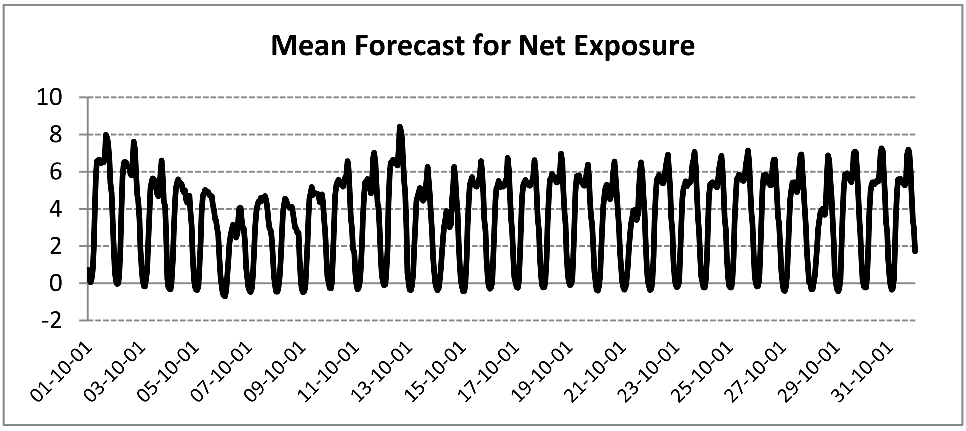


Figure 33: Net Exposure at DAM market Anode 2011 (Daily, randomized)

The exposures with their statistical properties and the price process together are used to calculate the optimal procurement quantities in both the stochastic and the deterministic demand case. Those results are shown in the table below.

		Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7
Stochastic	Baseload-only	6.5	4.7	4.7	4.4	3.9	5.2	5.6
	Baseload	4.4	2.8	2.4	2.3	1	3.2	3.7
	Peakload	3.0	3.3	4.0	3.8	5.3	3.7	3.4
	Average Load:	*5.9*	*4.5*	*4.4*	*4.2*	*3.7*	*5.0*	*5.4*
Deterministic	Baseload-only	6.3	4.9	4.9	4.6	4.4	5.3	5.7
	Baseload	4.9	2.9	2.8	2.3	2.4	3.5	4.0
	Peakload	2.0	3.4	3.8	3.8	3.5	3.4	3.4
	Average Load:	*5.9*	*4.7*	*4.7*	*4.2*	*4.2*	*5.2*	*5.7*
	Demand:	5.3	3.4	3.4	3.1	2.9	4.4	5.0

Table 6: The variance-minimizing base- and peakload quantities for the net exposure of Anode

From these results a number of conclusions can be drawn. First, the stochastic and the deterministic case do differ but still not substantially. In some situations a relatively large deviation occurs (period 5), but in most periods the deviation is relatively minor. However, this could be caused by the fact the minimum can be at a certain combination of peak- and baseload, but because of the low sensitivity the difference to adding some peakload to baseload has not a big effect. To check this hypothesis, the totals were also shown which were calculated as the baseload plus 0.5 times the peakload. These quantities show far less variation between the base- and B&P case.

It is surprising that the difference between deterministic and stochastic demand changes. While in the first monthly periods the total base and peak is lower than the deterministic case, this slowly changes in period 4 and 5, while in period 6 and 7 again the deterministic case is higher. Partially this could be explained by the different correlations with the pricing factors, where stochastic offsets each customer group thereby already reducing the overall variance compared to the one-customer cases in the previous section.

A clear trend is that more than the average demand should be procured, indicating a clear preference for off-peak exposure versus on-peak exposure. The price risk that is caused by price spikes makes that market participants have a preference for net long positions instead of short positions. The average demand is however not a really informative number since it does not give any information on the composition of the portfolio behind the net demand. A high demand, caused by stable producers or consumers would not result in higher hedging quantities, while a wind park could result in additional hedging. It is therefore not straightforward to compare one period with another period.

Summary Statistics of Cashflows from Net Exposure Overall						
	Base and Peak hedging		Base-only hedging		No hedging	
Mean	€	-405,800.00	€	-405,800.00	€	-405,800.00
Standard Deviation	€	40,100.00	€	51,780.00	€	419,000.00
Reduction SD		90.4%		87.6%		
Minimum	€	-591,300.00	€	-751,000.00	€	-2,580,000.00
Maximum	€	-219,200.00	€	-231,000.00	€	1,584,000.00
95% Confidence Interval						
Low	€	-471,965.00	€	-491,237.00	€	-1,097,150.00
High	€	-339,635.00	€	-320,363.00	€	285,550.00

Table 7: Summary Statistics of Cashflows from overall portfolio (not profit)

Below the different cash flow histograms are shown for the whole periods. It is clear that hedging reduces the risk of downward substantially, unfortunately at the cost of the positive scenarios as well. The most profitable outcome would result in a positive cashflow of 1.3M EUR, while the most negative cashflow would result in a negative cashflow of 2.5M EUR. It is clear that because most of the time the net exposure is negative (a short position) it is clear that the downward spikes are mainly directed in the negative direction. Hence, it is especially important that adequate hedging policies be implemented.

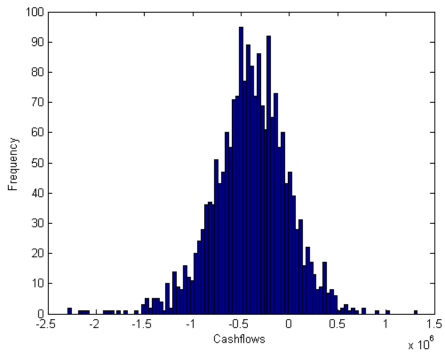


Figure 34: The distribution of cashflows from the total portfolio

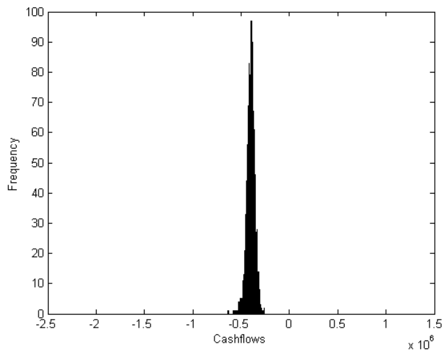


Figure 35: The distribution of cashflows from the total portfolio from hedged with base- and peakload

From the figures above, it is clear that that substantial negative cash flows can occur when not hedging price risk. The hedging reduces the spread of cashflows to a much less volatile range. The addition of the peak load contract does not have a large effect compared to the inclusion of a simple base load policy and is not shown above, because a large proportion of the portfolio consists of large customers and wind which each have a relatively flat expected demand curve. In addition, the wind factor greatly increases volatility in demand and can result in shortages where overages were expected and vice versa. Hence, the inclusion of these highly volatile customers have a large influence on the overall result and indicate that base- and peakload hedging does not add significant value in addition to the baseload only case.

9 Conclusion

For this thesis project analytical formulas are developed that solve the electricity hedging decision for an electricity trader with no generating capacity and both consumers and producers as customer. It is shown that hedging is important and that different customers require different strategies. Next, a number of managerial recommendations will follow based on the previous results, as well as suggestions for further research.

9.1 Managerial Recommendations

Risk management should be the main concern for any electricity trader. The fact that this trader has no direct control on the production or consumption of the customers makes traders especially vulnerable to both price and imbalance risk. First, an operational hedge was designed by pooling different customers with price or imbalance risk together. Next, financial hedges were incorporated to reduce exposures even further. Based on the algorithms described in chapter 5, the risk-minimizing quantities were calculated.

95% Confidence Interval for different Hedging Strategies and Customers

		Base & Peak	Base-only	No-Hedging
Wind Producers (1 month)	Low	€ 46,649.10	€ 46,642.50	€ -23,157.45
	High	€ 71,550.90	€ 71,557.50	€ 141,357.45
Profiled Customers (1 month)	Low	€ 84,610.45	€ 82,222.90	€ -50,907.35
	High	€ 102,849.55	€ 105,237.10	€ 238,367.35
Net Exposure (1 year)	Low	€ -471,965.00	€ -491,237.00	€ -1,097,150.00
	High	€ -339,635.00	€ -320,363.00	€ 285,550.00

Table 8: The 95% Confidence Intervals of Cashflows

It is shown that for profiled customers the risk minimizing quantities are substantially higher than the average demand. This is caused by the fact that most of the demand occurs in peak hours that have a high price risk. the risk-,minimizing quantity is a trade-off between a reduction of peak-hour exposures as well as an increase of exposure in the off-peak hours. Using the risk minimizing portfolio of hedging instruments the variance of cashflows could be reduced by 93%, which leads of a guaranteed profit for this group of customers, versus an uncertain profit or loss in the other cases. In the table 7 it can be seen that profiled customers reduce cashflows 95% CI to a certain profit, instead of loss/profit distribution.

The same procedure is applied to the wind producers, whose forecasted mean demand curve is flat. However, the variability of the demand and the correlation with prices is different in this case. It is shown that because of the flat forecasted demand curve, the optimal hedge is equal to the average demand. In addition, the peakload contract does not have any added value as there is no pattern in the mean forecasted demand curve. Hence, in both cases variance was reduced by 84%. The reduction leads to a guaranteed profit instead of an uncertain profit. For wind customers table 7 shows the resultant 95% confidence intervals.

Finally, the procedure was repeated for the net exposure curve which was constructed of the five customer groups. Each customer's demand was individually simulated and the result shows that a significant reduction of risk happens. The risk reduction is equal to 90% which is less than what is possible with only profiled customers, but more than only with risk. Again the difference in base- and peakload and baseload-only is equal to an additional 20% reduction. For the overall exposure such a reduction means that a cashflows' confidence interval can be reduced from -1M EUR to 350k EUR can be reduced to -400k to -300k EUR.

Note that this is the cash flow from the exposures and is not equal to the total profit to the firm. Finally, table 7 displays the 95% confidence interval for cashflows over the full year.

In general, we have shown that from a risk management perspective it is important to reduce the short exposures during peak hours. It is not necessarily so that is achieved by both peak- and baseload contracts. Hedging electricity price risk can be done with baseload contracts only, to capture 80% of the gains. Hence, this finding explains the interesting observation that peakload contracts are not heavily traded and considered illiquid. In addition, the risk-minimizing quantity is not very sensitive to deviations from the optimal baseload-only or any interchange between peak- and baseload quantities. This shows that rounding quantities to the nearest integer is a requirement that can easily be met at a relatively low cost.

These scenarios are built into a software tool that enables Anode to calculate these risk-minimizing quantity for any period and scenario that they would like to have investigated. A screenshot is shown in appendix N.

9.2 Suggestions for Further Research

This thesis has explored the management of price risk from a risk-minimizing perspective. As stated in the introduction, a lot literature focuses on the optimization of return given a risk limit. This is however, difficult to quantify for a number of reasons. First, the risk aversion of a firm is very difficult to state correctly. Further research could focus on the risk aversion of electricity traders.

In addition, it is important to investigate a better pricing model. Electricity prices are extremely hard to quantify because of the combination of stable returns and extreme spikes. A significant part of the literature is devoted to the modeling of electricity but no one model has proven to be universally applicable. Better pricing models would enable better risk management as the spikes are better captured.

Research could be put in the better prediction of demand curves. The current setting allows some simple stochasticity and correlation with the main pricing factor. Better understanding on price movements with respect to underlying demand curves would improve risk management in electricity traders as well.

The electricity market itself is also quite young. The liberalization of the European grids started 10-15 years ago, and the markets are continuously being further integrated. All market participants are still in the process of learning how the market functions and reliable price data is not yet available. In addition, because of the trend of increased production of electricity from renewable, the energy mix is being transformed and hence the price and demand patterns change accordingly. It is clear that the market is highly dynamic and that models and strategies that currently manage the risks accordingly are not necessarily the good strategies for the future.

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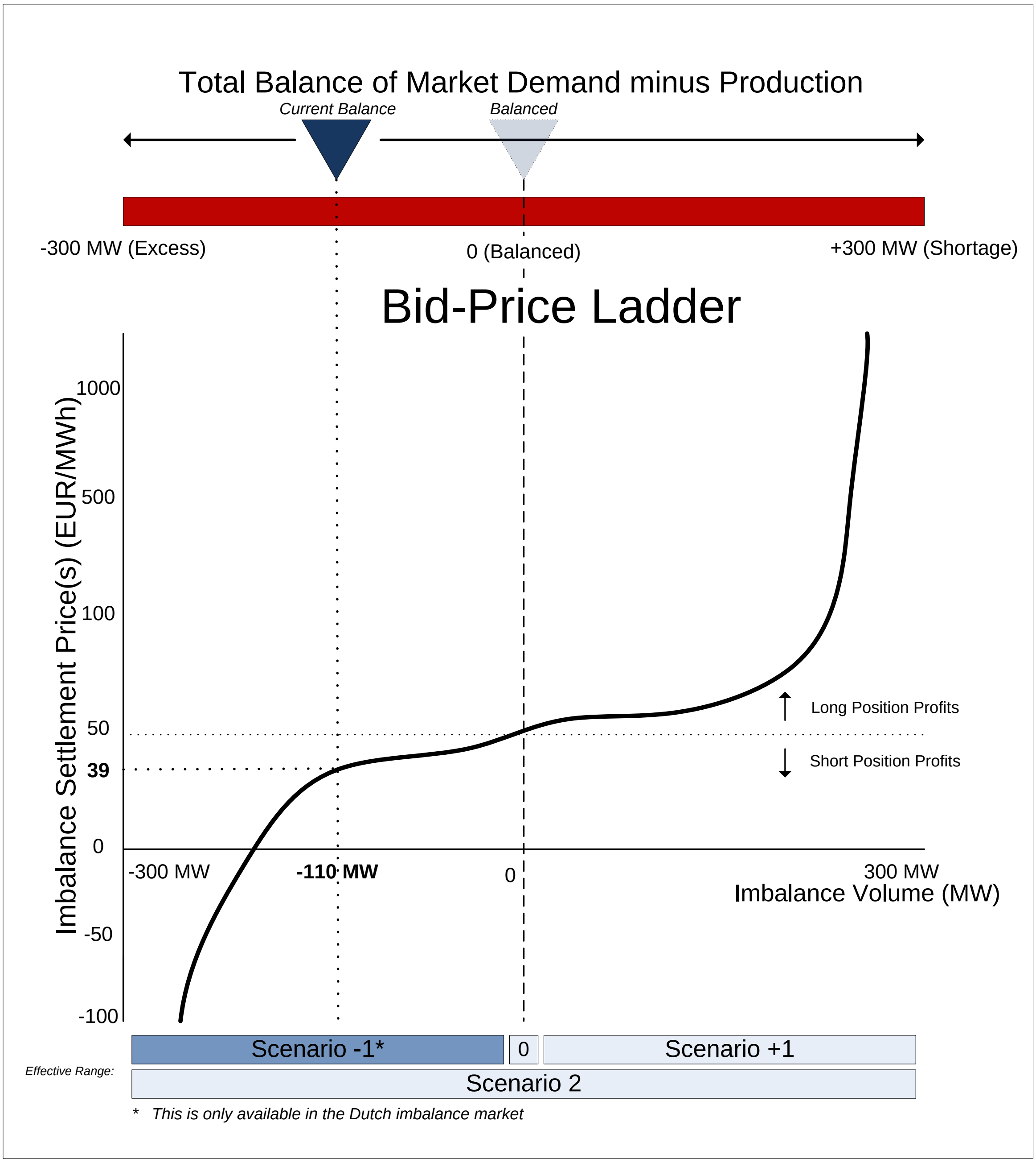
Appendix A: BID-PRICE LADDER



Appendix B: Proofs of Baseload – Deterministic Demand Formulas

Show that the expected profits are independent of Q

$$X = \sum_{i=1}^{I} D_i \mathrm{P} + (\mathrm{Q} - D_i)\tilde{S}_i - QF_{0,T}$$

$$E[X] = \sum_{h=1}^{H} QE[\tilde{S}_h] - \sum_{h=1}^{H} D_h E[\tilde{S}_h] - \sum_{h=1}^{H} QF_{0,T} - \sum_{h=1}^{H} D_h P$$

$$\sum_{h=1}^{H} QF_h - \sum_{h=1}^{H} D_h F_h - \sum_{h=1}^{H} QF_{0,T} - \sum_{h=1}^{H} D_h P$$

$$Q\left[\sum_{h=1}^{H}(F_h - F_{0,T})\right] = \sum_{h=1}^{H} D_h F_h - \sum_{h=1}^{H} D_h P$$

$$Q\left[\left(\sum_{h=1}^{H} F_h\right) - HF_{0,T}\right] = \sum_{h=1}^{H} D_h F_h - \sum_{h=1}^{H} D_h P$$

$$Q\left[\left(\sum_{h=1}^{H} F_h\right) - H\left(\frac{\sum_{h=1}^{H} F_h}{H}\right)\right] = \sum_{h=1}^{H} D_h F_h - \sum_{h=1}^{H} D_h P$$

$$Q\left[\left(\sum_{h=1}^{H} F_h\right) - \sum_{h=1}^{H} F_h\right] = \sum_{h=1}^{H} D_h F_h - \sum_{h=1}^{H} D_h P$$

$$E[X] = \sum_{h=1}^{H} D_h(\mathrm{P} - F_h)$$

Next, the derivation of the variance formula and the derivative with respect to Q.

$$Var(\tilde{X}) = \sum_{i=1}^{I}\sum_{j=1}^{I}(Q - D_i)(Q - D_j)Cov(\widetilde{S_i}, \widetilde{S_j})$$

$$Var(\tilde{X}) = \sum_{h=1}^{I}\sum_{i=1}^{I}(Q^2 - QD_i - QD_j + D_i D_j)Cov(\widetilde{S_i}, \widetilde{S_j})$$

$$\frac{dVar(\tilde{X})}{dQ} = \sum_{i=1}^{I}\sum_{j=1}^{I}(2Q - D_i - D_j)Cov(\widetilde{S_i}, \widetilde{S_j}) = 0$$

$$-2Q\sum_{i=1}^{I}\sum_{j=1}^{I} Cov(\widetilde{S_i}, \widetilde{S_j}) = \sum_{i=1}^{I}\sum_{j=1}^{I}(-D_i - D_j)Cov(\widetilde{S_i}, \widetilde{S_j})$$

$$Q = \frac{1}{2}\frac{\sum_{i=1}^{I}\sum_{j=1}^{I}(D_i + D_j)Cov(\widetilde{S_i}, \widetilde{S_j})}{\sum_{i=1}^{I}\sum_{j=1}^{I} Cov(\widetilde{S_i}, \widetilde{S_j})}$$

Appendix C: Proofs of Baseload – Stochastic Demand Formulas

$$E[X] = \sum_{i=1}^{I} \mathrm{E}[\widetilde{D}_i]\mathrm{P} + \left(\mathrm{Q} - \mathrm{E}[\widetilde{D}_i]\right)\mathrm{E}\left[\tilde{S}_i\right] - QF_{0,T}$$

$$E[X] = \sum_{i=1}^{I} \mathrm{PE}[\widetilde{D}_i] - \mathrm{E}\left[\widetilde{D}_i\tilde{S}_i\right] + \mathrm{QE}\left[\tilde{S}_i\right] - QF_{0,T}$$

$$\mathrm{E[X]} = \sum_{i=1}^{I} \mathrm{PE}[\widetilde{D}_i] + QE\left(\tilde{S}_i\right) - \left(Cov\left(\widetilde{D}_i, \tilde{S}_i\right) + \left(E\left(\widetilde{D}_i\right)E(\tilde{S}_i)\right) - QF_{0,T}\right.$$

$$\mathrm{E[X]} = \sum_{i=1}^{I} \mathrm{PE}[\widetilde{D}_i] + QE\left(\tilde{S}_i\right) - Cov\left(\widetilde{D}_i, \tilde{S}_i\right) - E\left(\widetilde{D}_i\right)E(\tilde{S}_i) - QF_{0,T}$$

$$\mathrm{E[X]} = \sum_{i=1}^{I} \mathrm{PE}[\widetilde{D}_i] + QF_i - Cov\left(\widetilde{D}_i, \widetilde{S}_i\right) - E(\tilde{S}_i)E\left(\widetilde{D}_i\right) - QF_{0,T}$$

$$\mathrm{E[X]} = \sum_{i=1}^{I} QF_i - \sum_{i=1}^{I} Cov\left(\widetilde{D}_i, \tilde{S}_i\right) - \sum_{i=1}^{I} E(\tilde{S}_i)E\left(\widetilde{D}_i\right) - \sum_{i=1}^{I} QF_{0,T}$$

$$\mathrm{E[X]} = Q\left[\sum_{i=1}^{I} F_i - F_{0,T}\right] + \sum_{i=1}^{I} \mathrm{PE}[\widetilde{D}_i] - Cov\left(\widetilde{D}_i, \tilde{S}_\mathrm{i}\right) - \sum_{i=1}^{I} E(\tilde{S}_i)E\left(\widetilde{D}_i\right)$$

$$\text{Total Expected Cost } (X) = \sum_{i=1}^{I} \mathrm{PE}[\widetilde{D}_i] - Cov\left(\widetilde{D}_i, \tilde{S}_i\right) - E(\tilde{S}_i)E\left(\widetilde{D}_i\right)$$

The derivation of the overall variance (excluding the constant term):

$$Cov\left(\sum \widetilde{X}_\iota\right) = \mathrm{Cov}\left(\sum_{i=1}^{I} \widetilde{D}_i\mathrm{P} + \mathrm{Q}\tilde{S}_i - \widetilde{D}_i\tilde{S}_i, \sum_{j=1}^{I} \widetilde{D}_j\mathrm{P} + \mathrm{Q}\tilde{S}_j - \widetilde{D}_j\tilde{S}_j\right)$$

$$\begin{aligned} Var\left(\sum \widetilde{X}_\iota\right) = \sum_{i=1}^{I}\sum_{\mathrm{j}=1}^{I} [&Q^2\mathrm{Cov}\left(\tilde{S}_i, \tilde{S}_j\right) - \mathrm{QCov}\left(\tilde{S}_i, \widetilde{D}_j\tilde{S}_j\right) + \mathrm{PQCov}\left(\tilde{S}_i, \widetilde{D}_j\right) - \mathrm{QCov}\left(\tilde{S}_\mathrm{j}, \widetilde{D}_i\tilde{S}_i\right) \\ &+ \mathrm{Cov}\left(\widetilde{D}_i\tilde{S}_i, \widetilde{D}_j\tilde{S}_\mathrm{j}\right) - \mathrm{PCov}\left(\widetilde{D}_i\tilde{S}_i, \widetilde{D}_\mathrm{j}\right) + \mathrm{PQCov}\left(\widetilde{D}_i, \tilde{S}_j\right) - \mathrm{PCov}\left(\widetilde{D}_i, \widetilde{D}_j\tilde{S}_j\right) \\ &+ \mathrm{P}^2\mathrm{Cov}(\widetilde{D}_i, \widetilde{D}_\mathrm{j}) = \end{aligned}$$

$$\text{solving the formula: } \frac{dVar\left(\sum \widetilde{X_h}\right)}{dQ} = 0$$

$$= \sum_{i=1}^{I}\sum_{\mathrm{j}=1}^{I} [2\mathrm{QCov}\left(\tilde{S}_i, \tilde{S}_j\right) - \mathrm{Cov}\left(\tilde{S}_i, \widetilde{D}_j\tilde{S}_j\right) + \mathrm{PCov}\left(\tilde{S}_i, \widetilde{D}_j\right) - \mathrm{Cov}\left(\tilde{S}_\mathrm{j}, \widetilde{D}_i\tilde{S}_i\right) + \mathrm{PCov}\left(\widetilde{D}_i, \tilde{S}_j\right)]$$

$$-\sum_{i=1}^{I}\sum_{j=1}^{I} 2QCov(\tilde{\mathrm{S}}_i, \tilde{S}_j)$$

$$= \sum_{i=1}^{I}\sum_{j=1}^{I} -Cov(\tilde{S}_i, \widetilde{D}_j\tilde{S}_j) - Cov(\widetilde{D}_i\tilde{S}_i, \tilde{S}_j) + \mathrm{PCov}(\tilde{S}_i, \widetilde{D}_j) + \mathrm{PCov}(\widetilde{D}_i, \tilde{S}_j)$$

$$Q = \frac{1}{2} \cdot \frac{\sum_{i=1}^{I}\sum_{j=1}^{I} Cov(\tilde{S}_i, \widetilde{D}_j\tilde{S}_j) + Cov(\widetilde{D}_i\tilde{S}_i, \tilde{S}_j) - \mathrm{PCov}(\tilde{S}_i, \widetilde{D}_j) - \mathrm{PCov}(\widetilde{D}_i, \tilde{S}_j)}{\sum_{i=1}^{I}\sum_{j=1}^{I} Cov(\tilde{S}_i, \tilde{S}_j)}$$

Appendix D: Proofs of Baseload and Peakload with Deterministic Demand Formulas

In order to find the optimal procurement quantities Q_B and Q_P under deterministic demand, the following notations are used, Q is replaced by ci.

$$X = \sum_{i=1}^{I} D_i \mathrm{P} + (\mathrm{Q} - D_i)\tilde{S}_i - QF_{0,T}$$

$$\sum_{i=1}^{I} D_i \mathrm{P} + c_i \tilde{S}_i - Q_B \mathrm{F}_{0,\mathrm{T,B}} - Q_P \mathrm{F}_{0,\mathrm{T,P}} \text{ where } c_i = \{ \begin{matrix} Q_B + Q_P - D_i & i \in P \\ Q_B - D_i & i \in B \end{matrix}$$

$$Cov(\tilde{X}_\iota, \tilde{X}_j) = \mathrm{Cov}\left(\sum_{i=1}^{I} D_i \mathrm{P} + c_i \tilde{S}_i - Q_B \mathrm{F}_{0,\mathrm{T,B}} - Q_P \mathrm{F}_{0,\mathrm{T,P}}, \sum_{i=1}^{I} D_i \mathrm{P} + c_j \tilde{S}_j - Q_B \mathrm{F}_{0,\mathrm{T,B}} - Q_P \mathrm{F}_{0,\mathrm{T,P}}\right)$$

$$Cov(\tilde{X}_\iota, \tilde{X}_j) = \sum_{i,j=1}^{I} c_i c_j \mathrm{Cov}(\tilde{S}_i, \tilde{S}_j)$$

To find the optimal Q, we take the two partial derivatives with respect to Q_B and Q_P, set them equal to zero, and then combine the two formulas the find the solution for Q_B

$$\frac{\partial Var(X)}{\partial Q_B} = \sum_{i,j}^{I} \frac{\partial}{\partial Q_B}[c_i c_j]\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) = \sum_{i,j}^{I}\left[\frac{\partial c_i}{\partial Q_B} c_j + c_i \frac{\partial c_j}{\partial Q_B}\right]\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) = 0$$

$$\frac{\partial c_\mathrm{i}}{\partial Q_B} = \{ \begin{matrix} 1 & i \in P \\ 1 & i \in B \end{matrix}$$

Then,

$$\frac{\partial Var(X)}{\partial Q_B} = \sum_{i,j}^{I} (c_i + c_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) = 0$$

$$\sum_{i,j \in P}^{I} (Q_B + Q_P - D_i + Q_B + Q_P - D_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) + \sum_{i,j \in B}^{I} (Q_B - D_i + Q_B - D_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) + 2\sum_{i \in P, j \in B}^{I} (Q_B + Q_P - D_i + Q_B - D_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) = 0$$

Note that the 2 is added to the PB case, since the reverse also needs to be evaluated BP

$$\sum_{i,j \in P}^{I} (2Q_B + 2Q_P - D_i - D_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) + \sum_{i,j \in B}^{I} (2Q_B - D_i - D_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) + 2\sum_{i \in P, j \in B}^{I} (2Q_B + Q_P - D_i - D_j)\mathrm{Cov}(\tilde{S}_i, \tilde{S}_j) = 0$$

$$\sum_{i,j\in P}^{I}(2Q_P - D_i - D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in B}^{I}(-D_i - D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + 2\sum_{i\in P,j\in B}^{I}(Q_P - D_i - D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) = -\sum_{i,j}^{I}2Q_B\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)$$

$$Q_B = \frac{1}{2}\frac{\left(\sum_{i,j\in P}^{I}(-2Q_P + D_i + D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in B}^{I}(D_i + D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + 2\sum_{i\in P,j\in B}^{I}(-Q_P + D_i + D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)\right)}{\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}$$

The same is then done for the QP

$$\frac{\partial Var(X)}{\partial Q_{\mathrm{P}}} = \sum_{i,j}^{I}\frac{\partial}{\partial Q_{\mathrm{P}}}[c_i c_j]\sigma_{i,j} = \sum_{i,j}^{I}\left[\frac{\partial c_i}{\partial Q_{\mathrm{P}}}c_j + c_i\frac{\partial c_j}{\partial Q_{\mathrm{P}}}\right]\sigma_{i,j} = 0$$

$$\frac{\partial c_i}{\partial Q_P} = \begin{cases}1 & i \in P\\ 0 & i \in B\end{cases}$$

Then,

$$\frac{\partial Var(X)}{\partial Q_P} = \sum_{i,j\in P}^{I}(c_i + c_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in B}^{I}(0)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + 2\sum_{i\in P,j\in B}^{I}(c_{\mathrm{j}})\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)$$
$$= \sum_{i,j\in P}^{I}(c_i + c_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + 2\sum_{i\in P,j\in B}^{I}c_j\,\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)$$

$$\sum_{i,j\in P}^{I}(Q_B + Q_P - D_i + Q_B + Q_P - D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + 2\sum_{i\in P,j\in B}^{I}(Q_B - D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) = 0$$

$$\sum_{i,j\in P}^{I}(2Q_B + 2Q_{\mathrm{P}} - D_i - D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in P,j\in B}^{I}(2Q_B - 2D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) = 0$$

$$\sum_{i,j\in P}^{I}(2Q_B - D_i - D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in P,j\in B}^{I}(2Q_B - 2D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) = -2Q_P\left(\sum_{i,j\in P}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)\right)$$

$$\frac{1}{2}\cdot\frac{\sum_{i,j\in P}^{I}(-2Q_B + D_i + D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in P,j\in B}^{I}2(-Q_B + D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}{\left(\sum_{i,j\in P}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)\right)} = Q_P$$

Now we put the formula for the optimal Q_P into the formula for the optimal QB

Because it is quite complicated, the formula is now split in parts, the first part where Q_P is substituted is here:

$$\sum_{i,j\in P}^{I}\left(-2Q_P + D_i + D_j\right)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)$$

$$= \sum_{i,j\in P}^{I}\left(-2\left(\frac{1}{2}\cdot\frac{\sum_{i,j\in P}^{I}(-2Q_B + D_i + D_j)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right) + \sum_{i\in P,j\in B}^{I} 2(-Q_B + D_j)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)}{\left(\sum_{i,j\in P}^{I}\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)\right)}\right) + D_i + D_j\right)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)$$

$$= \sum_{i,j\in P}^{I}\left(\left(\frac{\sum_{u,v\in P}^{I}(2Q_B)\sigma_{u,v} - \sum_{u,v\in P}^{I}(D_u + D_v)\sigma_{u,v} + \sum_{u\in P,v\in B}^{I}(2Q_B)\sigma_{u,v} - \sum_{u\in P,v\in B}^{I}(2D_\mathrm{v})\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}\right) + D_i + D_j\right)\sigma_{i,j}$$

$$= \sum_{i,j\in P}^{I}\left(\frac{\sum_{u,v\in P}^{I}(2Q_B)\sigma_{u,v} + \sum_{u\in P,v\in B}^{I}(2Q_B)\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)} + \frac{-\left[\sum_{u,v\in P}^{I}(D_u + D_v)\sigma_{u,v} - \sum_{u\in P,v\in B}^{I}(2D_v)\sigma_{u,v}\right]}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)} + D_i + D_j\right)\sigma_{i,j}$$

$$\sum_{i,j\in P}^{I}\left(\frac{\sum_{u,v\in P}^{I}(2Q_B)\sigma_{u,v} + \sum_{u\in P,v\in B}^{I}(2Q_B)\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}\right)\sigma_{i,j} + \sum_{i,j\in P}^{I}\left(\frac{\left[-\sum_{u,v\in P}^{I}(D_u + D_v)\sigma_{u,v} - \sum_{u\in P,v\in B}^{I}(2D_v)\sigma_{u,v}\right]}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)} + D_i + D_j\right)\sigma_{i,j}$$

The same is applied to the second term:

$$2\sum_{i\in P,j\in B}^{I}\left(-Q_P + D_i + D_j\right)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)$$

$$\sum_{i\in P,j\in B}^{I}\left(-2\left(\frac{1}{2}\cdot\frac{\sum_{i,j\in P}^{I}(-2Q_B + D_i + D_j)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right) + \sum_{i\in P,j\in B}^{I} 2(-Q_B + D_j)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)}{\left(\sum_{i,j\in P}^{I}\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)\right)}\right) + 2D_i + 2D_j\right)\sigma_{i,j}$$

$$\sum_{i\in P,j\in B}^{I}\left(\left(\frac{\sum_{u,v\in P}^{I} 2Q_B\sigma_{u,v} + \sum_{u\in P,v\in B}^{I} 2Q_B\sigma_{u,v} - \sum_{u,v\in P}^{I}(D_u + D_v)\sigma_{u,v} - 2\sum_{u\in P,v\in B}^{I}(D_\mathrm{v})\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}\right) + 2D_i + 2D_j\right)\sigma_{i,j}$$

$$2Q_B\sum_{i\in P,j\in B}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v} + \sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\sum_{u,v\in P}^{I}\sigma_{u,\mathrm{v}}}\right)\sigma_{i,j} + \sum_{i\in P,j\in B}^{I}\left(\frac{-\sum_{u,v\in P}^{I}(D_u + D_v)\sigma_{u,v} - 2\sum_{u\in P,v\in B}^{I}(D_v)\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)} + 2D_i + 2D_j\right)\sigma_{i,j}$$

Then, we take the original formula for QB and fill in the formula again of QP

$$Q_B = \frac{1}{2}\frac{\left(\sum_{i,j\in P}^{I}\left(-2Q_P + D_i + D_j\right)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right) + \sum_{i,j\in B}^{I}\left(D_i + D_j\right)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right) + 2\sum_{i\in P,j\in B}^{I}\left(-Q_P + D_i + D_j\right)\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)\right)}{\sum_{i,j}^{I}\mathrm{Cov}\left(\tilde{S}_i,\tilde{S}_j\right)}$$

$$\frac{\left(\sum_{i,j\in P}^{I}\left(\frac{\sum_{u,v\in P}^{I}(2Q_B)\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}(2Q_B)\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}\right)\sigma_{i,j}+\sum_{i,j\in P}^{I}\left(\frac{\left[-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u,v}-\sum_{u\in P,v\in B}^{I}(2D_v)\sigma_{u,v}\right]}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}+D_i+D_j\right)\sigma_{i,j}+\sum_{i,j\in B}^{I}(D_i+D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)+2Q_B\sum_{i\in P,j\in B}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\sum_{u,v\in P}^{I}\sigma_{u,\mathrm{v}}}\right)\sigma_{i,j}+\sum_{i\in P,j\in B}^{I}\left(\frac{-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u}\ldots}{\left(\sum_{u}^{I}\ldots\right.}\right.}{2\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}$$

$$Q_B-2Q_B\frac{\sum_{i,j\in P}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}\right)\sigma_{i,j}}{2\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}-2Q_B\frac{\sum_{i\in P,j\in B}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\sum_{u,v\in P}^{I}\sigma_{u,\mathrm{v}}}\right)\sigma_{i,j}}{2\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}$$
$$=\frac{\sum_{i,j\in P}^{I}\left(\frac{\left[-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u,v}-\sum_{u\in P,v\in B}^{I}(2D_v)\sigma_{u,v}\right]}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}+D_i+D_j\right)\sigma_{i,j}+\sum_{i,j\in B}^{I}(D_i+D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)+\sum_{i\in P,j\in B}^{I}\left(\frac{-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u,v}-2\sum_{u\in P,v\in B}^{I}(D_v)\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}+2D_i+2D_j\right)\sigma_{i,j}}{2\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}$$

$$Q_B-Q_B\left(\frac{\sum_{i,j\in P}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}\right)\sigma_{i,j}+\sum_{i\in P,j\in B}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\sum_{u,v\in P}^{I}\sigma_{u,\mathrm{v}}}\right)\sigma_{i,j}}{\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}\right)$$
$$=\frac{\sum_{i,j\in P}^{I}\left(\frac{\left[-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u,v}-\sum_{u\in P,v\in B}^{I}(2D_v)\sigma_{u,v}\right]}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}+D_i+D_j\right)\sigma_{i,j}+\sum_{i,j\in B}^{I}(D_i+D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)+\sum_{i\in P,j\in B}^{I}\left(\frac{-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u,v}-2\sum_{u\in P,v\in B}^{I}(D_v)\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}+2D_i+2D_j\right)\sigma_{i,j}}{2\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}$$

Use that Qb(1-2x)=y then Qb = y/(1-2x)

$$Q_B=\frac{\left\{\sum_{i,j\in P}^{I}\left(\frac{\left[-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u,v}-\sum_{u\in P,v\in B}^{I}(2D_v)\sigma_{u,v}\right]}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}+D_i+D_j\right)\sigma_{i,j}+\sum_{i,j\in B}^{I}(D_i+D_j)\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)+\sum_{i\in P,j\in B}^{I}\left(\frac{-\sum_{u,v\in P}^{I}(D_u+D_v)\sigma_{u,v}-2\sum_{u\in P,v\in B}^{I}(D_v)\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}+2D_i+2D_j\right)\sigma_{i,j}\right\}\left(\frac{1}{\sum_{i,j}^{I}2\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}\right)}{1-\left(\frac{\sum_{i,j\in P}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\left(\sum_{u,v\in P}^{I}\sigma_{u,v}\right)}\right)\sigma_{i,j}+\sum_{i\in P,j\in B}^{I}\left(\frac{\sum_{u,v\in P}^{I}\sigma_{u,v}+\sum_{u\in P,v\in B}^{I}\sigma_{u,v}}{\sum_{u,v\in P}^{I}\sigma_{u,\mathrm{v}}}\right)\sigma_{i,j}}{\sum_{i,j}^{I}\mathrm{Cov}(\tilde{S}_i,\tilde{S}_j)}\right)}$$

Appendix E: Proofs of Baseload and Peakload with Stochastic Demand Formulas

Start with the same problem set as in Appendix 9, however now demand is stochastic and hence covariance between demand and prices needs to be taken into account. We try to find the optimal Q_B and Q_P.

$$X = \sum_{i=1}^{I} D_i \mathrm{P} + (\mathrm{Q} - D_i)\tilde{S}_i - QF_{0,T}$$

$$\sum_{i=1}^{I} c_i\tilde{S}_i - \widetilde{D}_i\tilde{S}_i - Q_B\mathrm{F}_{0,\mathrm{T,B}} - Q_P\mathrm{F}_{0,\mathrm{T,P}} + \widetilde{D}_i\mathrm{P} \quad \text{where } c_i = \begin{cases} Q_B + Q_P & i \in P \\ Q_B & i \in B \end{cases}$$

$$\begin{aligned} Var\left(\sum \tilde{X}\right) = \sum_{i=1}^{I}\sum_{j=1}^{I} & c_ic_jCov(\tilde{S}_i,\tilde{S}_j) - c_iCov(\tilde{S}_i,\tilde{S}_j\widetilde{D}_j) - c_jCov(\tilde{S}_j,\tilde{S}_i\widetilde{D}_i) \\ & + Cov(\tilde{S}_i\widetilde{D}_i,\tilde{S}_j\widetilde{D}_j) + \mathrm{P}c_j\mathrm{Cov}(\widetilde{D}_i,\tilde{S}_j) + \mathrm{P}c_i\mathrm{Cov}(\widetilde{D}_j,\tilde{S}_i) - PCov(\widetilde{D}_i,\tilde{S}_j\widetilde{D}_j) \\ & - PCov(\widetilde{D}_\mathrm{j},\tilde{S}_\mathrm{i}\widetilde{D}_\mathrm{i}) + \mathrm{P}^2\mathrm{Cov}(\widetilde{D}_\mathrm{i},\widetilde{D}_\mathrm{j}) \end{aligned}$$

To find the optimal Q, we take the two partial derivatives with respect to Q_B and Q_P, set them equal to zero, and then combine the two formulas the find the solution for Q_B. Below excludes all terms of where the derivative is equal to zero.

$$\begin{aligned} \frac{\partial Var(X)}{\partial Q_B} &= \sum_{i,j}^{I} \frac{\partial}{\partial Q_B}[c_ic_j]Cov(\tilde{S}_i,\tilde{S}_j) - \frac{\partial}{\partial Q_B}c_iCov(\tilde{S}_i,\tilde{S}_j\widetilde{D}_j) - \frac{\partial}{\partial Q_B}c_iCov(\tilde{S}_j,\tilde{S}_i\widetilde{D}_i) \\ &\quad + \frac{\partial}{\partial Q_B}\mathrm{P}c_i\mathrm{Cov}(\widetilde{D}_j,\tilde{S}_i) + \frac{\partial}{\partial Q_B}\mathrm{P}c_j\mathrm{Cov}(\widetilde{D}_i,\tilde{S}_j) \\ &= \sum_{i,j}^{I}\left[\frac{\partial c_i}{\partial Q_B}c_j + c_i\frac{\partial c_j}{\partial Q_B}\right]Cov(\tilde{S}_i,\tilde{S}_j) - \frac{\partial}{\partial Q_B}c_iCov(\tilde{S}_i,\tilde{S}_j\widetilde{D}_j) \\ &\quad - \frac{\partial}{\partial Q_B}c_jCov(\tilde{S}_j,\tilde{S}_i\widetilde{D}_i) + + \frac{\partial}{\partial Q_B}\mathrm{P}c_i\mathrm{Cov}(\widetilde{D}_j,\tilde{S}_i) + \frac{\partial}{\partial Q_B}\mathrm{P}c_j\mathrm{Cov}(\widetilde{D}_i,\tilde{S}_j) = 0 \end{aligned}$$

$$\frac{\partial c_i}{\partial Q_B} = \begin{cases} 1 & i \in P \\ 1 & i \in B \end{cases}$$

Then,

$$\begin{aligned} \frac{\partial Var(X)}{\partial Q_B} = \sum_{i,j}^{I}&[c_j + c_i]Cov(\tilde{S}_i,\tilde{S}_j) - Cov(\tilde{S}_i,\tilde{S}_j\widetilde{D}_j) - Cov(\tilde{S}_j,\tilde{S}_i\widetilde{D}_i) + \mathrm{PCov}(\widetilde{D}_j,\tilde{S}_i) \\ & + \mathrm{PCov}(\widetilde{D}_i,\tilde{S}_j) = 0 \end{aligned}$$

$$\sum_{i,j\in B}^{I}[Q_B + Q_B]Cov(\tilde{S}_i,\tilde{S}_j) - Cov(\tilde{S}_i,\tilde{S}_j\widetilde{D}_j) - Cov(\tilde{S}_j,\tilde{S}_i\widetilde{D}_i) + \mathrm{PCov}(\widetilde{D}_j,\tilde{S}_i) + \mathrm{PCov}(\widetilde{D}_i,\tilde{S}_j)$$

$$+2\sum_{i\in B,j\in P}^{I}[Q_B + Q_B + Q_P]Cov(\tilde{S}_i,\tilde{\mathrm{S}}_j) - Cov(\tilde{S}_i,\tilde{S}_j\widetilde{D}_j) - Cov(\tilde{S}_j,\tilde{S}_i\widetilde{D}_i) + \mathrm{PCov}(\widetilde{D}_j,\tilde{S}_i) + \mathrm{PCov}(\widetilde{D}_i,\tilde{S}_j)$$

$$+\sum_{i,j\in P}^{I}[Q_B+Q_B+Q_P+Q_P]Cov(\tilde{S}_i,\tilde{S}_j)-Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j)-Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i)+\mathrm{PCov}(\tilde{D}_j,\tilde{S}_i)+\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j)=0$$

$$\sum_{i,j\in B}^{I}[2Q_B]Cov(\tilde{S}_i,\tilde{S}_j)+2\sum_{i\in B,j\in P}^{I}[2Q_B+Q_P]Cov(\tilde{S}_i,\tilde{S}_j)+\sum_{i,j\in P}^{I}[2Q_B+2Q_P]Cov(\tilde{S}_i,\tilde{S}_j)$$
$$=\sum_{i,j}^{I}Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j)+Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i)-\mathrm{PCov}(\tilde{D}_j,\tilde{S}_i)-\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j)$$

$$2Q_B\sum_{i,j}^{I}Cov(\tilde{S}_i,\tilde{S}_j)+2Q_P\left[\sum_{i\in B,j\in P}^{I}Cov(\tilde{S}_i,\tilde{S}_j)+\sum_{i,j\in P}^{I}Cov(\tilde{S}_i,\tilde{S}_j)\right]$$
$$=\sum_{i,j}^{I}Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j)+Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i)-\mathrm{PCov}(\tilde{D}_j,\tilde{S}_i)-\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j)$$

$$Q_B=\frac{1}{2}\frac{\left(\sum_{i,j}^{I}Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j)+Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i)-\mathrm{PCov}(\tilde{D}_j,\tilde{S}_i)-\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j)-2Q_P\left[\sum_{i\in B,j\in P}^{I}Cov(\tilde{S}_i,\tilde{S}_j)+\sum_{i,j\in P}^{I}Cov(\tilde{S}_i,\tilde{S}_j)\right]\right)}{\sum_{i,j}^{I}Cov(\tilde{S}_i,\tilde{S}_j)}$$

Now the same derivation is done to find the equation for the optimal Q_P.

$$\frac{\partial Var(X)}{\partial Q_P}=\sum_{i,j}^{I}\frac{\partial}{\partial Q_P}[c_ic_j]Cov(\tilde{S}_i,\tilde{S}_j)-\frac{\partial}{\partial Q_P}c_iCov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j)-\frac{\partial}{\partial Q_P}c_jCov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i)$$
$$+\frac{\partial}{\partial Q_P}\mathrm{P}c_i\mathrm{Cov}(\tilde{D}_j,\tilde{S}_i)+\frac{\partial}{\partial Q_P}\mathrm{P}c_j\mathrm{Cov}(\tilde{D}_i,\tilde{S}_j)$$
$$=\sum_{i,j}^{I}\left[\frac{\partial c_i}{\partial Q_P}c_j+c_i\frac{\partial c_j}{\partial Q_P}\right]Cov(\tilde{S}_i,\tilde{S}_j)-\frac{\partial}{\partial Q_p}c_iCov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j)$$
$$-\frac{\partial}{\partial Q_p}c_jCov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i)+\frac{\partial}{\partial Q_P}\mathrm{P}c_i\mathrm{Cov}(\tilde{D}_j,\tilde{S}_i)+\frac{\partial}{\partial Q_P}\mathrm{P}c_j\mathrm{Cov}(\tilde{D}_i,\tilde{S}_j)=0$$

$$\frac{\partial c_i}{\partial Q_P}=\begin{cases}1 & i\in P\\ 0 & i\in B\end{cases}$$

Then multiply the iBP term because the reverse is also included (it gets evaluated two times)

$$\sum_{i\in B,j\in P}^{I}[2Q_B]Cov(\tilde{S}_i,\tilde{S}_j) - 2Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) + 2\mathrm{PCov}(\tilde{S}_j,\tilde{D}_i) + \sum_{i,j\in P}^{I}[Q_B + Q_B + Q_P + Q_P]Cov(\tilde{S}_i,\tilde{S}_j) - Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j) - Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) + \mathrm{PCov}(\tilde{D}_j,\tilde{S}_i) + \mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) = 0$$

$$\sum_{i,j\in P}^{I}[2Q_B + 2Q_P]Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in B,j\in P}^{I}[2Q_B]Cov(\tilde{S}_i,\tilde{S}_j) = \sum_{i\in B,j\in P}^{I} 2Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - 2\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j) + Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - \mathrm{PCov}(\tilde{D}_j,\tilde{S}_i) - \mathrm{PCov}(\tilde{D}_i,\tilde{S}_j)$$

$$2Q_B\left(\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right) + 2Q_P\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) = \sum_{i\in B,j\in P}^{I} 2Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - 2\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j) + Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - \mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) - \mathrm{PCov}(\tilde{D}_j,\tilde{S}_i)$$

$$Q_P = \frac{1}{2}\frac{\sum_{i\in B,j\in P}^{I} 2Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - 2\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I}[Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j) + Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - \mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) - \mathrm{PCov}(\tilde{D}_j,\tilde{S}_i)] - 2Q_B\left(\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right)}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}$$

Now combine the two expressions: replace

$$W_{BP} = \sum_{i\in B,j\in P}^{I} 2Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - 2\mathrm{PCov}(\tilde{D}_i,\tilde{S}_j)$$

$$W_{PP} = \sum_{i,j\in P}^{I}\left[Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j) + Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - \mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) - \mathrm{PCov}(\tilde{D}_j,\tilde{S}_i)\right]$$

$$W_{MM} = \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)$$

$$\mathrm{Q_P} = \frac{1}{2}\frac{W_{BP} + W_{PP} - 2Q_B[W_{MM}]}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}$$

$$Q_B = \frac{1}{2}\frac{\left(\sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j) + Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - \mathrm{PCov}(\tilde{D}_j,\tilde{S}_i) - \mathrm{PCov}(\tilde{D}_i,\tilde{S}_j) - 2\left(\frac{1}{2}\frac{W_{BP} + W_{PP} - 2Q_B[W_{MM}]}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}\right)\left[\sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right]\right)}{\sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}$$

$$V = \sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j\tilde{D}_j) + Cov(\tilde{S}_j,\tilde{S}_i\tilde{D}_i) - \mathrm{PCov}(\tilde{D}_j,\tilde{S}_i) - \mathrm{PCov}(\tilde{D}_i,\tilde{S}_j)$$

$$Q_B = \frac{\mathrm{V}}{2\sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j)} + \left(\frac{-W_{BP} - W_{PP}}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)} + \frac{2Q_B[W_{MM}]}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}\right)\cdot\frac{\left[\sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right]}{2\sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}$$

$$Q_B - 2Q_B\left\{\frac{[W_{MM}]}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}\cdot\frac{\left[\sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right]}{\sum_{i,j}^{I} 2Cov(\tilde{S}_i,\tilde{S}_j)}\right\} = \frac{V}{2\sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j)} + \left(\frac{-W_{BP} - W_{PP}}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}\right)\cdot\frac{\left[\sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right]}{\sum_{i,j}^{I} 2Cov(\tilde{S}_i,\tilde{S}_j)}$$

Replace $Q_B - 2Q_B\{\ldots\}$ with $Q_B(1 - 2\{\ldots\}) and\ divide\ trough\ that$

$$Q_B = \frac{\left[\frac{V}{2\sum_{i,j}^{I} Cov(\tilde{S}_i,\tilde{S}_j)} + \left(\frac{-W_{BP} - W_{PP}}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}\right)\cdot\frac{\left[\sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right]}{\sum_{i,j}^{I} 2Cov(\tilde{S}_i,\tilde{S}_j)}\right]}{1 - 2\left\{\frac{[W_{MM}]}{\sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)}\cdot\frac{\left[\sum_{i\in B,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j) + \sum_{i,j\in P}^{I} Cov(\tilde{S}_i,\tilde{S}_j)\right]}{\sum_{i,j}^{I} 2Cov(\tilde{S}_i,\tilde{S}_j)}\right\}}$$

$$Q_B = \frac{\left[\frac{\sum_{i,j}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j \widetilde{D}_j\right) + Cov\left(\tilde{S}_j, \tilde{S}_i \widetilde{D}_i\right) - \mathrm{PCov}\left(\widetilde{D}_j, \tilde{S}_i\right) - \mathrm{PCov}\left(\widetilde{D}_i, \tilde{S}_j\right)}{2\sum_{i,j}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right)} + \left(\frac{\sum_{i \in B, j \in P}^{I} -2Cov\left(\tilde{S}_j, \tilde{S}_i \widetilde{D}_i\right) + 2\mathrm{PCov}\left(\widetilde{D}_i, \tilde{S}_j\right) + \sum_{i,j \in P}^{I}\left[-Cov\left(\tilde{S}_i, \tilde{S}_j \widetilde{D}_j\right) - Cov\left(\tilde{S}_j, \tilde{S}_i \widetilde{D}_i\right) + \mathrm{PCov}\left(\widetilde{D}_i, \tilde{S}_j\right) + \mathrm{PCov}\left(\widetilde{D}_j, \tilde{S}_i\right)\right]}{\sum_{i,j \in P}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right)}\right) \cdot \frac{\left[\sum_{i \in B, j \in P}^{I} Cov\left(\tilde{S}_i, \right.\right.}{\sum_{i,j}^{I} 2}}{1 - 2\left\{\frac{\left[\sum_{i,j \in P}^{\mathrm{I}} Cov\left(\tilde{S}_i, \tilde{S}_j\right) + \sum_{i \in B, j \in P}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right)\right]}{\sum_{i,j \in P}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right)} \cdot \frac{\left[\sum_{i \in B, j \in P}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right) + \sum_{i,j \in P}^{I} Cov\left(\tilde{S}_i, \tilde{S}_j\right)\right]}{\sum_{i,j}^{I} 2Cov\left(\tilde{S}_i, \tilde{S}_j\right)}\right\}}$$

Appendix F: Available contracts on Timeline

Start Month	Month	Quarter	Year	Periods for Algorithm
January	JAN-FEB-MRT	JAN-MRT APR-JUN JUL-SEP	JAN/DEC 11	[JAN] [FEB] [MRT] [APR-JUN] [JUL-SEP] [OCT-DEC]
February	FEB-MRT-APR	APR-JUN JUL-SEP OCT-DEC	JAN/DEC 12	[FEB] [MRT] [APR] [MAY-JUN] [JUL-SEP] [OCT-DEC] [JAN]
March	MRT-APR-MAY	APR-JUN JUL-SEP OCT-DEC	JAN/DEC 12	[MRT] [APR][MAY] [JUN] [JUL-SEP] [OCT-DEC][JAN-FEB]
April	APR-MAY-JUN	APR-JUN JUL-SEP OCT-DEC	JAN/DEC 12	[APR] [MAY] [JUN] [JUL-SEP] [OCT-DEC] [JAN-MRT]
May	MAY-JUN-JUL	JUL-SEP OCT-DEC JAN-MRT	JAN/DEC 12	[MAY] [JUN] [JUL] [AUG-SEP][OCT-DEC] [JAN-MRT] [APR]
June	JUN-JUL-AUG	JUL-SEP OCT-DEC JAN-MRT	JAN/DEC 12	[JUN] [JUL] [AUG] [SEP] [OCT-DEC] [JAN-MRT] [APR-MAY]
July	JUL-AUG-SEP	JUL-SEP OCT-DEC JAN-MRT	JAN/DEC 12	[JUL] [AUG] [SEP] [OCT-DEC] [JAN-MRT] [APR-JUN]
August	AUG-SEP-OCT	OCT-DEC JAN-MRT APR-JUN	JAN/DEC 12	[AUG] [SEP] [OCT] [NOV-DEC] [JAN-MRT] [APR-JUN] [JUL]
September	SEP-OCT-NOV	OCT-DEC JAN-MRT APR-JUN	JAN/DEC 12	[SEP] [OCT] [NOV] [DEC] [JAN-MRT] [APR-JUN] [JUL-AUG]
October	OCT-NOV-DEC	OCT-DEC JAN-MRT APR-JUN	JAN/DEC 12	[OCT] [NOV] [DEC] [JAN-MRT] [APR-JUN] [JUL-AUG]
November	NOV-DEC-JAN	JAN-MRT APR-JUN JUL-SEP	JAN/DEC 12	[NOV] [DEC] [JAN] [FEB-MRT] [APR-JUN] [JUL-SEP] [OCT]
December	DEC-JAN-	JAN-MRT	JAN/DEC 12	[DEC] [JAN] [FEB]

[redacted]	FEB	APR-JUN JUL-SEP	[MRT] [APR-JUN] [JUL-SEP] [OCT-NOV]

Table 1: The available contracts for hedging purposes

Appendix G: Graphs of Profiles Demands

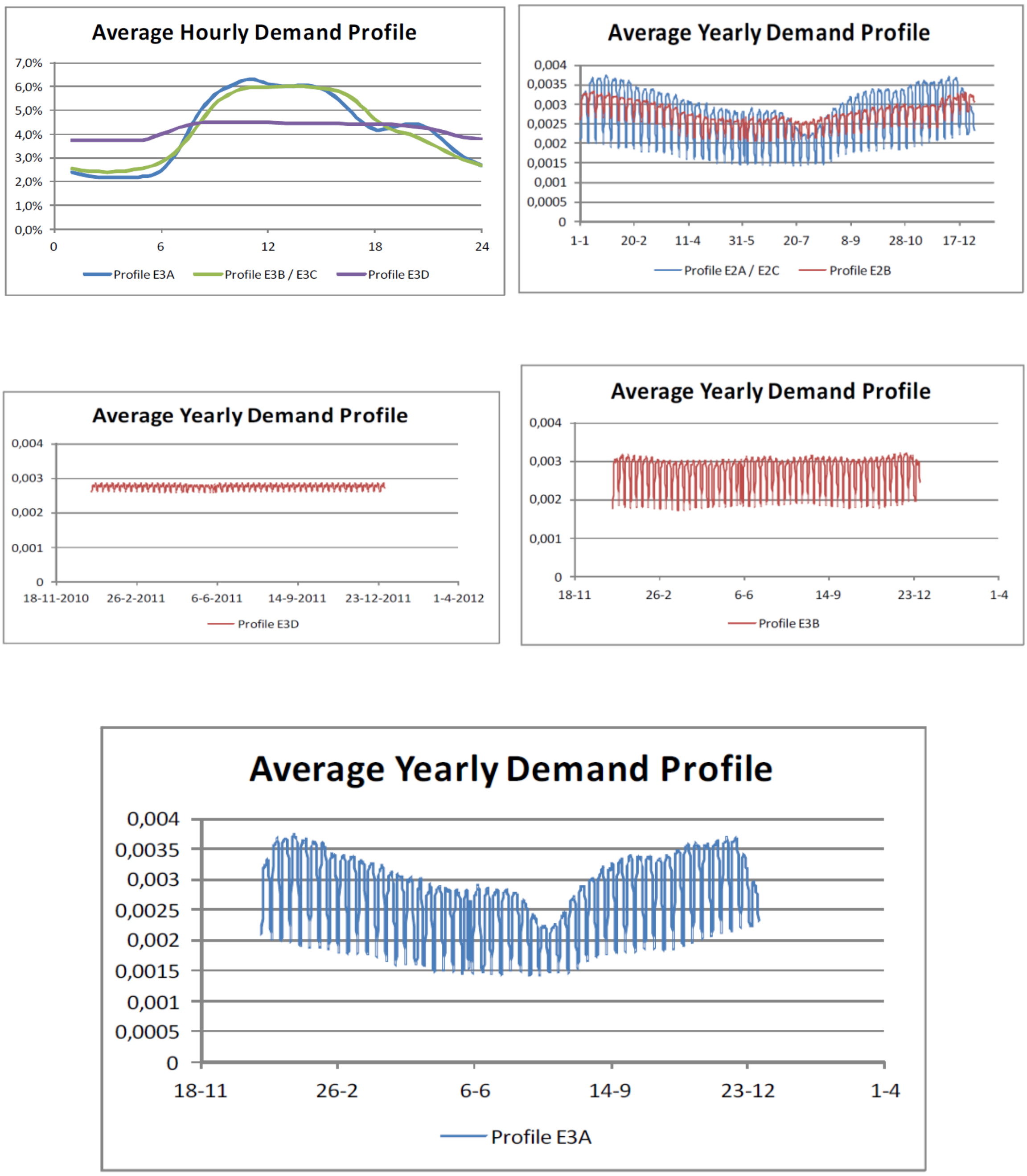


Figure 36: The demand profile of large profiled commercial customers

Appendix H: Summary Statistics of Electricity APX prices

stats	hour1	hour2	hour3	hour4	hour5	hour6	hour7	hour8	hour9	hour10	hour11	hour12
mean	38.52	34.01	30.65	27.20	26.55	30.85	37.42	50.26	56.65	64.04	68.45	74.30
sd	12.59	11.85	11.75	11.65	11.76	12.50	16.13	24.01	26.12	33.58	40.36	52.45
median	36.83	32.49	29.93	26.80	26.35	30.10	37.33	48.28	53.00	57.25	60.10	63.06
min	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	1.49	7.81	9.31
max	200.97	95.57	85.12	79.93	75.57	75.00	90.00	250.00	260.20	500.00	800.00	950.00
range	200.96	95.56	85.11	79.92	75.56	74.99	89.99	249.99	260.19	498.51	792.19	940.69
kurtosis	14.12	3.12	2.94	2.81	2.76	2.86	3.09	5.86	6.80	26.38	68.77	107.75
skewness	1.29	0.32	0.22	0.22	0.20	0.10	0.14	0.86	1.25	3.15	5.57	7.85
N	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438

stats	hour13	hour14	hour15	hour16	hour17	hour18	hour19	hour20	hour21	hour22	hour23	hour24
mean	65.44	63.68	60.07	55.47	55.09	65.67	66.70	61.60	56.83	50.05	48.39	43.11
sd	32.66	36.02	37.96	34.10	37.53	61.68	49.92	30.07	23.67	15.47	13.86	12.85
median	59.41	56.39	53.20	50.01	49.61	52.73	55.00	54.50	52.37	47.57	46.21	41.00
min	21.70	14.01	9.18	8.18	5.98	9.10	19.91	18.94	20.00	20.97	19.90	10.00
max	499.00	520.01	950.00	950.00	988.00	1,000.12	988.00	500.12	500.00	175.01	125.00	145.00
range	477.30	506.00	940.82	941.82	982.02	991.02	968.09	481.18	480.00	154.04	105.10	135.00
kurtosis	53.16	52.30	157.54	221.17	294.61	73.77	84.99	28.32	83.81	6.47	3.90	5.58
skewness	5.02	5.20	8.73	10.23	12.87	7.07	6.77	3.19	5.44	1.18	0.87	1.03
N	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438	2,438

Appendix I: PCA results

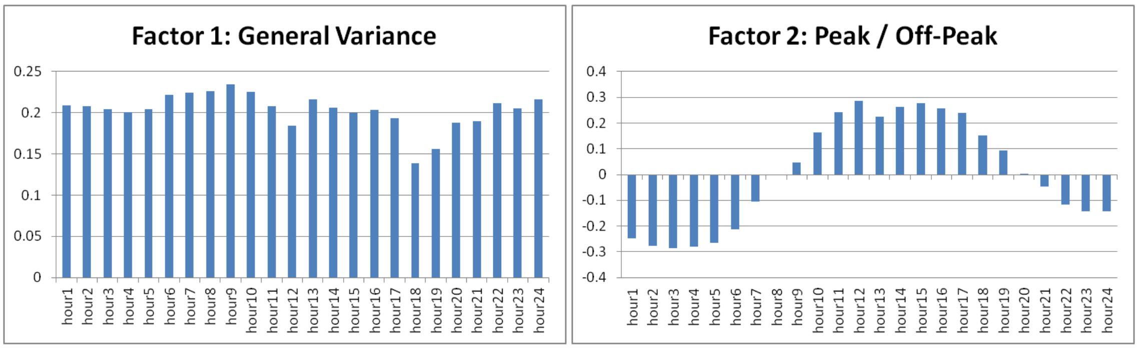


Figure 37: Factor loadings for the first and second factor (factor 1 left, factor 2 right)

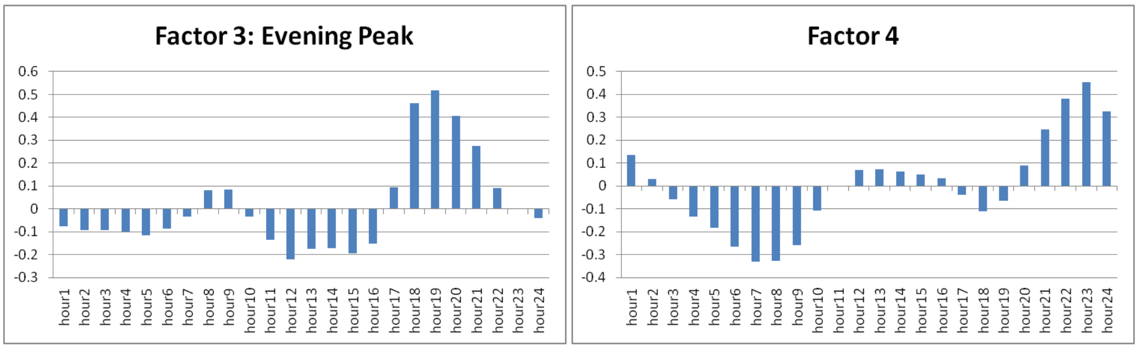


Figure 38: Factor loadings of factor 3 (left) and factor 4 (right)

PCA on PRICES

Component	Eigenvalue	Difference Eigenvalue	Proportion Variance	Cumulative Variance
Comp1	12.09	8.20	0.50	0.50
Comp2	3.89	1.96	0.16	0.67
Comp3	1.94	0.83	0.08	0.75
Comp4	1.10	0.28	0.05	0.79
Comp5	0.82	0.07	0.03	0.83
Comp6	0.75	0.16	0.03	0.86

Table 9: The Eigenvalues and other statistics from the PCA analysis of electricity price residuals

Appendix J: Results from Verification of Model

Principal Components Analysis on the Residuals

Component	Eigenvalue	Difference	Proportion	Cumulative
Comp1	1.19202	0.038006	0.0497	0.0497
Comp2	1.15402	0.0228038	0.0481	0.0978
Comp3	1.13121	0.0127058	0.0471	0.1449
Comp4	1.11851	0.00597341	0.0466	0.1915
Comp5	1.11253	0.0287357	0.0464	0.2378
Comp6	1.0838	0.00665552	0.0452	0.283
Comp7	1.07714	0.0170144	0.0449	0.3279
Comp8	1.06013	0.0156297	0.0442	0.3721
Comp9	1.0445	0.0201277	0.0435	0.4156
Comp10	1.02437	0.0191054	0.0427	0.4583
Comp11	1.00527	0.00718385	0.0419	0.5001
Comp12	0.998082	0.0200722	0.0416	0.5417
Comp13	0.978009	0.00263934	0.0408	0.5825
Comp14	0.97537	0.0172614	0.0406	0.6231
Comp15	0.958109	0.00124042	0.0399	0.663
Comp16	0.956868	0.00721221	0.0399	0.7029
Comp17	0.949656	0.0194379	0.0396	0.7425
Comp18	0.930218	0.011071	0.0388	0.7812
Comp19	0.919147	0.0132898	0.0383	0.8195
Comp20	0.905857	0.0178625	0.0377	0.8573
Comp21	0.887995	0.0276995	0.037	0.8943
Comp22	0.860296	0.0124392	0.0358	0.9301
Comp23	0.847856	0.0188196	0.0353	0.9655
Comp24	0.829037	.	0.0345	1

Dummy Variable	Description	Number
Year (2005-2010)	Captures yearly average electricity price	5
Month (Jan-Dec)	Captures the monthly price deviations which are caused by seasons and holiday (august)	12
Week (Mon-Sun)	Daily variations caused by general working/offwork and general utilization	7
HolidaysWk Wknd	An indicator dummy that captures variation caused by holiday. The effect of holidays probably differs based on whether it is a weekday or weekendday on which it occurs	2

Table 10: The variables used in the multiple regression

Standard Regression on HourPrice

Source	SS	df	MS
Model	28,569,310.30	203.00	140,735.52
Residual	38,344,627.20	58,308.00	657.62
Total	66,913,937.60	58,511.00	1,143.61

Number of obs	=	58512
F(203, 58308)	=	214.01
Prob > F	=	0
R-squared	=	0.427
Adj R-squared	=	**0.425**
Root MSE	=	25.644

Table 11: Results from the Multiple Regression

Appendix K: Other Distributions

Variable	Base-Deterministic	Base + Peak - Deterministic	Base-Stochastic	Base + Peak Stochastic
Baseload	13.70	10.86	13.73	11.16
Peakload	-	5.16	-	4.57

Again it can be seen that the stochastic model performs quite similar to the deterministic in the baseload only situation. Introducing stochasticity however, results in a change of 0.5MW. This looks like a relative small quantity, however, this implies a significant change in cashflows and risk reduction. Again the numbers are around the average of 13.

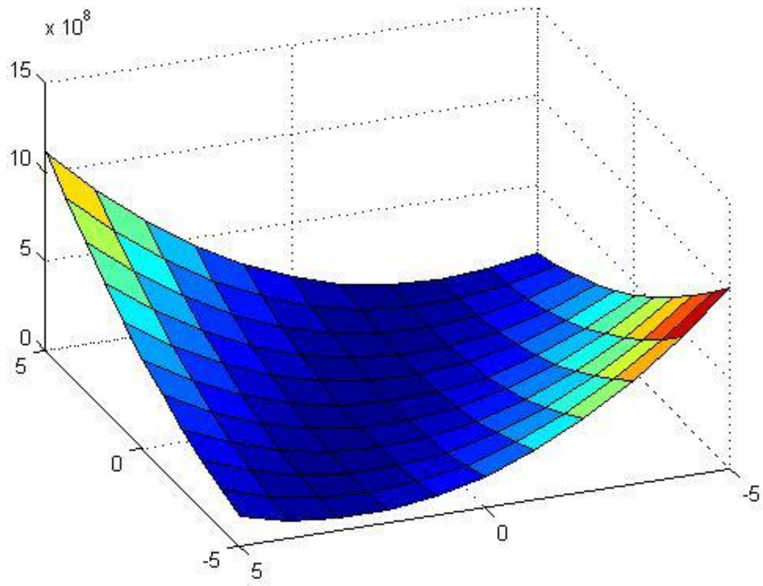
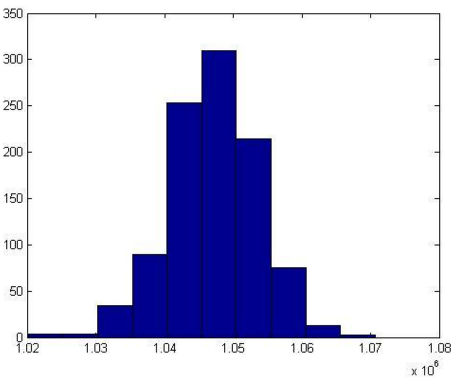


Figure 1: The surface reflecting the variance of cash flows for different hedge quantities (Student T)

The figure 6 shows that there is a plateau where the risk does not increase significantly between the switching between peak- and baseload contracts. However, deviations in the absolute amount will result in significant risk additions.



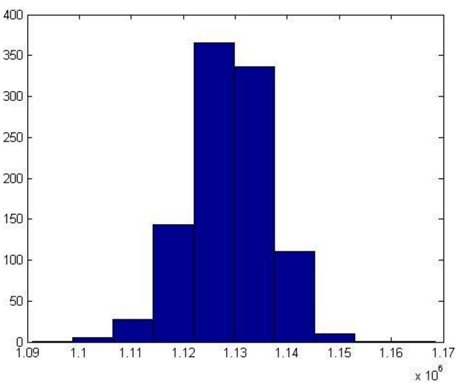


Figure 2: The Cashflows with Base- and Peakload hedge (T) Figure 3: The cashflows with base-only hedge

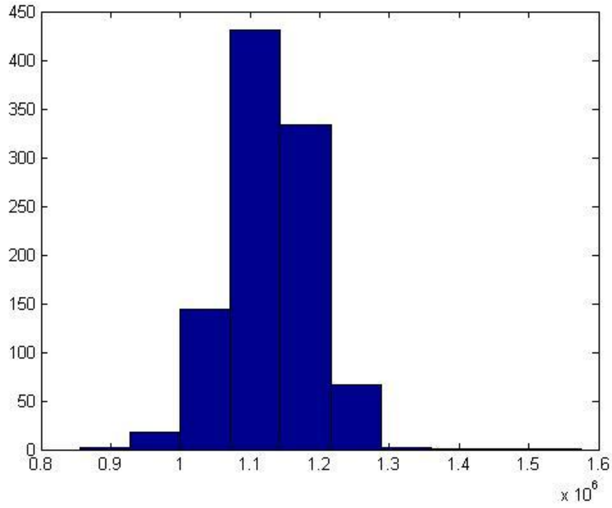


Figure 4: The Cashflow composition without a hedge (T)

Again it is clear that hedging reduces the risk significantly. The numbers are quite similar even though the distribution behind the innovations was changed. The seed has not been changed which results in a fair comparison between the three distribution models.

Variable	Base-Deterministic	Base + Peak - Deterministic	Base-Stochastic	Base + Peak Stochastic
Baseload	13.74	11.00	13.73	11.2395
Peakload	-	4.904	-	4.45

The Normal distribution was chosen for the final model. Again the numbers are very similar to the previous two models. The baseload-only model shows similar results, with or without stochasticity in demand. However, in this case the base- and peakload models are closer to each other.

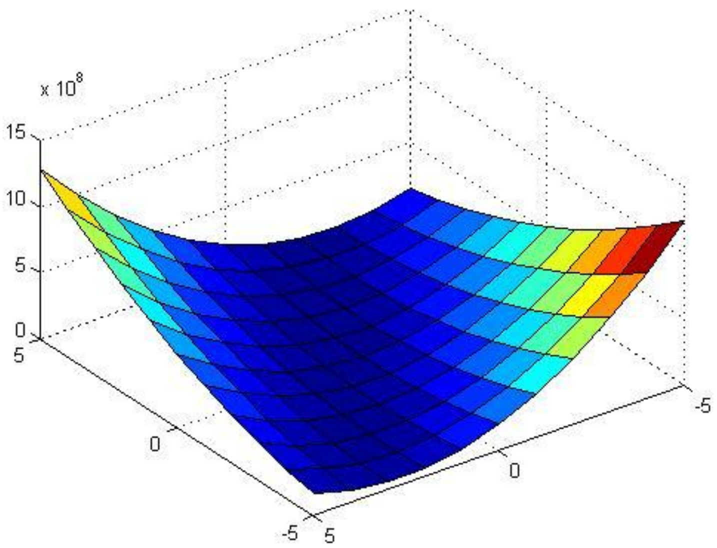


Figure 5: The surface reflecting the variance of cash flows for different hedge quantities (Normal)

The same plateau can be seen, where the switching of units from baseload to peakload or vice versa does not influence overall volatility that much.

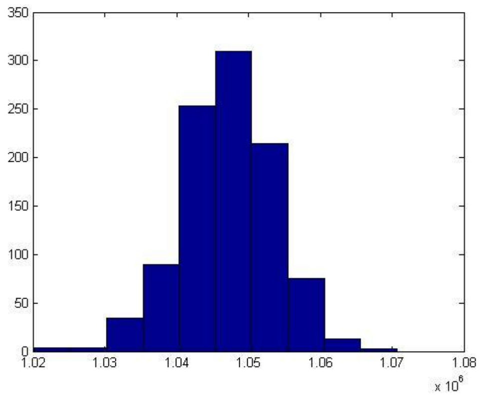


Figure 6: Base-only Cash Flow Diagram

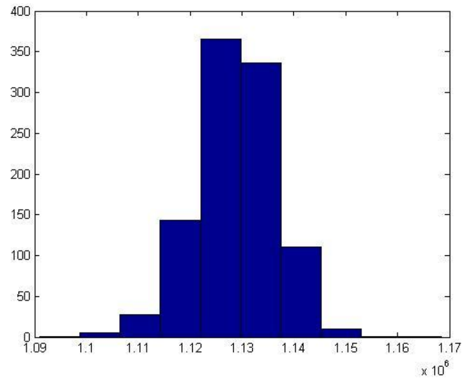


Figure 7: Optimal Cash Flow Histogram (Normal)

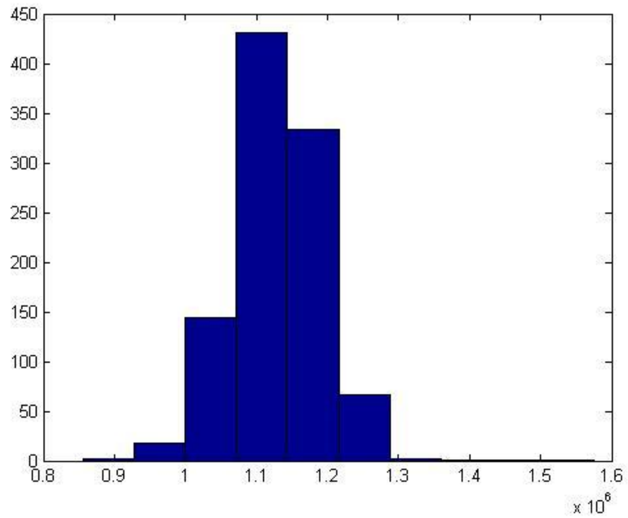


Figure 8: No Hedging Cash Flow Diagram (Normal)

These graphs are again similar to the previous charts. Cashflow volatility is significantly reduced with the introduction of hedging possibilities.

Appendix L: Price Risk Contract List

Contract Name	Exposure to Price Risk
Click Contract	Yes
Spot (+ OTC) Contract	No*
Uptime Contract	No
Wind Contract	Yes
CPVI Contract (Solar)	Yes
Fixed Price Contract	Yes
Reseller Contract	Yes
Fixed or Better Contract	Yes
Contract Gesplitste Verrekening (CGV)	No

** customer can however take forward positions*

Click Contract	With click contracts a customer can buy at most four times a year, at least 10% of total volume in the market. The price is based on the ENDEX index Calender future. Total consumption needs to lie between 80-120% of expected volume. Anode takes on imbalance risk as well.
Spot (+ OTC) Contract	The customer buys or sells electricity from Anode for the APX spot price with a certain fee subtracted. It is possible that customers buy a part of their demand forward in the markets.
Uptime Contract	In this contract a customer delivers energy to Anode for which it receives a certain payment. This payment is based on the APX Day-Ahead price summed with the net payment from a swap that lets the holder receive APX and pay the forward price. On the APX part Anode takes x% to cover imbalance and other costs/risks
Wind Contract	In the wind contract Anode will sell the energy the wind farm produces on the market. Customers can sell their electricity either for a fixed price a fixed price consisting of both a peak and trough price, or a price that is based on the ENDEX which requires that the total volume needs to be fixed five days in advance of 31/12. Anode takes on imbalance risk as well.
CPVI Contract (Solar)	A customer with solar energy production can sell electricity to Anode, for 1.1 times the baseload endex price. Anode takes all risks, no additional constraints.
Fixed Price Contract	The customer buys or sells a fixed quantity for a specified period for a fixed price. Optionally, based on available and reliable information, the customer is free to deviate up to 20% of total yearly production or consumption. Anode takes on imbalance risk as well.
Reseller Contract	The customer is a reseller that has its own customers or producers. All services related to the marketing, sales and finance are directed to the customer. Anode supplies the electricity required and takes on imbalance risk. The contract profile prices are set based on the ENDEX. For E1 and E2 profiles a fixed price can be generated for 1,2 or 3 years. For E3 and continuous connections the price are for up to five working days. With variable pricing, the yearly price is used that would be assigned for a contract written in the specific month with a certain fee. In addition resellers can opt for a forward deal.
Fixed or Better Contract	The customer buys or sells a fixed quantity for a specified period for a fixed price. Again the 20% deviation applies. Additionally, the customer can change the price at five moments in time for the new market situation. Anode takes on imbalance risk as well.

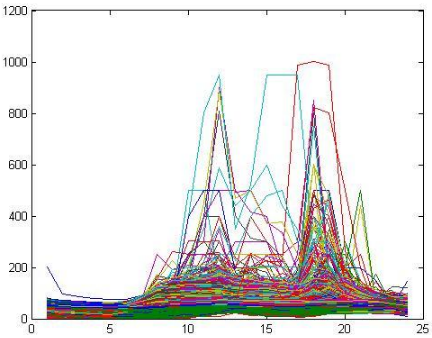
Contract Gesplitste Verrekening (CGV)	In this contract the customer transfers the PRP responsibility to Anode, while maintaining all other risks. It benefits from decreases in taxes and distribution costs.

Table 12: Overview of different contracts at Anode

Profile name	Description	Anode Type
E1	**Customers with a power throughput of at most 3 x 25A**	
E1A	Single tariff	small-use
E1B	Double tariff, night power	small-use
E1C	Double tariff, evening power	small-use
E2	**Customers with a power throughput between 3 x 25A and 3 x 80A**	
E2A	Single tariff	small-use
E2B	Double tariff	small-use
E3	**Customers with a power throughput higher than 3 x 80A and no continuous monitoring**	
E3A	Uptime less than or equal to 2000 hours	large-use
E3B	Uptime between 2000 and 3000 hours or equal	large-use
E3C	Uptime between 3000 and 5000 hours	large-use
E3D	Uptime higher than or equal to 5000 hours	large-use

Figure 39: The profiles in the Netherlands and their descriptions

Appendix M: Three Pricing Models charts



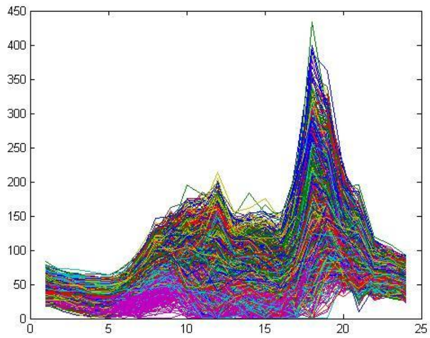
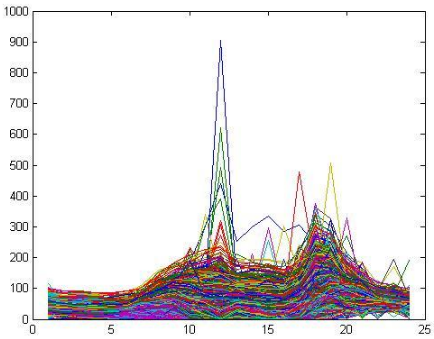


Figure 9: Figure of the Realized Prices on the APX DAM NL Figure 10: Simulation using the standard Gaussian distribution each line represents a single day in the period of 2005-2011



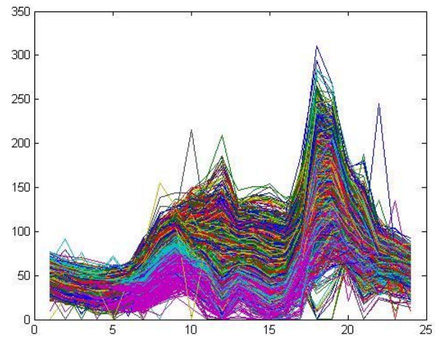


Figure 11: All Simulations with T=2000, student t3 Figure 12: Simulation using the student t distribution each line represents a single day in the period of 2005-2011

Appendix N: Anode Software Tool