MASTER

Arrival coordination
lust or loss?

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Arrival Coordination: Lust or Loss?

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Abstract

This master thesis provides an insight in the feasibility of partially en-route coordination of aircraft arrivals at a national airport. By issuing trajectory alterations to a part of the entire inflow, demand to the airport is (partially) regulated, reducing queuing time. To do so, two coordination concepts are developed; a ‘basic’ concepts which only distributes the coordinated part of the inflow evenly over time, and an ‘extended’ priority concept which distinguishes between arrivals; giving priority to coordinated movements over non-coordination movements. The viability of both of these concepts is proven. Supporting the general statement that coordination of only a part of the inflow will reduce waiting time. The conducted case study uses arrival data of a major national airport to gain insight into the applicability of partial en-route coordination in a real-world environment. The findings of this study indicate that partial control of the incoming movements will yield severe cost advantages for the participating aircraft.

Management Summary

This thesis addresses the feasibility of partially en-route coordination, aggravated to the aircraft arrival process. Currently, each aircraft approaching a runway operates an individually optimized trajectory, regardless of the available capacity. As aircraft movements converge during arrival, relative congestion may occur due to a mismatch between available capacity and the number of arriving aircraft. The repercussion of this congestion is that aircraft have to 'queue' in front of the runway, which is equivalent to extra flight time and track miles, increasing the operational costs, and affecting punctuality.

Research Question

By the issue of trajectory alterations to a part of the inflow, demand for runway capacity will partially be coordinated subject to the runway capacity. However, will coordination of only a part of the total inflow reduce the probability on congestion, and the consequential delay? This thought resulted in the following research question:

> *1A: "Can (partially) en-route coordination of incoming movements reduce the experienced overall delay of all movements, or at least benefits the concerned units, and.*
>
> *1B: What will be the minimal required percentage of the total movements to make this concept feasible?"*

The potential of en-route coordination

The objective of en-route arrival coordination is to use the current and future expected server status to modulate the inflow. To do so, trajectory alterations are issued to suppress the fluctuation in demand. Basically, to take precautionary measures against queueing time, aircraft have to spend supplementary time en-route such that demand will be spread evenly over time. En-route coordination is therefore only beneficial if ones trajectory is more (cost) efficient compared to waiting in a queue. The potential operational savings are expressed by two parameters: S_1 and S_2.

S_1 represents the cost difference between spending supplementary time en-route and spending time in a queue. The conversion of delay to additional time en route is comprehensive to a level where the entire inflow is well-ordered, in line with capacity. If the reduction in delay, due to a certain coordination policy, rises above the level achieved by only well-ordered inflow, the movement will, unless the conversion of queueing time to extra time en-route, be shortened, saving operational cost equal to $S_{2,}$

The framework used to address the potential of partially arrival coordination is a standard queueing system composed of an inflow, queue and server and gave rise to development of two coordination concepts: 'basic' arrival coordination and 'extended' arrival coordination

'Basic' arrival coordination

The 'basic' arrival coordination concept attempts to use en-route coordination only to enforce equal intervals between consecutive coordinated arrivals. The non-coordinated part of the arriving movements, the so called 'randoms', remain 'untouched' and will still arrive in certain random fashion. Assuming a First Come, First Serve production schedule, 'randoms' must be accommodated between evenly distributed coordinated arrivals, thereby 'disturbing' the handling of coordinated arrivals.

This ‘basic’ arrival coordination concept is modeled by a single server queuing system with two arrival streams; one (deterministic) arrival stream represents the coordinated arrivals and another (stochastic) arrival stream represents the non-coordinated movements. To assess the performance of such a system with a compounded arrival stream the algorithm of Albin (1986) is used. This coordination policy is characterized as follows:

- The relationship between the level of coordination and the reduction in delay is linear, depending strongly on the service distribution.
- The difference in waiting time reductions between random and coordinated arrivals is only minor.
- The entire system improves; giving an advantage to uncoordinated movements since en-route coordination reduces the experienced delay of all movements.

‘Extended’ arrival coordination

By elaborating the amount of control over incoming movements, the ‘extended arrival coordination concepts distinguishes between arrival types; favoring coordinated movements over ‘randoms’ by allocated the advantages of en-route coordination primarily to coordinated arrivals. This strategy resembles an appointment driven system with advanced access scheduling (Qu et al., 2007): coordinated arrivals are assigned to pre-planned ‘closed’ slots. To allow for uncoordinated arrivals, a special time window is reserved in which no (coordinated) arrivals are scheduled in advance, subdivided in ‘open’ slots, Figure 1. The number of consecutive ‘open’ slots depends on the choses strategy: exhaustive or non-exhaustive. By using a non-exhaustive strategy to handle non-coordinated movements the number of consecutive served random arrivals is limited to a certain number *k*. Contrary, an exhaustive strategy imposes the handling of all currently available ‘randoms’.

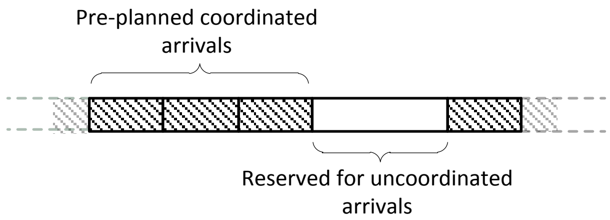

Figure 1: ‘Extend arrival coordination’

The performance of the ‘extended’ arrival concepts is approached by making use of ‘vacation queueing models’. This methodology is founded on the work of Vuuren & Adan (2007).

In general, extended arrival coordination almost completely dissolves the delay for coordinated arrivals when the participation level in coordination is low. However, the benefit of coordination fades away when the level of coordination rises. Hence, ‘extended’ coordination is only beneficial if the participation in coordination could be limited.

Since the delay for a non-coordinated movement will rise under a non-exhaustive policy, average system performance will decrease. Conversely, when using an exhaustive policy, delay could also be reduced for random arrivals. This comes with a cost; the performance of the exhaustive strategy is highly uncertain and therefore not (yet) suitable for implementation.

Figure 2 gives the combined results of ‘basic’ and non-exhaustive arrival coordination, clearly illustrating the potential savings per level of coordination and per type of coordination. Besides, is clearly visualizes the ‘tipping point’ in case of ‘extended arrival coordination.

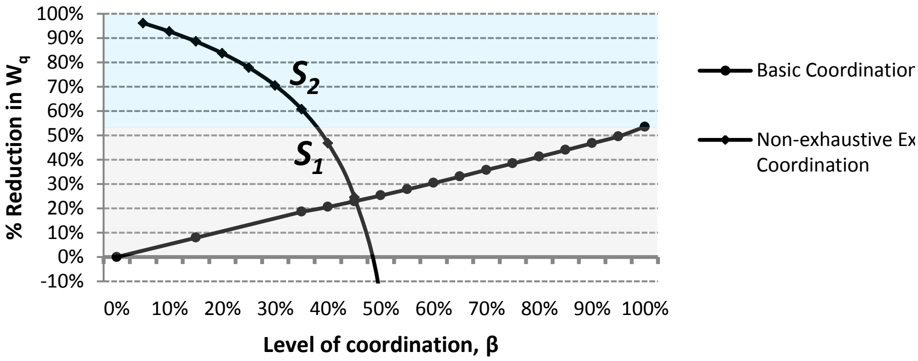

Figure 2: Basic and non-exhaustive extended coordination accompanied with (cost) saving parameters S_1 en S_2"

Case study: Amsterdam Airport Schiphol

A case study, based on arrival information of Amsterdam Airport Schiphol (AMS) supports the feasibility of partially en-route arrival coordination for the earlier discussed aircraft arrival process. The benefits and (dis)advantages differ per coordination strategy:

- 'Extended' coordination will yield the highest savings for coordinated arrivals only, but enforces tremendous additional expenditures for non-coordinated movements.
- During periods with reduced capacity the participating level in coordination is limited to only 20 percent when using 'extended' coordination.
- The 'basic' coordination policy will achieve savings at every level of coordination, irrespective of the utilization and improves the entire system performance.

Based on the enumeration of the 'pros' and 'cons', the most appropriate coordination concept to use at AMS is 'basic' arrival coordination. The potential annual pays-off to coordinated arrival only, per level of coordination, using 'basic' coordination are given by Figure 3.

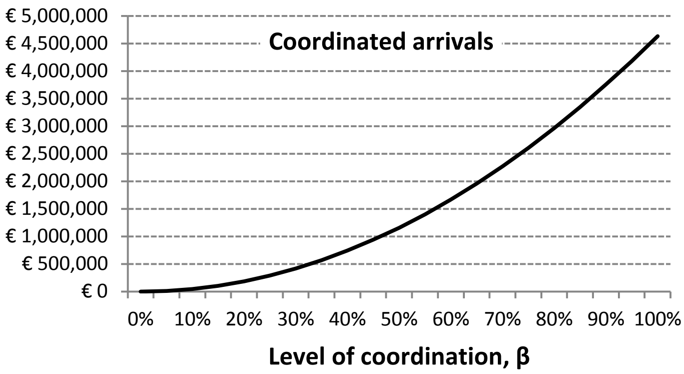

Figure 3: Annual cost savings for coodinated arrivals under 'basic' arrival coordination

Conclusion

Based on an elaborated analysis of both p coordination concepts and the conducted case study the main conclusions, and answers to the research question are:

- *"Partial en-route coordination will reduce experienced delay.*
- *There is no need for a minimal required participation level in coordination".*
- *The correlation between the level of coordination and the potential benefits is not always straightforward, resulting in the lack of an immediate 'Best Practice' to carry out (partial) arrival coordination.*

“Ah, ‘All things come to those who wait’.

They come, but often come too late.”

Mary M. Currie: Tout Vient à Qui Sait Attendre (1980)

Preface

This report is the result of a study initiated by Rockwell Collins, Inc. and represents the final part of my study in Operations Management and Logistics at Eindhoven University of Technology.

I would like to take this opportunity to express my gratitude towards the people that supported me throughout this graduation project. First of all, I would like to thank my company supervisor, Okko Bleeker, for providing me with the opportunity to apply my industrial engineers skills in a very challenging ´new´ environment. I also want to thank him for his excellent supervision; by providing me with challenging thoughts and valuable guidance he contributed not only to the project, but also to my personal development.

Furthermore, I would like to thank my primary university supervisor, dr. ir.Van Ooijen, for all his support, input and feedback during my entire project. His critical view and passion has been a source of inspiration.

I would like to thank my secondary university supervisor, dr. ir. Dellaert, and Ralf Luijt (NS) for their critical reflections and useful feedback on the project.

I thank my colleagues at Rockwell Collins, Inc for always proving me with assistance and information and all others who provided me with information regarding my study.

Finally, I would like to thank my family, friends and fellow students who have supported me during my time as student at the university, and especially during my graduation.

Mathijs Bestebreur

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List of abbreviations

AO	=	Airline Operators
AMS[1]	=	Amsterdam Airport Schiphol
ATM	=	Air Traffic Management
ATC	=	Air Traffic Control
ELS	=	E-Limited Service
FCTL	=	Fixed-Cycle-Traffic-Light
i.i.d.	=	independent and identically distributed
LST	=	Laplace Stieltjes Transformation
NS	=	Nederlandse Spoorwegen
Pdf	=	Probability density function
RCI	=	Rockwell Collins Inc.
SESAR	=	Single European Sky ATM Research

[1] Official abbreviation used by the International Air Transport Association (IATA)

List of Variables

a_1, a_2, p_1, p_2	=	Parameters of the Hyper-exponential distribution
$b\ \ x$	=	Distribution of the service time
b	=	First moment of the service time
b^2	=	Second moment of the service time
c_x	=	Coefficient of variation of variable *x*
c_a^2	=	Squared coefficient of variation of the asymptotic method
c_b^2	=	Squared coefficient of the service time distribution
c_c^2	=	Squared coefficient of the coordinated arrivals
c_h^2	=	Squared coefficient of variation of the Hybrid Approach
c_i^2	=	Squared coefficient of variation of inflow *i*
c_R^2	=	Squared coefficient of variation of random arrivals
c_s^2	=	Squared coefficient of variation of the stationary-interval method
c_x^2	=	Squared coefficient of variation of variable *x*
D_c	=	Busy period based on server vacation
$E\ \ X$	=	Expected value of random variable X
$g\ \ x$	=	General distribution *g*
$G^*\ \ s$	=	LST of general distribution *G*
h, d	=	Parameters of the exponential distribution shifted by a constant
k	=	Limit in an E-limited service queue
$\tilde{L}_j$	=	Expected number in the system seen by an arrival from stream *j*
$\tilde{L}_{aj}$	=	Expected number of stream *j* customers in the system seen by the arrival from stream *j*
L_{bj}	=	Expected number of customers from streams other than *j* in the system seen by the arrival from stream *j*
N_C	=	Amount of consecutive ‘closed slots’
N_R	=	Amount of consecutive ‘Open slots’
$\bar{N}$	=	Number of items in a standard M/G/1 system
$E\ \ Q$	=	Expected number of items in the system
S_i	=	Cost savings parameter *i*

$t_{s,C}$	=	Slot length for coordinated arrivals ('Closed slot')
$t_{s,R}$	=	Slot length for random arrivals ('Open slot')
$T_{s,C}$	=	Total time for all 'closed slots' in a cycle
$T_{s,R}$	=	Total time reserved for handling random arrivals per cycle
T_c	=	Length of one cycle
u	=	Numerical solution of LST
v	=	Length of a vacation period
v^*	=	LST of v
v^2	=	Second moment of the vacation period
W_h	=	Waiting time obtained by the hybrid approach
W_j	=	Expected delay for customers of component stream j
W_q	=	Waiting time in the queue
W_q^C	=	Waiting time in the queue for coordinated arrivals
W_q^R	=	Waiting time in the queue for random arrivals
$W_q^{\min}$	=	Lower bound for the expected waiting time in the queue
$W_q^{\max}$	=	Upper bound for the expected waiting time in the queue
W_q^{SIM}	=	Expected waiting time in the queue based on simulation
$W_q^{R,\max}$	=	Maximum delay experienced by random arrivals due to Extended arrival Coordination
β	=	Level of coordination (%)
γ_i	=	mean of inflow i
λ_j	=	Arrival rate of component j
λ_C	=	Rate of coordinated arrivals
λ_R	=	Rate of random arrivals
λ_T	=	Total arrival rate
θ_{RA}	=	Expected number of arrivals during interval D_c
μ	=	Service rate of the server
ρ	=	Utilization
ρ_j	=	Utilization solely based on component stream j
ρ_h	=	Traffic intensity in the queue

ρ_j	=	Utilization solely based on component stream j
ρ_c	=	Utilization based on coordinated arrivals only
ρ_o	=	Operational utilization of the entire system
ρ_s	=	Utilization under standard conditions (i.e. no coordination)
σ_j	=	Standard deviation of component *j*
σ_b^2	=	Variance of the service time distribution
σ_{RA}^2	=	Variance of the number of arrivals during interval D_c
σ_i^2	=	Variance of inflow i
ω	=	Weight factor of the Hybrid Approach
ξ	=	Buffer
Δ	=	Absolute difference (%)

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1 Introduction

Standing in line, waiting for some kind of service is part of everyday life. The unpredictability of demand and individual arrival times may “conflict” with the availability of resources so that queues of waiting customers will result. The length of the queue depends on the variability of the required service time and two aspects of the flow pattern: the average rate at which demands are placed upon the resource and the fluctuations of this rate.

A so called ‘queueing system’ is often an integrated part of an entire network of consecutive correlated processes. Waiting in front of a server, therefore, affects the entire system performance and may wipe out benefits obtained in the up and down stream processes of the network. More specifically, any unpredictability in the arrival process leads to inefficiency of an entire system.

1.1 Aircraft Arrival Process

The aircraft arrival process to an airport or, more specifically, to a runway, is an example of such a system where unpredictability affects the performance of the individual aircraft flight as well as that of the collective system. Each aircraft operates an individually optimized trajectory to the runway, Figure 1-1a. As aircraft movements converge during arrival (after Point A), relative congestion may occur due to late coordination amongst individual movements (i.e. multiple aircraft present themselves to pass a certain Point at the same time). The repercussion of this congestion is shown in Figure 1-1b; aircraft have to “wait” and waiting of an airplane is equivalent to extra flight time and track miles, increasing the operational cost. Moreover, this reduces the punctuality and predictability for succeeding aircraft movements, thus evaporating the relative efficiency realized during the individually optimized cruise flight.

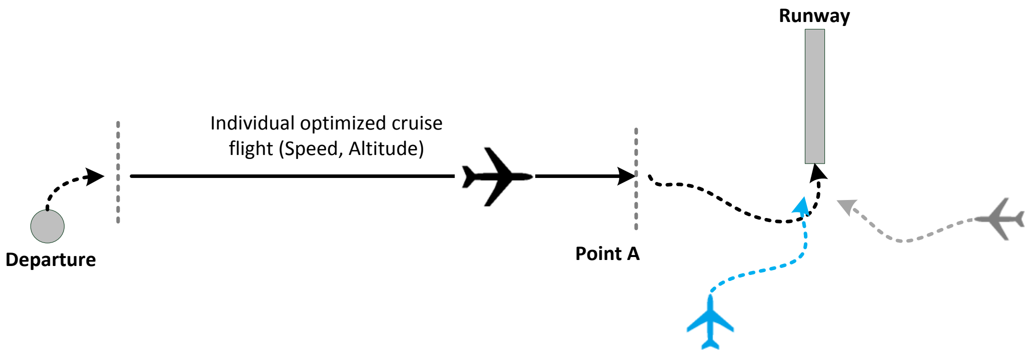

Figure 1-1a: Aircraft Arrival Process, uncongested

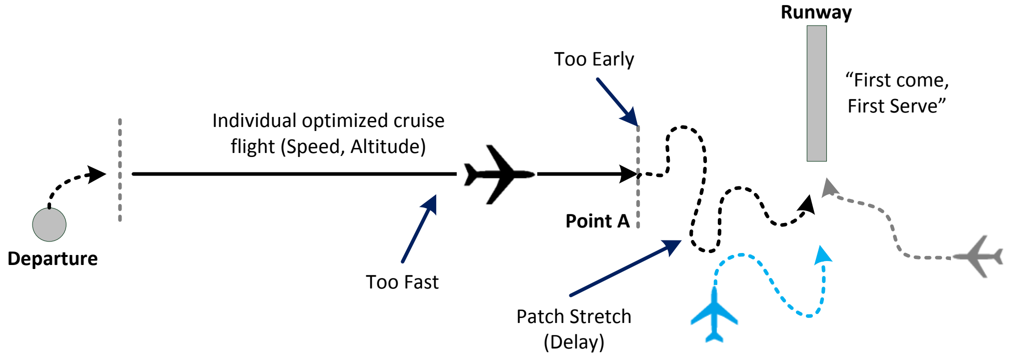

Figure 1-1b: Aircraft Arrival Process: Congestion

The need to wait before landing becomes evident only beyond Point A, from that moment on an 'arrival manager' is able to examine the status at of the 'server', i.e. the runway. Referring to Figure 1-1b, the airplane may actually be considered too early at Point A; slowing down earlier during its trajectory, ideally equal to the delay expected to be issued after Point A. Conversely one could argue it must have been flying too fast until that Point. Slowing down during its trajectory, ideally reaching Point A so that any delay beyond Point A would be obviated, will spread the arrivals over time, consistent with the acceptance rate of the runway, reducing the odds of congestion.

The current air traffic management is reactionary. Movements are 'detected' just before the actual moment of arrival (at Point A). The inability to estimate demand subsequently diminishes the capability to reduce fluctuation by spreading demand over time in line with capacity. The consequences for the entire system are substantial, the average delay per flight caused by airport congestion in Europe doubled to two minutes, despite a moderate traffic growth of only 0.9 percent. The average delay per delayed flight rose to 24 minutes, an increase of 3.6 minutes. With a total delay of almost 900 hours a day, and a standard minimum cost for one minute of delay of 73 euro, the annual cost exceeds 1.5 billion euros, giving a considerable cause for improvement. In addition, the forthcoming introduction of the Emissions Trading System[2] for aviation is another immediate cause enforcing airline operators to critically examine their operational efficiency (EUROCONTROL, 2011).

1.2 Generalization

The system under consideration generally consists of three parts:

- an individually optimized trajectory;
- a queue, and;
- a server.

The ability to examine the status of the queue and server is only possible after arriving at the queue, resulting in the probability of waiting in line due to a lack of capacity. Hence, there is no reference concerning the server and queue status at the expected arrival time during the individual trajectory. Deriving and coordinating a specific time slot for arrivals enables a time reference to a proper time of arrival during the individual trajectory, spreading demand evenly over time, Figure 1-2. That is, provide each arrival a specific time slot to arrive, long enough in advance such that the individual trajectory (i.e. en-route) could be re-optimized to arrive exactly at that time. Throughout this paper this particular concept is termed 'en-route arrival coordination', a general term, or 'trajectory based operations', often used in an aerospace context.

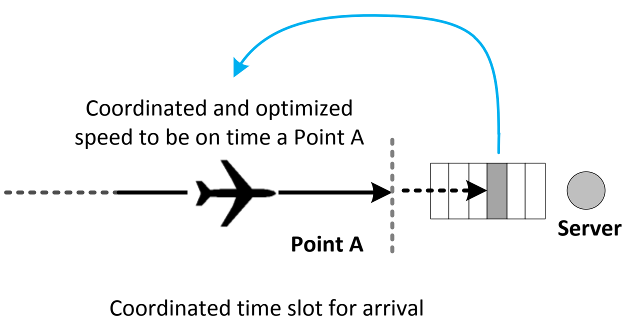

Figure 1-2: En-route arrival coordination

[2] http://ec.europa.eu/clima/policies/ets/index_en.htm

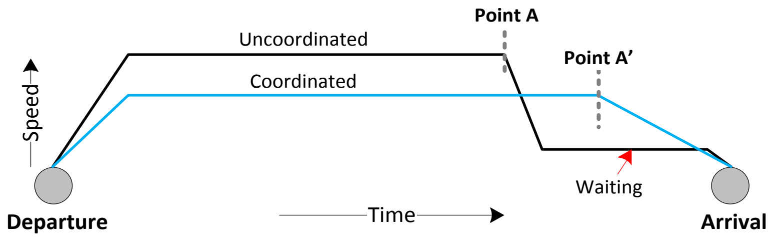

Figure 1-3: Trajectory speed-time graph

Figure 1-3 gives the individual speed-time relationships of (un)coordinated movements; uncoordinated movements have to wait beyond Point A, slowing down before entering service, while coordinated movements use their individual trajectory to prevent unplanned delay beyond Point A, by operating a reduced speed profile or, ultimately, by departing later.

A clear understanding of the term 'delay' is needed to fully understand the concept, as follows: the difference between the expected time of arrival and the actual time of arrival, triggered by any unexpected event. Remark that arriving earlier than expected is not assumed to benefit and is therefore not considered.

Not being aware of the situation beyond Point A, experiencing delay after Point A would then constitute an actual delay. Having insight in the situation beyond Point A before ever arriving there, would reveal the need to delay arrival at Point A, providing opportunity to take precautionary measures earlier on in the trajectory. The expected time of arrival (at Point A) will therefore change, but there will not be an experience of 'delay'. In fact, there could potentially be no gain in arrival time at all, only a shift in time of the moment of arriving at Point. The savings, however, obtained by using an en-route coordination concept may be twofold. First, choosing a trajectory with adapted speed profile will be cost efficient compared to waiting in the queue (baseline cost or better); second, by way of the process, the delay beyond Point A is taken out of the equation, meaning that extra track miles and fuel burn associated with that, as well as downstream unpredictability's (assets waiting for the flight to arrive), are correspondingly taken out of the equation.

Complexity and diversity of the arrival stream raises difficulties in implementing the coordination process, since coordination of the entire customer base (movements to be expected) will be hard to realize at a given time. Thus, it is likely that en-route trajectory coordination will be brought into operation in phases, with a percentage of 'participating' movements and a complementary percentage of movements that do not participate. The percentage of coordinated movements is therefore expected to vary over time.

1.3 Research significance

As introduced earlier, the need for improved air traffic management along with the considerable growth of air traffic, leads to increasingly congested airport and terminal area infrastructures. In line with this congestion, flight delays are evenly increasing, leading to the consequent need to reform the system. The current effort in Europe to develop a new European air traffic functionality by a joint undertaking of a large consortium of air transport stakeholders and EUROCONTROL (SESAR[3]), appears to evolve slowly and incrementally, resulting in a drive towards accelerating the process by several airline operators as well as industry.

[3] Single European Sky ATM Research

An obvious approach to prevent congestions is to increase throughput. However, the limits of the (European) infrastructure are in reach, and new infrastructural investments are costly and very time-consuming with lead times up to fifteen to twenty years. Using advanced scheduling concepts, which includes the use of individual spacing requirements or airline preferences in optimizing queueing, are proven concept to reduce delay but lack support from the European Air Traffic associations. To overcome these difficulties, a trajectory-based operation is being proposed, i.e. active coordination on the pre-planned route of the aircraft; diminishing the fluctuation in demand by increasing the predictability of arrivals.

The use of arrival coordination is already proposed in several papers in literature, often introduced as '4D-trajectory operations'. There are even some promising results regarding operationalization; albeit a purely theoretical framework is used Knorr et al. (2011) predicts estimated fuel savings up to forty percent. Affirmative results are also obtained by Coppenbarger et al. (2009), who discussed a successful implementation of trajectory based operations in an environment with only a single dominant carrier (Bestebreur, 2011; Wichman et al., 2007).

A trajectory-based approach is relevant to any environment where coordination is possible preceding the actual queueing system. The vision is to investigate the potential for an en-route coordination concept in a more general perspective, applicable to a broader range of (traffic) problems. Because of analogies between the aircraft arrival process and the railway arrival process, the Dutch national railway operator (Nederlandse Spoorwegen – NS) showed an interested in the potential of a similar en-route coordination concept, to diminish the effect of anomalies and to optimize railway network performance. The exact position of trains on the track is nowadays not known and, in combination with 4800 daily movements, high track utilization and absence of communication between individual movements, fluent train services depend heavily on the individual engine driver. Hence, the potential of using the real-time status of the network in support of active coordination could secure the feasibility of punctual connections, optimized use of platforms and information out to the travelling public. The analogy with aircraft arrivals is almost outright, on the understanding that the degree of freedom for railway movements is more constrained

1.4 Rockwell Collins

The opportunity of using en-route coordination to reduce delays and congestion in (future) European airspace arouses the interest of Rockwell Collins, Inc. (RCI): a global private company specialized in the defense and commercial avionics markets, and closely involved in developments concerning air transportations (e.g. enhancing efficiency or safety).

RCI expertise in communication, network, and navigation could be an eminent contribution to the development of trajectory based operations. Engagement in the early development phases could bring strategic advantages for future business opportunities; the roll out of trajectory based operations could, among other things, result in the need for of new communication and network functions by airline operators as well as capacity management and support systems by terminal operators. These future options are the incentive to facilitate this research to study the feasibility of arrival coordination.

1.5 Literature Review

The broad applicability of queueing models throughout the industry arouses the expectation that coordination and control of the arrivals, aimed on the reduction of congestion and prevention of delay, is discussed extensively in literature. The vast majority of research on arrival control mainly focuses on the condition when to reject or accept an order in idealized environments: optimizing computer networks and CPU task planning. The applicability of these theories outside the IT industry is however hardly considered, fostering the development of models accounting for stochastic demand and other environmental uncertainties. Two arrival control concepts where found in literature, applicable for general manufacturing environments; rejecting or accepting an arrival at its arrival epoch, based on either a critical number/state of the system or by adjusting the tolls levying on the arrivals. In the latter case, it is assumed that individual arrivals are utility maximizing and price and delay sensitive, avoiding the server in case of congestion. Distributed arrival time control is the other available concept, using an algorithm, which tries to minimize local due date deviation by equally penalizing earliness and lateness, the time at which parts start seeking the required machine are adjusted. This concept is initially designed for heterarchical manufacturing systems, but applicable outside its original scope.

Arrival coordination extended with allocation of specific time slots is equivalent to an appointment driven queueing system. Appointment driven systems are used in many service systems, especially in healthcare. Important measures of interest include the size of waiting list, the waiting time at the service facility and server utilization. The size of the waiting list refers to partition and amount of pre-planned appointment.

In addition, open access scheduling, an extension of regular appointment driven systems, is a concept used that seeks to schedule both the short notice and routine appointments. I.e. a certain percentage of appointment slots are planned in advance, and the remaining slots are "open", used for last moment emergencies. Hence, emergencies do not disrupt pre-planned appointments. Albeit the literature mainly focuses on using open access scheduling in appointments planning within the healthcare industry, a transition could be made to the manufacturing industry e.g. arrivals with minor arrival time deviation are planned in advance and arrivals with unknown, or uncertain arrival times are allocated to "open" slots.

At Louisville Standiford Airport, en-route control as described in section 1.3, is realized for nearly all incoming movements. Extensive coordination between air traffic control and the airline operator, trajectories and airport throughput is optimized. The main characteristic of this successful implementation is the presence of a single dominant carrier, simplifying the information sharing process. Nevertheless, it limits the expansion of the concept to other airports with multiple carriers. (Bestebreur, 2011)

1.6 Research Question

The preceding section reveals that coordination of the incoming movements to reduce congestion and delay in a general environment is only minor discussed in literature. Furthermore, only appointment scheduling methods are feasible to account in some manner for only partial coordination of the inflow, raising difficulties for implementation in complex environments and diversity among customers. Based on these conclusions and the problem generalization in section 1.3 the following research question is formulated.

> *1A: "Can (partially) en-route coordination of incoming movements reduce the experienced overall delay of all movements, or at least benefits the concerned units, and.*
>
> *1B: What will be the minimal required percentage of the total movements to make this concept feasible?"*

Founded on this research question a twofold research assignment is made up, including several sub arguments. The first assignment concerns research question one, the elaboration of a methodology to control multiple arrivals streams whereas the second assignment attempts, by acquiring the benefits of the developed algorithm, to gain insight in the effect of (partial) coordination.

1: Design an algorithm to allocate a mix of stochastic and deterministic arrivals to a set of pre-scheduled time slots.

- *a. Design a methodology to merge multiple different arrival flows;*
- *b. Design a mechanism to obtain the amount of 'open' slots for a given coordination level;*
- *c. Design a slot allocation mechanism to assign incoming units to a particular slot;*
- *d. Verify and validate the working of the algorithm.*

2: Determine the (cost) benefits and disadvantages of the algorithm under 1

- *a. Setup a benchmark performance level (i.e. what is the current performance level?)*
- *b. Address the performance, in basic measures, of the algorithm under different parameter values;*
- *c. Introduce case specific cost parameters;*
- *d. In-depth analyses of queue behavior; who experiences delay?*
- *e. Compare the performance of the algorithm with the earlier set benchmark performance level;*
- *f. Determine possible model improvements and extensions.*

1.7 Thesis Outline

The remaining part of this paper start in chapter 2 with a discussion on the potential of en-route coordination for future aviation, completed with an outline of the conceptual coordination model, ensuing in the introduction of a 'basic' and an 'extended' coordination concept. Chapters 3 and 4 will take a closer look on, respectively, the 'basic' and 'extended' coordination concept. Both chapters start with an outline of the methodology used to address the performance of the particular arrival concept, followed by elaborated discussion on their specific benefits and (dis) advantages. Chapter 5 presents a case study on the potential of en-route coordination for the aircraft arrival procedure at Amsterdam Schiphol Airport. The paper concludes with chapter 6, stating the general conclusion as well as the limitations and future research options.

2 En-route arrival coordination

This chapter introduces two en-route coordination concepts aggravated to prevent congestion. Section 2.1 starts with a discussion on the potential of en-route coordination, in particular for the aircraft arrival process. Section 2.2 defines the potential cost savings. Based on the generalized problem statement given in the preceding chapter, section 2.3 introduces a conceptual model and an accompanying theoretical framework. Based on general queueing theory, section 2.4 introduces a 'basic' arrival coordination' concept. By using coordination to distinguish between arrivals, a second 'extended' arrival coordination concept is introduced in section 2.5.

2.1 The potential of en-route coordination

En-route coordination is suggested to reduce delay, experienced in the final stage of the trajectory i.e. the moment of arrival. The definition for delay given in section 1.2 emphasizes on an "unexpected event", yielding the delay. The 'unexpected event' subject to discussion is congestion, caused by a mismatch between the arrival rate and server capacity. The aim of coordination is therefore to take precautions against possible delays ('queueing time') at the moment of arrival during the trajectory, based on the server status; diminishing demand fluctuations by dispersion of arrivals over time, preventing congestion.

The capacity of a runway (airport) is under surveillance of the local Air Traffic Control (ATC) authorities. If the amount of arriving aircraft exceeds the current available runway capacity, ATC is forced to generate a 'queue' (based on a First Come, First Serve principle). Actual 'queueing' of aircraft is equivalent to patch stretching in so called 'holding patterns', illustrated by Figure 2-1, and resulting in extra flight time and track miles. The origin of the discrepancy between demand and capacity could be reduced to two main causes: (1) a demand peak, and (2) reduced capacity due to an uncontrollable event e.g. weather issues.

Airline Operators (AO) schedule their flight operations autonomous and independent of other competitors, ATC, and even regularly without taking note of their own scheduled movements. The overall arrival timetable, listing all incoming movements, contains therefore overlap. Multiple aircraft are scheduled to arrive at the same time instance, even those of the same AO, Figure 2-2. Moreover, even when the overall pre-planned arrival schedule is in accordance with the capacity, the operational realization of the schedule often deviates from the preplanned activities. The actual flight path of an aircrafts is individually optimized according to a certain 'cost index' function in which the planned arrival time is a target value, not a constraint. E.g. if it appears to be more (cost) efficient, the aircraft will plan to arrive ten minutes late, or maybe, even ten minutes earlier. Ultimately, the demand for runway capacity is irregular, bringing on periods of excessive demand and the adverse effect of overrunning capacity, and moments with a capacity surplus.

Weather issues can impose reduced landing rates due to high winds, or the need for additional separation due to low visibility procedures. During such uncontrollable events capacity will become inadequate since service rates are decreasing while the number of arriving aircraft will not change due to these circumstances.

Figure 2-1: Two real life examples of ‘queueing’ aircraftt (patch stretching), nearby Amsterdam Airport schip at 02-11-2011:09.23 (www.flighrader24.com)

18:40	Barcelona	CZ 7634	China Southern Airlines	Expected
18:40	East Midlands	WW 5477	bmibaby	Expected
18:40	London Gatwick	EZY 8877	easyJet	Expected
18:40	Seoul Incheon	KL 0866	KLM Royal Dutch Airlines	Early
18:40	Barcelona	KL 1674	KLM Royal Dutch Airlines	Expected

Figure 2-2: Arbitrary random screenshot of planned arrivals at Schiphol Amsterdam Airport (http://www.schiphol.nl/Travellers/FlightInformation/Arrivals.htm)

An aircraft is, in the trajectory before the final approach of the runway, not informed about the runway status at the moment of arrival, and as a consequence, unaware of it will arrive during a demand peak, a capacity surplus, or a period with reduced landing rates. By individually optimizing the trajectory to a certain ‘cost index’ the aircraft indirectly assumes that it will not yield any delay in final phase of the flight. For example: due to a certain ‘cost index’ an aircraft plans to arrive ten minutes late compared to the earlier preplanned arrival time, unaware of the fact that it therefore arrives during a demand peak, and is forced to ‘queue’ for twenty minutes, vanishing all achieved efficiency gains and even increasing the operational cost. Because the aircraft was not acquainted of the server status, the occurrence of congestion at arrival was unforeseen. By suddenly spending twenty minutes in a holding pattern the actual arrival time will deviate from the expected arrival time, and is therefore classified as ‘delay[4]’

The objective of en-route arrival coordination is to use the current and future expected server status to modulate the inflow. Arriving aircraft are, during their trajectory, informed about the available runway capacity at their expected time of arrival, giving them the ability to take precautionary actions against arriving during a demand peak or capacity dispute. Referring to the example in the preceding paragraph, if the aircraft did know in advance that arriving ten minutes late would impose twenty minutes of ‘queuing’ time, it would have the option to, for example, slow down throughout the trajectory even further, arriving twenty five minutes late instead; during a capacity surplus with no liability on delay. The commission to arrive twenty five minutes behind the preset arrival time is known on forehand, issuing a new expected time of arrival. From the perspective of the AO, this postponed arrival time is the new ‘planned time of arrival’, and due to the absence of any congestion, ideally, equal to the actual time of arrival. The route extension of twenty five minutes is therefore not classified as ‘delay’. In essence, (unforeseen) queueing time is replaced by (known) additional time en-route, either by slowing down, a postponed departure or another conceivable way, giving the AO the ability to (re)-optimize the (downstream) operations in compliance with a newly defined moment of arrival.

[4] The definition of delay is given in section 1.2, page 3

An obvious approach to track current and, most important, future server (runway) capacity is to issue individual arrival times ('slots'). The amount 'slots' are chosen in line with the available capacity in order to minimize the liability on delay. To meet the imposed arrival time, individual movements could adapt their trajectory (i.e. slowing down), during the earlier phase of their route. Since the commission to adapt ones trajectory is known on forehand, arriving on a later point in time is in line with the expected time of arrival and is therefore not classified as a delay.

2.2 (Cost) Benefits

The success of arrival coordination in a real-world environment depends on the realized yield for the initial investors, but can cost vs. benefits be argued? Basically, queueing time is replaced by additional time en-route. En-route coordination is therefore only beneficial if ones trajectory is more (cost) efficient compared to waiting in a queue. The potential advantaged could be split out to a tactical and operational level. The latter mentioned benefits are related to the actual execution of the adapted movement and are expressed by two parameters: S_1 and S_2. Because it is difficult to give an unambiguous definition, the parameters are clarified by two examples:

S_1: Assume a single server and an average delay of ten minutes. Due to en-route arrival coordination four minutes of the total delay is absorb by trajectory. As a result, the delay in front of the server reduces to six minutes. The accumulation of queueing time and a lengthened trajectory is still ten minutes. The total savings is are equal to the cost difference between waiting four minutes in the queue and lengthening the trajectory by four minutes. These saving are denoted by $S_{1.}$

S_2: Now assume a comparable setup, and again due to en-route arrival coordination four minutes of the total delay is transformed to the trajectory. However, coordination enforces also a reduction of the total experienced delay by two minutes, resulting in a delay in front of the server of only four minutes. The resulting accumulation of queueing time and the adapted trajectory reduced to eight minutes. Hence, the total system performance improved with two minutes. The associated cost reduction, on top of the saving denoted by S_1, and related to the system improvement of two minutes, is denoted by S_2.

Regarding aircraft, flying on cruise speed is in general, assumed to be more efficient in terms of fuel consumptions (equivalent to the carbon food print), and other operational cost compared to flying in a holding pattern. In case of public train services, a stationary train will consume less energy compared to a moving train, but the energy required for speeding up a stationary train evaporates the difference, enlarging the total energy needed. Hence, the operational savings are dictated by the operational environment, requiring a detailed analysis for every singular case to determine the cost savings parameters S_1 and S_2 (Knorr et al., 2011).

Even without any operational benefits, considerable savings are achievable on a tactical level. Coordination issues a specific arrival time window, reducing the uncertainty for downstream processes. Increased predictability is an incentive to reconsider the robustness of operational schedules, which is analogue to optimized use of assets in down and upstream operations. Moreover, as demand is more evenly spread out over time, server throughput could be increased without any investment in tangible assets. It is almost impossible to capture these savings by a single general parameter, related to a reduction in delay. Moreover, to obtain these savings the entire operational organization must be thoroughly examined.

2.3 Theoretical framework

The framework used to address the feasibility of arrival coordination is a standard queueing system as given in Figure 2-1. The standard inflow/queue/server modeling approach is broadly used to model all kind of arrival processes, enhancing the range of application of the coordination concept outside its original scope.

By using this general framework, the primary objective reduces to the assessment of the potential benefits of using coordination for delay prevention, caused by congestion, from theoretical perspective only. The (technological) viability of coordination in terms of implementation and operationalization is therefore brushed aside.

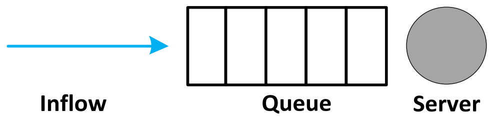

Figure 2-1: Standard queueing system

The use of a 'queueing' model implies the use of standard terminology to describe the individual parts of the model. Most discrepancy will arise while referring to the inflow i.e. the arrival stream or simplified to 'arrivals'. Moreover, the time between consecutive 'arrivals' is defined as the inter-arrival time. The server performance could be established by the service rate whereas service time corresponds to handling time of one unit. Throughout this document Kendall's notation will be used to describe and classify every queueing system (see Appendix A).

2.4 'Basic' arrival coordination concept

In a standard queueing system as given in Figure 2-1, there is no liability on any delay if both the inter-arrival times and service times are deterministic, a D/D/1 queue system. In addition, if the service times are generally distributed, average delay is minimized by deterministic inter-arrival times. The en-route coordination concept aims on avoiding congestion by trajectory adjustments before the actual 'entrance' of the queueing system, based on the allocation of individual time slots. That is, en-route coordination gives the ability to modulate the inter-arrival time distribution. Hence, if en-route coordination ensures the transformation of general distributed inter-arrival times into deterministic arrival times, delay is minimized.

To inquire the viability of this latter statement with only partial coordination chapter three introduces a 'basic' coordination concept'. Basic coordination attends to use en-route coordination to enforce equal intervals between consecutive coordinated arrivals. Spreading demand generated by those coordinated arrivals equally over time i.e. imposing deterministic inter-arrival times. Inter-arrival times between consecutive coordinated arrivals therefore change according to the level of coordination. The non-coordinated part of the movements, so called 'randoms', remain 'untouched' and will still arrive in a random fashion at the server, based on a certain general distribution. The consequential mixed arrival process of (un)coordinated arrivals is illustrated in Figure 2-2.

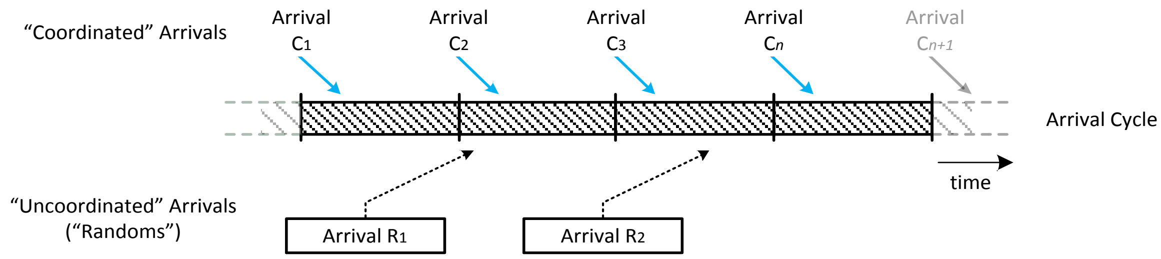

Figure 2-2: Coordinated arrival process "disturbed" by random arrivals

Assuming a First Come, First Serve production schedule, 'randoms' must be accommodated between evenly distributed coordinated arrivals, disturbing the flow of the deterministic arrivals; the production start of a coordinated arrival is expected to be delayed due to the arrival of uncoordinated a movements, which are served in between coordinated arrivals, Figure 2-3a. The resulting 'disturbed' production scheme is given in Figure 2-3b. This 'disturbing' effect will perhaps undo some of the advantages of coordination, and it is therefore likely that the system performance will depends heavily on the ratio of coordinated and random arrivals.

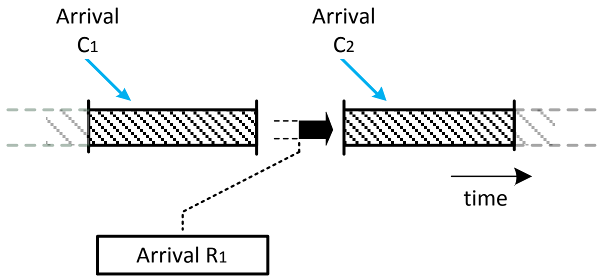

Figure 2-3a: The disturbing effect of random arrivals

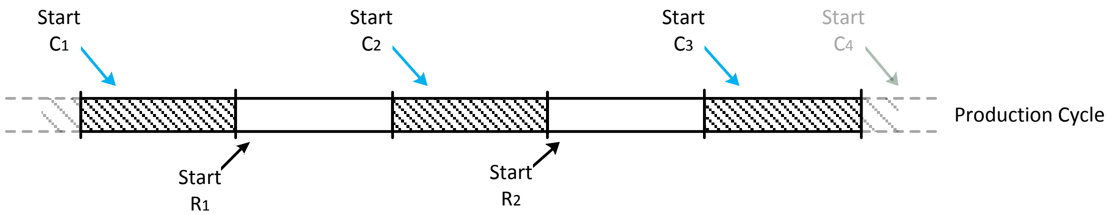

Figure 2-3b: Effect of random arrivals on the production cycle; accomodated between coordinated arrivals

2.5 'Extended' coordination concept

The basic coordination concept introduced in the preceding section only contemplates coordinated arrivals, without taking precautions against the probability of a 'disturbed' production schedule. Elaborating the amount of control over incoming movements gives the ability to distinguish between arrivals, eliminating the 'disturbing' effect of random arrivals. In addition, it also gives the possibility to allocate the advantages of en-route coordination primary to coordinated movements, alluring the investment in trajectory based operations.

In an M/G/1 system, the mean waiting time does not change due to any non-preemptive work-conserving queueing discipline (Kleinrock, 1976). It is therefore assumed that, if the delay for coordinated arrivals could be reduced, random arrivals automatically have to 'absorb' additional delay i.e. random uncoordinated arrivals are actually 'punished' for not participating in the coordination process.

Reducing the average delay for coordinated arrivals could be accomplished by giving coordination arrivals priority over random arriving movements. The subsequent proposed ‘extended’ coordination strategy resembles an appointment driven system with advanced access scheduling (Qu et al., 2007); coordinated arrivals are assigned to pre-planned ‘closed’ slots. The deterministic assumption for coordinated arrivals is preserved, but the interval between consecutive arrivals is now based on the partition of pre-planned slots. The number of pre-planned slots per time interval is equivalent to the arrival rate of coordinated movements.

To allow for uncoordinated arrivals, a special time window is reserved in which no (coordinated) arrivals are scheduled in advance, Figure 2-4a. This time period is subdivided into ‘open’ slots: a time frame to handle a single random arrival. The number of consecutive ‘open’ slots depends on the choses strategy: exhaustive or non-exhaustive, further explained in section 4.3. An uncoordinated arrival just have to wait till the start of such a period reserved for handling random arrivals before receiving any service. If the reserved period is not sufficient to handle all random arrivals currently in the system, some ‘randoms’ will have to wait till a next reserved period arrives. The resulting service schedule and a detailed overview of ‘extended arrival coordination concept is given in Figure 2-4b. Remark that, even if Arrival R_1 enters the system before Arrival C_1, R_1 still has to wait for the appearance of an ‘open’ slot, reserved for random arrivals (after C_N), before it actually receives service.

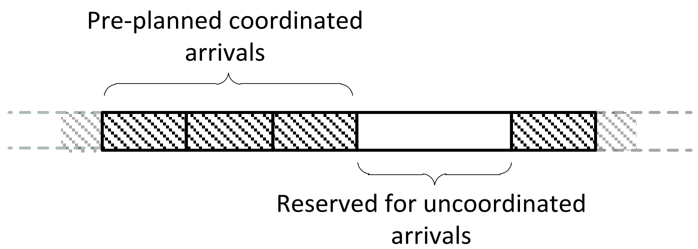

Figure 2-4a: ‘Extended’ arrival coordination concept

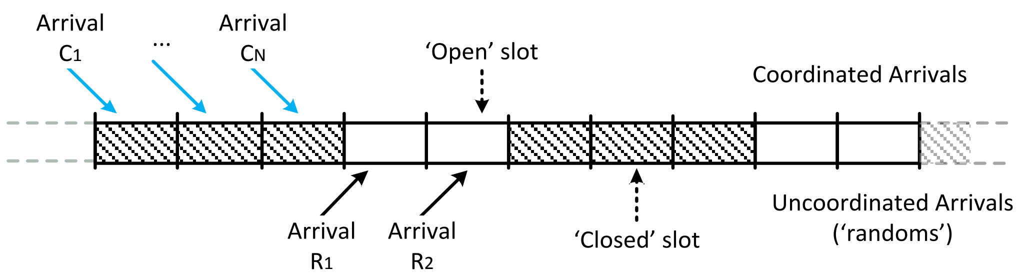

Figure 2-4b:Details view of the ‘extended’ arrival coordination concept

3 'Basic' arrival coordination

This chapter discusses the 'basic' arrival coordination concept, introduced in section 2.3 in detail. Section 3.1 starts with a review of the available literature to combine deterministic and stochastic arrivals into a single server queueing model, comparable to the 'basic' arrival coordination concept. Before elaborating the chosen methodology in detail in section 3.3, section 3.2 focusses upon theoretical performance boundaries. The subsequent part, section 3.4, verifies the validity of the algorithm followed by an outline of the obtained result for entire system and a sensitivity analysis in section 3.5. Section 3.6 discusses the consequences of basic arrival coordination for the individual components. The chapter ends in section 3.7 with an overview of the obtained results.

3.1 Merging of arrival streams

A system with partially 'basic' en-route coordinated arrivals is modeled by a single server queuing system with two arrival streams with independent and identically distributed (i.i.d) inter-arrival times (i.e. renewal processes), independent of one a another. The coordinated arrivals, with equal inter-arrival times between successive arrivals, are modeled by a degenerated inflow. A second arrival stream represents the uncoordinated random arrivals with stochastic i.i.d inter-arrival times, as is illustrated in Figure 3-3.

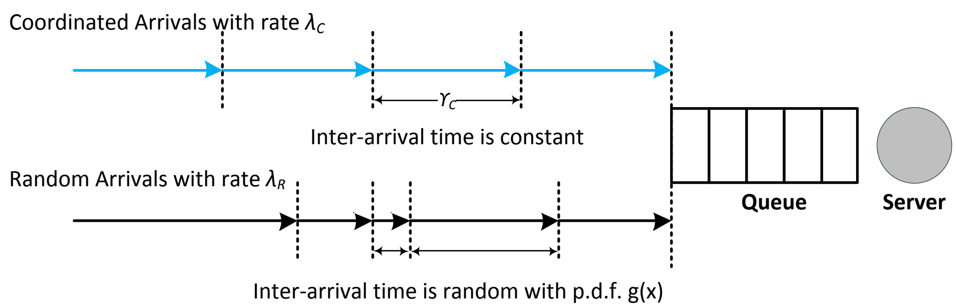

Figure 3-3: Queuing system with partial coordinated arrivals

To assess the performance of this multiple inflow system, the individual inter-arrival processes are merged into one composite inter-arrival time distribution, a so called "superposition". A superposition is only a renewal process itself if each individual arrival process is a Poisson process. The exact determination of the composite inter-arrival time distribution for this particular model, with at least one component without a Poisson distributed arrivals, is complex due to the mutual dependency of the both inflows. Hence, an approximation preferred to find the composite inter-arrival time distribution (Curry & Feldman, 2011).

If the composite arrival process is the superposition of many independent, relatively sparse component processes the superposition approaches a Poisson process as the number of component processes tends to go to infinity (Albin, 1982). However, only two component processes are involved. Albin (1984) developed, based on Whitt (1982), an algorithm capable of analyzing systems with only a few components and different component inter-arrival time distributions, thus more appropriate to use.

3.2 Upper and lower-bound

A system with a compounded arrival stream, based on two sources, holds two extremes; one out of the two arrival streams in responsible for all arriving units, while the arrival rate of the other inflow is equal to zero, and vice a versa. Resulting in either only coordinated arrivals, or only fully stochastic arrivals.

Due to the absence of any randomness in the distribution of the inter-arrival times for controlled arrivals, the extreme of fully controlled arrivals is defined as the lower-bound for the average waiting time in the queue, denoted by $W_q^{\min}$, whereas the case of fully stochastic arrivals is called the upper-bound for the average waiting time in the queue: $W_q^{\max}$.

In case of Exponential service times with rate μ and server utilization ρ, equation (3.1) is used to determine $W_q^{\min}$, whereby u represents the numeric solution of the Laplace Stieltjes Transformation (LST) of the inter-arrival time distribution, given by (3.2) (Appendix A).

$$W_q = \left(\frac{1}{\mu}\frac{1}{1-u}\right) - \frac{1}{\mu} \tag{3.1}$$

$$u = e^{-(1-u)/\rho} \tag{3.2}$$

When the total inflow is stochastic and distributed according to $pdf\ g(x)$, the upper-bound is again given by (3.1), but now the numerical solution u of the LST of $g(x)$ is used (3.3) (Kulkarni, 1999). Given these boundaries for the average waiting time in the queue the results of the algorithm of Albin should satisfy the criteria given in equation (3.4).

$$u = G^*(\mu(1-u)) \tag{3.3}$$

$$W_q^{\min} < E(W_q) < W_q^{\max} \tag{3.4}$$

3.3 Algorithm of Albin

Assuming that the composite stream of arrivals is independent and identically distributed, Albin (1984) introduced a procedure to construct an approximating renewal process for the superposition, approximately solved by the resulting G/G/1 system.

At first, the approximating renewal process is constructed by identifying the moments for the intervals between successive points by using the mean γ_i and variance σ_i^2 of the individual components. Based on both the squared coefficient of variation of the stationary–interval method, c_s^2, the squared coefficient of variation of the asymptotic method, c_a^2, and weight factor ω the resulting coefficient of variation of the approximated renewal interval, c_h^2 is determined.

The determination of c_a^2 is rather straightforward (Appendix B), whereas the stationary-interval method requires more attention, because a convenient distribution must be fitted founded on both component inter-arrival time distribution. For degenerated inter-arrival times, representing the coordinated arrivals, a convenient distribution to fit is an Exponential distribution, shifted by a constant and denoted by $EXPC(h,d)$. The coefficient of variation is zero (no randomness in the arrival process) resulting in simplified expressions for h and d, (3.5).

$$h = 0, \quad and \quad d = \gamma_i \tag{3.5}$$

To fit a distribution to the stochastic, random, part of the inflow, the nature of the underlying inter-arrival time distribution, by hands of the coefficient of variation, is examined. If $c_i^2 < 1$, an $EXPC(h,d)$ distribution is appropriate to use. When $c_i^2 \geq 1$, a Hyper-exponential distribution, denoted by $HVEX(a_1,a_2;p_1,p_2)$, is fitted. Detailed parameter estimations of those distributions and further deviation of c_a^2 and c_h^2 can be found in Appendix B.

Next, when all parameters for the approximated renewal process are known, the expected number of items in the system on an arbitrary moment in time, $E(Q)$, is approached by a *G/G/1* queueing system. The resulting expression for $E(Q)$, assuming general distributed service times with $pdf\, b(x)$ and squared coefficient of variation c_b^2 is determined by (3.6).

$$E(Q) = \frac{\rho}{\mu} + \frac{\rho^2 (c_h^2 + c_b^2)}{2(1-\rho)} \cdot \begin{cases} \exp\left[-2(1-\rho)(1-c_h^2)^2 / 3\rho(c_h^2+c_b^2)\right], & c_h^2 < 1 \\ \exp\left[-(1-\rho)(c_h^2-1) / (c_h^2+4c_b^2)\right], & c_h^2 \geq 1 \end{cases} \tag{3.6}$$

Finally, using Little's Law (Appendix A) and the value for $E(Q)$, the average time spend waiting in the queue is found by (3.7) (Kulkarni, 1999; Albin, 1984; Whitt, 1982).

$$E(W_q) = E(Q) \cdot \lambda^{-1} - \frac{1}{\mu} \tag{3.7}$$

3.4 Verification

To address the validity of the algorithm for our study, a verification study is conducted. The percentage of deterministic i.e. coordinated arrivals, indicated by β, is varied between zero and a hundred percent and numerous configurations are compared to simulation results. The level of coordination is related to the individual component rates and the total arrival rate λ_T as given in (3.8). Moreover, also the validity of the upper and lower bound criteria, stated in (3.4), is verified.

$$\begin{aligned} \lambda_C &= \beta \cdot \lambda_T \\ \lambda_R &= (1-\beta) \cdot \lambda_T \end{aligned} \tag{3.8}$$

Nyen, et al., (2004) argued that low variability of inter-arrival times will influence the reliability of the algorithm. The randomness of the stochastic inflow, denoted by c_i^2 is therefore set fairly low. To simplify calculation over various scenarios, the total arrival rate is set equal to one, letting the mean service time equalize the utilization. Moreover, the random inflow has exponential distributed inter-arrival times, and the service times are exponential too. Table 3-1 summarizes al parameter settings; detailed calculations along with the simulation characteristics are given in Appendix C.

Table 3-1: Parameters verification study

Inflow			***Server***		
i =	λ_i	c_i^2	*b*	μ	c_b^2
C (coordinated)	β*1	0	0,9	1,111...	1
R (random)	(1-β)*1	1			

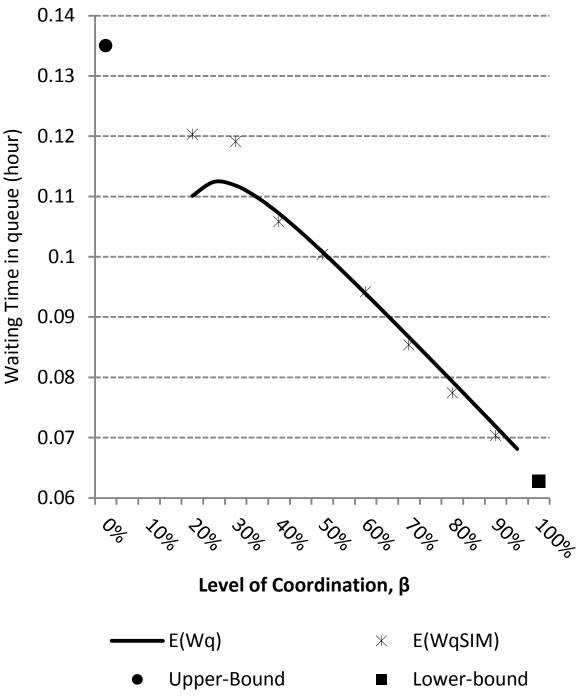

Figure 3-4: Results verification study

Table 3-2: Results verification study

β %	$E\ W_q$	W_q^{SIM}	Δ %
0%	0,1350 (Upper-bound)		
20%	0,1101	0,1203	-8,48%
25%	0,1124		
30%	0,1118	0,1191	-6,12%
35%	0,1098		
40%	0,1072	0,1058	1,29%
45%	0,1041		
50%	0,1008	0,1040	-0.42%
55%	0,0974		
60%	0,0939	0,0942	-0,35%
65%	0,0903		
70%	0,0867	0,0854	1.51%
75%	0,0830		
80%	0,0793	0,0774	2,47%
85%	0,0756		
90%	0,0719	0,0703	2,18%
95%	0,0681		
100%	0,0627(Lower-bound)	0,0616	1,82%

Table 3-2 shows the average waiting times for an arbitrary movement in the queue, obtained by the algorithm and by simulation, plus the percentage difference between both methods, denoted by Δ. First of all, al obtained values lie within the range set by the constraint given in (3.4). Second of all, for a β of 40 percent and higher is the reliability of the algorithm high with a maximum delta (on this particular range) of only 2.47. However, if the value for β drops below 40 percent, counterintuitive results, and ambivalent with the simulation outcome, appeared. From approximately $20\% < \beta < 40\%$ the slope of the graph, given in Figure 3-4, decreases, and when β falls below 20 percent, the value of $E\ W_q$, obtained by the algorithm, is even shortening again, while the randomness in the inflow still increases. A clear explanation for those deviant results is not found, but there are some clarifying allegations; the weight function used by the algorithm to determine c_h^2 is based on extensive simulation, and presumably tapered to a specific domain, e.g. telecommunication and other data networks, consequently less accurate in different environmental settings (Nyen, et al., 2004; Vuuren & Adan, 2007). Neglecting the outcome for low β values, the results of the algorithm are equivalent to the simulation results, and in line with the upper- and lower bound. The model is therefore classified as reliable, but prudence is called for low values of β.

3.5 Average waiting time

Neglecting the erratic results, Figure 3-4 gives a linear effect for the performance of the 'basic' coordination concept; an increased level of cooperation decreases the average waiting time in the queue for the entire inflow. To elucidate the results obtained by the algorithm, the values out of Table 3-2 are transformed by using (3.9), to represent the percentage of reduction in delay for different levels of coordination, compared to a situation without any coordination (the upper-bound).

Moreover, the odd values for β < 40 percent are corrected by using interpolation based on the upper-bound and the outcome for $E\ W_q$ of the algorithm for β > 40 percent, Figure 3-5. This figure shows that using coordination could more than halve the average queueing time i.e. a 54 percent reduction in delay if all movements are coordinated. The strict linear relation rebuts the need for a minimal required participation level in coordination.

$$Reduction\ in\ W_q = \frac{E\ W_q\ - W_q^{max}}{W_q^{max}} \cdot -100\% \tag{3.9}$$

To investigate the influence of different parameters settings, the level of utilization ρ, the squared coefficient of variation of the service times c_b^2, and the squared coefficient of variation of the non-coordinated part of the inflow c_R^2, are altered successively, while leaving other parameters unchanged.

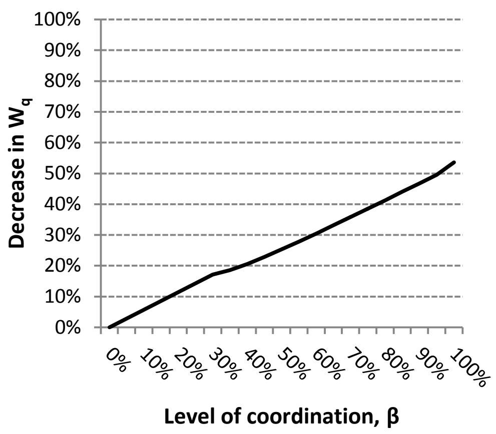

Figure 3-5: General results 'Basic' arrival coordination

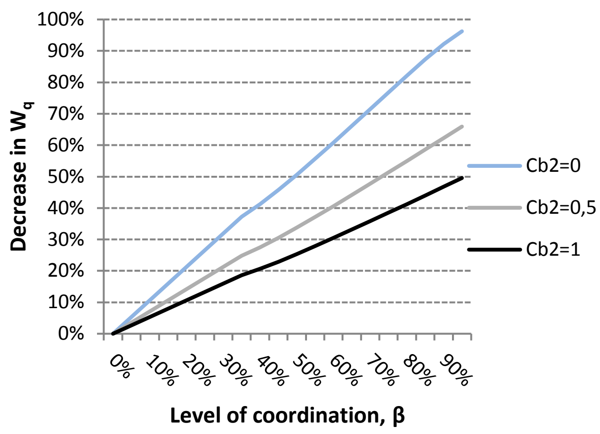

Figure 3-6: 'Basic' arrival coordination: Service distribution

Figure 3-7 gives the reduction in waiting time under different service conditions; the amount of variability in the service process, by means of the squared coefficient of variation of the service time, denoted by c_b^2, is lowered from: one, to a half, and finally to zero, thereby enlarging the benefits of coordination in respect to the time spent waiting in the queue.

The sensitivity analysis of the individual parameters revealed some new reliability issues concerning the methodology of Albin (1984). First of all, when utilization drops, the effect of ambivalent values for low beta values seems to worsens, as illustrated in 3-7. Probably explained by the general applicability of these kind of algorithms; high utilization levels are of particular interest, enforcing algorithms to perform best for highly utilized systems. Correcting the ambivalent values, by means of interpolation, gives Figure 3-8. This figure shows that the percentage reduction of W_q is nearly equal for different utilization levels. Second of all, when the squared coefficient of variance of random arrivals increased to two, a reversed effect as found earlier in section 3.4 appeared; the results also seems to deviate for $\beta > 85\%$, Figure 3-9. Consequently the reliability statement over the algorithm alters since also the average waiting time approximation for β values over 85% is potentially inaccurate. Figure 3-10 shows that, when the values are corrected, the reduction of W_q for a certain level of coordination does not change at all when randomness in the arrival process of uncoordinated movements changes.

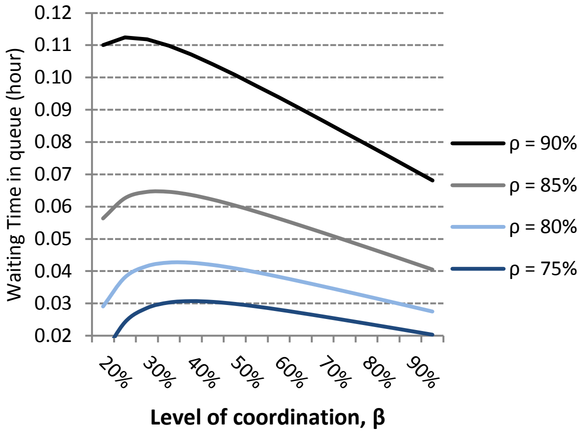

Figure 3-7: Decrease in reliability for different utilization levels

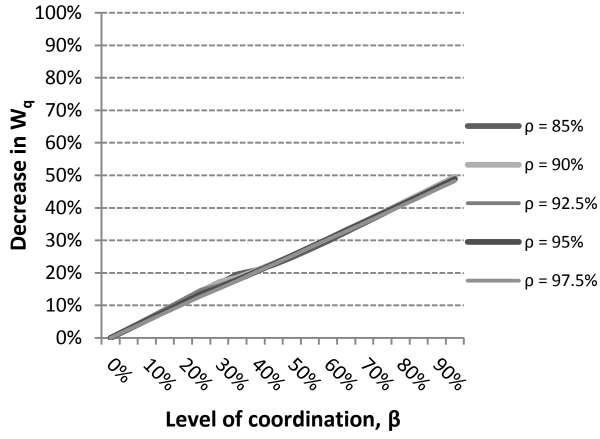

Figure 3-8: 'Basic' arrival coordination: Utilization

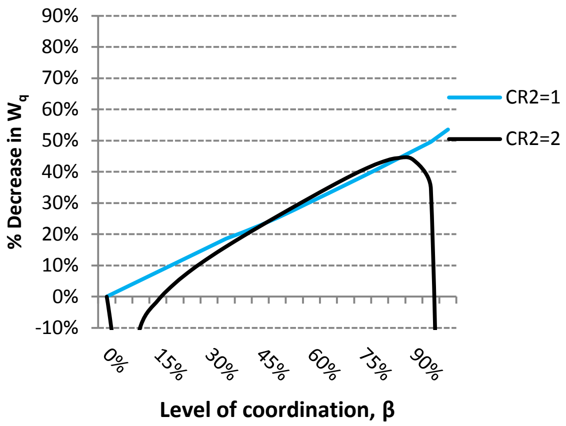

Figure 3-9: 'Basic' arrival coordination: c_R^2 uncorrected

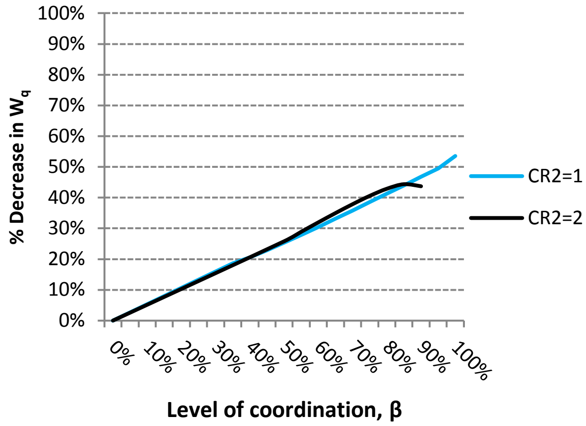

Figure 3-10: 'Basic' arrival coordination: c_R^2 corrected

3.6 Approximation of the component waiting time

The preceding section discussed the average waiting time in the queue for an arbitrary unit, without distinguishing between arrival type: coordinated or a random arrival. For a queue with an arrival process which is the superposition of several individual arrival processes is, in general, the expected delay not equal for arrivals from those different inflow components. Hence, the average delay experienced by coordinated arrivals could deviate from the average delay experienced by non-coordinated movements.

The objective is to investigate if there is a potential difference at all in experienced delay between both components of the inflow, resulting in the use of an approximation approach instead of a more exact simulation study. The methodology used is the 'Holtzman' approach presented in Albin (1986).

The first step of this approach is to establish a basic approximation value for the average delay for an arrival from component *j* by using (3.10). To improve consistency, the outcome of (3.10) is weighted by the relative arrival rate; if V_j is known for all components, the final value for the average waiting time in the queue of arrival from component *j* is given by (3.11). The methodology is described in more detail in Appendix D.

$$V_j = \frac{1}{\mu}\left[\sigma_j / (1-\sigma_j) + \rho_h / (1-\sigma_h)\right] \quad (3.10)$$

$$W_j \simeq (V_j / \bar{V}) . W_h \quad (3.11)$$

3.6.1 Results

The parameters used are equal to those used in section 3.4 and given in Table 3-1. Figure 3-11 gives the results, revealing only a minor difference in the average waiting time between the two components of approximately half minute, i.e. a difference of 5~10 percent. One of the input parameters of this approximation is the average waiting time of an arbitrary unit, affecting the reliability of the since W_h itself is an approximation. The unveiled difference is therefore assumed to be fractional. The 'Holtzman' estimation is founded on a G/M/1 queueing system, taking away the possibility of investigating the sensitivity of the service distribution; the parameter which influences the potential benefits of coordination most.

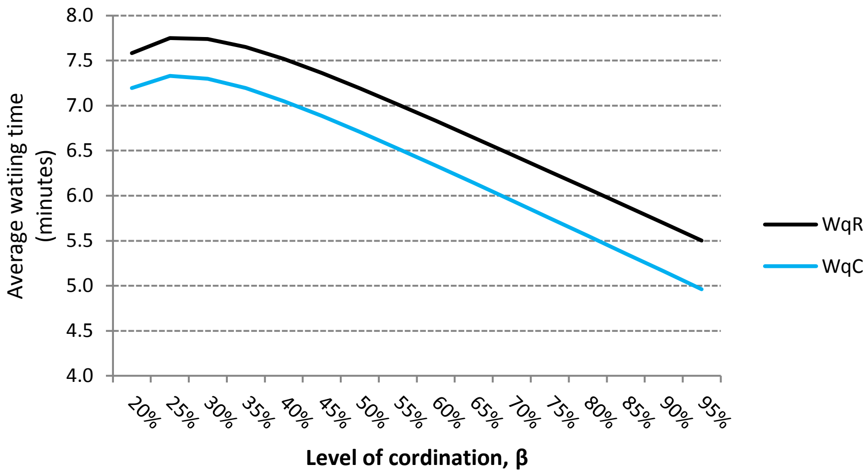

Figure 3-11: Individual Component waiting time

3.7 Conclusion

The 'basic' coordination concept introduced in this chapter is a simplified method of en-route control, realizing equal intervals between successive coordinated arrivals only. The system just accommodates uncoordinated random arrivals between pre-planned coordinated arrivals, without a predefined strategy. The analyses of this coordination concept showed a linear relationship between the level of coordination and the reduction of the average time spend in the queue, waiting to receive service, strongly depending on the service time distribution. If process variability could be reduced, the advantage of coordination rises.

The linear relationship between the level of coordination and average delay supports the feasibility of partial en en-route coordination; due to the lack of a minimal required installed base even 'basic' coordination of a small amount of the total movements will reduce the average delay of an arbitrary unit. Study of the component waiting times showed a minor difference between the individual waiting time reductions, giving an advantage to uncoordinated movements; en-route coordination reduces the experienced delay of all movements i.e. improving the entire system performance.

By adapting the inter-arrival time distribution, the ‘basic’ en-route coordination concept absorbs the expected waiting time in the queue during the trajectory i.e. delay is replaced by slowing down en-route. The potential savings are therefore equal to the cost difference between waiting in the queue and slowing down in the trajectory plus the benefits obtained by the enhanced predictability of downstream operations

To verify the result given by algorithm, used to examine the performance of the ‘basic’ coordination concept, a simulation study is conducted. In general the result obtained by the algorithm deviated only minor compared to the simulation results. Nonetheless, for low utilization levels and on the outer ranges of β the reliability of the algorithm declines. To overcome these difficulties interpolation (Appendix A) could be used, guided by the given performance boundaries of section 3.5.

4 'Extended' arrival coordination

In this chapter an extended arrival coordination concept is introduced which distinguishes between random and coordinated arrivals. Section 4.1 starts with an introduction and discussion of the methodology used to assess the performance of this 'extended' arrival coordination concept as well as an outline of the most important parameters. Next, a detailed analysis, regarding the advantages for coordinated arrivals is presented in section 4.2. In section 4.3 two handling strategies for uncoordinated arrivals are given, accompanied with an elaborated overview of the (dis)advantages for (un-)coordinated movements. The chapter concludes with a statement of all procured insights in section 4.4

4.1 Methodology

The conceptual model of 'extended' arrival coordination is closely related to a 'non-preemptive priority single server queueing systems' with deterministic arrivals for 'high' priority jobs, and stochastic inter-arrival times for the 'low' priority job. The difficulty of this priority queueing system is generated by the discrepancy in the statistical nature of both arrival streams. Many exact results are available for priority systems with Poisson arrivals. The amount of literature concerning priority queueing with a mixture of general arrivals is nevertheless quite limited.

Horvath (2005) proposed a matrix analytic approximation to address the waiting time in a G/G/1 queueing system with multiple priority classes. However, the applicability of this methodology is limited because it requires a squared coefficient of variation larger than 0.5 for all individual inter-arrival time distributions. Stanford (1997) analyzed the inter-departure and waiting times of a priority queueing system with a mixture of Poisson- and general arrivals; an arbitrary number of high priority Poisson-arrival classes and one lowest priority general arrival class, but not vice versa.

Vuuren & Adan (2007) presented a framework to analyze general preemptive priority queues; by decomposing the system into single queue vacation systems, the visit and inter-visit distributions where approximated by the idle and busy periods of a G/G/1 queue. Remark that the highest priority class is never interrupted, and could therefore be analyzed as a single G/G/1 queue without vacations.

The use of 'slots' implies that, besides single queue vacation systems, the use of a Fixed-Cycle-Traffic-Light (FCTL) queue approach will also be appropriate to address the waiting time distributions of the individual decomposed queues. Contrary to the rich availability of solutions to address the average delay in a FCTL queues, it is tough to examine the effect of different service time distributions and utilization levels by using this approach, raising difficulties to address all system characteristics under varying conditions.

The presence of only two priority classes and exact knowledge about the arrival times and busy period of the 'high' priority jobs encourage the use of the concept presented by Vuuren & Adan (2007), adapted for a non-preemptive priority policy: A single period of coordinated arrivals, composed of N_C individual jobs, is analyzed like a D/G/1 single queue system. The average waiting time of a random arrival is approximated by a single queue vacation system whereas the length of a vacation period equals the length of a busy period of serving high priority arrivals.

The setup of the extended coordination concepts depends basically on four parameters (neglecting the level of coordination): length of a 'closed' slot $t_{s,C}$, length of an 'open' slot $t_{s,R}$, and the amount of continuous open and closed slots N_C and N_R,. An initial value for N_c and N_R is found by estimating the expected number of coordinated and random arrivals in per time window and could vary depending on the known fluctuation in demand. The value of the other two parameters depends on the chosen strategy for handling 'randoms', section 4.3.

Directly related to these parameters are the length of a period reserved for coordinated arrivals $T_{s,C}$, and the length of a period reserved for random arrivals $T_{s,R}$, Figure 4-1. Besides the determination of these four parameters, several strategic issues must be assessed before actual operationalization of the concept could be realized. For example: if there are currently no random arrivals in the system, is it justified to start a period reserved for handling those random arrivals?

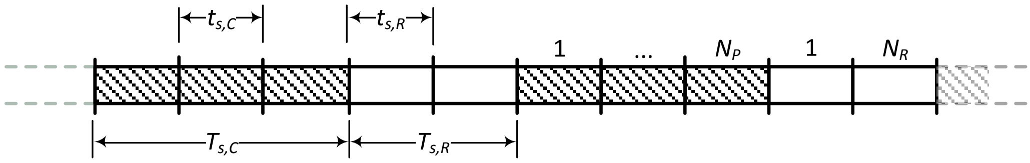

Figure 4-1: Extended Arrival Coordination Concept: Parameters

4.2 Coordinated Arrivals

The moment of arrival for coordinated arrivals is known, resulting in the exact determination of inter–arrival times and the length of the busy period since all coordinated arrivals are logically processed in reserved slots. The disturbing effect of random arrivals, section 3.1, is absent. Queueing is therefore only caused by the unpredictability in service times. The difference between the standard D/G/1 queuing model and an appointment system introduced earlier in this chapter is the use of slots. A slot is a single timeframe reserved to handle one customer, longer or equal to expected service time, Figure 4-2. The slack-time within a slot is directly related to the delay experienced by a new arrival i.e. the system's ability to cope with the uncertainty in service times. By using a single slot as unit of analyses, the arrival rate of the coordinated part of the inflow λ_C, is no longer involved in the performance assessment, but only substantial to determine the number of consecutive slots.

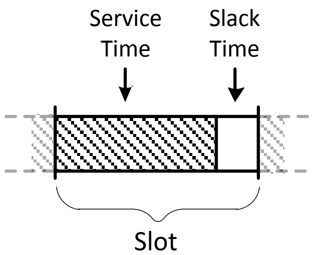

Figure 4-2: Slot structure

The utilization could normally be found by dividing the arrival rate by the service rate, while due to the use of slots, utilization now depends on service rate and slot length. The resulting utilization of the coordinated arrivals, ρ_C can be found by (4.1)

$$\rho_C = \frac{1}{t_{s,C} \cdot \mu}$$
$$with:$$
$$\rho_C < 1 \quad and \quad t_{s,C} > \frac{1}{\mu} \tag{4.1}$$

Lengthening the slot time will drop the utilization. The extra time allocated to handle coordinated arrivals consequently reduces the slack time for processing random arrivals. The operational utilization of the entire system ρ_O therefore limits the length of a slot for coordinated arrivals, because the system must be stable i.e. the accumulated arrival rate is less than the overall service rate. For a given coordination level β the operational utilization is given by (4.2) (Appendix E). Rewriting this equation using $\rho = 1$ (i.e. full utilization), the maximum slot length for coordinated arrivals is given by (4.3)

$$\rho_O = \lambda_T \left((1-\beta) b + \beta t_{s,C} \right) \tag{4.2}$$

$$\max t_{s,C} = \frac{\lambda_T^{-1} - (1-\beta) b}{\beta} \tag{4.3}$$

To pursuit general applicability of the en-route coordination concepts an approximation, based on a G/G/1 queueing system, is used to support a wide range of service distributions. Besides, the service time distribution is highly correlated to the average waiting time under a 'basic' arrival coordination policy. To reckon for this effect under an 'extended' coordination policy also a methodology, adaptable to multiple service distributions, is required.

The approximation used is the Krämer/Langenbach-Belz, and due to the deterministic inter-arrival time distribution, given by (4.4). This approximation is easy applicable and performs well (Appendix E) (Bolch et al., 2006).

$$W_q^C \approx \frac{\rho_C / \mu}{1-\rho_C} \cdot \frac{c_C^2 + c_b^2}{2} \cdot \left\{ \exp\left[-\frac{2}{3} \cdot \frac{1-\rho_C}{\rho_C} \cdot \frac{(1-c_C^2)^2}{c_C^2 + c_b^2} \right] \right\} \tag{4.4}$$

The queuing time determined by this approximation must be considered as a reasonable estimation. In a D/G/1 system, with a continuous flow of arriving units, delay is primarily caused by statistical fluctuations of the service time. The average delay calculated by (4.4) is the long-term average. Concerning coordinated arrivals, the sequence of consecutive arrivals is limited to an amount N_C, which could possibly affect the actual experienced delay; if $N_C = 2$, the first arrival will, ideally, not encounter any delay, and the delay for the second arrival depends only on the required service time of the first arrival, average delay will be minimal. A second example, an extreme value for the service time will have major consequences in a short sequence, due to the small probably of another extreme value in opposite direction. Hence, if N_c is relatively small, it is more likely that actual experienced delay will deviate from the long-term average due to (1) the relative large effect of the first arrival of the series and (2) ability to 'smoothen' the presence of extreme values.

4.2.1 Results

Neglecting the correlation between W_q^R and $t_{s,C}$, using maximum slot length gives the best possible performance in terms of waiting time reduction for coordinated arrivals. The reduction in delay will become less as the level of coordination rises; the amount of slack available in the system must be shared by more slots, lowering the amount of slack per individual slot and consequently reduces the ability to deal with uncertainty. In case of full coordination, the reduction in waiting time will be equal to the savings obtained by a hundred percent 'basic' coordination.

Using $\max t_{s,C}$ eliminates all slack in handling non-coordinated arrivals, leading to full utilization (100 percent) when handling random arrivals, and therefore ends up in tremendous delays for that type of arrivals. Even if $t_{s,C} \neq \max t_{s,C}$, the close relation amongst the 'closed' slot length and the delay for uncoordinated arrivals makes it unappealing to analyze the delay under an 'extended' coordination policy separately for both coordinated and uncoordinated arrivals.

The previous debate on the relation of 'closed' slot length and the consequences for W_q^R , revealed nevertheless and incentive for a new constraint concerning extended arrival coordination, (4.5). To prevent the occurrence of excessive delays for non-coordinated arrivals $W_q^{R,\max}$ is a predefined maximum for delay encountered by random arrivals due to an extend arrival coordination policy

$$W_q^R \leq W_q^{R,\max} \tag{4.5}$$

4.3 Uncoordinated arrivals

The extended coordination policy gives priority to coordinated arrivals. For that reason, random arrivals with general distributed inter-arrivals times have to wait for an 'open' slot to receive service. That is, a random, uncoordinated, arrival will frequently find the server 'blocked' due to high priority 'coordinated arrivals'. Such a server blockade is called a 'vacation' period with length *v*, with a length equal to a period of consecutively serving coordinated arrivals (a 'busy period'). Using slots for handling coordinated arrivals, the exact length of a busy period for coordinated arrivals, i.e. *v*, is given by (4.6)

$$v = N_C \cdot t_{s,C} \tag{4.6}$$

Moreover, due to coordination, the start of a vacation period is exactly known, allowing for accurate regulation of the vacation intervals. But also the other way round, since the start and end time the period reserved for handling random arrivals is dictated by the period reserved for handing coordinated arrivals, $T_{s,R}$ is set accurately. The accompanying slot length for 'randoms', $t_{s,R}$, is set equal to the expected service time, preventing any benefit for uncoordinated arrivals under 'extended' arrival control. This statement could nevertheless be relaxed on a later moment if it appears that the available slack time is sufficient to prevent delay for coordinated arrivals.

The actual handling of random arrivals can either be based on an exhaustive, or a non-exhaustive service policy; prescribing the length of the period reserved for handling random arrivals, and the amount of consecutive arrivals served, $T_{s,R}$ and N_R respectively. In case of a non-exhaustive policy the length of a period reserved for 'randoms' is limited by a predefined number of k service iterations, Figure 4-4a. On the contrary, when using an exhaustive service policy, all random arrivals currently in the system (or just arriving) are served before the server switches over to handling coordinated arrivals i.e. the start of a new 'vacation' period, Figure 4-4b. This will, intuitively, affect the benefit for coordinated arrivals.

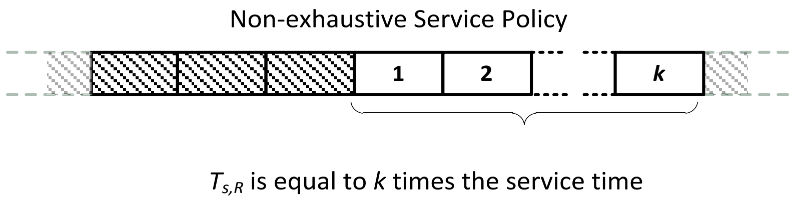

Figure 4-4a: Non Exhaustive Service Policy: k-limited

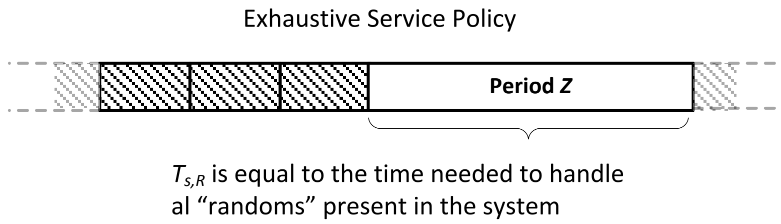

Figure 4-4b: Exhaustive Service Policy

4.3.1 Non-Exhaustive Service Policy

In the non-exhaustive service system the maximum number of service iterations of random arrivals, before the start of a new vacation period, is limited to a preset number k, after which the server switches over to serving pre-planned coordinated arrivals (Note: A comparable system using a time limit instead is not possible due to the non-preemptive service condition). If the number of random arrivals in the system is less than k, the system stays idle or/and handles new arriving 'randoms' till the planned start of a period reserved for handling coordinated arrivals. The performance of this system is approached by an E-Limited Service (ELS) vacation queueing system. In an ELS system, the server continues to serve until (1) k messages (including new arrivals) are served, or (2) the system empties, whichever occurs first. The difference between the modeling approach and the actual 'real' system is the state of action when the system becomes empty. The ELS system assumes that, when the system becomes empty, the server will start a new vacation. While in the 'real' system a changeover to coordinated arrivals will not take place before the planned arrival of a 'high' priority job, even if the system is empty. To overcome this issue, the limiting value k must be chosen relatively low to put 'pressure' on the system, minimizing the probability of less than k uncoordinated arrivals per service period (Takagi, 1991).

An exact analysis, using classical transform models, is possible for the ELS, but the computation of performance measures will involve finding k roots of a polynomial equation, combined with the need to analyze multiple configurations gives rise to the use of an approximation method.

Based on the statistical decomposition theorem (Fuhrmann & Cooper, 1985), which state: “the (stationary) number of customers present in a vacation system is the sum of two independent random variables, one of which is the (stationary) number of customer present in the corresponding standard M/G/1 queue” Zhang & Vickson (1993) approached the number of customer by (4.7) whereas v and $v^{(2)}$ represent the expected length and the second moment of the vacation period, respectively, EY is the expected number of customers present in the system at the server’s departure instant and is given by (4.8). $\bar{N}$ is the expected number of customers in the corresponding standard M/G/1 system (Appendix A). By using Little’s law, equation (3.7), the average waiting time in the could now be obtained.

To guarantee stability, a minimum number of customers must be served in the period between server vacations, see equation (4.9). This minimum number is also a guideline to ensure the earlier mentioned system ‘pressure’.

$$E[Q] = EY + \frac{\lambda_R v^{2}}{2v} + \bar{N} \tag{4.7}$$

$$EY \approx \left(\frac{1-\rho_R}{1-\rho_R-\frac{\lambda_R v}{k}} \right) \left(\frac{\lambda_R v^{2}}{2v} + \bar{N} \right) \left(\frac{\lambda_R v}{k^{2-\rho_R^2/2}(1-\rho_R) - (k^{1-\rho_R^2/2}-1)\lambda_R v} \right) \tag{4.8}$$

$$k > \frac{\lambda v}{1-\rho} \tag{4.9}$$

4.3.2 Results Non-Exhaustive Service Policy:

This section presents the results of the average waiting time in the queue for coordinated and random arrivals, W_q^C, W_q^R , respectively, under a non-exhaustive ‘extended’ coordination policy. To ensure a fair comparison between the different coordination concepts in this thesis, the service and arrival rates are set equal to those introduced in section 3.4, Table 3.1 (the utilization under standard conditions ρ_s, is 90%). A cycle of twenty minutes is considered, i.e. a total of twenty arriving units, with N_C and N_R based on the expected amount of arrivals, given a level of coordination β . The maximum delay for random arrivals, $W_q^{R,\max}$, is limited to 30 minutes, acting as a second restriction on slot length for coordinated arrivals, besides equation (4.3). A graphical presentation of the results is given in Figure 4-5. The companion table can be found in Appendix G.

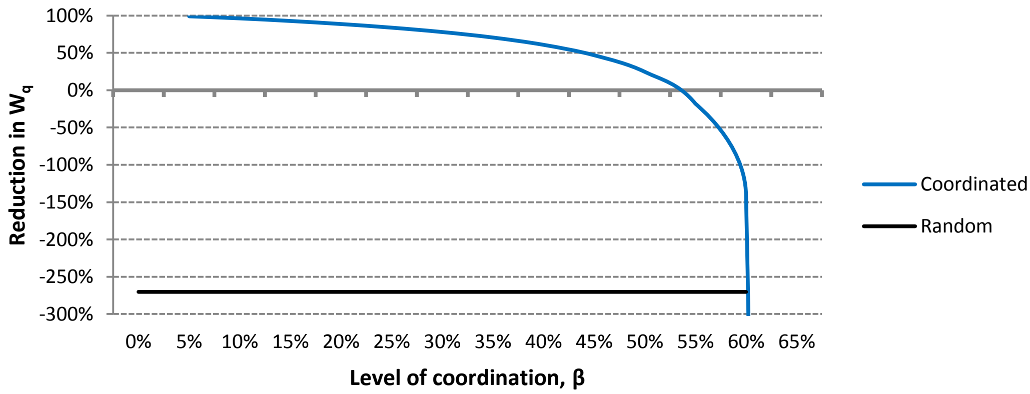

Figure 4-5: Non-exhaustive extended coordination

For relatively low values of coordination, delay is almost completely dissolved. The benefit of coordination, however, fades away when the level of coordination rises. When the level of coordination rises above 55% percent, a 'tipping point' arose from where the performance becomes worse compared to a situation without any coordination. The average waiting time in the queue for a random arrival is equal to the preset limit of 30 minutes, irrespective to the level of coordination, decreasing the average overall system performance.

W_q^R is equal to thirty minutes because the objective of the algorithm is to decrease the waiting time for coordinated arrivals as much as possible. The amount of slack time is not sufficient to completely reduce the delay for coordinated arrivals, therefore $t_{s,C}$ is maximized subject to $W_q^{R,\max}$ and $\max t_{s,C}$ whereby, in this particular case, $W_q^{R,\max}$ operates as the limiting factor. Moreover, lowering the value of k is for this reason also not desirable.

Contrary to basic coordination, the level of utilization will affect the performance of 'extended coordination, Figure 4-6 (Appendix F). This is explained by the direct relation between utilization, and the amount of available slack to increase slot length for coordinated arrivals; when the utilization rises, the benefit obtained by the extended coordination concept reduces because the amount of slack time available for lengthening $t_{s,C}$ reduces as the utilization rises.

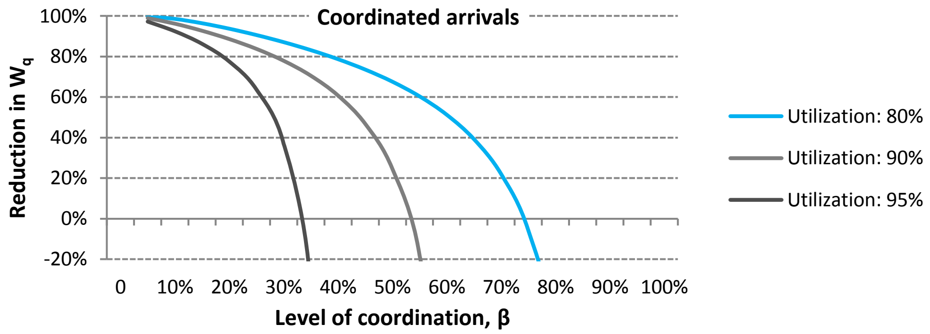

Figure 4-6: Non-exhaustive extended coordination: Utilization

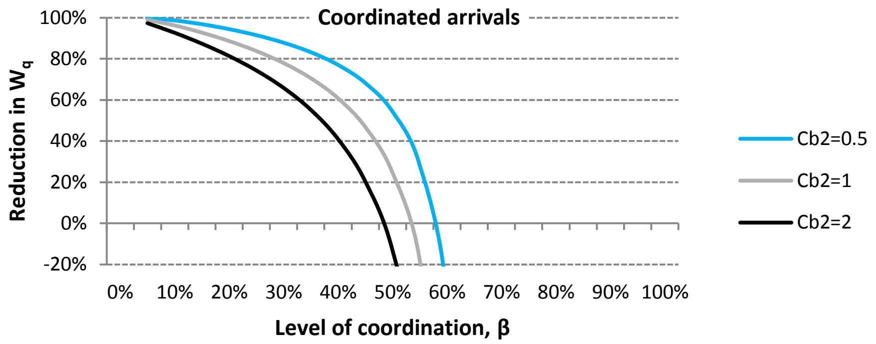

Figure 4-7: Non-exhaustive extended coordination: Service distribution

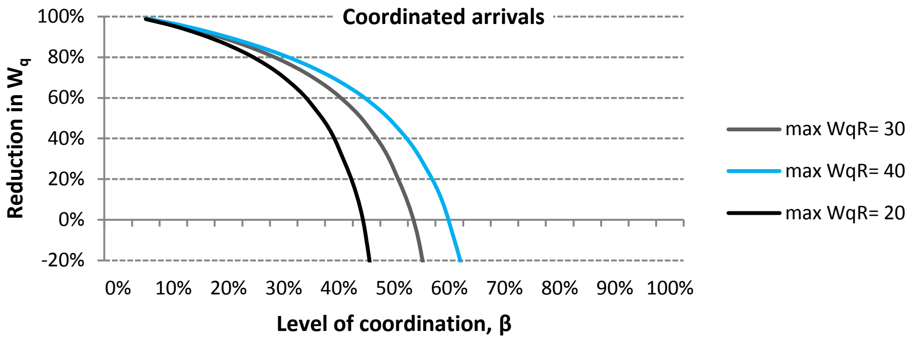

Figure 4-8: Non-exhaustive extended coordination: Limit W_q^R

The influence of the service distribution is given in Figure 4-7; discrepancy in performance between service distributions is highest just before the effect of coordination fades away. The squared coefficient of variation of the service distribution is not an input parameter of the formulas mentioned earlier in section 4.2 and 4.3.2, therefore Appendix A is used to convert c_b^2 to the companying second moment. Figure 4-8 shows the results of non-exhaustive extended coordination with varying limits on the maximum average delay assigned to 'randoms'; when the level of coordination increases, the difference in performance between the varying limits increases as well. Hence, the previous sensitivity analysis indicates that the performance of a non-exhaustive extended coordination policy heavily depends on individual parameter settings.

4.3.3 Exhaustive Service Policy

When using an exhaustive extended coordination policy, the server takes only a vacation i.e. starts serving coordinated arrivals, if all uncoordinated arrivals currently in the system are served. The expected length of the period reserved for serving random arrivals, identified as 'period z' with length $T_{s,R}$, is therefore equal to the expected length of a busy period serving random arrivals. The framework used to address the performance of this policy is a classical multi vacation model with the corresponding average waiting time given in (4.10). The length of 'period *z*' is equal to the length of the busy period, given by (4.11) with $\rho_R = \lambda_R / \mu$ (Tian & Zhang, 2006).

$$E(W_q) = \frac{\overline{v^2}}{2\bar{v}} + \frac{\lambda_R \overline{b^2}}{2(1-\rho_R)} \quad (4.10)$$

$$T_{s,R} = E(D_v) = \frac{\rho_R \bar{v}}{(1-\rho_R)(1-v^*(\lambda_R))} \quad (4.11)$$

The methodology used raises two difficulties, affecting the reliability of the results. (1): The classical multi vacation model assumes that, if no customers are present in the system, the next vacation will start immediately. This assumption is violated by the actual model since the start of the next vacation model is planned in advance, and could therefore not start early. (2): The length of the busy period is an 'expected value' based on the probability distribution of the inflow and the service times, and is therefore subject to vary. Moreover, the non-preemptive service assumption prevents the start of a new vacation period halfway through service. As a consequence, the actual length of the busy period could exceed the expected length, delaying all subsequent coordinated arrivals, see Figure 4-9

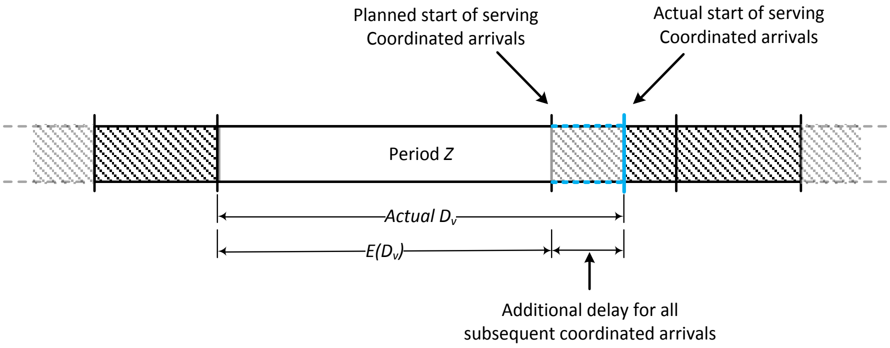

Figure 4-9: The actual D_v exceeds the expected D_v

To account for the first reliability issue: the start of a new vacation period directly after the preceding vacation period could be prevented if there is at least one customer in the system at the end of the vacation period, $P\left(X > v(0)\right)$ The probability of at least one arrival at the end of a vacation period is examined using (4.12), assuming Poisson arrivals. If this probability appears to be relatively low, enlarging of consecutive number of 'closed' slots is required (Montgomery & Runger, 2002).

$$P\left(X > v(0)\right) = 1 - e^{-\lambda_R v} \tag{4.12}$$

To prevent additional delay for coordinated arrival, due to the offset of the planned start of serving coordinated arrivals, buffer time, with length ξ, could be added. This 'buffer' is a time period between the expected end of 'Period *Z'* and the planned start of a period reserved for handling coordinated arrivals, and in which no service actions are pre-planned. Moreover, random arrivals, entering the system during a time window classified as 'buffer', are not serviced and have to wait till the next 'open' period arrives. If 'Period *Z*' overruns its expected length, the buffer absorbs, to some extent, this supplementary time, preventing additional delay for coordinated arrivals.

Determination of the buffer length is difficult; the probability of a delayed start of serving coordinated arrivals depends on the distribution of the busy period i.e. the length of 'Period *Z'*, which is generally assumed to be difficult[5], even for a standard M/M/1 queue. In addition, adding a buffer will affect the 'entire' system; length of 'Period *Z'*, closed slot length, and therefore the probability of exceeding the planned start time due to the mutual dependency between buffer size and length of 'Period *Z'* e.g. $P\left(X > Period\,Z^{(1)}\right) \neq P\left(X > Period\,Z^{(2)}\right)$, Figure 4-10.

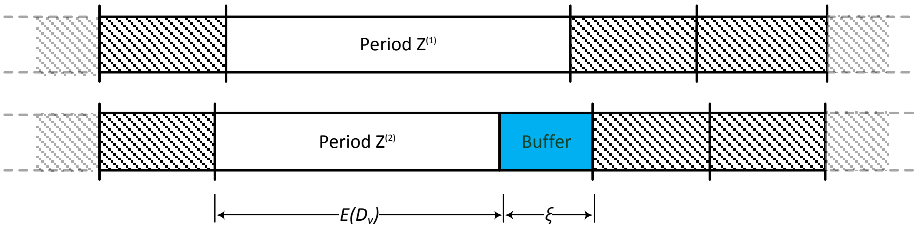

Figure 4-10: Relation between buffer time and length of Period *Z*

[5] Source: Prof. dr. ir. I.J.B.F. Adan, Eindhoven University of Technology

4.3.4 Results Exhaustive Service Policy

This section presents the result for the average waiting in the queue for coordinated and random arrivals under an exhaustive 'extended' coordination policy. The input parameters are again equal to those used in section 3.5 (Table 3-5). Closed slot length is maximized, reckoned with limitations due the limited maximum delay for 'randoms' and operational utilization. Actual maximization of the 'closed' slot length could be achieved by gradually increasing $t_{s,C}$ till either the constraint on W_q^R or on $t_{s,C}$ is violated.

The graph presented in Figure 4-11 gives the impression that exhaustive coordination benefits up to 85% for coordinated arrival, before fading away, and also gains a performance advantage for random arrivals up to a level of 70% coordination; the delay for 'randoms' is far below the constraint $W_q^{R,\max}$, which was set on thirty minutes (equal to the non-exhaustive case). The graph of Figure 4-11 gives, however, a biased view. The actual length of 'Period *Z'* is uncertain, affecting the start of serving coordinated arrival, which could lead to additional queueing time for coordinated and possible, even for random arrivals (Appendix G).

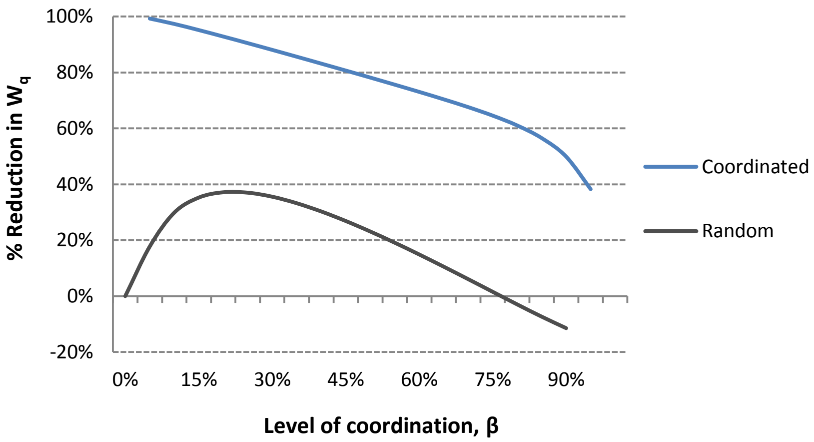

Figure 4-11: Exhaustive extended coordination

4.3.4.1 Fitting Process

The magnitude of the uncertainty, and the consequences for the reliability of the exhaustive 'extended' coordination model, are addressed by closely examining the variability in length of 'Period Z'. The length of 'Period Z' is equal to the busy period of a classical multi vacation model. The variability in the busy period length depends on the associated busy period distribution. Due to the complexity of this distribution, a phase type distribution is fitted, based on the first two moment of busy period, to represent the actual busy period distribution.

The first moment is equal to the expected value for the busy period, $E\ D_c$ and the second moment is given by (4.14), using the first two moments of the service time distribution and the first two moments of the number of arrivals during interval $D_c + T_{s,C}$ denoted by $\theta_{RA}, \sigma_{RA}^2$ respectively (Ross in Silver et al., 1998).

$$\text{var}\ D_c = \sigma_{RA}^2 \mu^2 + \theta_{RA} \sigma_b^2 \qquad (4.13)$$

The inter-arrival times of the uncoordinated arrivals are exponential distributed, thus the number of arrivals per interval follow a Poisson process. Therefore, $\theta_{RA} = \lambda_R (D_c + T_{s,C})$ and $\sigma_{RA}^2 = \theta_{RA}$, resulting in a coefficient of variation less than one, independent of the level of coordination (Appendix G, Table G-2).

An appropriate distribution to fit on these parameters (based on the coefficient of variation) is an Erlang*(k-1, k)* distribution. The cumulative distribution function, $E_{k-1,k}(x)$ describes the probability that a real valued random variable *x* will be found at a value less than or equal to *x*. The probability of a delayed start for coordinated arrivals is therefore given by (4.15) (Appendix A).

The probability of 'Period *Z'* overrunning its planned start time is substantial, ranging from 57 to 24 percent, Table 4-1. (Adan & Resing, 2002; Houtum, 2007)

$$P(X > Period\,Z) = 1 - E_{k-1,k}(D_c) \tag{4.14}$$

Table 4-1: Probabilty of a delayed start of Coordinated arrivals (Appendix G)

β	*P(X>Period Z)*	*β*	*P(X>Period Z)*
5%	57%	55%	52%
10%	57%	60%	51%
15%	57%	65%	50%
20%	57%	70%	49%
25%	56%	75%	48%
30%	56%	80%	45%
35%	55%	85%	42%
40%	54%	90%	37%
45%	54%	95%	24%
50%	53%		

Table 4-2: Probability of at least one arrival during a vacation period

β	*P(X>0)*	*β*	*P(X>0)*
5%	92%	55%	52%
10%	96%	60%	51%
15%	98%	65%	50%
20%	99%	70%	49%
25%	99%	75%	48%
30%	99%	80%	45%
35%	100%	85%	42%
40%	100%	90%	37%
45%	100%	95%	24%
50%	100%		

To re-open the first reliability issue, concerning the absence of an random arrival at the end of a 'vacation' period. The probability of no random arrival during a period reserved for coordinated arrivals is negligible, Table 4-2, and will therefore have almost no effect on the reliability of the prosed methodology.

4.3.4.2 Exhaustive Extended Coordination with buffer

As discussed earlier in section 4.3.3, buffer time ξ could be added to decrease uncertainty, reducing the probability on additional delay for coordinated arrivals: $P(X > Period\,Z + \xi)$. The actual length of the buffer time depends on individual preferences i.e. the 'cost' of uncertainty. General results of adding buffers are therefore not available; however an example is given to illustrate the difficulties of the circular argument while determining the buffer size (Figure 4-10) followed by a second example which shows the opportunity of adding buffers.

> ***Example 1:***
> Assume 45 percent of coordination and an accompanying probability of additional delay for coordinated arrivals of 54 percent (Table 4-1). Decreasing the probability of overrunning 'Period *Z'* to 20 percent requires solving equation (4.15), giving $\xi = 4{,}71842$. Actual implementation is nevertheless not possible due to utilization constraints. The required buffer size strokes with the time needed to serve all units.

$$1 - E_{k-1,k}(D_c + \xi) \geq 0{,}2 \tag{4.15}$$

A viable buffer size is 3, corresponding to $P\{X > Period\ Z\} = 0,35$. At implementation of this buffer, the length of 'Period *Z'* changes, altering the probability of additional delay for coordinated arrivals into +/- 27 percent (Table 3-3). Which gives as it is, rise to a smaller buffer size to obtain a reliability level of 0.35 percent. Moreover, note that buffer length is directly related to amount of slack available for handling coordinated arrivals. The addition of a buffer could decrease the average performance of coordinated arrivals, which is inconsistent with the intention is to improve performance. Regarding the example, W_q^C rises from 1.56 to 7.87 min, almost vaporizing the benefit of coordination (Table 4-3).

Table 4-3: Exhaustive 'extended' coordination with (out) buffer

General Parameters

	β	ξ	W_q^C	W_q^R	D_C	$v^{(1)}$	$Var(D_c)$	c_x^2	c_x
No Buffer	45%	0	1,56	5,92	9,92	10,08	17,82	0,18	0,426
Buffer	45%	3	7,87	5,16	8,45	8,55	15,15	0,21	0,460

Fitting Parameters

	k	$1/k$	$1/k-1$	q	λ	$E_{k-1,k}(D_c)$	$1-E_{k-1,k}(D+(\xi)_c)$
No Buffer	6	0,167	0,200	0,282	0,520	46%	54%
Buffer	5	0,200	0,250	0,155	0,518	73%	27%

Example 2:

The opportunity of applying an exhaustive coordination policy with buffers is illustrated by means of Figure 4-12. This figure gives the possible reduction in delay for a non-exhaustive policy, and exhaustive policy without a buffer and an exhaustive policy with buffer. The buffer size is set per level of coordination, using trial and error, to reach a certain performance level, neglecting $P\{X > Period\ Z\}$.

The red arrow shows the opportunity of using a buffer: At a coordination level of 55 percent, the benefit of non-exhaustive coordination is almost negligible. Contrary, an exhaustive coordination policy, realizes at a coordination level of 55 percent, an uncertain saving in queueing time of almost 80 percent. By adding buffer time, uncertainty could be curtailed to some extent, while still considerable savings are possible; the addition of buffer time will enforce the blue line to move up and down on the range indicated by the red arrow. Simultaneously, the uncertainty in the average queueing times rises and falls. Hence, if one is willing to coop with a certain level of uncertainty, 'extended' arrival coordination could be beneficial, extending the feasibility range of 'extended' en-route coordination, up to 75 percent.

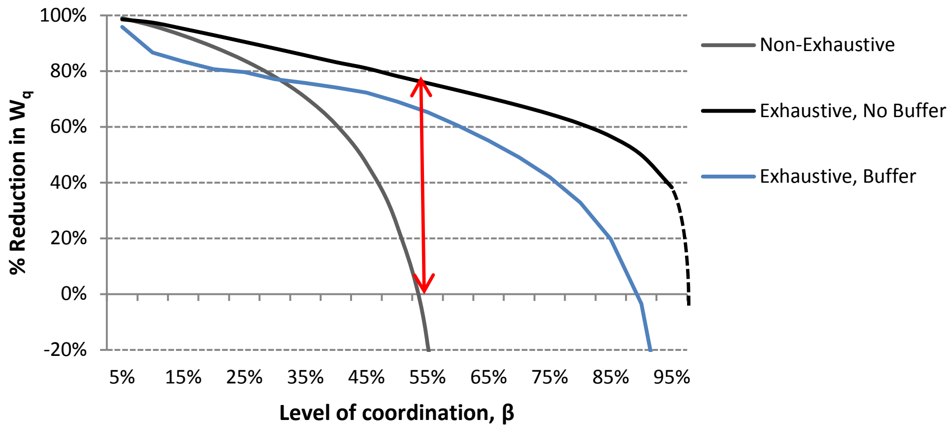

Figure 4-12: Opportunity of exhaustive extended coordination with buffer (Appendix G)

4.4 Conclusion

The extended coordination concepts introduced in this chapter uses en-route coordination to assign arrivals, based on their type, either to a set of consecutive 'closed' slots reserved for coordinated arrivals or a set of 'open' slots designated to accommodate random arrivals. Two policies where considered; an exhaustive policy and a non-exhaustive policy. In the latter case, the number of random arrivals served during a period reserved for handling random arrivals is limited to a certain number *k*. Coordinated arrivals experienced large benefits of this policy up to coordination levels of 55 percent, after which the benefit fades away. Uncoordinated arrivals had no benefit; their waiting time rises to a constant predefined maximum. It is therefore like that the overall system performance, based on the average queueing time for both coordinated and random arrivals, decreases compared to situation without coordination.

A sensitivity analysis revealed a large dependency on individual parameter settings. This will be important for future operationalization and implementations. Moreover, the increase in delay for non-coordinated movement will hugely influence operational feasibility, erecting resistance by non-coordinated movement. It is therefore important to closely examine the consequences of $W_q^{R,\max}$ for both coordinated and random movements.

An opposite effect was found by using an exhaustive extended coordination policy; the overall system performance increased, but yielded a considerable amount of 'uncertainty'. The planned start of serving coordinated arrivals could be delayed with probabilities up to 50 percent, adding additional waiting time to coordinated arrivals. Buffers could be introduced to deal with this issue, but appears to be a difficult due to the circular argument, and in relation to the average waiting time for coordinated arrivals. Hence, it could be argued if actual implementation of exhaustive 'extended' coordination will be feasible.

The conclusion, based on the results in this chapter, is that introducing an extended en-route coordination policy will not require the participation of all arrivals. Even a small amount of coordination could benefit, reducing delay almost completely with a coordination level between 5 to 35 percent. One could even argue that relative low levels of coordination are more desirable compared to substantial participation in the en-route coordination concept. Moreover, by the use of non-exhaustive policy a clear distinction could be made between arrivals who sign up to participate in a coordination concept, and those who do not, alluring the investment.

To be capable to address the feasibility of the 'extended' coordination concepts, it became a requirement to develop a methodology to assess the performance of an appointment system with advanced scheduling and a multi-distributed inflow. The development of this new approximation is thus a necessary enlargement of the initial research assignment, and therefore not extensively verified by means of a simulation study as customary. All components of the estimation are nevertheless verified by literature and checked for illogical results.

From a general (scientific) perspective, the major contribution of this chapter is therefore the unveiling of the existence of a 'tipping point' from which the advantage of coordinates fades away. A more accurate determination of the moment the advantage of coordination omits gives rise to future work. For example, using polynomial values to determine the length of the vacation period advantage under a ELS policy.

From an operational perspective, future work must contribute to development of enxtensive operational strategies. For example, both coordination concepts, introduced in this chapter, are starting the period reserved for handling 'randoms' after the last slot time is entirely expired (due to the computational methodology used). Another strategy could be to start this period directly after finishing the last consecutive coordinated arrival etc.

5 Case study: Aircraft Arrival Process

In this chapter, the previously introduced en-route coordination concepts are applied to a real world case, discussing the potential advantages for the aircraft arrival process at Amsterdam Airport Schiphol (AMS). This chapter starts in section 5.1 with a justification of the chosen inter-arrival time and service distribution as well as their corresponding parameters and an introduction of the benchmark performance levels and cost savings parameters. By means of two examples the benefits and (dis)advantages of en-route coordination of aircraft movements at AMS are uncovered in section 5.2. The results of both examples will be discussed in section 5.3, followed by an elaboration of the implementation issues in section 5.4.

5.1 Data gathering

This section will first start in 5.1.1 with the introduction of the benchmark performance levels required to evaluate the relative performance advantage of en-route arrival coordination. Section 5.1.2 and 5.1.3 discusses, respectively, the inter-arrival time distribution and the distribution of the service times. Section 5.1.4 discusses the derivation of the costs savings parameters S_1 and S_2.

5.1.1 Benchmark performance levels

A performance overview of AMS, published by the Dutch air traffic control authority LVNL, revealed an average delay of 0.9 minutes per dispatched flight over 2010. This long term average value includes moments of reduced capacity, excessive demand or times with hardly any arrival, and will function as input for the first example (section 5.2.1), representing normal circumstances. More detailed delay figures revealed that the average delay per delayed flight added up to 21 minutes. This indicates that, on average, substantial delays are quit exceptional, but when delays occur they are quit severe. It is therefore assumed that, if such notable delays are experienced, capacity is temporary an issue. Hence, the delay figure of 21 minutes is used as a benchmark value for a second example (section 5.2.2), assuming to represent a time frame with a capacity dispute (Luchtverkeersleiding Nederland, 2011).

5.1.2 Inter-arrival time distribution.

It is generally assumed that the inter-arrival times of aircraft arriving at an airport, before the final control actions are executed by the ATC authority, can be well-modeled by the exponential distribution. Already in the early seventies a number of theoretical and statistical arguments justified the Poissonian hypothesis for air traffic arrivals. Recently, based on studies at nine major U.S. airports, only a small difference was observed between the actual arrival process and the Poisson distribution. That is, the actual arrivals where slightly less random than according to a Poisson distribution (Balakrishnan & Chandran, 2006; Ndreca, 2009).

Due to the fact that aircraft arrivals are pre-scheduled, based on runway capacity and airline preferences, Ndreca (2009) questions the use of a Poisson distribution for modeling arriving movements. Instead, the use a multi parameter distribution i.e. a point process is proposes to account better for the ‘wave’ pattern of the inflow. The current lack of support in literature for this methodology, and aggravated computations favors the use of exponential inter-arrival times for modeling (uncoordinated) aircraft arrivals.

Representative parameters for the exponential inter-arrival time distribution are found by using data from four days during the summer period, Appendix I. The average demand of these four days is equal to 678 arrivals per day. A day represents an operational time window of 1000 minutes, resulting in $\lambda = 0.678$

5.1.3 Distribution of the service times

A general classification for the service time distributions could not be found in literature. The most plausible explanation of the lack of research in this area is that the variability of the individual service times itself are often subject to optimization. Neglecting the mix of aircraft, the actual landing times contains low variability. The danger of 'wake turbulence[6]' requires, however, a minimal spacing distance between succeeding aircraft, based on size and weight of the leading and trailing aircraft. A typical minimum separations matrix under normal conditions is given in Table 5-1. The asymmetry of this matrix is often used as an opportunity to develop advanced scheduling concepts to increase runway efficiency.

The actual inflow will be a mixture of different aircraft, and airline schedules vary day by day as well. It is therefore hard to obtain the actual distribution of the service times. The coordination concepts developed in preceding two chapters are well applicable when using exponential services times. .Hence, due to the obscurity of the actual service time distribution for simplicity exponential service times are assumed (Hu & Paolo, 2009; Balakrishnan & Chandran, 2006; Bestebreur, 2011).

The declared capacity of Amsterdam Airport Schiphol is set on 790 arrivals a day. The declared capacity is dictated by environmental constraints imposed by the Dutch authorities, limited to at most 68 arrival (slots) during an arrival peak. Moreover, the accounted service time (slot length) differs from the actual service time required, due to the inclusion of slack time for threshold crossings and clearance of the obstacle free zones around the runway. To summarize, the declared capacity is by no means related to the actual capacity of AMS. The parameters of the exponential distributed services are for that reason theoretical approach by using Little's Law and the known delay and arrival rate.

As stated before, the arrival rate is based on data from four days during the summer period, which is, compared to the winter period, busier i.e. a relative high arrival rate. Using Little's Law and the known delay, server capacity will be set relatively high as well, therefore, the outcome of en-route coordination may expected to be conservative compared to the real potential of the coordination concepts (Rockwell Collins, Inc.; www.slotcoordination.nl).

Table 5-1: Minimum time separation (in seconds) between landings (Balakrishnan & Chandran, 2006)

Leading Aircraft	*Trailing Aircraft*		
	Heavy	*Large*	*Small*
Heavy	96	157	196
Large	60	69	131
Small	60	60	82

[6] 'wake turbulence': turbulence behind an aircraft as it passes through the air

5.1.4 Cost savings

Section 2.2 introduces two cost parameters, S_1 and S_2, representing the potential cost savings of using en-route coordination on the operational level. The derivation of the values assigned to those parameters is difficult is case of the aircraft arrival process. Due to their dependency on the organizational structure of the OA, degree of delay, and the type of aircraft involved.

Basically, S_1 represents the difference in cost between being in a queue, i.e. flying in a holding pattern, or flying on a normal cruise speed-altitude configuration. Flying on a normal cruise speed altitude configuration is generally assumed to be more efficient compared to flying in a holding pattern, a detailed example (Knorr, et al., 2011):

> A Fokker 100, replacing one minute of time spend in a holding position by one minute flying on slightly lower cruise speed such that the aircraft will need one minute extra to cover the same distance will save 50 euros, and 75 kg of fuel (236 kg of CO^2 emission (ADSE[7]).

To come up with an industry average for S_1, independent of aircraft type a more generalized and conservative approach is chosen. By neglecting the interference in the correlation between speed and distance coverage in a holding position compared to flying on (slightly) cruise speed is assumed that one minute in the queue is exactly replaced by one additional minute en-route.

Flying in a holding pattern is equivalent to experiencing delay. The cost calculated for one minute of delay is 73 euros. This average value is determined by weighting the aircraft type, the known distribution of ATFM[8] delay, and includes reactionary delay cost, but does not consider any strategic cost. The cost per aircraft is composed of: fuel consumption, crew and maintenance cost, and cost of ownership, Table 5-2. Moreover, the determination of the cost of one minute delay is achieved by a cooperation including AO and aircraft manufactures, and was initiated by EUROCONTROL (Cook et al., 2004; EUROCONTROL, 2011).

Table 5-2: Strategic Airborne Costs

Strategic Airborne Costs	
	Delay costs per minute (€)
Fuel Costs:	38
Maintenance Costs:	12
Crew Costs:	9
Aircraft Ownership costs:	14
Direct cost to an Airline:	73

The cost of flying one minute at cruise speed is assumed to be equivalent to the average operational cost per minutes of the ten most used aircraft in Europe, and estimated on 44.20 euros (RCI; EUROCONTROL, 2009). The value of S_1, representing the cost difference between queueing and slowing down en-route, could now be easily calculated and is given by (5.1)

$$S_1 = 28.80 \quad € \tag{5.1}$$

7 An independent consulting and engineering company active in the market areas of Aerospace, Defense, Infrastructure and Rail Transport

8 Air Traffic Flow Management

S_2, briefly meaning; the cost saving regarding a reduction in the accumulated time of queueing and en-route time, is in general equal to a reduction of the operational time. If the operational time is reduced, the savings are given by the operational cost of 44.20 euros. Hence, S_2, is given by (5.2).

$$S_2 = 44.20 \quad € \tag{5.2}$$

The emission of greenhouse gasses like CO^2 is inherent to fuel consumption. It is however very difficult to extract the proportion of fuel usage from both cost savings parameters. To give some insight in potential of en-route coordination to reduce the pollution caused by aircraft movements a rough estimation is given:

> Regarding the earlier example, A Fokker 100, replacing one minute of queueing time by one additional minute en-route saves 236 kg of CO^2 emission. For a modern A320 the savings are even expected to be thirty percent more (ADSE). Hence, approximately 300 kg of CO^2 emission could be saved by interchanging one minute of queueing time by lengthening the flight path one single minute.

Direct cost savings on the strategic level, prompted by arrival coordination, are questionable. By adding slack into their operational schedules, AO encounter for delay on a strategic level. It is however, doubtful of AO are willing to decrease the robustness of their schedules simultaneous with the implementation of en-route arrival coordination. Besides, these potential savings will depend heavily on the internal organization of the involved OA and are difficult to capture by a single general parameter. The savings on the strategic level are therefore left out of consideration.

5.1.4.1 Cost saving for 'basic' arrival coordination

The 'basic' arrival coordination model will ensure deterministic inter-arrival times for coordinated arrivals, thereby actually interchanging queueing time caused by uncertainty in the arrival process by a more efficient pace en-route. The conversion of delay to a supplementary route is comprehensive to a level where the entire inflow is deterministic, a D/G/1 queueing system. Hence, the cost saving under the 'basic' coordination policy are equal to the reduction in delay times S_1.

5.1.4.2 Cost savings for 'extended' arrival coordination

If the reduction in delay rises above the level achieved by full deterministic arrivals (using extended coordination) the movement will, unless the conversion of queueing time to extra time en-route, be shortened (section 2.2). Till a performance level equivalent to an D/G/1 system, delay is assumed to be replaced by en-route activities, yielding savings S_1. If the reduction in delay rises above this level, the total accumulated operational time decreases from that moment on, represented by S_2.

5.2 En-route arrival coordination at AMS

The potential advantages of en-route coordination at AMS are demonstrated by use of two examples. The first example, introduced in section 5.2.1 represents a situation based on the long term average delay, equal to 0.9 minutes. The second example, given in section 5.2.2, illustrates a situation with reduced runway capacity with a corresponding average delay of 21 minutes.

5.2.1 Example 1: Long term average delay

This first example is based on the long term average for the delay of 0.9 minutes. Assuming 678 arrivals during a 1000 minute interval, the runway is, theoretical (Little's law), capable of handling 1271 arrivals. The service rate is therefore equal to $\mu = 1,271$, with a corresponding utilization level of 53%.

5.2.1.1 'Basic' arrival coordination

The utilization level of 53% affects the reliability of the algorithm of Albin (1984), section 3.5. Since the upper and lower-bound is known, and the linear relationship is proven in chapter three, interpolation (Appendix A) is used to determine the average queueing time of using 'basic' arrival coordination.

The savings per individual flight are given in Table 5-3. Remark that all savings are accountable to S_1. 'Basic' arrival coordination improves the entire system performance. The total potential annual cost savings, covering all arrival types, are therefore given in Figure 5-1. The participants in coordination are accountable for the investment; consequently, the pay-off to the coordinated arrivals only is given in Figure 5-2. This pay-off curve is slightly convex, contradictory to the linear pay-off curve given in Figure 5-1. Section 3.6 revealed a small discrepancy of 5~10 percent in the savings obtained by coordinated and random arrivals. The actual savings are therefore like to be slightly higher than given in Figure 5-2 (Appendix J).

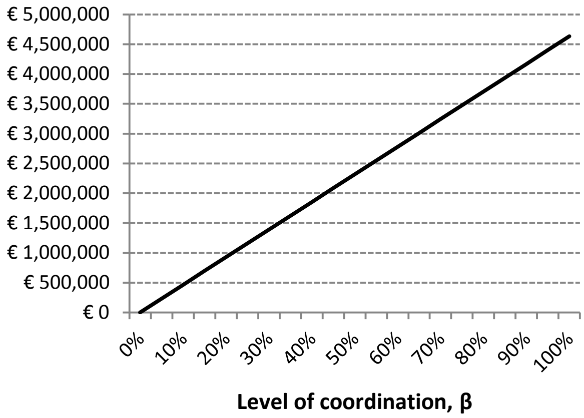

Figure 5-1: Annual cost savings under 'basic' arrival coordination

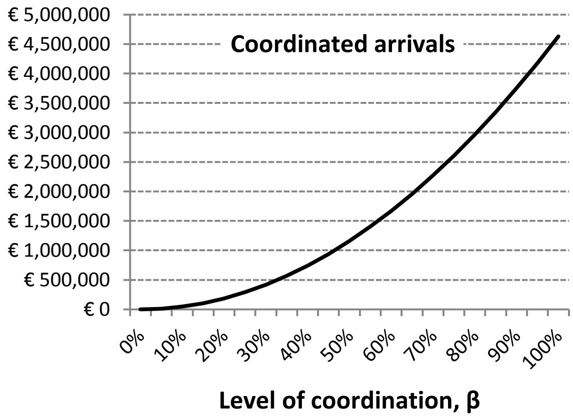

Figure 5-2: Annual cost savings for coodinated arrivals under 'basic' arrival coordination

Table 5-3: Savings for a random flight under a'basic' coordination policy

β	*Savings per flight*
0%	€ 0,00
5%	€ 0,94
10%	€ 1,87
15%	€ 2,81
20%	€ 3,74
%	€ 4,68
30%	€ 5,62
35	€ 6,55
40%	€ 7,49
45%	€ 8,42
50%	€ 9,36
55%	€ 10,30
60%	€ 11,23
65%	€ 12,17
70%	€ 13,10
75%	€ 14,04
80%	€ 14,97
85%	€ 15,91
90%	€ 16,8
95%	€ 17,78
100%	€ 18,72

5.2.1.2 ‘Extended’ arrival coordination

The relatively low utilization of 53 percent enables a sufficient enlargement of the time intervals reserved for coordinated arrivals. The maximum delay allocated to non-coordinated movements is limited to twenty minutes. This value is motivated by the variability of the reserve fuel carried by aircraft for (inter)continental movements.

The cost savings for a coordinated movement at a certain level of coordination is given by Table 5-4 and are illustrated by Figure 5-3. The average waiting time in the queue, with participation in ‘basic’ coordination is 0.25 minutes. The average waiting time in the queue, with full participation in ‘basic’ coordination, is 0.25 minutes (D/G/1 system). So, when the average waiting time drops below this level, the obtained savings are equal to S_2, instead of S_1.

The experienced delay rises from 0.9 minutes to 20 minutes for non-coordinated movements, regardless of the level of coordination; despite the low utilization, the amount of slack time in the system is not sufficient to keep the average delay for ‘randoms’ below the pre-set maximum. The 19.1 minutes of additional queueing time per flight comes with an additional cost of almost 1400 euros (i.e. 19.1 times 73 euros), and could be classified as the penalty for not participating in the coordination concept.

Average system performance decreases; on the one hand costs are saved by coordinated arrivals, on the other hand, operational expenditures rise extensively for non-coordinated movements. To prevent a biased view on the potential of coordination Figure 5-4 gives the annual savings obtained by AO who participate in coordination. This figure clearly visualizes the coordination level corresponding to the maximum payoff for coordinated arrivals (+/- 60 percent). Figure 5-5 gives, by contrast, the tremendous additional cost incurred by AO who do not participate in coordination, responsible for the random arrivals approaching the runway.

Table 5-4: Cost structure savings for coordinated arrivals under ‘extended’ arrival coordination

β	*Savings imposed by S_1*	*Savings imposed by S_2*
0%	€ 0,00	€ 0,00
5%	€ 7 2	€ 28,73
10%	€ 7,20	€ 28, 1
15%	€ 7,20	€ 28,56
20%	€ 7,20	€ 28,20
25%	€ 7,20	€ 27,62
30%	€ 7,20	€ 26,83
35%	€ 7,20	€ 25,82
40%	€ 7,20	€ 24,61
45%	€ 7,20	€ 23,16
5 %	€ 7,20	€ 21,46
55%	€ 7,20	€ 19,45
60%	€ 7,20	€ 17,03
65%	€ 7,20	€ 14,08
70	€ 7,20	€ 10,33
75%	€ 7,20	€ 5,35
80%	€ 0,21	€ 0,00
85%	€ 0 0	€ 0,00
90%	€ 0,00	€ 0,00
95%	€ 0,00	€ 0,00
100%	€ 0,00	€ 0,00

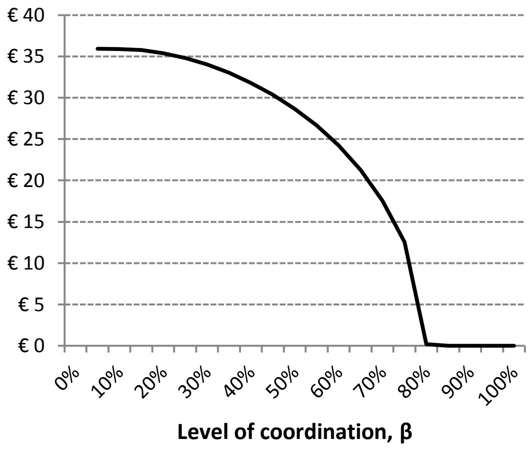

Figure 5-3: Range of the cost savings for a coordinated arrival under ‘extended’ arrival coordination.

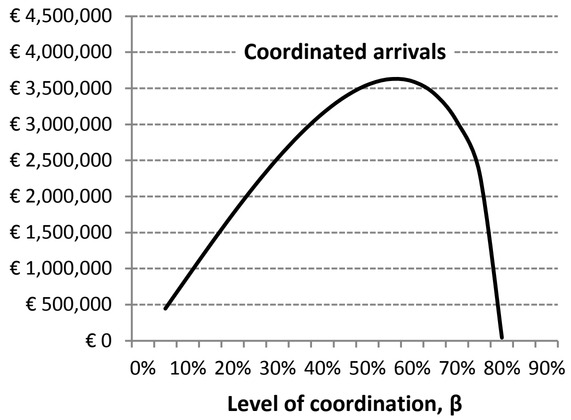

Figure 5-4: Annual cost savings for coordinated arrivals under 'extended' arrival coordination.

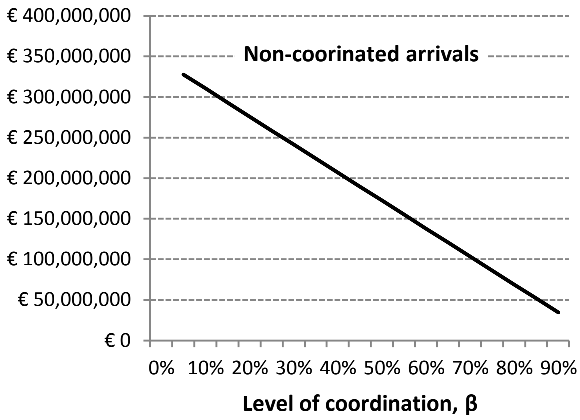

Figure 5-5: Annual additional costs for non-coordinated arrivals under 'extended' arrival coordination.

5.2.2 Example 2:Reduced capacity

During a period in which runway capacity is temporary reduced, average delay rises to 21 minutes. Giving the amount of delay and 678 arrivals during a 1000 minutes interval runway capacity is theoretically 723 arrivals a day. The resulting service time is almost twice as long compared to the first example, 83 seconds, corresponding to $\mu = 0.723$ and a $\rho = 0.94$. Since a capacity dispute is assumed to be temporary, and will be dissolved after some time, the advantages of coordination are presented on an hourly base, compared to annual savings in the preceding section.

5.2.2.1 'Basic' arrival coordination

The results given by the algorithm of Albin (1984) are assumed to be reliable due to the utilization of 94%. Comparing the results obtained by algorithm to results obtained by using interpolation on the lower-and upper bound, gives practically the same values with a maximum deviation of only three percent, again justifying the use of interpolation (Appendix K).

The benefits for the entire system, on an hourly base concerning all arrival types, are given in Figure 5-6. The cost savings are itemized per flight in Table 5-5. The hourly cost advantages, primary to those who bear the cost of coordination i.e. the coordinated arrivals, are given in Figure 5-7.

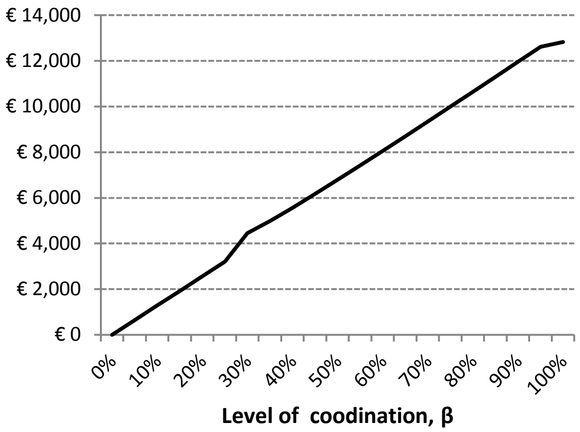

Figure 5-6: hourly cost savings under ‘basic’ arrival coordination

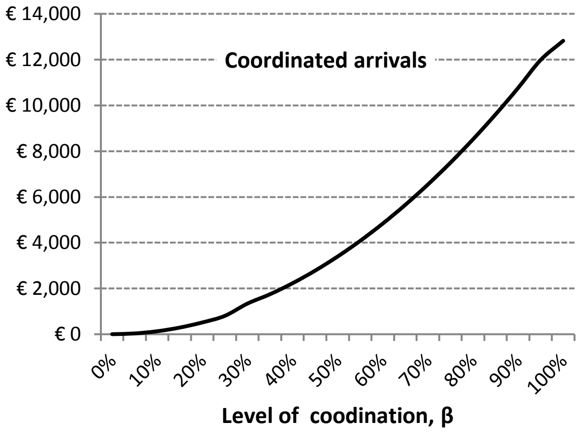

Figure 5-7: Annual cost savings for coodinated arrivals under ‘basic’ arrival coordination

Table 5-5: Savings for a random flight under a‘basic’ coordination policy

β	*Savings per flight*
0%	€ 0,00
5%	€ 15,76
10%	€ 31,53
15%	€ 47,29
20%	€ 63,06
25	€ 78,82
30%	€ 109,57
35%	€ 122,64
40%	€ 136,83
45%	€ 151,67
50%	€ 166,91
55%	€ 182,40
60%	€ 198,06
65%	€ 213,85
70%	€ 229,72
75%	€ 2 5 68
80%	€ 261,71
85%	€ 2 7,80
90%	€ 293,95
95%	€ 310,17
100%	€ 315,30

5.2.2.2 ‘Extended’ arrival coordination

Due to the utilization of 94 percent the slack time available in the system is limited, affecting the applicability of the ‘extended’ arrival coordination concept; prolongation of the slot time for coordinated arrivals is limited. The average delay in a comparable D/G/1 queueing system is 10.05 minutes. Hence, cost savings are given by S_2 if the experienced delay drops below this level of 10.05 minutes. Resulting in the cost savings presented in Table 5-6, and Figure 5-8 shows the (small) the application range of an ‘extended’ arrival policy in Figure 5-8.

The hourly cost savings, for both coordinated and random movements, are given in, Figures 5-8 and 5 9, respectively (Appendix K). The limited applicability and the coordination level at which pay-off is maximized for coordinated movements is again clearly visible.

Table 5-6: Cost structure savings for coordinated arrivals under ‘extended’ arrival coordination

β	*Savings imposed by S_1*	*Savings imposed by S_2*
0%	€ 0,00	€ 0,00
5%	€ 315,36	€ 390,73
10%	€ 315,36	€ 307,27
15%	€ 315,36	€ 154, 56
20%	€ 149,47	
25%	€ 0,00	

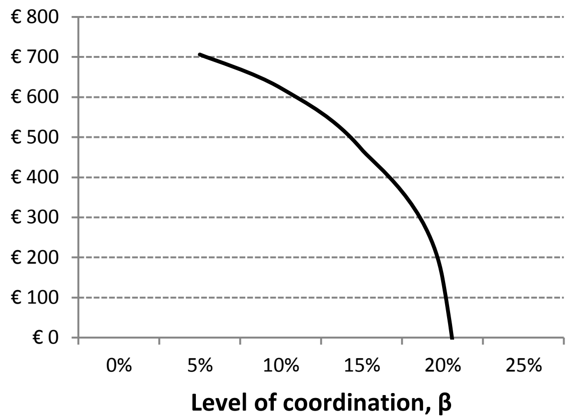

Figure 5-8 Range of the cost savings for a coordinated arrival under ‘extended’ arrival coordination.

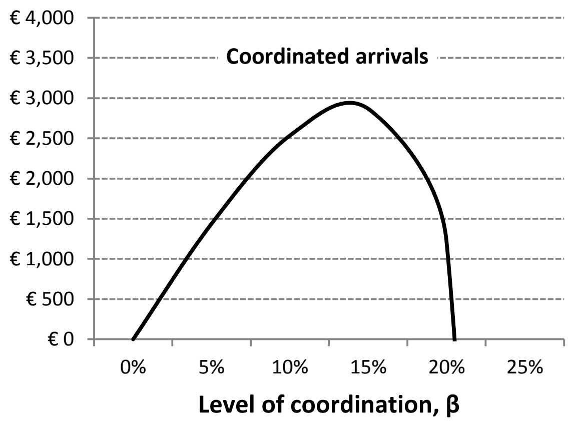

Figure 5-8: Hourlyl cost savings for coordinated arrivals under ‘extended’ arrival coordination.

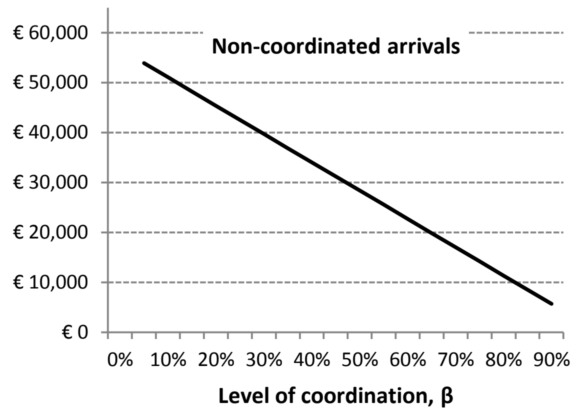

Figure 5-9: Hourly additional costs for non-coordinated arrivals under ‘extended’ arrival coordination

5.3 Discussion on results

This section summarizes and discusses the results concerning the case study on the arrival process at AMS, conducted to provide insights in the feasibility of arrival coordination in a real life environment. Sections 5.3.1 and 5.3.2 starting with an overview of the potential benefits and disadvantages of respectively, ‘basic’ coordination and ‘extended’ (non-exhaustive) coordination, after which a general statement about arrival coordination at AMS will be presented in section 3.3.3.

5.3.1 ‘Basic’ arrival coordination

The average long term delay for an arbitrary flight arriving at AMS is 0.9 minutes. The savings per individual flight by using ‘basic’ arrival coordination have a maximum of only 18.72 euros. This coordination concept reduces, however, the average queueing time for all movements. Hence the overall annual savings are substantial; e.g. if 60 percent of the movements is coordinated, the total operational savings will approach 2.8 million euros (or 456.83 euros per hour) and similarly, the annual reduction in CO^2 emission is roughly estimated on 300.000 metric ton.

The return to those who participate in coordination is, assuming 60 percent coordination, roughly 1.6 million euros. Due to the convex nature of the pay-off curve for coordinated arrivals, consideration of the total combined savings for both coordinated and 'randoms' only could lead to a biased view when assessing the financial feasibility of 'basic' coordination.

If the capacity is temporary at stake, savings could add up to 315.30 euros per flight, resulting in total hourly savings of almost 8000 euros when 60 percent of all movements are coordinated. The savings to coordinated arrivals only are nevertheless only 4834 euros.

5.3.2 Extended' arrival coordination

Using 'extended' arrival coordination is beneficial for those who participate, and will result in increased expenditures for those who are not willing to participate in coordination. To guard against only minor benefits or even deterioration of experienced delay, the level of coordination must be limited to some extent. Using the long term average delay of 0.9 minutes as an input, coordination is beneficial till a coordination level of almost 80 percent. The maximum pay-off is nevertheless already reached by a coordination level of 60 percent. The corresponding annual savings for coordinated arrivals are 3.6 million euros. The extra incurred operational cost by non-participating AO's, representing the remaining 40% the total movements, is tremendous with 138 million euros. Besides, due to the reduced overall system performance, total pollution will increase.

If runway capacity is temporary limited, utilization increases significantly, reducing the optimal coordination level to only 15 percent of all movements. This will correspond, on an hourly base, to potential saving of 3000 euros, compared to extra cost for non-coordinated movements of 48000 euros.

The additional cost for random arriving movements, as well as the optimal coordination level, depends, among other, on the pre-set limit on the maximum delay experienced by random arrivals. Lengthening this limit to, for example 30 minutes, will increase the performance, and the optimal participation level for coordinated movements, especially during a capacity dispute, but increase the additional cost for random movements half as much again. The limit $W_q^{R,\max}$ will therefor always be subject to discussion.

5.3.3 Conclusion

The implementation of en-route coordination at AMS will be feasible, but the realized and absolute advantages will depend on the chosen coordination concept. 'Extended' coordination will yield the highest saving under standard conditions for coordinated arrivals, but the overall costs increase is enormous and will possibly arouse resistance (as well as the participation limit). Moreover, during a capacity dispute the benefit of coordinated fades already away at a coordination level of 20 percent. This property will limit the applicability of 'extended' coordination during high utilization circumstances, hampering the actual implementation when capacity is subject to vary.

Under a 'basic' coordination concept savings will arise at every coordination level. During normal conditions, the savings per flight are quit limited. Nonetheless, the advantage of coordination is distributed over all arrivals, resulting in severe saving when examining the entire system. If capacity is subject to pressure, the advantage of coordination will increase as well, resulting in severe cost savings.

From an environmental point of view, ‘basic’ coordination is much more desirable. Since the overall system performance will be improved, a significant reduction in CO [2] emission is achievable. Using ‘extended’ coordination result in an opposite effect, pollution will increase due the deterioration of the overall system performance. Hence, this paper champions the use of ‘basic’ arrival coordination at Amsterdam Airport Schiphol.

Remark that the results are obtained by assuming exponential inter-arrival times between succeeding aircraft. The service times are derived purely theoretical. The potential cost savings presented by this case study are therefore only an indication of the potential savings.

5.4 Implementation

The findings about partial coordination, given on the conceptual level by the previous chapters, and by means of this case study on operational level, are expected to further contribute to find support for actual (large scale) implementation of trajectory based operations in order to reduce congestion at airports. Despite the positive results presented by this paper, still some major steps must be taken to overcome some (operational issues), vital for successful implementation of en-route aircraft arrival coordination.

The current structure of the aviation industry, with scattered responsibilities, impedes the startup of new incentives, like en-route coordination. The ‘queue’ to the runway is supervised by ATC authorities. The consequential delays of congestion are therefore issued by the ATC agencies. ATC is therefore, intuitively, the obvious organization to utilize the en-route coordination concept, improving future aviation. There is however, no incentive for ATC to do so; the AO ‘pays the bill’ of delay in terms of extra operational cost, while ATC has no direct gain of altering their operational structure.

AO have the ability to influence their own movements, and therefore the obvious means to introduce en-route arrival coordination. It will require inter-airline cooperation, but the major hurdle appears to be the fact that it is ‘counter intuitive’ to them; not directly in line with their current business profile, and from their perspective, the task of the capacity regulation authority i.e. ATC. Hence, an ‘impartial’ coordination service provider seems to be needed to further develop and implement en-route aircraft arrival coordination.

Using an impartial service provider, excluding ATC, will come with a risk. When using an impartial service provider, demand is (has to be) modulated before it will enters the area strictly controlled by the ATC authority. So, aircraft will enter this ATC area in line with the current runway capacity and spacing requirements, nullifying the need for any ATC intervention related to capacity or safety. ATC could issue some trajectory changes anyway, possible diminishing the benefits of coordination. It is could therefore be essential to inform, or even to some extent, involve ATC during development and implementation of en-route arrival coordination for the aircraft arrival process. This statement is currently subject of discussion.

En-route coordination is technological feasible in terms of multiple (standard) technologies (RCI). There is a large diversity in the (communication) equipment in use by the AO. It is therefore important to address the compatibility of the (advisable) technology, used to actually execute coordination, with the current equipment in use by the expected participants of coordination, and their willingness to invest in certain additional functions to actually realize arrival coordination.

By using en-route coordination, aircraft are assigned to specific arrivals slots prior to their actual arrival. Next, their entire trajectory is used to meet this slot time in the most efficient manner. An aircraft is, in general, capable of adapting its arrival time between three to five minutes per flight hour. Hence, the sooner an arrival slot is issued; the better aircraft could be directed to specific slots. The length of arbitrary flight is highly variable, continental vs. inter-continental flights, which consequently affects the ability to adapt ones moment of arrival: A continental flight of 1.5 hours could only delay its arrival time efficiently, with 8 minutes; while an inter-continental flight of 8 hours has the ability to delay the moment of arrival efficiently by 40 minutes. Hence, the exact moment of issuing slot times gives rise to future research, taking into account the consequences for departure times as well.

En-route coordination only contemplates the final part of the trajectory. However, due to uncertainties en-route an issued arrival time slot could be no longer feasible. Moreover, in case of a delayed departure another arrival time slot could be preferable. Procedures, accounting for those kinds of slot alterations, must be developed in a dialogue with the participants in coordination, thereby guaranteeing the impartiality of the service provider.

A final comment on implantation, the conducted case study in this paper is based on data from Amsterdam Airport Schiphol. The average delay at AMS of 0.9 is relatively low compared to other European airports. Hence, it will even be more beneficial to start the implementation of en-route coordination at airports with a considerably larger amount of experienced delay per arbitrary flight. .

6 Discussion

In this thesis, two conceptual models are developed, and a case study is conducted to verify the feasibility of partial en-route coordination in order to reduce experienced overall delay. Despite the fundamental differences between both concepts, solid support is found for an eminent conclusion regarding the research questions stated in chapter one: *"Partial en-route coordination will reduce experienced delay. Besides, both concepts showed already delay reductions with only minimal levels of coordinated movements, rejecting the need for a minimal required participation level"*. The correlation between the level of coordination and the potential benefits is nevertheless not always straightforward, resulting in the lack of an immediate 'Best Practice' to carry out (partial) arrival coordination. The remaining part of this chapter will start with an elaborated overview of the theoretical result obtain during this study in section 6.1, followed by a discussion of the case study results in section 6.2. Section 6.3 discusses the limitations and options for future research

6.1 The opportunity of en-route arrival coordination.

This study, guided by research assignment given in the first chapter, reaches the conclusion that partial coordination of the inflow will realize a reduction in experienced delay. At first, a simplified coordination concept was developed. To verify and validate this concept, it became eminent to determine the benefits, disadvantages and benchmark performance simultaneously, requiring working concurrently on both main components of the assignment. The assignment is passed through twice, albeit with some minor differences. The in-depth analyses of queue behavior, under the 'basic' coordination principle revealed the inability to distinguish between arrivals, given rise to the development of a second 'extended' coordination algorithm accompanied with the generation of an approximation algorithm. There is, however, a large discrepancy between both developed conceptual models and the distribution of their benefits. The upcoming sections, 6.1.1 and 6.1.2 therefore summarize the conclusions per type of coordination

6.1.1 'Basic Arrival coordination'

The 'Basic arrival coordination' concept disperses the coordinated part of the inflow evenly over time. Non-coordinated, random arrivals are just accommodated between coordinated arrivals on their moment of arrival, without a preset strategy. A positive linear correlation is found between the level of coordination and reduction in experienced delay. Hence, even a minor level of coordination will reduce queueing time. The pay-off curve to coordinated arrivals is nevertheless convex. The service time distributions significantly influences the possible gains in waiting time reduction; the smaller the variability in service times, the greater the advantage of coordination.

The advantages of basic coordination are not only reserved to those who participate in coordination; the distinction in delay reduction between both types of arrivals is almost negligible. This could make the initial investment unappealing, though equivalent to the overall system performance, individual performance will be improved. The desire to draw a distinction between benefits for coordinated and random arrivals induces the development of a second arrival coordination concept.

6.1.2 'Extended arrival coordination'

'Extended arrival coordination' distinguishes between arrivals, awarding an advantage to coordinated arrival over random arrivals. Available server capacity is divided into time slot, some of which are reserved to handle pre-planned coordinated arrivals and some to handle non-coordinated movements. Slack time is, as far as possible, allocated to handle coordinated arrivals. Two strategies were introduced to handle the open slots; exhaustive and non- exhaustive.

Apart from the chosen strategy or any parameter settings, the payoff curve concerning coordinated arrivals follow, in general, a similar pattern; for relatively low levels of coordination delay is almost completely resolved. However, when the level of coordination rises, a 'tipping point 'will arise from where the benefit of coordination fades away. The appearance of such a point argues for the use of this concept only when the percentage of participating movements could be regulated strictly.

Both strategies to accommodate random arrivals ensure that coordination is advantageous to coordinated arrivals. Under a non-exhaustive policy waiting will rise to preset maximum for non-coordinated arrivals, requiring a thorough analysis on the consequences of the set limit, since the average system performance decreases. Contrary, the use of an exhaustive policy also reduces the delay for random arrivals, improving the average system performance. This advantage comes to a price; a certain level of uncertainty will be introduced, probably affecting experienced delay of all movements and therefore hardly suitable (yet) for real-world implementation.

To conclude, an illustration of the delay reductions under both coordination concepts; basic and non-exhaustive extended, is given in Figure 6-1, clearly displaying the 'tipping point' for the extended coordination concept and the linear effect of the basic coordination concept. Besides, it demonstrates the responsibility of the service provider, willing to set up a coordination infrastructure. The desired performance benefit will limit the level of participants and the coordination concept to be used, and vice versa. E.g.: if the desired reduction in delay is 80%, a non-exhaustive extended coordination concept must be proposed with a maximum participation level of 30%.

6.1.3 Potential savings

En-route arrival coordination aims on using the entire trajectory to prevent congestion at the moment of arrival. By ensuring deterministic arrivals times instead of stochastic arrivals, demand is divided evenly, in line with capacity. The accompanying cost savings are expressed by two parameters: S_1 and S_2.

Saved queuing time is basically replaced by additional time en-route on a more (cost) efficient pace. The cost difference between supplementary time en-route and spending time in a queue is expressed by S_1. The conversion of delay to additional time en-route is comprehensive to a level where the entire inflow is deterministic (100 percent coordination), illustrated by the 'grey' area Figure 6-1.

If the reduction in delay rises above the level achieved by full deterministic arrivals (using extended coordination) the movement will, unless the conversion of queueing time to extra time en-route, be shortened, saving operational cost equal to S_2, and represented by the light blue area in Figure 6-1. To recapitulate: 'till a performance level equivalent to an D/G/1 system, delay is assumed to be replaced by en-route activities, yielding savings S_1, if the reduction in delay rises above this level, the required operational time decreases from that moment on, yielding savings of S_2.

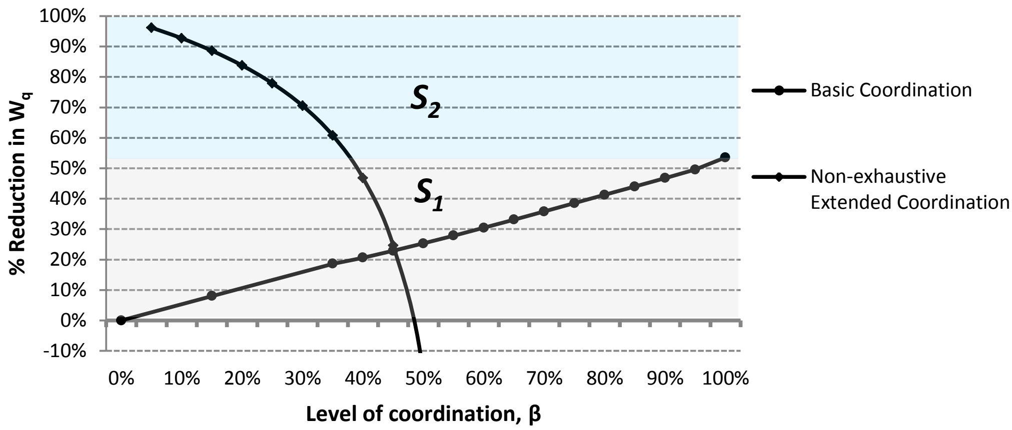

Figure 6-1: Basic and non-exhaustive extended coordination accompanied with (cost) saving parameters S_1 en S_2.

6.2 Case study

To address the feasibility of en-route arrival coordination and case study is conducted based on data from Amsterdam Airport Schiphol. The potential advantages will depend on the chosen coordination concept;

> 'Extended' coordination will yield the highest savings for coordinated arrivals only, enforcing tremendous additional expenditures for non-coordinated movements. Moreover, the optimal participation level reduces to only 20 percent during moments when capacity is at stake.
>
> The 'basic' coordination policy will achieve savings at every level of coordination, irrespective of the utilization. Contrary to the 'extended coordination concept', the overall system improves, which is desirable from an environmental perspective as well.

Hence, the most appropriate coordination concept to use at AMS is 'basic' arrival coordination.

6.3 Limitations and Future work

6.3.1 Limitations

This paper presents two conceptual coordination models and a case study. The main assumption, justification the use of en-route arrival coordination, is the gain in efficiency by transferring queueing time to supplementary time en-route. If this assumption is violated, arrival coordination will not be feasible at all since it will be more (cost) efficient to spend time in the queue.

The analysis regarding the first conceptual model, 'basic' arrival coordination, gives straightforward and reliable results. The methodology given for the analyzing an 'extended' arrival concept is, however, to some extent, limited its use. First of all, the presented methodologies, for the exhaustive and non-exhaustive policy, are approximations and do not represent exact results. Second of all, the methodology is limited in its use; the model is only capable of using exponential inter-arrival times to reproduce random arrivals.

Third of all, the average delay given by the exhaustive 'extended' arrival coordination is liable to a certain level of uncertainty. Additional research is required to gain in-depth insight in the real 'cost' of this uncertainty.

Regarding the case study, several assumptions are done to apply the developed coordination concepts in an aircraft arrival context. First of all, the arrival and service rate are assumed to be exponential. This is, especially for the service distribution not proven. Second of all, the service rate is theoretically approached, and not based on the actual runway capacity at AMS. At last, the cost saving are estimated based on average industry values. The actual cost savings for an AO, participation in coordination, will depend on the organization structure and age/composition of its fleet

This research is carried out in a purely theoretical framework contemplating the arrival process in its elementary form only. The trajectory priory to the moment of arrivals is not studied at all, lacking insight in the actual ability to carry out coordination. Hence, one must comply with the idealized environment while inferring from one of the proposed coordination concepts.

6.3.2 Future research

Based on the conclusions and limitations several recommendations for future research could be given. The potential of using en-route coordination to optimize server capacity is now proven. Giving rise to more profound research on the development of conceptual coordination models with enhanced performance and/or reliability. Moreover, research attention is required for implementation and operationalization; incentives must rise to clarify case specific parameters and advantages, as well as technological and practical feasibility.

The M/G/1 queueing system act as foundation for the conceptual models developed in this paper. This elementary queueing model, suitable to many applications, is extensively discussed in literature. A vast diversity of configurations, based on this model, is developed, and in use in industry. 'Widening' the foundations of the coordination concept or the introduction of additional restrictions will therefore largely expand the applicability to industries like manufacturing and healthcare: e.g. multi server configurations, or accountability for no shows etc. Moreover, In the case study, 'basic' coordination was in favor of 'extended' coordination due to the increase in delay assigned to non-coordinated movements. This rises the opportunity to extend the 'extended' coordination concept to the service industry, using this coordination concepts to distinguish in service contract offered to customers.

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Appendix A: Basic calculations

- Kendall's notation:

  In Queueing Theory Kendall's Notation is used to describe or classify the queueing models: A/B/C/K/N/D. Or more concise: A/B/C, assuming K = ∞, N = ∞ and D = FIFO.

  A: *Represent the arrival pattern of the request to the system*
  B: *Represent the servicing pattern of the system*
  C: *Represent the total number of servers in the system*
  K: *Represent the total number of request the can be queued in the system*
  N: *Represent the calling source (i.e. the customer base)*
  D: *Represent the service discipline of the how the jobs gets served by the system*

- Laplace Stieltjes Transformation:

$$g^*(s) = G(s) = \int_0^{\infty} e^{-sx} g(x)\, dx$$

- LST of Degenerate Distribution: Inter arrival times are constant and equal to c:

$$G^*(x) = \sum_{i=1}^{\infty} e^{-sx} p(x) = e^{-sc}$$

- Interpolation:

$$y = f(x) = y_2 + \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_2)$$

- Krämer/Langenbach-Belz Formula for a G/G/1 System (Bolch et al., 2006)

$$W_q \approx \frac{\rho/\mu}{1-\rho} \cdot \frac{c_i^2 + c_b^2}{2} \cdot \begin{cases} \exp\left[-\frac{2}{3} \cdot \frac{(1-\rho)}{\rho} \cdot \frac{(1-c_i^2)^2}{c_i^2 + c_b^2} \right], & 0 \le c_i \le 1 \\ \exp\left[-(1-\rho) \frac{c_i^2 - 1}{c_i^2 + 4c_b^2} \right], & c_i > 1, \end{cases}$$

- Little's Law:

$$\bar{N} = \lambda W$$

- Number of items in a standard M/G/1 system:

$$E[Q] = \bar{N} = \frac{\lambda b^2}{2(1-\rho)} + \rho$$

- Second moment:

$$b^2 = b^2 + \sigma^2$$
$$b^2 = c_b^2 \cdot b^2 + b^2$$

- Squared coefficient of variation:

$$c_b^2 = \frac{\sigma^2}{b^2}$$

- Erlang(k-1,k) distribution (Houtum, 2007):

$$E_{k-1,k}(x) = q\left(1-\sum_{j=0}^{k-2}\frac{(\lambda x)^j}{j!}e^{-\lambda x}\right) + (1-q)\left(1-\sum_{j=0}^{k-1}\frac{(\lambda x)^j}{j!}e^{-\lambda x}\right), \quad x \geq 0$$

$$\lambda = \frac{k-q}{\mu_x}$$

$$q = \frac{1}{1+c_x^2}\left[kc_x^2 - \sqrt{k(1+c_x^2) - k^2c_x^2}\right]$$

$$\frac{1}{k} < c_x^2 \leq \frac{1}{k-1}$$

Appendix B: Algorithm of Albin & Whitt

- Basic parameters:

$$\lambda_i = \gamma_i^{-1}$$
$$\lambda = \lambda_1 + \ldots + \lambda_n$$
$$\gamma = \lambda^{-1}$$
$$n^* = \left[\sum_{i=1}^{n} (\lambda_i / \lambda)^2\right]^{-1}$$

- Determination of c_s^2:

$$c_a^2 = \lambda^{-1} \sum_{i=1}^{n} \lambda_i c_i^2$$

- Distribution fitting:

$$if\ c_i^2 \geq 1: HVEX\ (a_1, a_2; p_1, p_2)$$
$$p_1 = \left(1 + \left[(c_i^2 - 1)/(c_i^2 + 1)\right]^{1/2}\right) / 2$$
$$p_2 = 1 - p_1;\ a_1 = 2p_1\lambda \text{ and } a_2 = 2p_2\lambda$$
$$if\ c_i^2 < 1: EXPH\ (h, d)$$
$$h = (c_i\gamma)^{-1} \text{ and } d = \gamma - h^{-1}$$

- Determination of c_s^2:

When $F_1 = \text{EXPC}(h_1, d_1)$ and $F_1\ \text{EXPC}(h_2, d_2)$ and $d_1 \leq d_2$, $c_s^2 = (I^2 - \gamma^2)/\gamma^2$

$$I^2 = 2(\gamma_1 + \gamma_2)^{-1} \begin{Bmatrix} \gamma_1\gamma_2(d_1 - d_1^2)(\gamma_1 + \gamma_2)/2 + d_1^3/3 + \gamma_2 h_1^{-2}\left[1 - e^{-h_1(d_2 - d_1)}\right] + h_1^{-3}\left[(1 + h_1 d_2)e^{-h_1(d_2 - d_1)} - (1 + h_1 d_1)\right] \\ + h_1^{-1}h_2^{-1}(h_1 + h_2)^{-1} e^{-h_1(d_2 - d_1)} \end{Bmatrix}$$

When $F_1 = \text{HYEX}(a_1, a_2; p_1, p_2)$ and $F_1\ \text{EXPC}(h_2, d_2)$, $c_s^2 = (I^2 - \gamma^2)/\gamma^2$

$$I^2 = e^{-a_1 d} p_1 / (a_1 h (a_1 + h)) + e^{-a_2 d} p_2 / (a_2 h (a_2 + h)) + p_1 a_1^{-3}(1 - e^{-a_1 d} - a_1 d e^{-a_1 d}) + p_2 a_2^{-3}(1 - e^{-a_2 d} - a_2 d e^{-a_2 d})$$
$$+ \left[h^{-1} + d\right]\left[p_1 a_1^{-2}(1 - e^{-a_1 d}) + p_2 a_2^{-2}(1 - e^{-a_2 d})\right]$$

- Determination of c_k^2:

$$c_h^2 = \omega c_a^2 + (1 - \omega) c_s^2$$
$$\omega = \left[1 + 6(1 - \rho)^{2.2} n^*\right]^{-1}$$

- Determination of $E(Q)$:

$$E(Q) = \frac{\rho}{\mu} + \frac{\rho^2 (c_h^2 + c_b^2)}{2(1 - \rho)} \cdot \begin{cases} \exp\left[-2(1 - \rho)(1 - c_h^2)^2 / 3\rho(c_h^2 + c_b^2)\right], & c_h^2 < 1 \\ \exp\left[-(1 - \rho)(c_h^2 - 1)/(c_h^2 + 4c_b^2)\right], & c_h^2 \geq 1 \end{cases}$$

Appendix C: Algorithm of Albin

Table C-1: Detailed calculation for Table 3-2

λ_1	γ_1	c_1^2	λ_2	γ_2	c_2^2	$n*$	c_a^2	I^2	c_s^2	c_k^2	ω_e	$E\ Q$	$E\ W_q$
0.2	5.000	0	0.8	1.250	1	1.471	0.8	-1.667	-2.6667	0.6172	0.9473	7.5058	0.1251
0.25	4.000	0	0.75	1.333	1	1.600	0.75	0.000	-1.0000	0.6501	0.9429	7.6461	0.1274
0.3	3.333	0	0.7	1.429	1	1.724	0.7	0.741	-0.2593	0.6412	0.9387	7.6085	0.1268
0.35	2.857	0	0.65	1.538	1	1.835	0.65	1.088	0.0884	0.6135	0.9350	7.4901	0.1248
0.4	2.500	0	0.6	1.667	1	1.923	0.6	1.250	0.2500	0.5762	0.9321	7.3302	0.1222
0.45	2.222	0	0.55	1.818	1	1.980	0.55	1.317	0.3169	0.5337	0.9303	7.1468	0.1191
0.5	2.000	0	0.5	2.000	1	2.000	0.5	1.333	0.3333	0.4883	0.9296	6.9494	0.1158
0.55	1.818	0	0.45	2.222	1	1.980	0.45	1.322	0.3223	0.4411	0.9303	6.7435	0.1124
0.6	1.667	0	0.4	2.500	1	1.923	0.4	1.296	0.2963	0.3930	0.9321	6.5320	0.1089
0.65	1.538	0	0.35	2.857	1	1.835	0.35	1.262	0.2623	0.3443	0.9350	6.3170	0.1053
0.7	1.429	0	0.3	3.333	1	1.724	0.3	1.224	0.2245	0.2954	0.9387	6.0994	0.1017
0.75	1.333	0	0.25	4.000	1	1.600	0.25	1.185	0.1852	0.2463	0.9429	5.8799	0.0980
0.8	1.250	0	0.2	5.000	1	1.471	0.2	1.146	0.1458	0.1971	0.9473	5.6589	0.0943
0.85	1.176	0	0.15	6.667	1	1.342	0.15	1.107	0.1073	0.1479	0.9516	5.4363	0.0906
0.9	1.111	0	0.1	10.000	1	1.220	0.1	1.070	0.0700	0.0987	0.9559	5.2125	0.0869
0.95	1.053	0	0.05	20.000	1	1.105	0.05	1.034	0.0342	0.0494	0.9598	4.9873	0.0831

Table C-2: Decrease in reliabily for different utilization levels, Table 3-7

ρ = 75% (E(W_q))/h	ρ = 80% (E(W_q))/h	ρ = 85% (E(W_q))/h	ρ = 90% (E(W_q))/h
0.0137	0.0291	0.0564	0.1101
0.0242	0.0381	0.0626	0.1124
0.0286	0.0416	0.0646	0.1118
0.0303	0.0427	0.0646	0.1098
0.0307	0.0426	0.0636	0.1072
0.0305	0.0419	0.0622	0.1041
0.0299	0.0409	0.0604	0.1008
0.0291	0.0396	0.0584	0.0974
0.0281	0.0383	0.0563	0.0939
0.0271	0.0368	0.0541	0.0903
0.0260	0.0353	0.0519	0.0867
0.0249	0.0338	0.0497	0.0830
0.0238	0.0322	0.0474	0.0793
0.0226	0.0307	0.0451	0.0756
0.0215	0.0291	0.0428	0.0719
0.0203	0.0275	0.0405	0.0681

Table C-3: Average waiting time for different utilization levels, Table 3-8

β %	ρ = 85%	ρ = 90%	ρ = 92.5%	ρ = 95%	ρ = 97.5%	ρ = 99%
	$(E(W_q))/h$	$(E(W_q))/h$	$((E(W_q))/h$	$(E(W_q))/h$	$(E(W_q))/h$	$((E(W_q))/h$
0%	0.0803	0.1350	0.1901	0.1185	0.1736	0.1020
20%	~~0.0564~~	~~0.1101~~	~~0.1625~~	~~0.0936~~	~~0.1460~~	0.0771
25%	~~0.0626~~	~~0.1124~~	0.1620	0.0959	0.1455	0.0794
30%	~~0.0646~~	0.1118	0.1592	0.0953	0.1427	0.0788
35%	0.0646	0.1098	0.1555	0.0933	0.1390	0.0768
40%	0.0636	0.1072	0.1512	0.0907	0.1347	0.0742
45%	0.0622	0.1041	0.1467	0.0876	0.1302	0.0711
50%	0.0604	0.1008	0.1419	0.0843	0.1254	0.0678
55%	0.0584	0.0974	0.1371	0.0809	0.1206	0.0644
60%	0.0563	0.0939	0.1321	0.0774	0.1156	0.0609
65%	0.0541	0.0903	0.1271	0.0738	0.1106	0.0573
70%	0.0519	0.0867	0.1221	0.0702	0.1056	0.0537
75%	0.0497	0.0830	0.1170	0.0665	0.1005	0.0500
80%	0.0474	0.0793	0.1119	0.0628	0.0954	0.0463
85%	0.0451	0.0756	0.1068	0.0591	0.0903	0.0426
90%	0.0428	0.0719	0.1017	0.0554	0.0852	0.0389
95%	0.0405	0.0681	0.0965	0.0516	0.0800	0.0351
100%	0.0357	0.0627	0.0902	0.0462	0.0737	0.0297

Table C-4: Average waiting time for different service time distributions, Table 3-6

β %	$c_b^2=0$	$c_b^2=0.5$	$c_b^2=1$
	$(E(W_q))/h$	$(E(W_q))/h$	$((E(W_q))/h$
0%	0.0675	0.10125	0.135
20%	0.0426	0.076347	0.110096
25%	0.044937	0.078685	0.112435
30%	0.04431	0.078058	0.111808
35%	0,04234	0,076087	0,109835
40%	0,039676	0,073421	0,107169
45%	0,036624	0,070366	0,104113
50%	0,033341	0,067078	0,100824
55%	0,029919	0,063648	0,097391
60%	0,026412	0,060127	0,093867
65%	0,022854	0,056548	0,090283
70%	0,01927	0,052928	0,086657
75%	0,015681	0,04928	0.082999
80%	0.012112	0.045609	0.079314
85%	0.008609	0.041919	0.075606
90%	0.005286	0.038213	0.071874
95%	0.002525	0.034495	0.068121

Table C-5: The effect of the squared coefficient of variation of the stochastic part of arrivals, Table 3-9

β %	$c_R^2=2$	$c_R^2=1$
	$(E(W_q))/h$	$(E(W_q))/h$
20%	0.1923	0.1101
25%	0.1829	0.11243
30%	0.1748	0.11181
35%	0.1676	0.10984
40%	0.1608	0.10717
45%	0.1543	0.10411
50%	0.1481	0.10082
55%	0.142	0.09739
60%	0.1361	0.09387
65%	0.1301	0.09028
70%	0.1243	0.08666
75%	0.1189	0.083
80%	0.1142	0.07931
85%	0.1113	0.07561
90%	0.1126	0.07187
95%	0.1307	0.06812

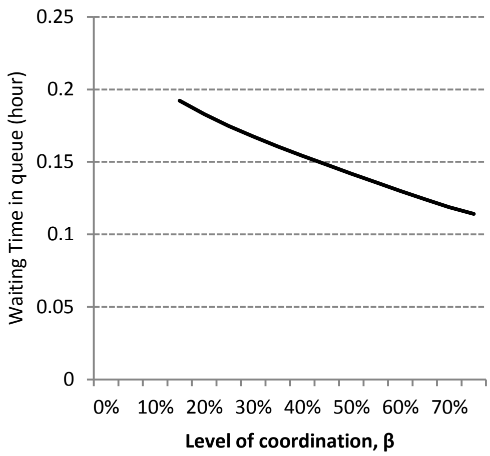


Figure C-2: W_q for $c_R^2 = 2$

Table C-6: Simulation parameters

β %	*Replications*	*Warm up period, (hours)*	*Run length, (hours)*	*Average time in system, (hours)*	*Average queueing time, (hours)*	*# waiting in queue*
0	10	350	2250	0.151	0.136	8.162
20%	10	350	2250	0.135	0.1203	7.218
30%	10	350	2250	0.134	0.1191	7.161
40%	10	350	2250	0.121	0.1058	6.348
50%	10	350	2250	0.115	0.1004	6.024
60%	10	350	2250	0.109	0.0942	5.653
70%	10	350	2250	0.100	0.08537	5.122
80%	10	350	2250	0.092	0.0774	4.640
90%	10	350	2250	0.085	0.07034	4.220

Appendix D: Holtzman Approach by Albin (1986)

- Variables:

$\tilde{L}_j$	=	*Expected number in the system seen by an arrival from stream j*
$\tilde{L}_{aj}$	=	*Expected number of stream j customers in the system seen by the arrival from stream j*
L_{bj}	=	*Expected number of customers from streams other than j in the system seen by the arrival from stream j*
λ_j	=	*Arrival rate of component j*
ρ_h	=	*Traffic intensity in the queue*
μ	=	*Service rate of the server*
W_j	=	*Expected delay for customers of component stream j*
ρ_j	=	*Utilization solely based on component stream j*

- Expected number in the system seen by an arrival from component stream *j*:

$$\tilde{L}_j = \tilde{L}_{aj} + L_{bj}$$

- Expected number of stream *j* customers in the system seen by the arrival from component stream *j:*

$$\tilde{L}_{aj} \cong \sigma_j \big/ \; 1 - \sigma_j$$
$$\bar{\lambda}_j = \lambda - \lambda_j$$
$$\sigma_j = A_j^* \;\; \mu \; 1 - \bar{\lambda}_j \;\; / \mu \;\; 1 - \sigma_j$$

- Expected number of customers from component streams other than *j* in the system seen by the arrival from component stream *j*:

$$L_{bj} \cong \rho_h \big/ \; 1 - \sigma_h$$
$$\rho_h = \bar{\lambda}_j \big/ \mu \; 1 - \rho_j$$
$$\rho_j = \lambda_j \big/ \mu$$
$$\sigma_h = A_h^* \;\; \mu \; 1 - \rho_j \;\;\; 1 - \sigma_h$$

- Average waiting time in the queue for a unit from component stream *j*:

$$W_j \cong V_j = \tfrac{1}{\mu}\Big[\sigma_j \big/ \; 1 - \sigma_j \;\; + \rho_h \big/ \; 1 - \sigma_h \;\; \Big]$$
$$W_j \cong \;\; V_j \big/ \bar{V} \;\; . W_h$$
$$\bar{V} = \sum_{j=1}^{n} \;\; \lambda_j \big/ \lambda \;\; . V_j$$

Table D-1: Component Waiting times, see Figure 2.5

β %	W_q^C	W_q^R	Δ %
20%	7,20	7,58	5%
25%	7,33	7,75	5%
30%	7,30	7,74	6%
35%	7,19	7,65	6%
40%	7,05	7,52	6%
45%	6,88	7,36	6%
50%	6,71	7,19	7%
55%	6,52	7,01	7%
60%	6,33	6,83	7%
65%	6,14	6,64	8%
70%	5,95	6,46	8%
75%	5,75	6,27	8%
80%	5,55	6,08	9%
85%	5,36	5,89	9%
90%	5,16	5,69	9%
95%	4,96	5,50	10%

Appendix E: Coordinated Arrivals

- Derivation of the operational utilization ρ_O:

$$\rho_O = \frac{\lambda}{\text{Average service rate}}$$

$$\rho_O = \frac{\lambda}{1-\beta \;\; \mu + \left(\beta \cdot {}^{1}\!/_{t_{s,P}}\right)} = \frac{\lambda}{\left(\frac{1-\beta}{b}\right) + \left(\frac{\beta}{t_{s,P}}\right)}$$

$$\rho_O = \lambda \;\; 1-\beta \;\; b + \beta t_{s,P}$$

- Justification of the Krämer/Langenbach-Belz Approximation:

  Many approximation formulas re mention in literature to address the performance of systems with general distributed arrivals and service time. Not all of them are, however, both simple and straightforward or a good approximation. The foundation of many approximations is the Cunneen approximation formula for G/G/m systems, giving the exact solution in case of M/G/1 systems. A very good approximation is the Krämer/Langenbach-Belz formual, which is a direct extension of the latter model. This formula is commonly used in literature, and therefore designated to use (Bolch et al., 2006)

Appendix F: Non-exhaustive Extended coordination

Table F-1: Non-exhaustive extended coordination, Figure 4-5

Coordinated Arrivals										
Reduction (%)	*β*	W_q *(min)*	$t_{s,P}$ *(min)*	N_P	*Max* $t_{s,C}$ *(min)*	ρ_C	ρ_0	ρ_s	C_b^2	λ_C
----	0%		1	0	0	0,9	0,9	0,9	1	0
99%	5%	0,08	2,47	1	2,90	0,36	0,98	0,90	1,00	0,05
96%	10%	0,31	1,66	2	1,90	0,54	0,98	0,90	1,00	0,10
93%	15%	0,59	1,3813	3	1,57	0,65	0,97	0,90	1,00	0,15
89%	20%	0,92	1,2414	4	1,40	0,72	0,97	0,90	1,00	0,20
84%	25%	1,31	1,1553	5	1,30	0,78	0,96	0,90	1,00	0,25
78%	30%	1,79	1,0960	6	1,23	0,82	0,96	0,90	1,00	0,30
71%	35%	2,38	1,0518	7	1,19	0,86	0,95	0,90	1,00	0,35
61%	40%	3,18	1,0169	8	1,15	0,89	0,95	0,90	1,00	0,40
47%	45%	4,31	0,9881	9	1,12	0,91	0,94	0,90	1,00	0,45
25%	50%	6,11	0,9633	10	1,10	0,93	0,93	0,90	1,00	0,50
-18%	55%	9,56	0,9411	11	1,08	0,96	0,92	0,90	1,00	0,55
-140%	60%	19,43	0,9205	12	1,07	0,98	0,91	0,90	1,00	0,60
-6258%	65%	515,02	0,9008	13	1,05	1,00	0,90	0,90	1,00	0,65
----	70%	-21,81	0,8812	14	1,04	1,02	0,89	0,90	1,00	0,70
----	75%	3,76	1,0000	15	1,03	0,90	0,98	0,90	1,00	0,75
----	80%	3,76	1,00	16	1,03	0,90	0,98	0,90	1,00	0,80
----	85%	3,76	1,00	17	1,02	0,90	0,99	0,90	1,00	0,85
----	90%	3,76	1,00	18	1,01	0,90	0,99	0,90	1,00	0,90
Random Arrivals										
Random	*1-β*	W_q	*k*	$v^{(1)}$	$v^{(2)}$	*k, Stability*	ρ_R	ρ_0	C_b^2	λ_R
----	100%	8,1	0	0	0	0	0,9	0,9	1	1
-270%	95%	30,00	19	2,47	6,09	16	0,86	0,98	1	0,95
-270%	90%	30,00	18	3,31	10,97	16	0,81	0,98	1	0,90
-270%	85%	30,00	17	4,14	17,17	15	0,77	0,97	1	0,85
-270%	80%	30,00	16	4,97	24,66	14	0,72	0,97	1	0,80
-270%	75%	30,00	15	5,78	33,37	13	0,68	0,96	1	0,75
-270%	70%	30,00	14	6,58	43,24	12	0,63	0,96	1	0,70
-270%	65%	30,00	13	7,36	54,21	12	0,59	0,95	1	0,65
-270%	60%	30,00	12	8,14	66,19	11	0,54	0,95	1	0,60
-270%	55%	30,00	11	8,89	79,09	10	0,50	0,94	1	0,55
-270%	50%	30,00	10	9,63	92,79	9	0,45	0,93	1	0,50
-270%	45%	30,00	9	10,35	107,17	8	0,41	0,92	1	0,45
-270%	40%	30,00	8	11,05	122,02	7	0,36	0,91	1	0,40
-270%	35%	30,00	7	11,71	137,13	6	0,32	0,90	1	0,35
-270%	30%	30,00	6	12,34	152,19	5	0,27	0,89	1	0,30
----	25%	----	5	15,00	225,00	5	0,23	0,98	1	0,25
----	20%	----	4	16,00	256,00	4	0,18	0,98	1	0,20
----	15%	----	3	17,00	289,00	3	0,14	0,99	1	0,15
----	10%	----	2	18,00	324,00	2	0,09	0,99	1	0,10

Table F-2: Non-exhaustive extended coordination: Utilization, Figure 4-6

	Coordinated Arrivals								
	ρ_s=80%			ρ_s=90%			ρ_s=95%		
β	Reduction(%)	W_q(min)	$t_{s,P}$ (min)	Reduction(%)	W_q (min)	$t_{s,P}$ (min)	Reduction(%)	W_q (min)	$t_{s,P}$ (min)
5%	100%	0.01	4.28	99%	0.08	2.47	97%	0.50	1.55
10%	99%	0.05	2.51	96%	0.31	1.66	92%	1.36	1.22
15%	96%	0.11	1.91	93%	0.59	1.38	86%	2.48	1.11
20%	94%	0.20	1.61	89%	0.92	1.24	78%	4.05	1.05
25%	91%	0.30	1.43	84%	1.31	1.16	64%	6.54	1.02
30%	87%	0.41	1.31	78%	1.79	1.10	37%	11.37	0.99
35%	83%	0.54	1.22	71%	2.38	1.05	-43%	25.89	0.97
40%	79%	0.68	1.15	61%	3.18	1.02	----	----	----
45%	74%	0.84	1.10	47%	4.31	0.99	----	----	----
50%	68%	1.04	1.05	25%	6.11	0.96	----	----	----
55%	60%	1.27	1.01	-18%	9.56	0.94	----	----	----
60%	51%	1.56	0.98	-140%	19.43	0.92	----	----	----
65%	39%	1.94	0.95	----	----	----	----	----	----
70%	22%	2.49	0.92	----	----	----	----	----	----
75%	-6%	3.38	0.89	----	----	----	----	----	----
80%	-61%	5.16	0.86	----	----	----	----	----	----
85%	-242%	10.95	0.83	----	----	----	----	----	----
90%	----	----	----	----	----	----	----	----	----

Table F-3: Non-exhaustive extended coordination: Service distribution, Figure 4-7

	Coordinated Arrivals								
	C_b^2 = 0.5			C_b^2 = 1			C_b^2= 2		
β	Reduction(%)	W_q (min)	$t_{s,P}$ (min)	Reduction(%)	W_q (min)	$t_{s,P}$ (min)	Reduction(%)	W_q (min)	$t_{s,P}$(min)
5%	100%	0.011	2.53	99%	0.08	2.47	97%	0.33	2.35
10%	99%	0.081	1.68	96%	0.31	1.66	93%	0.89	1.60
15%	97%	0.194	1.40	93%	0.59	1.38	87%	1.54	1.35
20%	94%	0.339	1.25	89%	0.92	1.24	81%	2.27	1.22
25%	91%	0.518	1.16	84%	1.31	1.16	74%	3.12	1.14
30%	88%	0.739	1.10	78%	1.79	1.10	66%	4.15	1.08
35%	83%	1.021	1.06	71%	2.38	1.05	55%	5.44	1.04
40%	77%	1.396	1.02	61%	3.18	1.02	41%	7.17	1.01
45%	68%	1.930	0.99	47%	4.31	0.99	20%	9.68	0.98
50%	54%	2.776	0.97	25%	6.11	0.96	-13%	13.75	0.96
55%	28%	4.371	0.94	-18%	9.56	0.94	-81%	21.94	0.94
60%	-44%	8.718	0.92	-140%	19.43	0.92	-298%	48.35	0.92
65%	----	----	----	----	----	----	----	----	----
70%	----	----	----	----	----	----	----	----	----

Table F-4: Non-exhaustive extended coordination: Limit W_q^R, Figure 4-8

	Coordinated Arrivals								
	max W_q^R= 20			*max W_q^R= 30*			*max W_q^R= 40*		
β	*Reduction(%)*	*W_q (min)*	*$t_{s,P}$(min)*	*Reduction(%)*	*W_q (min)*	*$t_{s,P}$ (min)*	*Reduction(%)*	*W_q (min)*	*$t_{s,P}$ (min)*
5%	99%	0.10	2.31	99%	0.08	2.47	99%	0.072	2.55
10%	95%	0.37	1.57	96%	0.31	1.66	97%	0.278	1.70
15%	91%	0.71	1.32	93%	0.59	1.38	93%	0.534	1.42
20%	86%	1.14	1.19	89%	0.92	1.24	90%	0.827	1.27
25%	79%	1.69	1.11	84%	1.31	1.16	86%	1.162	1.18
30%	70%	2.42	1.05	78%	1.79	1.10	81%	1.550	1.12
35%	57%	3.49	1.01	71%	2.38	1.05	75%	2.010	1.08
40%	35%	5.27	0.97	61%	3.18	1.02	68%	2.574	1.04
45%	-11%	8.95	0.94	47%	4.31	0.99	59%	3.297	1.01
50%	-171%	21.95	0.92	25%	6.11	0.96	47%	4.282	0.99
55%	----	----	----	-18%	9.56	0.94	29%	5.741	0.97
60%	----	----	----	-140%	19.43	0.92	-1%	8.219	0.95
65%	----	----	----	----	----	----	-68%	13.588	0.93
70%	----	----	----	----	----	----	----	----	----

Appendix G: Exhaustive Extended coordination

Table G-1: Exhaustive extended coordination, Figure 4-11

Coordinated Arrivals

β	Reduction (%)	W_q (min)	$t_{s,P}$ (min)	N_P	Max $t_{s,C}$ (min)	ρ_C	ρ_0	ρ_s	C_b^2	λ_C
0%	----	8,10	----	0,00	----	----	----		1	0
5%	99%	0,06	2,71	1,00	2,90	33%	99%	90%	1	0,05
10%	97%	0,21	1,84	2,00	1,90	49%	99%	90%	1	0,1
15%	95%	0,39	1,54	3,00	1,57	58%	100%	90%	1	0,15
20%	93%	0,58	1,39	4,00	1,40	65%	100%	90%	1	0,2
25%	90%	0,77	1,29	5,00	1,30	70%	100%	90%	1	0,25
30%	88%	0,97	1,23	6,00	1,23	73%	100%	90%	1	0,3
35%	86%	1,16	1,18	7,00	1,19	76%	100%	90%	1	0,35
40%	83%	1,36	1,15	8,00	1,15	78%	100%	90%	1	0,4
45%	81%	1,56	1,12	9,00	1,12	80%	100%	90%	1	0,45
50%	78%	1,77	1,10	10,00	1,10	82%	100%	90%	1	0,5
55%	76%	1,97	1,08	11,00	1,08	83%	100%	90%	1	0,55
60%	73%	2,18	1,06	12,00	1,07	85%	100%	90%	1	0,6
65%	70%	2,40	1,05	13,00	1,05	86%	100%	90%	1	0,65
70%	68%	2,62	1,04	14,00	1,04	87%	100%	90%	1	0,7
75%	65%	2,87	1,03	15,00	1,03	88%	100%	90%	1	0,75
80%	61%	3,15	1,02	16,00	1,03	88%	99%	90%	1	0,8
85%	57%	3,52	1,01	17,00	1,02	89%	99%	90%	1	0,85
90%	50%	4,06	0,99	18,00	1,01	91%	98%	90%	1	0,9
95%	38%	5,00	0,98	19,00	1,01	92%	97%	90%	1	0,95
100%	54%	3,76	1,00	20,00	1,00	90%	100%	90%	1	1

Random Arrivals

1-β	Reduction (%)	W_q	$v^{(1)}$	$v^{(2)}$	ρ_R	C_b^2	λ_R
100%	----	8,10	0,00	0,00	90%	1	1
95%	18%	6,66	2,71	7,34	86%	1	0,95
90%	30%	5,68	3,69	13,60	81%	1	0,9
85%	35%	5,24	4,63	21,43	77%	1	0,85
80%	37%	5,09	5,55	30,83	72%	1	0,8
75%	37%	5,10	6,47	41,80	68%	1	0,75
70%	36%	5,22	7,37	54,36	63%	1	0,7
65%	33%	5,41	8,28	68,52	59%	1	0,65
60%	30%	5,65	9,18	84,27	54%	1	0,6
55%	27%	5,92	10,08	101,61	50%	1	0,55
50%	23%	6,23	10,98	120,55	45%	1	0,5
45%	19%	6,55	11,88	141,06	41%	1	0,45
40%	15%	6,89	12,77	163,12	36%	1	0,4
35%	11%	7,25	13,66	186,69	32%	1	0,35
30%	6%	7,61	14,55	211,68	27%	1	0,3
25%	2%	7,97	15,43	237,93	23%	1	0,25
20%	-3%	8,34	16,28	265,12	18%	1	0,2
15%	-7%	8,69	17,11	292,68	14%	1	0,15
10%	-11%	9,03	17,88	319,57	9%	1	0,1

Table G-2: Probabilty of a delayed start of Coordinated arrivals, Table 4-1

β	$b^{(1)}$	σ_b^2	θ_{RA}	σ_{RA}^2	*Var(D_c)*	c_x^2	c_x	*k*	*1/k*	*1/k-1*	$E(D_c)$	*q*	*λ*	$E_{k-1,k}(D_c)$
5%	0,9	0,81	19,00	19,00	30,78	0,103	0,32	10,00	0,100	0,111	17,291	0,156	0,518	43%
10%	0,9	0,81	18,00	18,00	29,16	0,110	0,33	10,00	0,100	0,111	16,312	0,654	0,519	43%
15%	0,9	0,81	17,00	17,00	27,54	0,117	0,34	9,00	0,111	0,125	15,371	0,242	0,515	43%
20%	0,9	0,81	16,00	16,00	25,92	0,124	0,35	9,00	0,111	0,125	14,448	0,778	0,514	43%
25%	0,9	0,81	15,00	15,00	24,30	0,133	0,36	8,00	0,125	0,143	13,534	0,270	0,515	44%
30%	0,9	0,81	14,00	14,00	22,68	0,142	0,38	8,00	0,125	0,143	12,627	0,835	0,512	44%
35%	0,9	0,81	13,00	13,00	21,06	0,153	0,39	7,00	0,143	0,167	11,722	0,280	0,517	45%
40%	0,9	0,81	12,00	12,00	19,44	0,166	0,41	7,00	0,143	0,167	10,820	0,858	0,512	46%
45%	0,9	0,81	11,00	11,00	17,82	0,181	0,43	6,00	0,167	0,200	9,920	0,282	0,520	46%
50%	0,9	0,81	10,00	10,00	16,20	0,199	0,45	6,00	0,167	0,200	9,020	0,859	0,514	47%
55%	0,9	0,81	9,00	9,00	14,58	0,221	0,47	5,00	0,200	0,250	8,123	0,281	0,524	48%
60%	0,9	0,81	8,00	8,00	12,96	0,248	0,50	5,00	0,200	0,250	7,228	0,836	0,520	49%
65%	0,9	0,81	7,00	7,00	11,34	0,282	0,53	4,00	0,250	0,333	6,336	0,272	0,533	50%
70%	0,9	0,81	6,00	6,00	9,72	0,327	0,57	4,00	0,250	0,333	5,451	0,781	0,536	51%
75%	0,9	0,81	5,00	5,00	8,10	0,387	0,62	3,00	0,333	0,500	4,575	0,243	0,551	52%
80%	0,9	0,81	4,00	4,00	6,48	0,469	0,68	3,00	0,333	0,500	3,717	0,664	0,584	55%
85%	0,9	0,81	3,00	3,00	4,86	0,581	0,76	2,00	0,500	1,000	2,892	0,156	0,615	58%
90%	0,9	0,81	2,00	2,00	3,24	0,719	0,85	2,00	0,500	1,000	2,123	0,400	0,800	63%
95%	0,9	0,81	1,00	1,00	1,62	0,775	0,88	2,00	0,500	1,000	1,446	0,495	1,505	76%

Table G-3: Opportunity of exhaustive extended coordination with buffer, Figure 4-12

	Non-Exhaustive	***Exhaustive, No Buffer***	***Exhaustive, Buffer***						
β	*Reduction* W_q^C	*Reduction* W_q^C	*Reduction* W_q^C	*Reduction* W_q^R	W_q^C	$t_{s,P}$	W_q^R	D_c	c_x^2
5%	99%	99%	*96%*	25%	0,341	1,60	6,11	12,10	0,148
10%	96%	97%	*87%*	38%	1,081	1,20	5,04	11,57	0,152
15%	93%	95%	*83%*	43%	1,346	1,15	4,65	11,86	0,149
20%	89%	93%	*81%*	44%	1,564	1,12	4,55	11,85	0,150
25%	84%	90%	*80%*	43%	1,651	1,11	4,64	11,71	0,152
30%	78%	88%	*77%*	41%	1,852	1,09	4,80	11,25	0,159
35%	71%	86%	*76%*	38%	1,969	1,08	5,05	10,74	0,167
40%	61%	83%	*74%*	34%	2,100	1,07	5,34	10,11	0,177
45%	47%	81%	*72%*	30%	2,248	1,06	5,65	9,40	0,190
50%	25%	78%	*69%*	26%	2,416	1,05	5,99	8,64	0,208
55%	-18%	76%	*65%*	22%	2,816	1,03	6,28	7,76	0,231
60%	-140%	73%	*60%*	19%	3,208	1,02	6,60	6,91	0,259
65%	-6258%	70%	*55%*	14%	3,640	1,00	6,93	6,06	0,294
70%		68%	*49%*	10%	4,126	0,99	7,27	5,22	0,341
75%		65%	*42%*	6%	4,702	0,98	7,62	4,38	0,402
80%		61%	*33%*	2%	5,437	0,97	7,96	3,57	0,485
85%		57%	*20%*	-2%	6,504	0,96	8,30	2,79	0,597
90%		50%	*-4%*	-6%	8,385	0,95	8,61	2,06	0,729
95%		38%	*-60%*	-10%	12,961	0,93	8,88	1,42	0,767

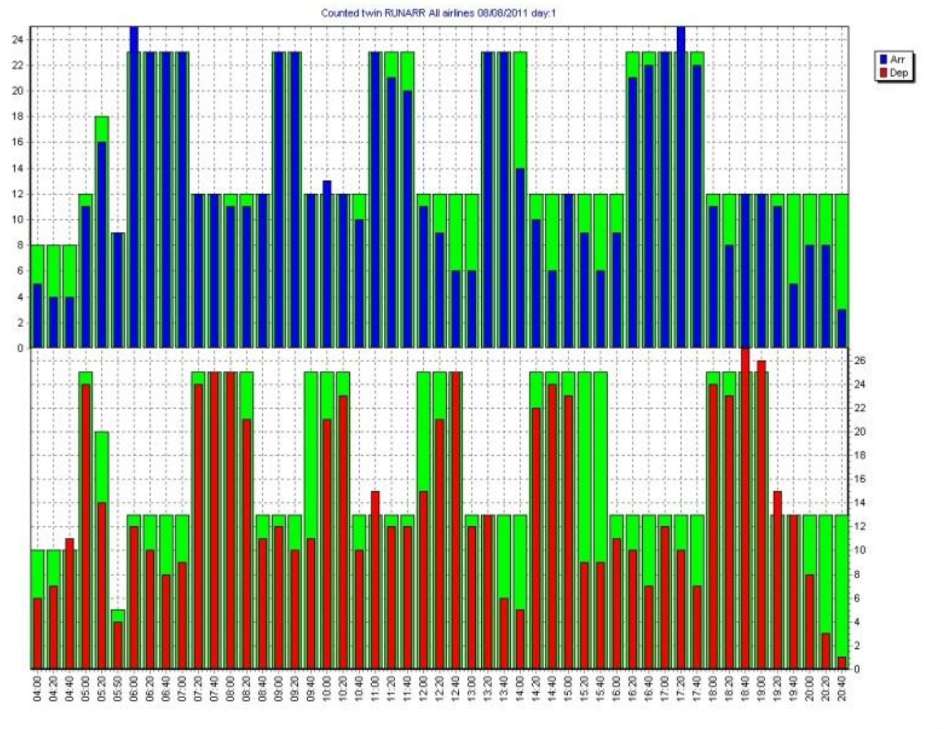
Appendix H: Cost calculations

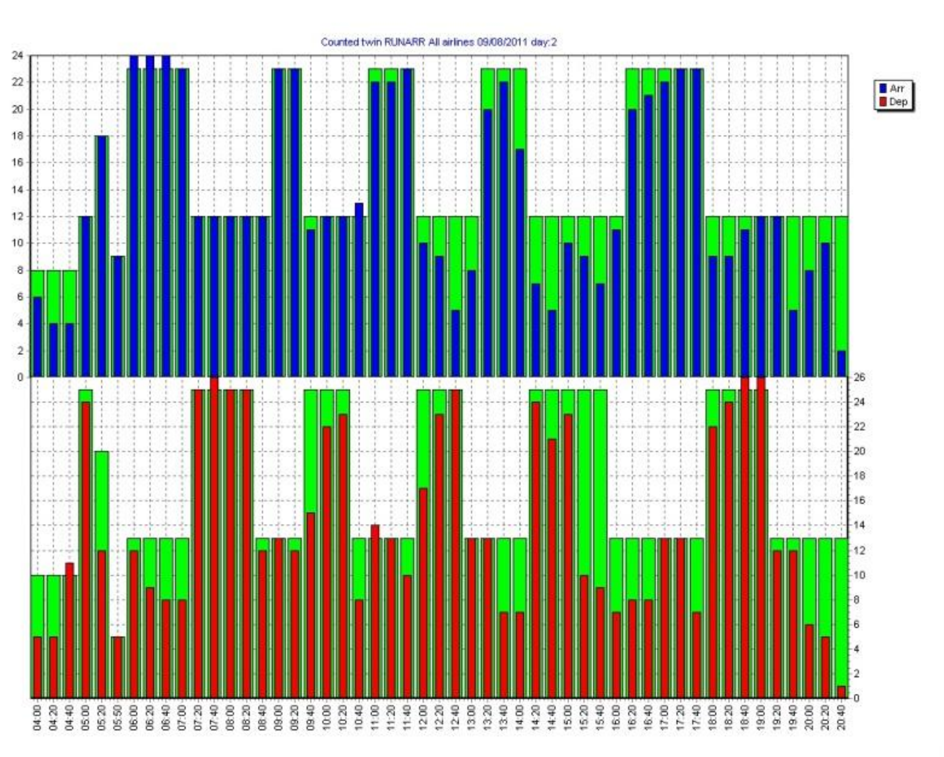
Table F-1: Average operating cost of aircraft

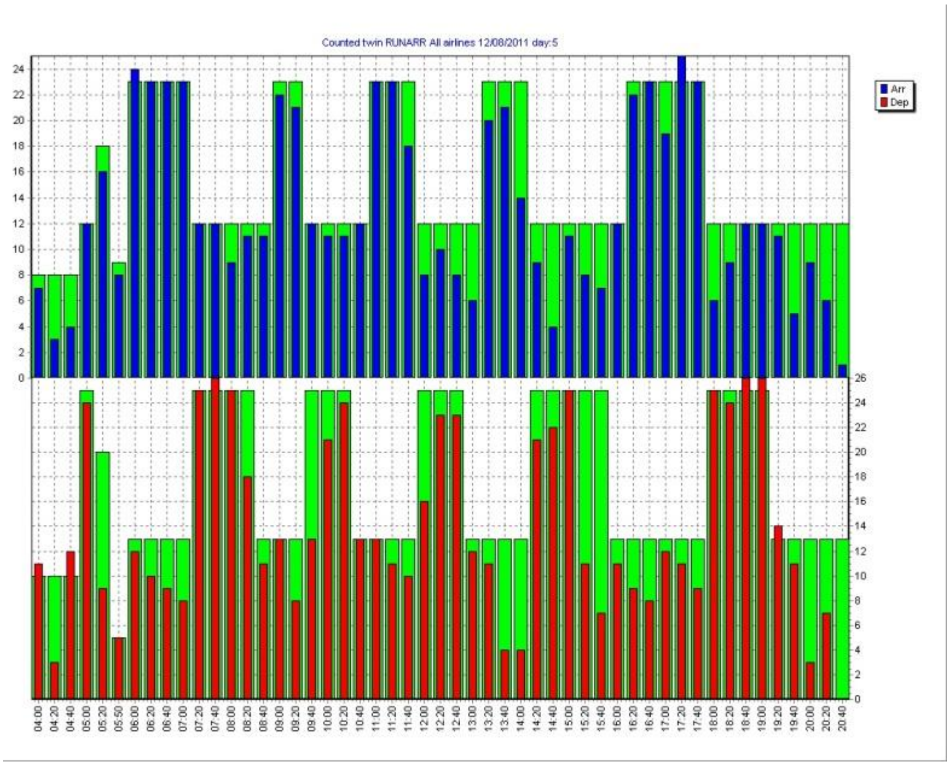
	Aircraft type	*Cumulative percentage*	*Number of flight*	*Total operating costs (€ per block hour)*
1	A320	11,3%	86861	3012
2	B738	20%	69815	2098
3	A319	29%	69725	2857
4	A321	33%	28877	3344
5	B733	37%	26697	2641
6	AT72	39%	21312	1992
7	B737	42%	17354	2641
8	DH8D	44%	15502	1430
9	B735	46%	15482	2751
10	B734	48%	15324	2982
			Weighted average cost per hour (€)	2651,90
			Cost per minute (€)	***44,20***

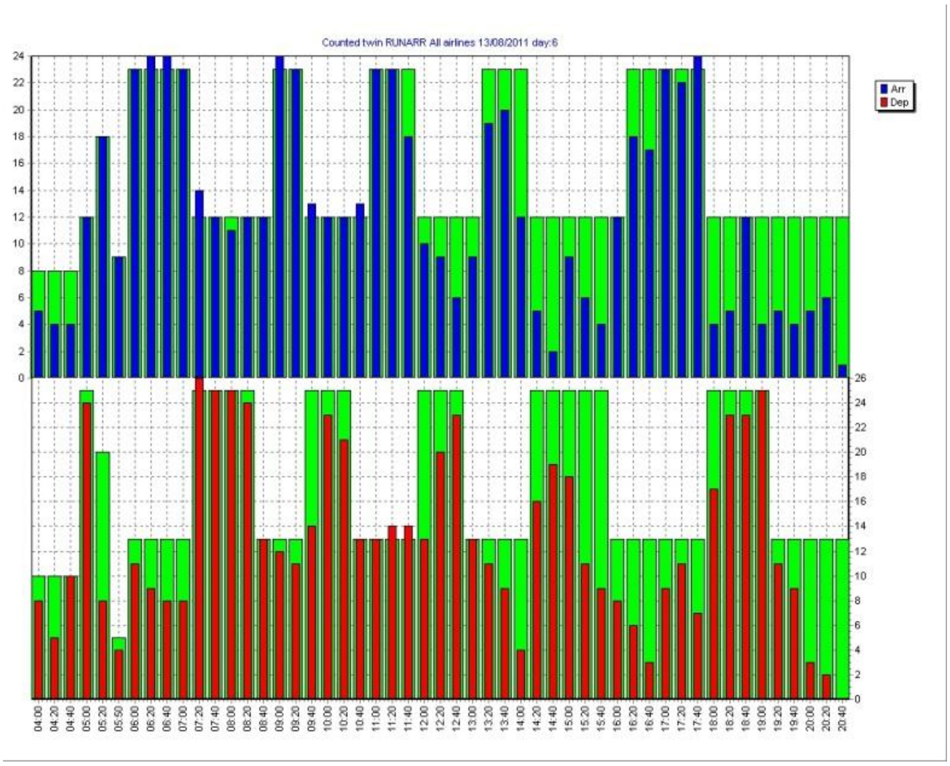
Appendix I: Demand and capacity AMS

Figure I-1: Four represtative random days of arrival and departure slots plus the actual arrivals and departures during the summer period at Schiphol Amsterdam Airport (www.slotcoordination.nl)






Counted twin RUNARR All airlines 12/08/2011 day:5
Arr
Dep


Counted twin RUNARR All airlines 13/08/2011 day:6
Arr
Dep

Appendix J: Case study; Example 1

Table J-1: Basic arrival coordination

β	W_q *(min)*	*Reduction in* W_q *(min)*	*Savings per flight (€)*	*Savings per day (€)*	*Annual Savings (€)*
0%	0,90	---	---	---	---
5%	0,87	0,03	€ 0,94	€ 634,55	€ 231.609,82
10%	0,84	0,06	€ 1,87	€ 1.269,09	€ 463.219,64
15%	0,80	0,10	€ 2,81	€ 1.903,64	€ 694.829,46
20%	0,77	0,13	€ 3,74	€ 2.538,19	€ 926.439,28
25%	0,74	0,16	€ 4,68	€ 3.172,74	€ 1.158.049,10
30%	0,71	0,19	€ 5,62	€ 3.807,28	€ 1.389.658,92
35%	0,67	0,23	€ 6,55	€ 4.441,83	€ 1.621.268,74
40%	0,64	0,26	€ 7,49	€ 5.076,38	€ 1.852.878,56
45%	0,61	0,29	€ 8,42	€ 5.710,93	€ 2.084.488,38
50%	0,58	0,32	€ 9,36	€ 6.345,47	€ 2.316.098,20
55%	0,54	0,36	€ 10,30	€ 6.980,02	€ 2.547.708,02
60%	0,51	0,39	€ 11,23	€ 7.614,57	€ 2.779.317,84
65%	0,48	0,42	€ 12,17	€ 8.249,12	€ 3.010.927,66
70%	0,45	0,45	€ 13,10	€ 8.883,66	€ 3.242.537,48
75%	0,41	0,49	€ 14,04	€ 9.518,21	€ 3.474.147,30
80%	0,38	0,52	€ 14,97	€ 10.152,76	€ 3.705.757,12
85%	0,35	0,55	€ 15,91	€ 10.787,31	€ 3.937.366,94
90%	0,32	0,58	€ 16,85	€ 11.421,85	€ 4.168.976,76
95%	0,28	0,62	€ 17,78	€ 12.056,40	€ 4.400.586,58
100%	0,25	0,65	€ 18,72	€ 12.690,95	€ 4.632.196,40

Table J-2: ‘Extended’ arrival coordination

β	W_q^C *(min)*	W_q^R *(min)*
0%	---	0,90
5%	0,00	20,00
10%	0,00	20,00
15%	0,00	20,00
20%	0,01	20,00
25%	0,03	20,00
30%	0,04	20,00
35%	0,07	20,00
40%	0,09	20,00
45%	0,13	20,00
50%	0,16	20,00

β	W_q^C *(min)*	W_q^R *(min)*
55%	0,21	20,00
60%	0,26	20,00
65%	0,33	20,00
70%	0,42	20,00
75%	0,53	20,00
80%	0,69	20,00
85%	0,96	20,00
90%	1,51	20,00
95%	---	20,00
100%	---	---

Appendix K: Case study; Example 2

Table K-1: 'Basic' arrival coordination

β	*W_q (min) (Albin, 1984)*	*W_q (min) (Interpolation)*	*Reduction in W_q (min)*	*Savings per flight*	*Savings per hour*
0%	21,00	21,00	---	---	---
5%		20,45	0,55	€ 15,76	€ 641,31
10%		19,91	1,09	€ 31,53	€ 1.282,63
15%		19,36	1,64	€ 47,29	€ 1.923,94
20%		18,81	2,19	€ 63,06	€ 2.565,26
25%		18,26	2,74	€ 78,82	€ 3.206,57
30%	17,20	17,72	3,80	€ 109,57	€ 4.457,21
35%	16,74	17,17	4,26	€ 122,64	€ 4.988,98
40%	16,25	16,62	4,75	€ 136,83	€ 5.566,39
45%	15,73	16,07	5,27	€ 151,67	€ 6.170,05
50%	15,20	15,53	5,80	€ 166,91	€ 6.789,82
55%	14,67	14,98	6,33	€ 182,40	€ 7.419,96
60%	14,12	14,43	6,88	€ 198,06	€ 8.057,10
65%	13,57	13,88	7,43	€ 213,85	€ 8.699,23
70%	13,02	13,34	7,98	€ 229,72	€ 9.345,17
75%	12,47	12,79	8,53	€ 245,68	€ 9.994,28
80%	11,91	12,24	9,09	€ 261,71	€ 10.646,23
85%	11,35	11,69	9,65	€ 277,80	€ 11.300,84
90%	10,79	11,15	10,21	€ 293,95	€ 11.958,05
95%	10,23	10,60	10,77	€ 310,17	€ 12.617,76
100%	10,05	10,05	10,95	€ 315,30	€ 12.826,29

Table K-2: 'Extended' arrival coordination

β	*W_q^C (min)*	*W_q^R (min)*
0%	21,00	20
5%	1,21	20
10%	3,10	20
15%	6,55	20
20%	15,81	20
25%	192,59	20