MASTER

Optimizing a gas-expansion system, to gain maximum flexibility of electricity production

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Optimizing a gas-expansion system, to gain maximum flexibility of electricity production

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Voorwoord

In 1997 heb ik mijn HTS studie op de T.H. Rijswijk afgerond. Na twee jaar gewerkt te hebben bij een middelgrote installateur, heb ik de overstap gemaakt naar een grote installateur, Croon elektrotechniek b.v. waar ik vervolgens engineer werd van hoogspanningsinstallaties. Na het bijspijkeren van mijn theoretische en praktische kennis op het gebied van aanleg van hoogspanningsinstallaties, had ik het gevoel toch diepgang te missen. Zodoende ben ik in 2001 gestart met de deeltijd opleiding elektrotechniek op de T.U. Eindhoven. In 2003 heb ik de overstap gemaakt naar mijn huidige werkgever E.ON Benelux.
Begonnen als elektro-procestechnicus op de Centrale Maasvlakte en sinds 2005 werkzaam bij de afdeling delivery and dispatch. Waar de opleiding een goede basis is voor mijn huidige functie binnen het bedrijf.

Deze opdracht heeft betrekking op het vak elektrotechniek, maar dan in ruimere zin. Er komt ook een groot stuk werktuigbouwkunde om de hoek kijken. Voor deze opdracht moest ik dan ook anders leren denken. Aanvankelijk had ik geen idee hoe te starten en hoe de vele losse ideeën in elkaar te laten vloeien. Op deze plaats wil ik dan ook Ing. Jan Kromhout Mbe bedanken voor zijn support en de discussies over de aanpak en opzet van het model.
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Ir. Henk Compter wil ik bedanken voor het creëren van een afstudeeropdracht. Tevens heeft hij mij de ruimte gegeven om deze opdracht tijdens het werk uit te voeren.
Ook wil ik prof. Ir. Wil Kling en Dr. Ir. Johanna Myrzik bedanken voor de mogelijkheid tot afstuderen buiten de faculteit. Het afstuderen binnen mijn werkomgeving was erg belangrijk. Zonder deze combinatie was het afstuderen voor mij niet mogelijk geweest.

Voor de universiteit is het werk nu afgerond. Binnen mijn eigen werk vormt dit een nieuwe basis die de komende jaren verder uitgewerkt gaat worden.

De studie op de TUe is lang en zwaar geweest. Er moest continu een balans gevonden worden tussen werk, studie en gezin. Er zijn nogal wat vakanties verzet, verkort of op een andere manier ingevuld, omdat er altijd wel tentamens waren. In het bijzonder wil ik daarom mijn vrouw Saskia bedanken, zonder haar steun zou ik niet zo ver zijn gekomen.

Table of Content

1. M.SC. ASSIGNMENT (DUTCH) 8

2. SUMMARY 9
2.1. SUMMARY 9
2.2. STRUCTURE OF THIS REPORT 10

3. INTRODUCTION 11
3.1. POWER GENERATION USING THE EXPANSION OF NATURAL GAS 12
3.2. THE PRINCIPLE 12
3.3. ORIGINAL DESIGN OF THE NRE GAS-EXPANSION SYSTEM 14
3.4. EXTENSION OF THE SYSTEM 15
3.5. LITERATURE SEARCH 15
3.6. LINEAR-PROGRAMMING 17
3.7. IMPLICIT ASSUMPTIONS IN LINEAR-PROGRAMMING 19

4. MATHEMATICAL MODEL 20
4.1. MODEL:.. 20
4.1.1. COMBINED HEAT AND POWER PLANT (CHP) 21
4.1.2. HEAT STORAGE TANK (ST) 21
4.1.3. GASEXPANDERS (GE) 21
4.1.4. BOILER (B) 22
4.1.5. ELECTRICITY (P) 22
4.1.6. GAS FUEL (F) 22
4.1.7. COOLING TOWER (CT) 23
4.1.8. HEAT BALANCE 24
4.1.9. START AND STOP COST 24
4.1.10. OBJECTIVE FUNCTION 24
4.2. OBJECTIVE FUNCTION IN STANDARD FORM 26

5. OPTIMIZATION 29
5.1. EQUIPMENT DATA 29
5.2. MODEL IN MATLAB 31
5.3. MARKET PRICE 32
5.4. GAS FLOW 33
5.5. EFFECT OF OPTIMIZATION PERIOD 34

6. CASE STUDIES 37
6.1. CASE 1: REFERENCE CASES 37
6.1.1. SUB CASE 1.1: USING ONLY A BOILER 37
6.1.2. SUB CASE 1.2: USING 2 CHP PLANTS AND A BOILER 38
6.1.3. SUB CASE 1.3: EXTENSION WITH 4 CHP PLANTS 39
6.2. CASE 2: 24H, 36H OR 48H MODEL 40
6.3. CASE 3: APX ON HOURLY BASIS OR PEAK AND OFF-PEAK 42
6.4. CASE 4: STORAGE STRATEGY 44
6.5. CASE 5: OPTIMAL STORAGE VOLUME 48

7. CONCLUSIONS AND RECOMMENDATIONS 50
7.1. CONCLUSIONS 50
7.2. RECOMMENDATIONS FOR FURTHER DEVELOPMENTS 51

8. REFERENCES 52

APPENDIX A. OVERVIEW CASES 55
APPENDIX B. M-FILES 57
APPENDIX C. LP FORMAT 65
APPENDIX D. APX GRAPH 2005 67
APPENDIX E. GAS FLOW GRAPH 2005 68
APPENDIX F. OVERVIEW RESULTS CASE 1 69
APPENDIX G. OVERVIEW RESULTS CASE 2 72
APPENDIX H. OVERVIEW RESULTS CASE 3 74
APPENDIX I. OVERVIEW RESULTS CASE 4 77
APPENDIX J. OVERVIEW RESULTS CASE 5 81

Nomenclature

Equipment Symbols

B = Gas fired Boiler
CT = Cooling tower for heat disposal
CHP = Combined Heat and Power plant
ST = Storage Tank

Quantities of energy

P = Electric power [kW]
Q = Heat flow rate [kW]
F = Gas Fuel consumption [GJ/h]

Other miscellaneous symbols

J = Objective function (daily operational cost)
K = Total number of sampling intervals
k = Index for each sampling time
N = Number of equipment installed
p_1, q_1, p_2, q_2 = Parameters related to performance characteristics of each piece of equipment
t_k = K^{th} time step
Δt = Time step interval [h]
V = Volume of heat storage tank
δ = Binary variable expressing on/off status of each piece of equipment
η = Efficiency
APX = Amsterdam Power eXchange, electricity market price
f = Price [€/GJ]
Φ = Flow [m^3/h]

Superscripts

in = Input to heat storage
out = Output from heat storage

Subscripts

d = Dummy
sell = Sale electricity
n = Index for each piece of equipment
gas = Gas
min = Minimum
max = Maximum

Abbreviation

APX = Amsterdam Power eXchange, electricity market price
LP = Linear-programming
ILP = Integer Linear-programming
MILP = Mixed Integer Linear-programming

1. M.Sc. Assignment (Dutch)

Introductie:
E.ON Benelux concentreert zich op de productie en levering van elektriciteit en warmte aan energiebedrijven en de energie-intensieve industrie in Nederland en België. E.ON Benelux is primair een elektriciteitsproductiebedrijf; het bedrijf kan (inter)nationaal inkopen en beschikt over een professionele verkooporganisatie. De onderneming is opgericht in 1941. Sinds 2000 is het onderdeel van E.ON Energie AG. De centrales van E.ON Benelux, met een totale elektrische capaciteit van 1.850 MW en een totale thermische capaciteit van 750 MW, staan opgesteld in de provincie Zuid-Holland. Het bedrijf heeft circa 600 medewerkers en is gevestigd in Rotterdam.
Het productiepark van E.ON Benelux bestaat uit zes gasgestookte centrales; Rotterdam (2x), Den Haag, Leiden, Delft en Maasvlakte en twee kolen gestookte centrales op de Maasvlakte. De elektriciteit die op deze zes locaties wordt geproduceerd, wordt via het hoogspanningsnet aan de distributiebedrijven en aan eindklanten geleverd.
Daarnaast leveren de centrales in Rotterdam, Den Haag en Leiden warmte voor de verwarming van huizen en kassen en de gasgestookte centrale op de maasvlakte, stoom aan een industriële plant. Bovendien levert de warmtekrachtcentrale RoCa3 kooldioxide (CO_2) in de vorm van gekoelde en gecomprimeerde rookgassen aan tuinders.
E.ON Benelux heeft sinds haar oprichting in 2000 hard gewerkt aan een fundament voor groei. De overname van NRE Energie (NV Nutsbedrijf Regio Eindhoven) is de eerste stap van E.ON naar verticale integratie, waarmee het ook in Nederland een goede balans wil krijgen tussen productie en levering van gas en elektriciteit.

Bij de overname van NRE zijn een aantal kleine energieopwekkers meegekocht, waaronder windturbines, Carbiogas, Whispergen en een gasexpansie installatie. Dit afstudeerproject focust alleen op het laatst genoemde project.

Opdracht omschrijving:
De student concentreert zich op de NRE gasexpansie installatie. De student moet het proces doorgronden en het proces omzetten in een model. Het hiervoor te gebruiken programma is geheel aan de student (MatLab, MathCad, Excel, etc.). Na verificatie van het model dient het proces geoptimaliseerd te worden voor het opwekken van elektriciteit en warmte tegen minimale kosten m.b.v. PowrSym. Indien PowrSym niet volstaat, zal de student een alternatief moeten zoeken.
Ook zal de student een aanbeveling moeten doen m.b.t. de procesaansturing (lokaal, vanuit NRE of vanuit E.ON).

Verwachte resultaten:
Verwacht wordt dat de student een advies/aanbeveling uitbrengt hoe de installatie optimaal “day ahead” te bedrijven.

Verwachte problemen:
PowrSym is een programma dat gebruikt wordt voor grote productieparken. Het programma kan wel omgaan met warmte en elektriciteit, maar is niet ingericht voor deze kleine complexe gecombineerde installaties van gasexpansie, warmteafgifte, wkk en hulpketels in een marktomgeving. De student zal bij het maken van zijn model enkele afwegingen moeten maken.

2. Summary

2.1. Summary

In this report a complex system of a gas-expansion facility, including 2 gas-expanders, 6 co-generation units, a boiler, heat storage and a heat dump facility, is considered.
The Netherlands has a deregulated electricity market. Within this electricity market, an important trade platform is active called the Amsterdam Power eXchange (APX). It is a forward market which means that prices are set on the day before. The electricity prices on this market vary from hour to hour and the electricity production from a Combined Heat and Power plant (CHP) or co-generation unit is more profitable during certain hours than others. A wise strategy would then be to produce the electricity during hours when the market price of electricity is high, store the heat obtained from the CHP and use it, when the CHP is out of operation, during the hours when the price is low. Storage can also be used to avoid starting a more expensive CHP during peak periods of the heat demand. When storing heat for that purpose, certain periods are more preferable than others. Therefore the optimization horizon, for the operational strategy of this gas-expansion system with storage, is also part of investigation.

To minimize the operational cost of the system, the developed model takes into account all operational restrictions, interactions between assets, and market prices.
In this thesis a model of the system is described mathematically. The minimal operational cost problem is shown to be a problem which can be solved with mixed integer programming. The solution is implemented in Matlab and the behavior of the model is investigated.

The most important conclusion is that the optimization period must take the low heat demand and low market prices during the night into account. This can be done by optimizing over a period of 36h or 48h. Another way of doing this is to optimize the storage heat content at a certain level at the end of a 24h optimization period; the best solution is 50% of the maximum storage capacity.
The developed model will help the dispatchers to operate the plant in an optimal way on the day ahead, with a forecasted heat demand and forecasted APX prices.

2.2. Structure of this report

Chapter 1 is a description of the master assignment.

The summary of this report is given in chapter 2.

In chapter 3 the principle of the gas-expansion facility is presented. The NRE system is described and the chapter ends with a short introduction of linear-programming.

The gas-expansion system is analyzed using a mixed integer linear program model. The setup of this mathematical model is presented in chapter 4.

In chapter 5 the data of the co-generation units, boiler, storage, and gas-expansion machines is presented. Secondly the data of the APX and gas flow is discussed. Also the optimization period is shortly evaluated.

To know the capabilities of the system, reference cases are needed. It will be discussed which period, which storage strategy and what kind of AXP form will give the best solution for this project. Therefore several case studies are studied in chapter 6.

Finally this report will end with the conclusions and recommendations in chapter 7.

3. Introduction

E.ON Benelux is concentrating on production and delivery of electricity and heat to energy companies and energy-intensive industries in the Netherlands and Belgium. The company was established in 1941 and has been part of E.ON Energy AG since 2000. E.ON Benelux's power stations with a total electricity capacity of 1,850 MW and total thermal capacity of 750 MW, are located in the province of South Holland. The company has approximately 600 personnel and is based in Rotterdam. The production portfolio conists of six gas fired plants, located at Rotterdam (2x), The Hague, Leiden, Delft and Maasvlakte-Rotterdam, and two coal fired plants located at Maasvlakte-Rotterdam. The electricity produced at these six locations is supplied to end-customers via the high voltage grid and the distribution companies.

In addition, the power stations in Rotterdam, The Hague and Leiden supply heating for homes and greenhouses and the gasfired plant at Maasvlakte-Rotterdam produce steam to a industrial plant. Meanwhile the RoCa3 combined heat and power plant supplies greenhouse operators with carbon dioxide (CO_2) in cooled and compressed flue-gases.

With the purchase of NRE Energy in 2005, E.ON also bought some small projects. One of these projects is the gas-expansion installation located at Kanaaldijk Noord in Eindhoven. The Netherlands have a deregulated electricity market. Within this deregulated electricity market, an important trade platform is active called the Amsterdam Power eXchange (APX). E.ON wants to know the opportunities for this gas-expansion facility based on this trade platform. The electricity prices vary from hour to hour and during certain hours it is more profitable to produce the electricity than other hours. Therefore E.ON is looking for a model which is able to simulate this system. This model must optimize the gas-expansion system whilst gaining maximum flexibility of electricity production.

The first thought was to use PowrSym3 (see M.Sc. Assignment) for this optimization, because of the experience within E.ON. After studying literature a own model was build using Mixed Integer Linear-programming (MILP) technology. For short period optimization this model is prefered above the PowrSym3 model, because of the speed to solve the problem. The flexibility to change the model in the (MI)LP technology is also a big advantage above PowrSym3.

The gas-expansion system is therefore analyzed, using a mixed integer linear program model to calculate the optimal operational strategy for the plants and the storage. Optimal operational strategy means that the model minimize the operational cost and take into account all operational restrictions, interactions between assets, and market prices. All calculations are based on deterministic cases. This means that the gas flow and spot prices are fully known, which is the case for historical data.
The optimization is done for each day throughout the whole year. The sum of the objective functions, the total cost of running the system over one year, is compared with reference cases.

Comparing the different load duration curves or other graphics one can see whether the system is performing better than the other cases. Doing this one can determine which optimization staregy is the best solution for this facility.

First a description will be given of what a gas-expansion system is and how it works.

3.1. Power generation using the expansion of natural gas

The Netherlands has a considerable supply of natural gas which is put to a variety of uses, both domestic and industrial. Gastransportservices, the national gas transport company, transports this gas through main and regional grids to the local utility companies. These local companies then distribute it among the end-users. The main grid operates in a pressure range of 4.0 to 6.2 MPa and the regional grids between 1.6 and 4.0 MPa. However, the local utility companies distribute the gas to end users at a maximum pressure of 0.8 MPa. The necessary pressure drops are generally carried out by Gastransportservices in a conventional reduction line. An alternative way to achieve the required pressure drop, whilst also generating electricity, is to use a gas-expansion turbine driving a generator. For this the gas must be pre-heated. An option for the production of this heat is to use a gas engine cogeneration unit.

3.2. The Principle

Figure 1 gives a schematic diagram of the installation. Natural gas is taken from the regional grid at a pressure of 4 MPa (40 bar), a temperature of 4 to 12°C and a flow varying, in our case, between 4,000 and 60,000 Nm^3/h.

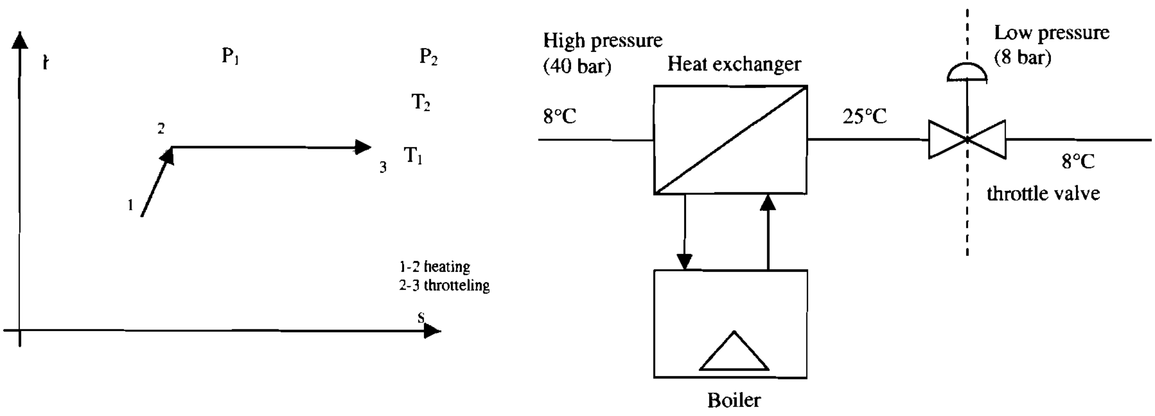

Figure 1: Schematic diagram of gas pressure reduction using a throttle valve [BAK97]

Standard, gas pressure reduction is accomplished in throttle-valves, where the isenthalpic expansion takes place without producing any energy. Most gases cool during the expansion (Joule-Thompson effect). The temperature drop in natural gas is approx. 4.5-6 K per 1 MPa, depending on gas composition and state. The gas must be pre-heated before the expansion to ensure that no liquid or solid phase condenses at the output temperature. When an expansion turbine driving a generator is used in place of the throttle valve, the energy in the gas can be used to produce electricity. The work the gas performs is gained from its internal energy (enthalpy) and the gas cools rapidly in the turbine. The temperature drop in the expansion turbine is approx. 15-20 K per 1 MPa of pressure drop in transmitting stations from transit pipeline depending on gas composition and state, and on the turbine's isentropic efficiency.

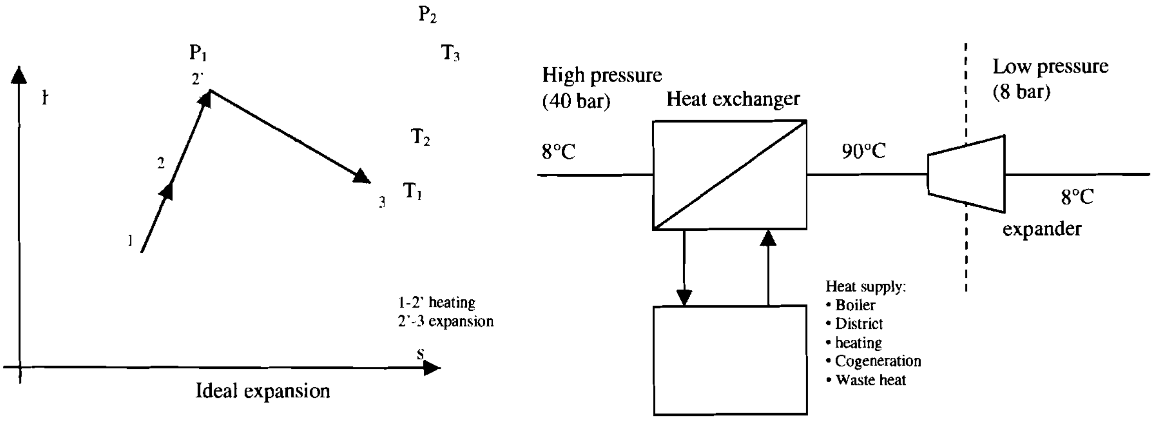

Figure 2: Schematic diagram of the gas-expansion and CHP process [BAK97]

When using an expansion turbine, gas outlet temperature must remain above hydrate zone and dew point. This means the gas must be pre-heated before it enters the turbine to temperatures higher than when using throttle valves, usually to 328 – 358 K (55-85°C). Reliability of the pressure regulating and reduction stations must be assured, and therefore the expansion turbines are installed parallel to existing conventional pressure reducing valves.

Expansion turbines are relatively small and compact, and are usually coupled with a generator in one power pack. The power output can be between hundreds of kW to several MW. Radial expansion turbines are, based on natural gas pressure drop, either one- or two-pressure stage constructions. A regulation valve is used to control the flow-rate of the gas; the used turbines, are high-speed with variable wheel speed (for smaller turbines with power output of hundreds of kW this is as much as 40.000 rpm), and therefore the produced alternate current must be converted to 50Hz in frequency converter. An alternate configuration is often encountered where a turbine with fixed shaft speed is used, connected to a gearbox that reduces it to 3.000 rpm to produce alternate current directly at 50 Hz. The waste heat produced in the system (generator, gearbox, frequency converter) is used to pre-heat the gas. An extensive detailed description of gas-expansion systems can be found in [BAK97], [BER82], [POZ04], [MIR86], [MIR88], [MIR89a], [MIR89b], [LEH01] and [VER90].

3.3. Original design of the NRE gas-expansion system

The original design of the NRE Energy gas-expansion system existed of two co-generation units, a heat boiler, two gas-expanders, emergency coolers and heat storage of $100m^3$. Figure 3 shows the original design of the NRE gas-expansion installation. In the original design the heat storage was somewhat bigger than usual for this kind of systems. The basic idea was to produce electricity during peak hours, store the heat obtained from the two co-generation units and discharge it during the hours when the price is low. Only the heat needed by the gas-expanders is continuously high, the maximum flow through the main gas street is 60.000 m^3 while the maximum capacity of the two gas-expanders together is 30.000 m^3. Because the system was autonomic locally controlled, which means that when the gasexpanders need heat, the co-generation units start automatically. In case these units are not available the heat is provided by the boiler. This means that the co-generation units are running most of the time to supply the heat. In case of high market prices the machines were switched on manually remotely.

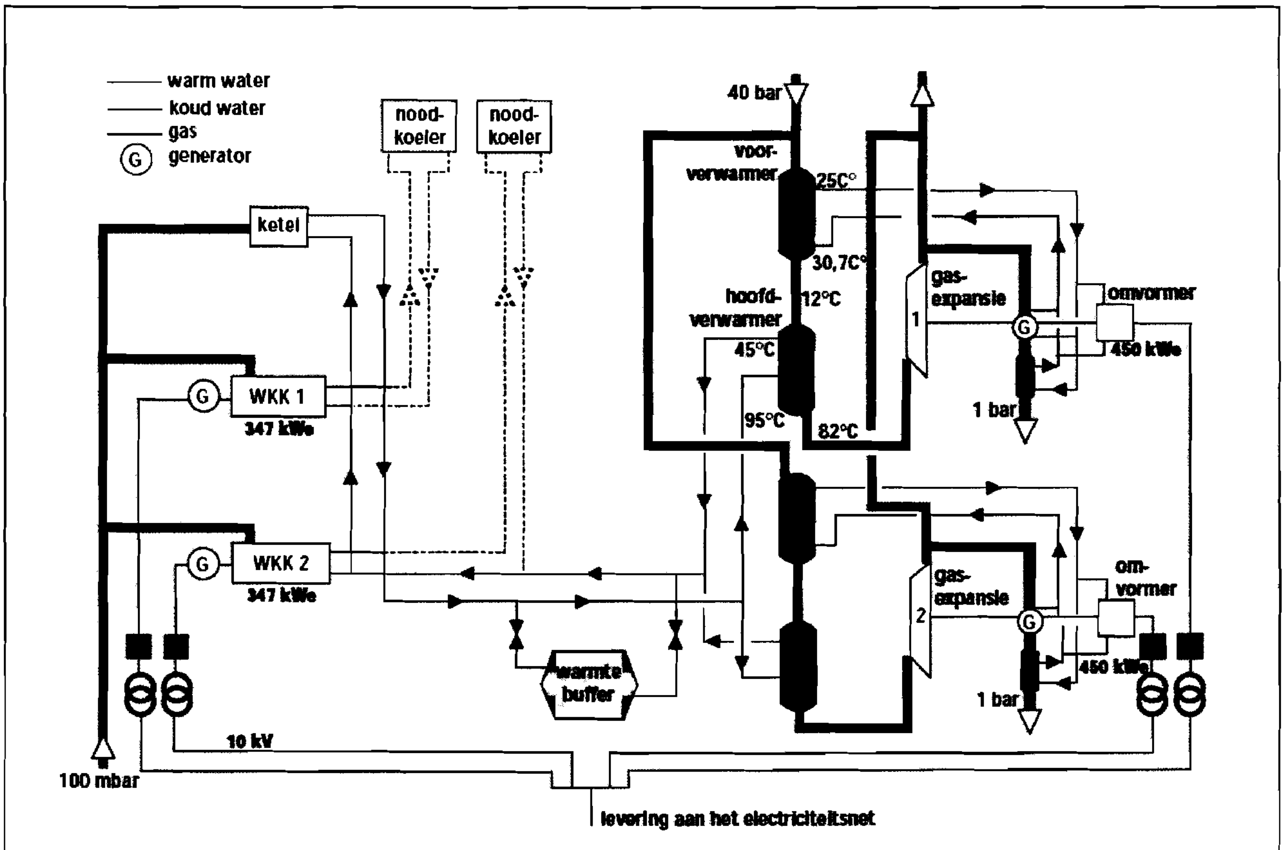

Figure 3: Original design of the NRE gas-expansion system [PTI00]

The CHP plants or co-generation units (in figure 3. called wkk) used in these kind of small facilities are existing of a gas engine. These gas engines operate mostly with only one working point, in bigger systems these machines can vary the power output between a P_{min} and a P_{max}. In this facility the co-generation units produce a fixed amound of electricity and heat. All equipment that can produce heat around these gas engines is used. Heat from exhausted, carter, oil-cooler etc. The efficiency of these engines in combination of producing electicity and heat is approx. 80%

Storage (buffer). The storage in this case is a hot water accumulator, of a simple design. Good insulated so there is little heat loss. For this system as described in this thesis the efficiency is approx. 99.5 %

Boiler (ketel): A boiler is used to provide the heat in case the production of the co-generation untis is not enough, or in case there is no production at all (back-up). The efficiency of a boiler is some where between the 85 and 95 %

Emergency cooler or heat dump facility (noodkoeler), will be activated when the return temperature of the system is getting to high. A heat dump facility is nothing more than a radiator with fans. The dimension of this cooler is big enough to dump the heat of all co-generation units can produce together.

3.4. Extension of the system

The NRE Company owned several co-generation units. These machines were placed under contracts at greenhouses or buildings to supply heat and electricity. At the end of 2004 some of these contracts stopped. NRE had the opportunity to sell the co-generation units, or to place them somewhere else. They chose the latter option, and the co-generation units were placed at the gas-expansion installation. The system was growing during this period from 2 to 6 co-generation units. The basic idea was still to produce electricity during day time, store the heat and use this heat during night time. NRE calculated that therefore a bigger storage was needed. Finally a 300 m^3 storage was placed.
To decide whether the extension is justified the standard literature doesn't give an answer. With the model described in this report an answer can be given for this question.
Before the mathematical model is presented, a short discussion of literature search is given.

3.5. Literature search

This particular gas-expansion facility with 6 co-generation untis, boiler, storage tank and cooling tower has not been covered in the literature. Also the approach of using hourly variation of the market price for a whole year is not done often in studies. Most literature describes large CHP plants and heat networks. When the literature is describing a small CHP plant the model is dedicated to that system. In this chapter the literature which was usefull for this thesis is shortly described.

Optimal operation of a co-generation unit with a heat-storage tank is studied in [ITO92] and [ITO94]. A combination of dynamic programming and mixed integer linear-programming is applied. The study covers 12 representative days for the whole year, and a winter and a summer price for the electricity is used, i.e. the optimization is not performed with respect to hourly variation in the price of electricity. The optimization is based on a deterministic situation. In [GUS91], as well as in [GUS92] and [GUS93] a CHP-network is studied using (mixed integer) linear-programming. The analysis model used is based on linear-programming, the time division is the 12 months of the year, with each month sub-divided into a high and a low electricity price period. The analysis is based on a deterministic situation. [RAV94] also deals with CHP and storage in a district-heating network. The model used is based on dynamic programming and includes some quite sophisticated modelling of start-and-stop costs for the various plants considered. All the units including the storage exist, i.e. no new investments are considered. The planning horizon is 48h and the situation is fully deterministic. Optimizing a district heating system based on linear-programming is discussed in [BEN94], a planning horizon of 48h is considered. In [HEA05] the impact of thermal storage on the operational behavior of CHP is studied. A planning horizon of 24h is used, and the storage has a heat content of 50% of its maximum capacity. The author of [HEN98] analyses a municipal energy system with heat storage among other things, using linear-programming. In the paper, typical winter weekdays with six time steps are used along with certain peak days and a rougher time-step division for the remainder of the year. The optimization is based on a deterministic situation. Long-term optimization of co-generation systems is studied in [THO05]. A combination of mixed integer linear-programming and Lagrangian relaxation is used. In [ROL04] a system a CHP plant and district heating in a deregulated electricity market is considered. The model used in this paper is optimizing over 24h. The analysis is done with deterministic data as well with forecast data. In [ZHA98] the optimal operation of a CHP with storage is discussed using a non-linear optimization model. A planning horizon of 48h is used.

In none of the above mentioned studies a cooling tower is involved. The maximum of the storage capacity is never reached or no issue at all. The system discussed in this report has a large number of heat production capacities and a small heat consumption. This means that a lot of heat can and need to be stored. When optimizing this system with a small storage capacity it is also important to have a heat dump possibility. Otherwise when the storage is full, the co-generation units will be stopped by the optimization program, which will have an effect on the flexibility of the system.

3.6. Linear-programming

In this chapter a short introduction of the theory of linear-programming will be discussed. For more and detailed information of the theory of Linear and Integer Programming the reader is refered to [SCH98] and [MOM01]. The text in this chapter is from the wikipedia site on linear-programming and can be found at http://en.wikipedia.org/wiki/linear_programming.

In mathematics, Linear-programming (LP) problems are optimization problems in which the objective function and the constraints are all linear. Linear-programming is an important field of optimization for several reasons. Many practical problems in operational research can be expressed as linear-programming problems. Certain special cases of linear-programming, such as network flow problems and multicommodity flow problems are considered important enough to have generated much research on specialized algorithms for their solution. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems. Historically, ideas from linear-programming have inspired many of the central concepts of optimization theory, such as duality, decomposition, and the importance of convexity and its generalizations. Likewise, linear-programming is heavily used in micro economics and business management, either to maximize the income or minimize the costs of a production scheme.

A Linear Program (LP) is a problem that can be expressed as follows (the so-called Standard Form):

$$\begin{aligned} &\text{Minimize } cx && (1)\\ &\text{Subject to } Ax = b && (2)\\ &\qquad\qquad x \geq 0 && (3) \end{aligned}$$

Where x is the vector of variables to be solved for, A is a matrix of known coefficients, and c and b are vectors of known coefficients. The expression "cx" is called the objective function, and the equations "Ax=b" are called the constraints. All these entities must have consistent dimensions, of course, and you can add "transpose" symbols to taste. The matrix A is generally not square, hence one doesn't solve a LP by just inverting A. Usually A has more columns than rows, and Ax=b is therefore quite likely to be under-determined, leaving great latitude in the choice of x with which to minimize cx.

Although all linear programs can be put into the Standard Form, in practice it may not be necessary to do so. For example, although the Standard Form requires all variables to be non-negative, most good LP software allows general bounds $l \leq x \leq u$, where l and u are vectors of known lower and upper bounds. Individual elements of these bound vectors can even be infinity and/or minus-infinity. This allows a variable to be without an explicit upper or lower bound, although of course the constraints in the A-matrix will need to put implied limits on the variable or else the problem may have no finite solution. Similarly, good software allows $b1 \leq Ax \leq b2$ for arbitrary b1, b2; the user need not hide inequality constraints by the inclusion of explicit "slack" variables, nor write $Ax \geq b1$ and $Ax \leq b2$ as two separate constraints. Also, LP software can handle maximization problems just as easily as minimization (in effect, the vector c is just multiplied by -1).

The importance of linear-programming derives in part from its many applications and in part from the existence of good general-purpose techniques for finding optimal solutions. These techniques take as input only a LP in the above Standard Form, and determine a solution without reference to any information concerning the LP's origins or special structure. They are fast and reliable over a substantial range of problem sizes and applications.

Two families of solution techniques are in wide use today. Both find a progressively improving series of trial solutions, until a solution is reached that satisfies the conditions for an optimum. Simplex methods, introduced by Dantzig about 50 years ago, found "basic" solutions computed by fixing enough of the variables at their bounds to reduce the constraints Ax = b to a square system, which can be solved for unique values of the remaining variables. Basic solutions represent extreme boundary points of the feasible region defined by Ax = b, $x \geq 0$, and the simplex method can be viewed as moving from one such point to another along the edges of the boundary. Barrier or interior-point methods, by contrast, visit points within the interior of the feasible region.

The related problem of integer programming (or integer linear-programming, strictly speaking) requires some or all of the variables to take integer (whole number) values. Integer programs (IPs) often have the advantage of being more realistic than LPs, but the disadvantage of being much harder to solve. The most widely used general-purpose techniques for solving IPs use the solutions to a series of LPs to manage the search for integer solutions and to prove optimality. Thus most IP software is built upon LP software.

Linear and integer programming have proved to be valuable for modelling many and diverse types of problems in planning, routing, scheduling, assignment, and design. Industries that make use of LP and its extensions include transportation, energy, telecommunications, and manufacturing of many kinds.

Standard Form is the usual and most intuitive form of describing a linear-programming problem. It consists of the following three parts:

- A linear function to be minimized

$$\text{e.g. minimize } c_1x_1 + c_2x_2 \tag{4}$$

- Problem constraints of the following form

$$\text{e.g. } a_{11}x_1 + a_{12}x_2 \leq b_1 \tag{5}$$
$$a_{21}x_1 + a_{22}x_2 \leq b_2 \tag{6}$$
$$a_{31}x_1 + a_{32}x_2 \leq b_3 \tag{7}$$

- Non-negative variables

$$\text{e.g. } x_1 \geq 0 \tag{8}$$
$$x_2 \geq 0 \tag{9}$$

The problem is usually expressed in matrix form, and then becomes:

$$\text{Minimize}\, C^T x \tag{10}$$
$$\text{subject to } Ax \leq b,\, x \geq 0 \tag{11}$$

Other forms, such as maximization problems, problems with constraints on alternative forms, as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form. If the unknown variables are all required to be integers, then the problem is called an integer programming (IP) or integer linear-programming (ILP) problem. In contrast to linear-programming, which can be solved efficiently in the worst case, integer programming problems are in the worst case undividable, and in many practical situations (those with bounded variables) NP-hard[*]. 0-1 integer programming is the special case of integer programming where variables are required to be 0 or 1 (rather than arbitrary integers). If only some of the unknown variables are required to be integers, then the problem is called a mixed integer programming (MIP) problem. These are generally also NP-hard.

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[*] NP stands for Nondeterministic Polynomial time, polynomial time refers to the computation time of a problem.

3.7. Implicit assumptions in linear-programming

As mentioned in [MOM01], one needs to make some notes by using MILP. There are certain assumptions utilized in linear-programming models that are intended to be only idealized representations of real problems. These approximations simplify the real-life problem to make it tractable. Adding too much detail and precision can make the model too unwieldy for useful analysis. All that is needed is that there is a reasonable high correlation between the prediction of the model and what would actually happen in reality.

4. Mathematical model

Since the operational policy of the plant has a considerable influence on its economics, the mathematical optimization method is adopted here. The plants operationally policy is determined optimally so as to minimize the operational cost for each daily heat demand. Therefore the objective function of the model is to minimize the operational cost of this co-generation unit system over all time steps. For each time step the model has bounds and constraints that need to be satisfied. A simplified scheme of the gas-expansion system is given in Figure 4.

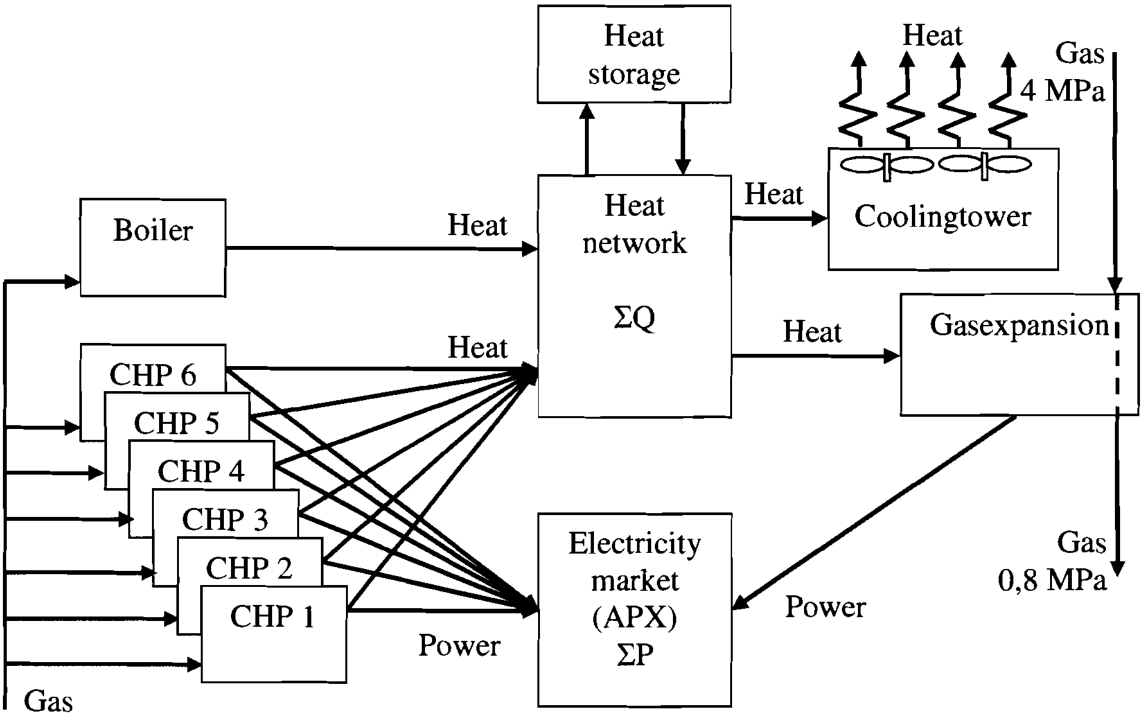

Figure 4: Simplified scheme of the gas-expansion system

4.1. Model:

First a time step definition must be given. Each k^{th} time step t_k is depending on time step zero t_0 and a multiplier k times the time step interval Δt. The time step interval Δt is the evaluated period N divided by the steps K to be taken within the evaluation period. In equation form:

$$t_k = t_0 + k\Delta t \quad (k = 1,..,K) \tag{12}$$

Where

$$\Delta t = \frac{N}{K} \tag{13}$$

The performance characteristics of equipment of each piece of equipment are considered as constraints in this optimization problem.

4.1.1. Combined Heat and Power Plant (CHP)

For each time step the performance characteristics of the gas engine generator can be expressed by linear relationships among generated electric power $P_{CHPn}(k)$, fuel consumption $F_{CHPn}(k)$ and produced heat $Q_{CHPn}(k)$ (see figure 4). In this case the equations for fuel and heat of the CHP are written as a function of electric power. The coefficients p and q are parameters related to the performance characteristics and are determined, respectively as gradient and offset. The delta sign $\delta_{CHPn}(k)$ is used to simulate if a CHP is on or off (committed or not).

$$\left.\begin{aligned} &F_{CHPn}(k) = (p_{1CHPn}P_{CHPn} + q_{1CHPn}) \cdot \delta_{CHPn}(k) \\ &Q_{CHPn}(k) = (p_{2CHPn}P_{CHPn} + q_{2CHPn}) \cdot \delta_{CHPn}(k) \\ &\delta_{CHPn}(k) \in \{0,1\} \end{aligned}\right\} (n = 1,\dots,N_{CHP}; k = 1,\dots,K) \tag{14}$$

The power of each CHP is limited between a minimum and a maximum, for each time step. This can be expressed as equation (15):

$$P_{CHPn\_\min}(k) \le P_{CHPn}(k) \le P_{CHPn\_\max}(k) \tag{15}$$

4.1.2. Heat Storage Tank (ST)

The storage tank can be expressed, as shown in figure 4, by relationships among heat in $Q_{ST}^{in}(k)$, heat out $Q_{ST}^{out}(k)$ and previous heat in storage $Q_{ST}(k-1)$. The storage efficiency η_{ST} needs also to be taken into account.

$$Q_{ST}(k) = Q_{ST}^{in}(k) - Q_{ST}^{out}(k) + \eta_{ST} \cdot Q_{ST}(k-1)\}(k = 1,\dots,K) \tag{16}$$

The storage capacity is limited between a minimum and a maximum, for each time step. This can be expressed as equation (17):

$$Q_{ST\_\min}(k) \le Q_{ST}(k) \le Q_{ST\_\max}(k) \tag{17}$$

4.1.3. Gasexpanders (GE)

The heat consumption $Q_{GEn}(k)$ and power production $P_{GEn}(k)$ can be expressed as a function of gas flow $\phi_{gas}(k)$. The coefficients p and q are parameters related to the performance characteristics and are determined, respectively as gradient and offset. The heat consumption and power production are approximated by linear functions, see paragraph 5.1

$$\left.\begin{aligned} &Q_{GEn}(k) = (p_{1GEn} \cdot \phi_{gas}(k) + q_{1GEn}) \cdot \delta_{GEn}(k) \\ &P_{GEn}(k) = (p_{2GEn} \cdot \phi_{gas}(k) + q_{2GEn}) \cdot \delta_{GEn}(k) \\ &\delta_{GEn}(k) \in \{0,1\} \end{aligned}\right\} (k = 1,\dots,K) \tag{18}$$

4.1.4. Boiler (B)

The fuel $F_{Bn}(k)$ used by the boiler is a linear relation between the heat $Q_{Bn}(k)$ that's being produced and its efficiency $\eta_{Bn}(k)$. The boiler can produce heat between zero and his maximum $Q_{Bn\max}$.

$$\left.\begin{array}{l} F_{Bn}(k) = \dfrac{3.6 \cdot Q_{Bn}(k)}{\eta_{Bn}} \\ 0 \le Q_{Bn}(k) \le Q_{Bn\max} \end{array}\right\} (k = 1, \ldots, K) \tag{19}$$

4.1.5. Electricity (P)

As shown in figure 4 the expression of electricity $P_{sell}(k)$ that can be sold to the market is the total power that is produced by the CHP plants $P_{CHP}(k)$ and the power produced by the gas expander $P_{GE}(k)$.

$$\left.\begin{array}{l} P_{sell}(k) = P_{CHP}(k) + P_{GE}(k) \\ P_{CHP}(k) = \sum_{n=1}^{N_{CHP}} P_{CHPn}(k) \cdot \delta_{CHPn} \\ P_{GE}(k) = \sum_{n=1}^{N_{GE}} P_{GEn}(k) \cdot \delta_{GEn} \end{array}\right\} (k = 1, \ldots, K) \tag{20}$$

4.1.6. Gas Fuel (F)

The amount of gas fuel used at each time step is the sum of gas fuel used by all CHP $F_{CHP}(k)$ and the fuel used by the boiler $F_B(k)$.

$$\left.\begin{array}{l} F_{CHP}(k) = \sum_{n=1}^{N_{CHP}} F_{CHPn}(k) \\ F_B(k) = \sum_{n=1}^{N_B} F_{Bn}(k) \end{array}\right\} (k = 1, \ldots, K) \tag{21}$$

4.1.7. Cooling tower (CT)

As mentioned in chapter 3.5 it is important to implement a cooling tower in this case. Otherwise the model is not working properly. It is possible that the storage reach the end of storage capacity at a certain time. If the model is not able to dump the heat, than the system will stop one or more co-generation units. This has effect on the flexibility of the system. The method which is used here has no influence on the object function. The total heat $Q_{tot}(k)$ is the sum of heat produced with the co-generation units $Q_{CHP}(k)$ plus the heat in storage $Q_{ST}(k-1)$.

$$Q_{tot}(k)=Q_{CHP}(k)+Q_{ST}(k-1)\ \ (k=1,...,K) \tag{22}$$

The total heat $Q_{tot}(k)$ can also be expressed by the sum of two dummy heat coefficients, dummy heat in storage $Q_{STd}(k)$ and dummy heat dumped with the cooling tower $Q_{CTd}(k)$.

$$Q_{tot}(k)=Q_{STd}(k)+Q_{CTd}(k) \tag{23}$$

Two extra coefficients are needed to make the system working. The first coefficient $\delta_{CTd}(k)$ is simulating on and off status of the dummy coefficient $Q_{CTd}(k)$ the other coefficient $\delta_{corr}(k)$ is to bring a correction factor $corr(k)$ into account. Only one of the two dummy coefficients can be selected, this is done by making the sum of the two on-off switches equal to 1.

$$\left.\begin{array}{l} Q_{CT}(k)=Q_{CTd}(k)\delta_{CTd}(k)-Q_{ST\max}(k)+corr(k)\delta_{corr}(k) \\ 0\le Q_{STd}\le Q_{ST\max} \\ 0\le Q_{CT}\le Q_{CT\max} \\ Q_{ST\max}\le Q_{CTd}\le Q_{ST\max}+Q_{CT\max} \\ corr(k)=Q_{ST\max} \\ \delta_{CTd}(k)+\delta_{corr}(k)=1 \\ \delta_{CTd}(k)\in\{0,1\} \\ \delta_{corr}(k)\in\{0,1\} \end{array}\right\}(k=1,...,K) \tag{24}$$

4.1.8. Heat balance

A balance of supply and demand must be achieved at each point of time. The heat balance therefore exists of a part of producing which is equal to a part of consuming, storing or dumping. Heat that is being produced with the co-generation units, boiler and from storage is equal the heat that is consumed by the gas expander, put into the storage and dumped. This is shown in equations below:

$$Q_{CHP}(k)+Q_B(k)+Q_{ST}^{out}(k)=Q_{GE}(k)+Q_{CT}(k)+Q_{ST}^{in}(k) \tag{25}$$

$$Q_{CHP}(k)=\sum_{n=1}^{N_{CHP}}Q_{CHPn}(k) \tag{26}$$

$$Q_B(k)=\sum_{n=1}^{N_B}Q_{Bn}(k) \tag{27}$$

$$Q_{GE}(k)=\sum_{n=1}^{N_{GE}}Q_{GEn}(k) \tag{28}$$

4.1.9. Start and stop cost

In this case the stop costs are neglected, stopping co-generation units of this size is instantly after giving the stop signal. Starting costs need to be taken into account to avoid unnecessary on and off switching of the co-generation units. A co-generation unit needs time to heat up and for this process an amount of gas is consumed. Therefore it is necessary to implement a start cost function. To realize start cost one need to know when the co-generation unit is switched on. This can be done with the equation given below:

$$\left.\begin{array}{l} CHPstart_{CHPn}(k) >= \delta_{CHPn}(k) - \delta_{CHPn}(k-1) \\ \delta_{CHPn}(k)\in\{0,1\} \end{array}\right\}(k=1,\ldots,K) \tag{29}$$

The start costs are implemented in the objective function and can be found in the next chapter

4.1.10. Objective function

The objective function of the model, called J, is to minimize the operational cost of this gas-expansion facility. As described before, are the co-generation units consuming gas to produce heat and electricity. The electricity can be sold to the market. Starting a co-generation unit means a start costs is incurred. The boiler is using gas for producing heat. Therefore the objective function is split up in four parts, called J_1 to J_4.

$$J=\min\sum_{k=1}^{K}\{J_1(k)+J_2(k)-J_3(k)+J_4(k)\}\Delta t, \tag{30}$$

The first part is the gas consumption of the chp plants multiplied with gasprice f_{gas}:

$$J_1(k)=f_{gas}\cdot F_{CHP}(k) \tag{31}$$

The second part is the gas consumption of the boiler multiplied with gasprice f_{gas} :

$$J_2(k) = f_{gas} \cdot F_B(k) \tag{32}$$

The third part is the power generated $P_{sell}(k)$ with the chp plants multiplied with the market price $APX(k)$.

$$J_3(k) = P_{sell}(k) \cdot APX(k) \tag{33}$$

The last part is the startcost of the chp plants.

$$J_4(k) = \text{startcost}_{CHPn} \cdot CHPstart_{CHPn}(k) \tag{34}$$

Fill in the equations (31) – (34), in equation (30):

$$J = \min \sum_{k=1}^{K} \left\{ \begin{array}{l} f_{gas} \cdot F_{CHP}(k) + \\ f_{gas} \cdot F_B(k) + \\ P_{sell}(k) \cdot APX(k) - \\ \text{startcost}_{CHPn} \cdot CHPstart_{CHPn}(k) \end{array} \right\} \Delta t, \tag{35}$$

The basics of the equations (14) – (16) and (20) – (21) can be found in [ITO92]

4.2. Objective function in standard form

As discussed in chapter 3.6 it is not necessary to write the objective function in a standard form. To prove that it is possible to write the objective function in a standard form, it is done in this chapter. For small problems it is easy, but for large problems it is difficult to oversee the problem when it is written in a matrix notation. To work to a standard form only one time step will be taken and P_{CHP} = constant. The objective function of equation (30) is used, only the part with the start cost (J_4) is left out.

Using equation (31) and filling in F_{CHP} from equation (21), results in:

$$\begin{aligned} J_1(k) &= f_{gas} \cdot F_{CHP}(k) \\ &= f_{gas} \cdot \sum_{n=1}^{N_{DG}} \left\{ \left(p_{1CHPn} P_{CHPn} + q_{1CHPn} \right) \cdot \delta_{CHPn}(k) \right\} \end{aligned} \tag{36}$$

Equation (32) and filling in F_B form equation (21), results in:

$$\begin{aligned} J_2(k) &= f_{gas} \cdot F_B(k) \\ &= f_{gas} \cdot \sum_{n=1}^{N_B} \left\{ \frac{3.6 \cdot Q_{Bn}(k)}{\eta_{Bn}} \cdot \delta_{Bn}(k) \right\} \end{aligned} \tag{37}$$

Equation (33) and filling in equation (20), results in:

$$\begin{aligned} J_3(k) &= P_{sell}(k) \cdot APX(k) = \left(P_{CHP} + P_{GE} \right) \cdot APX(k) \\ &= \left(\sum_{n=1}^{N_{CHP}} \left\{ P_{CHPn} \cdot \delta_{CHPn}(k) \right\} + \sum_{n=1}^{N_{GE}} \left\{ P_{GEn} \cdot \delta_{GEn}(k) \right\} \right) \cdot APX(k) \end{aligned} \tag{38}$$

Using equation (30), and fill in the equations (36) – (38):

$$J = \min \sum_{k=1}^{K} \left\{ \begin{array}{l} f_{gas} \cdot \sum_{n=1}^{N_{CHP}} \left\{ \left(p_{1CHPn} P_{CHPn} + q_{1CHPn} \right) \cdot \delta_{CHPn}(k) \right\} \ldots \\ + f_{gas} \cdot \sum_{n=1}^{N_B} \left\{ \frac{3.6 \cdot Q_{Bn}(k)}{\eta_{Bn}} \cdot \delta_{Bn}(k) \right\} \ldots \\ - \left(\sum_{n=1}^{N_{CHP}} \left\{ P_{CHPn} \cdot \delta_{CHPn}(k) \right\} + \sum_{n=1}^{N_{GE}} \left\{ P_{GEn} \cdot \delta_{GEn}(k) \right\} \right) \cdot APX(k) \end{array} \right\} \Delta t \tag{39}$$

As mentioned before P_{CHP} = constant, then variable $q_{1CHPn} = 0$. The commit variable of the boiler and the gas-expanders can be left out. This results in:

$$J = \min \sum_{k=1}^{K} \left\{ \begin{array}{l} f_{gas} \cdot \sum_{n=1}^{N_{CHP}} \{ (p_{1CHPn} P_{CHPn}) \cdot \delta_{CHPn}(k) \} \ldots \\ + f_{gas} \cdot \dfrac{3.6 \cdot Q_B(k)}{\eta_B} \ldots \\ - \left(\sum_{n=1}^{N_{CHP}} \{ P_{CHPn} \cdot \delta_{CHPn}(k) \} + \sum_{n=1}^{N_{GE}} \{ P_{GE}(k) \} \right) \cdot APX(k) \end{array} \right\} \Delta t, \tag{40}$$

Writing out the sum signs, then equation (40) becomes equation (41)

$$J = \min \sum_{k=1}^{K} \left\{ \begin{array}{l} f_{gas} \cdot \left(p_{1,CHP1} P_{CHP1} \cdot \delta_{CHP1}(k) + \ldots + p_{1,CHP6} P_{CHP6} \cdot \delta_{CHP6}(k) \right) \\ + f_{gas} \cdot \dfrac{3.6 \cdot Q_B(k)}{\eta_B} \ldots \\ - \left(P_{CHP1} \cdot \delta_{CHP1}(k) + \ldots + P_{CHP6} \cdot \delta_{CHP6}(k) + P_{GE}(k) \right) \cdot APX(k) \end{array} \right\} \Delta t, \tag{41}$$

Combine the constants to a constant called c:

$$= \min \sum_{k=1}^{K} \left\{ \begin{array}{l} c_{CHP1} \cdot \delta_{CHP1}(k) + \ldots + c_{CHP6} \cdot \delta_{CHP6}(k) \\ + c_B Q_B(k) \ldots \\ - \left(c_{apx\,CHP1} \cdot \delta_{CHP1}(k) + \ldots + c_{apx\,CHP6} \cdot \delta_{CHP6}(k) + c_{apx} P_{GE}(k) \right) \end{array} \right\} \Delta t, \tag{42}$$

Re-order the constants, in that way that the variable will be used once in the equation:

$$= \min \sum_{k=1}^{K} \left\{ \begin{array}{l} \left(c_{CHP1} - c_{apx\,CHP1} \right) \cdot \delta_{CHP1}(k) + \ldots + \left(c_{CHP6} - c_{apx\,CHP6} \right) \delta_{CHP6}(k) \\ + c_B Q_B(k) - c_{apx} P_{GE}(k) \end{array} \right\} \Delta t, \tag{43}$$

Write equation (44) in a matrix notation:

$$= \min \sum_{k=1}^{K} \begin{bmatrix} c_{CHP1} - c_{apx\,CHP1} \\ c_{CHP2} - c_{apx\,CHP2} \\ c_{CHP3} - c_{apx\,CHP3} \\ c_{CHP4} - c_{apx\,CHP4} \\ c_{CHP5} - c_{apx\,CHP5} \\ c_{CHP6} - c_{apx\,CHP6} \\ c_B \\ c_{GE,APX} \end{bmatrix}^T \begin{bmatrix} \delta_{CHP1}(k) \\ \delta_{CHP2}(k) \\ \delta_{CHP3}(k) \\ \delta_{CHP4}(k) \\ \delta_{CHP5}(k) \\ \delta_{CHP6}(k) \\ QB \\ 1 \end{bmatrix} \Delta t = \min \sum_{k=0}^{K-1} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \Delta t \tag{44}$$

Simplify:

$$= \min \sum_{k=1}^{K} c^T x(k) \Delta t \bigg|_{K=1} \tag{45}$$

Then finally one gets equation (47) which is the Standard Form notation:

$$= \min c^T x \tag{46}$$

5. Optimization

The problem in previous chapters is defined as a linear model. In this chapter, equipment data is given which is used for all optimizations. The optimization is done using MatLab in combination with the program LP-Solve. In chapter 5.2 a short description will be given of how this combination works. In paragraph 5.3 and 5.4 the gas flow and market price are discussed. In the last paragraph the effect of the optimizing period is shown.

5.1. Equipment data

The data of the co-generation units is given in Table 1. As formulated in equations (14), (15) and (29) one need electrical power, thermal power, gas consumption and start cost. For this optimization the start cost of all units are kept the same and are assumed to be 10 Euro. The average gas consumption = 118,5 m3/h, startup time is approx. 15 - 20 min. With a gasprice of 6,9 Euro/GJ and a heatingvalue of the gas of 35,17 MJ/m3 the start up cost are approx. 10 Euro.

Table 1: Co-generation units data

CHP nr.	Brand and Type	Pe [kW]	Pth [kW]	Gascons. [m^3/h]	Startcost [EUR]
1	Lescogen 350	347	518	111,4	10
2	Lescogen 350	347	518	111,4	10
3	Nutec 400	400	595	128,8	10
4	Zantec 312	312	447	102	10
5	Nutec 400	400	561	128,8	10
6	Nutec 400	400	561	128,8	10

For the boiler, only the maximum thermal power and the efficiency rate are needed.

Table 2: Boiler data

Pth [kW]	η [%]
1170	91

The heat storage is simulated, as formulated in equations (16) and (17), with its volume and efficiency.

Table 3: Heat storage data

Volume [m3]	η [%]
300	99.5

The gasexpanders both have the same characteristics as shown in figure 5. Before these characteristics can be used, they need to be linearized. Because the graphs are not convex, they can therefore be linearized with only a straight line. This is a good approximation as shown in figure 5. The coefficients of this linear presentation are then used for the optimization. These coefficients are presented in table 4.

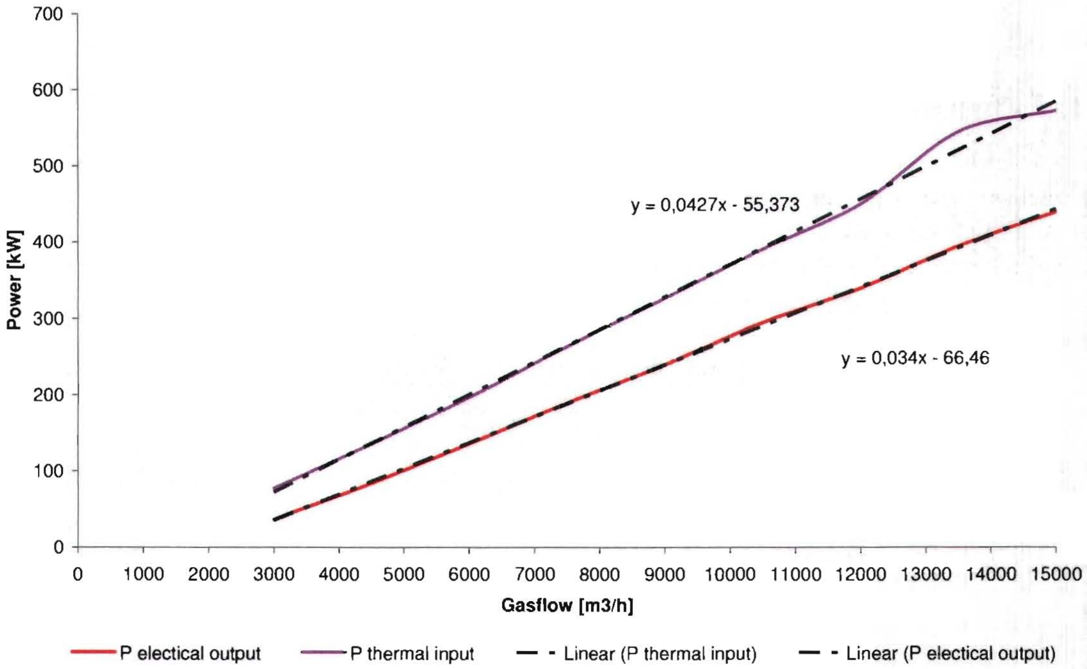

Figure 5: Piller MGT 400 gas flow as a function of (Power, Heat) [PIL00]

Table 4: Gas-expanders data

GE nr.	Brand and Type	Pe [kW]		Pth [kW]		Min. Gasflow	Max. Gasflow
		p1	q1	p2	q2		
1	Piller	0,034	-66,46	0,0427	-55,373	5.000	15.000
2	Piller	0,034	-66,46	0,0427	-55,373	15.000	30.000

5.2. Model in MatLab

In this paragraph a short description is given of how the mathematical model is implemented in MatLab. For solving the mixed integer linear problem MatLab uses the linear program engine LP-Solve. LP-Solve is a freeware program. For more detailed information of LP-Solve see http://groups.yahoo.com/group/lp_solve.

As shown in Figure 6 the m-file kanaaldijk.m is in fact the program from which all other m-files are called. First a basic initialization is done, and then the function write_lp_file.m is called. In this m-file all data is gathered and a LP-model is written (see Appendix C for a LP-model). From read_lp_file.m the LP-Model is called. In this m-file the LP-Solver is set and than the problem will be solved. Finally the (solved) matrix is transferred to graphics.m where a graphical presentation of the problem is given and the data is written to an excel-file where further analysis is possible. The model for 24h includes 672 constraints, 919 variables with 214 integers.

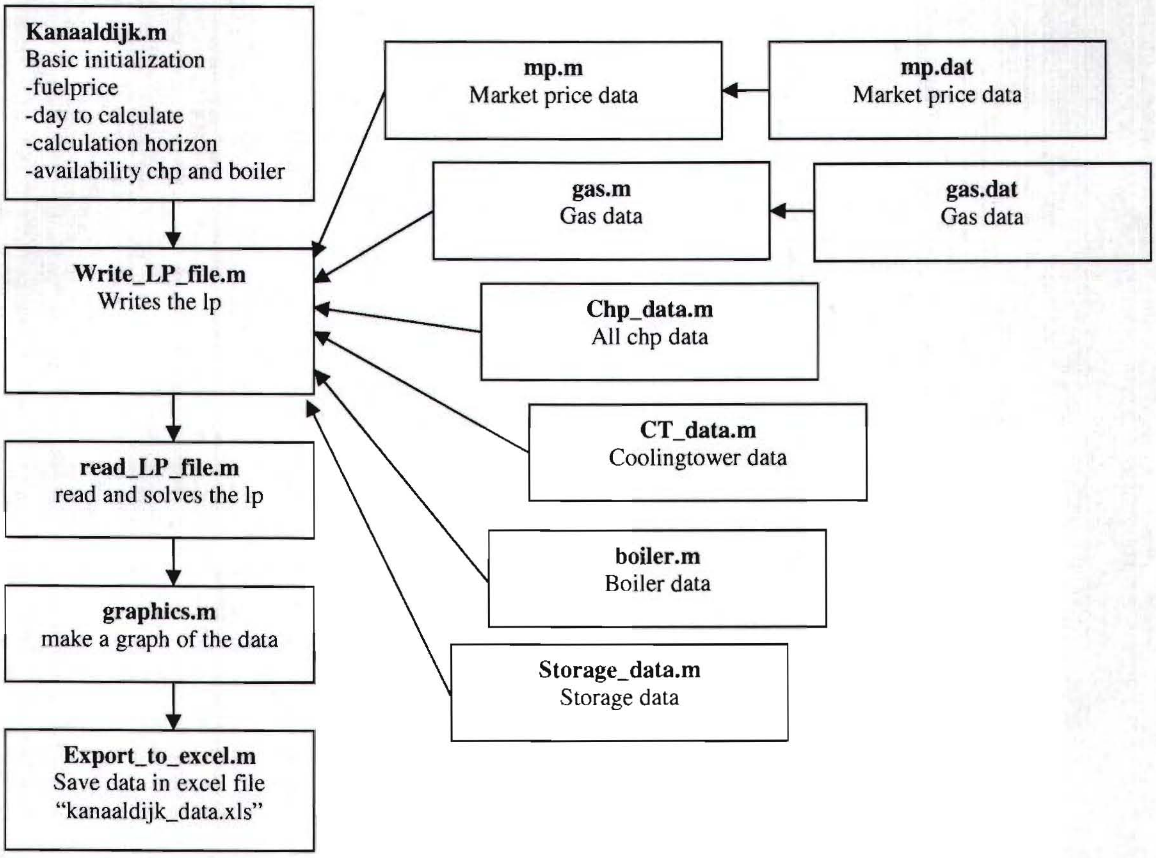

Figure 6: Overview of optimization program implemented in MatLab

5.3. Market price

As mentioned in chapter 3 E.ON wants to know the opportunities for this gas-expansion facility based on the APX market price. The electricity prices vary from hour to hour and during certain hours it is more profitable to produce the electricity than other hours. Using this market as a reference one knows the market value of this facility and if there will be electricity produced to sell to the market.

In Figure 7 the AXP market price of a random week is chosen. In this figure one can see the day and night pattern. This is important to know for optimization over a certain period. The first 36 hour period of this week is also given in Figure 8. The market price is given per hour, but it is also possible to split a day in two periods, named peak and off-peak period. This is also shown in Figure 8. A complete overview of the market price of 2005 is given in Appendix D. The market price data can be downloaded from the website www.apx.nl.

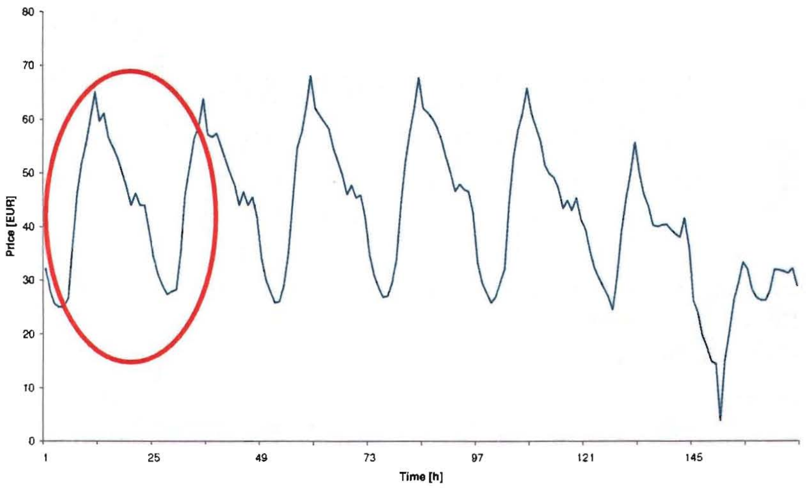

Figure 7: Market price pattern of a week

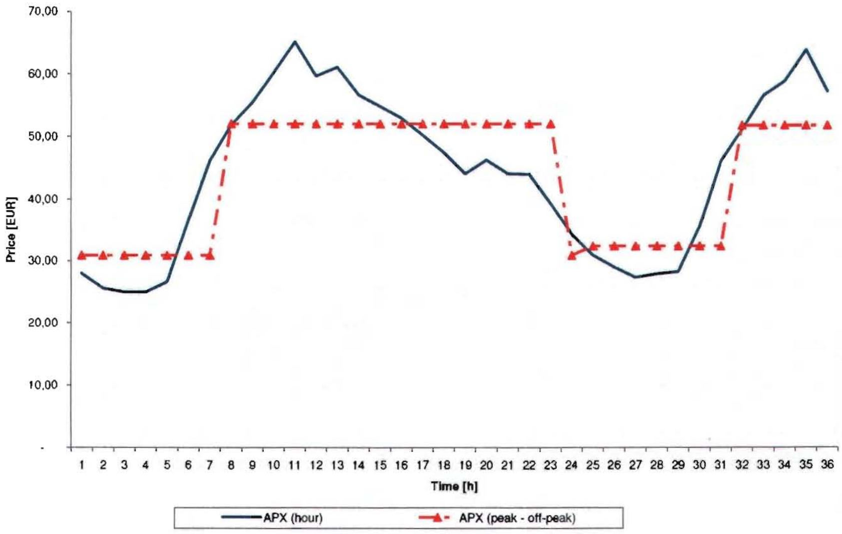

Figure 8: Market price pattern of 36h

5.4. Gas flow

In Figure 9 the gas flow of 2005 is given. The gas flow is passing the main gas street with a maximum capacity of 60.000 m^3/h. The maximum gas flow capacity the gas-expanders can handle together is 30.000m^3/h. In Figure 10 the gas flow of a random week is given. Also here the day and night pattern is seen. In Figure 11 a period of 36h is shown. In this figure you can see that during the night, from 23:00 – 06:00 the gas flow is low. So when an optimization is done for 24h it is clearly not the best optimization. Because, depending on strategy, no heat will be stored for the first hours of the next day. This heat will be produced with the boiler, because of low APX prices as shown in paragraph 5.3.
The gas flow of 2005 is also given in Appendix E.

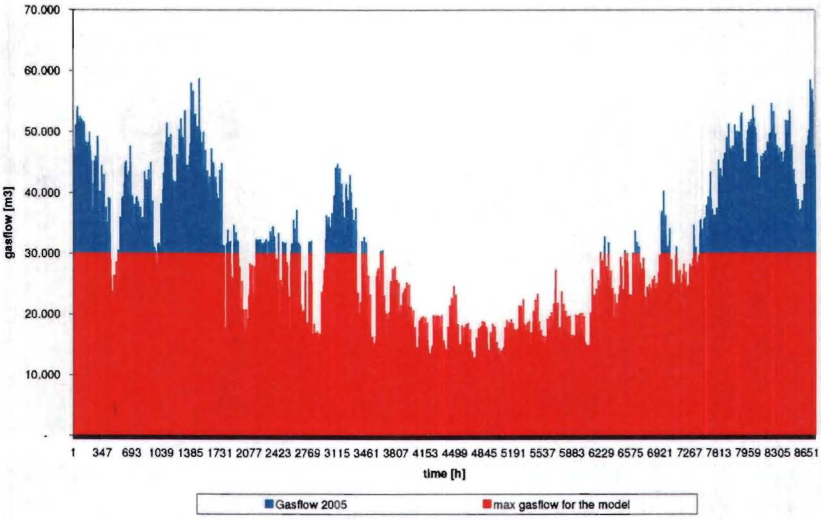

Figure 9: Gas flow of 2005

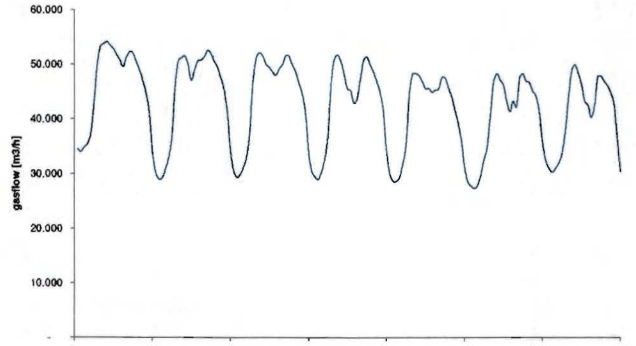

Figure 10: Gas flow pattern of a week

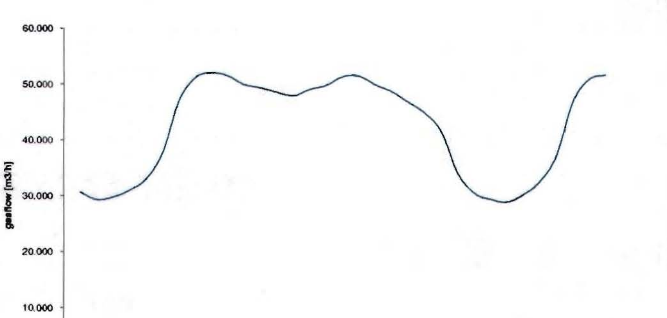

Figure 11: Gas flow pattern for 36h

5.5. Effect of optimization period

As described in chapter 3.5 most optimization models are optimizing over 24h. Some models are optimizing over 48h. In this report it is an item of investigation (see chapter 6.2) over which period the optimization must be done. In this chapter the optimization is done over 24, 36 and 48h for three days. This will show the effect of the different optimization periods.

In Figure 12 the optimization period is 24h. The end value of the storage and the status of the co-generation units will be transferred to the next day.

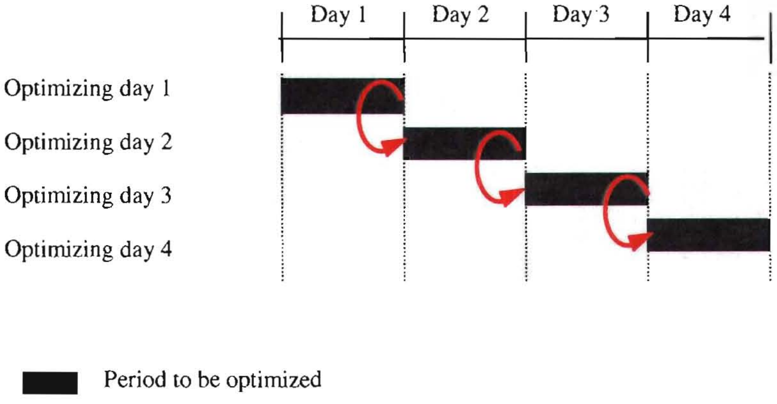

Figure 12: Illustration of optimizing 24h period

In Figure 13 the result of this optimization period is presented. On the left graphs of produced power with the co-generation units and the gas expander are presented, also the APX price is shown. On the right the heat graphs are given. On top the heat produced with the boiler is shown, second graph present the heat produced with the co-generation units, third graph is the heat that flows into the storage, fourth is the heat content of the storage, fifth graph is the heat that flows out of the storage, sixth graph is the heatdemand from the gas-expanders. This graph shows also how the heatdemand is fulfilled from the different sources. The last graph shows how much heat is dumped through the cooling tower. From the graphs one can read that during the day period the co-generation units are producing electricity and storing the heat. After the co-generation units stop producing, the heat will flow from the storage. In the last hour there is not enough heat in store and the boiler starts producing heat.

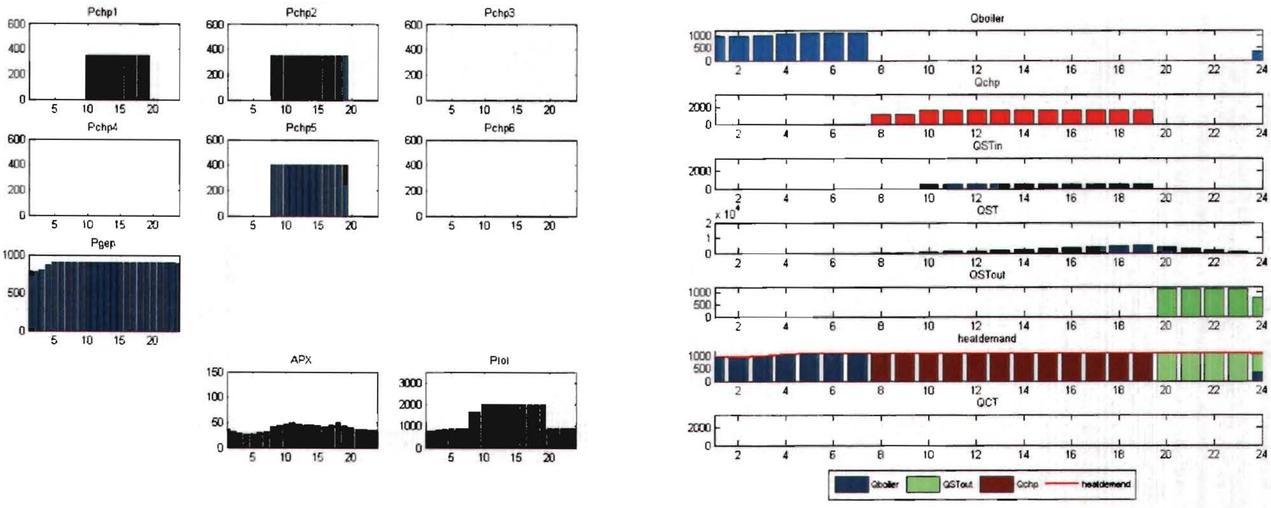

Figure 13: Result of optimizing over 24h

Optimizing over a period of 36h, as shown in Figure 14, the heat load of the night will be taken into account. Then the heat content of the storage of hour 24 will be transferred to the next day. In this case the heat in store is tuned for the first hours of the next day.

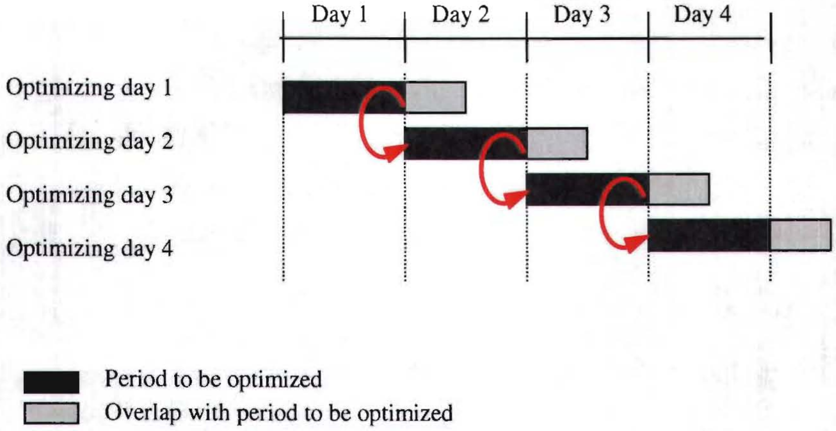

Figure 14: Illustration of optimizing 36h period

As a result of above mentioned overlap period, the optimization is much better as shown in Figure 15. In this figure one can see now that four co-generation units are producing electricity and heat instead of three as shown in the 24h optimization. The storage is filled up more than in the 24h model. The heat consumption of the gas expander can now be provided by the storage till hour 31 (07:00). So compared to the 24h optimization period this means a big advantage in optimization.

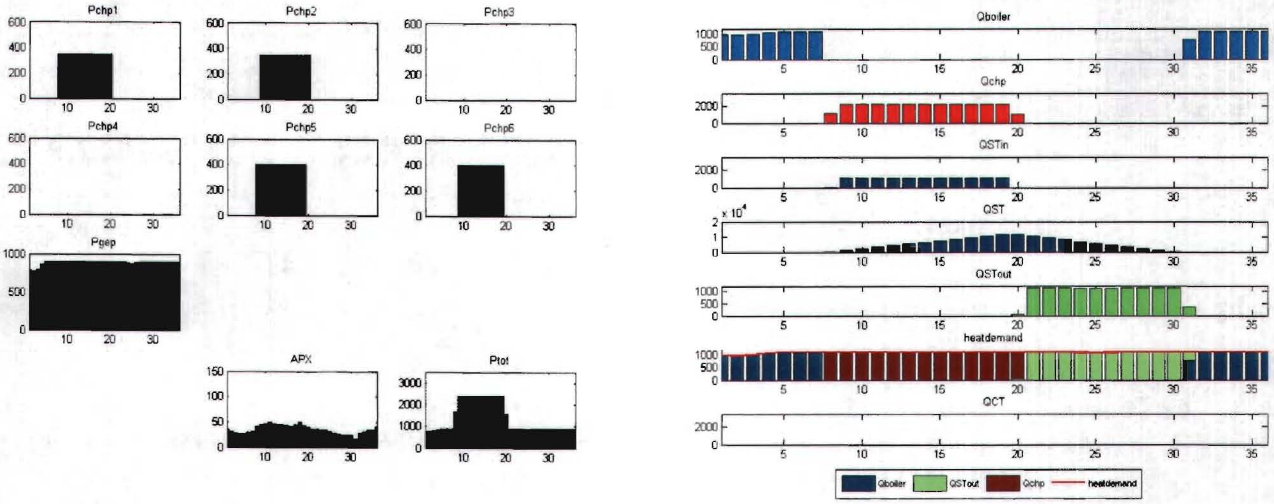

Figure 15: Result of optimizing over 36h

The last option is to optimize over 48h, which is shown in Figure 16. Now a longer period for optimizing is taken into account. Also here the storage content will be transferred.

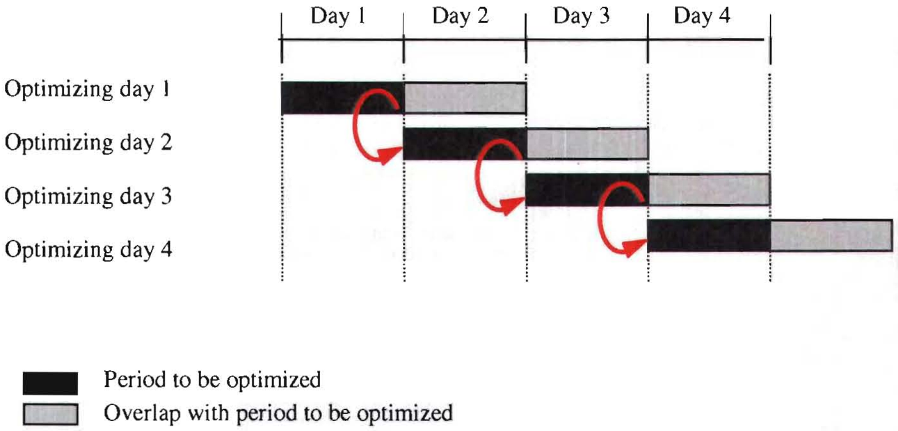

Figure 16: Illustration of optimizing 48h period

For the chosen period the overlap of 48 hour gives the same result as the 36h overlap (see Figure 17 and Figure 15).

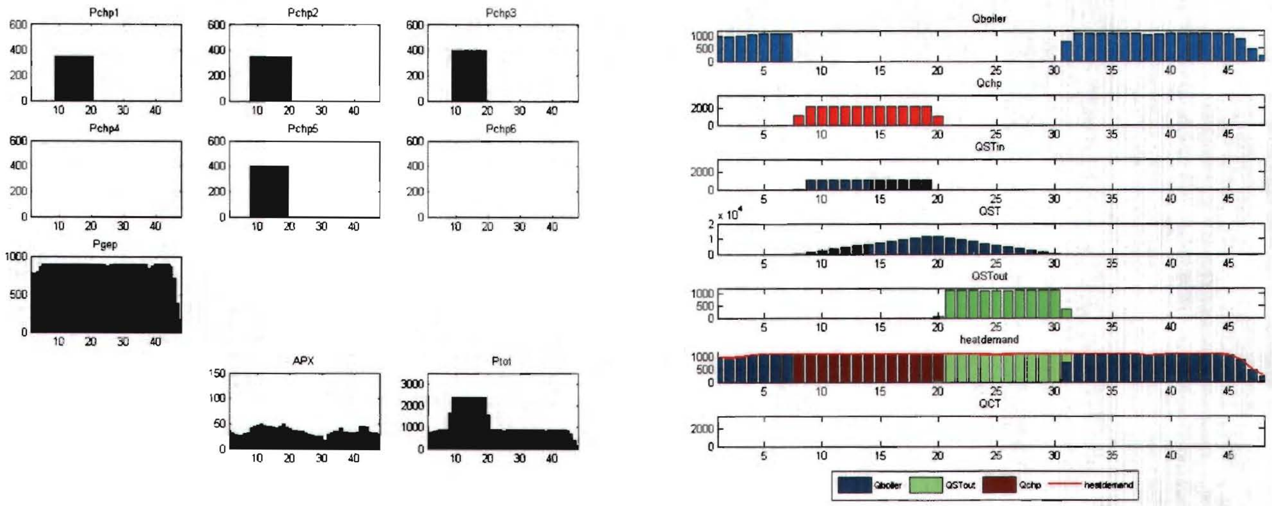

Figure 17: Result of optimizing over 48h

Conclusion:

- As shown in the above mentioned graphs, it is important to optimize over a longer period than 24h. During the night there is still a heatdemand and the market prices are low. When optimizing over 24h one forgets these items, which means that the heat during the night hours is produced with boilers and this is less efficient.
- One needs to take into account the storage efficiency. This efficiency is providing long period storage. It is also providing that heat produced with the boiler is put into the storage. It is not realistic to do so and therefore need to be avoided.
- Weather forecast over a long period is less accurate. The amount of gas that flows through the gas-expander is depending on the temperature in Eindhoven (see gas flow pattern in appendix E).
- In the graphs above it is shown that the 48h model is not better than the 36h model. But because the period is short and randomly chosen, it is not representative and therefore still under investigation in the next cases.

6. Case studies

In this chapter different cases are presented and interpreted. These cases give insight in the system and the best model for optimizing this problem. All cases are deterministic, which means that the gas flow and spot prices are fully known, which is the case for historical data. The gas flow and spot prices used for these optimizations are from 2005. The model is described by equations (12) – (35) in chapter 3. To compare the different cases, the objective values and the power produced are presented in each case. The following cases are studied:

1. Reference cases
2. 24h, 36h or 48h model
3. APX on hourly basis or peak – off-peak
4. Storage strategy
5. Optimal storage volume

6.1. Case 1: Reference cases

This case is divided into 3 sub cases. These sub cases must give insight in the original system, without actually optimizing the problem. These sub cases are the reference cases which are necessary to determine if the optimization is useful. In the first sub case, only a boiler is available, this means that there is no optimization possible at all. Then in the second sub case the two original co-generation units are in the system, only now the boiler is limited. The last sub case is based on the second sub case only now all 6 co-generation units are implemented in the system, including a 300m^3 storage. For the complete graphs see appendix F.

6.1.1. Sub case 1.1: Using only a boiler

In this sub case the heat will only be produced with the boiler. This means that there is no optimization possible. Heat produced with the boiler will not be put in storage because of heat losses of the storage. Therefore the heat needed for the gas-expansion is directly produced by the boiler. This simulation is done to know what the effect is of producing heat using co-generation units instead of a boiler. This case is therefore the reference case for the second sub case.

Table 5: Obj. value and produced power of sub case 1.1

Optimizing period	Produced power [MWh]	Obj. value [EUR]
24 h	5.609	- 120.932,04

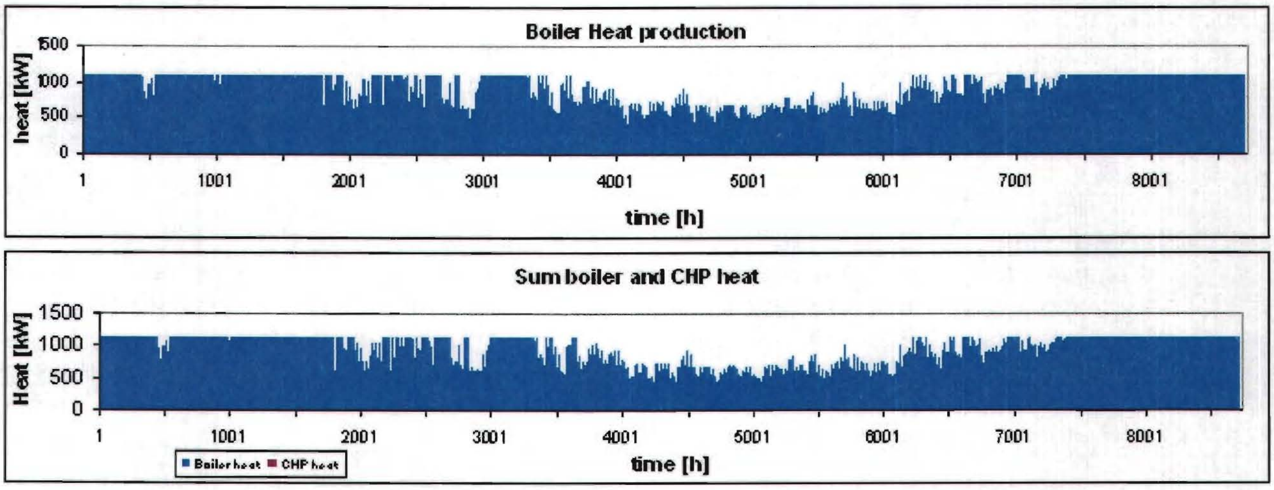

Figure 18: Heat produced with the boiler (top) is equal to the heat demand (bottom)

6.1.2. Sub case 1.2: Using 2 CHP plants and a boiler

The starting point is the original design with 2 co-generation units, a storage of 100m^3 and a boiler. Because the installation was working as a stand alone system, the co-generation units run when the heat was needed, also in unprofitable hours. To simulate this, the heat produced with the boiler is limited. This means that the heat will almost always be produced with the co-generation units.

Table 6: Obj. value and produced power of sub case 1.2

Optimized period	Produced power [MWh]	Obj. value [EUR]
24 h	10.024	- 204.873,41

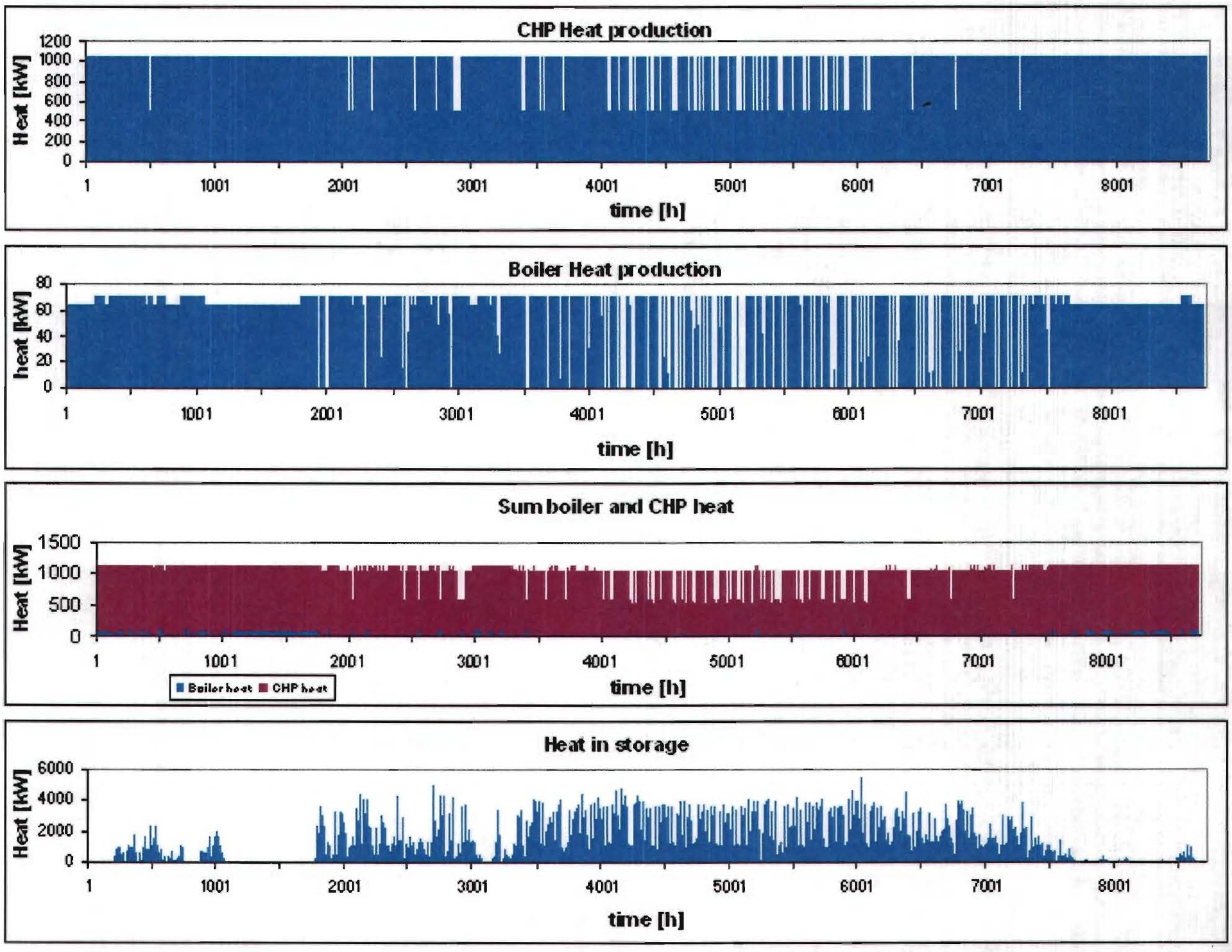

Figure 19: Heat production from CHP plants and boiler for sub case 1.2

Conclusion:
Compared to sub case 1.1 one can conclude that using co-generation units to produce the heat for this system is a good option. The system needs a heat storage, as shown in Figure 19 at the bottom.

6.1.3. Sub case 1.3: Extension with 4 CHP plants

In this case, the extension of only the 4 extra co-generation units is simulated. This done by using the model, but making the gas flow zero for all hours and switch unit 1 and 2 off. In this way the co-generation units produce electricity in those hours which are profitable. The heat is put in store. Combining case 1.2 with this case one get an idea of the area where NRE was thinking of. Doing this a fault is accepted. The heat produced with the 4 co-generation units is not taken into account. If it is assumed that the hours for high prices are only during day hours, and the heat from the 4 co-generation units are compensating the heat from the two original units during night hours, with prices of € 0,- then one can add the prices from the two cases together. The value is now the maximum what was expected before optimizing. The power produced with the 4 co-generation units is replacing the production of the two co-generation units and therefore not taken into account.

Table 7: Obj. value and produced power of sub case 1.3

Optimized period	Produced power [MWh]	Obj. value [EUR]
Sub case 1.2	10.024	- 204.873,41
Sub case 1.3	1.485	- 84.013,04
Total	10.024	- 287.886,45

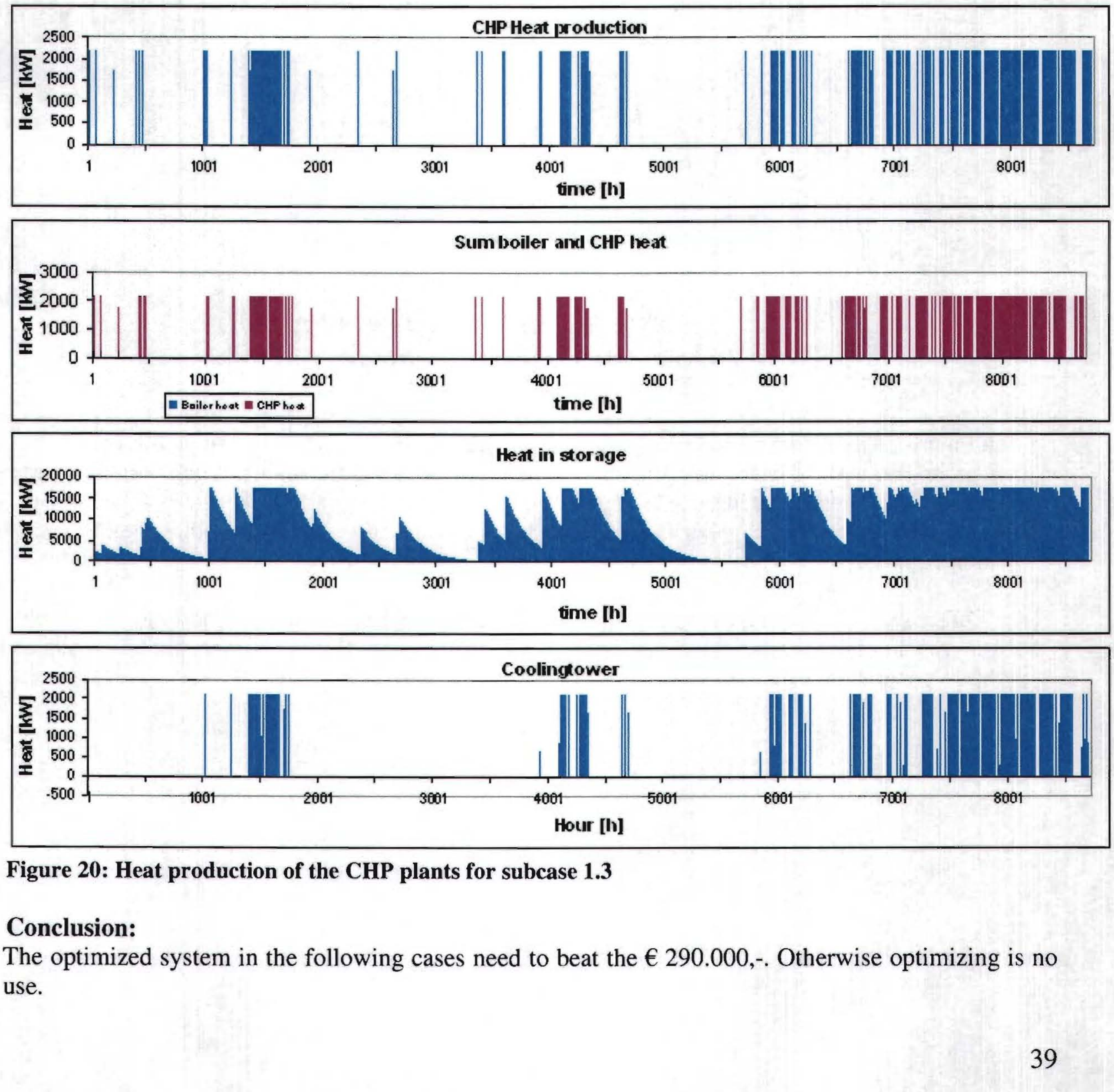

Figure 20: Heat production of the CHP plants for subcase 1.3

Conclusion:

The optimized system in the following cases need to beat the € 290.000,-. Otherwise optimizing is no use.

6.2. Case 2: 24h, 36h or 48h model

As described in paragraph 5.5, the optimization period has an effect on the outcome of the objective function. To have an idea of the impact of the optimization period for a calculation over a whole year, three optimization periods are evaluated. For these optimizations the system consists of 6 co-generation units and a storage capacity of 300m^3. For the complete graphs see appendix G.

Table 8: 24h, 36h or 48h model

Case	Optimized period	Produced power [MWh]	Obj. value [EUR]
2.1	24 h	9.474	- 328.440,81
2.2	36 h	10.284	- 336.132,88
2.3	48 h	10.454	- 337.337,06

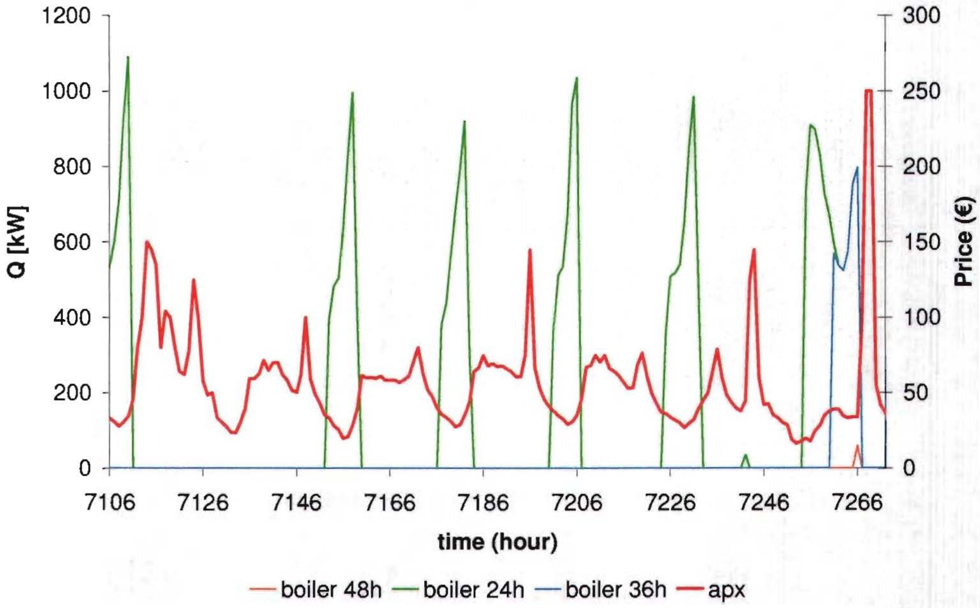

Figure 21: Boiler heat for optimization over 24h, 36h and 48h

In Figure 21 a random period is choosen and as shown, when optimizing over a 24h period, the boiler is producing a lot of heat during the night (green spikes during the night). When optimizing over 36h, the boiler is almost not used - in Figure 21 only in the last part of the week (blue spike). When optimizing over 48h the boiler is hardly used at all - only in the same period as the 36h model(little orange spike).

Another way of showing the improvement is using year-duration-curves. The following three graphs are year-duration-curves of the case described in this section. Boiler, storage and heat produced with the co-generation units.

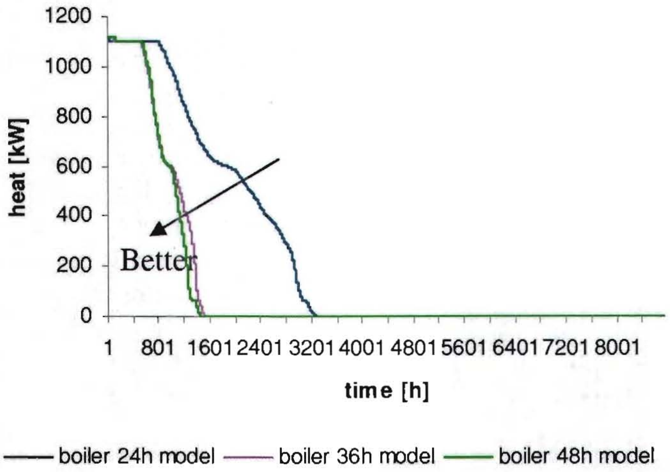

Figure 22: Duration curves of boiler heat for 24h, 36h and 48h model

Using less boilerheat means that more heat is used from the storage- and co-generation units.

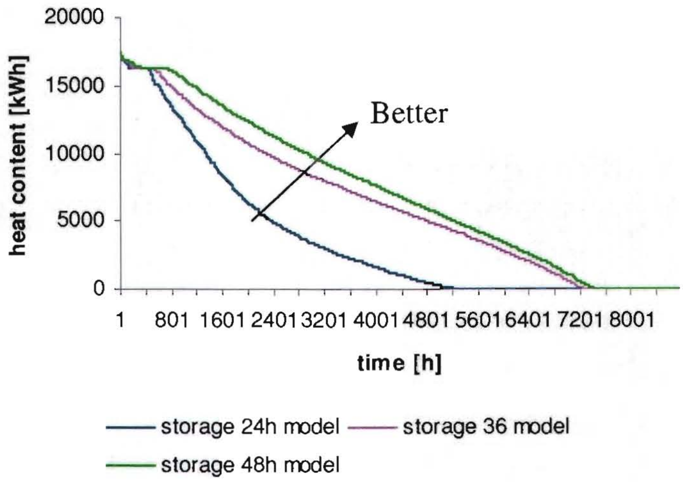

Figure 23: Duration curves of storage heat for 24h, 36h and 48h model

The storage is used more when the optimizing period is increasing.

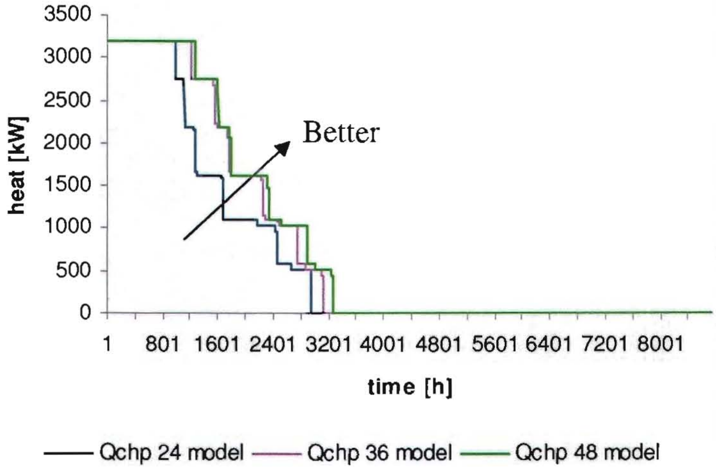

Figure 24: Duration curves of heat from CHP in 24h, 36h and 48h model

One can see in the above mentioned figures with the duration curves, that the 48h model is just a little bit better than the 36h model. Both models perform much better than the 24h model.

Conclusion:

- Optimizing over a period that includes the night has a big advantage above optimizing over 24h. The system increases in flexibility as more electricity is produced during day hours. Therefore optimizing over 36h or 48h is preferred above optimizing over 24h.
- Another item that should be taken into account is that, if the model is used for optimizing the system for future issues, then one needs a prediction of the gas flow. The gas flow is strongly correlated with the outside temperature. The gas flow in winter is much higher than in the summer, see Appendix E. Predicting the weather for periods longer that 24h is less accurate. This has an negative effect of optimizing the heat for the night hours and optimizing over long periods.

6.3. Case 3: APX on hourly basis or peak and off-peak

As described in paragraph 5.3 the APX can be expressed as a peak and off-peak period instead of hourly values. In this case these two methods are compared. With this case a decision can be made to let the system run automatically or controlled remotely. The optimization is done for a 24h and 36h period. The results are given in table 9.

Table 9: Different APX and optimization period

Case	Optimized period	Produced power [MWh]	Obj. value [EUR]
3.1	APX hourly, 24h model	9.474	- 328.440,81
3.2	APX hourly, 36h model	10.284	- 336.132,88
3.3	Peak and off-peak, 24h model	10.011	- 293.994,33
3.4	Peak and off-peak, 36h model	10.817	- 299.959,65

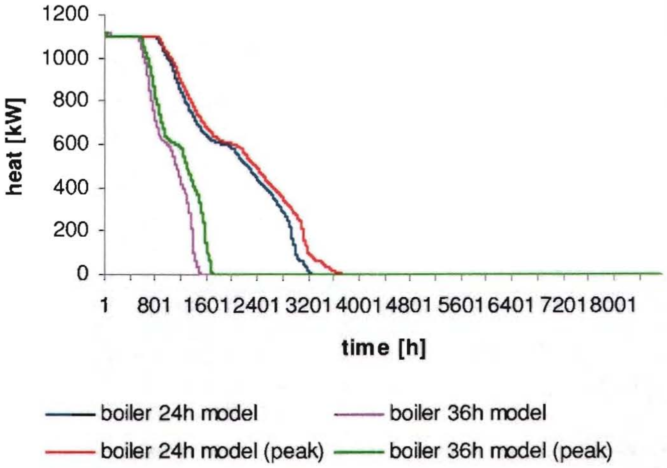

Figure 25: Duration curve of boiler heat for hourly and peak market price

The duration curve of boiler heat shows that in both cases, 24h and 36h model, the boiler is producing less heat with the hourly APX prices, than the models with the APX peak and off-peak prices. Both 36h models perform better than the 24h models.

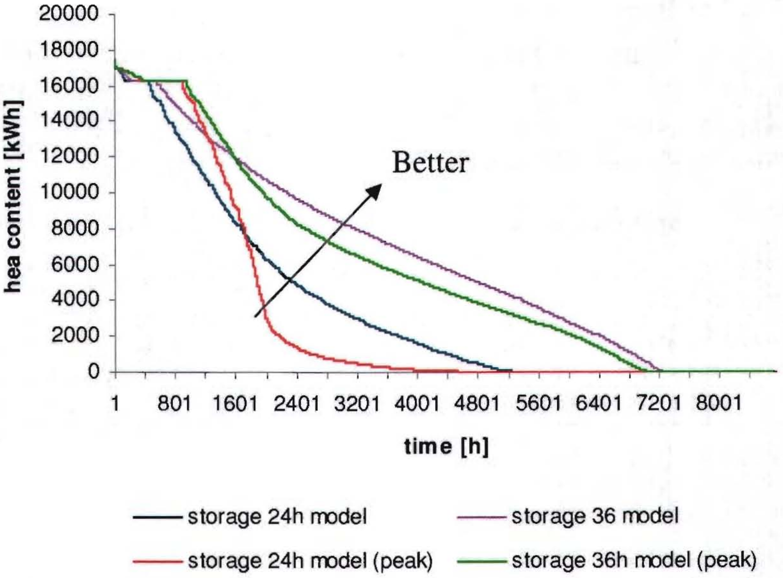

Figure 26: Duration curve of storage heat for hourly and peak market price

The storage usage within the 24h model with peak and off-peak APX prices is far from optimal. As one can see, the red-line is rapidly going down, which means that for a short time a lot of heat is stored and in other periods the storage is not used at all. Also in this figure the 36h models perform better.

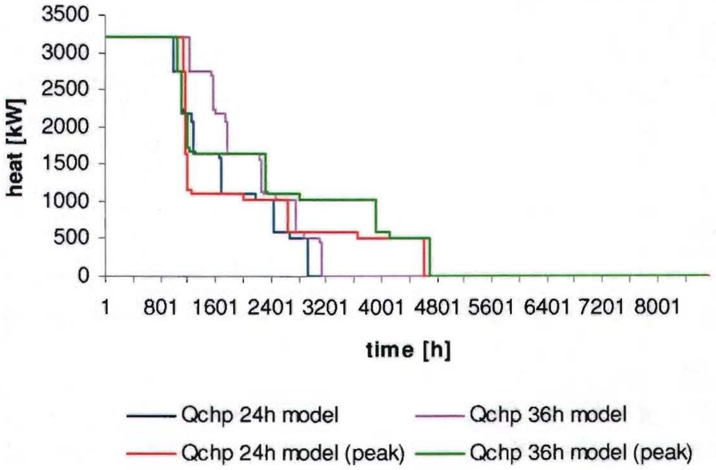

Figure 27: Duration curve of CHP heat for hourly and peak market price

For the heat produced with the co-generation units it is clear that in the model where the maket price is split in a peak and off-peak price, the units run less efficience than a market price based on hourly values. In Figure 27 both models with the peak prices run more hours.

Conclusion:
From the graphs mentioned in this paragraph, one can read that the hourly AXP data gives the best results. Because of these results one can also conclude that the system needs to be controlled remotely. In this case the system is operating more efficiently.

6.4. Case 4: Storage strategy

As described in paragraph 3.5 it is also possible to define the heat content of the storage at the end of a day. When one knows the impact of this on this system, then one can say something about this technique. For this case, 6 co-generation units and a storage of 300m^3 are evaluated. The storage capacity starts and ends with a minimum of 0, 50 or 95%.

Table 10: Different storage strategies for 6 co-generation units and 300m3 storage

Case	Storage end value	Produced power [MWh]	Obj. value [EUR]
4.1	>0%	9.474	- € 328.440,81
4.2	>50%	10.381	- € 337.957,71
4.3	>95%	10.580	- € 313.841,50

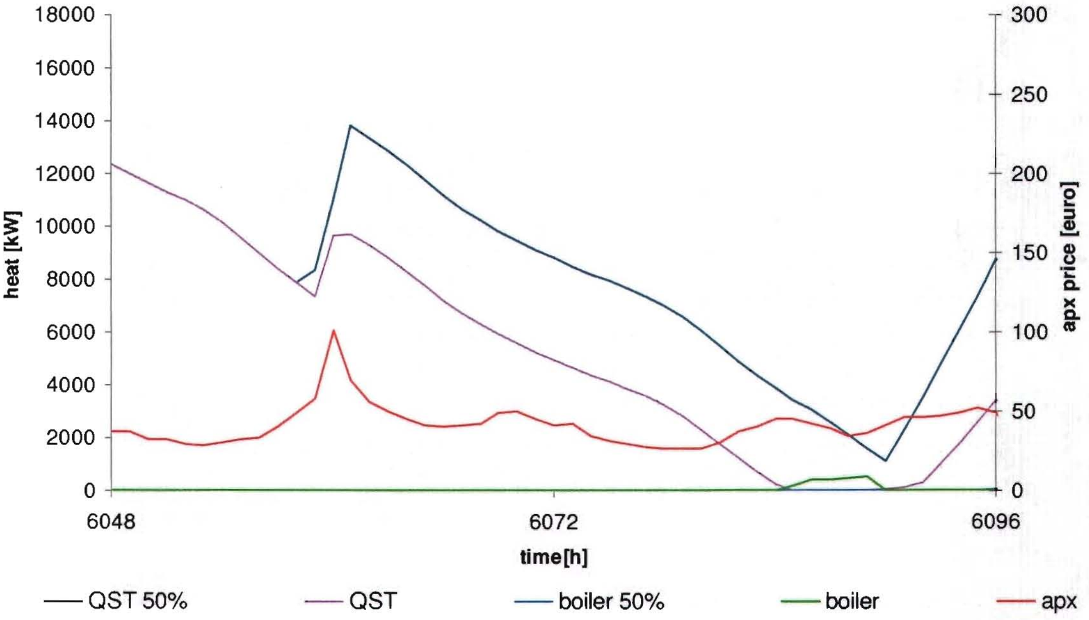

Figure 28: Storage heat content and boiler heat production for 36h model and 24h with heat content storage above 50%

In figure 28 a random period of 48h is shown. Within this period the two models differ from each other. The storage of the 36h model is ramping down (purple line) until it is empty, then a boiler (green line) is needed to provide some heat and the optimization will end with ramping up the heat in storage. The storage of the 50% model (blue line), is filled up more in the first 24h because at the end of that day the heat content needs to be mimimal 50% of his maximum capacity. After this the storage capacity is ramping down, until it is ramping up again. No boiler is needed in this case.

For this case also duration curves are made. These curves give insight in how the system reacts on the different storage strategies:

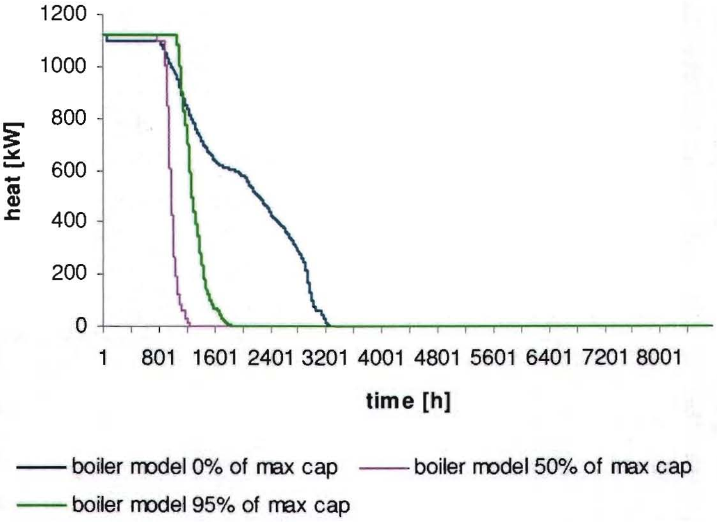

Figure 29: Duration curves of the boilers for different storage strategies

In Figure 29 the boiler of 0% and the 95% model is used more than the 50% model. This means that the 50% model is better than the other two models.

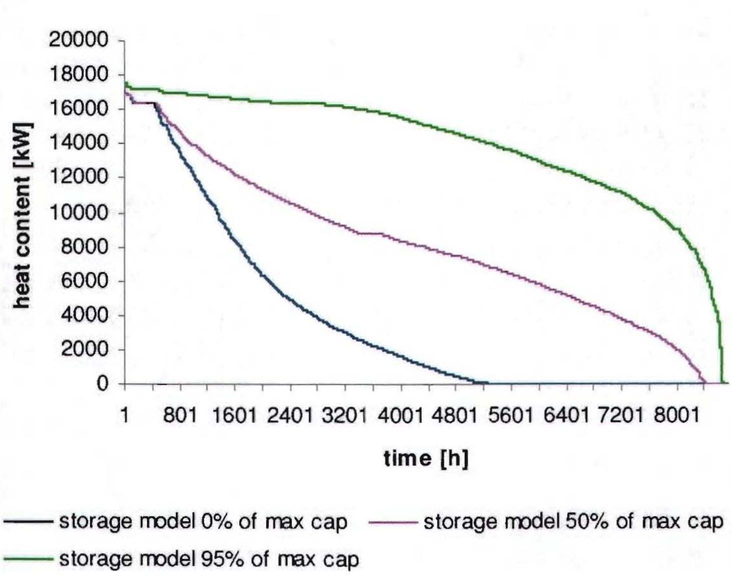

Figure 30: Duration curves of the storages for different storage strategies

In this figure one can see that the storage is used different in the different strategies. For the 95% and 50% model the storage is over the 3500h for 95% or 50% filled. For the 0% model the storage is used as seen before.

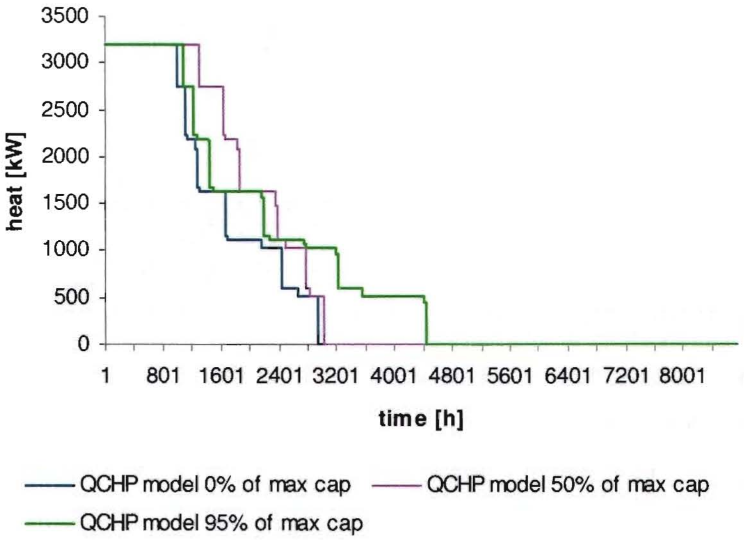

Figure 31: Duration curves of the co-generation units for different storage strategies

From Figure 31 one can see that the 95% model is far from optimal. The co-generation units for this case (green-line) are producing a lot more heat than the other two cases.

Table 10 shows that fixing the storage capacity at a value of 50% at the end of the day gives a good result on the outcome of the objective function. If this value is compared with the 36h model outcome, these values need to be compared with the 36h / 48h optimization. To determine if this value is only valid for this case, a few extra calculations were done. See table 11.

Table 11: Different storage strategies for 6 co-generation units

Case	Storage end value	Storage [m3]	Produced power [MWh]	Obj. value [EUR]
4.4	>50%	100	10.111	- 318.386,45
4.5	>50%	150	10.243	- 329.138,67
4.6	>50%	200	10.320	- 335.390,10
4.7	>50%	250	10.355	- 337.713,05
4.2	>50%	300	10.381	- 337.957,71

Table 12: 36h optimization obj. function values

Storage [m3]	Produced power [MWh]	Obj. value [EUR]
100	10.424	- 315.621,74
150	10.442	- 324.883,86
200	10.350	- 332.117,68
250	10.373	- 335.005,65
300	10.284	- 336.132,88

Comparing the values of the two models as mentioned in table 11 and 12, one get figure 22. In this figure one can see that the 50% model is quite better than 36h model. Solving the model for 24h with the storage filled up at the end of the day at a minimum level of 50% is faster than the 36h model.

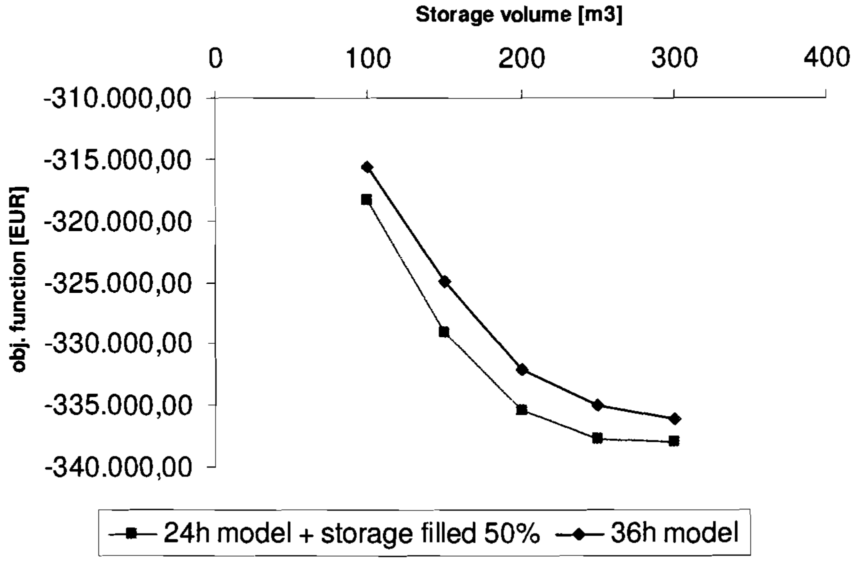

Figure 32: Optimizing with different storage size and strategy

Conclusion:

One can conclude from the figures presented in this paragraph, that optimizing this facility over a period of 24h with a storage strategy, that at the end of the day the storage is filled up for at least 50% of the maximum capacity, gives the best result.

6.5. Case 5: Optimal storage volume

This is a combination of the cases 2 and 3. All combinations from 2 co-generation units up to 6 co-generation units are considered, as are changing the storage capacity from 100m^3 to 300m^3 in steps of 50m^3, and calculating over 24h and 36h. For heat graphics see Appendix F.

Table 13: Overview of obj. value for 24h model with different number of co-generation units and different size of storage

		APX hourly value 24h model				
Case	Storage size	# co-generation units				
		2	3	4	5	6
5.1	100	- 217.566,13	- 249.718,67	- 267.837,73	- 292.097,64	- 314.940,63
5.2	150	- 217.267,72	- 252.588,85	- 272.914,48	- 298.300,10	- 321.952,75
5.3	200	- 217.566,14	- 252.931,27	- 276.205,53	- 301.825,79	- 326.585,65
5.4	250	- 217.566,13	- 252.995,57	- 276.994,84	- 303.568,00	- 328.360,99
5.5	300	- 217.566,13	- 252.332,39	- 277.023,06	- 303.978,35	- 328.440,81

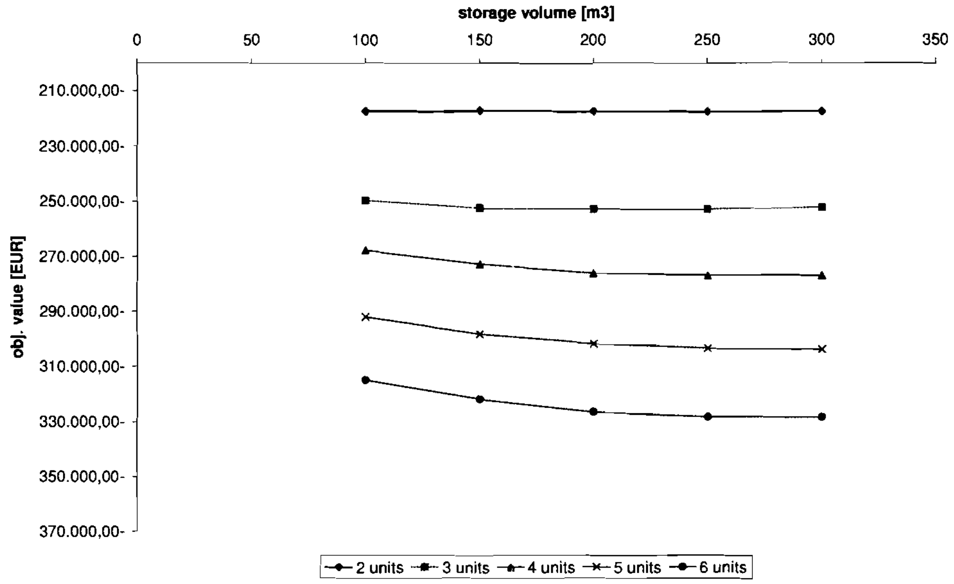

Figure 33: Obj. value as function of storage volume (24h model)

Table 14: Overview of obj. value for different number of co-generation units and different storage size (36h model)

		APX hourly value 36h model				
Case	Storage size	# co-generation units				
		2	3	4	5	6
5.6	100	- 222.222,58	- 255.091,64	- 271.308,46	- 294.461,14	- 315.621,74
5.7	150	- 221.871,00	- 262.971,69	- 280.323,38	- 301.838,75	- 324.883,86
5.8	200	- 222.403,91	- 264.461,88	- 286.389,09	- 309.764,74	- 332.117,68
5.9	250	- 222.459,95	- 264.708,87	- 288.494,84	- 312.871,50	- 335.005,65
5.10	300	- 221.860,04	- 264.197,30	- 288.238,70	- 313.019,15	- 336.132,88

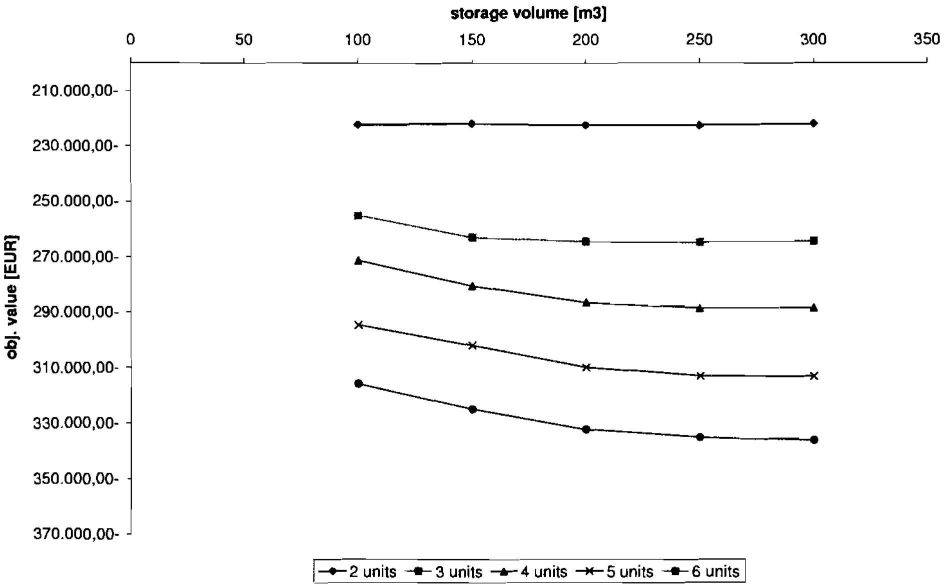

Figure 34: Obj. value as function of storage volume (36h model)

Conclusion:

- The optimal storage size for this system is somewhere between the 200m^3 and 250m^3.
- Because the investmentscosts of the different cases are not taken into account the curves bent to an asymptoot. If the investmentscosts are taken into account the curves show a minimum at a certain point. Than this will be the best storage volume for this facility.

7. Conclusions and recommendations

7.1. Conclusions

- For short period optimization this technique has proven that is efficient, fast and easy to use. This technique is prefered above PowrSym3 for short term optimization, because of the speed to solve the problem. Also the flexibility to change the model in the (MI)LP technology is a big advantage. If one can write the problem mathematically it can be implemented in the model.

- Comparing the basis case and the case with the best solution, which is the 24h model with the storage, filled 50% of his maximum capacity at the end of the day, one can conclude that the importance of having a model is proven. For the base case with the extension of 4 co-generation units the assumed profit for year 2005, which is an average year, is € 287.886 With the best optimization case the same facility can gain a profit of € 337.957 Which means that the model add € 50.000,-.to the profit. The co-generation units are running in those hours which are most profitable and the heat served from the boiler is limited to a minimum.

- Optimizing over a period that includes eighter fixed storage capacity or takes into account the heatdemand during the night, has a big advantage above optimizing over only 24h. The system increases in profitability as more electricity is produced during day hours. Therefore optimizing over 36h, 48h or 24h with a certain amount of heat content in storage at the end of the day, is preferred above optimizing over only 24h.

- Another item that should be taken into account is that, if the model is used for optimizing the system for future issues, then one needs a prediction of the gas flow. The gas flow is strongly correlated with the outdoor temperature. The gas flow in winter is much higher than in the summer, see Appendix E. Predicting the weather for periods longer that 24h is less accurate. This has an negative effect of optimizing the heat for the night hours and optimizing over long periods.

- It is seen that the heat capacity of the storage is utilized to move the heat production from periods with low prices for the produced electricity to periods when the price is higher. Therefore heat storage is important for these kind of system.

- The optimal storage size for this system is somewhere between the 200m^3 and 250m^3. Because the investmentscost of the different cases are not taken into account the curves bent to an asymptoot. If the investmentscost are taken into account the curves should show a minimum at a certain point. Than this will be the best storage volume for this facility.

- Minimum on- and off-time is not taken into account for the system described in this report. The co-generation units are small and the resolution that is used is 1 hour, which is the minimum on- and off-time of the units. When changing the resolution to 15 minutes or for large plants, which has longer on-off time periods, it is relevant to add the on- and off-time to the model.

- As mentioned in chapter 3.5 some optimization models uses a number of representative days. As described in chapter 5.5 one needs to be carefull to do this. In this report it is seen that based on a few days one can make the wrong decision concerning the optimization horizon for optimization in relation to improvement of the results. Best is to optimize over a whole year on hourly basis.

- Because of having the best performance of the model for hourly values of market price, one can conclude that the system needs to be controlled remotely for the best results, instead of a local simple on-off during peak and off-peak. An other issue is that when it is controlled remotely, dispatch can stop and run the machines when the spot- / imbalance-market is changing.

- The storage efficiency is important to simulate the facility more to reality. This efficiency rate is also limits long period storage. It also prevents that heat produced with the boiler is put into the storage, it is not realistic to do so and therefore need to be avoided in the model.

7.2. Recommendations for further developments

- Extend the model described in this report with a trading part to create a method for selling the power to the market.

- Implement this model in the company model of E.ON Benelux to evaluate the value of this system within the whole system of E.ON Benelux. The company model is build by Ing. Jan Kromhout together with Colin Sylvester during this project.

- When using the model for daily on-line use, add on- and off-time to the model and change the time step to a 15 min. basis. Then it is possible to react on the imbalance market.

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Appendix A. Overview cases

Table 15: Obj. value and produced power of sub case 1.1

Optimizing period	Produced power [MWh]	Obj. value [EUR]
24 h	5.609	- 120.932,04

Table 16: Obj. value and produced power of sub case 1.2

Optimized period	Produced power [MWh]	Obj. value [EUR]
24 h	10.024	- 204.873,41

Table 17: Obj. value and produced power of sub case 1.3

Optimized period	Produced power [MWh]	Obj. value [EUR]
Sub case 2	10.024	- 204.873,41
Sub case 1	1.485	- 84.013,04
Total	10.024	- 287.886,45

Table 18: 24h, 36h or 48h model

Case	Optimized period	Produced power [MWh]	Obj. value [EUR]
2.1	24 h	9.474	- 328.440,81
2.2	36 h	10.284	- 336.132,88
2.3	48 h	10.454	- 337.337,06

Table 19: Different APX and optimization period

Case	Optimized period	Produced power [MWh]	Obj. value [EUR]
3.1	APX hourly, 24h model	9.474	- 328.440,81
3.2	APX hourly, 36h model	10.284	- 336.132,88
3.3	Peak and off-peak, 24h model	10.011	- 293.994,33
3.4	Peak and off-peak, 36h model	10.817	- 299.959,65

Table 20: Different storage strategies for 6 co-generation units and 300m3 storage

Case	Storage end value	Produced power [MWh]	Obj. value [EUR]
4.1	>0%	9.474	- € 328.440,81
4.2	>50%	10.381	- € 337.957,71
4.3	>95%	10.580	- € 313.841,50

Table 21: Different storage strategies for 6 co-generation units

Case	Storage end value	Storage [m3]	Produced power [MWh]	Obj. value [EUR]
4.4	>50%	100	10.111	- 318.386,45
4.5	>50%	150	10.243	- 329.138,67
4.6	>50%	200	10.320	- 335.390,10
4.7	>50%	250	10.355	- 337.713,05
4.2	>50%	300	10.381	- 337.957,71

Table 22: 36h optimization obj. function values

Storage [m3]	Produced power [MWh]	Obj. value [EUR]
100	10.424	- 315.621,74
150	10.442	- 324.883,86
200	10.350	- 332.117,68
250	10.373	- 335.005,65
300	10.284	- 336.132,88

Table 23: Overview of obj. value for 24h model with different number of co-generation units and different size of storage

		APX hourly value 24h model				
Case	Storage size	# co-generation units				
		2	3	4	5	6
5.1	100	- 217.566,13	- 249.718,67	- 267.837,73	- 292.097,64	- 314.940,63
5.2	150	- 217.267,72	- 252.588,85	- 272.914,48	- 298.300,10	- 321.952,75
5.3	200	- 217.566,14	- 252.931,27	- 276.205,53	- 301.825,79	- 326.585,65
5.4	250	- 217.566,13	- 252.995,57	- 276.994,84	- 303.568,00	- 328.360,99
5.5	300	- 217.566,13	- 252.332,39	- 277.023,06	- 303.978,35	- 328.440,81

Table 24: Overview of obj. value for different number of co-generation units and different storage size (36h model)

		APX hourly value 36h model				
Case	Storage size	# co-generation units				
		2	3	4	5	6
5.6	100	- 222.222,58	- 255.091,64	- 271.308,46	- 294.461,14	- 315.621,74
5.7	150	- 221.871,00	- 262.971,69	- 280.323,38	- 301.838,75	- 324.883,86
5.8	200	- 222.403,91	- 264.461,88	- 286.389,09	- 309.764,74	- 332.117,68
5.9	250	- 222.459,95	- 264.708,87	- 288.494,84	- 312.871,50	- 335.005,65
5.10	300	- 221.860,04	- 264.197,30	- 288.238,70	- 313.019,15	- 336.132,88

Appendix B. M-files

Kanaaldijk.m

```
function [x_total,obj,err_array,iter_array,mp]=kanaaldijk()
% this program find the mimimum cost of operating the gas-expansion in
% combination whith the chp-plants. Using a MILP algoritm and the solver "LPSolve 5.5".
%It uses the following .m-files and .dat-files:
%- make_lp_file.m
%- read_lp_file.m
%- Graphics.m
%- mxlpsolve.lp, this is the matlab interface for LP_solve
%- gasflow.m, gas flow of the gas-expansion from gasflow.dat
%- mp.m, market price from the file mp.dat
%- wkk_data.m, data of al the chp
%- boiler_data.m
%- available.m, chp or boiler available yes or no
%- storage_data.m, data of the buffer
%- CT_data.m, data of the cooling tower

versionstring = mxlpsolve('lp_solve_version')

clc;
%gegevens afvragen
fbr=6.9; %Fuelprice, standard 6.9
hour=2; %standard 24
%calchour=48;
%week
MUT=1;% min. uptime =1 hour
MDT=1;% min. downtime = 1 hour
startday=135; % >= 1
days=1; %# days to calculate (max 364 days)
day_graph=13; %which day must be in the graph, incase more days are calculated
sw1=1; %chp1 available 1 = yes, 0 = no
sw2=1; %chp2 available 1 = yes, 0 = no
sw3=1; %chp3 available 1 = yes, 0 = no
sw4=1; %chp4 available 1 = yes, 0 = no
sw5=1; %chp5 available 1 = yes, 0 = no
sw6=1; %chp6 available 1 = yes, 0 = no
sw7=1; %Boiler available 1 = yes, 0 = no
sw8=1; %storage strategy
%- 0) Start value is zero
%- 1) Start value of buffer is end buffer of previous day
%- 2) fix start and end value of the buffer
%- 3) fix start value and end + above of the buffer
QST_end_v=0;

for day = startday:days+(startday-1);
  [mp,i] =
write_lp_file(fbr,hour,startday,day,sw1,sw2,sw3,sw4,sw5,sw6,sw7,sw8,QST_end_v,MUT,M
DT);
  %reading the file kanaaldijk.lp and solving the problem (MILP)
  day % just for verifing/folowing the process in matlab
  [x, obj, err, iter, gap]=read_lp_file();

  %generate a matrix incase days > 1.
  for var = 1:38*(hour*i)+7;
    x_total(var,day)=x(var);
  end
  if days == 1
    Graphics(startday, hour, x, mp); %generating some graphs of the data
  end

  %generate an obj, error,iter array
  obj_array(1,day)=obj;
  err_array(1,day)=err;
  iter_array(1,day)=iter;
  %incase days > 1 than endvalue of storage of prev.day, startvalue for
  %next day.
  if days >1 && sw8 > 0
    offset=14;
    offset2=24;
    QST_end_v=x_total(37+(offset*((hour*i)-1))+(hour-1)*offset2,day);
  end
end

if days > 1
  for var = 1:38*(hour*i)+7;
    x(var)=x_total(var,day_graph);
  end
  Graphics(day_graph, hour, x, mp);
end


export_to_excel(startday, days, hour, x_total, mp, obj_array, err_array,iter_array);

end
```

write_lp_file.m

```
function [mp,i] =
write_lp_file(fbr,hour,startday,day,sw1,sw2,sw3,sw4,sw5,sw6,sw7,sw8,QST_end_v,MUT,M
DT)

%This fuction creates a LP, and writng it in kanaaldijk.lp.
%The LP will be created thrue a fixed model of the plant, the data needed
%for creating this LP is gathered from different files, calculated
%and than written in the file kanaaldijk.lp

%multiplier for hour
i=1;

%uploaden van data (APX, Heatdemand, chp data, boiler data, available)
[Pchp1,Qchp1,fgas1,Pchp2,Qchp2,fgas2,Pchp3,Qchp3,fgas3,Pchp4,Qchp4,fgas4,Pchp5,Qchp
5,fgas5,Pchp6,Qchp6,fgas6] = chp_data();
[Qboiler_max,rendement] = boiler_data();
[apx]=MarketPrice();
[gasflow]=gas();
[QSTmax,QSTmin] = storage_data();
[QCTmax] = CT_data();

HSgas=35.17; %[MJ/m3]

%Coeff of CHP
co1chp1=fgas1*HSgas/(Pchp1*1000);
co2chp1=Qchp1/Pchp1;
co1chp2=fgas2*HSgas/(Pchp2*1000);
co2chp2=Qchp2/Pchp2;
co1chp3=fgas3*HSgas/(Pchp3*1000);
co2chp3=Qchp3/Pchp3;
co1chp4=fgas4*HSgas/(Pchp4*1000);
co2chp4=Qchp4/Pchp4;
co1chp5=fgas5*HSgas/(Pchp5*1000);
co2chp5=Qchp5/Pchp5;
co1chp6=fgas6*HSgas/(Pchp6*1000);
co2chp6=Qchp6/Pchp6;

co_boiler=3.6/(rendement*1000);

QST_end_lb=QSTmin;
QST_end_ub=QSTmax;
startcost_chp1=10; %min startcost 0.01
startcost_chp2=10; %min startcost 0.01
startcost_chp3=10; %min startcost 0.01
startcost_chp4=10; %min startcost 0.01
startcost_chp5=10; %min startcost 0.01
startcost_chp6=10; %min startcost 0.01

%QST start value incase of multiple days, last value of t-1
if sw8 == 0
  QST_start = 0;
  QST_end_lb = QSTmin;
  QST_end_ub = QSTmax;
elseif sw8 == 1 %- 1) start value is end value prev.day
  QST_start = QST_end_v;
  QST_end_lb = QSTmin;
  QST_end_ub = QSTmax;
elseif sw8 == 2
  %- 2) fix start and end value of the buffer
  QST_start = QST_end_v;
  QST_end_lb = QST_end_ub/2-1;
  QST_end_ub = QST_end_ub/2;
elseif sw8 == 3
  %- 3) fix start value and end + above of the buffer
  QST_start = QST_end_v;
  QST_end_lb = QST_end_ub/2;
  %QST_end_ub = QST_end_ub;
end

if sw7==0
  Qboiler_max=0;
end


% open the file kanaaldijk.lp
fid=fopen('c:\kanaaldijk\kanaaldijk.lp','wt');
%obj. functie maken
fprintf(fid, 'min: '); %minimalisation

%loop for time (time)
for time = 1:i*hour;
  fprintf(fid, '%f Pchp1_t%.0f + %f Pchp2_t%.0f + %f Pchp3_t%.0f + %f Pchp4_t%.0f + %f
Pchp5_t%.0f + %f Pchp6_t%.0f + %fQboiler_t%.0f -%f Ptot_t%.0f
\n',co1chp1*fbr,time,co1chp2*fbr,time,co1chp3*fbr,time,co1chp4*fbr,time,co1chp5*fbr,time
,co1chp6*fbr,time,co_boiler*fbr,time,apx(time+(day-1)*24,9),time);
  fprintf(fid, '%.2f CHPstart_chp1_t%.0f + %.2f CHPstart_chp2_t%.0f + %.2f
CHPstart_chp3_t%.0f + %.2f CHPstart_chp4_t%.0f + %.2f CHPstart_chp5_t%.0f + %.2f
CHPstart_chp6_t%.0f\n',startcost_chp1,time,startcost_chp2,time,startcost_chp3,time,startcost
_chp4,time,startcost_chp5,time,startcost_chp6,time);
```

```
end
fprintf(fid, ';\n\n');

% start conditions (beginvoorwaarden)
fprintf(fid, '/* start conditions */\n');
fprintf(fid, ' QST_t0 = %.0f;\n',QST_start);
fprintf(fid, ' CHPcommit_chp1_t0=0;\n');
fprintf(fid, ' CHPcommit_chp2_t0=0;\n');
fprintf(fid, ' CHPcommit_chp3_t0=0;\n');
fprintf(fid, ' CHPcommit_chp4_t0=0;\n');
fprintf(fid, ' CHPcommit_chp5_t0=0;\n');
fprintf(fid, ' CHPcommit_chp6_t0=0;\n');


%fixed model
%loop for time (time)
for time = 1:i*hour;
fprintf(fid, '\n/* t=%.0f */\n',time);
fprintf(fid, 'Ptot_t%.0f = Pchp1_t%.0f + Pchp2_t%.0f + Pchp3_t%.0f + Pchp4_t%.0f +
Pchp5_t%.0f + Pchp6_t%.0f + Pgep_t%.0f; /*sum of
power*/\n',time,time,time,time,time,time,time,time);
fprintf(fid, 'Qchp_t%.0f = Qchp1_t%.0f + Qchp2_t%.0f + Qchp3_t%.0f + Qchp4_t%.0f +
Qchp5_t%.0f + Qchp6_t%.0f; /*sum of heat*/\n',time,time,time,time,time,time,time);
fprintf(fid, 'Qchp_t%.0f + Qboiler_t%.0f + QSTout_t%.0f = heatdemand_t%.0f + QCT_t%.0f
+ QSTin_t%.0f; \n',time,time,time,time,time,time);
fprintf(fid, 'Qtot_t%.0f=Qchp_t%.0f + QST_t%.0f; /*heat total = sum heat of chp + heat in
storage (t-1)*/\n',time,time,time-1);
fprintf(fid, 'Qtot_t%.0f=QSTbr_t%.0f+QCTbr_t%.0f; /*heat total = heat storage bruto + heat
coolingtower bruto*/\n',time,time,time);
fprintf(fid, 'QST_t%.0f = QSTin_t%.0f - QSTout_t%.0f + 0.995QST_t%.0f; /*heat in storage
(t)= heat storage in (t) - heat storage out (t) + heat in storage (t-1)*/\n',time,time,time,time-1);
fprintf(fid, 'QCT_t%.0f <= Qchp_t%.0f; /*heat coolingtower <= sum heat chp*/\n',time,time);
fprintf(fid, 'QCT_t%.0f = QCTbr_t%.0f - %.0f + corr_t%.0f; /*heat coolingtower <= heat
coolingtower bruto - 17500 + correction factor*/\n',time,time,QSTmax,time);
fprintf(fid, '0 <= Qboiler_t%.0f <= %f; /*borders of the boiler*/\n',time,Qboiler_max);
fprintf(fid, '%.1f <= QSTbr_t%.0f <= %f; /*borders of heat storage
bruto*/\n',QSTmin,time,QSTmax);
fprintf(fid, '0 <= QSTin_t%.0f; /*heat storage in > 0*/\n',time);
if time < hour
   fprintf(fid, '%.1f <= QST_t%.0f <= %.1f; /*borders of heat
storage*/\n',QSTmin,time,QSTmax);
else
   fprintf(fid, '%.1f <= QST_t%.0f <= %.1f; /*borders of heat
storage*/\n',QST_end_lb,time,QST_end_ub);
end
fprintf(fid, '0 <= QCT_t%.0f <= %.1f; /*borders of the coolingtower*/\n',time,QCTmax);
```

```
fprintf(fid, '%.0f <= QCTbr_t%.0f <= %.1f; /*borders of the coolingtower bruto */
\n',QSTmax,time,QSTmax+QCTmax);
if gasflow(time+(day-1)*24)<15000
   heat_tot=0.0427*gasflow(time+(day-1)*24)-55.4;
   pgep_tot=0.034*gasflow(time+(day-1)*24)-66.46;
else
   heat_p1=0.0427*(gasflow(time+(day-1)*24)-(gasflow(time+(day-1)*24)-15000))-55.4;
   heat_p2=0.0427*(gasflow(time+(day-1)*24)-15000)-55.4;
   heat_tot=heat_p1+heat_p2;
   pgep_p1=0.034*(gasflow(time+(day-1)*24)-(gasflow(time+(day-1)*24)-15000))-66.46;
   pgep_p2=0.034*(gasflow(time+(day-1)*24)-15000)-66.46;
   pgep_tot=pgep_p1+pgep_p2;
end
fprintf(fid, 'heatdemand_t%.0f = %f; /*heatdemand (t)*/\n',time,heat_tot);
fprintf(fid, 'corr_t%.0f = %.0f; /*correction used for coolingtower*/\n',time,QSTmax);
fprintf(fid, 'CHPstart_chp1_t%.0f >= CHPcommit_chp1_t%.0f -
CHPcommit_chp1_t%.0f;\n',time,time,time-1);
fprintf(fid, 'CHPstart_chp2_t%.0f >= CHPcommit_chp2_t%.0f -
CHPcommit_chp2_t%.0f;\n',time,time,time-1);
fprintf(fid, 'CHPstart_chp3_t%.0f >= CHPcommit_chp3_t%.0f -
CHPcommit_chp3_t%.0f;\n',time,time,time-1);
fprintf(fid, 'CHPstart_chp4_t%.0f >= CHPcommit_chp4_t%.0f -
CHPcommit_chp4_t%.0f;\n',time,time,time-1);
fprintf(fid, 'CHPstart_chp5_t%.0f >= CHPcommit_chp5_t%.0f -
CHPcommit_chp5_t%.0f;\n',time,time,time-1);
fprintf(fid, 'CHPstart_chp6_t%.0f >= CHPcommit_chp6_t%.0f -
CHPcommit_chp6_t%.0f;\n',time,time,time-1);
fprintf(fid, 'Pgep_t%.0f = %f;\n',time,pgep_tot);
fprintf(fid, 'gasflow_t%.0f = %.f;\n',time,gasflow(time+(day-1)*24));
fprintf(fid, 'Pchp1_t%.0f = %.0f CHPcommit_chp1_t%.0f;\nPchp2_t%.0f = %.0f
CHPcommit_chp2_t%.0f;\nPchp3_t%.0f = %.0f CHPcommit_chp3_t%.0f;\nPchp4_t%.0f =
%.0f CHPcommit_chp4_t%.0f;\nPchp5_t%.0f = %.0f
CHPcommit_chp5_t%.0f;\nPchp6_t%.0f = %.0f
CHPcommit_chp6_t%.0f;\n',time,Pchp1,time,time,Pchp2,time,time,Pchp3,time,time,Pchp4,ti
me,time,Pchp5,time,time,Pchp6,time);
fprintf(fid, '%f Pchp1_t%.0f = Qchp1_t%.0f;\n%f Pchp2_t%.0f = Qchp2_t%.0f;\n%f
Pchp3_t%.0f = Qchp3_t%.0f;\n%f Pchp4_t%.0f = Qchp4_t%.0f;\n%f Pchp5_t%.0f =
Qchp5_t%.0f;\n%f Pchp6_t%.0f =
Qchp6_t%.0f;\n',co2chp1,time,time,co2chp2,time,time,co2chp3,time,time,co2chp4,time,time,
co2chp5,time,time,co2chp6,time,time);
if sw1==1
   fprintf(fid, '0<=CHPcommit_chp1_t%.0f<=1;\n',time);
else
   fprintf(fid, 'CHPcommit_chp1_t%.0f<=0;\n',time);
end
if sw2==1
```

```
  fprintf(fid, '0<=CHPcommit_chp2_t%.0f<=1;\n',time);
else
  fprintf(fid, 'CHPcommit_chp2_t%.0f<=0;\n',time);
end
if sw3==1
  fprintf(fid, '0<=CHPcommit_chp3_t%.0f<=1;\n',time);
else
  fprintf(fid, 'CHPcommit_chp3_t%.0f<=0;\n',time);
end
if sw4==1
  fprintf(fid, '0<=CHPcommit_chp4_t%.0f<=1;\n',time);
else
  fprintf(fid, 'CHPcommit_chp4_t%.0f<=0;\n',time);
end
if sw5==1
  fprintf(fid, '0<=CHPcommit_chp5_t%.0f<=1;\n',time);
else
  fprintf(fid, 'CHPcommit_chp5_t%.0f<=0;\n',time);
end
if sw6==1
  fprintf(fid, '0<=CHPcommit_chp6_t%.0f<=1;\n',time);
else
  fprintf(fid, 'CHPcommit_chp6_t%.0f<=0;\n',time);
end

end

%these values are integer
fprintf(fid, '\nint ');
for time = 1:i*hour
  fprintf(fid, 'CHPcommit_chp1_t%.0f CHPcommit_chp2_t%.0f CHPcommit_chp3_t%.0f
CHPcommit_chp4_t%.0f CHPcommit_chp5_t%.0f
CHPcommit_chp6_t%.0f\n',time,time,time,time,time,time);
end
fprintf(fid, ';\n');

%semicontinu is simulating delta = [0,1] times the power or boiler, the
%value is '0' or 'Pchp..' in case of the power
fprintf(fid, 'sec ');
for time = 1:i*hour;
fprintf(fid, 'Pchp1_t%.0f Pchp2_t%.0f Pchp3_t%.0f Pchp4_t%.0f Pchp5_t%.0f Pchp6_t%.0f
Qboiler_t%.0f QCTbr_t%.0f corr_t%.0f\n',time,time,time,time,time,time,time,time,time);
end
fprintf(fid, ';\n\n ');

fprintf(fid, '\nsos\n');
for time = 1:i*hour;
fprintf(fid, 'sos: QSTbr_t%.0f, QCTbr_t%.0f <= 1;\n',time,time);
fprintf(fid, 'sos: QCTbr_t%.0f, corr_t%.0f <= 1;\n',time,time);
end

%closing the file kanaaldijk.lp
fclose(fid);

mp=apx(:,9);

clear apx;
clear heatdemand;
clear gasflow;
clear QCTmax;

end
```

read_lp_file.m

```
function [x, obj, err, iter, gap]=read_lp_file()

%Read the file kanaaldijk.lp
lp = mxlpsolve('read_LP', 'c:\kanaaldijk\kanaaldijk.lp');

%max time setting 10 sec.
set_timeout=90;
set_verbose=3;
set_epsel=1e-12;

%settings for the lp
%standard settings of LP_solver
mxlpsolve('set_verbose', lp,set_verbose)%shows data of the LP
mxlpsolve('set_timeout', lp, set_timeout);%mxlpsolve('set_timeout', lp_handle, sectimeout)
%mxlpsolve('set_bb_rule',lp,17445);%specifies the branch and bound rules(standard)
mxlpsolve('set_bb_rule',lp,17461);
mxlpsolve('set_epsel', lp, set_epsel);%specifies the values set to be zero

err=0;

mxlpsolve('solve', lp);

x = mxlpsolve('get_variables', lp);
obj = mxlpsolve('get_objective', lp); %gives the result
iter = mxlpsolve('get_total_iter', lp);
time_elapsed = mxlpsolve('time_elapsed', lp);

if time_elapsed >= set_timeout;
```

```
    mxlpsolve('delete_lp', lp);
    lp = mxlpsolve('read_LP', 'c:\kanaaldijk\kanaaldijk.lp');
    %mxlpsolve('set_bb_rule',lp,17461); %change the branch and bound rules
    mxlpsolve('set_bb_rule',lp,17445);
    mxlpsolve('set_verbose', lp,set_verbose)%shows data of the LP
    mxlpsolve('set_timeout', lp, set_timeout);%mxlpsolve('set_timeout', lp_handle, sectimeout)
    mxlpsolve('set_epsel', lp, set_epsel);%specifies the values set to be zero
    %mxlpsolve('set_mip_gap', lp,1,1e-1);%set at 0,1, standard at 1e-11
    mxlpsolve('solve', lp);
    x2 = mxlpsolve('get_variables', lp);
    obj2 = mxlpsolve('get_objective', lp);
    iter2 = mxlpsolve('get_total_iter', lp);
    time_elapsed = mxlpsolve('time_elapsed', lp) + time_elapsed;
    if time_elapsed >= 2*set_timeout
      err=1;
    end
    if obj2 <= obj
       x=x2;
       obj=obj2;
       iter=iter2;
    end
end

gap=0;


%deleting the LP from the memory
mxlpsolve('delete_lp', lp);

end
```

graphics.m

```
function graphics(day, hour, x, mp)

offset=14;
offset2=24;
%grafisch
%data sorting / collecting
for time=1:1:hour;
    Pchp1(time)=x(1+(time-1)*offset);
    Pchp2(time)=x(2+(time-1)*offset);
    Pchp3(time)=x(3+(time-1)*offset);
    Pchp4(time)=x(4+(time-1)*offset);
    Pchp5(time)=x(5+(time-1)*offset);
    Pchp6(time)=x(6+(time-1)*offset);
    Qboiler(time)=x(7+(time-1)*offset);
    Ptot(time)=x(8+(time-1)*offset);
    QST_t0=x(hour*offset+1);%t=15
    Pgep(time)=x(22+(offset*(hour-1))+(time-1)*offset2);
    Qchp(time)=x(23+(offset*(hour-1))+(time-1)*offset2);
    Qchp1(time)=x(24+(offset*(hour-1))+(time-1)*offset2);
    Qchp2(time)=x(25+(offset*(hour-1))+(time-1)*offset2);
    Qchp3(time)=x(26+(offset*(hour-1))+(time-1)*offset2);
    Qchp4(time)=x(27+(offset*(hour-1))+(time-1)*offset2);
    Qchp5(time)=x(28+(offset*(hour-1))+(time-1)*offset2);
    Qchp6(time)=x(29+(offset*(hour-1))+(time-1)*offset2);
    QSTout(time)=x(30+(offset*(hour-1))+(time-1)*offset2);
    heatdemand(time)=x(31+(offset*(hour-1))+(time-1)*offset2);
    QCT(time)=x(32+(offset*(hour-1))+(time-1)*offset2);
    QSTin(time)=x(33+(offset*(hour-1))+(time-1)*offset2);
    Qtot(time)=x(34+(offset*(hour-1))+(time-1)*offset2);
    QST(time)=x(37+(offset*(hour-1))+(time-1)*offset2);
end

%Power
scrsz = get(0,'ScreenSize');
figure('Position',[10 60 scrsz(3)/1.2 scrsz(4)/1.2]) %rect = [left, bottom, width, height]

%figure('Color','white','Toolbar','none');
time=1:1:hour;
subplot(4,3,12);
bar(Ptot(time));
title 'Ptot'
axis([1 hour 0 3500]);%axis([xmin xmax ymin ymax])

subplot(4,3,1);
bar(Pchp1(time));
title 'Pchp1'
axis([1 hour 0 600]);%axis([xmin xmax ymin ymax])

subplot(4,3,2);
bar(Pchp2(time));
title 'Pchp2'
axis([1 hour 0 600]);%axis([xmin xmax ymin ymax])

subplot(4,3,3);
bar(Pchp3(time));
title 'Pchp3'
axis([1 hour 0 600]);%axis([xmin xmax ymin ymax])

subplot(4,3,4);
```

```
bar(Pchp4(time));
title 'Pchp4'
axis([1 hour 0 600]);%axis([xmin xmax ymin ymax])

subplot(4,3,5);
bar(Pchp5(time));
title 'Pchp5'
axis([1 hour 0 600]);%axis([xmin xmax ymin ymax])

subplot(4,3,6);
bar(Pchp6(time));
title 'Pchp6'
axis([1 hour 0 600]);%axis([xmin xmax ymin ymax])

subplot(4,3,7);
bar(Pgep(time));
title 'Pgep'
axis([1 hour 0 1000]);%axis([xmin xmax ymin ymax])

subplot(4,3,11);
bar(mp(time+(24*(day-1)))*1000);
title 'APX'
axis([1 hour 0 150]);%axis([xmin xmax ymin ymax])


%Heat
figure('Position',[10 60 scrsz(3)/1.2 scrsz(4)/1.2])

subplot(7,1,1);
bar(Qboiler,'b');
title 'Qboiler'
axis([1 hour 0 1200]);%axis([xmin xmax ymin ymax])

subplot(7,1,2);
bar(Qchp,'r');
title 'Qchp'
axis([1 hour 0 3500]);%axis([xmin xmax ymin ymax])

subplot(7,1,3);
bar(QSTin);
title 'QSTin'
axis([1 hour 0 3500]);%axis([xmin xmax ymin ymax])

subplot(7,1,4);
bar(QST);
title 'QST'
axis([1 hour 0 20000]);%axis([xmin xmax ymin ymax])

subplot(7,1,5);
bar(QSTout,'g');
title 'QSTout'
axis([1 hour 0 1200]);%axis([xmin xmax ymin ymax])

subplot(7,1,6);
Qserverd(time,1)=Qboiler(time);
Qserverd(time,2)=QSTout(time);
Qserverd(time,3)=Qchp(time)-QSTin(time)-QCT;
hold on;
bar(Qserverd,'stack')
plot(heatdemand,'r','LineWidth',2)
title 'heatdemand'
axis([1 hour 0 1200]);%axis([xmin xmax ymin ymax])
legend({'Qboiler','QSTout','Qchp','heatdemand'},...
 'FontName','Arial',...
 'FontSize',8,...
 'Orientation','horizontal',...
 'Location',[0.372 0.013 0.273 0.054]);


subplot(7,1,7);
bar(QCT);
title 'QCT'
axis([1 hour 0 3500]);%axis([xmin xmax ymin ymax])
hold off;

end
```

export_to_excel.m

```
function export_to_excel(startday, days, hour, x, mp, obj, err, iter)

offset=14;
offset2=24;
%grafisch
%data sorting / collecting
for day=1:1:days;
   for time=1:1:hour;
      Pchp1(time+24*(day-1))=x(1+(time-1)*offset,day + (startday-1));
      Pchp2(time+24*(day-1))=x(2+(time-1)*offset,day + (startday-1));
      Pchp3(time+24*(day-1))=x(3+(time-1)*offset,day + (startday-1));
      Pchp4(time+24*(day-1))=x(4+(time-1)*offset,day + (startday-1));
      Pchp5(time+24*(day-1))=x(5+(time-1)*offset,day + (startday-1));
      Pchp6(time+24*(day-1))=x(6+(time-1)*offset,day + (startday-1));
      Qboiler(time+24*(day-1))=x(7+(time-1)*offset,day + (startday-1));
      Ptot(time+24*(day-1))=x(8+(time-1)*offset,day + (startday-1));
      QST_t0=x(hour*offset+1,day + (startday-1));%t=15
      Pgep(time+24*(day-1))=x(22+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qchp(time+24*(day-1))=x(23+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qchp1(time+24*(day-1))=x(24+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qchp2(time+24*(day-1))=x(25+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qchp3(time+24*(day-1))=x(26+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qchp4(time+24*(day-1))=x(27+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qchp5(time+24*(day-1))=x(28+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qchp6(time+24*(day-1))=x(29+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      QSTout(time+24*(day-1))=x(30+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      heatdemand(time+24*(day-1))=x(31+(offset*(hour-1))+(time-1)*offset2,day + (startday-
1));
      QCT(time+24*(day-1))=x(32+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      QSTin(time+24*(day-1))=x(33+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      Qtot(time+24*(day-1))=x(34+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      QST(time+24*(day-1))=x(37+(offset*(hour-1))+(time-1)*offset2,day + (startday-1));
      APX(time+24*(day-1))= mp(time+24*(day-1)+(startday-1)*24);
   end
end

%clear excelsheet
%9000 x 30
rowi=1:hour*days+1;
coli=1:22+1;
clear_data (rowi,coli) = ('0');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls',clear_data,'data', 'A1');
name_array = {'hour', 'day', 'Pchp1', 'Pchp2', 'Pchp3', 'Pchp4', 'Pchp5', 'Pchp6', 'Pgep',
'Qboiler', 'Ptot', 'Qchp', 'Qchp1', 'Qchp2', 'Qchp3', 'Qchp4', 'Qchp5', 'Qchp6', 'QSTin', 'QCT',
'Qtot', 'QST', 'QSTout', 'heatdemand', 'apx'};
time_array = 1:hour*day;
i=1;
for daynr = startday:days+startday
   for time = i:i*hour
      day_array (time)= daynr;
   end
   i=i+24;
end
data_array = [Pchp1; Pchp2; Pchp3; Pchp4; Pchp5; Pchp6; Pgep; Qboiler; Ptot; Qchp; Qchp1;
Qchp2; Qchp3; Qchp4; Qchp5; Qchp6; QSTin; QCT; Qtot; QST; QSTout; heatdemand;
APX*1000];
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', name_array, 'data', 'A1');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', time_array', 'data', 'A2');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', day_array', 'data', 'B2');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', data_array', 'data', 'C2');

day_array=1:day;
name_array2 = {'day', 'obj', 'err', 'iter'};
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', name_array2, 'data2', 'A1');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', day_array', 'data2', 'A2');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', obj', 'data2', 'B2');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', err', 'data2', 'C2');
s = xlswrite('c:\kanaaldijk\kanaaldijk_data.xls', iter', 'data2', 'D2');

end
```

gas.m

```
function [gasflow]=gas()

%read the gas flow data
 gas flow = load('c:\kanaaldijk\gasflow.dat')*1000;
```

storage_data.m

```
function [QSTmax,QSTmin] = storage_data()

Volume=300; %m3
DeltaT=50;%gr K
Cp=4.2;

QSTmin=0;
QSTmax=Volume*DeltaT*Cp/3.6;
```

chp_data.m

```
function
[Pwkk1,Qwkk1,fgas1,Pwkk2,Qwkk2,fgas2,Pwkk3,Qwkk3,fgas3,Pwkk4,Qwkk4,fgas4,Pwkk5
,Qwkk5,fgas5,Pwkk6,Qwkk6,fgas6] = chp_data()

%wkk1, lescogen
Pwkk1=347; %[kW]
Qwkk1=518; %[kW]
fgas1=111.4; %[m3/h]

%wkk2, lescogen
Pwkk2=347; %[kW]
Qwkk2=518; %[kW]
fgas2=111.4; %[m3/h]

%wkk3, Gevers-donkers
Pwkk3=400; %[kW]
Qwkk3=561; %[kW]
fgas3=128.8; %[m3/h]

%wkk4, Van Dijk
Pwkk4=312; %[kW]
Qwkk4=447; %[kW]
fgas4=102; %[m3/h]

%wkk5, Raijmakers
Pwkk5=400; %[kW]
Qwkk5=595; %[kW]
fgas5=128.8; %[m3/h]

%wkk6, Keijsers
Pwkk6=400; %[kW]
Qwkk6=561; %[kW]
fgas6=128.8; %[m3/h]
```

boiler_data.m

```
function [Qmax,rendement] = boiler_data();

Qmax=1170.2; %[kW]
rendement=0.91; %rendement
```

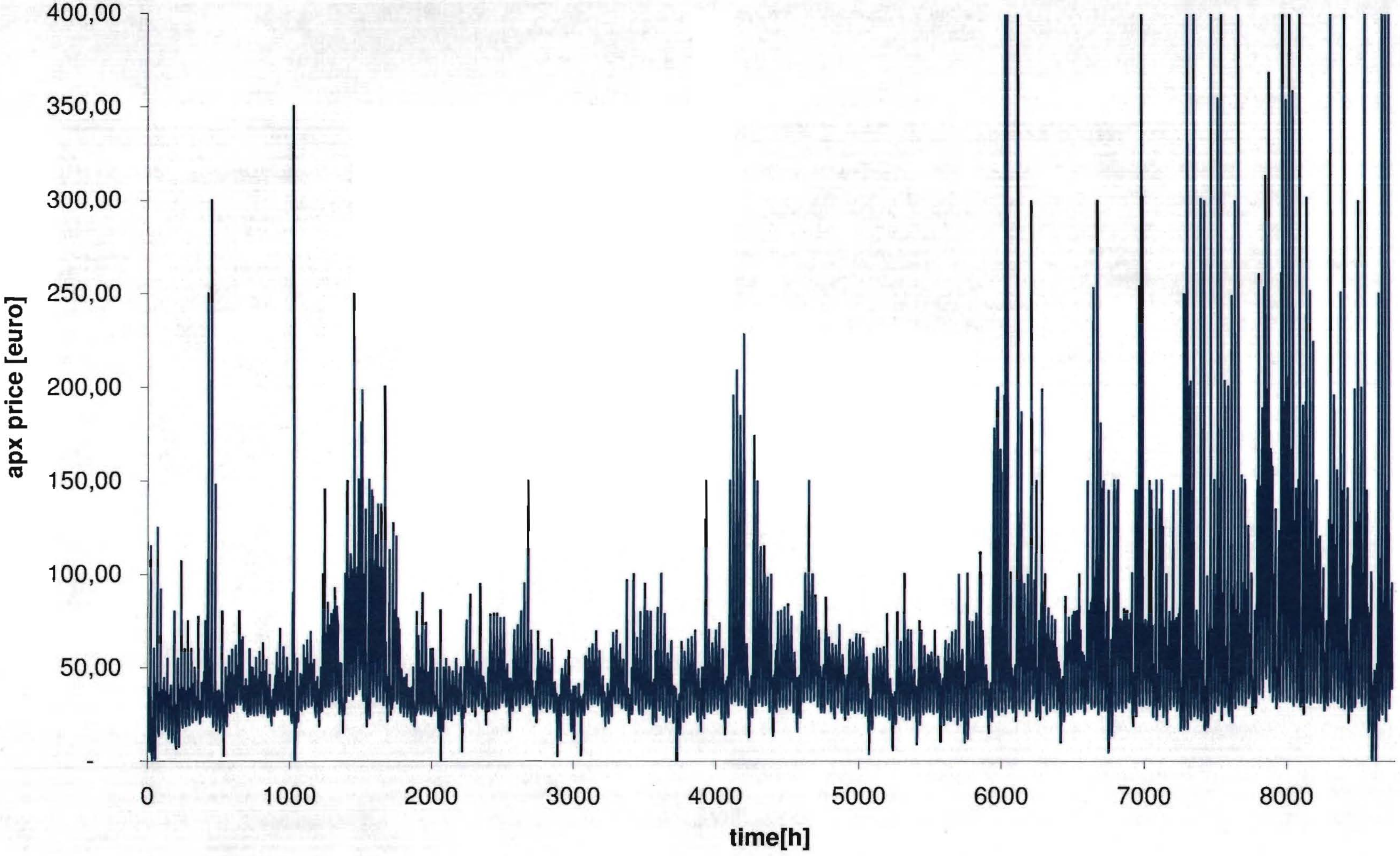
Appendix C. LP format

In this appendix the LP format of a 2 hour optimization is shown. The LP format is a good readable format. Per time step the constraints and bounds are put together.

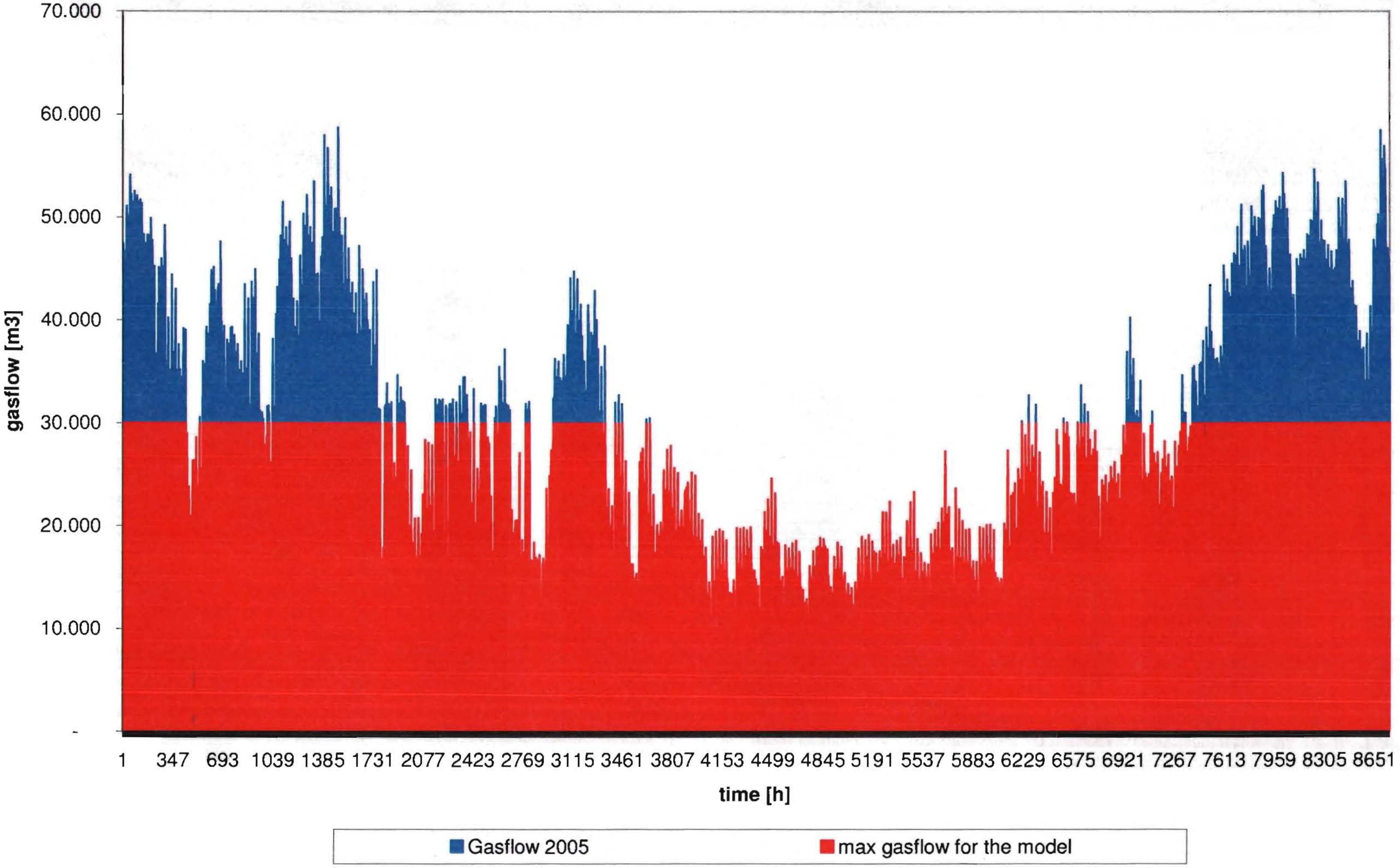
```
min: 0.077907 Pchp1_t1 + 0.077907 Pchp2_t1 + 0.078141 Pchp3_t1 + 0.079335 Pchp4_t1 +
0.078141 Pchp5_t1 + 0.078141 Pchp6_t1 + 0.027297Qboiler_t1 -0.033000 Ptot_t1
10.00 CHPstart_chp1_t1 + 10.00 CHPstart_chp2_t1 + 10.00 CHPstart_chp3_t1 + 10.00
CHPstart_chp4_t1 + 10.00 CHPstart_chp5_t1 + 10.00 CHPstart_chp6_t1
0.077907 Pchp1_t2 + 0.077907 Pchp2_t2 + 0.078141 Pchp3_t2 + 0.079335 Pchp4_t2 +
0.078141 Pchp5_t2 + 0.078141 Pchp6_t2 + 0.027297Qboiler_t2 -0.030000 Ptot_t2
10.00 CHPstart_chp1_t2 + 10.00 CHPstart_chp2_t2 + 10.00 CHPstart_chp3_t2 + 10.00
CHPstart_chp4_t2 + 10.00 CHPstart_chp5_t2 + 10.00 CHPstart_chp6_t2
;

/* start conditions */
 QST_t0 = 0;
 CHPcommit_chp1_t0=0;
 CHPcommit_chp2_t0=0;
 CHPcommit_chp3_t0=0;
 CHPcommit_chp4_t0=0;
 CHPcommit_chp5_t0=0;
 CHPcommit_chp6_t0=0;

/* t=1 */
Ptot_t1 = Pchp1_t1 + Pchp2_t1 + Pchp3_t1 + Pchp4_t1 + Pchp5_t1 + Pchp6_t1 + Pgep_t1;
/*sum of power*/
Qchp_t1 = Qchp1_t1 + Qchp2_t1 + Qchp3_t1 + Qchp4_t1 + Qchp5_t1 + Qchp6_t1; /*sum of
heat*/
Qchp_t1 + Qboiler_t1 + QSTout_t1 = heatdemand_t1 + QCT_t1 + QSTin_t1;
Qtot_t1=Qchp_t1 + QST_t0; /*heat total = sum heat of chp + heat in storage (t-1)*/
Qtot_t1=QSTbr_t1+QCTbr_t1; /*heat total = heat storage bruto + heat coolingtower bruto*/
QST_t1 = QSTin_t1 - QSTout_t1 + 0.995QST_t0; /*heat in storage (t)= heat storage in (t) -
heat storage out (t) + heat in storage (t-1)*/
QCT_t1 <= Qchp_t1; /*heat coolingtower <= sum heat chp*/
QCT_t1 = QCTbr_t1 - 17500 + corr_t1; /*heat coolingtower <= heat coolingtower bruto -
17500 + correction factor*/
0 <= Qboiler_t1 <= 1170.200000; /*borders of the boiler*/
0.0 <= QSTbr_t1 <= 17500.000000; /*borders of heat storage bruto*/
0 <= QSTin_t1; /*heat storage in > 0*/
0.0 <= QST_t1 <= 17500.0; /*borders of heat storage*/
0 <= QCT_t1 <= 3500.0; /*borders of the coolingtower*/
17500 <= QCTbr_t1 <= 21000.0; /*borders of the coolingtower bruto */
heatdemand_t1 = 778.214000; /*heatdemand (t)*/
corr_t1 = 17500; /*correction used for coolingtower*/
CHPstart_chp1_t1 >= CHPcommit_chp1_t1 - CHPcommit_chp1_t0;
CHPstart_chp2_t1 >= CHPcommit_chp2_t1 - CHPcommit_chp2_t0;
CHPstart_chp3_t1 >= CHPcommit_chp3_t1 - CHPcommit_chp3_t0;
CHPstart_chp4_t1 >= CHPcommit_chp4_t1 - CHPcommit_chp4_t0;
CHPstart_chp5_t1 >= CHPcommit_chp5_t1 - CHPcommit_chp5_t0;
CHPstart_chp6_t1 >= CHPcommit_chp6_t1 - CHPcommit_chp6_t0;
Pgep_t1 = 574.960000;
gasflow_t1 = 20820;
Pchp1_t1 = 347 CHPcommit_chp1_t1;
Pchp2_t1 = 347 CHPcommit_chp2_t1;
Pchp3_t1 = 400 CHPcommit_chp3_t1;
Pchp4_t1 = 312 CHPcommit_chp4_t1;
Pchp5_t1 = 400 CHPcommit_chp5_t1;
Pchp6_t1 = 400 CHPcommit_chp6_t1;
1.492795 Pchp1_t1 = Qchp1_t1;
1.492795 Pchp2_t1 = Qchp2_t1;
1.402500 Pchp3_t1 = Qchp3_t1;
1.432692 Pchp4_t1 = Qchp4_t1;
1.487500 Pchp5_t1 = Qchp5_t1;
1.402500 Pchp6_t1 = Qchp6_t1;
0<=CHPcommit_chp1_t1<=1;
0<=CHPcommit_chp2_t1<=1;
0<=CHPcommit_chp3_t1<=1;
0<=CHPcommit_chp4_t1<=1;
0<=CHPcommit_chp5_t1<=1;
0<=CHPcommit_chp6_t1<=1;

/* t=2 */
Ptot_t2 = Pchp1_t2 + Pchp2_t2 + Pchp3_t2 + Pchp4_t2 + Pchp5_t2 + Pchp6_t2 + Pgep_t2;
/*sum of power*/
Qchp_t2 = Qchp1_t2 + Qchp2_t2 + Qchp3_t2 + Qchp4_t2 + Qchp5_t2 + Qchp6_t2; /*sum of
heat*/
Qchp_t2 + Qboiler_t2 + QSTout_t2 = heatdemand_t2 + QCT_t2 + QSTin_t2;
Qtot_t2=Qchp_t2 + QST_t1; /*heat total = sum heat of chp + heat in storage (t-1)*/
Qtot_t2=QSTbr_t2+QCTbr_t2; /*heat total = heat storage bruto + heat coolingtower bruto*/
QST_t2 = QSTin_t2 - QSTout_t2 + 0.995QST_t1; /*heat in storage (t)= heat storage in (t) -
heat storage out (t) + heat in storage (t-1)*/
QCT_t2 <= Qchp_t2; /*heat coolingtower <= sum heat chp*/
QCT_t2 = QCTbr_t2 - 17500 + corr_t2; /*heat coolingtower <= heat coolingtower bruto -
17500 + correction factor*/
0 <= Qboiler_t2 <= 1170.200000; /*borders of the boiler*/
0.0 <= QSTbr_t2 <= 17500.000000; /*borders of heat storage bruto*/
```

```
0 <= QSTin_t2; /*heat storage in > 0*/
0.0 <= QST_t2 <= 17500.0; /*borders of heat storage*/
0 <= QCT_t2 <= 3500.0; /*borders of the coolingtower*/
17500 <= QCTbr_t2 <= 21000.0; /*borders of the coolingtower bruto */
heatdemand_t2 = 818.565500; /*heatdemand (t)*/
corr_t2 = 17500; /*correction used for coolingtower*/
CHPstart_chp1_t2 >= CHPcommit_chp1_t2 - CHPcommit_chp1_t1;
CHPstart_chp2_t2 >= CHPcommit_chp2_t2 - CHPcommit_chp2_t1;
CHPstart_chp3_t2 >= CHPcommit_chp3_t2 - CHPcommit_chp3_t1;
CHPstart_chp4_t2 >= CHPcommit_chp4_t2 - CHPcommit_chp4_t1;
CHPstart_chp5_t2 >= CHPcommit_chp5_t2 - CHPcommit_chp5_t1;
CHPstart_chp6_t2 >= CHPcommit_chp6_t2 - CHPcommit_chp6_t1;
Pgep_t2 = 607.090000;
gasflow_t2 = 21765;
Pchp1_t2 = 347 CHPcommit_chp1_t2;
Pchp2_t2 = 347 CHPcommit_chp2_t2;
Pchp3_t2 = 400 CHPcommit_chp3_t2;
Pchp4_t2 = 312 CHPcommit_chp4_t2;
Pchp5_t2 = 400 CHPcommit_chp5_t2;
Pchp6_t2 = 400 CHPcommit_chp6_t2;
1.492795 Pchp1_t2 = Qchp1_t2;
1.492795 Pchp2_t2 = Qchp2_t2;
1.402500 Pchp3_t2 = Qchp3_t2;
1.432692 Pchp4_t2 = Qchp4_t2;
1.487500 Pchp5_t2 = Qchp5_t2;
1.402500 Pchp6_t2 = Qchp6_t2;
0<=CHPcommit_chp1_t2<=1;
0<=CHPcommit_chp2_t2<=1;
0<=CHPcommit_chp3_t2<=1;
0<=CHPcommit_chp4_t2<=1;
0<=CHPcommit_chp5_t2<=1;
0<=CHPcommit_chp6_t2<=1;

int CHPcommit_chp1_t1 CHPcommit_chp2_t1 CHPcommit_chp3_t1 CHPcommit_chp4_t1
CHPcommit_chp5_t1 CHPcommit_chp6_t1
CHPcommit_chp1_t2 CHPcommit_chp2_t2 CHPcommit_chp3_t2 CHPcommit_chp4_t2
CHPcommit_chp5_t2 CHPcommit_chp6_t2
;
sec Pchp1_t1 Pchp2_t1 Pchp3_t1 Pchp4_t1 Pchp5_t1 Pchp6_t1 Qboiler_t1 QCTbr_t1 corr_t1
Pchp1_t2 Pchp2_t2 Pchp3_t2 Pchp4_t2 Pchp5_t2 Pchp6_t2 Qboiler_t2 QCTbr_t2 corr_t2
;


sos
sos: QSTbr_t1, QCTbr_t1 <= 1;
sos: QCTbr_t1, corr_t1 <= 1;
sos: QSTbr_t2, QCTbr_t2 <= 1;
sos: QCTbr_t2, corr_t2 <= 1;
```

Appendix D. APX graph 2005



Appendix E. Gas flow graph 2005



Appendix F. Overview results case 1

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	-	-	-	-	-	-	5609337
Run hours	-	-	-	-	-	-	6232,596

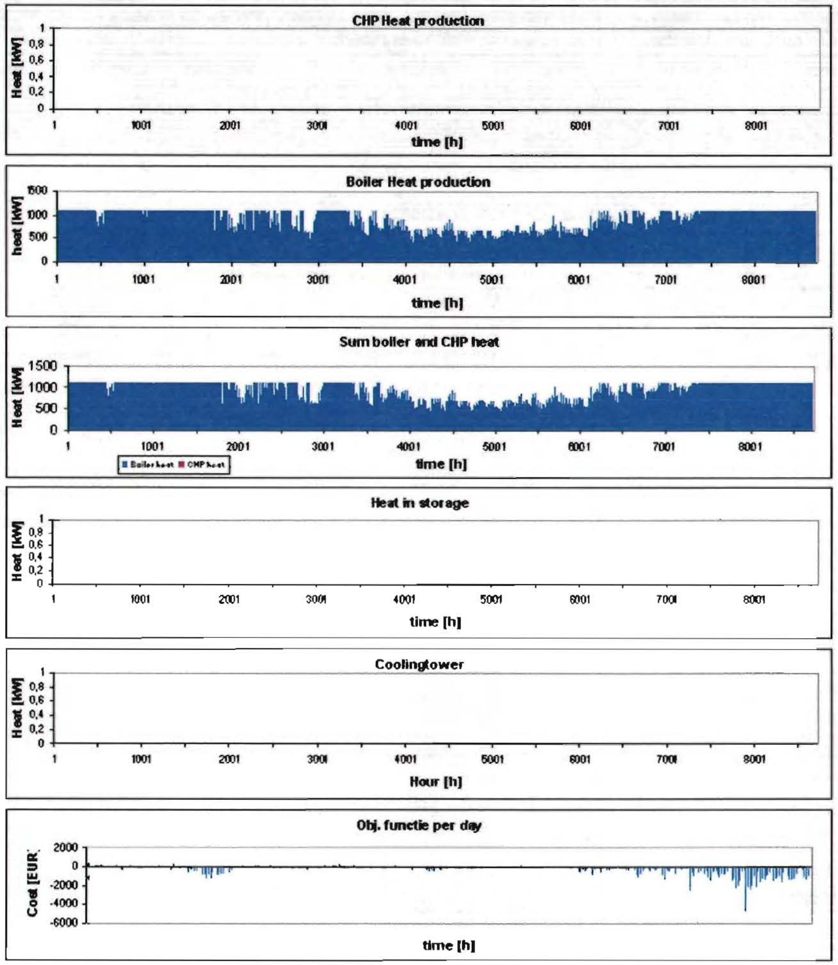


Figure F 1: Results of producing heat with only boiler

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	2311367	2103861	0	0	0	0	5609337
Run hours	6661	6063	0	0	0	0	6232,596

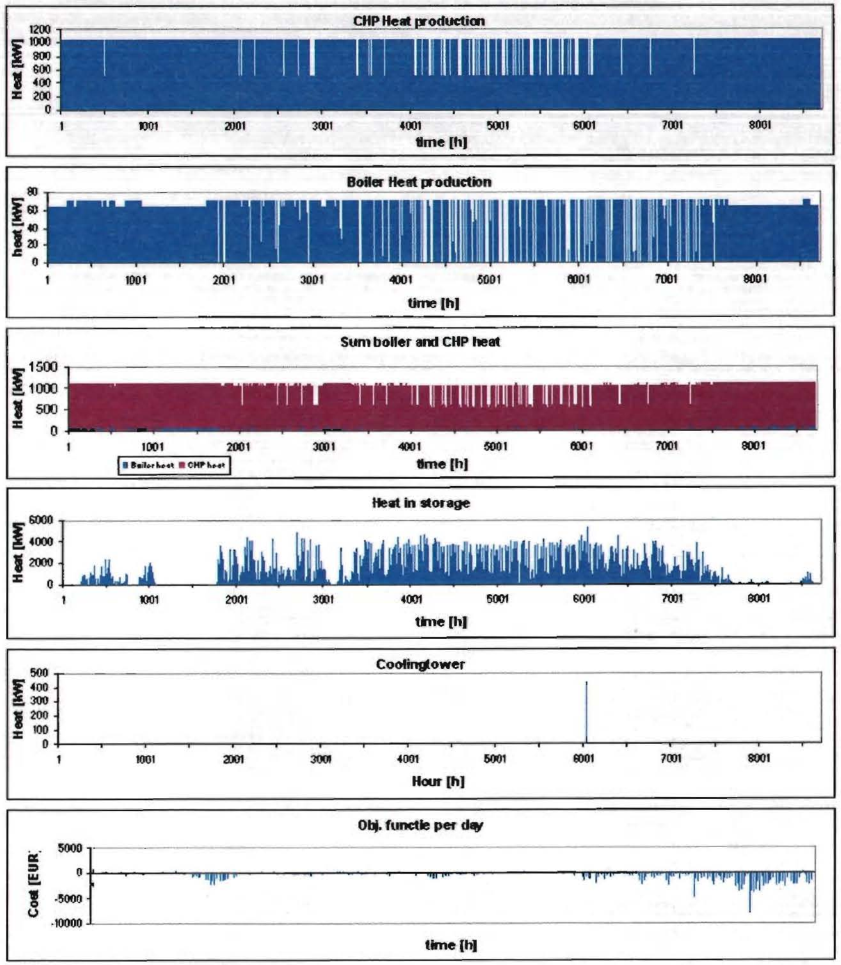


Figure F 2: Results using 2 co-generation units for heat production and a limited boiler

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	0	0	396000	296712	396400	396000	0
Run hours	0	0	990	951	991	990	0

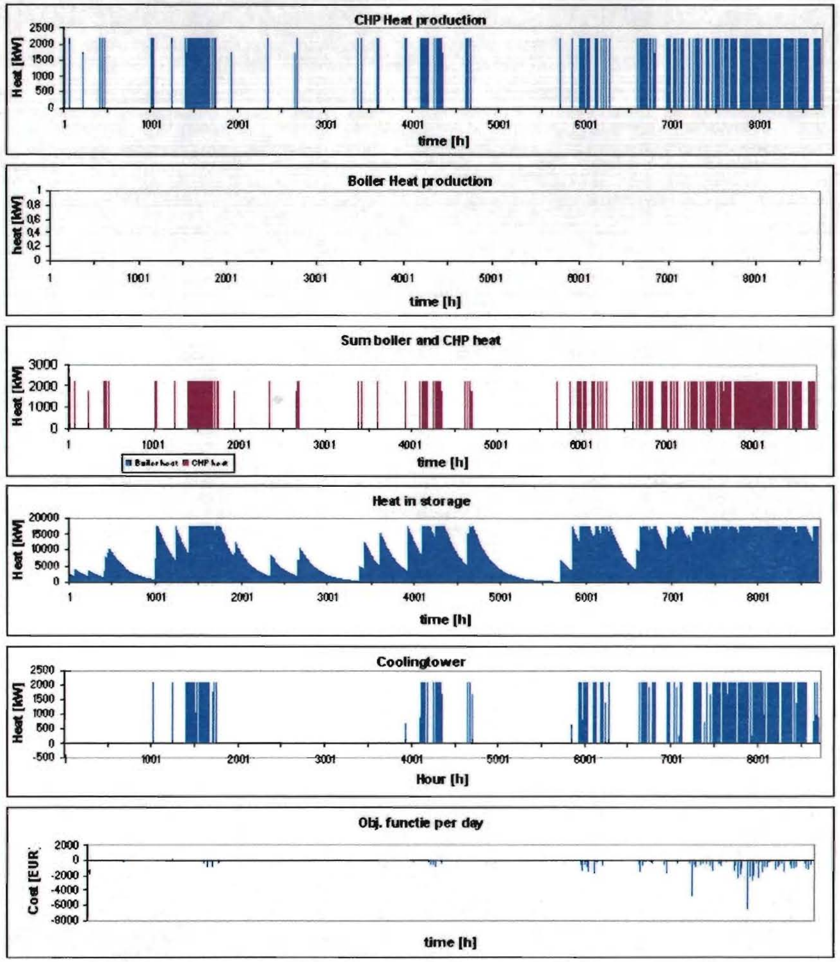


Figure F 3: Results using 4 co-generation units only for electricity production.

Appendix G. Overview results case 2

Table 25: 24h, 36h or 48h model

Optimizing period	Produced power [MWh]	Obj. value [EUR]
24 h	9.474	- 328.440,-
36 h	10.284	- 336.132,-
48 h	10.454	- 332.337,-

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	789.078	796.018	499.600	316.368	947.600	494.400	5.609.337
Run hours	2.274	2.294	1.249	1.014	2.369	1.236	6.233

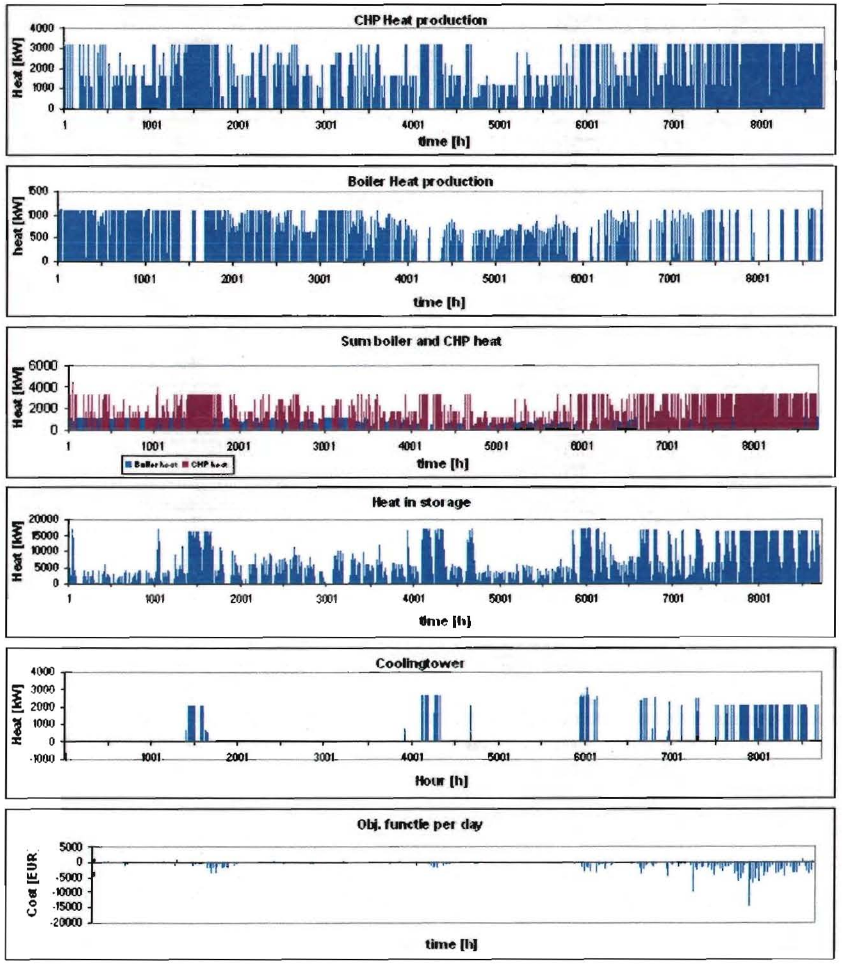


Figure G 2: Results optimizing 6 co-generation units and 300m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	932.389	948.004	693.200	407.784	1.003.200	689.600	5.609.337
Run hours	2.687	2.732	1.733	1.307	2.508	1.724	6.233

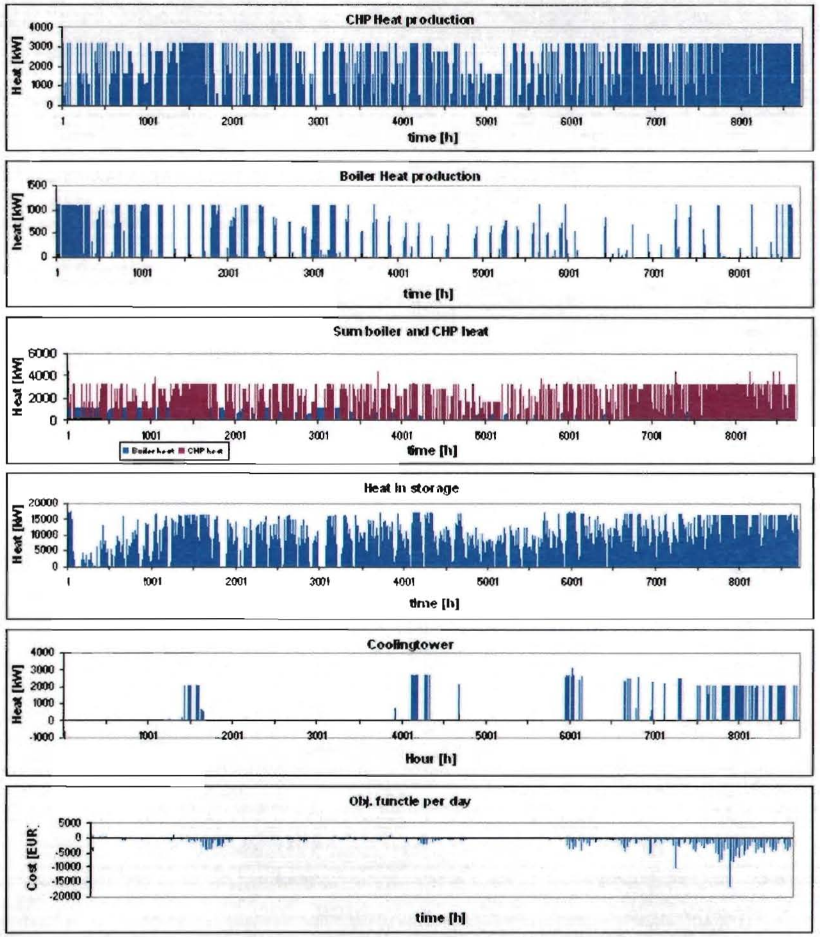


Figure G 2: Results optimizing 6 co-generation units and 300m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	995890	1009423	701600	423072	1011600	704000	5609337
Run hours	2870	2909	1754	1057,68	3242,308	1760	6232,596

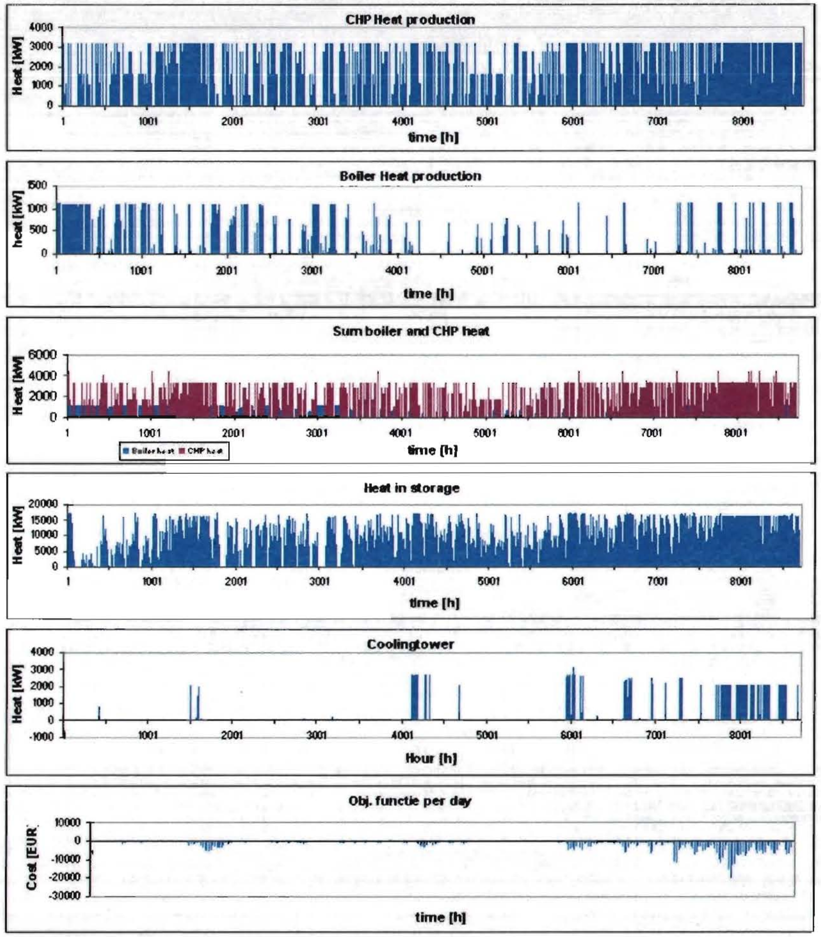


Figure G 3: Results optimizing 6 co-generation units and 300m3 storage over a period of 48h

Appendix H. Overview results case 3

Storage size	# CHP plants	APX hourly value		APX peak / off peak	
		24 uur	36 uur	24 uur	36 uur
300	6	328.440,81-	336.132,88-	293.994,33-	299.959,65-

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.367.874	1.353.300	-	-	-	-	5.630.937
Run hours	3.942	3.900	-	-	-	-	6.257

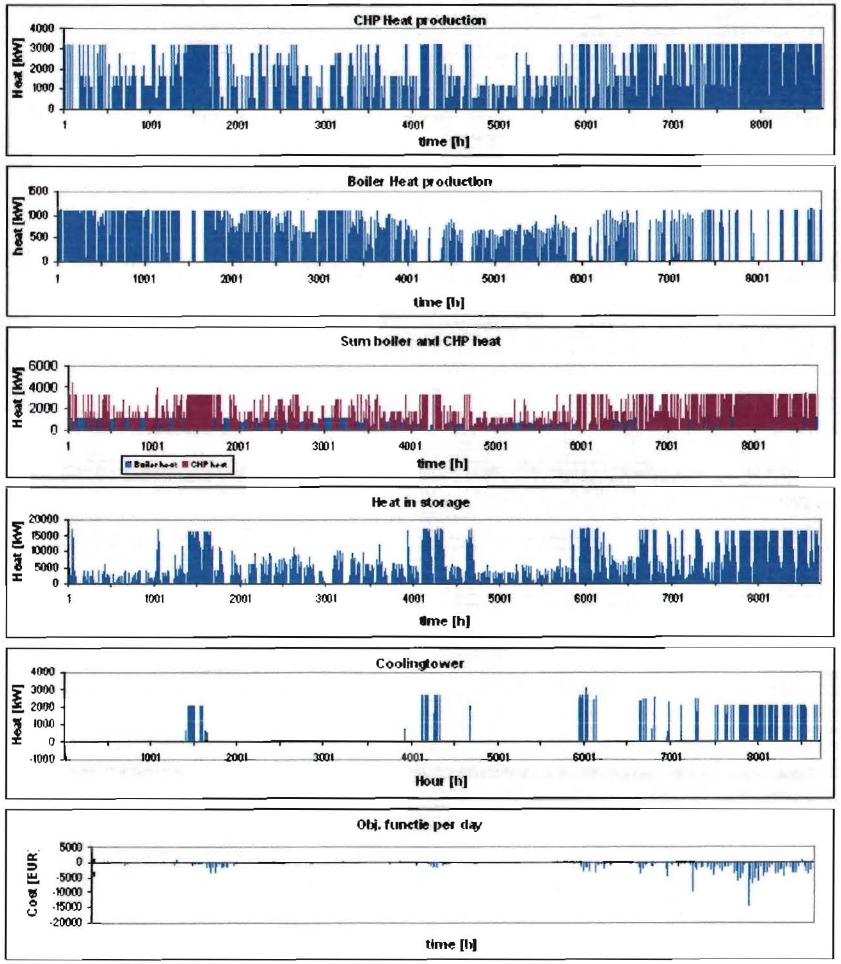


Figure H 3: Results optimizing 6 co-generation units and 300m3 storage over a period of 24h and using APX hourly values

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.367.874	1.353.300	-	-	-	-	5.630.937
Run hours	3.942	3.900	-	-	-	-	6.257

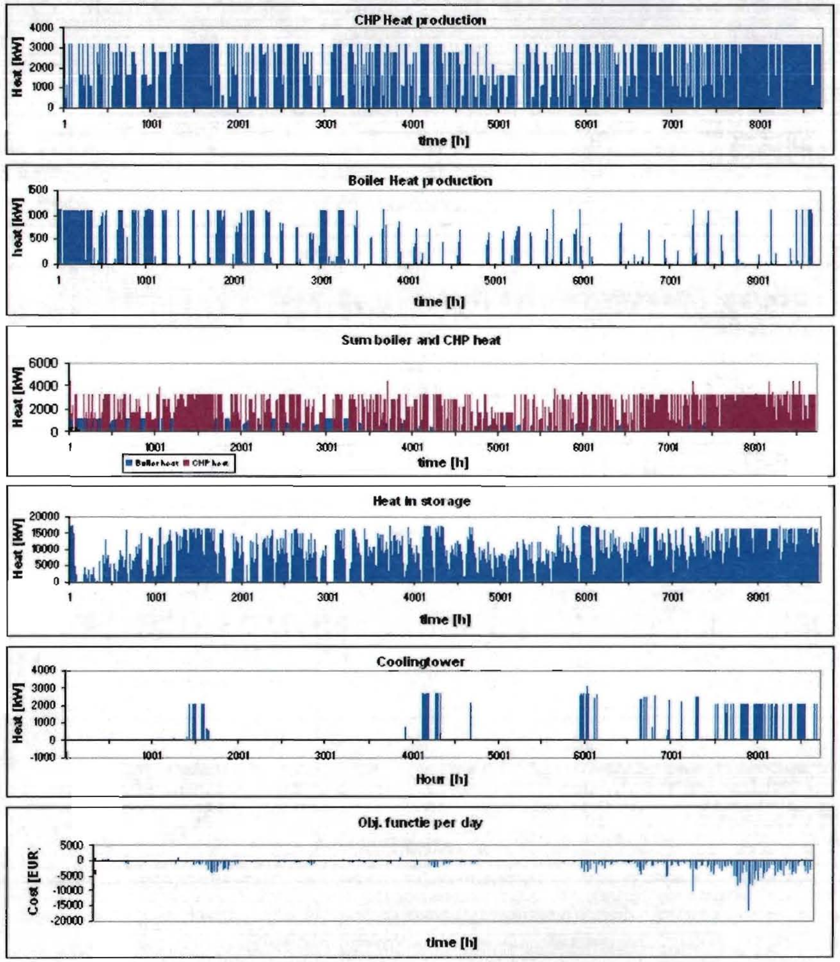


Figure H 2: Results optimizing 6 co-generation units and 300m3 storage over a period of 36h and using APX hourly values

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.367.874	1.353.300	-	-	-	-	5.630.937
Run hours	3.942	3.900	-	-	-	-	6.257

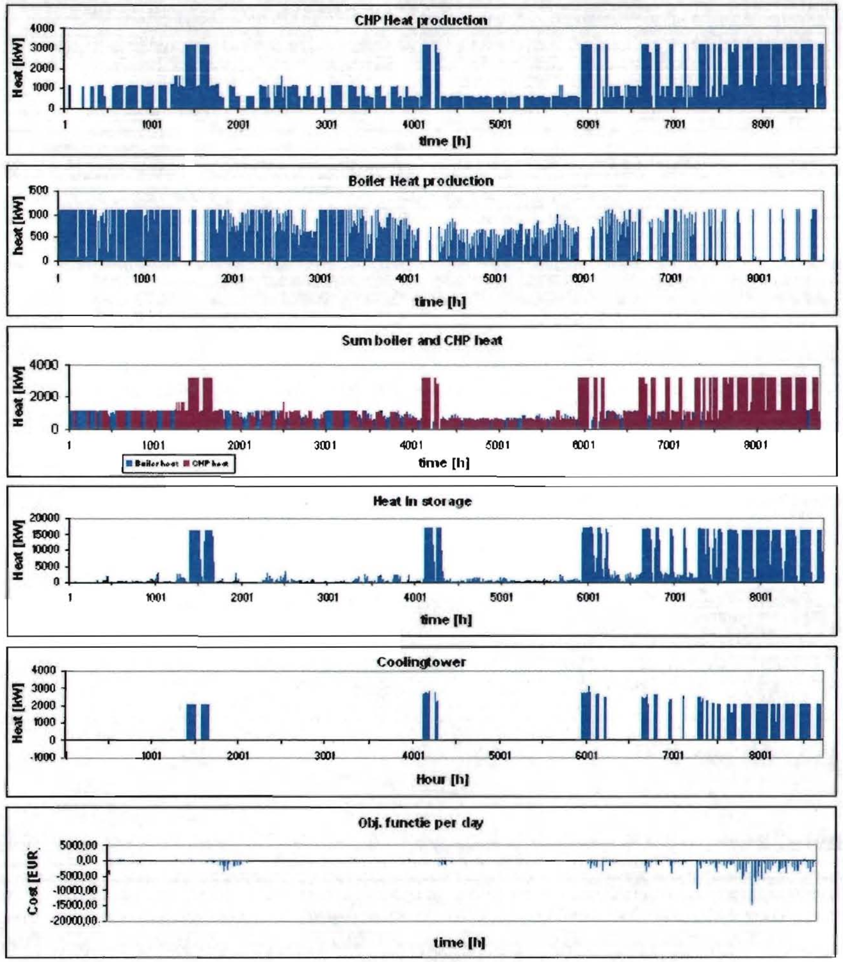


Figure H 3: Results optimizing 6 co-generation units and 300m3 storage over a period of 24h and using APX peak and off-peak values

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.367.874	1.353.300	-	-	-	-	5.630.937
Run hours	3.942	3.900	-	-	-	-	6.257

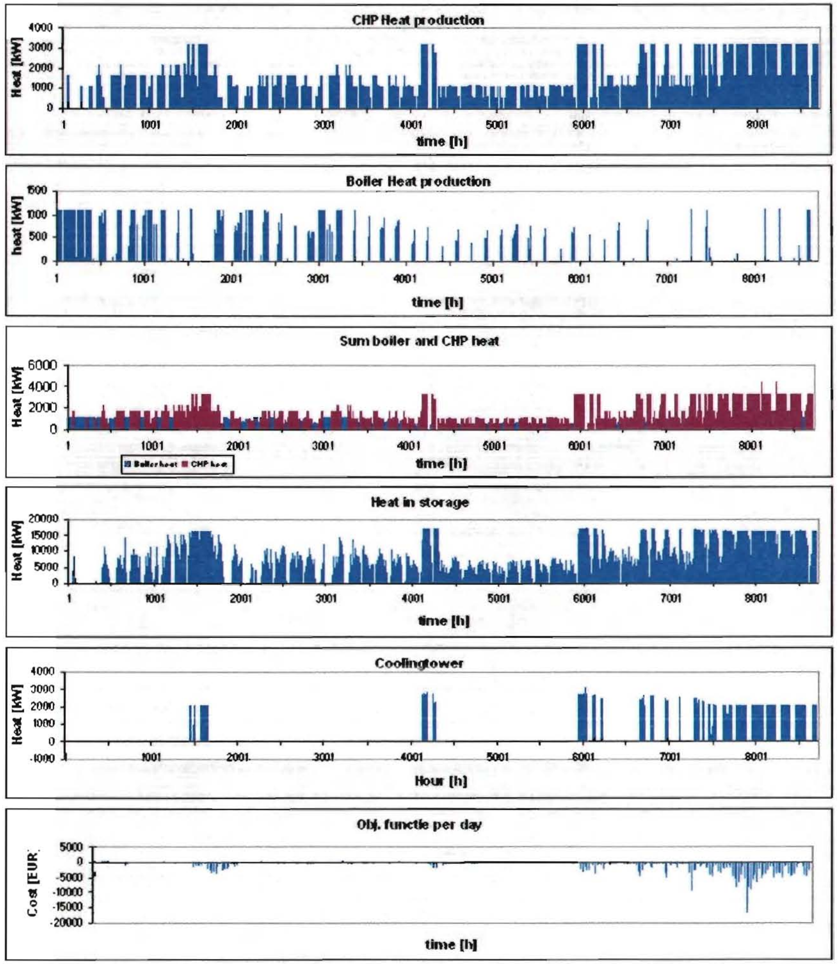


Figure H 4: Results optimizing 6 co-generation units and 300m3 storage over a period of 36h and using APX peak and off-peak values

Appendix I. Overview results case 4

Storage size	# CHP plants	Percentage of max capacity	hourly value 24 uur	[MWh]
100	6	50%	318.386,45-	10.111
150	6	50%	329.128,67-	10.243
200	6	50%	335.390,10-	10.320
250	6	50%	337.713,05-	10.355
300	6	50%	337.957,71-	10.381
300	6	0%	328.440,81-	9.474
300	6	95%	313.841,50-	10.580

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1046552	1039612	480400	323544	1120000	491200	5609337
Run hours	3016	2996	1201	809	3590	1228	6232,596

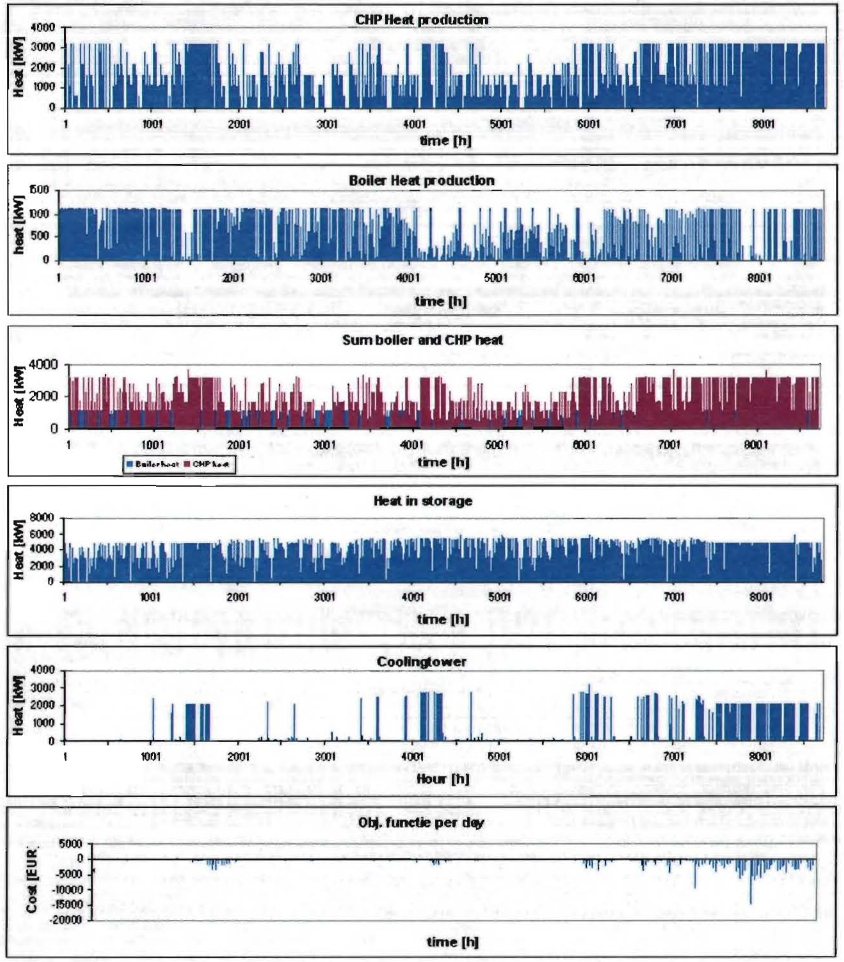


Figure I 4: Results optimizing 6 co-generation units and 100m3 storage minimum 50% filled at the end of the day, over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1039612	1057656	563200	335712	1076000	561200	5609337
Run hours	2996	3048	1408	839	3449	1403	6232,596

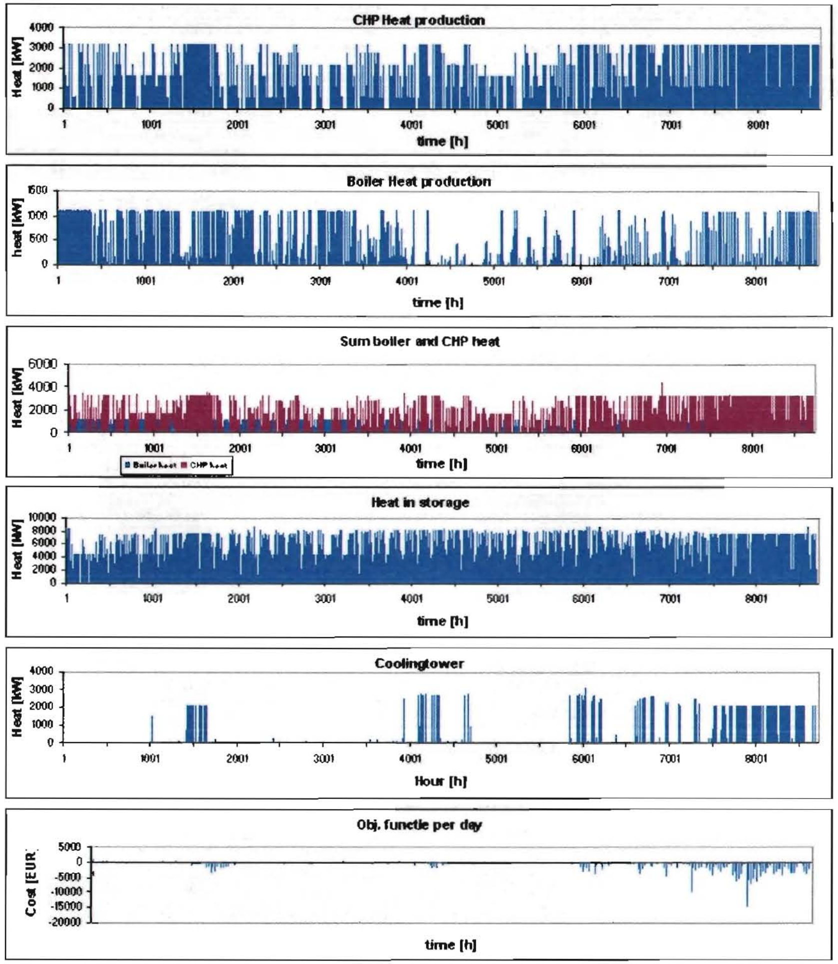


Figure I 2: Results optimizing 6 co-generation units and 150m3 storage minimum 50% filled at the end of the day, over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1034754	977846	644400	373152	1036400	644000	5609337
Run hours	2982	2818	1611	933	3322	1610	6232,596

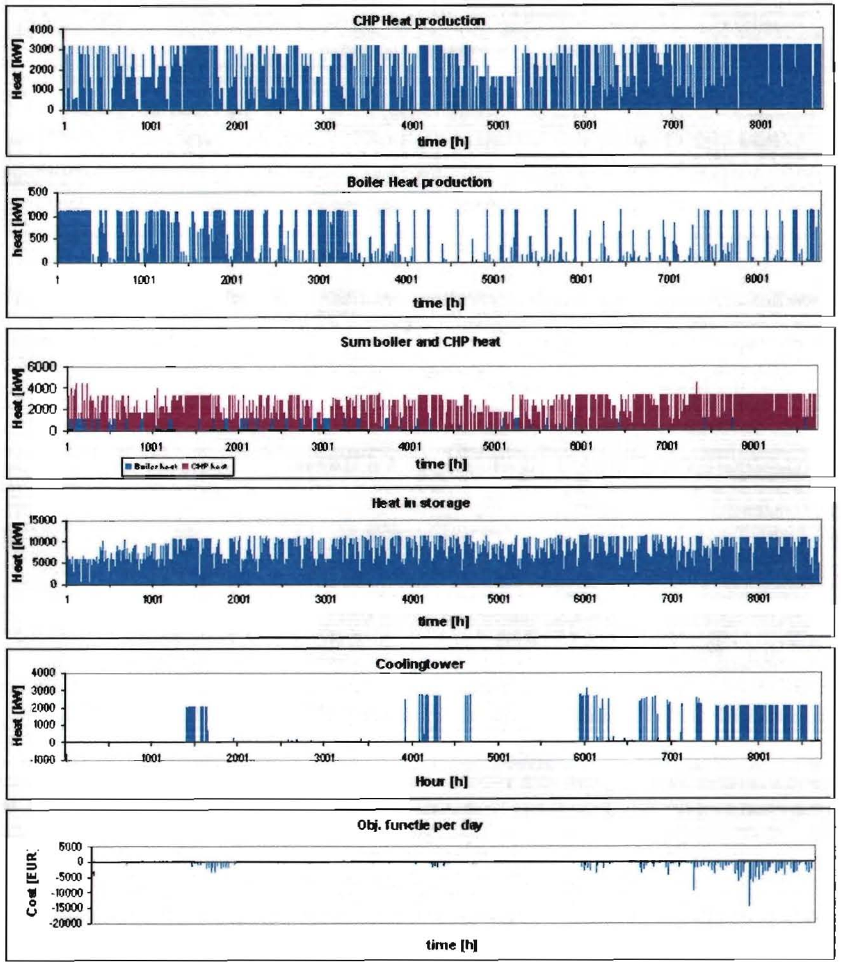


Figure I 3: Results optimizing 6 co-generation units and 200m3 storage minimum 50% filled at the end of the day, over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	979581	968477	681200	399048	1025600	691600	5609337
Run hours	2823	2791	1703	998	3287	1729	6232,596

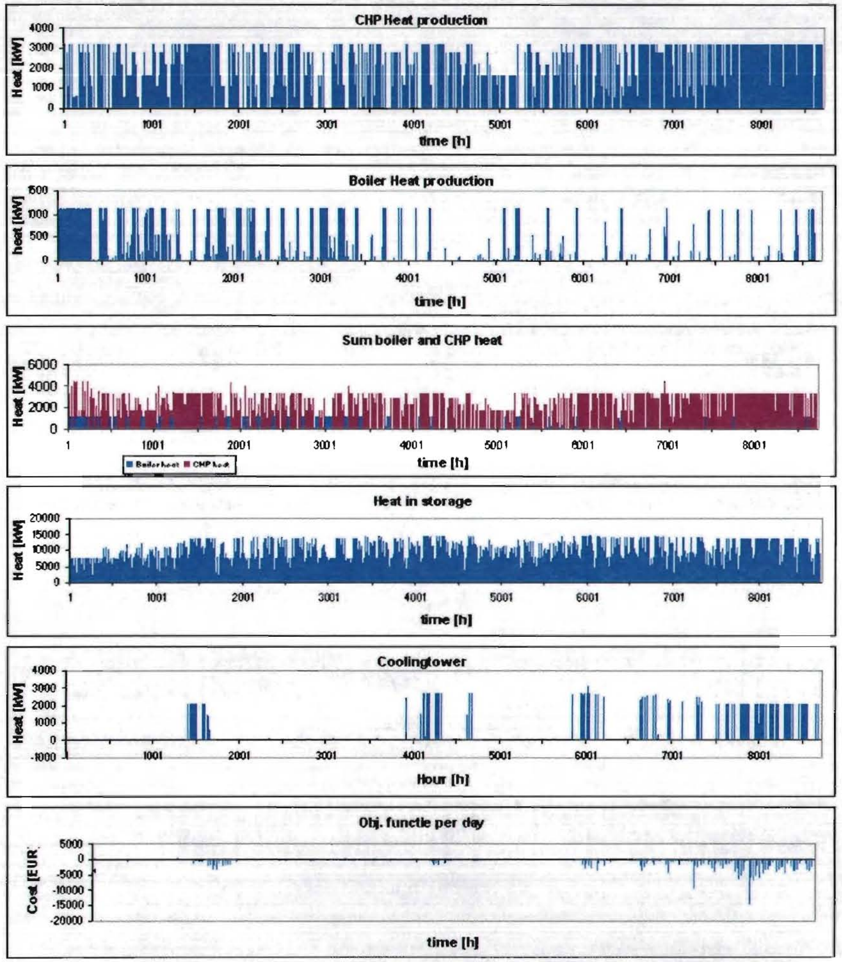


Figure I 4: Results optimizing 6 co-generation units and 250m3 storage minimum 50% filled at the end of the day, over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	968824	970906	701200	417456	1012800	700000	5609337
Run hours	2792	2798	1753	1044	3246	1750	6232,596

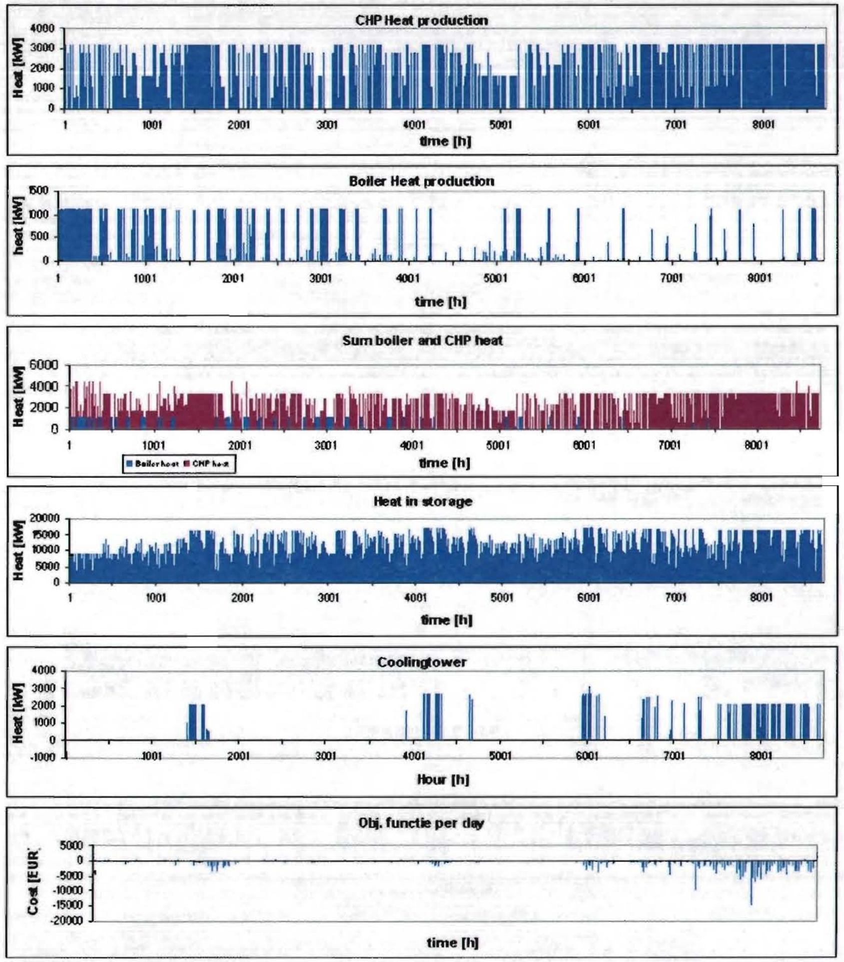


Figure I 5: Results optimizing 6 co-generation units and 300m3 storage minimum 50% filled at the end of the day, over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	789.078	796.018	499.600	316.368	947.600	494.400	5.609.337
Run hours	2.274	2.294	1.249	1.014	2.369	1.236	6.233

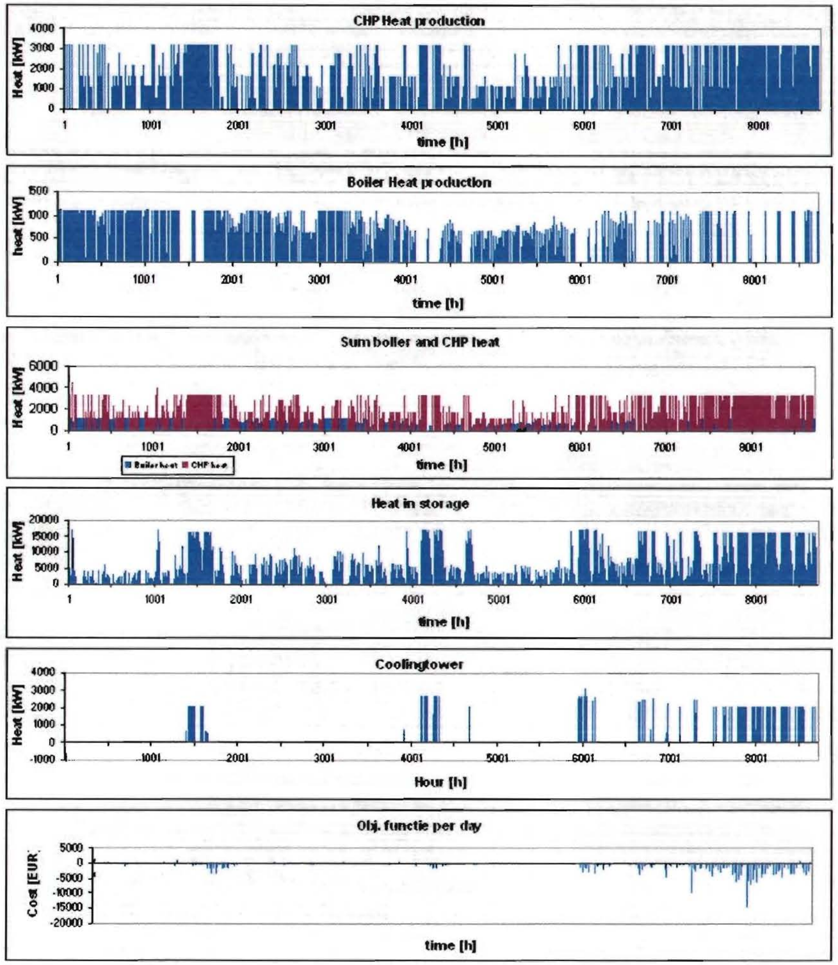


Figure I 6: Results optimizing 6 co-generation units and 300m3 storage which is filled for minimum 0% at the end of the day, over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1164185	1079864	578800	365664	1216800	587200	5587958
Run hours	3355	3112	1447	1172	3042	1468	6208,842

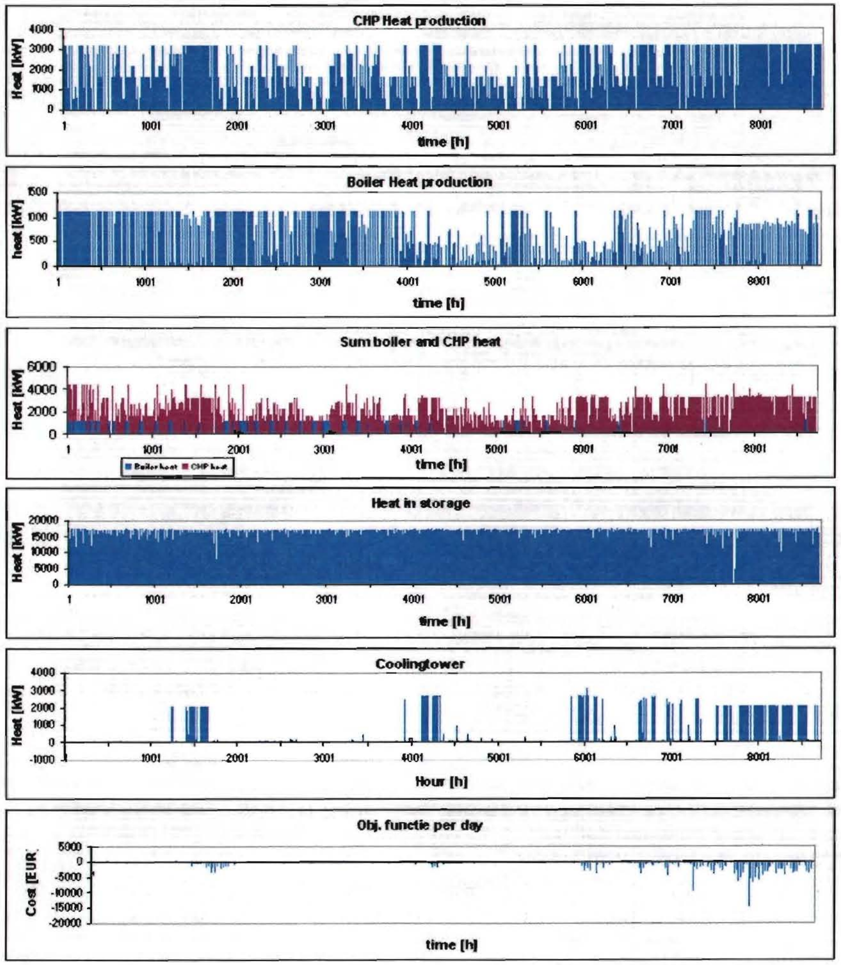


Figure I 7: Results optimizing 6 co-generation units and 300m3 storage which is filled for minimum 95% at the end of the day, over a period of 24h

Appendix J. Overview results case 5

Storage size	# CHP plants	APX hourly value 24 uur	36 uur
100	2	217.566,13-	222.222,58-
150	2	217.267,72-	221.871,00-
200	2	217.566,14-	222.403,91-
250	2	217.566,13-	222.459,95-
300	2	217.566,13-	221.860,04-
100	3	249.718,67-	255.091,64-
150	3	252.588,85-	262.971,69-
200	3	252.931,27-	264.461,88-
250	3	252.995,57-	264.708,87-
300	3	252.332,39-	264.197,30-
100	4	267.837,73-	271.308,46-
150	4	272.914,48-	280.323,38-
200	4	276.205,53-	286.389,09-
250	4	276.994,84-	288.494,84-
300	4	277.023,06-	288.238,70-
100	5	292.097,64-	294.461,14-
150	5	298.300,10-	301.838,75-
200	5	301.825,79-	309.764,74-
250	5	303.568,00-	312.871,50-
300	5	303.978,35-	313.019,15-
100	6	314.940,63-	315.621,74-
150	6	321.952,75-	324.883,86-
200	6	326.585,65-	332.117,68-
250	6	328.360,99-	335.005,65-
300	6	328.440,81-	336.132,88-

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.367.874	1.353.300	-	-	-	-	5.609.337
Run hours	3.942	3.900	-	-	-	-	6.233

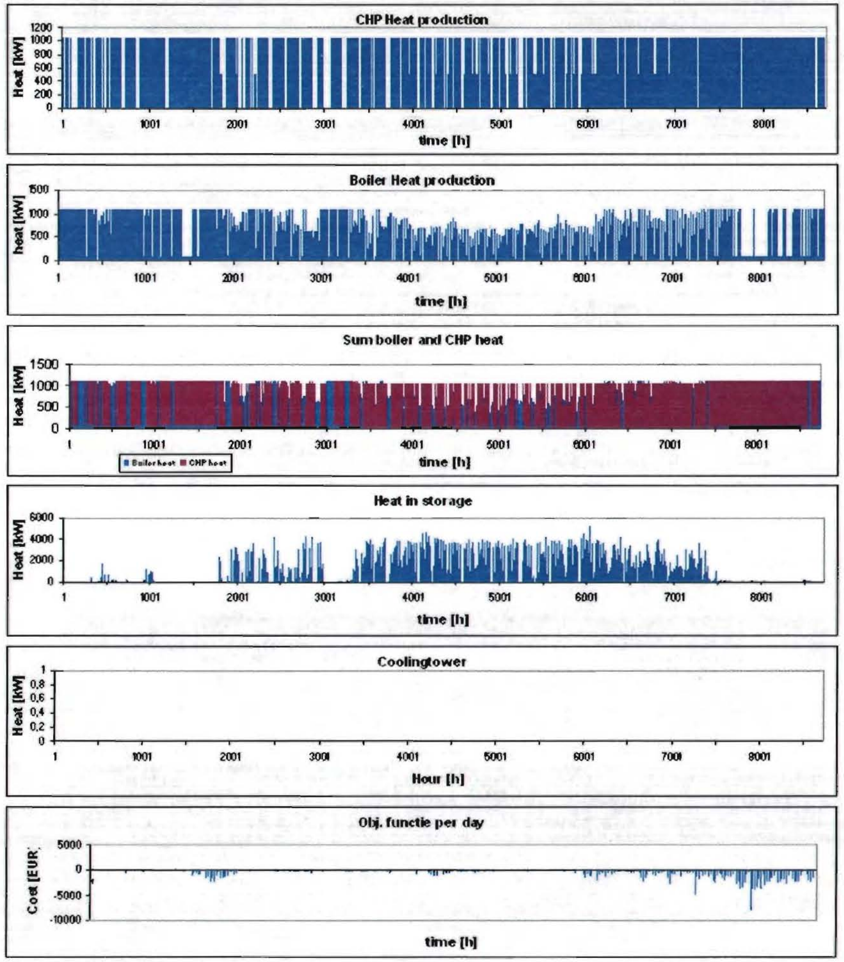


Figure J 5: Results optimizing 2 co-generation units and 100m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.380.713	1.346.707	-	-	-	-	5.609.337
Run hours	3.979	3.881	-	-	-	-	6.233

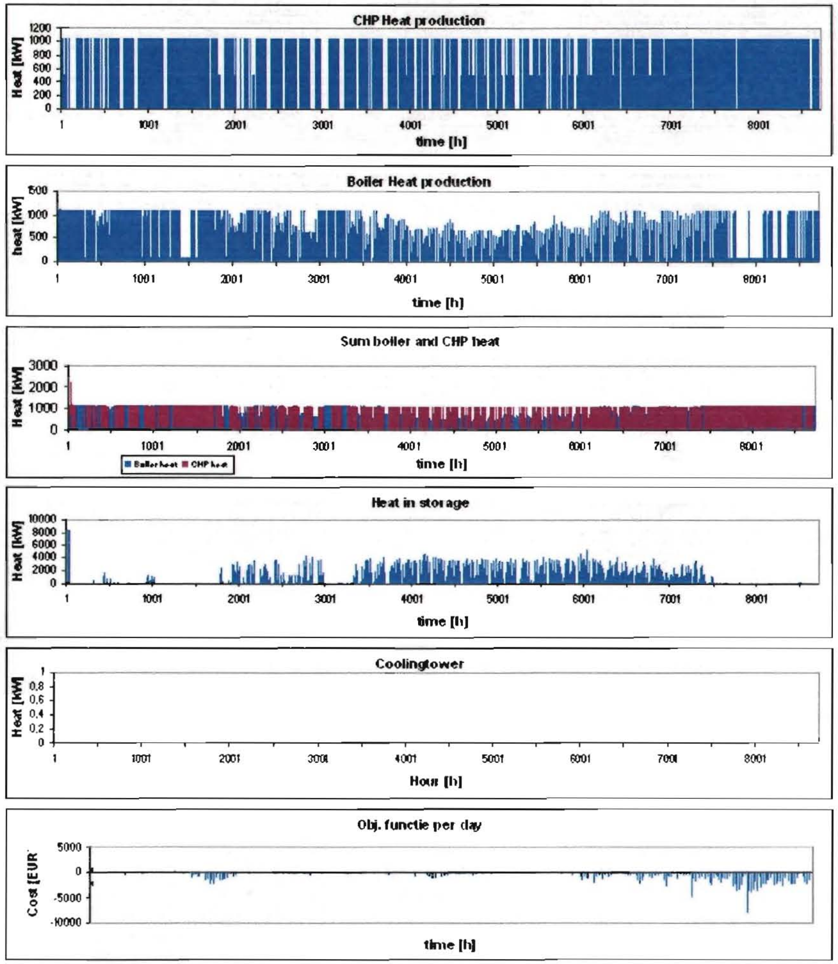


Figure J 6: Results optimizing 2 co-generation units and 150m^3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.372.732	1.348.442	-	-	-	-	5.609.337
Run hours	3.956	3.886	-	-	-	-	6.233

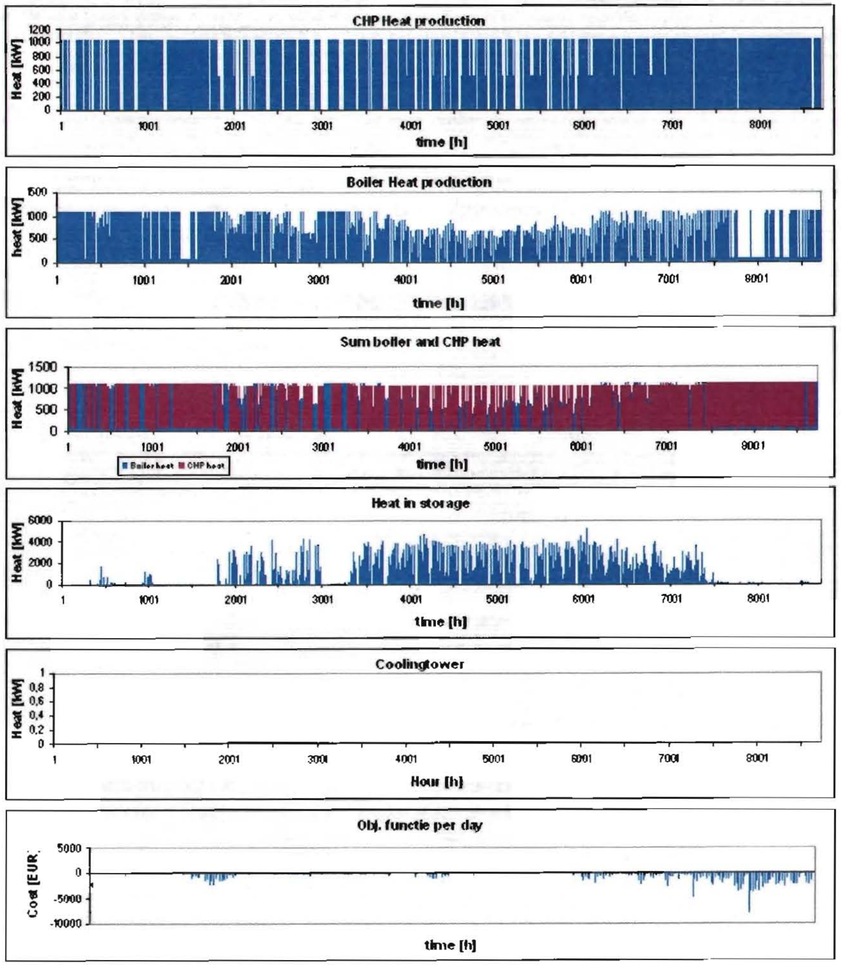


Figure J 7: Results optimizing 2 co-generation units and 200m^3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.371.691	1.349.483	-	-	-	-	5.609.337
Run hours	3.953	3.889	-	-	-	-	6.233

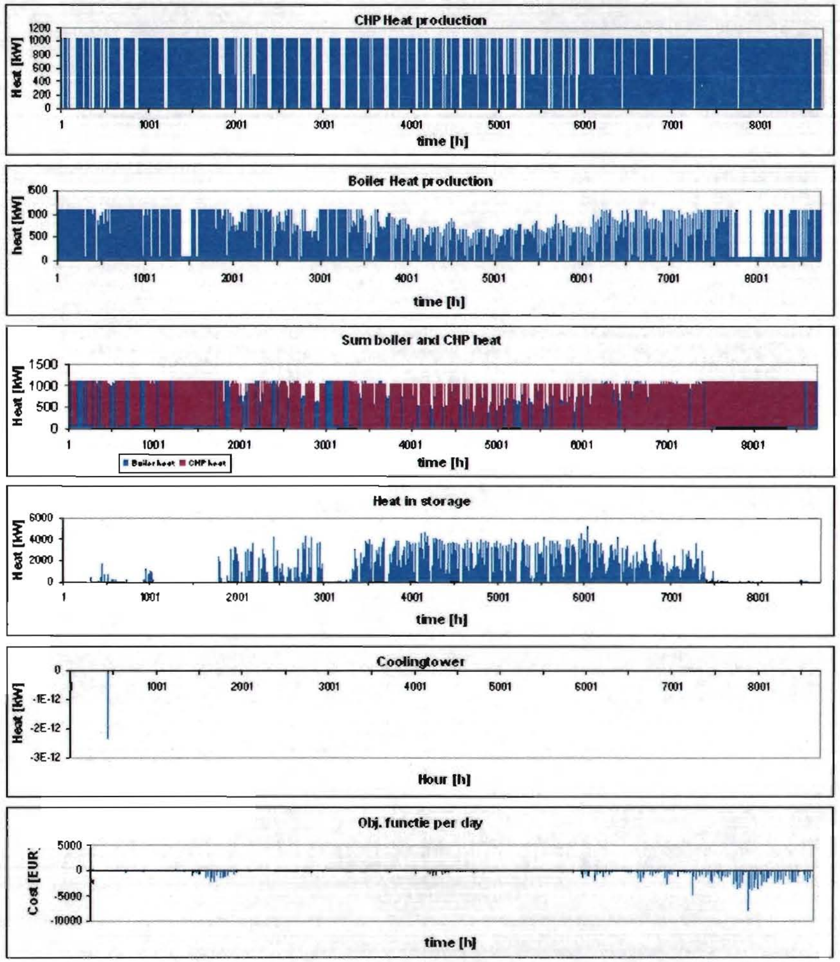


Figure J 8: Results optimizing 2 co-generation units and 250m^3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.372.385	1.348.789	-	-	-	-	5.609.337
Run hours	3.955	3.887	-	-	-	-	6.233

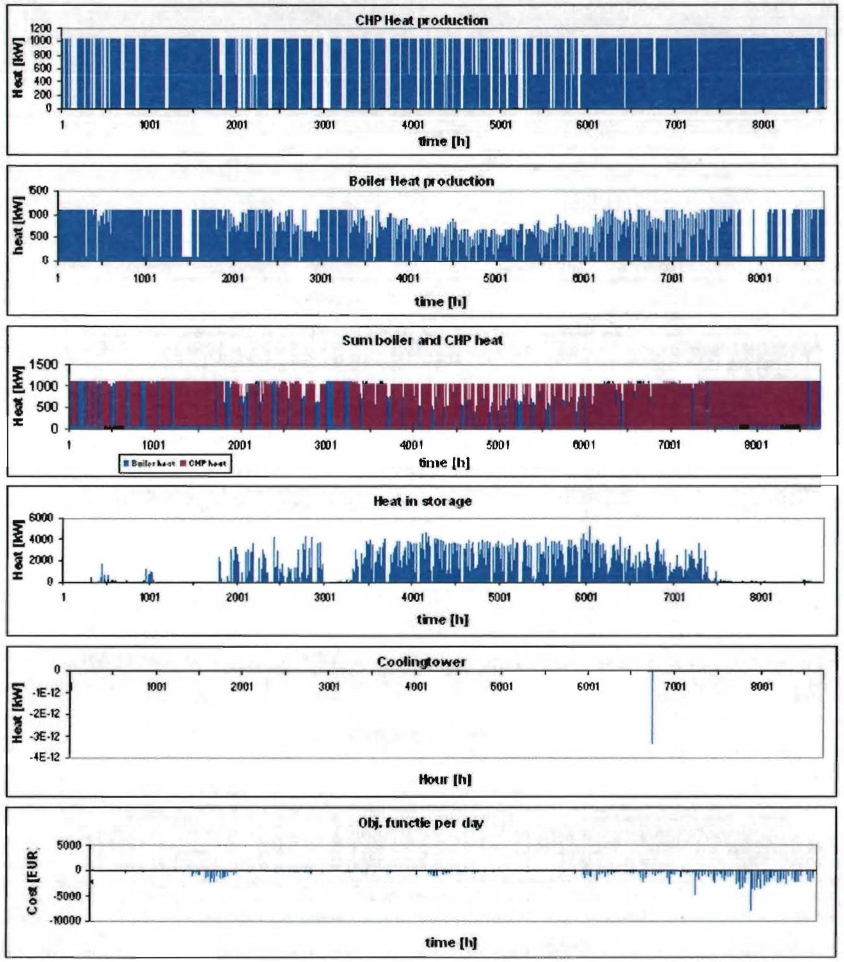


Figure J 9: Results optimizing 2 co-generation units and 300m^3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.163.491	1.110.400	748.400	-	-	-	5.609.337
Run hours	3.353	3.200	1.871	-	-	-	6.233

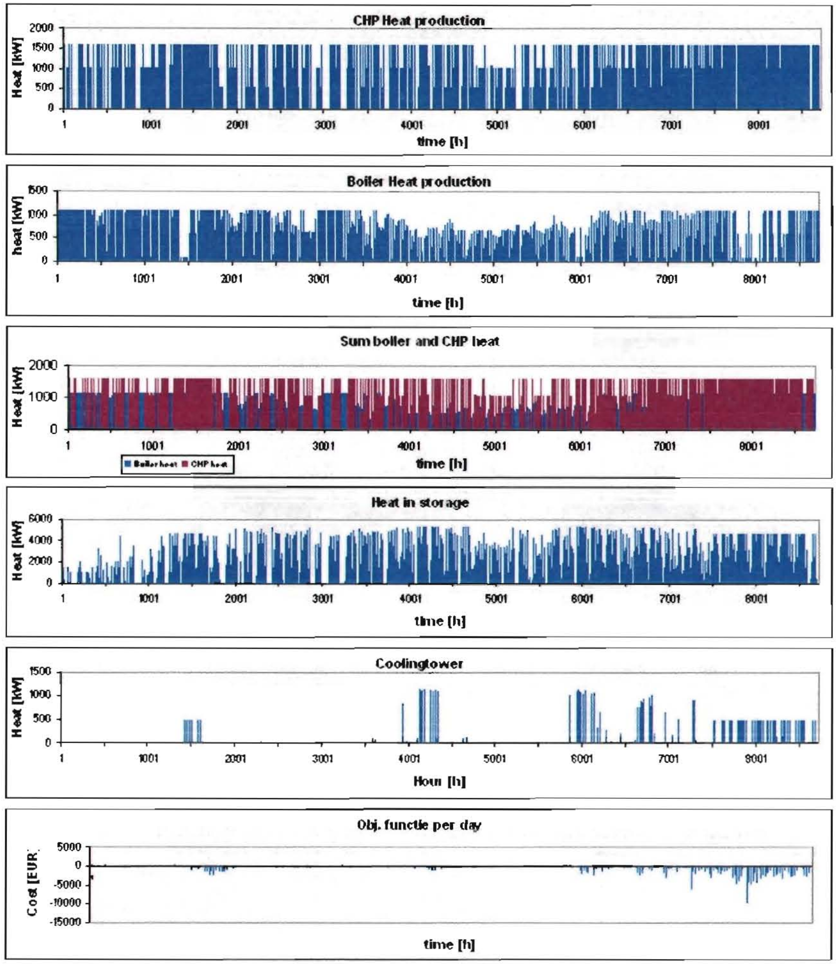


Figure J 6: Results optimizing 3 co-generation units and 100m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.122.892	1.079.170	789.200	-	-	-	5.609.337
Run hours	3.236	3.110	1.973	-	-	-	6.233

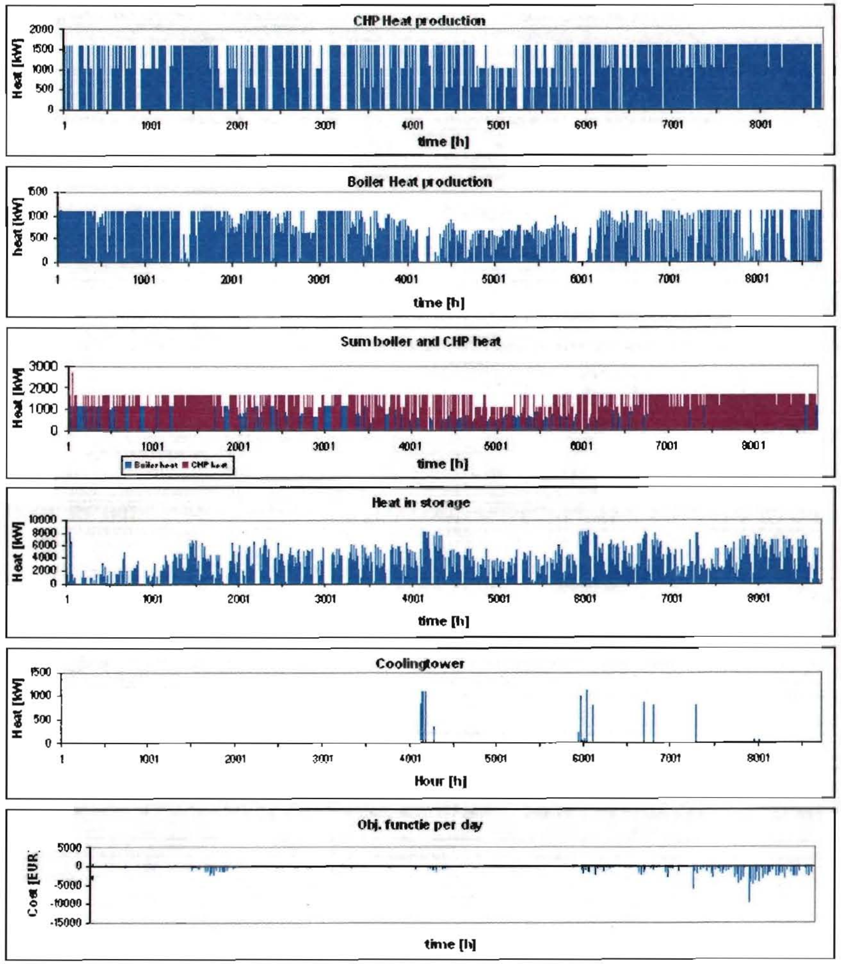


Figure J 7: Results optimizing 3 co-generation units and 150m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.123.586	1.070.495	784.000	-	-	-	5.609.337
Run hours	3.238	3.085	1.960	-	-	-	6.233

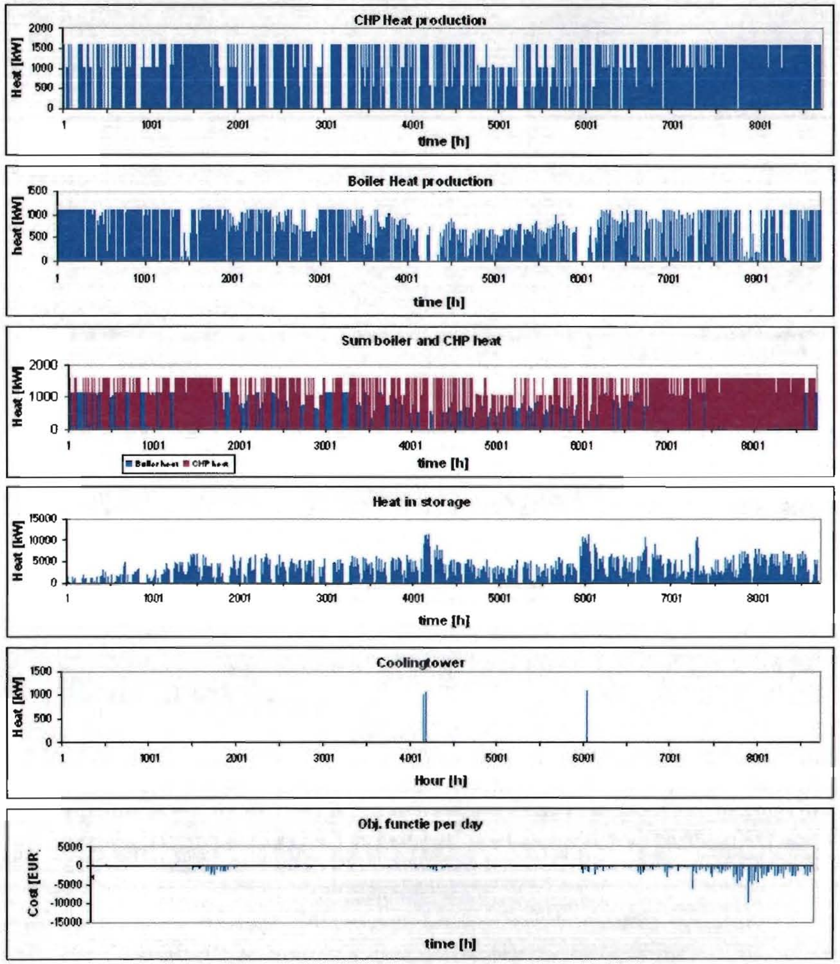


Figure J 8: Results optimizing 3 co-generation units and 200m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.118.728	1.074.312	782.400	-	-	-	5.609.337
Run hours	3.224	3.096	1.956	-	-	-	6.233

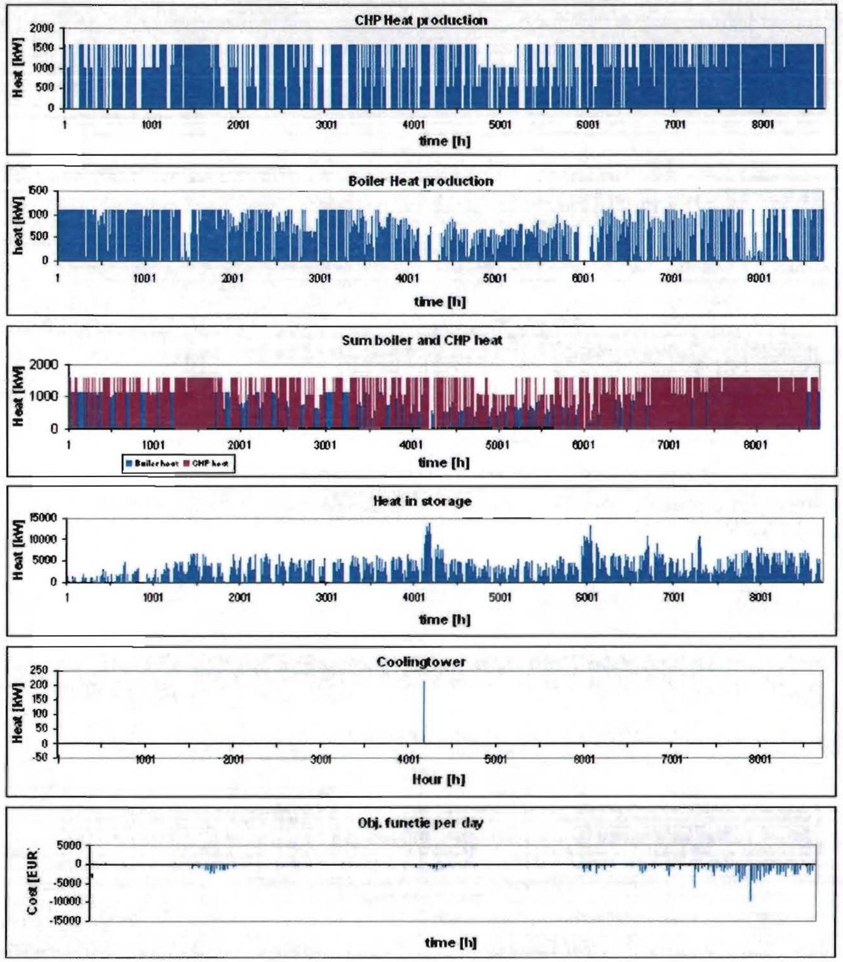


Figure J 9: Results optimizing 3 co-generation units and 250m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.114.911	1.081.599	790.400	-	-	-	5.609.337
Run hours	3.213	3.117	1.976	-	-	-	6.233

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.138.160	1.087.845	702.800	361.296	-	-	5.609.337
Run hours	3.280	3.135	1.757	1.158	-	-	6.233

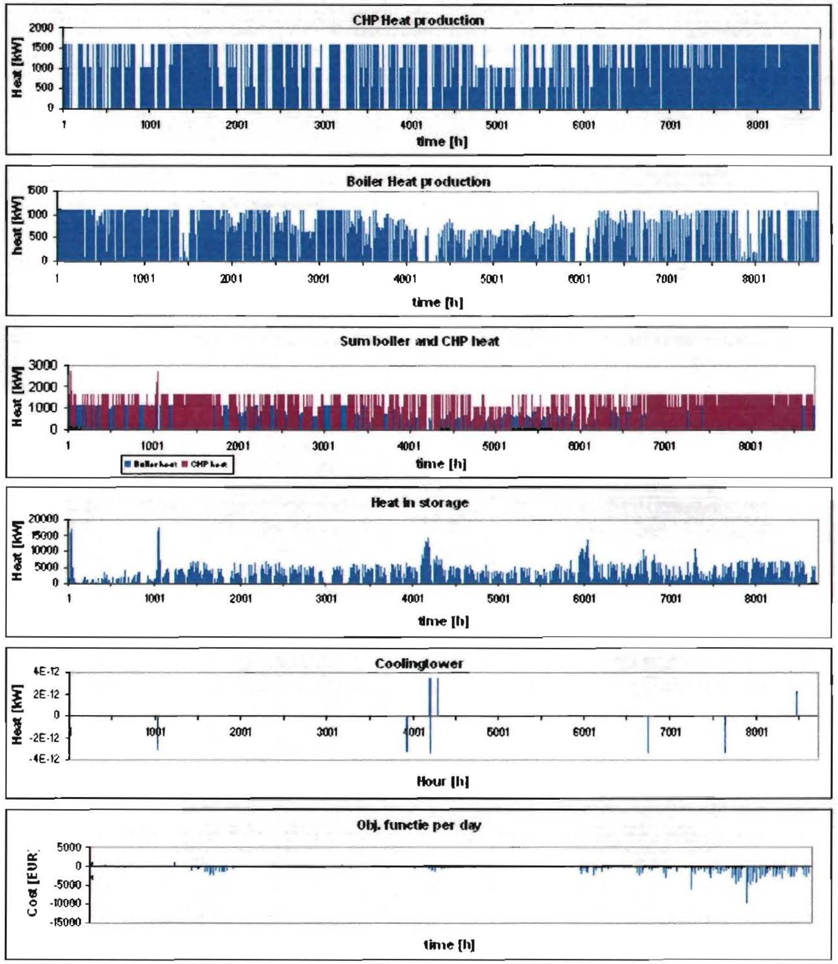


Figure J 10: Results optimizing 3 co-generation units and 300m3 storage over a period of 24h

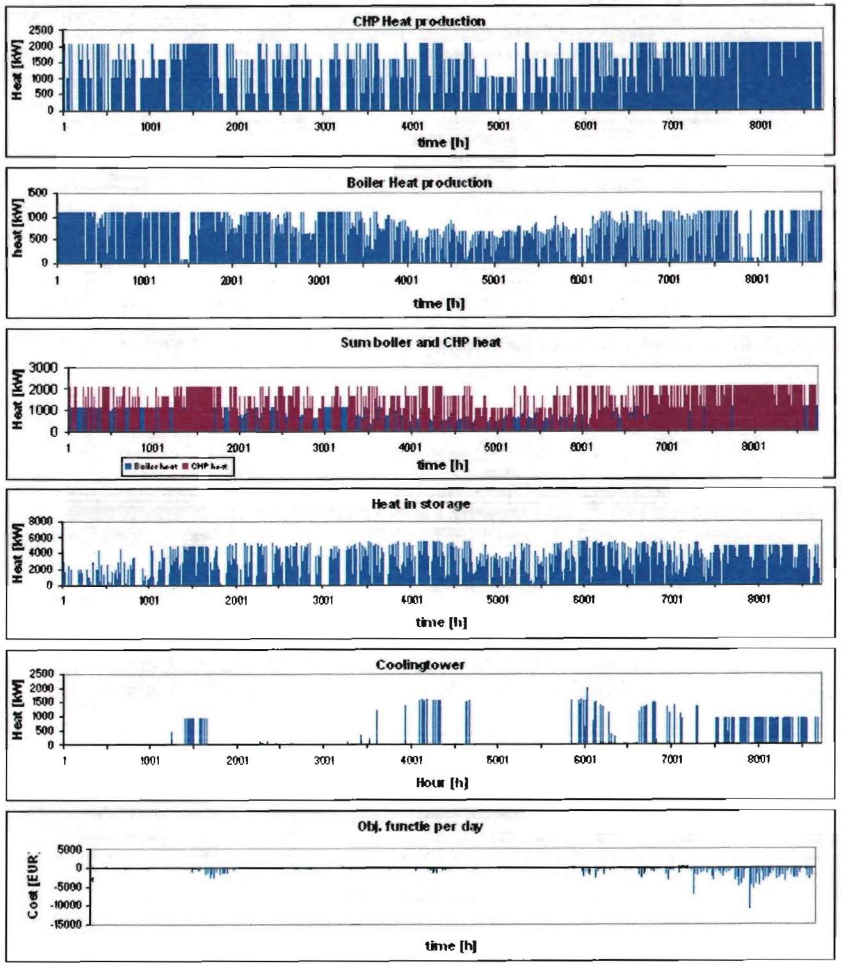


Figure J 11: Results optimizing 4 co-generation units and 100m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.071.883	1.034.060	726.000	407.784	-	-	5.609.337
Run hours	3.089	2.980	1.815	1.307	-	-	6.233

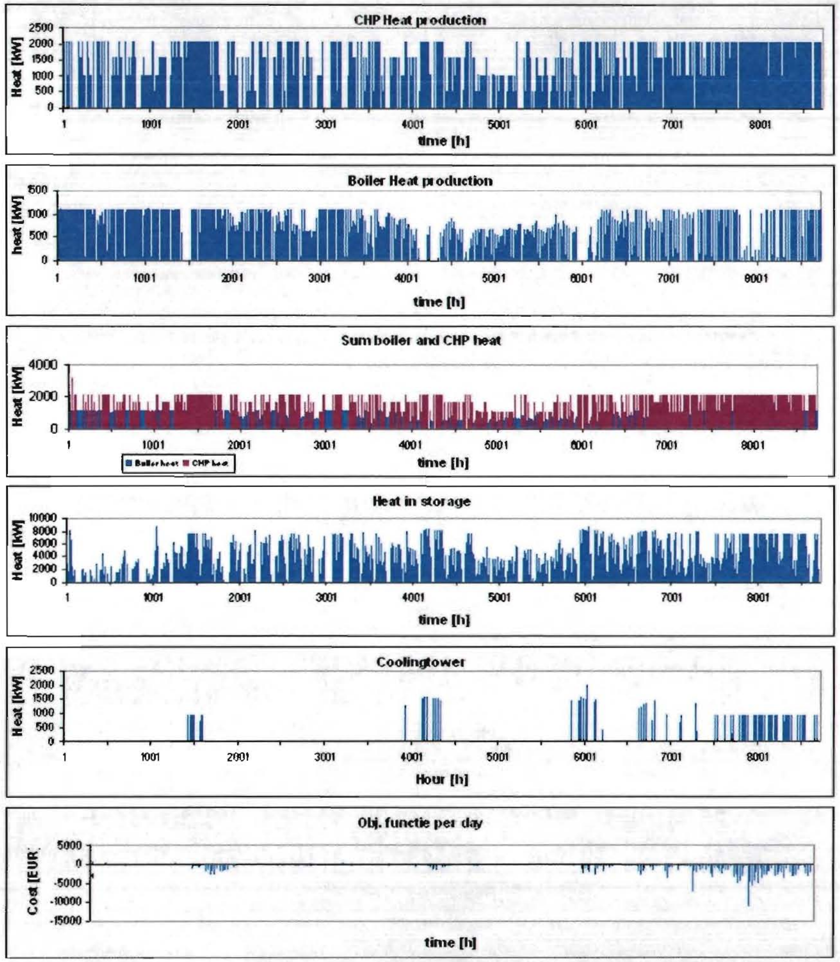


Figure J 12: Results optimizing 4 co-generation units and 150m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.040.653	1.013.240	722.800	405.600	-	-	5.609.337
Run hours	2.999	2.920	1.807	1.300	-	-	6.233

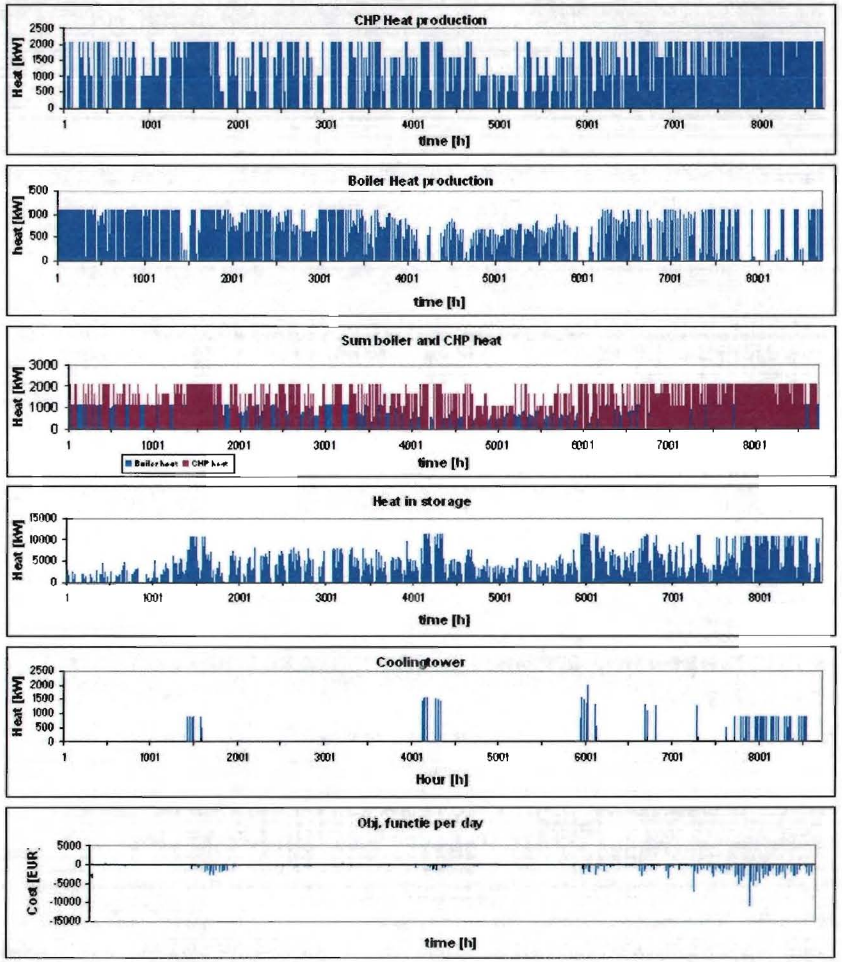


Figure J 13: Results optimizing 4 co-generation units and 200m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.030.243	1.005.606	717.200	404.976	-	-	5.609.337
Run hours	2.969	2.898	1.793	1.298	-	-	6.233

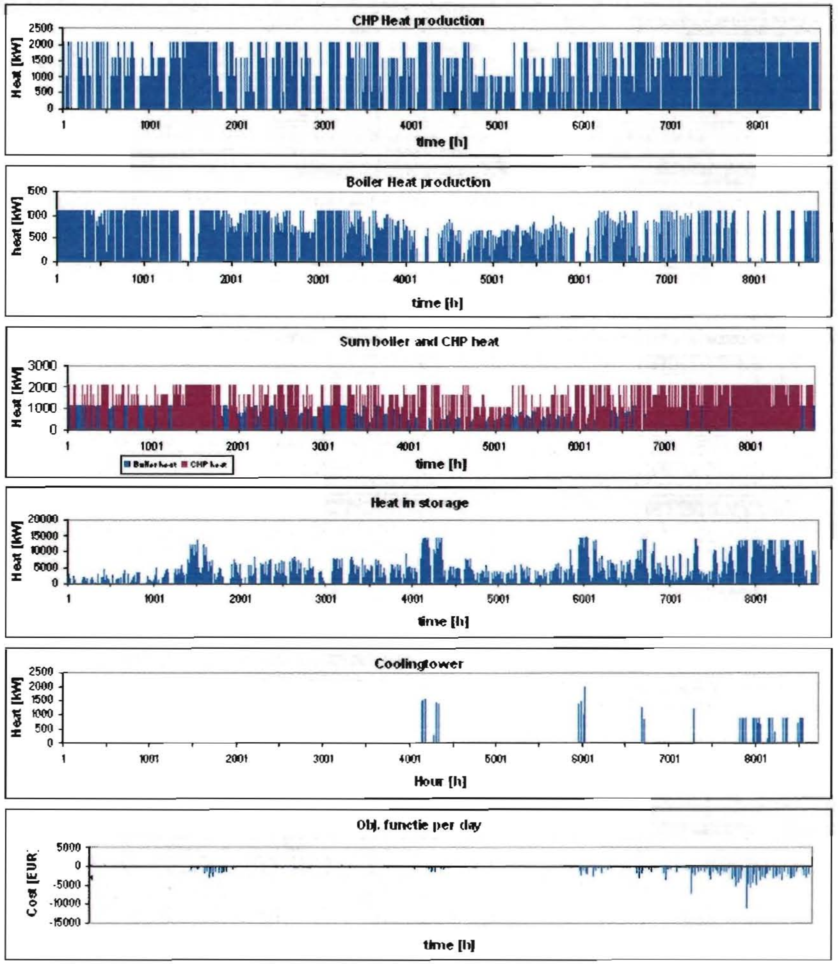


Figure J 14: Results optimizing 4 co-generation units and 250m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.029.549	1.002.830	714.800	406.848	-	-	5.609.337
Run hours	2.967	2.890	1.787	1.304	-	-	6.233

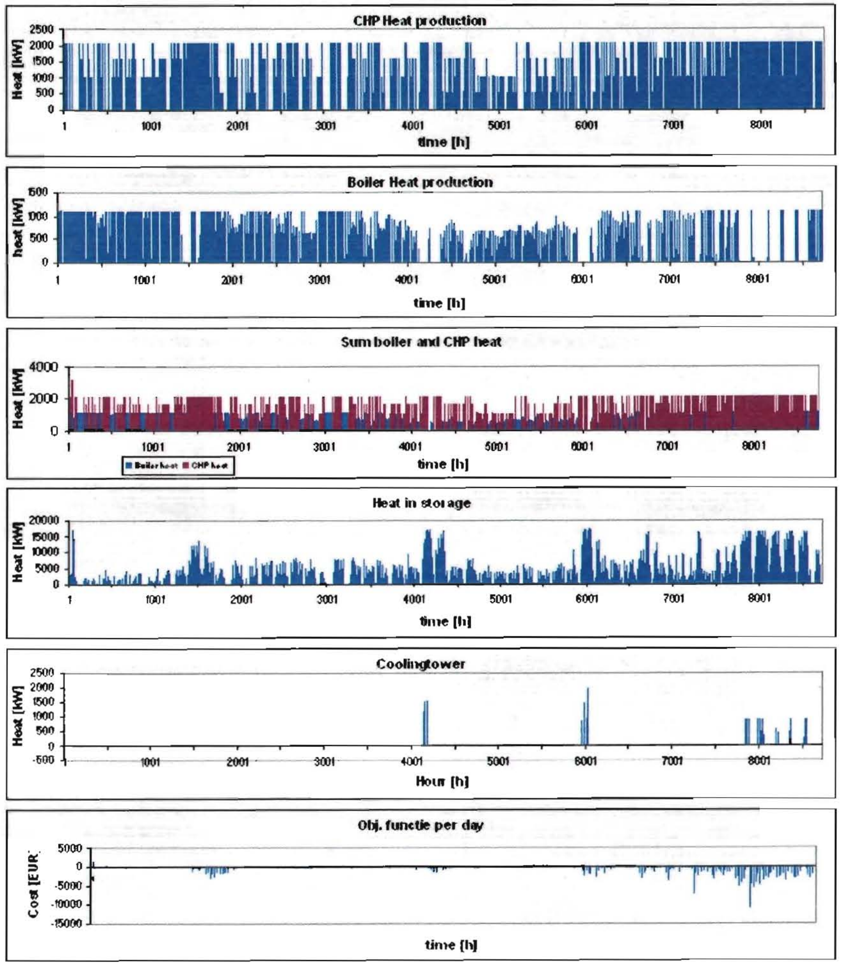


Figure J 15: Results optimizing 4 co-generation units and 300m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	920.591	882.768	490.800	316.056	1.058.000	-	5.609.337
Run hours	2.653	2.544	1.227	1.013	2.645	-	6.233

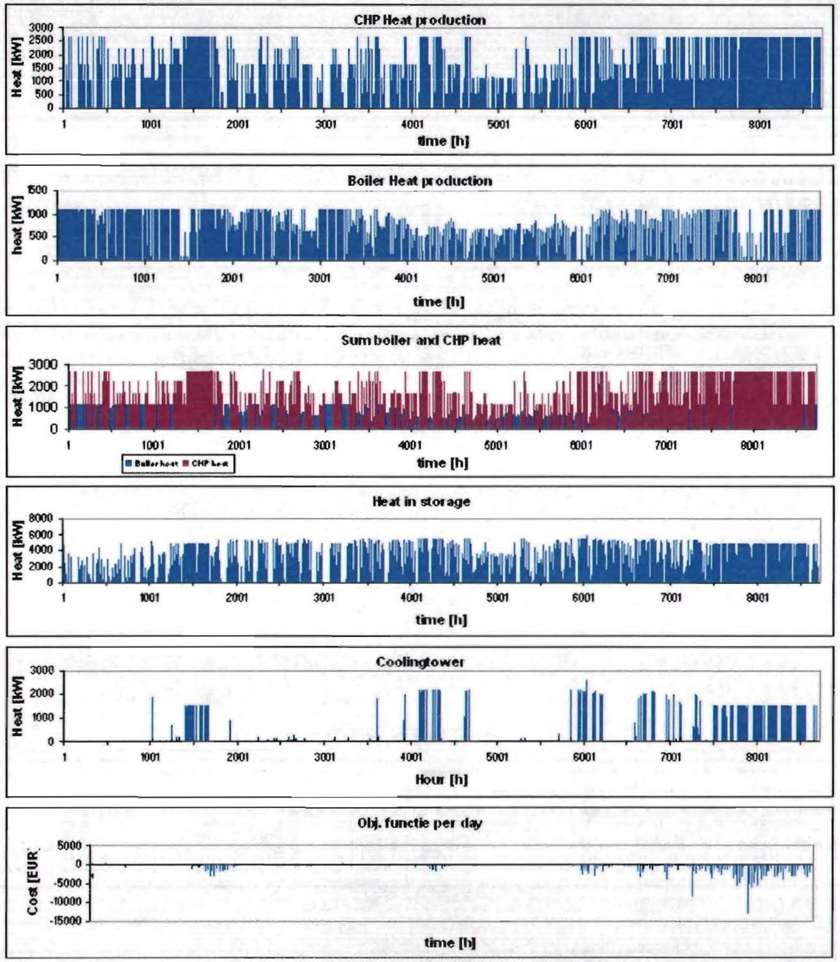


Figure J 16: Results optimizing 5 co-generation units and 100m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	869.929	866.459	535.200	325.104	1.019.200	-	5.609.337
Run hours	2.507	2.497	1.338	1.042	2.548	-	6.233

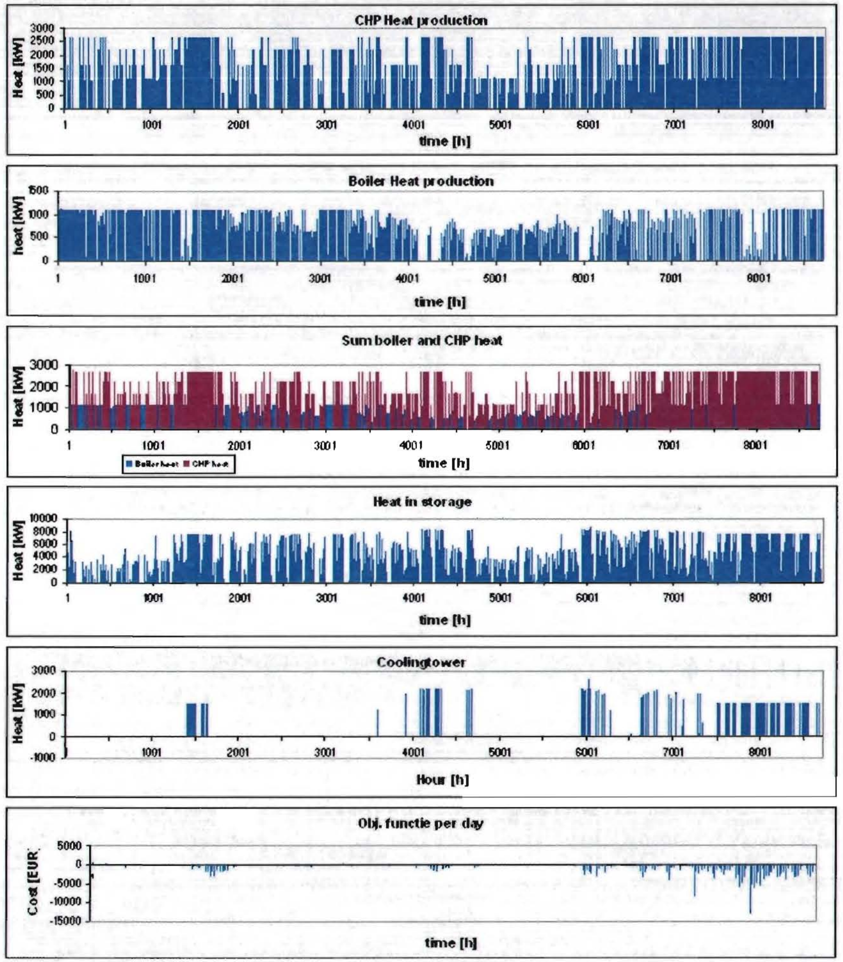


Figure J 17: Results optimizing 5 co-generation units and 150m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	842.169	840.087	536.800	331.656	993.600	-	5.609.337
Run hours	2.427	2.421	1.342	1.063	2.484	-	6.233

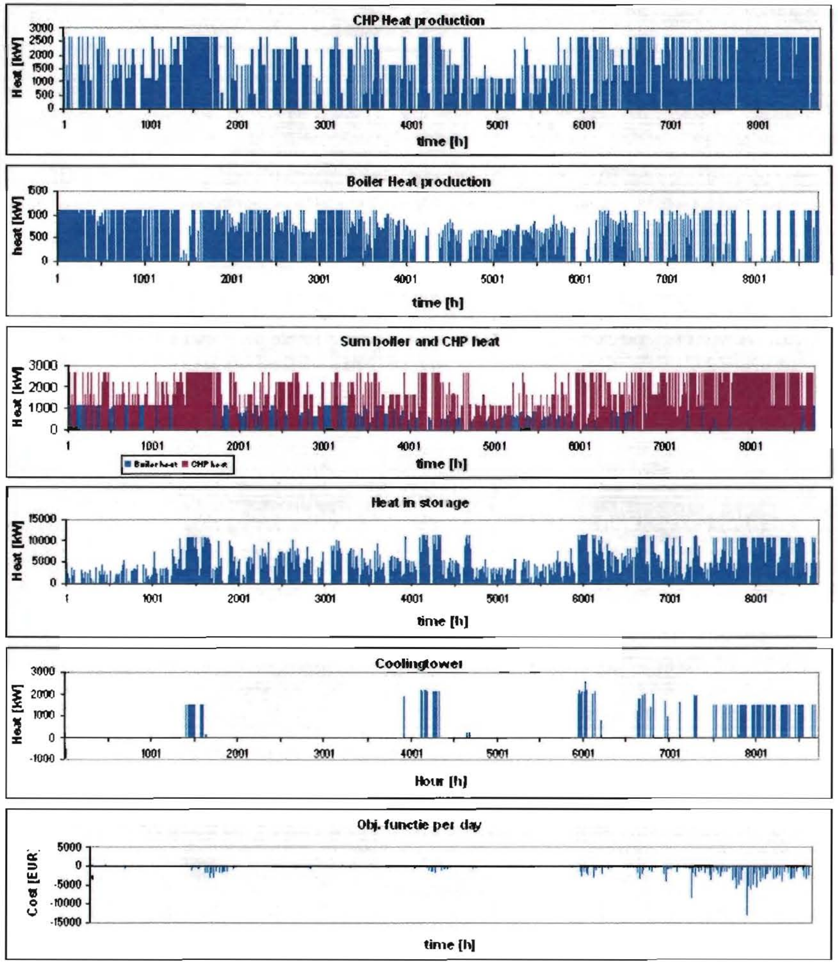


Figure J 18: Results optimizing 5 co-generation units and 200m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	841.822	816.491	536.000	330.720	968.800	-	5.609.337
Run hours	2.426	2.353	1.340	1.060	2.422	-	6.233

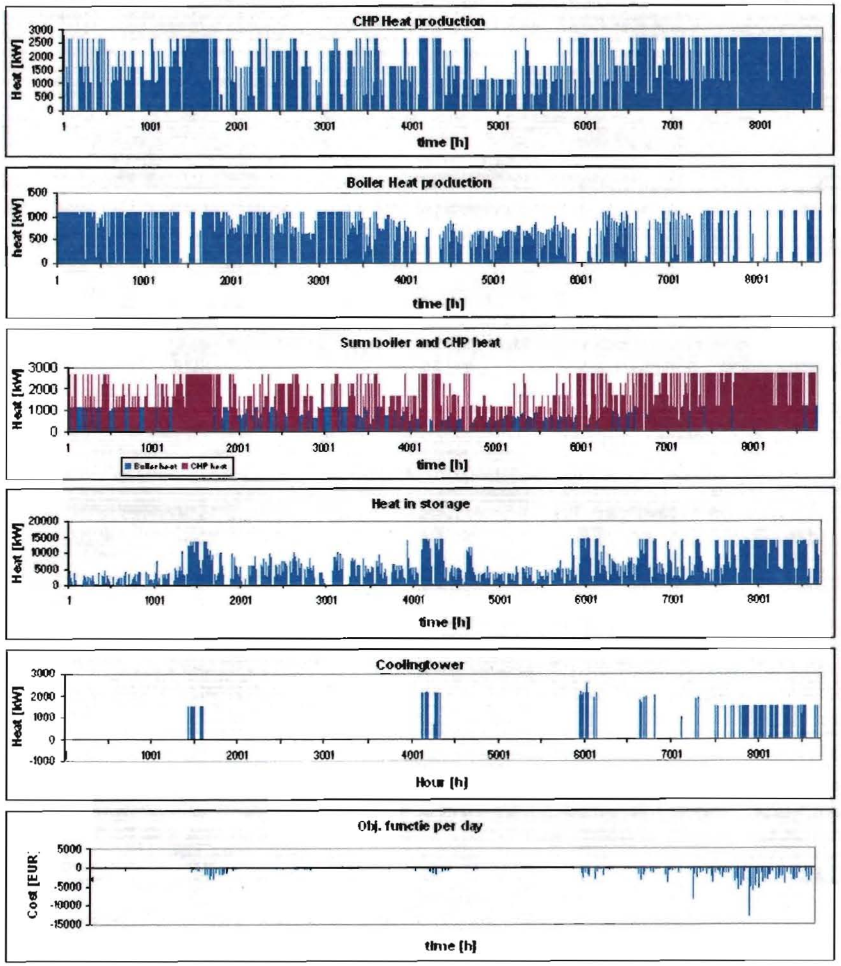


Figure J 19: Results optimizing 5 co-generation units and 250m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	849.456	805.040	538.000	333.216	962.800	-	5.609.337
Run hours	2.448	2.320	1.345	1.068	2.407	-	6.233

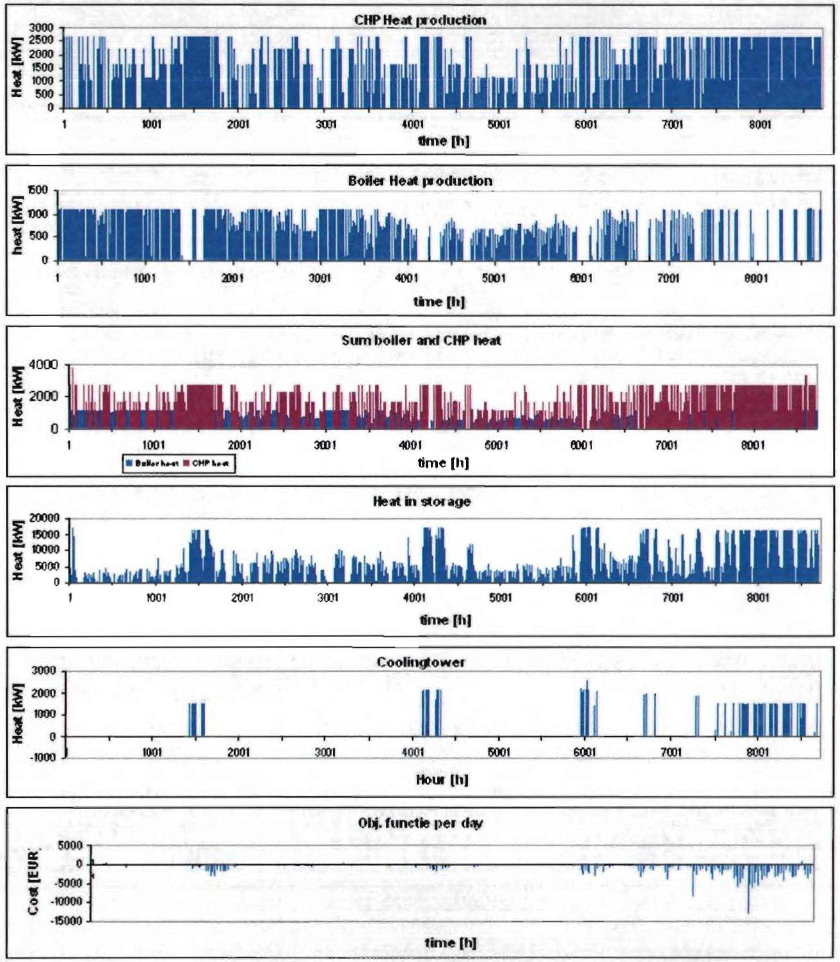


Figure J 20: Results optimizing 5 co-generation units and 300m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	894.566	876.175	455.200	316.368	1.052.400	473.600	5.609.337
Run hours	2.578	2.525	1.138	1.014	2.631	1.184	6.233

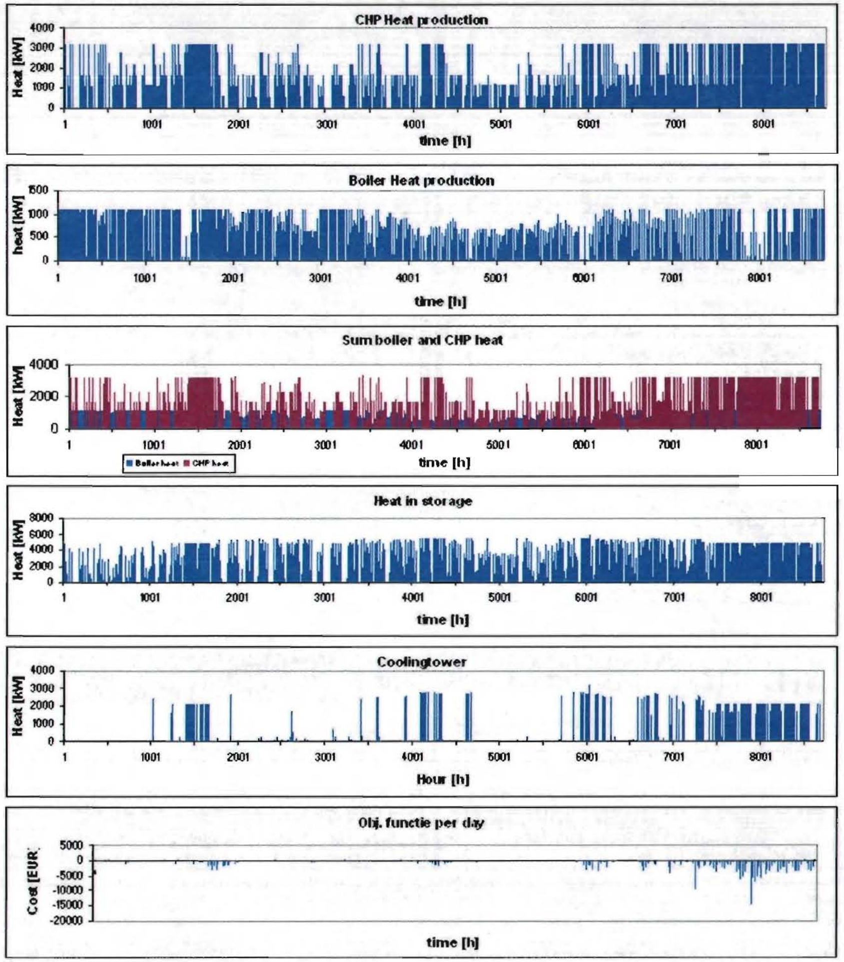


Figure J 21: Results optimizing 6 co-generation units and 100m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	865.765	834.882	484.800	313.560	1.000.800	497.200	5.609.337
Run hours	2.495	2.406	1.212	1.005	2.502	1.243	6.233

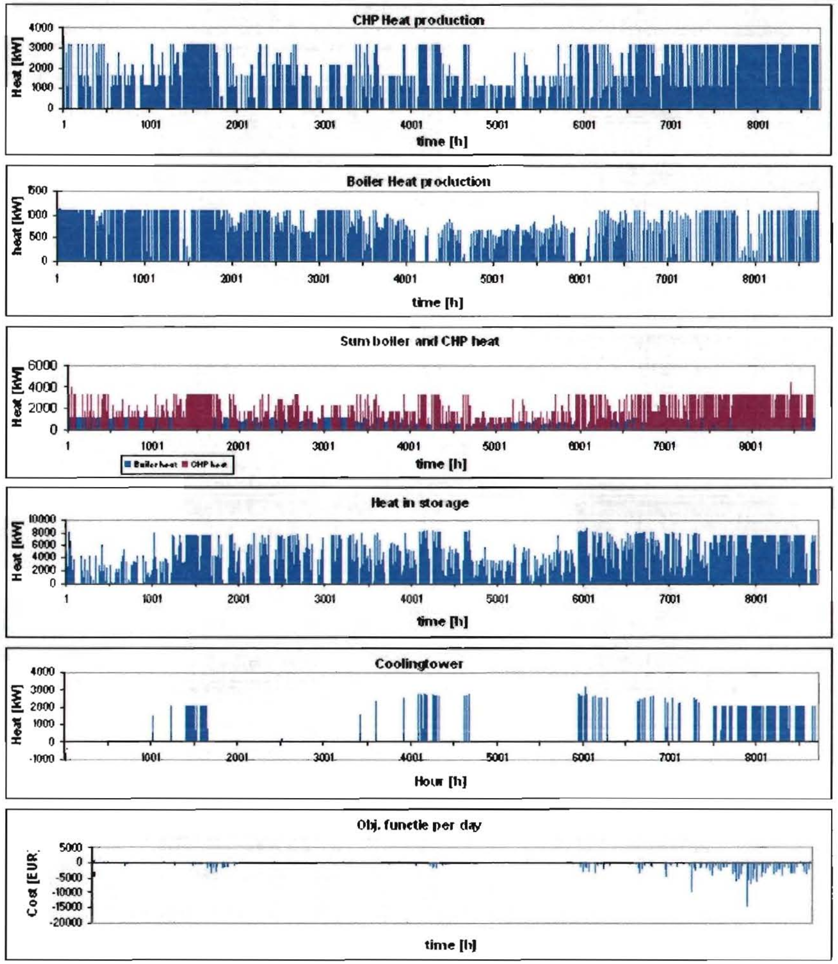


Figure J 22: Results optimizing 6 co-generation units and 150m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	825.513	808.857	495.200	319.176	982.000	497.600	5.609.337
Run hours	2.379	2.331	1.238	1.023	2.455	1.244	6.233

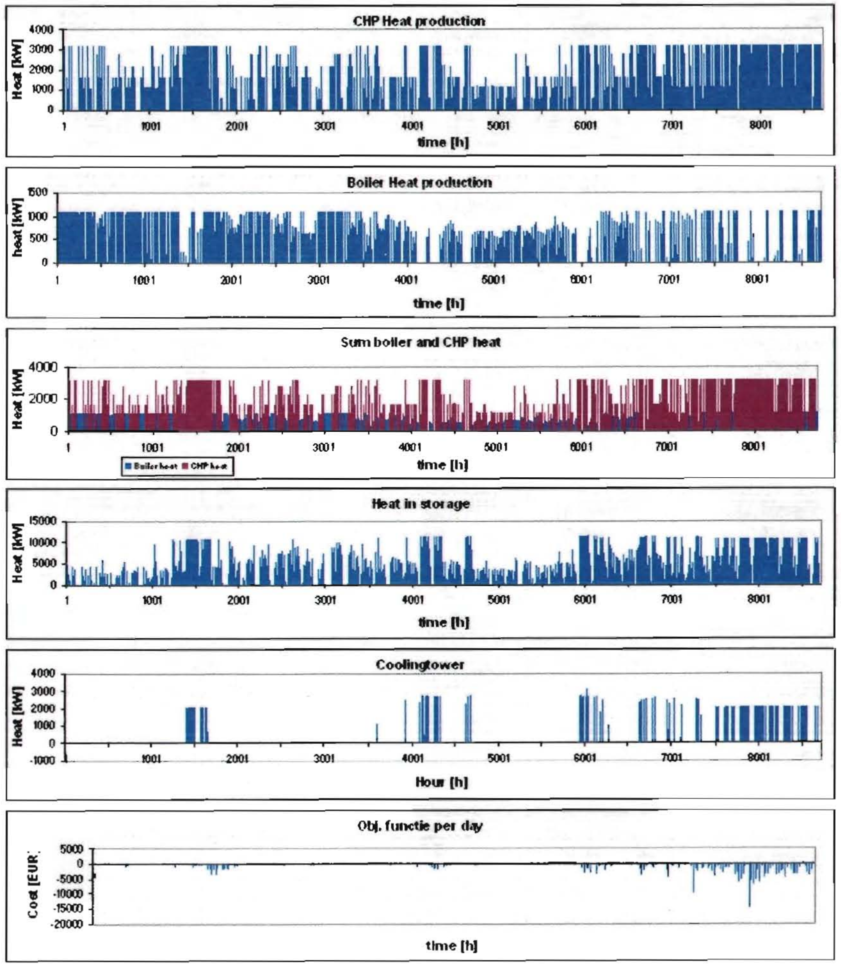


Figure J 23: Results optimizing 6 co-generation units and 200m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	793.936	801.570	498.000	312.936	956.000	498.800	5.609.337
Run hours	2.288	2.310	1.245	1.003	2.390	1.247	6.233

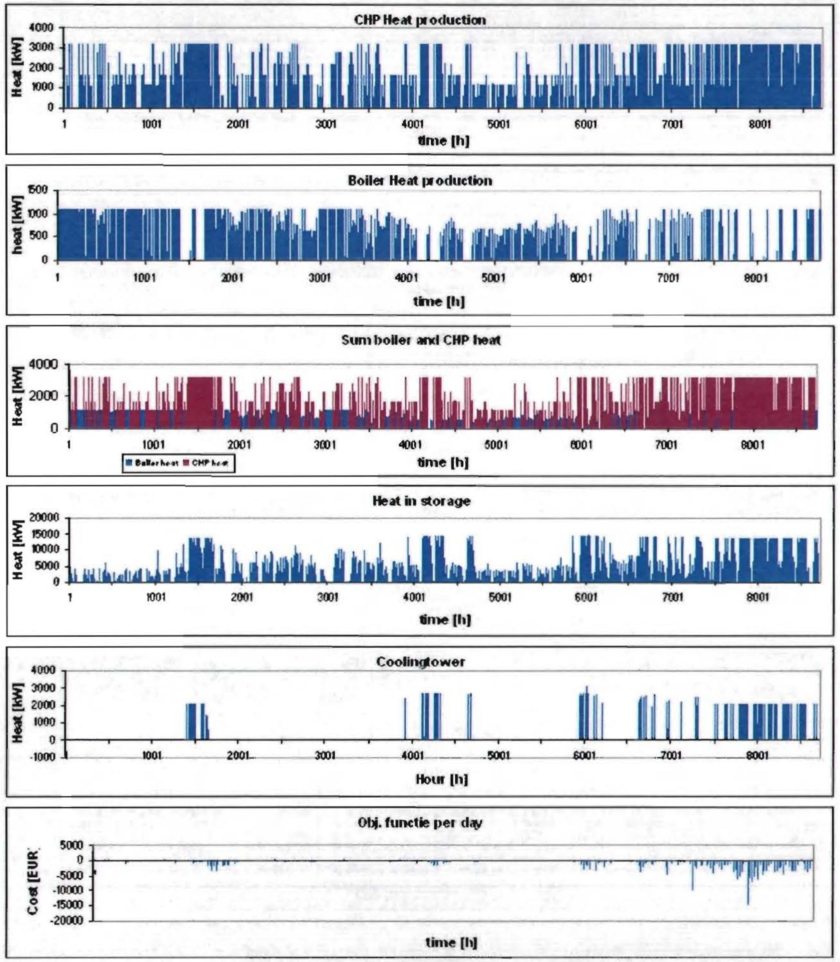


Figure J 24: Results optimizing 6 co-generation units and 250m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	789.078	796.018	499.600	316.368	947.600	494.400	5.609.337
Run hours	2.274	2.294	1.249	1.014	2.369	1.236	6.233

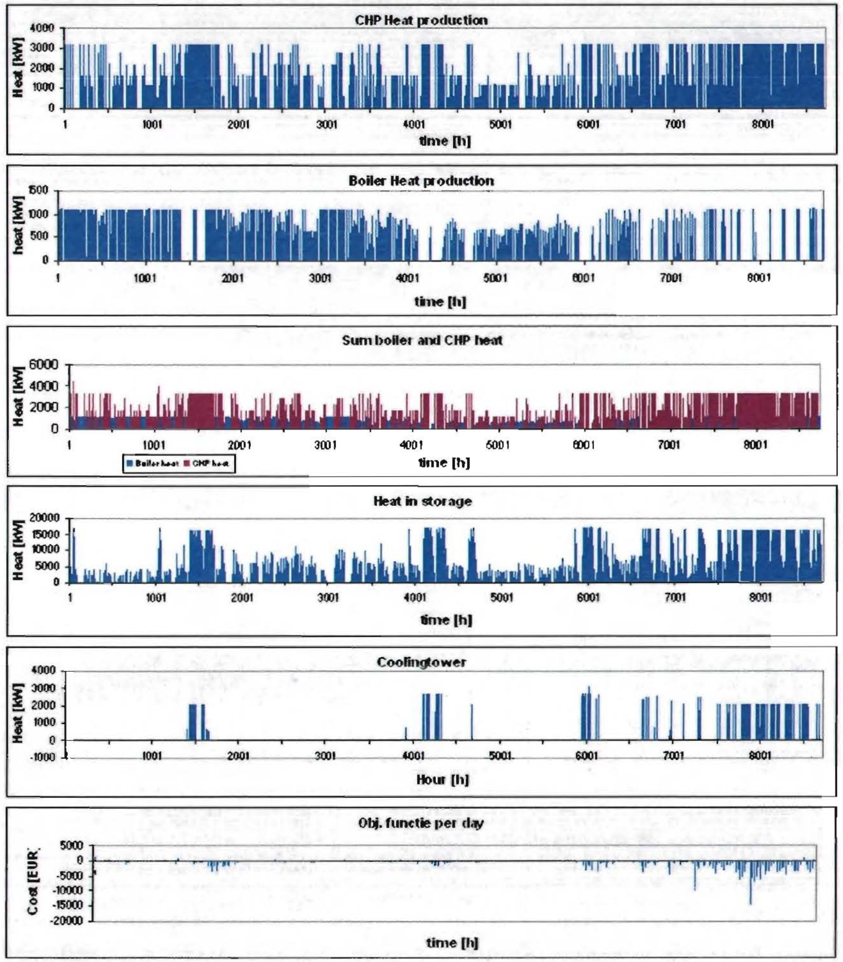


Figure J 25: Results optimizing 6 co-generation units and 300m3 storage over a period of 24h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.566.705	1.575.727	-	-	-	-	5.609.337
Run hours	4.515	4.541	-	-	-	-	6.233

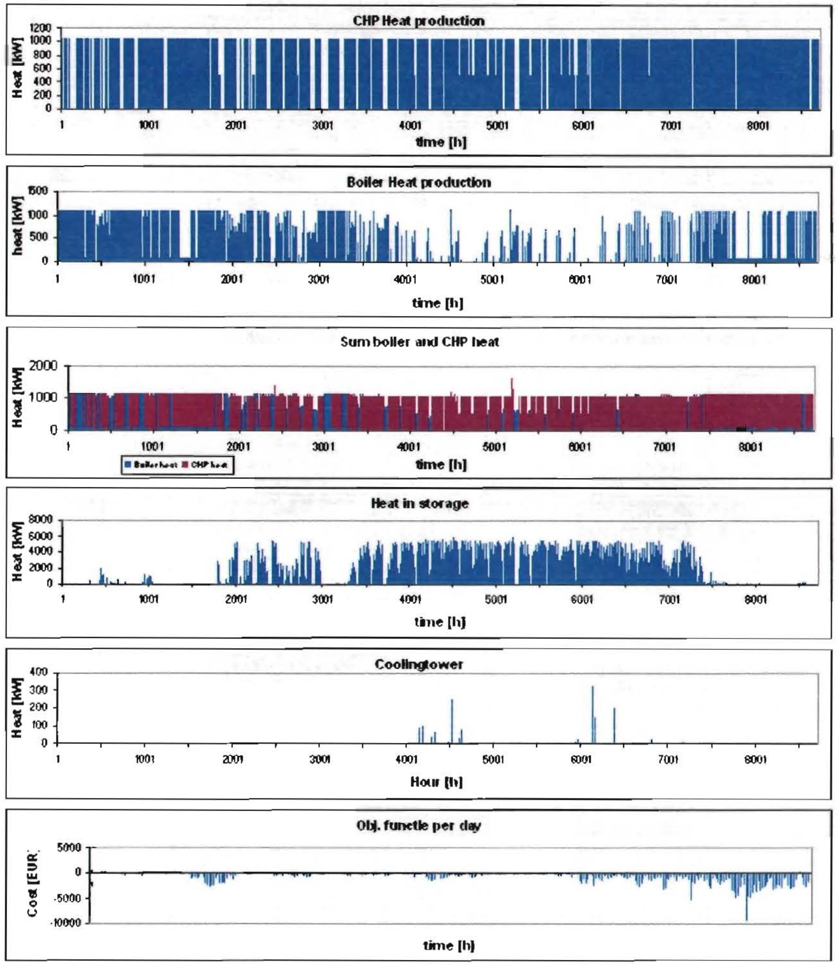


Figure J 26: Results optimizing 2 co-generation units and 100m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.596.894	1.588.566	-	-	-	-	5.609.337
Run hours	4.602	4.578	-	-	-	-	6.233

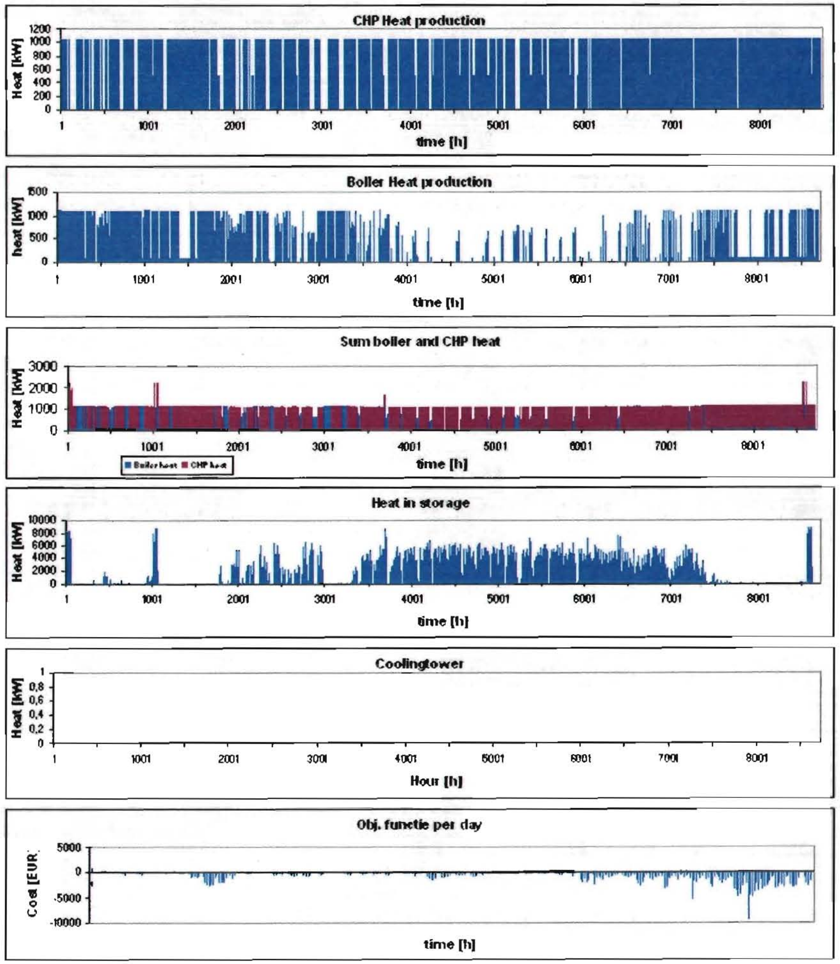


Figure J 27: Results optimizing 2 co-generation units and 150m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.575.380	1.579.891	-	-	-	-	5.609.337
Run hours	4.540	4.553	-	-	-	-	6.233

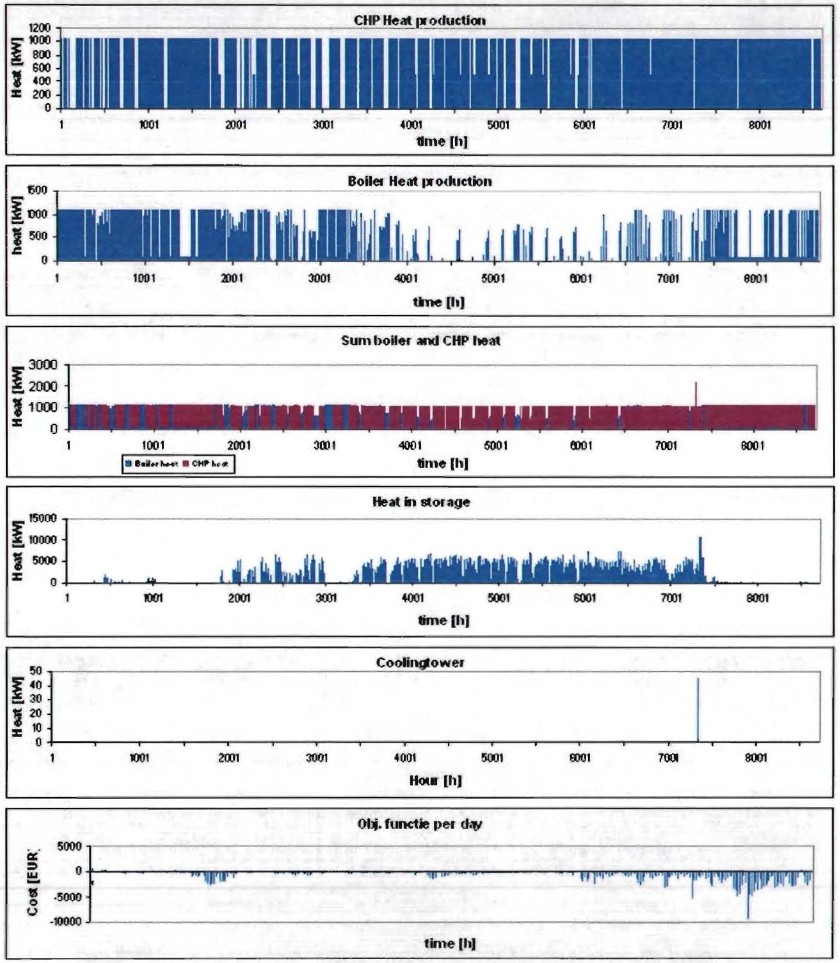


Figure J 28: Results optimizing 2 co-generation units and 200m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.592.730	1.561.500	-	-	-	-	5.609.337
Run hours	4.590	4.500	-	-	-	-	6.233

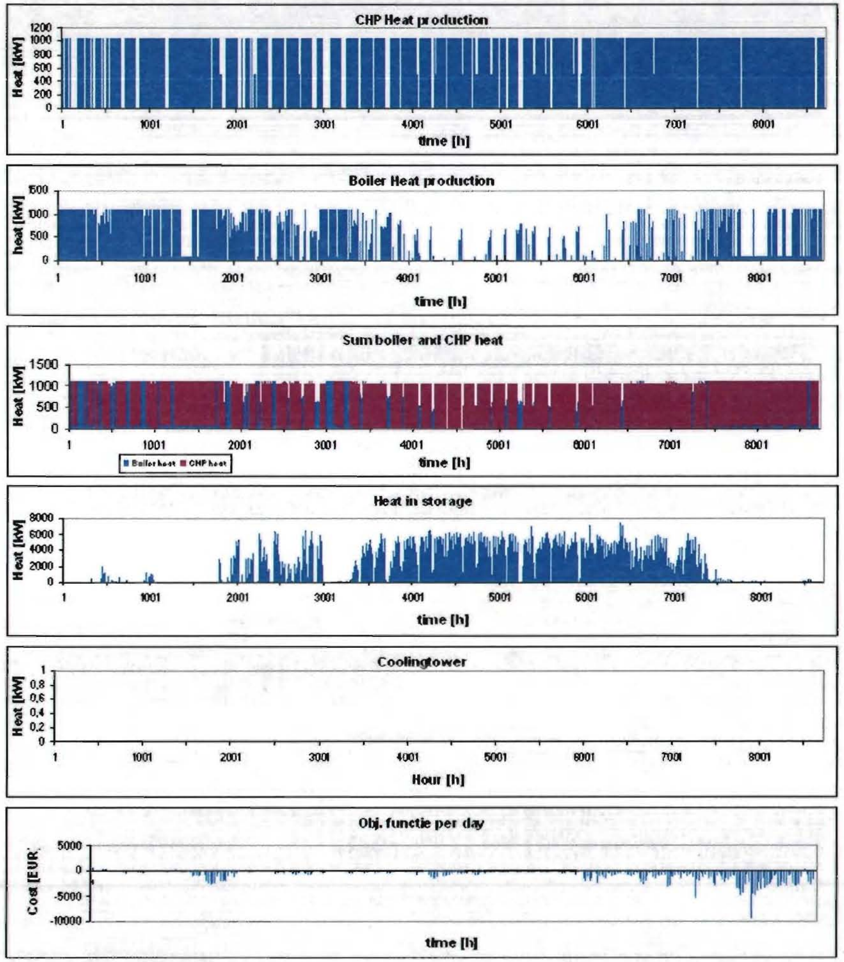


Figure J 29: Results optimizing 2 co-generation units and 250m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.589.607	1.571.563	-	-	-	-	5.609.337
Run hours	4.581	4.529	-	-	-	-	6.233

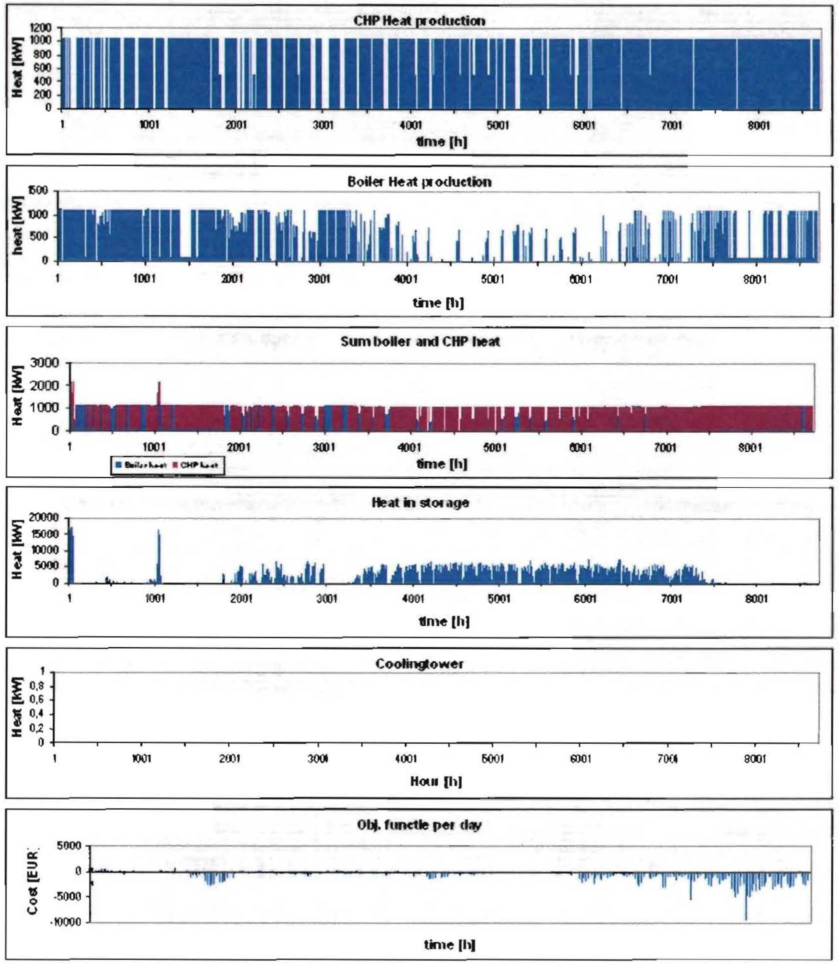


Figure J 30: Results optimizing 2 co-generation units and 300m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.383.836	1.364.404	857.600	-	-	-	5.609.337
Run hours	3.988	3.932	2.144	-	-	-	6.233

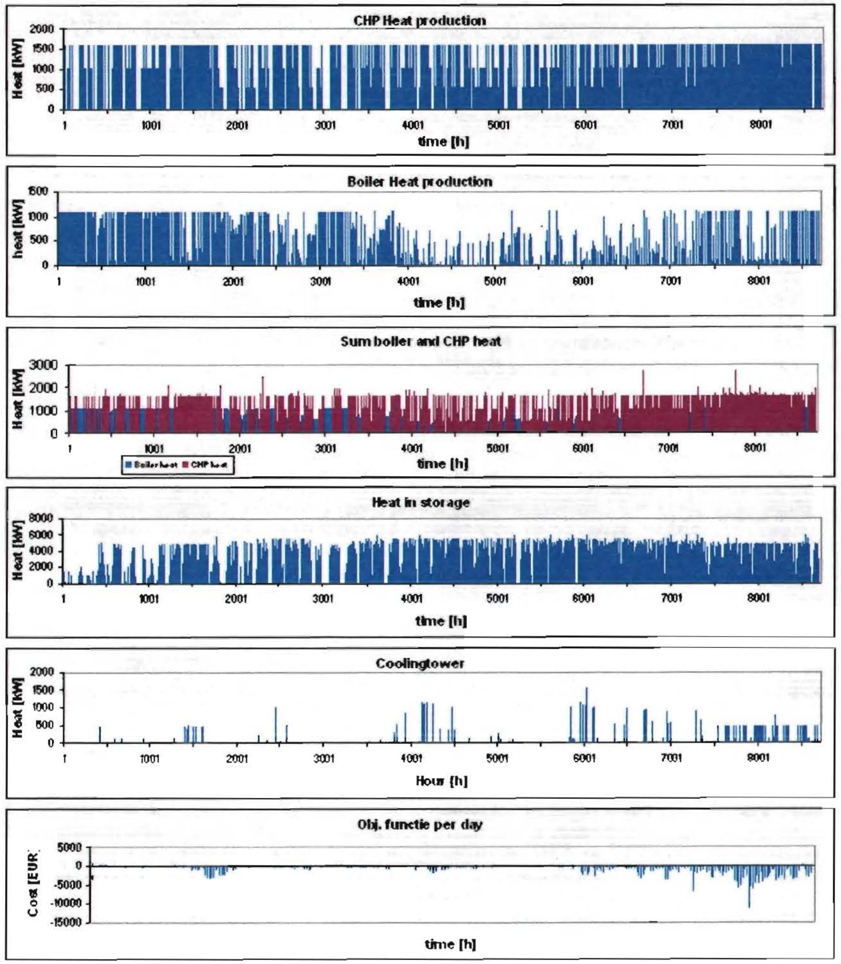


Figure J 31: Results optimizing 3 co-generation units and 100m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.312.354	1.341.502	1.141.600	-	-	-	5.609.337
Run hours	3.782	3.866	2.854	-	-	-	6.233

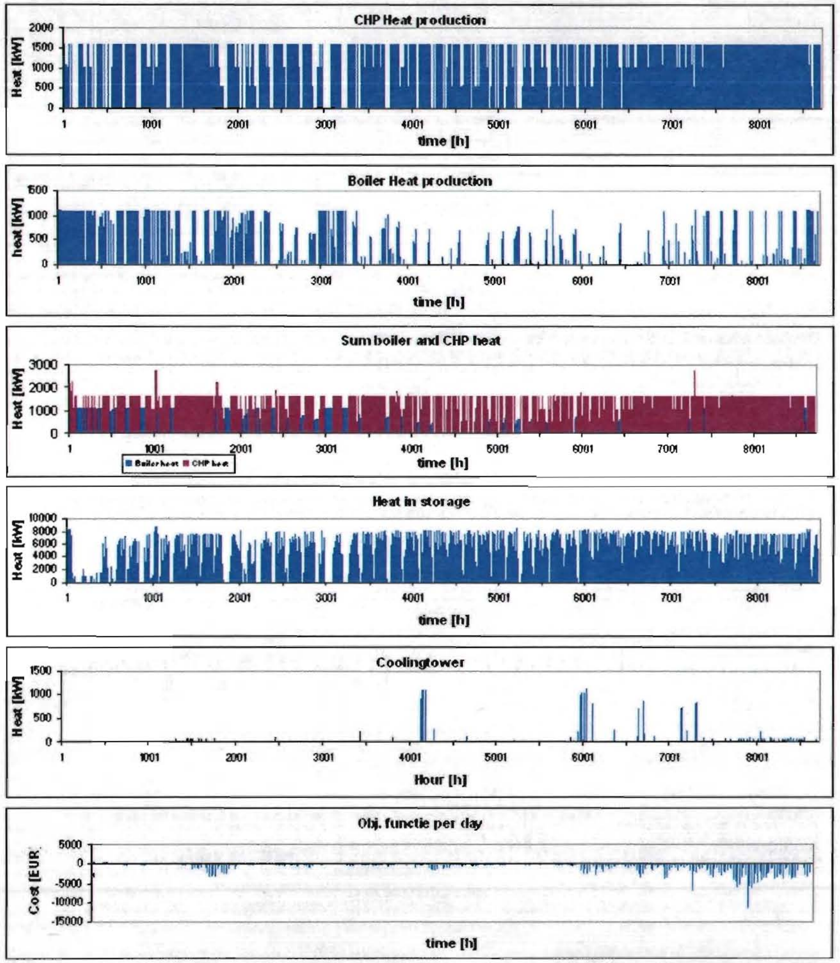


Figure J 32: Results optimizing 3 co-generation units and 150m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.317.212	1.302.985	1.182.000	-	-	-	5.609.337
Run hours	3.796	3.755	2.955	-	-	-	6.233

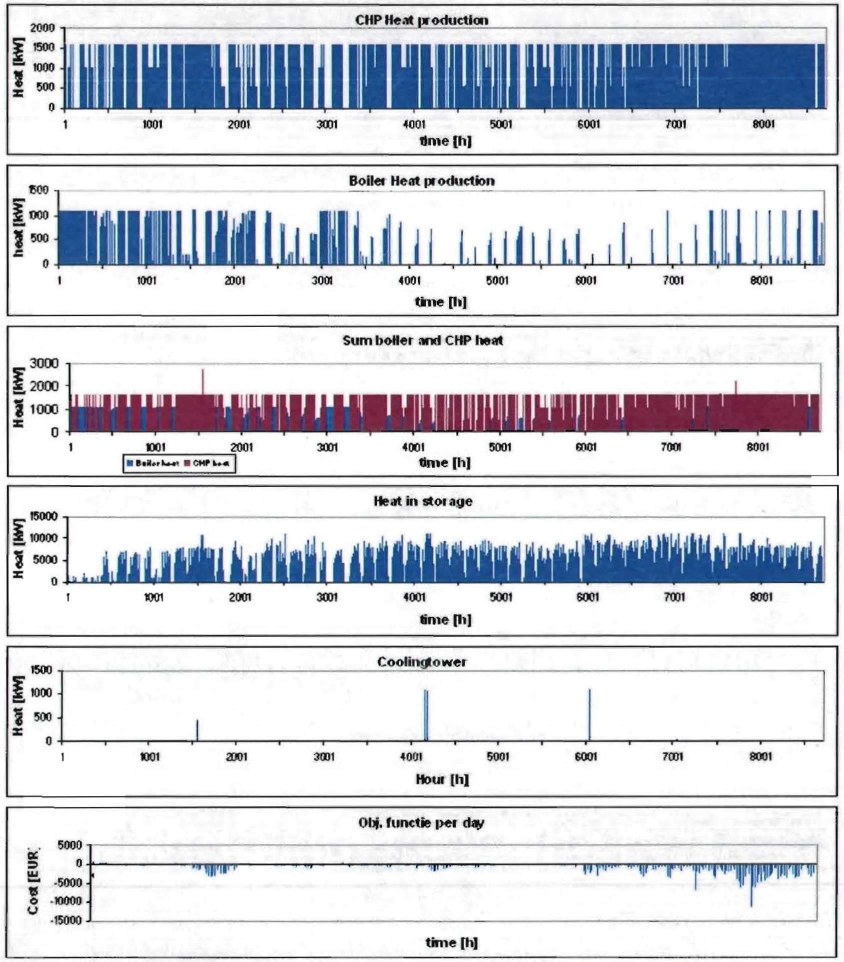


Figure J 33: Results optimizing 3 co-generation units and 200m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.312.701	1.306.108	1.185.600	-	-	-	5.609.337
Run hours	3.783	3.764	2.964	-	-	-	6.233

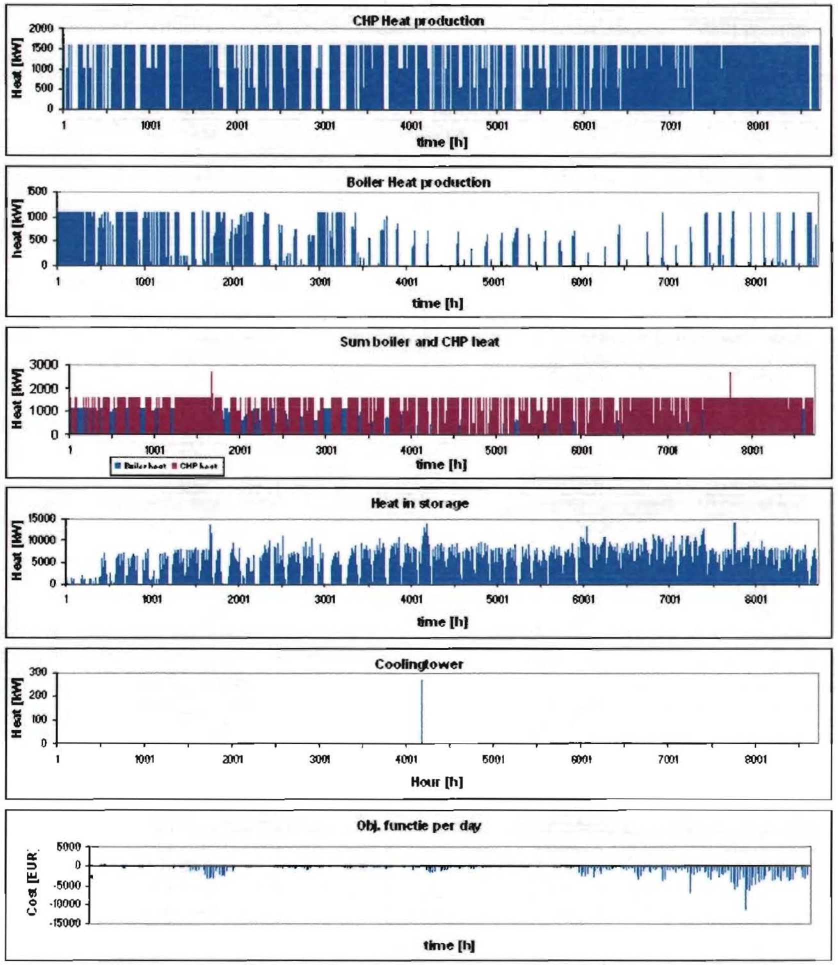


Figure J 34: Results optimizing 3 co-generation units and 250m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.325.193	1.298.474	1.195.600	-	-	-	5.609.337
Run hours	3.819	3.742	2.989	-	-	-	6.233

35

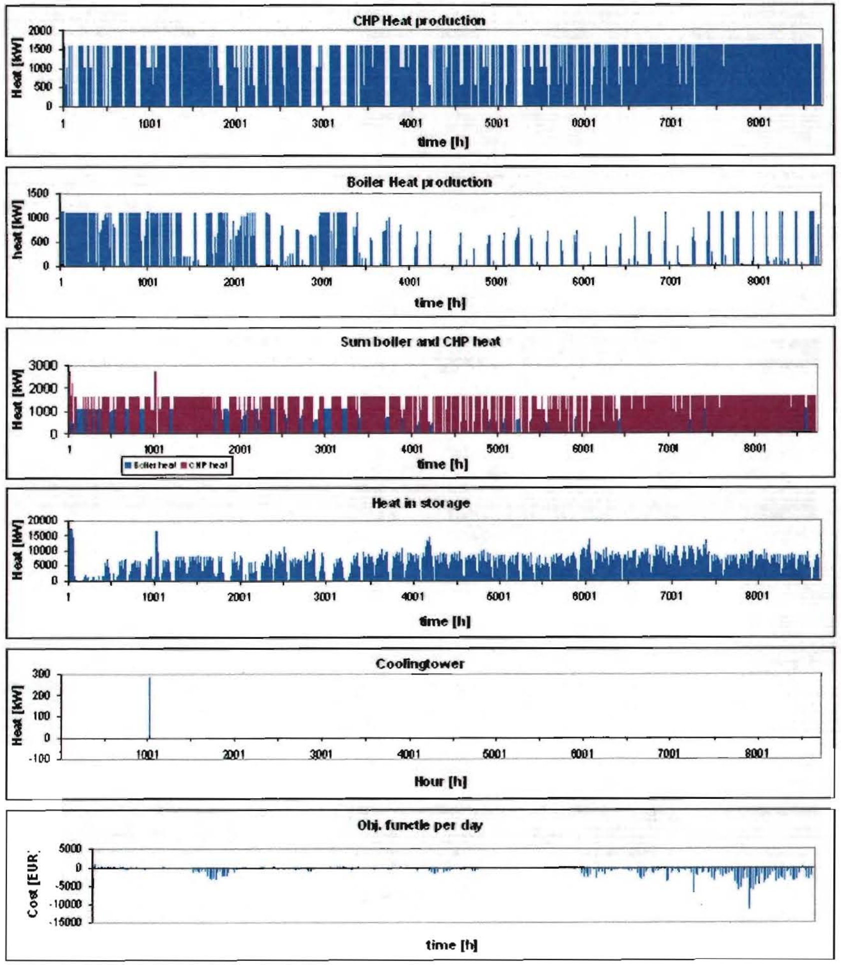


Figure J 35: Results optimizing 3 co-generation units and 300m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.359.546	1.368.221	751.200	411.216	-	-	5.609.337
Run hours	3.918	3.943	1.878	1.318	-	-	6.233

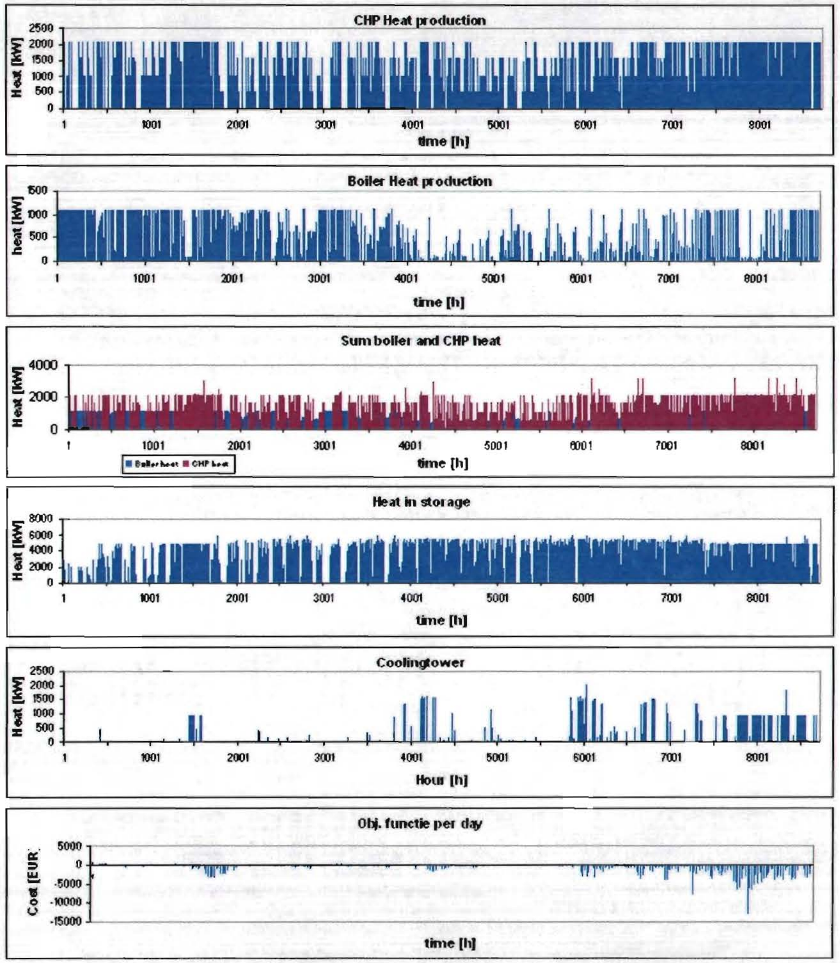


Figure J 36: Results optimizing 4 co-generation units and 100m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.270.020	1.271.061	957.600	520.104	-	-	5.609.337
Run hours	3.660	3.663	2.394	1.667	-	-	6.233

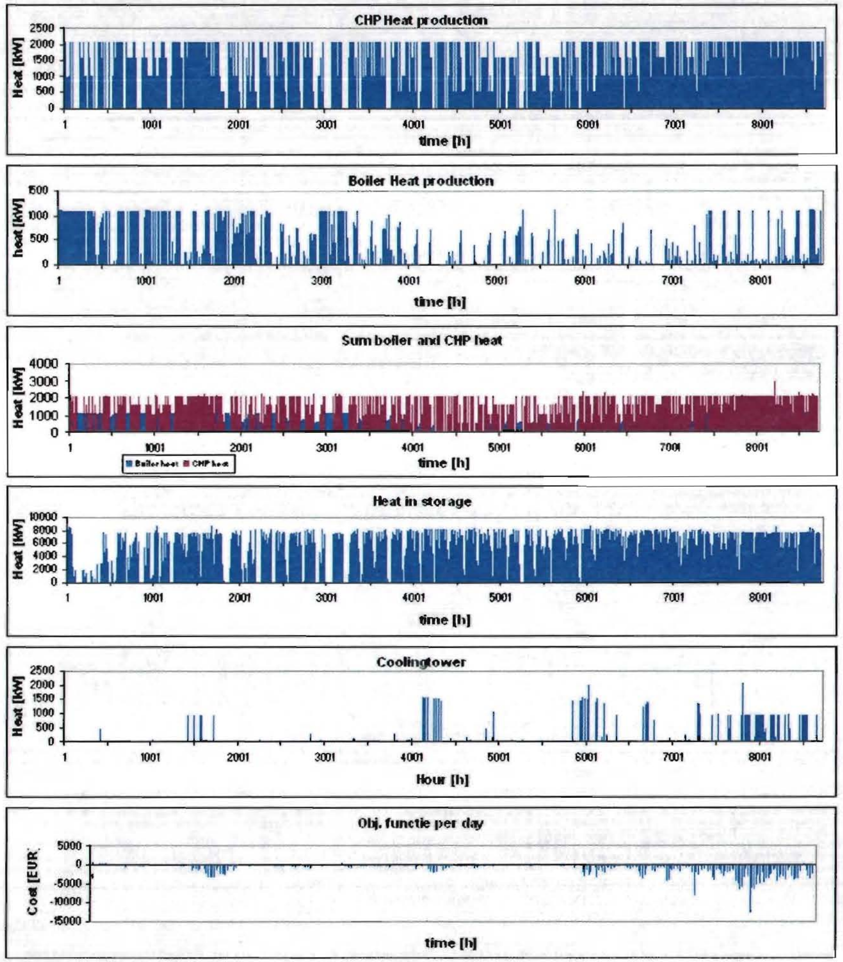


Figure J 37: Results optimizing 4 co-generation units and 150m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.191.251	1.169.737	1.000.400	645.840	-	-	5.609.337
Run hours	3.433	3.371	2.501	2.070	-	-	6.233

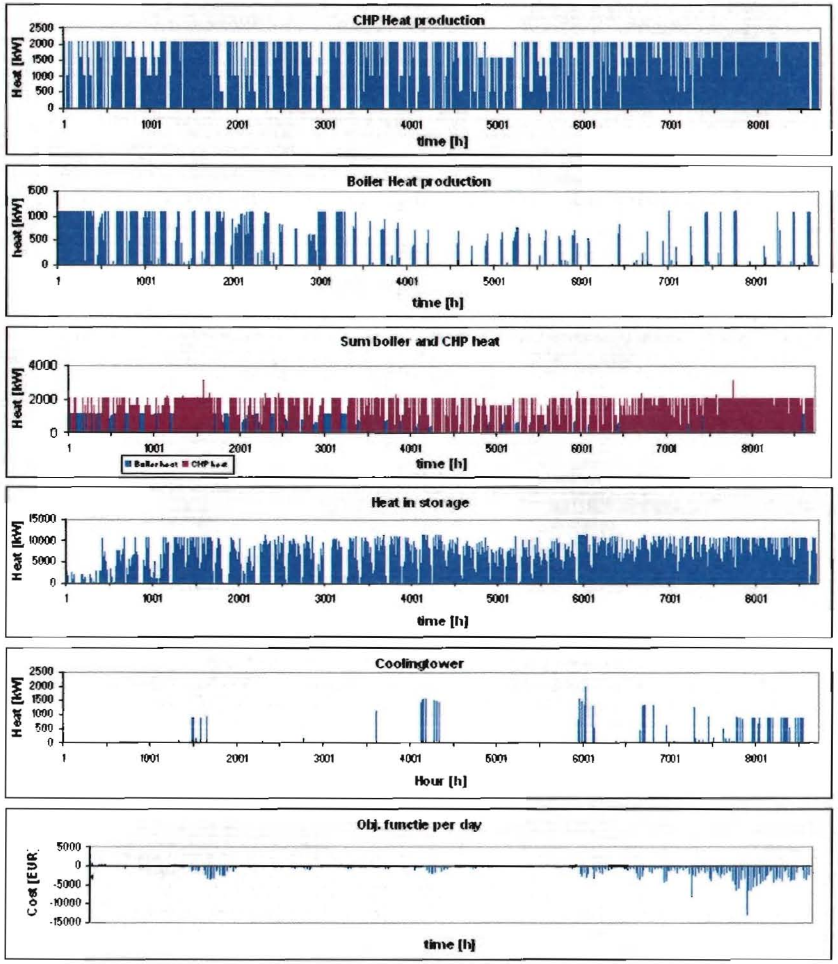


Figure J 38: Results optimizing 4 co-generation units and 200m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	1.164.532	1.156.551	1.027.200	665.808	-	-	5.609.337
Run hours	3.356	3.333	2.568	2.134	-	-	6.233

39

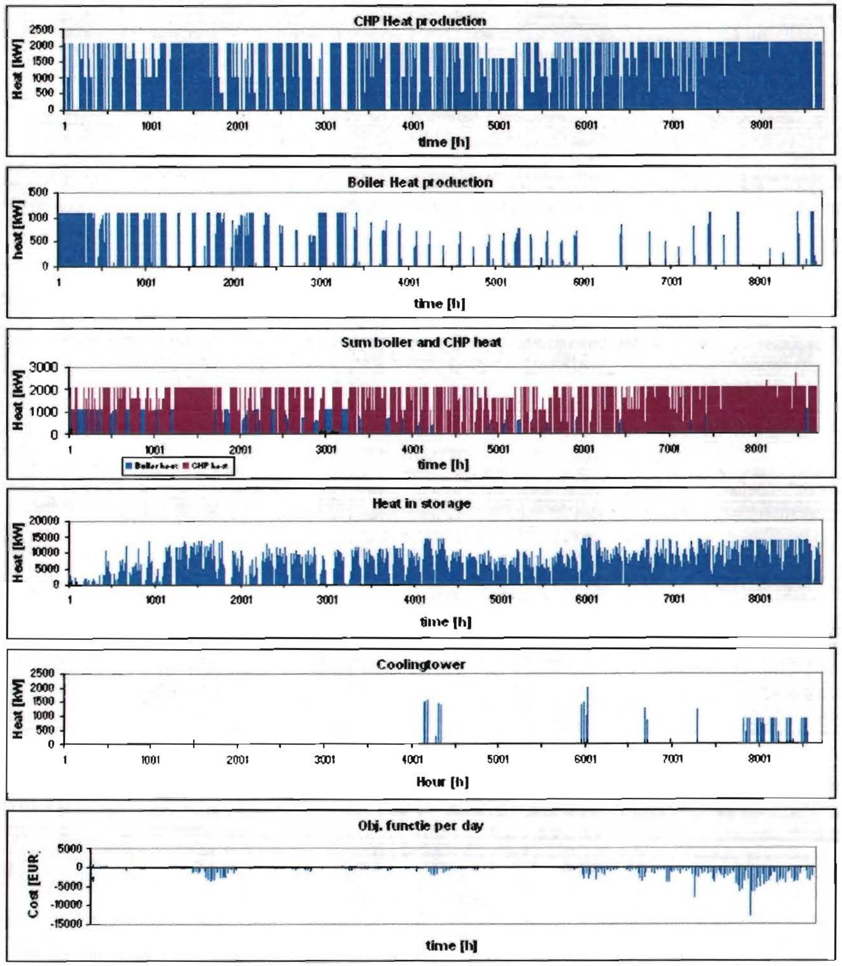


Figure J 39: Results optimizing 4 co-generation units and 250m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.158.633	1.155.857	1.033.600	672.360	-	-	5.609.337
Run hours	3.339	3.331	2.584	2.155	-	-	6.233

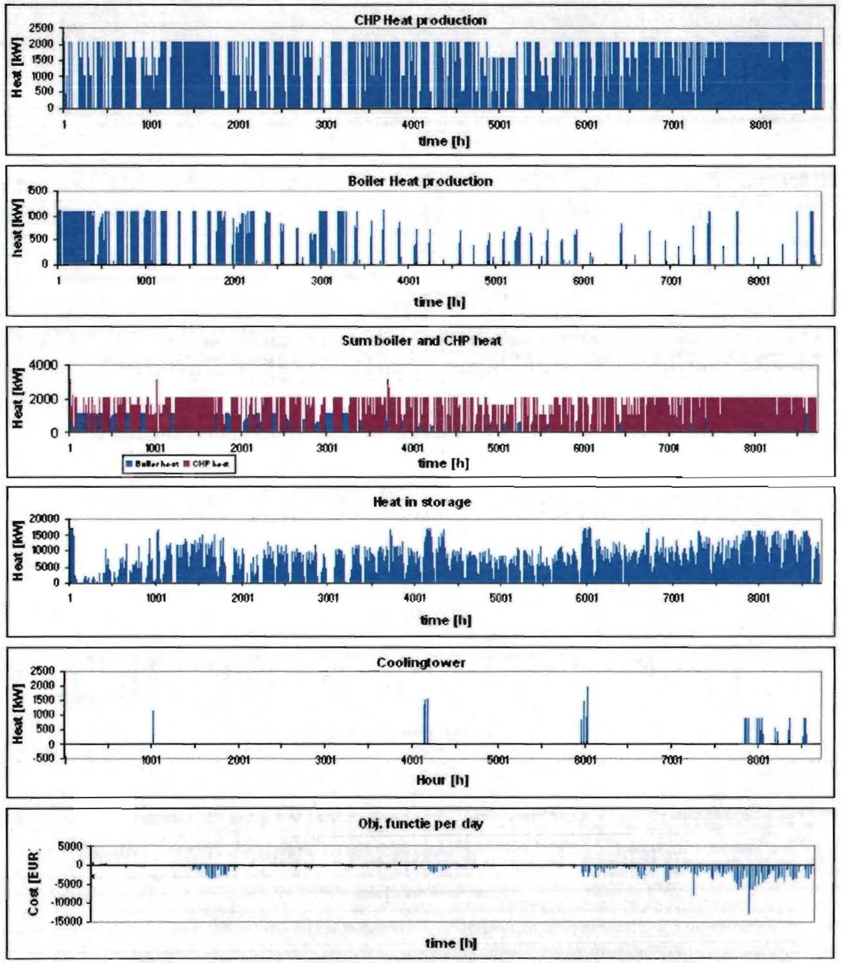


Figure J 40: Results optimizing 4 co-generation units and 300m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.204.784	1.124.974	536.800	359.736	1.177.200	-	5.609.337
Run hours	3.472	3.242	1.342	1.153	2.943	-	6.233

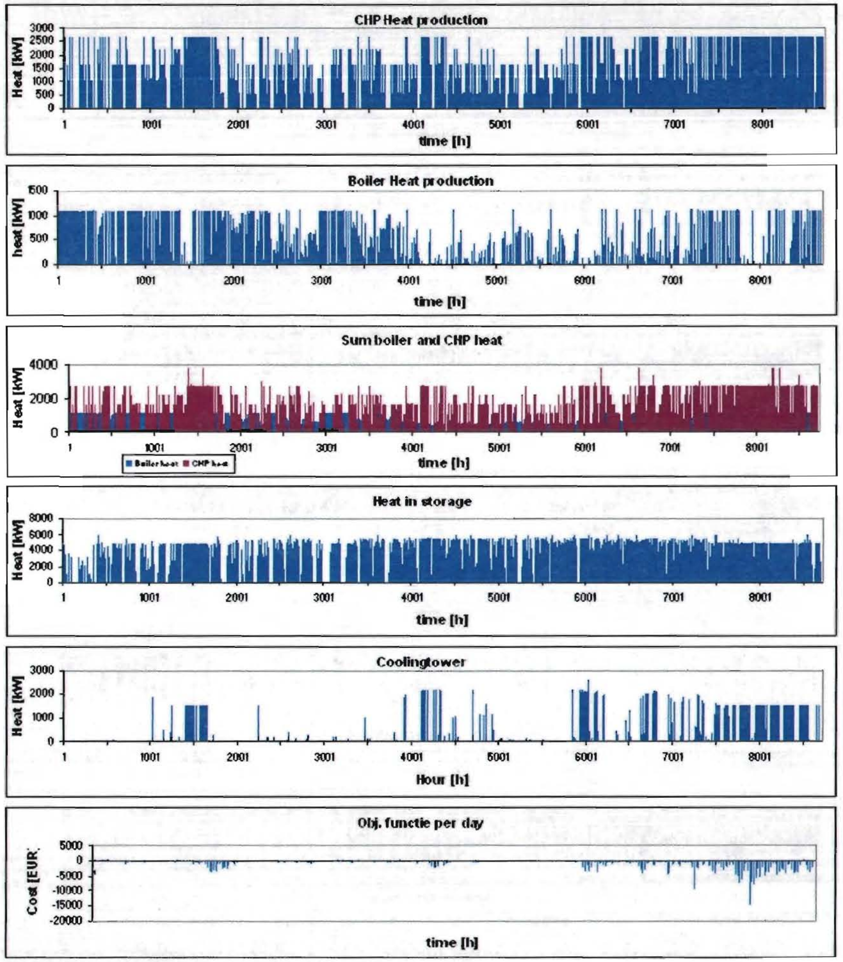


Figure J 41: Results optimizing 5 co-generation units and 100m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.166.961	1.117.687	623.600	394.056	1.128.800	-	5.609.337
Run hours	3.363	3.221	1.559	1.263	2.822	-	6.233

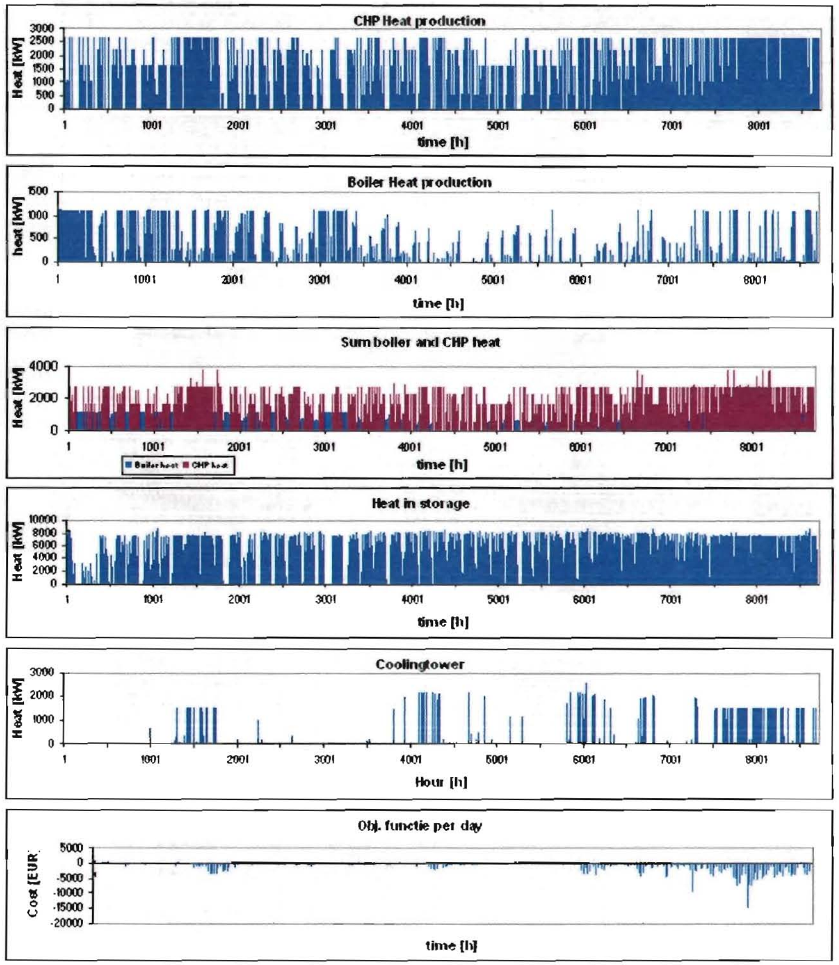


Figure J 42: Results optimizing 5 co-generation units and 150m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.061.473	1.049.328	734.400	425.256	1.078.800	-	5.609.337
Run hours	3.059	3.024	1.836	1.363	2.697	-	6.233

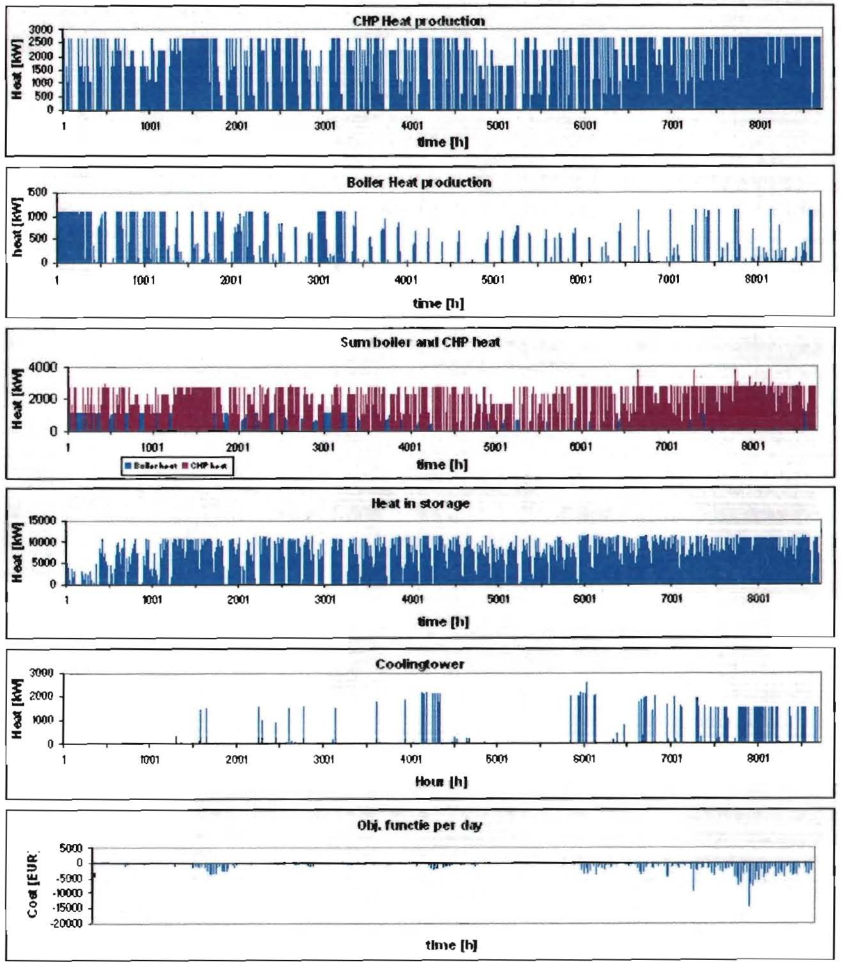


Figure J 43: Results optimizing 5 co-generation units and 200m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.018.445	1.005.259	768.800	482.976	1.065.600	-	5.609.337
Run hours	2.935	2.897	1.922	1.548	2.664	-	6.233

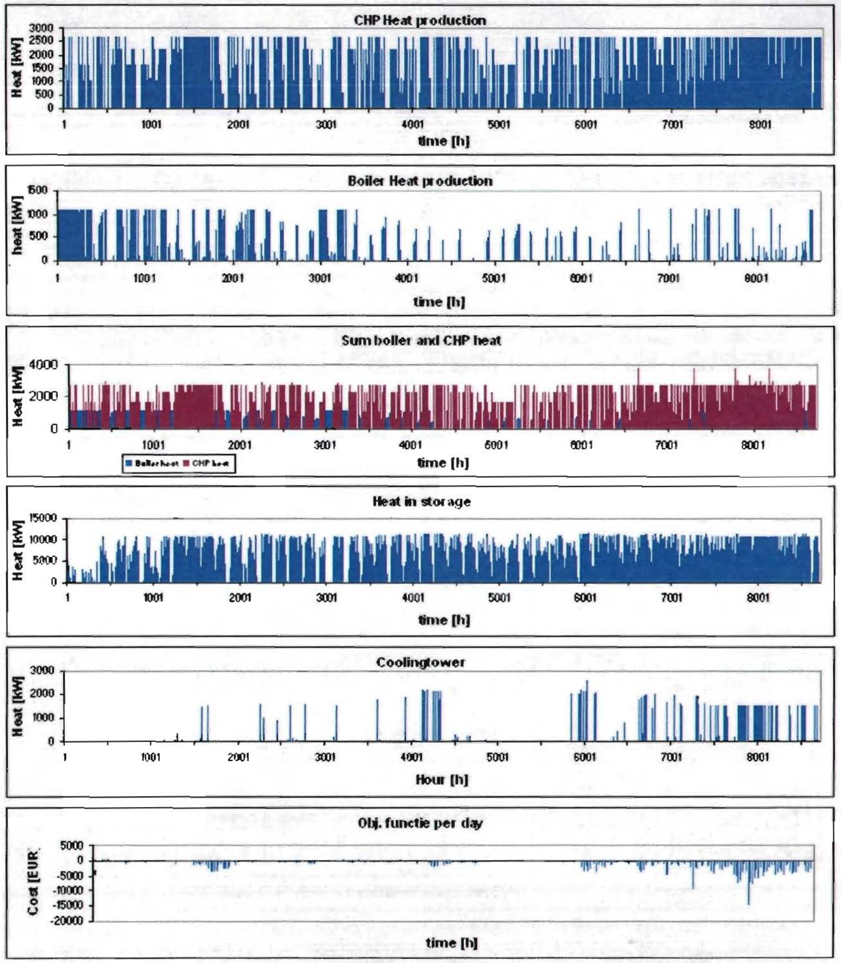


Figure J 44: Results optimizing 5 co-generation units and 250m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.017.057	989.991	784.000	476.424	1.060.000	-	5.609.337
Run hours	2.931	2.853	1.960	1.527	2.650	-	6.233

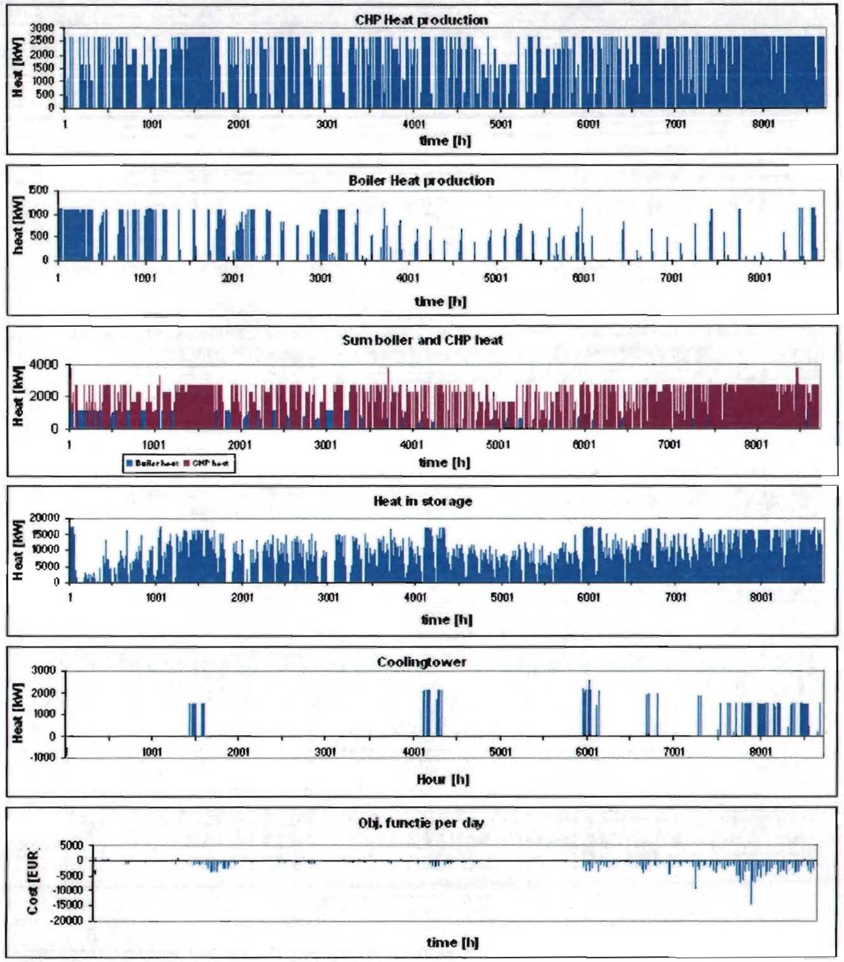


Figure J 45: Results optimizing 5 co-generation units and 300m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.152.040	1.148.917	506.000	353.184	1.142.000	512.800	5.609.337
Run hours	3.320	3.311	1.265	1.132	2.855	1.282	6.233

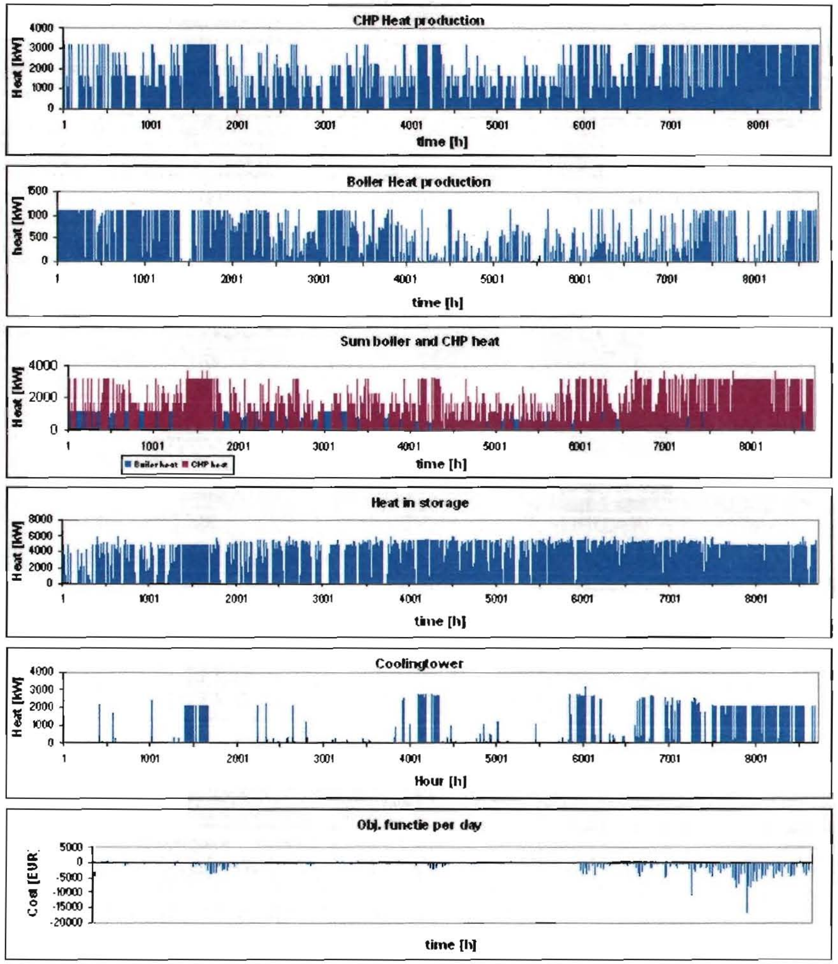


Figure J 46: Results optimizing 6 co-generation units and 100m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.112.482	1.116.299	567.600	345.696	1.101.200	589.200	5.609.337
Run hours	3.206	3.217	1.419	1.108	2.753	1.473	6.233

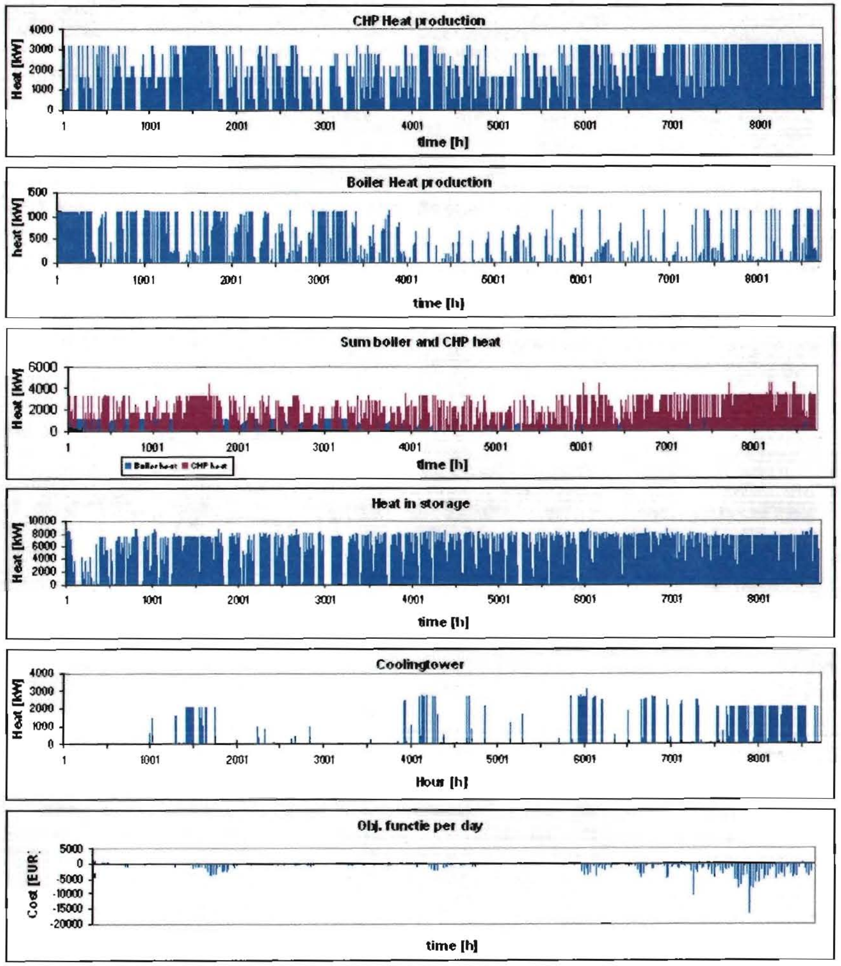


Figure J 47: Results optimizing 6 co-generation units and 150m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.059.391	1.011.505	642.000	355.056	1.029.600	643.200	5.609.337
Run hours	3.053	2.915	1.605	1.138	2.574	1.608	6.233

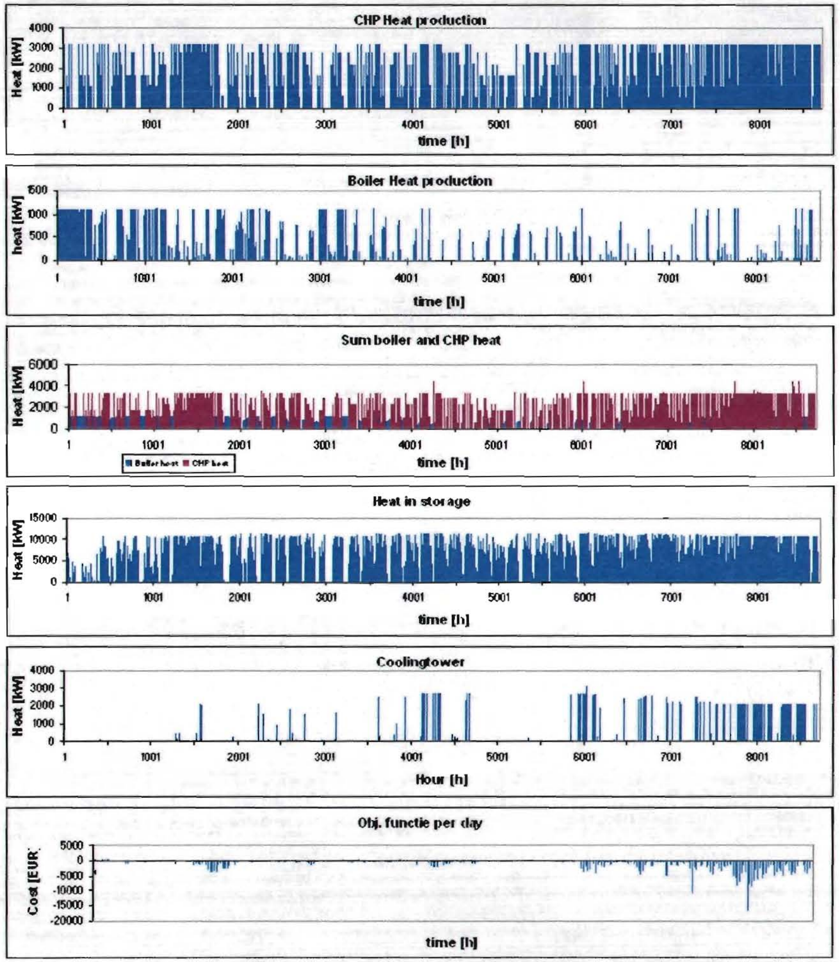


Figure J 48: Results optimizing 6 co-generation units and 200m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	347	347	400	312	400	400	900
kWh	1.007.341	964.660	701.600	381.264	1.042.400	666.400	5.609.337
Run hours	2.903	2.780	1.754	1.222	2.606	1.666	6.233

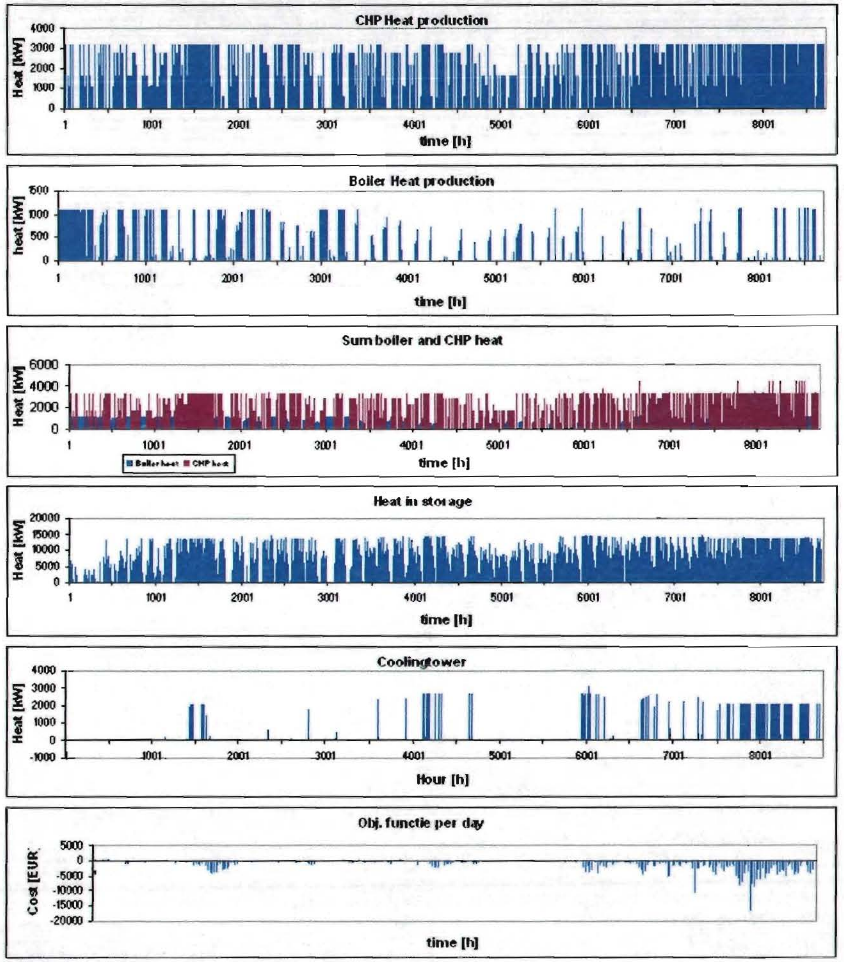


Figure J 49: Results optimizing 6 co-generation units and 250m3 storage over a period of 36h

Unit	1	2	3	4	5	6	GE
P	**347**	**347**	**400**	**312**	**400**	**400**	**900**
kWh	932.389	948.004	693.200	407.784	1.003.200	689.600	5.609.337
Run hours	2.687	2.732	1.733	1.307	2.508	1.724	6.233

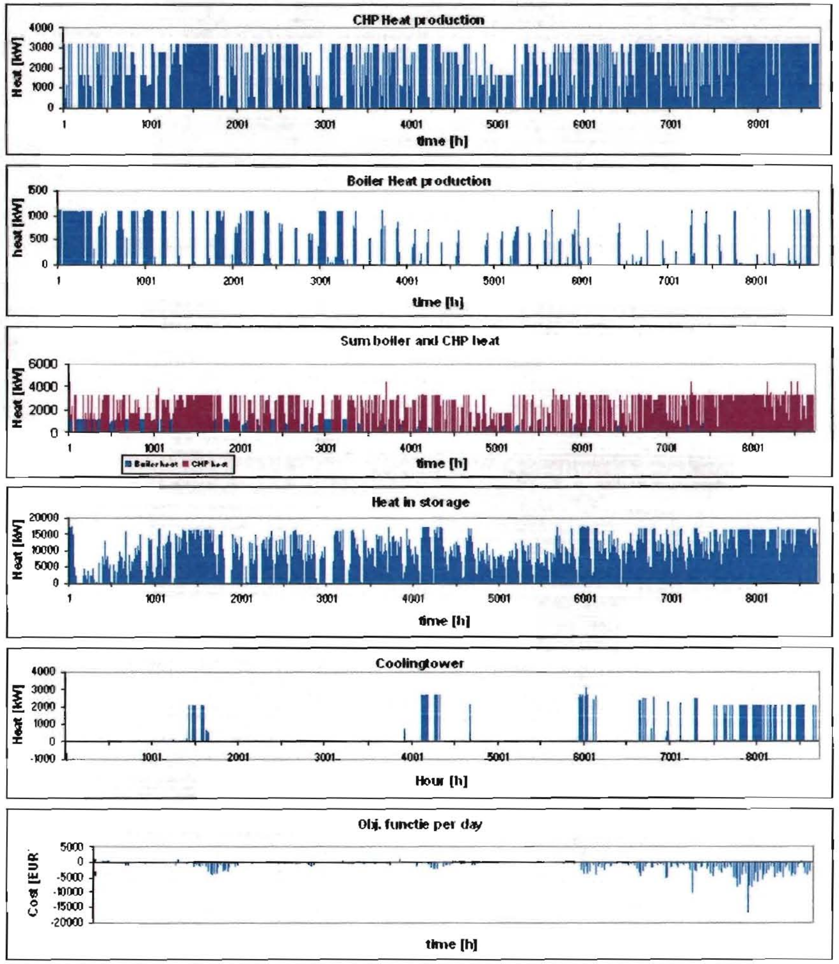


Figure J 49: Results optimizing 6 co-generation units and 300m3 storage over a period of 36h