MASTER

The final order problem for non-repairable items at Vanderlande Industries

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The Final Order Problem for Non-Repairable Items at Vanderlande Industries

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I. Abstract

In this master thesis project the final order problem for non-repairable items is researched. Two new aspects of the final order problem are addressed: substitution of items and uncertainty in the length of the time period that the final order should cover. This problem is researched at Vanderlande Industries. By means of a cost model, the optimal strategy to cover demand during the planning horizon can be found.

II. Management Summary

This report is the result of a master thesis project conducted at Vanderlande Industries (VI). VI provides automated material handling systems and accompanying services. She is a global player that is active in several markets: baggage handling, distribution, parcel and postal, and services. At VI there are difficulties regarding the 'final order problem'. The Supply Chain Management Services (SCMS) department of VI is responsible for the spare parts. Unfortunately, it occurs that suppliers of those spare parts stop supplying the item. Based on the preparation phase of this master thesis project [Bakx, 2010], in which a literature study regarding this type of problem and in which the problem at VI was analyzed in more depth, the problem that is researched in this master thesis project can be defined as follows:

It is unclear how to solve the final order problem for non-repairable items when substitution of items should be taken into account. Substitution of items leads to a 'transition time', where the length of this transition time can be uncertain. Also, the transition costs and the costs for one unit of the alternative item can be uncertain. It is not known how to determine the optimal final order quantity for this (uncertain) length of the transition time, where this uncertainty in costs should be taken into account.

In the master thesis project a cost model should be developed for the final order problem that takes into account substitution. Using this model, VI can determine the optimal strategy to cover the demand during the entire planning horizon. The following 'basic idea' regarding the transition time is used to develop the model. Regarding an end-of-supply problem VI, or any company, can face the following events:

1) For some reason, an EOS notification for a specific service item (say item 1) is received. When the EOS notification concerns an item that is normally bought from an external supplier, the supplier will often offer an alternative item, e.g. an upgrade of item 1.
2) At the same time as event 1), or (preferably) at some later time, item 1 is end-of-supply. This means it is not possible to order item 1 anymore or VI decided that from this moment on, the item is not ordered anymore. However, when the EOS occurs, it is possible to place a final order. Note that in the worst-case scenario, event 2) occurs before event 1), which means that the company simply does not know that the item is EOS and it may only find this out by trying to place an order while the item is not available anymore (this situation is not considered in this project, during this project it is always possible to place a final order).
3) After a certain lead time the final order quantity arrives. This final order quantity should cover the demand during the so-called 'Required Transition Time (RTT)'. Here, the required transition time is defined as the time it takes the company to be able to use item 2 (which is the alternative for item 1) to satisfy demand.
4) After the RTT, the company can use item 2 to satisfy demand.

The events mentioned above can be depicted as follows:

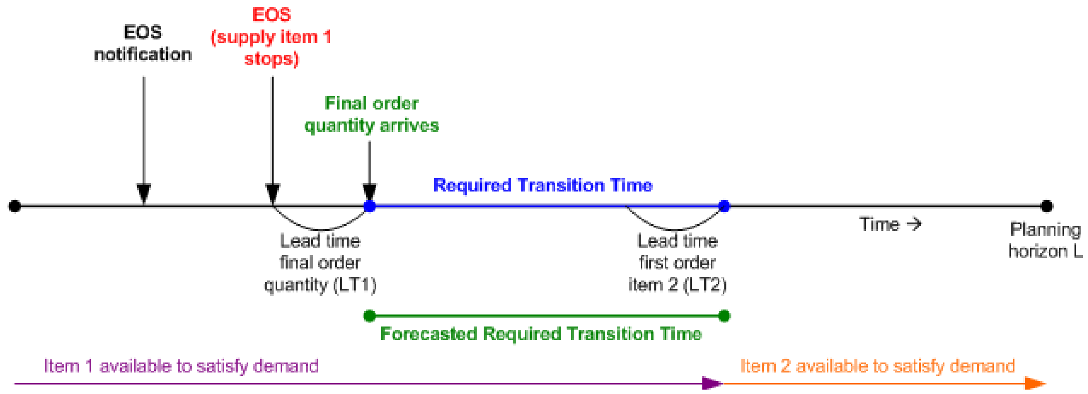


Figure II.1: Events that can occur when an item becomes an EOS issue.

Note that this figure depicts the ideal situation: the EOS notification takes place before the moment of the real end-of-supply and the forecasted RTT is equal to the real RTT. In case of an EOS the RTT depends on the following: the moment in time at which the alternative item is available, the moment in time at which the alternative is really usable (which depends on the fact whether the alternative item is FFF or not), and the lead time of the alternative item. This means that the length of the RTT can be uncertain and/or hard to estimate. The total cost model should be able to cope with this uncertainty in the length of the RTT.

Two policies are considered:

1) *The simple policy*: Here, only a final order quantity is placed to cover demand during the RTT.
2) *The remove policy*: Here, a final order quantity is placed to cover demand during the RTT. Next to that, the inventory on hand is reviewed at the end of each period during the RTT. If there are too many items on stock in comparison with the remaining forecasted demand during the RTT, VI can dispose units until the 'remove-down-to level' of that period is reached.

The total cost model that is developed to solve the problem situations at VI differs from the models in literature as follows:

- The total cost model can cope with uncertainty in the length of the RTT, where a discrete probability distribution for the length of the RTT is taken into account.
- The total cost model can cope with uncertainty in the expected transition costs and uncertainty in the expected cost of one unit of the alternative item.
- The total cost model does not take penalty costs (which are incurred each time that demand occurs that cannot be satisfied) into account, instead it uses a fillrate restriction. This is done because it is impossible to VI to estimate the penalty costs per item that is short.
- The total cost model takes into account substitution. Instead of covering the entire planning horizon with a final order quantity for one item, the planning horizon can now be covered by an alternative item as well. Here, substitution is defined such that the alternative item cannot be used before the end of the RTT.

Regarding the final order quantity, the model takes into account the following cost parameters: purchasing costs of the original item, holding costs, fixed order costs and disposal costs. The larger the final order quantity is, the higher the expected costs will be. Therefore, the optimal final order quantity is the first final order quantity that meets all restrictions in the model (e.g. the fillrate restriction). Regarding the remainder of the planning horizon, i.e. the time periods after the RTT, the following cost parameters are taken into account:

- The expected purchasing costs of the alternative item. Here, uncertainty of the price is taken into account, since it is possible that the price of the alternative item is not known exactly at the moment the final order is placed.
- The expected setup costs of the alternative item. By setup costs the fixed costs that are incurred to be able to use the alternative item instead of the original item are meant. Here, uncertainty of the setup costs is taken into account, since it is possible that the setup costs are not known exactly at the moment the final order is placed.
- The total cost model has been implemented in a tool that is programmed in VBA in Excel 2007. This tool can be used by VI to determine the optimal final order quantity (and the optimal remove-down-to levels) for the original item. The model is validated and verified and a sensitivity analysis is conducted to check the effects of some of the input parameters.

The following academic conclusions can be drawn after this master thesis project:

- The models that are developed during this project concern a unique combination of substitution of items, uncertainty in the transition time (and transition costs) and a fillrate restriction. The models were found to be valid, which makes them a contribution to the existing academic literature regarding the final order problem.

- Based on the models developed during this project, the following can be concluded regarding the effects of the input parameters:
  - *When the original item cannot be used after the RTT, the following holds:*
    - Cost parameters regarding original item and the alternative item do not influence the optimal final order quantity when the length of the RTT is fixed or when it follows a probability distribution. The optimal final order quantity solely depends on the fillrate restriction and the other restrictions regarding the order size (e.g. the minimum order quantity should be taken into account). Better cost estimates only improve the estimation of the total expected discounted costs that result from the model, and thus improve the decision regarding which alternative item to choose (if several alternative items are available).
    - The fillrate restriction directly influences the optimal final order quantity. The larger the fillrate restriction, the higher the final order quantity will be.
    - The length of the RTT (or its probability distribution) and the characteristics of the demand per period have a major influence on the optimal final order quantity. Estimating values for these parameters should deserve great attention.
  - *When the original item can be used after the RTT, the following holds:*
    - General statements about the effects of most of the input parameters of the model on the outcomes of the model cannot be made. The effects of the input parameters are dependent on the specific settings of the parameters.
    - Based on the above point it can be concluded that estimating the input parameters should deserve a lot of attention. The more reliable the cost parameters are estimated, the more reliable the results are.
    - The expected setup costs and the fixed order costs for the final order do not influence the decision regarding the final order quantity: no matter what the size of the final order is, these costs are always incurred. Estimating these cost parameters is important to get better insights in the total costs that can be expected to cover the demand during the entire planning horizon and they influence the decision regarding which alternative item to choose (if several alternative items are available).
- During this project a heuristic has been developed for the remove policy. This was done because the exact method that exists in academic literature was too time-consuming. The heuristic for the remove policy results in costs that are equal to or higher than the costs that result from the exact methods in literature. The heuristic will always result in a solution that satisfies the fillrate restriction.

The following conclusions that are relevant to VI can be drawn after this master thesis project:

- Before this master thesis project, it was unclear to the department Supply Chain Management Services (SCMS) whether a final order should be placed or not and what the optimal final order quantity should be. During this project a solution for this problem has been designed. This solution is implemented in a tool in Excel.
- This tool enables SCMS to determine the optimal final order quantity for the original item and the total expected discounted costs that are incurred to cover demand during the entire planning horizon. Here, the tool takes into account a fillrate restriction during the so-called required transition time.
- The availability of this Excel tool is an improvement of the situation before the master thesis project, since back then there was no appropriate method to determine when to place a final order and what the size of this final order would be. Note however that this tool includes a quantitative model only. Qualitative issues regarding the EOS issues should be taken care of by the SOP-Team.
- A rolling horizon strategy with the simple policy seems to be more appropriate than a remove policy. Using the rolling horizon strategy, updated information can be used to estimate the

parameters in the tool more accurately. Also, the rolling horizon strategy enables SCMS to calculate a ‘remove-down-to level’ at any moment they want.

The following recommendations for VI can be made after this master thesis project:

- It is recommended that the tool that has been developed during this project will be included in the EOS procedure that is present at VI.
- Related to the previous recommendation, it is advised that the EOS procedure at VI is revised. The following recommendations are given that are related to the EOS procedure:
    - SCMS should have a more distinct role in the EOS procedure: The users of the tool developed during this project are the people from Team Worldwide, so they can provide the (quantitative) advice about what to do regarding the final order.
    - In the current EOS procedure, responsibilities are not always clear to all the stakeholders. Moreover, most stakeholders see the EOS issues as a problem for the Business Unit Services, so why would they have to solve it?
    - Management support should be created regarding the EOS procedure, such that people will understand that solving EOS issues is important for all departments.

Regarding the Excel tool and its input parameters, the following is recommended:

- Demand characteristics have a major influence on the optimal final order quantity. Currently, there is no clear method for forecasting demand. It is recommended to devote a project to improve demand forecasting, since this will also make the results of the tool more reliable.
- Although the estimations of the cost parameters for both items do not influence the decision on the optimal final order quantity when the original item cannot be used after the RTT, these estimations do affect the decision when the original item can be used after the RTT. Therefore, it is recommended to keep the information regarding items as up to date as possible. Next to that, time should be devoted to estimating the cost parameters, such as the holding costs per item (e.g. by storing data regarding the volume of an item) and the expected price of the alternative item.
- VI can use different parameter settings in the tool to see the effects of changing these parameters. Also, parameter settings for different alternative items can be used to calculate the total expected discounted costs during the planning horizon for all different alternative items. Knowing these costs, VI can choose the ‘optimal’ alternative item.
- The more reliable the information that is available is, the more reliable the results of the tool are. Therefore, attention should be given to estimating the length of the RTT and the fillrate restriction.
- It is recommended that VI uses the rolling horizon strategy with the simple policy instead of the remove policy. The rolling horizon strategy seems more appropriate to VI, especially when the length of the RTT is uncertain.

III. Preface

You are reading a report that is the result of my master thesis project that I conducted at Vanderlande Industries in Veghel. After about four years of following courses and doing projects, this master thesis project represents the final part of my study Industrial Engineering and Management Sciences at Eindhoven University of Technology.

Vanderlande Industries provides automated material handling systems and accompanying services in several markets, such as baggage handling and parcel and postal. As a graduate student in Operations, Planning, Accounting, and Control, or more specifically the research field of Capital Goods Operations, the project was conducted at the Supply Chain Management Services department of Vanderlande Industries. This department is responsible for the world wide delivery of spare parts, in order to provide service after sales. During my master thesis project I was able to design a tool that can be used by this department to help decision making regarding end-of-supply issues. I.e. in case an item is end-of-supply, the tool enables them to calculate the size of the final order for that item, even when they are not sure when they are able to use an alternative item to replace the item that became an end-of-supply issue.

Of course, I would like to thank several people that helped me throughout this master thesis project. First of all, I would like to thank Simme Douwe Flapper, my primary university supervisor, for all his enthusiasm and support during my entire project. We had regular meetings that were always interesting and helped me to overcome the challenges I experienced during my project. Furthermore I would like to thank Geert-Jan van Houtum, my secondary university supervisor, for his critical view on the project and his useful feedback.

At Vanderlande Industries I would like to thank Katja Kleinveld, who gave me the opportunity to conduct my master thesis project at Vanderlande Industries. Next to that, I would like to thank her and Willeke van der Rijt as company supervisors for the time and effort they invested in my project. We always had nice discussions about the project and their opinion about the issues I faced helped me a lot. Furthermore, I would like to thank Erlend Hessel, who is often involved in end-of-supply issues at Vanderlande Industries. His experience with end-of-supplies learned me how the company tries to tackle end-of-supply issues and what could be improved. Also, I would like to thank all the other colleagues at Vanderlande Industries who I interviewed throughout my project for their feedback and ideas. Besides, I would like to thank my nearest colleagues at Supply Chain Management Services for making my stay a pleasant one and involving me in the meetings they had.

Finally, I would like to thank my family and friends who have supported me during my entire study and especially during this master thesis project. Last but not least I would like to thank my boyfriend Martijn for his support and feedback, and for helping me with programming the tool.

Lonneke Bakx,
June, 2010

IV. List of Abbreviations

EOL	End-of-Life (service) period
EOS	End-of-supply
FFF	Form-Fit-Function
MOQ	Minimum Order Quantity
PLS	Product Lifecycle Support
RSP	Remaining Service Period
SCMS	Supply Chain Management Services
SOP	Steeringcommittee Operational Products
VI	Vanderlande Industries

V. List of Definitions

End-of-Life (service) period	The End-of-Life (service) period (EOL) is defined as the remaining period that the company has to satisfy her customers' demand. Note that the terms EOL, planning horizon, and remaining service period (RSP) are used interchangeably in this report.
Form-Fit-Function (FFF)	When an alternative item is Form-Fit-Function (FFF), this means that the original can be replaced by the alternative item without changing anything on the existing system. The alternative item fulfills the same function as the original item.
Lead time	The lead time is the time it takes the supplier to deliver service items to the SCW.
Planning horizon	See the definition of End-of-Life (service) period (EOL).
Remaining Service Period (RSP)	See the definition of End-of-Life (service) period (EOL).
Remove Policy	Policy where the company places a final order and reviews its inventory level by using remove-down-to levels that are effective at the end of each period during the required transition time.
Required Transition Time (RTT)	The time it takes the company to change from the original item (item 1) to an alternative item (item 2). The required transition time depends on: - The moment at which the alternative item is available (i.e. the alternative item can be ordered) - The moment at which the alternative item is really usable (if the alternative item is FFF it is usable when it can be ordered, no changes are required) - The lead time of the alternative item.
Simple Policy	Policy where the company only places a final order. There are no intermediate reviews of the inventory levels.

VI. Table of Contents

1 Introduction

In this report my master thesis project and its results are described. This master thesis project is conducted at Vanderlande Industries (VI), one of the key companies in among others the baggage handling systems market. Within Vanderlande Industries a problem exists regarding the end-of-supplies she faces. The master thesis project was executed for the department Supply Chain Management Services (SCMS).
To discover what the problem exactly is and ultimately, to solve this problem, this master thesis project was preceded by a preparation phase. During this preparation phase, the problem at VI was analyzed in more detail. Next to this a literature study regarding the type of problem, the final order problem, was conducted. Using the knowledge from academic literature, the specific problem at VI could be defined and a solution could be developed. This report will elaborate the problem at VI and how this problem is solved. For more details about the preparation of the master thesis project we refer to the research proposal for this master thesis project [Bakx, 2010].

In this chapter first the main conclusions of the literature study that is conducted in the preparation phase are given in section 1.1. Especially the existing literature regarding the final order problem and the relationship of this research project with this existing literature is elaborated. After this, the problem definition will be given in section 1.2. Next, a description of VI is given in section 1.3. The relevance of the problem to VI is discussed in section 1.4. This is followed in section 1.5 by a description of the procedure that is currently used at VI to tackle the problem. Finally, the outlay of the remainder of this report is given in section 1.6.

1.1 Literature Study

To prepare for this master thesis project a literature study regarding the final order problem is conducted. The complete literature study can be found in [Bakx, 2010]. The final order problem can be classified using several dimensions. The issues one should consider when discussing the final order problem are:

- Maximizing expected net profit versus minimizing expected total cost.
- One-period versus multi-period.
- Repairable versus non-repairable items.
- With returns (reuse of returned items) versus without returns.

Since the final order is a rather specific type of problem and there is not overwhelmingly much literature about this, it is chosen to try to review the literature in such a way that a complete overview of the existing final order literature is obtained. This means that all of the above mentioned issues are considered relevant, e.g. articles about repairable items are reviewed as well as articles about non-repairable items. In this literature study a broad view of the final order problem is researched, since for example general service items literature (e.g. regarding inventory policies) is considered as well. Again, this has been done to obtain a complete overview of the type of problem and its aspects. In the next section, the most important articles regarding the final order problem are addressed shortly. Most of these articles concern a quantitative model for the final order problem. Finally, the relationship of this research project with the existing literature is discussed.

1.1.1 Final Order Problem Literature

In this section the articles that specifically address the final order problem are considered. For each article, a very short description of the contents of that article is given. More details can be found in Appendix A.

The first article that addresses the forecasting of the ‘all-time requirement’ is the article by Moore [1971]. Here, a final production run is used to obtain items to satisfy cumulative demand for all time in the future. Ritchie & Wilcox [1979] developed a demand forecasting model for the all-time future demand that

explicitly takes the criticality of the service item into account. The less critical a service item is, the sooner the demand for such a service item decreases. In the article by Fortuin [1980] a quantitative model is developed to determine the optimal final order quantity to be able to meet a certain availability degree. This availability degree should be met for the all-time future demand. In [Fortuin, 1981] the model of [Fortuin, 1980] is revised, where a fictitious reduction of the remaining service period is used to lower the all-time requirement of spare parts. Teunter & Klein Haneveld [1998] used the model of Fortuin [1980], but instead they use a discrete probability distribution for demand instead of a continuous one. Next to that they use shortage costs instead of an availability degree. Also, they consider decomposing the multi-component final order problem to single component final order problems. Teunter & Fortuin [1999] developed two quantitative models for the final order problem: one where only the final order quantity is calculated and one where the final order quantity and intermediate 'remove-down-to levels' are calculated. In order to calculate the optimal final order quantity with both types of the model estimates of the cost parameters (e.g. purchasing costs, penalty costs) and estimates for the probability distribution of the demand are needed. In [Teunter & Fortuin, 1998] a case study is done for the models described in [Teunter & Fortuin, 1999]. Here, a new method for gathering and storing data is proposed which should ease the estimation of the parameters (e.g. costs, demand) and the calculation of the final order quantity.
Feng [2007] developed a model that is an extended version of the model described in [Teunter & Fortuin, 1999]. It can handle more complex situations, because it can handle multi-part systems concurrently rather than one part at a time. Also, it accounts for uncertainties in the demand and penalty inputs. The article by Kooten & Tan [2007] specifically addressed the final order problem including the risk of condemnation. The model can calculate the final order quantities for repairable spare parts considering a predefined service level, where condemnation is explicitly taken into account. Bradley & Guerrero [2009] developed a model to determine the lifetime buy for multiple service parts that become obsolete over its lifecycle. Cattani & Souza [2003] consider the final order quantity and focus on its optimal timing for a fixed and known remaining number of periods of demand. They quantify the benefits of delaying the final order.

Next to placing the final order extra order possibilities after the final order are considered as well in literature. The article by Geurts & Moonen [1992] considers the final order quantity, but it considers other possibilities to acquire service items as well. The authors try to find out what the influence of having the possibility to order service parts after the final order is, where the price of the service item that is ordered after the final order is relatively high. This looks similar to a situation where quantity discounts can be obtained. In [Klein Haneveld & Teunter, 1997] and [Teunter & Klein Haneveld, 2002] the same kind of problem as in [Geurts & Moonen, 1992] is addressed. However, they revised the model and they found an approach that determines the initial order-up-to-level and a subsequent series of decreasing order-up-to-levels for various intervals of the planning horizon. In [Inderfurth & Mukherjee, 2008] three possibilities to acquire service items are addressed: a final order, extra production, and remanufacturing. In the article by Krikke & van der Laan [2009] an additional way of acquiring service items is considered. Next to spare parts management and last time buy they also consider reverse logistics. Sprengler & Schröter [2003] used a system dynamics approach and modeled an integrated production and recovery system for supplying spare parts to evaluate the possible strategies for meeting spare parts demand for electronic equipment in the end-of-life phase.

An article by Li et al. [2010] addresses the final order problem and product substitution. In their article, inventory planning decisions for product upgrades when there is no replenishment opportunity during the transition period are considered, where the authors allow for product substitution. The transition time ends when the new-product demand rate stabilizes and that of the old product drops to a negligible level. In the models they consider a start date for the transition and a finite time horizon, where the transition time is the time interval between the transition start date and the finite time horizon. Next to this, during this transition time the company sells both the old and the new product. Thus, when a shortage of the old product occurs, the new product can be used as a substitute. The authors show that the optimal substitution decision is a time-varying threshold policy. Also, they find that substitution reduces the need to hold the old product and can increase the profitability over the transition. Next to this, whereas the total stock increases when the variability in the transition start date increases, the division between the old and new product inventory is not monotonic. Regarding the delay of a new product release they find that one should try to address excess inventory in the old product first through the initial stock decisions, and if that is not sufficient, subsequently delaying the release of the new product.

1.1.2 Literature Study Conclusions

Based on the existing academic literature regarding the final order problem as summarized in section 1.1.1 and Appendix A, the following can be concluded:

- Most articles only consider a fixed planning horizon for which the all-time requirement should be determined. Only Fortuin [1981] and Cattani & Souza [2003] consider changing the length of the time period that should be covered by the final order quantity.
- Next to only using a final order quantity to satisfy the all-time cumulative demand, other possibilities of acquiring service items are addressed in literature as well. [Geurts & Moonen, 1992], [Klein Haneveld & Teunter, 1997], [Teunter & Klein Haneveld, 2002], [Inderfurth & Mukherjee, 2008], [Krikke & van der Laan, 2009], and [Sprengler & Schröter, 2003] all consider additional possibilities to acquire spare parts. In the models in these articles the final order quantity does not necessarily have to satisfy the all-time cumulative demand. However, all articles address very specific possibilities to acquire spare parts, e.g. Inderfurth & Mukherjee [2008] consider three options to acquire service items.
- Some articles, namely [Fortuin, 1980 & 1981] and [Kooten & Tan, 2003], consider quantitative models in which an availability level or service level is included. In articles where the model does not include a service level, a parameter for shortage costs or penalty costs is taken into account.
- In some articles stochastic dynamic programming is used to model the final order quantity and its costs. Other methods are used as well in literature, for example a system dynamics approach is used by Sprengler & Schröter [2003].
- Only one article was found that specifically addressed substitution of items, namely the one by Li et al. [2010]. Here, a (stochastic) transition start date is taken into account and during the transition time the new product can be used when a shortage of the old product occurs.

A topic that deserved almost no attention in final order problem literature is substitution of items. Although this may look similar to the situation in which other possibilities to acquire spare parts (e.g. extra production) are considered, there is a fundamental difference: other possibilities to acquire spare parts lead to exactly the same spare parts as the ones bought in the final order. E.g. used spare parts in the field are repaired, which means that the company has the same spare part again. However, substitution of items means that the 'original' item for which the final order can be placed is replaced by an alternative item. This alternative item may be almost equal to the original item; it differs at least in one characteristic. This means that substitution of items can resemble the situations in which other possibilities of acquiring items are used. For example, if it is possible to have extra production runs, the only difference between the original item and the alternative item may be the unit cost. However, substitution is a broader concept in the sense that the alternative item can be completely different from the original item. Although it may not be desired to have a large difference between the two items, it is possible that the original item is replaced by a completely different alternative item.

Substitution of items means that a transition has to take place from a situation where the original item is used to satisfy demand to a situation where the alternative item is used to satisfy demand. In literature, the only article that was found regarding the final order problem that addresses substitution is [Li et al., 2010]. However, in this article the alternative item can be used when a shortage of the old product occurs during the transition time. Here, the transition time is defined as the time interval between the transition start date and the moment the new-product demand rate stabilizes and that of the old product drops to a negligible level. In this project, we consider the situation where the final order quantity is the only opportunity to obtain units of the original item and demand during the transition time can be satisfied by this original item only. We define the transition time such that it is not possible to use the alternative item before the end of the transition time. In other words: the transition time is the time that is needed by the company to change from the original item to the alternative item. This is fundamentally different than the definition of the transition time in [Li et al., 2010]. In that article it is possible to use the alternative item to satisfy demand during the transition period, while this is not the case in this project.

We try to develop a quantitative model for the final order quantity in which one or more possible alternative items are taken into account. Using this model, decisions regarding the final order quantity and choosing the ‘best’ alternative item from all possible alternative items can be supported quantitatively. This means that the model explicitly takes into account (an) alternative item(s). By taking this into account, this automatically means that the model should take into account a ‘transition time’.
However, determining the exact length of the time period that the transition from the original item to a specific alternative item takes may be very difficult. Because the length of the transition time is uncertain, the model has to take into account a probability distribution for the length of the transition time.

This discussion leads to the following aspects of the final order problem that were not extensively addressed in literature, but that we would like to elaborate in more detail during this master thesis project:

- Substitution of items (which automatically takes other possibilities to acquire items into account). Here, substitution leads to a required transition time where the demand during the transition time can only be satisfied by the original item. After the end of the transition time, it is possible to use the alternative item to satisfy demand.
- The time the transition from one item to another takes is uncertain, which means that a probability distribution for the transition time needs to be taken into account. Also, uncertainty regarding the transition costs should be taken into account.
- The combination of using a service level (instead of penalty costs) and substitution, where the transition time can be uncertain and thus the moment from which the company can use the alternative item is uncertain, is a new combination of aspects of the final order problem that has not been addressed in academic literature yet according to our knowledge. Therefore, we would like to address this kind of final order problem in this master thesis project.

Having this background knowledge regarding the final order problem in academic literature and the aspects we would like to contribute to this literature, the final order problem at VI is analyzed in more detail during the preparation phase of this master thesis project. The resulting problem definition is given in the next section.

1.2 Problem Definition

In this section the conclusions regarding the problem situations at VI that were identified during the preparation phase of this master thesis project are given. For more details regarding the problem situations, we refer to [Bakx, 2010]. Also, more details about the methodology that is used during this master thesis project can be found in [Bakx, 2010]. Based on this preparation phase, in which the literature study was conducted (see section 1.1) and the final order problem at VI was analyzed, the following can be concluded:

- An end-of-supply (EOS) can be initiated by a supplier, but by VI herself as well (e.g. because the supplying conditions become disadvantageous in the nearby future).
- When an item becomes EOS, most often it will be substituted by an alternative item that fulfills the same functionality as the original item. When the EOS issue concerns an EOS that is initiated by the external supplier, the supplier should offer an alternative item to VI.

- If there is an alternative item available directly, this does not necessarily mean that VI can use this item directly to satisfy demand. If the alternative item is not form-fit-function (FFF), this means that the alternative item does not have the same functionality and/or dimensions as the original item. Using the alternative item requires changes on the drawings then, and maybe even changes in the systems in the field.
- The problem that VI experiences regarding end-of-supplies is a common problem: it applies to any kind of item, so the items that are considered do not necessarily have to be service items.
- No matter what the source of the alternative item is, the final order problem always narrows down to a change from the original item to the alternative item. Note that there can be several alternative items from which VI can choose.
- The transition time that is needed to change from one item to another is hard to estimate: it depends on issues such as the availability of an alternative item, the characteristics of the alternative item, how long it takes to find an alternative item, etc.
- It is not possible to use the alternative item before the end of the transition time.
- It is impossible to make a sound estimation of the penalty costs that are incurred when VI does not have an item available. Therefore, it is more appropriate for VI's situation to include a service level in the model instead of penalty costs.
- At VI, only non-repairable items are considered. According to the supervisors' knowledge there are only very few repairable items, but there is no data about (returns of) these items.

This means that VI is a company where the final order problem exists including a combination of all the aspects that were not addressed (simultaneously) in literature yet: The final order problem, substitution of items, an uncertain transition time, and a service level should be taken into account during this project, which is a unique combination that is not addressed in literature yet to our knowledge.

Based on the above, the problem that is researched in this master thesis project can be defined as follows:

It is unclear how to solve the final order problem for non-repairable items when substitution of items should be taken into account. Substitution of items leads to a 'transition time', where the length of this transition time can be uncertain. Also, the transition costs and the costs for one unit of the alternative item can be uncertain. It is not known how to determine the optimal final order quantity for this (uncertain) length of the transition time, where this uncertainty in costs should be taken into account.

As mentioned above, this problem concerns a 'transition time', which resembles the time it takes a company to change from the original item to the alternative item. Therefore, the model that is developed in this master thesis project will be based on this transition time. In this model, the expected costs during the entire planning horizon should be considered, where the objective of the model is to minimize these costs while meeting all restrictions. How this model is developed exactly will be described in detail in chapter 2 to chapter 4.

The model that will be developed to solve the problem will be applied to Vanderlande Industries. Therefore, a description of VI is given in the next section.

1.3 Description of Vanderlande Industries

"Vanderlande Industries provides automated material handling systems and accompanying services. It focuses on improving its customers' business processes and strengthening their competitive position." [Annual Report 2009, p.7]. VI, which has her headquarters located in Veghel, the Netherlands, is a company that is active in several markets, namely:

- *Baggage handling*: Baggage handling systems at airports, from check-in through destination sorting to baggage reclaim.
- *Distribution*: Automated logistic solutions in warehouses and distribution centers.

- *Parcel and Postal*: Automated systems to deliver parcels and documents on time and in perfect condition.
- *Services*: A full range of services to ensure lifetime reliability of logistic operations and systems.

VI started as an equipment supplier, but now it focuses on total solutions, not only the equipment or systems. This development is depicted in Figure 1.1.

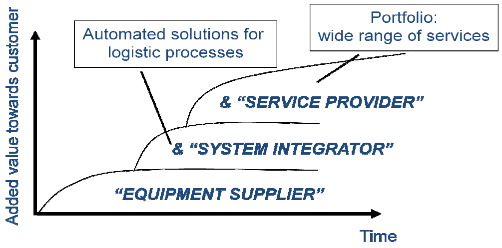


Figure 1.1: Development of Vanderlande Industries.

Lifecycle support and its accompanying services are one of the main pillars in the company's approach to customers.

1.3.1 Mission Statement

The mission of VI is described as follows [Annual Report 2009, p.5]:
"To support our customers worldwide in significantly improving their competitive position by designing, implementing and servicing automated material handling systems."

VI's business heavily relies on reputation. This is also put forward in their core values:
"Vanderlande Industries' core values are that we always strive to deliver what we promise to our customers, and that we believe in the people who work for our company. Our reputation is based on the professionalism and personal integrity of each of our employees and those with whom we do business. Our business success depends on it. Our continued growth, profitability and prosperity are linked to our employees' ability to make decisions that are consistent with our business ethics and code of conduct, which forms an intrinsic part of the way in which we work." [www.vanderlande.com, September 2009].

1.3.2 Organizational Structure

The headquarters and the factory that produces parts are located in Veghel, the Netherlands. VI is a global player which is present in all key regions of the world by operating locally through Customer Centers. VI has Customer Centers in the Netherlands, Belgium, Germany, France, Great Britain, Spain, Canada, PR China, South Africa and the USA. These Customer Centers handle key business functions and maintain direct contacts with the local customers. Each Customer Centre can be seen as an individual small company. Customers are allocated to a Customer Centre based on the type of system and the geographical location of the customer. The headquarters are the backbone of the organization and support its Customer Centers. The organizational groups that are located in Veghel are based on the four markets VI is in: Baggage Handling, Distribution, Postal and Parcel, and Services. Because this research project is conducted at the Business Unit Services, this business unit is described in more detail in section 1.3.3.

1.3.3 Business Unit Services

The Business Unit Services is a relatively young business unit of VI. From Figure 1.1 it can be seen that Services is of increasing concern for VI. VI does not see itself as an equipment provider, but as a company that provides total solutions including services. In the short while that the Business Unit Services exists (around two years), already 14% of the total order intake in the financial years 2005-2009 is accounted for by Services.

The Business Unit Services has a wide-spread portfolio of services, which can basically be divided into the following categories: maintenance services (preventive and corrective), spare parts services (parts supply, consignment services), hotline services, lifecycle services, training (fundamental, customer specific), process management services (process analysis and optimization), and site based services (full maintenance, operations staffing). Here, the operational service activities are executed through the Customer Centers and on site organizations.

The organizational structure of the Business Unit Services is depicted in Figure B.1 in Appendix B. Of special concern is the department Supply Chain Management Services (SCMS), since this is the department for which this research project is conducted. SCMS is responsible for the spare parts services mentioned above. The orders for spare parts that come via Customer Centers around the world or via Team BNL or Team International are handled by Team Worldwide.

1.4 Relevance of the Type of Problem to Vanderlande Industries

As described in section 1.2 , the final order problem, substitution of items, an uncertain transition time, and a service level is a combination of aspects of the final order quantity that are not addressed (simultaneously) in literature yet to our knowledge.VI struggles with all these issues, which means that VI is an interesting case company for this type of problem. In this section the relevance of this problem is discussed.

Since VI operates in fast moving, highly technical markets, end-of-supplies can occur regularly because old parts are updated. Substitution of old service items by new versions of the service items is rather a rule than an exception. When substitution occurs, a final order can be placed for the original service item. If the final order quantity cannot cover the entire remaining service period, the alternative service item can replace the original one and thereby fulfill the remaining demand. The final order quantity of the original item can be reduced by the existence of an alternative item: When the original item is replaceable one on one by the alternative item (e.g. the original and alternative item only differ in color), a final order is even not necessary. When it takes some time to be able to replace the original item by the alternative item (e.g. the R&D department has to come up with new system documents in which the alternative item is used), the final order quantity of the original item has to cover at least this time period.

VI is responsible for keeping the systems in its installed base running for a long service period, namely 10 years. End-of-supplies can cause major problems in such a situation, because VI is responsible for the systems for this long period of time and thus for the supply of items. When the supply of a specific item is discontinued, VI has to find another way to be able to keep the systems that use this item running. If they are not able to solve this problem, VI risks a possible penalty cost. Moreover, they have a great risk of losing their good reputation as a full service provider.

During approximately the last 1,5 years, 40 end-of-supply issues were registered in the databases of VI. Unfortunately, this registration of end-of-supplies is not completely up-to-date, but it gives a good indication. The end-of-supplies that are registered consider very diverse types of items, from control boxes to connecting bolts, and the items differ greatly in price (prices range between €2,30 and €1449,60). Although for each end-of-supply issue it is tracked what the status of that issue is, most issues are 'on hold'. This means that there still is no appropriate solution found for the issue. Regarding the issues that are 'solved' or ended, information is available regarding the solution of the issue. E.g. a possible solution is an alternative item that is FFF. Unfortunately, when the (temporary) solution was a final order, it is not registered how large this final order was.

Next to the end-of-supply issues, the Operations department of VI offers their obsolete items to the SCMS as if it is a final order. When SCMS buys these items from the Operations department instead of external suppliers, it is likely that the price that Operations charges is lower than the price from external suppliers.

Conclusively it can be said that VI experiences end-of-supplies for various types of items, sometimes including very expensive items. Sometimes these items are replaced by updates of the original item, sometimes another solution has to be found to solve the end-of-supply problem. In the latter case, possible solutions are placing a final order, producing the item, or finding alternative suppliers. No matter what the source of the alternative item is, the item that became EOS will be substituted by an alternative item somehow. If it will not be substituted at all, a final order for the original item should be placed. Note that a combination of a final order and using an alternative item is also possible. Next to that, the Operations department offers items to SCMS as if it is a final order. Therefore, the final order problem including substitution, which can consider end-of-supply situations and quantity discount situations, is a very relevant problem to VI.

1.5 End-of-Supplies at Vanderlande Industries

Currently, end-of-supply (EOS) issues are solved at VI by following the so-called End-of-Supply procedure. In this section the End-of-Supply procedure that is used at VI is analyzed. The procedure is depicted in Appendix C. This procedure is of interest because it describes how employees should handle an end-of-supply issue and it can give insight in how to implement the solution that is designed for the problem. Also, it shows how VI thinks about EOS issues and how to tackle them, which may result in interesting inputs for the solution. This way of thinking may also indicate on which aspects resistance to change is likely to occur, which is of importance for the implementation of the solution.

In short, the procedure as depicted in Appendix C can be summarized as follows:

1) Any VI employee can signal an end-of-supply (EOS) issue and should report this to the item controller and hand over the needed information (e.g. by a so-called ‘change request’)
2) The item controller checks the EOS issue quickly and determines among other things the urgency, problem owner, affected products, running projects and a planning of replacement. In case the urgency is really high, a so-called SOP-Team is asked to meet and decide on the actions to take.
3) The problem owner (a SOP-Teammember from the R&D department) performs a detailed analysis of the EOS issue, in which FFF, alternatives, and risks during usage are investigated. A decision is made on whether FFF is respected or not. Here, two outcomes are possible:
    a. Yes, the alternative item is FFF. Go to step 5).
    b. No, the alternative item is not FFF. Go to step 4).
4) Since the available alternative item is not FFF the problem owner should search for an alternative item. The new alternative item should always be approved by the SOP-Team and the Product Teams (which are responsible for the products).
   The new components/technology is handed over by the problem owner to the involved SOP-Teammembers. Next to that the Product Manager Services informs the Customer Centres about the change, the Product Manager Engineering changes documentation of the items (such as the drawings) and the item controller changes the statuses of the items in the databases.

In the meanwhile several documents are created to keep track of the different items and the decisions and changes regarding those items.

In Appendix D a more detailed analysis of the EOS procedure at VI is given. Here, the involved departments and the qualitative issues regarding end-of-supply issues are discussed.

1.6 Report Outlay

The model that is developed during this project to contribute to academic literature (see section 1.1) and to solve the problem at VI as described in section 1.2 is a quantitative decision support model for decisions regarding the final order quantity and substitution. Therefore, the structure of most articles that consider quantitative decision support models (e.g. the articles described shortly in section 1.1) is followed throughout this master thesis. In short, this structure starts with an introduction, a literature review and a problem definition. After this the mathematical model that is used to solve the problem is described. Once the mathematical model is defined, the model is used to obtain results. These results can be used to get insights in how the model operates and how the problem can be solved. Finally, these insights are used to draw conclusions about the model and how it solves the problem.

The first steps of this structure have been described in section 1.1 to section 1.5. More details about the literature study and the problem definition can be found in the research proposal that resulted from the preparation phase of this master thesis project [Bakx, 2010]. The remainder of this report is organized as follows:
In chapter 2 the solution to the problem described in section 1.2 is designed. After this, the models that are used to solve the final order problem are given in chapter 3 and chapter 4. In chapter 5.2.3 a new way of looking to the problem is addressed. Here, the situation is considered where VI may consider switching to the alternative item earlier than is required. In chapter 6 the research and the models are evaluated, i.e. the model is validated and verified. Next, the model for the simple policy and its principles are discussed in chapter 7. These principles lead to conclusions regarding the effects of the input parameters. In chapter 4 the heuristic for the remove policy that is developed during this master thesis project is compared to the exact method from literature. After this, the implementation of the model at VI is discussed in chapter 9. Finally, conclusions and recommendations are given in chapter 10.

Some parts of this report may be less interesting for practitioners (i.e. the employees at VI), such as the description of the mathematical models that are used to solve the problem. However, these models are very important for the contribution of this research project to the academic literature. Therefore, it is decided to give a ‘quick walkthrough table’, or a ‘way to navigate through this report’, which indicates what parts of this report are interesting for academics and what parts of the report are interesting for practitioners. The way to navigate through the report is given below. In Table 1.1 an ‘x’ indicates what is interesting to the academic and/or practitioner. Sometimes additional information about a chapter is given, especially when a chapter itself is interesting for a certain reader but certain parts of that chapter are less interesting.

Table 1.1: How to navigate through this report (an ‘x’ indicates what is interesting).

Chapter	Academics	Practitioners
1	x	x
2	x	x
3	x	Only 3.1 and 3.2
4	x	Only 4.1 and 4.2
5	x	Only 5.1
6	x	x
7	x	x
8	x	Only 8.2
9		x
10	Only 10.1	Only 10.2

2 Development of Solutions

In this chapter the solution that is developed to solve the problem described in section 1.2 is described. First, the main idea behind the solution, the required transition time, is described in section 2.1. In section 2.2 two policies that can be used to deal with the final order problem are described. Finally, in section 2.3 a new strategy regarding one of these policies, the simple policy, is described.

2.1 The Required Transition Time

2.1.1 Introduction

In this section a solution is designed that covers the problem situation as described in section 1.2. To design this solution, the general idea that is used to solve the problem situations is described in this section first. The idea is rather general in the sense that it can cover many different situations, e.g. it covers the situation where the supplier offers an alternative item vs. the situation where the supplier does not offer an alternative item, or the situation where the company causes the end-of-supply (EOS) vs. the situations where a supplier causes the EOS.

Regarding the end-of-supply problem a company can face the following events:

1) For some reason, an EOS notification for a specific item (say item 1) is received. When the EOS notification concerns an item that is normally bought from an external supplier, the supplier will often offer an alternative item, e.g. an upgrade of item 1.
2) At the same time as event 1), or (preferably) at some later time, item 1 is end-of-supply. This means it is not possible to order item 1 anymore or the company decided that from this moment on the item is not ordered anymore. However, when the EOS occurs, it is possible to place a final order. Note that in the worst-case scenario, event 2) occurs before event 1), which means that the company simply does not know that the item is EOS and it may only find this out by trying to place an order while the item is not available anymore.
3) After a certain lead time (which is longer than or equal to 0), the final order quantity arrives. This final order quantity should cover the demand during the so-called ‘Required Transition Time (RTT)’. Here, the required transition time is defined as the time it takes the company to be able to use item 2 (which is the alternative item for item 1) to satisfy demand.
4) After the RTT, the company can use item 2 to satisfy demand.

These events are depicted in Figure 2.1:

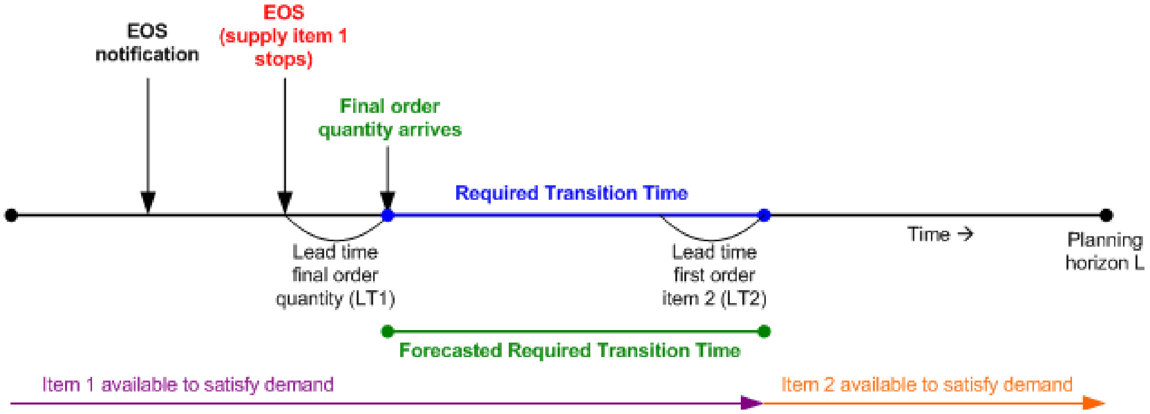


Figure 2.1: Events that occur when an item becomes end-of-supply (EOS).

Note that in real life not all of these events have to occur each time the company faces an EOS. For example, it is possible that the company faces an EOS because a supplier stops supplying a certain item and that there will never be an alternative item that is one-on-one replaceable with the original item. This means that the company cannot simply replace item 1 by item 2, the replacement requires changes in the system's drawings, the systems in the field, or anything like that. In the situation where no alternative item will be available during the planning horizon, the required transition time would be the time period between the moment that supply stops and the end of the planning horizon.

In the upcoming sections each of the events given above are described in more detail..

2.1.2 Occurrence of Events

In this section the events that the company can face when a certain item (say item 1) is replaced by an alternative item (say item 2), are described. Item 1 will become an end-of-supply issue, which is replaced by item 2. The events that occur are depicted in Figure 2.1. In the following sections, all relevant issues regarding these events will be discussed.

2.1.2.1 EOS Notification

At a certain moment in time the company receives an EOS notification. This EOS can originate from two sources:

1) *The supplier*: In this case the supplier decides that he does not want to supply the service item anymore. Preferably the supplier notifies the company that he will stop supplying the item. However, in reality this notification is not always sent by the supplier, which means the company itself discovers that the item becomes end-of-supply. In the worst case, the company finds out that the item even already is end-of-supply because the supplier stopped supplying the item and the company is not able to order the item anymore. Especially the latter situation is dangerous: the EOS is noticed too late by the company since the company is not able to order the item anymore.
2) *The company*: In this case the company decides that it does not want to order the service item anymore. This can have several reasons:
    a. *The supplier indicated that its supplying conditions will change in the nearby future.* When these changes are disadvantageous for the company (e.g. longer lead times, higher prices, lower reliability), the company may decide to stop ordering that item at the supplier.
    b. *The company's R&D department decided that the original item is not sufficient anymore for the end products and replaces the original item by an upgraded or totally new item.* Main reasons to do this are to keep ahead of competition, to comply with new regulations or the company's drive to keep improvements and developments going.

Ideally, the company knows that an item will become EOS before it truly is EOS. When the EOS is caused by a supplier this requires that the supplier notifies the company timely and that the company processes this notification as well. Also, when an EOS is caused internally (e.g. by R&D using new items instead of the older version), this should be communicated as soon as possible. Even if the company does not know anything about the upcoming EOS, at a certain point in time it will occur. In the next section, the moment that the supply of the original item stops is discussed.

2.1.2.2 EOS – Supply of the Original Item Stops

At a certain moment in time the supplier will really stop supplying item 1, or the company decides to not order item 1 anymore. Note that in the latter situation supply does not really stop since the company can still order the item from the supplier if it wants to. However, for some reason the company thinks that buying the item from that supplier is unattractive, and it decides to stop the supply by that specific supplier from that moment on. Whatever the reason may be, this moment indicates the time at which the company is going to switch from item 1 to item 2. At this moment, the company can decide to place a final order for item 1. This final order quantity can be used to satisfy demand, e.g. demand from customers who really only want item 1 instead of item 2. Moreover, the final order quantity can be used to 'buy time'. The time that is 'bought' here is the so-called Required Transition Time (RTT), which is discussed in the following section. Note that the lead time of the final order quantity (let's say $LT1$) can be positive (see Figure 2.1). When this is the case, it is assumed that the company has ordered enough of item 1 to cover the demand during this lead time. The final order quantity is only used to cover demand during the RTT.

2.1.2.3 Required Transition Time

When an EOS occurs (whether it is originated by a supplier or by the company does not matter), item 1 cannot or should not be ordered anymore. If item 1 is an item that is normally bought from a supplier and this supplier caused the EOS issue, this supplier can offer an alternative item. It is most likely that he will do this, because he does not want to lose sales. Moreover, the company has contracts with its suppliers that state that they should supply the item, or an alternative for the item that is equal in functionality and price, for at least a certain time period (depending on the type of item). However, if the company is only a relatively small customer to the supplier, it can occur that he does not offer an alternative item. In such a situation, it may be possible to find an alternative item from some other source, e.g. from a different supplier or from the company's own factory. This alternative item, item 2, will thus replace item 1. The company should make itself ready to be able to use item 2. Here, the following situations can occur:

1) *The alternative item is form-fit-function (FFF)*: This means that item 1 is one-on-one replaceable by item 2. Thus, replacing item 1 by item 2 does not require any changes in the drawings of the end products nor changes in the products in the field. The only change that has to be made is that the new item number for item 2 is registered well in all databases and drawings. However, the effort that this change takes is negligible.
2) *The alternative item is not (entirely) FFF*: This means that item 1 cannot be replaced one-on-one by item 2, e.g. item 2 has different dimensions than item 1. Using item 2 requires changes in the drawings and/or changes in the products in the field.

When searching for an alternative item, the company will always try to find an alternative item that is FFF, since this item fulfills all functions of the original item. Next to that, it requires minimal changes in the database and drawing. Finding an alternative item that is FFF immediately solves the problem: the company can use the alternative item to satisfy demand because it does exactly the same thing as the original item. If the supplier that causes the EOS issue offers an alternative item, the company should check whether this alternative item is FFF. If it is not, the company can decide to search for some other source for the alternative item. Finding an alternative item that is 100% FFF can be very time-consuming, so the company may decide to accept an alternative that is not FFF (and thus requires changes) because it wants to solve the problem fast. This can be depicted as is shown in Figure 2.2.

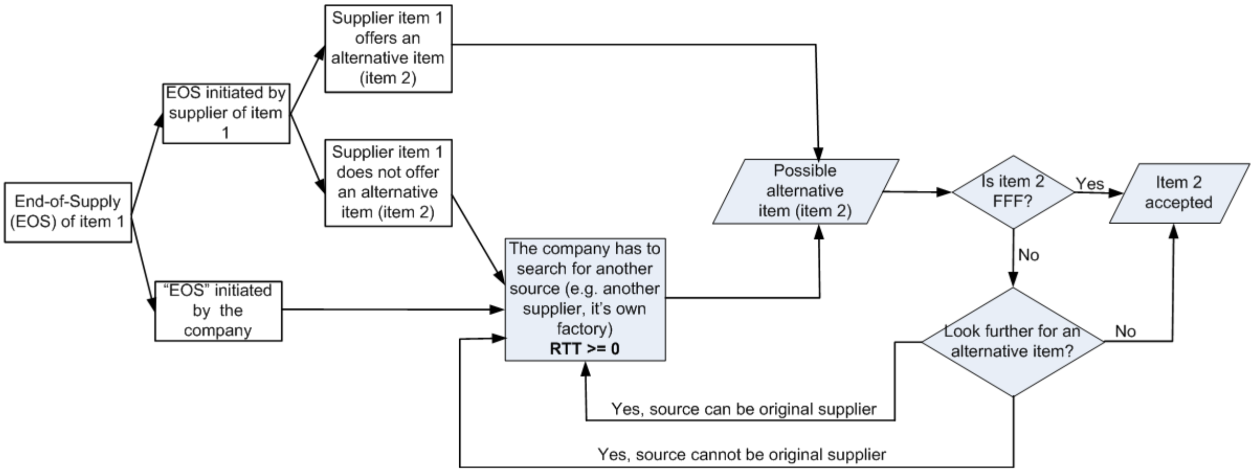


Figure 2.2: Searching for an alternative item.

Regarding Figure 2.2, note the following:

1) When the company initiates the EOS, this is not really an end-of-supply since the supplier does not stop supplying the item. However, for some reason the company thinks it is more attractive to discontinue ordering from that supplier, e.g. because the supplier's delivery performance is very

bad or his price is very high compared to other suppliers. Note that it may be possible that when the company initiates the EOS, the R&D department may already have developed an alternative item. In the case where this alternative item is FFF, the RTT would be 0.

2) An alternative item can be any item, as long as it differs on at least one characteristic from item 1. This means that the concept of RTT is also useful for quantity discounts situations, since in this situation the company basically compares item 1 (with price 1) to item 2 (with price 2). Here, the items are exactly the same, they only differ in price. Price 1 indicates the price when the quantity discount holds, price 2 indicates the price when the order quantity would be below the threshold to obtain the quantity discount.
3) Determining whether to accept an alternative item or not is not as easy as it looks like in Figure 2.2. The more complex an item is, the more research employees have to do before they can argue to accept an alternative item or not. 'FFF' is a measure that includes many different aspects of an item, including its functions, dimensions, etc. All these aspects have to be considered to be able to make a sound decision.
4) Although an alternative item may not be FFF, it can be decided to not look further for a better alternative item. This is mainly done when there is little time available to search for better options, or when employees expect that only little improvements can be found by intensive search efforts.

The search for an alternative item and the measure of FFF lead to a so-called Required Transition Time (RTT). Simply said, the RTT is defined as the time it takes the company to be able to use the alternative item to satisfy demand. Here, the RTT starts from the moment the original item cannot be ordered anymore (so the supply of item 1 has stopped). The RTT depends on:

1) *The moment in time at which the alternative item is available (i.e. the alternative item can be ordered).* This can be immediately at the moment of EOS (or even before that moment) when the supplier that causes the EOS offers an alternative item. However, the moment the alternative item is available can also be after the EOS, because the company has to search for an alternative item.
2) *The moment in time at which the alternative item is really usable.* If the alternative item is 100% FFF, the alternative item can be used immediately from the moment it is available. If it is not 100% FFF, the R&D department will have to change the drawings of the end products (e.g. by making work-arounds) and/or changes have to be made in the systems in the field. In some cases even tests (e.g. durability tests) have to be executed before these changes are made. If making these changes is very time-consuming and/or expensive, the company may consider searching for an alternative item that is 'more FFF' and/or less expensive.
3) *Lead time of the alternative item* (let's say $LT2$, see Figure 2.1). Even when an alternative item is available (i.e. it can be ordered) and it is usable in the field, it is possible that the company cannot use it yet because of a positive lead time of the alternative item. Therefore, the RTT includes the lead time of the first order of the alternative item, because only after this lead time the alternative item is really available to the company to satisfy customer demand.

If the alternative item is immediately available at the EOS moment (see section 2.1.2.2), the alternative item is 100% FFF, and the lead time for item 2 is 0, the required transition time will be 0 because the alternative item is available and does not require any changes. However, if the alternative item is not immediately available (e.g. the company is still searching for a suitable alternative) or if the alternative is not 100% FFF and thus requires changes in the drawings or systems in the field, the required transition time will be positive. If the RTT has passed, the company is able to use item 2 instead of item 1. It is assumed that all demand during the remainder of the planning horizon can be satisfied with item 2, i.e. there is no EOS for item 2 during the planning horizon. Next to this, note that the source of item 2 can be anything: it can be the same supplier as the supplier of item 1, it can be a different supplier, and it can be the company's own factory. No matter what the source of item 2 is, it is assumed that item 2 is always available during the remainder of the planning horizon. Also note that the concept of required transition time can also be used when the supply of item 1 is discontinued by a supplier, but there is no alternative item needed and the company simply has to find a new supplier for item 1. In such a case, the required transition time simply represents the time the company needs to find this new supplier.

Since the RTT depends on the moment of availability of the alternative item, the moment at which the company can really use the alternative item to satisfy demand (i.e. after the changes in the drawings and/or the systems in the field are made) and the lead time of the alternative item ($LT2$), the length of the RTT is uncertain. In some cases the RTT will be 0 because the item that is EOS is a standard item that any supplier can supply, in other cases the RTT will be long because the item that is EOS is unique and finding an appropriate alternative costs time. Next to that, when changes in drawings or existing products in the field are required, the time this takes depends among other things on the effort the employees put into this problem. If they do not perceive the EOS as a big problem, they will prefer to work on tasks that they perceive as more important, which lengthens the RTT. Thus, the time it takes to be able to use an alternative item can be varying a lot, depending on the specific EOS issue. Therefore, the model should take into account a probability distribution for the length of the RTT.

2.1.2.4 The Remainder of the Planning Horizon

The required transition time is the critical time period for the EOS problem. When the company 'survives' this period, the EOS issue is basically solved since the demand during the remainder of the planning horizon can be satisfied by item 2. As mentioned previously, it does not matter what the source of item 2 is (e.g. it can be a new supplier, or the company's own factory). The RTT is needed to find the source of item 2 and to eventually change drawings and systems to be able to use item 2. Once item 2 and its source are found and usable, the RTT is over since item 2 can be used to satisfy demand. It is assumed that the availability of item 2 during the remainder of the planning horizon will not be discontinued, i.e. no EOS will occur for item 2. This assumption is made because an EOS of item 2 basically repeats the concept of RTT: in such a case a transition has to be made from item 2 to item 3. Repeating the RTT concept does not add any value in the understanding of the RTT, because it is simply a repeat of what we already know. Therefore, it is assumed that item 2 will always be available during the remainder of the planning horizon.

Since this graduation project is focused on the EOS problem, the project will focus on 'surviving' the RTT instead of making a detailed order planning for the entire planning horizon. It is logical to assume that the company knows how to order wisely when supply is not discontinued. Since it is assumed that the supply is never discontinued during the remainder of the planning horizon, it is assumed that this remainder of the planning horizon is not problematic since the company knows how to do the ordering.

2.1.3 How to survive the Required Transition Time

In the previous section the events that occur when item 1 is replaced by item 2 are discussed. A critical period is the so-called required transition time (RTT). As explained in the previous section, the RTT is the period of time the company needs to find item 2 and eventually to make changes such that item 2 can be used to satisfy demand. Note that the concept of RTT does not necessarily mean that there must be an alternative item: when there is no alternative item the RTT simply resembles the time period between the moment supply of the original item stops and the end of the planning horizon. Once the company 'survived' the RTT, item 2 is assumed to be always available to satisfy the demand during the remainder of the service period. So, when the company passed the RTT, it solved the EOS problem because item 1 is successfully replaced by item 2. But how do you 'survive' the RTT?

If the supplier initiated the EOS, the only possible way to 'survive' the RTT is by placing a final order for item 1. At the start of the RTT, item 2 is not available yet (the company cannot order it), it is not applicable yet, or it is not delivered yet. Even if item 2 may be available from any of the possible sources (e.g. a new supplier, the company's factory), it cannot be used by the company to satisfy demand yet (otherwise the RTT would not be positive). The only option to satisfy demand during the RTT is by using item 1. However, at the start of the RTT item 1 is EOS, which means it cannot be ordered anymore by the company. Normally, the supplier offers the company the option to place a final order for item 1. Although there may be other suppliers who are able to supply item 1, these suppliers should be found first and

finding them takes time. This leads to a positive RTT (although it may be short), and also this searching time should be covered somehow.
Note that the final order quantity for item 1 is the only option to survive the RTT only in the case when the EOS is caused by the supplier. If the EOS is caused by the company itself (e.g. because the R&D department changed the specifications of some items), supply is not discontinued and item 1 can be ordered normally until the end of the RTT. However, considering a 'final' order that covers the RTT in such a situation instead of using the normal ordering procedure during the RTT may be interesting, since a final order is an order of a larger quantity than normal, which may result in quantity discounts. Next to that, a final order may be required due to unattractive supplying conditions.

Hereafter, we focus on the situation in which the company places a final order to survive the RTT. Ideally, the final order quantity covers demand during the RTT exactly, i.e. the company estimates the length of the RTT correctly and it estimates the demand during the RTT correctly. In such a case, the final order quantity is exactly large enough to cover all demand during the RTT, and there is no item 1 left at the end of the RTT (see Figure 2.1). However, demand forecasting is a very difficult task. Next to this, as explained in the previous section, the length of the RTT can be uncertain and follows a probability distribution. Therefore, the following situations can occur:

1) The company is optimistic and estimates the RTT to be shorter than it actually is.
2) The company is pessimistic and estimates the RTT to be longer than it actually is.

These two situations are discussed in the following sections.

2.1.3.1 The Optimistic Scenario

In the optimistic scenario, the company estimates the RTT to be shorter than it actually is. In such a situation, the company faces the risk of having shortages. This is depicted in Figure 2.3.

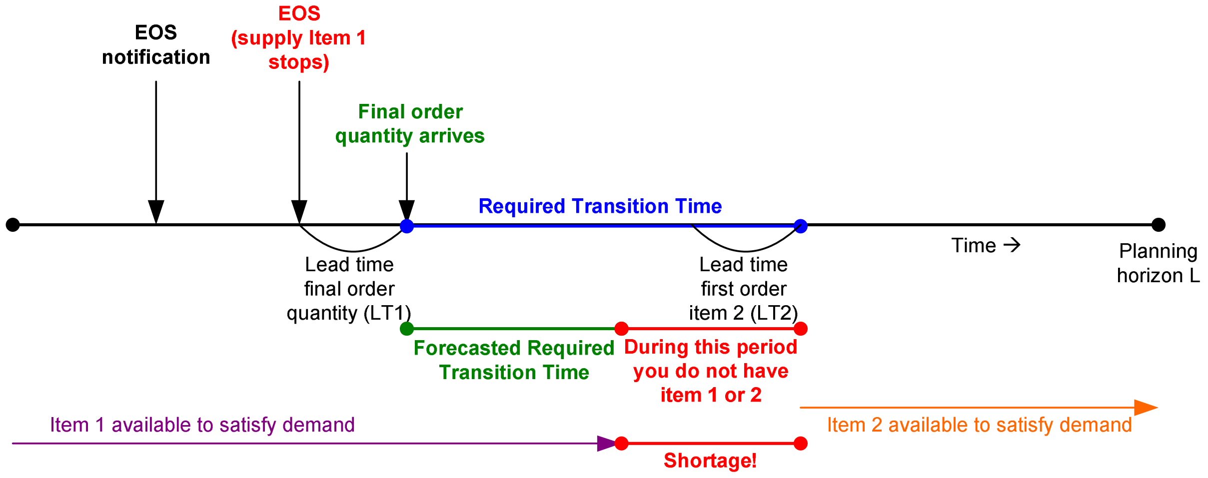


Figure 2.3: The optimistic scenario regarding the estimation of the length of the RTT.

During the period that the company does not have enough item 1's on stock (i.e. it has a shortage), the following can happen:

1) The customer accepts the fact that the company cannot deliver item 1 and waits for item 2. This causes a backlog.
2) There is some 'magical' option to obtain units of item 1. For example, it may be possible to still order items from the original supplier of item 1 (at very high cost). In this case the items are available immediately when they are demanded. This option is a very expensive solution in comparison to ordering the item in a final order.

Although both situations sound very unrealistic, it is assumed that option 2) holds when the RTT turns out to be longer than expected. Thus, when the company does not have enough units of item 1, it simply orders an item 1 from the 'magical supplier' as soon as it is demanded, and the item is immediately available. An assumption like this is common in final order literature. Next to this option 1) is even more unrealistic, mainly because a customer cannot simply wait until item 2 is available to solve his problem (especially when his system broke down due to the defect in item 1).

When a shortage occurs, the company faces penalty costs. These penalty costs consist of the following:

1) Penalty costs that the company should pay to the customer, or:
2) Costs that the company pays to its 'magical' supplier for an item.

When determining the final order quantity, these penalty costs should be taken into account. However, in practice it can be very difficult to estimate these penalty costs. In such a situation it can be better to use a fillrate restriction to restrict the number of out-of-stocks, instead of adding costs to each out-of-stock.

2.1.3.2 The Pessimistic Scenario

In the pessimistic scenario, the company estimates the RTT to be longer than it actually is. In such a situation, the company faces the risk of having excess inventories. This is depicted in Figure 2.4.

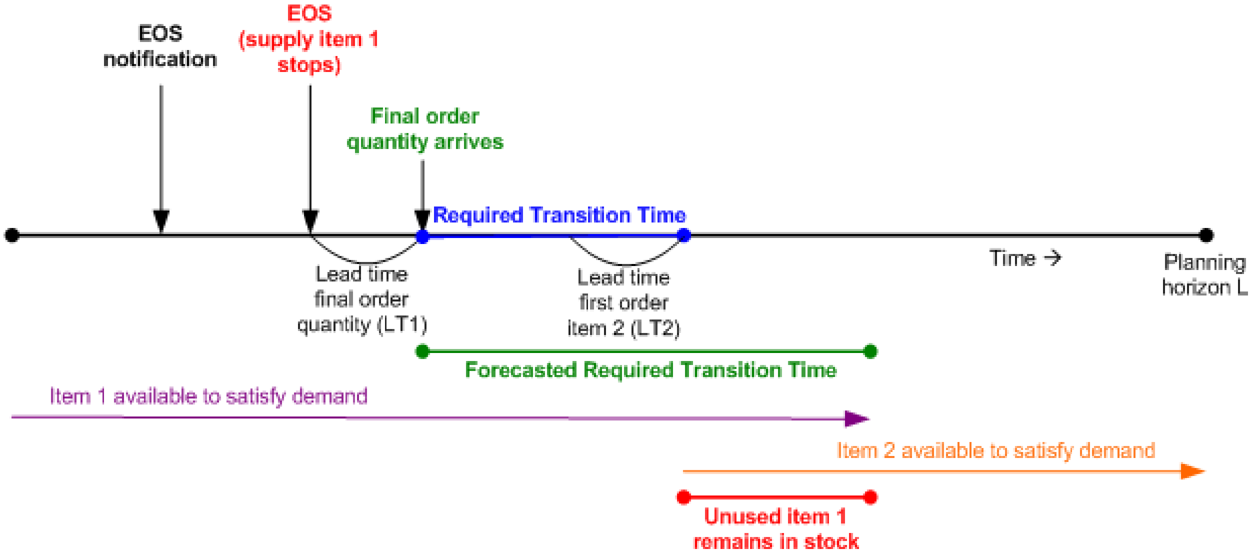


Figure 2.4: The pessimistic scenario regarding the estimation of the length of the RTT.

In the pessimistic scenario the company faces the risk of having unused units of item 1 in stock, while item 2 is already available to satisfy demand. Regarding the excess inventory a choice has to be made between:

1) The units that are in excess are disposed. A reason for doing this can be that using item 2 in the end product results in better sales chances, or improves the performance of the system. It may also be possible that it is prohibited by law to use item 1 instead of item 2 (e.g. because the materials in the item are damaging to the environment). Note that disposing items can also result in revenues, e.g. when items can be sold to another company or when the raw materials of the item can be recycled by the supplier.
2) The units that are in excess are used to satisfy demand. Item 2 is not used to satisfy demand until item 1 is completely out-of-stock.

Although this decision does not affect the final order quantity, it does affect the order policy during the remainder of the planning horizon and the accompanying expected costs. When it is decided to keep the units of item 1 on stock, the company has to buy fewer units of item 2 to satisfy demand after the RTT. It may be beneficial to keep units of item 1 on stock, e.g. when the alternative item is relatively expensive in comparison to the original item and its associated costs. When units of item 1 that are in excess are disposed, all demand during the remainder of the planning horizon has to be satisfied by buying item 2.

2.1.4 Conclusions Required Transition Time

In section 2.1 the concept ‘required transition time (RTT)’ and its elements are described. This RTT will form the basis for the model that is designed to solve the problem that is defined in section 1.2. Simply said, the RTT is the time it takes the company to change from the original item (item 1) to the alternative item (item 2). At VI, there are four different problems that can be linked to the final order problem. For a graphical overview of these problems see Appendix E. In Bakx [2010] more details can be found about these problem situations. Below these four problem situations are related to the RTT:

1) *End-of-supply*: No matter whether there is an alternative item available or not, the concept RTT is applicable to this problem in the following way:
    a. If there is an alternative item, the RTT can range between 0 and the EOL.
    b. If there is no alternative item, the RTT can range between any positive value (e.g. the expected searching time to find an alternative item) and the EOL.
    c. Uncertainty in the length of the RTT (thus a stochastic RTT) resembles the situation in which the company does not know for sure when they can use item 2.
2) *Operations offers a last buy option for items*: This resembles point 1), since this last buy option from Operations can be seen as an internal end-of-supply.
3) *Quantity discounts*: The concept of RTT can be used to decide how large the ‘final’ order for item 1 (item 1 benefits from the quantity discount) should be. Basically item 1 (with quantity discounts) is compared to item 2 (without quantity discounts), and the only difference between the items is the price. So, there is an ‘alternative item’ available immediately and the RTT can range between 0 and the EOL.
4) *Operations offers items randomly*: Basically this situation resembles point 3): when the Operations department offers items randomly the Business Unit Services could consider placing a bigger ‘final’ order to obtain quantity discounts. On the other hand, an item offered by Operations could be compared to an alternative item that is offered by an external supplier. In this situation the RTT can range between 0 and the EOL.

Obviously, the required transition time is a very useful concept since it covers many different kinds of situations. Therefore the remainder of the research project will focus on the modeling of this RTT and the total expected costs during the entire EOL. Basically, the following is done:

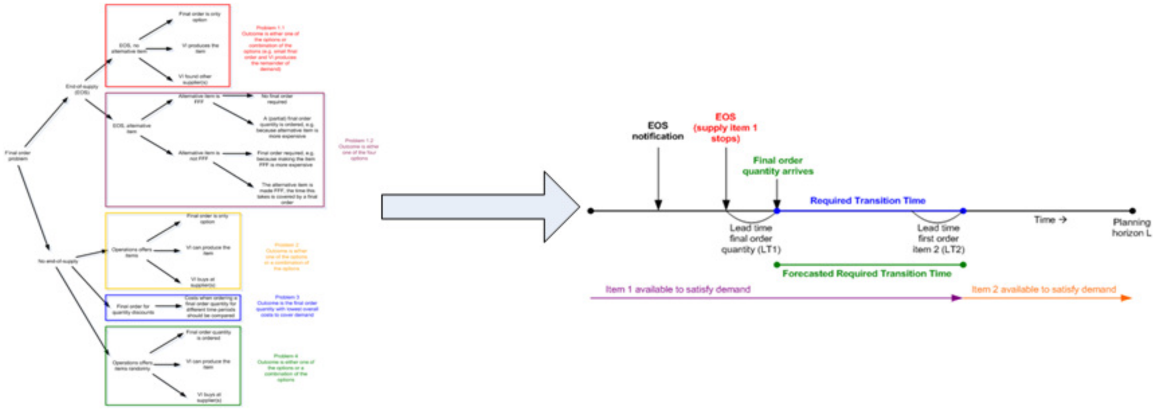


Figure 2.5: From four problem situations to one concept: the required transition time.

So, instead of modeling four different problem situations where each problem situation can be solved by different options (e.g. final order, in-house production), only one model is required now that is applicable to all these situations.

Note that the model that includes the required transition time is comparing an original item to only one alternative item. This is done because the required transition time can be dependent on the alternative item that is considered. For example, an alternative item that is FFF may have a very short RTT (or even no RTT at all), while another alternative item can be completely different than the original item, which leads to a longer RTT. In case there is no alternative item available at all (and it will never be found as well), the RTT is equal to the entire planning horizon since all demand during the planning horizon must be satisfied by the original item. In case there is more than one alternative item to consider, the model should be used several times. I.e. when there are three alternative items, the model should be used three times as well: The costs and final order quantity are calculated one time for the original item and alternative item 1, one time for the original item and alternative item 2, and one time for the original item and alternative item 3. After using the model three times, the company can choose the best alternative item from the available alternative items (for example, they can choose the alternative item that leads to the lowest costs).

2.2 The Simple Policy & Remove Policy

In this section two policies that can be used to deal with the final order problem are described. These two possible policies are [Teunter & Fortuin, 1999]:

- *Simple policy*: When the company uses a simple policy this means that only a final order quantity is placed. The total expected discounted costs during the entire planning horizon only depend on the final order quantity that the company decides to order.
- *Remove policy*: When the company uses a remove policy this means that a final order quantity is placed. This is similar to the simple policy. However, when using a remove policy the optimal remove-down-to levels are calculated as well. At the end of each period the remaining demand is forecasted and the inventory on hand is reviewed. If there are too many items on stock, the company can dispose units until the inventory level equals the remove-down-to level. In the case of a remove policy the total expected discounted costs during the entire planning horizon depend on the final order quantity and the remove-down-to levels for all periods during the RTT.

These two different policies can be depicted as follows:

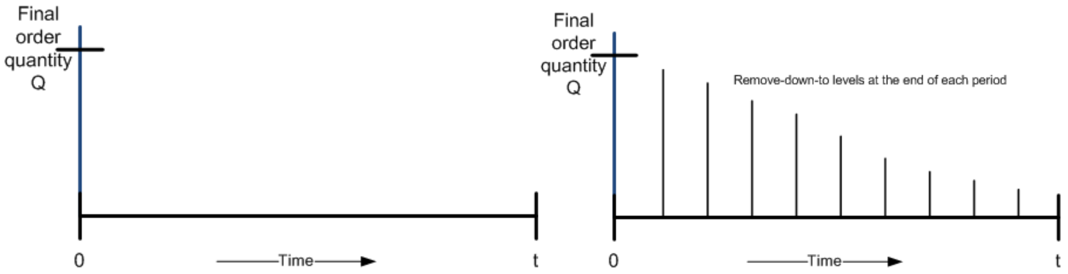


Figure 2.6: The simple policy (left) and the remove policy (right).

Each policy has its advantages and disadvantages. The remove policy reduces holding costs and is interesting when storage capacity is low. However, it requires more administration than the simple policy because the company has to keep track of inventory levels [Teunter & Fortuin, 1999]. Another disadvantage of the remove policy is that the company may dispose items that may be needed in later periods.

Teunter & Fortuin [1999] modeled both the simple and the remove policy. However, their model is applicable for a fixed EOL, and there is no demand after that anymore. Therefore, they assume that remove-down-to level of the last period equals 0. However, in this project, the final order quantity should cover the RTT, which may very well be a shorter period of time than the EOL. This means that demand can still occur after the RTT, and therefore it may be reasonable to have a remove-down-to level at the end of the RTT that is not equal to 0. If one would simply remove all excess stock at the end of the RTT, it may be possible that one would have to purchase new items in the first period of the remainder of the planning horizon. This is not optimal, since then one would have to pay disposal costs and purchasing costs, while keeping an item on stock may be much more cost efficient. There are two ways to handle this:

1) It is possible to have a positive remove-down-to level (and thus a positive end inventory) at the end of the RTT. This means that items are not necessarily disposed, because they may be of use after the RTT.
2) Instead of using remove-down-to levels during the RTT and at the end of the last period of the RTT, it may be better to use the simple policy again at each time the company wants to review the inventories of all items. If the company would review the inventory at the end of the last period in the RTT, it calculates the final order quantity that is needed from that moment on. It can compare this quantity with the quantity on hand, and dispose items if it turns out there are too many items on stock. This basically resembles a rolling horizon problem, which will be described in more detail below.

2.3 Simple Policy with Rolling Horizon

As explained in the previous section, the remove policy is a policy in which a final order quantity is ordered at the beginning of the RTT and at the end of each time period during the RTT the inventory level is reviewed and when needed lowered to the remove-down-to level. At the moment the decision regarding the final order quantity is made, the remove-down-to levels for the upcoming periods during the RTT are calculated as well. One can imagine that when the RTT is a long period of time, e.g. 10 years, the remove-down-to levels that were once calculated at the start of those 10 years are not very reliable. After all, at the moment that remove-down-to levels were calculated, all demand during those 10 years had to be forecasted. Next to that, calculating the remove-down-to levels for all periods at the same time as the final order quantity has another drawback. This drawback will be explained by using a simple example:

Consider two items: item A and item B. Item A became end-of-supply in January 2010 and item B will be end-of-supply in September 2010. The RTT that is considered for both items is 5 years and the length of each period is 1 year. Now let’s say the company acted fast in both cases and for both items the decision regarding the final order quantity and remove-down-to levels is made in the same month as the item became EOS. Thus, the final order quantity and the remove-down-to levels for item A are calculated in January 2010 and for item B this is done in September 2010. The results of the model (a final order quantity and remove-down-to levels for each period during the RTT) can be depicted on a timeline as follows:

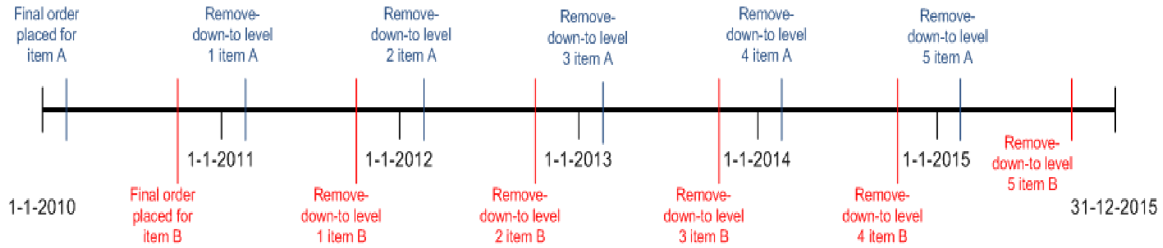


Figure 2.7: Remove-down-to levels for item A and item B.

On this timeline one can see at what moment in time a remove-down-to level for either item A or item B is effective. Although both items are modeled using the same length of a period (1 year), the remove-down-to level of both items are effective at different moments in time! This would mean that the company has to review the inventory of each item at different times. In case there are many different items on stock, it is more logical that the inventories of all items are reviewed at the same moment. But if one would review all inventories at the same moment, the calculated remove-down-to levels make no sense anymore because those are effective on different moments!

Thus, in case the company wants to review all inventories of all items at the same moment, calculating remove-down-to levels at the same moment as the decision regarding final order quantity is made can be disadvantageous for two reasons:

1) Only information available at the decision moment is used in the calculation of the remove-down-to levels. At that moment there may be great uncertainty regarding the demand, the price of the alternative item, etc.
2) The remove-down-to levels of different items that are calculated at different moments, are effective at different moments. This makes reviewing all different items in stock at once impossible.

All the above drawbacks of the remove policy hold for a situation where the length of the RTT is fixed to a known value. In the situation where the length of the RTT is fixed, this means that the number of remove-down-to levels is fixed too. For example, if there are 5 periods during the RTT there will be 5 remove-down-to levels as well. However, an important aspect of this master thesis project is that the length of the RTT is not always fixed: it is very well possible that the exact length of the RTT is unknown and that it follows a probability distribution (see section 2.1.2.3). But when the length of the RTT is variable, this means that the number of remove-down-to levels that should be calculated by the model is variable too! For example, if the length of the RTT follows a Uniform distribution where the minimum RTT is 0 periods, and the maximum RTT is 5 periods, it is not known how many remove-down-to levels should be calculated. One could say that 5 remove-down-to levels should be calculated, since this is the maximum number of remove-down-to levels that is required. However, in such a situation the remove-down-to levels of the latter periods will approach 0 very fast. After all, the probability that the RTT is shorter than 5 periods is positive, which means that the probability that there are less than 5 remove-down-to levels is positive as well. If the RTT is for example equal to 2 periods, this would automatically mean that the remove-down-to levels in the periods 3 to 5 are 0. This means that when the length of the RTT is variable the calculation of a fixed number of remove-down-to levels is impossible, simply because it is not known how many remove-down-to levels are required.

All of these problems can be overcome by using the simple policy with a rolling horizon. By this the following is meant: At the start of the RTT, the company calculates the optimal final order quantity by using the total cost model that is described in chapter 3. At the moment the company wants to review the inventories of all items, it runs this model again for all of these items. This leads to ‘new’ optimal final order quantities for all items, where each new final order quantity is based on updated information regarding the demand and regarding the availability of the alternative item. After all, time has passed by and the company knows more about demand, the alternative item, the remaining length of the RTT, etc. Note that this new final order quantity, only represents the expected final order quantity that is needed to meet the fillrate restriction, based on the updated information. The new final order quantity does not need to be the final order quantity that minimizes the expected costs for the remaining periods, because the rolling horizon strategy brings some difficulties regarding the costs that should be taken into account. E.g. the initial purchasing costs, the fixed order costs and the setup costs are sunk at the moment the new final order quantity is calculated. Also, the values for the cost parameters and demand parameters may have changed, and the net present values should be taken into account. This is very difficult and doing this correctly would be an entire project itself.

However, running the model again with the updated information does tell the user what final order quantity is expected to be needed to meet the fillrate restriction. Once this new final order quantity is known, the company can compare the new final order quantity with the inventory on hand for each item. If the inventory on hand is larger than the new optimal final order quantity, the company can dispose these items. Also, the company can use the updated demand information to calculate the maximum cumulative demand for the periods up to and until the end of the planning horizon. If this maximum cumulative demand is lower than the inventory on hand, the company certainly can dispose the excess units since these are not needed anyway. Also, these items should be disposed from a cost perspective: keeping these units results in holding costs, while these units have to be disposed anyway at the end of the planning horizon since they are never demanded. Therefore, it is always better to dispose the number of units that one knows of that they will never be asked for. This strategy can be used at any review moment in time, thus also at the end of the RTT. This means that the company itself can decide to dispose items at the end of the RTT or not. If items are not disposed, these items can be used to satisfy demand after the RTT.

The rolling horizon strategy has one drawback in comparison with the remove policy: using the rolling horizon strategy requires the company to run the model at each review moment for all items again. If the number of items increases, this can be very time-consuming and costly. However, the company's supervisors indicated that they would re-run the model at such a moment anyway, just to be sure about the values they calculated previously or to use updated information. Therefore, the rolling horizon strategy is more appropriate for the company's practice. The company's supervisors indicated that they were interested in calculating remove-down-to levels right away as well, instead of only using a simple policy at different moments in time. This is mainly to enable them to get insights in what the model would advice them to do and what the effects of updated information are. For this reason, both the simple policy and the remove policy are modeled. The model for the simple policy can be found in chapter 3 and the model for the remove policy can be found in chapter 4.

3 Model Description for the Simple Policy

In this chapter the detailed model for the simple policy will be described. First, the assumptions that are used to model the simple policy are given in section 3.1. Next, the objective function of the model will be given in section 3.2. Also, an overview is given of the inputs the user of the model should give and the outputs that 'black box' (in which the model operates) results in. The remainder of this chapter concerns the contents of this black box, and will thus contain mathematical descriptions of the model and its parameters. In section 3.3 the mathematical description of the objective of the model will be given. In section 3.4 the decision variables will be described. After this, the restrictions and input parameters will be discussed in section 3.5 and section 3.6. After this all these elements are used to develop the cost model for the simple policy. This model is given in the sections 3.7 to 3.10. Finally, in section 3.11 it is explained how the optimal final order quantity can be calculated. Those who are not interested in this mathematical model can skip the sections 3.3 up to and until 3.11.

3.1 Modeling Assumptions for the Simple Policy

Regarding the modeling of the total expected costs for the simple policy and the remove policy several assumptions are made. Below these modeling assumptions are given. Note that these are applicable for both policies.

1) Returns of used items are not included in the model because this is not applicable to the company's situation. The model concerns non-repairable items, so returns (and thus the remanufacturing) of used parts are not included.

   The total expected costs consist of (item 1 is the original item and item 2 is the alternative item):
   - Initial purchasing costs for item 1
   - Expected purchasing costs for item 2
   - Holding costs for item 1
   - Disposal costs for item 1
   - Fixed ordering costs for item 1
   - Expected setup costs (e.g. R&D costs) for item 2.
2) All costs (except for the purchasing costs of the final order quantity) are discounted with a fixed yearly discounting factor.
3) Penalty costs are not taken into account because it is impossible to make a sound estimation of these costs. The most important 'penalty' in case of system downtime and/or unavailability of items is that customers lose their trust in the company. Downtime costs are hard to estimate and if it would be possible to estimate them, they are hardly ever incurred by the company's customers; the relationship between the company and its customers is based on trust. Loosing trust by not having an item available is very hard to quantify. Therefore, a fillrate restriction is used instead of penalty costs.
4) If disposing a unit of item 1 results in revenues, it is assumed that these revenues are always less than the unit cost of item 1.
5) It is assumed that all the units of item 1 that a company wants to dispose at a certain moment can be disposed immediately.
6) The expected setup costs for item 2 are constant. This means these costs are independent of the length of the RTT. Even if the RTT is equal to 0 time periods, these setup costs are incurred.
7) It is assumed that the expected setup costs for item 2 occur at the start of the RTT.
8) The planning horizon is an integer number of periods.
9) The required transition time RTT has a discrete probability distribution, which is known.
10) The holding costs are incurred at the end of a period. This way, holding costs are not overestimated (if they would be incurred at the beginning of a period, one would have to include

holding costs for the entire final order quantity as well since this is the inventory at the start of the first period).

11) Holding costs are incurred after excess stock is disposed.
12) The demand distributions are known for all periods.
13) Demand in different periods is independent.
14) By demand we mean demand for a certain functionality that both item 1 and item 2 offer. This means that demand is not specific for either one of the items; the demand distribution represents the demand for a functionality and it does not matter whether this demand is satisfied by item 1 or item 2.
15) The price of item 2 does not affect the demand distribution.
16) When the lead time of the final order quantity is positive (see $LT1$ in Figure 2.1), it is assumed that the company has sufficient inventory to cover the demand during this lead time. This means that it is impossible to have a negative stock just before the final order quantity is delivered.
17) The model always calculates the costs for one given alternative item, it is impossible to evaluate all possible alternative items at once. If there are no alternative items available, the RTT is equal to the EOL. If there are more than one alternative items available, the model should be used as many times as there are alternative items (e.g. if there are 3 alternative items, the model should be used 3 times). The company can choose from all alternative items the one that they think is best (e.g. the one that leads to the lowest costs).
18) It is assumed that the fillrate restriction is only applicable to the required transition time. After the RTT the company immediately receives the appropriate number of units to fulfill demand (see assumption 19), which means that there will be no shortages during the period after the RTT. Note that when the RTT is larger than the EOL, the fillrate restriction is applicable to the entire EOL instead of the entire RTT (the periods after the EOL do not matter to the company).
19) In the periods after the RTT, the company orders exactly all the units that are needed to satisfy demand and the supplier is able to deliver all these items with a lead time of 0.
20) Regarding the expected costs for the intervals after the RTT, only expected purchasing costs for item 2, holding costs for item 1 (the original item) and disposal costs for item 1 are taken into account. These costs represent the minimum for the real total expected costs after the RTT (e.g. ordering costs for item 2 are not considered). However, the focus of this project is on the calculation of the final order quantity for item 1. The company does not have any real problems when they can normally order units of item 2 at their suppliers, so it is not important to take all the detailed costs into account.
21) The end inventory at the end of the EOL should be equal to 0. The company does not have to satisfy any demand after the EOL, so it makes no sense to keep items on stock.
22) If item 1 can be used to satisfy demand during the periods after the RTT, no remove-down-to levels during these periods after the RTT are considered. Then, the only moment at which excess inventory is disposed, is at the end of the EOL.
23) If item 1 cannot be used to satisfy demand during the periods after the RTT (e.g. because of regulations), all stock of item 1 will be disposed at the end of the RTT.
24) Supply capacity is assumed to be ample: this means there is no restriction on the final order quantity and any final order quantity can be delivered by the supplier.
25) There is no storage restriction: any final order quantity can be kept on stock.
26) For the models given in chapter 3 and chapter 4 it is assumed that there is no initial stock of item 1.

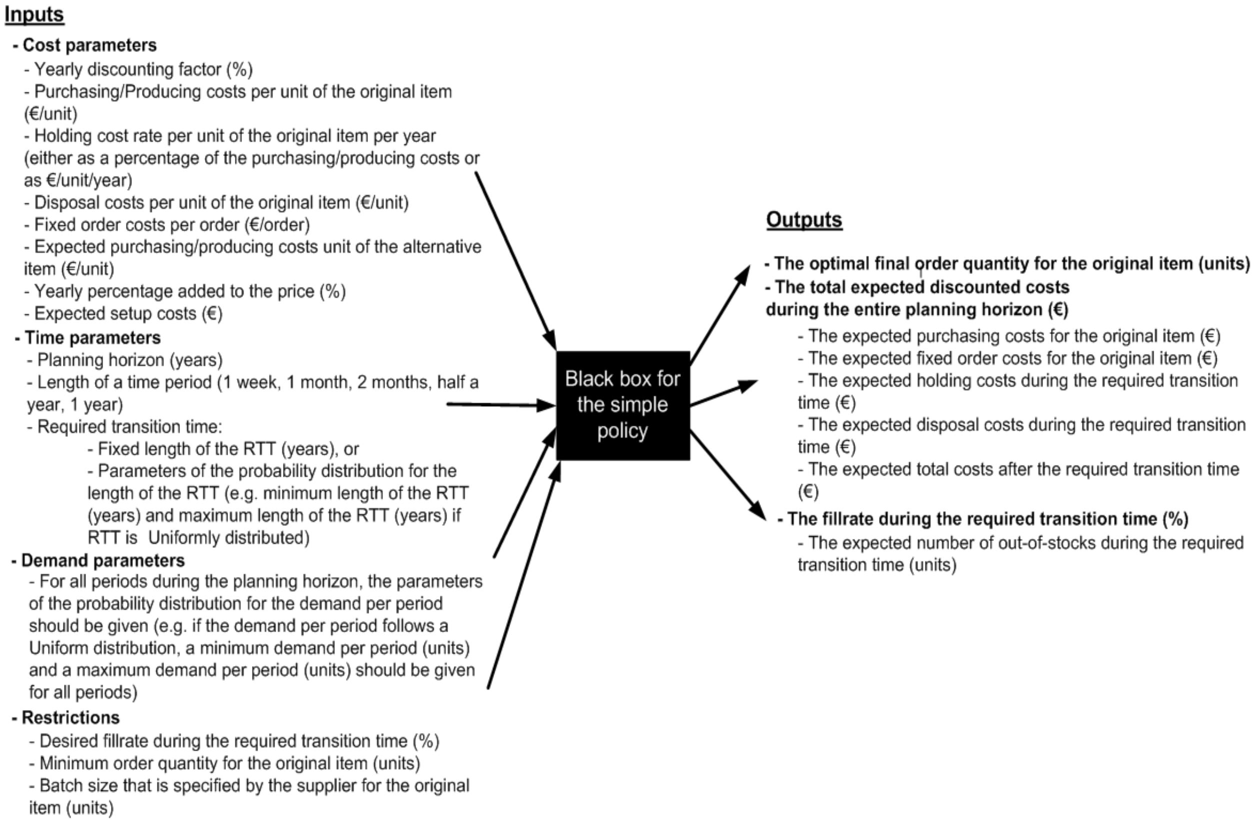
3.2 Objective of the Model for the Simple Policy

The goal of the model is to provide a quantitative advice to the company about how to cover the entire planning horizon. The planning horizon should be taken into account completely, since the company has to satisfy demand during the entire planning horizon. To be able to provide this quantitative advice it is decided to build a total cost model. Using this total cost model, one can optimize the final order quantity. Here, the objective of the total cost model is as follows:

The objective of the model is to minimize the total expected discounted costs during the entire planning horizon.

Discounting is taken into account because the planning horizon can be long. The optimal final order quantity is the final order quantity that leads to the lowest costs possible to cover the entire planning horizon.

To calculate the total expected discounted costs during the entire planning horizon the model should take into account decision variables, restrictions and input parameters. The model itself should 'decide' on the decision variables, where its objective is to minimize the total expected discounted costs during the entire planning horizon. The values for the decision variables that the model results in should meet all restrictions. Basically, the mathematical model behind the total cost calculation acts like a 'black box', where the user of the model provides some inputs, the black box does its 'trick' and it gives some outputs. For the simple policy, the inputs that the black box needs and the outputs that the black box generates are depicted in Figure 3.1. Details about these input parameters, the mathematical model and the outputs of the model can be found in the remainder of this chapter. If you do not want to read the detailed modeling of the black box for the simple policy, the remainder of this chapter can be skipped.



Inputs

- **Cost parameters**
  - Yearly discounting factor (%)
  - Purchasing/Producing costs per unit of the original item (€/unit)
  - Holding cost rate per unit of the original item per year (either as a percentage of the purchasing/producing costs or as €/unit/year)
  - Disposal costs per unit of the original item (€/unit)
  - Fixed order costs per order (€/order)
  - Expected purchasing/producing costs unit of the alternative item (€/unit)
  - Yearly percentage added to the price (%)
  - Expected setup costs (€)
- **Time parameters**
  - Planning horizon (years)
  - Length of a time period (1 week, 1 month, 2 months, half a year, 1 year)
  - Required transition time:
    - Fixed length of the RTT (years), or
    - Parameters of the probability distribution for the length of the RTT (e.g. minimum length of the RTT (years) and maximum length of the RTT (years) if RTT is Uniformly distributed)
- **Demand parameters**
  - For all periods during the planning horizon, the parameters of the probability distribution for the demand per period should be given (e.g. if the demand per period follows a Uniform distribution, a minimum demand per period (units) and a maximum demand per period (units) should be given for all periods)
- **Restrictions**
  - Desired fillrate during the required transition time (%)
  - Minimum order quantity for the original item (units)
  - Batch size that is specified by the supplier for the original item (units)

Outputs

- **The optimal final order quantity for the original item (units)**
- **The total expected discounted costs during the entire planning horizon (€)**
  - The expected purchasing costs for the original item (€)
  - The expected fixed order costs for the original item (€)
  - The expected holding costs during the required transition time (€)
  - The expected disposal costs during the required transition time (€)
  - The expected total costs after the required transition time (€)
- **The fillrate during the required transition time (%)**
  - The expected number of out-of-stocks during the required transition time (units)

Figure 3.1: Inputs, black box, and outputs for the simple policy.

3.3 Objective Function for the Simple Policy

The simple policy is a policy where only a final order quantity is used (see section 2.2). This means that the final order quantity is the only decision variable in the model for the simple policy. Using this information, the objective function (see section 3.2) for the simple policy can be formulated mathematically as shown below.

For the total expected discounted costs during the entire planning horizon, given a final order quantity Q, the following holds:

$$E[TC(Q)] = \sum_{t=minRTT_a}^{EOL} E'[TC(Q,t)] * P(RTT_a = t) + \sum_{t=EOL+1}^{maxRTT_a} E'[TC(Q,t)] * P(RTT_a = t) \qquad \textbf{[1]}$$

Where:

$E[TC(Q)]$ = Total expected discounted costs during the entire planning horizon for a final order quantity that is equal to Q *(€)*

EOL = End-of-Life (service) period / length of the entire planning horizon *(# periods)*

$E'[TC(Q,t)]$ = Total expected costs for a final order quantity Q given that the required transition time (RTT) is equal to t periods *(€)*

$P(RTT_a = t)$ = The probability that the RTT for a given alternative item a is equal to t periods.

$minRTT_a$ = The minimum possible value for the length of the RTT for a given alternative a *(# periods)*

$maxRTT_a$ = The maximum possible value for the length of the RTT for a given alternative a *(# periods)*

Equation [1] is minimized to find the optimal value for the final order quantity. This automatically results in the optimal simple policy, because the only decision variable to take into account when considering a simple policy is the final order quantity.

In the remainder of this section the mathematical model will be described that is used to calculate the total expected discounted costs during the entire planning horizon for a given final order quantity Q, a given alternative item a, and a given value t for the length of the RTT. These costs are denoted by $E'[TC(Q,t)]$. As can be seen from equation [1], t can range between $minRTT_a$ and $maxRTT_a$ and each possible value for t has a probability $P(RTT_a = t)$. For all these possible values for the RTT the total expected discounted costs are calculated for a given final order quantity Q. Using all values for $E'[TC(Q,t)]$ and $P(RTT_a = t)$, the total expected discounted costs for a given final order quantity can be calculated by using equation [1].

Regardless of the probability distribution for the demand per period, it is possible to distinguish three different situations when calculating $E'[TC(Q,t)]$:

1) $t = EOL$
2) $t > EOL$
3) $t < EOL$

Relating these three situations to the objective function in equation [1] can be done as follows:

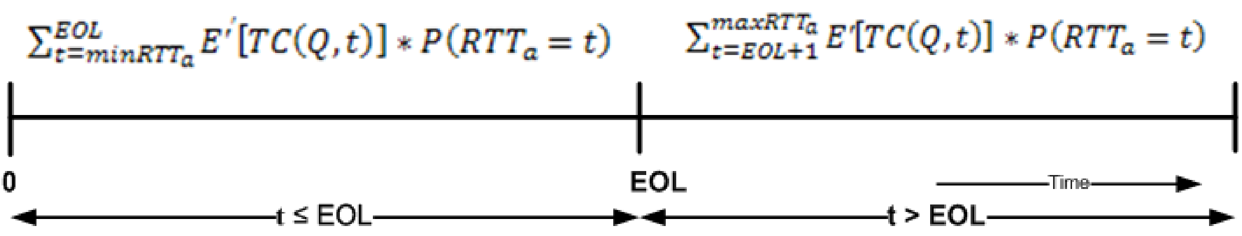


Figure 3.2: Relating the objective function of the simple policy to the three situations.

For each of these three situations a detailed model for the simple policy has been developed. The kind of modeling that is used for this is stochastic dynamic programming. Using the models all possible values for $E'[TC(Q,t)]$ can be calculated. These values are used in equation [1] to calculate the total expected discounted costs during the entire planning horizon for a given final order quantity Q and a given alternative item a. To be able to formulate these models, the decision variables, restrictions and input parameters for the model of the simple policy are described first in the upcoming sections (also see Appendix F for an overview of used notation). In section 3.7 to 3.9 the models for the three situations are described.

3.4 Decision Variables for the Simple Policy

As explained in the previous section, the only decision variable to consider in case of the simple policy is the final order quantity:

Q *The final order quantity (units)*

The final order quantity is the quantity of the original item that is ordered to cover the expected demand during the RTT. Equation [1] (see section 3.2) shows how this decision variable is included in the objective function.

3.5 Restrictions for the Simple Policy

In this section the restrictions that the results of the model for the simple policy should meet are given.

Fillrate restriction $fillrate \geq desiredfillrate$

Where: $fillrate = 1 - \frac{Expected\ number\ of\ out{-}of{-}stocks\ during\ the\ RTT}{Expected\ demand\ during\ the\ RTT}$

[2]

In equation [2] the expected number of out-of-stocks during the RTT is the expected number of items that are demanded during the RTT but are not available, i.e. it is the expected number of shortages. The ‘desired fillrate’ is a value that is specified by the user of the model, which can range between 0 and 1 (0% and 100%).

Minimum order quantity (units) $Q \geq MOQ$

The MOQ is the minimum order quantity (units). It is impossible to order less units of item 1 than the minimum order quantity that the supplier of item 1 requires. Orders that are smaller than the MOQ are simply not accepted by the supplier, the supplier delivers at least MOQ units. The final order quantity should be at least equal to the MOQ.

The optimal final order quantity should always be a multiple of a batch size specified by the supplier, BS_s.

It is possible that the supplier of item 1 only allows orders that are a multiple of a certain batch size that he specified. For example, he may only want orders that are a multiple of 10. Placing a final order quantity that is not a multiplication of 10 is not allowed in this example. Thus, the optimal final order quantity should always be a multiplication of this batch size. If the batch size is equal to 0, this means that there is no restriction regarding the batch size and the final order quantity can have any size.

3.6 Input Parameters

In this section the input parameters that need to be taken into account are given. Most of these input parameters are based on the article by Teunter & Fortuin [1999]. However, for the company some input parameters were not applicable (e.g. penalty costs) and some input parameters had to be added (e.g. a yearly price increase). The input parameters are divided into several categories: cost parameters, time parameters, demand parameters, and initial stock.

3.6.1 Cost Parameters

$CN1$ *Initial purchasing cost of one unit of item 1, the original item (€/unit)*

The model should decide on the final order quantity that should be ordered for item 1, the original item. The costs that are incurred immediately at the start of the RTT are the purchasing costs of this final order quantity. These costs are equal to $CN1 * Q$, where Q is the final order quantity and $CN1$ is the known price of item 1 at the start of the RTT.

$E[CN2_k]$ *Expected value of the purchasing cost of one unit of item 2, the alternative item, in period k (€/unit)*

Sine item 2 is bought after the RTT, there can be uncertainty in the price of item 2 at the moment the decision about the final order quantity is made. Especially when the RTT is a long period of time, the uncertainty about the unit cost of item 2 can be high. Therefore, one could expect that a probability distribution for the price of item 2 should be taken into account when calculating the expected costs in a period after the RTT. For any period after the RTT (but before the end of the EOL) this would roughly be as follows (assuming that there is no inventory of item 2 on hand at the beginning of period $k = RTT + 1, \ldots, EOL$):

$$E[TC_k] = \sum_{Price_k = minimumprice_k}^{maximumprice_k} Price_k * expecteddemand_k * P(CN2_k = Price_k)$$

[3]

Where:
$E[TC_k]$ = Expected costs for period k *(€)*
$CN2_k$ = The price of one unit of item 2 in period k, which can range between a minimum price and a maximum price *(€/unit)*
$expecteddemand_k$ = The expected demand in period k *(units)*
$P(CN2_k = Price_k)$ = The probability that the price of item 2 in period k is equal to $Price_k$ *(dimensionless)*

However, when one looks closely to equation [3], one can see that this can be written as follows:

$$E[TC_k] = \left(\sum_{Price_k = minimumprice_k}^{maximumprice_k} Price_k * P(CN2_k = Price_k) \right) * expecteddemand_k$$

Where it holds that:

$$\sum_{Price_k = minimumprice_k}^{maximumprice_k} Price_k * P(CN2_k = Price_k) = E[CN2_k]$$

Thus, equation [3] can be re-written into:

$$E[TC_k] = E[CN2_k] * expecteddemand_k$$

[4]

From equation [4] we see that it does not matter what distribution holds for the price of item 2 in period k, it is only the expected value of this price that matters.

Next to this useful finding, the company indicated that the price of an item depends on the year an item is bought, since most suppliers use a yearly price increase for the items. Therefore, the following parameter should be taken into account too:

$\widehat{PI}$ *Expected yearly price increase (%)*

Using the expected price for one unit of item 2 in period $RTT + 1$ ($E[CN2_{RTT+1}]$), thus the expected price for one unit of item 2 in the first period after the RTT, and the expected yearly price increase $\widehat{PI}$, one can calculate the expected price of item 2 in the periods $k = RTT + 1, \ldots EOL$ as follows:

```
Step = 1
years = 1
For k = RTT+1 To EOL
    $E[CN2_k] = (E[CN2_{RTT+1}] * ((1 + \widehat{PI})^{Step-1})$
    If k = (years * 1 / m) Then
        Step = Step + 1
        years = years + 1
    Else
        Step = Step
        years = years
    End If
Next
```

Here m is the length of a period that is specified by the user of the model (e.g. month, year), see section 3.6.2.

$\widehat{CH1_k}$ *Estimated holding cost rate for item 1 in period k (€/unit/period)*

The company does not have data regarding the volume of an item, which makes it impossible to calculate the exact holding costs per unit of that item. Therefore, the holding cost rate is calculated as a percentage of the initial purchasing costs of one unit of item 1.

$\widehat{CD1_k}$ *Estimated disposal cost of item 1 in period k (€/unit)*

At the end of the RTT it can be decided to dispose items. When remove-down-to levels are taken into account for all the periods in the RTT, it is even possible to dispose items every period. If any items are left in stock at the end of the EOL, those items have to be disposed since there is no use in keeping them. Thus, no matter what situation is modeled (e.g. the simple policy or the remove policy), disposal costs have to be taken into account since at some point in time it may be possible that the company has to get rid of excess inventory. If $\widehat{CD1_k} > 0$, disposing costs money. If $\widehat{CD1_k} < 0$, disposing results in revenues (e.g. when the item is recyclable).

$CO1$ *Fixed order costs per order for item 1 (€/order)*

It is common practice that a supplier charges fixed order costs every time the company orders items. These costs should be taken into account in the total cost calculation. Notice that $CO1$ will occur only once, immediately at the beginning of the RTT.

$E[CO2]$ *Expected setup costs for item 2 (€)*

Since item 2 is the alternative item, several things can happen regarding the time it takes before the company can use this item. For example, it is possible that item 2 is FFF, and therefore no systems in the field have to be changed and the changes on the drawings of systems are minimal. However, if the item is not FFF, these changes on the drawings will be more difficult and it may be possible that changes in the field are required as well. Also, the company may want to search for a better alternative item. Thus, the expected setup costs for item 2 consist of costs such as the costs of changing the drawings of the items, which are made by the R&D department to find an alternative item or to make an available alternative item fit into existing systems. The R&D department should make the estimation for this parameter. However, it is very likely that this cost parameter is very difficult to estimate. There may be great uncertainty regarding the costs that are made before item 2 can be used. Similarly to the price of one unit of item 2, a probability distribution for the setup costs for item 2 is in place. However, similar to $E[CN2_k]$, one can show that only the expected value of the setup costs matters:

$$E[Costs\_after\_RTT] = \sum_{Setupcosts=minimumsetupcosts}^{maximumsetupcosts} CO2 * P(CO2 = Setupcosts) + \cdots \ldots \qquad \textbf{[5]}$$

Note that equation [5] is only considering the setup costs, while in the total cost model (see sections 3.7 to 3.10) other costs are considered as well (e.g. the purchasing costs of item 2). Using basic probability theory knowledge, one can rewrite equation [5]:

$$E[Costs\_after\_RTT] = E[CO2] + \cdots \ldots$$

Thus, again, only the expected value for the setup costs matters and the probability distribution of these setup costs does not have to be taken into account.

α *Fixed yearly discounting factor (dimensionless)*

Since the final order problem typically concerns relatively long planning horizons, it is reasonable to take a discounting factor into account. All costs are discounted with a yearly continuous discounting factor. Note that the costs of purchasing the final order quantity (the fixed order costs $CO1$ and the purchasing costs $CN1 * Q$) are not discounted since these costs occur at time 0. The yearly discounting factor is related to the discounting factor per period (parameter a in the calculations) as follows:

$$\alpha = -\ln\left(a^{\frac{1}{m}}\right)$$

Here, m is the length of a period (e.g. month, year), see section 3.6.2.

3.6.2 Time Parameters

EOL *Length of the planning horizon (# periods)*

The End-of-Life (service) period (EOL) is defined as the remaining period that the company has to satisfy its customers' demand. Calculating the number of periods during the EOL is done by dividing the EOL expressed in years by m. Note that the terms EOL, planning horizon, and remaining service period (RSP) are used interchangeably in this report.

RTT *The required transition time (# periods)*

The required transition time is the time the company needs to be able to use the alternative item instead of the original item. A more detailed description is given in section 2.1 (and section 2.1.2.3). If the RTT is given in years then calculating the number of periods during the RTT is done by dividing the RTT expressed in years by m.

m *User specified length of periods (years)*

Parameter m indicates the length of the periods that the user of the model wants to use in the calculation. The user can choose the length of the period that he/she thinks is appropriate. In the case of VI, typical lengths of periods to consider are 'month', 'quarter', 'half a year' and 'year'. Respectively, this means that $m = 1/12$, $m = 1/4$, $m = 1/2$, and $m = 1$. The unit of m is 'years': when the length of the periods is equal to 6 months, this means that $m = 1/2\ years$.

3.6.3 Demand Parameters

d_k *Demand in period* $k = 1, \ldots, EOL$ *(units)*

For each period during the EOL, the parameters of the probability distribution for demand in that period are forecasted and given in the model by the user of the model. This way, the model is applicable in any phase of the life-cycle of the item. For example, when the item is in the final phase the user can fill in a mean demand per period that declines as time goes by.

3.6.4 Initial Stock

$S[1]$ *The initial amount of items on stock at the start of the RTT (units)*

When the initial amount of items on stock at the start of the RTT is positive, this means that the optimal final order quantity can be decreased with this value. For simplicity reasons, it is chosen that in the descriptions of the model the initial stock is assumed to be 0. Using this assumption the final order quantity is calculated, and if the initial stock turns out to be positive this value is simply subtracted from the optimal final order quantity. Note that when the optimal final order quantity is smaller than the initial stock, no final order should be placed and the excess stock should be disposed.

3.7 Detailed Model for the Simple Policy where t = EOL

In the situation when t is equal to the EOL, the entire EOL should be covered by the final order quantity Q. Since t equals the EOL, and after the EOL the company does not have to satisfy demand anymore, the model will aim at an inventory at the end of t that is equal to 0 (all inventory that is left at the end of t has to be disposed).

In this section a division is made between the following situations:

1) $t = 0$
2) $t = 1$
3) $t \geq 2$

This division is made because the equations used to calculate the total expected discounted costs are different for these different situations. In the next sections the detailed model for the simple policy will be given for all these situations.

3.7.1 Simple Policy, $t = EOL = 0$

When $t = EOL = 0$, and both of these periods equal 0, this means that nothing should be done. There simply is no problem because the company does not have to satisfy any demand anymore. After all, the company considers a planning horizon of 0 periods and it is impossible to have positive demand during 0 periods. Note that it is assumed that the fillrate during 0 periods is always 1 (100%).

3.7.2 Simple Policy, $t = EOL = 1$

When $t = EOL = 1$ the final order quantity should cover the expected demand during this 1 period. Here, the objective of the model is to find the final order quantity that minimizes the total expected discounted costs, but also meets the restrictions. Note that the model will always aim to an end inventory at the end of t that is equal to 0. This is done because no demand after t ($= EOL$) should be satisfied.

To calculate the total expected discounted costs for a certain final order quantity Q, the following equation is used:

$$E'[TC(Q, t = 1)] = CO1 + CN1 * Q + \sum_{j=0}^{Q} \widehat{CD1}_1(Q - j) * P(d_1 = j) + \sum_{j=Q+1}^{\infty} 0 * P(d_1 = j)$$

[6]

Where (for more details see section 3.6 and Appendix F):

$E'[TC(Q, t = 1)]$	The total expected discounted costs, given a final order quantity Q and t is 1 time period *(€)*
$CN1$	The price of one unit of item 1 *(€)*
$CO1$	The fixed order costs per order for item 1 *(€)*
$\widehat{CD1}_1$	The estimated disposal cost per unit of item 1 that is disposed at the end of period 1 *(€)*
$P(d_1 = j)$	The probability that the demand in period 1 is equal to j
a	The discounting factor per period, which is related to the yearly discounting factor, α, as follows: $\alpha = -\ln(a^{\frac{1}{m}})$

Note that there is no expression for the holding costs in equation [6]. The model will not include any holding costs for the inventory during t ($= 1$), since following from the assumptions (see section 3.1) the end inventory of that period is equal to 0 (which leads to no holding costs).

For the expected number of out-of-stocks, which is used to calculate the fillrate during the RTT, the following holds:

$$E'[OOS(Q, t = 1)] = \sum_{j=0}^{Q} 0 * P(d_1 = j) + \sum_{j=Q+1}^{\infty} (j - Q) * P(d_1 = j)$$

[7]

Where:

$E'[OOS(Q, t = 1)]$	= The expected number of out-of-stocks, given a final order quantity Q and t is 1 time period.

Using equation [7], the fillrate during this period, given a final order quantity Q, ($fillrate(Q, t = 1)$) can be calculated as follows:

$$fillrate(Q, t = 1) = 1 - \frac{E'[OOS(Q, t = 1)}{\sum_{j=0}^{\infty} j * P(d_1 = j)}$$

[8]

Note that when the expected demand during the period is 0, the company automatically satisfies all demand and the fillrate is 1 (100%).

Using equations [6] to [8] it is possible to calculate the total expected costs and the fillrate during the RTT, given a final order quantity Q and an RTT that is equal to 1 period. Since the situation where the RTT is equal to the EOL is considered in this section, it is known that there are no other possible values for t. This means that the objective function in equation [1] can be re-written to:

$$E[TC(Q)] = E'[TC(Q, t = 1)] \tag{9}$$

The optimal final order quantity Q^* is the final order quantity that minimizes the total expected costs in equation [9]. To find the optimal final order quantity Q^*,the following algorithm is used:

Algorithm to find the optimal final order quantity Q^* in a simple policy with $t = EOL$

Step 1) Set $Q = MOQ$ or $Q = 1 * BS_s$

Step 2) Calculate the total expected discounted costs using equation [6] and [9] and calculate the fillrate using equation [7] and [8].

Step 3) If $fillrate(Q, t = 1) < desiredfillrate$ then go to step 4), else go to step 7)

Step 4) Set $Q = Q + 1$

Step 5) Calculate the total expected discounted costs using equation [6] and [9] and calculate the fillrate using equation [7] and [8].

Step 6) If $fillrate(Q, t = 1) \geq desiredfillrate$ then go to step 7), else go to step 4).

Step 7) The optimal final order quantity $Q^* = Q$ and the accompanying minimal costs are $E[TC(Q^*)]$. If $Q^* \neq n * BS_s$, where $n = 1,2, \dots$ (see section 3.5), the costs and fillrate for the first final order quantity that meets this restriction are calculated.

This algorithm leads to the optimal final order quantity for the following reason: If one takes a close look at the total cost function (e.g. the one in equation [6]), one can see that the costs can only increase when Q is increased. For each additional unit in the final order quantity, one extra unit cost and additional disposal costs are incurred. In most articles in literature the total cost models take penalty costs into account. In such a model, increasing Q can actually lead to cost decreases: the higher Q, the lower the expected number of out-of-stocks and the lower the penalty costs. However, in this project it is decided to take into account a fillrate restriction instead of penalty costs, because estimating the penalty costs is too difficult.
When modeling the total expected discounted costs and a fillrate restriction, the behavior of the total expected discounted costs can be described as follows: The higher the final order quantity is, the higher the total expected costs are. It also holds that the higher the final order quantity is, the higher the fillrate is. The algorithm keeps increasing the final order quantity until the fillrate restriction is met. Exactly at the first final order quantity that leads to a fillrate that is equal to or larger than the desired fillrate, is the optimal final order quantity. After all, if the final order quantity would be increased further, this only leads to additional costs while the fillrate restriction is already met. The only possibility where the final order quantity is increased further is in the case that the first final order quantity that meets the fillrate restriction is not a multiple of BS_s. Therefore, the algorithm described above leads to the optimal final order quantity that minimizes the total expected discounted costs during the entire planning horizon while meeting all restrictions.

3.7.3 Simple Policy, $t = EOL$, $t \geq 2$

When $t = EOL$ the final order quantity should cover the expected demand during the entire EOL. Here, the objective of the model is to find the final order quantity that minimizes the total expected discounted costs, but also satisfies the restrictions. Note that the model aims for an end inventory at the end of the RTT that is equal to 0. In this section the model to find the optimal final order quantity is described for the situation in which the RTT and EOL are at least equal to 2 periods.

To calculate the total expected discounted costs for a certain final order quantity Q, the following equation is used:

For the last period $k = t$:

$$E[TC_t(I_t)] = a\left(\widehat{CH1_{t-1}} * I_t + \sum_{j=0}^{I_t} \widehat{CD1_t} * (I_t - j) * P(d_t = j) + \sum_{j=I_t+1}^{\infty} 0 * P(d_t = j)\right)$$

For the periods $k = t-1, \ldots, 2$:

$$E[TC_k(I)] = a\left(\widehat{CH1_{k-1}} * I_k + \sum_{j=0}^{I_k} E[TC_{k+1}(I_k - j)] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} E[TC_{k+1}(0)] * P(d_k = j)\right)$$

For the first period $k = 1$:

$$E[TC_1(Q)] = CO1 + CN1 * Q + \sum_{j=0}^{Q} E[TC_2(Q - j)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E[TC_2(0)] * P(d_1 = j)$$

Here, I_k denotes the inventory of item 1 directly at the beginning of period k. Note that the calculation starts at the last period during the RTT instead of the first. This is common practice in stochastic dynamic programming.

The total expected discounted costs for a given final order quantity Q during an RTT that is equal to t are:

$$E'[TC(Q, RTT = t)] = E[TC_1(Q)] \tag{10}$$

Note that all equations in equation [10] can be executed for any $t \geq 2$.

For the expected number of out-of-stocks the following holds:

For the last period $k = t$:

$$E[OOS_t(I_t)] = \sum_{j=0}^{I_t} 0 * P(d_t = j) + \sum_{j=I_t+1}^{\infty} (j - I_t) * P(d_t = j)$$

For the periods $k = t-1, \ldots, 2$:

$$E[OOS_k(I_k)] = \sum_{j=0}^{I_k} (0 + E[OOS_{k+1}(I_k - j)]) * P(d_k = j) + \sum_{j=I_k+1}^{\infty} \big((j - I_k) + E[OOS_{k+1}(0)]\big) * P(d_k = j)$$

For the first period $k = 1$:

$$E[OOS_1(Q)] = \sum_{j=0}^{Q} (0 + E[OOS_2(Q - j)) * P(d_1 = j) + \sum_{j=Q+1}^{\infty} ((j - Q) + E[OOS_2(0)) * P(d_1 = j)$$

Then, for the total number of expected out-of-stocks during an RTT that is equal to t periods the following holds:

$$E'[OOS(Q, RTT = t) = E[OOS_1(Q)] \tag{11}$$

The fillrate during the entire RTT, given a final order quantity Q and an RTT that is equal to t, equals:

$$fillrate(Q, RTT = t) = 1 - \frac{E'[OOS(Q, RTT = t)}{\sum_{k=1}^{t}\left(\sum_{j=0}^{\infty} j * P(d_k = j)\right)} \tag{12}$$

Using equations [10] to [12] it is possible to calculate the total expected costs and the fillrate during the RTT, given a final order quantity Q and an RTT that is equal to t periods. Since the situation where $t = EOL$ is considered in this section, it is known that there are no other possible values for t. This means that the objective function in equation [1] can be re-written to:

$$E[TC(Q)] = E'[TC(Q, RTT = t)]$$

Finding the optimal final order quantity Q^* is done using a similar algorithm as described in section 3.7.2.

3.8 Detailed Model for the Simple Policy where $t > EOL$

In the situation where $t > EOL$, the demand after the EOL (the demand during the period $(EOL, t]$) is not taken into account in the calculation of the total expected discounted costs. This is done because the company is only interested in satisfying demand during the EOL. When $t > EOL$ the entire EOL is covered by the final order quantity Q. Since the company does not have to satisfy demand anymore after the EOL, the model aims at an end inventory at the end of the EOL that is equal to 0 (so all excess inventory that is left at the end of the EOL has to be disposed).

Based on the above it can be concluded that the model for the situation where $t > EOL$ is exactly the same as the model for the situation where $t = EOL$. In both situations the same number of periods is considered, namely the periods during the EOL. Therefore, the appropriate model can be found in section 3.7.

3.9 Detailed Model for the Simple Policy where $t < EOL$, Original Item cannot be used after RTT

In the situation where $t < EOL$, the alternative item (item 2) must be used to satisfy demand during the remainder of the planning horizon. In the remainder of this section the detailed models for the simple policy is described for the situation in which the required transition time is shorter than the EOL. Again, a division is made between the following situations:

1) $t = 0$
2) $t = 1$
3) $t \geq 2$

This division is made because the equations used to calculate the total expected discounted costs are different for these different situations. In the next sections the detailed model for the simple policy will be given for all these situations.

Note that the models given in section 3.9.1 to section 3.9.3 consider the situation where the original item cannot be used after the RTT. All units of the original item that are left at the end of the RTT are disposed. In section 3.10 the model for the situation where the original item can be used after the RTT is given. Note that this division in modeling is only necessary for the case where $t < EOL$. After all, if $t = EOL$ or $t > EOL$ all units of the original item that are left at the end of the RTT are disposed because no demand has to be satisfied anymore. In case $t < EOL$, demand can occur after the end of the RTT, which means a decision has to be made about disposing excess units of the original item.

3.9.1 Simple Policy, $t < EOL$, $t = 0$, no Item 1 after RTT

When t is 0 periods, this means that no final order quantity is needed to cover demand. However, because the EOL is at least equal to 1 in this situation, $t = 0$ does not mean that no expected costs should be calculated. The total expected discounted costs to cover the planning horizon in the situation where $t < EOL$ namely exist of the total expected discounted costs that follow from the final order quantity, and the total expected discounted costs that follow from the 'normal' order policy after the end of t.

The total expected discounted costs for the remainder of the planning horizon (the periods after t) are calculated as follows:

For the last period $k = EOL$:

$$E[TC_{EOL}(I_{EOL})] = a\left(\widehat{CH1_{EOL-1}} * I_{EOL} + \sum_{j=0}^{I_{EOL}} \widehat{CD1_{EOL}} * (I_{EOL} - j) * P(d_{EOL} = j) + \sum_{j=I_{EOL}+1}^{\infty} E[CN2_{EOL}] * (j - I_{EOL}) * P(d_{EOL} = j)\right)$$

For the periods $k = EOL - 1, \dots, t + 2$:

$$E[TC_k(I_k)] = a\left(\widehat{CH1_{k-1}} * I_k + \sum_{j=0}^{I_k} E[TC_{k+1}(I_k - j)] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} (E[CN2_k] * (j - I_k) + E[TC_{k+1}(0)]) * P(d_k = j)\right)$$

For the first period in the remainder of the planning horizon, $k = t + 1 = 1$ ($t = 0$):

$$E[TC_1(Q)] = \sum_{j=0}^{Q} E[TC_2(Q - j)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} (E[CN2_1] * (j - Q) + E[TC_2(0)]) * P(d_1 = j) \qquad \textbf{[13]}$$

Here all parameters are defined as shown in section 3.6 and in the overview of used notation in Appendix F.

Although $t = 0$, this does not mean that Q has to be equal to zero as well. As shown in the objective function in equation [1], the calculation of the total expected discounted costs for a given final order quantity Q is calculated by setting Q to a specific value. After this, the expected total discounted costs for this Q, given an RTT that equals t periods is calculated ($E'[TC(Q,t)]$). Thus, even though the RTT is equal to 0, this does not mean that Q is zero as well. A simple example shows the importance of this statement:

Imagine that the purchasing costs are €5 per item. All other cost parameters are equal to €0. Assume that the possible values for the RTT are 0, 1 and 2 time periods, where $P(RTT = 0) = P(RTT = 1) = P(RTT = 2) = 1/3$. Note that this section concerned the calculation of the total expected discounted costs for RTT = 0 only. The other possibilities are discussed in the next sections. However, as can be seen from the objective function in equation [5], $t = 0$ is only a possible length of the RTT. Depending on the probability distribution of the length of the RTT, the RTT can be bigger than 0 as well. The purpose of this example is to show that Q can be positive while $t = 0$, because for the calculation of the costs for that same Q other values for t have to be taken into account as well due to the probability distribution for the length of the RTT.

Now imagine we want to calculate the expected costs for Q = 10. Then the expected costs E[TC(Q = 10)] are:

$$E[TC(Q = 10)] = E'[TC(Q = 10, RTT = 0)] * P(RTT = 0) + E'[TC(Q = 10, RTT = 1)] * P(RTT = 1) + E'[TC(Q = 10, RTT = 2)] * P(RTT = 2)$$

$$E[TC(Q = 10)] = (10 * 5 * 1/3) + (10 * 5 * 1/3) + (10 * 5 * 1/3) = €50$$

Now imagine that one would assume that Q is 0 when the RTT is 0, since there is no time that the final order quantity should cover. This would result in the following expected costs:

$$E[TC(Q = 10)] = (0 * 5 * 1/3) + (10 * 5 * 1/3) + (10 * 5 * 1/3) = €33{,}33$$

This example shows that even though $t = 0$, this does not mean that $Q = 0$ needs to be the optimal final order quantity. However, this section concerns the situation where the original item cannot be used after the RTT. This means that if any final order quantity would be placed, these units are disposed right away because they cannot be used after the RTT. Therefore, the expected total discounted costs for any Q, given $t = 0$ is:

$$E'[TC(Q, t = 0)] = CO1 + CN1 * Q + \widehat{CD1}_0 * Q + E[CO2] + E[TC_1(0)] \tag{14}$$

Note that the fillrate during the RTT (which equals 0 periods) is 1 (100%). During 0 time periods it is impossible to have any out-of-stocks, which means that the fillrate is 1.

3.9.2 Simple Policy, $t < EOL$, $t = 1$, no Item 1 after RTT

When $t = 1$, this automatically means that the EOL is at least 2 periods. One could think that when $t = 1$, this means that the models described in section 3.7.2 can be used here. However, since $t < EOL$, the total expected costs for the periods after t should be taken into account. Therefore, the equations in the above sections have to be adapted a bit. To calculate the total expected discounted costs during the entire planning horizon, the following is done:

$$\begin{aligned} E'[TC(Q, t = 1)] \\ &= CO1 + CN1 * Q + E[CO2] + \sum_{j=0}^{Q} \left(\widehat{CD1}_1 * (Q - j) + E[TC_2(0)]\right) * P(d_1 = j) \\ &+ \sum_{j=Q+1}^{\infty} E[TC_2(0) * P(d_1 = j) \end{aligned} \tag{15}$$

Here, $E[TC_2(I_2)]$ is calculated as follows:

For the last period $k = EOL$:

$$\begin{aligned} E[TC_{EOL}(I_{EOL})] = a\Bigg(& \widehat{CH1}_{EOL-1} * I_{EOL} + \sum_{j=0}^{I_{EOL}} \widehat{CD1}_{EOL} * (I_{EOL} - j) * P(d_{EOL} = j) \\ & + \sum_{j=I_{EOL}+1}^{\infty} E[CN2_{EOL}] * (j - I_{EOL}) * P(d_{EOL} = j)\Bigg) \end{aligned}$$

For the periods $k = EOL - 1, \ldots, t + 2$:

$$\begin{aligned} E[TC_k(I_k)] = a\Bigg(& \widehat{CH1}_{k-1} * I_k + \sum_{j=0}^{I_k} E[TC_{k+1}(I_k - j)] * P(d_k = j) \\ & + \sum_{j=I_k+1}^{\infty} (E[CN2_k] * (j - I_k) + E[TC_{k+1}(0)]) * P(d_k = j)\Bigg) \end{aligned}$$

For the first period after the RTT $k = t + 1$:

$$\begin{aligned} E[TC_{t+1}(I_{t+1})] = a\Bigg(& \widehat{CH1}_t * I_{t+1} + \sum_{j=0}^{I_{t+1}} E[TC_{t+2}(I_{t+1} - j)] * P(d_{t+1} = j) \\ & + \sum_{j=I_{t+1}+1}^{\infty} (E[CN2_{t+1}] * (j - I_{t+1}) + E[TC_{t+2}(0)]) * P(d_{t+1} = j)\Bigg) \end{aligned} \tag{16}$$

The expected number of out-of-stocks during t and the fillrate are calculated as follows:

$$E'[OOS(Q,t=1)] = \sum_{j=0}^{Q} 0 * P(d_1 = j) + \sum_{j=Q+1}^{\infty} (j - Q) * P(d_1 = j)$$

[17]

$$fillrate(Q,t=1) = 1 - \frac{E'[OOS(Q,t=1)}{\sum_{j=0}^{\infty} j * P(d_1 = j)}$$

[18]

Note that when the expected demand during the period is 0, the company automatically satisfies all demand and the fillrate is 1 (100%).

3.9.3 Simple Policy, $t < EOL$, $t \geq 2$, no Item 1 after RTT

When t is at least equal to 2 periods, this automatically means that the EOL is at least 3 periods. One could think that when $t \geq 2$, this means that the models described in section 3.7.3 can be used here. However, since $t < EOL$, the total expected costs for the periods after t should be taken into account. Therefore, the equations in the above sections have to be adapted a bit. To calculate the total expected discounted costs during the entire planning horizon, the following is done:

For the periods $k = EOL, \dots, t+1$: See equation [16]

For the last period in the RTT $k = t$:

$$E[TC_t(I_t)] = a\left(\widehat{CH1_{t-1}} * I_t + \sum_{j=0}^{I_t}\left(\widehat{CD1_t} * (I_t - j) + E[TC_{t+1}(0)]\right) * P(d_t = j) + \sum_{j=I_t+1}^{\infty} E[TC_{t+1}(0)] * P(d_t = j)\right)$$

For the periods $k = t-1, \dots, 2$:

$$E[TC_k(I_k)] = a\left(\widehat{CH1_{k-1}} * I_k + \sum_{j=0}^{I_k} E[TC_{k+1}(I_k - j)] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} E[TC_{k+1}(0)] * P(d_k = j)\right)$$

For the first period $k = 1$:

$$E[TC_1(Q)] = CO1 + CN1 * Q + E[CO2] + \sum_{j=0}^{Q} E[TC_2(Q - j)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E[TC_2(0)] * P(d_1 = j)$$

Thus, the total expected discounted costs during the entire planning horizon are:

$$E'[TC(Q, RTT = t)] = E[TC_1(Q)]$$

[19]

The expected number of out-of-stocks and the fillrate during t are calculated as follows:

For the last period in the RTT $k = t$:

$$E[OOS_t(I_t)] = \sum_{j=0}^{I_t} 0 * P(d_t = j) + \sum_{j=I_t+1}^{\infty} (j - I_t) * P(d_t = j)$$

For the periods $k = t - 1, \dots, 2$:

$$E[OOS_k(I_k)] = \sum_{j=0}^{I_k} E[OOS_{k+1}(I_k - j)] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} \big((j - I_k) + E[OOS_{k+1}(0)]\big) * P(d_k = j)$$

For the first period $k = 1$:

$$E[OOS_1(Q)] = \sum_{j=0}^{Q} E[OOS_2(Q - j)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} \big((j - Q) + E[OOS_2(0)]\big) * P(d_1 = j)$$

The total expected number of out-of-stocks is:

$$E'[OOS(Q, RTT = t)] = E[OOS_1(Q)]$$

[20]

For the fillrate for a given final order quantity Q and an RTT that is equal to t, the following holds:

$$fillrate(Q, RTT = t) = 1 - \frac{E'[OOS(Q, RTT = t)}{\sum_{k=1}^{t}\big(\sum_{j=0}^{\infty} j * P(d_k = j)\big)}$$

[21]

3.10 Detailed Model for the Simple Policy where $t < EOL$, Original Item can be used after RTT

In the situation where $t < EOL$, the alternative item (item 2) must be used to satisfy demand during the remainder of the planning horizon. In the remainder of this section the detailed models for the simple policy is described for the situation in which the required transition time is shorter than the EOL. Here, the same division is made as in section 3.9. The difference in comparison to section 3.9 is that section 3.9 shows the model for the situation where the original item cannot be used after the RTT, where this section concerns the model for the situation where the original item can be used after the RTT.

3.10.1 Simple Policy, $t < EOL$, $t = 0$, Item 1 after RTT

When t is 0 periods, this means that no final order quantity is needed to cover demand. However, because the EOL is at least equal to 1 in this situation, $t = 0$ does not mean that no expected costs should be calculated. The total expected discounted costs to cover the planning horizon in the situation where $t < EOL$ namely exist of the total expected discounted costs that follow from the final order quantity, and the total expected discounted costs that follow from the ‘normal’ order policy after the end of t. For more details regarding this discussion see section 3.9.1.

In the situation that is modeled here, the original item can be used after the RTT. This means that the units that are left after the RTT do not need to be disposed. The expected total discounted costs for any Q, given $t = 0$ are calculated as follows:

For the periods $k = EOL, \dots, t + 1$: See equation [13]

Since $t = 0$, no demand can occur during the RTT. However, similar to the model in section 3.9.1, the total expected discounted costs for any Q given $t = 0$:

$$E'[TC(Q, t = 0)] = CO1 + CN1 * Q + E[CO2] + E[TC_1(Q)]$$

[22]

Note that the fillrate during the RTT (which equals 0 periods) is 1 (100%). During 0 time periods it is impossible to have any out-of-stocks, which means that the fillrate is 1.

3.10.2 Simple Policy, $t < EOL$, $t = 1$, Item 1 after RTT

Here, the model given in section 3.9.2 has to be adapted a bit. For the situation where the original item can be used after the RTT, the total expected discounted costs for any Q given $t = 1$ are:

$$E'[TC(Q, t = 1)] = CO1 + CN1 * Q + E[CO2] + \sum_{j=0}^{Q} E[TC_2(Q - j)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E[TC_2(0)] * P(d_1 = j)$$

[23]

Here, $E[TC_2(I_2)]$ is calculated as shown in equation [16]. Equation [17] and [18] can be used to calculate the expected number of out-of-stocks and the fillrate during the RTT.

3.10.3 Simple Policy, $t < EOL$, $t \geq 2$, Item 1 after RTT

Since this section concerns the model for the situation where the original item can be used after the RTT, the equations as shown in section 3.9.3 have to be adapted a bit. The total expected discounted costs for any Q given $t \geq 2$ are:

For the periods $k = EOL, \ldots, t + 1$: See equation [16]

For the last period in the RTT $k = t$:

$$E[TC_t(I_t)] = a\left(\widehat{CH1_{t-1}} * I_t + \sum_{j=0}^{I_t} E[TC_{t+1}(I_t - j)] * P(d_t = j) + \sum_{j=I_t+1}^{\infty} E[TC_{t+1}(0)] * P(d_t = j)\right)$$

For the periods $k = t - 1, \ldots, 2$:

$$E[TC_k(I_k)] = a\left(\widehat{CH1_{k-1}} * I_k + \sum_{j=0}^{I_k} E[TC_{k+1}(I_k - j)] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} E[TC_{k+1}(0)] * P(d_k = j)\right)$$

For the first period $k = 1$:

$$E[TC_1(Q)] = CO1 + CN1 * Q + E[CO2] + \sum_{j=0}^{Q} E[TC_2(Q - j)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E[TC_2(0)] * P(d_1 = j)$$

Thus, the total expected discounted costs during the entire planning horizon are:

$$E'[TC(Q, RTT = t)] = E[TC_1(Q)]$$

[24]

Equation [20] and [21] can be used to calculate the expected number of out-of-stocks and the fillrate during the RTT.

3.11 Calculating the Optimal Final Order Quantity for the Simple Policy

In the previous sections the models for the following parameters were described:

- The total expected discounted costs for a <u>given</u> final order quantity Q and a <u>given</u> length of the required transition time t.
- The fillrate during a <u>given</u> length of the required transition time t for a <u>given</u> final order quantity Q.

The objective of the total cost model is to minimize the total expected discounted costs for a certain final order quantity Q. The objective function can be found in equation [1]. Using equation [1] and the equations in sections 3.7 to 3.9, one can calculate the total expected discounted costs for a final order quantity Q, $E[TC(Q)]$. Similarly, one can calculate the expected fillrate given a final order quantity Q, $E[fillrate(Q)]$, as follows:

$$E[fillrate(Q)] = \sum_{t=minRTT_a}^{EOL} fillrate(Q,t) * P(RTT_a) + \sum_{t=EOL+1}^{maxRTT_a} fillrate(Q,EOL) * P(RTT_a = t)$$

[25]

To find the optimal final order quantity Q^*, which minimizes the total expected discounted costs but also meets all restrictions, the algorithm described is used. Note that this algorithm holds for both the situation where the original item cannot be used after the RTT and the situation where the original item can be used after the RTT. If one solely wants to consider the situation where the original item cannot be used after the RTT, an algorithm similar to the one given in section 3.7.2 can be used. The algorithm in section 3.7.2 is faster than the one given below, since it stops the calculation immediately when the final order quantity meets all restrictions. The algorithm below calculates the costs etc. for all possible values for the final order quantity, which is more time-consuming. However, the algorithm given below is applicable to both situations, the one in section 3.7.2 is only applicable to the situation where the original item cannot be used after the RTT.

Algorithm to find the optimal final order quantity Q* for the simple policy

For all $Q = MOQ$ or $Q = 1 * BS_s$ to $Qmax = \sum_{k=1}^{EOL} maxdemand(k)$ (where $Qmax$ is the maximum possible value for the final order quantity), calculate the total expected discounted costs using equation [1] and calculate the fillrate using equation [25] .The optimal final order quantity $Q^* = Q$ is the final order quantity that is accompanied by lowest total expected discounted costs during the planning horizon that meets all restrictions.

4 Model Description for the Remove Policy

In this chapter the detailed model for the remove policy will be described. First, the assumptions that are used to model the remove policy are given in section 4.1. Next, the objective function of the model will be given in section 4.1. Also, an overview is given of the inputs the user of the model should give and the outputs that 'black box' (in which the model operates) results in. The remainder of this chapter concerns the contents of this black box, and will thus contain mathematical descriptions of the model and its parameters. In section 4.3 the mathematical description of the objective of the model will be given. In section 4.4 the decision variables will be described. After this, the restrictions and input parameters will be discussed in section 4.5 and section 4.6. After this all these elements are used to develop the cost model for the remove policy. This model is given in the sections 4.7 to 4.9. Finally, in section 4.10 it is explained how the optimal final order quantity and remove-down-to levels can be calculated. Those who are not interested in this mathematical model can skip the sections 4.3 up to and until 4.10.

4.1 Modeling Assumptions for the Remove Policy

The modeling assumptions that are used to model the remove policy are the same assumptions as the ones for the simple policy. See section 3.1 for these assumptions.

4.2 Objective of the Model for the Remove Policy

The objective of the total cost model for the remove policy is the same as the objective of the total cost model for the simple policy (see section 3.2). The objective of the total cost model for the remove policy is as follows:

The objective of the model is to minimize the total expected discounted costs during the entire planning horizon.

Discounting is taken into account because the planning horizon can be long. The optimal remove policy is the remove policy that leads to the lowest costs possible to cover the entire planning horizon. Here, remove-down-to levels are only calculated for the periods during the required transition time. A remove-down-to level, which is effective at the end of a period, is an inventory level which you desire to have at that moment. If the stock at that moment is lower than or equal to the remove-down-to level, no items are disposed. If the stock at that moment is higher than the remove-down-to level, the excess units are disposed. After the required transition time, the alternative item must be used to satisfy demand.

To calculate the total expected discounted costs during the entire planning horizon the model should take into account decision variables, restrictions and input parameters. The model itself should 'decide' on the decision variables, where its objective is to minimize the total expected discounted costs during the entire planning horizon. The values for the decision variables that the model results in should meet all restrictions. Basically, the mathematical model behind the total cost calculation acts like a 'black box', where the user of the model provides some inputs, the black box does its 'trick' and it gives some outputs. For the simple policy, the inputs that the black box needs and the outputs that the black box generates are depicted in Figure 4.1. Details about these input parameters, the mathematical model and the outputs of the model can be found in the remainder of this chapter. If you do not want to read the detailed modeling of the black box for the remove policy, the remainder of this chapter can be skipped.

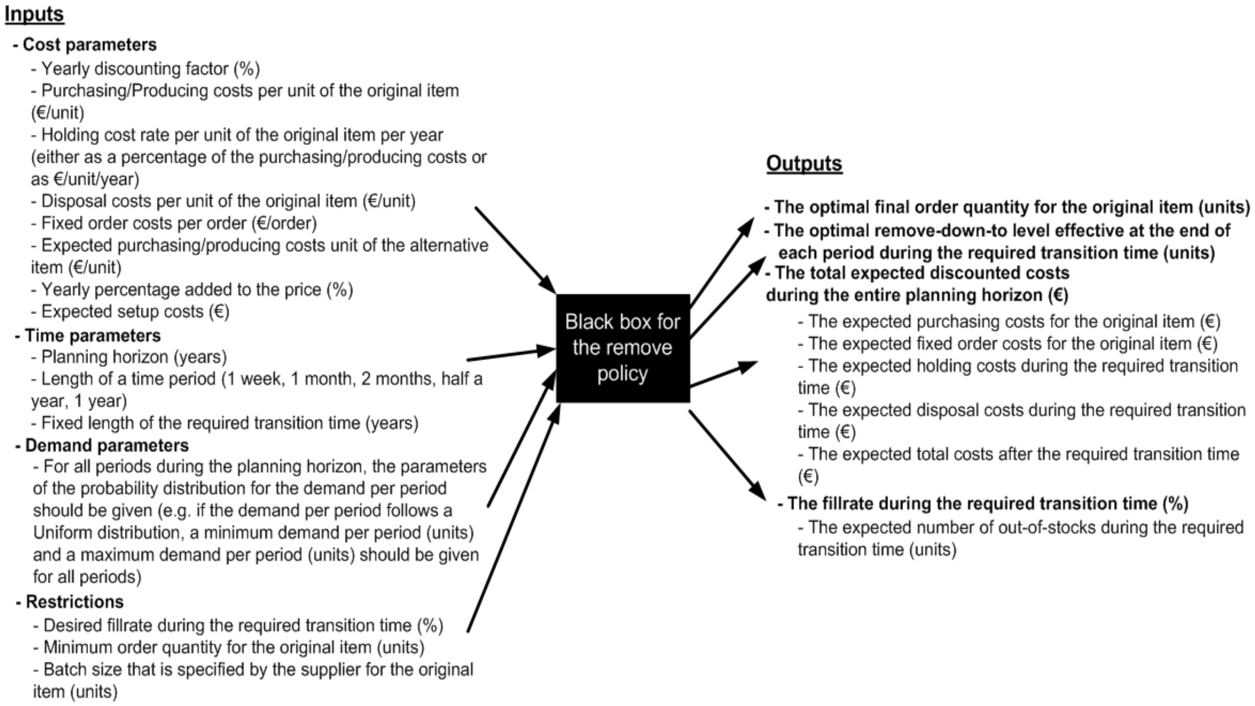


Figure 4.1: Inputs, black box, and outputs for the remove policy.

4.3 Objective Function for the Remove Policy

The remove policy is a policy where a final order quantity and remove-down-to levels at the end of all periods during the RTT are used (see section 2.2). Using this information, the objective function for the remove policy can be formulated mathematically as shown below.

For the total expected discounted costs during the entire planning horizon, given a final order quantity Q and remove-down-to levels, the following holds:

$$E[TC(Q,U_1,U_2,\dots,U_{maxtime})] = \sum_{t=minRTT_a}^{EOL} E'\,[TC(Q,U_1,\dots,U_t,t)] * P(RTT_a = t)$$
$$+ \sum_{t=EOL+1}^{maxRTT_a} E'\,[TC(Q,U_1,\dots,U_{EOL},t)] * P(RTT_a = t)$$

[26]

Where:

$E[TC(Q,U_1,U_2,\dots,U_{maxtime})]$	Total expected discounted costs during the entire planning horizon for a remove policy using a final order quantity Q and remove-down-to levels $U_1,\dots,U_{maxtime}$ *(€)*
$E'[TC(Q,U_1,\dots,U_t,t)]$	Total expected discounted costs during the entire planning horizon for a remove policy using a final order quantity Q and remove-down-to levels $U_1,\dots,U_t$, given that the RTT is equal to t periods *(€)*
EOL	Length of the End-of-Life (service) period (EOL) / planning horizon *(# periods)*
$maxRTT_a$	The maximum possible value for the length of the RTT for a given alternative item a *(# periods)*
$maxtime$	The minimum of the EOL and $maxRTT$. This minimum is the maximum number of periods for which remove-down-to levels should be calculated. If $t < EOL$, then remove-

	down-to levels are needed for t periods. If $t \geq EOL$, then remove-down-to levels are needed for EOL periods.
$minRTT_a$	The minimum possible value for the length of the RTT for a given alternative item a *(# periods)*
$P(RTT_a = t)$	The probability that the RTT for a given alternative item a is equal to t periods
t	A specific value for the length of the RTT, which can range between $minRTT_a$ and $maxRTT_a$ *(# periods)*

The optimal remove policy is the combination of the final order quantity and the remove-down-to levels that minimizes the total expected discounted costs as expressed in equation [26]. In the remainder of this section the mathematical model will be described that is used to calculate the total expected discounted costs for a given final order quantity Q, remove-down-to levels, and a given value t for the length of the RTT. These costs are denoted by $E'[TC(Q, U_1, U_2, \ldots, U_t, t)]$. As can be seen from equation [26], t can range between $minRTT_a$ and $maxRTT_a$ and each possible value for t has a probability $P(RTT_a = t)$. For all these possible values for t the total expected discounted costs are calculated for a given final order quantity Q and a given set of remove-down-to levels. Using all values for $E'[TC(Q, U_1, U_2, \ldots, U_t, t)]$ and $P(RTT_a = t)$, the total expected discounted costs for a given final order quantity and a given set of remove-down-to levels can be calculated (see equation [26]).

Note that equation [26] includes a probability distribution for the length of the RTT. As explained in section 2.3 calculating remove-down-to levels for a stochastic RTT is impossible since the number of periods during the RTT is not fixed, the number of remove-down-to levels is not fixed as well. If the RTT can range between 0 and 5 periods, the number of remove-down-to levels also ranges between 0 and 5. Calculating the optimal remove policy during the uncertain RTT is impossible then, since it is not known what the optimal number of remove-down-to levels is. An uncertain RTT leads to an uncertain number of periods during the RTT. By definition, remove-down-to levels are effective at the end of each period during the RTT. If it is not known for sure how many periods there are during the RTT, it is not known as well how many remove-down-to levels are needed. Thus, the number of decision variables that should be decided on is uncertain.

This means that the model that will be described in the next sections only holds for a fixed length of the RTT. When it is uncertain what this length of the RTT will be (i.e. it follows a certain probability distribution), it is advised to use the rolling horizon strategy that is described in section 2.3). After all, when the length of the RTT is uncertain, the number of remove-down-to levels that are needed is uncertain as well. Instead of calculating remove-down-to levels when the length of the RTT is uncertain, using the simple policy at each moment the inventory levels are reviewed is better. This way, updated information is used and more reliable results are obtained.

Since calculating remove-down-to levels only makes sense when the number of periods for which this should be done is known, the objective function in equation [26] can be re-written to the following equation:

$$E[TC(Q, U_1, U_2, \ldots, U_{maxtime})] = E'[TC(Q, U_1, \ldots, U_{maxtime}, t)]$$

[27]

In equation [27] the probability distribution for the length of the RTT (as shown in equation [26]) is removed, since remove-down-to levels can only be calculated for a fixed length of the RTT. Regardless of the probability distribution for the demand per period, it is possible to distinguish three different situations when calculating $E'[TC(Q, U_1, U_2, \ldots, U_{maxtime}, t)]$:

1) $t = EOL$
2) $t > EOL$
3) $t < EOL$

For each of these three situations a detailed model for the remove policy has been developed. The kind of modeling that is used for this is stochastic dynamic programming. To be able to formulate these models, the decision variables, restrictions and input parameters for the model of the simple policy are described first in the upcoming sections (also see Appendix F for an overview of used notation). In section 4.7 to 4.9 the models for the three situations are described.

4.4 Decision Variables for the Remove Policy

As mentioned in section 4.3, the total expected discounted costs for the remove policy depend on the final order quantity and the remove-down-to levels that are effective at the end of each period during the RTT. This means that the decision variables for the remove policy are:

Q *The final order quantity (items)*

The final order quantity is the quantity of the original item that is ordered to cover the expected demand during the RTT. Equation [26] (see section 4.1) shows how this decision variable is included in the objective function.

This is similar to the simple policy. However, when considering the remove policy, the remove-down-to levels that are effective at the end of each period in the RTT are additional decision variables:

U_k *Remove-down-to level effective at the end of period* $k = 1, \ldots., t$ *(items)*

Here, t is the last period in the RTT (expressed in the number of periods). Using remove-down-to levels means that at the end of each period during the RTT the stock level is reviewed and if the number of units in stock is higher than the remove-down-to level at that moment, the excess units in stock will be disposed. Equation [26] shows how these decision variables are included in the objective function.

4.5 Restrictions for the Remove Policy

Regarding the remove policy, the same restrictions hold as the ones for the simple policy (see section 3.5). However, an additional restriction should be taken into account:

The remove-down-to level of period k *should be equal to or smaller than the remove-down-to level of period* $k - 1$.

This restriction must be met for the following reason: The only way the inventory was ever increased was by placing a final order. This final order arrived at the beginning of the first period. In the periods during the EOL, there are no returns of items. This means that the inventory will decrease over time due to demand. Due to these characteristics, the remove-down-to level of a certain period must be equal to or lower than the remove-down-to level of the previous period.

Mathematically, this restriction can be denoted as follows:
$U_k \leq U_{k-1}$

4.6 Input Parameters for the Remove Policy

The input parameters for the model for the remove policy are the same as the input parameters for the model for the simple policy. See section 3.6 for the input parameters.

4.7 The Remove Policy when $t = EOL$

In this section the same situations as in section 3.7 are described, but now this is done for the remove policy.

4.7.1 Remove Policy, $t = EOL = 0$

As in the case of the simple policy, when t and EOL are equal to 0 the company does not have to satisfy any demand. Thus, in this situation there is no final order problem and no costs have to be calculated because the time period considered is 0. Note that it is assumed that in a time period of 0 the fillrate is always 1 (100%).

4.7.2 Remove Policy, $t = EOL = 1$

Since t and EOL are equal to only one period, the calculation of the total expected discounted costs for the remove policy is exactly the same as the calculation for the simple policy given in section 3.7.2. Because there is only 1 period to consider, it is impossible to have any intermediate remove-down-to levels, which means that the remove policy and the simple policy coincide.

4.7.3 Remove Policy, $t = EOL, t \geq 2$

When $t = EOL$ and thus both of these time periods are at least equal to 2, this means that the final order quantity should cover the expected demand during these time periods. Here, the objective of the model is to find the final order quantity and the remove-down-to level for each period that minimize the total expected discounted costs, but also satisfy the restrictions. Note that the model aims at an end inventory at the end of t that is equal to 0, since this is also the end of the EOL.

An important remark should be made regarding the calculation including the intermediate remove-down-to levels. Since the article of Teunter & Fortuin [1999] forms the basis for the equations in this project, one could expect that the optimal remove-down-to levels for each period are calculated in a similar way as in that article. The method that is used in Teunter & Fortuin [1999] can shortly be described as follows:

1) The time period that is considered is fixed, e.g. 10 years are considered where each period has the length of a month → $t = EOL = 120$ periods.
2) A 'smart' set of vectors is generated. A vector is any combination of a value for the final order quantity Q and values for the remove-down-to levels for each period in the RTT, which can be written down as follows for $t = 120$:
$$vector(i) = \{Q, U_1, U_2, U_3, \dots, U_{118}, U_{119}, 0\}$$
Here, i is the index for a given vector. Note that the remove-down-to level of the last period is always equal to 0, since Teunter & Fortuin [1999] consider a fixed RTT which is equal to the entire EOL and at the end of the EOL you do not want to have any stock left.
3) For all vectors the total expected discounted costs are calculated.
4) The vector with the lowest costs is selected as the optimal vector, which means that an optimal final order quantity and optimal remove-down-to levels are found.

This method by Teunter & Fortuin [1999] finds the optimal combination of the final order quantity and remove-down-to levels, which makes it a very strong method to solve the final order problem. A drawback of the method is that as the number of periods increases, so will the number of vectors. Similarly, the number of vectors will increase when the demand per period increases. This is because when the number of periods and/or the demand per period increases, the maximum value for the final order quantity Q will also increase. A larger maximum value for Q means that the remove-down-to levels can be larger as well. The larger the number of possible values for Q and the remove-down-to levels, the larger the number of vectors. The number of vectors that should be evaluated increases rapidly, which makes this method a very time consuming method. The company is most likely going to use the model during decision making meetings, which means the model should calculate the optimal costs as fast as possible (i.e. within 15 minutes).

The company wants a convenient model that generates results as fast as possible. This means that the method described in Teunter & Fortuin [1999] is not applicable to the company's case, mostly because its calculation time is very long.

To solve this problem a heuristic has been developed to calculate the remove-down-to levels. The purpose of this heuristic is that it generates good results faster than the exact method. However, this heuristic may result in sub-optimization: it optimizes the remove-down-to level for each period in the RTT given a certain inventory at the beginning of that period. This may lead to higher costs than the exact method described above for the 'total' problem, since in the optimal solution the combination of all decision variables (for all periods) is optimized simultaneously instead of each decision variable one by one. To get insights in the performance of the heuristic in comparison to the exact method, the results of this heuristic will be compared for a fixed number of periods to the method described in Teunter & Fortuin [1999]. The comparison of the heuristic to the exact method can be found in chapter 8.

The logic behind the heuristic will be described first in section 4.7.3.1. After the logic of the heuristic has been described, the mathematical equations that are used in the heuristic are given in section 4.7.3.2.

4.7.3.1 Logic of the Heuristic for the Remove Policy

The optimal remove policy is the combination of the final order quantity and remove-down-to levels for all periods during the RTT that minimizes the total expected discounted costs. To find this optimal combination, all the decision variables have to be optimized. In the case of the simple policy this optimization process was easy, since only one decision variable had to be optimized. Now, remove-down-to levels for all periods during the RTT have to be optimized as well. Due to the restrictions that hold for the remove policy (see section 4.5), it is known that all remove-down-to levels are smaller than or equal to the final order quantity Q. This means that the larger the final order quantity is, the more possible values there are for the remove-down-to levels. For each value for the final order quantity, there should be an optimal set of remove-down-to levels.

Based on the above there are two optimization processes necessary to find the optimal remove policy:

1) The optimization of the final order quantity Q: the algorithm that is used to find the optimal final order quantity is similar to the one for the simple policy (see section 3.7.2)
2) For each final order quantity Q, the optimal set of remove-down-to levels should be found.

The algorithm in section 3.7.2 can be used here to find the optimal final order quantity, because we only consider the remove policy for the situation where the original item cannot be used after the RTT. This is done because using a model including remove-down-to level aims at an end inventory at the end of the RTT that is equal to 0.

Below a small example is given that considers three time periods. For this example, the heuristic is executed step by step to illustrate how the heuristic works. After this example, the principles behind the heuristic are described in more detail.

Consider a planning horizon of three time periods. In each time period k, demand d_k can only have two possible values: 0 and 1. Here, $P(d_k = 0) = P(d_k = 1) = 1/2$. We would like to know whether a final order quantity Q would result in lower costs than a final order quantity $Q - 1$ and we would like to know the remove-down-to levels when the inventory at the start of the first period is equal to Q units. Using this information, the situation we consider can be depicted as follows:

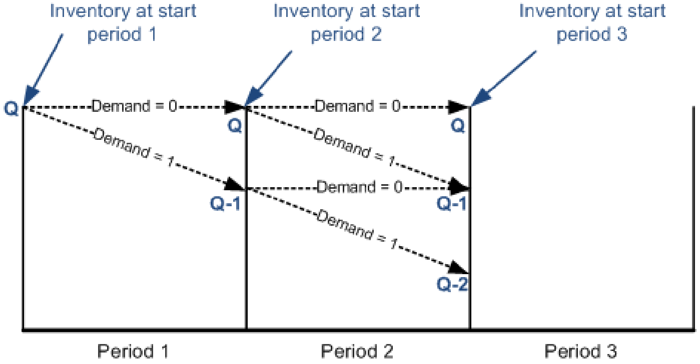


Figure 4.2: Example to illustrate the idea of the heuristic for the remove policy.

Step 1) We start in the last period, period 3. For all possible values for the inventory at the start of period 3 we know what the optimal remove-down-to level will be; no matter what the inventory at the start of the period is, the remove-down-to level is always equal to 0. This means that $U_3(Q) = U_3(Q-1) = U_3(Q-2) = 0$. Here, $U_3(I_3)$ denotes the remove-down-to level effective at the end of period 3 for a given inventory I_3 at the start of period 3.

Step 2) We move to period 2. Here, it is possible that the inventory at the start of period 2, I_2 is equal to Q or to $Q-1$. Next, we do the following:

1) Set $I_2 = Q$
    a. Set $U_2(Q) = Q$. Here, $U_2(Q)$ denotes the remove-down-to level effective at the end of period 2 given that the inventory at the start of period 2 is Q. Q is the maximum possible value for this remove-down-to level. Calculate the total expected discounted costs during period 2 and 3, $E\left[TC_2\left(Q, U_2(Q)\right)\right]$, and the fillrate during period 2 and 3, $fillrate_2(Q, U_2(Q), 3)$ (see Appendix F for an overview of the used notation and see section 4.7.3.2 for the equations that are used to calculate these costs and fillrate).
    b. Set $U_2(Q) = U_2(Q) - 1$ and calculate the total expected discounted costs during period 2 and 3, $E\left[TC_2\left(Q, U_2(Q)\right)\right]$, and the fillrate during period 2 and 3, $fillrate_2(Q, U_2(Q), 3)$, for this new value for the remove-down-to level.
    c. If $fillrate_2(Q, U_2(Q), 3) \geq desiredfillrate$ and $E\left[TC_2\left(Q, U_2(Q)\right)\right] \leq E[TC_2(Q, U_2(Q) + 1)]$, move to *b*, else, move to *d*.
    d. The optimal remove-down-to level effective at the end of period 2, given that the inventory at the start of period 2 is equal to Q units, is $U_2^*(Q) = U_2(Q) + 1$ (the '+1' is added because a remove-down-to level that is equal to $U_2(Q)$does not meet the fillrate restriction, otherwise the algorithm would not have moved to step *d*).
2) The steps in 1) are repeated for $I_2 = Q - 1$. This results in an optimal remove-down-to level effective at the end of period 2, given that the inventory at the start of period 2 is equal to $Q - 1$ units: $U_2^*(Q-1)$.

Step 3) We move to period 1. We know that there is only one possible level for the inventory at the start of period 1: $I_1 = Q$. Using the same algorithm as described above we calculate the optimal remove-down-to level effective at the end of period 1, given that the inventory at the start of period 1 is equal to Q units: $U_1^*(Q)$.

Step 4) Now we have the optimal remove-down-to levels for all possible values for the inventory at the start of a specific period, and this is done for all periods. However, this can result in more than 1 remove-down-to level per period, since there can be more than 1 possible value for the inventory at the start of the period. Therefore, we have to transform all the $U_k^*(I_k)$ for all periods k into one remove-down-to level per period: U_k. This is done as follows:

1) We know that the inventory at the start of period 1 is equal to Q units. Therefore, the optimal remove-down-to level for period 1, U_1, is known too: $U_1 = U_1^*(Q)$.
2) We move to period 2. For period 2 it is not known what the inventory at the start of the period is. The possible values for the inventory at the start of period 2 depends on U_1:
   Imagine that $U_1 = Q$, then I_2 can be equal to Q and it can be equal to $Q - 1$ (see Figure 4.2). But, if $U_1 = Q - 1$, then the only possible value for I_2 is $Q - 1$.
   Therefore, we calculate the expected value for this inventory as follows (for the used notation see Appendix F, see section 4.7.3.2 for more details about this equation):

$$E[I_2] = \sum_{j=0}^{Q-U_1} U_1 * P(d_1 = j) + \sum_{j=Q-U_1+1}^{Q} (Q-j) * P(d_1 = j) + \sum_{j=Q+1}^{\infty} 0 * P(d_1 = j)$$

However, this equation can result in an expected value for the inventory at the start of period 2 that is non-integer. Therefore, the following is done:

a. The expected value is rounded up: This is denoted by I_2^u. This means that the remove-down-to level for period 2 is equal to $U_2 = U_2^*(I_2^u)$.
b. The expected value is rounded down: This is denoted by I_2^d. This means that the remove-down-to level for period 2 is equal to $U_2 = U_2^*(I_2^d)$.

So, there are actually two possible values for U_2, for which we do not know which one is best.

3) We move to period 3. For period 3 it is known that the remove-down-to level will always be equal to 0, no matter what the inventory at the start of period 3 is. Therefore, $U_3 = 0$

Step 5) This results in two vectors: $(Q, U_1^*(Q), U_2^*(I_2^u), 0)$ and $(Q, U_1^*(Q), U_2^*(I_2^d), 0)$. However, it is possible that the heuristic results in sub-optimization (the heuristic can result in the optimal solution as well sometimes), which means that it can estimate the remove-down-to levels and the final order quantity to be larger than is actually required to meet the fillrate restriction. Therefore, step 4 is repeated for $I_1 = Q - 1$. This results in two additional vectors: $(Q-1, U_1^*(Q-1), U_2^*(I_2^u), 0)$ and $(Q-1, U_1^*(Q-1), U_2^*(I_2^d), 0)$

Step 6) For all four vectors, the actual total expected discounted costs and fillrate are calculated (see section 4.7.3.2). The vector which results in the lowest total expected discounted costs and also meets the fillrate restriction is selected as the solution of the heuristic.

Now the logic behind the this example is explained in more detail. As in the case of the simple policy, stochastic dynamic programming is used to model the remove policy. This means that we start the calculation of the costs etc. in the last period during the RTT, period t. Then we move to period $t-1$, $t-2$,…,1, so we work our way back to the first period. There are several characteristics we already know:

- The inventory at the beginning of the first period is equal to Q units.
- The inventory at the beginning of period $k = 2, \dots, t$, which is denoted by I_k, can range between 0 and U_{k-1}. Here U_{k-1} is the remove-down-to level that is effective at the end of period $k-1$.
- The remove-down-to level that is effective at the end of the RTT, U_t, is always 0. This is because we only consider the remove policy for the situation where the original item cannot be used after the RTT.
- The remove-down-to level that is effective at the end of period $k = 1, \dots, t-1$, which is denoted by U_k, can range between 0 and $maxdemand(k+1)$. Here, $maxdemand(k+1)$ is the maximum possible demand during the periods $k+1, k+2, \dots, EOL$. Having a remove-down-to level that is larger than $maxdemand(k+1)$ is useless: holding costs are incurred for items that will never be consumed. It is better to dispose items as early as possible to prevent paying holding costs for items that are not needed to satisfy demand. This means that in the optimal solution the remove-down-to level will never be larger than $maxdemand(k+1)$.

The problem with working back from the last period during the RTT to the first, is that it is not known what the inventory at the start of the period that is considered will be. All that is known is that it ranges between the values given above. The basic principle behind the heuristic is as follows:

For each period, all possible values for the inventory at the start of that period are considered and for each possible inventory at the start of that period an optimal remove-down-to level for that period is calculated.

This principle was illustrated by the example given earlier in this section. E.g. there were two possible values for the inventory at the start of period 2, and for each possible value for the inventory a remove-down-to level was calculated. But how do you calculate the optimal remove-down-to level for a given inventory at the beginning of a certain period? The algorithm that is used to do this is given below. Note that this algorithm is executed for *all* periods and for *all* possible values for the inventory at the beginning of a period, where we start with period $maxtime - 1$. If $t < EOL$, then $maxtime = t$ and if $t > EOL$ then $maxtime = EOL$. We do not have to consider the calculation of the remove-down-to level for the last period $maxtime$, since this is equal to 0 anyway. See Appendix F for an overview of the notation that is used in the algorithm.

Algorithm to find the optimal remove-down-to level in a certain period given a certain inventory directly at the beginning of that period

Step 1) Set $U_k(I_k) = \min(I_k, maxdemand(k+1))$

Step 2) Calculate the total expected discounted costs for period k, which are denoted by $E\left[TC_k\left(I_k, U_k(I_k)\right)\right]$ and the expected number of out-of-stocks and the fillrate.

Step 3) If $fillrate_k(I_k, U_k(I_k), t) \geq desiredfillrate$ and $U_k(I_k) > U_{k+1}(I_k)$ then go to step 4), else go to step 7).

Step 4) Set $U_k(I_k) = U_k(I_k) - 1$

Step 5) Calculate the total expected discounted costs for period k and the expected number of out-of-stocks and the fillrate.

Step 6) If $fillrate_k(I_k, U_k(I_k), t) \geq desiredfillrate$ and $U_k(I_k) > U_{k+1}(I_k)$ and $E\left[TC_k\left(I_k, U_k(I_k)\right)\right] \leq E[TC_k(I_k, U_k(I_k) + 1)]$ then go to step 4), else go to step 7).

Step 7) The optimal remove-down-to level for period k given a starting inventory I_k is $U_k(I_k) + 1$.

Important: Note that the optimal remove-down-to level does not necessarily meet the fillrate restriction: If I_k is that low that even $U_k(I_k) = \min(I_k, maxdemand(k+1))$ (which is equal to $U_k(I_k) = I_k$ then) does not meet the fillrate restriction, the 'optimal' remove-down-to level does not meet the fillrate restriction. The model will recognize this since $fillrate_1(Q, U_1(Q), t)$ will be too low then as well and it will eventually increase Q.

In words, the algorithm works as follows: The remove-down-to level is first set equal to the minimum of the inventory at the start of that period and the maximum total remaining demand, since that is the maximum possible value for the remove-down-to level. The remove-down-to level is decreased as long as it keeps satisfying the fillrate restriction and as long as costs are reduced. The optimal remove-down-to level for a specific value for the inventory at the start of a period is the remove-down-to level that minimizes the costs and still meets the fillrate restriction.

After the model evaluated all possible inventories for all periods during the RTT, this results in a matrix with all optimal remove-down-to levels $U_k(I_k)$ for all k and all I_k. If the fillrate restriction is not met, the model increases Q with one unit and the entire process is repeated again. At a certain point it will stop with this loop: the optimal final order quantity Q^* and all accompanying optimal remove-down-to levels are found.

Note however that the remove-down-to levels are still dependent on the inventory at the start of the period. This means that the remove-down-to levels that are found deviate from the true optimal solution: In real life it is impossible to have more than one remove-down-to level per period. There is only one remove-down-to level in a period and these remove-down-to levels should be used to calculate the 'real' fillrate and expected costs. The model as described up to now uses more than one remove-down-to level per period: for each possible inventory the optimal remove-down-to level is known and this is used in the calculation of the costs and fillrate. But this is not possible in real life! Therefore, the following is done:

The heuristic up to now results in an optimal final order quantity and all accompanying optimal remove-down-to levels, $U_k(I_k)$, for all possible values for the inventory at the start of a period k, I_k, in all periods. We know that the resulting costs and fillrate deviate from the real costs and fillrate, since it is impossible to have more than one remove-down-to level per period. Therefore, it is decided to calculate the 'expected remove-down-to levels' based on the final order quantity Q^*. How these expected remove-down-to levels are calculated is shown in the next section. These expected remove-down-to levels, U_k for $k = 1, \dots, t$ are then used again to calculate the 'real' total expected discounted costs and the fillrate. During the testing of this heuristic it turned out that the fillrate that resulted from $(Q^*, U_1, \dots, U_t)$ was always larger than the desired fillrate: i.e. the heuristic resulted in a very safe solution that has higher costs than the exact optimal solution. Therefore, the following step was also included in the heuristic:

The expected remove-down-to levels are calculated for a final order quantity Q^* and for a final order quantity $Q^* - 1$. After all, the heuristic results in a 'safe' solution, so maybe $Q^* - 1$ is also safe enough and this could result in lower costs. Reducing the final order quantity further most probably results in not meeting the fillrate restriction. So there are two vectors: one with Q^* and the accompanying expected remove-down-to levels, and one with $Q^* - 1$ and the accompanying remove-down-to levels. For both vectors the 'real' costs and fillrate are calculated. The one which meets the fillrate restriction at lowest costs is the best solution. Note that the second vector with Q^* is only introduced when there is no restriction regarding BS_s (see section 4.5). If there is a restriction regarding this, the only vector that is considered is the one with Q^*.

The entire optimization process has been illustrated by the example given earlier in this section. Next to this, the entire optimization process can be depicted as follows:

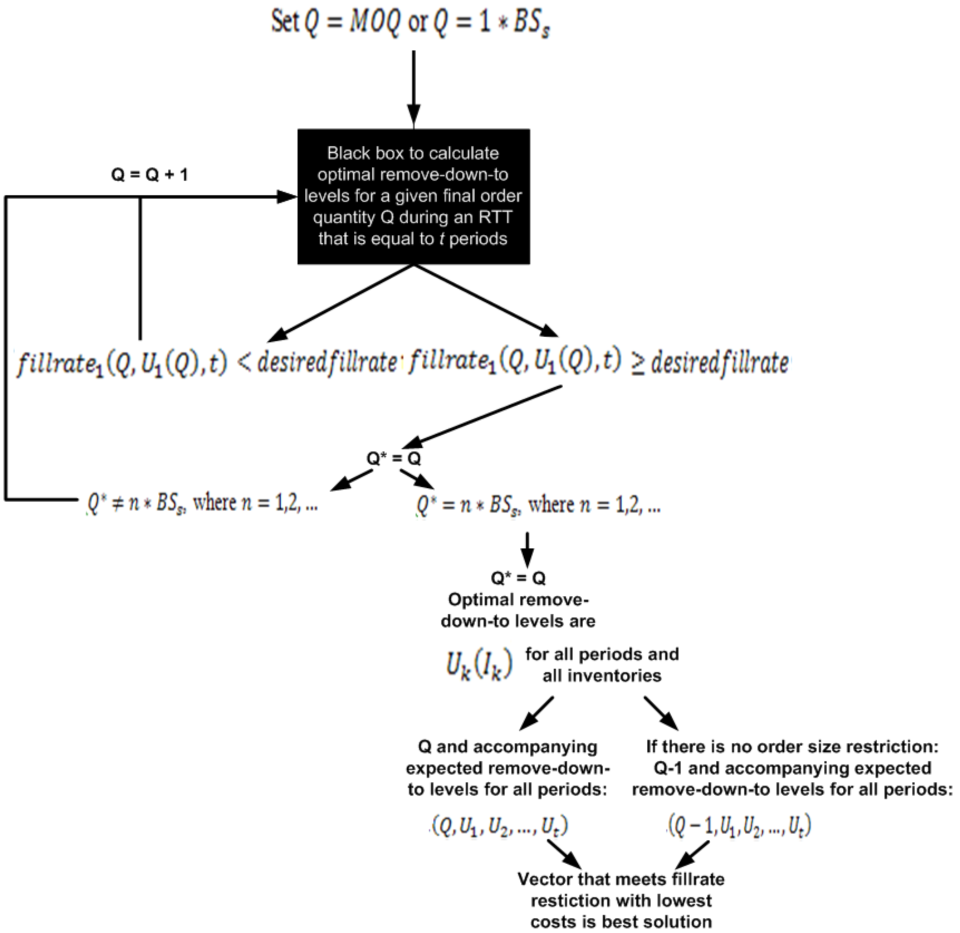


Figure 4.3: Complete optimization process for the remove policy.

The heuristic is developed to calculate remove-down-to levels faster than the exact method that is proposed by Teunter & Fortuin [1999]. But why would the heuristic result in shorter calculation times?

As illustrated by the above example, the heuristic calculates the optimal remove-down-to levels for a specific period and for a specific inventory at the start of that period, $U_k(I_k)$, first. All these optimal remove-down-to levels are later transformed to one remove-down-to level per period. However, to calculate all $U_k(I_k)$'s, the heuristic does not need any information about the periods before the period for which $U_k(I_k)$ is calculated. E.g. if the inventory at the start of period 2 is equal to 4 units, the heuristic uses this information to calculate the optimal remove-down-to level. It does not matter how the inventory at the start of period 2 became 4 units. Using the process described above this leads to one remove-down-to level period. Contradictory to the heuristic, the exact method considers a vector of remove-down-to levels for all periods at once, instead of considering each remove-down-to level per period independently. This means that all vectors with possible combinations of the final order quantity and the remove-down-to level have to be considered. The number of vectors can increase rapidly, especially when the number of periods and/or the maximum demand per period increases. The heuristic results in only four vectors, where all the remove-down-to levels were calculated independently first and transformed to one remove-down-to level per period later. This process is very likely to be faster than the exact method. This is tested in chapter 8.

4.7.3.2 Equations used in the Heuristic when $t = EOL, t \geq 2$

In the previous section the idea behind the heuristic for the remove policy is given. In this section, the equations that should be used in this heuristic when $t = EOL$ and $t \geq 2$ are given. The notation that is used in the equations can be found in Appendix F. To calculate the total expected discounted costs for a certain final order quantity Q, the following is done:

For the last period $k = t(= EOL)$:

$$E[TC_t(I_t, 0)] = a\left(\widehat{CH1_{t-1}} * I_t + \sum_{j=0}^{I_t} \widehat{CD1_t} * (I_t - j) * P(d_t = j) + \sum_{j=I_t+1}^{\infty} 0 * P(d_t = j)\right)$$

For the periods $k = t-1, \dots, 2$:

$$\begin{aligned} E\big[TC_k\big(I_k, U_k(I_k)\big)\big] = a\Bigg(& \widehat{CH1_{k-1}} * I_k \\ &+ \sum_{j=0}^{I_k - U_k(I_k)} \left(\widehat{CD1_k} * \big(I_k - j - U_k(I_k)\big) + E\left[TC_{k+1}\left(U_k(I_k), U_{k+1}\big(U_k(I_k)\big)\right)\right]\right) * P(d_k = j) \\ &+ \sum_{j=I_k - U_k(I_k)+1}^{I_k} E\big[TC_{k+1}\big(I_k - j, U_{k+1}(I_k - j)\big)\big] * P(d_k = j) \\ &+ \sum_{j=I_k+1}^{\infty} E\big[TC_{k+1}\big(0, U_{k+1}(0)\big)\big] * P(d_k = j)\Bigg) \end{aligned}$$

For the first period $k = 1$:

$$\begin{aligned} & E\big[TC_1\big(Q, U_1(Q)\big)\big] \\ &= CO1 + CN1 * Q + \sum_{j=0}^{Q - U_1(Q)} \left(\widehat{CD1_1} * \big(Q - j - U_1(Q)\big) + E\left[TC_2\left(U_1(Q), U_2\big(U_1(Q)\big)\right)\right]\right) * P(d_1 = j) \\ &+ \sum_{j=Q-U_1(Q)+1}^{Q} E\big[TC_2\big(Q - j, U_2(Q - j)\big)\big] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E\big[TC_2\big(0, U_2(0)\big)\big] * P(d_1 = j) \end{aligned}$$

The total expected discounted costs for a given final order quantity Q and remove-down-to levels for all possible inventories at the beginning of all periods during the entire RTT are:

$$E\left[TC_1\big(Q, U_1(Q)\big)\right]$$

[28]

Here all parameters are defined as done previously (see also Appendix F), besides:

$E\left[TC_k\big(I_k, U_k(I_k)\big)\right]$	Total expected discounted costs during the periods $k, \dots, t$, where $k = 1, \dots, t$ *(€)*. These costs are calculated for a specific value for I_k and $U_k(I_k)$.
I_k	The inventory of item 1 directly at the beginning of period k *(units)*. Here, $I_1 = Q$ and $I_k = 0, \dots, maxdemand(k)$. $maxdemand(k)$ is the total maximum demand during the periods $k, k+1, \dots, EOL$. The inventory in a period can never be larger than this total maximum remaining demand because having more items in stock only costs money. If $I_k > maxdemand(k)$ all the excess items cost extra money (i.e. those items were once bought, holding costs are incurred, disposal costs are incurred), while it is known that these items will never be needed to satisfy demand. Therefore, it is impossible to have an inventory at the beginning of a period that is larger than the total maximum demand during the remainder of the EOL.
$U_k(I_k)$	The remove-down-to level effective at the end of period k *(units)*, where the remove-down-to level is dependent on I_k. $U_k(I_k)$ is used to calculate U_k (the remove-down-to level of period k that is independent of I_k).

To calculate the optimal remove-down-to levels, the fillrate should be taken into account. The fillrate of a certain period for a given I_k and the remove-down-to level $U_k(I_k)$ should be calculated. If this fillrate is below the desired fillrate, then either the inventory directly at the beginning of the period was too low, or the remove-down-to level $U_k(I_k)$ is too low. To be able to calculate the fillrate, the expected number of out-of-stocks is calculated first:

For the last period $k = t$:

$$E[OOS_t(I_t, 0)] = \sum_{j=0}^{I_t} 0 * P(d_t = j) + \sum_{j=I_t+1}^{\infty} (j - I_t) * P(d_t = j)$$

Note again that the remove-down-to level in the last period is equal to 0.

For the periods $k = t-1, \dots, 2$:

$$E\left[OOS_k\big(I_k, U_k(I_k)\big)\right] = \sum_{j=0}^{I_k - U_k(I_k)} E\left[OOS_{k+1}\Big(U_k(I_k), U_{k+1}\big(U_k(I_k)\big)\Big)\right] * P(d_k = j)$$
$$+ \sum_{j=I_k - U_k(I_k)+1}^{I_k} E\left[OOS_{k+1}\big(I_k - j, U_{k+1}(I_k - j)\big)\right] * P(d_k = j)$$
$$+ \sum_{j=I_k+1}^{\infty} \Big((j - I_k) + E\left[OOS_{k+1}\big(0, U_{k+1}(0)\big)\right]\Big) * P(d_k = j)$$

For the first period $k = 1$:

$$E\left[OOS_1\big(Q, U_1(Q)\big)\right] = \sum_{j=0}^{Q - U_1(Q)} E\left[OOS_2\big(U_1(Q), U_2(U_1(Q))\big)\right] * P(d_1 = j)$$
$$+ \sum_{j=Q - U_1(Q)+1}^{Q} E\left[OOS_2\big(Q - j, U_2(Q - j)\big)\right] * P(d_1 = j)$$
$$+ \sum_{j=Q+1}^{\infty} \Big((j - Q) + E\left[OOS_2\big(0, U_2(0)\big)\right]\Big) * P(d_1 = j)$$

The total number of expected out-of-stocks during the RTT are:

$$E'[OOS(Q, RTT = t)] = E\left[OOS_1\left(Q, U_1(Q)\right)\right]$$

[29]

For all periods, the fillrate is based on the expected number of out-of-stocks that occur in the periods k up to and until the last period t. Using equation [29], the calculation of the fillrate in period k is as follows:

$$fillrate_k(I_k, U_k(I_k), t) = 1 - \frac{E\left[OOS_k\left(I_k, U_k(I_k)\right)\right]}{\sum_{i=k}^{t} \sum_{j=0}^{\infty} j * P(d_i = j)}$$

[30]

This means that $fillrate_1(Q, U_1(Q), t)$ denotes the fillrate during the entire RTT.

To calculate the total expected discounted costs as shown in equation [25], the optimal remove-down-to level in period 1 given a certain starting inventory Q in that period should be calculated. Equation [25] can be used for any set of remove-down-to levels. However, we want to obtain the minimal total expected discounted costs, which means we have to search for the optimal remove-down-to levels. To find the optimal remove-down-to level for all periods during the RTT, the following is done:

For the last period $k = t$:
Here, $U_t(I_t) = 0$ for all $I_t \in \{0, \dots, maxdemand(t)\}$ since this last period is also the last period of the EOL. Here, $maxdemand(t)$ is the maximum demand during the last period t.

The following algorithm is executed for *all* periods and for *all* possible values for the inventory at the beginning of a period, where we start with period $t - 1$ (after all the optimal remove-down-to levels of the last period are already known).

Algorithm to find the optimal remove-down-to level in a certain period given a certain inventory directly at the beginning of that period

Step 1) Set $U_k(I_k) = \min(I_k, maxdemand(k + 1))$

Step 2) Calculate the total expected discounted costs for period k, which are denoted by $E\left[TC_k\left(I_k, U_k(I_k)\right)\right]$ using equation [25] and the expected number of out-of-stocks and the fillrate using equation [26] and [27].

Step 3) If $fillrate_k(I_k, U_k(I_k), t) \geq desiredfillrate$ and $U_k(I_k) > U_{k+1}(I_k)$ then go to step 4), else go to step 7).

Step 4) Set $U_k(I_k) = U_k(I_k) - 1$

Step 5) Calculate the total expected discounted costs for period k using equation [25] and the expected number of out-of-stocks and the fillrate using equation [26] and [27].

Step 6) If $fillrate_k(I_k, U_k(I_k), t) \geq desiredfillrate$ and $U_k(I_k) > U_{k+1}(I_k)$ and $E\left[TC_k\left(I_k, U_k(I_k)\right)\right] \leq E[TC_k(I_k, U_k(I_k) + 1)]$ then go to step 4), else go to step 7).

Step 7) The optimal remove-down-to level for period k given a starting inventory I_k is $U_k(I_k) + 1$.

For more details about this algorithm see section 4.7.3.1.

Using these equations and the algorithm described above, one obtains the optimal remove-down-to levels for all periods and for each period the optimal remove-down-to level is calculated for each possible inventory level at the beginning of that period. Using these results, the following is done to obtain remove-down-to levels per period (which are not dependent on a specific value for the inventory directly at the beginning of the period anymore!):

Unlike in the cost calculation, we start in period 1 instead of in the last period of the RTT. This is done because the value for the remove-down-to level in period 1 is known: you know for sure that the inventory directly at the beginning of period 1 is equal to the final order quantity Q.

This means that the following holds for the first period ($k = 1$):

$$U_1 = U_1(Q)$$

Depending on the demand distribution of period 1, the starting inventory of period 2 can be any value between 0 and U_1. Each possible starting inventory has an accompanying remove-down-to level. For a low number of periods and a low maximum demand per period it is possible to exactly calculate the probability that the starting inventory in period k equals a specific value, which results in a specific remove-down-to level. However, one can imagine that a problem occurs when the number of periods increases and the maximum demand per period increases: The possible ‘paths’ to a specific starting inventory increase and calculating the probability that you will be on inventory I_k at the start of period k is very difficult due to the different demand probabilities per period and due to the remove-down-to levels of each period. Therefore, it is chosen to calculate an ‘expected’ starting inventory for each period. For period 2, the expected value for the starting inventory of period 2 can be calculated as follows:

$$E[I_2] = \sum_{j=0}^{Q-U_1} U_1 * P(d_1 = j) + \sum_{j=Q-U_1+1}^{Q} (Q-j) * P(d_1 = j) + \sum_{j=Q+1}^{\infty} 0 * P(d_1 = j)$$

Now it is assumed that the starting inventory of period 2 actually equals the expected value $E[I_2]$. Two values for $E[I_2]$ are considered: one is $E[I_2]$ rounded down to the closest integer value, the other is $E[I_2]$ rounded up to the closest integer value. By assuming that the starting inventory resembles $E[I_2]$ (rounded up or down), it is known what the ‘expected’ remove-down-to level of period 2 will be: $U_2 = U_2(E[I_2])$.

This line of reasoning is followed for all periods $k = 3, \ldots, t$:

$$E[I_k] = \sum_{j=0}^{E[I_{k-1}]-U_{k-1}} U_{k-1} * P(d_{k-1} = j) + \sum_{j=E[I_{k-1}]-U_{k-1}+1}^{E[I_{k-1}]} (E[I_{k-1}] - j) * P(d_{k-1} = j) + \sum_{j=E[I_{k-1}]+1}^{\infty} 0 * P(d_{k-1} = j)$$

The resulting ‘expected’ remove-down-to level is:

$$U_k = U_k(E[I_k])$$

[31]

After the ‘expected’ remove-down-to levels for all periods during the RTT have been calculated, these values form a ‘vector’ and the costs of this vector are calculated as follows:

For the last period $k = t$:

$$E[TC_t(I_t, 0)] = a\left(\widehat{CH1}_{t-1} * I_t + \sum_{j=0}^{I_t} \widehat{CD1}_t * (I_t - j) * P(d_t = j) + \sum_{j=I_t+1}^{\infty} 0 * P(d_t = j)\right)$$

For the first period $k = t-1, \ldots, 2$:

$$E[TC_k(I_k, U_k)] = a\left(\widehat{CH1}_{k-1} * I_k + \sum_{j=0}^{I_k-U_k} \left(\widehat{CD1}_k * (I_k - j - U_k) + E[TC_{k+1}(U_k, U_{k+1})]\right) * P(d_k = j) + \sum_{j=I_k-U_k+1}^{I_k} E[TC_{k+1}(I_k - j, U_{k+1})] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} E[TC_{k+1}(0, U_{k+1})] * P(d_k = j)\right)$$

For the first period $k = 1$:

$$E[TC_1(Q,U_1)] = CO1 + CN1 * Q + \sum_{j=0}^{Q-U_1} \left(\widehat{CD1}_1 * (Q - j - U_1) + E[TC_2(U_1,U_2)]\right) * P(d_1 = j)$$
$$+ \sum_{j=Q-U_1}^{Q} E[TC_2(Q-j,U_2)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E[TC_2(0,U_2)] * P(d_1 = j)$$

Thus, the total expected discounted costs during the entire planning horizon are:

$$E[TC(Q,U_1,U_2,\dots,U_t)] = E'[TC(Q,U_1,\dots,U_t,t)] = E[TC_1(Q,U_1)]$$

[32]

4.8 Detailed Model for the Remove Policy where $t > EOL$

As explained in section 3.8, there is no difference between the modeling for the situation $t = EOL$ and $t > EOL$. This is because in both situations the same number of periods is considered, namely EOL periods. Therefore, the appropriate models and algorithms for the remove policy can be found in section 4.7.

4.9 Remove Policy when $t < EOL$

In this section the detailed models for the remove policy is described for the situation in which the required transition time is shorter than the EOL. In this situation, item 2 is needed to satisfy demand after the RTT (see section 3.9). Again, a division is made between the following situations:

1) $t = 0$
2) $t = 1$
3) $t \geq 2$

This division is made because the equations used to calculate the total expected discounted costs are different for these different situations. In the next sections the detailed model for the remove policy will be given for all these situations.

4.9.1 Remove Policy, $t < EOL$, $t = 0$

As in the case of the simple policy, when the RTT is equal to 0 the total expected discounted costs consist of purchasing costs for the final order quantity and expected costs for the remainder of the planning horizon. The equations that are used to calculate the costs for the time periods after the RTT can be found in section 3.9.1.

4.9.2 Remove Policy, $t < EOL$, $t = 1$

When $t < EOL$ and t is only one period, this means that the final order quantity should cover the expected demand during this 1 period. Here, the objective of the model is to find the final order quantity that minimizes the total expected discounted costs during the entire planning horizon, but also satisfies the fillrate restriction. Since the RTT is equal to only one period, the calculation of the total expected costs for the remove policy is exactly the same as the calculation for the simple policy given in section 3.9.2. Because there is only 1 period to consider during t, there are no intermediate remove-down-to levels. This means that the remove policy and the simple policy coincide.

4.9.3 Remove Policy, $t < EOL$, $t \geq 2$

When $t < EOL$ and t is at least equal to two periods, this means that the final order quantity should cover the expected demand during the entire RTT. Also, intermediate remove-down-to levels should be calculated. To calculate the total expected discounted costs during the entire planning horizon, both the costs for the final order quantity and the expected costs for the remainder of the planning horizon should be taken into account.

To find the optimal remove-down-to levels, the heuristic as described in section 4.7.3.1 is used. The equations that should be used in this heuristic are the following:

For the periods $k = EOL, \dots, t+1$: See equation [16] in section 3.9.2

For the last period in the RTT $k = t$:

$$E[TC_t(I_t, 0)] = a\left(\widehat{CH1_{t-1}} * I_t + \sum_{j=0}^{I_t}\left(\widehat{CD1_t} * (I_t - j) + E[TC_{t+1}(0)]\right) * P(d_t = j) + \sum_{j=I_t+1}^{\infty} E[TC_{t+1}(0)] * P(d_t = j)\right)$$

For the periods $k = t-1, \dots, 2$:

$$\begin{aligned} E\left[TC_k\left(I_k, U_k(I_k)\right)\right] = a\Bigg(& \widehat{CH1_{k-1}} * I_k \\ &+ \sum_{j=0}^{I_k - U_k(I_k)} \left(\widehat{CD1_k} * \left(I_k - j - U_k(I_k)\right) + E\left[TC_{k+1}\left(U_k(I_k), U_{k+1}\left(U_k(I_k)\right)\right)\right]\right) * P(d_k = j) \\ &+ \sum_{j=I_k - U_k(I_k)+1}^{I_k} E\left[TC_{k+1}\left(I_k - j, U_{k+1}(I_k - j)\right)\right] * P(d_k = j) \\ &+ \sum_{j=I_k+1}^{\infty} E\left[TC_{k+1}\left(0, U_{k+1}(0)\right)\right] * P(d_k = j)\Bigg) \end{aligned}$$

For the first period $k = 1$:

$$\begin{aligned} E\left[TC_1\left(Q, U_1(Q)\right)\right] &= CO1 + CN1 * Q + E[CO2] \\ &+ \sum_{j=0}^{Q - U_1(Q)} \left(\widehat{CD1_1} * \left(Q - j - U_1(Q)\right) + E\left[TC_2\left(U_1(Q), U_2\left(U_1(Q)\right)\right)\right]\right) * P(d_1 = j) \\ &+ \sum_{j=Q-U_1(Q)+1}^{Q} E\left[TC_2\left(Q - j, U_2(Q - j)\right)\right] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E\left[TC_2\left(0, U_2(0)\right)\right] * P(d_1 = j) \end{aligned}$$

Note that for the last period the remove-down-to level is not dependent on the inventory directly at the beginning of that period; it is always equal to 0 since that is what the model is aiming for. The total expected discounted costs during the entire planning horizon for a given final order quantity Q and remove-down-to levels for all possible inventories at the beginning of all periods during the entire RTT are:

$$E\left[TC_1\left(Q, U_1(Q)\right)\right]$$

[33]

To find the optimal remove-down-to levels for each period given an inventory level directly at the beginning of that period that is equal to I_k, the algorithm described in section 4.7.3.1 can be used. To be able to use this algorithm, the expected number of out-of-stocks and the fillrate should be calculated as follows:

For the periods $k = t$:

$$E[OOS_t(I_t, 0)] = \sum_{j=0}^{I_t} 0 * P(d_t = j) + \sum_{j=I_t+1}^{\infty} (j - I_t) * P(d_t = j)$$

For the periods $k = t - 1, \dots, 2$:

$$\begin{aligned} & E\big[OOS_k\big(I_k, U_k(I_k)\big)\big] \\ & = \sum_{j=0}^{I_k - U_k(I_k)} E\Big[OOS_{k+1}\Big(U_k(I_k), U_{k+1}\big(U_k(I_k)\big)\Big)\Big] * P(d_k = j) \\ & + \sum_{j=I_k-U_k(I_k)+1}^{I_k} E\big[OOS_{k+1}\big(I_k - j, U_{k+1}(I_k - j)\big)\big] * P(d_k = j) \\ & + \sum_{j=I_k+1}^{\infty} \Big((j - I_k) + E\big[OOS_{k+1}\big(0, U_{k+1}(0)\big)\big]\Big) * P(d_k = j) \end{aligned}$$

For the first period $k = 1$:

$$\begin{aligned} & E\big[OOS_1\big(Q, U_1(Q)\big)\big] \\ & = \sum_{j=0}^{Q-U_1(Q)} E\big[OOS_2\big(U_1(Q), U_2(U_1(Q))\big)\big] * P(d_1 = j) \\ & + \sum_{j=Q-U_1(Q)+1}^{Q} E\big[OOS_2\big(Q - j, U_2(Q - j)\big)\big] * P(d_1 = j) \\ & + \sum_{j=Q+1}^{\infty} \Big((j - Q) + E\big[OOS_2\big(0, U_2(0)\big)\big]\Big) * P(d_1 = j) \end{aligned}$$

The total number of expected out-of-stocks during the RTT are:

$$E'[OOS(Q, RTT = t)] = E\big[OOS_1\big(Q, U_1(Q)\big)\big]$$

[34]

For all periods, the fillrate is based on the expected number of out-of-stocks that occur in the periods k up to and until the last period t. Using equation [34], the calculation of the fillrate in period k is as follows:

$$fillrate_k(I_k, U_k(I_k), t) = 1 - \frac{E'\big[OOS_k\big(I_k, U_k(I_k)\big)\big]}{\sum_{i=k}^{t} \sum_{j=0}^{\infty} j * P(d_i = j)}$$

[35]

This means that $fillrate_1(Q, U_1(Q), t)$ denotes the fillrate during the entire RTT. Using these equations and the algorithm described in section 4.7.3.1, one obtains the optimal remove-down-to levels for all periods and for each period the optimal remove-down-to level is calculated for each possible inventory level at the beginning of that period. Using these results, ‘expected’ remove-down-to levels per period (which are not dependent on a specific value for the inventory directly at the beginning of the period anymore!) are calculated as shown in equation [31] in section 4.7.3.2. After the ‘expected’ remove-down-to levels for all periods during the RTT have been calculated, these values form a ‘vector’ and the costs of this vector are calculated as follows:

For the periods $k = EOL, \dots, t + 1$: See equation [16] in section 3.9.2

For the last period in the RTT $k = t$:

$$E[TC_t(I_t,0)] = a\left(\widehat{CH1}_{t-1} * I_t + \sum_{j=0}^{I_t}\left(\widehat{CD1}_t * (I_t - j) + E[TC_{t+1}(0)]\right) * P(d_t = j) \right.$$

$$\left. + \sum_{j=I_t+1}^{\infty} + E[TC_{t+1}(0)] * P(d_t = j)\right)$$

For the first period $k = t - 1, \ldots, 2$:

$$E[TC_k(I_k,U_k)] = a\left(\widehat{CH1}_{k-1} * I_k + \sum_{j=0}^{I_k-U_k}\left(\widehat{CD1}_k * (I_k - j - U_k) + E[TC_{k+1}(U_k,U_{k+1})]\right) * P(d_k = j) \right.$$

$$\left. + \sum_{j=I_k-U_k+1}^{I_k} E[TC_{k+1}(I_k - j, U_{k+1})] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} E[TC_{k+1}(0,U_{k+1})] * P(d_k = j)\right)$$

For the first period $k = 1$:

$$E[TC_1(Q,U_1)] = CO1 + CN1 * Q + E[CO2]$$

$$+ \sum_{j=0}^{Q-U_1}\left(\widehat{CD1}_1 * (Q - j - U_1) + E[TC_2(U_1,U_2)]\right) * P(d_1 = j) + \sum_{j=Q-U_1}^{Q} E[TC_2(Q-j,U_2)] * P(d_1 = j)$$

$$+ \sum_{j=Q+1}^{\infty} E[TC_2(0,U_2)] * P(d_1 = j)$$

Thus, the total expected costs during the entire planning horizon are:

$$E[TC(Q,U_1,U_2,\ldots,U_t)] = E'[TC(Q,U_1,\ldots,U_t,t)] = E[TC_1(Q,U_1)] \qquad \textbf{[36]}$$

4.10 Calculating the Optimal Final Order Quantity and Remove-Down-To Levels for the Remove Policy

See section 4.7.3.1 for the logic behind the heuristic and what steps are followed to come to the best combination of a final order quantity and remove-down-to levels for all periods during the RTT.

5 Changing to the Alternative Item before the End of the RTT

In this chapter a new way of looking to the final order problem discussed so far is addressed. In short, the situation in which there is a maximum required transition time after which the company must use the alternative item is addressed, but now it is the question whether the company should switch to the alternative item earlier than the end of the maximum RTT. This is discussed in more detail in section 5.1. In section 5.2 the mathematical model for this situation is given. This model is not used to generate results, but those interested in numerical results can use the equations in section 5.2 to obtain these results.

5.1 Shortening the Required Transition Time

Up to now, there was a required transition time after which the company could use the alternative item to satisfy demand. There were two possible situations: a situation where the original item cannot be used after the RTT (which means all units of the original item that are left at the end of the RTT are disposed) and a situation where the original item can be used after the RTT. In the latter situation it is possible that the optimal final order quantity is larger than the final order quantity that is needed to meet all restrictions.

In this chapter we look to this transition time in a different way: Let's say there is a 'maximum required transition time' after which the company must use the alternative item. However, the company does not necessarily need this entire 'maximum required transition time' to be able to use the alternative item. It may be possible that the company works harder to shorten the transition time, which would mean that the alternative item can be used earlier as well. In this situation the transition time that is realized by the company is shorter than the maximum transition time that is allowed, and the units of the original item that are left after the realized transition time can still be used after the end of the realized transition time. However, at the end of the maximum transition time, these units of the original item should be disposed. This can be depicted as follows:

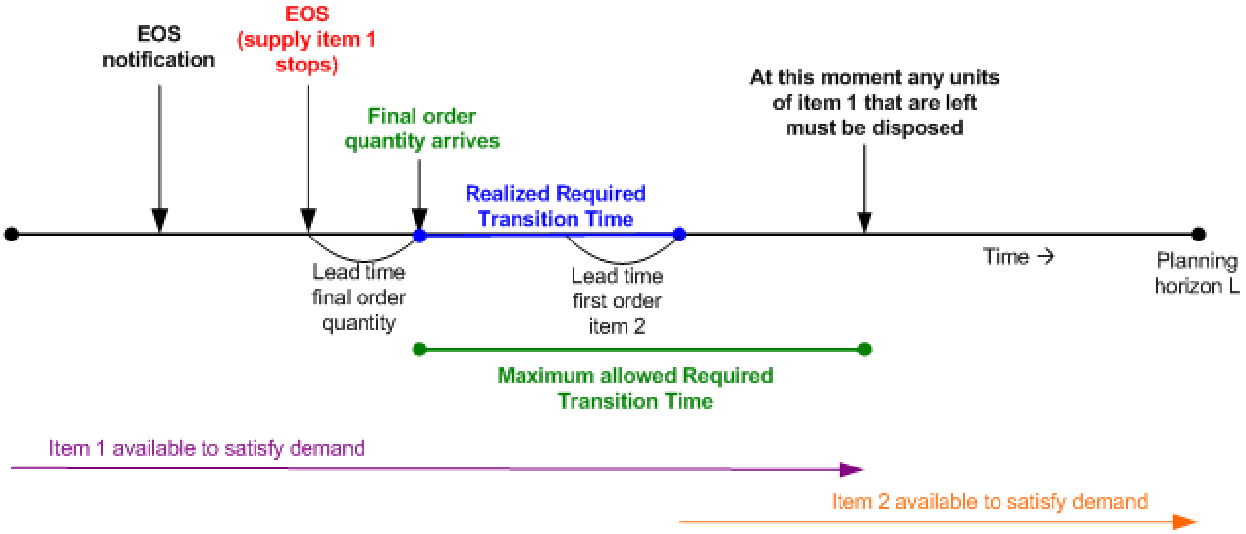


Figure 5.1: Maximum allowed required transition time and the realized required transition time.

The models up to now implicitly assumed that the entire 'maximum' required transition time was needed to switch to the alternative item. In this chapter the situation is considered where the company may shorten this required transition time. E.g. by using more resources, the expected setup costs for the alternative item may increase, but the required transition time may be shorter. This can be very attractive, especially when the alternative item is much cheaper than the original item.

The objective of the model in this chapter is to find the length of the RTT and the final order quantity for the original item that minimizes the total expected discounted costs during the entire planning horizon. To do this, the following logic is used:

The maximum required transition time is a given number of periods, i.e. it is specified by law when the company must use the alternative item. The real required transition time for the company can be any value between the number of periods during the minimum required transition time ($minimumRTT$) and the number of periods during the maximum required transition time ($maximumRTT$). Note that the minimum RTT can be 0 periods as well. In case the realized RTT is shorter than $maximumRTT$, the units of the original item that are left at the end of the realized RTT do not have to be disposed. In case the realized RTT is equal to $maximumRTT$, the units of the original item that are left at the end of the realized RTT have to be disposed. For all possible values of the realized RTT the final order quantity and the total expected discounted costs during the entire planning horizon can be calculated. The value for the realized RTT that minimizes these total expected discounted costs is the optimal value for the realized RTT. This optimal RTT works as a due date to the company: it is cost-optimal to be finished with changing to the alternative item at the end of the optimal RTT.

The equations that are used to calculate the total expected discounted costs during the entire planning horizon are given in the next section.

5.2 Mathematical Model for the Maximum Required Transition Time

Regarding the mathematical model for the situation described in section 5.1, only the models are given for the situation in which $maximumRTT < EOL$. When the maximum RTT is equal to or larger than the EOL, the models in chapter 3 and chapter 4 can also be used. However, there is a difference between the model in chapter 3 and the one described here when the maximum RTT is equal to or larger than the EOL; the model described in this section is only used for a discrete number of periods in the RTT that are covered by the final order quantity (e.g. the RTT is 1 year, 2 years, 3 years, etc), while the models in chapter 3 can also result in a final order quantity that covers demand during for example 1,5 years. Therefore, it is better to use the model in chapter 3 when the maximum RTT is equal to or larger than the EOL. Because of this it is decided to only give the equations for the situation where the maximum RTT is smaller than the EOL in the remainder of this section.

In the situation where $maximumRTT < EOL$, the alternative item (item 2) must be used to satisfy demand during the remainder of the planning horizon. In the remainder of this section the detailed models for the simple policy is described for the situation in which the maximum required transition time is shorter than the EOL. A division is made between the following situations:

1) $maximumRTT = 0$
2) $maximumRTT = 1$
3) $maximumRTT \geq 2$

This division is made because the equations used to calculate the total expected discounted costs are different for these different situations. In the next sections the equations that are used to calculate the total expected discounted costs during the entire planning horizon are given. The equations used to calculate the fillrate are not different than the ones in chapter 3, so we refer to chapter 3 for those equations.

5.2.1 $maximumRTT < EOL, maximumRTT = 0$

When the maximum RTT is equal to 0 periods, this means that all demand during the entire planning horizon has to be satisfied by the alternative item. Also, the RTT that is actually realized by the company can only be equal to 0, since the maximum RTT that is allowed is 0 as well. Buying units of the original item is useless, since these cannot be used after the RTT. Therefore, the equations as given in section 3.9.1 can be used for this situation.

5.2.2 $maximumRTT < EOL, maximumRTT = 1$

When the maximum RTT is equal to 1 period, the RTT that the company realizes can be either equal to 0 periods (if the minimum RTT is equal to 0 periods) or it can be equal to 1 period.

In case the RTT is equal to 0 periods a final order quantity for the original item can still be placed, since it is allowed to use the original item for 1 period (after all, the maximum RTT is 1). The equations for the total expected discounted costs during the entire planning horizon are:

For the last period $k = EOL$:

$$E[TC_{EOL}(I_{EOL})] = a\left(\widehat{CH1_{EOL-1}} * I_{EOL} + \sum_{j=0}^{I_{EOL}} \widehat{CD1_{EOL}} * (I_{EOL} - j) * P(d_{EOL} = j) + \sum_{j=I_{EOL}+1}^{\infty} E[CN2_{EOL}] * (j - I_{EOL}) * P(d_{EOL} = j)\right)$$

For the periods $k = EOL - 1, \dots, t + 2$:

$$E[TC_k(I_k)] = a\left(\widehat{CH1_{k-1}} * I_k + \sum_{j=0}^{I_k} E[TC_{k+1}(I_k - j)] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} (E[CN2_k] * (j - I_k) + E[TC_{k+1}(0)]) * P(d_k = j)\right)$$

For the first period in the remainder of the planning horizon, $k = t + 1 = 1$ ($t = 0$):

$$E[TC_1(Q)] = \sum_{j=0}^{Q} \left(\widehat{CD1_1} * (Q - j) + E[TC_2(0)]\right) * P(d_1 = j) + \sum_{j=Q+1}^{\infty} (E[CN2_1] * (j - Q) + E[TC_2(0)]) * P(d_1 = j)$$

The total expected discounted costs during the entire planning horizon are:

$$E'[TC(Q, t = 0)] = CO1 + CN1 * Q + E[CO2] + E[TC_1(Q)]$$

[37]

The total expected discounted costs during the entire planning horizon when the RTT that is realized by the company is equal to 1 period, can be calculated by the equations given in section 3.9.2.

5.2.3 $maximumRTT < EOL, maximumRTT \geq 2$

The RTT that the company realizes can be any value between 0 (if the minimum RTT is equal to 0 periods) and the maximum RTT. When the realized RTT is shorter than the maximum RTT, the units of the original item that are left at the end of the RTT do not have to be disposed. At the end of the maximum RTT, these units have to be disposed since the alternative item must be used hereafter to satisfy demand during the remainder of the planning horizon. The equations for the total expected discounted costs during the entire planning horizon are:

For the periods $k = EOL, \dots, t + 1$: See equation [16]

For the last period in the RTT $k = t$:

If $t < maximumRTT$ then:

$$E[TC_t(I_t)] = a\left(\widehat{CH1_{t-1}} * I_t + \sum_{j=0}^{I_t} E[TC_{t+1}(I_t - j)] * P(d_t = j) + \sum_{j=I_t+1}^{\infty} E[TC_{t+1}(0)] * P(d_t = j)\right)$$

If $t = maximumRTT$ then:

$$E[TC_t(I_t)] = a\left(\widehat{CH1_{t-1}} * I_t + \sum_{j=0}^{I_t}\left(\widehat{CD1_t} * (I_t - j) + E[TC_{t+1}(0)]\right) * P(d_t = j) + \sum_{j=I_t+1}^{\infty} E[TC_{t+1}(0)] * P(d_t = j)\right)$$

For the periods $k = t - 1, \dots, 2$:

$$E[TC_k(I_k)] = a\left(\widehat{CH1_{k-1}} * I_k + \sum_{j=0}^{I_k} E[TC_{k+1}(I_k - j)] * P(d_k = j) + \sum_{j=I_k+1}^{\infty} E[TC_{k+1}(0)] * P(d_k = j)\right)$$

For the first period $k = 1$:

$$E[TC_1(Q)] = CO1 + CN1 * Q + E[CO2] + \sum_{j=0}^{Q} E[TC_2(Q - j)] * P(d_1 = j) + \sum_{j=Q+1}^{\infty} E[TC_2(0)] * P(d_1 = j)$$

Thus, the total expected discounted costs during the entire planning horizon are:

$$E'[TC(Q, RTT = t)] = E[TC_1(Q)]$$

[38]

6 Research and Model Evaluation

In this chapter the quality of the research conducted during this project is evaluated. The most important research-oriented quality criteria are [Swanborn, 1996 and Yin, 1994 in Van Aken, et al., 2006, p.125]:

1) Controllability
2) Reliability
3) Validity

Each of these criteria is discussed in this chapter. Next to that the model verification will be addressed, since this is required to establish credibility [Law, 2007].

6.1 Controllability

Controllability is a prerequisite for the evaluation of validity and reliability. In order to make research results controllable, researchers should reveal how they executed a study. A detailed description enables others to replicate it and to check the outcomes. In this project a case study and a scenario analysis is executed. For both the case study and the scenario analysis the input parameters are explicitly given. Next to that, the models that are used to calculate the outcomes are given in chapter 3 and chapter 4. By using this model and the given input parameters anyone could replicate the outcomes. Therefore it can be concluded that the research conducted during this project is controllable.

6.2 Reliability

"The results of a study are reliable when they are independent of the particular characteristics of that study and can therefore be replicated in other studies." [Yin, 1994, Swanborn, 1996 in van Aken, et al., 2006, p. 127]. Research results should not depend on the researches who conducted the research, the respondents and measuring instrument that were used during the study, and the specific situation in which the research was conducted. A common strategy to determine the reliability of a measurement is to repeat it. Although this graduation project does not involve respondents, the model that is developed and implemented in an Excel tool should be reliable. After all, running the tool at different moments in time using the same input parameters should result in the same results. During the project the tool is tested extensively. E.g. while trying to find out the limitations of the tool (regarding calculation time and memory use), the results that were obtained were compared with previous results. When comparing runs of the Excel tool that were executed at different moments, but using the same set of input parameter values, the results that were obtained are exactly the same. Next to that, for the situation in which the remove-down-to levels are calculated, the heuristic developed during this study (see chapter 4) is compared to the exact method that is used in Teunter & Fortuin [1999], see chapter 8. Although the heuristic is suboptimal when comparing it to the exact method, it sometimes results in exactly the same results (e.g. when the fillrate restriction is set to 100% both methods must result in the same outcome, and this actually happened). This means that the way the heuristic is modeled is correct, otherwise it was never possible to have the same results. Based on the above discussion, it can be said that the model and the tool developed during this graduation project are reliable.

Note that only reliability of the model and the Excel tool has been tested in this study. From Van Aken et al. [2006] it is known that reliability involves different kinds of reliability:

- *Researchers and reliability*: To test the reliability of the research regarding the researcher, the study should be repeated by someone else. In this case, this would mean that someone else would have to implement the model in Excel to check whether the results are the same. This is a very time-consuming task and therefore this is not done during the project.
- *Instruments and reliability*: To test the reliability of the research regarding the instruments that are used, the model developed during this project should be implemented in some other tool than Excel. Since this is very time-consuming and Excel is the only tool that is available to the company, this is not done during the project.
- *Respondents and reliability*: This project did not concern any respondents that were needed to generate results, so this kind of reliability is irrelevant.

6.3 Validity

“A research result is valid when it is justified by the way it is generated. The way it is generated should provide good reasons to believe that the research result is true or adequate.” [Audi, 1998 in Van Aken et al., 2006, p.130]. When a model is valid, it is an accurate representation of the actual system being studied (for the particular objectives of the study) [Law, 2007]. There are different types of validity [Swanborn, 1996, Yin, 1994 in Van Aken et al., 2006]:

1) Construct validity
2) Internal validity
3) External validity

All these kinds of validity are discussed below.

6.3.1 Construct Validity

Construct validity is the extent to which an instrument measures what it is intended to measure. Construct validity refers to the quality of the operationalization of a concept [De Groot, 1969 in Van Aken et al., 2006]. Construct validity includes the following two issues:

1) The concept should be covered completely.
2) The measurement should have no components that do not fit the meaning of the concept.

Construct validity can be improved by repairing flaws that were detected. The concept that is ‘measured’ in this project is the planning horizon and its associated costs. Maybe a better name for this kind of validity in this project is ‘model validity’ [Law, 2007], since this study concerns developing a simulation model instead of constructs that are used to measure attributes of respondents.

The model developed in this study concerns all periods during the planning horizon and all cost parameters that were thought to be relevant are included. Although the cost model for the periods after the required transition time could be more detailed than the way it is included now, the purpose of the cost model for the periods after the required transition time is to be a lower bound to the expected costs. Including more details does not enhance the decision making process, while enhancing decision making is the purpose of the cost model that covers the entire planning horizon. Regarding the cost model for the periods during the required transition time more details are included. All included parameters are justified by literature or by discussions with the company’s supervisors. For example, including penalty costs is justified in literature but it did not fit the company’s situation. Therefore, it was decided to include a fillrate restriction. During the development of the model the model and the included parameters were discussed repeatedly. Some parameters were decided to be excluded (e.g. penalty costs, a restriction to the end inventory after the required transition time) because these parameters did not fit the situation. If a parameter was missing (e.g. the fillrate), this was included in the model.

Another important aspect of model validity is the fact that assumptions are made to be able to model the real system. There is no such thing as absolute model validity, a model is always an approximation of reality. In this study a list of assumptions was used to be able to model the total expected discounted costs (see section 3.1). These assumptions are in line with assumptions in literature and are made in cooperation with the supervisors.

Based on the above discussion, it can be said that the model for the total expected discounted costs is complete and adequate. Therefore it can be concluded that the model is valid.

6.3.2 Internal Validity

The results of a study are internally valid when conclusions about relationships are adequate and complete. In order to establish the internal validity of a proposed relationship, it has to be made sure that there are no plausible competing explanations.

To test the internal validity of the model extreme tests were executed. E.g. when the planning horizon is 0 or the demand during the planning horizon is 0, the resulting final order quantity should also be 0. Next to that, the results that were obtained during the testing of the tool were always checked to be logical. Also, a scenario analysis is conducted for several input parameters to check the effects of changing these input parameters on the results.

6.3.3 External Validity

External validity refers to the generalizability of the results and conclusions. Although the model that is developed during this project should fit the situation at VI in the first place, the idea behind the model is quite general. Recall the idea behind the model is the required transition time (see section 2.1). No matter whether there is an alternative item or not, what the source of an alternative item may be, or what initiated the end-of-supply issue, the idea of the required transition time is applicable. Although the cost parameters that are included in the total cost model may be specific to VI, the largest deviation from existing cost models for the final order quantity in literature is that penalty costs are excluded. Instead, a fillrate restriction is used. However, it is very well possible that this fillrate restriction is more applicable than the inclusion of penalty costs, because penalty costs are hard to estimate. Although the specific model developed in this study fits best to the situation for VI, it can very well be applicable to other situations as well.

6.4 Verification

Next to checking whether the model is an accurate representation of the actual system, it should also be checked whether the assumptions and the model are correctly translated into a computer program. Basically, this means that the computer program should be debugged. This process of debugging the computer program, which is the verification of the model, can be done using several techniques [Law, 2007]:

1) Write and debug the computer program in modules or subprograms.
2) Let more than one person review the computer program.
3) Run the simulation under a variety of settings of the input parameters, and check to see that the output is reasonable.
4) To debug a discrete-event simulation program, a trace can be used. In a trace, the state of the simulated system are displayed just after each event occurs and this state is compared with hand calculations to see if the program is operating as intended.
5) The model should be run, when possible, under simplifying assumptions for which its true characteristics are known or can easily be computed.

In this project the model is programmed in VBA in Excel 2007. Several different models were programmed, i.e. the model with and without remove-down-to levels, the model where the original item can be used after the required transition time, and so on. During the programming, especially technique 1, 3, 4, and 5 were used to debug the program:

1) Each different kind of model (e.g. including remove-down-to levels or not, allowing the original item after the RTT or not, etc.) is programmed in a different module. This means that when an error occurs in a specific situation, it is known exactly in what module this bug exists. Next to that, the basic elements of the model were first programmed rather simplistically in a ‘test program’, simply to check whether the mechanism was working as desired. E.g. first a simple cost calculation and optimization of the final order quantity was programmed. Once this worked, the cost calculations were extended step by step to the ones as described in chapter 3 and chapter 4. If there were still bugs in the code, the VBA ‘debug’ or ‘trace’ option (see 4) was used to find the line of code that was wrong.
2) Only one person was involved in writing the code. The only way that the code is ‘checked’ is by the fact that the heuristic that is developed for the remove-down-to levels (see chapter 4) and the exact model for remove-down-to levels as described by Teunter & Fortuin [1999] were both programmed and compared. Although the heuristic is suboptimal and did not always result in the same outcomes as the exact method, the expected costs that the model calculated were exactly the same in circumstances where they must be the same (e.g. when the fillrate restriction is 100%). Similarly, the model for the final order quantity only was checked by running a calculation with

exactly the same input parameters as a case in the article by Teunter & Fortuin [1999]. To be able to do this, a version of the model which includes penalty costs was used to check whether the mechanism of the program works or not. After all, the models in Teunter & Fortuin [1999] included penalty costs instead of a fillrate restriction, so to be able to check the code the model should include penalty costs as well. Again, the same expected costs as in the article were obtained. Although this does not involve an extra person to check the code, it does confirm that the way the costs are programmed is right.

3) In this project a scenario analysis is conducted to check the effects of changing certain input parameters. The results of running a simulation are always checked to be reasonable. E.g. when trying to find out the limits of the program (regarding calculation time) all outcomes were evaluated. Also, extreme values for input parameters were used to check how the program responds on that. E.g. a planning horizon that is equal to 0 periods should result in a final order quantity of 0.
4) In VBA it is possible to go through each step in the code one-by-one. For short calculations the code was traced and the reasonableness of each change in expected costs, expected number of out-of-stocks, etc. was checked. Also, when it was known that there was some kind of bug in the code of a specific module, the trace button was used to find out in which line of code this mistake actually occurred.
5) Like explained in 3), the program was used to calculate the expected costs for simple examples. These simple examples typically concerned only a few periods and very low demand (e.g. demand ranging between 0 and 4 items per period). Also, some cost parameters are set to 0 in those examples, simply to be able to calculate the costs easily by hand. For these simple examples, the optimal final order quantity and the accompanying costs were calculated by hand. These calculations were compared with the output of the program.

7 Results for the Simple Policy

In this chapter the model that is developed in chapter 3 is used to get insights in how the input parameters affect the outcomes of the model. This is done for two different models: the model where the original item cannot be used after the RTT and the model where the original item can be used after the RTT. For both situations the behaviour of the models is explained. Based on the behaviour of a model, one can see which input parameters affect the outcomes of the model and how these input parameters affect the outcomes. In section 7.1 the behaviour of the model for the situation where the original item cannot be used after the RTT is discussed. In section 7.2 the behaviour of the model for the situation where the original item can be used after the RTT is discussed.

7.1 Results Simple Policy, Original Item cannot be used after RTT

In this section the situation in which the original item cannot be used after the RTT is considered. The behaviour of the model for this situation is described in section 7.1.1. The findings that are based on this behaviour are given in section 7.1.2.

7.1.1 Behaviour of the Model where the Original Item cannot be used after the RTT

When the original item cannot be used after the RTT, e.g. because of regulations, the company must satisfy demand after the RTT with the alternative item. During the RTT, the demand should be satisfied by the original item because the alternative item is not available yet. Since the original item is an end-of-supply issue, the company should place a final order for the original item to be able to cover the demand during the RTT. Here, a fillrate restriction is taken into account which states what fillrate during the RTT should be achieved. The objective of the model is to find the optimal final order quantity that minimizes the total expected discounted costs during the entire planning horizon while meeting all restrictions.

Without looking into the equations of the model that are given in section 3.7 to section 3.9, the above characteristics of the situation that is discussed here indicates how the model behaves:
To be able to meet the fillrate restriction (and if applicable the restriction regarding the batch size that is specified by the supplier and the restriction regarding the minimum order quantity), the model should increase the final order quantity. For each value for final order quantity, the total expected discounted costs during the entire planning horizon and the fillrate during the RTT is calculated. If the final order quantity results in a fillrate during the RTT that meets the fillrate restriction, and all other restrictions are met as well, the model immediately stops increasing the final order quantity. Increasing the final order quantity with one more unit than is needed to meet all restrictions results in extra costs: for this extra unit of the original item purchasing costs, holding costs and disposal costs are incurred. After all, it is known that the extra unit is not needed to meet the restriction, thus it is expected that it needs to be disposed at the end of the RTT because the original item cannot be used after the RTT. Buying an extra unit then is useless: you buy an extra unit of which you know that it is expected that you have to dispose it anyway. This means that buying even only one more unit than is necessary to meet the final order quantity results in extra costs, while the objective of the model is to minimize the costs while all restrictions are met. Thus, the model will stop increasing the final order quantity immediately when the restrictions are met.

7.1.2 Results of the Model where the Original Item cannot be used after the RTT

This behaviour of the model immediately tells us what input parameters affect the outcomes of the model and what their effect is. Remember that the outcomes of the model are the optimal final order quantity, the total expected discounted costs and the fillrate during the RTT (see Figure 3.1). These findings are described below.

7.1.2.1 Effects of the Cost Parameters when the Original Item cannot be used after the RTT

Regarding the cost parameters it holds that none of the cost parameters affect the size of the optimal final order quantity. After all, the final order quantity is increased until all restrictions are met. None of the restrictions are related to cost parameters, so the cost parameters do not affect the optimal final order quantity. This automatically means that the cost parameters do not affect the fillrate during the RTT that is achieved by placing an order with the size of the optimal final order quantity.

The cost parameters do affect the total expected discounted costs during the entire planning horizon. The direction of the effect of a cost parameter on the total costs can be derived from the equations given in section 3.7 to section 3.9. Note that not all cost parameters are discussed below. Only the parameters that were thought to be interesting based on the company's supervisors' opinion are discussed here. The effects of the parameters that are not discussed here can be derived similarly as the ones shown below. Regarding the cost parameters, the following effects can be found:

- *The higher (lower) the expected setup costs are, the higher (lower) the total expected discounted costs during the planning horizon are*: The expected setup costs are simply added to the total costs since they are incurred immediately at the start of the planning horizon.
- Similarly, *the higher (lower) the fixed order costs are, the higher (lower) the total expected discounted costs during the planning horizon are*: The fixed order costs are also incurred immediately at the start of the planning horizon.
- *The higher (lower) the holding cost rate is, the higher (lower) the total expected discounted costs during the planning horizon are*: In the situation where the original item cannot be used after the RTT, holding costs are only incurred during the RTT (due to assumptions there are no holding costs after the RTT). If it costs more to keep a unit on stock, this will logically increase the total costs as well.
- *The higher (lower) the expected price of one unit of the alternative item is, the higher (lower) the total expected discounted costs during the planning horizon are*: All demand after the RTT has to be satisfied by the alternative item. If it costs more to buy a unit of the alternative item, the total costs will be higher as well. Note that this also means that when the price increase per year is increased (decreased), the total expected discounted costs during the planning horizon increase (decrease) as well.

7.1.2.2 Effects of the Fillrate Restriction when the Original Item cannot be used after the RTT

The higher (lower) the fillrate restriction is, the higher (lower) the optimal final order quantity is: Since the model keeps increasing the final order quantity until the fillrate restriction is met, it is logical that when the fillrate restriction increases the optimal final order quantity will have to increase as well.

The higher (lower) the fillrate restriction is, the higher (lower) the total expected discounted costs during the planning horizon are: A higher fillrate restriction leads to a higher optimal final order quantity. A higher final order quantity does not mean that the company can buy less units of the alternative items to satisfy demand after the RTT, since the original item cannot be used after the RTT. The final order quantity is needed to cover the demand during the RTT, so if this final order quantity increases (decreases) the total costs will increase (decrease) as well.

7.1.2.3 Effects of the Characteristics of the RTT when the Original Item cannot be used after the RTT

The longer (shorter) the fixed length of the RTT is, the higher (lower) the optimal final order quantity is: When the fixed length of the RTT is longer, this means that the optimal final order quantity should meet the fillrate restriction during a longer time period. After all, the fillrate restriction requires that the fillrate during the RTT is above a specific level. To meet the fillrate restriction during a longer (shorter) time period requires more (less) units to be ordered in the final order.

The effect of the fixed length of the RTT on the total expected discounted costs during the planning horizon depends on the cost parameters for both items: When the length of the RTT is longer, this means that more units of the original item have to be bought to meet the fillrate restriction during the RTT. On the other hand, there are fewer periods after the RTT up to the end of the planning horizon, so the company has to buy less units of the alternative item to satisfy demand after the RTT. In case the original item is relatively expensive in comparison to the alternative item, a longer (shorter) RTT would result in higher (lower) total costs. However, it is impossible to give a general statement about the exact effect of the length of the RTT on the total costs, since this depends on all the values for the cost parameters. An example of an effect that the length of the RTT can have on the total costs can be found in Figure 7.1. This example is based on a parameter setting as given in Appendix G. The only parameter that is varied is the length of the RTT (it ranges between 0 years and 4 years, where the base case setting is 2 years).

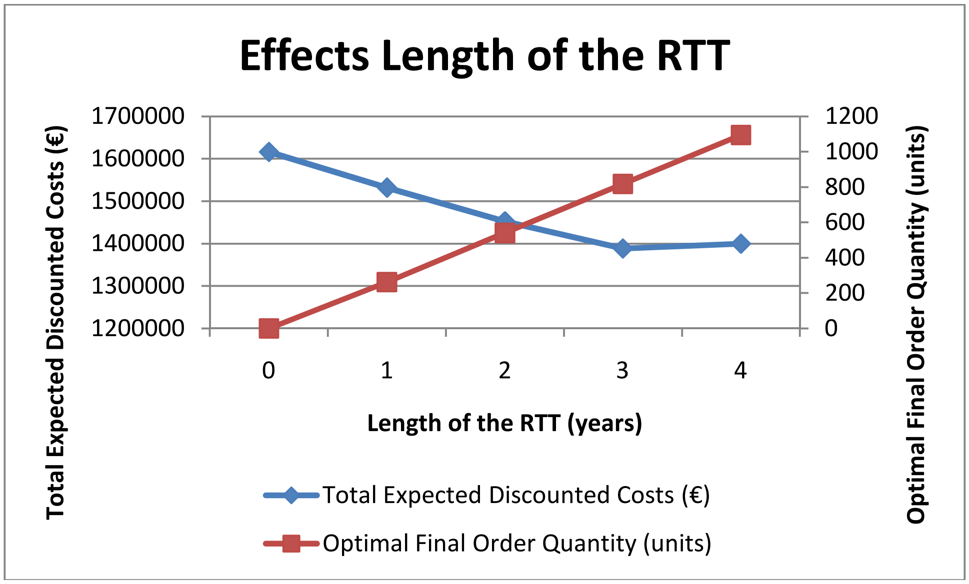


Figure 7.1: Effects of the length of the RTT where all other parameters are set equal to the base case values.

This figure confirms that the longer (shorter) the fixed length of the RTT is, the higher (lower) the optimal final order quantity is. Making such a statement for the total expected discounted costs is not possible, since this is dependent on all the settings of the cost parameters.

The larger (smaller) the spread in the possible values for the length of the RTT is, the higher (lower) the optimal final order quantity is: If there is uncertainty regarding the length of the RTT, the Uniform distribution can be a probability distribution that is used to capture this uncertainty. An advantage of the Uniform distribution is that it can be easier for the company to come up with a range: they can easier come up with a minimum and maximum RTT between which each length of the RTT has the same probability, then they can come up with the expected length of the RTT for e.g. a Poisson distribution (since then probabilities are not spread 'evenly' anymore). If the spread in possible lengths of the RTT is large, then there is a possibility that the RTT is relatively long as well. We know that when the RTT is longer (shorter), the optimal final order quantity will be larger (smaller) as well. If the spread in possible values for the RTT is large, there is a larger chance that the RTT will be long and a higher final order quantity is needed. If the spread is rather small, then it is more certain what the actual length of the RTT will be and there are less possible values for the length of the RTT (thus also less values that are 'extremely' high or low). This will result in a lower optimal final order quantity than when the spread in possible lengths for the RTT is large, since it is known for better for how many periods the fillrate restriction should be met and one does not have to take into account meeting the fillrate for much longer periods of time.

The effect of the spread in the possible values for the length of the RTT on the total expected discounted costs during the planning horizon depends on the cost parameters for both items: Similar to the discussion above for the fixed length of the RTT, the effect of having a larger spread in the possible values for the length of the RTT depends on the cost parameters for both items. If the original item is relatively expensive in comparison to the alternative item, a higher chance on a long length of the RTT (i.e. there is a large spread in the possible values for the length of the RTT) would result in higher costs. However, it is impossible to give a general statement about the exact effects of the probability distribution for the length of the RTT on the total costs, since this depends on all the values for the cost parameters (e.g. the holding cost rate also affects the total costs). An example of an effect that the spread in the possible values for the length of the RTT can have can be found in Figure 7.2. This example is based on the parameter setting as given in Appendix G. The only parameter that is varied is the spread of the probability distribution for the length of the RTT, where the expected of the probability distribution remains equal.

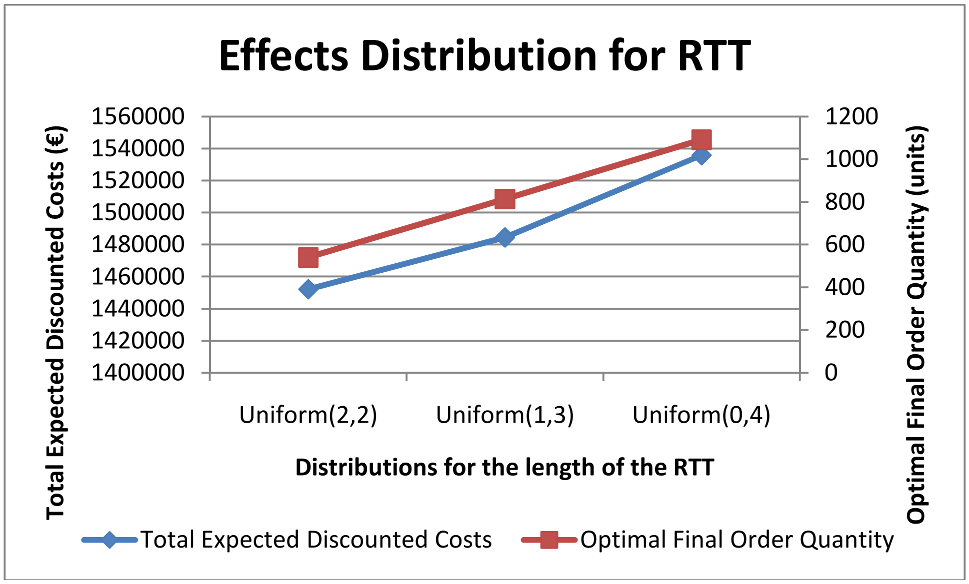


Figure 7.2: Effects of the probability distribution for the length of the RTT where all other parameters are set equal to the values for the base case.

This figure confirms that the larger (smaller) the spread in possible values for the length of the RTT is, the higher (lower) the optimal final order quantity is. Making such a statement for the total expected discounted costs is not possible, since this is dependent on all the settings of the cost parameters.

7.2 Results Simple Policy, Original Item can be used after RTT

When the original item can be used after the RTT, the company can satisfy demand after the RTT with both the original item and the alternative item. During the RTT, the demand should be satisfied by the original item because the alternative item is not available yet. Since the original item is an end-of-supply issue, the company should place a final order for the original item to be able to cover the demand during the RTT. Here, a fillrate restriction is taken into account which states what fillrate during the RTT should be achieved. The objective of the model is to find the final order quantity that minimizes the total expected discounted costs during the entire planning horizon while meeting all restrictions. Since the original item can be used after the RTT to satisfy demand, it is possible that the optimal final order quantity is larger than the final order quantity that is required to meet the restrictions. E.g. when the alternative item is relatively expensive, it may be better to buy more units of the original item since then the company has to buy less units of the relatively expensive alternative item.

Without looking into the equations of the model that are given in section 3.7 to section 3.9, the above characteristics of the situation that is discussed here indicates how the model behaves:

To be able to meet the fillrate restriction (and if applicable the restriction regarding the batch size that is specified by the supplier and the restriction regarding the minimum order quantity), the model should increase the final order quantity. For each value for final order quantity, the total expected discounted costs during the entire planning horizon and the fillrate during the RTT is calculated. If the final order quantity results in a fillrate during the RTT that meets the fillrate restriction, and all other restrictions are met as well, the model will only stop increasing the final order quantity when this is disadvantageous for the total expected discounted costs during the entire planning horizon. E.g. if the alternative item is very cheap in comparison to the original item, it is better to buy the number units of the original item that are necessary to meet all restrictions. After all, you want to switch to the cheap alternative item as soon as possible, so stocking extra units of the expensive original item is not cost-optimal. The other way around a similar behaviour occurs: if the alternative item is very expensive in comparison to the original item, it is better to buy more units of the original item than is necessary to meet all restrictions. After all, if you would buy just the number of units of the original item that is needed to meet all restrictions you have to buy an expensive unit of the alternative item every time demand occurs after the RTT.

To illustrate this behaviour, the model is run for the situations where the costs of the original item (item 1) are smaller than, equal to, and larger than the costs for the alternative item (item 2). The base case settings of Appendix G are used, except all cost parameters are set to 0 and the prices of the two items are varied. E.g. when costs item 1 = costs item 2, this means that the prices of both items are equal and all other cost parameters are 0. The results for these three situations can be depicted as shown in Figure 7.3.

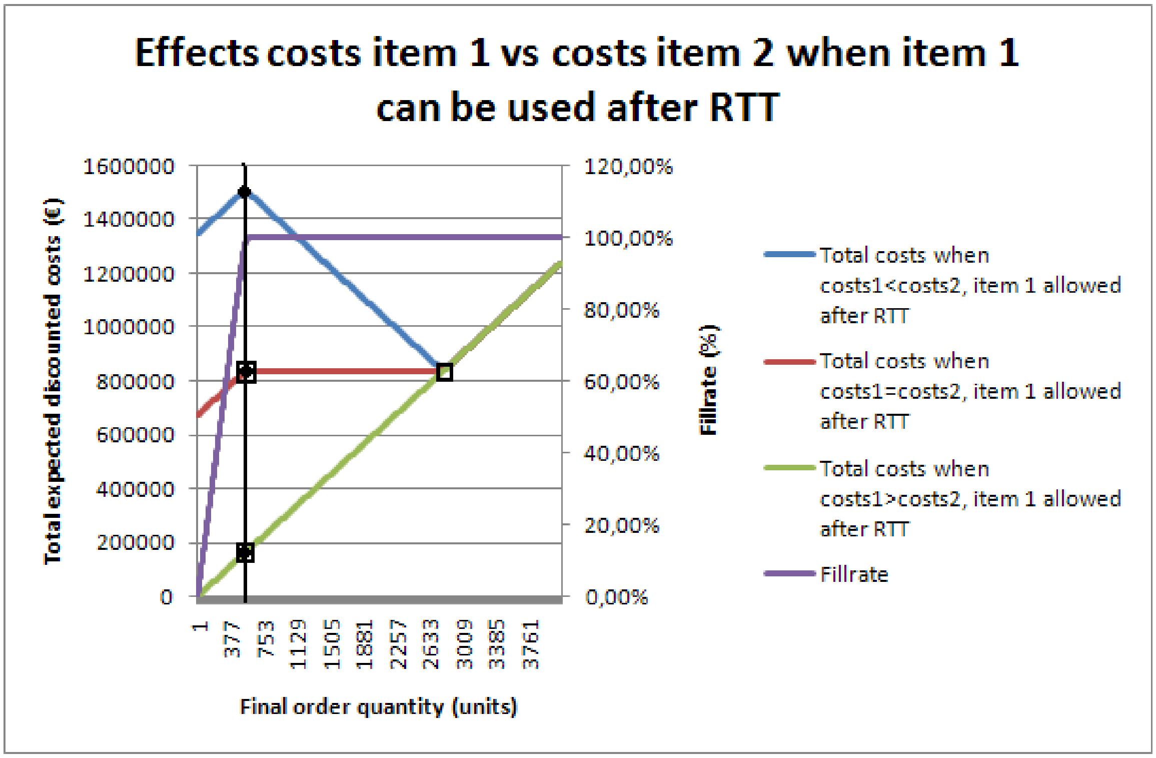


Figure 7.3: Effects of the costs for item 1 and the costs for item 2 when item 1 can be used after the RTT.

The dots in Figure 7.3 indicate the final order quantity that meets the fillrate restriction. This would be the optimal final order quantity in the situation where the original item cannot be used after the RTT (see section 7.1). The squares in Figure 7.3 indicate the optimal final order quantity when the original item can be used after the RTT.

When the original item can be used to satisfy demand after the RTT the following holds: In case the costs for the original item are equal to the costs of the alternative item, the total costs will not increase when the final order quantity is increased. After all, both items have equal costs and it does not matter whether demand is satisfied by the original item or the alternative item. In case the costs for the original item are larger than the costs for the alternative item, the optimal final order quantity is the first final order quantity that meets all restrictions. After all, you want to switch to the cheaper alternative item as soon as possible, so buying more units of the original item is not cost-optimal. In case the costs for the original item are smaller than the costs for the alternative item, the optimal final order quantity is equal to the expected total demand during the entire planning horizon (which is 2787 units in this case). Increasing the final order even more is useless, since you do not expect that you sell these items (this is also indicated by the increase in total costs when the final order quantity is higher than 2787 units in Figure 7.3).

Unfortunately, the price of the original item and the alternative item are not the only cost parameters that should be taken into account. Holding costs, disposal costs, and so on, are all cost parameters that should be taken into account when determining the optimal final order quantity. This means that Figure 7.3 is too simplistic. However, it does show how the model behaves when the original item can be used after the RTT. The difficulty that arises when the other cost parameters are taken into account is that it is not possible to make general statements about the effects of the input parameters on the optimal final order quantity and the total expected discounted costs during the planning horizon. All that is known is that the optimal final order quantity will be somewhere between the final order quantity that is needed to meet all restrictions and the total expected demand during the entire planning horizon. The model should be run for the specific parameter setting every time to get to know what the optimal final order quantity and the associated costs exactly are.

The only general statements about cost parameters that can be made here concern the expected setup costs and the fixed order costs. Both of these costs are incurred immediately, so these costs are incurred no matter what the size of the final order is. This automatically means that changing these two cost parameters will not affect the optimal final order quantity. Next to this it holds that the higher (lower) these two cost parameters are, the higher (lower) the total expected discounted costs during the planning horizon are.

Regarding the effects of the other input parameters general statements cannot be made when the original item can be used after the RTT. However, to get some insights in way the model behaves and the effects of the input parameters experiments are run for several input parameters. The results of these experiments can be found in Appendix H.

8 Heuristic vs. Exact Method for the Remove Policy

In this chapter the heuristic that is developed for the remove policy (see section 4.7.3.1) is compared to the exact method for the remove policy by Teunter & Fortuin [1999]. The purpose of the heuristic is to provide results faster than the exact method. Although the results may be sub-optimal, the exact method takes too long to be interesting for VI. However, it is interesting to check the performance of the heuristic in comparison to the exact method. Therefore, first some changes have to be made to the exact method. These changes are explained in section 8.1. After this, hypotheses are stated and tested on the basis of the case study settings in Appendix G. These hypotheses are discussed in section 8.2.

8.1 Changing the Exact Method to be able to Compare Both Methods

As mentioned above, the purpose of the heuristic is to provide results (i.e. the final order quantity, remove-down-to levels for all periods during the RTT and the accompanying costs) faster than the exact method. The exact method can take considerable time because many different combinations for the final order quantity and the remove-down-to level are considered. The heuristic considers fewer combinations, and therefore it is expected that this heuristic generates results faster. See section 4.7.3.1 for more details about the mechanisms of the heuristic and the exact method.

Note that the exact method of Teunter & Fortuin [1999] takes into account penalty costs. In this project, it was impossible to make sound estimations of the penalty costs. Therefore, a fillrate restriction is included in the models for the simple policy and the remove policy. This means that the exact method by Teunter & Fortuin [1999] should be adapted before it can be used to compare it to the heuristic. Therefore, the model proposed by Teunter & Fortuin was changed such that the penalty costs were left out and a fillrate restriction was included.
Next to this, the model of Teunter & Fortuin [1999] only considers a fixed time period that should be covered by the final order quantity. This fixed time period is equal to the entire planning horizon, which means that demand will never be satisfied by an alternative item. Therefore, to be able to compare the heuristic with the exact method, the length of the RTT is set equal to the planning horizon.

After these two changes are made, the two methods can be compared. This is done in the next section.

8.2 Hypotheses and Results for Comparing the Heuristic to the Exact Method

In this section several hypotheses are stated regarding the performance of the heuristic in comparison to the exact method. The hypotheses concern the difference in calculation time, the difference in fillrate, and the difference in expected costs between both methods.

As mentioned before, the heuristic should result in shorter calculation times than the exact method. This leads to the following hypothesis:

Hypothesis 1: The calculation time of the heuristic is shorter than the calculation time of the exact method.

Next to this, the mechanism behind the heuristic works in such a way that remove-down-to levels could be higher than necessary. After all, the heuristic can result in sub-optimization because it optimizes remove-down-to levels per period per possible inventory level at the start of a period, while the exact method solves the problem for all periods at once. In the heuristic, the remove-down-to levels per period in the RTT have to be calculated after the optimization process because all that results from the optimization process are optimal remove-down-to levels for a given inventory level at the start of a specific period. Once these optimal remove-down-to levels are translated in a 'sub-optimal' remove-down-to level per period, the real fillrate and costs are calculated. In the heuristic, this translation to remove-down-to levels per period is done in four different ways (see section 4.7.3). The translation can result in higher remove-down-to levels than that are necessary to meet the fillrate restriction. In the exact method, there is only one remove-down-to level for each period (instead of several different remove-down-to levels for all possible inventory levels at the start of a period). This means that the fillrate that results from the optimization process automatically is the ‘real’ fillrate, because the remove-down-to levels do not have to be transformed. Therefore, it is known exactly which combination of remove-down-to levels meets the fillrate restriction. This leads to the following hypothesis:

Hypothesis 2: The heuristic will result in a fillrate that is equal to or higher than the fillrate that results from the exact method.

Because the heuristic most likely results in a combination of a final order quantity and remove-down-to levels that has a higher fillrate than the combination that results from the exact method, it is likely that the heuristic results in higher total expected discounted costs as well. After all, the remove-down-to levels of the heuristic can be higher than the remove-down-to levels of the exact method, which means that fewer items are disposed and higher holding costs are incurred. This leads to the following hypothesis:

Hypothesis 3: The heuristic will result in total expected discounted costs during the entire planning horizon that are equal to or higher than the total expected discounted costs during the entire planning horizon that result from the exact method.

To test these hypotheses, only a very short planning horizon is considered since increasing the number of periods during the planning horizon drastically increases the number of combinations that should be calculated and evaluated in the exact method. Similarly, only a very small maximum demand per period is considered, since increasing the demand per period also drastically increases the number of calculations that should be done in the exact method. By trial and error it was found that calculating the optimal solution for parameter settings that result in around 140000 combinations (i.e. 140000 vectors with different combinations of the final order quantity and the remove-down-to levels for all periods) take around half an hour. Calculating the solution for a larger number of periods or larger spread in demand, which results in more than 140000 combinations, is very time-consuming. It is expected that with the set of experiments given below the differences between the heuristic and the exact method become clear as well, which makes running time-consuming experiments unnecessary.

Based on the above it is chosen to take the 140000 combinations, or a calculation time of 30 minutes, as a threshold. To test the hypotheses the number of periods that are considered and the spread in demand per period is increased. By trial and error it was found that a parameter setting in which 5 periods are considered and the demand per period follows a Uniform(0,6) distribution results in around 140000 combinations. For 4 or less periods the demand per period can have a larger spread, since the number of combinations is smaller then due to the fact that fewer periods are considered. However, increasing the number of periods to 6 or more results in more than 140000 combinations. I.e. when 6 periods are considered, only a maximum demand per period of 2 units can be considered. A higher maximum demand per period results in more than 140000 combinations. Therefore, the heuristic and exact method are compared for the following settings for the number of periods and the distribution for the demand per period:

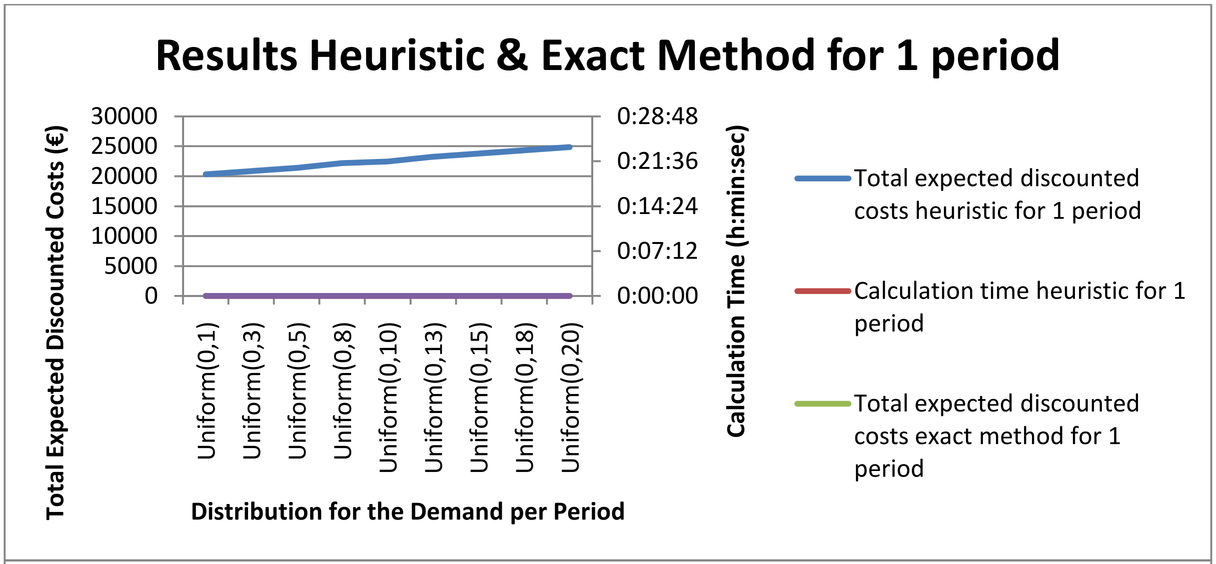
Table 8.1: Settings for the number of periods and the demand per period to compare the heuristic and exact method.

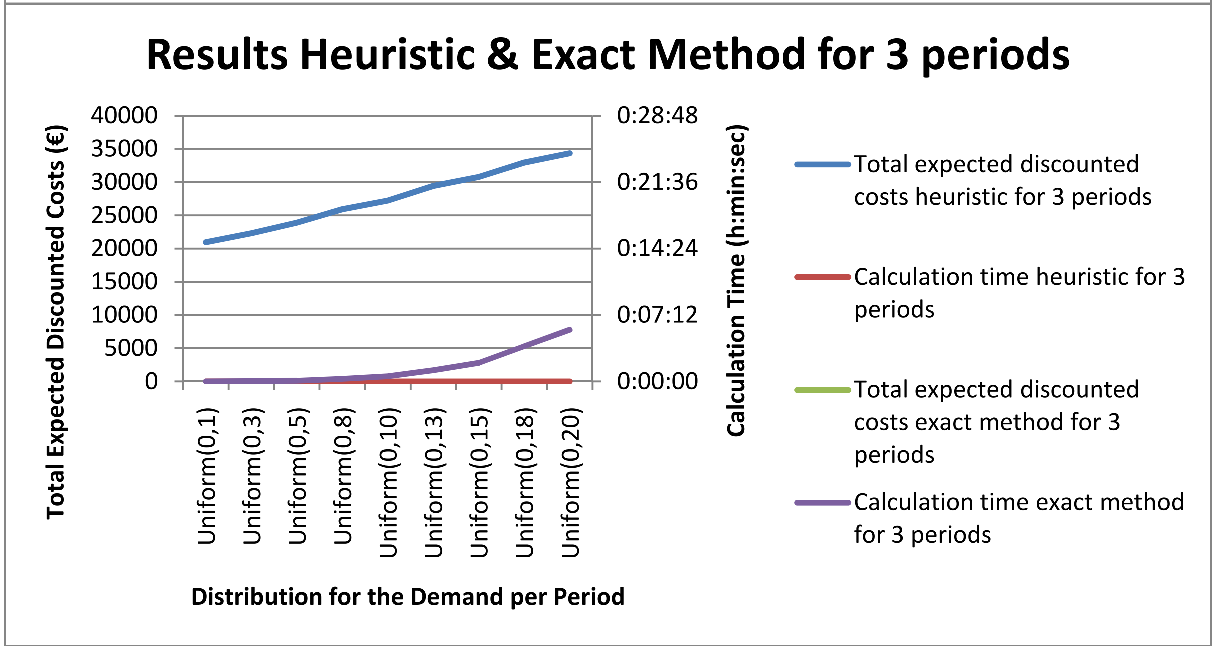
1 period	3 periods	5 periods
Demand per period ~ Uniform(0,1)	Demand per period ~ Uniform(0,1)	Demand per period ~ Uniform(0,1)
Demand per period ~ Uniform(0,3)	Demand per period ~ Uniform(0,3)	Demand per period ~ Uniform(0,2)
Demand per period ~ Uniform(0,5)	Demand per period ~ Uniform(0,5)	Demand per period ~ Uniform(0,3)
Demand per period ~ Uniform(0,8)	Demand per period ~ Uniform(0,8)	Demand per period ~ Uniform(0,4)

Demand per period ~ Uniform(0,10)	Demand per period ~ Uniform(0,10)	Demand per period ~ Uniform(0,5)
Demand per period ~ Uniform(0,13)	Demand per period ~ Uniform(0,13)	Demand per period ~ Uniform(0,6)
Demand per period ~ Uniform(0,15)	Demand per period ~ Uniform(0,15)	
Demand per period ~ Uniform(0,18)	Demand per period ~ Uniform(0,18)	
Demand per period ~ Uniform(0,20)	Demand per period ~ Uniform(0,20)	

All other input parameters are set equal in all experiments to the values of the base case (see Appendix G). However, note that we set the length of the RTT equal to the planning horizon, since this is also the case in Teunter & Fortuin [1999]. This means we have a fixed length of the RTT.

The results from the experiments that compared the heuristic for the remove policy with the exact method are given in detail in Appendix I. The results regarding the total expected discounted costs and the calculation time are depicted below.





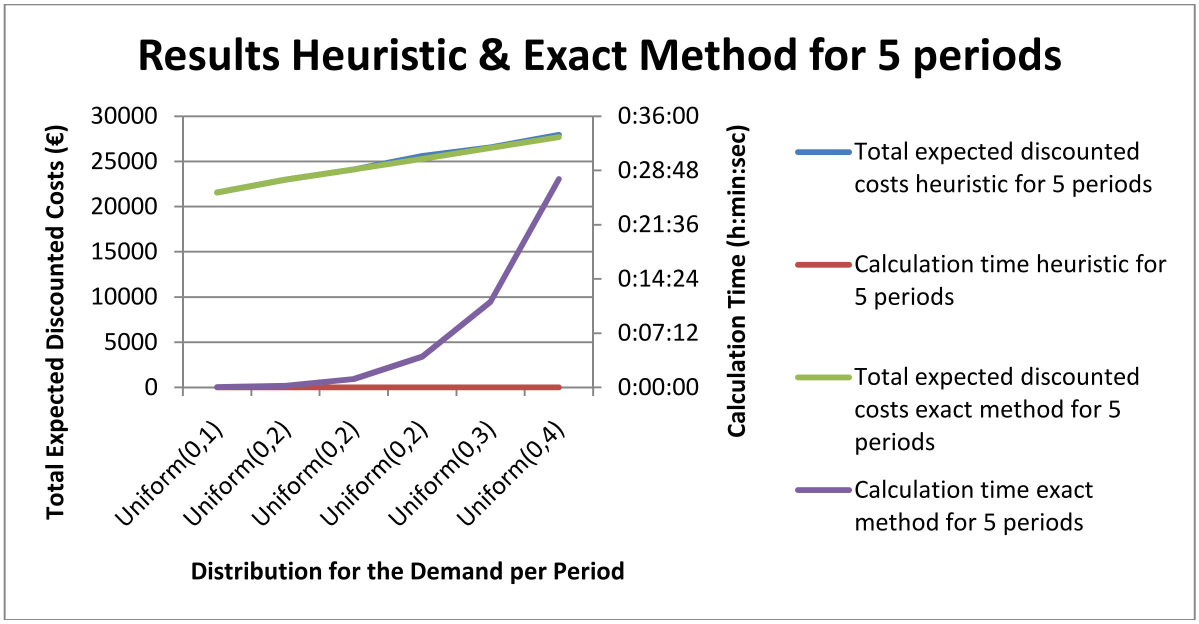


Figure 8.1: Results for the heuristic and exact method for the remove policy for 1 period (upper figure), 3 periods (middle figure), and 5 periods (bottom figure).

From the tables in Appendix I it can be seen that if the heuristic does not result in exactly the same solution as the exact method, the solution of the heuristic will always have a higher fillrate and a higher total expected discounted cost than the exact method. This is illustrated in Figure 8.1 for the total expected discounted costs as well. As shown in this figure, the total expected discounted costs do not deviate much between both methods; in the upper and middle figure it is even impossible to see any differences because the lines that represent the total expected discounted costs for the heuristic and for the exact method coincide.

Regarding the calculation time it can be seen from Figure 8.1 that the larger the number of periods is, the larger the calculation time for both methods. However, the calculation time for the exact method increases rapidly in comparison to the calculation time for the heuristic. The same behaviour of the calculation time holds when the spread in demand per period is increased.

Based on the information in the tables in Appendix I and Figure 8.1, the following can be concluded:

1) In almost all cases (except for the cases where 1 period is considered), the heuristic calculates a solution much faster than the exact method. Especially when the number of periods that is considered or the spread in demand per period is increased the heuristic outperforms the exact method regarding calculation time. Based on this it can be concluded that hypothesis 1 is true when more than 1 period is considered.
2) From the tables in Appendix I it can be seen that the solution of the heuristic always has a fillrate that is equal to or higher than the fillrate of the solution of the exact method. Based on these results, which are based on the case study input parameters (except for the number of periods and the demand per period), it can be concluded that hypothesis 2 is true.
3) Similarly, the solution of the heuristic always has total expected discounted costs that are equal to or higher than the total expected discounted costs of the solution of the exact method. Based on these results, it can be concluded that hypothesis 3 is true.

The maximum difference in the total expected costs between both methods for these experiments is 1,163%. If only the costs during the RTT are taken into account, this maximum difference is 5,560%. For these results, the benefits in calculation time for using the heuristic can be 100%.

9 Implementation

Regarding the implementation of the designed solution a change plan should be made. A change plan specifies the various actions to be taken, the actors that are to execute those actions, and the actors that should get involved in the process [Van Aken et al., 2006]. The contents of a change plan are the following [Van Aken et al., 2006, p.84]:

- The objectives of the change process and a delta-analysis: A delta-analysis is an analysis of the major differences between the present business system and the redesigned business system.
- A specification of the actions to be taken to realize the planned changes.
- On the basis of the delta-analysis and the actions that should be taken, a stakeholder analysis should be executed. This analysis results in a specification of the people who have to execute these actions (direct stakeholders) and of the people who have to get involved in the change process (indirect stakeholders). After the stakeholders are identified, the possible resistance to the proposed changes should be analysed for each group of direct stakeholders.
- A design of the change organization, which is a temporary structure within which the above-mentioned people will work.
- A communication plan.

All of these issues are addressed in the next sections.

9.1 Objectives of the Change Process & Delta-Analysis

The objective of the change process is that the Excel tool in which the solution that has been developed during this master thesis project is successfully implemented. Moreover, the overall objective is that the decision making process regarding end-of-supply problems (or any kind of problem where an original item is compared to an alternative item) is improved.

The major difference between the present business system and the redesigned business system is that in the latter one the Excel tool is implemented. Currently, end-of-supply issues are tackled by the end-of-supply procedure as described in section 1.5, Appendix C and Appendix D. In this procedure people from different departments have different responsibilities and based on one or more meetings it is decided what to do with this EOS issue. However, when it is decided that a final order should be placed, the determination of this final order mostly is a ‘wild guess’. In the redesigned business system the Excel tool is implemented in such a way that anyone who joins that meeting understands the tool’s output to help the decision making process. Using the tool results in a quantitative evaluation of the problem and it provides a quantitative model to determine the optimal final order quantity. Using the tool also means that there should be clear responsibilities regarding who delivers what input to the tool. On the one hand this may be more ‘work’ for the involved people, on the other hand it distinguishes people’s responsibilities better and people are more obliged to do the work than in the current situation. During the project it became clear that obtaining accurate information regarding final order quantities for EOS issues in the past is not as easy as it may seem. Because of this lack of accurate information it was not possible to compare the outcomes of the tool with the real decisions that were once made.

The main action to be taken is a presentation in which the Excel tool and its functionality are explained to all direct stakeholders. If not all direct stakeholders can be present, at least one stakeholder per stakeholder group (or department) should be present. The people from Team Worldwide, who will be the users of the tool, will be trained in using the Excel tool. The tool is relatively small and requires no advanced computer knowledge, so this training will mainly focus on the input parameters that are required and how these should be entered. Also, the options of the tool are discussed and the outcomes of several examples are discussed. Next to that, a user manual is developed in which all steps that should be executed to run the tool correctly are described. This user manual is checked with the company supervisors and it was agreed that the user manual provides a clear guide regarding how to use the tool.

Since using the tool only changes the end-of-supply procedure within the company and only a limited number of people are involved in the decision making regarding EOS issues, a presentation regarding this project seems to be sufficient. However, the presentation should convince all direct stakeholders of the usefulness of the tool, since they should provide the information that is needed in the tool. Therefore, all stakeholders and their roles are analysed. Next to that, possible reasons for resistance are analyzed. This is done in the next sections.

9.1.1 Stakeholder Analysis

Regarding stakeholders a division is made between direct stakeholders and indirect stakeholders. Direct stakeholders are people whose work processes, roles or vital interests are directly affected by the problem or by the possible solutions. Indirect stakeholders are people who are to cooperate with direct stakeholders and therefore have to know about the problem and about the changes in roles and processes of the direct stakeholders [van Aken et al., 2006, p.83]. The current EOS-process at VI (see section 1.5, Appendix C and Appendix D) gives a clear indication of the direct and indirect stakeholders. These are listed below.

Direct stakeholders

The direct stakeholders are the members of the SOP-Team. The SOP-Team is a team of employees from different departments that meet to discuss EOS issues. The tool that has been developed during this project should be used before these meetings to help the decision making process. The departments that are always involved in the SOP-Team are:

- *The Business Unit Services*: The business unit Services ‘suffers’ from the EOS problem since they have to provide service to the customers which is almost impossible if an item is EOS. Thus, they are most eager to solve the problem. The responsibility of an employee from the business unit Services is to forecast the demand for the upcoming periods, based on historical data and/or data about the installed base.
- *The R&D and/or Product Lifecycle Department*: This department is an essential part in any EOS issue that may require an alternative item. It is the responsibility of an employee of PLS to search for an alternative item and to test the item (e.g. is the item form-fit-function?). An employee from this department is the ‘problem owner’ for a specific EOS issue. This means that he/she is responsible for solving the EOS issue.

Because SOP-Teammembers play an important role in EOS issues, SOP-Teammembers from both departments should be present at the presentation regarding this project.

However, the SOP-Teammembers are not the employees who will use the tool to calculate the optimal final order quantity. This will always be done by one of the employees from Team Worldwide in SCMS. Currently, these employees are involved with EOS issues when the SOP-Team needs information about demand of a specific item. Also, these employees are operationally responsible for ordering spare parts. In case a final order decision has to be made, they should determine the final order quantity because they have the best information regarding demand and the suppliers. The SOP-Team should provide Team Worldwide with information about an EOS issue and the proposed alternative item (or several alternative items). Team Worldwide will feed back to them what to do regarding this alternative item (or what alternative item is best to chose) and what the optimal final order quantity would be. To be able to do this these employees should be instructed about the tool and how it exactly works. Therefore, a short training session will be organized in which the tool will be explained and several case studies will be analyzed by using the Excel tool.

Indirect stakeholders

The direct stakeholders are the members of the SOP-Team and Team Worldwide. However, for the SOP-Team to work, they should have sufficient information. Next to that, their decision should be processed to other departments. E.g. if the SOP-Team decides to switch to an alternative item, the Engineering department should change the drawings and specifications. Regarding the indirect stakeholders, the following stakeholder groups can be identified:

- *Employees from the Purchasing department*: The Purchasing department is responsible for ordering items that are needed in running projects. Therefore, they should provide the SOP-Team and SCMS with forecasts of the demand for running projects for the upcoming periods.
- *Employees from the Engineering department*: If it is decided to switch to an alternative item, the Engineering department should update this in the specifications.
- *Employees from the business unit Services that are not in the SOP-Team or in Team Worldwide*: No matter what is decided regarding an EOS issue, the other employees of the business unit Services should be informed about this so they know what to tell the customer (and Customer Centres) about it.
- *Non-SOP-Teammembers from the R&D department*: No matter what the SOP-Team decided, the other employees of the R&D department should change the drawings etc. if the EOS issue was related to an item they are responsible for. Next to that, the R&D department can be the cause of an EOS issue, since they decided to switch to an alternative item. If this is the case, they should immediately update the item controller about that.
- *Item controllers*: Item controllers keep the databases about item information updated. As soon as an item controller identifies an EOS issue, he performs a quick scan and determines a problem owner. After a decision has been made about the EOS issue, the item controller should update the information in the database.
- *Any employee within VI*: Any employee within VI could signal an EOS issue. As soon as he/she identifies an EOS issue, this should be reported to the item controller immediately.
- *Management of the involved departments*: Although it is likely that they never have to solve an EOS issue themselves, they will hear about such issues and therefore they should know how those issues are solved. Next to that these managers are important to create organizational support.

Although the indirect stakeholders theoretically never have to use the tool, they should know what to do in case there is an EOS-issue. Regarding the indirect stakeholders a presentation about the tool seems unnecessary, since they will never use it anyway. Also, presenting them their responsibilities will most likely cause resistance. Therefore, it is decided relatively early during the project that the ‘qualitative side’ of the EOS problem is a problem on itself that is outside the scope of this project. However, during the project VI has made big steps in its procedures, among which the EOS procedure (see Appendix C and Appendix D). Actually at the end of this project discussions took place regarding the appropriateness of the procedure as it is now, and how the tool that is developed during this project could be combined with this procedure. The resulting updated version of the EOS procedure is not finished yet, but it is good to see that VI is reconsidering this procedure and is actually taking action to improve it.

9.1.2 Resistance to Change

Regarding each group of direct stakeholders, an analysis should be made regarding possible sources for resistance. Van Aken et al. [2006, p.85] identify the following sources of resistance:

- *Lack of understanding*: It may be possible that employees do not understand that there is a problem, or they do not understand the new business system and/or what their own role in that new business system would be.
- *Differences in opinion*: It may be possible that an employee may understand the problem, but disagrees with the solution on technical, economic or private grounds.
- *Lack of trust*: It may be possible that employees do not trust members of the change organization, either in their intentions or in their competences.
- *Low willingness to change*: Direct stakeholders may not want to change because they fear the unknown, do not want to change a familiar environment, etc. An employee can have an inherent low willingness to change, which can be related with the history of the organization or department, or with the type of people. An employee can also have induced low willingness to

change, which can be caused by an unprofessional approach of the change process. In this case the employee once wanted to change, but due to unprofessional actions the employee's willingness to change decreased.
- *Conflicts of interest*: Organizational changes normally are not neutral with respect to the material or immaterial interests of the various stakeholders.

Now each of these sources of resistance is evaluated for the direct stakeholders:

- *Lack of understanding*: SOP-Teammembers from the business unit Services and employees from SCMS most likely understand that there is a problem, since the business unit Services most often 'suffers' from the EOS issues. After all, the business unit Services is responsible for providing service to the company's customers, thus when there is an EOS issue they may have a serious problem. Also, it will be clear to them what their role is when the tool is implemented, since the tool and what they should do is discussed several times with them. Regarding the SOP-Teammembers from the R&D/PLS department lack of understanding can be expected. Although they do see that an EOS issue is a real problem, they really have to be pushed to work on it. After all, it is the problem of the business unit Services. Next to that, they may not understand how the tool helps the decision making process since they are more concerned with finding and evaluating alternatives on a 'qualitative' basis (e.g. is the alternative item form-fit-function?). However, it is expected that they will understand the usefulness of the tool once it is presented to them: the tool is an additional and quantitative method to evaluate the alternative item, which can be very interesting to them.
- *Differences of opinion*: Some people may think that the tool is not always applicable, since it requires pretty much information and it is thought that in most EOS issues it is impossible to place a (large) final order. However, the tool does not require more information than one would expect to be needed during a 'normal' SOP-meeting without the tool. Next to that, the tool's functionality is to help decision making: the output of the tool simply is an advice and not a strict order. If it is impossible to order the final order the tool advices (e.g. because it is larger than the number of units that the supplier can deliver), then you should order the amount of units that comes closest to this final order quantity. Finally, some people think that the real problem regarding EOS issues lies in the EOS process. Although this qualitative side of the problem certainly is important, improving the EOS process was out of the scope of this project. Currently, the SOP-Teams meet on a regular basis, so the tool can actually be of added value as a preparation for those meetings even when the entire EOS process is not perfect yet.
- *Lack of trust*: During the project there was no reason to think that there was a lack of trust.
- *Low willingness to change*: Implementing the tool does actually not require that much change. Most information that should be used in decision making regarding EOS issues is already gathered in the current business system, the extra actions to take in the new business system are to enter this information in the tool and to run it. Since the tool is simply an additional help, instead of an entirely new business system, there is no low willingness to change.
- *Conflicts of interests*: Since the change is not radical, it is not expected that there are conflicts of interests.

9.2 Change Organization & Communication Plan

A change organization is a temporary structure within which the stakeholders will work to change from the current business system to the redesigned business system. However, the changes that are required in this project are relatively small: the SOP-Team changes from working without output from the tool to working with output from the tool. Team Worldwide changes from working without the tool to working with the tool. Therefore, there is no need for a change organization. Regarding the communication the following is planned:

- As much as possible direct stakeholders are contacted and informed about a presentation about this project. The problem and the solution are shortly described.
- The presentation about this project takes place. In this presentation, the tool will be showed and a quick walkthrough will be shown.
- A short training session with Team Worldwide will be done to teach them how to use the tool. Several case studies will be discussed to show how the tool works. Next to that, a user manual is developed that describes how the tool works.
- If any other stakeholders have problems with the tool, it is expected that the stakeholders that were present during the presentation can help them out. Regarding the indirect stakeholders no presentation is required, since they will mostly provide information to the SOP-Team or use the output of the SOP-meetings. This means that when the SOP-Team needs (more specific) information from an indirect stakeholder in the SOP-meetings or when Team Worldwide needs more information to be able to use the tool, the SOP-Teammembers and Team Worldwide simply ask for it.

9.3 Intervention Strategy

The analysis of the resistance (see section 9.1.2) is the basis for the design of the intervention strategy. This intervention strategy is built on Tichy's TPC model [Tichy, 1983 in Van Aken et al., 2006]. In this model, one should manage organizational change processes simultaneously in three intertwined aspect systems:

1) The *technical system* (T): This concerns technical and economical issues, such as the business problem, its strategic context, the new business system.
2) The *political system* (P): This concerns the material and immaterial interests and how people protect these interests.
3) The *cultural system* (C): This concerns the culture of the organization, departments, and individuals and their emotions.

In each aspect system there are interventions to deal with the issues in that system:

1) The typical *technical intervention* is the report: In the report all details can be found about the problem, what solution was designed for it etc.
2) The typical *political intervention* is the formal order: Appointments and dismissals are an important type of formal order. People who are against the change may be dismissed or transferred and their successors should have the mission to make the change a success.
3) The typical *cultural intervention* is participation: (Potential) stakeholders can be involved in the definition of the problem, the design of solutions, etc. Participation can improve the quality of the new system, the understanding of it and it can give a sense of ownership.

"The intervention strategy is the in main lines planned series of steps to be taken, plus the mix of interventions to be used in the change process." [Van Aken et al., 2006, p.86]. The mix of interventions depends on the expected resistance. From the analysis of resistance we know that if there is any resistance, this will most likely be a lack of understanding or a difference in opinion. For these two types of resistance, one can rely on technical and political interventions. Technical interventions can be used to explain the problem and the solution and political interventions can be used to decide on the remaining issues.

The intervention strategy used in this project is quite similar to the communication plan given in section 9.2. The technical interventions are this report, the announcement of the presentation (in which a short summary of the problem and solution is given) and the presentation itself. Next to that, the user manual helps the employees to understand the tool. The political interventions are the discussion after the presentation. It is important to convince managers of the direct stakeholder groups, since they can create organizational support. This organizational support and sense of ownership are enhanced throughout the project as well, since different stakeholders were involved in the problem definition and in the design of the solution.

9.4 Using the Tool

The people who will use the tool are the employees in Team Worldwide. Therefore, it is important that they know how the tool works and how they should gather the necessary information. Regarding the tool a presentation will be given (see section 9.2 and 9.3) in which the tool will be explained. Next to that, a short training session will be given. Also, a user manual is available that explains how the tool works. Regarding the information that is used in the tool Team Worldwide is responsible themselves: if they want the tool to give more detailed output, or a more accurate solution, then they should also provide more accurate information. If they do not have this information, they should involve the SOP-Team or indirect stakeholders to obtain better information. Still, making accurate forecasts for costs and demand can be very difficult, especially when the planning horizon is very long. Therefore, it is better to revise the decision that was once made periodically. In this way, one can use updated information in the tool to generate a more accurate solution. This principle of the 'rolling horizon' is explained in more detail in section 2.3. To be able to use this rolling horizon strategy as good as possible, there should be a small database in which the information is stored and updated. This way, the user of the tool can see the old information on which the old outcome of the tool was based, and the user can see the new information and the new outcome.

10 Conclusions & Recommendations

In this chapter the conclusions and recommendations that result from the master thesis project are given. Here, a division is made between academic conclusions and recommendations (i.e. future research opportunities) and the conclusions and recommendations for VI. The academic conclusions and recommendations are given in section 10.1. The conclusions and recommendations for VI are given in section 10.2.

10.1 Academic Conclusions & Recommendations

In section 10.1.1 the academic conclusions that can be drawn from this master thesis project are discussed. Next to this, the academic recommendations, or future research opportunities, are given in section 10.1.2.

10.1.1 Academic Conclusions

Based on this master thesis project, the following conclusions can be drawn that are relevant to academics:

At the start of this master thesis project a literature study was conducted. Based on this literature study it was found that some aspects of the final order problem should deserve more attention (see section 1.1.2). The aspects of the final order problem that we would like to elaborate in more detail during this master thesis project:

- Substitution of items (which automatically takes other possibilities to acquire items into account, such as extra production runs). Here, substitution leads to a required transition time where the demand during the transition time can only be satisfied by the original item. After the end of the transition time, it is possible to use the alternative item to satisfy demand.
- The time the transition from one item to another takes is uncertain, which means that a probability distribution for the transition time needs to be taken into account. Also, uncertainty regarding the transition costs should be taken into account.
- The combination of using a service level (instead of penalty costs) and substitution, where the transition time can be uncertain and thus the moment from which the company can use the alternative item is uncertain, is a new combination of aspects of the final order problem that has not been addressed in academic literature yet according to our knowledge. Therefore, we would like to address this kind of final order problem in this master thesis project.

During this master thesis project models were developed that take all the above aspects of the final order problem into account. The basis for these models, the so-called required transition time, is discussed extensively and to our knowledge the combination of the final order substitution of items has never been modeled in a way as has been done during this project. Next to that a fillrate restriction has been taken into account instead of penalty costs, which makes the combination of aspects that is modeled during this project even more unique. During the project the models that were developed were found to be valid. This makes the models that are developed during this project a contribution to the existing academic literature regarding the final order problem.

Based on the models developed during this project, the following can be concluded regarding the effects of the input parameters:

- *When the original item cannot be used after the RTT, the following holds:*
  - Cost parameters regarding original item and the alternative item do not influence the optimal final order quantity when the length of the RTT is fixed or when it follows a probability distribution. The optimal final order quantity solely depends on the fillrate restriction and the other restrictions regarding the order size (e.g. the minimum order quantity should be taken into account). This means that estimating for example the holding costs per item more accurately (e.g. based on the volume of an item) is not

required for the decision on the final order quantity itself. Better cost estimates only improve the estimation of the total expected discounted costs that result from the model, and thus improve the decision regarding which alternative item to choose (if several alternative items are available).
    - The fillrate restriction directly influences the optimal final order quantity. The larger the fillrate restriction, the higher the final order quantity will be.
    - The length of the RTT (or its probability distribution) and the characteristics of the demand per period have a major influence on the optimal final order quantity. Estimating values for these parameters should deserve great attention.
- *When the original item can be used after the RTT, the following holds:*
    - General statements about the effects of most of the input parameters of the model on the outcomes of the model cannot be made. The effects of the input parameters are dependent on the specific settings of the parameters. I.e. if the alternative item is relatively expensive in comparison to the original item, the model will result in a relatively high optimal final order quantity (i.e. a higher final order quantity than is needed to meet the fillrate restriction). However, no clear statements can be made on what this optimal final order quantity would be exactly.
    - Based on the above point it can be concluded that estimating the input parameters should deserve a lot of attention. The more reliable the cost parameters are estimated, the more reliable the results are.
    - The expected setup costs and the fixed order costs for the final order do not influence the decision regarding the final order quantity: no matter what the size of the final order is, these costs are always incurred. Estimating these cost parameters is important to get better insights in the total costs that can be expected to cover the demand during the entire planning horizon and they influence the decision regarding which alternative item to choose (if several alternative items are available).

During this project a heuristic has been developed for the remove policy. This was done because the exact method that exists in academic literature was too time-consuming. The heuristic for the remove policy results in costs that are equal to or higher than the costs that result from the exact methods in literature. The heuristic will always result in a solution that satisfies the fillrate restriction.

10.1.2 Academic Recommendations / Future Research

This master thesis project concerns the final order problem including substitution of items for non-repairable items. A logical direction for future research is to research this type of problem for repairable items. When considering repairable items instead of non-repairable items additional challenges arise:

- Because items are repairable, it is possible that there are returns of items in each period during the planning horizon. These returns are most often not known beforehand. Next to that, returned items are not necessarily usable items: it can be possible that a returned part is damaged that much that repairing it is impossible, the returned item may require testing before it can be repaired, etc. Also, one can or even should distinguish between returns of the original item and returns of the alternative item.
- Due to possible returns of items, an additional cost parameter should be taken into account: remanufacturing/repair costs.
- Including returns in a model that also includes substitution gives rise to new trade-offs, for example:
    - Repairing returned original items vs. purchasing (or maybe ever changing to) the alternative item.
    - Repairing returned original items vs. repairing returned alternative items.

These issues regarding repairable items make the final order problem including substitution for repairable items a very interesting challenge and direction for future research.

10.2 Conclusions & Recommendations for Vanderlande Industries

In section 10.2.1 the conclusions that are relevant to VI that can be drawn from this master thesis project are discussed. Next to this, recommendations for VI are given in section 10.2.2.

10.2.1 Conclusions for Vanderlande Industries

Based on this master thesis project, the following conclusions can be drawn that are relevant to VI:

Before this master thesis project, it was unclear to SCMS whether a final order should be placed or not and what the optimal final order quantity should be. During this project a solution for this problem has been designed. This solution is implemented in a tool in Excel. This tool enables SCMS to determine the optimal final order quantity for the original item and the total expected discounted costs that are incurred to cover demand during the entire planning horizon. Here, the tool takes into account a fillrate restriction during the so-called required transition time. Unfortunately, it is impossible due to a lack of data to compare the results of the tool with real decisions regarding EOS issues in the past. The availability of this Excel tool is an improvement of the situation before the master thesis project, since back then there was no appropriate method to determine when to place a final order and what the size of this final order would be. Note however that this tool includes a quantitative model only. Qualitative issues regarding the EOS issues should be taken care of by the SOP-Team.

A rolling horizon strategy with the simple policy seems to be more appropriate than a remove policy. Using the rolling horizon strategy, updated information can be used to estimate the parameters in the tool more accurately. Also, the rolling horizon strategy enables SCMS to calculate a ‘remove-down-to level’ at any moment they want.

10.2.2 Recommendations for Vanderlande Industries

Based on the master thesis project and its conclusions, the following recommendations can be made:

It is recommended that the tool that has been developed during this project will be included in the EOS procedure that is present at VI. Although running the tool will always be the responsibility of Team Worldwide, it is important that all direct stakeholders (such as the SOP-Teammembers) know about the tool and understand its outcomes. The stakeholders should provide Team Worldwide with the information that is necessary to run the tool. This does not differ much from the current situation, since this information is required now too in making decisions regarding an EOS issue. Including the tool in the EOS procedure may enhance the implantation of the tool in decision making regarding EOS issues.

Related to the previous recommendation, it is advised that the EOS procedure at VI is revised. The following recommendations are given that are related to the EOS procedure:

- SCMS should have a more distinct role in the EOS procedure: Currently, they are not involved formally at all. However, SCMS is responsible for providing spare parts, so placing final orders is their responsibility. Next to that, they are already indirectly involved in EOS issues, since they have information regarding historical demand and they cannot order the item anymore when it is EOS. It is advised that when a SOP-Team is needed to solve the EOS issue, that the SOP-Team will involve Team Worldwide regarding the final order quantity that may be needed. The users of the tool developed during this project are the people from Team Worldwide, so they can provide the (quantitative) advice about what to do regarding the final order.
- In the current EOS procedure, responsibilities are not always clear to all the stakeholders. Moreover, most stakeholders see the EOS issues as a problem for the Business Unit Services, so why would they have to solve it?
- Management support should be created regarding the EOS procedure, such that people will understand that solving EOS issues is important for all departments.

Regarding the Excel tool and its input parameters, the following is recommended:

- Demand characteristics have a major influence on the optimal final order quantity. Currently, there is no clear method for forecasting demand. It is recommended to devote a project to improve demand forecasting, since this will also make the results of the tool more reliable.
- Although the estimations of the cost parameters for both items do not influence the decision on the optimal final order quantity when the original item cannot be used after the RTT, these estimations do affect the decision when the original item can be used after the RTT. This means that using the tool to help decision making regarding the final order requires the best information that is available. Using better and/or more reliable information will result in more reliable output of the tool. Therefore, it is recommended to keep the information regarding items as up to date as possible. Next to that, time should be devoted to estimating the cost parameters, such as the holding costs per item (e.g. by storing data regarding the volume of an item) and the expected price of the alternative item.
- VI can use different parameter settings in the tool to see the effects of changing these parameters. Also, parameter settings for different alternative items can be used to calculate the total expected discounted costs during the planning horizon for all different alternative items. Knowing these costs, VI can choose the ‘optimal’ alternative item.
- The more reliable the information that is available is, the more reliable the results of the tool are. Therefore, attention should be given to estimating the length of the RTT and the fillrate restriction. Currently, these parameters are not estimated by clear methods. These methods should be searched to get more reliable results. If this is impossible, the users of the tool should perform a scenario analysis and choose the optimal solution or the solution that they think is most appropriate.
- It is recommended that VI uses the rolling horizon strategy with the simple policy instead of the remove policy. The rolling horizon strategy seems more appropriate to VI, especially when the length of the RTT is uncertain.

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The other literature that is used during the preparation phase of this master thesis project can be found in Bakx [2010].

Appendix A – Final Order Problem Literature

The final order problem consists of two parts: the determination of the final order quantity and the demand forecast on which this final order quantity is based. The article by Moore [1971] is the first article that addresses the forecasting of the 'all-time requirement' and the translation of these forecasts into production schedules. Moore found that transforming historical sales date to a logistic scale resulted in three demand patterns (elliptical, parabolic and linear patterns) that describe 85% of the parts that were considered. When reliable forecasts of the all-time requirements are available, a quantity sufficient to satisfy cumulative demand for all time in the future could be produced and inventoried and the manufacturing capability disposed of. Such a production run, an all-time production run, can have benefits such as:

- Reduced file maintenance since drawings and instructions are discarded.
- Scrap payments and tax write-offs for discarded production tools.
- Reduced purchased component inventories.
- Floor space can be used for new items.

Demand forecasting during the final phase of a service part is not easy. Uncertainty is high because the final phase typically is very long (e.g. 10 years or longer) and because there is lack of knowledge about the demand distribution [Inderfurth & Mukherjee, 2008]. Next to that, Fortuin [1980] gives some examples of factors that are hard to quantify and influence the demand during the final phase:

- The customer can prefer to buy a new system instead of having the defective system repaired.
- The customer can decide to not to repair a system, because the service item that is defective is non-functional or it concerns a function that the customer does not really need.
- Service items are often used in other systems than the ones that were originally intended.
- New service items with improved quality can enter the market, leading to substitution of 'old' service items.

Next to this, Ritchie & Wilcox [1976] notice that the less critical a service item is, the sooner the demand of such a service item decreases. Therefore, they developed a demand forecasting model for the all-time future demand, explicitly taking criticality of the service item into account.

Regarding the second part of the final order problem, the determination of the final order quantity, several quantitative methods can be found in literature. Fortuin's article [1980] regarding the 'All-time requirement of spare parts for service after sales' is one of the first articles that addresses a quantitative model to determine the final order quantity. In this article a method is developed to determine the optimal final order quantity to be able to meet a certain availability degree. This method is revised in [Fortuin, 1981], where a fictitious reduction of the remaining service period is used to lower the all-time requirement of spare parts. This fictitious reduction namely causes the desired value of the availability degree to be reached before the end of the real remaining service period, and thus this reduction lowers the all-time requirements. In both articles Fortuin assumes that demand per year can be estimated with a non-stationary, uncorrelated Gaussian process with a mean that decreases exponentially over time (the discounting factor he assumed is 0.7). Basically, in both articles relationships between the availability degree, safety stock, length of the remaining service period, shortage risk and obsolescence risk are derived, taking into account the properties of the demand process for service items in the final phase. Teunter & Klein Haneveld [1998] use the model described in [Fortuin, 1980], but they use a discrete probability distribution for demand instead of a continuous one. Next to that, they show that for problems with large out-of-order costs, the multi-component final order problem can be decomposed (approximately) into single component final order problems.

Teunter & Fortuin [1999] developed two models for calculating the final order quantity. In the first model a final order is placed at the beginning of the final phase and at the end of the service period all remaining stock is removed. So all service items remain stocked until the end of the service period, although it may be the case that stocked items are not needed to satisfy demand during the final phase. This can lead to higher inventory holding costs than necessary. Therefore they developed a second model, in which the possibility to remove stock at the end of each period is added. A drawback of applying this remove-down-to strategy is that administration costs can increase since stock levels and demand predictions have to be reviewed continuously. Their results show that the difference in expected cost between the two final order strategies is small. Therefore the simple strategy (without the remove-down-to levels) seems to be sufficient. In order to calculate the optimal final order quantity with both types of the model estimates of the cost parameters (initial purchasing cost, penalty cost, holding cost and removing cost) and estimates of the probability distribution of the demand are needed. A case study in Teunter & Fortuin [1998] shows that finding these estimates can be very hard. Therefore, they proposed a new method for gathering and storing data, which should ease the calculation of the final order quantity.

The master thesis by Feng [2007] considers a tool for determining the final order quantity, or as they call it the 'lifetime buy', for electronic parts. This tool, the Life of Type Evaluation tool (LOTE) is an extended version of the model described in [Teunter & Fortuin, 1999]. LOTE can handle more complex situations than the model of Teunter & Fortuin, because it can handle multi-part systems concurrently rather than one part at a time. Next to that it also accounts for uncertainties in the demand and penalty inputs. This master thesis is summarized in Feng et al. [2007] for the Aging Aircraft Conference 2007.

Kooten & Tan [2007] extended the existing literature regarding the final order problem by including the risk of condemnation in their model. Their model makes it possible to calculate final order quantities for repairable spare parts considering a predefined service level, where condemnation is explicitly taken into account. Another extension of the final order problem literature is done by Bradley & Guerrero [2009]. They developed a model to determine the lifetime buy for multiple service parts that become obsolete over its lifecycle. In most other articles regarding the lifetime buy, only one service part at a time is considered. However, a drawback of this article is that the writers assume that the demand forecast is known beforehand and does not change.

The literature mentioned above concerns the final order problem in its most basic form: At the beginning of the final phase of the life cycle of a service item a final order quantity is ordered that should cover demand during the remaining service period. The primary results of the articles mentioned above are the calculations of the final order quantity itself (except for Teunter & Fortuin [1999, 1998], where remove-down-to levels are included as well). However, there exists literature regarding other aspects of the final order problem too. An example is the article by Cattani & Souza [2003]. They consider the final order quantity, or as they call it the 'end-of-life build', and focus on its optimal timing for a fixed and known remaining number of periods of demand. They basically quantify the benefits of delaying the end-of-life build. Obviously, the company (or buyer) would like to delay this decision as much as possible, since a later decision means more demand certainty. On the contrary, the supplier would prefer no delay for two major reasons:

1) The supplier has costs of maintaining a production line of a declining product
2) The company is forced by a final order to buy more units as a safety stock than otherwise would have been required with no final order. Delaying a final order results in lower demand and thus lower sales for the supplier.

Obviously, both parties have opposing interests regarding the delay of the final order decision.
Next to this, extra order possibilities after the final order are considered as well. An example is the article by Geurts & Moonen [1992]. This article does not only consider the final order as the only possibility to acquire service items. The writers try to find out what the influence is of having the possibility to order service parts after the final order. Here, the price of a service item that is ordered after the final order is higher than the price of a service item that is ordered in the final order. Thus, placing a final order can be seen as placing an order to get discounts. They found a near-optimal solution for determining the final

order quantity and the subsequent series of order-up-to levels. By dividing the planning horizon into equal discrete periods and applying dynamic programming they found optimal policies for insurance type service parts. However, they assume a zero replenishment lead time and a zero holding cost rate. Klein Haneveld and Teunter [1997] address the same kind of problem as Geurts & Moonen [1992]. In their model a zero replenishment lead time and a fixed penalty each time a demand occurs and no service item is in stock are assumed. They found an approach that determines the initial order-up-to-level at time 0, and a subsequent series of decreasing order-up-to-levels for various intervals of the planning horizon. However, their model results in an order-up-to level after the beginning of the final phase that is either zero or one. Therefore, they revised this model in [Teunter & Klein Haneveld, 2002]. The new model can result in order-up-to levels ranging from -∞ to any positive discrete number.
However, in 'real' final order problem situations there is no possibility to re-order the service item. The supplier simply stops delivering the service item to its customer and the customer has one last opportunity to buy the service item. However, this final order is not the only way of acquiring the service item. Inderfurth & Mukherjee [2008] there are three possibilities to acquire service items:

- *A final order*: Here, the final order is added to the last lot of production. After this lot, the production is stopped. Advantages of this option are that the final order lot is combined with the last production lot, which leads to low cost price per product and no extra setup costs. A disadvantage is the large uncertainty in demand during the remaining service period. Another disadvantage is that items are kept on stock for a long period, leading to a large inventory investment.
- *Extra production*: This refers to manufacturing or outside procurement of service items after the end of the product life cycle. Trough additional setups with small lots this option is very costly. If the company lacks production capabilities, demand is met by procuring the service items from outside suppliers at high cost. However, there is relatively low uncertainty in demand since the production planning is based on short-term forecasts.
- *Remanufacturing*: Remanufacturing is initiated once the final order production lot has been dispatched. The production costs through remanufacturing are moderate, given that remanufacturing facilities are available. However, there may be a restriction regarding a limited amount of available returns. Next to this, the acquisition of returns is uncertain.

A recent article by Krikke & van der Laan [2009] considers an additional way of acquiring service items. Next to spare parts management and a last time buy they also consider reverse logistics. This does not mean they consider only items that are returned by customers that are to be repaired, but they also consider so-called 'phase-out return flows'. These phase-out return flows are products that a certain customer does not need anymore because he stops using the products, but those same products can be used by other customers who still use them. So basically this article considers the final order problem including several return flows. An interesting finding is that their case-study shows some counter-intuitive results: Last time buy quantity does not make the decisive difference; rather it is the disposal and repair control policy that is of major consequence. This finding emphasizes the need to consider disposal policies when the final order problem is addressed.

A totally different approach to the final order problem is described in an article by Sprengler & Schröter [2003]. The writers take a system dynamics approach and modeled an integrated production and recovery system for supplying spare parts to evaluate the possible strategies for meeting spare-parts demand for electronic equipment in the end-of-life phase. To come to the final lot size they used the Vensim Optimization Tool, where Vensim is a simulation program that simulates the causal loop diagram the writers identified.

An article by Li et al. [2010] addresses the final order problem and product substitution. In their article, inventory planning decisions for product upgrades when there is no replenishment opportunity during the transition period are considered, where the authors allow for product substitution. The transition time ends when the new-product demand rate stabilizes and that of the old product drops to a negligible level. In the models they consider a start date for the transition and a finite time horizon, where the transition time is the time interval between the transition start date and the finite time horizon. Next to this, during this transition time the company sells both the old and the new product. Thus, when a shortage of the old product occurs, the new product can be used as a substitute. Next to this, the authors address three problems regarding a product transition [Li et al., 2010, p.144]:

1) Given a deterministic transition start date, what are the optimal planning quantities for the old and new products?
2) Given a stochastic transition start date, what are the optimal planning quantities for each product?
3) Given an initial scheduled transition start date and the initial inventory position of the old product, should the firm delay the transition? If so, by how much?

The authors show that the optimal substitution decision is a time-varying threshold policy. Also, they find that substitution reduces the need to hold the old product and can increase the profitability over the transition. Next to this, whereas the total stock increases when the variability in the transition start date increases, the division between the old and new product inventory is not monotonic. Regarding the delay of a new product release they find that one should try to address excess inventory in the old product first through the initial stock decisions, and if that is not sufficient, subsequently delaying the release of the new product.

Appendix B – Organizational Structure Business Unit Services

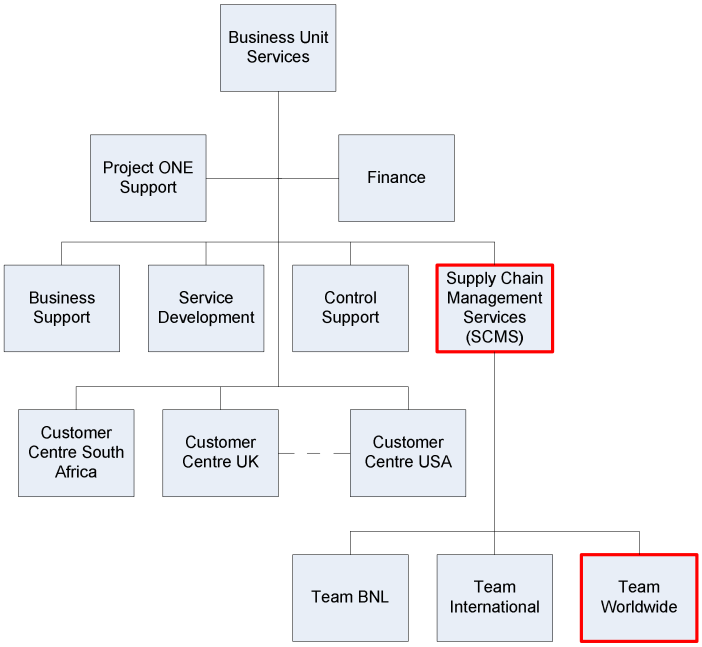
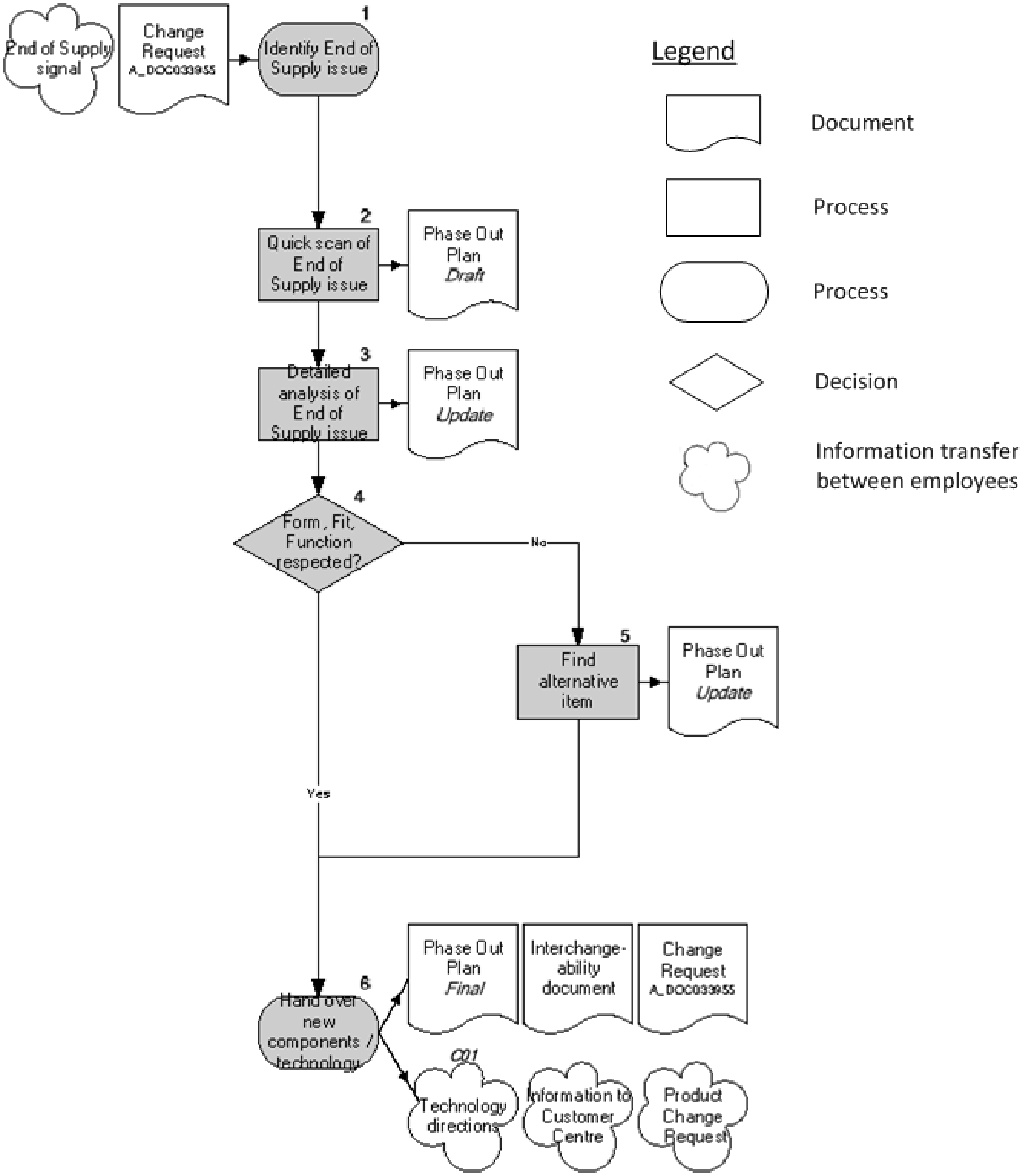


Figure B.1 Organizational structure Business Unit Services.

Appendix C – Procedure End-of-Supply



1. **Identify End of Supply issue**
   Any VI Employee
   - signals an EOS issue;
   - reports this to the item controller (by sending e-mail to the 'itemcontrol' mailbox), submitting information on the affected item / article, supplier and a copy of the notification (preferably by using an Item Change Request [1]).
2. **Quick scan of End of Supply issue**
   Item Controller
   - accepts the EOS issue;
   - changes the item's status in SmarTeam and JDE with reference to the EOS notification;
   - conducts Quick Scan;
   - determines urgency, problem owner, severity, affected products, running projects and installed base (where used information, see [2]);
   - determines planning of replacement of the item (last buy, last supply, first buy of alternative);
   - informs SOP Tech chair holder by e-mail;
   - in case of unclear problem ownership, initiates meeting with the SOP (Tech) members;
   - drafts a Phase Out Plan [3] and sends it to the problem owner;

   SOP (Tech) members
   - in case of urgency, determine actions how to deal with the EOS issue.
3. **Detailed analysis of End of Supply issue**
   Problem Owner
   - investigates the EOS issue on the following aspects with all relevant parties:
     - Form, Fit Function;
     - Alternatives and their impact on products and installed base;
     - Risk during usage;
   - coordinates actions for analyzing the End of Supply issue using the action list in the Phase Out Plan;
   - initiates short-term supply actions (including stock taking) and informs supply stakeholders;
   - updates Phase Out Plan.
4. **Form, Fit, Function respected?**

   Problem Owner
   - decides whether form, fit, function is respected.
5. **Find alternative item (when Form, Fit, Function can not be respected)**
   Problem Owner
   - coordinates actions for releasing new components/ technology based on End of Supply issue;

   SOP (Tech) members
   - decide on releasing new components/ technology based on investigation results for use within VI products and systems (Note: final decision on actually using new components/technology is with Product Teams);
   - determine actions how to release new components / technology;

   Problem Owner
   - updates Phase Out Plan.
6. **Hand over new components/ technology**
   Problem Owner
   - hands over the released components / technology to the involved SOP-IG members;
   - if required, writes interchangeability document (using [4]) and Item Change Request [1];
   - provides training advice and support in using the new components / technology where needed;
   - initiates item status in JDE and SmarTeam;
   - initiates stock taking for spare parts when needed;
   - updates and finalizes the Phase Out Plan;

   Product Manager Service
   - informs CC's;

   Product Manager Engineering
   - submits product change requests (CR) according to instruction: product change management process [5];
   - submits item (change) requests when needed;

   Item Controller
   - changes item status in JDE and SmarTeam and closes EOS.

Appendix D – Analysis Procedure End-of-Supply

As shown in Appendix C, the EOS procedure involves many departments. One can easily imagine that it is not this easy to solve an EOS issue by simply suggesting a quantitative method to calculate the size of the final order quantity. As can be seen from the EOS procedure at VI, the EOS problem concerns more events and decisions which can be less controlled and in which the human aspect of the problem plays an important role.

The additional issues that play an important role in the EOS problem at VI, but that are hard to model or to solve, are discussed below. First, the departments that are involved in the EOS issues are described. Next, issues regarding the notification of the EOS are described. Also, some additional issues that influence an EOS issue (such as employees and their habits regarding the moment of starting to solve the EOS problem) are discussed.

D.1 The Involved Departments

As mentioned before, the EOS problem concerns many departments at VI. One could basically describe the roles of the involved departments as follows:

- *Business Unit Services*: The Business Unit Services has to be able to keep its customers' systems running for a long time. When an EOS occurs, Services basically suffers from this problem in the sense that they have to have this problem solved, otherwise they cannot provide service to their customers. When they cannot provide service, customer goodwill and reputation is lost. Next to this, the Business Unit Services is in control of ordering the items that are termed as spare parts within the company. When there is an EOS regarding a spare part, SCMS cannot or should not order the item anymore.
- *The Purchasing department*: The Purchasing department is in charge of ordering items that are needed in projects that the company is doing. Sometimes they order items for the Business Unit Services as well, but this basically only happens when the item the Business Unit Services needs is also needed for projects. In such situations, the Business Unit Services can benefit from the larger order quantities that the Purchasing department places.
- *The R&D department and the PLS department*: R&D can cause an EOS by using new versions of an item in the new products instead of the 'old' items. Since the company is specialized in delivering customized systems, the R&D department will do everything to satisfy the customer and to provide the most attractive system. This leads to a very low amount of standardization in the items used in systems. As soon as it is known that there is an EOS problem and an alternative item needs to be searched, it is the task of PLS to search for this alternative item. Since they are part of the R&D department, they know best what function the alternative item should be able to provide. It is their responsibility to find and approve an alternative item, e.g. when the Purchasing department thinks they found an appropriate alternative item at another supplier, the PLS department has to approve this item first.
- *The Engineering department*: This department is closely related to the R&D department in the sense that they make the specifications of the systems. Simply put, R&D invents some new item or (part of a) system, and Engineering has to come up with the specifications that make this new item work.
- *Other departments*: Next to the departments mentioned above several other employees are involved in the EOS problem as well. Any employee could signal the EOS issue. Next to that, an item controller has some duties regarding the EOS issue as well (see Appendix C).

So, there are many players involved and each player has its own role. Unfortunately, it became clear by interviewing employees from the different departments that the procedure described in Appendix C is not always well known (or even never heard of). This can slow down the process or even lead to faulty actions.

The decision to use an alternative item is not solely based on the opinion of someone from the R&D department. Each EOS issue that is perceived as urgent enough (e.g. it involves a large amount of money, the problem has to be solved really fast, etc.) is discussed within a group of people from all different departments described above, where each employee provides a certain input (e.g. Business Unit Services indicates the expected demand, R&D comes with a new supplier, etc.). This group is called the SOP (Steeringcommittee Operational Products) Team and its members decide what to do. Even if the alternative item that is found is FFF, it may be decided that R&D and/or Purchasing have to look for another alternative item because the one they found is simply too expensive.

EOS issues most often occur in the item group Controls. Controls mostly involves electronic equipment, such as power supplies. Next to that, the order of departments involved in a project normally is as follows:

1) Sales: The Sales department tenders at customers and tries to sell the company's system. Basically, if the tender is won, the Sales department sold a functionality rather than a product to the customer.
2) When a project is sold, the Engineering department is going to make the specifications which are needed to provide the promised functionality. This is done in close collaboration with the R&D department. In this phase, it is decided what items will be used in a project to be able to provide the functionality.
3) After the specifications are defined, the focus moves to the sourcing of items. Here, the Purchasing department or SCMS is involved. Because the needed items are already given, these departments are going to search for ways to source these items. It is in this phase that end-of-supplies are found out about.

One could say that this order only causes more EOS problems: In step 2) the specifications are made and the sourcing of the items in those specifications is not taken into account at all. Employees simply look in the database of items and pick the first item that meets their specifications. The database does not register well enough whether the item is still available or not: there are many different item statuses and unfortunately they are inconsistent as well. This inconsistency is being solved at this moment, but it is hard since VI just changed to a new IT system. Next to that, even if it would be registered well enough, employees who make the specifications simply do not think about the possibility of an end-of-supply. Thus, now items are used in specifications, while it is possible that in step 3) employees find out that an item in those specifications is not available anymore. This means that the drawings have to be changed again. It would be more logical that the sourcing part of a project is involved earlier in the project, because this would lead to specifications which include only items that truly are available. This can save considerable time and costs, because drawings do not have to be redrawn in such a situation. Luckily, this is done already for the Resale category. Regarding Resale items (e.g. scanners, scales), the order of events is as follows:

1) Regarding Resale items, the Purchasing department knows what the market offers.
2) Based on what is available in the market, the Sales department knows what is possible to offer to the customer.
3) Since the Sales department already knows what is possible and what is not, the specifications will always include items that are available in the market.
4) If, in a subsequent project, it turns out that a certain item is not available in the market anymore, the Purchasing department will look for alternative items. These alternative items then form the basis for Sales, etc.

Because the sourcing part of the process is involved way earlier in the project when considering Resale items than when considering items of other categories, there are almost no EOS problems in the Resale category. Luckily, the company tries to involve the Purchasing department earlier now in all projects for all kinds of items, so that the process for all items changes more and more to the one described for Resale items. This is also elaborated in the new process map that is implemented in the company. Previously, Engineering was done first and then Supply Chain (including Purchasing) was done. Now, both activities are depicted in parallel, which means that the company acknowledges that both activities should start at the same time and interact a lot to prevent problems such as the EOS problem. This new process map is depicted in Figure D.1.

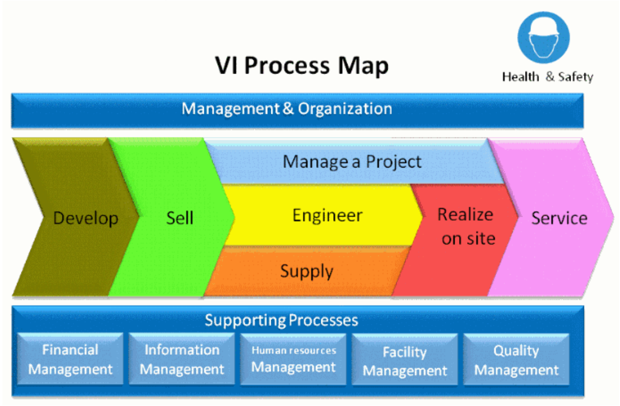


Figure D.1: Process map of Vanderlande Industries.

D.2 Issues regarding the EOS Notification

As mentioned in chapter 2, an EOS can be initiated by a supplier or by VI herself. When the supplier is the cause of the EOS, the following issues can occur:

- The supplier does not give a notification that the item will become an EOS. It is hard to make sure that a supplier will always give such a notification.
- Even if the supplier gives a notification, the employee who receives it does not communicate it to the right people and/or the item's status is not updated in the database. From interviews it appeared that employees do not really know who to inform about such a notification.
- There is no clear contact point for the supplier to give the notification to. A supplier can contact either the Purchasing department, the Business Unit Services, or both, because these departments are involved with the procurement of items. One can imagine that leaving an EOS notification at Purchasing may not be digested through the entire organization, which means that the Business Unit Services is still unaware of the (upcoming) EOS.
- Even if the right people know that an item will become EOS, it is possible that they do not act to solve the problem. For example, they do not perceive the EOS issue as important because it considers a standard product, or they think the EOS issue is very far ahead in the future which means it does not require any attention or effort already.
- As explained in the previous section, specifications of items are sometimes made before it is checked whether the items are still available or not. Extra work can be prevented by involving the Purchasing department during the specification of the items.

Unfortunately it is hard to measure in how many cases the supplier truly gave a notification and in how many cases the notification was communicated timely within VI. This is difficult because the notification is mostly done by phone or by email, which is hard to track back. Also, it is impossible to find out in how many cases the notification came too late, because hardly any notifications are registered in a database.
When VI is the cause of the EOS, the following issues can occur:

- It is not communicated well enough which item is not going to be used anymore.
- The status of the item is not updated in the database, which means employees will not see that the item is not going to be used anymore.
- When R&D decides to use a new version of the item, while there is still inventory of the original item, it should be decided what to do with the old versions of the items (are they still used to satisfy demand or are they scrapped?).

No matter what initiated the EOS situation, communication is key. A question that is relevant in both EOS situations, is whether the company proactively goes to the customer and says ‘we are going to replace item 1 in your system by item 2’ or first simply uses item 1 until all stock of item 1 is depleted and then the change in the customer’s systems is made to item 2.

D.3 Additional Issues

When there is a serious EOS issue, VI will have to find some source of an item. This alternative item can be FFF or not, it can be expensive or cheap, etc. The SOP-Team decides, based on these kinds of characteristics, whether to accept the alternative item or not. This decision, the searching for an alternative item (if needed) and maybe even changes to drawings or systems in the field take time. During this time, the original item is not available anymore because it is an EOS issue. Thus, a final order is needed to ‘buy time’. The longer this time period is, the larger the final order quantity will have to be. Also, the uncertainty in the length of this time period may play a role in the size of the final order quantity. Several issues may occur that influence the length of the time period and its uncertainty:

- The involved employees may not perceive the EOS issue as important. Somehow it should be proven to them, e.g. in costs that the company faces when not having an item available, that the problem is important enough to put time in.
- The involved employees see EOS issues as some additional task they have to do sometimes, but their core activities have nothing or little to do with EOS issues. So, when an EOS occurs this basically means extra work for them, which they most often do not like and want to finish as quickly as possible because they want to work on their core activities. This means for example that the R&D department searches for an alternative item and accepts the first item that provides the function they want, while the alternative item is not acceptable when price and delivery characteristics are taken into account.
- If employees would acknowledge the importance of EOS issues and put effort in it from the moment it is known that an item will become EOS, this means that the RTT can be shortened. E.g. if the EOS notification occurs before the true stop of supply, and employees start working on the EOS issue as soon as they receive the notification, they may have solved (part of) the EOS issue before the supply really stops. Thus by investing time immediately instead of waiting until the EOS comes really close, reduces the length of the time period that is necessary to find an alternative item and to decide on this item. This reduces the final order quantity.
- Even if the activities to solve the EOS issue would start at the moment of EOS (instead of immediately at the moment the notification is received), if the importance of the problem is clear to the employees it is likely that time period necessary to solve the EOS issue will be shorter than when they rather do their core activities because they think the problem is not important.
- When it is expected time period is really long, e.g. because it is very hard to change the existing systems in such a way that the alternative item can be used, it may be beneficial to not to invest time in trying to change to an alternative item. In such a situation it may be cheaper to cover the entire planning horizon by a final order for the original item.

Appendix E – Graphical Overview of the Problem Situations

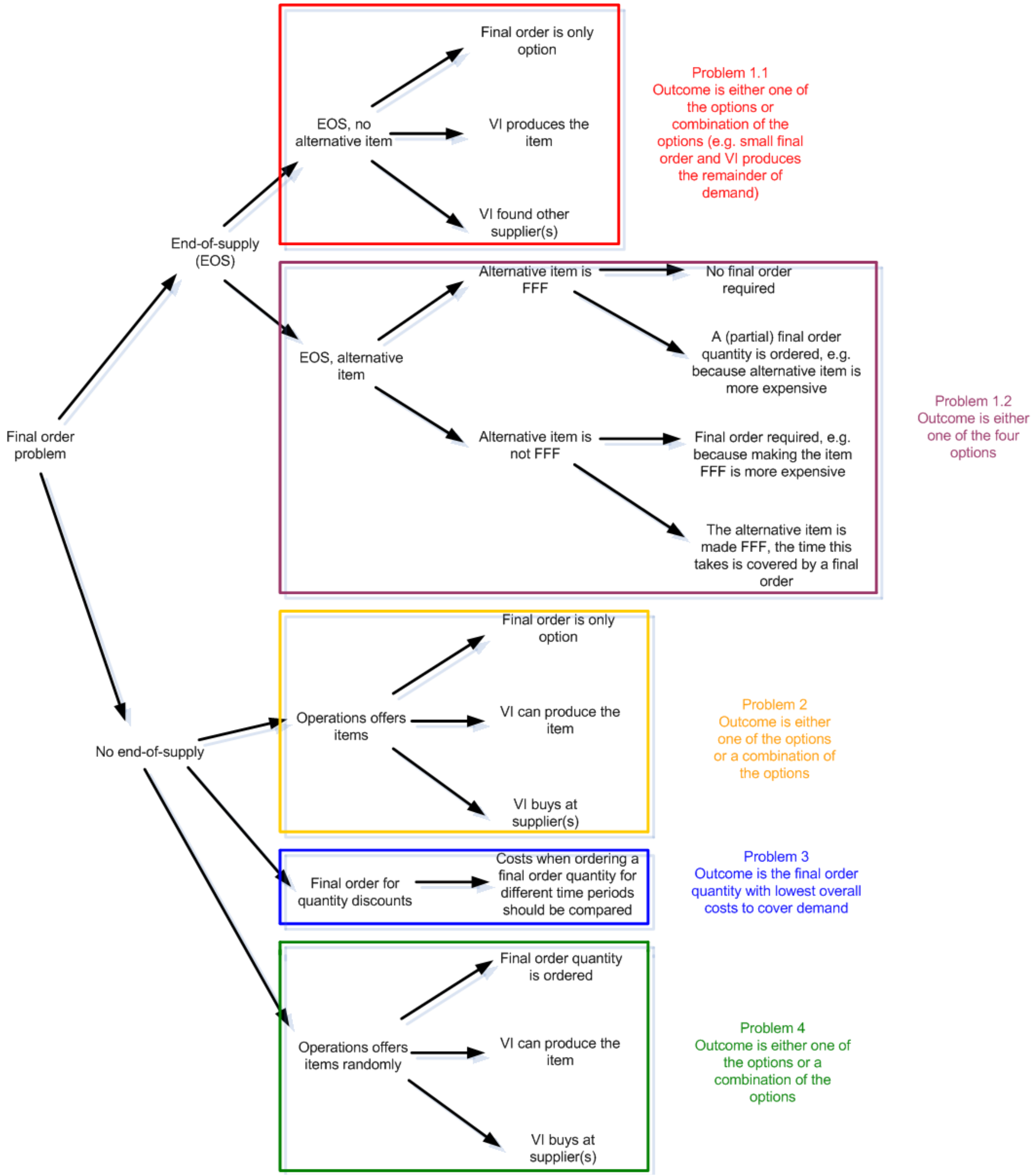


Figure E.1 Overview of the problem situations at Vanderlande Industries.

Appendix F – Overview of Used Notation

Note that item 1 resembles the original item and item 2 resembles the alternative item.

F.1 Simple Policy

F.1.1 Objective Function Simple Policy

The objective is to minimize the total expected discounted costs during the entire planning horizon:

$$E[TC(Q)] = \sum_{t=minRTT_a}^{EOL} E'\,[TC(Q,t)] * P(RTT_a) + \sum_{t=EOL+1}^{maxRTT_a} E'\,[TC(Q,t)] * P(RTT_a = t)$$

$E[TC(Q)]$	Total expected discounted costs during the entire planning horizon for a simple policy using a final order quantity Q *(€)*
$E'[TC(Q,t)]$	Total expected discounted costs for a simple policy using final order quantity Q, given that the RTT is equal to t periods *(€)*
EOL	Length of the End-of-Life (service) period (EOL) / planning horizon *(# periods)*
$maxRTT_a$	The maximum possible value for the length of the RTT for a given alternative a *(# periods)*
$minRTT_a$	The minimum possible value for the length of the RTT for a given alternative a *(# periods)*
$P(RTT_a = t)$	The probability that the RTT for a given alternative a is equal to t periods
t	A specific value for the length of the RTT, which can range between $minRTT_a$ and $maxRTT_a$ *(# periods)*

To calculate $E'[TC(Q,t)]$, the following variables are used:

$E[TC_k(I_k)]$	The total expected discounted costs during the periods $k, \dots, t$, where $k = 1, \dots, t$. These costs are calculated for a specific value for I_k.
I_k	The inventory of item 1 directly at the beginning of period k, where $k = 1, \dots, t$.

$E'[TC(Q,t)] = E[TC_1(Q)]$

F.1.2 Decision Variables Simple Policy

Q	The final order quantity, i.e. the quantity of item 1 that is ordered to satisfy demand during the RTT *(units)*

F.1.3 Input Parameters Simple Policy

F.1.3.1 Cost Parameters

α	Fixed yearly discounting factor *(dimensionless)*
a	Fixed discounting factor per period *(dimensionless)* $\propto = -ln(a^{1/m})$
$\widehat{CDi_k}$	Estimated disposal cost of item i in period k *(€)* $i \in \{1,2\}$, where $i = 1$ denotes item 1 and $i = 2$ denotes item 2 $k = 1, \dots, EOL$
$\widehat{CHi_k}$	Estimated holding cost rate for item i in period k *(€/unit/year)* $i \in \{1,2\}$, where $i = 1$ denotes item 1 and $i = 2$ denotes item 2 $k = 1, \dots, EOL$
$CN1$	Purchasing cost of item 1, i.e. the price that is paid per unit of item 1 *(€)*
$CO1$	Fixed order costs per order for item 1 *(€)*
$E[CN2_k]$	Expected value of the purchasing cost of item 2 in period k *(€)*
$E[CO2]$	Expected value of the setup costs for item 2 *(€)*
$\widehat{PI}$	Estimated yearly price increase *(%)*

F.1.3.2 Time Parameters

EOL	Length of the End-of-Life (service) period (EOL) / planning horizon *(# periods)*
RTT	Length of the required transition time (RTT) *(# periods)*
m	User specified length of a period *(years)* Example: when each period is 6 months, $m = 1/2$
$P(RTT_a = t)$	The probability that the RTT for a given alternative a is equal to t periods
$minRTT_a$	The minimum possible value for the length of the RTT for a given alternative a *(# periods)*
$maxRTT_a$	The maximum possible value for the length of the RTT for a given alternative a *(# periods)*
$meanRTT_a$	The mean value for the length of the RTT for a given alternative a *(# periods)*

F.1.3.3 Demand Parameters

d_k	Demand in period $k = 1, \dots, EOL$ *(units)*

F.1.3.4 Initial Stock

$S[1]$	The initial amount of units of item 1 on stock at the start of the RTT *(units)* *Note that this variable is not included in the model descriptions. If the initial stock is positive the optimal final order quantity is simply subtracted by this value.*

F.1.4 Restrictions Simple Policy

BS_s	The batch size that is specified by the supplier of item 1 *(units)* Q should always be a multiplication of BS_s.
$E[fillrate(Q)]$	The expected fillrate for a given final order quantity Q
$E[OOS_k(I_k)]$	The expected number of out-of-stocks during the periods k up to and until t, given an inventory at the start of period k that is equal to I_k. This number is used to calculate $E'[OOS(Q,t)]$.
$E'[OOS(Q,t)]$	The expected number of out-of-stocks during an RTT of length t. This number is used to calculate $fillrate(Q,t)$.
$fillrate(Q,t)$	The fillrate during an RTT of length t, given a final order quantity Q The fillrate during the RTT should always be equal to or bigger than the desired fillrate: $fillrate(Q,t) \geq desiredfillrate$.
MOQ	The minimum order quantity for item 1 *(units)* $Q \geq MOQ$

F.2 Remove Policy

F.2.1 Objective Function Remove Policy

The objective is to minimize the total expected discounted costs during the entire planning horizon:

$$E[TC(Q,U_1,U_2,\dots,U_{maxtime})] = \sum_{t=minRTT_a}^{EOL} E'\left[TC(Q,U_1,\dots,U_t,t)\right] * P(RTT_a)$$

$$+ \sum_{t=EOL+1}^{maxRTT_a} E'\left[TC(Q,U_1,\dots,U_{EOL},t)\right] * P(RTT_a = t)$$

$E[TC(Q,U_1,U_2,\dots,U_{maxtime})]$	Total expected discounted costs during the entire planning horizon for a remove policy using a final order quantity Q and remove-down-to levels $U_1,\dots,U_{maxtime}$ *(€)*
$E'[TC(Q,U_1,\dots,U_t,t)]$	Total expected discounted costs during the entire planning horizon for a remove policy using a final order quantity Q and remove-down-to levels $U_1,\dots,U_t$, given that the RTT is equal to t periods *(€)*

EOL	Length of the End-of-Life (service) period (EOL) / planning horizon *(# periods)*
$maxRTT_a$	The maximum possible value for the length of the RTT for a given alternative a *(# periods)*
$maxtime$	The minimum value of the EOL and $maxRTT$. This minimum is the maximum number of periods for which remove-down-to levels should be calculated. If $t < EOL$, then remove-down-to levels are needed for t periods. If $t \geq EOL$, then remove-down-to levels are needed for EOL periods.
$minRTT_a$	The minimum possible value for the length of the RTT for a given alternative a *(# periods)*
$P(RTT_a = t)$	The probability that the RTT for a given alternative a is equal to t periods
t	A specific value for the length of the RTT, which can range between $minRTT_a$ and $maxRTT_a$ *(# periods)*

This objective function is reduced to the following objective function, because it is impossible to calculate useful remove-down-to levels when the length of the RTT is uncertain:

$$E[TC(Q, U_1, U_2, \ldots, U_t)] = E'[TC(Q, U_1, U_2, \ldots, U_t, t)]$$

To calculate $E'[TC(Q, U_1, \ldots, U_t, t)]$ the following variables are used:

$E\left[TC_k\left(I_k, U_k(I_k)\right)\right]$	Total expected discounted costs during the periods $k, \ldots, t$, where $k = 1, \ldots, t$. These costs are calculated for a specific value for I_k and $U_k(I_k)$.
I_k	The inventory of item 1 directly at the beginning of period k. Here, $I_1 = Q$ and $I_k = 0, \ldots, maxdemand(k)$. $maxdemand(k)$ is the maximum demand during the periods $k+1, \ldots, EOL$.
$U_k(I_k)$	The remove-down-to level effective at the end of period k, where the remove-down-to level is dependent on I_k. $U_k(I_k)$ is used to calculate U_k (the remove-down-to level of period k that is independent of I_k).

To calculate U_k, the expected inventory of item 1 at the start of period k is calculated. This is denoted by:

$E[I_k]$	The expected inventory of item 1 directly at the beginning of period k, where $E[I_1] = I_1 = Q$, because the inventory of item 1 at the start of the first period is exactly equal to the final order quantity Q.

Then it holds that $U_k = U_k(E[I_k])$. These remove-down-to levels are the input to the final cost calculation. Up to now all calculations have been done with remove-down-to levels that were dependent on I_k. Now, the costs are calculated based on U_k instead of $U_k(I_k)$. To calculate these costs the following variables are used:

$E[TC_k(I_k, U_k)]$	Total expected discounted costs during the periods $k, \ldots, t$, where $k = 1, \ldots, t$. These costs are calculated for a specific value for I_k and U_k.

The total expected costs for the remove policy during a RTT that is equal to t periods is: $E'[TC(Q, U_1, \ldots, U_t, t)] = E[TC_1(Q, U_1)]$. Note that, due to the fact that the model is a stochastic dynamic programming model, the remove-down-to levels $U_2, \ldots, U_t$ are included in the cost calculation for $E[TC_1(Q, U_1)]$.

F.2.2 Decision Variables Remove Policy

Q	The final order quantity, i.e. the quantity of item 1 that is ordered to satisfy demand during the RTT *(units)*
U_k	The remove-down-to level effective at the end of period $k = 1, \ldots, maxRTT$ *(units)*

F.2.3 Input Parameters Remove Policy

See section E.1.3.

F.4.2 Restrictions Remove Policy

BS_s	The batch size that is specified by the supplier of item 1 *(units)* Q should always be a multiplication of BS_s.
$E[fillrate(Q, U_1, U_2, \ldots, U_{maxtime})]$	The expected fillrate for a given final order quantity Q and a given set of remove-down-to levels $U_1, \ldots, U_{maxtime}$.
$E\left[OOS_k\left(I_k, U_k(I_k)\right)\right]$	Total expected number of out-of-stocks during the periods $k, \ldots, t$, where $k = 1, \ldots, t$. These costs are calculated for a specific value for I_k and $U_k(I_k)$.
$fillrate_k(I_k, U_k(I_k), t)$	Total fillrate during the periods $k, \ldots, t$, where $k = 1, \ldots, t$. These costs are calculated for a specific value for I_k and $U_k(I_k)$. The fillrate during the entire RTT is given by $fillrate_1\left(Q, U_k(Q)\right)$. The fillrate during the RTT should always be equal to or bigger than the desired fillrate: $fillrate_1\left(Q, U_k(Q)\right) \geq desiredfillrate$.
MOQ	The minimum order quantity for item 1 *(units)* $Q \geq MOQ$
$U_k \leq U_{k-1}$	The remove-down-to level of period k should be equal to or smaller than the remove-down-to level of period $k - 1$.

Appendix G – Case Study & Results

G.1 Case Study Selection

The case study concerns an item that has been an end-of-supply (EOS) issue for several years now, and still no decision has been made about it. The information regarding the item can be used in the Excel tool to check which decision would have been made based on the model that is developed during this project.

The EOS issue considered for this case study concerns a so-called QCB-box. The QCB is end-of-supply since September 1st, 2008. At that moment VI had about 30 QCBs on stock. Therefore, VI decided to search for an alternative item. The supplier of the QCB is not forced to provide a possible alternative item, because this supplier was already an emergency solution. Before, the QCB was supplied by another supplier. After that supplier announced the EOS, VI switched to the next supplier. However, now that the components of the QCB are hard to get, which leads to unattractive supply conditions, VI has a problem again. Although the item became EOS several years ago, the EOS issue is still not solved. There still is no alternative item found that fits all requirements. In the meantime the QCB is not ordered anymore, because of the reasons given above.

The fact that the QCB is an EOS issue results from another EOS issue at VI: the microcontroller of the QCB is not supplied anymore. VI found this out on June 12th, 2007. However, the supplier already stopped supplying this item on March 31st, 2007. This meant that VI could not order the microcontroller anymore. Without this microcontroller, the QCB does not work and this also holds the other way around.

This means that this case study actually considers two items that are EOS:

1) The QVeyor Controller Box (QCB)
2) The microcontroller of the QCB.

Currently, there is an alternative available but this is not form-fit-function. This alternative item is a complete solution for both the QCB-box and the microprocessor. So replacing either a QCB-box or a microcontroller with this alternative item means that the other item is replaced as well. Therefore, it is decided to continue the case study regarding the combination of the QCB and the microcontroller. For both of these items the following is known:

Table G.1: Information regarding the QCB and the microcontroller.

Item	Unit Cost (€)	Stock at moment EOS (units)	Safety Stock (units)	Demand 2009 (units)	Demand 2008 (units)	Demand 2007 (units)	Demand 2006 (units)	Demand 2005 (units)
QCB-box	250,55	30	25	125	141	159	112	165
Microcontroller	19,16	0	0	133	200	149	85	170
Combination	269,71	0	0	258	341	308	197	335

To obtain the demand for the 'combination' the demand of the QCB and the microcontroller are summed and rounded to the closest integer. The demands of both items are summed, since no matter which item of the two needs to be replaced, the alternative item replaces both. Next to this, the item was discussed with employees at VI. Together with them, the input parameters of the model (see section 3.6 and Appendix F) are set as follows:

- The planning horizon to be considered is 10 years. Since only information regarding demand per year is available, the length of a period is 1 year.
- The EOS decision for the combination of both items has been made on January 1st 2009. This means that demand has to be forecasted up to and until 2018.

- The MOQ for the combination of the QCB and the microcontroller is 0. Also, there is no batch size specified by the supplier (see section 3.5 for an explanation of these restrictions).
- The fixed order costs are €20.
- The holding cost rate per unit per period is 20% of the unit cost of the item.
- The disposal cost per unit disposed is €0.
- The yearly discounting factor is 4%.
- The yearly price increase of the alternative item is expected to be 3%.
- At the moment the decision has been made, the expected price for one unit of the alternative item was €600.
- It is expected that the alternative can be used after 2 years, so it can be used to satisfy demand from January 1st 2011. This means that the case study concerns a fixed required transition time that is equal to 2 years.
- The expected setup costs for the alternative item are estimated to be €20000.

Since a planning horizon of 10 years is considered, demand has to be forecasted. Regarding the expected demand in the upcoming years, this forecasting is done by drawing a linear trend line through the known demand information and following this trend for the upcoming years. This can be depicted as follows:

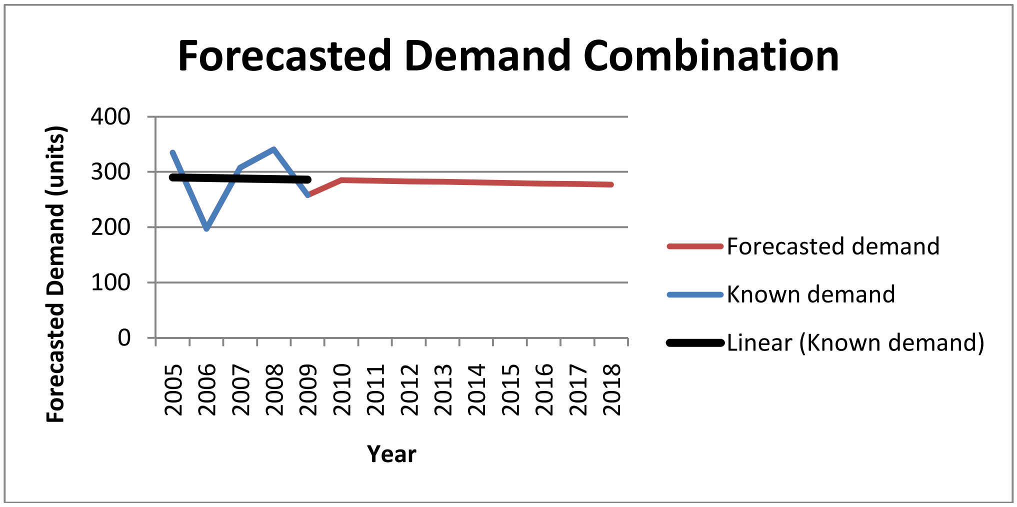


Figure G.1: Demand forecasts for the combination of both items, based on a linear trend line.

Following the linear trend line, it is expected that the mean demand for the combination of both items will slightly decline over time. Note that this is a simplistic way of forecasting demand. The demand in Figure seems to be seasonal, which means that a linear trend line would be inappropriate. However, forecasting demand is not the focus of this project. These simple forecasts are done to illustrate the case study. If employees have more information regarding future demand or expect different demand patterns, this should obviously be taken into account when forecasting demand. For now, all the available information that is available is demand during 2005-2009, so all this information is used to forecast demand.

The forecasted values for the expected demand are rounded to the next integer, since the model can only handle integer values for demand. Next to that, the Uniform distribution is chosen to be the probability distribution for the demand per period. Here, the minimum demand per period is set to 90% of the expected demand, and the maximum demand per period is set to 110% of the expected demand. The resulting forecasted demand characteristics are:

Table G.2: Demand forecasts for the combination during the entire planning horizon (demand 2009 is known).

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Expected demand	258	285	284	283	282	281	280	279	278	277
Minimum demand	232	257	256	255	254	253	252	251	250	249
Maximum demand	284	313	312	311	310	309	308	307	306	305

Finally, the company only wants to know a final order quantity and does not need remove-down-to levels. This means that the simple policy is in place. In the next section all the above information used is in the model to generate results.

G.2 Case Study Results

The parameter setting given in section G.1 is used in two different models: the model for the situation in which the original item cannot be used after the RTT and the model for the situation in which the original item can be used after the RTT. Running these models results in the following outcomes:

Table G.3: Results case study.

	Original item cannot be used after RTT	*Original item can be used after RTT*
Optimal final order quantity	540 units	1924 units
Expected total discounted costs during RTT	€160278,58	€605285,63
Expected purchasing costs during RTT	€145643,39	€518922,03
Expected ordering costs during RTT	€20	€20
Expected holding costs during RTT	€14165,19	€86343,62
Expected disposal costs during RTT	€0	€0
Expected number of out-of-stocks during RTT	10,75	0
Fillrate during RTT	98,019%	100%
Expected total discounted costs after RTT	€1291871,30	€689786,38
Expected total discounted costs during entire planning horizon	€1452149,88	€1295072

Note that an optimal final order quantity of 540 units (for the situation where the original item cannot be used after the RTT) in this case study means that there should be 540 ‘combinations’ of the QCB and the microcontroller. After all, it is the complete functionality of the two items combined together that is EOS and for which an alternative exists. This means that the model advises VI to buy 540 QCBs and 540 microcontrollers to cover the demand during the required transition time. After the required transition time, which is equal to two years, the alternative item can be used to satisfy demand.

Running the model for the situation in which the original item can be used after the RTT resulted in an optimal final order quantity of 1924 units and the total expected discounted costs during the planning horizon are €1295072. In the base case, the alternative item is relatively expensive in comparison to the original item. Therefore, the optimal final order quantity is much larger than the final order quantity that is needed to meet the fillrate restriction (which is 540 units). Indeed, buying more units of the original item resulted in lower total costs than when all demand after the RTT should be satisfied by the expensive alternative item.

Appendix H – Effects Input Parameters for Simple Policy when Original Item can be used after RTT

In this appendix the effects of some of the input parameters on the optimal final order quantity and the total expected discounted costs during the entire planning horizon are researched for the situation where the original item can be used after the RTT. Note that this appendix merely gives examples of the possible effects: all the outcomes are based on the base case as given in Appendix G. Therefore, no general conclusions can be based on these results. The results merely indicate what can happen when the value for an input parameter is changed.

In this appendix one cost parameter is researched for each item (e.g. for the original item we vary the holding cost rate to check the effects). Investigating more cost parameters per item will result in similar effects (e.g. increasing the holding cost rate will have a similar effect as increasing the price of the item). Next to that the effects of the fillrate restriction and the characteristics of the length of the RTT are discussed. For the original item, the effects of the holding cost rate will be analyzed in section H.1. In section H.2, the effects of the expected price of the alternative item are described. After this, the effects of the fillrate restriction are addressed in section H.3. In section H.4 the effects of varying the fixed length of the RTT are discussed. Finally, the characteristics of the probability distribution for the length of the RTT are discussed in section H.5.

H.1 Holding Cost Rate

In this section the base case settings as given in Appendix G are used. However, the value for the holding cost rate is varied. The base case setting for the holding cost rate is 20%. When this holding cost rate is varied from 0% to 40%, the following results are obtained:

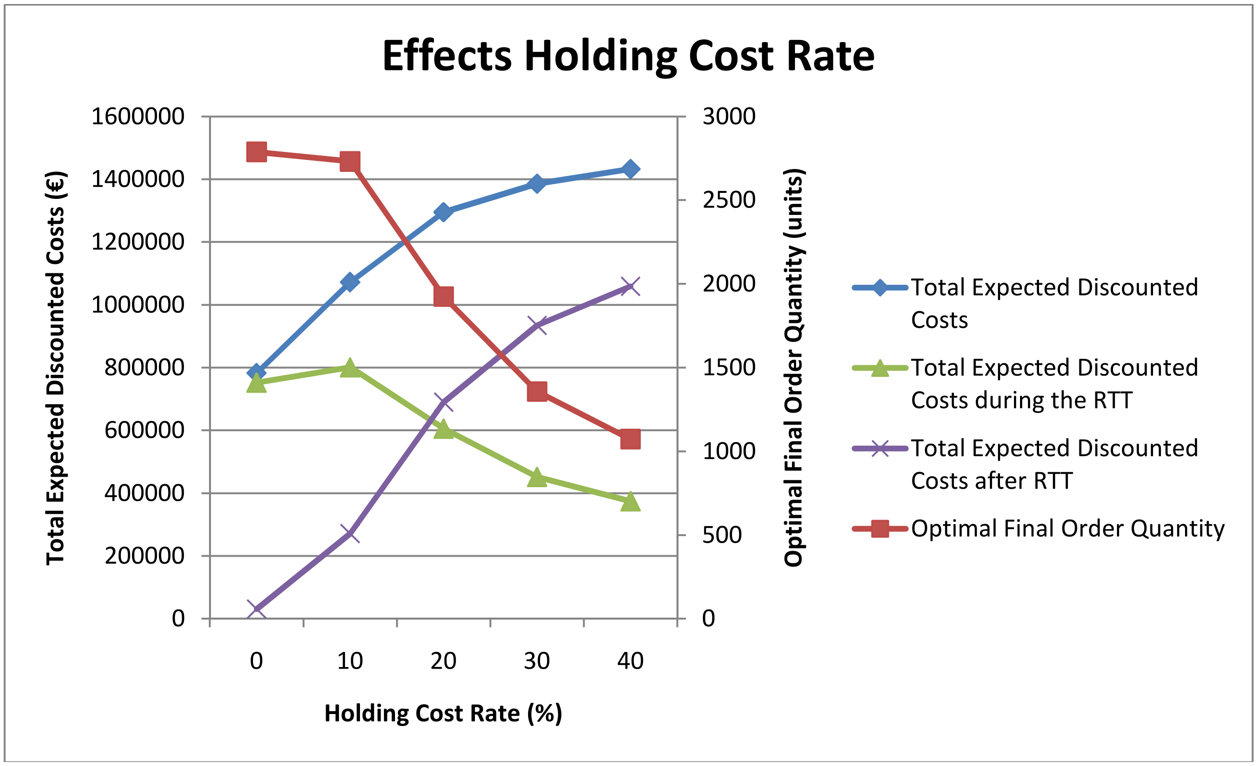


Figure H.1: Effects of the holding cost rate on the optimal final order quantity and the total expected discounted costs during the planning horizon.

From Figure it can be seen that when the holding cost rate increases (decreases), the optimal final order quantity will decrease (increase). This is the expected behavior: after all, when the holding cost rate is increased (decreased), keeping units of the original item in stock costs more (less), which is unattractive (attractive). Because the alternative item is relatively expensive in comparison to the original item in the base case, the total expected discounted costs increase when the final order quantity decreases.

H.2 The Expected Price of One Unit of the Alternative Item

In this section the base case settings as given in Appendix G are used, but the value for the expected price of the alternative item is varied. The base case setting for the expected price of the alternative item is €600. When this expected price of the alternative item is varied from €0 to €1200, the following results are obtained:

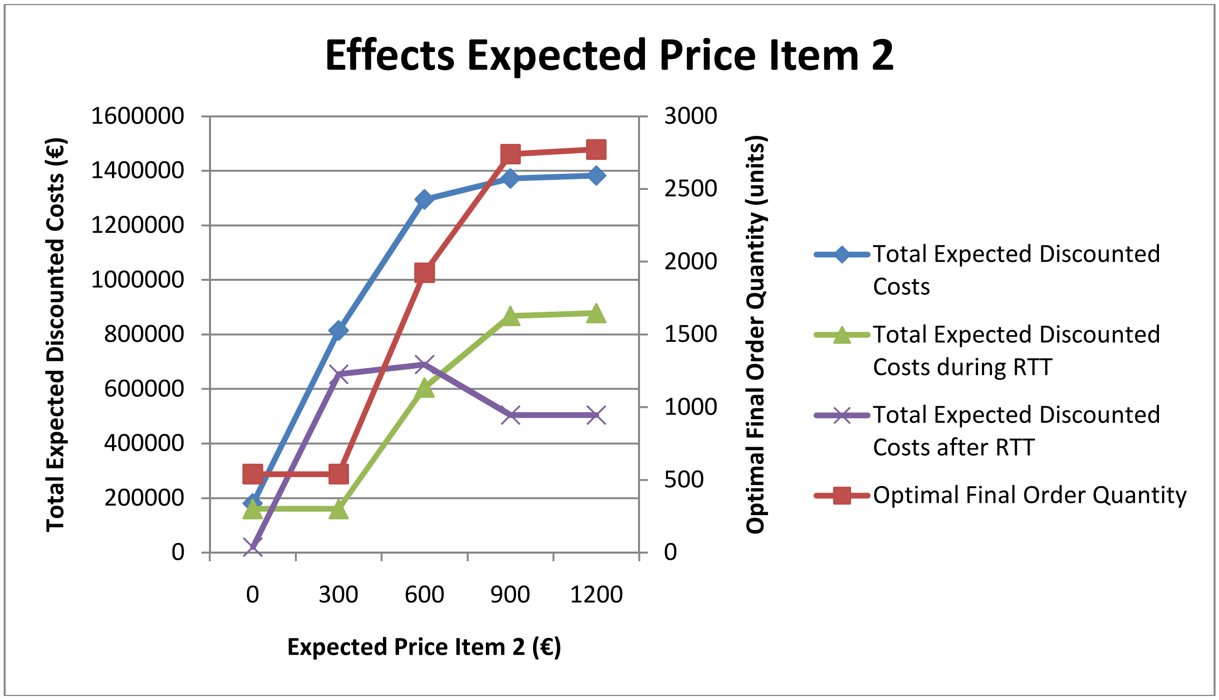


Figure H.2: Effects of the expected price of the alternative item on the optimal final order quantity and the total expected discounted costs during the planning horizon.

From Figure H.2 it can be seen that when the expected price of the alternative item increases (decreases), the optimal final order quantity increases (decreases) as well. After all, when the alternative item becomes cheaper, it is more attractive to switch earlier to the alternative item. However, the optimal moment to switch to the alternative item, which depends on the optimal final order quantity for the original item, depends on the cost parameters for the original item as well. Note that the total costs after the RTT decrease when the price of the alternative item is higher than €600. This is because the alternative item is that expensive then, that the final order quantity for the original item is increased and less units of the expensive alternative item have to be bought to satisfy demand.

H.3 Fillrate Restriction

In this section the base case settings as given in Appendix G are used, but the value for the fillrate restriction is varied. The base case setting for the fillrate restriction is 98%. Regarding the fillrate restriction, the following values are used in the model: 80%, 90%, 95%, 97%, 98%, 99% and 100%. The results for these settings can be depicted as follows:

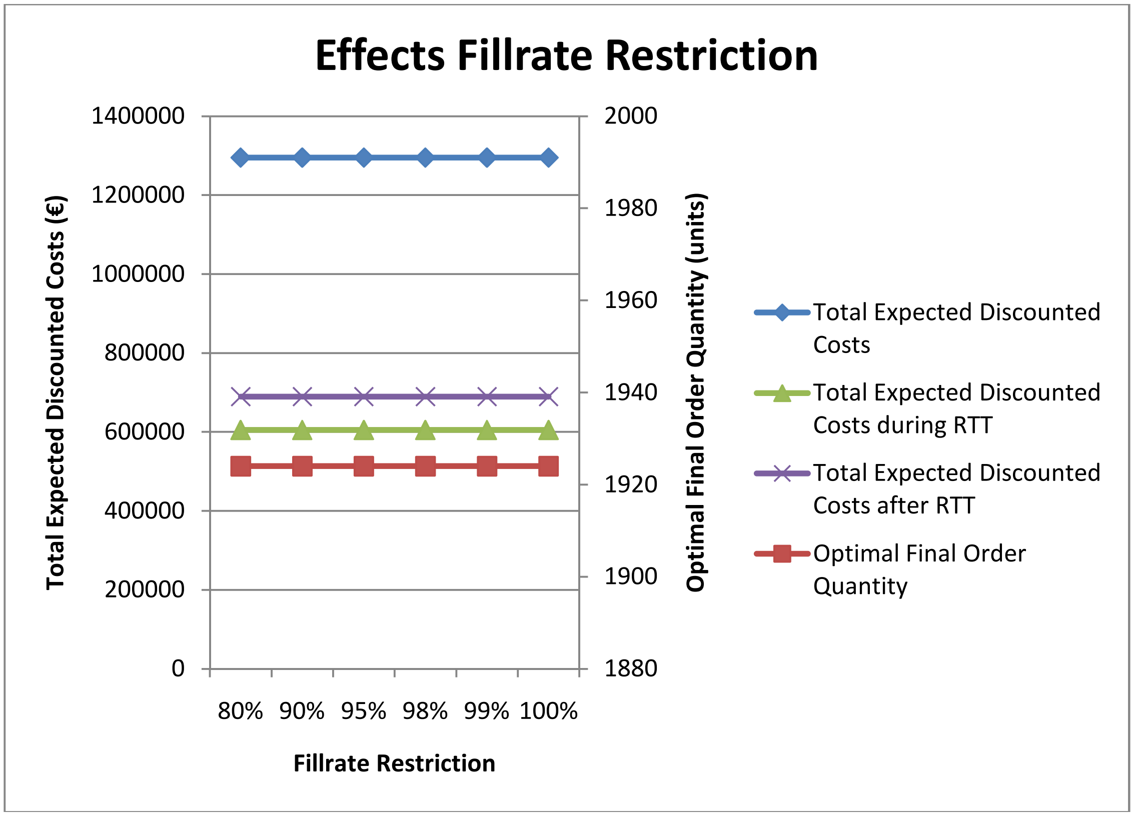


Figure H.3: Effects of the fillrate restriction on the optimal final order quantity and the total expected discounted costs during the planning horizon.

From Figure H.3 it can be seen that the fillrate restriction does not influence the optimal final order quantity and the total expected discounted costs during the planning horizon. Note that this can only be said for these specific parameter settings! In the parameter settings of the base case (see Appendix G) the alternative item is relatively expensive in comparison to the original item. Therefore, the model chooses for an optimal final order quantity that is much larger than the final order quantity that is needed to meet the fillrate restriction. Apparently it is cheaper to stock more units of the original item than to satisfy all demand after the RTT by buying the alternative item. Since the difference between the optimal final order quantity (1924 units) and the final order quantity that is needed to meet all the restrictions (540 units, see section G.2) is large, the fillrate restriction does not affect the optimal final order quantity and the total costs. One can imagine that when the values for the input parameters differ from the base case setting, the results that are obtained from the model can be very different from the ones described above as well.

H.4 Fixed Length of the RTT

In this section the base case settings as given in Appendix G are used, but the value for the fixed length of the RTT is varied. The base case setting for the fixed length of the RTT is 2 years. In this example, the fixed length of the RTT is varied between 0 years and 4 years. The results for these settings can be depicted as follows:

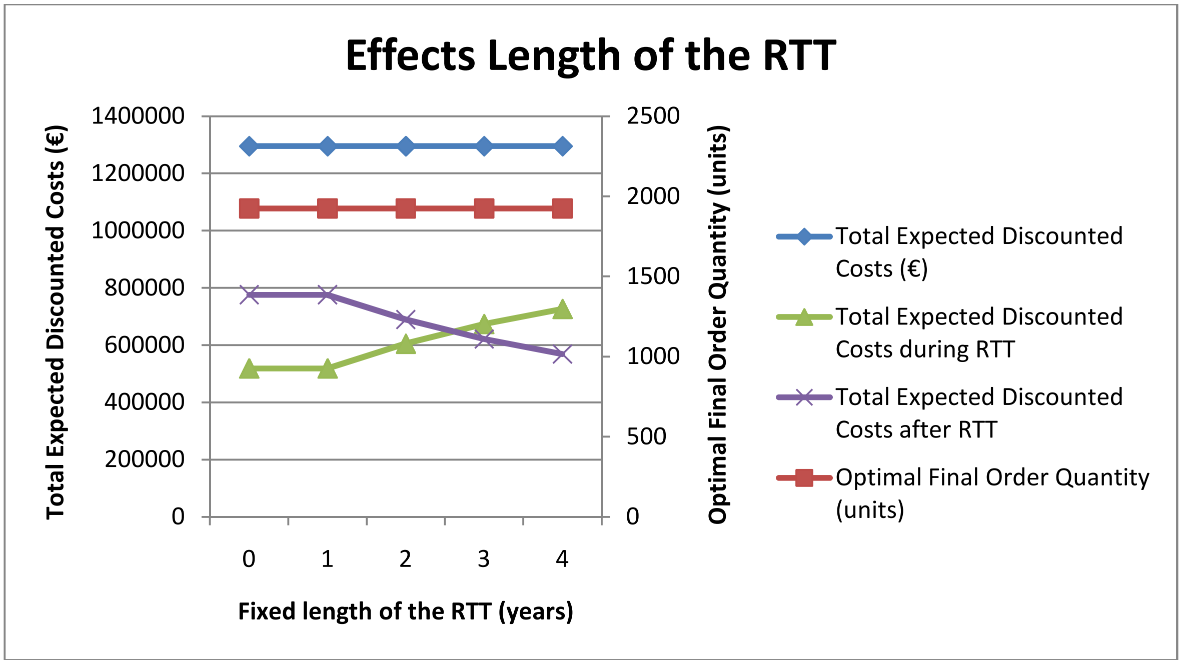


Figure H.4: Effects of the fixed length of the RTT on the optimal final order quantity and the total expected discounted costs during the planning horizon.

The effects of the fixed length of the RTT are similar to the effects of the fillrate restriction (see section H.3). In the example of the base case, the alternative item is relatively expensive in comparison to the original item. Therefore, the model increases the optimal final order quantity to a value that is much higher than the final order quantity that is needed to meet all restrictions. Because the optimal final order quantity is that large, varying the RTT between 0 and 4 years does not affect the optimal final order quantity at all; the time period that is covered by the optimal final order quantity is much longer than 4 years. When the optimal final order quantity does not change, this automatically means that the total expected discounted costs do not change as well. Note that the total expected discounted costs during an RTT that is equal to 0 years are positive. This is because the fixed order costs, the expected setup costs, and the purchasing costs of the final order are incurred immediately. Also note that the results in Figure H.4 only hold for the base case settings. One can imagine that when the values for the input parameters differ from the base case settings, the results that are obtained from the model can be very different from the ones described above as well.

H.5 Characteristics of the Probability Distribution for the length of the RTT

In this section the base case settings as given in Appendix G are used, but now a probability distribution for the length of the RTT is used. The base case setting for the fixed length of the RTT is 2 years. Now the model is used where the length of the RTT follows a Uniform(2,2) distribution, a Uniform(1,3) distribution, or a Uniform(0,4) distribution. Here, the values between the brackets are the minimum length of the RTT (years) and the maximum length of the RTT (years). Note that all probability distributions have the same expected value, which is equal to 2 years. The only thing that is varied for these experiments is the spread in the possible values for the length of the RTT. Note that a Uniform(2,2) distribution has no spread at all: its minimum and maximum value are equal to 2, so the length of the RTT must be 2 years. This means that the results of the Uniform(2,2) should be equal to the results of the base case. The results for these probability distributions for the length of the RTT can be depicted as follows:

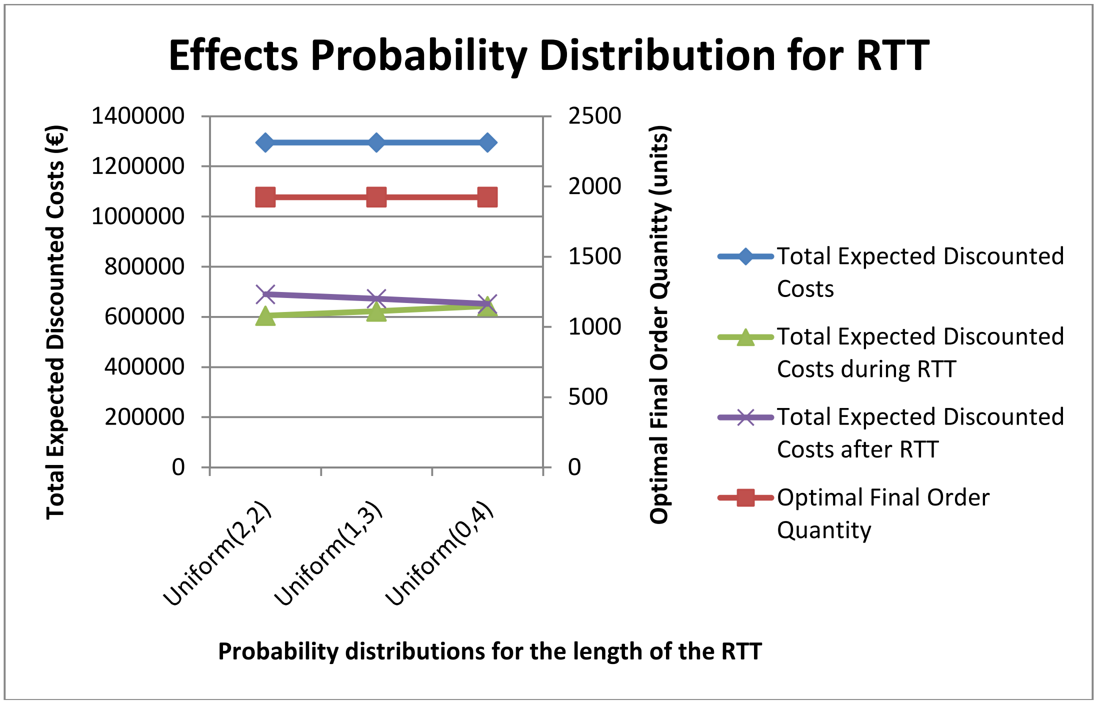


Figure H.5: Effects of probability distributions for the length of the RTT on the optimal final order quantity and the total expected discounted costs during the planning horizon.

The effects of the probability distribution for the length of the RTT are similar to the effects of varying the fixed length of the RTT (see section H.4). In the example of the base case, the alternative item is relatively expensive in comparison to the original item. Therefore, the model increases the optimal final order quantity to a value that is much higher than the final order quantity that is needed to meet all restrictions. Because the optimal final order quantity is that large, varying the probability distribution for the length of the RTT such that its maximum possible value is 4 years does not affect the optimal final order quantity at all; the time period that is covered by the optimal final order quantity is much longer than 4 years. When the optimal final order quantity does not change, this automatically means that the total expected discounted costs do not change as well. Note that the total expected discounted costs during an RTT that is equal to 0 years are positive. This is because the fixed order costs, the expected setup costs, and the purchasing costs of the final order are incurred immediately. Also note that the results in Figure H.5 only hold for the base case settings. One can imagine that when the values for the input parameters differ from the base case settings, the results that are obtained from the model can be very different from the ones described above as well.

Appendix I – Results Heuristic and Exact Method Remove Policy

Table I.1: Results for the heuristic and the exact method for the remove policy for a planning horizon of 1 period.

Distribution demand per period	Uniform(0,1)		Uniform(0,3)		Uniform(0,5)		Uniform(0,8)	
Heuristic (H) / Exact method (E)	H	E	H	E	H	E	H	E
Optimal Final Order Quantity (units)	1	1	3	3	5	5	8	8
Costs during RTT (€)	289,71	289,71	829,13	829,13	1368,55	1368,55	2177,68	2177,68
Costs after RTT (€)	20000	20000	20000	20000	20000	20000	20000	20000
Total Costs (€)	20289,71	20289,71	20829,13	20829,13	21368,55	21368,55	22177,68	22177,68
Fillrate (%)	100	100	100	100	100	100	100	100
Calculation time	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00
Remove-down-to level 1(units)	0	0	0	0	0	0	0	0
Difference in total costs (%)	0		0		0		0	

Distribution demand per period	Uniform(0,10)		Uniform(0,13)		Uniform(0,15)	
Heuristic (H) / Exact method (E)	H	E	H	E	H	H
Optimal Final Order Quantity (units)	9	9	12	12	14	14
Costs during RTT (€)	2447,39	2447,39	3256,52	3256,52	3795,94	3795,94
Costs after RTT (€)	20000	20000	20000	20000	20000	20000
Total Costs (€)	22447,39	22447,39	23256,52	23256,52	23795,94	23795,94
Fillrate (%)	98,182	98,182	98,901	98,901	99,167	99,167
Calculation time	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00
Remove-down-to level 1(units)	0	0	0	0	0	0
Difference in total costs (%)	0		0		0	

Distribution demand per period	Uniform(0,18)		Uniform(0,20)	
Heuristic (H) / Exact method (E)	H	E	H	E
Optimal Final Order Quantity (units)	16	16	18	18
Costs during RTT (€)	4335,36	4335,36	4874,78	4874,78
Costs after RTT (€)	20000	20000	20000	20000
Total Costs (€)	24335,36	24335,36	24874,78	24874,78
Fillrate (%)	98,246	98,246	98,571	98,571
Calculation time	0:00:00	0:00:01	0:00:00	0:00:01
Remove-down-to level 1(units)	0	0	0	0
Difference in total costs (%)	0		0	

Table I.2: Results for the heuristic and the exact method for the remove policy for a planning horizon of 3 periods.

Distribution demand per period	Uniform(0,1)		Uniform(0,3)		Uniform(0,5)		Uniform(0,8)	
Heuristic (H) / Exact method (E)	H	E	H	E	H	E	H	E
Optimal Final Order Quantity (units)	3	3	7	7	12	12	18	18
Costs during RTT (€)	982,58	982,58	2317,00	2317,00	3882,42	3882,42	5927,89	5861,93
Costs after RTT (€)	20000	20000	20000	20000	20000	20000	20000	20000
Total Costs (€)	20982,58	20982,58	22317,00	22317,00	23882,42	23882,42	25927,89	25861,93
Fillrate (%)	100	100	98,264	98,264	98,148	98,148	98,525	98,045
Calculation time	0:00:00	0:00:01	0:00:00	0:00:00	0:00:00	0:00:00	0:00:00	0:00:17
Remove-down-to level 1 (units)	2	2	6	6	8	8	15	13
Remove-down-to level 2 (units)	1	1	3	3	5	5	8	8
Remove-down-to level 3(units)	0	0	0	0	0	0	0	0
Difference in total costs (%)	0		0		0		0,255	

Distribution demand per period	Uniform(0,10)		Uniform(0,13)		Uniform(0,15)	
Heuristic (H) / Exact method (E)	H	E	H	E	H	E
Optimal Final Order Quantity (units)	22	22	29	28	33	32
Costs during RTT (€)	7231,60	7201,87	9440,46	9198,70	10785,42	10544,37
Costs after RTT (€)	20000	20000	20000	20000	20000	20000
Total Costs (€)	27231,60	27201,87	29440,46	29198,70	20785,42	30544,37
Fillrate (%)	98,272	98,122	98,206	98,045	98,214	98,009
Calculation time	0:00:00	0:00:33	0:00:00	0:01:14	0:00:01	0:02:01
Remove-down-to level 1 (units)	18	17	21	23	26	28
Remove-down-to level 2 (units)	10	10	13	13	14	15
Remove-down-to level 3(units)	0	0	0	0	0	0
Difference in total costs (%)	0,109		0,828		0,789	

Distribution demand per period	Uniform(0,18)		Uniform(0,20)	
Heuristic (H) / Exact method (E)	H	E	H	E
Optimal Final Order Quantity (units)	40	39	44	43
Costs during RTT (€)	12973,33	12724,15	14365,73	14057,74
Costs after RTT (€)	20000	20000	20000	20000
Total Costs (€)	32973,33	32724,15	34365,73	34057,74
Fillrate (%)	98,051	98,004	98,197	98,01
Calculation time	0:00:01	0:03:50	0:00:01	
Remove-down-to level 1 (units)	28	30	35	34
Remove-down-to level 2 (units)	18	17	18	19
Remove-down-to level 3(units)	0	0	0	0
Difference in total costs (%)	0,761		0,904	

Table I.3: Results for the heuristic and the exact method for the remove policy for a planning horizon of 5 periods.

Distribution demand / period	Uniform(0,1)		Uniform(0,2)		Uniform(0,3)	
Heuristic (H) / Exact method (E)	H	E	H	E	H	E
Optimal Final Order Quantity (units)	4	4	8	8	11	11
Costs during RTT (€)	1549,97	1549,97	2991,14	2967,52	4126,56	4126,56
Costs after RTT (€)	20000	20000	20000	20000	20000	20000
Total Costs (€)	21549,97	21549,97	22991,14	22967,52	24126,56	24126,56
Fillrate (%)	98,75	98,75	98,519	98,272	98,047	98,047
Calculation time	0:00:00	0:00:02	0:00:00	0:00:13		
Remove-down-to level 1 (units)	4	4	6	6	9	9
Remove-down-to level 2 (units)	3	3	6	5	7	7
Remove-down-to level 3(units)	2	2	4	4	6	6
Remove-down-to level 4 (units)	1	1	2	2	3	3
Remove-down-to level 5(units)	0	0	0	0	0	0
Difference in total costs (%)	0		0,103		0	

Distribution demand / period	Uniform(0,4)		Uniform(0,5)		Uniform(0,6)	
Heuristic (H) / Exact method (E)	H	E	H	E	H	E
Optimal Final Order Quantity (units)	15	14	17	17	21	20
Costs during RTT (€)	5585,68	5291,48	6550,71	6484,78	7929,67	7675,83
Costs after RTT (€)	20000	20000	20000	20000	20000	20000
Total Costs (€)	25585,68	25291,48	26550,71	26484,78	27929,67	27675,83
Fillrate (%)	98,304	98,003	98,264	98,059	98,262	98,026
Calculation time	0:00:00	0:04:05	0:00:00	0:11:20	0:00:00	0:27:39
Remove-down-to level 1 (units)	12	12	17	15	17	19
Remove-down-to level 2 (units)	9	10	14	13	16	16
Remove-down-to level 3(units)	8	7	10	10	12	12
Remove-down-to level 4 (units)	4	4	5	5	6	6
Remove-down-to level 5(units)	0	0	0	0	0	0
Difference in total costs (%)	1,163		0,249		0,917	