MASTER

Fuzzy logic
an introduction

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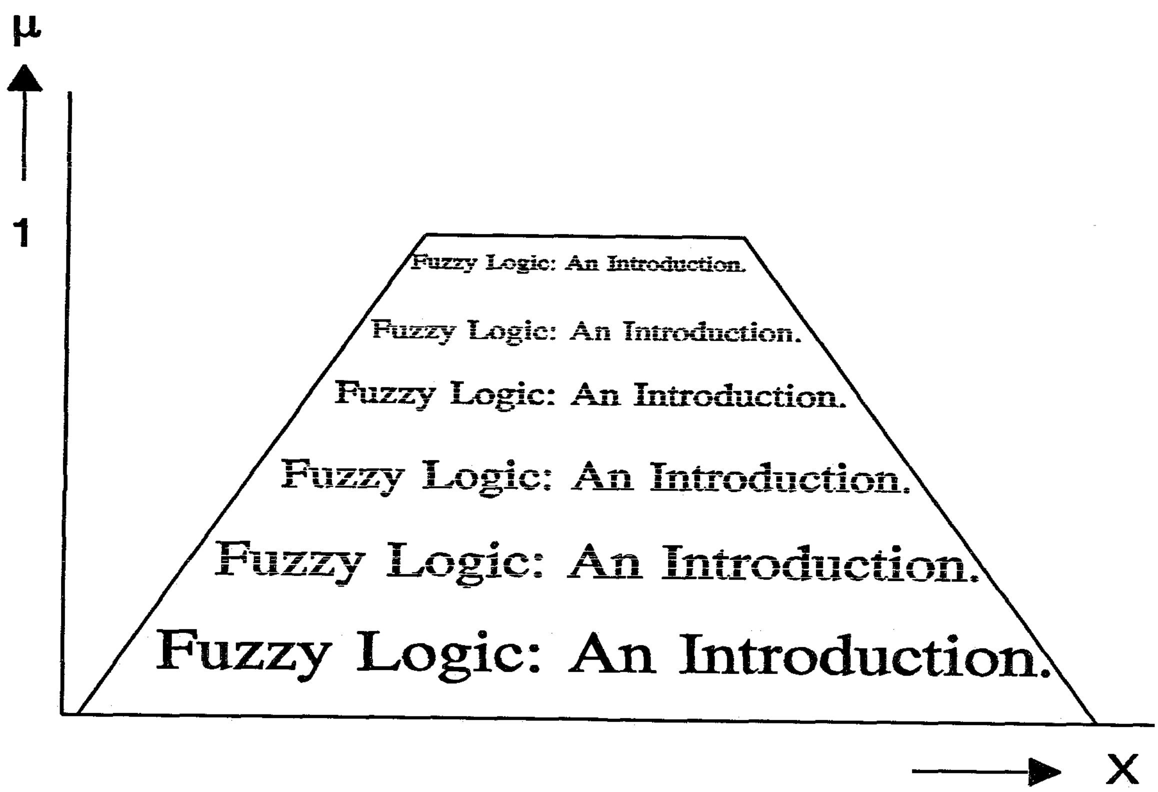

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Fuzzy Logic: An Introduction.

Graduation Report

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The great tragedy of Science, the slaying of a beautiful hypothesis by an ugly fact.

T.H. Huxley

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GLOSSERY OF SYMBOLS

$\{x, y, ...\}$	Set of elements x, y,...
$\{x \mid p(x)\}$	Set determined by property p
$[a, b]$	Closed interval of real numbers between a and b
$[a, b)$	Interval of real numbers closed in a, open in b
X	Universele set
$A, B, C, ...$	Arbitrary sets (crisp or fuzzy)
$x \in X$	Member of a set
$A = B$	Set equality
$A \neq B$	Set inequality
$A - B$	Difference between sets
$A \subseteq B$	Set inclusion
$A \subset B$	Proper set inclusion
$\emptyset$	Empty set
$A \cap B$	Set intersection
$A \cup B$	Set union
$A \times B$	Cartesian product of two sets
A^2	Cartesian product of A x A
$X \to Y$	Function of X into Y
$R(X, Y)$	Relation on X x Y
$R \circ Q$	Max-min composition of binairy fuzzy relations R and Q
$<$	Less then
$\leq$	Less than or equal to
$x \mid y$	x given y
$\forall$	For all
$\max(x_1, x_2, ..., x_n)$	Maximum of $x_1, x_2, ..., x_n$
$\min(x_1, x_2, ..., x_n)$	Minimum of $x_1, x_2, ..., x_n$
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^+$	Set of all nonnegative real numbers

SUMMARY

Subject of this graduation report is fuzzy logic. It's aim is to provide the reader such an insight in this matter, that he/she will be able to fathom the working of a fuzzy controller and eventualy to apply the (base-)theory.

Contrary to her name, which leads one to think that the theorie is not clear, fuzzy set theory is least of all vague. This buzzword is at most meant for the big freedom one has at using the many possible definitions. Fuzzy logic can be seen as a more general form of the traditional binary logic. Can classical logic only be characterized as being 'black or white' (consider for example a set, then an element either **is** or **isn't** a member), fuzzy logic is familiar with greyvalues (an element can also partly belong to a set). One of the advantages that come along with that is that owing to this we are in are able to decribe linguïstic notions mathematically. Consider, for example, the notion "long", then it will be clear that we hardly can describe this with classical sets, after al where should we draw the line? Is a person of 1,80 meters "long"? If so, is then what about a someone of 1,79 meters, is that person not "long"? In the fuzzy set "long" a person of 1,80 meters could for instance belong to that set with a value of, let's say 0.8 and someone who is 1,79 meters with a value of 0.75. Thanks to this particular property we can control processes on the basis of, from classical logic, the familiar *if* ... *then* ... rules, in which the conditional part and the conclusion part are now allowed to be linguïstic notions. An example of such a rule is: *if* 'temperature' 'high' *then* 'close valve', in which the conditional part 'temperature high' and the conclusion part 'close valve' consequently are characterized by fuzzy sets. All rules together shape the so-called rule-base of the controller.
The ultimate output of the controller is obtained by extracting one crisp (= exact) value out of the combination of the conclusion-parts of all rules. For this operation several methods have been suggested. The one most frequently used computes the centre of gravity of the fuzzy set that comes into being by the combination of all, from the rules resulting, fuzzy sets.

In this report will, after treatment of a few basic definitions and operations, be explained how the degree of membership in the fuzzy set from the conclusion part is determined and how fuzzy sets can be aggregated. Some methods to come to one crisp outputvalue will be presented, after which the theory wich we came forward with will be used in an application.
That fuzzy set theory isn't soul-saving will be demonstrated in the second part of the report, where some disadvantages are reviewed.

As this report is build up in a different way as usual, in that sense that here is mainly-spoken about a theory and simulations merely formed a very small part in this investigation, conclusions are hardly to be drawn from the report itself. Hence that merely some general advantages and disadvantages of this method of control will be discussed. However, recommendations can (and will) be made in behalf of an eventual continuation of the investigation of this promissing field of control.

PART I

BASIC THEORY

Chapter 1 INTRODUCTION

Most methods for modelling, reasoning and calculating are by nature *crisp*, deterministic and precise. By crisp I mean dichotomous, which means **yes** or **no** instead of more or less. In conventional binary logic, for instance, a proposition is either **true** or **not true** but cannot be something in between. In the set theory an element either **is** or **is not** a member of the set. Precision means that the parameters of a model exactly depict either our conception of the modelled object or the properties of the real system. At the same time precision (usually) also means that a model is unequivocal and thereby clear. This does not correspond to the real world, which is full of ambiguities. For example, let us take the set of "animals". Dogs, birds and horses are unmistakably members of this set. It is also obvious that rocks, liquids and plants cannot be regarded as members of this set. However, starfish and bacteria present us with a problem because it is hard to make out whether or not they belong to this set. A similar ambiguity can be discerned with regard to the number 10 if we look at the set of "real numbers much greater than one", and if we examine the set of "tall people", in which "tall" depends on the height of the observer and the culture to which he belongs. Nevertheless such unclearly defined "sets" play an important role in human thinking and feeling (intuition). The mere fact that we have several synonyms for one word already illustrates that the force of our thoughts and feelings far exceeds that of spoken language. If we compare spoken language with the language of logic, it appears that the latter is even more inadequate. A one-to-one translation of that which we have in mind into mathematical or logic language therefore seems impossible. It must be remarked, however, that mathematical language is not rejected. Its usefulness is still undisputed. But in certain cases it is found wanting because of its dichotomous nature.

This report will discuss how we can handle the above types of sets as well as the (im)possibilities involved. The concept we are dealing with is referred to as a *fuzzy set*, which is a set of which the membership may have any value in the interval [0,1]. This is a contrast to the traditional sets, of which the elements can only have a membership value of one or zero (to indicate that they **belong** or **don't belong** to the set). As will be shown in the course of this report, the notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework wich parallels in many respects the framework used in the case of ordinary sets but is more general and, potentially, may prove to have a much wider scope of applicability. Essentially such a framework provides a natural way of dealing with problems in wich the source of imprecisions is the absence of sharply defined criteria of class membership (boundaries) [Zadeh 1965, p. 339].

After an introduction of the concept of membership function by means of classical sets, Chapter 2 will give a few definitions concerning fuzzy sets and will introduce a few properties we have not encountered in classical sets.

1.2 Survey of other chapters

The report is divided into two parts. The first one is the principal part of the report, in the latter we will dig a little deeper in the theory which was already presented in the first part. A short description of the report is given below.

In this core part we will merely discuss those subjects and definitions which are needed to gather a good notion of the way in which fuzzy controllers are build up and how they work.

After an introduction of the concept of membership function on the basis of classical sets, Chapter 2 will give a few definitions concerning fuzzy sets. Next we shall discuss a few properties we have not encountered in classical sets.

Chapter 3 deals with operations on fuzzy sets, after first having executed these operations on classical sets. A few distinct operations will also be mentioned. Then the specific properties of the three major operators will be discussed: intersection, union and complement. In the final section of this chapter we shall consider fuzzy relations and fuzzy rules.

Chapter 4 explains how the knowledge obtained in the previous chapters can be used to control a process. On the basis of a simple example, this chapter will illustrate how fuzzy rules can help us to get from a measuring value to an output value.

In the last chapter of the first part we shall use the devices gathered from the previous chapters to work towards a solution, on the basis of the phase plane, for one more or less classical problem (for fuzzy controllers).

In this second part we shall focus on the limitations of the theory, described in the first four chapters, and some important possiblities (which are not necessary for some basic umderstanding of the theory). Chapter 6 will present one of the most important properties of fuzzy logic, the ability to reason. We shall consider two inference rules. After having discussed a number of disadvantages of the max and min operator, Chapter 7 will provide an alternative for these two operators. Next a classification of all possible operators will be given. A disadvantage of the most used inference methods will be dealt with, after which the chapter is concluded with the presentation of a possible solution.

Chapter 8 will examine fuzzy ranking methods. In certain cases it is required to rank fuzzy sets and, because of the gradated concepts used in fuzzy logic, this does not appear to be a trivial matter. Based on a few criteria, an improved ranking method is presented.

Finally, this second part of the report will introduce a few alternative fuzzy control methods. After which some (general) conclusions and recommendations shall conclude this report.

Chapter 2 FUZZY SETS

2.1 Introduction

This chapter contains the basic definitions of fuzzy sets. In addition, we shall indicate which notation systems are used in literature for fuzzy sets. This report consistently uses the same terminology.

Although the reading of this report does not require a special mathematical background, some elementary knowledge concerning classical sets will be needed. Concepts such as set, subset, elements, intersection, union, cartesian product etc. are supposed to be familiar. Mathematical proof is omitted from this report and anyone who is interested in it will be referred to literature.

2.2 Ordinary sets

A **classical** (crisp) **set A** is a set of elements or objects **x** in universe of discourse (domain) **X**. A set can be finite, infinite, countable or uncountable. Any **x** from **X** either belongs or does not belong to **A**.

Classical set **A** can be described in several ways. Sometimes we can enumerate or give an analytic description of the set, for instance, by stating the conditions under which an element belongs to the set ($A = \{x \mid x \leq 5\}$). We can also indicate whether or not an element is a member of a set by means of a so-called *characteristic function* μ_A, in which $\mu_A(x) = \mathbf{1}$ if x is an element of A and $\mu_A(x) = \mathbf{0}$ if x is not an element of **A**. This characteristic function μ_A can therefore have only **two** values and mathematically looks as follows:

$$\mu_A : X \rightarrow \{0, 1\}$$

$$\mu_A(x) = \begin{cases} 0, & \text{if } x \notin A \\ 1, & \text{if } x \in A \end{cases}$$

The characteristic function can be regarded as a membership function: if $\mu_A(x) = 1$ then **x** is a member of the set, otherwise it is not.

The membership function of empty set $\emptyset$ is identical to zero, so $\mu_\emptyset(x) = 0, \forall x \in X$.

2.3 Fuzzy sets: a few definitions

The boundaries of classical set **A** are sharply defined: each $\mathbf{x} \in \mathbf{X}$ is either an element of **A** or it is not. In fuzzy sets these boundaries are blurred in the sense that the membership function not only has values between zero and one, but can in fact assume any non-negative value! In concurrence with the definition of the empty set in the crisp situation, the value of the membership function of a fuzzy empty set of elements from the universe is zero.

Before giving a number of examples (in various notations), the formal definition of a fuzzy set will be given [Zadeh 1965, p. 339].

Definition 2-1

Let **X** be a space of points (objects). An arbitrary element of **X** is indicated as **x**. A *fuzzy set* **A** in **X** is characterised by a *membership function* μ_A, Which adds a real number from [0,1] to each $\mathbf{x} \in \mathbf{X}$. The value $\mu_A(x)$ indicates the membership degree of **x** in **A**: the closer $\mu_A(x)$ is to one, the more **x** belongs to **A**. **A** can be indicated as a collection of ordened pairs:

$$\mathbf{A} = \{ (\mathbf{x}, \mu_A(\mathbf{x})) \mid \mathbf{x} \in \mathbf{X} \}$$

Elements with a membership degree of zero are normally not indicated. If **A** is a classical set, $\mu_A(x)$ can only have two values, **0** and **1**, which reduces $\mu_A(x)$ to the characteristic function of the non-fuzzy set. The range of the membership function is a subset of the non-negative real numbers with a finite supremum[1].

In literature different notations are used for fuzzy sets. They can roughly be divided into two groups:

I A fuzzy set is notated as a set of **pairs** of which the first part indicates the elements and the second part the membership degree of the element in the set.

Examples:

A = "the real numbers close to 10"

$$A = \{ (x, \mu_A(x) \mid x \in X \},$$
$$X = \mathbb{R}\ ; \quad \mu_A(x) = (1+(x-10)^2)^{-1}$$

A = "the real numbers greater than 10"

$$A = \{ (x, \mu_A(x)) \mid x \in X \}, \qquad \textit{in wich :}$$

$$X = \mathbb{R}\ ; \quad \mu_A(x) = \begin{cases} 0\,, & x \le 10, \\ (1+(1+(x-10))^{-2})^{-1}, & x > 10. \end{cases}$$

1 The supremum $\sup_{x \in X} \mu_A(x)$ is the smallest number which is larger than or equal to $\mu_A(x)$ for all $\mathbf{x} \in \mathbf{X}$. So $\sup_{x \in \mathbb{R}^+}(1-e^{-\alpha x}) = 1$ if $\alpha > 0$.

A = "comfortable shower temperature"

A = { (39,0.1), (40,0.3), (41,0.5), (42,0.8), 43(1.0), (44,0.5), (45,0.1) }
X = [0,100].

Zadeh suggests a simpler notation. If **U** represents a finite set $\{x_1,...,X_n\}$, he proposes to notate **A** as:

$$A = \mu_A(x_1)/x_1 + ... + \mu_A(x_n)/X_n = \sum_{i=1}^{n} \mu_A(x_i)/x_i$$

In this formula **+** has nothing to do with a summation but represents an operation which meets a/x+b/x = max(a,b)/x. Furthermore, **a/ω** stands for a pair and **not** for a mathematical division. If **U** is continuous (i.e. uncountable) we write the formula as follows:

$$A = \int_U \mu_A(x)/x, = \{ (x, \mu_A(x)) \quad | \; x \in U \}$$

In this case the integral sign stands for the combination of all separate elements and does **not** represent the usual mathematical operation.

II A fuzzy set is only represented by its membership function.

Example:

A = " The real numbers close to 10"

$$A = \int_{\mathbb{R}} \frac{1}{1+(x-10)^2}/x$$

Instead of μ_A, f_A or χ_A are also frequently used in literature.

As has already been mentioned before, the value of the membership function is not limited to the closed interval [0,1]. If the supremum $\sup_{x \in X} \mu_A(x) = 1$, fuzzy set **A** is called *normal.* Moreover, any fuzzy set **A** which is not empty, that is to say a set of at least one **x ∈ X** so that $\mu_A(x) > 0$, can be *normalised* by dividing $\mu_A(x)$ by $\sup_{x \in X} \mu_A(x)$. After which $\mu_A: X \rightarrow [0,1]$.

In the rest of this report we shall, for the sake of convenience, start from normalised membership functions, unless stated otherwise.

In the fuzzy set theory we also have subsets which are defined as follows:

Definition 2-2

If the membership value of each element from universal set **X** in fuzzy set **A** is smaller than or equal to the membership degree in fuzzy set **B**, then **A** is called a *subset* of **B**, which is mathematically expressed as follows:

$$\text{If } \mu_A(x) \le \mu_B(x), \quad \forall x \in X, \quad \text{then: } A \subseteq B$$

Definition 2-3

The *support* of fuzzy set **A**, **S(A)**, is the crisp set of all **x** $\in$ **X**, to which applies that $\mu_A(x) > 0$.

A slightly more general and useful concept is that of the *α-level set.*

Definition 2-4

The (crisp) set of elements **x** $\in$ **X** with $\mu_A(x) \ge \alpha$ is called the α-level set $\mathbf{B}_\alpha$ of **A**:

$$B_\alpha = \{ x \in X \mid \mu_A(x) \ge \alpha \}$$

Convexity plays an important role in the fuzzy set theory. By contrast to the classical set theory, the convexity conditions are defined in terms of membership function instead of in terms of support.

Definition 2-5

A fuzzy set is convex if and only if:

$$\forall x,y \in X \quad \forall \lambda \in [0,1]: \quad \mu_A(\lambda x + (1-\lambda)y) \ge \min(\mu_A(x), \mu_A(y))$$

In other words, a fuzzy set is convex if all its α-level sets are convex.
Examples of both a convex and a non-convex fuzzy set are given below.

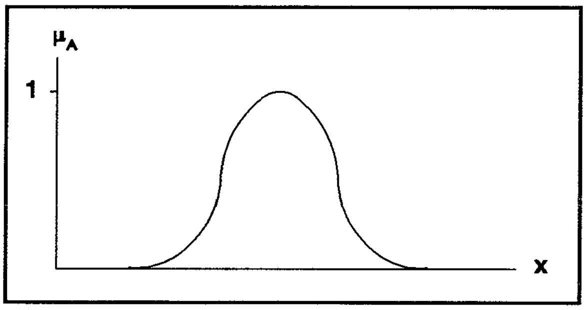


Figure 2-1: Convexe fuzzy set A.

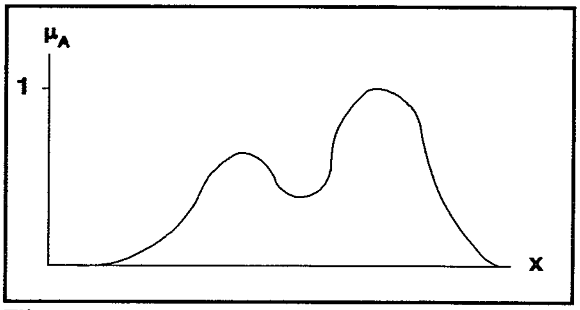


Figure 2-2: A non-convex fuzzy set.

2.4 Membership functions

A few linguistic concepts which are frequently used as examples in the fuzzy set theory are old, tall, heavy and red. They are respectively defined on the domains of age, height, weight and colour. What these four concepts have in common is that they are very hard to describe by means of sharply defined boundaries.

The fuzzy set theory tries to solve the ensuing problems by means of its fuzzy sets. The problem of giving an exact definition of each linguistic concept is avoided by aiming at the most general membership functions possible.

The above Figure (2-1) of the convex fuzzy set gives one possible shape of a membership function. The freedom of choice as regards the course of this function is very great, but is more or less restricted by the meaning of the fuzzy set to be characterised. The above does not correctly depict a concept like "old". After all, a person is old above a certain age and the membership function should not decrease beyond this point.
However, there is one requirement which should be met by all membership functions, and that is that the membership value can **never** be negative. Moreover, normalised membership functions must lie in the interval [0,1].

Zadeh has formulated two types of membership functions which cover a large group of linguistic concepts.
The first of these is the S-function, which is defined as follows:

$$S(x;\alpha,\beta,\gamma) = \begin{cases} 2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right)^2, & \textit{for } \alpha < x \\ 1-2\left(\dfrac{x-\alpha}{\gamma-\alpha}\right)^2, & \textit{for } \beta < x \\ 1, & \textit{for } x > \gamma \end{cases}$$

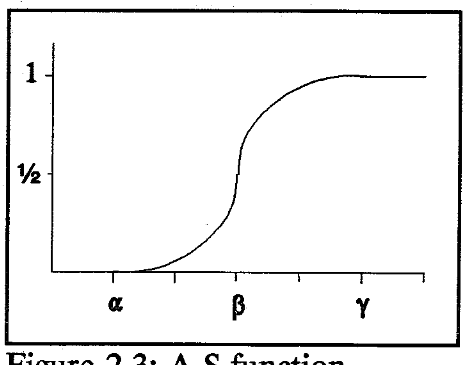


Figure 2-3: A S-function.

in which $\beta = (\alpha+\gamma)/2$ is the "turning point".

Please note that the shape of the S-function is selected arbitrarily. This also holds for Zadeh's bell-shaped π-function, which is defined as:

$$\pi(x;\beta,\gamma) = \begin{cases} S(x;\gamma-\beta,\gamma-\frac{\beta}{2},\gamma), & \text{for } x \le \gamma \\ 1-S(x;\gamma,\gamma+\frac{\beta}{2},\gamma+\beta), & \text{for } x \ge \gamma \end{cases}$$

It is very hard to indicate why an S- or π-function should be used instead of a possible alternative. At the same time it is very difficult to demonstrate what the membership degree really means for an object. According to some authors this is not relevant, as fuzzy sets are intrinsically vague and need only give an indication or tendency of the corresponding linguistic concept.
We may (want to) have a function which strongly resembles the π-function or a function which more or less approximates the S-function, depending on the choice of the fault function. Hence the striving for a membership function which is as universally applicable as possible. This function should also give us the opportunity to represent trapezoidal sets (fuzzy sets with membership degree 1 in interval $[\beta,\gamma]$).
An example of such a function is the "Quadruple" introduced by Bonisonne:

$$Q(x;\alpha,\beta,\gamma,\delta) = \begin{cases} 0, & \text{for } x \le \alpha, \\ \frac{(x-\alpha)}{(\beta-\alpha)}, & \text{for } \alpha \le x \le \beta, \\ 1, & \text{for } \beta \le x \le \gamma, \\ \frac{(\delta-x)}{(\delta-\gamma)}, & \text{for } \gamma \le x \le \delta, \\ 0, & \text{for } x > \delta. \end{cases}$$

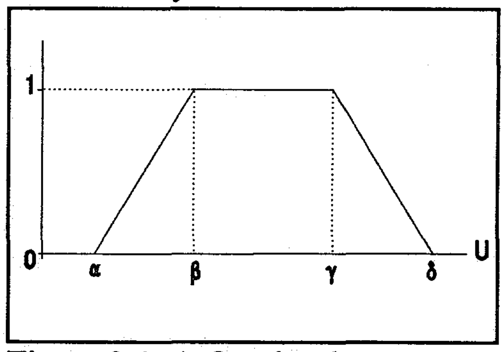


Figure 2-4: A Quadruple.

If $\beta \neq \gamma$, this function cannot be displayed by an S-function, nor by a π-function.

We shall call the non-decreasing function, the Γ-*function* because the shape of the Greek letter Γ shows some resemblances with the shape of an increasing function. (The S was already reserved by Zadeh). This function is defined as follows:

$$\Gamma(x;\alpha,\beta) = \begin{cases} 0, & \text{for } x \le \alpha \\ \left(\frac{x-\alpha}{\beta-\alpha}\right)^2, & \text{for } \alpha < x \le \beta \\ 1, & \text{for } x > \beta. \end{cases}$$

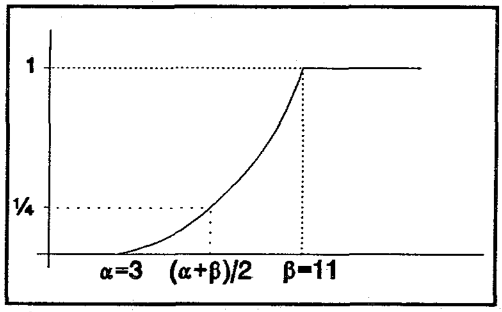


Figure 2-5: A Gamma function.

The Γ-function is used to represent concepts like old, hot, tall, high, etc.

For the decreasing function we have the L-function, which owes its name to its shape which resembles an L. This function is defined as follows:

$$L(x;\gamma,\delta) = \begin{cases} 1, & \text{for } x \le \gamma \\ 1 - \left(\dfrac{x-\gamma}{\delta-\gamma}\right)^2, & \text{for } \gamma < x \le \delta \\ 0, & \text{for } x > \delta. \end{cases}$$

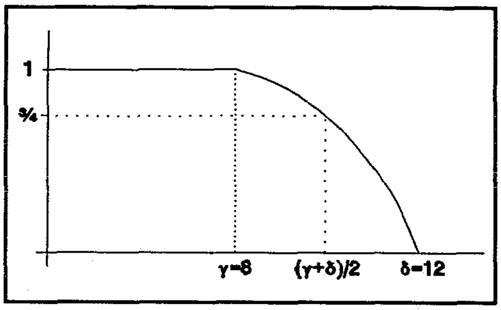


Figure 2-6: A L-function.

So the decreasing function is the complement of the non-decreasing function with one, $L(x;\alpha,\beta) = 1 - \Gamma(x;\alpha,\beta)$. It can be used to describe concepts like young, cold, low, etc.

A general trapezoidally shaped **Π**-function is defined by $\Pi(x;\alpha,\beta,\gamma,\delta)$ and is equal to $\Gamma(x;\alpha,\beta)$ to the left of β and to $L(x;\gamma,\delta)$ to the right of γ. Between β and γ the membership value is **1**. The membership function is used for concepts like middle age, normal weight, etc. Slightly more limited is the bell-shaped **Λ**-function, which equals $\Pi(x;\alpha,\beta,\beta,\gamma)$ and is used to represent "approximate" concepts. A few examples of these are approximately zero, about three years, around 75 kilograms.

2.5 Hedges

Apart from concepts like long and old referred to above, our language also makes use of modifiers such as very, rather, more or less, etc. How does fuzzy logic deal with those? Such modifiers are called *hedges*.
Zimmerman [1991, p. 137] defines these hedges as follows:

"A linguistic hedge is an operation which changes the meaning of a term and creates a new fuzzy set."

In order to represent hedges we need the following operations: **normalisation, concentration, dilatation,** and **contrast intensification.** By contrast to operations like intersection, union and complement (Chapter 3), these operations do not have counterparts in classical set theory.

A non-empty fuzzy set can, as has been stated before, be *normalised* in such a way that at least one element is **fully** part of the set.
The elements can also be *concentrated* by reducing the membership value of all elements which only belong partly to the (fuzzy) set (so **0** and **not 1**) in such a way that the elements which belong least to the set are the most reduced.

As a result, the membership curve will become steeper.
Conversely, we can also *dilatate* the set by increasing the membership values of elements which hardly belong to the set. This will make the membership curve less steep. Finally we can *intensify* the curve by increasing the contrast between the elements that belong for more than fifty percent to the set and those that do not. Mathematically the operators mentioned, which act on fuzzy set A, are defined as follows (for a graphic representation, see Appendix I):)

$$Norm(A) = \int_U \frac{\mu_A(u)}{hgt(A)}/u; \qquad hgt(A) = sup_{x \in U}\mu_A(x)$$

$$Con(A) = A^2 = \int_U \mu_A{}^2(u)/u;$$

$$Dil(A) = A^{0.5} = \int_U \sqrt{\mu_A(u)}/u;$$

$$Int(A) = \int_{0 \le \mu_A(u) \le 0.5} 2(\mu_A(u))^2/u + \int_{0.5 \le \mu_A(u) \le 1} 1-2(1-\mu_A(u))^2/u.$$

Each of these functions is regularly used in the fuzzy set theory to represent hedges.

A few examples are the following, by Zadeh proposed, hedges, which have meanwhile become generally accepted and are often used in applications:

$$\begin{aligned} \textit{very } A &= Con(A) = A^2, \\ \textit{more or less } A &= Dil(A) = A^{0.5}, \\ \textit{plus } A &= A^{1.25}, \\ \textit{min } A &= Int(\textit{ plus } A \textit{ and not } (\textit{very } A)); \end{aligned}$$

A few other examples are :

$$\begin{aligned} \textit{very very } A &= A^4, \\ \textit{minus } A &= A^{0.75}. \end{aligned}$$

2.6 Shifted hedges method

Now let us examine the linguistic concept **old** in connection with the hedge **very**. Let us assume that **old** represents the following fuzzy set:

$$Old = \int_0^{120} \Gamma(x;60,80)\ /x\ ;$$

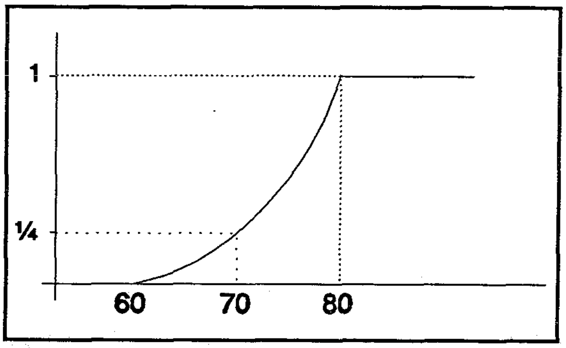


Figure 2-7: The fuzzy set 'old'.

This means that people younger than 60 are not old, while people older than 80 are. As we have just seen, **very(A) = Con (A) = A²**, and we can conclude that **very^n(A) =** A^{2n}. This is called a powered hedge approach. From this it follows that in our example someone of 60 is **not** old, nor is he very old etc. In fuzzy set **old** someone of 70 would have a membership value of ½, while in fuzzy set **very old** he would have a membership value of ¼.

So far everything is alright. However, all functions **very^n(A)** will reach the value of 1 at the same point, i.e. 80 years. In other words, someone of 80 is both old and very old etc., which is **not** logical.

For this reason a different approach, referred to as the shifted hedges method, is frequently chosen. This approach assumes that the following formula is a better representation of, for instance, the concept **very old**:

$$Very\ old = \int_0^{120} \Gamma(x;70,90)\ /x\ ;$$

Because a person of 70 is by no means very old.

As this name already indicates, this method involves an adaptation of the boundaries or hedges!

Chapter 3 OPERATIONS ON FUZZY SETS

3.1 Introduction

As the previous chapter will have made clear, the membership function is **the** crucial characteristic of the fuzzy set. It will therefore not come as a surprise that operations involving fuzzy sets are defined by means of their membership functions. In this chapter we shall examine the concepts proposed by Zadeh as regards the basic operations. We shall also pay attention to a few alternatives.

3.2 Operations on crisp sets

In the classical set theory various operations on sets have been defined. For instance, we can calculate the intersection and union of two sets. The determination of the complement (negation) of a set is another example of such a basic operation.

Let us assume that we are looking at a space which consists of the integers 1 to 6, so **X = {1,2,3,4,5,6}** and at sets **A** and **B** within this space.

A = {1,3,5,6} and **B = {2,4,5,6}**

3.2.1 Intersection

Intersection **A∩B** of **A** and **B** equals **{5,6}**. If there is an element in the intersection, it is part of **A** *and* **B**. This is nothing new. We can also determine the intersection by means of the characteristic functions of both sets, which look as follows:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in \{1,3,5,6\} \\ 0 & \text{otherwise} \end{cases} \quad \textit{and} \quad \mu_B(x) = \begin{cases} 1 & \text{if } x \in \{2,4,5,6\} \\ 0 & \text{otherwise} \end{cases}$$

In this case $\mu_{A \cap B}(x) = \min(\mu_a(x), \mu_B(x)$ applies to all $\mathbf{x \in X}$.

Note that we can obtain the same result for $\mu_{A \cap B}$, if we define this function by means of $\mu_{A\cap}(x) = \mu_A(x).\mu_B(x)$ for all $\mathbf{x \in X}$.

The intersection of an arbitrarily chosen set **A⊂X** with **X** yields the same set and the intersection with the empty set leads to an empty set.

3.2.2 Union

Union **A∪B** of **A** and **B** is the set of all elements which are elements of **A** and \ *or* of **B**.

In terms of characteristic functions this operation can be recorded by means of the *maximum operator*. In that case $\mu_{A \cup B}(x) = \max (\mu_A(x), \mu_B(x))$ holds good for $x \in X$.

Here too there are several ways to obtain the same result for $\mu_{A \cup B}$, for example by means of $\mu_{A \cup B}(x) = \min (1, \mu_A(x) + \mu_B(x))$.

3.2.3 Complement

The complement of classical set **A** forms, together with its original, the total space within which the set is defined. Elements of **A** do *not* belong to complement **A'**. In the case of set **A**, complement **A'** = {2,4}. This property is called the *'law of excluded middle'* [Klir J., 1988 p. 7]. The complement can also be determined with the help of the characteristic function. The following holds: $\mu_{A'}(x) = 1 - \mu_A(x)$ for all $x \in X$.

As a crisp set by definition does not have elements in common with its complement, the intersection with the complement also yields and empty set. This property is called the *'law of contradiction'*.
A few important characteristics of intersection, union and complements are mentioned in Appendix II.

3.3 Operations on fuzzy sets

The clarity of the operations on crisp sets just discussed is the direct result of the fact that they are **and, or** and **not** operations with clearly defined semantics. In the fuzzy set theory their interpretation is not so easy because of the gradated concepts used. Moreover, the operations are no longer straightforward, as there may be different definitions for the various operations. Which definition is used also depends on (among other things) the application. Still, all definitions meet certain requirements. One of these requirements is that any operation working on a crisp set yields the ordinary operation for crisp sets.

3.3.1 Fuzzy operations

The intersection of two fuzzy sets **A** and **B** is fuzzy set **A∩B**, of which membership function $\mu_{A \cap B}$ is for instance defined by:

$$\mu_{A \cap B}(x) = \min (\mu_A(x), \mu_B(x)), \quad \forall x \in X.$$

As is required of a intersection operation, the resulting set is a subset of both **A** and **B**.

The union of two fuzzy sets **A** and **B** is fuzzy set **A∪B** with membership function $\mu_{A \cup B}$ which is for instance defined by:

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)), \quad \forall x \in X$$

Note that **A** and **B** are subsets of fuzzy set **A∪B.**

The membership function of complement **A'** of normalised set **A** is defined by:

$$\mu_{A'}(x) = 1 - \mu_A(x), \quad \forall x \in X.$$

It should be noted, however, that the alternative operators suggested for the crisp intersection and the crisp union can also be applied in the fuzzy situation.
But the results in that case are quite different!

3.3.2 Graphic representation of fuzzy operations

To indicate the difference with classical sets, we shall here sketch the results of the three operations mentioned, applied to two fuzzy sets **A** and **B.** In this explanation we assume $\mathbf{A} = \{ Q(x;2,3,5,6) \mid x \in \mathbb{R} \}$ and $\mathbf{B} = \{ Q(x;5,6,8,9) \mid x \in \mathbb{R} \}$, in which **Q** stands for the Quadruple introduced in section 2.4.

The fuzzy intersection and the fuzzy union of **A** and **B** and the complement of **A** will look as follows (note that an element can be member of **A** as well as of **A'**!):

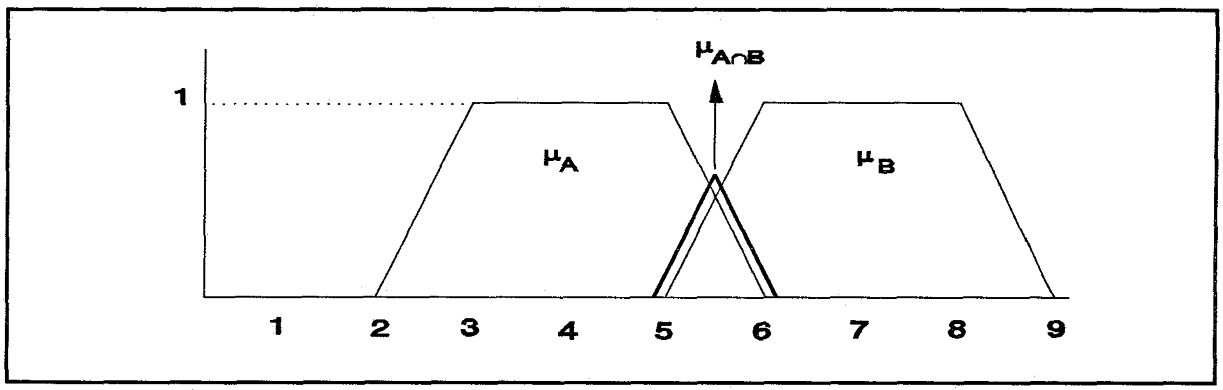


Figure 3-1: Intersection **A∩B** of two fuzzy sets.

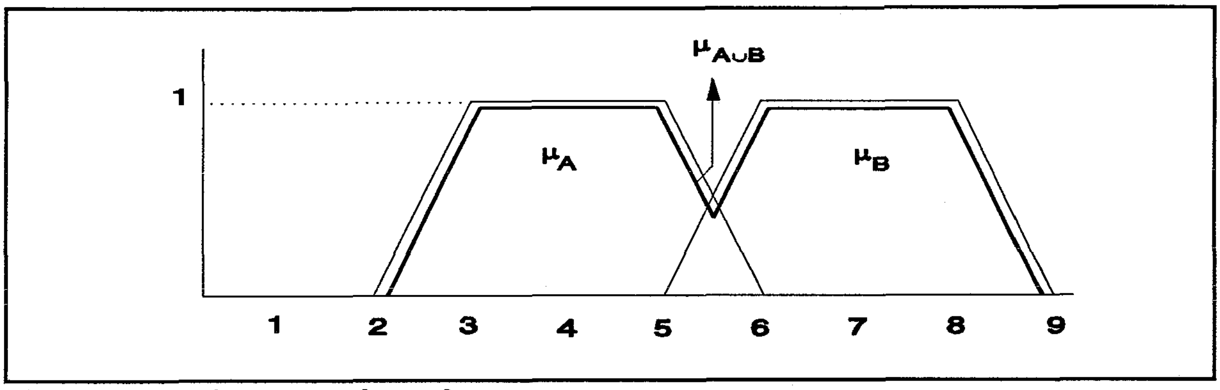


Figure 3-2: Union **A∪B** of two fuzzy sets.

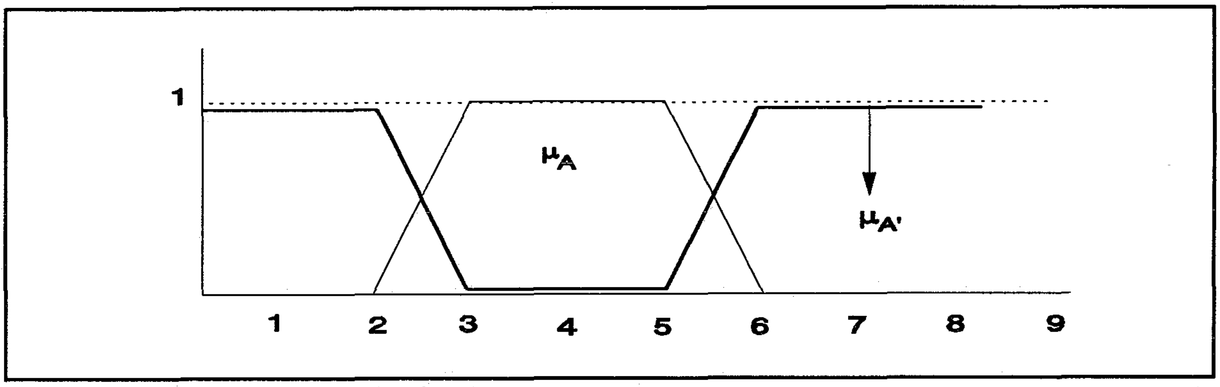


Figure 3-3: Complement **A'** of a fuzzy set **A**.

3.3.3 Other algebraic operations on fuzzy sets

By way of supplement to the operations union, intersection and complement we can define various other operations to combine and link fuzzy sets. Below we shall discuss some of the most important operations.

In terms of membership functions of A and B, the *algebraic product* **A.B** of **A** and **B** can be defined by the relation:

$$\mu_{A.B} = \mu_A(x).\mu_B(x) \quad \text{for all } \mathbf{x \in X}$$

The *algebraic sum* **A+B** of **A** and **B** is defined by:

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x) \text{ for all } \mathbf{x \in X}$$

The *bounded sum* **A ⊕ B** is defined by:

$$\mu_{A \oplus B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\} \text{ for all } \mathbf{x \in X}$$

The *bounded difference* **A ⊖ B** is defined by:

$$\mu_{A\ominus B}(x) = \max\{0, \mu_A(x) + \mu_B(x) -1\} \quad \text{for all } \mathbf{x \in X}$$

The *absolute difference* **|A-B|** of **A** and **B** is defined by:

$$\mu_{|A-B|}(x) = |\mu_A(x) - \mu_B(x)| \quad \text{for all } \mathbf{x \in X}$$

In case of ordinary sets the absolute difference is reduced to the relative complement of **A∩B** in **A∪B**.

Let $A_1,...,A_n$ be fuzzy sets in $X_1,...,X_n$. The *Cartesian product* will then be a fuzzy set in the product space $X_1*X_1*,...,*X_n$ with membership function:

$$\mu_{(A_1*A_2,...,*A_n)}(x) = \underset{i}{Min}\{\mu_{A_i}(x_i) \mid x = (x_1,...,x_n), \quad x_i \in X_i\}$$

The m^{th} power of fuzzy set A is a fuzzy set with membership function

$$\mu_{A^m}(x) = [\mu_A(x)]^m, \qquad x \in X$$

EXAMPLES:

If: A = { (3,0.5), (5,1), (7,0.6) } and B = { (3,1), (5,0.6) }

Then above definitions lead to the following results:

A.B	=	{ (3,0.5), (5,0.6) }
A+B	=	{ (3,1), (5,1), (7,0.6) }
A⊕B	=	{ (3,1), (5,1), (7,0.6) }
A⊖B	=	{ (3,0.5), (5,0.6) }
|A-B|	=	{ (3,0.5), (5,0.4) }
A*B	=	{ [(3;3),0.5], [(5;3),1], [(7;3),0.6], [3;5),0.5], [(5;5),0.6], [(7;5),0.6]}
A^2	=	{ (3,0.25), (5,1), (7,0.36) }

3.4 Fuzzy relations

Binary crisp relation **R** on Cartesian product $X_1 * X_2$ of two sets X_1 and X_2 links up elements of these sets in one way or another. Formally speaking **R** is a subset of $X_1 * X_2$ in such a way that $(x_1, x_2) \in$ **R**, if the relation is correct, and $(x_1, x_2) \notin$ **R**, if this is not the case. By way of example we shall examine relation **y > x** with domain **R** for both **x** and **y**. In that case the following holds:

$$\mathbf{R} = \{ (x,y) \in \mathbb{R}^2 \mid x<y \}$$

In other words: **R** consists of the part of the plane above line **y=x**.

This relation can, of course, also be characterised in terms of function $\mu_R(x,y)$. In which case:

$$\mu_R(x,y) = \begin{cases} 1, & \forall\, (x,y) \in \mathbb{R}^2 \text{ with } x<Y \\ 0, & \forall\, (x,y) \in \mathbb{R}^2 \text{ with } x \geq y \end{cases}$$

So a crisp relation indicates whether or not there is some kind of relation between two or more sets.

This concept can be generalised by allowing a relation to have various degrees. The degrees of such a relation can be represented by the membership values in a *fuzzy relation*, in the same way in which an element which is member of a set is represented by a fuzzy set. A fuzzy relation is a fuzzy set based on the Cartesian product of crisp sets in which the elements may assume different membership values.
The formal definition of a fuzzy relation is as follows [Zimmerman, H.-J., 1991]

Let **X**, **Y** be universal sets, then

$$\mathbf{R} = \{ ((x,y), \mu_R(x,y)) \mid (x,y) \in X * Y \}$$

is called the fuzzy relation in **X*Y**.

The problem is how to determine the membership function μ_R of the fuzzy relation. We can represent this membership function by means of a formula or by means of a table. By way of illustration we shall give a few examples of a crisp relation and a few fuzzy relations [Klir G.J., 1988].

Let R be a **(crisp)** relation between three sets X = {English, French}, Y = {Dollar, Pound, Mark, Franc} and Z = { United States, France, Canada, Great Britain, Germany}, which links a currency and a language to a country in the following way:

R(X,Y,Z) = {(English,Dollar,U.S.), (French,Franc,France), (English,Dollar,Canada), (French, Dollar, Canada), (English, Pound, G.B.)}.

This relation can also be indicated by means of a three-dimensional matrix:

	U.S.	Fr	Can	G.B.	Ger
Dollar	1	0	1	0	0
Pound	0	0	0	1	0
Franc	0	0	0	0	0
Mark	0	0	0	0	0

ENGLISH

	U.S.	Fr	Can	G.B	Ger
Dollar	0	0	1	0	0
Pound	0	0	0	0	0
Franc	0	1	0	0	0
Mark	0	0	0	0	0

FRENCH

Let **R** be a **fuzzy relation** between two sets **X** = {New York, Paris} and **Y** = {Peking, New York, London}, representing the concept "very far". The membership function of this relation can be depicted as follows:

	New York	Paris
Peking	1	.9
New York	0	.7
London	.6	.3

Another example of a **fuzzy relation** is the following:

If **X, Y** belong to ℝ and **R** and the fuzzy relation is "considerably greater than", the membership function of the fuzzy relation, which is also a fuzzy set (on **X*Y**), could be the following:

$$\mu_R(x,y) = \begin{cases} 0 & \text{for } x \leq y \\ \dfrac{(x-y)}{10y} & \text{for } y < x \leq 11y \\ 1 & \text{for } x > 11y \end{cases}$$

With the help of fuzzy relations we are able to represent so-called *if...then* rules. The relation has membershipvalue 1 if antecedent and consequent are in complete agreement with each other, and membershipvalue 0 if they don't fit. Membership-values between 0 and 1 point to a fuzzy relation.

As has already been stated, a fuzzy relation is a fuzzy set in a product space (and not a mathematical function). We can therefore also define algebraic operations in analogy to the definitions for "ordinary" fuzzy sets defined in this chapter.

Definition 3-1

Let **R** and **Z** be two fuzzy sets in the same product space. Then the intersection and union of **R** and **Z** are defined as follows:

$$\mu_{R\cap Z}(x,y) = \min\{\ \mu_R(x,y),\ \mu_Z(x,y)\ \},\quad (x,y) \in X*Y$$
$$\mu_{R\cup Z}(x,y) = \max\{\ \mu_R(x,y),\ \mu_z(x,y)\ \},\quad (x,y) \in X*Y$$

Although we are also using the minimum and maximum for the definitions of intersection and union in this case, there are alternatives as regards projection and cylindrical extension of fuzzy relations [Zimmerman H.-J., 1991 p.72].

Fuzzy relations in various product spaces can be combined with the help of the *composition* operation. Various definitions have been proposed for this operation. These definitions differ as regards results and as regards mathematical properties. The max-min composition is the most familiar and the most frequently used operation.

Definition 3-2

Max-min composition: Let $\mathbf{R}_1$(x,y), (x.y) $\in$ **X*Y** and $\mathbf{R}_2$(y,z), (y,z) $\in$ **Y*Z** be two fuzzy relations. Then the max-min composition is fuzzy set:

$$R_1 \circ R_2 = \{[(x,z),\ \max_y\{\ \min\{\mu_{R_1}(x,y),\ \mu_{R_2}(y,z)\ \}\ \}\ ]\ \ |\ x\in X,\ y\in Y,\ z\in Z\ \}$$

Alternative compositions are, for instance the max-prod and the max-av composition (product and average).

Fuzzy relations also play a role in fuzzy preferential methods and in *approximate reasoning* (*compositional rule of inference*), which will both be discussed later in this report.

3.5 Rules, implications

In classical binary logic not only operators such as ***and***, ***or*** and ***not*** (which are, in terms of sets, associated with intersection, union and complement) are used, but also implications are applied. These implications, also called rules, usualy look like

R: ***if*** antecedent ***then*** consequence.

A simple example of such a rule is: ***if*** it rains ***then*** the streets will become wet. Rules play a very important part in fuzzy logic, which aims itself at reasoning in terms of fuzzy quantities. In fuzzy logic both the antecedent and the consequence can be formulated in terms of fuzzy quantities. The antecedent for instance might be of the form "**x** belongs to **P**" and the consequence, in the same way, could run "**y** belongs to **Q**" in which **P** and **Q** are fuzzy sets on universe **X** respectively on universe **Y**.

It will be clear that this antecedent and this consequence are the equivalent of $\mu_P(x)>0$, respectively $\mu_Q(y)>0$, so that the rule R can be formulated as follows:

R: ***if*** $\mu_P(x)>0$ ***then*** $\mu_Q(y)>0$

We say that this rule is *active* if the antecedent is true, consequently if $\mu_P(x)>0$.
Just like in binary logic only when the rule is active sentence can be passed about the consequence. Provided that a rule is active, the fuzzy situation is much more complicated than the one in the binary case. After all in binary logic applies that whenever a rule is active $\mu_P(x)=1$, and it follows that $\mu_Q(y)=1$. In fuzzy logic unfortunately this does not hold stand since $\mu_P(x)\in[0,1]$ and $\mu_Q(y)\in[0,1]$. For a given value $\mathbf{x\in X}$ with $\mu_P(x)>0$ (so rule is active) not only we would infer that $\mu_Q(y)>0$ but also we would like to be able to pronounce upon the magnitude of $\mu_Q(y)$. For that purpose a more detailed specification of rule R is necessary. To be able to provide this the notion *transferrelation* is introduced. This is in fact an operation working on the cartesian product of both fuzzy sets **P** and **Q**. A transferrelation, belonging to rule R, is a fuzzy set on X*Y with membershipfunction μ_R: $X*Y \rightarrow [0,1]$, in which

$$\mu_R(x,y) = \underset{R}{f}(\mu_P(x),\mu_Q(y)) \;, \qquad \forall\,(x,y) \in X*Y$$

where $f_R : [0,1]*[0,1] \rightarrow [0,1]$ is a, further to be specified, function.
This function has to fulfill a certain number of demands [Klir G.J., 1988], for instance

$$\underset{R}{f}(\alpha,\beta)>0 \;, \quad \forall\,\alpha\in(0,1] \;; \qquad \underset{R}{f}(1,1)=1 \;, \qquad \underset{R}{f}(0,\beta)=0$$

Frequently used definitions in literature for this function are [Kouatli en Negouita]:

$$\underset{R}{f}(\alpha,\beta)=min(\alpha,\beta) \;, \qquad \forall\,(\alpha,\beta)\in[0,1]*[0,1]$$

$$\underset{R}{f}(\alpha,\beta)=\alpha.\beta \;, \qquad \forall\,(\alpha,\beta)\in[0,1]*[0,1]$$

For a given *input* $x\in X$ for rule R, say $x=a$, the transferfunction $\mu_R(x,y)$ passes into a function $\mu_R(a,y)$ of the *output* y.
We will conceive this function as the membershipfunction $\mu_{Q'}$ of fuzzy set Q′ on Y. This fuzzy set Q′ is a subset of Q! It should be noted that in general this function will *not* be normalised.

At the interpretation of rule R given above, an input $x=a$ with $a\in X$ and $\mu_P(a)>0$ does *not* lead to one value for the output $y\in Y$ but to a fuzzy set Q′ on Y, of which the membership function $\mu_{Q'}$ ensues from $\mu_{Q'}(y)=\mu_R(a,y)$.
Stated otherwise: the result of the rule is a fuzzy set *not* one value for y. For a grahical presentation see Figure 3-4. In this picture the shaded area represents the result Q′ of the rule R (a fuzzy set).

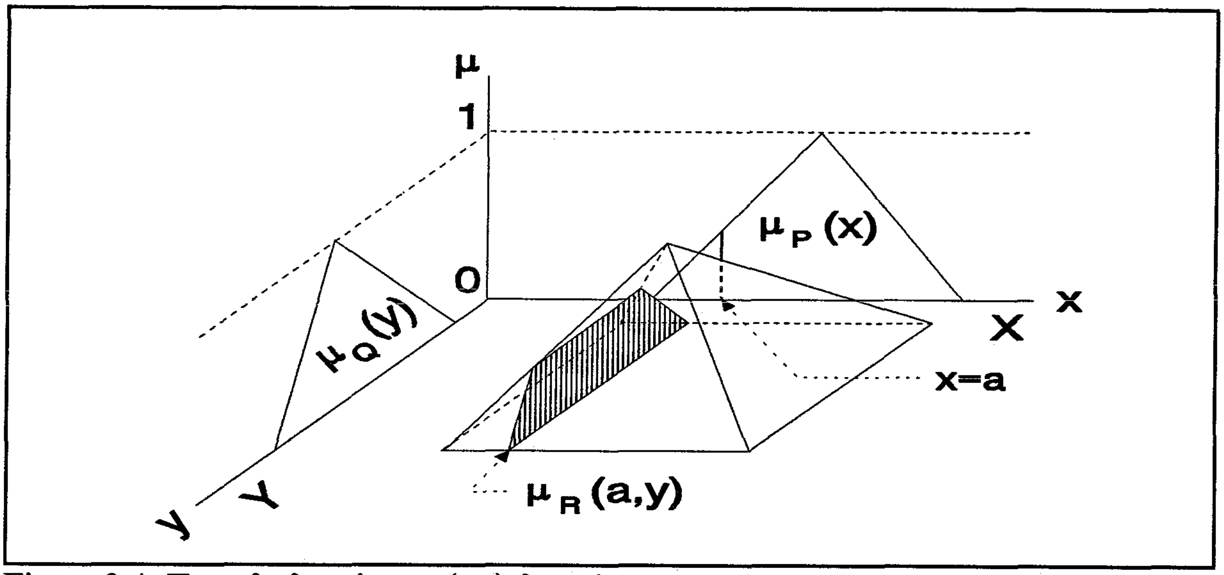


Figure 3-4: Transferfunction μ_R(a,y) for crisp antecedent x=a.

In some cases, a.o. with fuzzy controllers, a such-like result is not merely useful and one value for the output y is wanted. This process is called *defuzzification*, for which, several alternatives are put forward in literature. A very simple and obvious method is to take **that** value for y wich maximizes $\mu_{Q'}=\mu_{Q'}(y)$ (the so-called "first of maxima"-method). However more in use is the "Centre of Area"-method, where

$$y = [\int_Y s.\mu_{Q'}(s)ds] . [\int_Y \mu_{Q'}(s)ds]^{-1}$$

We are permitted to interpret this value being the y-coordinate of the geometrical centre of the surface beneath the graph of membership function $\mu_{Q'}$.

In many cases we will have to deal with more then one rule. Let n (n>1) be the number of rules and let rule i (i=1,2,...,n) be of the following type:

R_i : *if* $\mu_{P_i}(x)>0$ *then* $\mu_{Q_i}(y)>0$

The transferrelation, which goes with rule R_i, is a fuzzy set on X*Y with membership function μ_{R_i}. For the determination of μ_{R_i} we can, assuming that the functions μ_{P_i} and μ_{Q_i} are known, for instance make use of

$$\mu_{R_i}(x,y) = min(\mu_{P_i}(x), \mu_{Q_i}(y))$$

For a given value for the input x∈X, say x=a, we would like to derive one fuzzy set Q′ on Y from these rules $R_1,R_2,...,R_n$. This can be accomplished in several ways. One possible procedure is to combine first all rules $R_1,R_2,...,R_n$ to one new rule R and subsequently proceed with that new rule in the way described before. In fact it comes down in combining the transferrelations $\mu_{R_1},\mu_{R_2},...,\mu_{R_n}$ to one transferrelation μ_R. However, at this moment there is no optimal method known to execute this. Often the following formula is applied:

$$\mu_R(x,y) = max(\mu_{R_1}(x,y),...,\mu_{R_n}(x,y)) \ , \qquad \forall\ (x,y) \in X*Y$$

Further if x=a, the saught-after fuzzy set Q′ can be tied up by its membership function:

$$\mu_{Q'}(y) = \mu_R(a,y) \ , \quad \forall\ y \in Y$$

A different approach is to determine, for each rule R_i (i=1,2,...,n), first all the belonging fuzzy sets Q'_i and next combine those sets $Q'_1, Q'_2,...,Q'_n$ to one new fuzzy set Q'. One of many ways of doing this is to appoint the membershipfunction $\mu_{Q'}(y)$ of Q' wich can be derived from:

$$\mu_{Q'}(y) = max(\mu_{Q'_1}(y),...,\mu_{Q'_n}(y)).$$

While working out the rule another problem may occur: the antecedent can contain more antecedents. For example we consider an antecedent of the form "x_1 belongs to P_1 *and* x_2 belongs to P_2" in which P_1 and P_2 are fuzzy sets on universe X_1, respectively X_2. In order to be able to make use of the earlier given elaboration we have to rebuild this to one antecedent with two fuzzy variables. For this fuzzy set P on X_1*X_2 can be defined from wich membershipfunction μ_P depends on a further to be specified way of the membership functions μ_{P_1} and μ_{P_2}.
the membershipfunctions μ_{P_1} and μ_{P_2}. For the given composite antecedent we can for instance make use of

$$\mu_P(x_1,x_2) = min(\mu_{P_1}(x_1),\ \mu_{P_2}(x_2))$$

Another frequently applied definition is

$$\mu_P(x_1,x_2) = \mu_{P_1}(x_1).\mu_{P_2}(x_2)$$

For a composite antecedent of the form "x_1 belongs to P_1 *or* x_2 belongs to P_2" we can go to work in a similar way, now μ_P can for instance be defined by

$$\mu_P(x_1,\ x_2) = max(\mu_{P_1}(x_1),\ \mu_{P_2}(x_2))$$

Here we are also free too use one of the many alternative definitions.

A last complication when elaborating rules in fuzzy logic occurs at the moment that input x of the antecedent in the rule

R: *if* x belongs to P *then* y belongs to Q

is not given as a precise (crisp) value, but if that input is also a fuzzy quantity.
In this way x can be the result of a measurement and be given as "x is about 1,80 m.". This can be taken into account by defining a fuzzy set S on universe X for this quantity with membershipfunction μ_S representing the description "about 1,80 m.".

Subsequently, in the same way as described above, a transferrelation (that is to say a fuzzy set) with membershipfunction μ_R: $X*Y \rightarrow [0,1]$ upon rule R can be added. However fuzzy set Q′ on Y for output y, which goes with this rule R and a specified input x, can at this point not be determined from $\mu_{Q'}(y) = \mu_R(x,y)$ because x is now only given in terms of fuzzy sets. More precise: x is specified by the membershipfunction μ_S. A possibility for determining $\mu_{Q'}$ is provided by:

$$\mu_{Q'}(y) = \max_{x \in X} (\min (\mu_R(x,y), \mu_S(x)))$$

In this way for every crisp $x \in X$ first the minimum $\mu_R(x,y)$ and $\mu_S(x)$ is computed for all $y \in Y$. This will result in a function ψ of x and y (the bold printed "frame tent" within the pyramide, see Figure 3-5). The next step then is the computation of the maximum of that function for each $y \in Y$ wile x accepts all possible values on X (the, in our picture shaded, fuzzy set for **that** value of x, which $\min(\mu_P(x), \mu_S(x))$ is maximal). In (the particulair) case that $\mu_S(a)=1$ and $\mu_S(x)=0$ for all $x \in X$ with $x \neq a$, so if for the input x crisp value a is given, then this relation for $\mu_{Q'}$ transforms into the above decribed relation.

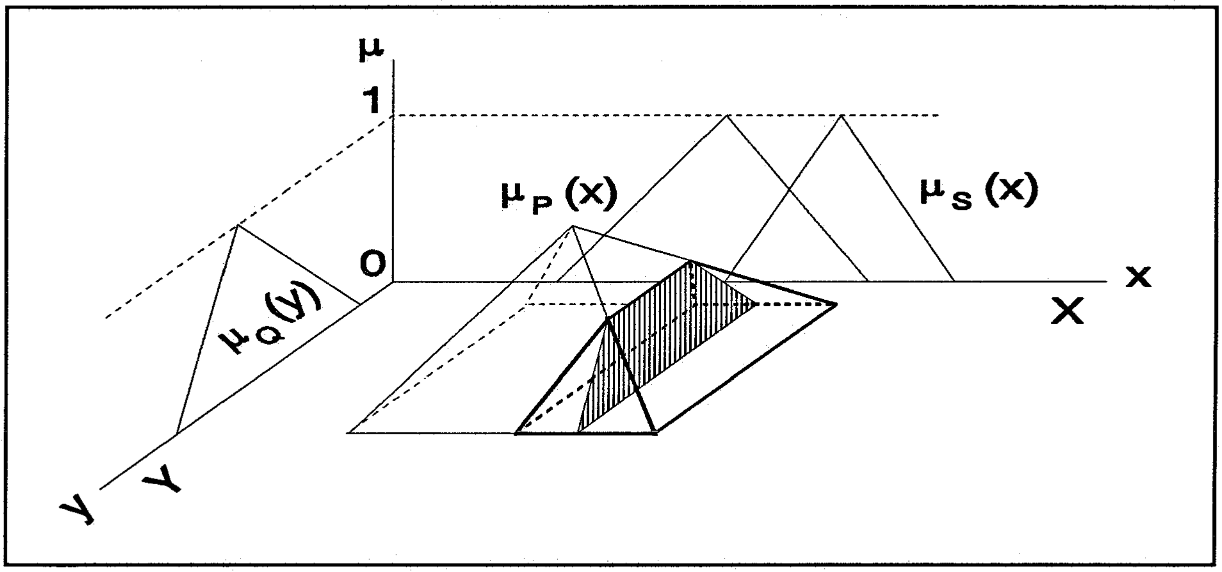


Figure 3-5: Determination of fuzzy set Q′ given a fuzzy input.

Along the way we just described how too work out one rule and how to do that when more rules are involved we have made choises on several places.

Those choises have (great) impact on the final result of the rule(s), however at this moment, unfortunately enough, little is known about guidelines and fist-rules to come to optimal or at least acceptable choises in a certain situation.

Roughly we can state that the theory as has been dicussed just now builds up the foundation on wich fuzzy logic is based. The next chapter will describe how one thing and another fits into a fuzzy controller.

Chapter 4 FUZZY CONTROL

4.1 Introduction

Fuzzy logic can, for instance, be used in control technology (fuzzy control). Conventional control methods start from descriptions of the processes to be controlled. The controllers are determined on the basis of a description. These methods have a number of disadvantages [Giessel van, G.P. 1991, p.12]:

- In some cases a process which seems to be control easily will result in a highly complex mathematical model.
- Small changes in the process to be controlled sometimes call for the complete redimensioning of the controller. It is in general impossible to add the knowledge about the change to the existing controller.
- With non-linear processes it is often hard to determine a controller which leads to an acceptable closed-loop behavior in the entire working-space. In this case the controller is frequently designed for one or a few previously defined operation points.

The big difference with conventional control technology is that fuzzy control does not describe the process to be controlled by means of a more or less detailled mathematical model. Fuzzy control tries to include expert know-how in the controller. It starts from a number of more or less intuitively rules.

For humans who have been controlling a process manually for some time, these rules are relatively easy to compile. All fuzzy rules together form the knowledge-based system of the controller. What this controller looks like schematically will be discussed in section 4.2.

4.2 Architecture of a fuzzy controller

The first step while designing a fuzzy controller for a process consists, quite conventional, of defining the quantities wich need to be controlled and determining the input-quantities of that process. For simplicity we take for granted that all quantities to be controlled are being measured without appreciable errors, so that the acquired measurement values can be regarded as crisp quantities. Besides it is assumed that the desired value is known for all quantities to be controlled. The difference between the desired value and the real value, the error signal, is the inputsignal for the fuzzy controller. The crisp output signals of the controller are the input of the process.

In a fuzzy controller three functional blocks can be distinguished: the fuzzification-block, the rules-block and the defuzzification-block. These blocks are very globaly discussed in the next paragraphs and explained on basis of a very simple example. This example concerns a chemical process of which the temperature ν and the pressure p have to be controlled. The input of this process is the adjustable culvert-opening ϕ of a fuel-valve.

Consequently the controller consists of two inputs viz.:

- the difference $e_v = v_d - v$ between the desired and the real temperature &
- the difference $e_p = p_d - p$ between the desired and the real pressure

The (only) output is the variable culvert ϕ or the variable $\Delta\phi$ of that culvert.

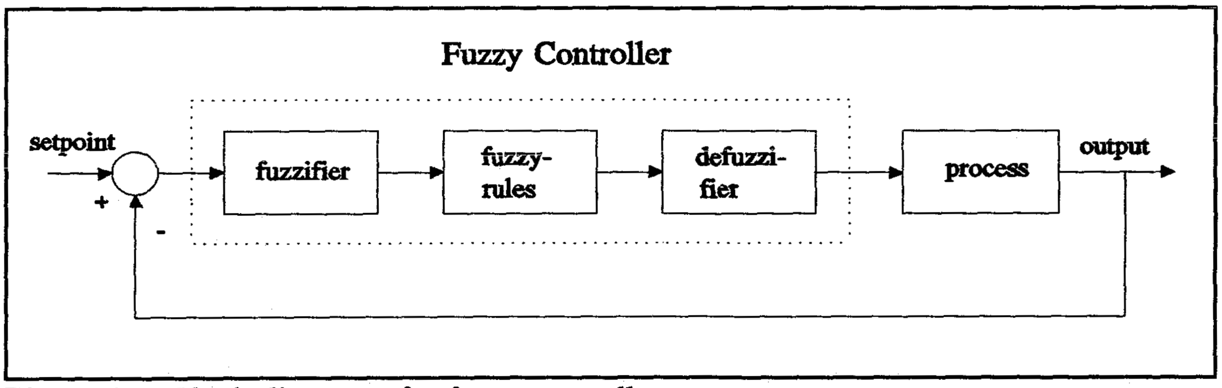


Figure 4-1: Block diagram of a fuzzy controller.

4.2.1 Fuzzifier

In the first block, the Fuzzifier, the inputsignals are being converted into membership-values in a number of fuzzy sets who are previously to be chosen. Therefore first the universe has to be chosen for each inputsignal, next some fuzzy sets have to be defined on that universe. Generally five to zeven fuzzy sets are used for each inputsignal, for example five fuzzy sets named B (positive big), S (positive small), Z (about zero), N (negative small) and L (negative big). We must also establisch what the membership functions of the fuzzy sets look like. In general we have to take care that the fuzzy sets partly overlap each other for each inputsignal so that each inputsignal at least belongs to two of those sets. The number, the shape and the position of the membership functions (on their respective universes) for any of the input quantities are for the, by the controller attainable, result of great importance: while the number is increasing the controller will behave itself ever flexible and deliver a beter result altough its complexity shall increase very drastic.
At a given value of an inputsignal its possible to calculate the membership value in each of the, for that signal introduced, fuzzy sets. This has to be done for everyone of the inputsignals, consequently in our example both for e_v as for e_p. This is called fuzzification.
When for both e_v and e_p five fuzzy sets of the above given type are introduced this process returnes five membership values for e_v as well as for e_p at given crisp input values. Some examples for e_v are:

$\mu_{P_v}(e_v)$: membership value of e_v in the fuzzy set P_v
" desired temperature much bigger then the real one"

$\mu_{Z_v}(e_v)$: membership value of e_v in the fuzzy set P_v
"desired temperature about the same as the real one"

Although there isn't a connection with the block "Fuzzifier", discussed in this paragraph, we remark here that not only the universe has to be defined for anyone of the outputsignals of the controller but also that the fuzzy sets who are situated on that universe need to be defined by choosing shape and position of the corresponding membership functions. In our example the output, so the change $\Delta\phi$ of the culvert, can for instance contain the following five fuzzy sets G_ϕ (much further open), P_ϕ (little more open), Z_ϕ (almost don't alter the opening), N_ϕ (close a little more) and L_ϕ (close much further).

4.2.2 Fuzzy Rules

In the block "Fuzzy rules" (Figure 4-1) the rules, describing how the process should be controlled, are worked out. At this point we restrict ourselfs to rules of the type which already has been discussed in Chapter 3. So to rules of the form

R_i: *if* antecedent ***then*** consequence.

In our example i.a. the following rules might appear:

R_1: *if* (v much too high ***and*** p much too high) ***then*** close valve much further

R_2: *if* (v is about v_d *and* p much too high) ***then*** close valve a little more

Because the elaboration of these and similar rules already extensively came up for discussion in Chapter 3 we won't go in any deeper on this. The final result is a fuzzy set Q' for the output $\Delta\phi$ of the controller.
In principle we have to formulate a rule for every possible antecedent. Suppose that m is the number of inputsignals of the controller and that there are in all n_i $(i=1,2,...,m)$ fuzzy sets defined for inputsignal x_i. Then the number of all possible antecedents is equal to $n = n_1 \cdot n_2 \cdot ... \cdot n_m$
Considering more then one inputsignal rapidly leads to a very large number of antecedents (and consequently to a large number of rules to be formulated), especially when many fuzzy sets are introduced per inputsignal. Luckely enough many of these rules often deal with situations which don't hapen in practice (or anyhow aren't allowed to occur!). Also due to this the number of rules often can drasticaly be reduced. Formulating an as small as possible rule-base therefor only seems possible for experts with a very good insight in the process to be controlled!

4.2.3 Defuzzifier

It has in the previous section already been mentioned that the output of the block "Fuzzy rules" is a fuzzy set Q' for the output. However, such a result can't be offered to for instance a servomotor that has to realise the input of the process. For that purpose it's necesary to deduct one crisp value out of the fuzzy set Q' wich can be passed to the output.

This can be done in one of the already in Chapter 3 outlined ways, for example with the "Centre of Area"-method. However, here we will introduce the "Centre of Gravity"-method as an alternative, because it will be made use of in Chapter 5. In this method the x-coordinate (Z) of the centre of gravity from the, out of the total rule-base, resulting fuzzy set is computed, by using the following formula:

$$z = \frac{(\sum_{n=1\ldots N} t_n * Z(t_n))}{(\sum_{n=1\ldots N} Z(t_n))}$$

In which t_n represents the local centre of gravity of the (local) area $Z(t_n)$.

This crisp value Z is well suited for being used as control signal for the servomotor. In Figure 4-2 this method is being illustrated; the shaded part represents the resulting fuzzy set (the output set, which is obtained by adding up all fuzzy sets resulting from the different rules).

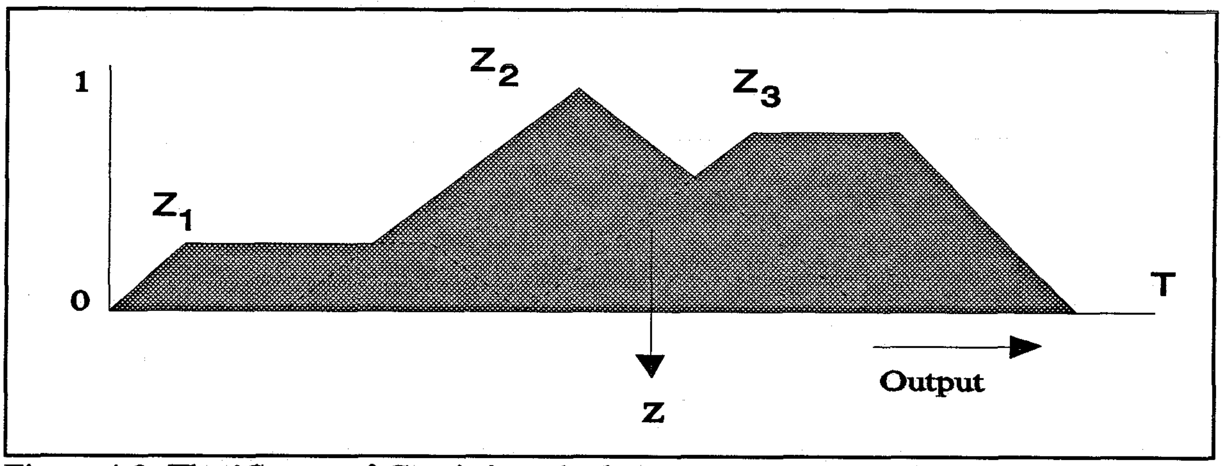


Figure 4-2: The 'Centre of Gravity' method.

Chapter 5 'BALL IN GROOVE' PROBLEM

5.1 Introduction

Comparable to the (traditional) inverted pendulum problem, we shall here try to use fuzzy logic to steer a ball in a groove from a random initial position to a (random) end position in a groove. The fuzzy control method used in this case is the method based on the phase plane. In section 8.3 we will shortly get back on this type of fuzzy control.

5.2 Mathematical model

In order to be able simulate or to generate measuring values, we need a mathematical description of the system. This system (see Figure 5-1) is fairly complex.

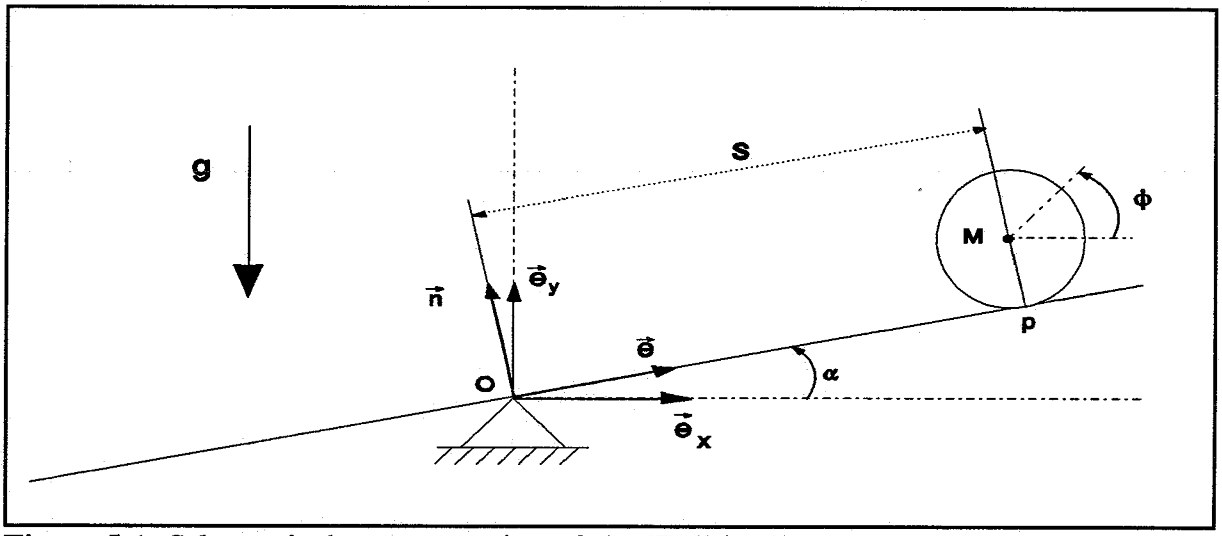


Figure 5-1: Schematical representation of the 'Ball in the groove' problem.

By determining the body-related vectors and their derivatives, we can calculate the position, speed and acceleration of mass centre x_M of the ball (as a function of the body-related vectors):

$$\vec{e} = \cos\alpha.\vec{e}_x + \sin\alpha.\vec{e}_y$$
$$\vec{n} = -\sin\alpha.\vec{e}_x + \cos\alpha.\vec{e}_y$$
$$\Rightarrow \dot{\vec{e}} = \dot{\alpha}\vec{n} \; ; \qquad \dot{\vec{n}} = -\dot{\alpha}\vec{e}$$

$$\vec{x}_M = s\vec{e} + R\vec{n}$$
$$\Rightarrow \dot{\vec{x}}_M = \dot{s}\vec{e} + s\dot{\vec{e}} + R\dot{\vec{n}} = (\dot{s} - R\dot{\alpha})\vec{e} + s\dot{\alpha}\vec{n}$$
$$\ddot{\vec{x}}_M = (\ddot{s} - R\ddot{\alpha})\vec{e} + (\dot{s} - R\dot{\alpha})\dot{\alpha}\vec{n} + \dot{s}\dot{\alpha}\vec{n} + s\ddot{\alpha}\vec{n} -$$
$$= (\ddot{s} - R\ddot{\alpha} - s\dot{\alpha}^2)\vec{e} + (s\ddot{\alpha} - R\dot{\alpha}^2 + 2\dot{s}\dot{\alpha})\vec{n}$$

This means that we will find the following relations for the speed of the contact point:

$\vec{v}_P$: *speed of the contact point, viewed as a point of the ball*

$$\Rightarrow \vec{v}_P = \dot{\vec{x}}_M + R\dot{\varphi}\vec{e} = (\dot{s} - R\dot{\alpha} + R\dot{\varphi})\vec{e} + s\dot{\alpha}\vec{n}$$

$\vec{v}_P{}^*$: *speed of the contact point, viewed as a point of the groove*

$$\Rightarrow \vec{v}_P{}^* = s\dot{\alpha}\vec{n}$$

Condition for accurate rolling $\rightarrow \vec{v}_P = \vec{v}_P{}^*$

$$\Rightarrow \dot{\varphi} = \dot{\alpha} - \frac{\dot{s}}{R}$$

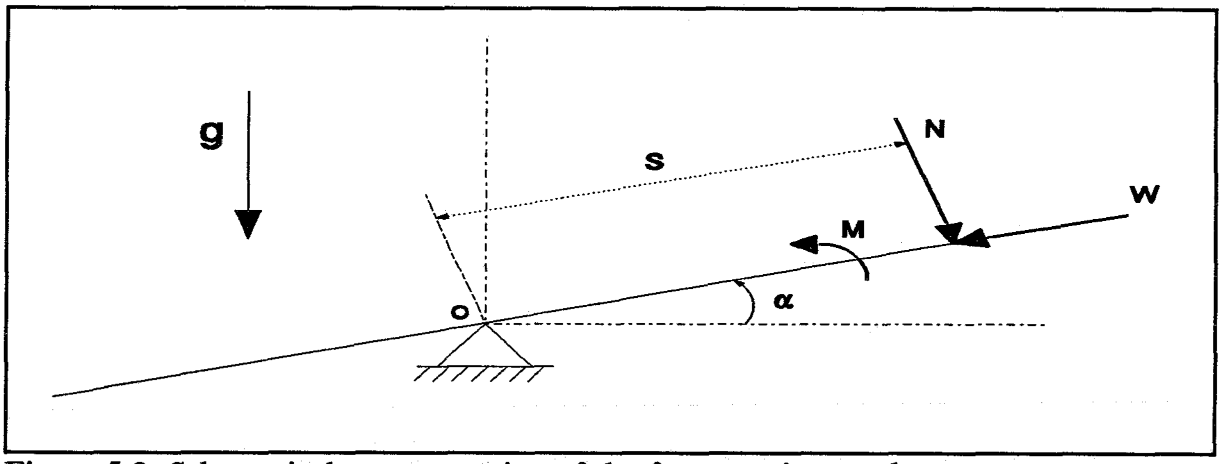


Figure 5-2: Schematical representation of the forces acting on the system.

If J_g is the mass moment of inertia of the groove with regard to O, while O is the mass centre of that groove, then the following holds (see Figure 5-2):

$$\rightarrow J_g.\ddot{\alpha} = M - s.N$$

With m_k (mass of ball) and J_k (moment of inertia of ball with regard to mass centre M) the forces become (see Figure 5-3):

$$m_k\ddot{\vec{x}}_M = W\vec{e} + N\vec{n} - m_k g\vec{e}_y$$

$$J_k\ddot{\varphi} = RW = J_k.\frac{1}{R}.\ddot{s}$$

$$\Rightarrow W = \frac{J_k}{R}\ddot{\alpha} - \frac{J_k}{R^2}\ddot{s}$$

$$\Rightarrow N = m_k\ddot{\vec{x}}_M.\vec{n} + m_k g\vec{e}_y.\vec{n}$$

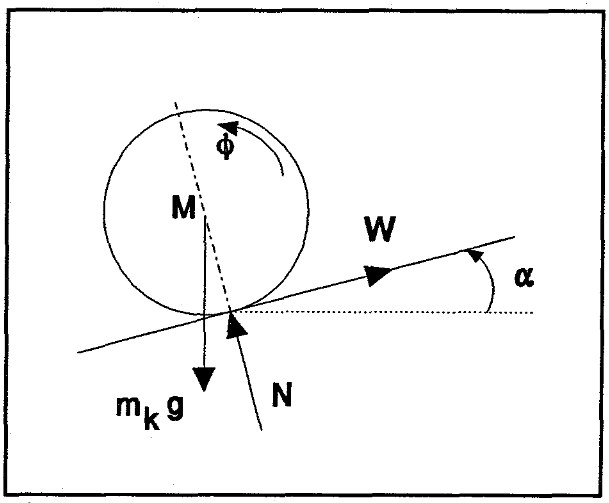


Figure 5-3: Detailed drawing of the ball.

This means that the relevant motion equations will look as follows:

$$J_g \ddot{\alpha} = M - s.m_k(\ddot{\vec{x}}_M.\vec{n}) - sm_k g(\vec{e}_y.\vec{n})$$

$$m_k \ddot{\vec{x}}_M.\vec{e} = \frac{J_k}{R}\ddot{\alpha} - \frac{J_k}{R^2}\ddot{s} - m_k g(\vec{e}_y.\vec{e})$$

Which after some rephrasing yield the following system:

$$(J_g + m_k s^2)\ddot{\alpha} = sm_k\left[R\dot{\alpha}^2 - 2\dot{s}\dot{\alpha} - g\cos\alpha\right] + M$$

$$(m_k + \frac{J_k}{R^2})\ddot{s} = \frac{1}{R}(J_k + m_k R^2)\ddot{\alpha} + m_k s\dot{\alpha}^2 - m_k g\sin\alpha$$

And we can therefore use the following (non-linear) system equations to generate our measuring values (F_α is an auxiliary variable):

$$F_\alpha = \frac{1}{J_g + m_k s^2}\left[\, m_k s(R\dot{\alpha}^2 - 2\dot{s}\dot{\alpha} - g\cos\alpha) + M \,\right];$$

$$\Rightarrow \ddot{\alpha} = F_\alpha \,;$$

$$\Rightarrow \ddot{s} = R.F_\alpha + \frac{m_k R^2}{J_k + m_k R^2}(s.\dot{\alpha}^2 - g\sin\alpha)$$

5.3 Fuzzy control on the basis of the phase plane

The system which has just been described can be controlled by (traditional) PID controllers, despite its highly non-linear character. However, the system may be expected to be rather slow in view of this non-linearity. This problem could be avoided by using the fuzzy control described below, i.e. a control on the basis of the phase plane for a second-order system. (Note: this system has not been calculated with a traditional PID controller so that no reference material is available.)

5.3.1 Controller

As has already been stated before, the controller has been developed on the basis of a phase plane of a second-order system. For the fault in position **e**, the fault in speed **v** (=ė), and the
control action **U** to be executed, the following fuzzy sets have been defined:

Positive Big **e** = PEB	Negative Big **e** = NEB	
Positive Small **e** = PES	Negative Small **e** = NES	
Positive Big **v** = PVB	Negative Big **V** = NVB,	(I)
Positive small **v** = PVS	Negative Small **v** =NVS	
Positive Big **U** = PUB	Negative Big **U** = NUB	
Positive Small **U** = PUS	Negative Small **U** = NUS	

By means of these linguistic variables and using the familiar *if* A *then* B rules, the control has been arranged as follows.

The phase plane is divided into 6 parts A,...,F [Palm, R. 1988], see Figure 5-4. The line $e+v = 0$ is the switching line of the sign of control action **U**.

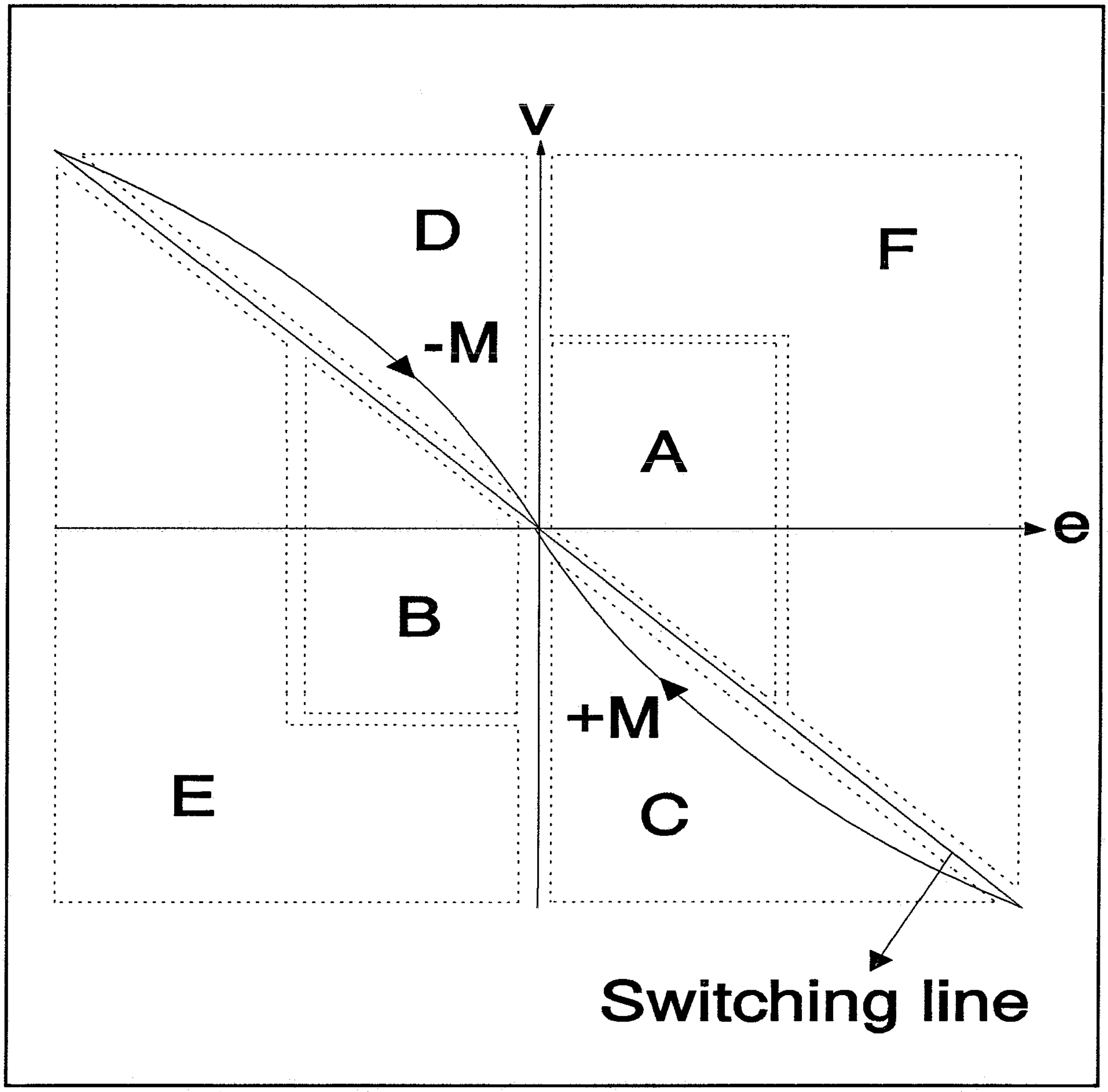


Figure 5-4: The phase plane divided into six parts.

Combination of the various parts of the phase plane with the resulting control action to be executed leads to the following rule base:

	PART		ACTION	
IF	F	THEN	NUB	
IF	A OR D	THEN	NUS	(II)
IF	B OR C	THEN	PUS	
IF	E	THEN	PUB	

If we now combine this rule base (II) with the fuzzy sets (I) defined before, using the logic *and* and the logic *of* operator, then we can describe the parts A,...,F as follows:

A:	$(v > -e)$	AND PES AND (PVS OR NVS)	
B:	$(v \leq -e)$	AND NES AND (PVS OR NVS)	
C:	$(v \leq -e)$	AND (PES OR PEB)	(III)
D:	$(v > -e)$	AND (NES OR NEB)	
E:	$(v \leq -e)$	AND (NES OR NEB) AND (NVB OR NEB)	
F:	$(v > -e)$	AND (PES OR PEB) AND (PVB OR PEB)	

And in combination with (II) this ultimately produces the rule base, consisting of two rules:

IF $v > -e$ THEN
IF (PES OR PEB) AND (PVB OR PEB) THEN NUB
IF (NES OR NEB) OR (PES AND (PVS OR NVS)) THEN NUS

IF $v \leq -e$ then
IF (NES OR NEB) AND (NVB OR NEB) THEN PUB
IF (PES OR PEB) OR (NES AND (PVS OR NVS)) THEN PUS

It must be noted that premise parts $v > -e$ and $v \leq -e$ are used in their normal mathematical meaning and are therefore not fuzzy.
The result of these rules is a fuzzy set of the (new) moment **M** which is to be exercised and from which a scalar value is determined by means of the centre of gravity method.

5.4 Conclusion

Based on the results (Appendix VI) it can be concluded that the controller is robust regarding variation in initial position as well as regarding variation in desired end position. The controller goes relatively quickly to the desired end position, considering the fact that only two rules are being used.

As has already been pointed out, the ball in the groove problem discussed here is similar to the inverted pendulum problem. Controlling the inverted pendulum in this problem, which can also be described by means of two chained differential equations, by means of traditional fuzzy control (as described in Part I) will require a rule base consisting of at least seven rules [Yamakawa T, 1989]. Therefore it seems rather interesting to find out to which amount the number of rules used, in combination with the fuzzy control method, effects

- the required calculation time (for each cycle)
- the speed by which the systeem converges towards its desired state.

Notes:

- The values of **e**, **v** and **M** have been normalised by means of heuristically obtained values (these values also make the variables dimensionless).

- The well-known oscillating behaviour (sliding mode), which can also be observed here, can be eliminated, if required, by applying a boundary layer on both sides of the switching line.

- If a different sampling frequency is used (for instance, 0.05 sec.) the program appears to be unstable. This is probably attributable to the integration program (ode23) used within Matlab. Another cause might be that the used sample frequency is a multiple of one of the own frequencies of the system (or is almost equal to one).

- The system doesn't settle down. However that doesn't matter here, because that wasn't the goal of our experiment. But if desired, one can accomplish this by switching to a conventional controller as soon as the system enters a (to be defined) boundary layer.

PART II

BACKGROUNDS

Chapter 6 FUZZY REASONING

6.1 Introduction

Central theme with fuzzy reasoning is the dealing with inaccuracies. Suppose that an implication (rule) of the following form is given:

if the temperature is too high *then* decrease fuel flow.

Now lets assume that the temperature is not too high but that it's much too high. Then what should be concluded, that the fuel flow has to be decreased a lot? Drawing conclusions of this form is in general no problem at all for human beings. We do it all the time and are often unconscious of doing this. However an useful mathematical translation of it is least of all trivial, and is one of the subjects in the field of "approximate reasoning". Notions like fuzzy sets and fuzzy relations which have been spoken about earlier on, and the inference rules, to be discussed in this chapter, play a very important role near it. In "approximate reasoning" we make use of several inference rules, such as the *modus ponens*, the *modus tollens* and the *contraposition*. Hereafter some of those rules are reviewed. In that discussion we will make use of a couple of universus X_1 and X_2, and of the fuzzy sets P_1 and Q_1, defined on X_1, and the fuzzy sets P_2 and Q_2, defined on X_2.

6.2 Generalised modus ponens

The *generalised modus ponens* (G.M.P.) is an extended form of the modus ponens commonly used in traditional logic and which is as follows for fuzzy sets:

Premise	$x \epsilon X_1$ belongs to Q_1, consequently $\mu_{Q1}(x_1) > 0$
Implication	*if* x belongs to P_1 *then* $x_2 \epsilon X_2$ belongs to P_2
Conclusion	x_2 belongs to Q_2

By way of comment we will consider the example out of the introduction again. In that case x_1 is representing the temperature while x_2 is representing the fuel flow.
P_1 and Q_1 are the fuzzy sets "temperature too high" and "temperature much too high" while P_2 and Q_2 are the fuzzy sets "decrease fuel flow" and "decrease fuel flow a lot". For this example the implication translates into:

if $\mu_{P_1}(x_1) > 0$ *then* $\mu_{P_2}(x_2) > 0$

Suppose now that a value for the temperature, so for x_1 is given. Then $\mu_{Q1}(x_1)$ can be determined. Then the GMP has to supply us a prescription which helps us to compute $\mu_{Q2}(x_2)$, of course under the condition that $\mu_{Q1}(x_1) > 0$. When $\mu_{Q1}(x_1) = 0$ no sentence can be passed about $\mu_{Q2}(x_2)$ (in general).

Some of the criteria that should be met by the G.M.P. are stated below [Hellendoorn 1991, p. 68]:

- If Q_1 is the opposite of P_1, we cannot say anything about Q_2, any value is possible.
- If Q_1 is *slightly smaller* than P_1, then Q_2 will also be slightly smaller than P_2, though it is not known how much smaller. So this is a departure from classical logic, in which an antecedent is either true or not true. If it is true we can draw a conclusion, if it is not, we can't.
- If Q_1 is *slightly larger* than P_1, then Q_2 is slightly larger than P_2 (see above).
- If two fuzzy sets Q_1 and Q_1' are *almost identical*, then the corresponding fuzzy sets P_1 and P_2' will also be almost identical.

By means of these criteria (which have not all been mentioned) we can define a function with P_1, P_2 and Q_1 as inputs, and Q_2 as output. This is rather difficult, as fuzzy sets P_1 and Q_1 have to be compared with each other. This comparing of fuzzy sets will be discussed in Chapter 7.

6.2 Compositional rule of inference

An alternative for this generalised modus ponens is the *compositional rule of inference* proposed by Zadeh in 1973, a rule which does not occur in ordinary logic. This rule combines fuzzy sets with fuzzy relations. Fuzzy relations are, as has already been stated in section 3.4, fuzzy subsets defined on **X*Y**.

In the implication "if **A** then **B**", a relation **R(x,y)** is assumed between the two fuzzy sets **A** in domain **X** and **B** in domain **Y**.

Several translation rules have been proposed by, among others, Zadeh, Madani and Mizumoto in order to translate the fuzzy rule *if* '**x** is **A**' *then* '**y** is **B**' into a fuzzy relation. Zadeh [1973, p. 148] assumes that the relation is such that the consequent is the max-min composition of fuzzy set **A** with fuzzy relation **R(x,y)**, which can be represented as follows in a formula:

$$\mu_Q(x_2) = \max_x \min \{ \mu_P(x_1), \mu_R(x_1,x_2) \}$$

An example of such an inference rule is:	Its general appearance:
$\mathbf{x_1}$ is high,	$\mathbf{x_1}$ is **P**,
$\mathbf{x_1}$ and $\mathbf{x_2}$ are almost equal,	$\mathbf{x_1}$ is $\mathbf{Rx_2}$,
$\mathbf{x_2}$ is more or less high	$\mathbf{x_2}$ is **Q**.

So, instead of the *if..then* rule customary in fuzzy logic, there is a fuzzy relation R. In general it appears to be impossible to represent this *if..then* rule by means of a fuzzy relation, in any case not in such a way that the exact relation between the antecedent and the consequent of the *if..then* rule is indicated.

6.3 Problems in approximate reasoning

In view of what has been said in subsections 6.2.1 and 6.2.2 it will be clear that fuzzy sets allow us to make models which simulate *approximate reasoning*, i.e. models which enable us to deal with problems in the way we humans tend to deal with problems.
The flexibility obtained by this linguistic approach is only valuable if it is efficient. This efficiency can be greatly affected if the output (of the model) must also be linguistic.
The model which receives input in the form of fuzzy sets generates a fuzzy set as output. The problem, frequently referred to as "*linguistic approximation*", consists in finding a label (a sentence or word from a language) of which the meaning, that is to say the representation of that label in the form of a fuzzy set, is identical to (or corresponds strongly to) the unlabelled fuzzy set which results from the model.
The fuzzy ranking problem, as this comparison of fuzzy sets is called, will be discussed in more detail in Chapter 7.

Another problem may be that the conclusion obtained fails to coincide with what we intuitively expect. For this reason a number of criteria have been defined in the course of time to which inference methods should conform. This has resulted in alternative translation rules as well as a number of alternative compositions [Mizumoto M, 1982].

In order to keep my account readable I shall not go further into these alternative inference methods. This would make the account so mathematical that it would become entirely unreadable (and incomprehensible).

Chapter 7 DISADVANTAGES OF CLASSICAL FUZZY LOGIC

7.1 Introduction

Although the fuzzy set theory which has just been discussed will prove adequate in most cases, it must be said that this will not always be the case. It has certain drawbacks. This chapter will present an alternative for the max. and the min. operator, after having demonstrated the flaws of these classical operators. Next a classification of the various operators will be given. The chapter will be concluded with a brief description of the disadvantage of inference methods, which has already been hinted at earlier, and the presentation of a solution to this problem.

7.2 Disadvantage of the max and the min operator

As has already been stated before, in most of the current application of fuzzy logic the minimum, operator is used as a logic (= natural) ***and***, whilst the maximum operator is used as a logic ***or***. However, these operators are only a rough approximation of the linguistic concepts they represent.
Let us take another look at a rule from the controller in the chemical plant (4.2.2):

LINGUISTIC

IF	Combustion chamber temp. (x)	=	low (μ_{low})
AND	Pressure in the advanced ignition chamber (y)	=	high (μ_{high})
THEN	Methane valve (z)	=	wider open ($\mu_{wider\ open}$)

MATHEMATICAL

IF	$\{ (x, \mu_{low}(x))$	$\mid x \in \text{Temp} \} > 0$
AND	$\{ (y, \mu_{high}(y))$	$\mid y \in \text{Pressure} \} > 0$
THEN	$\{ (z, \mu_{wider\ open}(z)) \mid z \in \text{Angular displacement} \} > 0$	

in which z represents the new state which follows from the antecedent of the rule via the membership value of the currently valid fuzzy set within the linguistic variable. If the antecedent consists of several premisses, then the membership value of the consequence will be a combination of the membership values of the premisses (the antecedents), which are calculated in the manner explained in 3.5 In many cases the consequence (in case of two conditions x and y) is notated as follows: $Z \rightarrow (X \circ Y)$.

The above rule ensures that the methane valve is opened further when the combustion temperature is too low and the pressure in the advanced ignition chamber too high. In other words, if the condition is critical, the methane supply will be increased. To what extent the valve will be opened is calculated by means of the *logic and* operator.

Let us consider the following conditions in the plant:

#	Temperature	μ_{low}	Pressure	μ_{high}
1	920°C	0.3	40 bar	0.4
2	920°C	0.3	51 bar	0.8
3	910°C	0.2	69 bar	1.0

If we apply the minimum operator, state #1 and state #2 are assessed as equally critical by the fuzzy controller, whereas we would regard #2 as the most critical state. If state #3 is compared with state #1, the fuzzy controller will judge state #1 as the most critical one, whereas we evaluate #3 as the most critical state. This difference in evaluation is due to the fact that the minimum operator cannot make a compensatory distinction, the way we humans can. If a human being combines two concepts by means of linguistic ***and***, the following usually applies: "much more of the one will compensate a (small) shortage of another". The maximum operator is subject to the same flaw.

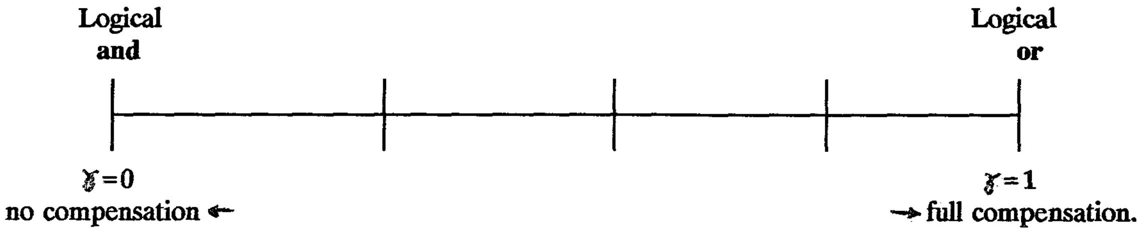
This problem can be solved in two ways. One way is to define more fuzzy sets (within the linguistic variables). This solutions has two disadvantages. In the first place, it would lead to a dramatic increase in the number of rules (in the rule base). This will not only hamper the design process and result in a less clear system structure, but it will also require more effort from the computer. Secondly, if more terms are defined for the linguistic variable than were originally considered in the concept (at the time of development), this will rapidly reduce the system's interpretability.

The second solution entails the definition of a so-called compensatory operator.

7.3 The γ-operator

How this compensatory ***and*** can be represented has been established by means of empirical investigations, which examined the applicability of the various operators. This research showed that the gamma-operator in particular provides an excellent representation of the linguistic concept of ***and*** in various meanings.

By means of a selection of γ, to which this operator owes its name, this operator can vary infinitely between *logic **and*** (no compensation) and *logic **or*** (full compensation), depending on its meaning (see drawing).
A disadvantage of this operator is that it is unattractive from a mathematical point of view. Contrary to the minimum operator, for instance, it is not associative and leads to substantially higher computing costs.



Mathematically the (compensatory) gamma-operator looks as follows (if $\mu_M(x)$ represents the membership value of x in fuzzy set M):

$$\mu_{More\ open}(X \circ Y) = (\mu_{Low}(X) \cdot \mu_{High}(Y))^{1-\gamma} \cdot (1-[(1-\mu_{Low}(X)) \cdot (1-\mu_{High}(Y))])^{\gamma}$$

The γ-operator therefore is a combination of the algebraic product, modelling the natural ***and***, and the algebraic sum, modelling ***of***. With this operator entirely different conclusions are possible (of course, depending on the value of γ), as is clearly shown by the table below with regard to the same example as used before.

State	Temp.	Pressure	γ = 0.0	γ = 0.3	γ = 0.5	γ = 0.7	γ = 1
# 1	910°C	40 bar	0.12	0.19	0.26	0.36	0.58
# 2	920°C	40 bar	0.24	0.35	0.45	0.59	0.86
# 3	920°C	60 bar	0.20	0.32	0.45	0.62	1.00

Note: **The case γ = 0 corresponds to the *logic* and which is calculated with the alternative of the max-min operator, the so-called max-prod. inference method. Both will extensively be come up for discussion in Appendix III.**

7.4 Classes of operators

Although the min. and max. operators are the most commonly used operators in everyday practice, other possibilities for the intersection and union of fuzzy sets have been suggested. These suggestions vary both as regards universality and adaptability of the operators and as regards the degree in which they are justified for the various problems.

Whether an operator is justified (and to what extent) can be intuitively, empirically or axiomatically be determined. The adaptability ranges from uniquely defined (i.e. non-adaptive) concepts via parametered 'families' of operators to highly universal classes of operators which need only comply with a few requirements.

We distinguish two classes of operators: operators for intersection and union — referred to as triangular norms or t-norms and conorms — and the class of averaging operators (for example, the γ-operator). Each class contains both parametered and unparametered operators.

7.4.1 Triangular norms

t-norms. For the intersection of fuzzy sets Zadeh suggested the min. operator and the algebraic product. Both belong to the t-norms or triangular norms, which owe their name to the shape of the resulting membership function. Operators belonging to this class are associative and it is therefore possible to calculate the membership values of the intersection of more than two fuzzy sets by applying the t-norm operator recursively (for a mathematical definition, see Appendix IV).

t-norms (or s-norms). Zadeh introduced the max. operator and the algebraic sum for the union of fuzzy sets. These intersection operators belong to the group of fuzzy set aggregating operators, the so-called t-conorms or s-norms (Appendix IV).

It may be required to adapt the operator to be used to the context in which it is used. To this end different authors (e.g. Hamacher, Yager) have proposed parametrized families of t-norms and conorms, often maintaining their associativity property. Such parametrized t-norms can vary between the min. operator, the product operator and a few alternative intersection operators, depending on the value of the parameter.

7.4.2 Averaging operators

Operators giving a value between the value of logic *and* and that of logic *of* are called *averaging operators*. They combine the *max* and *min* operators with the mathematical average (of the two membership values). Dubois and Prade [1984] have done a great deal of research on these non-parametric averaging operators.

Just as is the case with the triangular norms, there are also parametered operators in the class of averaging operators. These operators may vary between logic *and* and logic *of*, depending on the value of the parameter. An example of such an operator was already discussed: the gamma-operator. Alternatives have been suggested, but in all cases research has shown that the 'compensatory and' operator is the closest approximation to the way human beings tend to take their decisions.

The relations between the various operators (for the union of two fuzzy sets **A** and **B**) are illustrated in Figure 7-1.

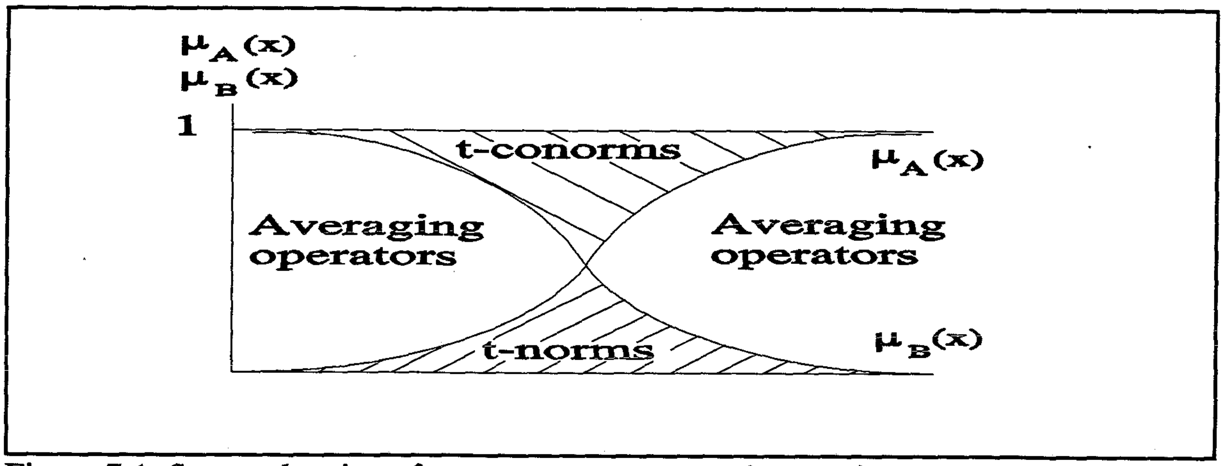


Figure 7-1: Survey drawing of t-norms, t-conorms and averaging operators.

The great variety of possible operators can be very confusing and makes the selection of the most suitable one very difficult. For this reason Zimmerman has formulated a number of criteria which might facilitate a selection. I shall not mention all criteria but only briefly sketch the most obvious ones.

- The operator must not only meet certain mathematical requirements (it must, for instance be associative or commutative), it should also be empirically suitable. In other words its must not be in conflict with the real system.

- It stands to reason that the operator must be numerically efficient (enough). A gamma-operator could, for instance, be very satisfactorily employed in case of a simple rule base with few rules. With rather large rule bases, containing two to eight hundred rules, application of the gamma-operator is out of the question on account of the number of calculations to be executed.

- The degree in which an operator must be compensatory distinctive, as well as the range within which the operator should lie if use of a compensatory operator is to be allowed (some operators require normalised membership functions, so the [0,1] range), obviously also play a role in the selection.

- The last criterion I want to put forward is the behaviour of the operator. If we combine several fuzzy sets by means of the product operator, the degree of the resulting fuzzy set will be reduced every time another set is added. This can be a desired result as well as an undesired one.

7.5 Disadvantage of the max-min inference method

In order to explain the disadvantage of the traditional inference methods, we shall first describe the usual procedure concerning process control.

A problem is usually roughly described by a number of logic rules. These are the rules which come to mind first when we consider the problem. Later the rule strategy is further refined. Some rules are eliminated and others are added.

Finally we reach the stage in which adding or not adding a rule is too coarse a step to optimise the rule strategy.

We could in that case adapt the membership functions in order to obtain the required behaviour. This would, however, be inconsistent with the concept of fuzzy control. The membership functions would have to represent the linguistic concepts of (physical) data in such a way that they become clear.

Fiddling with the membership functions to fine tune the rule strategy appears to be counterproductive. The more complex the problem is, the more counterproductive it becomes. The solution is quite simple: in order to fine tune a rule strategy we should adapt the **rules** and not the membership functions. However, only permitting a rule to be defined or not defined is dichotomous i.e. not fuzzy.

An advanced inference method consists of assigning weighing factors to the rules. So the tuning entails the adjustment of the weighing factors of the various rules. In practice, the values 0 and 1 are to be used first. Then for very accurate adjustments, values between 0 and 1 can be used.

Chapter 8 FUZZY RANKING METHODS

8.1 Introduction

As has already been briefly mentioned a few times, it will in certain cases be necessary to compare fuzzy sets. A specific example of this is the following case. Let us assume that we have a controller, which not only has fuzzy sets as input(s), but which must also supply them to the output, for instance, in the form of a linguistic concept like 'greatly reduce the valve opening'. The problem in this case is that the fuzzy controller must compare its own output set with the fuzzy sets corresponding to the linguistic concepts. This comparison of fuzzy sets is called a fuzzy ranking problem. After having discussed a few criteria to which the fuzzy ranking method should conform, the ranking method worked out by Yuan [1991] will be presented.

8.2 Required properties of a fuzzy preferential method

Suppose our problem is the following: the choice between **n** alternatives, each with a different result, indicated by means of fuzzy set $\{A_1: i = 1...n\}$.
For instance, if several rules are suitable for "firing", but they appear to have different resulting fuzzy sets, and we have to make a choice, which rule is the best? We limit ourselves to fuzzy sets with membership values between 0 and 1, and which are indicated by normal (the support is completely part of the domain), convex (no 'dips' in the membership function, see Chapter 2) and separate continuous membership functions.

In order to be able to compare the alternatives a preferential relation $P(A_i,A_j)$ is determined for each ranked pair $(A_i,A_j),i,j=1,..,n$. From these preferential relations the most suitable set can be derived. According to Yuan, four properties are important in case of such a ranking method, viz.:

1) fuzzy preferential presentation
2) rationality of preferential ranking
3) distinguishability
4) robustness.

These properties will be dealt with in the following subsections.

8.2.1 Fuzzy preference presentation

A fuzzy ranking method should be able to indicate the preference relations in linguistic terms. For instance, instead of simply concluding that **A** is to be preferred to **B** or not, the method should be able to represent the degree of preference by means of a (fuzzy) membership function. This property is desired in situations in which fuzzy sets represent uncertainty, in which case the decision to be taken must inherently also be vague.

It is not fitting to give the ultimate choice an artificial precision. Decisions should be linguistic rather than numeric. Employing this criterion, all methods can be divided into two groups.

The first method projects all fuzzy figures in single points on the real line with the help of ranking function **F**. The order is then determined on the basis of this real line. This approach is relatively simple and easy to implement. However, by reducing the entire analysis to one single figure, we lose a great deal of information.

The second method uses fuzzy relations to rank fuzzy numbers. With this approach a fuzzy relation is determined for each pair of fuzzy numbers. Then the linguistic meaning of the relation can be interpreted on the basis from the membership of the relation [Baas and Kwaakernaak]. This method allows us to represent preference by means of a preferential degree. In this case it may occur, however, that we are unable to determine a consistent total ranking of all alternatives.

8.2.2 Rationality of preference ordering

A ranking method ought to be able to represent rational human behaviour in terms of consistency and coherence. Consistency in this case can point to antimetry, i.e. if **A** is preferred over **B**, then **B** should not be preferred over **A**. Coherence may refer to transitivity, i.e. if **A** is preferred to **B**, and **B** over **C**, then **A** is preferred to **C**. If fuzzy relations are used to compare alternatives, the rationality requirements must be sufficiently met in order to obtain a consistent fuzzy ranking.

8.2.3 Distinguishability

By distinguishability Yuan means the ability to distinguish fuzzy figures in terms of preferential degree, if the differences represented by those fuzzy figures are important for taking a decision. As this is difficult to express in a mathematical formula, it is common to use several criteria, which together provide a definite answer. The four indices used by Dubois and Prade [1983] represent the possibility of dominance, the possibility of strict dominance, the necessity of dominance and the necessity of strict dominance. If one index is unable to distinguish between two alternatives, another will perhaps be able to do so. The problem is that all four indices may lead to different conclusions so that we still have to make a choice after all.

8.2.4. Robustness

A fuzzy ranking method should tolerate minor estimation faults concerning (fuzzy) membership functions. This means that the degree of relation between the two functions should not undergo a radical change if the shift in membership function is sufficiently small. This is a desired property because it is frequently difficult to make an accurate estimate of the membership function in a fuzzy environment.

The robustness can be formulated as follows: if **A′** greatly resembles **A** for all **B**, then the preferential degree of **A′** compared to **B** will lie very close to the preferential degree of **A** compared to **B**.
In mathematical terms:

$$|\mu_p(A,B) - \mu_p(A',B)| \leq \eta, \quad \forall A' \epsilon U, \quad d(A,A') < \delta$$

in which **A′** is an approximation of **A**, and **d(A, A′)** is the maximum difference between **A** and **A′**.

It must be remarked, however, that distinguishability and robustness may be inconsistent with each other, as the former requires a very sensitive method of comparison, whereas the latter does not.

A great deal of research has been conducted into 'ranking methods', for instance by Baas and Kwakernaak, and by Nakamura. Based on earlier methods, Yuan [1991] presents an improved method.

8.3 The improved ranking method

For a definition of the preferential relation of A_i with regard to A_j, Yuan does not look to the membership functions of A_i and A_j, but to the membership function of the difference A_i-A_j and then compares these with 0. It will be clear that in a crisp situation this would be of no importance (the difference between two real figures A_i and A_j is equivalent to the difference between A_i and A_j and 0), but in the fuzzy situation they are not the same. By using the fuzzy set of the difference, we are able to determine the differences between A_i and A_j for all possible combinations of A_i and A_j instead of simply calculating the difference between the best (worst) of A_i and the best (worst) of A_j. For this purpose we define $Q(A_i, A_j)$ as a fuzzy preferential relation between A_i And A_j with membership function:

$$\mu_Q(A_i, A_j) = \mu_p(A_i\text{-}A_j, Z_o)$$

which represents the preference of A_i with regard to A_j.
In order to keep this definition simple and make it easy to calculate, the following equivalent definition is introduced:

For each A_i, A_j ϵU, $Q(A_i, A_j)$ is defined as the fuzzy preferential relation between A_i and A_j by means of the following membership function:

$$\mu_Q(A_i,A_j) = \begin{cases} (S_1+S_2)/S, & S>0, \\ \frac{1}{2}, & S=0, \end{cases}$$

in wich $S = S_1 + S_2 + S_3 + S_4$, with:

$$S_1 = \int_{\{\beta:\rho_{A_i-A_j}(\beta)>0\}} \rho_{A_i-A_j}(\beta)d\beta,$$

$$S_2 = \int_{\{\beta:l_{A_i-A_j}(\beta)>0\}} l_{A_i-A_j}(\beta)d\beta,$$

$$S_3 = \int_{\{\beta:\rho_{A_i-A_j}(\beta)<0\}} \rho_{A_i-A_j}(\beta)d\beta,$$

$$S_4 = \int_{\{\beta:l_{A_i-A_j}(\beta)<0\}} l_{A_i-A_j}(\beta)d\beta,$$

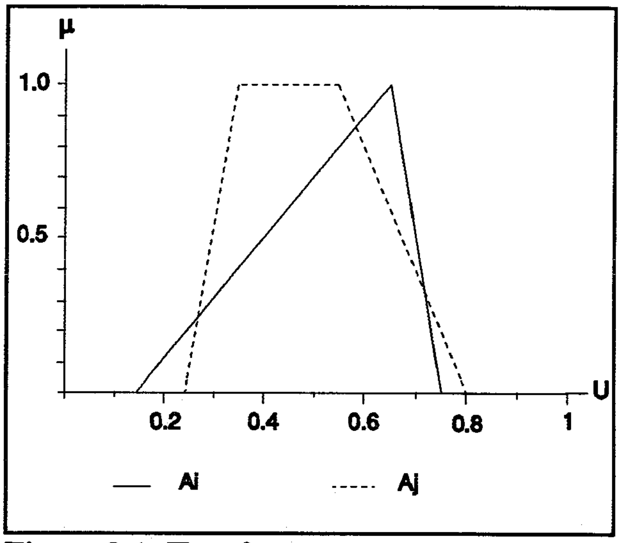


Figure 8-1: Two fuzzy sets.

and

$$\rho_{A_i-A_j}(\beta) = \max_{\mu_{A_i-A_j}(z)\geq\beta}(z),$$

$$l_{A_i-A_j}(\beta) = \min_{\mu_{A_i-A_j}(z)\geq\beta}(z).$$

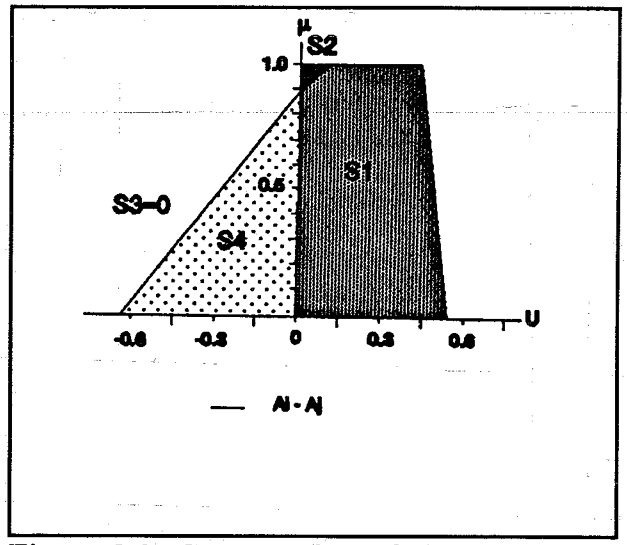


Figure 8-2: Computation of the preference degree of 2 fuzzy sets.

Figures III and IV are an illustration of the calculation of $\mu_Q(A_i, A_j)$. In this case $S_1 = 0.375$, $S_2 = 0.006$, $S_3 = 0$, $S_4 = 0.281$ and $\mu_Q(A_i, A_j) = 0.575$.

Intuitively we can interpret this as follows. S_1 indicates the portion where A_i is preferred to A_j in the most favorable situation. S_2 indicates the extent to which A_i is preferred to A_j in the most unfavourable case. S_3 indicates the portion where A_j is preferred to A_i in the most favorable position.
S_4 indicates the portion where A_j is preferred to A_i in the most unfavourable position. $\mu_Q(A_i, A_j) = (S_1 + S_2)/(S_1+S_2+S_3+S_4)$ indicates the overall degree of preference of A_i over A_j.

The major difference between this method and Nakamura's method is that in the most favorable case, we compare the best of A_i with the worst of A_j instead of the best of A_i with the best of A_j, and in the most unfavourable situation, we compare the worst of A_i with the best of A_j instead of the worst of A_i with the worst of A_j.
See Figure 8-2 for an illustration of the calculation of $\mu_Q(A_i, A_j)$.

Using the preference relation defined for each ordered pair, it's each to rank **n** alternatives $\{A_1, A_2,...,A_n\}$. The procedure is the following:

(1) Calculate $\mu_Q(A_i, A_j)$ for **i = 1,...,n, j = 1,...,n,** which represents a **n*n** matrix. Making use of the fact that preferential relation Q is reciprocal, i.e. $\mu_Q(A_i,A_j) = 1 - \mu_Q(A_1, A_j)$, we need only determine **½n*(n-1)** membership values.

(2) Sort $\{A_1, A_2,...,A_n\}$ in $\{A_{k1}, A_{k2},...,A_{kn}\}$ so that for all **i<j**, $\mu_Q(A_i, A_j) \geq ½$. The feasibility of this operation is guaranteed by the fact that Q is transitive (see required properties). In the ultimate ranking, A_{k1} is the most favourable.

(3) Give a linguistic interpretation of the fuzzy preferential relation. The linguistic interpretation can be defined as the fuzzy set 'calculated preferential degree', the membership value of this set points to the truth value of the interpretation.

According to Yuan, it can be demonstrated that this method is robust. And practice has meanwhile shown that the performances of this method exceed those of earlier methods.

Note: If necessary, we can add a weighing factor to the formulas for S_1 to S_4.

Chapter 9 ALTERNATIVE METHODS OF FUZZY CONTROL

9.1 Introduction

In Chapter 4 we already discussed a fuzzy control method. There are, however, other methods, of which a few will be examined in this chapter. The first one to be considered is the method developed by Sugeno and Takagi, which is based on a different control structure. The next method to be dealt with is the one based on the phase plane. This method is very interesting and will be further examined in Part III (Chapter 9), where this method will be used to control the ball in the groove. The last method to be discussed is the so-called fuzzy predictive controller. This method evaluates rules in advance, compares the results with reality and fires the most suitable rule.

9.2 Sugeno and Takagi

Sugeno and Takagi [1988] tackle a problem in a way which differs from the method usually employed. Usual in this case means that it is made of a relatively simple controller which works with simple rules, max. or min. operator and centre of gravity defuzzification. Sugeno and Takagi also start from a rule set, but their rules have an entirely different structure:

$$R1:\ if\ x_1\ is\ A_1^i\ and ... and\ x_k\ is\ A_k^i,\ then\ u^i = p_0^i + p_1^i x_1 + ... + p_k^i x_k$$

$A_1...A_k$ are fuzzy sets like highly negative, moderately negative, highly positive, while $x_1...x_k$ are crisp input values (real figures), such as fault **e** and derivative **ė** of the fault. The values $p_0...p_k$ are parameters and **u** is the ultimate result. For a system with **n** rules, there are **n** output values u^1 to u^n.

In these fuzzy control systems the fuzzification does not depend on the result of the rule. This means that feasibility value $\mu_i \in [0,1]$ is determined for each rule R_i, but this value does not affect the value of u^i. Sugeno and Takagi use the following formula to determine the ultimate result:

$$u = \frac{\sum_{i=1}^{n} \mu_i u_i}{\sum_{i=1}^{n} \mu_i}$$

So their method does not employ an explicit defuzzification operation. This fuzzy control method is highly suitable for application in so-called machine learning techniques because the parameters can be derived from examples (e.g. by means of neural networks, see also Appendix V). Moreover, it is very easy to see which rules contribute to the result and which do not.

9.3 Phase plane

Another method was suggested by Østergaard, King and Palm in 1978. It is common to use the phase plane to design fuzzy controllers for second-order systems (see for instance the 'Ball in Groove' problem in Chapter 5).

The rule set is usually symmetric in the switching line, which divides the phase plane into two. In one part of the plane the output value is positive, in the other part it is negative. The output value of each quadrant depends on, for instance, the distance to the switching line. The quality and robustness of fuzzy control systems developed in this way are based on the system entering the so-called 'sliding mode'. In this sliding mode, the system is robust with regard to parameter changes and other interferences.

The continuous distribution of the fuzzy output values in the phase plane greatly resembles the sliding plane with boundary layers [Jager, de et al., 1991].
As a result the system is highly susceptible to input changes, but performs very well as regards (trajectory) tracking quality and robustness.
The switching line in the normalised phase plane is usually described by means of the following formula: $\lambda e + \delta e/\delta = 0, \lambda > 0$. Such fuzzy applications can also be used in the case of non-linear second-order problems.

9.4 Fuzzy predictive controller

Finally, we have the so-called fuzzy predictive controller. This method calculates and evaluates all possibilities in advance. The rule with the best score in the evaluation is subsequently applied.

This fuzzy control method is used in the public transport system of Sendai, Japan. On account of the enormous traffic congestion in this metropolis, city planners wanted to have an underground railway system which would offer the city's inhabitants a high degree of comfort, safety and efficiency. Fuzzy control appeared to be the most suitable technology because it allows the trains to accelerate and brake smoothly.

Conventional fuzzy controllers were shown to have disadvantages in this special application on account of their inability to provide feedback on selected control actions. A solution to this problem turned out to be the application of *fuzzy predictive control.*

In this *Automatic Train Operation* system (ATO) a control action is selected by evaluating all rules every 100 ms. The fuzzy ATO outperforms conventional ATOs as regards travelling comfort and the accuracy with which trains are made to pull up at the platform (in Japan trains etc. always stop in exactly the same spot). The ride on the train and its braking to stop is so smooth that passengers in the usually overcrowded carriages of the Sendai underground trains consider the hanging straps to be totally superfluous! Moreover, fuzzy control appears to have reduced energy consumption by more than ten percent (or the travelling time has been reduced).

An example of a rule from the ATO fuzzy controller is the following: "If the control degree is not changed when the train stops within the preset zone, then the controller will not change the control conditions". A graphical representation of the controller can be found in Figure 9-1.

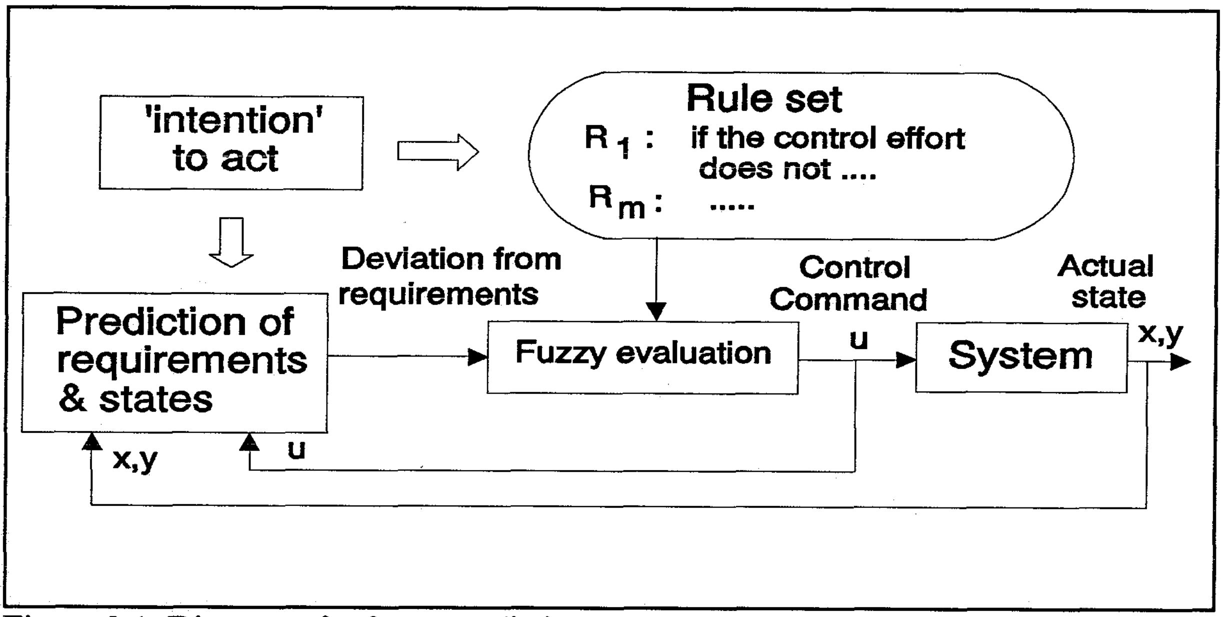


Figure 9-1: Diagram of a fuzzy predictive controller.

The controller has the following algorithm. In the fuzzy predictive controller the value of a control command is a crisp figure, $\mathbf{u} = \mathbf{c}_1, \mathbf{c}_2,...,\mathbf{c}_n$, and **x** and **y** are performance indices. Evaluations of **x** and **y** — for instance, as 'good' and 'bad'— are fuzzy sets, characterised by $\mu_{A_i}(x)$ and $\mu_{B_i}(y)$. The fuzzy controller (periodically) evaluates the effectiveness of (linguistic) rules like "if the performance index **x** is $\mathbf{A}_i$ index **y** is $\mathbf{B}_i$ when control action **u** is $\mathbf{C}_i$ at that moment, then this control action is selected and $\mathbf{C}_i$ becomes the output of the controller". In actual fact this rule looks as follows:

$$R_i: = \textbf{IF } (\mathbf{u} \text{ is } C_i \Rightarrow \mathbf{x} \text{ is } A_i \text{ and } \mathbf{y} \text{ is } B_i), \textbf{ THEN } \mathbf{u} \text{ is } C_i$$

The performance indices, $X(C_i,t)$ and $Y(C_i,t)$ are predicted on the basis of a model of the system.

In other words: On the basis of the actual state of the system we ascertain a certain deviation from the desired state. Next, all rules will be passed through, these will all result in a (possible to be fired) control action u_i. The actions are being fed back to the state predictor. Finally **that** rule that reduces the deviation (with regard to the desired state) the most will be selected and fired.

This method is therefore particularly suitable if we possess a dynamic model of the system and the number of (control) alternatives is relatively small.

Chapter 10 CONCLUSIONS & RECOMMENDATIONS

10.1 Introduction

As has already been announced in the introduction of this report conclusions are, in view of the course of the study, hardly to be drawn. That's why we will here merely discuss the biggest disadvantages and advantages, these can for that matter be regarded as general conclusions (since they don't implicitly infer from the report itself).

10.2 Disadvantages of fuzzy logic

- Although fine results are obtained in theory (in the field of simulations), only few serious applications are known. Even of famous applications (like for instance the fuzzy autofocus camera) the precise working of the used controller is not revealed. This might have something to do with what is know as 'the law of the restraining lead', through which companies will hardly incline to publish the exact construction c.q. working of their fuzzy controllers.

  More likely though is that there are always problems with the unprovable stability of the controller during realization in practice. Stability (unfortunately) is a property which for fuzzy controllers can't be proved in the way as we became acquainted with out of the classical control technology. although in a few cases it is let go at (for exampled in case of the metro in Sendai, see Chapter 9).

- When the rule-base is rather big (more then eighthundred rules) the process becomes quite fast very reckon-intensive, and demands a large memory, for the variables.

- In order to be able to construct a controller not only knowledge of fuzzy set theory is required, but the engineer also needs to have a very good insight in the process.

- The knowledge of a process, captured by the rules, doesn't necessarely lead to an optimal process control. In other words, a fuzzy regulation is not an optimizing one, yet it's lonely describing a process.

- The absence of fist-rules and directives to come to the right, or at least acceptable, choises, given a certain situation.

10.3 Advantages of fuzzy logic

- Fuzzy logic enables us to control very strong non-linear processes. This due to the fact that we are, with the help of fuzzy rules, able to build anykind of control law.

- A fuzzy controller can be implemented quickly, also thanks to the fact that a process merely has to be described roughly, this can be done without a complex analysis or model building.

- A fuzzy controller is (in software) very structured, and through that is easily adaptable (or expandable). Alterations not too big (like for example a heavier motor) will in general only require the adjustment of a few variables.

- Fuzzy control is robust because it can deal with uncertain and inaccurate inputs. More over, a controller on the basis of fuzzy control is able to draw correct conclusions everywhere in the state spacee for the output, not just in some previously defined working points.

10.4 Recommendations

Interesting to find out would be to which amount a fuzzy controller performs better with regard to a traditional PID controller (in scope of calculation time and stability). A nice problem to perform this on is the one described in Chapter 5.

At the same time one could study the effect of the great many possible definitions for the various operators. What, for instance, is the effect whenever we apply $\mu_A(x).\mu_B(x)$ for the *logical and* in stead of $\min(\mu_A(x), \mu_B(x))$?

Finally it is an advantage to compare the various methods of fuzzy control with each other, like for instance the 'traditional' method as has been discussed in Part I with regard to the one used in Chapter 5 (on the basis of the phase plane).

One of the disadvantages of fuzzy control is that we have to generate the rules of the controller ourselfs. Altough it is still in its infancy, it seems possible to join fuzzy logic and neural nets (Appendix V). In such a Neuro-Fuzzy controller the rules can (more or less) be acquired with help of a neural net. This seems very promising, we should, in the furure (after more is known about fuzzy logic, and it's usefulness for our problems) pay attention to it!

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APPENDICES

Appendix I

Graphic representation of the operators, as discussed op page 10.
In the figures the original membershipfunction always will be denoted as $\mu_F(x)$.
The adapted fuzzy set is represented by the dotted line.

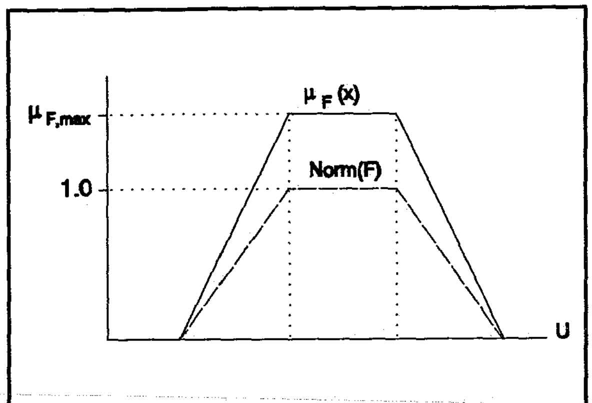


Figure I-1: Normalisation

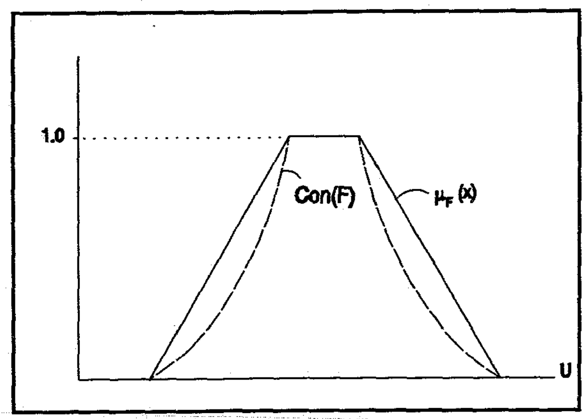


Figure I-2: Concentration

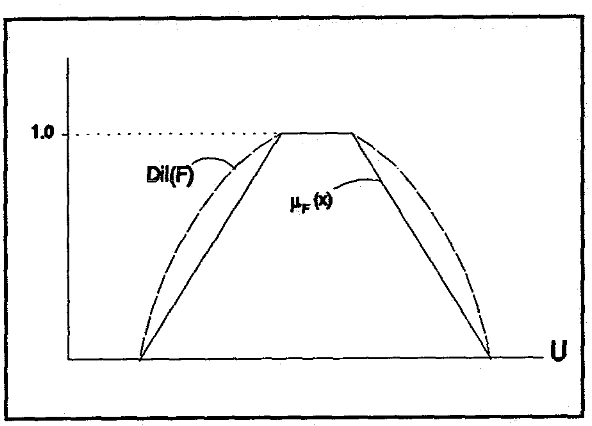


Figure I-3: Dilatation

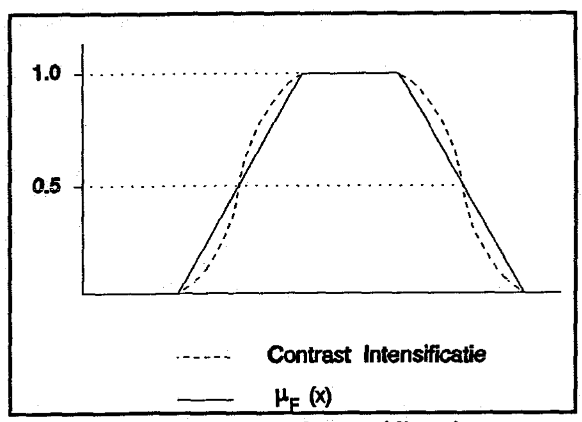


Figure I-4: Contrast Intensification

Appendix II

SOME PROPERTIES OF INTERSECTION, UNION AND COMPLEMENT

By means of the operations intersection, union and complement, which have been defined before, many of the properties applying to ordinary (classical) sets can be extended to fuzzy sets.
A few examples:

$$\left.\begin{aligned} (A \cup B)' &= A' \cap B' \quad &(1)\\ (A \cap B)' &= A' \cup B' \quad &(2)\end{aligned}\right\} \textit{Laws of De Morgan}$$

$$\left.\begin{aligned} C \cap (A \cup B) &= (C \cap A) \cup (C \cap B) \quad &(3)\\ C \cup (A \cap B) &= (C \cup A) \cap (C \cup B) \quad &(4)\end{aligned}\right\} \textit{Distributive laws}$$

These and comparable equalities can be determined by showing that the corresponding relations for the membership functions of A, B and C are identical. In case (1), for example, we get:

$$1 - Max(\mu_A, \mu_B) = Min\,(1-\mu_A, 1-\mu_B)$$

of which the equality can be simply established by testing it in two possible cases: $\mu_A(x) > \mu_B(x)$ and $\mu_A(x) < \mu_B(x)$.

In the same way the corresponding equation of (4) in terms of μ_A, μ_B, and μ_C will become:

$$Max(\mu_C, Min(\mu_A, \mu_B)) = Min\,(Max\,(\mu_C, \mu_A), Max(\mu_C, \mu_B))$$

Which can be verified by considering the following six cases:

$$\mu_A(x) > \mu_B(x) > \mu_C(x),\ \mu_A(x) > \mu_C(x) > \mu_B(x),\ \mu_B(x) > \mu_A(x) > \mu_C(x),$$

$$\mu_B(x) > \mu_C(x) > \mu_A(x),\ \mu_C(x) > \mu_A(x) > \mu_B(x),\ \mu_C(x) > \mu_B(x) > \mu_A(x).$$

Appendix III

The two most used inference methods.

As has already been pointed out in Chapter 2, there are numerous operators which can be used, depending on the application. Moreover, these operators can also be combined, which is referred to as inference. It will be clear that such a aggregation (of several fuzzy sets) can be realised in various ways. For instance, compare the Max-min with the Max-dot inference method.

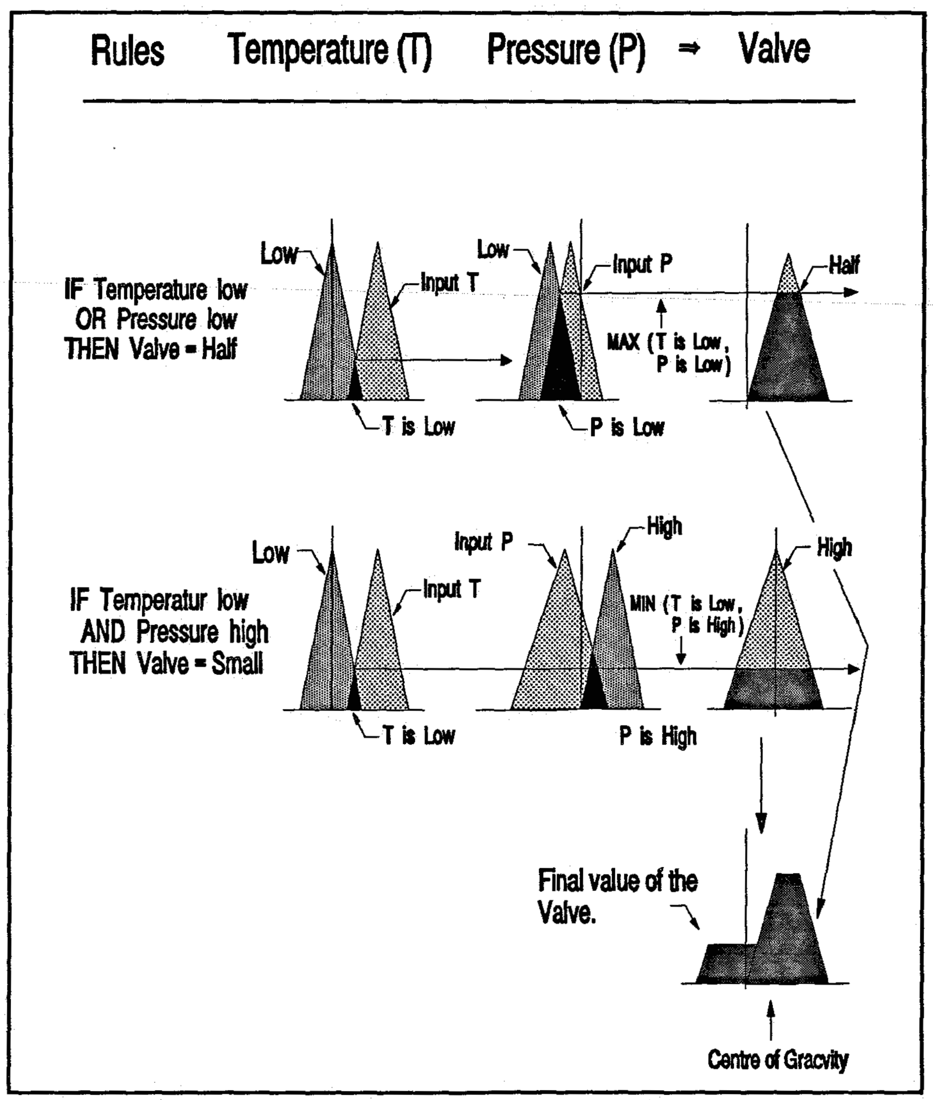


Figure III-1: The Max-Min inference method.

In the max-min inference method the ultimate membership function of the output (for each output) is the union of all fuzzy sets assigned to this output (in the consequences of the rules), after these fuzzy sets have been cut off at the membership degree of the corresponding antecedent(s), as is shown in Figure III-1.

Formulated in another way: In Figure III-1 the input of the controller consist of the fuzzy quantities "Input T" and "Input P" respectively representing the measured temperature and the measured pressure. The dark hatched area "T is low" indicates in how far the "Input T" belongs to the fuzzy set "Low", this value is calculated by min("Input T", "Low"). Likewise "P is Low" represents the value in which "Input P" belongs to the fuzzy set "Low". Of these two fuzzy sets (P and T), for each, the biggest value is calculated with the help of max("T is Low") respectively max("P is Low"). Hence the name of the method: Max-Min! In case that both antecedents in the rule are coupled by the *logical and* the smallest of the two crisp values is taken (Chapter 2). After which the fuzzy set defined on the output (for example "Half") is cut of at that value.

In case of the Max-Prod (also known as Max-Dot) method we go to work in exactly the same way, however now we don't cut off the (for the output defined) membership function at the smallest value in the final step, but we multiply the complete membership function with this value (see Figure III-2). Consequently the name Max-Prod is in a certain way misleading because the Max-min inference is applied in the beginning!

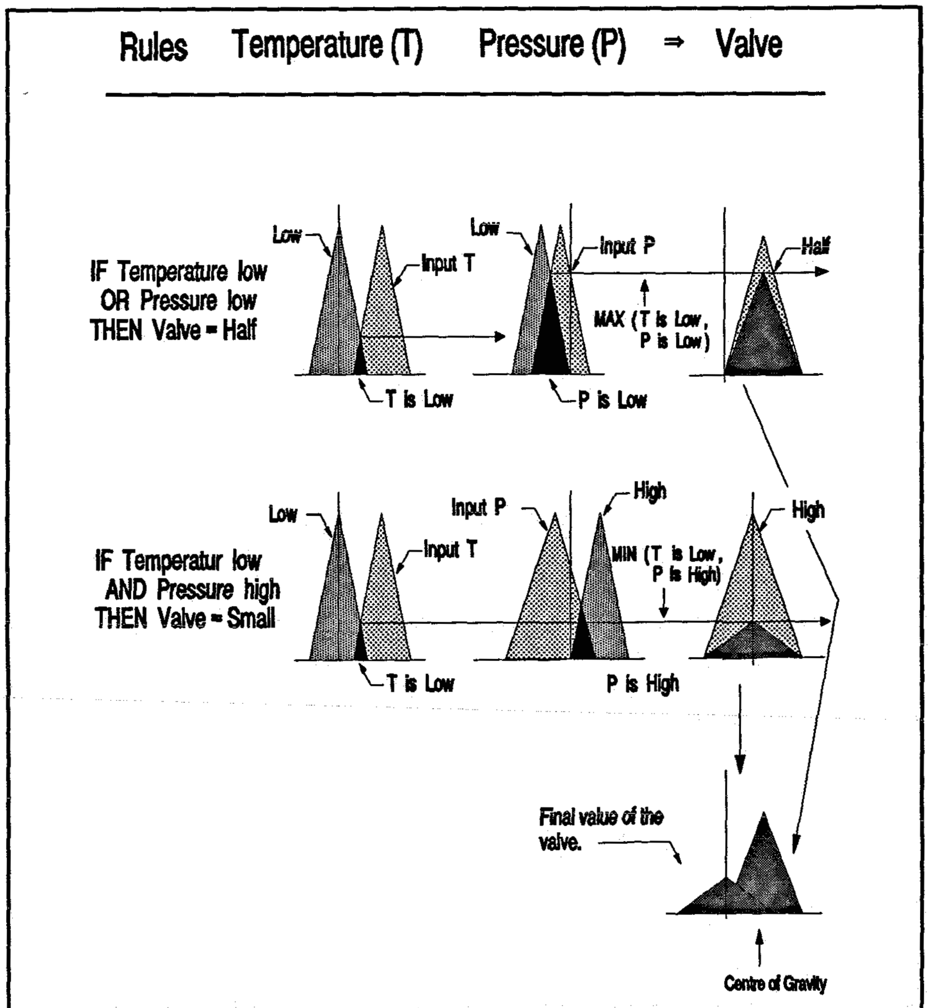


Figure III-2: The Max-Prod. inference method.

In the max-dot or max-product inference method the ultimate membership function of the output (for each output) is the union of all fuzzy sets assigned to this output in a consequent, after these fuzzy sets have been multiplied by the (maximum) membership value of the antecedent(s), as is illustrated in Figure III-2.

Appendix IV

Demands with relation to t-norms and t-conorms [Dubois and Prade 1980, p. 17]:

t-norms are functions which satisfy the following demands:

1. $t(0,0) = 0;\ t(\mu_A(x),1) = t(1,\mu_A(x)) = \mu_A(x),\quad x \in X$
2. $t(\mu_A(x),\mu_B(x)) \le t(\mu_C(x),\mu_D(x))$
   als $\mu_A(x) \le \mu_C(x)$ *en* $\mu_B(x) \le \mu_D(x)$ (*monotonicity*)
3. $t(\mu_A(x),\mu_B(x)) = t(\mu_B(x),\mu_A(x))$ (*commutativity*)
4. $t(\mu_A(x),t(\mu_B(x),\mu_C(x))) = t(t(\mu_A(x),\mu_B(x)),\mu_C(x))$ (*associativity*)

t-conorms are functions which satisfy the following demands (analogous):

1. $s(1,1) = 1;\ s(\mu_A(x),0) = s(0,\mu_A(x)) = \mu_A(x),\quad x \in X$
2. $s(\mu_A(x),\mu_B(x)) \le s(\mu_C(x),\mu_D(x))$
   als $\mu_A(x) \le \mu_C(x)$ *en* $\mu_B(x) \le \mu_D(x)$ (*monotonicity*)
3. $s(\mu_A(x),\mu_B(x)) = s(\mu_B(x),\mu_A(x))$ (*commutativity*)
4. $s(\mu_A(x),s(\mu_B(x),\mu_C(x))) = s(s(\mu_A(x),\mu_B(x)),\mu_C(x))$ (*associativity*)

APPENDIX V

NEURAL NETWORKS

A neural network is a structure consisting of relatively simple elements (neurons) which have been linked up by means of various connections. Via these connections a neuron can influence the activities of other neurons. The degree in which this takes place depends on the variable weighing factors determining the strength of the influence.
Neural networks are highly simplified copies of the human brain. They possess corresponding properties like parallelism, adaptive capacity, self-organising ability, the capacity to learn by means of examples and associative powers.

The main properties distinguishing a neural network from classical artificial intelligence technology is its self-learning capacity and parallelism. Thanks to its self-learning capacity, the neural network can detect relations in the information supplied with the help of a learning algorithm. As the neurons in the network operate parallel to each other, a high processing speed can be achieved. Another advantage of parallelism is that, if one of the neurons breaks down, this does not immediately disrupt the entire network. Data transmission in the network proceeds via several neurons so that the breaking down of one part need not be fatal to the functioning of the network.
Neural networks are suitable for a number of applications:

- classification of data (detection of relations)
- data storage (compact and rapid-access memory)
- description of complicated functions (as complement of classical function theory)
- pattern recognition.

Apart from advantages, neural networks unfortunately also have a number of disadvantages. The proper learning in of a neural network calls for a large quantity of data. Learning in is also a slow process, requiring a great deal of calculation. This means that application of a neural network in a continuous learning system will considerably slow down the operation of the system.
Moreover, it is not or hardly possible to establish how a learned-in network "operates". The way in which the information is stored in the network, and especially the accuracy of the output, the sensitivity to malfunctions, tolerances and the risk of an incorrect output value are hard to determine.
As a result of the above advantages and disadvantages, the worldwide research efforts are mainly focused on applications which cannot or can hardly be achieved with classical methods and require parallelism. Pattern recognition is a good example of this.

Appendix VI

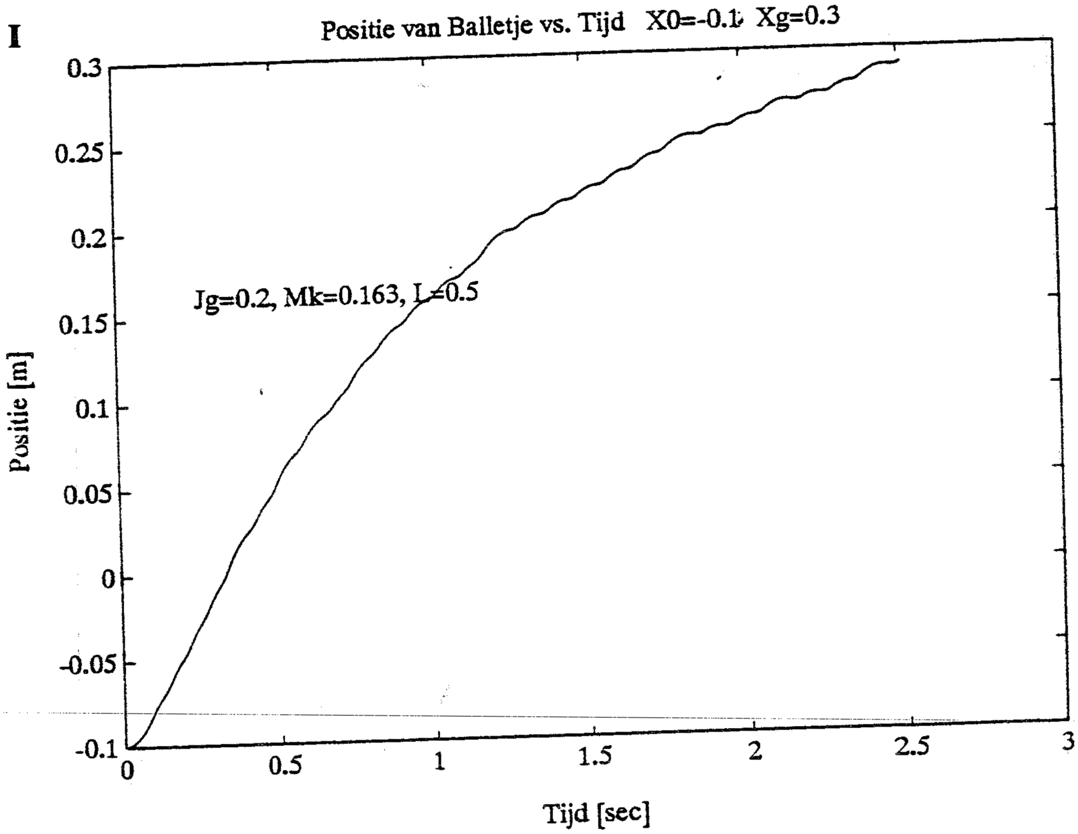
Comment on the results:

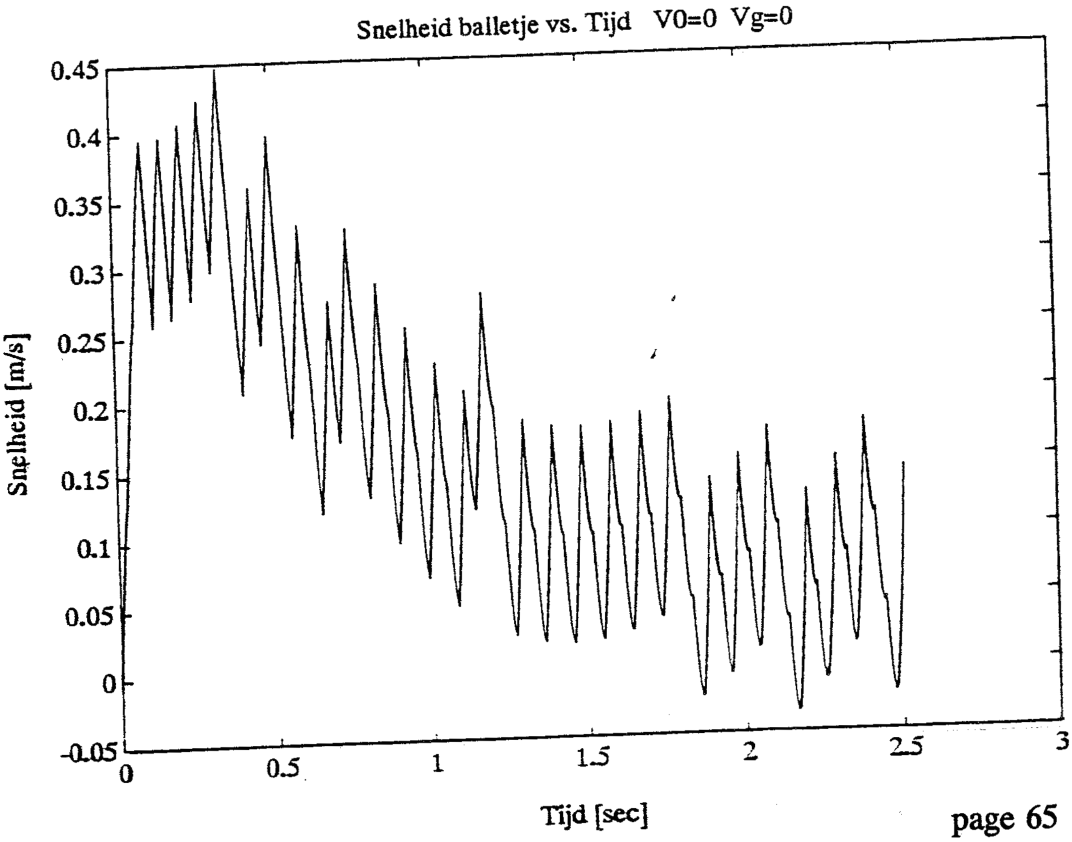
In this appendix some results shall be presented, which are obtained from simulations of the, by means of fuzzy control regulated, "Ball in Groove" problem. The control strategy, as well as the (non-linear) differential equations, describing the system, have detailled been under discussion in Chapter 5. At the same time some remarks have been made concerning the results, a.o. the strong switching of the sign of the output (the moment) came up for discussion. Although this is a phenomenon where fuzzy logic should be of some assistence to us, it appears that finding a solution for this switching is, in practice, very difficult. Simple introducing a boundary layer didn't turn out to be working, on the contrary, the solution diverged owing to this! Remark, several sample-frequencies were tried. Perhaps that the non-converging has something to do with the used fuzzy control method.

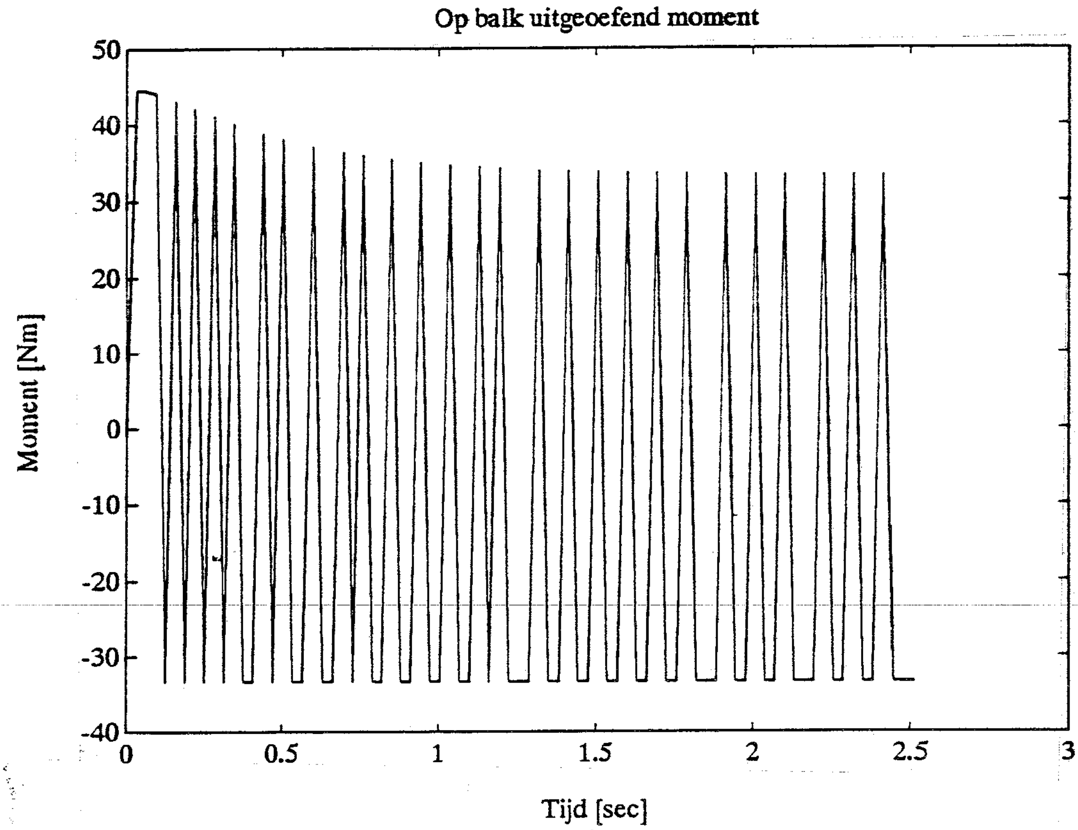
The results of the simulations are least of all perfect. However, to demonstrate that a fuzzy controller can be implemented relative simple, and rather quick, and that it besides can deliver quite acceptable results in the entire working space, the plots are yet inserted in this report. One should realize that, when beholding the graphs, the goal of this graduation study was slightly different as usual. Drawing up some model, carrying out simulations with that, and pulling conclusions on that basis was **not** the objective of this investigation. This report merely has been trying to provide the reader some insight in the building and working of a controller operating on the basis of fuzzy logic.

Comment on the graphs: X0 is the starting position of the ball on the groove, Xg is the desired ending position. By the way, the groove is one meter long (L is half the lenght!), and thus excels half a meter to both sides, with regard to the centre (0,0). The desired ending speed equals the starting speed (in all cases considered) and amounts to 0 m/s. The mass of the ball is represented by Mk, while Jg represents the moment of inertia of the groove.

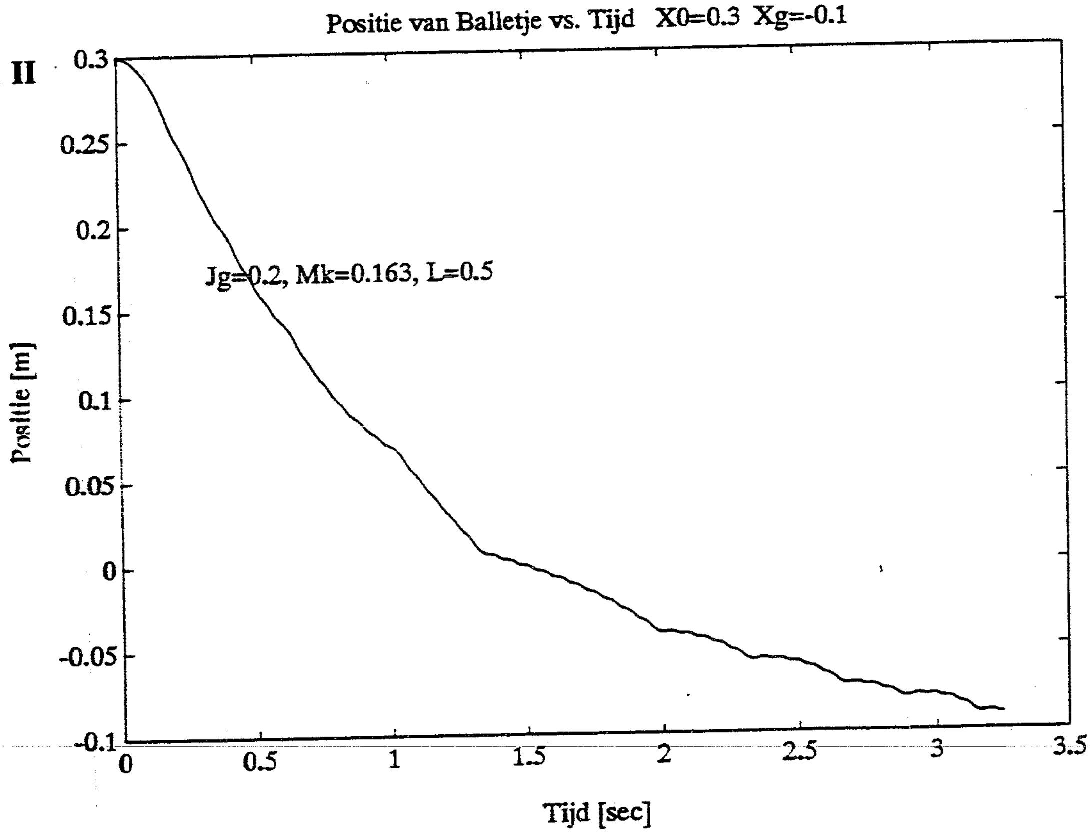
Simulation I

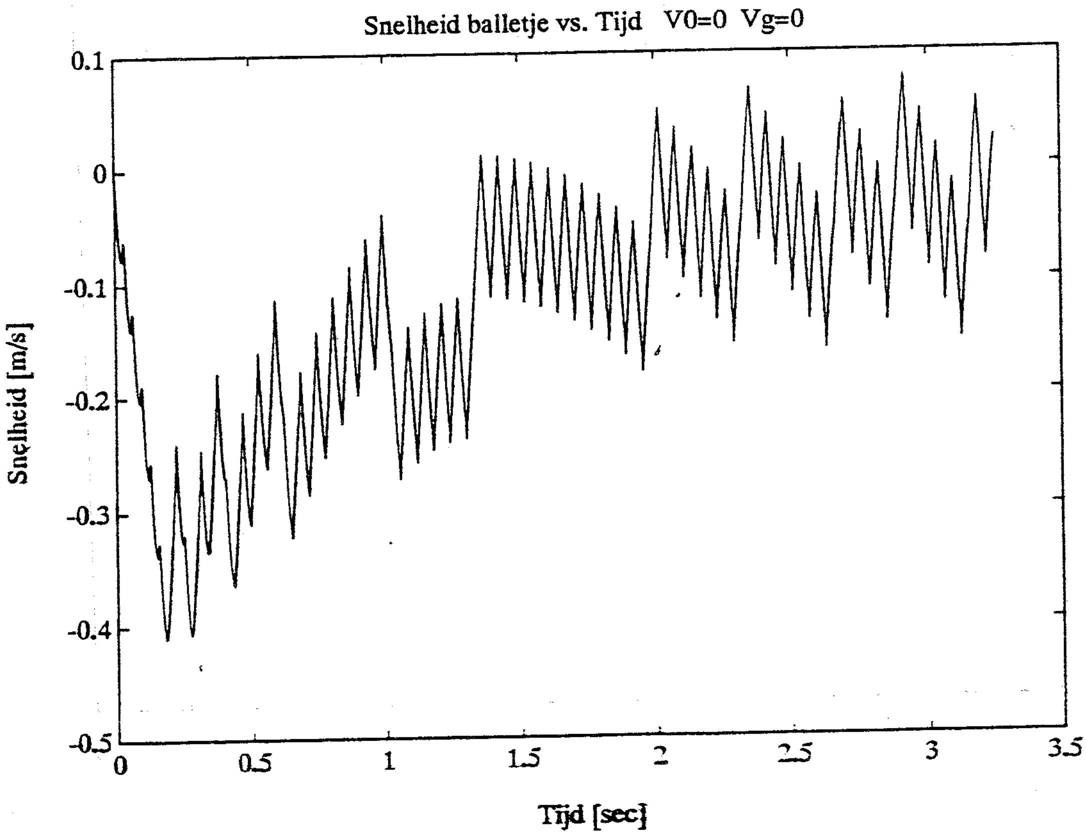


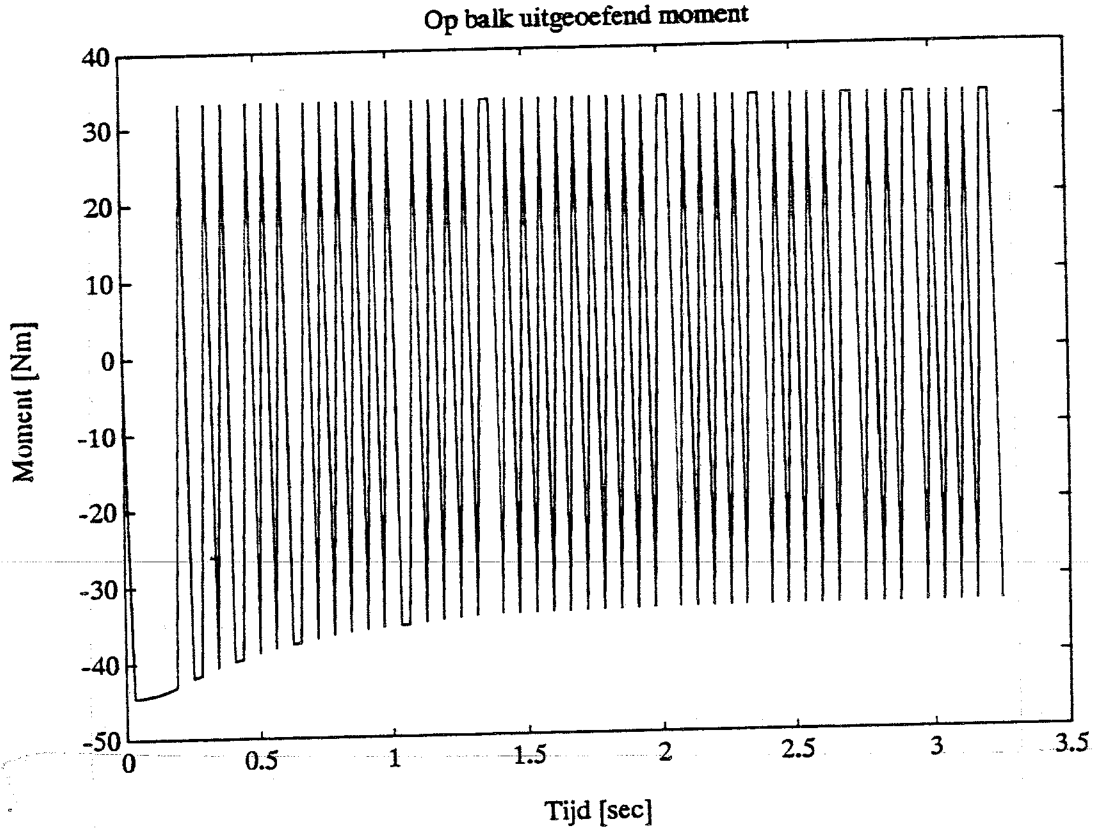



Op balk uitgeoefend moment
Moment [Nm]
50
40
30
20
10
0
-10
-20
-30
-40
0
0.5
1
1.5
2
2.5
3
Tijd [sec]

Simulation II

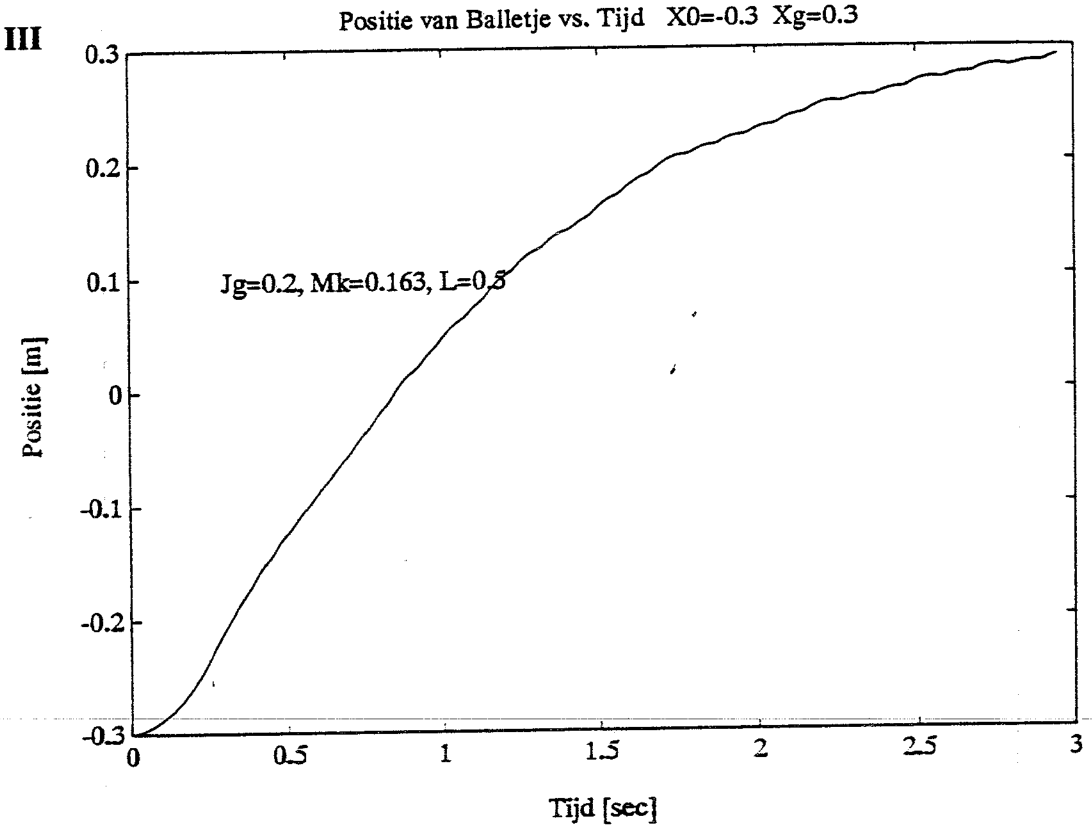


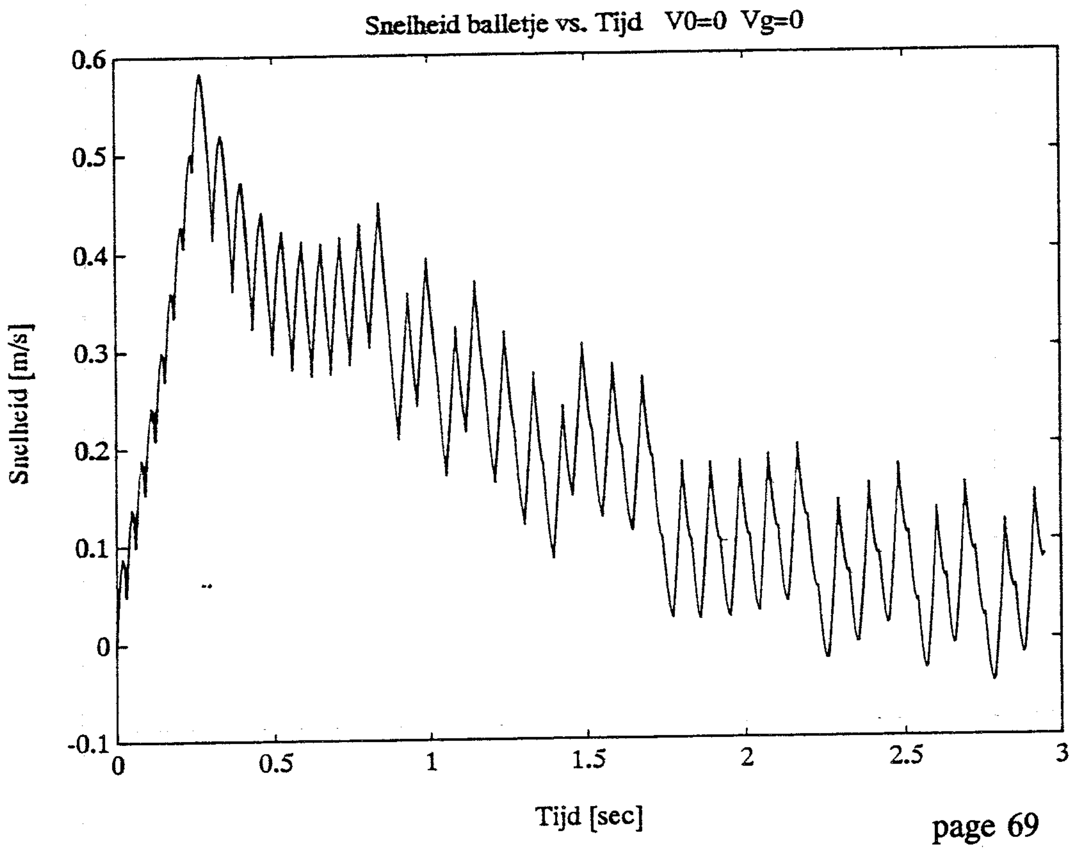


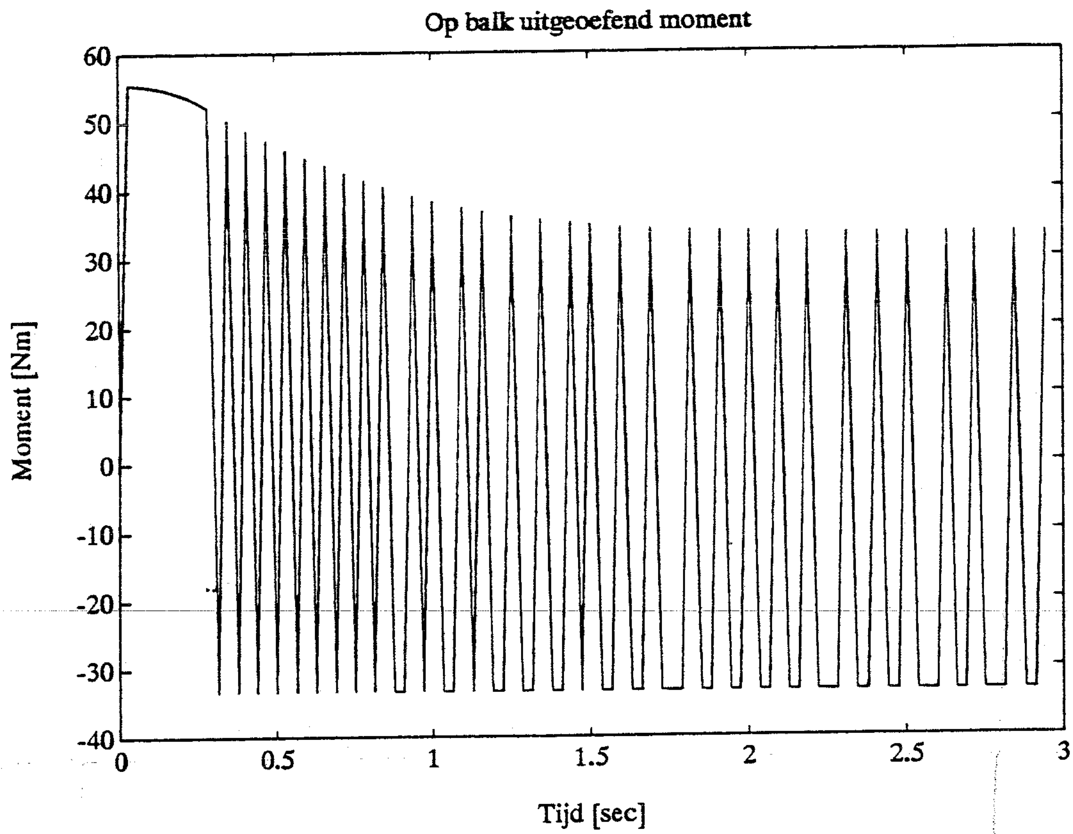


Op balk uitgeoefend moment
Moment [Nm]
Tijd [sec]
40
30
20
10
0
-10
-20
-30
-40
-50
0
0.5
1
1.5
2
2.5
3
3.5

Simulation III






Op balk uitgeoefend moment
Moment [Nm]
60
50
40
30
20
10
0
-10
-20
-30
-40
0
0.5
1
1.5
2
2.5
3
Tijd [sec]

APPENDIX VII

Listing of the used Matlab program for simulating the 'Ball in Groove' problem, as discussed in Chapter 5.

FUZZY.M

```
clear
pack
global R mk Jg g l M
R=0.025;mk=0.1625;Jg=0.2;g=9.81;        % system constants
umax=100;                                % maximum practised moment
l=0.5;vref=6;                            % Ref. length and speed in
                                         % orden to normalise E and V.
xg=-0.1;vg=0;                            % desired end position
x0=[0.3 0 0 0];                          % start conditions
t0=0;deltat=0.031;                       % integration interval

m1=num2str(mk);                          % Strings with values of the
m2=num2str(Jg);                          % constants, for use in
m3=num2str(l);                           % graphs.
m4=num2str(x0(:,1));
m5=num2str(x0(:,2));
m6=num2str(xg);

M=0;z1=0;z2=0;
Mom=[];Z1=[];Z2=[];
T=[];X1=[];X2=[];X3=[];X4=[];            % Matrices for state
E=-xg+x0(:,1);                           % position error on t=0
V=-vg+x0(:,2);                           % speed error on t=0
E1=[E];V1=[V];                           % Matrices for memorising all positi-
                                         % on and speed errors

%      *** LOOP ***

tb=0;tf=tb+deltat;xb=x0;

while abs(E/(2*l))>0.01                  % As long as the desired state isn't
                                         % accomplished we keep controlling.
                                         % Membership values are (at
PES=0;PEB=0;NES=0;NEB=0;                 % each control action
PVS=0;PVB=0;NVS=0;NVB=0;                 %  initialy equal to zero.
u1=0;u2=0;u3=0;u4=0;
A1=0;A2=0;A3=0;A4=0;
g1=0;g2=0;g3=0;g4=0;
Mom=[Mom M];
```

```
%       *** Lidmaatschapsfuncties voor positie ***
if E>0
        PES = -1/(2*l)*E+1;                    % P is short for Positive
        PEB =  1/(2*l)*E;                      % E is the error in position
  else if E<=0                                 % S short for 'small'
        NES =  1/(2*l)*E+1;                    % B short for 'big'
        NEB = -1/(2*l)*E;                      % N short for Negative
  end                                          % V is the error in speed
end
%       *** Membership function for speed ***
if V>0
        PVS = -1/(vref)*V+1;
        PVB =  1/(vref)*V;
  else if V<=0
        NVS =  1/(vref)*V+1;
        NVB = -1/(vref)*V;
  end
end
%       *** RULE BASE ***

if V>-E
        u1=min([max([PES PEB]) max([PVB PEB])]);
        u2=max([max([NES NEB]) min([PES max([PVS NVS])])]);
        u3=0;
        u4=0;
end
if V<=-E
        u1=0;
        u2=0;
        u3=min([max([NES NEB]) max([NVB NEB])]);
        u4=max([max([PES PEB]) min([NES max([PVS NVS])])]);
end

%       *** Fixing the next control action ***
%               That is to say the new Moment (=M), to be occupied on
%               the groove.

if V>-E
        p=-umax*u2/(u1+u2);                    % intersection (p,q) of NUS and
                                               NUB
        q=u1*u2/(u1+u2);
        A1=umax*q;                 % A1 t/m A3 form the area beneath the
        A2=-0.5*(u2-q)*p;          % Resulting membership functions, necessary
        A3=-0.5*(u1-q)*(-umax-p);  % for the determination of the C.o.G.
        g1=-umax/2;                % centre of gravity of A1 (x-coordinate)
        g2=1/3*p;                  % centre of gravity of A2 (x-coordinate)
        g3=p+2/3*(-umax-p);        % centre of gravity of A3 (x-coordinate)
```

```
        M=(g1*A1+g2*A2+g3*A3)/(A1+A2+A3); % centre of gravity of total m.s.f.
        else if V<=-E
        p=umax*u4/(u3+u4);          % intersection (p,q) of PUS and PUB
        q=u3*u4/(u3+u4);
        A1=umax*q;                  % A1 t/m A3 form the area beneath the
        A2=0.5*(u4-q)*p;            % Resulting membership function, necessary for
        A3=0.5*(u3-q)*(umax-p);     % the determination of the C.o.G.
        g1=umax/2;                  % centre of gravity of A1 (x-coordinate)
        g2=1/3*p;                   % centre of gravity of A2 (x-coordinate)
        g3=(p+2/3*(umax-p));        % centre of gravity of A3 (x-coordinate)
        M=(g1*A1+g2*A2+g3*A3)/(A1+A2+A3); % centre of gravity of total m.s.f.
  end
end

[t,x]=ode23('fuzzyrhs',tb,tf,xb);
y=x(max(size(t)),:);                % last state
E=-xg+y(:,1)                        % position error
V=-vg+y(:,2);                       % speed error
E1=[E1 E];V1=[V1 V];
T=[T' t']';                         % expand time vector
X1=[X1' x(:,1)']';X2=[X2' x(:,2)']'; % expand state vector
X3=[X3' x(:,3)']';X4=[X4' x(:,4)']';

Z1=[Z1 z1];Z2=[Z2 z2];
z1=y(:,3);
z2=y(:,4);

tb=tf;tf=tf+deltat;                 % alter the time interval
xb=[y(:,1) y(:,2) 0 0];             % new start condition

end                                 % End of the WHILE loop

T1=0:T(max(size(T)),1)/(max(size(Z1))-1):T(max(size(T)),1);

plot(T,X1),title(['Position of Ball vs. Time   X0=',m4,'  Xg=',m6]),...
xlabel('Tijd [sec]'),ylabel('Position [m]'),text(.1*max(T),...
0.65*(max(X1(:,1))-min(X1(:,1)))+min(X1(:,1)),['Jg=',m2,...
', Mk=',m1,', L=',m3]),pause
% meta plot1
plot(T,X2),title(['Speed ball vs. Time   V0=',m5,'  Vg=0']),...
xlabel('Tijd [sec]'),ylabel('Speed [m/s]'),pause
% meta plot1
%plot(T1,Z1),title('Angle distortion of groove vs. Time   θ0=0  θg=0'),...
%xlabel('Time [sec]'),ylabel('Angle θ [rad]'),pause
% meta plot2
%plot(T1,Z2),title('Angle speed of ball vs. Time   δθ0=0  δθg=0'),...
%xlabel('Tijd [sec]'),ylabel('Angle speed [rad/s]'),pause
```

```
% meta plot2
plot(T1,Mom),title('exercised moment on groove'),xlabel('Time [sec]'),...
ylabel('Moment [Nm]'),pause
% meta plot2
plot(E1,V1),title('Phaseplane'),xlabel('Position error'),ylabel('Speed error'),pause
```

Listing of the Matlab program in behalf of the calculation of the new state of the system (out of two coupled differential equations), being the 'measured' values of the state.

FUZZYRHS.M:

```
function rhs=fuzzyrhs(t,x)

Fa=(mk*x(1)*((R*x(4)-2*x(2))*x(4)-g*cos(x(3)))+M)/(Jg+mk*x(1)*x(1));
% Fa is de hulpvariabele uit Hoofdstuk 5.

rhs(1)=x(2);
rhs(2)=R*Fa+5*(x(1)*x(4)*x(4)-g*sin(x(3)))/7;
rhs(3)=x(4);
rhs(4)=Fa;
```

% the column rhs(i) (i= 1,2,3,4) represents the derivated state.
% the derived state therefor is: $[\dot{s}, \ddot{s}, \dot{\alpha}, \ddot{\alpha}]^T$.