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The effect of imperfect inspections on cost optimal maintenance scheduled under a reliability constraint

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The effect of imperfect inspections on cost optimal maintenance schedules under a reliability constraint

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Die schönste Formgebung nutzt nichts,
wenn es an der Ausführung mangelt.

Martin Winterkorn

Preface

Before you lies the report which is the result of my graduation project in completion of the Master Operations Management & Logistics at the Eindhoven University of Technology. The project has been conducted at the Maintenance Development department of NedTrain B.V. in Utrecht. On this note I would like to address my gratitude to all people who have made this project possible. Some people, however, still deserve some additional acknowledgement.

First, I would greatly like to thank my first supervisor Hao Peng. Your guidance and the helpful meetings have truly been of tremendous help in bringing me this far. Furthermore, I would like to thank you for the flexibility and patience you have had with me during the long meetings and during my internship abroad, amongst others. Second, I would like to thank Geert-Jan van Houtum, my second supervisor, for his useful comments and suggestions to this thesis work in order to improve its quality.

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Next, I would like to thank my friends who have encountered me, mainly in the last few weeks of the project. Thank you for keeping interest in my project despite my occasional obsession for it, and making resources available when I (kindly) asked you to.

None of this would have been possible without the infinite support of my parents and sister. Family time has not been family time for quite some occasions in the last few months, but you kept on supporting me, kept showing interest and kept me motivated.

Last but not least, I would like to thank my girlfriend Guusje. You have endured the lonesome days, evenings and weekends. In addition even, came all the troubles and stress I dragged home. Despite all, your love, support, guidance and help has never been less than infinite. Thank you so much!

Joni Driessen

Abstract

In this report we consider a single-component asset which can be in three different states: normal, defective and failed. The asset is subject to imperfect inspections and perfect maintenance. In case the inspection result yields the requirement of a maintenance action, this action is performed instantaneously. In addition, the inspections are performed under a fixed time interval. Imperfect inspections are classified into two categories, false positives and false negatives. A false positives denotes the event where the asset is unnecessarily maintained, whereas a false negatives corresponds to the asset's defect remaining undetected upon inspection. By modelling the probability of false positives and false negatives in three distinct ways, their effects are illustrated. We propose a cost model and reliability model for each of the three model approaches for the probabilities of false positives and false negatives. Subsequently, the cost and reliability model are used in optimising the maintenance schedule with respect to costs over an infinite time horizon, under a reliability constraint.

We present a numerical study illustrating the effects of differently modelling the probabilities of false positives and false negatives on the cost optimal maintenance schedules. Under the premises of Weibull distributed state durations, we conclude that the modelling approach for non-constant probabilities of false positives and false negatives determines the effects on the optimal maintenance schedules, compared to the constant probability modelling. In addition to the modelling approaches, the general inspection quality affects the optimal maintenance schedules and costs as well. As a final insight, we observe the effects of the reliability constraint. In satisfying this constraint the inspection interval length plays a determinant role. The reliability constraint further affects the cost effects of the non-constant probabilities of false positives and false negatives.

Keywords: Condition-based Maintenance, Delay Time Model, False Negative, False Positive, Imperfect Inspections, Maintenance, Optimisation, Reliability

Executive Summary

Cost minimisation of the Total Cost of Ownership plays a major role in all industries across our globe. In the framework of Total Cost of Ownership, maintenance is a key element contributing to the total costs and their minimisation. Mathematical modelling might present a solution to the focus on the minimisation of both, Total Costs of Ownership and maintenance costs. As many assets are inspected before the maintenance action is performed, assumptions on these inspections underlie the solutions obtained for the optimal maintenance schedules. This work considers a maintenance schedule to be comprised of two elements. The first states when inspections are done and the second denotes the inspection instance of preventive asset replacement.

Most models used in maintenance modelling assume the inspections to be perfect. Yet, imperfect inspections in maintenance are highly realistic and affect the maintenance schedules obtained. Furthermore, the way these imperfect inspections are included in the mathematical maintenance modelling might even further alter the optimal maintenance schedules and the associated costs.

In addition to the cost aspect of maintenance modelling, asset reliability has gained importance in the European Legislation encountered by NedTrain. This ever more demanding legislation requires European companies as NedTrain to include the reliability aspects in their analyses done.

Both aspects bring NedTrain to the challenge of understanding the effects of imperfect inspections on optimal maintenance schedules, when including the asset reliability as well. The main research question initiating this research is then defined as.

How does the imperfectness of inspections affect cost optimal maintenance schedules under a reliability constraint?

In this research we focus on a single-component asset characterised by three states: the normal state, the defective state and the failed state. The asset is subject to inspections which are performed according to a fixed inspection interval. Dependent on the inspection outcome, a perfect maintenance action is required and is performed instantaneously upon request.

Because the results of the inspections are considered imperfect, we differentiate two distinct events of imperfect inspections: false positives and false negatives. The former denotes the asset unnecessarily undergoing maintenance, whereas the latter corresponds to the asset's defect remaining undetected. We present three distinct ways of modelling the probabilities of false positives and false negatives. By developing a cost model, reliability model and using both in an optimisation model for each of the model approaches, the effects of imperfect inspections on the cost optimal maintenance schedules under a reliability constraint are compared.

The first of the model approaches includes the probabilities of false positives and false negatives to be independent on any variable, i.e. the probabilities of both inspection errors are constant. The second approach considers the probabilities of false positives and false negatives as being non-constant and dependent on the defect progress and degradation progress, respectively. The final model, the third model approach, relates the probabilities of false positives and false negatives to the inspection frequency and the Mean Time To Failure, implying non-constant probabilities.

By using the models developed, we present a numerical study illustrating the effects of differently modelling the probabilities of imperfect inspections, i.e. false positives and false negatives, on the cost optimal maintenance schedule under a reliability constraint. For the numerical study, parameter values are based on a fictive asset, which has characteristics resembling real life assets. A sensitivity analysis presents more insights in comparing the three different models to one another and their respective effects on cost optimal maintenance schedules under a reliability constraint.

We derive conclusions under the premises of Weibull distributed state durations. Firstly, we conclude that the three distinct ways of modelling the probabilities of false positives and false negatives yield different optimal maintenance schedules and costs. Compared to the constant probabilities, the way the non-constant probabilities for false positives and false negatives are modelled, determines the effects of imperfect inspections on optimal maintenance schedules and costs under a reliability constraint. The numerical study shows that the non-constant modelling of imperfect inspections can be worthwhile, as cost decreases of up to $23,6\%$ are obtained, with respect to the constant probabilities.

In addition to the way the probabilities of false positives and false negatives are modelled, we also denote the general effect of the quality of inspections. A low inspection quality decreases the frequency of inspections and increases costs, whereas a high inspection quality yields an increase in the inspection frequency and decreases costs. Therefore, we conclude that is cost beneficial to improve on the quality of inspections.

Furthermore, we obtain insights with respect to the reliability constraint. We conclude that the inspection interval length is highly determinant in satisfying the reliability constraint, compared to the inspection instance of preventive asset replacement. This implies that the inspection interval length dominates the preventive asset replacement in constraint satisfaction. In addition, the reliability constraint further affects the cost effects that the non-constant probabilities of false positives and false negatives have compared to the constant probabilities.

Based on the conclusions made and the models developed, recommendations are made to NedTrain. The main recommendation stemming from this research includes

Compare results from current practice to the models' results
We recommend NedTrain to compare the costs and reliability of current practice to the models' results. This can be done by using the maintenance schedule from current practice to theoretically calculate the costs and reliability. Upon these two results, it is then recommended to compare both.

Next to the main recommendation, other recommendations to NedTrain are made. Each of these recommendations are included in the enumeration below.

- **Investigate and estimate the probabilities of false positives and false negatives**
  This work assumes certain probabilities for false positives and false negatives in

conducting the numerical study. However, we recommend NedTrain to further investigate how the probabilities of false positives and false negatives can be derived in more detail.

- **Derive model parameters based on quantitative data**
  In the numerical study from this work, various parameter values have been based on the expert opinions from NedTrain employees. Despite the fact that this method is valid, we would still recommend to quantitatively derive the parameter values and verify whether the qualitative judgement fits the results from data.

- **Develop additional model approaches for the modelling of imperfect inspections**
  Three model approaches are posed in this work, but this does not imply that these cover all the ways on which probabilities of false positives and false negatives can be modelled. Hence, it is recommended to further develop other model approaches. Once developed, the author highly recommends comparing the results from the model approaches to practical results.

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Introduction

Cost minimisation is a core aspect in all industries on this globe. This problem is tackled in the various research and industrial fields such as procurement or maintenance. One of the terms often considered is the Total Cost of Ownership (TCO) of systems, which is attempted to be minimised. Within the TCO, maintenance plays a key role in contributing to the costs and their minimisation. Therefore, research in the field of maintenance is not only driven by academia but by industry as well. By making a trade-off between the maintenance frequency (and its corresponding costs), renewal frequency (and correspondingly its costs) and the non-operating asset costs, the TCO with respect to maintenance can be minimised.

A core assumption of the trade-off between maintenance costs, renewal or replacement costs and the non-operating costs is that the inspections, which are part of the maintenance, are assumed to be perfect. This implies that every judgement of the maintenance staff is always perfect, e.g. sensors are never falsely read or the subjective judgement always perfectly matches the asset's state. Yet, imperfectness in inspections are highly realistic and may alter the cost trade-off. In order to optimise this trade-off, the inclusion of imperfect inspections offers the potential of decreasing the asset's maintenance related costs even further.

Additionally, the ever more demanding European legislation forces companies such as NedTrain to explicitly include their asset's risks in their analyses. One of the important aspects of asset risk is denoted by asset reliability. The pressure on accurate reliability estimations is ever increasing and therefore further stimulates NedTrain in facilitating this research.

Based on both trends indicated above, i.e. the cost minimisation and legislation, this research will investigate the effects of imperfect inspections on cost optimal maintenance schedules under a reliability constraint. This chapter will introduce the company facilitating this research, NedTrain B.V. (NedTrain), in Section 1.1. In the succeeding Section 1.2, the concept of maintenance, its associated terminology and the applications of maintenance at NedTrain are discussed. Section 1.3 motivates on the inclusion of imperfect inspections in modelling and states the maintenance problem currently faced. Finally, this chapter will conclude with an outline of the complete report in Section 1.4.

1.1 Company description

Formerly known as NS Materieel, NedTrain B.V. (NedTrain) is the locomotive and rolling stock maintenance and repair company of the Dutch railways, de Nederlandse Spoorwegen (NS). Headquarted in Utrecht, this full subsidiary of NS splits its business activities in the fields of Maintenance & Service, Modernisation, Repair, Cleaning and Technical Services. By their way of operating, an approximated turnover of €500 million is generated with the employment of over 3000 full time employees.

As briefly discussed, NedTrain is a full subsidiary of NS, next to some daughter companies such as NS Stations and NS Reizigers. The corporate structure of the NS Group is illustrated in Figure 1.1 along with the different departments within NedTrain.

Formally, NedTrain has been founded as a separate entity within NS in the early 1990s when NS was divided into three different divisions: ProRail, NS Reizigers (NSR) and NedTrain. The latter division started out as the company NS Materieel but was renamed to NedTrain B.V. in 1999. Historically, NedTrain has two main locations for the actual refurbishment and overhaul (R&O) of trains: Tilburg (NedTrain Componentenbedrijf, NCB), where components are revised and repaired, and Haarlem (R&O Workshop) where the overhaul and modernisation of trains is done. In addition to these two locations, its headquarters are located in Utrecht and NedTrain owns four maintenance depots at which (un)scheduled maintenance activities are performed. Furthermore, 30 service sites next to the rails are operated by NedTrain. These service sites focus on daily maintenance and cleaning activities, such as daily checks and minimal repairs. With these maintenance and service locations, NedTrain not only operates as a maintenance, repair and operations (MRO) service provider for NS, but includes Arriva, Syntus and Veolia in their clientele as well.

1.2 Maintenance at NedTrain

To keep the report tractable and readable, this section will introduce the concept of maintenance, its associated terminology and the maintenance applications at NedTrain. Maintenance is defined as ”the function that monitors and keeps plant, equipment, and facilities working. It must design, organize, carry out, and check the work to guarantee nominal functioning of the item during working times ’Ti’ (uptimes) and to minimize stopping intervals (downtimes) caused by breakdowns or by the resulting repairs” (Manzini, Regattieri, Pham, & Ferrari, 2009).

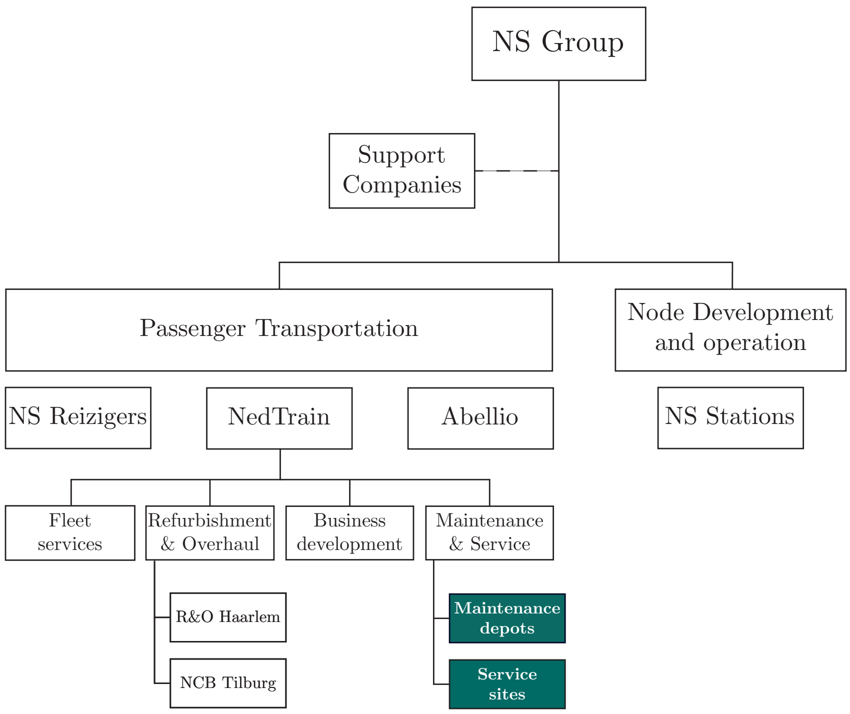


Figure 1.1: Hierarchical decomposition of the NS Group and NedTrain

This concept of maintenance is relevant for both industry and academia in its power to increase asset reliability over a predefined time span and to decrease an asset's TCO. This section distinguishes three general and distinct types of maintenance (Nakagawa, 2005; Jardine, Lin, & Banjevic, 2006; Niu, Yang, & Pecht, 2010).

1. **Corrective maintenance:** this type of maintenance is often referred to as the fail-and-fix maintenance policy. This implies that the maintenance action, i.e. the restoring of the failed asset, is only done in case the asset has physically failed. Such type of maintenance policy is often suitable when failure costs are low compared to the costs of preventively maintaining the asset.

2. **Planned maintenance:** planned maintenance has the asset maintained every predetermined T time units in order to prevent asset failure. This maintenance interval T can be derived qualitatively, but a more quantitative approach is to model the time dependent failure behaviour of the asset considered and based on this expression one can derive an optimal maintenance interval. Note that an asset does not need to survive the time interval T, i.e. it can break down just before a planned maintenance action. In this case, the asset will be maintained correctively. Therefore, most assets facing a planned maintenance schedule also include the corrective maintenance policy for the cases in which the asset does not survive the time intervals.

   Planned maintenance can be classified into a block policy, which fixes the maintenance interval and an age policy which does not. To further illustrate, in the block policy when an asset is maintained every T time units, but fails in between, corrective maintenance is executed but the schedule for the planned maintenance remains untouched. In case of the age policy, when the asset breaks down, corrective maintenance is performed and the maintenance schedule changes, i.e. the next planned maintenance is scheduled after T time units. Figure 1.2 illustrates the block and age policies graphically, where the cross denotes the time instance of asset failure.

3. **Condition-based maintenance:** condition-based maintenance is centred around the concept of inspecting the asset's condition and based on this inspection, decide whether or not to include the asset for maintenance. It attempts to prevent the asset from failing. This inspection can be done on three levels: continuous inspections, periodic inspections and aperiodic inspections. The former would imply the use of sensors, which is becoming ever more popular in modern society and industries. Periodic inspections indicate a fixed and periodic inspection scheme. This type of inspection is mostly done by maintenance staff inspecting the asset. The latter level of inspection is the aperiodic inspection which has the asset inspected at aperiodic time epochs, mostly by maintenance staff. Because the maintenance is scheduled based on the current asset condition, the number of unnecessary maintenance activities is reduced. Based on the information obtained by the condition monitoring or inspection, the degradation state of the asset can be estimated, which in turn can be used to optimise the maintenance schedule.

Due to the fact that NedTrain is the maintenance and rolling stock company of NS, maintenance activities are one of the core competences of NedTrain. In their maintenance schedules, NedTrain applies all three different types of maintenance, as discussed above. It classifies its maintenance activities as follows.

1. **Overhaul:** during the overhaul maintenance the trains and locomotives are completely renewed, revised and refurbished. This type of maintenance is executed roughly every 15 years and is classified as planned maintenance under a block policy.

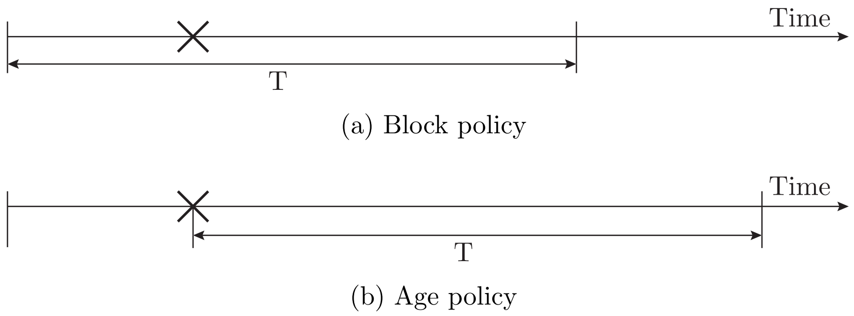


Figure 1.2: Planned maintenance policies

2. **Quarterly maintenance:** As the name already suggests, quarterly maintenance is executed roughly every 3 months, i.e. the inspection interval is periodic. In this type of maintenance, the trains are extensively tested and maintained when necessary, thereby reflecting the condition-based maintenance with periodic inspections. Such an extensive test can be a Non Destructive Test for an asset which is rather time and resource consuming and is therefore scheduled in the quarterly maintenance. In case any defects are found, the particular asset is replaced or revised during this type of maintenance.

3. **Daily maintenance:** daily maintenance is somewhat contradictory with the actual maintenance interval. In reality the daily maintenance is done every 2 or 8 days also implying a constant inspection interval for this type of maintenance. The trains are cleaned every 2 days, referred to as A class daily maintenance. The B class daily maintenance (8 days) includes the inspection and maintenance of the basic functions of the trains and locomotives, e.g. inspections and maintenance on the power conductors from the train to the high current net. This type of maintenance corresponds to condition-based maintenance under periodic inspections.

Due to the fact that condition-based maintenance is the focus of this research, this topic is considered in the remainder of this research. Condition-based maintenance (maintenance in the remainder) includes two aspects: the first is the aspect of inspecting an asset's state. In case any form of actual maintenance action is necessary, the maintenance action is planned or directly performed which is the second aspect of maintenance. Please be aware of the difference between maintenance and the maintenance action, where the former corresponds to the complete process including the inspection and the latter merely to the maintenance action in case the inspection results require the asset to be maintained. Because the inspection is always performed in maintenance, whereas the maintenance action is not, the maintenance frequency or interval of maintenance is defined by the inspection frequency or interval.

Because daily maintenance and quarterly maintenance both consider condition-based maintenance, this work mainly affects these two maintenance classes of NedTrain. Note that this corresponds to the maintenance activities performed at NedTrain's maintenance depots and service sites.

1.3 Problem description

The quarterly and daily maintenance require the maintenance staff to inspect the condition of specific assets of NS trains or locomotives. Most modelling approaches in maintenance, however, consider these inspections to be perfect, i.e. the inspection outcome always matches the true asset's state. As NedTrain carefully notices, imperfect inspections are present, realistic and affect costs and reliability aspects in maintenance modelling. When such inspections are imperfect, they can yield cost inefficient maintenance schedules and can result in a reliability decrease of the asset under consideration (Scarf, Cavalcante, Dwight, & Gordon, 2009). In other words, imperfectness in inspections is highly realistic and may alter the the optimal maintenance schedule under a reliability constraint for the asset considered.

Additionally, under the strong industrial focus on cost minimisation, inspection intervals are stretched to such extends that it becomes of vital importance to find the defect upon the inspection. Stretching the interval largely, ultimately leaves no other inspection which might reveal the defect. Hence, one has to be sure that the defect is detected upon the first inspection after defect arrival, when the inspection intervals are stretched largely. Since inspections are not perfect in practice, this further motivates on the inclusion of imperfect inspections in maintenance modelling.

The complete problem encountered by NedTrain is decomposed into two aspects. The former relates to the way of including imperfect inspections in modelling, whereas the latter corresponds to explicitly including reliability aspects in the models.

In order to derive a cost efficient maintenance schedule, the problem of how to include imperfect inspections in the maintenance modelling arises. Since multiple ways for including imperfectness in inspections exist, the various ways of inclusion might affect the optimal maintenance schedules obtained. Hence, NedTrain faces the challenge of further understanding the effects of including imperfect inspections in maintenance modelling on various ways. This comprises the first aspect of the complete problem encountered by NedTrain.

Additionally, NedTrain is faced with the ever more demanding and prescribing European Legislation explicitly requiring risk related aspects into their analyses. This implies considering asset reliability in the maintenance analyses done. For the derivation of optimal maintenance schedules under the various ways of modelling imperfect inspections, the second aspect of the problem encountered considers the inclusion of such reliability aspects in the analyses.

Both aspects yield the complete problem encountered by NedTrain. This denotes the issue of how to obtain the optimal maintenance schedule under a reliability constraint, when considering various ways of modelling imperfect inspections. In other words, for various ways of modelling imperfect inspections, what is the most cost efficient maintenance schedule that satisfies a preset asset reliability.

1.4 Report outline

The report is structured in the following way. Chapter 2 introduces the current status of the academic literature on this topic of imperfect inspections in maintenance modelling and subsequently presents the research questions underlying this work. Chapter 3 will then present a description of the models which are developed in the remainder and will elaborate on the assumptions and the various ways on how imperfect inspections can be modelled. In the following chapter, Chapter 4, three mathematical models are presented which are abstract in their nature thereby increasing their generalisability. Chapter 5 presents, based on the abstract models from Chapter 4, the three detailed models used for the numerical study and concludes by presenting the numerical results and implications. Chapter 6 presents various aspects related to model implementation and points of attention. This report finalises with Chapter 7 presenting the conclusions and recommendations which are made based on the research.

2

Literature and Research Framework

This chapter presents a literature overview on maintenance modelling and the inclusion of imperfect inspections in Section 2.1. Next, the research questions on which this work is based are stated and the subsequent research framework is presented in Section 2.2. Section 2.3 will then demarcate the research. In Section 2.4 the used methodology is discussed and in Section 2.5 an adjusted form of the Delay Time Model is explained in more detail. The chapter concludes with the contributions this project has to NedTrain and current literature in Section 2.6.

2.1 Relevant literature

This section briefly summarizes and extends the literature study conducted by Driessen (2013). In current literature, multiple models are suggested to use for condition-based maintenance with periodic inspections. The two most prominent in literature are the Markovian based models and the Delay Time Model (DTM). Both models are briefly discussed and compared to one another in order to support a proper model choice for this research.

A Markov model is a mathematical modelling technique in which an asset is characterised by states and the state transition rates. Markov models use a probability distribution for the states in which the asset can be in (Kulkarni, 1995). The state transition rates define the intensity of the asset's state changing to succeeding or preceding states. An important point to be made is that the Markov models possess the Markov property of exponential arrival and departure rates from their states. Due to this Markov property, real life application of the Markov and Semi Markov models remains rather challenging (Choi, 1997). In overcoming this hurdle one is able to use Hidden or Semi Hidden Markov Models.

Despite the limited applicability of the pure Markov models, Welte, Vatn, and Heggest (2006) have considered a single-component asset which is modelled by a Markov chain. By approximating the accelerated life during degradation, a reduction factor derived from a gamma process is introduced to the standard Markovian exponential distribution for state arrival and departure. The imperfectness of inspections is modelled as a binary variable, i.e. the observation of the inspection either matches the actual asset state or not. The model evaluates a cost criterion under a fixed inspection interval but does not explicitly evaluate the asset's reliability. In contrast to Welte et al. (2006), Hokstad, Langseth, Lindqvist, and Vatn (2005) do consider a fixed inspection interval and model the imperfect inspections in twofold: false positives, where the asset is maintained unnecessarily, and false negatives, where the asset's defect remains undetected. These type of imperfect inspections are modelled as distinct asset states yielding a reliability evaluation of the model. An optimisation as well as a cost evaluation are omitted in their work.

To include the non-constant arrival and departure rates from the Markov states, the Hidden and Semi Hidden Markov Models (HMM and HSMM) are used. The literature on prognostics using the HMM and HSMM is extremely limited, but faces increasing popularity in literature (Baruah & Chinnam, 2005). The work by Baruah and Chinnam (2005) is one of the earlier works in using a HMM to make prognoses on the asset's remaining useful life. In their early work, maintenance costs are not included and therefore no cost optimisation is considered. Other research by Neves, Santiago, and Maia (2011) extends the HMM by including the probability of imperfect inspections. Based on the two layers of a HMM a cost minimisation problem is solved over an infinite time horizon.

The second class of models very commonly used in maintenance modelling is the Delay Time Model (DTM) developed by Christer and Waller (1984). This model has been developed to include inspections in maintenance modelling. In principle, the DTM considers one dimension: time. This implies that the DTM decomposes an asset's life into multiple states analogous to Markov models, see Figure 2.1. The vertical seperator lines in the figure indicate that the time axis is interrupted and the time to degradation initiation (the lightning symbol) is random. Furthermore, the dashed decaying line in Figure 2.1 denotes the fact that the time from degradation initiation until failure is also random. Figure 2.1 identifies four states, but the DTM only defines three. The difference lies in the second state (interval between the degradation initiation and the detection level) which indicates the asset having started to deteriorate, but it is still unobservable. For the DTM this state is not distinguished because one cannot observe the degradation and therefore the asset is still considered free of defects. This reduces the number of states to three.

The first state is then denoted by the asset operating without any observable degradation, the second state by the asset suffering from an observable deterioration and a final state in which the asset has failed. Figure 2.1 illustrates these states of the DTM where the first state corresponds to the time period before point u labelled as the normal state, the second state to the period between u and failure, defined as the defective state and the last state to the failed state, which is denoted as such. To clarify the DTM's notation: point u is the time epoch upon which the deterioration of the asset becomes observable, i.e. the asset becomes notably defective. This point u is defined as the time to defect. The time length h then denotes the time from point u to the actual time of failure and is referred to as the delay time.

As one might notice, Figure 2.1 also includes the dimension of the asset's load carrying capacity (Tinga, 2013), thereby reflecting the concept of the potential failure (P-F) interval in condition-based maintenance (Moubray, 2001). The concept of the DTM is highly related to the concept of the P-F interval, but the DTM enables mathematical modelling whereas the P-F concept serves conceptualising purposes instead of enabling mathematical modelling (Wang, 2012). Nevertheless, the relationship with the P-F concept is depicted in Figure 2.1 due to the familiarity of many engineers in industry and research academia with the concept of the P-F interval.

To return to the mathematical maintenance modelling, the maintenance should be planned under the DTM during the interval $(u, u + h)$ preventing asset failure from occurring. From point u onwards the defect of the asset is observable, thereby bounding the lower limit of the maintenance action time window. The point $u + h$ defines the upper bound of the maintenance action time window since the asset fails at this time epoch. To determine if a maintenance action is necessary at all, inspections are done to judge whether the asset has entered its defective state or that it still remains in its normal state. In the former scenario a maintenance action is required, whereas in the latter it is not. Note that in this distinction lies one of the main assumptions of the DTM: a maintenance action is required directly upon defect detection. When the inspection interval is kept very small, the probability of finding the defect before

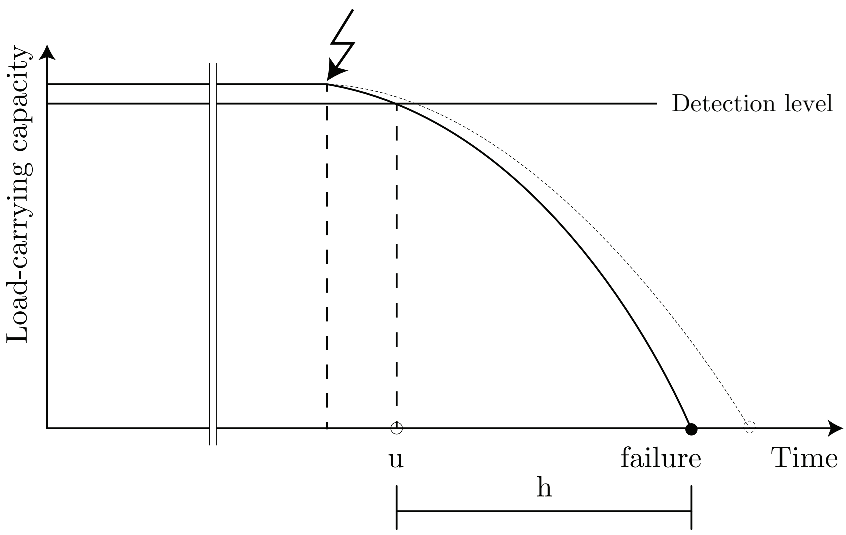


Figure 2.1: The delay time model concept

point $u + h$ is rather large, whereas a large inspection interval implies a lower probability of finding the defect before $u + h$, thereby risking a larger probability of asset failure. For a more detailed elaboration on the DTM the reader is referred to Wang (2008).

Before discussing the current state on imperfect inspections in the DTM, a vital issue specific to the DTM is to be noted. The DTM differentiates two distinct types of assets which can be modelled, whereas the Markov models remain equal for both cases. The first is the single-component asset, which encounters only a single failure mode. The second type of DTM is the multi-component asset, which has multiple failure modes, e.g. a train is analysed as whole and can fail due to its power conductor or motor etcetera. Hence, the modelling for both types of models differs. When including imperfect inspections, one should take into account the distinction between both types of models. For both types, the current literature status will be discussed with an emphasis on the aspect of imperfect inspections.

The first class of Delay Time Models considers the single-component assets. Baker and Wang (1993) were the first to include imperfect inspections in the DTM for a single-component asset. This model has included a probability for a binary imperfectness, i.e. the inspection can either be perfect or not. To further specify the imperfect inspections, Okumura, Jardine, and Yamashina (1996) introduced two types of imperfect inspections: false positives and false negatives. Both early papers focus on cost evaluation and inspection interval optimisation with respect to costs. The asset's reliability is less considered in literature. The work of Berrade, Scarf, Cavalcante, and Dwight (2013) considers both aspects in an evaluating fashion, i.e. evaluating the asset's reliability and costs. Their optimisation is unconstrained and minimises the average costs over an infinite time horizon under two types of imperfect inspections. In contrast to the unconstrained problem, a constrained optimisation problem for a single-component asset is considered by Aven and Castro (2009). The authors minimise the costs over an infinite time horizon under a constraint, but assume perfect inspections.

All models discussed assume maintenance actions to be performed directly upon defect detection, which is also one of the key assumptions of the DTM. However, doing so might be cost sub-optimal. van Oosterom, Elwany, Çelebi, and van Houtum (2014) therefore present a model for cost minimisation of a single-component asset including the postponement of a maintenance action upon perfect defect detection, i.e. the inspections are assumed perfect. Moreover, due to their assumption of perfect maintenance, these maintenance actions equal to preventive replacement which definition is used throughout their work. To return to the focus on the inspections themselves, Flage (2013) extends the mathematical modelling on inspections by including the fact that inspections can be failure inducing in addition to being imperfect. He uses these inspection characteristics in optimising the interspection intervals with respect to the costs over an infinite time horizon. As Wang, Banjevic, and Pecht (2010) indicate, the work done on single-component assets might be practically less relevant, but it offers a solid basis for gaining insights in the models and offers a building block for the extension to multi-component asset models.

Imperfect inspections for multi-component assets were first introduced by Christer, Wang, Baker, and Sharp (1995). By including the imperfectness of inspections as being binary, i.e. either perfect or imperfect, with an associated probability, they derive the expected number of failures during an inspection interval. This expected number of failures is then used for extending the original DTM (Christer & Waller, 1984). Due to the property of a multi-component asset, the assumption of a homogeneous Poisson process (HPP) for the defect arrival has been made. Under this HPP assumption, Cunningham et al. (2011) present a methodology and two case studies including imperfect inspections under a Monte Carlo simulation. Based on the Monte Carlo simulation, the DTM's analysis is made more practical for industry, offering an increased utility value for engineers in practice. Their research also offers extension possibilities to a single-component asset. Zhao, Chan, Roberts, and Madelin (2007) alter the HPP assumption by introducing a non-homogeneous Poisson arrival process for the defects, resulting in sequential inspection intervals, i.e. non-constant intervals. The way Zhao et al. (2007) model the imperfect inspections, however, is no different from Christer et al. (1995), i.e. the inspection outcome is either perfect or imperfect. Inspection intervals are optimised to obtain the maximum reliability of the asset under consideration. Costs of inspecting and maintaining the asset are not considered by Zhao et al. (2007). All multi-component DTMs discussed so far assume a constant probability of imperfect inspections, which might be illogical because defects are easier to determine when the asset's age or degradation increases (Wang, 2012). Note that Wang (2012) hereby only refers to the occurrence of false negatives. To tackle non-constant nature of the probability of false negatives, Wang (2010) introduces a temporal imperfect inspection probability for multi-component assets. Based on this temporal probability, the costs over the infinite time horizon are minimised.

Markovian based models as well as the Delay Time Models are used in academic literature to model imperfect inspections in maintenance. However, the DTM is generally more flexible and has a higher practical value. In comparing the DTM to the Markov models, the first issue that is very prominent is the complexity of Markov models compared to the rather simple DTM, resulting in lower computation times for Delay Time Models compared to Markovian based models (Christer, Wang, Choi, & Schouten, 2001). A second advantage of using a Delay Time Model over a Markovian based model is that the DTM has the possibility of optimisation, whereas the Markov models lack this feature (Choi, 1997). With respect to the research framework from Figure 2.2, the DTM type of model is more appropriate when taking into account that optimisation is the research's objective.

2.2 Research questions

This section presents the main research question, which in turn is aided by the sub questions posed. The main research question that will be answered in the proposed work is:

How does the imperfectness of inspections affect cost optimal maintenance schedules under a reliability constraint?

In order to properly tackle this research question, supporting research questions are introduced. These type of questions give a better overview on the problem decomposition and provide guidance in solving the problem from the main research question. A two layer enumeration is included, because several sub questions have their own sub questions in turn.

1. How does the modelling of perfect inspections affect the reliability model under the maintenance schedule?
    - How is the reliability model defined and conceptualised?
    - What variables determine the maintenance schedule?
    - What are the assumptions for the reliability model under perfect inspections?
    - What theories are useful for including imperfect inspections in reliability modelling?
    - How is the reliability modelled under perfect inspections?
2. How to extend the perfect inspection reliability model to include imperfect inspections under the maintenance schedule?
    - What variables determine the maintenance schedule?
    - What factors may affect the imperfectness of inspections?
    - What assumptions underlie the imperfect inspections model?
    - What theories are useful for including imperfect inspections in reliability modelling?
    - How can imperfect inspections be modelled in the reliability model?
3. How does the modelling of perfect inspections affect the cost model under the maintenance schedule?
    - How is the cost model defined and conceptualised?
    - What variables determine the maintenance schedule?
    - What assumptions are made for the costs model under perfect inspections?
    - What theories are useful for including imperfect inspections in the cost modelling?
    - How are the costs modelled in case of perfect inspections?
4. How to extend the perfect inspection cost model to imperfect inspections under the maintenance schedule?
    - What variables determine the maintenance schedule?
    - What factors may affect the imperfectness of inspections?
    - What assumptions underlie the imperfect inspections model?
    - What theories are useful for including imperfect inspections in the cost modelling?
    - How can imperfect inspections be modelled in the cost model?
5. How can the maintenance schedule be optimised with respect to costs and under a reliability constraint?
    - What variables determine the maintenance schedule?
    - What is the objective function for the inspection interval optimisation problem?
    - What are the constraints for the inspection interval optimisation problem?
    - What type of mathematical modelling can be used in order to solve this question?

Many supportive research questions have been posed in the enumeration above. All of the main sub questions are highly related to one another and build up a framework for the optimisation of the maintenance schedule when imperfect inspections are taken into account. The reliability model and the cost model are both used in deriving the optimal maintenance schedule, as Figure 2.2 indicates.

Furthermore, we would briefly like to address the conceptualisation of the reliability and the cost model. The former denotes a mathematical expression which defines the probability of asset survival of a predefined time period, from 0 to the given time t. The latter, the cost model, corresponds to a mathematical expression that denotes the average costs per time unit.

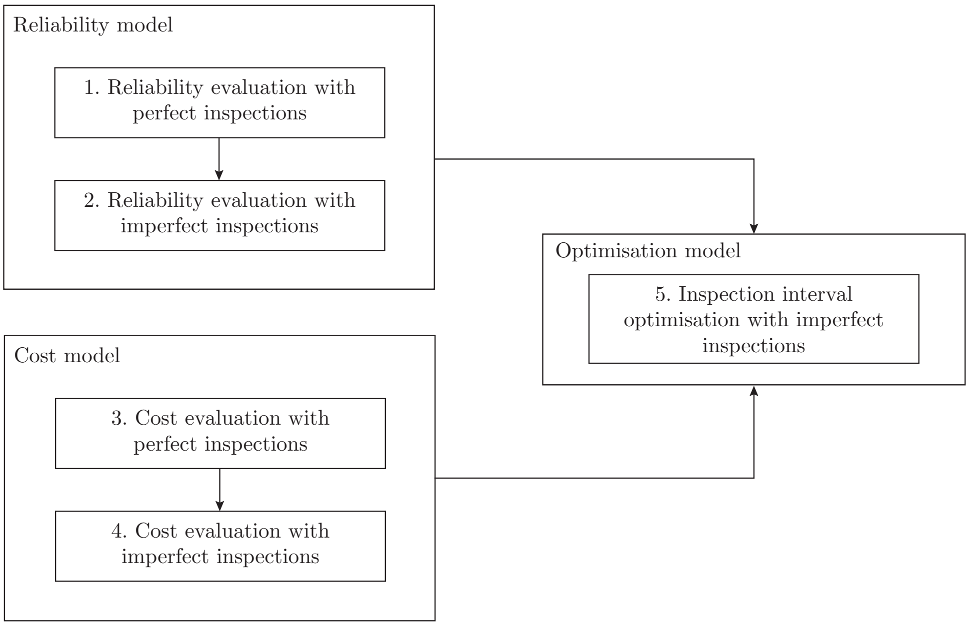


Figure 2.2: Overview of complete research framework and research questions

2.3 Scope

This research does not consider all aspects associated to maintenance. Therefore, its scope is discussed here. The work limits itself to considering the imperfect inspections in maintenance. The fact that maintenance actions itself can be imperfect is not considered, also to decrease model complexity. Such inclusion of imperfect maintenance actions offers a possible extension to the models developed.

Furthermore, the study will not consider any statistical analysis which attempts to update or even initiate distribution parameters, e.g. Bayesian updating. The distribution parameters for the required distributions will be either assumed or be provided by the Risk Based Maintenance project within NedTrain.

The research will only cover one type of modelling. This implies that only one modelling concept will be used, i.e. a Delay Time Model (DTM) type. No comparison of the different types of models, which are able to include the imperfect inspections, will be made. The focus of this research is to investigate the effects of imperfect inspections and comparing their results. Therefore, only one type of modelling is used. For further research, however, a different type of modelling can be used to compare results.

Finally, the study will not consider any physical implementation of the maintenance schedules. It will present some aspects and issues related to the implementation of the models developed in this research.

2.4 Methodology

Due to the fact that the focus of this research lies on imperfect inspections and their effects on the optimal maintenance schedule, this work assumes three distinct ways for modelling the imperfectness of inspections.

For the reliability and cost evaluation, a DTM type of model will be used. Based on these evaluations, an optimisation problem is derived that minimises the costs over an infinite time horizon under a reliability constraint. Note that a DTM type of model enables one in optimisation, whereas the Markovian based models do not (Choi, 1997). The DTM type of model will consider a single-component asset offering a solid basis for an extension to a multi-component model. Additionally, renewal theory will be applied in the DTM type of model. The optimisation model will be stated in terms of a mixed integer problem.

For the numerical study, which will further illustrate the effects of imperfect inspections on the cost optimal maintenance schedule under a reliability constraint, the optimisation problem is programmed in the numerical program MATLAB.

2.5 Relating practice to Delay Time Models

The concept of the DTM has already been explained and its use justified in the previous paragraphs. However, the DTM is rather basic in its nature and simplifies asset's actual degradation pattern under consideration, by solely considering the time aspect of the asset's life. In Figure 2.1 this basic DTM concept is illustrated and it distinguishes three different asset states. For an overview on the state definitions, see Section 2.1.

This section presents the asset behaviour in practice in the first paragraph. The second paragraph then introduces the behaviour of inspections in maintenance. Based on the behavioural aspects of both, the asset and the inspections in maintenance, the DTM is altered which brings us to the third paragraph on the Adjusted Delay Time Model (DTMa). All of the paragraphs explain their respective elements by using a realisation of a degrading asset, i.e. in the figures included, no random aspects are drawn to enhance further understanding of the concepts.

Asset degradation behaviour

In practice, the asset can be decomposed into more than the three states distinguished by the DTM. The first state denotes the state in which the asset remains free of any defects, i.e. no degradation is present in the asset. This state is depicted in Figure 2.3 by the plot's first part until the lightning symbol. The lightning symbol indicates the asset starting to degrade.

The asset's degradation initiation might have occurred, but engineers or equipment is not yet able to detect it. Therefore, the asset can be in a second state. This second state corresponds to the asset which has started to deteriorate but its degradation cannot be detected by the maintenance staff. To illustrate the level from which the asset's degradation can be detected, a detection level is included in Figure 2.3. This level denotes the load carrying capacity of the asset from which the maintenance staff is able to detect asset degradation.

As Section 2.1 on current literature has indicated, direct maintenance actions upon the detection of an asset's degradation might not be economically worthwhile. Therefore, a rejection level is introduced in practice. Such a rejection level is a practical guideline for the maintenance staff to judge whether the inspected asset requires a maintenance action or not. The rejection level is a decision variable for NedTrain, which affects how much the asset's life is used. To further elaborate, when the asset's degradation is detectable, the maintenance staff does not engage in a maintenance action until the rejection level is reached. In case the asset's load carrying capacity becomes lower than the rejection level, a maintenance action is performed to the asset. This rejection level is included in Figure 2.3 as well. The third state of the asset corresponds to the asset's load carrying capacity indicating that a defect is detectable, but economically not worthwhile to perform a maintenance action, i.e. the load carrying capacity lies between the detection level and the rejection level.

Since the asset faces a random load, a critical level is introduced in practice to cope with the load's random behaviour. This critical level is set by the designers of the asset, i.e. in practice this critical level is a decision variable. The critical level corresponds to the level indicating that the asset is not allowed to be operated anymore. Because this critical level is set by the designers, it divides responsibility for the maintenance department and the design department. The maintenance department is responsible for a cost optimal maintenance schedule that does not violate the critical level and the design department is responsible for setting the critical level at proper values to cope with the stochastic load. For the maintenance department, this type of level is a operating constraint denoting the lowest load carrying capacity an asset may have during operation.

The fourth state is then defined by the asset's load carrying capacity being between the rejection level and the critical level. In case the load carrying capacity lies between these two levels, the asset is allowed to be operated, but is degraded beyond the rejection level, which implies that a maintenance action is required. Therefore maintenance action costs are incurred in the fourth asset state upon an inspection.

When the asset's load carrying capacity becomes equal to or less than the critical level, but remains higher than the load encountered, the asset is not allowed to be operated anymore, but has not yet failed. This denotes the asset's fifth state. In this state the operating constraint (critical level) is violated, but no extra costs are incurred for this state, since the asset has not yet failed. In practice three different costs can be incurred: inspection costs, maintenance action costs and asset failure costs. In case the asset is in the fifth state upon inspection, only inspection and maintenance action costs are incurred, but more importantly the operating constraint is violated.

The final asset state corresponds to the asset's load carrying capacity equalising the load at a point in time. When the load carrying capacity equals the load, the asset physically fails denoting the last and sixth state of the asset. In this last state, the cost of asset failure are incurred.

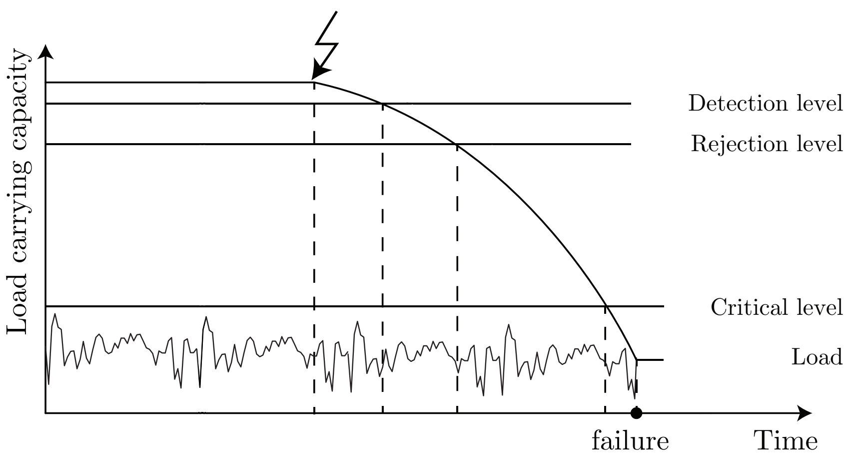


Figure 2.3: Asset degradation in practice

To further enlighten the asset's states present in practice, let us consider an example. Consider a steel construction suffering from degradation in the form of crack development. The state in which the asset remains completely without degradation, then corresponds to a construction without any form of crack in it.

At a certain time epoch, a crack starts to occur in the construction's steel. Such a crack might be so small that it cannot be observed by the maintenance staff and equipment used, i.e. the crack size is so small that it is undetectable. Yet still, the crack has started to degrade. Therefore the construction enters its second state which denotes the asset's degradation being present but undetectable.

From a certain load carrying capacity on, the crack in the construction can be detected by the maintenance staff. Say that the crack can be detected from 1 millimeter (mm) on. One might think of the load carrying capacity being defined by the crack size. When the crack can be detected, it might not be economically worthwhile to immediately restore the steel construction. Therefore, the construction is not maintained until the crack has reached a certain crack size, the rejection level. Say the construction undergoes maintenance when the crack size exceeds 5 mm, i.e. the rejection level is 5 mm. For detectable crack sizes in between 1 mm and 5 mm, then, no maintenance is performed, thereby denoting the construction's third state.

When the crack size becomes larger or equal to 5 mm, maintenance should be performed on the steel construction. Say the critical level corresponds to 10 mm. In case the crack grows larger than 10 mm, the asset is not allowed to be operated anymore. Therefore, the fourth state in this example corresponds to the allowable crack sizes in the construction when maintenance is done, i.e. crack sizes between 5 and 10 mm.

In case the asset's crack size has becomes larger than the critical level of 10 mm, the construction is not allowed to be operated anymore. Therefore, the operating constraint (the critical level) is violated. Note that if a crack size larger than 10 mm is found upon inspection and the construction has not failed, no failure costs are incurred.

Failure costs are incurred when the construction reaches its final state, in which it has failed. This occurs when the load carrying capacity, that has decreased due to the presence of the crack, meets the load encountered. In other words, in case the construction does not have sufficient load carrying capacity left to cope with the load encountered, the construction fails.

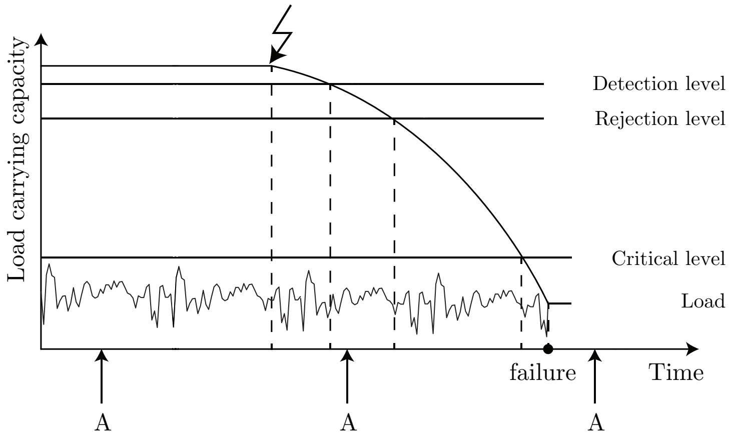
Inspection behaviour

In order to prevent the asset from progressing in its degradation, inspections and maintenance activities are scheduled which restore the asset, thereby preventing the asset from violating the critical level and failing. Inspections are done to determine whether the asset requires any maintenance action, i.e. whether the asset's load carrying capacity lies between the rejection level and the load, see Figure 2.3. The maintenance actions are namely required when the asset's load carrying capacity exceeds the rejection level but has not failed. So the asset's states that require maintenance actions correspond to states four and five.

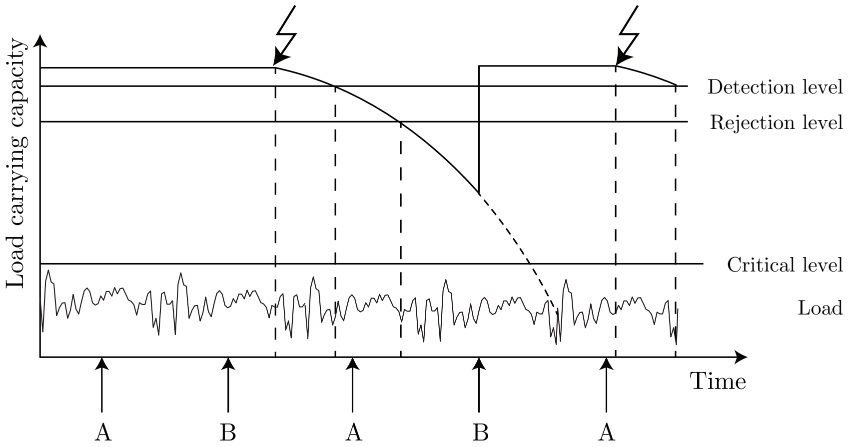
To determine whether a maintenance action is required, inspections are performed. Let us first illustrate the basic idea by considering perfect inspections, i.e. the inspection result always matches the asset's true state. Under these perfect inspections, the inspections are scheduled to reveal that the asset has degraded sufficiently to undergo maintenance.

In case only a few inspections are planned, the probability of critical level violation and asset failure will increase, i.e. when the time between inspections is long the probability of the asset violating the critical level and the probability of asset failure increases. Such few inspections correspond to the case where inspections are performed only on the time instances A from Subfigure 2.4a. As Subfigure 2.4a presents, the inspection outcomes of the first two inspections A yield no requirement for maintenance actions and the last inspection A comes to late to prevent asset failure.

In contrast, a high inspection frequency will correspond to a situation in which the probability of the asset violating the critical level and the probability of asset failure will both decrease. This situation corresponds to the case in which inspections are performed on time instances A and B from Subfigure 2.4b. Due to more inspections and subsequently less time between inspections, the degradation will be detected earlier resulting in a maintenance action, which decreases the probability of critical level violation and the probability of asset failure. In the specific case of Subfigure 2.4b, introducing additional inspections at time points B, the asset gets maintained upon the inspection result of the second instance of B. Due to the maintenance, the asset's load carrying capacity increases thereby reducing the probability of the asset reaching the critical level and the probability of asset failure. The dashed line in Subfigure 2.4b illustrates the effect of omitting inspections at time instances B.



(a) Asset degradation under few inspections A



(b) Asset degradation under many inspections A and B

Figure 2.4: Asset degradation in practice related to inspections

As Chapter 1 has already presented, the inspections might be imperfect, thereby affecting the probability of violating the critical level and asset failure. When the inspections are imperfect, the asset's actual load carrying capacity might be different from the inspection outcome. We consider two types of imperfectness in inspections, false positives and false negatives. The former denotes the event in which the asset is maintenance too early and unnecessarily, i.e. the asset is maintained before it has reached the rejection level. The latter, false negative, denotes the event in which the asset requires a maintenance action, but the inspection outcome yields that the asset does not.

The case of a false positive occurs when the asset's load carrying capacity has not reached the rejection level and the asset undergoes a maintenance action. In this case, the asset's probability violating the critical level and the probability of failure both decrease. The costs, however, increase as the asset is maintained too early. To illustrate, let us assume that inspections are performed on time points A and B, see Subfigure 2.4b. A false positive might occur at all inspection instances A, or at the first inspection instance of B. In these cases, the asset does not require a maintenance action but the inspections which are performed might present the outcome that the asset does, i.e. a false positive occurs. In case such unnecessary maintenance action is performed, the asset's probability of violating the critical level decreases as well as the probability of asset failure. Since the asset's life is not fully used, the costs increase.

The second type of imperfect inspections denotes the case of false negatives. This case might be catastrophic to the asset. When the asset requires a maintenance action but the inspection outcome does not yield this result, no maintenance action is performed. This increases the probability of the asset violating the critical level and increases the risk of asset failure which negatively affects costs. Analogous to the case of false positives, let us illustrate this event by using Subfigure 2.4b. A false negative can only occur in case the asset's load carrying capacity lies between the rejection level and the load. Note that when the asset's load carrying capacity becomes less than the critical level, the operating constraint is violated, but the asset still requires a maintenance action, since it has not yet failed. With respect to Subfigure 2.4b, a false negative can only occur on the second inspection instance of B. Upon this inspection, the asset actually requires a maintenance action, but when a false negative occurs, such an action is not performed resulting in asset failure before the next inspection. The effect of such a false negative is then depicted by the dotted line of the asset's degradation path in Subfigure 2.4b.

As been illustrated, the time between inspections and the imperfectness of inspections affect the probability of the asset reaching the critical level and the probability of asset failure. For practice, therefore, a challenge lies in deciding on the inspection interval length to decrease the probability of reaching the critical level and asset failure, and to cope with the imperfectness of inspections. Besides deciding on the time between inspections, practice also applies the technique of preventively replacing the asset considered. Such a strategy also copes with imperfectness of inspections. This is illustrated by an extreme example of considering the number of inspections after which preventive asset replacement occurs, to equal to 1. Note that this corresponds to the fact that inspection outcomes are irrelevant. In this case, the asset is preventively replaced at each inspection preventing the asset from degrading and thereby reducing the probability of the asset reaching the critical level and the probability of asset failure. In case the inspections are imperfect and the the asset is not preventively replaced, the probability of the asset's load carrying capacity reaching the critical level increases as well as the probability of asset failure compared to quickly replacing the asset preventively. For a graphical illustration of both situations, see Appendix F.1.

By applying both strategies, i.e. setting the inspection interval length and the instance of preventive asset replacement, practice is able to reduce maintenance costs and still satisfying reliability objectives. In this work, the practical combination of the inspection interval length and the inspection instance of preventive asset replacement defines the maintenance schedule.

Adjusted Delay Time Model

The practical situation sketched up to here has been highly detailed. This makes the mathematical modelling of the process described above highly complex. To be able to mathematically derive an optimal maintenance schedule which minimises the long term costs under a reliability constraint, the situation from practice is simplified. This simplification is based on the reasoning of the DTM type of models.

This study will limit itself to considering three asset states: normal, defective and failed. In the first state no maintenance action is required, in the second state a maintenance action is required and in the last state the asset has failed and needs to be replaced by a new asset.

To define the first state in more detail, we will start by considering when a maintenance action is performed. According to the description from practice, a maintenance action is executed when the asset's load carrying capacity is equal to or less than the rejection level. When the asset's load carrying capacity is higher than the rejection level, no maintenance action is required, and therefore the asset is considered to be operating normally. When the load carrying capacity becomes equal to or less than the rejection level, the asset is considered defective. The time the asset takes to become defective is referred to as the time to defect and denoted by x, see Figure 2.5.

Note that this definition of a defective asset differs from the original DTM's definition, since this original DTM defines the defective state as the state in which the asset's degradation is detectable, i.e. the original DTM considers the asset to be defective when the load carrying capacity reaches the detection level (Wang, 2008), see Figure 2.5. Recall from Section 2.1 that the time to defect for the original DTM is denoted by the variable u. Due to this conceptual difference in the definition of the defective state, we propose an Adjusted Delay Time Model (DTMa). Furthermore, in this DTMa, we consider the rejection level to be a constant and given to simplify further analysis.

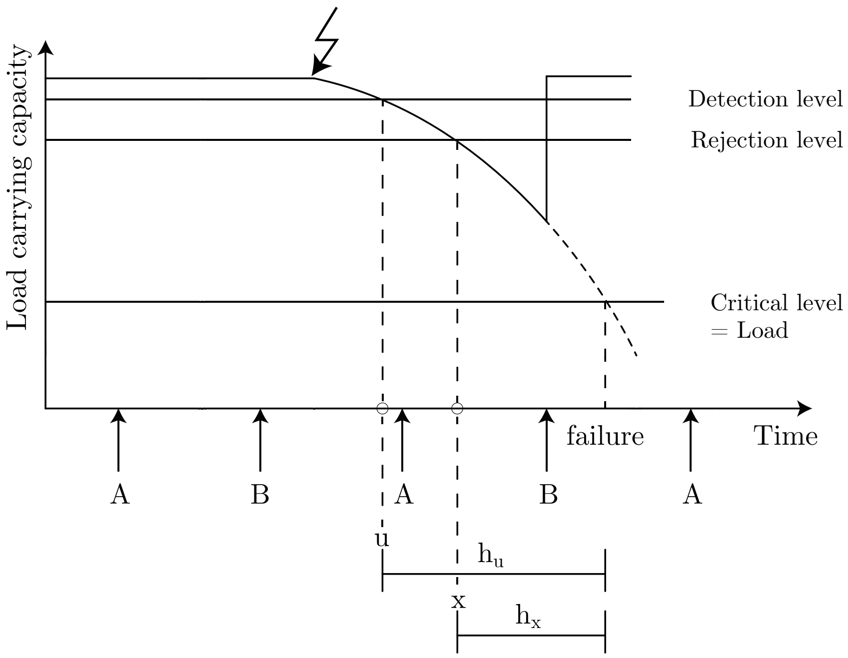


Figure 2.5: Simplified practical situation of asset behaviour

When considering the defective state, we derived after what time the defective state starts in the DTMa, after x time units. To determine the time the asset is in its defective state, the delay time, we shall first discuss the issue of when the asset is considered failed.

In the description of the asset behaviour we identified a critical level, which was introduced to cope with the random load pattern and denotes the operating constraint for the asset. This relates to the aspect of reliability, as the asset's load carrying capacity reaching the critical level yields an asset too unreliable for safe operation. Since we want to reduce the number of states for modelling feasibility, we will consider a worst case scenario in which the asset immediately fails when it reaches its critical level. This implies that the load encountered by the asset is assumed to be constant and equal to the critical level. Note that in this case, the costs for the asset failing are incurred when the asset's load carrying capacity reaches the critical level. Therefore, the costs for this model might be higher compared to the costs incurred in practice.

The delay time now denotes the time from the asset's defect occurrence, at time x in the DTMa and at time u in the original DTM, until the asset's load carrying capacity reaches its critical level and immediately fails. In the DTMa, the length of the delay time is denoted by h_x, whereas the delay time's length in the original DTM is denoted as h_u, see Figure 2.5.

Inspections are now performed to prevent the asset's load carrying capacity from reaching the critical level and therefore failing. When only a few inspections are done, the probability of the asset reaching the critical level and failing increases, whereas many inspections correspond to a decrease in the probability. Furthermore, in case the inspections are imperfect the probability of the asset reaching its critical level and failing is affected as well.

In addition to the inspections, the asset gets preventively replaced after a number of inspection instances. Analogous to practice, the combination of preventive asset replacement and the inspection interval length builds up the maintenance schedule considered in this work.

Since this research is based on the re-conceptualised state definitions from the DTMa, we will use this model in the remainder of the research. For brevity we will denote the delay time under the DTMa h_x as the variable h.

Next to the notation, we would like to address the fact that the time to defect x and the delay time h have been considered realisations up to now. However, since we do not know the time to defect and the delay time in advance, both variables are random variables. The remainder of this report will therefore take the random aspect of the time to defect and delay time into account.

2.6 Research contributions

The thesis project contributes with the knowledge obtained in twofold. The first contributions are for NedTrain as host of this project. The insights gained throughout this research with respect to reliability, cost and the optimal maintenance schedule contribute to NedTrain's program in further reducing costs and meeting European legislation. Costs can be further reduced by a more detailed and careful assessment of the reliability and the costs associated to an asset's maintenance schedule. The models developed support NedTrain in this assessment and quantification of the results.

Besides the contributions to NedTrain, this research also contributes to the academic literature in two ways. This work underlies the re-conceptualisation from the asset's defect and asset failure as discussed in Section 2.5. However, the mathematical modelling remains equal for the DTMa and the DTM. Hence, this research contributes to the literature on the DTM as well. The first contribution is that it presents insights in minimising costs over an infinite time horizon of a single-component asset being subject to a reliability constraint and imperfect inspections. Other work found which minimises costs over an infinite time horizon subject to a constraint is the work from Aven and Castro (2009), who present a reliability constraint as well. However, Aven and Castro (2009) consider the inspections to be perfect, whereas this work includes imperfect inspections.

The second aspect of this work's contribution to current literature denotes the inclusion of non-constant probabilities of false positives and false negatives. In literature, research already has been done with respect to false positives and false negatives. Baker and Wang (1993) and Wang (2010) developed models including solely false negatives. This is referred to as 'Single type inspections' in Table 2.1. The latter research extends the work of the former by including non-constant probabilities of false negative, which is defined in Table 2.1 as 'Non-constant single type imperfect inspections'.

Because false positives exist in inspections as well, Okumura et al. (1996) and Berrade et al. (2013) both consider these aspects in their works next to the false negatives. Both papers assume constant probabilities of false positives and false negatives. This constant probability of false positives and false negatives is denoted in Table 2.1 as 'Constant two type imperfect inspections'. This research extends current research by considering non-constant probabilities for both false positives and false negatives.

For an overview of the contributions this research has to current literature, Table 2.1 is included. This table graphically presents the added value of this work to literature.

Table 2.1: Comparison of papers

Attribute	Christer (1987)	Baker and Wang (1993)	Okumura et al. (1996)	Aven and Castro (2009)	Wang (2010)	Berrade et al. (2013)	This work
Single-component asset	X	X	X	X		X	X
Multi-component asset					X		
Reliability evaluation	X				X	X	X
Cost evaluation			X	X	X	X	X
Maintenance schedule optimisation	X			X		X	X
Constrained maintenance schedule optimisation				X			X
Perfect inspections	X	X	X	X	X	X	X
Constant single type imperfect inspections		X	X		X	X	X
Constant two type imperfect inspections			X			X	X
Non-constant single type imperfect inspections					X		X
Non-constant two type imperfect inspections							X

3

Description of Models

This chapter presents the description of the models developed in this research. This work focuses on the model development and comparison of the different ways of modelling imperfect inspections. This is the sole difference between the models presented. Hence, the models are identical with respect to the model description, as described in Section 3.1. Concerning the assumptions, all are common for the models. These assumptions are discussed in Section 3.2. The single difference between the various models is the way imperfect inspections are modelled. The final section, Section 3.3 presents three distinct approaches for modelling the imperfectness in inspections.

3.1 Description of the models

Based on the DTMa from Chapter 2 we present a description of the models developed. We will not explicitly include the asset's load carrying capacity, but we will focus on the time aspect of the defect arrival and the asset's delay time. Only the time dimension is considered because this decreases model complexity. Figure 3.1 presents such a representation of the DTMa that merely includes the time dimension, under a realisation of the time to defect and delay time. Note that the inspections are now denoted by integer numbers instead of A and B.

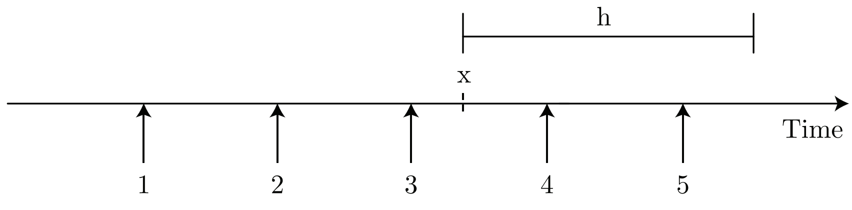


Figure 3.1: Illustrating the DTMa only considering the time dimension

Based on Figure 3.1, the models are described in more detail. Let us consider an asset which is characterised by three different states, as identified by the DTMa. The first denotes the state in which the asset operates normally, i.e. it is free of defects. The second state denotes the asset being defective and the final state corresponds to the asset having failed. The time to defect is denoted by the random variable X which is non-negative and continuous. The asset's delay time H then corresponds to the time the asset is in the defective state and also is a non-negative continuous random variable. By summing both, the time at which the asset fails is obtained, i.e. $X + H$. Because both times X and H are random variables the probability density function (pdf) and the cumulative density function (cdf) associated to both X and H are represented by $f_X(x)$ and $F_X(x)$ for the time to defect, and $f_H(h)$ and $F_H(h)$ for the delay time H respectively.

The asset is subject to condition-based maintenance with periodic inspections. The maintenance schedule of the condition-based maintenance applies consists of inspections and preventive asset replacement. The former aspect of the maintenance schedule is denoted by the inspection interval length T and the latter by the inspection instance M after which the asset is preventively replaced. The variables T and M are decision variables and comprise the maintenance schedule for the asset, i.e. these variables are optimised to yield lowest costs under the satisfaction of a reliability constraint.

Inspections to the asset are done each T time units. Hence, T denotes the fixed inspection interval length. Such inspections are considered imperfect. The imperfectness of inspections is decomposed into false positives and false negatives. The probability of the occurrence of false positives is denoted by α, whereas the probability of the false negatives corresponds to β. A false positive denotes the case in which an asset is considered defective when in fact it is operating normally and a false negative corresponds to the asset being considered normal when in fact it is defective. Table 3.1 presents an overview on the different errors which may occur. Note that only two errors are possible and four cells are included in the table. The remaining two cells indicate the probability of non erroneous behaviour.

Table 3.1: Classification of errors

	Asset's state	
	Normal	Defective
Inspection result Normal	$1-\alpha$	β
Inspection result Defective	α	$1-\beta$

We would like to stress the fact that multiple terms are used throughout literature denoting false positives and false negatives. The former is also referred to as a Type I error, whereas the latter is often denoted as a Type II error.

In addition to inspections, the asset is preventively replaced after M inspections, i.e. the inspection instance upon which preventive asset replacement occurs is denoted by M. Recall that M is a decision variable next to T. Both variables M and T build up the maintenance schedule. At each inspection, inspection costs c_0 are incurred and in case the inspection yields a maintenance action, costs c_p are incurred as well. The costs for the preventive asset replacement equal the costs for a maintenance action. The last cost aspect which is linked to the asset is the cost of failure denoted by c_f. All three costs are used in deriving the models in this research. Note that c_p corresponds to preventive maintenance costs and c_f to corrective maintenance costs.

The goal of the models developed is to minimise the average cost per time unit over an infinite time horizon under a reliability constraint, by allocating values to the maintenance schedule parameters M and T. Let us first consider the description of the cost aspect of the model.

The cost expression is based on the principle of the average cost per time unit (Ross, 1983). This average cost per time unit is denoted by $C_w(M,T)$ and is dependent on the maintenance schedule parameters, M and T, since these are decision variables. Because we introduce multiple model approaches for the probability of false positives and false negatives, the index w is included to indicate the model approach considered, e.g. the model of the first approach has index $w = 1$. $C_w(M,T)$ is based on the concept of cycles and is defined by the fraction between the expected cycle costs and the expected cycle length (Ross, 1983), see Equation 3.1.

$$C_w(M,T) = \frac{E_w(\textit{cycle costs})}{E_w(\textit{cycle length})} \tag{3.1}$$

A cycle may end in a maintenance action due to the inspection outcome, in preventive asset replacement or in asset failure. It is assumed that the costs of a maintenance action equal the costs of preventive asset replacement and are denoted by c_p. This assumption implies that a cycle can end in the event of incurring maintenance action costs c_p or in asset failure, upon which the corrective maintenance costs c_f are incurred.

The expected cycle costs are then comprised of the expected number of inspections in a cycle $E_w(K)$ multiplied by the costs per inspection c_0, the probability of incurring costs c_p in a cycle $P_w(c_p)$ multiplied by the maintenance action costs c_p and the probability of the asset failing multiplied by the costs associated to asset failure c_f. The latter probability corresponds to $1-P_w(c_p)$ because a cycle either ends in incurring maintenance action costs or in asset failure. Since $P_w(c_p)$ denotes the probability of the cycle ending in the event of incurring maintenance action costs, $1 - P_w(c_p)$ denotes the probability of the cycle ending in asset failure. This yields the total expected cycle costs corresponding to the numerator from Equation 3.1.

$$E_w(\textit{cycle costs}) = c_0 E_w(K) + c_p P_w(c_p) + c_f(1 - P_w(c_p))$$

By inserting the expression for the expected cycle costs into Equation 3.1, three terms are to be evaluated for the cost expression: $E_w(K)$ and $P_w(c_p)$, and the expected cycle length $E_w(\textit{cycle length})$. Since the goal of the models developed is to minimise the costs under a reliability constraint, we will continue this section with the discussion on the reliability description.

In considering asset reliability, first thing to do is to define when the asset is considered reliable. The asset is considered reliable when it has not failed, i.e. the asset is in its normal or defective state. Note that the reliability constraint corresponds in a way to the critical level from Figure 2.5, i.e. the asset is considered reliable when it has not reached its critical level.

The second aspect to clearly underline, is the notation of the reliability expression. A standard reliability function $R(t)$ denotes the probability of an unmaintained asset surviving up to its argument, the given time t. Since we include inspections and maintenance actions, we extend the notation of the reliability expression by including a subscript MT indicating the maintenance schedule of the asset. In addition, we add an index w denoting the model considered, e.g. $w = 1$ corresponds to the model of the first modelling approach, see Section 3.3. The final aspect of the reliability expression for the inspected and maintained asset, is the timespan over which the reliability is defined. We consider the time span of the asset to be characterised by the given variable t by $(0, t)$. In the models developed, the variable t is an input and is therefore given. The asset's reliability expression now corresponds to $R_{MT,w}(t)$. The reliability expression is non-conditional and therefore non-increasing by definition. Since we consider the reliability as a constraint, the reliability objective over the same timespan $(0, t)$ is considered. This reliability objective is denoted by $R_{obj}(t)$.

3.2 Assumptions for the models

The models discussed in this work underlie several common assumptions. All of these assumptions are listed below. Due to the fact that the models developed in this research only differ with respect to the way imperfect inspections are modelled, the common assumptions can be listed in a single enumeration.

- **The asset's state is classified into three states**
  The asset's life is decomposed into three states, as discussed in Section 3.1. The first is

the normal state, the second corresponds to the defective state and the last is the failed state.

- **A single-component asset is considered**
  The models consider one asset consisting of a single component, i.e. a single failure mode. The author notices that most assets consist of multiple components, but this model only considers the single component asset.

- **An asset's failure is instantaneously detected and the asset is immediately replaced upon failure detection**
  Upon asset failure, the failure is directly observed and the asset is immediately replaced by the maintenance staff. This replacement of the failed asset corresponds to a corrective maintenance type of activity, for which costs c_f are incurred.

- **The asset's defective state is observable, but can only be revealed by inspections**
  The asset's defective state can only be revealed by inspections. This implies that no remote monitoring is done.

- **Upon defect detection, the maintenance action is performed directly**
  When the asset's defect is being detected upon an inspection, a maintenance action is directly performed. This implies that no postponement of the maintenance action is possible for the models developed.

- **The maintenance action itself is perfect**
  The maintenance action itself is assumed to be perfect, corresponding to restoring the asset to the 'as new' condition. This assumption implies that maintenance in this case equals to asset replacement.

- **The costs for preventive asset replacement equal the costs of a maintenance action**
  The costs for preventive asset replacement, which is done at inspection M, equal the costs for a maintenance action done before inspection M. These costs are represented by c_p. The costs c_p correspond to preventive maintenance costs, i.e. the preventive asset replacement and the maintenance upon defect detection are preventive maintenance activities, and therefore incur the same costs c_p.

- **In case of asset replacement the new asset comes from a homogeneous population**
  When the asset is replaced, the replacement unit comes from a homogeneous population with the same probability distributions for the time to defect and delay time as the defective or failed asset.

- **The inspection interval length T is fixed for every interval**
  The inspection interval length T is fixed and does not change for each interval.

- **The inspection schedule restarts after asset failure**
  The inspection schedule with fixed T restarts after asset failure. This implies that in case the asset fails, the next inspection is done T time units from the time of asset failure.

- **Two types of imperfect inspections are present**
  False positives and false negatives can occur in the model which are both a type of imperfect inspections. The former has a probability of occurrence α, whereas the latter has probability β.

- **Upon the last inspection M the system is still inspected**
  At the last inspection M moment, the asset is still inspected after which it is preventively replaced. This replacement is then independent on the inspection outcome.

3.3 Modelling imperfect inspections

In the previous two sections the common description for the models and the common assumptions have been presented. This section discusses three distinct ways of modelling imperfect inspections. As we differentiate imperfect inspections into false positives and false negatives, three ways of modelling the probabilities α and β are considered. This paragraph first discusses the most basic form of conceptualising the probabilities α and β and continues with two extensions that model α and β as being non-constant.

This first and most basic way of modelling the probabilities of false positives α and false negatives β is to model these probabilities as constants, i.e. α and β are independent on any other variable, see Figure 3.2. This modelling approach is referred to in the remainder of the work as A1.

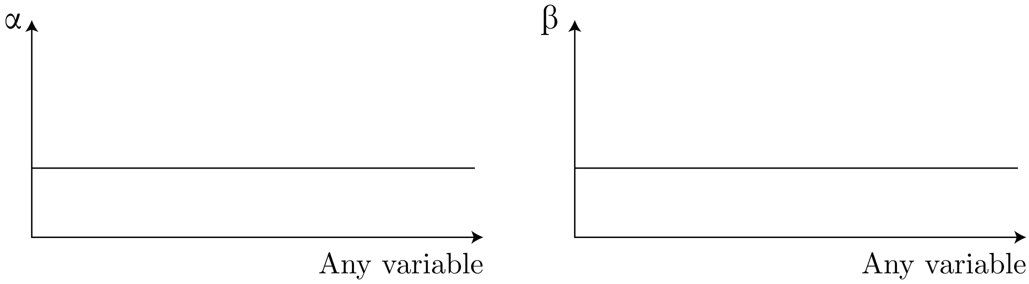


Figure 3.2: Illustration of α and β of model approach 1

The A1 approach underlies the work of Okumura et al. (1996) and Berrade et al. (2013), but the question is whether this way of modelling the probabilities of false positives and false negatives captures any of the dynamics of imperfect inspections. It might be reasonable to assume non-constant probabilities of imperfect inspections. In doing so, this section presents two other model approaches which both relate the probabilities α and β to various other variables.

The probabilities α and β are build up in such a way that a part of the probability is inevitable and the other part is dependent on particular variables. The reasoning for including the inevitable probability of false positives and false negatives lies in the fact that one cannot completely reduce the probability of imperfect inspections, because measurement equipment might return biased or wrong sensor values, or maintenance staff accidentally misjudges the asset's state. This inevitable probability for false positives and false negatives is denoted by α_0 and β_0 respectively. The other dependent part, which is assumed, for the false positives and false negatives is denoted by α_A and β_A respectively. By combining both, the general form for α and β is obtained. Note that this differentiation has not been made for the first model approach because it provides no added value due to the constant nature of α and β.

$$\alpha = \alpha_0 + \alpha_A \qquad \beta = \beta_0 + \beta_A$$

In addition, when modelling the probabilities α and β as two component expressions, the model approaches can be easily compared to the first model approach. The second and third model approach posed in the remainder take into account this two piece nature of α and β and differ in the way α_A and β_A are modelled.

To model the probability of a false positive α_A as being non-constant one may consider a psychological effect. When the asset has been free of defects for a long period of time compared to its time to defect, the engineer might be more prone to falsely judge the system as being defective when in fact it is still operating normally, i.e. the engineer engages in a false positive. Note that this claim underlies the premise of the engineer recognising the asset, he or she inspects at that moment. Otherwise the asset's inspection would be memoryless and the engineer does not know how long the asset has been defective. Under this premise, we assume an increase in the probability of a false positives when the time of inspection relative to the asset's time to defect increases. The ratio between the time of inspection and the assets time to defect is referred to as the asset's defect progress and is formally conceptualised as:

$$Defect\ progress = \frac{Time\ instance\ of\ inspection}{Time\ to\ defect}$$

Hence, we assume the probability of false positives to increase with increasing values of the defect progress. This type of modelling of the defect progress only considers inspections which occur before defect arrival, i.e. the time instance of inspection is strictly less or equal to the time to defect.

Based on the same type of reasoning, the probability of a false negative β_A is likely to decrease when the asset has been in its defective state for a longer period of time relative to its delay time. This approach is based on the idea presented by Wang (2010) but additionally includes the asset's delay time. In this case, one takes the asset's duration in the defective state relative to the delay time of the specific asset. This relative fraction is denoted as the degradation progress. This ratio represents how degraded the asset is and is defined as:

$$Degradation\ progress = \frac{Duration\ in\ defective\ state}{Delay\ time}$$

Therefore, we assume decreasing probabilities of false negatives for increasing values of the degradation progress. Both assumed probabilities of false positives and false negatives are graphically illustrated in Figure 3.3 presenting an intuitive understanding of the relationship between the probabilities and the asset's age related to the inspections.

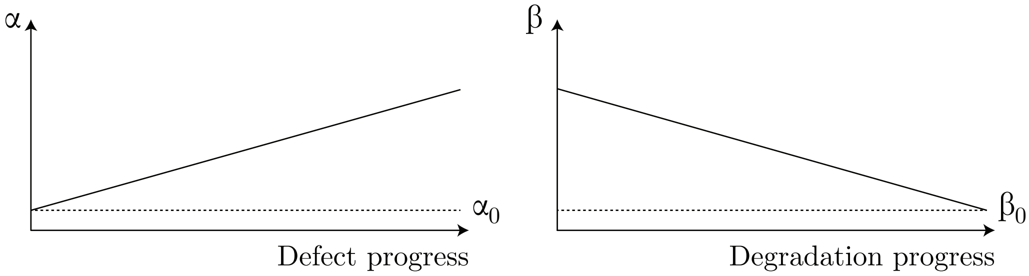


Figure 3.3: Illustration of potential α and β functions of model approach 2

Combining both assumed aspects of the probabilities of false positives and false negatives, the second model approach A2 is obtained, i.e. the probability of false positives increases with increasing values of the defect progress and the probability of false negatives decreases with increasing values of the degradation progress. Furthermore, the probability of both, false positives and false negatives, cannot drop below the inevitable probabilities α_0 and β_0 respectively.

Besides the probabilities of α and β being dependent on the defect progress and the degradation progress respectively, the probabilities of imperfect inspections can also be modelled in a different way to capture a different psychological phenomenon. The probabilities of false positives and false negatives might be related to the inspection frequency and the Mean Time To Failure (MTTF). The MTTF denotes the average time an unmaintained asset takes until its failure. As a reliability characteristic, the MTTF is chosen instead of the failure rate due to fact that it does not vary over time. When one were to use a temporal characteristic it would be harder to derive expressions for the cost and reliability models under imperfect inspections. Additionally, due to the two phase characteristic of the asset's life, the MTTF can rather easily be determined for any distribution function, whereas the failure rate faces larger problems in algebraically determining its form for any distribution function, e.g. algebraically determining the convolution of two Weibull distributions is highly complex.

To illustrate how the probabilities of false positives and false negatives might be affected by the MTTF and the inspection frequency, two extreme cases are considered. The first case consists of the fact in which a high inspection frequency is combined with a high MTTF. This would yield, on average, many inspections in which nothing is found. Assets namely fail infrequently, but are still often inspected. In case this happens it is expected that the maintenance staff is less inclined to do a thorough inspection resulting in a high value for β and a low value for α.

The second case consists of a low MTTF combined with a low inspection frequency. In this scenario assets fail frequently but are subject to a low inspection frequency. It is assumed that the maintenance staff in this case will be careful, because it is of vital importance to find a defect when the asset fails frequently and only few numbers of inspections are done. Subsequently the probabilities of false positives α and false negatives β become high and low respectively.

A third and fourth case are also recognised which include the combination of a low MTTF with a high inspection frequency and a high MTTF combined with a low inspection frequency. Because the probabilities α and β for these cases are highly unpredictable and complex, these are not included in the third model approach. This results in the fact that one can illustrate both extreme cases in a table presenting an overview on the relationship between the probabilities of false positives, false negatives, inspection frequency and the asset's MTTF. This overview is depicted in Table 3.2.

Table 3.2: Assumed α and β dependent on MTTF and inspection frequency

		Mean Time To Failure	
		Low	High
Inspection frequency	Low	High α Low β	
	High		Low α High β

Note that the total probability of false positives and false negatives cannot drop below the inevitable probabilities of α_0 and β_0, analogous to the A2 model approach. To graphically enhance the third approach, the data from Table 3.2 is drawn in Figure 3.4.

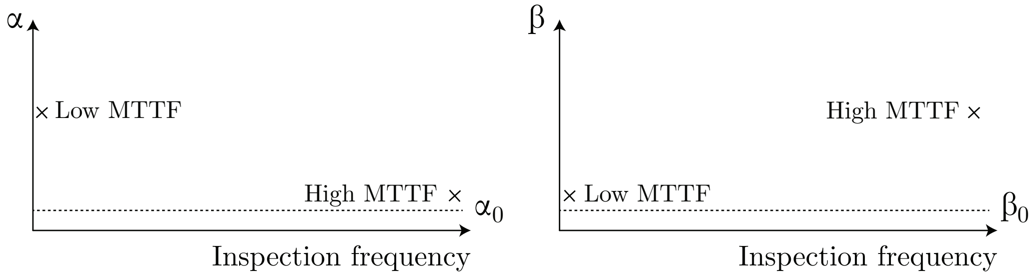


Figure 3.4: Illustration of potential α and β functions of model approach 3

In Figure 3.4 no specific type of relationship between α, β, the MTTF and the inspection frequency is included. This is not done due to the fact that specifying such type of relationship is highly complex. The interesting notion stemming from Figure 3.4 is that α is negatively related to the combination of the MTTF and the inspection frequency. Additionally, β is positively related to the combination of the MTTF and the inspection frequency. This yields the third model approach A3 in which the probability of false positives α is negatively related to the combination of the inspection frequency and the MTTF, and the probability of false negatives β is positvely related to the combination of the inspection frequency and the MTTF. Furthermore α and β cannot drop below α_0 and β_0, respectively.

4

Model Evaluation and Optimisation

This chapter presents three different models which each correspond to the model approaches of the probabilities of false positives and false negatives, as posed in Chapter 3. The models are labelled according to the model approaches' number from the previous chapter, i.e. the model corresponding to the first approach is denoted by A1 model etcetera.

All the models developed in this chapter are based on the concept of the DTMa and all consist of the same general form in their evaluation and optimisation. First, the evaluation of the models will gain insights in the way the costs and reliability function are modelled. Subsequently these evaluated expressions are used for the optimisation of the inspection interval length T and the last inspection instance M upon which preventive replacement takes place, to yield lowest costs over an infinite time horizon and satisfying a reliability constraint. This yields the formal optimisation model, as presented in Equation 4.1.

$$\begin{aligned} \min_{M,T} \quad & C_w(M,T) = \frac{c_0 E_w(K) + c_p P_w(c_p) + c_f(1 - P_w(c_p))}{E_w(\textit{cycle length})} \\ \text{subject to} \quad & \\ & R_{MT,w}(t) \geq R_{obj}(t) \\ & T > 0,\ M \in \mathbb{N} \end{aligned} \tag{4.1}$$

The model from Equation 4.1 restricts the values for M in such way that these are only natural numbers, integers. Additionally, the optimisation model offers a comparative basis for comparing the effects of differently modelling the probabilities of false positives and false negatives. Such a comparison is presented in Chapter 5. This chapter will focus on the evaluation and derivation of the optimisation models for the three distinct ways of modelling the probabilities of false positives and false negatives. Section 4.1 will present the cost and reliability evaluation, and the optimisation model for the model corresponding to the first model approach, labelled the A1 model. This model is based on the work from Berrade et al. (2013) and is the basis for the other more advanced models, the A2 model and the A3 model. The first of the advanced models, the A2 model, includes the probability of α to be dependent on the defect progress and the probability of β to be dependent on the degradation progress. The cost and reliability expressions are evaluated and used in deriving the optimisation model. This is done in Section 4.2. This chapter concludes with Section 4.3, in which the A3 model is presented by deriving a cost and reliability evaluation and optimisation model for the situation in which the probability of false positives and false negatives are both dependent on the Mean Time to Failure (MTTF) and the inspection frequency. Recall that in the third model approach the MTTF of an unmaintained asset is considered.

4.1 A1 model

This section presents the evaluation for the cost and the reliability expression under constant probabilities of false positives α and false negatives β. The subscript 1 is included to indicate that the A1 model is considered, i.e. $w = 1$.

4.1.1 Model evaluation

Cost evaluation

Three expressions are evaluated which build up the complete expression for the average costs per time unit for the A1 model: $E_1(K)$, $P_1(c_p)$ and $E_1(\textit{cycle length})$. For a detailed derivation of each of the mathematical expressions, the reader is referred to Appendix D.1.1. The expected number of inspections in a cycle under the A1 model $E_1(K)$ equals to Equation 4.2.

$$\begin{aligned}
E_1(K) &= \sum_{j=1}^{M-1} jP(K=j) + MP(K=M) \\
&= \sum_{j=1}^{M-1} j \Bigg\{ \sum_{i=1}^{j} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx \\
&+ \sum_{i=1}^{j} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \int_{jT-x}^{(j+1)T-x} \beta^{j-i+1} f_H(h)dhf_X(x)dx \\
&+ (1-\alpha)^j \int_{jT}^{(j+1)T} F_H((j+1)T-x)f_X(x)dhdx \\
&+ \alpha(1-\alpha)^{j-1}(1-F_X(jT)) \Bigg\} \\
&+ M \Bigg\{ \sum_{i=1}^{M} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{M-i}(1-F_H(MT-x))f_X(x)dx \\
&+ (1-\alpha)^{M-1}(1-F_X(MT)) \Bigg\}
\end{aligned} \tag{4.2}$$

Equation 4.2 roughly consists of two parts, the first corresponds to the expected value of the number of inspections per cycle due to j inspections in a cycle, where $j = 1, \ldots, M-1$. The latter part denotes the contribution of the expected value when M inspections in a cycle occur.

The first term $P(K = j)$ consists of four scenarios, of which the first denotes the probability that asset has not undergone false positives up to and including inspection $i-1$. Furthermore, it becomes defective in the interval $((i-1)T, iT)$, where $i = 1, \ldots, j$. The asset does not fail and the defect is then found upon the j^{th} inspection with probability $1-\beta$, leaving $j-i$ inspections upon which false negatives occur. The second term of $P(K = j)$ corresponds to the probability of no false positives having occurred before the asset has become defective. The asset becomes defective in $((i-1)T, iT)$, where $i = 1, \ldots, j$, and the defect is not being found up to and including the j^{th} inspection. This leaves $j-i+1$ number of inspections in which false negatives occur. After the j^{th} inspection the asset fails in the interval $(jT, (j+1)T)$. Term three yields the probability of the asset remaining in its normal operating state up to and including the j^{th} inspection and no false positives have occurred. The asset becomes defective and fails after inspection j and before inspection $j+1$. The final term then corresponds to a false positive occurring upon the j^{th} inspection with probability α, and no false positives having occurred before the j^{th} inspection.

The probability of M inspections in a cycle $P(K = M)$ consists of two terms. The first denotes the probability that the asset becomes defective before inspection M in interval $((i-$

$1)T, iT)$, where $i = 1, \ldots, M$, and the asset survives to time MT. The defect, however, is not found corresponding to $M - i$ false negatives occurring. Until defect arrival, no false positives have occurred. The second term of $P(K = M)$ concludes the probability expression by representing the probability of the asset remaining free of defects until the M^{th} inspection and no false positives have occurred before inspection M.

To evaluate the probability of incurring maintenance action costs $P_1(c_p)$, a brief recapitulation to the assumptions is made. Recall that we assume the costs for preventive asset replacement to equal to the costs of a maintenance action. This implies that upon the M^{th} inspection, costs c_p are incurred for preventive asset replacement. For a detailed derivation of $P_1(c_p)$ the reader is referred to Appendix D.1.1. This section merely presents the evaluated expression and elaborates briefly upon it. The probability of incurring costs c_p corresponds to Equation 4.3.

$$
\begin{aligned}
P_1(c_p) = &\sum_{i=1}^{M-1} \sum_{j=i}^{M-1} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx \\
&+ \sum_{j=1}^{M-1} \alpha(1-\alpha)^{j-1}(1-F_X(jT)) \\
&+ \sum_{i=1}^{M}(1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{M-i}(1-F_H(MT-x))f_X(x)dx \\
&+ (1-\alpha)^{M-1}(1-F_X(MT))
\end{aligned}
\tag{4.3}
$$

The first term denotes the probability of asset becoming defective in $((i-1)T, iT)$, where $i = 1, \ldots, M-1$. Until the asset becomes defective no false positives have occurred. The asset survives up to its defect detection, which occurs upon inspection j with probability $1-\beta$, leaving $j - i$ inspections subject to false negatives, where $j = i, \ldots, M-1$.

The second term corresponds to the probability of the asset remaining in its normal operating state until the j^{th} inspection, no false positives have occurred before inspection j but upon this inspection a false positive occurs, where $j = 1, \ldots, M-1$.

The third term equals to the probability of the asset becoming defective in $((i-1)T, iT)$, where $i = 1, \ldots, M$. Before the asset's defect arrival no false positives have occurred. The asset then survives to inspection M but its defect is not revealed until the M^{th} inspection corresponding to $M - i$ false negatives. Upon this inspection M the asset is then preventively replaced which corresponds to incurring cost c_p.

The final term denotes the probability of the asset operating normally up to inspection M and no false positives have occurred. Due to the preventive replacement, costs c_p are incurred.

The final term evaluated for the cost evaluation is the expected cycle length $E_1(\textit{cycle length})$. A detailed derivation of the expression for the expected cycle length is included in Appendix D.1.1. Here the expression is presented in Equation 4.4 and briefly discussed.

$$
\begin{aligned}
E_1(cycle\ length) = &\sum_{i=1}^{M}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\int_{0}^{iT-x}(x+h)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\int_{jT-x}^{(j+1)T-x}\beta^{j-i+1}(x+h)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-\alpha)^{i-1}jT(1-\beta)\int_{(i-1)T}^{iT}\beta^{j-i}(1-F_H(jT-x))f_X(x)dx \qquad (4.4) \\
&+\sum_{i=1}^{M}(1-\alpha)^{i-1}MT\beta^{M-i}\int_{(i-1)T}^{iT}(1-F_H(MT-x))f_X(x)dx \\
&+(1-\alpha)^{M-1}MT\,(1-F_X(MT))+\alpha T\sum_{j=1}^{M-1}j(1-\alpha)^{j-1}(1-F_X(jT))
\end{aligned}
$$

The first term from Equation 4.4 denotes the expected cycle length when a cycle ends in asset failure. The asset becomes defective and fails in the interval $((i-1)T, iT)$, where $i = 1, \ldots, M$. No false positives have occurred before defect arrival. In case the cycle ends in asset failure, its length equals to $x + h$. Because both variables x and h are realisations of the random variables X and H, these are included in the probability integrals over the time to defect and delay time.

The second term denotes the cycle also ending in a failure, but the asset now becomes defective in the interval $((i-1)T, iT)$, where $i = 1, \ldots, M-1$. The defect is not found in any of the succeeding inspections j, where $j = i, \ldots, M-1$, and the asset fails in the interval $(jT, (j+1)T)$. This corresponds to the asset's delay time being in the interval $(jT-x, (j+1)T-x)$. No false positives occur before the defect arrives, i.e. $i-1$ times no false positives occur. In addition, $j-i+1$ number of false negatives occur before asset failure. The cycle length for this term equals to $x+h$. Analogous to the first term, $x+h$ is included in the integrals due to the fact that they are realisations of the random variables X and H.

Term three corresponds to the cycle ending in the asset not failing and the defect being found. The asset becomes defective in $((i-1)T, iT)$ but the defect is found upon inspection j with probability $1-\beta$, where $j = i, \ldots, M-1$. This implies $j-i$ false negatives to occur. Before the defect arrival no false positives have occurred. Because the defect is found upon inspection j, the cycle ends at this inspection which is performed at time jT.

The fourth term denotes the cycle ending in preventive asset replacement. The asset has become defective in $((i-1)T, iT)$, but survives to inspection M. The defect is not found until the M^{th} inspection after which the preventive asset replacement is done. This yields $M-i$ false negatives, i.e. β^{M-i}. Additionally, no false positives have occurred before the defect arrival and the cycle length for this term corresponds to MT.

The fifth term corresponds to length of the cycle ending in preventive asset replacement. No defect and no false positives have occurred until the M^{th} inspection.

The final term denotes the cycle which ends in a false positive. Such a false positive occurs at inspection j with probability α, where $j = 1, \ldots, M-1$. In order for a false positive to occur, the asset must be free of defects until inspection j and no false positives have occurred before inspection j. The length of the cycle in this case equals the time on which inspection j is performed: jT.

All three evaluated expressions $E_1(K)$, $P_1(c_p)$ and $E_1(cycle\ length)$ are inserted in Equation 3.1 to yield the expected cost per time unit $C_1(M, T)$. The cost expression is not presented due to the large form it takes.

Reliability evaluation

This paragraph presents the evaluation for the reliability function under the A1 model. For a more detailed overview on this reliability function the reader is referred to Appendix D.1.2. Here, the reliability evaluation is presented briefly. We will derive a general reliability expression for any value for the given time argument t, i.e. we derive $R_{MT,w}(t)$ for any given value of t. This will be done in a stepwise fashion. We will start by considering t to be smaller than the time instance of preventive asset replacement MT. From the moment of preventive asset replacement, a recurrence of the reliability function is included to model the reliability expression for any value of t. This recurrence is included since the asset gets preventively replaced.

As the asset undergoes imperfect inspections, the time interval from 0 to the moment of preventive asset replacement MT is decomposed into intervals bounded by the inspections. Let us consider an inspection s denoting the interval $((s-1)T, sT)$, where $s = 1, \dots, M$. We first derive an expression for the reliability when $(s-1)T \leq t < sT$. This expression is then used to derive the reliability expression when one considers time t, where $0 \leq t < MT$. The reliability function for the A1 model, under $(s-1)T \leq t < sT$ is expressed by Equation 4.5. Note that the interval characterised by inspection s is included as a superscript in the notation.

$$
\begin{aligned}
R^{(s)}_{MT,1}(t) = &\sum_{i=1}^{s-1}\sum_{j=i}^{s-1}(1-\alpha)^{i-1}(1-\beta)\beta^{j-i}\int_{(i-1)T}^{iT}(1-F_H(jT-x))f_X(x)dx\, R^{(s-j)}_{MT,1}(t-jT)\\
&+\sum_{i=1}^{s-1}(1-\alpha)^{i-1}\beta^{s-i}\int_{(i-1)T}^{iT}(1-F_H(t-x))\,f_X(x)dx\\
&+\sum_{j=1}^{s-1}(1-\alpha)^{j-1}\alpha\,(1-F_X(jT))\,R^{s-j}_{MT,1}(t-jT)\\
&+(1-\alpha)^{s-1}\int_{(s-1)T}^{t}(1-F_H(t-x))\,f_X(x)dx\\
&+(1-\alpha)^{s-1}\,(1-F_X(t))
\end{aligned}
\tag{4.5}
$$

where $(s-1)T \leq t < sT$

The first term of the reliability expression for interval $((s-1)T, sT)$ denotes the probability of the asset becoming defective in $((i-1)T, iT)$, where $i = 1, \dots, s-1$. No false positives have occurred before inspection i. After the defect arrival the asset remains defective until inspection j, where the defect is found with probability $1-\beta$, where $j = i, \dots, s-1$. This leaves $j-i$ false negatives to occur. When the defect is found, the asset is maintained and due to the assumption of perfect maintenance, the recursive reliability expression is included. From inspection j on, the asset is considered new and independent of its wear before time jT. Hence, recurrent expression for the reliability denoting the reliability of the maintained asset from time jT to time t, for the remaining $s-j$ intervals is included.

The second term equals the probability of the asset becoming defective in $((i-1)T, iT)$, where $i = 1, \dots, s-1$, and remaining defective up to time t. Before defect arrival no false positives have occurred and after defect arrival $s-i$ false negatives occur, because the defect is not found upon any inspection preceding time t.

Term three corresponds to the probability of a false positive occurring upon inspection j. For the occurrence of such a false positive, the asset has to remain free of defects until inspection j and no false positives may have occurred before inspection j. Due to the maintenance action done in this term, the recursive element of the reliability is included denoting the probability of the maintained asset surviving up to time t from the time instance of the false positive jT on for the $s-j$ intervals remaining.

The fourth term denotes the probability of the asset becoming defective after inspection $s-1$ and before time t, but it does not fail until time t. Recall that $(s-1)T \leq t < sT$. The inspections up to $s-1$ have not encountered a false positive.

The final term of the reliability expression corresponds to the probability of the asset operating normally up to time t and no false positives have occurred before time t.

Note that the summation signs in Equation 4.5 equal to a value of 0 when the upper bound is less than he lower bound, because it contains no integer values which can be summed (Koltun, 2008).

The reliability expression for a given time argument t lying between 0 and MT is comprised of all the different reliability expressions for s, where $s = 1, \ldots, M$. This implies the following relationship for $0 \leq t < MT$.

$$R_{MT,1}(t) = R^{(s)}_{MT,1}(t), \quad (s-1)T \leq t < sT, \quad s = 1, \ldots, M$$

We use the results from the expression above to yield the reliability expression for any value of t. We will present and briefly discuss the general reliability expression $R_{MT,1}(t)$ for any t. For more details on the derivation, the reader is referred to Appendix D.1.2.

Analogous to the inclusion of the recurrent reliability expression in Equation 4.5, a recurrent term of the reliability is included when considering the general reliability expression for any value for t. In this case the asset does not undergo perfect maintenance but it gets replaced, i.e. the recurrent reliability is included due to preventive asset replacement upon MT. This yields the general expression for the reliability under the A1 model for any t.

$$R_{MT,1}(t) = R^{(aMT+s)}_{MT,1}(t) = R^{(M)}_{MT,1}(MT)^{a} R^{(s)}_{MT,1}(t - aMT), \quad (aMT+s-1)T \leq t < (aMT+s)T, \quad s = 1, \ldots, M, \quad a = 0, 1, \ldots, \infty \tag{4.6}$$

The first term of the expression above, $R^{(M)}_{MT,1}(MT)^{a}$, denotes the probability that the maintained asset survives a times the time between preventive asset replacement instances. The latter term corresponds to the reliability of the asset at time t, which lies in the interval characterised by inspection s, from the a^{th} MT on.

4.1.2 Model optimisation

Both evaluations for the costs $C_1(M,T)$ and the reliability $R_{MT,1}(t)$ are used in the optimisation model for determining the optimal values for M and T. The evaluated expression for $E_1(K)$, $P_1(c_p)$, $E_1(\textit{cycle length})$ and $R_{MT,1}(t)$ are not inserted into the optimisation model from Equation 4.7, because this yields a hard to read model.

$$\begin{aligned} \min_{M,T} \quad & C_1(M,T) = \frac{c_0 E_1(K) + c_p P_1(c_p) + c_f(1 - P_1(c_p))}{E_1(\textit{cycle length})} \\ \text{subject to} \quad & \\ & R_{MT,1}(t) \geq R_{obj}(t) \\ & T > 0, \ M \in \mathbb{N} \end{aligned} \tag{4.7}$$

4.2 A2 model

This section discusses the cost and reliability evaluation models, and the optimisation model under the probability of false positives being dependent on the defect progress, and the probability of false negatives being dependent on the degradation progress. A subscript 2 is added to each of the model's terms to indicate that the terms derived correspond to the A2 model, i.e $w = 2$.

4.2.1 Model evaluation

This subsection presents an evaluation of the costs and the reliability expressions for the model incorporating the assumed relationship between the probability of a false positive and the defect progress, and the relationship between β and the asset's degradation progress. The evaluation of the A2 model is based on the evaluation of the A1 model from Section 4.1. Before evaluating the cost and reliability expressions, the probability expressions for α and β are evaluated.

Evaluating α and β

As assumed in the second model approach from Chapter 3, the probability of false positives is dependent on the defect progress. This defect progress has been conceptualised as the fraction between the time of inspection and the asset's time to defect x, given that the asset is still operating normally upon inspection. Note that this latter condition is met by definition when a false positive occurs.

$$\textit{Defect progress} = \frac{\textit{Time instance of inspection}}{\textit{Time to defect}}$$

Recall that the probability α is decomposed into $\alpha = \alpha_0 + \alpha_A$, where α_0 denotes the inevitable probability of a false positive. As the A2 approach from Chapter 3 poses, α_A is dependent on the defect progress. This implies that the total probability of false positives α is dependent on the defect progress as well.

To derive a mathematical expression for the defect progress, let us first consider numerator in the fraction, the time instance of the inspection. The time of an inspection r then corresponds to rT, when a fixed inspection interval T is considered. Recall that this time instance rT always occurs before the time to defect, meeting the condition that the asset operates normally upon inspection r. To obtain the defect progress, rT is divided by the asset's time to defect x yielding $\frac{rT}{x}$. This fraction is then subsequently used as the argument of α_A and in turn as the argument for α. All yields the following expression for the probability of false positives:

$$\alpha\left(\textit{Defect progress}\right) = \alpha\left(\frac{rT}{x}\right)$$

In addition to deriving the expression for the probability of false positives, we also derive an expression for β becoming dependent on the degradation progress. Analogous to α, β has been decomposed into the inevitable part β_0 and the assumed part β_A. This assumed part is dependent on the degradation progress resulting in the total expression for β being dependent on the degradation progress, i.e. $\beta\left(\textit{Degradation progress}\right)$. This degradation progress has been conceptualised as:

$$\textit{Degradation progress} = \frac{\textit{Duration in defective state}}{\textit{Delay time}}$$

Note that, by definition, the duration in the defective state is strictly less or equal to the delay time and non-negative. The denominator, delay time, is denoted by the variable h. The numerator, the duration in the defective state, is represented differently. Let us consider an inspection q and recall that false negatives can only occur when the asset has already become defective. In this case, the asset has to become defective before inspection time qT. The time instance at which the asset becomes defective is denoted by the time to defect x. The duration the asset has been in the defective state upon inspection q then equals to $qT - x$. To conceptualise Figure 4.1 is included.

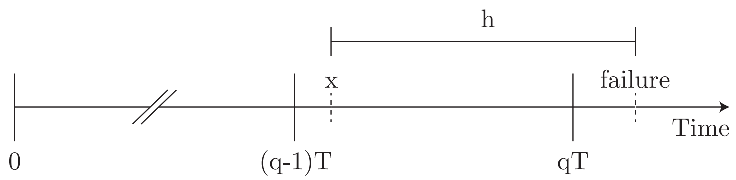


Figure 4.1: Illustrating the degradation progress

In Figure 4.1 the time between point 0 and $(q-1)T$ is interrupted by two skewed lines to indicate that the time between 0 and $(q-1)T$ varies for different values of q. Furthermore, note that Figure 4.1 is merely an illustration and that the asset's time to defect does not necessarily need to be in the interval $((q-1)T, qT)$. Additionally, Figure 4.1 presents the different elements of the degradation progress and visually enhances their relation to one another. When the duration in the defective state is defined as $qT - x$ and the delay time by h, the general expression of β being dependent on the degradation progress upon inspection q becomes.

$$\beta\left(Degradation\ progress\right) = \beta\left(\frac{qT - x}{h}\right)$$

The definitions of $\alpha\left(\frac{rT}{x}\right)$ and $\beta\left(\frac{qT-x}{h}\right)$ are used in the evaluation of the costs $C_2(M, T)$ and the reliability expression in the following paragraphs.

Cost evaluation

As mentioned before, the cost evaluation of the A2 model is heavily based on the A1 model. The difference between both models lies in the modelling of the probability of false positives and false negatives. The first step in deriving the cost evaluation for this A2 model is to rewrite the original A1 model, i.e. Equations 4.2, 4.3 and 4.4. By rewriting these equations it becomes clear how the A1 model can be altered to include the probability of false positives being dependent on the defect progress, and the probability of false negatives being dependent on the degradation progress.

The thing to notice is that the probabilities of false positives and false negatives under the A2 model are no longer constant during evaluation. As been discussed, first thing to do to include this non-constant nature is to rewrite the A1 model. This implies rewriting the probabilities for false positives and false negatives to their product expressions, e.g. $(1-\alpha)^{i-1} = \prod_{n=1}^{i-1}(1-\alpha)$, and rewriting the cdf expressions to their corresponding pdf integral expressions. Note that these operations do not change the A1 model, but merely alter the notation which makes model adjustment for the A2 model easier. For an overview on the procedure of rewriting, see Appendix D.2.1.

After rewriting the A1 model to the one including the product signs instead of powers and pdf integrals instead of the cdf expressions, the model can actually be altered. We will start with the alteration of the non-constant probability of false positives. Let us first consider the occurrence of a false positive. As been discussed in Section 4.1, the false positive can only occur once per cycle, upon inspection j yielding the rewriting of the probability of a false positive upon inspection j to $\alpha\left(\frac{jT}{x}\right)$. In addition to a false positive occurring, many scenarios describe no false positives occurring by the inclusion of the product for false positives, which is obtained after rewriting the A1 model. To include the defect progress dependency for these cases, the product sign's index is used, which corresponds to the inspections that have preceded the asset's defect arrival. When these inspections are denoted by the product sign's index n, the probability of a false positive not occurring upon inspection n equals to $1-\alpha\left(\frac{nT}{x}\right)$. By rewriting the constant expression to the non-constant expression in the product sign, the non-constant probability for false positives not occurring is obtained. Note that the expressions for both, the occurrence of a false positive and the non-occurrence of false positives, include the realisation x of the random variable X, implying the inclusion of the terms in the integral over the time to defect.

The same logic applies for the false negatives, but vice versa. A false negative, namely, may occur multiple times per cycle, whereas a false positive can only occur once per cycle. The non-occurrence of a false negative, however, can occur only once per cycle because this denotes the ending of a cycle, i.e. the defect which has occurred is found and the cycle ends. By the same logic as applied to the false positives, the non-occurrence of a false negative only happens upon inspection j, yielding the non-constant probability of $1-\beta\left(\frac{jT-x}{h}\right)$. In an analogous fashion as the non-occurring false positives, the probability of false negatives occurring includes the index variable of the product of false negatives, which is denoted by k. This variable denotes the inspection k, upon which a false negative occurs with a corresponding probability of $\beta\left(\frac{kT-x}{h}\right)$.

When including all of the derived expressions for the occurring and non-occurring false positives and false negatives, the expressions for the expected number of inspections per cycle under the A2 model, the probability of incurring maintenance action costs under the A2 model and the expected cycle length under the A2 model are obtained, as Equations 4.8, 4.9 and 4.10 present, respectively. No further explanation on the individual terms is presented here, because these correspond to the A1 model. For the summarised elaboration the reader is referred to Section 4.1. A detailed explanation on the derivation of the terms can be found in Appendix D.2.1.

$$
\begin{aligned}
E_2(K) = \sum_{j=1}^{M-1} j \Bigg\{ & \sum_{i=1}^{j} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right) \int_{jT-x}^{\infty} \left(\prod_{k=i}^{j-1} \beta\left(\frac{kT-x}{h}\right)\right) \\
& \cdot \left(1-\beta\left(\frac{jT-x}{h}\right)\right) f_H(h)dh f_X(x)dx \\
& + \sum_{i=1}^{j} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right) \int_{jT-x}^{(j+1)T-x} \left(\prod_{k=i}^{j} \beta\left(\frac{kT-x}{h}\right)\right) f_H(h)dh f_X(x)dx \\
& + \int_{jT}^{(j+1)T} \left(\prod_{n=1}^{j}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right) F_H((j+1)T-x) f_X(x)dx \\
& + \int_{jT}^{\infty} \left(\prod_{n=1}^{j-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right) \alpha\left(\frac{jT}{x}\right) f_X(x)dx \Bigg\} \\
& + M \Bigg\{ \sum_{i=1}^{M} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right) \int_{MT-x}^{\infty} \left(\prod_{k=i}^{M-1} \beta\left(\frac{kT-x}{h}\right)\right) f_H(h)dh f_X(x)dx \\
& + \int_{MT}^{\infty} \left(\prod_{n=1}^{M-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right) f_X(x)dx \Bigg\}
\end{aligned}
\tag{4.8}
$$

$$
\begin{aligned}
P_2(c_p) = & \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{jT-x}^{\infty} \left(\prod_{k=i}^{j-1} \beta \left(\frac{kT-x}{h} \right) \right) \\
& \cdot \left(1 - \beta \left(\frac{jT-x}{h} \right) \right) f_H(h) f_X(x) dh dx \\
& + \sum_{j=1}^{M-1} \int_{jT}^{\infty} \alpha \left(\frac{jT}{x} \right) \left(\prod_{n=1}^{j-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) f_X(x) dx \\
& + \sum_{i=1}^{M} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{MT-x}^{\infty} \left(\prod_{k=i}^{M-1} \beta \left(\frac{kT-x}{h} \right) \right) f_H(h) f_X(x) dh dx \\
& + \int_{MT}^{\infty} \left(\prod_{n=1}^{M-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) f_X(x) dx
\end{aligned}
\tag{4.9}
$$

$$
\begin{aligned}
E_2(cycle\ length) = & \sum_{i=1}^{M} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{0}^{iT-x} (x+h) f_H(h) f_X(x) dh dx \\
& + \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{jT-x}^{(j+1)T-x} \left(\prod_{k=i}^{j} \beta \left(\frac{kT-x}{h} \right) \right) \\
& \cdot (x+h) f_H(h) dh f_X(x) dx \\
& + T \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} j \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{jT-x}^{\infty} \left(\prod_{k=i}^{j-1} \beta \left(\frac{kT-x}{h} \right) \right) \\
& \cdot \left(1 - \beta \left(\frac{jT-x}{h} \right) \right) f_H(h) dh f_X(x) dx \\
& + \sum_{i=1}^{M} MT \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{MT-x}^{\infty} \left(\prod_{k=i}^{M-1} \beta \left(\frac{kT-x}{h} \right) \right) \\
& \cdot f_H(h) dh f_X(x) dx \\
& + MT \int_{MT}^{\infty} \left(\prod_{n=1}^{M-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) f_X(x) dx \\
& + T \sum_{j=1}^{M-1} j \int_{jT}^{\infty} \left(\prod_{n=1}^{j-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \alpha \left(\frac{jT}{x} \right) f_X(x) dx
\end{aligned}
\tag{4.10}
$$

Each of the three general expressions above, Equations 4.8, 4.9 and 4.10, are used in the derivation of the cost evaluation by inserting them into Equation 3.1 yielding the average cost per time unit for the A2 model, $C_2(M, T)$. The complete cost expression is not presented here due to the large form it takes.

The cost evaluation faces one major issue due to this way of modelling the probabilities of false positives and false negatives. It can namely not be algebraically evaluated, since product signs are included in the integrals. This implies that only a numerical evaluation of the cost model can be done.

Reliability evaluation

For the reliability evaluation of the A2 model the exact same logic is applied as been done for the cost evaluation above. This means rewriting the powers to the product expressions and cdf terms to their pdf integral expressions for the A1 model. By including the dependency of the probability of false positives on the defect progress and the dependency of the probability of false negatives on the degradation progress, the reliability evaluation model under the A2 model is obtained, presented in Equation 4.11.

The expression for the A2 model focuses on the expression for $(s-1)T \leq t < sT$, where $s = 1, \ldots, M$. The derivation procedure for the general reliability expression for any given value of t, $R_{MT,2}(t)$, equals the one from the A1 model. Hence, the reader is referred to Section 4.1.1 for an overview on the procedure. Furthermore, for the procedural details of including the non-constant probabilities under the A2 model, the reader is referred to the cost evaluation of the A2 model above or to Appendix D.2.2.

$$
\begin{aligned}
R^{(s)}_{MT,2}(t) = &\sum_{i=1}^{s-1}\sum_{j=i}^{s-1}\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{jT-x}^{\infty}\left(1-\beta\left(\frac{jT-x}{h}\right)\right)\\
&\cdot\left(\prod_{k=i}^{j-1}\beta\left(\frac{kT-x}{h}\right)\right)f_H(h)dhf_X(x)dx\ R^{(s-j)}_{MT,2}(t-jT)\\
&+\sum_{i=1}^{s-1}\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{t-x}^{\infty}\left(\prod_{k=i}^{s-1}\beta\left(\frac{kT-x}{h}\right)\right)f_H(h)dhf_X(x)dx\\
&+\sum_{j=1}^{s-1}\int_{jT}^{\infty}\left(\prod_{n=1}^{j-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\alpha\left(\frac{jT}{x}\right)f_X(x)dxR^{s-j}_{MT,2}(t-jT)\\
&+\int_{(s-1)T}^{t}\left(\prod_{n=1}^{s-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\left(1-F_H(t-x)\right)f_X(x)dx\\
&+\int_{t}^{\infty}\left(\prod_{n=1}^{s-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)f_X(x)dx
\end{aligned}
\tag{4.11}
$$

where $(s-1)T \leq t < sT$

As a note, we would like to point out that, analogous to the summation, a product of which the upper bound is less than the lower bound yields an empty product which reflects the value of 1 (Koltun, 2008).

By using Equation 4.11, the general reliability expression for the A2 model is obtained, as Equation 4.12 presents. Note that the derivation procedure is not included, since this equals the one from the A1 model. Hence, the reader is referred to Section 4.1.1 for a procedural overview.

$$
R_{MT,2}(t) = R^{(aMT+s)}_{MT,2}(t) = R^{(M)}_{MT,2}(MT)^{a}R^{(s)}_{MT,2}(t-aMT), \quad \begin{aligned}&(aMT+s-1)T \leq t < (aMT+s)T,\\ &s = 1, \ldots, M,\\ &a = 0, 1, \ldots, \infty\end{aligned}
\tag{4.12}
$$

Analogous to the cost evaluation, the reliability evaluation faces the same issue of the algebraic evaluation, i.e. the reliability expression cannot be evaluated algebraically due to the inclusion of the products in the integrals of the time to defect and the delay time.

4.2.2 Model optimisation

Both evaluations for the A2 model are used in the optimisation model in determining the optimal values for M and T under the probability of false positives being dependent on the defect progress and the probability of false negatives dependent on the degradation progress.

$$\begin{aligned} \min_{M,T} \quad & C_2(M,T) = \frac{c_0 E_2(K) + c_p P_2(c_p) + c_f(1 - P_2(c_p))}{E_2(\mathit{cycle\ length})} \\ \text{subject to} \quad & \\ & R_{MT,2}(t) \geq R_{obj}(t) \\ & T > 0,\ M \in \mathbb{N} \end{aligned} \tag{4.13}$$

The evaluated expression for $E_2(K)$, $P_2(c_p)$, $E_2(\mathit{cycle\ length})$ and $R_{MT,2}(t)$ are not inserted into the optimisation model from Equation 4.13 because this yields a very hard to read model.

4.3 A3 model

The model developed in this section corresponds to the third model approach on the relationship between the probability of false positives α, the MTTF and the inspection frequency, as well the relationship between the probability of false negatives β, the MTTF and the inspection frequency. Recall that the MTTF denotes the mean time to failure of the unmaintained asset. Analogous to the previous Sections 4.1 and 4.2, this section differentiates between model evaluation and model optimisation. In the evaluation step, not only the cost and reliability evaluation are discussed, but the evaluation of α and β is taken into account as well. All evaluated expressions include a subscript 3 indicating the A3 model, i.e. $w = 3$.

4.3.1 Model evaluation

Evaluating α and β

As been assumed in Chapter 3, the probabilities of false positives and false negatives are dependent on the MTTF and the inspection frequency $\frac{1}{T}$. In order to assume no formal and mathematical relationship in this chapter between MTTF and $\frac{1}{T}$, α and β are dependent on both variables.

$$\alpha\left(MTTF, \frac{1}{T}\right) \qquad \beta\left(MTTF, \frac{1}{T}\right)$$

When both variables, the probability of false positives and the probability of false negatives, are dependent on the MTTF and the inspection frequency $\frac{1}{T}$, these probabilities become constant under evaluation. The evaluation, namely, is based on the inputs for the distributions and distribution parameters for the time to defect and the delay time, which together can be evaluated to the MTTF. Additionally, the evaluation is based on given values for M and T. This implies that both arguments for α and β are constant under evaluation. By changing the variables α and β to $\alpha\left(MTTF, \frac{1}{T}\right)$ and $\beta\left(MTTF, \frac{1}{T}\right)$ respectively, a model for including the MTTF and the inspection frequency can be obtained. Further details are presented in the following sections, as well as the implications of $\alpha\left(MTTF, \frac{1}{T}\right)$ and $\beta\left(MTTF, \frac{1}{T}\right)$ being constant under evaluation.

The asset might not be the only asset inspected and maintained by the maintenance staff. In other words, the maintenance engineer is not necessarily concerned with one specific asset, but might check multiple assets per inspection and maintenance. Let η denote the average number of assets inspected and maintained by a single engineer. When η assets are considered, the MTTF over all assets becomes $\frac{MTTF}{\eta}$ and the average inspection frequency, based on all assets, becomes $\frac{\eta}{T}$. When modelling the probabilities of false positives and false negatives as being dependent on the scaled variants of the MTTF and the inspection frequency the following expressions are obtained.

$$\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right) \qquad \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right) \tag{4.14}$$

Note that $\eta = 1$ corresponds to the case in which the engineer is faced with only one specific asset. As previously discussed, the probabilities of false positives and false negatives are constant in evaluation, and remain to be constant after including η as well.

Cost evaluation

As already been presented in the previous section, the expressions for the probabilities of false positive and false negative remain constant under evaluation. Therefore, the A1 model does need to be rewritten, i.e. the power functions from the A1 model do not need to be rewritten to the product terms, and because no realisations of the random variable X or H are included in the arguments for both the false positives and false negatives, the cdf expressions do not need to be rewritten to the pdf integrals. Under evaluation, the probabilities of false positives and false negatives remain constant which enables the substitution of the expressions for α and β by $\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)$ and $\beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)$, respectively. This yields the cost model for the A3 model.

After modifying the probabilities of false positives and false negatives, the expressions for the expected number of inspections per cycle, the probability of incurring maintenance action costs in a cycle and the expected cycle length under the A3 model are obtained and presented in Equations 4.15, 4.16 and 4.17, respectively.

$$\begin{aligned}
E_3(K) = &\sum_{j=1}^{M-1} j \left\{ \sum_{i=1}^{j} \left(1-\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{j-i} \left(1-\beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)\right. \\
&\cdot \int_{(i-1)T}^{iT} (1-F_H(jT-x)) f_X(x) dx \\
&+ \sum_{i=1}^{j} \left(1-\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{j-i+1} \\
&\cdot \int_{(i-1)T}^{iT} \int_{jT-x}^{(j+1)T-x} f_H(h) dh f_X(x) dx \\
&+ \left(1-\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{j} \int_{jT}^{(j+1)T} F_H((j+1)T-x) f_X(x) dh dx \\
&\left. + \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right) \left(1-\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{j-1} (1-F_X(jT)) \right\} \\
&+ M \left\{ \sum_{i=1}^{M} \left(1-\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{M-i} \right. \\
&\cdot \int_{(i-1)T}^{iT} (1-F_H(MT-x)) f_X(x) dx \\
&\left. + \left(1-\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{M-1} (1-F_X(MT)) \right\}
\end{aligned} \tag{4.15}$$

$$
\begin{aligned}
P_3(c_p) = & \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{j-i} \left(1 - \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right) \\
& \cdot \int_{(i-1)T}^{iT} (1 - F_H(jT - x)) f_X(x) dx \\
& + \sum_{j=1}^{M-1} \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right) \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{j-1} (1 - F_X(jT)) \\
& + \sum_{i=1}^{M} \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{M-i} \int_{(i-1)T}^{iT} (1 - F_H(MT - x)) f_X(x) dx \\
& + \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{M-1} (1 - F_X(MT))
\end{aligned} \tag{4.16}
$$

$$
\begin{aligned}
E_3(cycle\,length) = & \sum_{i=1}^{M} \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} \int_{(i-1)T}^{iT} \int_{0}^{iT-x} (x+h) f_H(h) f_X(x) dh dx \\
& + \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{j-i+1} \\
& \cdot \int_{(i-1)T}^{iT} \int_{jT-x}^{(j+1)T-x} (x+h) f_H(h) f_X(x) dh dx \\
& + \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} jT \left(1 - \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right) \\
& \cdot \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{j-i} \int_{(i-1)T}^{iT} (1 - F_H(jT - x)) f_X(x) dx \\
& + \sum_{i=1}^{M} \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{i-1} MT\beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)^{M-i} \\
& \cdot \int_{(i-1)T}^{iT} (1 - F_H(MT - x)) f_X(x) dx \\
& + \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{M-1} MT\,(1 - F_X(MT)) \\
& + \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right) T \sum_{j=1}^{M-1} j \left(1 - \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)\right)^{j-1} (1 - F_X(jT))
\end{aligned} \tag{4.17}
$$

Where $MTTF = \int_0^\infty x f_X(x) dx + \int_0^\infty h f_H(h) dh$

The integral notation of the $MTTF$ is not included in the cost evaluation model to improve readability of the model. For the proof of the MTTF expression, see Appendix H. As been done in all previous sections, the three terms obtained are inserted into Equation 3.1, which yields the average cost per time unit for the A3 model. However, due to the large form these average costs per time unit takes by inserting Equation 4.15, 4.16 and 4.17 into Equation 3.1, this is not presented here. Note that the cost expression has the advantage that it can be algebraically evaluated in contrast to the expressions derived under the A2 model from Section 4.2.1

Reliability evaluation

The reliability evaluation follows the same logic as the cost evaluation from the A3 model, i.e. substituting the constant by the non-constant probabilities of the third model approach from Chapter 3, which are constant under evaluation. For the reasoning behind this substitution the reader is referred to Section 4.3.1. The evaluated reliability expression is therefore heavily based on the reliability expression from the A1 model.

Furthermore, note that, similar to the A2 model's reliability expressions, the reliability expression for $(s-1)T \leq t < sT$ and the general reliability expression $R_{MT,3}(t)$ under the A3 model are included in this paragraph, where $s = 1, \ldots, M$. The derivation procedure for $R_{MT,3}(t)$ is identical to the one from Section 4.1.1.

$$\begin{aligned}
R^{(s)}_{MT,3}(t) = &\sum_{i=1}^{s-1}\sum_{j=i}^{s-1}\left(1-\alpha\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)\right)^{i-1}\beta\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)^{j-i}\left(1-\beta\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)\right)\\
&\cdot\int_{(i-1)T}^{iT}(1-F_H(jT-x))f_X(x)dx\; R^{(s-j)}_{MT,3}(t-jT)\\
&+\sum_{i=1}^{s-1}\left(1-\alpha\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)\right)^{i-1}\beta\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)^{s-i}\int_{(i-1)T}^{iT}(1-F_H(t-x))f_X(x)dx\\
&+\sum_{j=1}^{s-1}\left(1-\alpha\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)\right)^{j-1}\alpha\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)(1-F_X(jT))\,R^{s-j}_{MT,3}(t-jT)\\
&+\left(1-\alpha\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)\right)^{s-1}\int_{(s-1)T}^{t}(1-F_H(t-x))\,f_X(x)dx\\
&+\left(1-\alpha\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)\right)^{s-1}(1-F_X(t))
\end{aligned} \tag{4.18}$$

where $(s-1)T \leq t < sT$, $MTTF = \int_0^\infty x f_X(x)dx + \int_0^\infty h f_H(h)dh$

Because the reliability expression for the A3 model is highly related to the reliability expression of the A1 model, the reader is referred to Section 4.1.1 for a brief overview on the terms included in the expression. For a more detailed elaboration and derivation on the terms included, see Appendix D.1.2.

The general reliability expression under the A3 model corresponds to the expression from Equation 4.19. The derivation procedure for $R_{MT,3}(t)$ for any given value of t is identical to the one from the A1 model. Hence, the reader is referred to Section 4.1.1 for an overview on the derivation procedure.

$$R_{MT,3}(t) = R^{(aMT+s)}_{MT,3}(t) = R^{(M)}_{MT,3}(MT)^{a} R^{(s)}_{MT,3}(t-aMT), \quad \begin{aligned}&(aMT+s-1)T \leq t < (aMT+s)T,\\ &s = 1,\ldots,M,\\ &a = 0,1,\ldots,\infty\end{aligned} \tag{4.19}$$

The main advantage this type of model has over the model from Subsection 4.2.1 is that it can be algebraically evaluated. This evaluation can be done because the probabilities of false positives and false negatives are independent on the time to defect X, the delay time H and are not included in a product which is integrated.

4.3.2 Model optimisation

The derived cost and reliability expressions for the A3 model are now, analogously to the other two models (A1 and A2), used for the general optimisation model. The separate terms of the optimisation are not inserted due to readability issues arising.

$$\begin{aligned}
\min_{M,T} \quad & C_3(M,T) = \frac{c_0 E_3(K) + c_p P_3(c_p) + c_f(1 - P_3(c_p))}{E_3(\mathit{cycle\ length})} \\
\text{subject to} \quad & \\
& R_{MT,3}(t) \geq R_{obj}(t) \\
& T > 0,\ M \in \mathbb{N}
\end{aligned} \tag{4.20}$$

5

Numerical Results

This chapter presents the numerical study illustrating the models developed and compares the effects of differently modelling the probabilities of imperfect inspections. To start this chapter, the testbed on which the analyses are based is discussed Section 5.1. Section 5.2 presents the way the probabilities of false positives and false negatives are modelled in detail, for each of the model approaches. In order to compare the A1 model to the A2 and A3 models, parameter values for the probabilities of false positives and false negatives need to be set accordingly. This issue is addressed in Section 5.3. The insights and implications gained from the numerical study are presented in Section 5.4 together with the insights obtained from the sensitivity analysis.

5.1 Testbed

All three models are tested based on a testbed offering the basis for the numerical study. The data source for the distribution parameters, the cost parameters and the reliability parameters is a fictive, critical asset from which the characteristics are comparable to actual components. Since no quantitative dataset is available from which the distributions for the time to defect and the delay time can be derived, the parameter values for the time to defect and the delay time are approximated based on expert opinions from NedTrain. The asset considered suffers from gradual degradation starting from the initial point of operation, i.e. the defect arrives not completely random. To model this characteristic a Weibull distribution is used for describing the time to defect. The delay time is modelled as a Weibull distributed random variable as well, because the degradation after defect arrival develops gradually, i.e. it is not completely random as well.

To differentiate between the Weibull parameters of the time to defect and the delay time, the following notation is introduced. The shape parameter for the time to defect is denoted by δ_X, whereas the shape parameter for the delay time corresponds to δ_H. The scale parameter for the time to defect then equals to θ_X and for the delay time to θ_H.

Since no quantitative dataset is available, the shape parameter for the time to defect is assumed to equal to $\delta_X = 2,5$ for working purposes. The value for the scale parameter θ_X of the time to defect is determined based on the estimated mean time to defect and the assumed shape parameter. Based on expert opinions from the NedTrain staff, we estimate that the asset's mean time to defect equals to 3 years or 1095 days. Under a shape parameter of $2,5$ the scale parameter for the time to defect's Weibull distribution equals to $\theta_X = 1234$ to yield the estimated mean value.

Analogous to the time to defect's shape parameter, the delay time's shape parameter cannot be based on quantitative data. Hence, the value for this parameter is assumed to equal to $\delta_H = 2,5$ as well. Based on an estimation for the mean delay time and the shape parameter, the scale parameter θ_H is obtained. On average we estimate that the asset considered has a delay time of 180 days or a half year. This is based on interviews with NedTrain staff. When combining this estimated mean delay time with the assumed shape parameter, the scale parameter equals to $\theta_H = 203$.

The cost derivation for the inspection costs is derived based on the hours spent on the activities multiplied by the hourly wage of the maintenance engineers. The hourly wage is set to €100. The inspection costs are set to €100 since this take approximately 1 hour. The maintenance action costs consist of labour and material spent. The maintenance action costs c_p are set notably higher than the inspection costs, to €1.000, because the maintenance action consists of the asset's disassembly from the complete train, the maintenance of the asset, which requires material, and the assembly of the asset back into the train. Considering the failure costs, these are set to €100.000, since asset is replaced upon failure and the failure might inflict damage to other constructions as well.

The cost parameters highly determine the optimal maintenance schedule for each of the three models. The main issue with such cost parameters is the value they are assigned and how they are calculated. This research has derived these costs in a simplified way, since the focus of this work does not lie on the highly detailed determination of the cost parameters.

The optimisation problems considered include a reliability constraint. This reliability constraint captures the aspects which are hard to quantify, e.g. damage to company image. In practice, the Mean Time Between Failures (MTBF) is often considered as a reliability objective. This MTBF can follow various different distributions, e.g. a Weibull distribution or a Lognormal distribution. Based on the distribution together with the MTBF, the reliability objective can be obtained in terms of the probability of asset survival over a given time t.

For this work it is assumed that, for the derivation of the reliability objective, practice applies a Poisson process for the asset's failure behaviour for simplicity reasons. Due to the assumed Poisson process, the MTBF is exponentially distributed with rate λ. Since the objective from practice for the reliability is stated in terms of the MTBF and that the time between failures is exponentially distributed under a Poisson process, one is able to express the MTBF in terms of the survival probability from 0 to t: $R_{obj}(t)$.

The time span over which the reliability objective is defined is characterised by the lifespan of the train, which equals to 40 years or 14.600 days, i.e. $t = 14.600$. Because the unit of analysis in this numerical study equals to days, the lifespan is stated in days. The MTBF requirement for the asset equals to 300.000 years or 109.500.000 days. Note that this is a working assumption based on expert opinions from NedTrain staff. Under the exponential distribution, the scale parameter equals to $\lambda = 1/109.500.000$. Generally, the exponential distribution relates in the following way to the reliability at time t:

$$R_{obj}(t) = e^{\int_0^t -\lambda dx} = e^{-\lambda t} \tag{5.1}$$

By inserting all numerical values, the reliability objective is obtained in terms of the asset's survival probability over the lifespan of the train.

$$R_{obj}(14.600) = e^{-14.600/109.500.000} = 0,99987 \tag{5.2}$$

Since this reliability objective considers a lifespan of the train of 40 years, so will the reliability expression in the optimisation models, i.e. $R_{MT,w}(14.600)$.

For the probabilities of false positives and false negatives, no data is available. Therefore, the values for these probabilities are assumed. The assumptions made are working assumptions derived in cooperation with NedTrain. Furthermore, the probabilities for false positives and false negatives are in line with previous literature assumptions on these probabilities, see Berrade et al. (2013). For the the probability of false positives, α is set to $0,2$ as well as the probability of false negatives, $\beta = 0,2$.

When considering the A2 and A3 models, the inevitable probability of false positives and false negatives is assumed to be equal for both models. For this research, the inevitable probabilities are assumed to equal to half of the constant probabilities. This is a working assumption implying $\alpha_0 = 0,1$ and $\beta_0 = 0,1$.

5.2 Modelling non-constant α and β

The models presented in Chapter 4 are rather abstract and high level models. In order to present a numerical study, these models need to be further specified. A key focus on specifying certain model functions lies on modelling the functions of both α and β. The A1 model with constant values for α and β does not require any more detail its modelling. However, the models that consider non-static probabilities of imperfect inspections do need more detailing for α and β, i.e. models A2 and A3. This section presents the functions for α and β for the A2 and A3 model.

5.2.1 A2 Model

This subsection presents the functions for the probability of false positives and false negatives, which are dependent on the defect progress and the degradation progress, respectively. First, the probability of the false positives is discussed. The probability of false positives has been decomposed into a constant, inevitable term α_0 and a non-constant, assumed term α_A. The latter term α_A is dependent on the defect progress resulting in the total probability of false positives being dependent on the defect progress $\frac{rT}{x}$. The inevitable probability α_0 is constant, non-negative and strictly less or equal to the total probability of false positives, i.e. $0 \leq \alpha_0 \leq \alpha\left(\frac{rT}{x}\right)$. This yields the following expression for the probability of false positives:

$$\alpha\left(\frac{rT}{x}\right) = \alpha_0 + \alpha_A\left(\frac{rT}{x}\right)$$

The latter, assumed part is modelled as being linearly dependent on the defect progress. A linear relationship is modelled due to its simplicity and its effective way of illustrating the non-constant effect of the probability of false positives. Due to the constraint that rT is strictly less or equal to x and is non-negative, the fraction of $\frac{rT}{x}$ returns a value in the interval $(0,1)$. Such result is directly usable in modelling the probability of false positives. Recall that the probability of false positives is increasing with increasing values for the defect progress. However, due to the fact that the total probability of false positives $\alpha\left(\frac{rT}{x}\right)$ is constrained to the interval $(0,1)$, $\alpha_A\left(\frac{rT}{x}\right)$ cannot exceed $1-\alpha_0$. To meet this constraint, a scaling variable c_α is introduced to scale the fraction $\frac{rT}{x}$ to feasible values. This implies $0 \leq c_\alpha \leq 1-\alpha_0$. Both aspects discussed, i.e. the increasing linear relationship between the probability of false positives and the defect progress, and the constraining aspect of $\alpha_A\left(\frac{rT}{x}\right)$ comprise the complete expression for the probability of false positives.

$$\alpha\left(\frac{rT}{x}\right) = \alpha_0 + c_\alpha\frac{rT}{x}$$

As been discussed in Subsection 4.2.1, β is modelled as being dependent on the degradation progress. Similarly to the false positives case, the probability of a false negative β consists of a constant, inevitable part β_0 and a non-constant term β_A. When the non-constant part β_A is dependent on the degradation progress, the complete probability of false positives β is dependent on this progress as well. Similar to α_0, β_0 is constrained by the non-negative characteristic and the fact that it is strictly less or equal to $\beta\left(\frac{qT-x}{h}\right)$, i.e. $0 \leq \beta_0 \leq \beta\left(\frac{qT-x}{h}\right)$. By combining the inevitable probability and the non-constant probability, the probability expression for false negatives is obtained.

$$\beta\left(\frac{qT-x}{h}\right) = \beta_0 + \beta_A\left(\frac{qT-x}{h}\right)$$

The latter term from the equation above is modelled as being linearly dependent on the degradation progress. A linear relationship is chosen due to its simple form and that illustrates the effects of a non-constant probability of false negatives. Combining this linear dependency with the second model approach from Chapter 3, stating that with increasing values of the degradation progress the probability of false negatives decreases, the non-constant term for false negatives is modelled as being negatively related to the degradation progress: $1 - \frac{qT-x}{h}$. In a same fashion as the false positives case, a scaling variable, now denoted by c_β is introduced under $0 \leq c_\beta \leq 1 - \beta_0$. This scaling parameter prevents the total probability of false negatives to exceed the value of 1. This yields the final formulation for the probability of false negatives dependent on the degradation progress.

$$\beta\left(\frac{qT-x}{h}\right) = \beta_0 + c_\beta\left(1 - \frac{qT-x}{h}\right)$$

Due to the fact that the probability of false positives and false negatives are non-constant under the A2 model, the cost results can be verified by determining a lower and upper cost bound. Appendix E.1 presents such a verification method for verifying the costs obtained under the A2 model.

5.2.2 A3 Model

This section presents the detailed functions used for the probability of false positives and false negatives in the numerical study of the A3 model. It assumes and elaborates on a relationship between α, the MTTF and the inspection frequency, and on the relationship between β, the MTTF and the inspection frequency. The general form of the probability expressions are depicted below.

$$\alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right) = \alpha_0 + \alpha_A\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)$$
$$\beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right) = \beta_0 + \beta_A\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)$$

Analogous to the properties of α_0 and β_0 from the A2 model, these variables are characterised by the following: $0 \leq \alpha_0 \leq \alpha\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)$ and $0 \leq \beta_0 \leq \beta\left(\frac{MTTF}{\eta}, \frac{\eta}{T}\right)$. For the procedural overview on the inevitable probabilities, see Section 5.2.1. When considering Table 3.2 and Figure 3.4 two important cases are distinguished, a low inspection frequency combined with a low MTTF and a high inspection frequency combined with a high MTTF. To be able to reflect this differentiable characteristic in the function for both α and β, both aspects are modelled as being multiplicative, i.e. a high inspection frequency combined with a high MTTF yields a high value, whereas a low inspection frequency combined with a low MTTF yields a low value. Note that a ratio between the MTTF and inspection frequency would yield results hard to distinguish.

$$\alpha\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)=\alpha_0+\alpha_A\left(\frac{MTTF}{\eta}\frac{\eta}{T}\right)=\alpha_0+\alpha_A\left(\frac{MTTF}{T}\right)$$
$$\beta\left(\frac{MTTF}{\eta},\frac{\eta}{T}\right)=\beta_0+\beta_A\left(\frac{MTTF}{\eta}\frac{\eta}{T}\right)=\beta_0+\beta_A\left(\frac{MTTF}{T}\right)$$

Based on the third model approach from Chapter 3, the probability of false positives α is decreasing with increasing values of the product, whereas the probability of false negatives β is increasing with increasing values of $\frac{MTTF}{T}$. Additionally, from henceforth it is assumed that the relationships between the probabilities and $\frac{MTTF}{T}$ are linear, under same reasoning as the A2 model. Under this linear relationship between the probability and $\frac{MTTF}{T}$ one issue arises: the fraction can take values from 0 to infinity because $0 < T \leq \infty$. A linear relationship is then hard to derive. Hence, T is bounded by $1 \leq T \leq \infty$ which implies that at most one inspection per time unit is performed. This assumption is reasonable when considering the time units to be days, because multiple inspections per day are not done at NedTrain. When a different time unit is used, e.g. months, the same assumption of at most one inspection per day holds, i.e. T becomes bounded by dividing 1 by the average length of the months considered. This analysis is based on the time unit of days and will therefore not address a fractional lower bound on T. Subsequently, constraining T in such way, i.e. $1 \leq T \leq \infty$, yields values for $\frac{MTTF}{T}$ from 0 to MTTF. Based on this bounded characteristic, one is able to derive a linear relationship for the probabilities of false positives and false negatives, which are both dependent on $\frac{MTTF}{T}$ over the interval $(0, MTTF)$. Figure 5.1 illustrates this bounded characteristic of $\frac{MTTF}{T}$ under the third approach from Chapter 3.

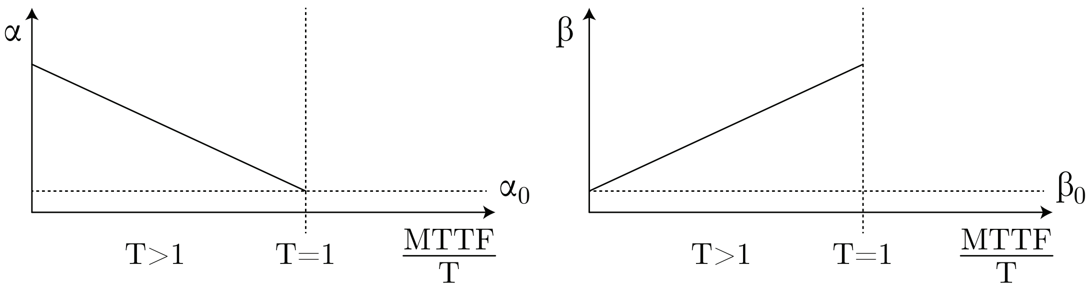


Figure 5.1: Illustration of the bounded behaviour of α and β for the A3 model

To link the values of $\frac{MTTF}{T}$ bounded over the interval $(0, MTTF)$ directly to a probability, one may take the fraction $\frac{MTTF}{T}$ relative to its maximum value of MTTF. In this case, one obtains values in the interval $(0, 1)$, which can directly be used for probability modelling. When $\frac{MTTF}{T}$ is taken relative to its maximum value, the expression reduces to a single variable dependent function, i.e. $\frac{\frac{MTTF}{T}}{MTTF} = \frac{1}{T}$, which is used as the argument for the α_A and β_A terms. Subsequently, the argument for the total probability of false positives and false negatives also changes to $\frac{1}{T}$. Due to the decreasing probability of false positives with an increasing argument value, $\alpha_A\left(\frac{1}{T}\right)$ is being modelled linearly as $1 - \frac{1}{T}$. Additionally, probability of false negatives $\beta_A\left(\frac{1}{T}\right)$ is increasing linearly with increasing argument values and is modelled as $\frac{1}{T}$.

Because the total probability of false positives and false negatives may only take values from the interval $(0, 1)$ and both include an inevitable probability α_0 and β_0 respectively, $\alpha_A\left(\frac{1}{T}\right)$ and $\beta_A\left(\frac{1}{T}\right)$ are scaled by the variables γ_α and γ_β, respectively. These scaling variables may take values of $0 \leq \gamma_\alpha \leq 1 - \alpha_0$ and $0 \leq \gamma_\beta \leq 1 - \beta_0$. Note that the derivation procedure equals the one from Section 5.2.1 and therefore the reader is referred to this section for more details. By summing the inevitable probabilities α_0 and β_0 with the A3 model's non-constant probabilities of false positives and false negatives, the probability expressions from Equation 5.3 are obtained.

$$\begin{aligned}\alpha\left(\frac{1}{T}\right) &= \alpha_0 + \alpha_A\left(\frac{1}{T}\right) = \alpha_0 + \gamma_\alpha\left(1 - \frac{1}{T}\right)\\ \beta\left(\frac{1}{T}\right) &= \beta_0 + \beta_A\left(\frac{1}{T}\right) = \beta_0 + \gamma_\beta\frac{1}{T}\end{aligned} \tag{5.3}$$

Analogous to the A2 model, the costs obtained for the maintenance schedule can be verified by deriving a lower and upper bound for the costs. The verification expressions are presented in Appendix E.2 and are not further discussed here.

5.3 Model comparison

To compare all three models, the relevant parameters need to be set accordingly. These relevant parameters include c_α, c_β, γ_α and γ_β. The inevitable probabilities α_0 and β_0 are assumed to be strictly less or equal to the A1 model probabilities α and β. Furthermore, the inevitable probabilities α_0 and β_0 are assumed to be identical for the A2 and A3 model. The A1 model will be compared to the A2 model and to the A3 model.

In comparing the A1 model to the A2 model, first the parameter setting for the false positive case is considered. The A1 model's total probability of false positives should equal the A2 model's total probability of false positives over the complete value range of the defect progress $\frac{rT}{x}$. As discussed in Section 5.2.1, the defect progress can only take values from 0 to 1. Hence, the total probabilities of the A1 and A2 model are considered over the range $(0, 1)$. The goal, then, is to determine the value of c_α satisfying the equality requirement.

Both expressions are equalised over the interval of $\frac{rT}{x}$ which is $(0, 1)$. This implies that the integral of the A1 model's α from 0 to 1 has to equal the integral of the A2 model's false positive expression over the same interval. By solving the equations for the scaling parameter c_α, its expression is obtained which enables comparison. The integration is performed over the defect progress which is denoted by $\frac{rT}{x}$. For brevity, this defect progress is denoted as the variable g and is integrated over.

$$\begin{gathered}\int_0^1 \alpha\, dg = \int_0^1 \alpha_0\, dg + \int_0^1 c_\alpha g\, dg\\ c_\alpha = 2\left(\alpha - \alpha_0\right)\end{gathered}$$

In exactly the same manner, the A1 model's total probability of false negatives has to equal the A2 model's total probability of false negatives over the complete value range of the defect progress $\frac{qT-x}{h}$, which is $(0, 1)$. Analogous to the false positives, the integral of the A1 model's β is taken with respect to the degradation progress and is integrated over $(0, 1)$. It is then equalised to the integral of the A2 model's false negative probability with respect to the degradation progress over $(0, 1)$. For evaluating these integrals the degradation progress $\frac{qT-x}{h}$ is denoted by v resulting in the expression for the scaling parameter c_β.

$$\begin{gathered}\int_0^1 \beta\, dv = \int_0^1 \beta_0\, dv + \int_0^1 c_\beta(1 - v)\, dv\\ c_\beta = 2\left(\beta - \beta_0\right)\end{gathered}$$

For the comparison of the A1 model to the A3 model, the same technique is used for setting the parameter values for γ_α and γ_β. For the former, the A1 model's total probability of false positives has to equal the A3 model's probability of false positives over the value range of the inspection frequency $\frac{1}{T}$, corresponding to $(0, 1)$. Note that the probability expression for the false positives under the A3 model is solely dependent on $\frac{1}{T}$ as discussed in Section 5.2.2. In addition, the inspection interval length T has been constrained to $1 \leq T \leq \infty$ yielding the value range of integration of $(0, 1)$. For the analysis, the inspection frequency $\frac{1}{T}$ is denoted by the variable u. This substitution can be done because of the linear relationship between the probability of false positive and the inspection frequency. The equation for setting the proper value for γ_α then corresponds to:

$$\int_0^1 \alpha du = \int_0^1 \alpha_0 du + \int_0^1 \gamma_\alpha (1-u) du$$
$$\gamma_\alpha = 2(\alpha - \alpha_0)$$

Similar to the expression of γ_α, the expression for γ_β is derived. The probability of false negatives of the A3 model is dependent the inspection frequency $\frac{1}{T}$ as well. Hence, the derivation procedure is similar, as well as the integration variable and its integration interval. Note that the integration variable is the inspection frequency denoted by u. This yields the expression for γ_β used in comparing the A3 model to the A1 model.

$$\int_0^1 \beta du = \int_0^1 \beta_0 du + \int_0^1 \gamma_\beta u du$$
$$\gamma_\beta = 2(\beta - \beta_0)$$

5.4 Results and insights

Under the functions presented in Section 5.2 and the parameter values for proper comparison from Section 5.3, the numerical study is conducted. This section is divided into two subsections. The first presents the results based on the testbed and the last subsection includes the sensitivity analysis for the models developed.

The optimisation problems considered are computationally intensive, i.e. optimising the costs under a reliability constraint to both variables M and T requires much computational efforts. To tackle this computational issue, the optimal inspection interval length T^* is numerically calculated for given values of M. M is limited in this case to keep the problem computationally feasible by $M \in \{1, \ldots, 25\}$, where M is integer. For each value of M we determine the optimal inspection interval length T^*. Based on all combinations obtained, we determine the optimal combination of M and T that yields lowest costs under the reliability constraint. This procedure is followed for every model, i.e. the A1, A2 and A3 models. Note that we denote the optimal value for the last inspection instance by M^* and the optimal inspection interval length by T^*.

5.4.1 Numerical results

By applying the procedure of enumerating M and subsequently determining T^* for the enumerated M, the numerical results from Table 5.1 are obtained. For each enumerated M, the optimal T^* for each model is obtained by solving the optimisation problem in MATLAB. The constrained minimisation solver *fmincon* is used, which applies the *interior-point* algorithm.

Table 5.1 presents the three models, A1, A2 and A3. For the A2 and A3 model, three model variants are considered because both models include two effects, the non-constant probability of false positives and false negatives. When considering the effect of false positives and false negatives in isolation, one is better able to observe and explain the models' behaviour. Hence, the A2 model considers one variant only including the non-constant probability of false positives, one variant only including non-constant probabilities of false negatives and one variant which includes both. The A3 model follows the same logic.

Note that the results presented in Table 5.1 do not consider integer values for the optimal inspection interval length T^*. Non-integer values for T^* are considered because constraining T^* to integer values increases model complexity and makes the problem computationally even more intensive, which is undesirable. In case one has the desire to obtain an integer value for the optimal inspection interval length T^*, the models developed present an initial point from which one is able to search for an integer valued T^* yielding lowest costs, i.e. the non integer T^* values present a point of departure for the search of the optimal integer T^* value.

Table 5.1: Numerical results

Case	**Model**	**Distribution parameters**				**Reliability parameter**	**Cost parameters**			**Probability parameters**								**Optimal Maintenance schedule**		
		δ_X	θ_X	δ_H	θ_H	R_{obj}	c_0	c_p	c_f	α	β	α_0	β_0	c_α	c_β	γ_α	γ_β	M^*	T^*	$C(M^*,T^*)$
1	A1	2,5	1234	2,5	203	0,00000	100	1.000	100.000	0,2	0,2							2	162,18	5,21
2	A2	2,5	1234	2,5	203	0,00000	100	1.000	100.000			0,1	0,2	0,2				5	96,37	4,82
3	A2	2,5	1234	2,5	203	0,00000	100	1.000	100.000			0,2	0,1		0,2			1	271,71	5,24
4	A2	2,5	1234	2,5	203	0,00000	100	1.000	100.000			0,1	0,1	0,2	0,2			6	88,37	4,84
5	A3	2,5	1234	2,5	203	0,00000	100	1.000	100.000			0,1	0,2			0,2		1	271,71	5,24
6	A3	2,5	1234	2,5	203	0,00000	100	1.000	100.000			0,2	0,1				0,2	4	113,42	4,97
7	A3	2,5	1234	2,5	203	0,00000	100	1.000	100.000			0,1	0,1			0,2	0,2	2	169,32	5,23
8	A1	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000	0,2	0,2							3	20,23	25,20
9	A2	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,2	0,2				22	11,58	19,30
10	A2	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,2	0,1		0,2			4	17,28	25,39
11	A2	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,1	0,2	0,2			22	11,61	19,25
12	A3	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,2			0,2		2	26,25	26,12
13	A3	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,2	0,1				0,2	5	17,24	23,07
14	A3	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	22,29	24,77

The results from Table 5.1 denote seven unconstrained model variants, cases 1-7, and seven constrained variants, cases 8-14. Note that the latter are the focus of this numerical study, but the unconstrained cases are considered since these provide a basis for understanding the models' behaviour. Hence, these unconstrained models will be discussed first. Furthermore, note that the change in parameter value is highlighted in Table 5.1 by the bold font used.

Unconstrained models

To start, we provide a sensibility check for the optimal values M^* and T^* for the unconstrained cases 1-7 from Table 5.1. Figure 5.2 is included to present such an evaluation. Figure 5.2 indicates that the optimal inspection interval lengths T^* for the unconstrained cases 1-7 from Table 5.1, are sensible under their M^*. A sensibility check that considers other values for M combined with T for each of the models is included in Appendix F.2. Due to the fact that seven figures are obtained, the reader is referred to Appendix F.2 for a sensibility check on various M and their T for each of the models for the unconstrained cases.

The results for the optimal M^* and T^* of the unconstrained cases are considered plausible due to the cost evaluation done. Hence, we start by comparing the results of the different models, corresponding to cases 1-7 from Table 5.1.

Comparing the A2 model to the A1 model, unconstrained

As one might expect, for the A2 model that only considers non-constant probabilities of false positives, case 2, more inspections are performed. This is done because this implies a decrease in T^*, which reduces the probability of false positives, that in turn is cost beneficial. Furthermore, a cost decrease compared to case 1 is obtained since the first few inspections have low probabilities of false positives. This characteristic stems from the definition of the defect

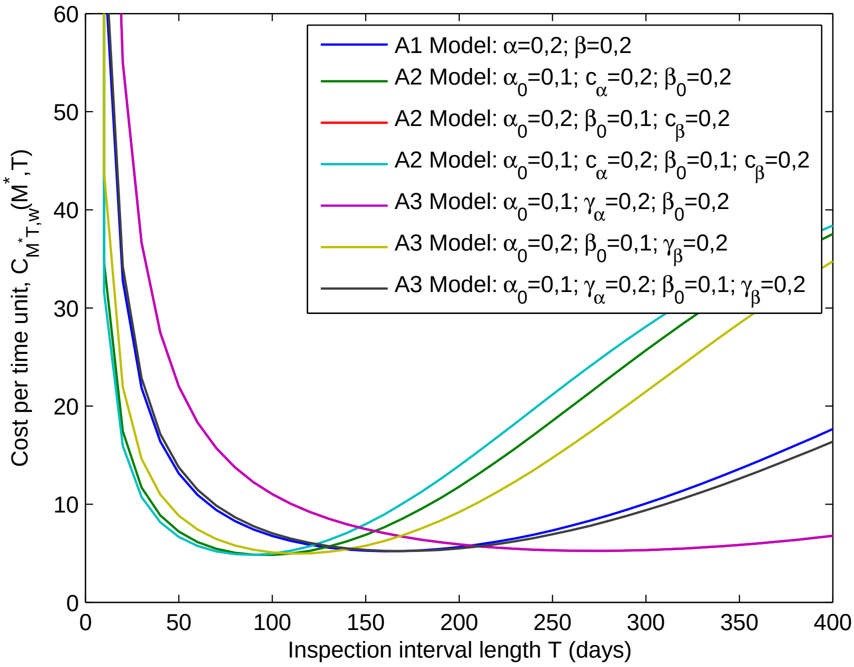


Figure 5.2: Cost evaluation for T under M^* for unconstrained cases 1-7

progress $\frac{rT}{x}$. In other words, the first few inspections, corresponding to low values of r yield low values for the fraction and therefore low probabilities of false positives. Additionally, these first few inspections, low r, with low probabilities of false positives are included more often in the cost expression compared to later inspections, high r. Hence, the costs are decreased compared to the costs of case 1.

For case 3, we expect less inspections done, i.e. a decrease in M^* and an increase in T^*. Such decrease in M^* and increase in T^* is observed in Table 5.1. However, the result corresponds to focusing completely on preventive asset replacement, because $M^* = 1$. Since the constant probabilities model, the A1 model, reports $M^* = 2$, only one inspection is included on which a false negative can occur. Such a number of inspections is rather troublesome for the model of case 3. For the result of the A1 model, the probability of false negatives is too high to be cost optimal. Furthermore, doing more frequent inspections by decreasing T^* would yield an increase in the probability of false negatives, which is not optimal. To remedy the false negatives, the optimal maintenance schedule focuses completely on preventive asset replacement, i.e. decreasing M^* and increasing T^*. Note that the costs in this case increase with respect to the A1 model, as preventive asset replacement is a more expensive solution because this does not use the asset's life.

When considering case 4, let us compare this case to the individual cases 2 and 3. For case 4 it might be reasonable to expect that the optimal response will be to increase T^*, compared to the results from case 2. The reasoning behind this is that the effect of T^* on the non-constant probability of false negatives is taken into account as well in case 4. Since increasing values for T^* yield lower probabilities of false negatives, we would expect the optimal T^* under case 4 to increase compared to the results of case 2. However, this is not what occurs. It appears that for the unconstrained A2 model, case 4, it is optimal to increase the number of inspections instead of increasing T^*. This behaviour is explained by the fact that increasing T^* to decrease the probability of false negatives comes at the expense of an increase in the probability of false positives, which are costly. To still remedy false negatives, more inspections are done instead of increasing T^*. Doing more inspections then corresponds to increasing M^* and decreasing T^*. Note that the costs for this fourth case exceed the costs of solely including the non-constant probability of false positives from case 2. This slight cost increase stems from doing more inspections compared to case 2.

Comparing the A3 model to the A1 model, unconstrained

For the A3 model only considering the non-constant probability of false positives, case 5, one expects the cost optimal maintenance schedule to change with respect to case 1 by reducing the number of inspections. The cause for this behaviour lies in the non-linear relationship between the inspection interval length T and the probability of false positives, i.e. the probability of false positives is linearly related to the inspection frequency by $\alpha_0 + \gamma_\alpha(1 - 1/T)$ and is therefore non-linearly related to T. From this function it can be seen that the largest incremental increases in the probability of false positives are obtained for low values of T. Note furthermore, that for larger values of T, the probability of false positives is high and even approaches $\alpha_0 + \gamma_\alpha$. Only for changes for low values of T the probability is affected notably. However, such low values of T do not occur due to the Weibull distributed delay time. Hence, higher values for T are considered facing the problem of a high probability of false positives, that is nearly not affected by altering T. Consider the probability of false positives then to be rather constant and high. In such case, doing less inspections becomes cost optimal (Berrade et al., 2013), i.e. a decrease in M^* and an increase in T^*. This is explained by the fact that in such case many false positives occur upon inspections and therefore, doing less inspections creates less opportunity for engineers to engage in false positives. The results from Table 5.1 correspond to this behaviour.

This increase in the optimal inspection interval length T^* is highly related to its associated costs. The costs of case 5 increase with respect to the costs of case 1. The optimal inspection interval length T^* for the A3 model yields high probabilities of false positives, i.e. a probability of false positives for case 5 of $\alpha_0 + \gamma_\alpha\,(1 - 1/T^*) = 0,1 + 0,2(1 - 1/26,25) = 0,29$, whereas the probability of false positives of case 1 equals to $0,2$. Because these false positives are costly, the costs are higher compared to the costs from case 1.

For the A3 model that only includes the non-constant probability of false negatives, the opposite occurs, i.e. a decrease in T^* and an increase in M^*. This behaviour also has to do with a non-linear relationship. In this case the inspection interval length T is non-linearly related to the probability of false negatives by $\beta_0 + \gamma_\beta 1/T$. This relationship implies high incremental decreases for the probability for low values of T and low incremental decreases for high values of T. Furthermore, be aware of the fact that high values for T imply a low probability of false negatives. To obtain lower probabilities of false negatives, and thereby reducing costs, one expects T^* to increase. However, low probabilities of false negatives imply more inspections to be cost optimal, since the inspections that are done reveal the defect with a higher probability and therefore the preventive asset replacement is less required.

To reduce the probability of false negatives, one would be inclined to increase T^*, but for such low probabilities of false negatives doing more inspections is becomes beneficial. This implies that one might consider decreasing T^* to such values that the beneficial effect of more inspections is obtained and still low probabilities for false negatives are realised. This yields more inspections for the sixth case, i.e. an increase in M^* and a decrease in T^*, see Table 5.1.

In comparing the costs of case 6 to case 1, a cost decrease is noted. Based on the optimal inspection interval length T^* under case 6, the probability of false negatives is low compared to case 1. The probability of a false negative for case 6 namely has become $\alpha_0 + \gamma_\alpha/T^* = 0,1 + 0,2/17,24 = 0,11$, whereas the probability of false negatives for case 1 equals to $0,2$. This change in the probability of false negatives causes the costs of case 6 to be lower than the costs from case 1.

When considering both the non-constant probabilities, i.e. case 5 and case 6, the numerical results from case 7 are obtained. This seventh case's maintenance schedule parameters are a trade-off between the maintenance schedules of the isolated cases, case 5 and case 6. The optimal inspection interval length T^* from case 7 is explained by the following. When only considering case 5 compared to case 1, an increase in the inspection interval length due to the non-constant probability of false positives has been observed and discussed. The sixth case presents a decrease in T^*. When combining case 5 with the decreasing effects on T^* from case 6, or vice versa, one expects the optimal inspection interval length for case 7 to be in between the two values, i.e. a trade-off between the optimal T^* of cases 5 and 6. This is exactly what Table 5.1 reports. The same reasoning holds for the optimal value of M^* for case 7, which is also a trade-off between the optimal M^* of case 5 and case 6.

Due to the fact that the costs from case 5 are higher compared to the costs from the A1 model (case 1) and the costs from case 6 are lower compared to case 1, the costs for case 7 are expected to be in between the costs of case 5 and 6. Table 5.1 confirms this expectation by reporting costs for the seventh case which are between the costs from case 5 and case 6.

Constrained models

Now that the unconstrained behaviour of the models has been illustrated, let us continue by introducing the reliability constraint. The constrained cases correspond to cases 8-14 from Table 5.1. A cost evaluation is done for these cases as well. The figures for these cost evaluations are included in Appendix F.3. Note that the figures in the Appendix consider the evaluation for T given a value for M, and only include the allowed values for T, i.e. the plots are constrained. From the cost evaluation figures from the Appendix it is concluded that the models are rather insensitive to changes in M, around the optimal M^*. Furthermore, the cost evaluation figures for the A2 model present inconsistent results for M^*, but this is caused by the step size taken for the evaluation, which equals to 1. When a smaller step size is taken, the optimal M^* from Table 5.1 should equal to the optimal value for M under cost evaluation. The remainder of the cost evaluation results is consistent with Table 5.1, when a step size of 1 is considered.

In addition, let us check whether the reliability for the optimal maintenance schedules does not violate the reliability constraint at any point in time. For each of the cases 8-14 from Table 5.1, a reliability function is generated together with the reliability objective function such that it provides a check that the reliability constraint is not violated. The reader is referred to Appendix F.4 for the reliability check. Appendix F.4 confirms that the optimal maintenance schedules under a reliability constraint behave properly by not violating the reliability constraint at any point in time of the finite horizon, $(0, 14600)$.

On a general note, we would like to highlight an issue observable from Table 5.1. Upon the inclusion of a reliability constraint, one is able to meet such a constraint in two ways, or in a combination of both ways. The first is to schedule more frequent inspections, i.e. decreasing T^*, whereas the second is decreasing M^* implying earlier preventive asset replacement. However, when considering Table 5.1 only increasing values for M^* are observed, instead of decreasing values for M^*, upon comparing the constrained to the unconstrained cases. Furthermore, note that the values for T^* decrease for all constrained cases compared to the unconstrained ones, to meet the constraint. This indicates that T outperforms M in meeting the reliability constraint. In other words, the effects of T on the reliability dominate the effects of M and therefore decreases in T^* and increases in M^* are observed for all cases 8-14, compared to cases 1-7.

In addition to the effects of the reliability constraint on the optimal maintenance schedule parameters, the costs are affected notably as well. Since the reliability constraint enforces more frequent inspections the costs increase strongly. Furthermore, the cost differences of the A2 and A3 models compared to the A1 are also affected by the introduction of the reliability constraint. This implies that such a reliability constraint affects the cost effects of the A2 and A3 model compared to the A1 model. Cost differences compared to the A1 model might increase notably, e.g. the cost difference between case 1 and 4 $(7,1\%)$ compared to the difference between case 8 and 11 $(23,6\%)$ increases by $16,5\%$.

Comparing the A2 model to the A1 model, constrained

The A2 model that only includes the non-constant probability of false positives, case 9, shows a similar picture than its unconstrained variant, case 2. This implies that the optimal number of inspections M^* increases, and the optimal inspection interval length T^* decreases compared to the A1 model, to meet the reliability constraint and to decrease costs. The cost difference between case 9 and case 11 becomes larger compared to the unconstrained models, as the pressure on decreasing T^* in satisfying the reliability constraint has economical benefits compared to the A1 model, since the probability of false positives is affected by T^* under the A2 model.

For the A2 model considering the non-constant probability of false negatives, case 10, a different behaviour compared to the A1 model is observed in Table 5.1, with respect to the unconstrained cases. A decrease in the optimal inspection interval length T^* and an increase for M^* is observed for case 10. Such behaviour stems from the fact T is highly determinant in meeting the reliability constraint. Instead of increasing T^* and therefore decreasing the probability of false negatives, more inspections are done to remedy for false negatives and meet the constraint, i.e. T^* is decreased and M^* is increased. This particular response yields a slight cost increase of case 10 compared to case 8, as the probability of false negatives increases with decreasing values for T^*.

When considering case 11, little changes are observed compared to case 9. This implies a strong effect of the non-constant probability of false positives under the A2 model, for case 11. For the unconstrained models, the complete A2 model (case 4) denotes a decrease in T^* and an increase in M^* compared to the A2 model only including the non-constant probability of false positives, case 2.

For the constrained A2 model (case 11) a different behaviour is observed compared to the constrained A2 model only including the non-constant probability of false positives (case 9). In this case T^* increases slightly. This slight increase in T^* is observed as many inspections are already done. Slightly increasing T^* decreases the probability of false negatives and increases the probability of false positives slightly. However, note that the increase in T^* is so small that this would not alter a maintenance schedule in practice.

The costs for case 11 become lower than the costs of both individual cases, 9 and 10. This cost decrease is observed because an optimal T^* is found that find an optimal trade-off value for reducing the probability of false negatives and reducing the probability of false positives compared to the individual cases. However, note that the cost decrease is present but very small $(0,26\%)$.

Comparing the A3 model to the A1 model, constrained

For all A3 models, the same dominant behaviour of T over M in satisfying the reliability constraint is observed. Under the inclusion of the reliability constraint, all A3 model variants, cases 12-14, denote a decrease in T^* and an increase in M^* to satisfy the constraint. The behaviour of each of the cases 12-14 compared to the A1 model does not change upon including the reliability constraint. Hence, no further explanation on their behaviour compared to the A1 model is presented here. For an elaboration, the reader is referred to the discussion on comparing the unconstrained A3 model to the unconstrained A1 model. Furthermore, note that the costs increase as the reliability constraint enforces a decreasing T^* and an increasing M^*.

5.4.2 Sensitivity analysis

For the sensitivity analysis, the input parameters from the models are all altered by either increasing the original value with 50% or decreasing it with 50%. The reliability objective over the train's lifespan is altered by increasing and decreasing the MTBF by 50%. This corresponds to a decreased MTBF of 54.750.000 days and an increased MTBF of 164.250.000. By the same logic as been applied to the determination of the reliability objective based on the MTBF, the decreased and increased reliability objective values are obtained. These equal to $0,99973$ and $0,99991$, respectively.

To sole exception to this rule of increasing and decreasing parameter values by 50%, are the shape parameters of the Weibull distributions, which are altered to properly compare the effects of lowering or heightening variance in the distributions. Because a shape parameter of a value 1 denotes the special case of an exponential distribution, we will consider this distribution for the decreased shape parameter for both the time to defect and the delay time, i.e. a decreased shape parameter for the time to defect of $\delta_X = 1,0$ and a decreased shape parameter for the delay time of $\delta_H = 1,0$. Note that for these decreases in both shape parameters, the scale parameters are altered as well to yield the same mean time to defect and the mean delay time, respectively. The values for these scale parameters equal to $\theta_X = 1095$ and $\theta_H = 180$.

Analogous to decreasing the shape parameter for the time to defect and the delay time, the shape parameters δ_X and δ_H are increased as well. We increase the shape parameters with the same absolute value as they have been decreased, i.e. an increase in δ_X and δ_H of $1,5$ corresponds to the increased shape parameters $\delta_X = 4,0$ and $\delta_H = 4,0$ for the time to defect and the delay time, respectively. Similar to the decreases in the shape parameters, the scale parameters are affected as well to yield the same mean time to defect and the same mean delay time as the original values. This corresponds to $\theta_X = 1208$ and $\theta_H = 199$.

The results for the changing the parameter values are presented in Table 5.2. Note that Table 5.2 follows the same structure as Table 5.1. Furthermore, the altered parameter values are highlighted by the bold fonts. This subsection will further enlighten each change in parameter values to present model robustness. As we have already concluded, the inspection interval length T dominates M in satisfying the reliability constraint. Hence, we will mainly focus on the discussion of T.

Table 5.2: Sensitivity analysis results

Case	**Model**	**Distribution parameters**				**Reliability parameter**	**Cost parameters**			**Probability parameters**								**Optimal Maintenance schedule**		
		δ_X	θ_X	δ_H	θ_H	R_{obj}	c_0	c_p	c_f	α	β	α_0	β_0	c_α	c_β	γ_α	γ_β	M^*	T^*	$C(M^*,T^*)$
15	A1	2,5	1234	2,5	203	0,99987	**50**	1.000	100.000	0,2	0,2							6	14,47	22,19
16	A2	2,5	1234	2,5	203	0,99987	**50**	1.000	100.000			0,1	0,2	0,2				25	11,46	14,97
17	A2	2,5	1234	2,5	203	0,99987	**50**	1.000	100.000			0,2	0,1		0,2			6	14,45	22,22
18	A2	2,5	1234	2,5	203	0,99987	**50**	1.000	100.000			0,1	0,1	0,2	0,2			25	11,50	14,92
19	A3	2,5	1234	2,5	203	0,99987	**50**	1.000	100.000			0,1	0,2			0,2		3	20,78	24,15
20	A3	2,5	1234	2,5	203	0,99987	**50**	1.000	100.000			0,2	0,1				0,2	8	14,62	19,86
21	A3	2,5	1234	2,5	203	0,99987	**50**	1.000	100.000			0,1	0,1			0,2	0,2	5	18,11	22,27
22	A1	2,5	1234	2,5	203	0,99987	**150**	1.000	100.000	0,2	0,2							2	25,82	27,33
23	A2	2,5	1234	2,5	203	0,99987	**150**	1.000	100.000			0,1	0,2	0,2				19	11,73	23,60
24	A2	2,5	1234	2,5	203	0,99987	**150**	1.000	100.000			0,2	0,1		0,2			1	41,79	27,52
25	A2	2,5	1234	2,5	203	0,99987	**150**	1.000	100.000			0,1	0,1	0,2	0,2			19	11,77	23,54
26	A3	2,5	1234	2,5	203	0,99987	**150**	1.000	100.000			0,1	0,2			0,2		1	41,79	27,52
27	A3	2,5	1234	2,5	203	0,99987	**150**	1.000	100.000			0,2	0,1				0,2	4	18,91	25,86
28	A3	2,5	1234	2,5	203	0,99987	**150**	1.000	100.000			0,1	0,1			0,2	0,2	2	27,31	26,94
29	A1	2,5	1234	2,5	203	0,99987	100	**500**	100.000	0,2	0,2							1	41,79	14,36
30	A2	2,5	1234	2,5	203	0,99987	100	**500**	100.000			0,1	0,2	0,2				4	17,87	13,82
31	A2	2,5	1234	2,5	203	0,99987	100	**500**	100.000			0,2	0,1		0,2			1	41,79	14,36
32	A2	2,5	1234	2,5	203	0,99987	100	**500**	100.000			0,1	0,1	0,2	0,2			10	13,00	13,85
33	A3	2,5	1234	2,5	203	0,99987	100	**500**	100.000			0,1	0,2			0,2		1	41,79	14,36
34	A3	2,5	1234	2,5	203	0,99987	100	**500**	100.000			0,2	0,1				0,2	2	26,79	14,10

Case	**Model**	**Distribution parameters**				**Reliability parameter**	**Cost parameters**			**Probability parameters**								**Optimal Maintenance schedule**		
		δ_X	θ_X	δ_H	θ_H	R_{obj}	c_0	c_p	c_f	α	β	α_0	β_0	c_α	c_β	γ_α	γ_β	M^*	T^*	$C(M^*,T^*)$
35	A3	2,5	1234	2,5	203	0,99987	100	**500**	100.000			0,1	0,1			0,2	0,2	1	41,79	14,36
36	A1	2,5	1234	2,5	203	0,99987	100	**1.500**	100.000	0,2	0,2							5	15,63	34,95
37	A2	2,5	1234	2,5	203	0,99987	100	**1.500**	100.000			0,1	0,2	0,2				25	11,46	24,62
38	A2	2,5	1234	2,5	203	0,99987	100	**1.500**	100.000			0,2	0,1		0,2			5	15,59	35,05
39	A2	2,5	1234	2,5	203	0,99987	100	**1.500**	100.000			0,1	0,1	0,2	0,2			25	11,50	24,55
40	A3	2,5	1234	2,5	203	0,99987	100	**1.500**	100.000			0,1	0,2			0,2		2	26,25	37,28
41	A3	2,5	1234	2,5	203	0,99987	100	**1.500**	100.000			0,2	0,1				0,2	7	15,26	31,44
42	A3	2,5	1234	2,5	203	0,99987	100	**1.500**	100.000			0,1	0,1			0,2	0,2	4	19,70	34,69
43	A1	2,5	1234	2,5	203	0,99987	100	1.000	**50.000**	0,2	0,2							3	20,23	25,20
44	A2	2,5	1234	2,5	203	0,99987	100	1.000	**50.000**			0,1	0,2	0,2				22	11,58	19,30
45	A2	2,5	1234	2,5	203	0,99987	100	1.000	**50.000**			0,2	0,1		0,2			3	20,11	25,35
46	A2	2,5	1234	2,5	203	0,99987	100	1.000	**50.000**			0,1	0,1	0,2	0,2			22	11,61	19,25
47	A3	2,5	1234	2,5	203	0,99987	100	1.000	**50.000**			0,1	0,2			0,2		2	26,25	26,12
48	A3	2,5	1234	2,5	203	0,99987	100	1.000	**50.000**			0,2	0,1				0,2	5	17,24	23,06
49	A3	2,5	1234	2,5	203	0,99987	100	1.000	**50.000**			0,1	0,1			0,2	0,2	3	22,29	24,77
50	A1	2,5	1234	2,5	203	0,99987	100	1.000	**150.000**	0,2	0,2							3	20,23	25,20
51	A2	2,5	1234	2,5	203	0,99987	100	1.000	**150.000**			0,1	0,2	0,2				22	11,58	19,30
52	A2	2,5	1234	2,5	203	0,99987	100	1.000	**150.000**			0,2	0,1		0,2			4	17,28	25,39
53	A2	2,5	1234	2,5	203	0,99987	100	1.000	**150.000**			0,1	0,1	0,2	0,2			22	11,61	19,26
54	A3	2,5	1234	2,5	203	0,99987	100	1.000	**150.000**			0,1	0,2			0,2		2	26,25	26,12
55	A3	2,5	1234	2,5	203	0,99987	100	1.000	**150.000**			0,2	0,1				0,2	5	17,24	23,07
56	A3	2,5	1234	2,5	203	0,99987	100	1.000	**150.000**			0,1	0,1			0,2	0,2	3	22,29	24,77
57	A1	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000	0,2	0,2							3	24,30	20,98
58	A2	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,2	0,2				20	14,00	16,27
59	A2	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,2	0,1		0,2			4	20,76	21,14
60	A2	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,1	0,2	0,2			22	13,91	16,24
61	A3	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,2			0,2		2	31,54	21,75
62	A3	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,2	0,1				0,2	6	19,38	19,15
63	A3	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	26,83	20,61
64	A1	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000	0,2	0,2							3	18,45	27,63
65	A2	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,2	0,2				23	10,54	21,04
66	A2	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,2	0,1		0,2			4	15,76	27,83
67	A2	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,1	0,2	0,2			25	10,50	20,99
68	A3	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,2			0,2		2	25,26	27,14
69	A3	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,2	0,1				0,2	5	15,69	25,33
70	A3	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	20,31	27,17
71	A1	2,5	1234	2,5	203	0,99987	100	1.000	100.000	**0,3**	0,2							2	26,29	26,18
72	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,15**	0,2	**0,3**				17	13,99	19,87
73	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,3**	0,1		0,2			1	41,79	26,32
74	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,15**	0,1	**0,3**	0,2			17	14,00	19,85
75	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,15**	0,2			**0,3**		1	41,79	26,32
76	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,3**	0,1				0,2	3	22,37	24,89
77	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,15**	0,1			**0,3**	0,2	2	28,30	26,18
78	A1	2,5	1234	2,5	203	0,99987	100	1.000	100.000	**0,1**	0,2							8	12,05	22,87
79	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,05**	0,2	**0,1**				24	9,18	19,18
80	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,1**	0,1		0,2			8	12,06	22,86
81	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,05**	0,1	**0,1**	0,2			25	9,15	19,10
82	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,05**	0,2			**0,1**		6	13,99	24,08
83	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,1**	0,1				0,2	10	12,42	20,42
84	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			**0,05**	0,1			**0,1**	0,2	8	13,95	21,63
85	A1	2,5	1234	2,5	203	0,99987	100	1.000	100.000	0,2	**0,3**							2	24,94	26,28
86	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,3**	0,2				21	10,01	22,27
87	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,2	**0,15**		**0,3**			1	41,79	26,32
88	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,15**	0,2	**0,3**			23	10,00	22,11
89	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,3**			0,2		1	41,79	26,32
90	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,2	**0,15**				**0,3**	4	17,89	24,53
91	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,15**			0,2	**0,3**	2	26,67	25,71
92	A1	2,5	1234	2,5	203	0,99987	100	1.000	100.000	0,2	**0,1**							6	16,36	22,69
93	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,1**	0,2				25	13,52	16,55
94	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,2	**0,05**		**0,1**			6	16,32	22,74
95	A2	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,05**	0,2	**0,1**			25	13,51	16,56
96	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,1**			0,2		4	19,92	24,55

Case	Model	Distribution parameters				Reliability parameter	Cost parameters			Probability parameters								Optimal Maintenance schedule		
		δ_X	θ_X	δ_H	θ_H	R_{obj}	c_0	c_p	c_f	α	β	α_0	β_0	c_α	c_β	γ_α	γ_β	M^*	T^*	$C(M^*,T^*)$
97	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,2	**0,05**				**0,1**	10	12,42	20,42
98	A3	2,5	1234	2,5	203	0,99987	100	1.000	100.000			0,1	**0,05**			0,2	**0,1**	8	13,95	21,63
99	N/A	2,5	1234	2,5	203	0,99987	100	1.000	100.000	**0**	**0**							15	11,55	14,45
100	A1	2,5	**617**	2,5	203	0,99987	100	1.000	100.000	0,2	0,2							3	13,11	38,90
101	A2	2,5	**617**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2	0,2				19	7,58	30,33
102	A2	2,5	**617**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1		0,2			4	11,20	39,18
103	A2	2,5	**617**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			20	7,56	30,25
104	A3	2,5	**617**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2			0,2		2	17,00	40,25
105	A3	2,5	**617**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1				0,2	5	11,08	35,87
106	A3	2,5	**617**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	3	14,36	38,28
107	A1	2,5	**1851**	2,5	203	0,99987	100	1.000	100.000	0,2	0,2							3	26,09	19,54
108	A2	2,5	**1851**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2	0,2				25	14,83	14,82
109	A2	2,5	**1851**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1		0,2			4	22,29	19,69
110	A2	2,5	**1851**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			25	14,87	14,78
111	A3	2,5	**1851**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2			0,2		2	33,87	20,26
112	A3	2,5	**1851**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1				0,2	6	20,81	17,83
113	A3	2,5	**1851**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	3	28,81	19,20
114	A1	2,5	1234	2,5	**101**	0,99987	100	1.000	100.000	0,2	0,2							3	13,09	38,96
115	A2	2,5	1234	2,5	**101**	0,99987	100	1.000	100.000			0,1	0,2	0,2				25	7,46	29,10
116	A2	2,5	1234	2,5	**101**	0,99987	100	1.000	100.000			0,2	0,1		0,2			4	11,18	39,24
117	A2	2,5	1234	2,5	**101**	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			25	7,48	29,02
118	A3	2,5	1234	2,5	**101**	0,99987	100	1.000	100.000			0,1	0,2			0,2		2	16,97	40,33
119	A3	2,5	1234	2,5	**101**	0,99987	100	1.000	100.000			0,2	0,1				0,2	5	11,06	35,95
120	A3	2,5	1234	2,5	**101**	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	3	14,34	38,35
121	A1	2,5	1234	2,5	**305**	0,99987	100	1.000	100.000	0,2	0,2							4	22,39	19,60
122	A2	2,5	1234	2,5	**305**	0,99987	100	1.000	100.000			0,1	0,2	0,2				21	14,95	15,23
123	A2	2,5	1234	2,5	**305**	0,99987	100	1.000	100.000			0,2	0,1		0,2			4	22,29	19,69
124	A2	2,5	1234	2,5	**305**	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			20	15,06	15,19
125	A3	2,5	1234	2,5	**305**	0,99987	100	1.000	100.000			0,1	0,2			0,2		2	33,87	20,26
126	A3	2,5	1234	2,5	**305**	0,99987	100	1.000	100.000			0,2	0,1				0,2	6	20,82	17,83
127	A3	2,5	1234	2,5	**305**	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	3	28,82	19,20
128	A1	**1**	**1095**	2,5	203	0,99987	100	1.000	100.000	0,2	0,2							21	1,88	160,98
129	A2	**1**	**1095**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2	0,2				25	2,31	93,55
130	A2	**1**	**1095**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1		0,2			22	1,90	159,41
131	A2	**1**	**1095**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			25	2,33	92,76
132	A3	**1**	**1095**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2			0,2		25	1,87	157,60
133	A3	**1**	**1095**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1				0,2	19	1,85	164,26
134	A3	**1**	**1095**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	24	1,84	159,86
135	A1	**4**	**1208**	2,5	203	0,99987	100	1.000	100.000	0,2	0,2							1	119,61	9,20
136	A2	**4**	**1208**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2	0,2				1	119,61	9,20
137	A2	**4**	**1208**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1		0,2			1	119,61	9,20
138	A2	**4**	**1208**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			1	119,61	9,20
139	A3	**4**	**1208**	2,5	203	0,99987	100	1.000	100.000			0,1	0,2			0,2		1	119,61	9,20
140	A3	**4**	**1208**	2,5	203	0,99987	100	1.000	100.000			0,2	0,1				0,2	1	119,61	9,20
141	A3	**4**	**1208**	2,5	203	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	1	119,61	9,20
142	A1	2,5	1234	**1**	**180**	0,99987	100	1.000	100.000	0,2	0,2							1	9,84	111,78
143	A2	2,5	1234	**1**	**180**	0,99987	100	1.000	100.000			0,1	0,2	0,2				25	2,40	87,97
144	A2	2,5	1234	**1**	**180**	0,99987	100	1.000	100.000			0,2	0,1		0,2			1	9,84	111,78
145	A2	2,5	1234	**1**	**180**	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			25	2,39	88,07
146	A3	2,5	1234	**1**	**180**	0,99987	100	1.000	100.000			0,1	0,2			0,2		1	9,84	111,78
147	A3	2,5	1234	**1**	**180**	0,99987	100	1.000	100.000			0,2	0,1				0,2	2	5,92	110,77
148	A3	2,5	1234	**1**	**180**	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	1	9,84	111,78
149	A1	2,5	1234	**4**	**199**	0,99987	100	1.000	100.000	0,2	0,2							2	45,19	14,51
150	A2	2,5	1234	**4**	**199**	0,99987	100	1.000	100.000			0,1	0,2	0,2				20	20,13	11,59
151	A2	2,5	1234	**4**	**199**	0,99987	100	1.000	100.000			0,2	0,1		0,2			3	34,87	14,63
152	A2	2,5	1234	**4**	**199**	0,99987	100	1.000	100.000			0,1	0,1	0,2	0,2			18	20,47	11,52
153	A3	2,5	1234	**4**	**199**	0,99987	100	1.000	100.000			0,1	0,2			0,2		2	45,54	15,08
154	A3	2,5	1234	**4**	**199**	0,99987	100	1.000	100.000			0,2	0,1				0,2	7	27,82	12,70
155	A3	2,5	1234	**4**	**199**	0,99987	100	1.000	100.000			0,1	0,1			0,2	0,2	3	39,31	14,10

Changing the cost parameters

Let us start with discussing the changes of the models' cost parameters. As one might expect, decreasing the inspection costs c_0 yields more inspections planned, because it becomes cheaper to do inspections, i.e. a decrease in T^* for all cases 15-21 from Table 5.2 is observed. In addition, the costs decrease for each of the cases considered compared to the original c_0 value, because the inspections become cheaper. Note that the value for the preventive asset replacement instance M^* also increases for all cases, as this allows T^* to decrease further.

Upon increasing the inspection costs, the opposite effect on the optimal maintenance schedule parameters occurs. Less inspections are done because the inspections become more expensive. This implies increasing values for T^* and decreasing values for M^* for each of the cases 22-28 compared to cases 8-14. The associated costs increase due to the increased value of c_0. One remark is made here corresponding to the A2 model only considering the non-constant probability of false negatives, case 24. Since inspections become even less attractive, the reliability constraint is satisfied by avoiding inspections instead of increasing the inspections done.

For decreasing the maintenance action costs c_p, a stronger focus on preventive asset replacement is noted in Table 5.2. Since preventive asset replacement becomes cheaper, the optimal maintenance schedule takes this preventive replacement aspect more into account. This is observed by decreasing values for M^* and increasing values T^* for all cases 29-35 compared to cases 8-14. The costs of the maintenance schedules also decrease compared to cases 8-14, since the maintenance action costs c_p decrease. Case 32 denotes quite a strong increase in M^* compared to case 30, due to the dominant effect of T over M in meeting the reliability constraint.

The increase in the maintenance action costs c_p yields the observation that a stronger focus lies on performing inspections, i.e. T^* decreases and M^* does not decrease, for all the cases considered, cases 36-42. Since preventive asset replacement becomes more expensive, this event is postponed by doing more inspections. This implies that T^* decreases and M^* does not decrease. Considering costs, these increase for all cases 36-42 compared to cases 8-14. More inspections are done and upon the occurrence of a maintenance action, the incurred costs c_p are higher compared to the original parameter value.

The models are rather insensitive to changes in the failure costs. The reported optimal maintenance schedules, and subsequently the associated costs, for cases 43-56 do not differ from the results from cases 8-14, except for case 45. Since a tight reliability constraint is included, the failure costs play less of role in obtaining the optimal maintenance schedules, for both the increased and decreased c_f. Because case 45 denotes a change in maintenance schedule we will briefly discuss this case in more detail.

For case 45, the effect of T in meeting the reliability constraint compared to M becomes less. Since the value for M^* decreases compared to case 10 and the value for T^* increases, this becomes evident. Nevertheless, the effect of T remains large compared to the effects of M. The schedule obtained meets the reliability constraint, but changes in M^* and T^* compared to case 10 are obtained because the low failure cost alters the cost trade-off. This alteration in cost trade-off reduces the dominant behaviour of T over M. Generally, the models are rather insensitive to changes in the failure cost parameter due to the tight reliability constraint encountered.

Changing the reliability objective

Let us first consider cases 57-63 which denote the decreased value for the reliability objective parameter. As one might expect, decreasing the reliability objective yields results which decrease the frequency of the inspections, i.e. T^* increases. This makes sense as the relaxed reliability objective allows one to wait longer with inspections which becomes more cost advantageous. The changes in M^* are rather small, when comparing cases 57-63 to cases 8-14 implying that the models' optimal M^* is rather insensitive to decreasing the reliability, and mainly T^* is affected by the reliability constraint. The loosening of the reliability constraint yields cost benefits by decreasing the costs for all cases 57-63 compared to cases 8-14.

The increase in the reliability objective presents the opposite from the decreased reliability objective value, see cases 64-70. This implies that increasing the reliability objective yields more frequent inspections, i.e. decreasing values for T^* compared to cases 8-14. This makes sense because asset reliability is increased by decreasing T^*. Mainly for the A2 models considering the non-constant probability of false positives, cases 65 and 67, the dominant behaviour of T becomes stronger, as T^* decreases further compared to cases 9 and 10, and M^* increases further. However, the changes in M^* are small when comparing cases 64-70 to cases 8-14 implying insensitive model behaviour to the optimal M^* under an increased reliability objective. Since the inspections are performed more frequently, the costs increase of all cases 64-70, see Table 5.2.

To illustrate the effects of increasing the reliability objective on the costs of the optimal maintenance schedules for each model, Figure 5.3 is included. For a more detailed overview on the optimal maintenance schedules under a certain value for the reliability objective, the reader is referred to Table G.1 in Appendix G. Figure 5.3 presents cost increases for all models when the reliability objective is increased. This makes sense as increasing values for the reliability objective imply that more frequent inspections are required to prevent the asset from failing thereby inducing larger costs. In Figure 5.3 no distinction can be made between the A2 model including both non-constant probabilities and the A2 model solely including the non-constant probability of false positives, since the cost values are nearly identical.

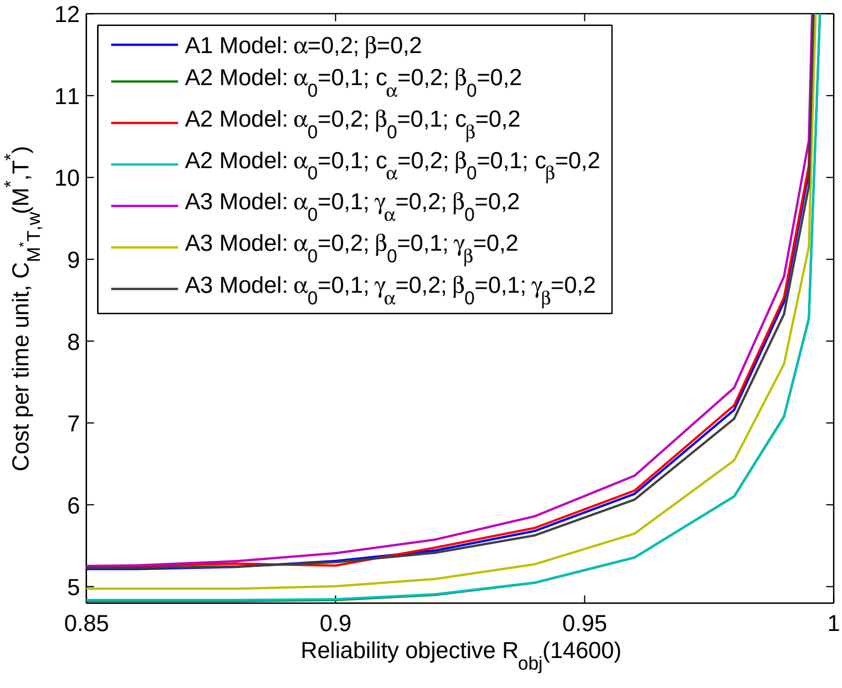


Figure 5.3: Cost optimal maintenance schedule costs versus the reliability objective R_{obj}

From Figure 5.3 the models' sensitivity with respect to the reliability objective is illustrated. This implies that the models become more sensitive to changes in the reliability objective, when the reliability objective takes higher values.

Changing the probability of imperfect inspections

For altering the probabilities of imperfect inspections, the same logic is applied as the one from cases 1-14. This implies that half of the total probability for false positives and false negatives is considered inevitable and the remainder is modelled as being non-constant.

Let us start by considering increasing and decreasing the probability of false positives. Increasing the probability of false positives yields less frequent inspections, i.e. increasing values for T^*. Furthermore the inspection instance of preventive asset replacement M^* decreases. This behaviour is observed in Table 5.2 because doing less and less frequent inspections, avoids the opportunity for engineers to engage in false positives. Furthermore, the costs of all cases 78-84 increase due to the increased probability of false positives, which are expensive.

For the decrease in the probability of false positives, more inspections are planned, i.e. T^* decreases and M^* increases. Less false positives are encountered during inspections and therefore the inspections become less costly on average yielding the optimal response of planning more inspections. The result of case 80 is related to this increase in the number of inspections. As the number of inspections increases due to the decrease of the probability of false positives, the reliability constraint is satisfied by increasing T^* slightly compared to case 78. Increasing T^* decreases the probability of false negatives, which slightly decreases costs. However, the costs and the maintenance schedule differences between cases 78 and 80 are so small that they are negligible for practice.

For increasing the probability of false negatives, the focus on preventive asset replacement becomes larger, i.e. generally increasing T^* and decreasing M^* compared to cases 8-14. With respect to cases 8-14 the dominant behaviour of T over M becomes less, for all cases except 86 and 88. This decrease in dominant behaviour, when the probability of false negatives increases, is observed by the values for M^*, which do not increase compared to the unconstrained cases. For cases 86 and 88, this dominant effect of T over M, however, remains very strong due to the fact that reducing T^* not only satisfies the reliability constraint, but also decreases costs. The costs of all cases increase with respect to cases 8-14, because the inspection quality decreases and earlier preventive asset replacement is done.

In case the probability of false negatives is reduced, more inspections are done and these are performed more frequently. As one is better able to detect a defect, the focus on preventive asset replacement decreases and therefore inspections become more important in the optimal maintenance schedules. Furthermore, the dominant effect of T over M increases, as the optimal values for M^* increase for all cases 92-98 compared to cases 8-14. Since reducing the probability of false negatives yields a lower probability of asset failure and therefore lower costs, the costs reported by Table 5.2 are lower for cases 92-98 compared to cases 8-14.

We also include case 99 denoting perfect inspections. This presents a point for illustrating the effect imperfect inspections have on costs and the optimal maintenance schedule. Case 99 presents that more inspections are planned compared to cases 8-14, when these inspections are considered perfect. This implies that the optimal maintenance schedule for case 99 yields quite different results to the ones from the imperfect inspections. Furthermore, the costs calculated are also substantially lower compared to the cases of imperfect inspections. Compared to the A1 model from case 8, the costs decrease by nearly 43%. Compared to the non-constant models from cases 9-14 of Table 5.1, the cost differences between perfect and imperfect inspections lie between $24,9\%$ and $44,7\%$. Since the optimal maintenance schedules also differ quite largely when comparing cases 8-14 with case 99, the importance of including imperfectness of inspections into analyses is illustrated.

Changing the scale parameter of the time to defect's distribution

For the decrease in the mean time to defect, the shape parameter remains unaffected and only the scale parameter is altered to yield a decrease of 50%. The shape parameter remains unaffected because this yields a non-changing coefficient of variation for the time to defect's distribution. The scale parameter's value is set to 617. We expect inspections to be done more frequently, i.e. a decrease in T^* for cases 100-106 compared to cases 8-14. As the mean time to defect decreases, a higher inspection frequency is required to meet the reliability constraint. Furthermore, decreasing M^* aids cases 101 and 103 in satisfying the reliability constraint further. This behaviour for the optimal maintenance schedule is observed for cases 100-106 in Table 5.2. Furthermore, decreasing the mean time to defect yields substantial cost increases compared to cases 8-14, since inspections are required more frequently and the asset is preventively replaced earlier to meet the reliability constraint.

For an increase in the mean time to defect, the scale parameter is set to $\theta_X = 1851$. Under the same reasoning as for the decrease in the parameter value, only the scale parameter is altered. For an increased θ_X we expect T^* to increase as the time to defect takes more time on average and therefore the reliability constraint can be met by less frequent inspections. Table 5.2 confirms this increase in T^* for all cases 107-113 compared to cases 8-14. Furthermore, in considering M^* we note that its optimal value increases for cases 108, 110 and 112. This indicates that the dominant behaviour of decreasing T over decreasing M in meeting the reliability constraint becomes larger. The costs of cases 107-113 decrease notably compared to cases 8-14 because the inspections are done less frequently.

Based on the rather large changes for all cases 100-113 in the costs and the optimal maintenance schedules, when altering the mean time to defect, the models are considered to be sensitive to the value set for the scale parameter of the time to defect's distribution.

Changing the scale parameter of the delay time's distribution

Analogous to considering an increase and decrease in the mean time to defect, an increase and decrease in the mean delay time is considered by $\theta_H = 101$ and $\theta_H = 305$ respectively. Note that the shape parameter δ_H does not change to yield the same coefficient of variation. By lowering the mean delay time, we expect the inspections to be performed more frequently to meet the reliability constraint, i.e. decreasing T^*. Table 5.2 confirms this expectation of decreasing values for T^* when comparing cases 114-120 to cases 8-14. The costs associated to the optimal maintenance schedules increase compared to cases 8-14, because the inspections are done more frequently. Note that the reliance on T increases for cases 115 and 117, compared to cases 9 and 11, because T^* has to decrease further to meet the reliability constraint. This implies that M^* is increased to be able to reduce T^* further, thereby meeting the reliability constraint.

In accordance with expectations, increasing the mean delay time yields the opposite results, i.e. less frequent inspections and lower costs, see cases 121-127 in Table 5.2. The asset's delay time becomes longer on average yielding an increase in T^*. This makes sense as one is able to wait longer with the inspections and still satisfying the constraint, because the asset will also take longer, on average, to fail from defect arrival. In compliance with the decrease in the mean delay time, an increased value for the mean delay time yields rather large cost decreases for all cases 121-127 compared to cases 8-14. This stems from the fact that inspections are done less frequently, thereby reducing costs.

Analogous to the changes in the mean time to defect, changing the asset's mean delay time affects all models notably. These effects consider not only the optimal maintenance schedules obtained, but the associated costs as well. This implies that the cases 114-127 are considered sensitive to the scale parameter of the delay time's distribution.

Changing the shape parameter of the time to defect's distribution

For cases 128-134, the exponential distributed time to defect is considered. Since the defect now arrives completely random, more inspections are required to meet the reliability constraint implying a decreased in T^* and an increase in M^* for all cases 128-134 compared to cases 8-14. Subsequently, this exponential distributed time to defect increases costs tremendously for all cases 128-134 with respect to cases 8-14. Note that the dominant effect of T over M in meeting the constraint becomes very large, as M increases strongly compared to cases 8-14. This increase in the dominant effect stems from the fact that is has become harder to preventively replace the asset due to the exponentially distributed time to defect. However, the response of the cases 128-134, compared to the A1 model, differs from the behaviour observed for a Weibull distributed time to defect, see cases 8-14.

Due to the reliability constraint, the A2 model that solely includes the non-constant probability of false positives, case 129, increases T^* compared to case 128. We might expect T^* to decrease in order to reduce the probability of false positives, but since the inspections are already scheduled very frequently, the probability of false positives is already low. Hence, the optimal response becomes to increase T^*, which increases the probability of false positives thereby affecting the reliability positively. The costs associated to case 129 decrease strongly compared to the A1 model as the probability of false positives is low compared to case 128.

For case 130, the A2 model with the sole inclusion of the non-constant probability of false negatives denotes a small increase in T^*, which might be expected as this decreases the probability of false negatives and therefore has the ability to decrease costs and increase reliability slightly. This cost decrease compared to case 128 is observable in Table 5.2, but is very small.

By combining the characteristics of case 129 and case 130, the results from case 131 are obtained. Increasing T^* compared to case 129 is beneficial for meeting the reliability constraint as it increases the probability of false positives and decreases the probability of false negatives. Hence further increasing T^* for the complete A2 model makes sense. Because the increase in T^* decreases the probability of false negatives, the costs are further reduced compared to the individual cases, 129 and 130. Note that the cost difference with case 129 is very small, but still present.

Case 132 includes the A3 model's non-constant probability of false positives, i.e. the inspection interval length T is non-linearly related to the probability of false positives by $\alpha_0 + \gamma_\alpha(1 - 1/T)$. Since the exponential distribution for the time to defect combined with the reliability constraint implies low values for T^*, the effect of altering T^* becomes visible. A decrease in T^* compared to case 128 is observed, because this now yields a decrease in the probability of false positives. Due to the decreasing probability of false positives, the costs compared to case 128 are positively affected. This is reflected in Table 5.2 as well.

The A3 model only including the non-constant probability of false negatives, case 133, denotes a decrease in T^* compared to the results from case 128. We would expect an increase in T^*, because this implies a lower probability of false negatives thereby increasing the asset reliability. However, the optimal response in meeting the reliability objective is to reduce M^* and T^* corresponding to early preventive maintenance, instead of reducing the probability of false negatives by increasing T^*. The costs of case 133 increase with respect to case 128 because the preventive asset replacement is done so early that maintenance action costs c_p are often incurred.

The full A3 model including both non-constant probabilities (case 134) denotes a further decrease in T^* as the individual cases 132 and 133 decrease T^* compared to the A1 model from case 128. Since case 132 increases M^* and case 133 decreases M^* with respect to the A1 model, the complete A3 model reports a balanced value for M^* in between the values for case 132 and 133. Such balanced behaviour is observed for costs as well.

When we consider cases 135-141, the increased shape parameter for the time to defect's

Weibull distribution, inspections become obsolete for all cases. As the variance is strongly decreased for the time to defect, one is able to focus purely on preventive asset replacement in satisfying the reliability constraint, i.e. decreasing M^* to 1. Inspections become obsolete, as well as the difference between the A2 and the A1, and the difference between the A3 and A1 model. Furthermore, due to this decrease in the variance of the time to defect, the costs are drastically decreased compared to cases 8-14.

The changes in the shape parameter of the time to defect's distribution indicate that the models are highly sensitive to these changes. This models' sensitivity makes sense as the distributions are characterisable for the asset considered. A distribution for the time to defect subject to a high level of variance implies many inspections and high costs, whereas low levels of variance denotes less inspections and lower costs.

Changing the shape parameter of the delay time's distribution

The decreased value for the shape parameter of the delay time's distribution is set to $\delta_H = 1,0$, denoting the special case of a Weibull distribution: an exponentially distributed delay time. This corresponds to cases 142-148 from Table 5.2. For all cases, T^* is strongly decreased compared to cases 8-14. This decrease in T^* stems from the fact that the asset fails completely random upon defect arrival and the reliability constraint enforces a high reliability. To prevent the asset from entering its defective state a strong focus lies on preventive asset replacement for all cases, except 143 and 145. This strong focus on preventive asset replacement stems from the low values for M^*. Furthermore, the value for T^* is decreased as well to meet the reliability constraint. In other words, lowering M^* is not powerful enough in satisfying the reliability constraint, and therefore T^* is decreased as well for all cases, except 143 and 145.

For the A2 models that consider the non-constant probability of false positives, cases 143 and 145, T^* is decreased strongly compared to cases 9 and 11. Because a decrease in T dominates a decrease in M so strongly in meeting the reliability constraint, for the A2 models including the non-constant probability of false positives, the value for M^* for cases 143 and 145 grows large to enable further decreases in T^* that satisfy the reliability constraint.

The costs for all cases 142-148 are substantially higher compared to the costs of cases 8-14 where a Weibull distributed delay time was considered. This cost increase stems from the reliability constraint combined with the delay time's distribution. For the exponential distributed delay time, more maintenance action cost and inspection costs are incurred to prevent the asset from reaching its highly uncertain defective state.

When the shape parameter of the delay time's distribution is increased, see cases 149-155, the inspections are planned less frequently, i.e. an increase in T^* compared to cases 8-14. Since the asset's delay time becomes more certain, one is able to do inspections less frequently and still satisfying the reliability constraint. The decrease in T^* yields large cost benefits for cases 149-155 compared to 8-14, since the inspections are done less frequently. To still meet the reliability constraint, the optimal value for M^* is decreased compared to cases 8-14. This implies that the dominant effect of reducing T to meet the reliability constraint over decreasing M is weakened. Nevertheless, T is still more determinant in meeting the reliability constraint, as M decreases slightly compared to cases 8-14. Note that case 154 is the single case for which an increase in M^* is observed, corresponding to the fact that the reliability constraint is met by decreasing T^*.

Analogous to changes in the shape parameter of the time to defect's distribution, changes in the shape parameter of the delay time's distribution affect the optimal maintenance schedules and subsequently its costs largely. This makes sense as the asset's life characteristics are altered. Nevertheless, the model behaves properly, but is considered sensitive to changes in the distribution's shape parameter.

6

Implementing the Model

This chapter discusses the implementation issues and aspects which are of importance for model implementation. We present situations in which the models are applicable and how they are applicable. Furthermore we will denote some aspects with respect to the derivation of parameter values for using the models.

The models developed consider a single-component asset which can be categorised into three states: normal, defective and failed. Therefore the models are to be used for assets which fit this definition of the states. In addition to the asset's state definition, the models' use is intended for degrading assets. One might think of rotating parts, moving parts or wear-out parts. Let us further enlighten this asset characteristic which highly determines model applicability.

Since the models developed include a preventive maintenance instance M, which is optimised, completely random behaviour of the time to defect or the delay time, makes it hard for the models to find a solution within acceptable computation times. This issue is mainly troublesome when the mean time to defect or mean delay time are long. In the specific case of completely random defect arrival, high values for M have to be evaluated, which are highly time consuming. Especially when critical assets are considered, of which a defect occurs very infrequently, the parameter setting becomes too extreme for the models. Therefore we recommend implementing the models developed for assets that are characterised by an increasing defect arrival rate.

Another aspect, related to the asset's states and the associated distributions, includes the parameter values chosen. The models are rather sensitive to changes in the parameter values. We have tested this for a Weibull distributed time to defect and a Weibull distributed delay time, under various shape and scale parameter values. For practical model implementation it is of importance that the distribution parameters for the time to defect and delay time are properly set. Before implementing the model, we therefore urge users to quantitatively derive these distributions' parameter values.

A different aspect, not related to the the asset itself, corresponds to imperfect inspections. In order to obtain most from the models developed, one should first determine the probabilities of false positives and false negatives before implementing the models themselves. It is important to determine how the probabilities are build up, implying the potential identification of an inevitable probability of false positives and false negatives and a non-constant term. One might think of further investigating imperfect inspections due to erroneous behaviour of the measurement equipment, which might be inevitable. Another option and aspect of imperfect inspections relates to human behaviour. However, determining this human error is hard and challenging, but important nevertheless to be able to implement the models developed.

Since the models include a reliability constraint, we would like to discuss some implementation aspects for such a constraint. The models can be implemented for assets that are subject to a reliability constraint. Moreover, the models developed are also applicable to assets which are not subject to a reliability constraint, because the reliability objective parameter can be set to such a value that the optimisation problem considered neglects the reliability constraint.

The way the reliability is formulated in the models should be taken into account in considering a reliability constraint. Since the reliability objective is defined by the minimal probability of asset survival over a predefined time period t, the reliability objective has to be stated in these terms. The models are not capable of reformulating reliability indicators such as the MTTF, MTBF or failure rate to the probability of asset survival. In case one encounters a reliability objective defined in terms of the MTTF etcetera, this characteristic should be rewritten to the probability of asset survival over a predefined time period t in order to use the models from this research.

Finally, we would like to make a last notion on the aspect of the cost parameters for the models. Since the models are economic optimisation models under a reliability constraint, the cost parameters have an effect on the optimal maintenance schedule obtained. Upon model implementation the user should be well aware of the fact that the costs for inspections, maintenance actions, and asset failure can be determined on various ways and on different aggregation levels. Before implementing the models, the procedure for deriving the numerical values for the cost parameter values should be clear.

7

Conclusions & Recommendations

This chapter discusses the main conclusions found in this research and presents answers to the main research question from Chapter 2. Section 7.2 will conclude the chapter by elaborating on the recommendations for NedTrain and further research.

7.1 Conclusions

The goal of the research has been to investigate the effect of imperfect inspections on the optimal maintenance schedules under a reliability constraint, for which the main research question was posed. This research has conceptualised the maintenance schedule based on two variables, the inspection interval length and the inspection instance after which a preventive replacement of the asset is done. The starting point for the research has been the Delay Time Model (DTM), which includes inspections in maintenance. Based upon this principle of the DTM, this work has re-conceptualised the definitions of a defective asset and the failed asset. This has resulted in the adjusted DTM labelled DTMa. The mathematical analysis and derivations remain equal to one from the original DTM.

The inspections, which are included in the DTMa, have been considered imperfect. For the modelling of imperfect inspections, two types of imperfect inspections have been differentiated: false positives and false negatives. The occurrence of false positives and false negatives are assumed to occur with a certain probability. Three model approaches have been developed that distinctly model the probabilities of false positives and false negatives. The first model approach denotes constant probabilities for the false positives and false negatives. The second model approach considers non-constant probabilities of false positives and false negatives. It models the probability of false positives as being dependent on the defect progress and the probability of false negatives as being dependent on the degradation progress. The third model relates the probability of both false positives and false negatives to the inspection frequency and the MTTF, implying non-constant probabilities. For each of the three approaches, a cost expression and reliability expression have been developed, and subsequently been used in developing an optimisation model. Such optimisation model minimises the average costs over an infinite time horizon subject to a reliability constraint.

Based on these three models developed, the main question is answered. In the remainder of this section, the main research question is posed and answered along with the implications that follow from the research. Recall the main research question to be:

How does the imperfectness of inspections affect cost optimal maintenance schedules under a reliability constraint?

For this research we have considered a single-component asset, for which three states are distinguished: normal, defective and failed. For the numerical study, we considered a Weibull distributed time to defect and a Weibull distributed delay time, i.e. Weibull distributed state durations for the first two states. Here, we derive conclusions for the effects of imperfect inspections on cost optimal maintenance schedules under a reliability constraint, which are valid under the premises of the asset's state definitions and distributions.

The three modelling approaches yield different results for the cost optimal maintenance schedules under a reliability constraint, and their costs. Compared to the constant probabilities model, labelled the A1 model, both non-constant probability models, A2 and A3, differ in the way and extent to which the cost optimal maintenance schedules are affected. This implies that the modelling approach of the non-constant probability of false positives and false negatives determines the effects on the optimal maintenance schedules and associated costs. Nevertheless, the numerical study illustrates the worthiness of modelling non-constant probabilities of false positives and false negatives by reporting cost decreases up to $23,6\%$, with respect to the constant probabilities.

In line with earlier work on imperfect inspections by Berrade et al. (2013), this work concludes that the optimal maintenance schedules are affected by the imperfectness of inspections under a reliability constraint as well. This implies that an increase in inspection quality yields more frequent inspections and cost decreases, whereas a decrease in inspection quality yields a maintenance schedule of the inspections being performed less frequently and the costs increase. To link this conclusion to the industry focus of stretching inspection intervals, increasing the inspection interval length becomes more cost inefficient when the inspection quality becomes worse. Therefore, improving inspection quality is beneficial with respect to cost minimisation under a reliability constraint.

An additional insight gained from this research relates to the reliability constraint. We conclude that the inspection interval length is highly determinant for satisfying the reliability constraint. In constraint satisfaction the inspection interval length dominates the inspection instance of preventive asset replacement, i.e. the reliability constraint is satisfied by doing more frequent inspections instead of earlier preventive maintenance. The numerical study illustrates this behaviour for various parameter settings by increasing the instance of preventive asset replacement and decreasing the inspection interval lengths when a reliability constraint is considered. Furthermore, the reliability constraint contributes to further affecting the cost effects the non-constant probabilities of false positives and false negatives have, compared to constant probabilities.

We would briefly like to make a note on the models' sensitivity to the distribution parameters for the asset's life distributions. Since the models are sensitive to the distribution parameters we suggest to pay attention to setting the values as well as possible.

7.2 Recommendations

In this section recommendations are made to the host of this research, NedTrain, and recommendations are discussed for further research.

7.2.1 Recommendations for NedTrain

This work has focused on differently modelling the probabilities of false positives and false negatives in three distinct ways. However, this work does not consider comparing the results from the models to practice. This brings us immediately to the main recommendation for NedTrain listed below.

- **Compare results from current practice to the models' results**
  To further check how plausible the models' results are with respect to costs and reliability, the author recommends NedTrain to compare the costs and reliability of current practice to the models' results. This can be done by using the maintenance schedule from current practice to theoretically calculate the costs and reliability. Upon these two results, it is then recommended to compare both.

Next to the main recommendation, some other helpful recommendations for NedTrain are made. Each of these recommendations are included in the enumeration below.

- **Investigate and estimate the probability false positives and false negatives**
  This work has assumed certain probabilities for false positives and false negatives in conducting the numerical study from Chapter 5. However, we recommend NedTrain to further investigate how the probabilities of false positives and false negatives can be derived more in more detail.
- **Derive model parameters on quantitative data**
  In the numerical study from this research, the distributions for the time to defect and the delay time have been based on the expert opinions from NedTrain employees. Despite the fact that this method is valid, we would still recommend to quantitatively derive the distribution parameters for the time to defect and delay time. Additionally, Weibull distributions have been assumed, but this does not need to be the case, i.e. other distributions might better describe the asset's time to defect and delay time, thereby contributing to more worthwhile numerical results of the models considered.
- **Develop additional model approaches for the modelling of imperfect inspections**
  In this work, three model approaches have been posed, but this does not imply that these cover all the ways on which probabilities of false positives and false negatives can be modelled. Hence, it is recommended to further develop other model approaches on the relationships between various variables and the probability of false positives and false negatives. Once developed, the author recommends comparing the results from the model approaches to practical results.

7.2.2 Recommendations for further research

With respect to further research several recommendations are presented here. No main recommendation is distinguished from others. Hence, all recommendations for further research are listed in a single enumeration as included below.

- **Consider equal time horizons in the optimisation model**
  The optimisation model in this research has focused on the minimisation of the costs over an infinite time horizon subject to a reliability constraint. The way in which the reliability has been included in the optimisation model has followed literature. However, the reliability is considered over a finite horizon, whereas the cost element is considered over an infinite time horizon. We recommend to improve model consistency in further research by either considering the reliability constraint over an infinite time horizon or considering the cost expression over a finite time horizon.

- **Include an asset state denoting a non failed asset which is not allowed to be operated**
  The research has considered an asset to fail immediately when the asset's load carrying capacity reaches the critical level. However, in practice this does not occur. In such case, the asset is not allowed to be operated anymore upon reaching the critical level, but it has not yet failed. This research has relaxed this aspect and therefore we recommend further research for including such an additional state.

- **Include asset redundancy**
  The models developed include a single one-component asset. However, including redundancy of the one-component asset might present an interesting study. Mainly when reliability constraint compared to the asset's failure behaviour is tight, including asset redundancy might contribute to further cost reductions and might alter the cost optimal maintenance schedules.

- **Extend the current models to multi-component asset models**
  As the current models are all subject to the assumption that a single-component asset is considered, considering an asset consisting of multiple components might be more realistic and present new insights. The single-component asset has been considered in this work due to the fact that it offers a building block for the multi-component asset models. Hence, the recommendation for further research includes considering a multi-component asset, e.g. a complete rotable of a train which has multiple failure modes.

- **Exclude the inspection upon the preventive asset replacement**
  In the models developed, an inspection is performed before the preventive asset replacement, which is executed on inspection instance M. Further research could alter this assumption by excluding the inspection before the preventive asset replacement. Because all three models from this research underlie this same assumption of the inspection occurring before the preventive asset replacement, all three models will have to be altered.

- **Include asset replacement costs**
  In this work we consider three different cost aspects, i.e. inspection costs, maintenance action costs and failure costs. When the asset is preventively replaced, at time MT, maintenance action costs are incurred for the asset replacement. The author recommends to introduce another cost type, the preventive replacement costs, which are incurred only when the asset gets preventively replaced instead of incurring maintenance action costs upon such an event.

- **Include imperfect maintenance actions**
  Although our models assume imperfect inspections, the maintenance actions itself are assumed to be perfect. In practice, perfect maintenance actions will not exists. Therefore, we recommend the inclusion of the imperfect maintenance actions in the models developed.

- **Use Markov Models for modelling imperfect inspections under a reliability constraint**
  This research has used a DTM type of model for modelling imperfect inspections in maintenance under a reliability constraint. However, as the literature study has suggested, Markovian based models are also applicable for modelling such imperfect inspections. Hence, the author recommends further research to model the imperfect inspections under a reliability constraint for Markov Models as well, and compare the effects to the DTM type of modelling.

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A

List of Abbreviations

cdf	Cumulative Density Function
DTM	Delay Time Model
DTMa	Adjusted Delay Time Model
HMM	Hidden Markov Model
HPP	Homogeneous Poisson Process
HSMM	Hidden Semi Markov Model
MTBF	Mean Time Between Failures
MTTF	Mean Time To Failure
NS	Nederlandse Spoorwegen
pdf	Probability Density Function
P-F	Potential Failure
TCO	Total Cost of Ownership

B

List of Definitions

Asset	A unit comprised of one or more components
Asset failure (DTMa)	The asset's load carrying capacity equals the critical level
Asset defect (DTMa)	The asset's load carrying capacity is less than the rejection level, i.e. a maintenance action is required
Asset failure (DTM)	The load exceeds the asset's load carrying capacity, i.e. physical failure
Asset defect (DTM)	The asset's load carrying capacity is less than the detection level
Condition-based Maintenance	Maintenance performed preventively based on the asset's condition
Corrective Maintenance	Maintenance done upon asset failure
Cost model	A mathematical expression representing the average costs per time unit
Critical level (DTMa)	The load carrying capacity level which can never be violated. This level is set by asset designers
Delay time	Time between the asset's defect arrival and asset failure
Detection level	The load carrying capacity level after which degradation can be visually observed
Failure costs	The costs of asset failure
False positive	The inspection results yields a defective asset when in fact the asset is operating normally without defects
False negative	The inspection results yields a normal operating asset when in fact the asset defective
Imperfect inspection	Inspection results might deviate from the actual asset's state
Inspection costs	The costs for inspecting the asset
Load carrying capacity	The load the asset can endure at a time instance
Maintenance	All activities involved in the complete maintenance, including inspections and the actual maintenance itself
Maintenance action costs	The costs for maintaining the asset
Maintenance modelling	The mathematical modelling of maintenance

Maintenance schedule	The schedule on which the maintenance is based consisting of the inspection interval length and the inspection instance of preventive asset replacement
Optimisation model	A mathematical model that minimises the Cost Model under a constraint comprised of the Reliability Model
P-F Interval	Interval between degradation initiation and physical asset failure
Perfect inspection	No single inspection result deviates from the actual asset's state
Planned Maintenance	Maintenance performed preventively based on a predetermined time interval
Rejection level (DTMa)	The load carrying capacity level after which the asset is considered defective
Reliability	The probability of the asset operating up to a certain time instance
Reliability model	A mathematical expression representing the reliability on any time t
Time to defect	The time instance upon which the asset becomes defective
Type I error	See false positive
Type II error	See false negative

C

List of Variables

a	The number of preventive asset replacement instances
c_α	Scaling variable of the non-constant probability of false positives under the A2 model
c_β	Scaling variable of the non-constant probability of false negatives under the A2 model
c_0	Inspection costs
c_f	Failure costs
c_p	Maintenance action costs
$C_w(M,T)$	Average cost per time unit given M and T under model w
$E_w(\textit{cycle cost})$	Expected costs per cycle under model w
$E_w(\textit{cycle length})$	Expected length of the cycle under model w
$E_w(K)$	Expected number of inspections K done in a cycle under model w
$f_H(h)$	pdf of asset's delay time
$F_H(h)$	cdf of asset's delay time
$f_X(x)$	pdf of asset's time to defect
$F_X(x)$	cdf of asset's time to defect
H	The random variable denoting the asset's delay time
h	A realisation of the asset's time to defect
i	The inspection bounding the interval in which the asset becomes defective. The interval is bounded by $((i-1)T, iT)$
j	The inspection upon which the asset's defect is detected
K	The number of inspections done in a cycle
k	The inspection instance upon which a false negative occurs
M	Inspection instance upon which preventive replacement occurs
n	The inspection instance upon which no false positive occurs
$P_w(c_p)$	Probability of incurring maintenance action costs c_p under model w
q	The inspection instance after the asset's defect arrival and before asset failure
r	The inspection instance before the asset's defect arrival
$R_{MT,w}(t)$	The reliability of the asset on time t under model w and preventive replacement on time MT
$R^{(s)}_{MT,w}(t)$	The reliability of the asset on time t, where $((s-1)T \leq t < sT)$, under model w and preventive replacement on time MT
$R_{obj}(t)$	The reliability objective from time 0 to time t

s	The inspection under consideration for the reliability function that bounds the reliability function's argument t
T	Inspection interval length
t	Time variable used in the reliability expression
w	The index indicating the model approach under consideration
X	The random variable denoting the asset's time to defect
x	A realisation of the asset's time to defect
α	Constant probability of false positive
α_0	Inevitable probability of false positive under non-constant probability of false positive
$\alpha_A()$	The assumed non-constant term of the probability of false positives being dependent on the argument
β	Constant probability of false negative
β_0	Inevitable probability of false negative under non-constant probability of false negative
$\beta_A()$	The assumed non-constant term of the probability of false negatives being dependent on the argument
δ_H	Weibull shape parameter of the delay time for the numerical study
δ_X	Weibull shape parameter of the time to defect for the numerical study
η	The average number of assets inspected and maintained by a single engineer
γ_α	Scaling variable of the non-constant probability of false positives under the A3 model
γ_β	Scaling variable of the non-constant probability of false negatives under the A3 model
θ_H	Weibull scale parameter of the delay time for the numerical study
θ_X	Weibull scale parameter of the time to defect for the numerical study

D

Additional Model Derivations

D.1 A1 model

This section further elaborates on the derivation for the cost and the reliability expression under the A1 model. Because the cost expression consists of three separate terms, these are al individually derived in Section D.1.1. The section concludes with the detailed derivation of the reliability expression in Section D.1.2. Furthermore, note that when the probabilities α and β both equal to 0, a model under perfect inspections is obtained.

D.1.1 Deriving the cost evaluation

The cost evaluation for all models is based on the concept of the average costs per time unit (Ross, 1983). By using cycles the average costs per time unit are obtained. Recall the relation for the average cost expression based on the fraction between the expected cycle costs and the expected cycle length.

$$C_1(M,T) = \frac{E_1(\textit{cycle costs})}{E_1(\textit{cycle length})}$$

The expected cycle costs are comprised of the expected number of inspections in a cycle $E_1(K)$ multiplied by the costs per inspection c_0, the probability of incurring costs c_p in a cycle $P_1(c_p)$ multiplied by the maintenance action costs c_p and the probability of the asset failing multiplied by the costs associated to asset failure c_f. The latter probability corresponds to $1 - P_1(c_p)$ because a cycle either ends in the incurring of maintenance action costs or in asset failure. Since $P_w(c_p)$ denotes the probability of the cycle ending in the event of incurring maintenance action costs, $1 - P_1(c_p)$ denotes the probability of the cycle ending in asset failure. This yields the total expected cycle costs.

$$E_1(\textit{cycle costs}) = c_0 E_1(K) + c_p P_1(c_p) + c_f(1 - P_1(c_p))$$

Three terms are to be evaluated to obtain the average costs per time unit: $E_1(K)$ and $P_1(c_p)$, and the expected cycle length $E_1(\textit{cycle length})$.

Deriving $E_1(K)$

The first term in the expected cycle cost equation to evaluate, is the expected number of inspections in a cycle $E_1(K)$. In this case it is assumed that at the last inspection instance

M an inspection is done. The number of inspections in a cycle K can then vary between $0, 1, \ldots, M$. To derive the expression for the expected number of inspections in a cycle, the probability of each value for K is calculated and multiplied by the number of inspections done. By summing all of the outcomes the expected value for the number of inspections in a cycle is obtained. In the probability expressions for the number of inspections in a cycle K three cases are identified: $P(K = 0)$, $P(K = 1, \ldots, M - 1)$ and $P(K = M)$.

P(K=0)
The first case of $P(K = 0)$ denotes the probability that no inspections are done. This implies that the asset becomes defective and fails before the first inspection at time T. Let us first consider the expression when the asset's time to defect x is given and lies in the interval $(0, T)$, i.e. given that $0 \leq x < T$. Under the condition of x, the asset fails before the inspection at time T when its delay time is shorter than the time left from the defect arrival, i.e. $h < T - x$. Because the delay time is non-negative, this failure event occurs with probability $P(0 \leq h < T - x)$. As this case considers no inspections to occur in the cycle, false positives and false negatives are excluded from the analysis. The probability of the asset failing before the first inspection, given $0 \leq x < T$ now corresponds to:

$$P(K = 0 | 0 \leq x < T) = P(0 \leq h < T - x)$$

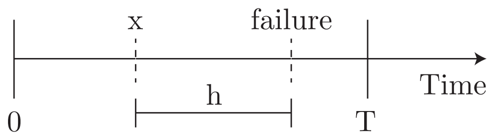


Figure D.1: Possible realisation of $K = 0$

To illustrate this equation, Figure D.1 is included presenting a possible realisation when the asset fails before the first inspection. The expression above $P(0 \leq h < T - x)$ denotes the cdf of the delay time with argument $T - x$ and can therefore be rewritten to the following.

$$P(K = 0 | 0 \leq x < T) = \int_0^{T-x} f_H(h) dh = F_H(T - x)$$

The goal is to determine $P(K = 0)$ without the condition of $0 \leq x < T$. Furthermore, the time to defect X is a random variable implying that a realisation of the asset's time to defect x, lies in the interval $(0, T)$ with an associated probability. To yield the expression for $P(K = 0)$ Bayes' rule is applied.

$$\begin{aligned} P(K = 0) &= P(0 \leq x < T) P(K = 0 | 0 \leq x < T) \\ &= P(0 \leq x < T) F_H(T - x) \end{aligned}$$

The expression for the probability that the time to defect x lies in $(0, T)$ corresponds to the pdf integral of the time to defect with a lower bound of 0 and an upper bound of T. The cdf expression cannot be written due to the fact that the delay time's cdf is also dependent on x. Therefore, the delay time's cdf is also included in the integral of the time to defect. Hence,

$$\begin{aligned} P(K = 0) &= P(0 \leq x < T) F_H(T - x) \\ &= \int_0^T F_H(T - x) f_X(x) dx \end{aligned}$$

P(K=j)
For the probability case where K can have the values of $1, \ldots, M - 1$ the expression becomes

harder. Let j denote the number of inspections in the cycle, i.e. we derive the expression for $P(K = j)$ where $j = 1, \dots, M-1$. Four possible probability events may occur which yield the case that $K = j$. The first corresponds to the probability that the asset enters its defective state before the j^{th} inspection and is detected at the j^{th} inspection thereby ending the cycle. The second scenario denotes the probability of the asset becoming defective before j^{th} inspection, the defect has not been identified in any of the inspections and the asset will fail in the time interval following the j^{th} interval, i.e. failure before inspection $j+1$. The third probability case represents that the asset is still operating normally upon the j^{th} inspection but gets defective and fails before inspection $j+1$. Finally the last probability event consists of the probability of the asset being maintained when it is still operating normally, i.e. a false positive. To explain each of the cases into more detail the enumeration below is included.

- **Scenario 1:** In this scenario, the asset becomes defective in any of the intervals preceding inspection j, upon which the defect is detected. Let variable i denote the inspection characterising the interval in which the asset becomes defective, where $i = 1, \dots, j$. For illustrative purposes on variable i, Figure D.2 is included.

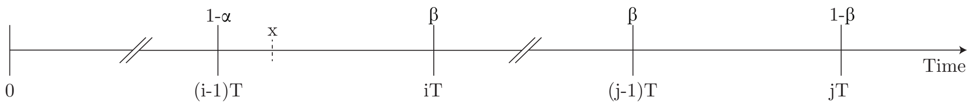


Figure D.2: Illustrating interval i preceding the j^{th} inspection

Say the asset becomes defective at the given time x in the interval $((i-1)T, iT)$, i.e. $((i-1)T \leq x < iT)$. The asset has to survive in its defective state from the time to defect x to the j^{th} inspection at time jT. This probability is denoted by $P(h \geq jT - x)$. Due to the fact that the asset has become defective in interval i, it has been free of defects for $i-1$ inspections. Furthermore, in the remaining inspections from i to j, the defect is not detected with probability β, i.e. the defect is not detected for $j-i$ inspections yielding β^{j-i}. Note that in case $j - i = 0$ the asset has become defective in the interval before the j^{th} inspection. Additionally, the defect is found upon the j^{th} inspection with probability $1-\beta$ to end the scenario. Combining all aspects under the condition of $(i-1)T \leq x < iT$, the scenario is now denoted by:

$$(1-\alpha)^{i-1}\beta^{j-i}(1-\beta)P(h \geq jT - x)$$

The probability expression for the delay time $P(h \geq jT - x)$ can be rewritten to $1 - P(0 \leq h < jT - x)$. This obtained expression for the delay time can subsequently be expressed in one minus the cdf of the delay time with argument $jT - x$. This yields:

$$\begin{aligned}(1-\alpha)^{i-1}\beta^{j-i}P(h \geq jT - x)& \\ &= (1-\alpha)^{i-1}\beta^{j-i}(1-\beta)(1 - P(0 \leq h < jT - x)) \\ &= (1-\alpha)^{i-1}\beta^{j-i}(1-\beta)(1 - F_H(jT - x))\end{aligned} \tag{D.1}$$

The scenario's expression is derived based on the condition that the asset's time to defect x is given and lies in the interval $((i-1)T, iT)$. To obtain the expression of the first scenario for a given i and j and without the condition, Equation D.1 is multiplied by the probability of the time to defect x being in the interval $((i-1)T, iT)$, i.e. $P((i-1)T \leq x < iT)$. This probability equals to the integral over the time to defect's pdf over the range $((i-1)T, iT)$. Note that the delay time's cdf is also included in the integration since it is also dependent on the random variable X.

$$
\begin{aligned}
&(1-\alpha)^{i-1}\beta^{j-i}(1-\beta)P((i-1)T \leq x < iT)(1-F_H(jT-x)) \\
&\qquad = (1-\alpha)^{i-1}\beta^{j-i}(1-\beta)\int_{(i-1)T}^{iT}(1-F_H(jT-x))f_X(x)dx
\end{aligned}
$$

The expression derived up to here considers the probability for one specific interval i, preceding the j^{th} inspection. To capture all intervals i, the expression is summed over all possible values resulting in the final expression for the first scenario, where $i = 1, \ldots, j$. This yields the final expression for the first scenario of $P(K = j)$.

$$
\sum_{i=1}^{j}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx
$$

- **Scenario 2:** The second scenario relates to the first. However, instead of doing maintenance upon the j^{th} inspection, the defect is not detected at the j^{th} inspection and the asset fails before the next inspection $j+1$. Note that both scenarios have equal numbers of inspections, namely j. Let us again start with the given value for x which lies in $((i-1)T, iT)$, where $i = 1, \ldots, j$. Under this condition, the process until the j^{th} inspection equals the process of the first scenario implying the same expression for the probability of false positives not occurring, $(1-\alpha)^{i-1}$. In comparison to the first scenario the defect is not found, so instead of a probability $1-\beta$, an additional probability of β occurs resulting in an omission of the term $1-\beta$ and a multiplication of the term β^{j-i} by β, which corresponds to the term β^{j-i+1}. The main difference to the first scenario lies in the fact that the asset fails after the j^{th} inspection and before the $(j+1)^{th}$ inspection. Given that the asset has become defective at time x, its delay time has to lie in the interval $(jT-x, (j+1)T-x)$ which occurs with probability $P(jT-x \leq h < (j+1)T-x)$. The latter probability can be written in terms of the delay time's pdf integral. This results in the following expression.

$$
\begin{aligned}
&(1-\alpha)^{i-1}\beta^{j-i+1}P(jT-x \leq h < (j+1)T-x) \\
&\qquad = (1-\alpha)^{i-1}\beta^{j-i+1}\int_{jT-x}^{(j+1)T-x}f_H(h)dh
\end{aligned}
$$

The expression above underlies the condition of x, which lies in the interval $((i-1)T, iT)$. To include the random nature of the time to defect X, the equation above is multiplied with the probability that $(i-1)T \leq x < iT$, i.e. $P((i-1)T \leq x < iT)$. This probability expression is captured by the pdf integral of the time to defect from $(i-1)T$ to iT.

$$
\begin{aligned}
&(1-\alpha)^{i-1}\beta^{j-i+1}P((i-1)T \leq x < iT)\int_{jT-x}^{(j+1)T-x}f_H(h)dh \\
&\qquad = (1-\alpha)^{i-1}\beta^{j-i+1}\int_{(i-1)T}^{iT}\int_{jT-x}^{(j+1)T-x}f_H(h)dhf_X(x)fx
\end{aligned}
$$

The integral expression for the delay time is included in the integral over the time to defect's pdf as well, because the delay time's integral includes the variable x in its boundaries. The expression obtained only holds for one specific interval i in which the asset becomes defective before inspection j. To derive the probability of the asset becoming defective in any interval preceding inspection j, the sum over the equation above is taken from 1 to j, where j corresponds to the interval just before the j^{th} inspection.

$$\sum_{i=1}^{j}(1-\alpha)^{i-1}\beta^{j-i+1}\int_{(i-1)T}^{iT}\int_{jT-x}^{(j+1)T-x} f_H(h)dhf_X(x)dx$$

- **Scenario 3:** The third case consists of the probability of the asset remaining free of detects until the j^{th} inspection. Before the $j+1^{th}$ inspection the asset then becomes defective and fails. In this case also j inspections are performed. Consider the condition that the time to defect x is given and lies in the interval $(jT,(j+1)T)$. Due to the fact that the asset remains without defects until the j^{th} inspection and no false positives have occurred the term $(1-\alpha)^j$ is included indicating the probability that the asset has been free of defects and inspected as such j times. Given the time to defect x lying in $(jT,(j+1)T)$, the asset fails before the inspection at time $(j+1)T$, i.e. the asset's delay time h is shorter than $(j+1)T-x$ but non-negative. This event occurs with a probability $P(0 \le h < (j+1)T-x)$, which can be rewritten to the cdf expression of the delay time. Under the condition that the time to defect x is given and lies in $(jT,(j+1)T)$, the probability expression for the third scenario is obtained.

$$\begin{aligned}(1-\alpha)^j P(0 \le h < (j+1)T-x) \\ = (1-\alpha)^j F_H((j+1)T-x)\end{aligned}$$

To include the random nature of the variable X, the time to defect, the expression above is multiplied with the probability that the time to defect actually lies in the interval $(jT,(j+1)T)$ denoted by $P(jT \le x < (j+1)T)$. This latter probability is subsequently represented by the integral expression of the time to defect's pdf from jT to $(j+1)T$.

$$\begin{aligned}(1-\alpha)^j P(jT \le x < (j+1)T)F_H((j+1)T-x) \\ = (1-\alpha)^j \int_{jT}^{(j+1)T} F_H((j+1)T-x)f_X(x)dx\end{aligned}$$

Note that the probability expression for the delay time is included in the integral over the time to defect, as the delay time's cdf $F_H((j+1)T-x)$ includes the variable x.

- **Scenario 4:** All scenarios up to here have considered the case in which no false positives have occurred. However a false positive may occur at the j^{th} inspection causing the cycle to end and in that case j inspections are performed. For such a false positive to occur, the asset must be free of defects implying the time to defect to exceed the inspection time of the j^{th} inspection, jT. This event takes place with a probability of $P(x \ge jT)$. Note that this probability expression can be rewritten as $P(x \ge jT) = 1 - P(0 \le x < jT)$. Furthermore, the asset has been inspected $j-1$ times without a false positive yielding a probability of $(1-\alpha)^{j-1}$. Upon inspection j, the asset then undergoes a false positive with a probability α. Combing all three aspects yields the probability expression of the fourth scenario.

$$\alpha(1-\alpha)^{j-1}(1-P(0 \le x < jT))$$

The probability expression $(1 - P(0 \le x < jT))$ can be represented by the time to defect's cdf with argument jT. Including this characteristic into the equation yields the final expression for the fourth scenario.

$$\alpha(1-\alpha)^{j-1}(1-F_X(jT))$$

The probability of the occurrence of j inspections in a cycle $P(K=j)$ has been decomposed into four cases, where $j=1,...,M-1$. By summing each of the four scenarios, the complete expression for the probability of j inspections being performed in a cycle is obtained. This summation of all four cases results in the final probability expression for the probability of j inspections in a cycle $P(K=j)$.

$$\begin{aligned} P(K=j) = &\sum_{i=1}^{j}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx \\ &+\sum_{i=1}^{j}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\int_{jT-x}^{(j+1)T-x}\beta^{j-i+1}f_H(h)dhf_X(x)dx \\ &+(1-\alpha)^{j}\int_{jT}^{(j+1)T}F_H((j+1)T-x)f_X(x)dhdx \\ &+\alpha(1-\alpha)^{j-1}(1-F_X(jT)) \end{aligned} \tag{D.2}$$

From Equation D.2 it becomes evident that the case for $P(K=0)$ cannot be captured by the expression from Equation D.2. Equation D.2 namely includes a term which loses its realistic representation in case j is set to 0. This term is the last term of Equation D.2. In case $K=0$, the fourth term of Equation D.2 includes $(1-\alpha)^{-1}$ which violates reasoning. Therefore the $K=0$ case is considered separately.

P(K=M)
The case at which the number of inspections equals to M, $K=M$, can be derived from Equation D.2. The first and last term remain intact, whereas the situations corresponding to the second and third part of the equation cannot occur. These second and third parts of Equation D.2 take into account the fact that the asset will fail in between inspections j and $j+1$, but when $j=M$ this event cannot occur because M denotes the last inspection performed. Hence, the first part and the last part are considered. Both parts are altered slightly when setting $j=M$. The first part is changed in such a way that no maintenance action is done, i.e. the probability on a false negative $1-\beta$ is not considered. This stems from the property of preventive asset replacement upon inspection M. Moreover, the last and fourth part of Equation D.2 is altered. A false positive cannot occur on the M^{th} inspection because the asset is replaced, independent on the inspection outcome. This yields the following equation for $K=M$.

$$\begin{aligned} P(K=M) = &\sum_{i=1}^{M}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\beta^{M-i}(1-F_H(MT-x))f_X(x)dx \\ &+(1-\alpha)^{M-1}(1-F_X(MT)) \end{aligned}$$

All three probability cases, $P(K=0)$, $P(K=1,...,M-1)$ and $P(K=M)$, are used as inputs in deriving the expected number of inspection in a cycle for the A1 model, $E_1(K)$. By multiplying the probabilities of a number of inspections with the number of inspections and summing all, yields the expected value for $E_1(K)$. To illustrate, consider $M=2$.

$$E_1(K) = 0P(K=0)+1P(K=1)+2P(K=2)$$

This technique for calculating the expected number of inspections in a cycle $E_1(K)$ can also be applied to the general equations derived for all three scenarios $P(K=0)$, $P(K=$

$1, ..., M-1)$ and $P(K=M)$. In multiplying the first case $P(K=0)$ with 0, yields a value of 0 corresponding to omitting this term from the expected value expression $E_1(K)$. The second case has presented a general equation for calculating the probability of j inspections in a cycle, where $j = 1, ..., M-1$. By multiplying the probability of j inspections by the j inspections and summing, the expected number of inspections for the second case is obtained. However, to derive the complete expression for the expected number of inspections, the multiplication of $P(K=M)$ and M is added to the sum of $jP(K=j)$ over j. These operations yield the complete expression for the expected number of inspections per cycle, see Equation D.3.

$$
\begin{aligned}
E_1(K) &= \sum_{j=1}^{M-1} jP(K=j) + MP(K=M) \\
&= \sum_{j=1}^{M-1} j \left\{ \sum_{i=1}^{j} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx \right. \\
&+ \sum_{i=1}^{j} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \int_{jT-x}^{(j+1)T-x} \beta^{j-i+1} f_H(h)dhf_X(x)dx \\
&+ (1-\alpha)^j \int_{jT}^{(j+1)T} F_H((j+1)T-x)f_X(x)dhdx \\
&\left. + \alpha(1-\alpha)^{j-1}(1-F_X(jT)) \right\} \\
&+ M \left\{ \sum_{i=1}^{M} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{M-i}(1-F_H(MT-x))f_X(x)dx \right. \\
&\left. + (1-\alpha)^{M-1}(1-F_X(MT)) \right\}
\end{aligned}
\tag{D.3}
$$

Deriving $P_1(c_p)$

The next step in deriving the expected cycle costs for the A1 model is determining the probability of incurring maintenance action costs $P_1(c_p)$. The maintenance action costs c_p are incurred in a cycle when certain events occur. These events consists of the following:

- **Scenario 1:** This event corresponds to the probability of the asset becoming defective before the moment of preventive replacement MT, the asset does not fail and its defect is found before MT. Note that this scenario relates to the first term from Equation D.2.

  Consider the asset to become defective at time x, when $(i-1)T \leq x < iT$, where $i = 1, \ldots, M-1$. The asset's defect is found upon inspection j, where $j = i, \ldots, M-1$. In order for the asset's defect to be found upon inspection j, its delay time has to exceed the time the asset has been defective, i.e. $h \geq jT - x$. This event occurs with probability $P(h \geq jT-x) = 1 - P(0 \leq h < jT-x) = 1 - F_H(jT-x)$. In between asset's defective arrival and defect detection, $j-i$ inspections occur with probability β^{j-i}. Before, defect arrival no false positives have occurred and the defect is detected upon inspection j with probability $1-\beta$.

  $$(1-\alpha)^{i-1}\beta^{j-i}(1-\beta)(1-F_H(jT-x))$$

  The expression above holds for a given x. However, the asset becomes defective in $((i-1)T, iT)$ with a certain probability $P((i-1)T \leq x < iT)$, due to the fact that the time to defect is a random variable X. To include the random nature of the time to defect:

  $$(1-\alpha)^{i-1}\beta^{j-i}(1-\beta)P((i-1)T \leq x < iT)(1-F_H(jT-x))$$

The probability of $P((i-1)T \leq x < iT)$ is written in terms of the integral of the time to defect's pdf. Note that the expression for the delay time $1 - F_H(jT - x)$ is included in the integration as well, since this term includes the variable x. This yields the following expression:

$$(1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx$$

Because the asset can become defective in any interval $((i-1)T, iT)$, all possible intervals are included by summing over all values for i. In addition, the defect can be found upon any inspection j yielding a summation over all possible j as well, where $j = i, \ldots, M-1$. This yields the final expression for the first scenario for the probability of incurring maintenance costs c_p.

$$\sum_{i=1}^{M-1} \sum_{j=i}^{M-1} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx$$

- **Scenario 2:** The asset may also undergo a false positive at any of the inspections preceding inspection M, i.e. at inspections $1, \ldots, M-1$. This probability situation directly relates to the fourth scenario from the expression for $P(K = j)$. This fourth scenario from $P(K = j)$ includes the exact same problem, but it considers one specific value for inspection j upon which the false positive occurs. To consider any inspection upon which a false positive may occur one needs to consider $j = 1, ..., M-1$. This implies that the exact expression for the fourth scenario from $P(K = j)$ should be summed from 1 to $M-1$ to yield the probability of a maintenance action due to a false positive.

$$\sum_{j=1}^{M-1} \alpha(1-\alpha)^{j-1}(1-F_X(jT))$$

- **Scenario 3:** The third scenario of incurring costs c_p is defined when the asset becomes defective before the last inspection time MT, but remains in its defective state until MT and is then preventively replaced. Recall that upon preventive asset replacement, maintenance action costs c_p are incurred as well. This probability expression is identical to the first part of the probability expression for $P(K = M)$. Hence,

$$\sum_{i=1}^{M}(1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \beta^{M-i}(1-F_H(MT-x))f_X(x)dx$$

- **Scenario 4:** The last probability scenario denotes the probability of the asset to remain free of defects until the last inspection M. In all preceding inspections no false positives have occurred. Upon the last inspection M the asset is preventively replaced thereby incurring costs c_p. This probability event is identical to the second part of the expression for $P(K = M)$.

$$(1-\alpha)^{M-1}(1-F_X(MT))$$

By summing all four probability scenarios one obtains the complete expression for the probability of incurring maintenance action costs under the A1 model $P_1(c_p)$, see Equation D.4.

$$\begin{aligned}P_1(c_p) = &\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\beta^{j-i}(1-\beta)(1-F_H(jT-x))f_X(x)dx \\ &+ \sum_{j=1}^{M-1}\alpha(1-\alpha)^{j-1}(1-F_X(jT)) \\ &+ \sum_{j=1}^{M}(1-\alpha)^{j-1}\int_{(j-1)T}^{jT}\beta^{M-j}(1-F_H(MT-x))f_X(x)dx \\ &+ (1-\alpha)^{M-1}(1-F_X(MT))\end{aligned} \tag{D.4}$$

Both equations, Equation D.3 and Equation D.4, are used to derive the expected cycle costs. The remainder of this subsection will discuss the derivation of the expected cycle length $E_1(\textit{cycle length})$.

Deriving $E_1(\textit{cycle length})$

To derive the expected cycle length for the A1 model, the derivation is split into three conditional expected cycle length expressions. Each of the expressions differ in their number of terms included and are therefore separately considered. The first part derives an expression for the conditional expected cycle length given the asset's time to defect x, which lies in the interval between inspection $i-1$ and inspection i, where $i = 1, \dots, M-1$. The second expression then derives the conditional expression for a time to defect, in the interval $((M-1)T, MT)$. The final expression derives an equation given the asset's time to defect exceeding the last inspection time epoch MT. By multiplying each of the three expressions with the probability of their respective conditions, the separate contribution of each of the terms to the expected cycle length under the A1 model is determined. Summing over all the obtained terms then yields the complete equation for the expected cycle length $E_1(\textit{cycle length})$.

Deriving $E_1(\textit{cycle length}|(i-1)T \leq x < iT)$, for $i = 1, \dots, M-1$
This part of the subsection derives an expression for the expected cycle length given the asset's time to defect x, which lies between time $(i-1)T$ and iT, where $i = 1, \dots, M-1$. The variable i denotes the inspection which comprises the interval in which the asset becomes defective. This conditional expected cycle length is mathematically represented as $E_1(\textit{cycle length}|(i-1)T \leq x < iT)$ for $i = 1, \dots, M-1$. The expression for $E_1(\textit{cycle length}|(i-1)T \leq x < iT)$ is decomposed into five scenarios as discussed in the enumeration below.

- **Scenario 1:** The first case consists of the probability that the asset fails in the interval $((i-1)T, iT)$. For asset to fail in this interval, its delay time h has to be smaller than the remaining time, from point x to inspection iT. Because the delay time is non-negative, the failure event occurs with a probability of $P(0 \leq h < iT - x)$. In addition, no false positive has occurred before the i^{th} inspection. Otherwise, the cycle would have ended already due to a maintenance action. Its probability is represented by $(1-\alpha)^{i-1}$. This first scenario only occurs when all conditions are met implying multiplication of both probability expressions. Additionally, the scenario takes $x + h$ time units. The expected cycle length contribution for this scenario is then defined as:

$$(1-\alpha)^{i-1}P(0 \leq h < iT - x)(x + h)$$

  The probability of the delay time being non negative and smaller than $iT - x$ cannot be expressed in terms of the delay time's cdf because the variable h is included in the equation, i.e. the inclusion of $x + h$. Hence, the integral expression is used integrating the delay time's pdf from 0 to $iT - x$.

$$(1-\alpha)^{i-1} \int_0^{iT-x} (x+h) f_H(h) dh$$

- **Scenario 2:** Given that the asset becomes defective in $((i-1)T, iT)$ the defect is not found upon the inspections from i on. Additionally, the asset then fails in interval characterised by inspection j, where $j = i+1, ..., M$ implying that the asset fails after the i^{th} inspection and before the point of preventive replacement MT. $j = M$ is also included because this denotes the last interval in which the asset may fail. Before defect arrival, no false positives occur, which corresponds to a probability of $(1-\alpha)^{i-1}$. Additionally, the asset then fails in any of the remaining intervals characterised by inspection j. The probability of the asset failing in interval $((j-1)T, jT)$ can be described by $P((j-1)T - x \leq h < jT - x)$. The probability of the number of times the asset has then undergone false negatives is then represented as β^{j-i}. However, the asset may fail in any interval $((j-1)T, jT)$ implying that one has to sum over all possible values for j. Since we consider the cycle length we include the length of the cycle for this scenario. As the second scenario considers the asset to fail, the cycle ends in failure occurring at time $x + h$. The expression now becomes:

$$(1-\alpha)^{i-1} \sum_{j=i+1}^{M} \beta^{j-i} P((j-1)T - x \leq h < jT - x)(x+h)$$

The probability of the delay time $P((j-1)T - x \leq h < jT - x)$ is rewritten to the integral expression, since the probability is not bounded by either 0 or ∞. Furthermore, due to the inclusion of the realisation h of the random variable H in the expression, the integral expression of the delay time has to be included. The delay time's pdf is integrated from $(j-1)T - x$ to $jT - x$ yielding:

$$(1-\alpha)^{i-1} \sum_{j=i+1}^{M} \beta^{j-i} \int_{(j-1)T-x}^{jT-x} (x+h) f_H(h) dh$$

For clarity reasons the lower bound of the summation sign is changed to $j = i$ yielding an upper bound of $M - 1$, a β term of β^{j-i+1} and finally altering the integration boundaries, the lower bound from $(j-1)T - x$ to $jT - x$ and the upper from $jT - x$ to $(j+1)T - x$. By this minor alteration the final form for the expression of the second scenario is acquired.

$$\begin{aligned}(1-\alpha)^{i-1} \sum_{j=i+1}^{M} \beta^{j-i} \int_{(j-1)T-x}^{jT-x} (x+h) f_H(h) dh \\ = (1-\alpha)^{i-1} \sum_{j=i}^{M-1} \beta^{j-i+1} \int_{jT-x}^{(j+1)T-x} (x+h) f_H(h) dh\end{aligned}$$

- **Scenario 3:** This scenario does not consider the asset to fail anymore. The asset namely becomes defective in $((i-1)T, iT)$ but then remains defective until inspection j, where $j = i, ..., M-1$. Upon the j^{th} inspection the defect is found with probability $1 - \beta$. The cycle now ends at the j^{th} inspection, which occurs at time jT. For the defect to be found upon inspection j. The asset's delay time has to exceed the time from the defect until the inspection time jT occurring with probability $P(h \geq jT - x)$. All inspections between i

and j then undergo a false negative with probability β^{j-i}. The derivation of terms up to here assumes the asset's defect to be detected upon a specific inspection j. However, the defect can be detected at any inspection j implying a summation over j from i to $M-1$. Finally, in this scenario no false positive occur before defect arrival, i.e. no false positives up to and including inspection $i-1$. This yields a probability of $(1-\alpha)^{i-1}$. Combining all yields the contribution for the third scenario to the conditional expected cycle length for $i = 1, ..., M-1$.

$$\begin{aligned}(1-\alpha)^{i-1}\sum_{j=i}^{M-1}(1-\beta)\beta^{j-i}jTP(h \geq jT-x)\\ = (1-\alpha)^{i-1}(1-\beta)T\sum_{j=i}^{M-1}\beta^{j-i}j(1-P(0 \leq h < jT-x))\end{aligned}$$

The probability expression for the delay time h exceeding the remaining time from the time to defect to the j^{th} inspection $P(h \geq jT-x)$ has already been rewritten in terms of a standard non-negative cdf expression $(1-P(0 \leq h < jT-x))$. This implies that this latter term can be represented by the cdf of the delay time with the argument $jT-x$ resulting in the expression for the third scenario.

$$\begin{aligned}(1-\alpha)^{i-1}(1-\beta)T\sum_{j=i}^{M-1}\beta^{j-i}j(1-P(0 \leq h < jT-x))\\ = (1-\alpha)^{i-1}(1-\beta)T\sum_{j=i}^{M-1}\beta^{j-i}j(1-F_H(jT-x))\end{aligned}$$

- **Scenario 4:** This scenario highly relates to the previous one, with respect to the fact that the asset becomes defective in $((i-1)T, iT)$ but remains defective until an inspection. Instead of considering any inspection until which the asset remains defective, this scenario considers the last inspection, i.e. the asset remains defective until inspection M. This implies a probability of $P(h \geq MT-x)$ and $M-i$ inspections do not reveal the defect with a probability of β per inspection corresponding to a combined probability of false negatives β^{M-i}. In this case the cycle's duration equals to MT. The asset namely survives up to time MT and is not maintained until then. Finally the notion of no false positives occurring in all inspections up to and including inspection $i-1$, results in the probability of $(1-\alpha)^{i-1}$. Combining all aspects yields the conditional expected cycle length contribution for the fourth case.

$$\begin{aligned}(1-\alpha)^{i-1}MT\beta^{M-i}P(h \geq MT-x)\\ = (1-\alpha)^{i-1}MT\beta^{M-i}(1-P(0 \leq h < MT-x))\end{aligned}$$

Analogous to the third case the probability $P(h \geq MT-x)$ is rewritten as $1-P(0 \leq h < MT-x)$. The lower bound of 0 is included because the delay time is non-negative. The latter term, $1-P(0 \leq h < MT-x)$, is represented by subtracting the cdf of the delay time from 1, as $1-F_H(MT-x)$. Rewriting the equation above yields the expression for scenario 4.

$$\begin{aligned}&(1-\alpha)^{i-1}MT\beta^{M-i}(1-P(0\leq h<MT-x))\\&\qquad=(1-\alpha)^{i-1}MT\beta^{M-i}(1-F_H(MT-x))\end{aligned}$$

- **Scenario 5:** The final scenario corresponds to false positives occurring. All preceding scenarios included the probability that no false positives have occurred. However, the cycle may also end preventively in a false positive. Given the time to defect x, which lies in $((i-1)T, iT)$, the false positives can only occur before inspection i, i.e. from the first inspection up to inspection $i-1$. Let variable j now denote the inspection on which a false positive may occur, where $j = 1, ..., i-1$. When the false positives occur on inspection j with probability α, all preceding inspections cannot have undergone false positive corresponding to a probability of $(1-\alpha)^{j-1}$. The cycle length will be jT in case a false positive occurs upon inspection j. In addition, we do not consider only one specific false positive inspection j, but any inspection j on which a false positive occurs. This implies summing over all j from 1 to $i-1$ yielding the complete expression for the fifth scenario.

$$\alpha\sum_{j=1}^{i-1}jT(1-\alpha)^{j-1}$$

In summing all the scenarios, the complete expression for the conditional expected cycle length given the asset's time to defect that is bounded by inspections $i-1$ and i is obtained, see Equation D.5.

$$\begin{aligned}E_1(\textit{cycle length}|(i-1)T\leq x<iT)&=(1-\alpha)^{i-1}\int_0^{iT-x}(x+h)f_H(h)dh\\&+(1-\alpha)^{i-1}\sum_{j=i}^{M-1}\int_{jT-x}^{(j+1)T-x}\beta^{j-i+1}(x+h)f_H(h)dh\\&+(1-\alpha)^{i-1}T(1-\beta)\sum_{j=i}^{M-1}j\beta^{j-i}(1-F_H(jT-x))\\&+(1-\alpha)^{i-1}MT\beta^{M-i}(1-F_H(MT-x))\\&+\alpha T\sum_{j=1}^{i-1}j(1-\alpha)^{j-1}\end{aligned}\tag{D.5}$$

Deriving $E_1(\textit{cycle length}|(M-1)T \leq x < MT)$
Analogous to the derivation of the conditional expected cycle length given $(i-1)T \leq x < iT$, the conditional expected cycle length given $(M-1)T \leq x < MT$ can similarly be derived which is represented by $E_1(\textit{cycle length}|(M-1)T \leq x < MT)$. In the latter, less terms are included compared to the former. Only three scenarios are distinguished. All three scenarios are presented in the enumeration below.

- **Scenario 1:** Given the asset's time to defect x, which is in $((M-1)T, MT)$, the asset also fails in this interval. This implies that the asset's delay time should be less than $MT - x$. Due to the non-negative characteristic of the delay time, the probability of this asset's delay time length is denoted by $P(0 \leq h < MT - x)$. As it is given that the asset becomes defective in $((M-1)T, MT)$, $M-1$ inspections have not encountered false positives, which corresponds to a probability of $(1-\alpha)^{M-1}$. As the asset fails upon time $x+h$, this also denotes the end of the cycle. By multiplying all aspects the probability expression for the first scenario is obtained.

$$(1-\alpha)^{M-1}P(0 \leq h < MT - x)(x+h)f_H(h)dh$$

Since the term $x+h$ is included in the expression above, the probability $P(0 \leq h < MT - x)$ cannot be written in terms of its cdf. Hence, the integral over the delay time's pdf from 0 to $MT-x$ is used, resulting in the final expression for the first scenario.

$$\begin{aligned}(1-\alpha)^{M-1}&P(0 \leq h < MT - x)(x+h)f_H(h)dh \\ &= (1-\alpha)^{M-1}\int_0^{MT-x}(x+h)f_H(h)dh\end{aligned}$$

- **Scenario 2:** The second scenario contributing to the conditional expected cycle length for $(M-1)T \leq x < MT$, includes the asset to survive in its delay time to inspection time MT. Given the asset's time to defect x, the asset's delay time has to exceed $MT-x$. This occurs with probability $P(h \geq MT - x) = 1 - P(0 \leq x < MT - x)$. Note that the non-negative characteristic of the delay time is included. In case the asset survives up to the last inspection M, the cycle ends in preventive asset replacement which occurs after MT time units. Additionally, no false negatives have occurred. All aspects combined yields:

$$(1-\alpha)^{M-1}MT(1-F_H(MT-x))$$

- **Scenario 3:** The last scenario represents the case in which a false positive is found in any of the inspections preceding the time to defect x, i.e. the inspections from 1 to $M-1$. Let j denote the inspection upon which the false positive occurs, where $j = 1, \ldots, M-1$. Upon inspection j, a false positive occurs with probability α and before inspection j no false positives have occurred with probability $(1-\alpha)^{j-1}$. Furthermore, upon a false positive the cycle ends. Since the false positive occurs on inspection j performed at time jT, the cycle ends on this time as well. This yields the expression for a specific j on which a false positive occurs.

$$\alpha(1-\alpha)^{j-1}jT \tag{D.6}$$

Because the false positive can occur in any inspection j, a summation over all possible values of j is included, resulting in the final expression for the third scenario under $E_1(\textit{cycle length}|(M-1)T \leq x < MT)$.

$$\alpha T\sum_{j=1}^{M-1} j(1-\alpha)^{j-1}$$

By summing all three scenarios the total probability for $E_1(\textit{cycle length}|(M-1)T \leq x < MT)$ is obtained, as depicted in Equation D.7.

$$
\begin{aligned}
E_1(\textit{cycle length}|(M-1)T \leq x < MT) = (1-\alpha)^{M-1} & \int_0^{MT-x} (x+h) f_H(h) dh \\
& + (1-\alpha)^{M-1} MT(1 - F_H(MT-x)) \\
& + \alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1}
\end{aligned} \tag{D.7}
$$

Deriving $E_1(\textit{cycle length}|x \geq MT)$
The final conditional expected cycle length denotes the condition of the time to defect, which exceeds the time epoch upon which the last inspection M is performed. Under this condition, only two scenarios can occur.

- **Scenario 1:** Given the asset's time to defect, which exceeds the time of the last inspection MT, no false positives have occurred in the $M-1$ preceding inspections corresponding to a probability of $(1-\alpha)^{M-1}$. The cycle in this scenario takes MT time units because the asset remains free of defects until the preventive replacement. Combining both yields the expression for the contribution of scenario 1 to the conditional expected cycle length $E_1(\textit{cycle length}|x \geq MT)$:

$$(1-\alpha)^{M-1} MT$$

- **Scenario 2:** This scenario corresponds to the third scenario from $E_1(\textit{cycle length}|(M-1)T \leq x < MT)$. This case namely denotes the asset undergoing a false positives when the asset is still free of defects. The false positives can occur on inspections $1, ..., M-1$. Let the inspection at which the false positive occurs be denoted by j, where $j = 1, \ldots, M-1$. A false positive then occurs with probability α and can only occur when no false positives have occurred before inspection j. This latter condition occurs with a probability of $(1-\alpha)^{j-1}$. When a false positive takes place on inspection j, the cycle ends which corresponds to time jT. All three events denote the contribution to the expected cycle length under the condition of $(M-1)T \leq x < MT$ and that a false positive occurs upon inspection j.

$$\alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1}$$

  Because a false positive can occur upon any inspection j, the sum over all possible values for j is taken, i.e. the sum from 1 to $M-1$. Hence, the expression for this scenario equals to:

$$\alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1}$$

The complete expression for $E_1(\textit{cycle length}|x \geq MT)$ is now comprised of the sum of both scenarios resulting in:

$$E_1(\textit{cycle length}|x \geq MT) = (1-\alpha)^{M-1} MT + \alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1} \tag{D.8}$$

Combining all three parts in deriving $E_1(\textit{cycle length})$
To yield the complete expression for the expected cycle length $E_1(\textit{cycle length})$ under the A1 model, each of the three separate terms derived above, i.e. $E_1(\textit{cycle length}|(i-1)T \leq x < iT)$, $E_1(\textit{cycle length}|(M-1)T \leq x < MT)$ and $E_1(\textit{cycle length}|x \geq MT)$, are multiplied by the probabilities corresponding to their conditions. This implies that the former is multiplied by the probability $P((i-1)T \leq x < iT))$, the second by the probability $P((M-1)T \leq x < MT))$ and the latter by $P(x \geq MT))$.

Each of the multiplied terms denotes the contribution of each interval, in which the asset can become defective, to the expected cycle length. By summing all terms the total expected cycle length expression under the A1 model is obtained. Note that $E_1(\textit{cycle length}|(i-1)T \leq x < iT)$ multiplied by $P((i-1)T \leq x < iT))$ yields the contribution for the interval $((i-1)T, iT)$, where $i = 1, \ldots, M-1$. To consider every interval bounded by the inspection i, $E_1(\textit{cycle length}|(i-1)T \leq x < iT)P((i-1)T \leq x < iT))$ is summed over all possible values for i. Overall, the complete expression for the expected cycle length under the A1 model $E_1(\textit{cycle length})$ is comprised as follows:

$$\begin{aligned} E_1(\textit{cycle length}) = &\sum_{i=1}^{M-1} E_1(\textit{cycle length}|(i-1)T \leq x < iT)P((i-1)T \leq x < iT)) \\ &+ E_1(\textit{cycle length}|(M-1)T \leq x < MT)P((M-1)T \leq x < MT)) \\ &+ E_1(\textit{cycle length}|x \geq MT)P(x \geq MT)) \end{aligned} \tag{D.9}$$

Let us insert the expressions for $E_1(\textit{cycle length}|(i-1)T \leq x < iT)$, $E_1(\textit{cycle length}|(M-1)T \leq x < MT)$ and $E_1(\textit{cycle length}|x \geq MT)$ into Equation D.9. Furthermore, rewrite the probability $P((i-1)T \leq x < iT))$ to the integral of the time to defect's pdf from $(i-1)T$ to iT, the probability $P((M-1)T \leq x < MT))$ to its corresponding integral from $(M-1)T$ to MT and the probability $P(x \geq MT)$ to the integral from MT to infinity. This yields Equation D.10. Since some terms in the conditional expressions include the variable x, these are included in the integrations as well.

$$
\begin{aligned}
E_1(\mathit{cycle\ length}) = & \sum_{i=1}^{M-1}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\int_{0}^{iT-x}(x+h)f_H(h)f_X(x)dhdx \\
& + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\int_{jT-x}^{(j+1)T-x}\beta^{j-i+1}(x+h)f_H(h)f_X(x)dhdx \\
& + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-\alpha)^{i-1}T(1-\beta)\int_{(i-1)T}^{iT}j\beta^{j-i}(1-F_H(jT-x))f_X(x)dx \\
& + \sum_{i=1}^{M-1}(1-\alpha)^{i-1}MT\beta^{M-i}\int_{(i-1)T}^{iT}(1-F_H(MT-x))f_X(x)dx \\
& + \sum_{i=1}^{M-1}\sum_{j=1}^{i-1}\alpha Tj(1-\alpha)^{j-1}\int_{(i-1)T}^{iT}f_X(x)dx \\
& + (1-\alpha)^{M-1}\int_{(M-1)T}^{MT}\int_{0}^{MT-x}(x+h)f_H(h)dhf_X(x)dx \\
& + (1-\alpha)^{M-1}MT\int_{(M-1)T}^{MT}(1-F_H(MT-x))f_X(x)dx \\
& + \alpha T\sum_{j=1}^{M-1}j(1-\alpha)^{j-1}\int_{(M-1)T}^{MT}f_X(x)dx \\
& + (1-\alpha)^{M-1}MT\int_{MT}^{\infty}f_X(x)dx \\
& + \alpha T\sum_{j=1}^{M-1}j(1-\alpha)^{j-1}\int_{MT}^{\infty}f_X(x)dx
\end{aligned}
\tag{D.10}
$$

Note that the summation signs have been rearranged for clarity reasons. Additionally, the first and sixth term can be captured in a single term. The same holds for the fourth and the seventh term. The eighth and tenth terms can be combined as well. The ninth term is rewritten to the time to defect's cdf expression. After rearranging the terms the following Equation D.11 is obtained.

$$
\begin{aligned}
E_1(\textit{cycle length}) = &\sum_{i=1}^{M}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\int_{0}^{iT-x}(x+h)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-\alpha)^{i-1}\int_{(i-1)T}^{iT}\int_{jT-x}^{(j+1)T-x}\beta^{j-i+1}(x+h)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-\alpha)^{i-1}T(1-\beta)\int_{(i-1)T}^{iT}j\beta^{j-i}(1-F_H(jT-x))f_X(x)dx \\
&+\sum_{i=1}^{M}(1-\alpha)^{i-1}MT\beta^{M-i}\int_{(i-1)T}^{iT}(1-F_H(MT-x))f_X(x)dx \\
&+(1-\alpha)^{M-1}MT(1-F_X(MT)) \\
&+\sum_{i=1}^{M-1}\sum_{j=1}^{i-1}\alpha Tj(1-\alpha)^{j-1}\int_{(i-1)T}^{iT}f_X(x)dx \\
&+\alpha T\sum_{j=1}^{M-1}j(1-\alpha)^{j-1}\int_{(M-1)T}^{\infty}f_X(x)dx
\end{aligned}
\tag{D.11}
$$

The sixth and seventh term from Equation D.11 can be combined into a single term. However, in doing so, the double summation under different orders from the sixth term is rewritten, to be combined with the seventh term. By mathematical definition the sixth term equals to the following:

$$
\begin{aligned}
\sum_{i=1}^{M-1}\sum_{j=1}^{i-1}\alpha Tj(1-\alpha)^{j-1}\int_{(i-1)T}^{iT}f_X(x)dx &= \sum_{j=1}^{M-1}\sum_{i=j+1}^{M-1}\alpha Tj(1-\alpha)^{j-1}\int_{(i-1)T}^{iT}f_X(x)dx \\
&= \alpha T\sum_{j=1}^{M-1}j(1-\alpha)^{j-1}\sum_{i=j+1}^{M-1}\int_{(i-1)T}^{iT}f_X(x)dx
\end{aligned}
$$

The term αT can be extracted from the double summation as it is not dependent on the summations. Analogously, $j(1-\alpha)^{j-1}$ is taken out of the sum over the index i. The latter sum $\sum_{i=j+1}^{M-1}\int_{(i-1)T}^{iT}f_X(x)dx$ can now be evaluated:

$$
\alpha T\sum_{j=1}^{M-1}j(1-\alpha)^{j-1}\sum_{i=j+1}^{M-1}\int_{(i-1)T}^{iT}f_X(x)dx = \alpha T\sum_{j=1}^{M-1}j(1-\alpha)^{j-1}\int_{jT}^{(M-1)T}f_X(x)dx
$$

Combining the rewritten form for the sixth term from Equation D.11 with the seventh term from Equation D.11, a single expression is obtained capturing both. The expression is fully simplified by writing the probability integral to its corresponding cdf expression, when possible.

$$\begin{aligned}
&\alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1} \int_{jT}^{(M-1)T} f_X(x)dx + \alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1} \int_{(M-1)T}^{\infty} f_X(x)dx \\
&\quad = \alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1} \int_{jT}^{\infty} f_X(x)dx \\
&\quad = \alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1}(1-F_X(jT))
\end{aligned}$$

By replacing the sixth and seventh term from Equation D.11 by the above, the final form for the expected cycle length is obtained as depicted in Equation D.12.

$$\begin{aligned}
E_1(\textit{cycle length}) = &\sum_{i=1}^{M}(1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \int_{0}^{iT-x} (x+h) f_H(h) f_X(x) dh dx \\
&+ \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} (1-\alpha)^{i-1} \int_{(i-1)T}^{iT} \int_{jT-x}^{(j+1)T-x} \beta^{j-i+1}(x+h) f_H(h) f_X(x) dh dx \\
&+ \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} (1-\alpha)^{i-1} T(1-\beta) \int_{(i-1)T}^{iT} j\beta^{j-i}(1-F_H(jT-x)) f_X(x) dx \\
&+ \sum_{i=1}^{M}(1-\alpha)^{i-1} MT\beta^{M-i} \int_{(i-1)T}^{iT} (1-F_H(MT-x)) f_X(x) dx \\
&+ (1-\alpha)^{M-1} MT(1-F_X(MT)) \\
&+ \alpha T \sum_{j=1}^{M-1} j(1-\alpha)^{j-1}(1-F_X(jT))
\end{aligned} \tag{D.12}$$

D.1.2 Deriving the reliability evaluation

Before going into detail on the derivation of the reliability evaluation under the A1 model, let us first define the when the asset is considered reliable. The asset is considered reliable when it has not failed, i.e. the asset is either operating normally or it is defective.

Another point highly important to clearly underline, is the notation of the reliability expression. A standard reliability function $R(t)$ denotes the probability of the asset surviving up to its argument, given time t. Since we include inspections and maintenance actions, we extend the notation of the reliability expression by including a subscript MT indicating the maintenance schedule of the asset. In addition, we add an index w denoting the model considered, e.g. $w = 1$ corresponds to the model with the first model approach, see Section 3.3. The final aspect of the reliability expression for the inspected and maintained asset is the timespan over which the reliability is considered. We consider the time span of the asset to be characterised by the given variable t by $(0, t)$. The asset's reliability expression now corresponds to $R_{MT,w}(t)$. The reliability expression is non conditional and therefore non increasing by definition.

In the derivation procedure for the reliability expression of the A1 model for any given t, we will start by considering the reliability expression for the case in which $0 \leq t < MT$, i.e. the reliability expression for time instances before the first preventive asset replacement. This derivation is presented in Subsection D.1.2. The expression obtained from Subsection D.1.2 is then used to derive the general expression for the reliability for any given value of t under the A1 model, see Subsection D.1.2.

Deriving $R_{MT,1}(t)$, $0 \leq t < MT$

Due to the inclusion of inspections and maintenance actions, the asset can be maintained before time MT which alters the reliability function for each interval between inspections. Therefore the reliability function is expressed per interval. Such an interval is bounded by the inspection s under consideration by $((s-1)T, sT)$ implying the time variable to be bounded by $(s-1)T \leq t < sT$. Because we consider preventive asset replacement at time MT, variable s can only take values from 1 to M. To denote the reliability expression for the interval $((s-1)T, sT)$, variable s is added as a superscript to the reliability expression yielding the notation for the asset's reliability to be denoted as $R^{(s)}_{MT,w}(t)$, which is based on the notation used by Berrade et al. (2013).

Before deriving the general expression for the reliability function, the conceptual derivation idea is explained in Figure D.3. The preventive asset replacement instance M is set to 2. This implies that we can consider inspections $s = 1, 2$.

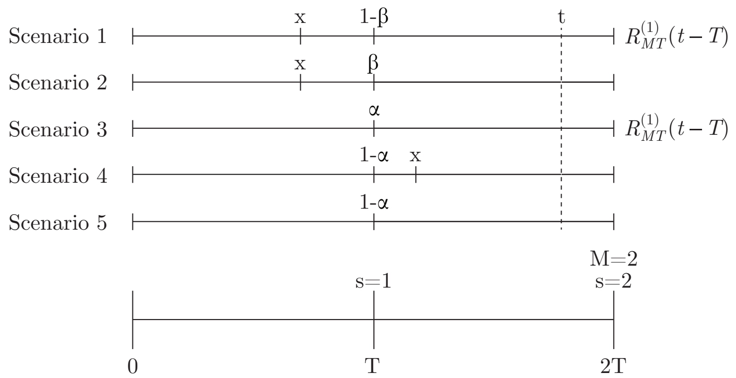


Figure D.3: Various scenarios of the DTM under imperfect inspections

Figure D.3 distinguishes five scenarios which may occur in case of imperfect inspections for the second interval bounded by inspection $s = 2$, i.e. $(T \leq t < 2T)$. Note that the finite horizon of $M = 2$ is considered, i.e. at the second inspection the asset is preventively replaced. The first scenario represents the case in which the asset becomes defective in the first interval $(0, T)$ at time x in Figure D.3. The asset then survives to the inspection at time T, i.e. its delay time exceeds the time from x to inspection time T. Upon inspection the defect is found with a probability of $1 - \beta$ and the asset is maintained. As the asset is perfectly maintained, the asset is restored to an 'as-new' condition. This implies that the maintained asset at time $T \leq t < 2T$ is independent on the wear until time T. Hence, the reliability of this first scenario then corresponds to the product of the asset's reliability up to the first inspection T, and the asset's reliability for the remaining time $R^{(1)}_{MT,1}(t-T)$. Note that the recurrent reliability expression $R^{(1)}_{MT,1}(t-T)$ includes the number of intervals remaining for the reliability, i.e. (1) in case $T \leq t < 2T$ and the defect is found upon T.

The second case represents the asset still becoming defective in the first interval, $(0, T)$ at time x, and surviving to the inspection. At the inspection, however, the defect is not detected with a probability β. For the reliability function, the asset still has to survive in its defective state until time t, i.e. its delay time has to exceed $t - x$.

The third scenario corresponds to the asset undergoing unnecessary maintenance, i.e. a false positive occurs. The asset namely has not yet become defective but upon inspection the maintenance staff still judges the asset as being defective with probability α and correspondingly has the asset maintained. Analogous to the first scenario, the asset is restored to the 'as-new' condition and therefore the recurrent reliability expression for the remaining time $t - T$ and the remaining interval 1 is included.

The fourth term denotes the fact that the considered asset becomes defective after the last inspection, $s-1$, and before time t. In Figure D.3 this time to defect is denoted by x. Upon the first inspection the asset was still free of any defects and has been judged as such with a probability of $1-\alpha$. Furthermore, the asset does not break down until time t, i.e. its delay time is longer than $t-x$.

Finally, the asset may still remain in its normal state at time t. This implies that the asset has been in its normal state at the first inspection at time T and has been considered as being operating normally with a probability of $1-\alpha$.

All scenarios combined yield the reliability expression, as depicted below, of an asset when considering $s=2$. As a small remark, the order of scenarios from Figure D.3 corresponds to the order of the terms in the equation below for $s=2$.
If $T \leq t < 2T$

$$\begin{aligned} R^{(2)}_{MT,1}(t) = &\int_0^T (1-\beta)\left(1-F_H(T-x)\right) R^{(1)}_{MT,1}(t-T) f_X(x)dx + \beta \int_0^T \left(1-F_H(t-x)\right) f_X(x)dx \\ &+ \alpha\left(1-F_X(T)\right) R^{(1)}_{MT,1}(t-T) + (1-\alpha)\int_T^t \left(1-F_H(t-x)\right) f_X(x)dx + (1-\alpha)\left(1-F_X(t)\right) \end{aligned} \tag{D.13}$$

This derivation procedure can be followed for any value of s. In case $s=1$, a remark should be made, because in this case no inspection is done, i.e. $0 \leq x < T$. The time variable t namely never equals the actual inspection time T. For $s=1$ only the fourth and fifth scenario may occur without the inclusion of the α probability.
If $0 \leq t < T$

$$R^{(1)}_{MT,1}(t) = \int_0^t \left(1-F_H(t-x)\right) f_X(x)dx + \left(1-F_X(t)\right) \tag{D.14}$$

By the same reasoning as for the $s=2$ case, the reliability evaluation function for the third inspection interval $s=3$ can be derived. In deriving this equation one should pay attention to the fact that imperfect inspections can occur multiple times, i.e. the maintenance staff can falsely judge the asset at the first and second inspection.
If $2T \leq t < 3T$

$$\begin{aligned} R^{(3)}_{MT,1}(t) = &\int_0^T \left(1-F_H(T-x)\right)(1-\beta) R^{(2)}_{MT,1}(t-T) f_X(x)dx \\ &+ \int_0^T \left(1-F_H(2T-x)\right)\beta(1-\beta) R^{(1)}_{MT,1}(t-2T) f_X(x)dx \\ &+ \int_0^T \left(1-F_H(t-x)\right)\beta^2 f_X(x)dx \\ &+ (1-\alpha)\int_T^{2T} \left(1-F_H(2T-x)\right)(1-\beta) R^{(1)}_{MT,1}(t-2T) f_X(x)dx \\ &+ (1-\alpha)\int_T^{2T} \left(1-F_H(t-x)\right)\beta f_X(x)dx + (1-\alpha)^2 \int_{2T}^t \left(1-F_H(t-x)\right) f_X(x)dx \\ &+ (1-\alpha)^2\left(1-F_X(t)\right) + \alpha\left(1-F_X(T)\right) R^{(2)}_{MT,1}(t-T) \\ &+ (1-\alpha)\alpha\left(1-F_X(2T)\right) R^{(1)}_{MT,1}(t-2T) \end{aligned} \tag{D.15}$$

The number of terms is increasing with increasing values of s. This is due to the fact that more occasions for defects, false positive event and false negatives events are introduced.

The three specific cases $s=1,2,3$ have presented an intuitive reliability expression for the asset under constant probabilities of false positives and false negatives. To derive a general reliability expression for $(s-1)T \leq t < sT$, we consider the five possible scenarios that may occur. Each denotes a probability of the asset being reliable up to time t. The enumeration below extensively presents each of these scenarios which finally result in the reliability function $R^{(s)}_{MT,1}(t)$, when $(s-1)T \leq t < sT$, by summing all five scenarios.

- **Scenario 1:** The first probability scenario denotes the probability of the asset becoming defective in any of the inspection intervals preceding time t, i.e. the intervals characterised by inspections 1 to $s-1$. Let us first consider the case in which the asset's time to defect is given and lies in a specific interval. Such interval $((i-1)T, iT)$ is characterised by the inspection i, where $i = 1, ..., s-1$. In all inspections that precede the asset's time to defect, no false positives have occurred, i.e. no false may have occurred from inspection 1 to $i-1$ corresponding a probability of $(1-\alpha)^{i-1}$. The defect is then found upon inspection j with probability $1-\beta$, where $j = i, ..., s-1$. This leaves $j-i$ false negatives to occur with probability β at each inspection denoting a total probability of β^{j-i}. Furthermore, given the asset becoming defective at time x, in $((i-1)T, iT)$, it still has to survive in its defective state until the defect is detected at inspection j which is performed at time jT. This results in the time the asset has to survive defectively of $jT-x$ and subsequently relates to a probability of the asset's delay time h exceeding this $jT-x$ denoted by $P(h \geq jT-x) = 1 - P(0 \leq x < jT-x)$.

  When the asset is maintained upon inspection j, its condition is restored to the 'as-new' condition. Therefore, the asset is independent at time $t \geq jT$ on the wear history before time jT. This implies that the reliability at time jT can be multiplied by remaining reliability (Lewis, 1987). This remaining reliability considers the time $t-jT$ over $s-j$ intervals. This yields the inclusion of the recurrent reliability term of $R^{(s-j)}_{MT,1}(t-jT)$.

  Under the condition that the time to defect is given and $(i-1)T \leq x < iT$, the reliability expression's contribution for the first scenario is obtained by multiplying the terms derived above. Note that we consider specific inspections i and j.

$$
\begin{aligned}
&(1-\alpha)^{i-1}(1-\beta)\beta^{j-1}P(h \geq jT-x)R^{(s-j)}_{MT,1}(t-jT) \\
&\quad = (1-\alpha)^{i-1}(1-\beta)\beta^{j-1}(1-P(0 \leq h < jT-x))R^{(s-j)}_{MT,1}(t-jT)
\end{aligned}
$$

  Now we introduce the random nature of the time to defect, which has been assumed given up to here. By multiplying the expression above by the probability that the time to defect actually meets $(i-1)T \leq x < iT$, the unconditioned reliability expression for the first scenario under specific i and j is obtained. The probability of $(i-1)T \leq x < iT$ is then denoted by $P((i-1)T \leq x < iT)$ and subsequently can be denoted as the integral over the time to defect's pdf from $(i-1)T$ to iT. Additionally, all terms which including the variable x are included in the integral expression as well.

$$
\begin{aligned}
&(1-\alpha)^{i-1}(1-\beta)\beta^{j-1}P((i-1)T \leq x < iT)(1-P(0 \leq h < jT-x))R^{(s-j)}_{MT,1}(t-jT) \\
&\quad = (1-\alpha)^{i-1}(1-\beta)\beta^{j-i}\int_{(i-1)T}^{iT}(1-F_H(jT-x))f_X(x)dx\, R^{(s-j)}_{MT,1}(t-jT)
\end{aligned}
$$

  In the derivation procedure up to here, specific inspections i and and j have been assumed. However, the asset can become defective in any interval characterised by i implying a summation over all i. Moreover, the defect can be detected at any inspection j also implying a summation over all possible j. By including both summations, the final expression for the first probability scenario is obtained as depicted below.

$$\sum_{i=1}^{s-1}\sum_{j=i}^{s-1}(1-\alpha)^{i-1}(1-\beta)\beta^{j-i}\int_{(i-1)T}^{iT}(1-F_H(jT-x))f_X(x)dx\, R_{MT,1}^{(s-j)}(t-jT)$$

- **Scenario 2:** The second scenario denotes the probability that the asset becomes defective in any of the intervals $((i-1)T, iT)$ preceding t, where $i = 1, ..., s-1$. Analogous to scenario 1, let us consider a given time to defect x which lies in the interval $((i-1)T, iT)$, characterised by inspection i. No false positives occur before defect arrival, i.e. no false positive upon inspections $1, ..., i-1$ yielding a probability of $(1-\alpha)^{i-1}$. The defect remains undiscovered up to time t. This implies false negatives occurring from inspection i to inspection $s-1$. Recall $(s-1)T \leq t < sT$. In this case the number of false negatives occurring equals to $s-i$ and subsequently has a probability of β^{s-i}. When the defect is not found the asset still has to survive to time t to be considered reliable. The time the asset has to survive in its defective state is then represented by $t-x$. The probability that the asset then remains reliable, equals the probability that the asset's delay time h exceeds the remaining time the asset has to survive defectively $t-x$, i.e. $P(h \geq t-x) = 1 - P(0 \leq h < t-x)$. Because all of the conditions have to be met, the following mathematical relation denotes the second scenario under the given $((i-1)T \leq x < iT)$, for a specific interval characterised by inspection i.

$$\begin{aligned}&(1-\alpha)^{i-1}\beta^{s-i}P(h \geq t-x)\\ &\quad = (1-\alpha)^{i-1}\beta^{s-i}(1-P(0 \leq h < t-x))\end{aligned}$$

The term $1 - P(0 \leq h < t-x)$ is written in terms of the delay time's cdf with argument $t-x$ yielding:

$$\begin{aligned}&(1-\alpha)^{i-1}\beta^{s-i}(1-P(0 \leq h < t-x))\\ &\quad = (1-\alpha)^{i-1}\beta^{s-i}(1-F_H(t-x))\end{aligned}$$

Since we started with the conditional expression for the second reliability scenario under a specific i, let us now introduce the probabilistic element of the time to defect. The condition $((i-1)T \leq x < iT)$ is met with a probability $P((i-1)T \leq x < iT)$, which can be rewritten to the integral expression. This integral expression corresponds to the integral of the time to defect's pdf from $(i-1)T$ to iT.

$$\begin{aligned}&(1-\alpha)^{i-1}\beta^{s-i}P((i-1)T \leq x < iT)(1-F_H(t-x))\\ &\quad = (1-\alpha)^{i-1}\beta^{s-i}\int_{(i-1)T}^{iT}(1-F_H(t-x))f_X(x)dx\end{aligned}$$

Under the same premise as the first scenario, this case has merely considered the expression for a given value of i. However, the asset may fail before any inspection i yielding a summation over all i for the complete expression of the second scenario.

$$\sum_{i=1}^{s-1}(1-\alpha)^{i-1}\beta^{s-i}\int_{(i-1)T}^{iT}(1-F_H(t-x))f_X(x)dx$$

- **Scenario 3:** Scenario three includes the probabilities that the asset is maintained too early, i.e. a false positive, in any interval before time t. The interval considered is characterised by inspection j, where $j = 1, ..., s-1$, by $((j-1)T, jT)$. Note that upon inspection $s-1$ a false positive may also occur. Again, consider a specific inspection j upon which the false positive occurs with probability α. This implies that $j-1$ times no false positives have occurred which corresponds to a probability of $(1-\alpha)^{j-1}$. For a false positive to occur, the asset must be free of defects implying that its time to defect has to exceed the time upon which inspection j is performed, relating to a probability of $P(x \geq jT)$. Note that this probability can be rewritten as the non-negative probability conceptualisation $P(0 \leq x < jT)$.

  The final aspect included here is the fact that when a false positive occurs the asset is maintained and restored to the 'as new' condition. Analogous to the first scenario this corresponds to the recursive reliability function. The number of intervals over which this reliability function is taken is then characterised by $s-j$, i.e. from the inspection of the false positive j to the inspection which bounds the time t. Similarly the remaining time is then conceptualised as the time t minus the time upon which the asset is maintained jT. This yields a recurrent reliability function of $R_{MT,1}^{s-j}(t-jT)$. In meeting all four probability conditions the following relation is obtained.

$$\begin{aligned}&(1-\alpha)^{j-1}\alpha P(x \geq jT) R_{MT,1}^{s-j}(t-jT)\\&\quad = (1-\alpha)^{j-1}\alpha(1-P(0 \leq x < jT)) R_{MT,1}^{s-j}(t-jT)\end{aligned}$$

  In rewriting the abstract probabilities one has to consider the probability $1-P(0 \leq x < jT)$ which can be rewritten to the time to defect's cdf with argument jT yielding:

$$\begin{aligned}&(1-\alpha)^{j-1}\alpha(1-P(0 \leq x < jT)) R_{MT,1}^{s-j}(t-jT)\\&\quad = (1-\alpha)^{j-1}\alpha(1-F_X(jT)) R_{MT,1}^{s-j}(t-jT)\end{aligned}$$

  Because only one specific inspection of the false positive j is considered and we want to consider the false positive to occur upon any inspection preceding time t, one has to sum the expression above to obtain the expression for scenario three.

$$\sum_{j=1}^{s-1}(1-\alpha)^{j-1}\alpha(1-F_X(jT)) R_{MT,1}^{s-j}(t-jT)$$

- **Scenario 4:** In the fourth probability scenario, let us start by considering the asset's time to defect x to be given and satisfying $(s-1)T \leq x < t)$. To include this scenario in the reliability function the asset will have to survive up to time t in its defective state. Given that the asset becomes defective upon x and it has to survive to time t, it's delay time h will have to exceed $t-x$. This relates to a probability of $P(h \geq t-x)$, which equals $1-P(0 \leq h < t-x)$. Moreover, this probability expression can be expressed in terms of the delay time's cdf by $1-F_H(t-x)$. In addition no false positives have occurred upon all inspections preceding time t with probability $(1-\alpha)^{s-1}$. When meeting all three conditions the probability expression for this scenario becomes:

$$\begin{aligned}&(1-\alpha)^{s-1}P(h \geq t-x)\\&\quad = (1-\alpha)^{s-1}(1-P(0 \leq h < t-x))\\&\quad = (1-\alpha)^{s-1}(1-F_H(t-x))\end{aligned}$$

Analogous to the first and second scenario from this reliability analysis, we started by considering a given time to defect x, which lies between particular time instances. However, the time to defect is a random variable and therefore its value lies in between certain time epochs with a probability. For this fourth scenario we now include the probability of the condition, which equals to the probability $P((s-1)T \leq x < t)$. By multiplying the expression above, which is conditioned, by the probability of the condition, the unconditioned reliability for the fourth scenario is obtained.

$$\begin{aligned}&(1-\alpha)^{s-1}P((s-1)T \leq x < t)(1-F_H(t-x))\\&= (1-\alpha)^{s-1}\int_{(s-1)T}^{t}(1-F_H(t-x))f_X(x)dx\end{aligned}$$

Note that the expression $P((s-1)T \leq x < t)$ is already rewritten to its corresponding integral of the time to defect's pdf from $(s-1)T$ to time t. Additionally, all the terms which include the variable x are integrated over as well due to their dependence on the time to defect.

- **Scenario 5:** The final probability corresponds to the case in which no false positives have occurred at all inspections done, i.e. from inspections $1, ..., s-1$ no false positives have occurred. This denotes the probability $(1-\alpha)^{s-1}$. Furthermore, the asset will still remain in its normal state at time t with probability $P(x \geq t)$. Note that the difference with the fourth scenario is rather subtle, but in the fourth the asset becomes defective in the interval $((s-1)T, t)$, whereas the asset remains free of defects in this scenario until time t. The probability of this fifth scenario is then denoted by:

$$\begin{aligned}&(1-\alpha)^{s-1}P(x \geq t)\\&= (1-\alpha)^{s-1}(1-P(0 \leq x < t))\end{aligned}$$

In the exact same manner as in the previous probability events, the probability of $P(x > t)$ is rewritten to $1 - P(0 \leq x < t)$ which is subsequently rewritten the cdf of the time to defect with argument t resulting in the final form for the fifth scenario:

$$\begin{aligned}&(1-\alpha)^{s-1}(1-P(0 \leq x < t))\\&= (1-\alpha)^{s-1}(1-F_X(t))\end{aligned}$$

Each of the scenarios denote the the probability of the asset being reliable on time t. The reliability function then in turn corresponds to the probability of the asset being reliable at time t and therefore includes all scenarios. This implies that summing all probability scenarios yields the final expression for the general reliability function which is presented in Equation D.16.

$$
\begin{aligned}
R_{MT,1}^{(s)}(t) = & \sum_{i=1}^{s-1}\sum_{j=i}^{s-1}(1-\alpha)^{i-1}(1-\beta)\beta^{j-i}\int_{(i-1)T}^{iT}(1-F_H(jT-x))f_X(x)dx\, R_{MT,1}^{(s-j)}(t-jT) \\
& + \sum_{i=1}^{s-1}(1-\alpha)^{i-1}\beta^{s-i}\int_{(i-1)T}^{iT}(1-F_H(t-x))\, f_X(x)dx \\
& + \sum_{j=1}^{s-1}(1-\alpha)^{j-1}\alpha\,(1-F_X(jT))\, R_{MT,1}^{s-j}(t-jT) \\
& + (1-\alpha)^{s-1}\int_{(s-1)T}^{t}(1-F_H(t-x))\, f_X(x)dx \\
& + (1-\alpha)^{s-1}\,(1-F_X(t))
\end{aligned} \tag{D.16}
$$

where $(s-1)T \leq t < sT$

Note that a summation of which the upper bound is less than he lower bound yields a value of 0 because it contains no integer values which can be summed (Koltun, 2008).

The reliability expression when the time argument t lies between 0 and MT is then comprised of all the different reliability expressions for s. This implies the following relationship for $0 \leq t < MT$.

$$
R_{MT,1}(t) = R_{MT,1}^{(s)}(t), \quad (s-1)T \leq t < sT, \quad s = 1, \ldots, M \tag{D.17}
$$

Deriving $R_{MT,1}(t)$

To derive a reliability expression for any value for t, we use the reliability expression for $0 \leq t < MT$ from Subsection D.1.2. Since the asset is preventively replaced at time MT a recurrent pattern in the reliability expression for any t is obtained.

Let us start by considering the reliability when $MT \leq t < 2MT$. Upon preventive asset replacement at time MT, a new asset is introduced which has a 'new' condition. This new asset is not dependent on the asset which has been replaced at MT and therefore the total reliability at time $MT \leq t < 2MT$ is independent on the reliability before time MT. Due to this property the complete reliability expression at time $MT \leq t < 2MT$ equals to the product of the probability that the asset has survived to time MT, and the probability that the new asset will survive from time MT to time t, $R_{MT,1}(t - MT)$. In mathematical terms:

$$
R_{MT,1}(t) = R_{MT,1}(MT)R_{MT,1}(t-MT), \quad MT \leq t < 2MT
$$

The latter term $R_{MT,1}(t - MT)$ can be rewritten by using the expression from Equation D.17. This yields the expression for the reliability when $MT \leq t < 2MT$:

$$
\begin{aligned}
R_{MT,1}(t) = R_{MT,1}(MT)R_{MT,1}^{(s)}(t-MT), \quad & (s-1)T \leq t - MT < sT, \\
& s = 1, \ldots, M \\
R_{MT,1}(t) = R_{MT,1}(MT)R_{MT,1}^{(s)}(t-MT), \quad & (M+s-1)T \leq t < (M+s)T, \\
& s = 1, \ldots, M
\end{aligned}
$$

When we consider $2MT \leq t < 3MT$, the same technique can be applied. In this case, the asset is preventively replaced by a new one at times T and $2T$. The new asset now becomes independent on the reliability before time $2MT$. This yields the multiplication of the reliability at time $2MT$ by the reliability for the remaining time $t - 2MT$.

$$R_{MT,1}(t) = R_{MT,1}(2MT)R_{MT,1}(t-2MT), \quad 2MT \leq t < 3MT$$

Since the asset has been replaced at time MT and time $2MT$, one is able to write the reliability at time $2MT$ as the reliability at time MT squared. Recall that upon asset replacement, the asset's reliability is independent on the reliability before the replacement instance.

$$R_{MT,1}(t) = R_{MT,1}(MT)^2 R_{MT,1}(t-2MT), \quad 2MT \leq t < 3MT \tag{D.18}$$

By inserting the expression from Equation D.17 into Equation D.18, Equation D.19 is obtained.

$$R_{MT,1}(t) = R_{MT,1}(MT)^2 R_{MT,1}(t-2MT), \quad \begin{array}{l}(2M+s-1)T \leq t < (2M+s)T, \\ s = 1,\dots,M\end{array} \tag{D.19}$$

This procedure of deriving the reliability expression for every value for t then subsequently yields the general reliability expression for any t. This relation is depicted in Equation D.20.

$$R_{MT,1}(t) = R_{MT,1}^{(aMT+s)}(t) = R_{MT,1}^{(M)}(MT)^a R_{MT,1}^{(s)}(t-aMT), \quad \begin{array}{l}(aMT+s-1)T \leq t < (aMT+s)T, \\ s = 1,\dots,M, \\ a = 0,1,\dots,\end{array} \tag{D.20}$$

D.2 A2 model

This section presents a detailed derivation for the cost model and the reliability model under the A2 model. The cost model consists of the identical terms as the A1 model. Now we consider a different model, i.e. w changes to 2 yielding:

$$C_2(M,T) = \frac{E_2(\textit{cycle costs})}{E_2(\textit{cycle length})}$$

$$E_2(\textit{cycle costs}) = c_0 E_2(K) + c_p P_2(c_p) + c_f(1 - P_2(c_p))$$

This leaves three separate terms to be evaluated: $E_2(K)$, $P_2(c_p)$ $E_2(\textit{cycle length})$. Each of these terms are individually derived in Section D.2.1. The section concludes with the detailed derivation of the reliability model in Section D.2.2.

Because the terms of both the cost and reliability evaluation model have been extensively discussed in Appendix D.1, the terms will not be elaborated upon here. This section in the Appendix assumes the reader to first have read Appendix D.1. The cost and reliability evaluation will therefore focus on rewriting the expressions in such forms that they can be easily altered to yield the A2 model's cost and reliability expressions.

D.2.1 Deriving the cost evaluation

Analogous to the A1 model's detailed cost evaluation, this cost evaluation considers the three cost elements separately, i.e. the expected number of inspections per cycle $E_2(K)$, the probability of incurring maintenance action costs $P_2(c_p)$ and the expected cycle length $E_2(\textit{cycle length})$.

Deriving $E_2(K)$

The derivation of the expected number of inspections per cycle under the A2 model $E_2(K)$ is done in a two step procedure. We start by rewriting the A1 model, which enables us to alter the model more easily to yield the A2 model.

First thing done is to rewrite the cdf of the delay time of the first term of $E_1(K)$ to its corresponding integral expression. This implies integrating the delay time's pdf over the cdf's argument $jT - x$ to infinity ∞. Because the probabilities α and β become dependent on other variables these may not be constant anymore implying that one cannot raise them to powers. In tackling this issue the power raised α and β from Equation 4.2 are rewritten to their product expressions, e.g. α^{i-1} equals $\prod_{n=1}^{i-1} \alpha$. Equation D.21 is obtained after these minor alterations.

$$\begin{aligned}
E_1(K) = \sum_{j=1}^{M-1} j \Bigg\{ & \sum_{i=1}^{j} \left(\prod_{n=1}^{i-1} (1-\alpha) \right) \int_{(i-1)T}^{iT} \int_{jT-x}^{\infty} \left(\prod_{k=i}^{j-1} \beta \right) (1-\beta) f_H(h) f_X(x) dh dx \\
& + \sum_{i=1}^{j} \left(\prod_{n=1}^{i-1} (1-\alpha) \right) \int_{(i-1)T}^{iT} \int_{jT-x}^{(j+1)T-x} \left(\prod_{k=i}^{j} \beta \right) f_H(h) dh f_X(x) dx \\
& + \left(\prod_{n=1}^{j} (1-\alpha) \right) \int_{jT}^{(j+1)T} F_H((j+1)T - x) f_X(x) dh dx \\
& + \left(\prod_{n=1}^{j-1} (1-\alpha) \right) \alpha (1 - F_X(jT)) \Bigg\} \\
& + M \Bigg\{ \sum_{i=1}^{M} \left(\prod_{n=1}^{j-1} (1-\alpha) \right) \int_{(i-1)T}^{iT} \left(\prod_{k=i}^{M-1} \beta \right) (1 - F_H(MT - x)) f_X(x) dx \\
& + \left(\prod_{n=1}^{M-1} (1-\alpha) \right) (1 - F_X(MT)) \Bigg\}
\end{aligned} \tag{D.21}$$

After rewriting the A1 model to the one including the product signs instead of powers and pdf integrals instead of the cdf expressions, the model can actually be altered. We will start with the alteration of the non-constant probability of false positives. Let us first consider the occurrence of a false positive. The false positive can only occur once per cycle, upon inspection j yielding the rewriting of the probability of a false positive upon inspection j to $\alpha\left(\frac{jT}{x}\right)$. In addition to a false positive occurring, many scenarios describe no false positives occurring by the inclusion of the product for false positives, which is obtained after rewriting the A1 model. To include the defect progress dependency for these cases, the product sign's index is used, which corresponds to the inspections that have preceded the asset's defect arrival. When these inspections are denoted by the product sign's index n, the probability of a false positive not occurring upon inspection n equals to $1 - \alpha\left(\frac{nT}{x}\right)$. By rewriting the constant expression to the non-constant expression in the product sign, the non-constant probability for false positives not occurring is obtained. Note that the expressions for both, the occurrence of a false positive and the non-occurrence of false positives, include the realisation x of the random variable X, implying the inclusion of the terms in the integral over the time to defect.

The same logic applies for the false negatives, but vice versa. A false negative, namely, may occur multiple times per cycle, whereas a false positive can only occur once per cycle. The non-occurrence of a false negative, however, may only occur once per cycle because this denotes the ending of a cycle, i.e. the defect which has occurred is found and the cycle ends. By the same logic as applied to the false positives, the non-occurrence of a false negative can only happen upon inspection j yielding the non-constant probability of $1 - \beta\left(\frac{jT-x}{h}\right)$. In an analogous fashion as the non occurring false positives, the probability of false negatives occurring includes the product index variable from Equation D.21, which is denoted by k. This variable k denotes the inspection k upon which a false negative occurs with corresponding probability of $\beta\left(\frac{kT-x}{h}\right)$. Moreover, note that due to the inclusion of the variables h and x the probability of false negatives is included in the integrals of both the time to defect and the delay time.

When writing the derived non-constant probability expressions for the false positives and false negatives instead of the constant probabilities, the A2 model is obtained, see Equation D.22.

$$
\begin{aligned}
E_2(K) = \sum_{j=1}^{M-1} j \Bigg\{ & \sum_{i=1}^{j} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha\left(\frac{nT}{x}\right)\right)\right) \int_{jT-x}^{\infty} \left(\prod_{k=i}^{j-1} \beta\left(\frac{kT-x}{h}\right)\right) \\
& \cdot \left(1 - \beta\left(\frac{jT-x}{h}\right)\right) f_H(h) dh f_X(x) dx \\
& + \sum_{i=1}^{j} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha\left(\frac{nT}{x}\right)\right)\right) \int_{jT-x}^{(j+1)T-x} \left(\prod_{k=i}^{j} \beta\left(\frac{kT-x}{h}\right)\right) f_H(h) dh f_X(x) dx \\
& + \int_{jT}^{(j+1)T} \left(\prod_{n=1}^{j} \left(1 - \alpha\left(\frac{nT}{x}\right)\right)\right) F_H((j+1)T - x) f_X(x) dx \\
& + \int_{jT}^{\infty} \left(\prod_{n=1}^{j-1} \left(1 - \alpha\left(\frac{nT}{x}\right)\right)\right) \alpha\left(\frac{jT}{x}\right) f_X(x) dx \Bigg\} \\
& + M \Bigg\{ \sum_{i=1}^{M} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha\left(\frac{nT}{x}\right)\right)\right) \int_{MT-x}^{\infty} \left(\prod_{k=i}^{M-1} \beta\left(\frac{kT-x}{h}\right)\right) f_H(h) dh f_X(x) dx \\
& + \int_{MT}^{\infty} \left(\prod_{n=1}^{M-1} \left(1 - \alpha\left(\frac{nT}{x}\right)\right)\right) f_X(x) dx \Bigg\}
\end{aligned}
\tag{D.22}
$$

Deriving $P_2(c_p)$

Analogous to the derivation of the expected number of inspections per cycle under the A2 model $E_2(K)$, the probability of incurring maintenance action costs c_p under the A2 model $P_2(c_p)$ is derived by rewriting the A1 model's power functions to the product expressions and the delay time's cdf integral to its pdf integral expression. Note that this still equals the A1 model under constant probabilities of false positives and false negatives. Based on this rewritten A1 model, the non-constant probability expressions for false positives and false negatives included to yield the probability of incurring maintenance action costs c_p under the A2 model $P_2(c_p)$.

By rewriting Equation D.4 to the expression including the product signs instead of power raised α and β and expressing the delay time's cdf in its integral expression, Equation D.23 is obtained.

$$
\begin{aligned}
P_1(c_p) = & \sum_{i=1}^{M-1} \sum_{j=1}^{i} \left(\prod_{n=1}^{i-1} (1-\alpha) \right) \int_{(i-1)T}^{iT} \int_{jT-x}^{\infty} \left(\prod_{k=i}^{j-1} \beta \right) (1-\beta) f_H(h) f_X(x) dh dx \\
& + \sum_{j=1}^{M-1} \alpha \left(\prod_{n=1}^{j-1} (1-\alpha) \right) (1 - F_X(jT)) \\
& + \sum_{i=1}^{M} \left(\prod_{n=1}^{i-1} (1-\alpha) \right) \int_{(i-1)T}^{iT} \int_{MT-x}^{\infty} \left(\prod_{k=i}^{M-1} \beta \right) f_H(h) f_X(x) dh dx \\
& + \left(\prod_{n=1}^{M-1} (1-\alpha) \right) (1 - F_X(MT))
\end{aligned}
\tag{D.23}
$$

In the exact same fashion as the derivation procedure for $E_2(K)$, rewriting the constant probabilities of false positives and false negatives to the non-constant ones in the product signs and integrating over the proper variables, final expression for $P_2(c_p)$ is obtained, see Equation D.24. For an elaboration on the expressions for the non-constant probabilities of false positives and false negatives, see the Subsection D.2.1.

$$
\begin{aligned}
P_2(c_p) = & \sum_{i=1}^{M-1} \sum_{j=i}^{M-1} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{jT-x}^{\infty} \left(\prod_{k=i}^{j-1} \beta \left(\frac{kT-x}{h} \right) \right) \\
& \cdot \left(1 - \beta \left(\frac{jT-x}{h} \right) \right) f_H(h) f_X(x) dh dx \\
& + \sum_{j=1}^{M-1} \int_{jT}^{\infty} \alpha \left(\frac{jT}{x} \right) \left(\prod_{n=1}^{j-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) f_X(x) dx \\
& + \sum_{i=1}^{M} \int_{(i-1)T}^{iT} \left(\prod_{n=1}^{i-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) \int_{MT-x}^{\infty} \left(\prod_{k=i}^{M-1} \beta \left(\frac{kT-x}{h} \right) \right) f_H(h) f_X(x) dh dx \\
& + \int_{MT}^{\infty} \left(\prod_{n=1}^{M-1} \left(1 - \alpha \left(\frac{nT}{x} \right) \right) \right) f_X(x) dx
\end{aligned}
\tag{D.24}
$$

Deriving $E_2(\textit{cycle length})$

To derive the expression $E_2(\textit{cycle length})$, first thing to do is to rewrite Equation 4.4 to an expression in which the power raised terms are written in their product expressions, e.g. $(1-\alpha)^{i-1} = \prod_{n=1}^{i-1}(1-\alpha)$. Secondly, the cumulative distribution function of the delay time h is written to its equivalent integral expression, which integrates the delay time's pdf. In applying notational alterations Equation D.25 is obtained, which is a rewritten form of the expected cycle length under the A1 model.

$$
\begin{aligned}
E_1(\textit{cycle length}) = &\sum_{i=1}^{M}\left(\prod_{n=1}^{i-1}(1-\alpha)\right)\int_{(i-1)T}^{iT}\int_{0}^{iT-x}(x+h)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}\left(\prod_{n=1}^{i-1}(1-\alpha)\right)\int_{(i-1)T}^{iT}\int_{jT-x}^{(j+1)T-x}\left(\prod_{k=i}^{j}\beta\right)(x+h)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}\left(\prod_{n=1}^{i-1}(1-\alpha)\right)T(1-\beta)\int_{(i-1)T}^{iT}\int_{jT-x}^{\infty}j\left(\prod_{k=i}^{j-1}\beta\right)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M}\left(\prod_{n=1}^{i-1}(1-\alpha)\right)MT\left(\prod_{k=i}^{M-1}\beta\right)\int_{(i-1)T}^{iT}\int_{MT-x}^{\infty}f_H(h)f_X(x)dhdx \\
&+\left(\prod_{n=1}^{M-1}(1-\alpha)\right)MT\left(1-F_X(MT)\right) \\
&+\alpha T\sum_{j=1}^{M-1}j\left(\prod_{n=1}^{j-1}(1-\alpha)\right)\left(1-F_X(jT)\right)
\end{aligned}
\tag{D.25}
$$

The non-constant expressions for the probabilities of false positives and false negatives can now be used and combined with the model from Equation D.25. The expected cycle length under the A2 model is then obtained. For the non-constant expressions for the probability of false positives and false negatives, the reader is referred to Subsection D.2.1. When rewriting the probabilities of Equation D.25 to their non-constant expressions, note that the probability of false positives includes the variable x implying that it has to be integrated over x. Analogously, the non-constant probability of a false negatives includes both variables x and h and is therefore integrated over h and x. This yields the final expression for the expected cycle length under the A2 model $E_2(\textit{cycle length})$, see Equation D.26.

$$
\begin{aligned}
E_2(\textit{cycle length}) = &\sum_{i=1}^{M}\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{0}^{iT-x}(x+h)f_H(h)f_X(x)dhdx \\
&+\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{jT-x}^{(j+1)T-x}\left(\prod_{k=i}^{j}\beta\left(\frac{kT-x}{h}\right)\right) \\
&\cdot(x+h)f_H(h)dhf_X(x)dx \\
&+T\sum_{i=1}^{M-1}\sum_{j=i}^{M-1}j\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{jT-x}^{\infty}\left(\prod_{k=i}^{j-1}\beta\left(\frac{kT-x}{h}\right)\right) \\
&\cdot\left(1-\beta\left(\frac{jT-x}{h}\right)\right)f_H(h)dhf_X(x)dx \\
&+\sum_{i=1}^{M}MT\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{MT-x}^{\infty}\left(\prod_{k=i}^{M-1}\beta\left(\frac{kT-x}{h}\right)\right)f_H(h)dhf_X(x)dx \\
&+MT\int_{MT}^{\infty}\left(\prod_{n=1}^{M-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)f_X(x)dx \\
&+T\sum_{j=1}^{M-1}j\int_{jT}^{\infty}\left(\prod_{n=1}^{j-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\alpha\left(\frac{jT}{x}\right)f_X(x)dx
\end{aligned}
\tag{D.26}
$$

D.2.2 Deriving the reliability evaluation

We focus here on the reliability expression under the A2 model when $(s-1)T \leq t < sT$, where $s = 1, \ldots, M$. This reliability expression is denoted by $R^{(s)}_{MT,2}(t)$. This term is then used for deriving the general reliability expression for the A2 model for any t value, $R_{MT,2}(t)$. The procedure of using the reliability under $(s-1)T \leq t < sT$ to derive the general reliability expression equals the method used for the derivation of the general reliability expression of the A1 model, see Appendix D.1.2. Hence, this section of the Appendix will only discuss the derivation of the reliability expression $R^{(s)}_{MT,2}(t)$ under the A2 model when $(s-1)T \leq t < sT$, where $s = 1, \ldots, M$, and present the general reliability expression $R_{MT,2}(t)$ for any given t.

In deriving the expression for $R^{(s)}_{MT,2}(t)$, the same logic is applied as been done by the cost evaluation under the A2 model, i.e. the A1 model's reliability expression is rewritten and subsequently the expression is altered to include the non-constant probabilities of false positives and false negatives. This section will not present the expressions for the non-constant probabilities of false positives or false negatives. For a discussion on these expressions, see Subsection D.1.1.

To start with, let us rewrite the power functions to their equivalent product expressions, and the delay time's cdf to its equivalent pdf integral expression. This yields the rewritten reliability model under the A1 model, see Equation D.27.

$$
\begin{aligned}
R^{(s)}_{MT,1}(t) = & \sum_{i=1}^{s-1}\sum_{j=i}^{s-1}\left(\prod_{n=1}^{i-1}(1-\alpha)\right)(1-\beta)\left(\prod_{k=i}^{j-1}\beta\right)\int_{(i-1)T}^{iT}\int_{jT-x}^{\infty} f_H(h)dh f_X(x)dx\, R^{(s-j)}_{MT,1}(t-jT) \\
& + \sum_{i=1}^{s-1}\left(\prod_{n=1}^{i-1}(1-\alpha)\right)\left(\prod_{k=i}^{s-1}\beta\right)\int_{(i-1)T}^{iT}\int_{t-x}^{\infty} f_H(h)dh f_X(x)dx \\
& + \sum_{j=1}^{s-1}\left(\prod_{n=1}^{j-1}(1-\alpha)\right)\alpha\left(1-F_X(jT)\right) R^{s-j}_{MT,1}(t-jT) \\
& + \left(\prod_{n=1}^{s-1}(1-\alpha)\right)\int_{(s-1)T}^{t}\left(1-F_H(t-x)\right) f_X(x)dx \\
& + \left(\prod_{n=1}^{s-1}(1-\alpha)\right)\left(1-F_X(t)\right)
\end{aligned}
\tag{D.27}
$$

where $(s-1)T \leq t < sT$

The rewritten reliability model under the A1 model from Equation D.27 is now used to yield the reliability model under the A2 model $R^{(s)}_{MT,2}(t)$. To yield the reliability expression under the A2 model, the non-constant probabilities of false positives and false negatives are included and are integrated over their appropriate variables, i.e. the probability of false positives is integrated over x and the probability of false negatives is integrated over x and h. This yields the final expression for the reliability model under the A2 model, $R^{(s)}_{MT,2}(t)$, see Equation D.28.

$$\begin{aligned} R^{(s)}_{MT,2}(t) = &\sum_{i=1}^{s-1}\sum_{j=i}^{s-1}\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{jT-x}^{\infty}\left(1-\beta\left(\frac{jT-x}{h}\right)\right) \\ &\cdot\left(\prod_{k=i}^{j-1}\beta\left(\frac{kT-x}{h}\right)\right)f_H(h)dhf_X(x)dx\ R^{(s-j)}_{MT,2}(t-jT) \\ &+\sum_{i=1}^{s-1}\int_{(i-1)T}^{iT}\left(\prod_{n=1}^{i-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\int_{t-x}^{\infty}\left(\prod_{k=i}^{s-1}\beta\left(\frac{kT-x}{h}\right)\right)f_H(h)dhf_X(x)dx \\ &+\sum_{j=1}^{s-1}\int_{jT}^{\infty}\left(\prod_{n=1}^{j-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)\alpha\left(\frac{jT}{x}\right)f_X(x)dxR^{s-j}_{MT,2}(t-jT) \\ &+\int_{(s-1)T}^{t}\left(\prod_{n=1}^{s-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)(1-F_H(t-x))\,f_X(x)dx \\ &+\int_{t}^{\infty}\left(\prod_{n=1}^{s-1}\left(1-\alpha\left(\frac{nT}{x}\right)\right)\right)f_X(x)dx \end{aligned} \tag{D.28}$$

where $(s-1)T \leq t < sT$

The expression $R^{(s)}_{MT,2}(t)$ from Equation D.28 is then used for deriving the general reliability expression under the A2 model for any t, $R_{MT,2}(t)$. For the derivation procedure, see Section D.1.2. This yields the general reliability expression $R_{MT,2}(t)$ for any t:

$$R_{MT,2}(t) = R^{(aMT+s)}_{MT,2}(t) = R^{(M)}_{MT,2}(MT)^{a} R^{(s)}_{MT,2}(t-aMT), \quad \begin{aligned} &(aMT+s-1)T \leq t < (aMT+s)T, \\ &s = 1,\ldots,M, \\ &a = 0,1,\ldots, \end{aligned}$$

E

Cost Verification

E.1 A2 Model

For the verification of the cost expression and the cost values obtained in the numerical study from Chapter 5, this section of the Appendix presents expressions to derive an upper bound and a lower bound on the costs under the A2 model.

In modelling the probabilities of false positives as $\alpha_0 + c_\alpha \frac{rT}{x}$, the probability is constrained by the interval $(\alpha_0, \alpha_0 + c_\alpha)$. Analogously, the probability of false negatives being modelled by $\beta_0 + c_\beta \left(1 - \frac{qT-x}{h}\right)$ yields probability values in the interval $(\beta_0, \beta_0 + c_\beta)$. The bounded nature of the probabilities implies that the model terms of the A2 cost model are also constrained. The expected inspections per cycle $E_2(K)$ has a minimum value and a maximum value due to its dependency on false positives and false negatives which are both constrained. Analogously, the probability of incurring maintenance action costs $P_2(c_p)$ and the expected cycle length $E_2(\textit{cycle length})$ are constrained as well. Each of these lower and upper bounds of these three terms are denoted by $E_2^{min}(K)$, $E_2^{max}(K)$, $P_2^{min}(c_p)$, $P_2^{max}(c_p)$, $E_2^{min}(\textit{cycle length})$ and $E_2^{max}(\textit{cycle length})$ respectively.

The bounded characteristic of each of the individual terms yields bounds the average costs per time unit of the A2 model. The lower and upper bound of the average costs per time unit under the A2 model then equal to the expressions below, respectively.

$$\begin{aligned} C_2^{min}(M^*, T^*) &= \frac{c_0 E_2^{min}(K) + c_p P_2^{max}(c_p) + c_f(1 - P_2^{max}(c_p))}{E_2^{max}(\textit{cycle length})} \\ C_2^{max}(M^*, T^*) &= \frac{c_0 E_2^{max}(K) + c_p P_2^{min}(c_p) + c_f(1 - P_2^{min}(c_p))}{E_2^{min}(\textit{cycle length})} \end{aligned} \tag{E.1}$$

The values for the variables $E_2^{min}(K)$, $E_2^{max}(K)$, $P_2^{min}(c_p)$, $P_2^{max}(c_p)$, $E_2^{min}(\textit{cycle length})$, $E_2^{max}(\textit{cycle length})$, and subsequently $C_2^{min}(M^*, T^*)$ and $C_2^{max}(M^*, T^*)$ are calculated by the numerical study for M^* and T^*. The individual terms from the cost expressions from Equation E.1 are calculated by inserting M^* and T^*, into the A1 model with constant probabilities of α_0 combined with β_0. The same values for M^* and T^* are then also inserted into the A1 model with probabilities $\alpha_0 + c_\alpha$ combined with β_0. The same process is then also done for the A1 model under α_0 combined with $\beta_0 + c_\beta$ and for the A1 model with $\alpha_0 + c_\alpha$ and $\beta_0 + c_\beta$. From all four A1 model instances the minimum and maximum value of each of the three cost elements is taken, e.g. the minimum value $E_2^{min}(K)$ from the four A1 model instances is taken as well as the maximum value $E_2^{max}(K)$ from the same four A1 model instances. Based on these constrained values, the cost bounds are determined. Note that the optimal M and T values stem from the optimisation model of the A2 model. In other words, based on the results from the A2 model its bounds are calculated.

E.2 A3 Model

Analogous to the A2 model, the cost expression and the numerical cost values for the A3 model can be verified as well. The verification procedure follows the one from the A2 model, i.e. deriving an upper and lower boun for the cost expression.

Similar to the A2 model, the cost function of the A3 model is bounded as well. The way in which the lower and upper bounds are determined, equals the procedure from the A2 model. The probability of false positives is bounded for the A3 model by $(\alpha_0, \alpha_0 + \gamma_\alpha)$ and the probability of false negatives is bounded by $(\beta_0, \beta_0 + \gamma_\beta)$. The remainder of the derivation of the cost bounds remains equal to the one discussed for the A2 model. The sole difference lies in the subscript which now equals 3 to indicate the A3 model. Therefore, the derivation is not presented here. This yields the following expressions for the upper and lower bound of the cost expression.

$$C_3^{min}(M^*, T^*) = \frac{c_0 E_3^{min}(K) + c_p P_3^{max}(c_p) + c_f(1 - P_3^{max}(c_p))}{E_3^{max}(cycle\ length)}$$
$$C_3^{max}(M^*, T^*) = \frac{c_0 E_3^{max}(K) + c_p P_3^{min}(c_p) + c_f(1 - P_3^{min}(c_p))}{E_3^{min}(cycle\ length)}$$

The method of determining the lower and upper cost bounds numerically, follows the one from the A2 model. For more details, see Section 5.2.1. In sum, the optimal M^* and T^* are calculated for the A3 model and both values are inserted into the four A1 models to obtain the bounds of the A3 model's cost expression. The four A1 models consists all possible combinations of α_0, $\alpha_0 + \gamma_\alpha$, β_0 and $\beta_0 + \gamma_\beta$. The obtained cost bounds enhance further understanding of the numerical results, whether the results are plausible.

F

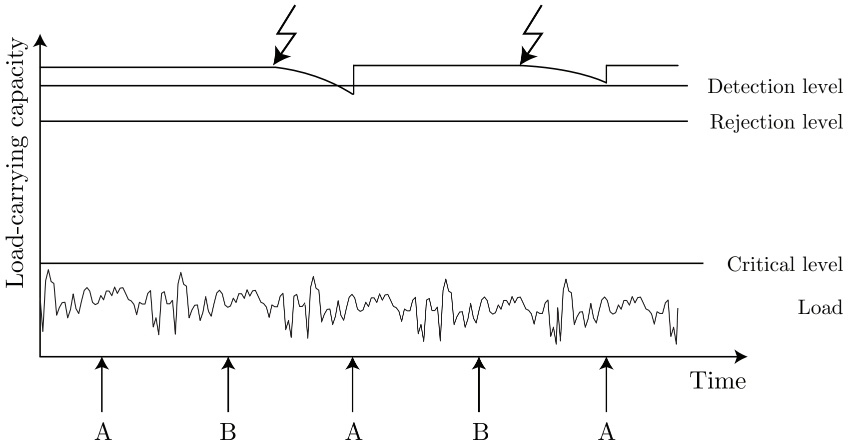
Additional Figures

In this chapter of the Appendix, all additional figures are presented. Section F.1 presents the illustration of the effect of preventive asset replacement on the asset's load carrying capacity. Section F.2 will present the cost evaluation figures for the unconstrained models and variants from Table 5.1. Section F.3 then presents the cost evaluation for the constrained models of Table 5.1, cases 8-14. This Appendix will conclude with Section F.4, which presents the reliability evaluation for the unconstrained cases 8-14 from Table 5.1.

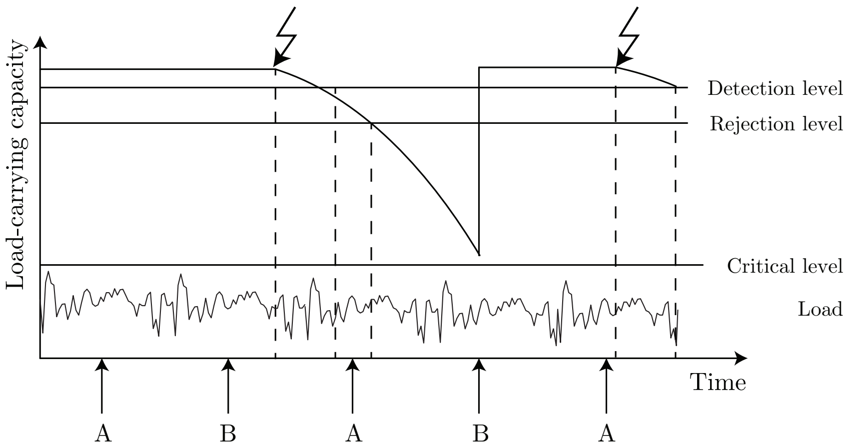
F.1 Preventive asset replacement in practice

In this section of the Appendix, two additional figures are included to illustrate the effect of preventively replacing the asset. Subfigure F.1a illustrates preventive asset replacement at each inspection. In case the asset is preventively replaced at each inspection, the asset's load carrying capacity will remain at rather high levels.

Subfigure F.1b denotes the opposite effect, i.e. the asset is not preventively replaced. This implies that the inspections have to reveal the defect. Subsequently this corresponds to a larger probability of the asset's load carrying capacity reaching the critical level and the probability of asset failure. Subfigure F.1b presents this effect by the load carrying capacity which nearly reaches the critical level due to the fact that the asset is not preventively replaced.



(a) Asset degradation under preventive asset replacement upon each inspection



(b) Asset degradation under no preventive asset replacement

Figure F.1: Asset degradation in practice related to preventive asset replacement

F.2 Unconstrained cost evaluation figures

For each unconstrained case from Table 5.1, a separate figure is generated illustrating whether the solutions for the optimal maintenance schedules are sensible, i.e. a sensibility check for the optimal values of M and T for each of the first seven cases of Table 5.1.

Each of the figures illustrates the sensibility check for individual cases. In other words, each case has a dedicated figure to illustrate whether the results from Table 5.1 are sensible. For each of the seven cases, the results are concluded to be sensible and plausible as the figures illustrate. The sensibility check for the first case is presented in Figure F.2, the second case in Figure F.3, the third case in Figure F.4, the fourth case in Figure F.5, the fifth case in Figure F.6, the sixth case in Figure F.7 and the final, seventh case in Figure F.8.

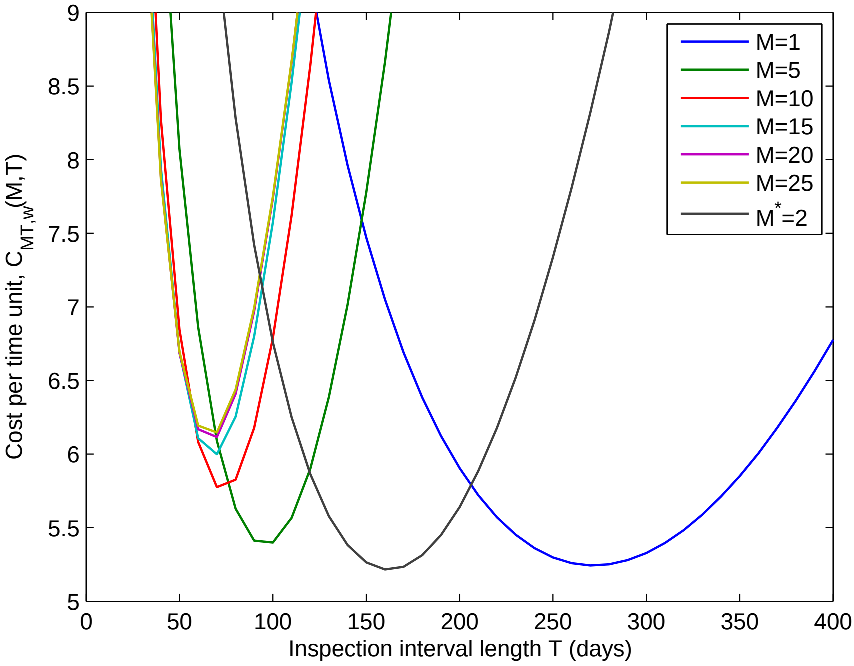


Figure F.2: Cost evaluation for the unconstrained A1 model with $\alpha = 0,2$ and $\beta = 0,2$ under various M and T values

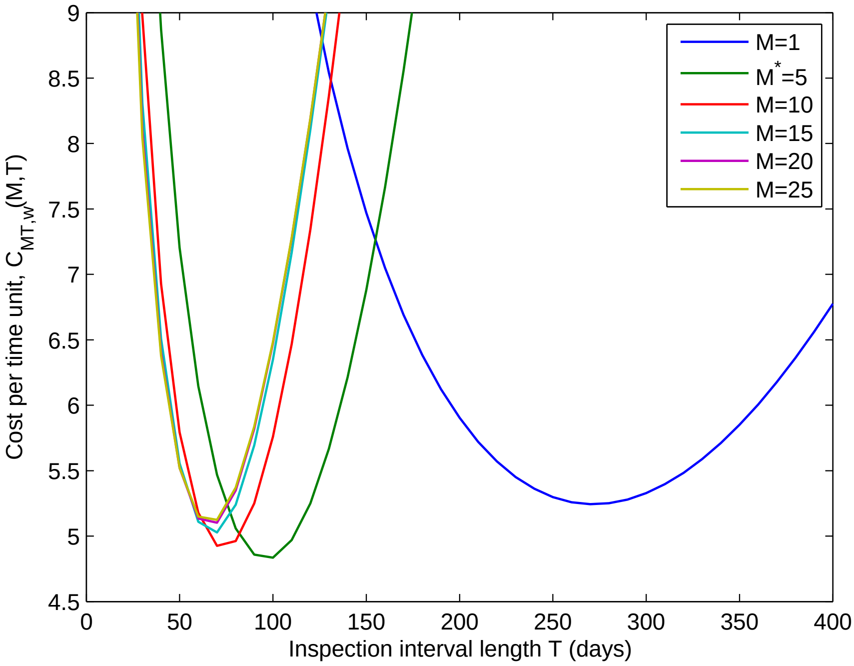


Figure F.3: Cost evaluation for the unconstrained A2 model with $\alpha_0 = 0,1$, $c_\alpha = 0,2$ and $\beta_0 = 0,2$ under various M and T values

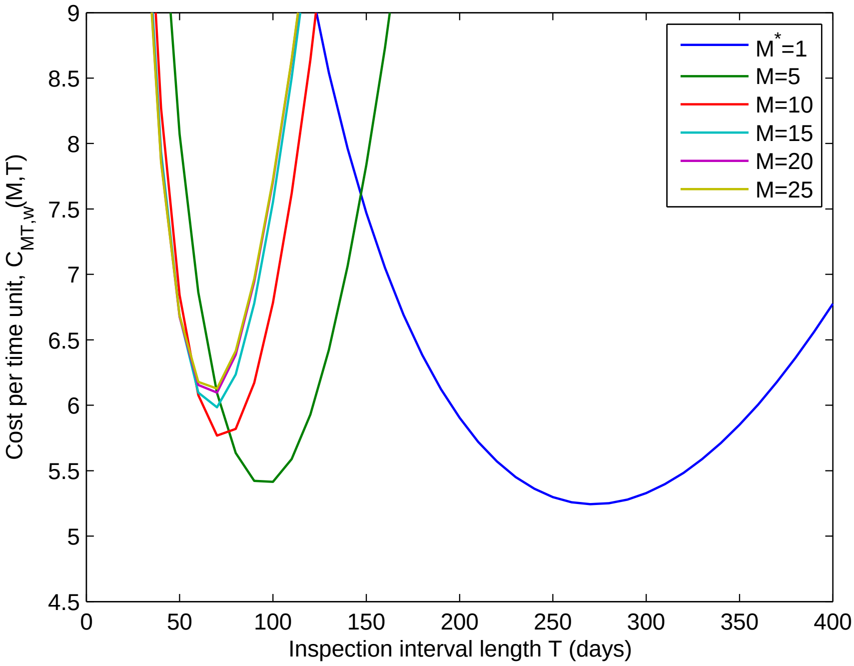


Figure F.4: Cost evaluation for the unconstrained A2 model with $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$ under various M and T values

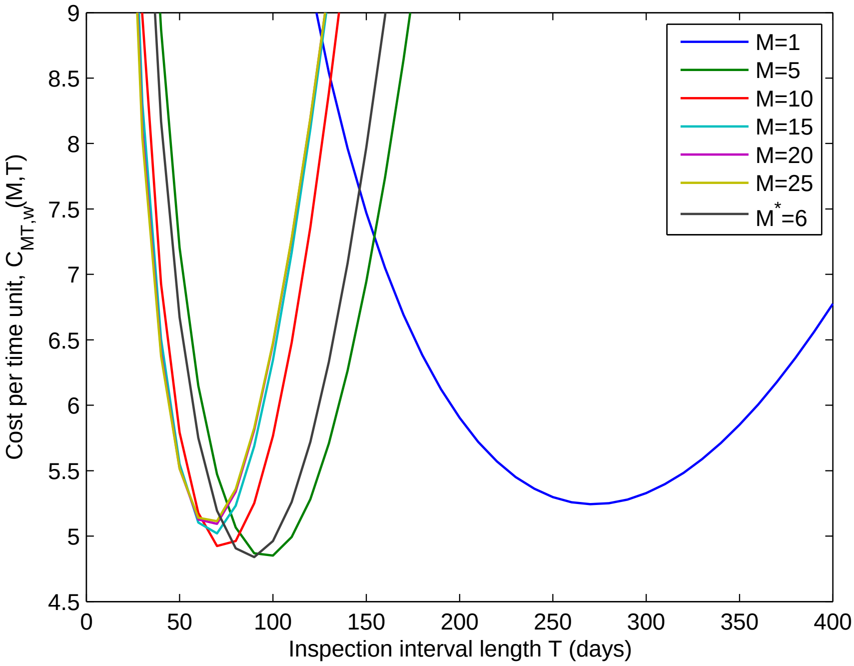


Figure F.5: Cost evaluation for the unconstrained A2 model with $\alpha_0 = 0,1$, $c_\alpha = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$ under various M and T values

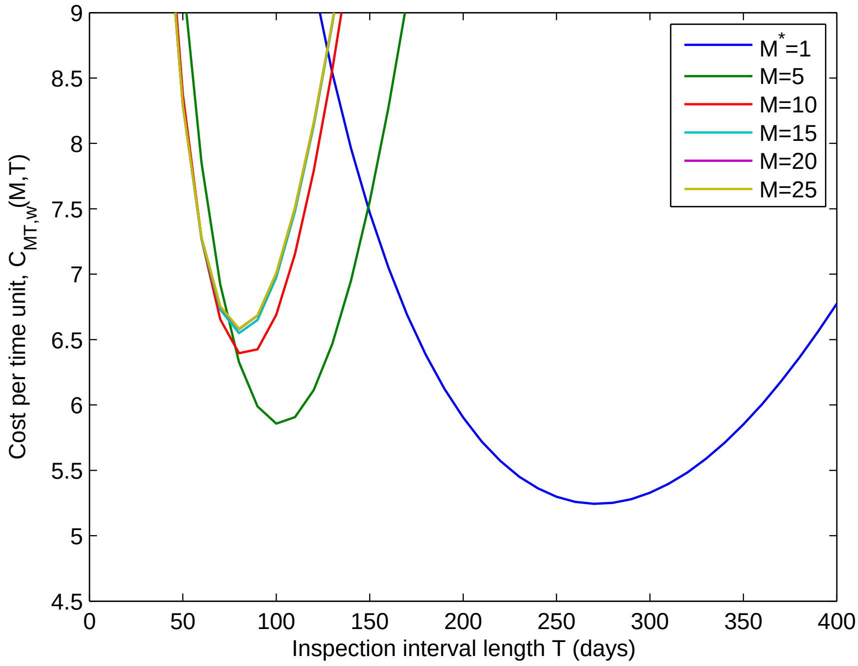


Figure F.6: Cost evaluation for the unconstrained A3 model with $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$ and $\beta_0 = 0,2$ under various M and T values

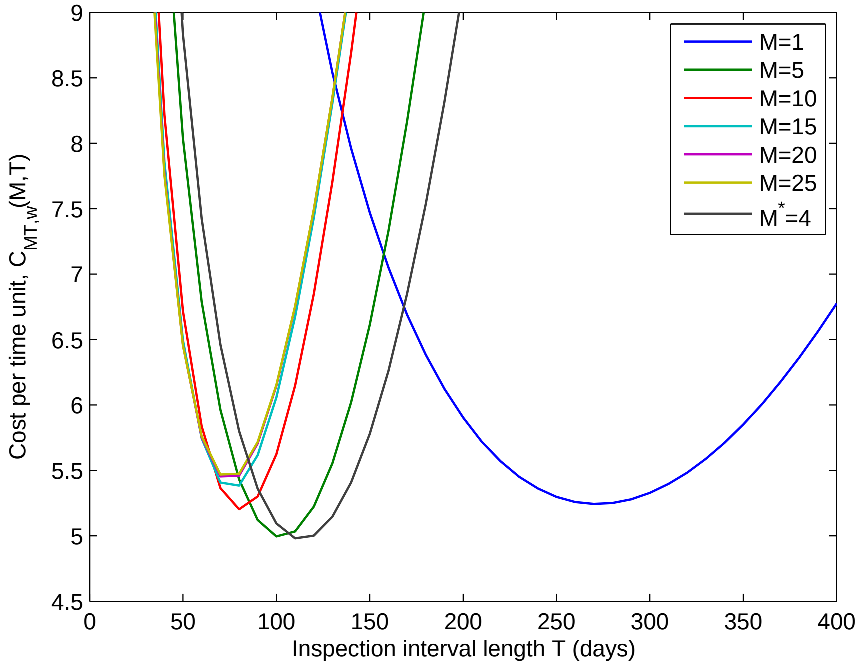


Figure F.7: Cost evaluation for the unconstrained A3 model with $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$ under various M and T values

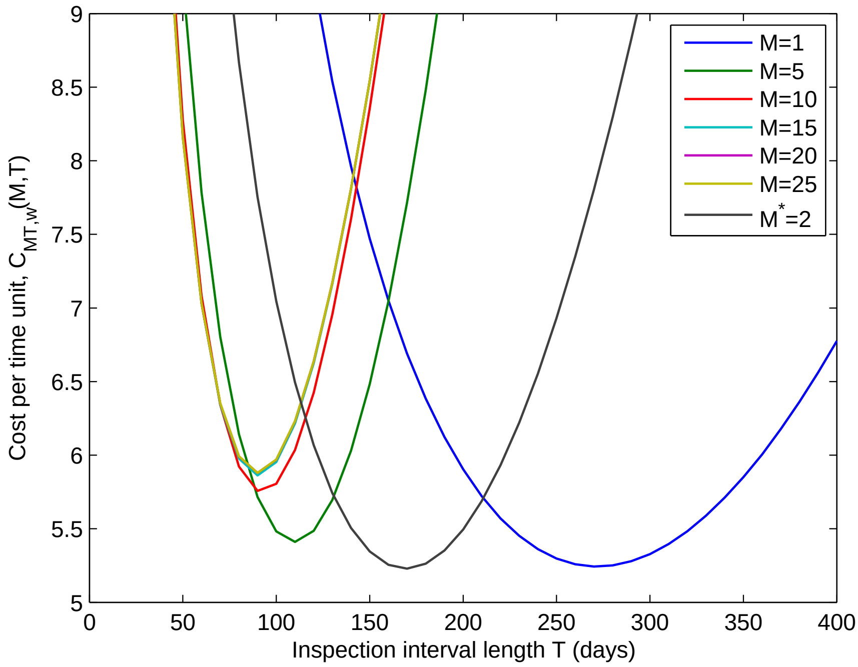


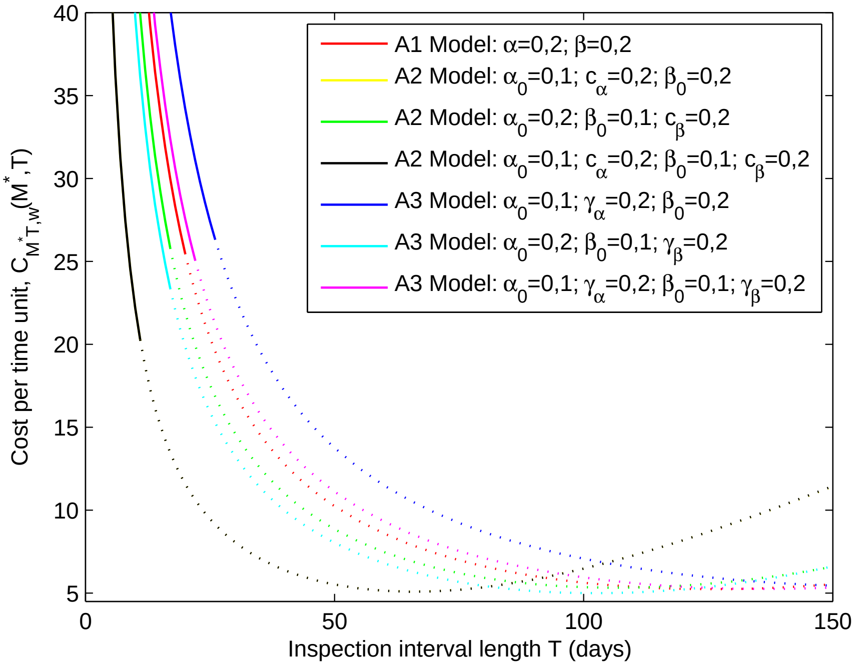
Figure F.8: Cost evaluation for the unconstrained A3 model with $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$ under various M and T values

F.3 Constrained cost evaluation figures

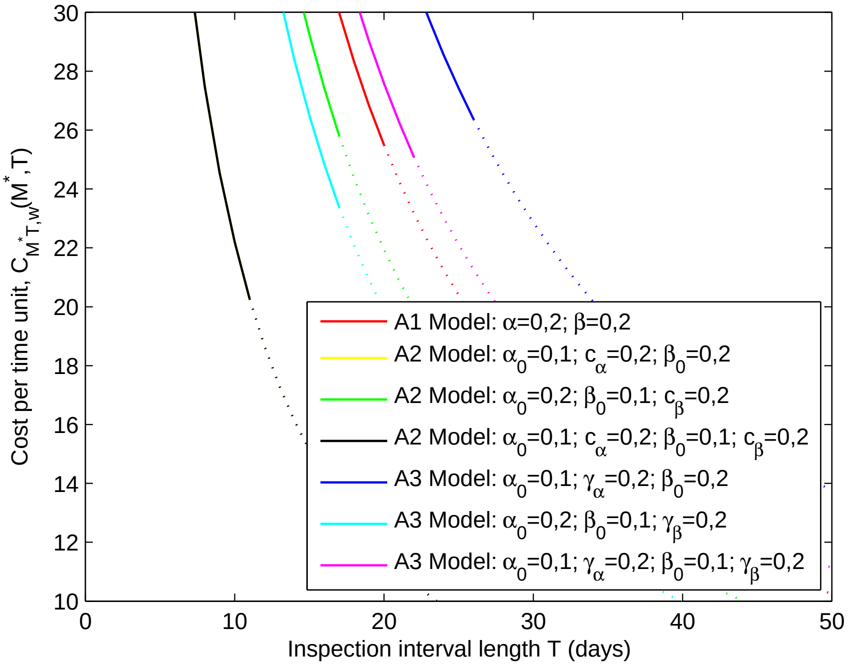
This section of the Appendix presents all the cost figures for the unconstrained cases of Table 5.1, corresponding to cases 8-14. All of the figures are decomposed into two sub-figures because some results might be hard to distinguish. Figure F.9 presents a cost evaluation for T under the optimal M^* of each of the cases, 8-14. The other, remaining, figures illustrate the cost evaluation for other values of M as well.

Since the evaluation figures have been generated with an evaluation step size of 1, i.e. the inspection interval length T is evaluated with a step size of 1, the results in Figures F.11 and F.13 present the fact that a different value for M is ought to be optimal. However, this is observed due to the step size of the analysis chosen. If this step size were to be very small, the optimal M^* yields the lowest cost under evaluation.

From the evaluation figures, especially the figures corresponding to the A2 model, it can be concluded that the cost differences for the near optimal M, slightly differ from the result for M^*. Hence, the effect of M around the optimal solution is very small.

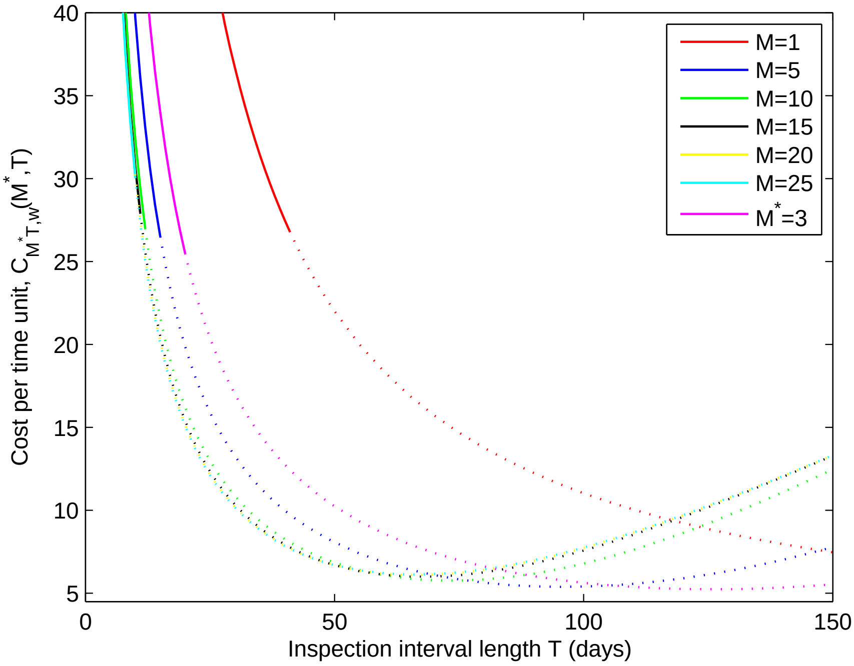


(a) Cost evaluation for cases 8-14

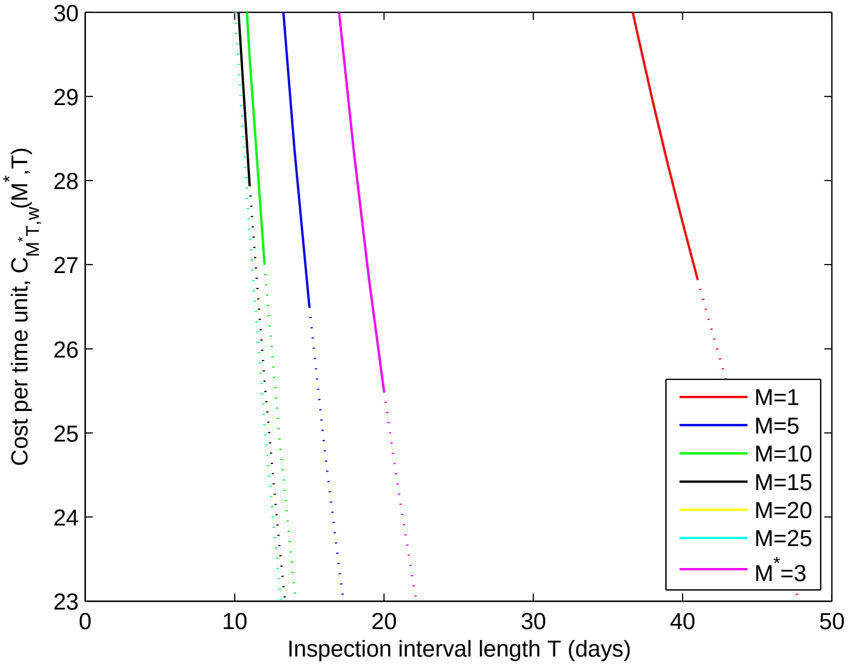


(b) Cost evaluation for cases 8-14 (zoomed)

Figure F.9: Asset reliability evaluation for constrained cases 8-14

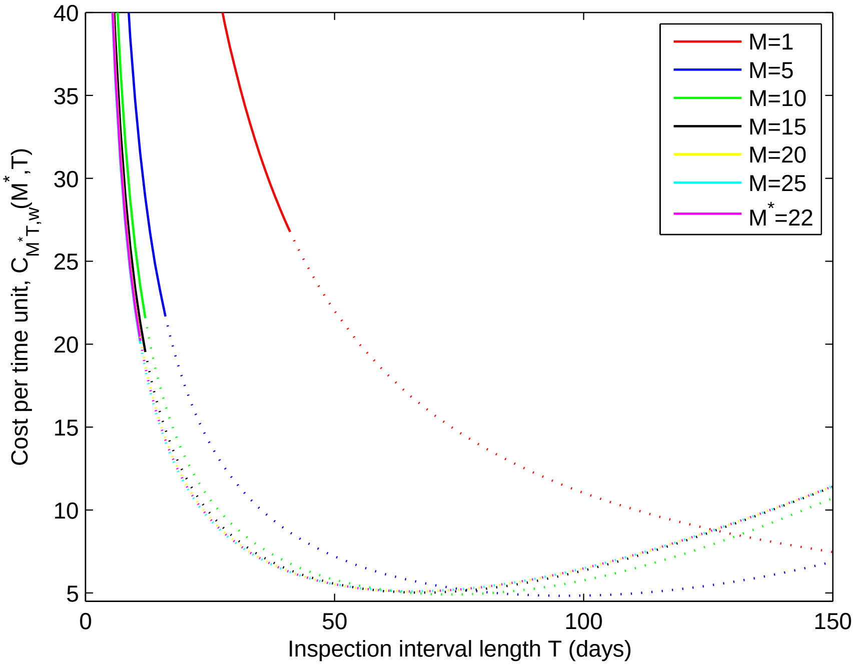


(a) Cost evaluation A1 model with $\alpha = 0,2$ and $\beta = 0,2$ under various M and T values

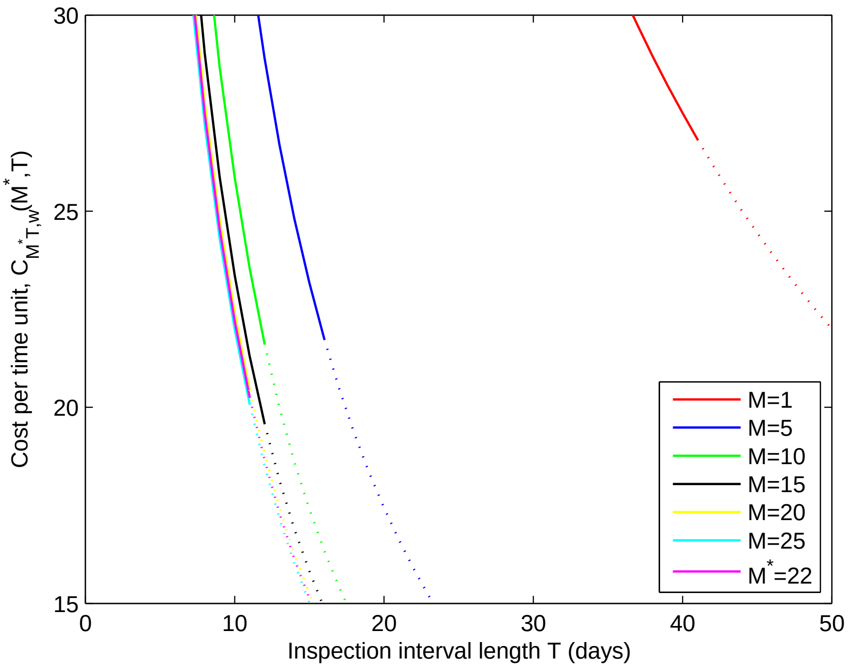


(b) Cost evaluation A1 model with $\alpha = 0,2$ and $\beta = 0,2$ under various M and T values (zoomed)

Figure F.10: Cost evaluation for the constrained A1 model, $\alpha = 0,2$ and $\beta = 0,2$

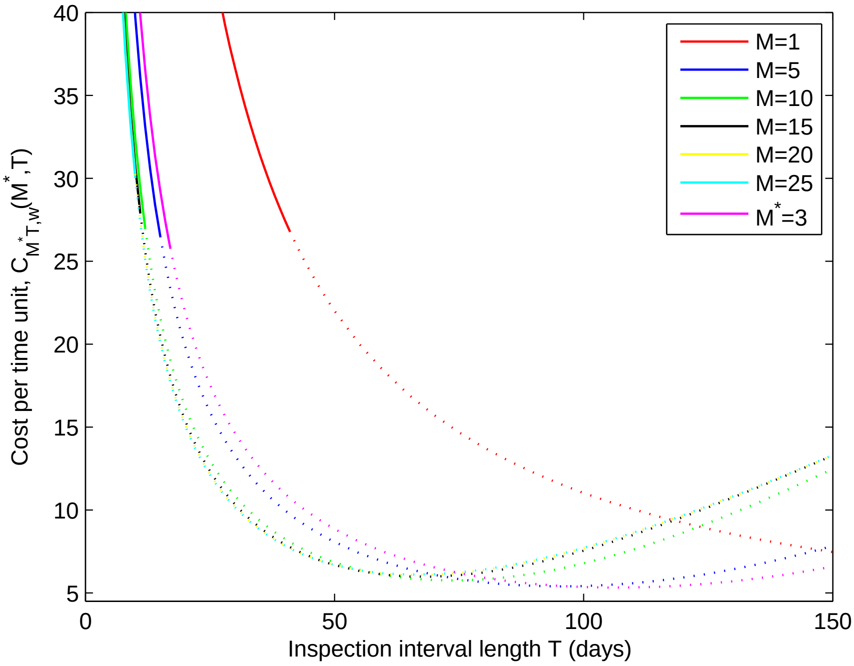


(a) Cost evaluation A2 model with $\alpha_0 = 0,1$, $c_\alpha = 0,2$ and $\beta_0 = 0,2$ under various M and T values

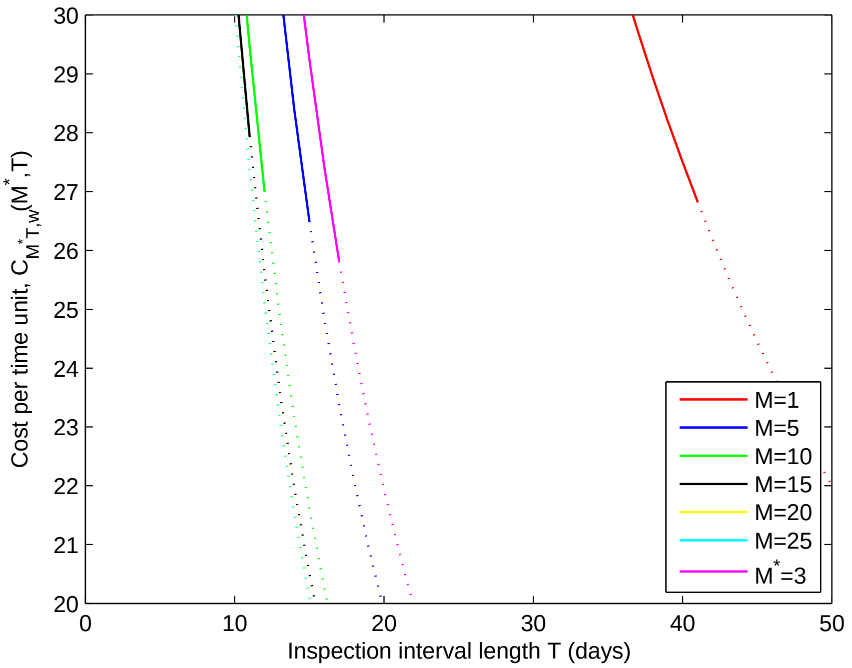


(b) Cost evaluation A2 model with $\alpha_0 = 0,1$, $c_\alpha = 0,2$ and $\beta = 0,2$ under various M and T values (zoomed)

Figure F.11: Cost evaluation for the constrained A2 model, $\alpha_0 = 0,1$, $c_\alpha = 0,2$ and $\beta_0 = 0,2$

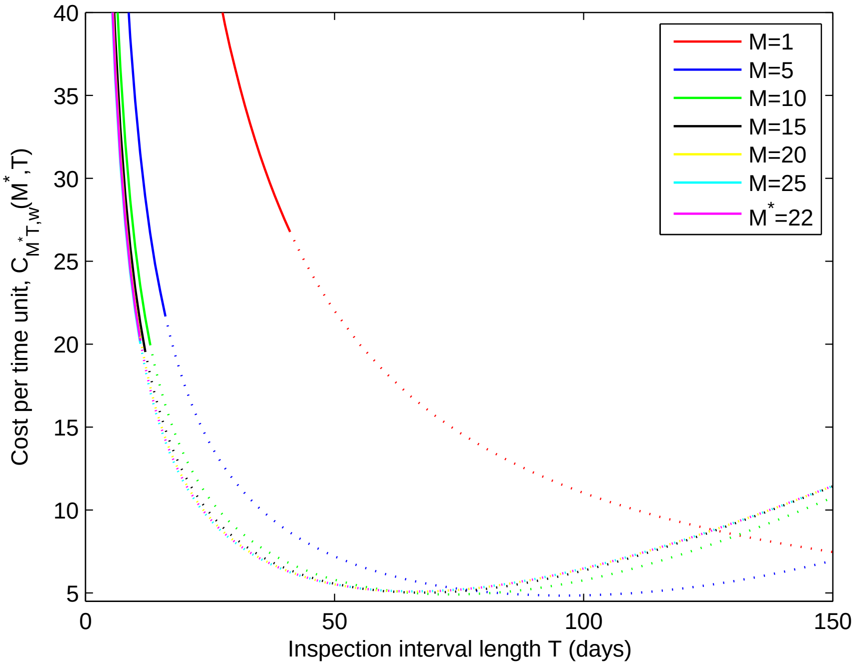


(a) Cost evaluation A2 model with $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$ under various M and T values

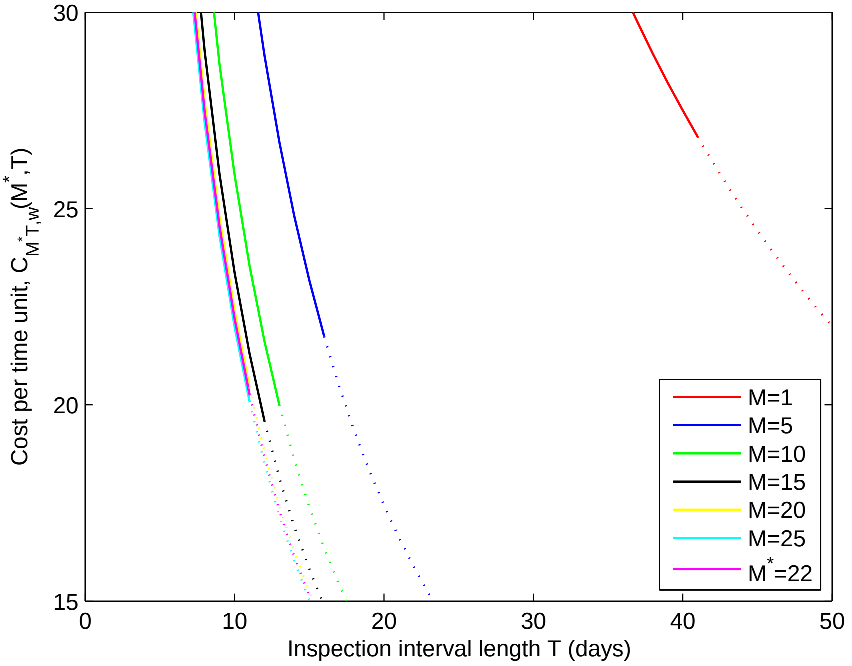


(b) Cost evaluation A2 model with $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$ under various M and T values (zoomed)

Figure F.12: Cost evaluation for the constrained A2 model, $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$

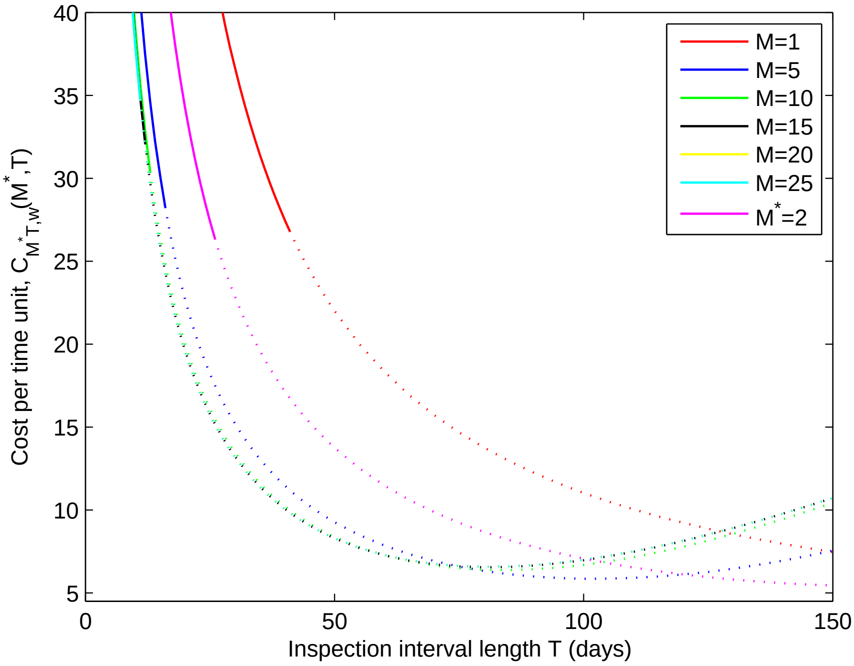


(a) Cost evaluation A2 model with $\alpha_0 = 0,1$, $c_\alpha = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$ under various M and T values

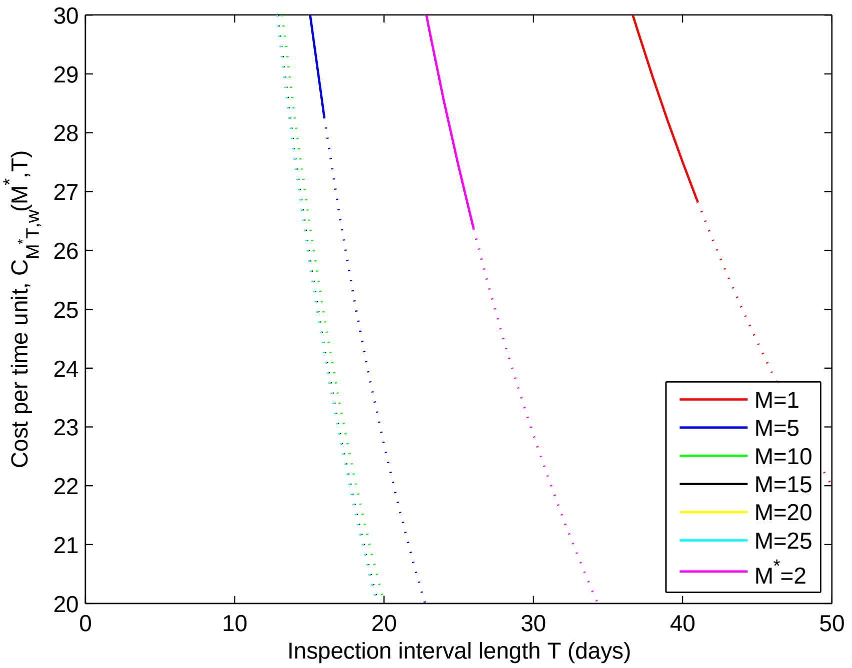


(b) Cost evaluation A2 model with $\alpha_0 = 0,1$, $c_\alpha = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$ under various M and T values (zoomed)

Figure F.13: Cost evaluation for the constrained A2 model, $\alpha_0 = 0,1$, $c_\alpha = 0,2$, $\beta_0 = 0,1$ and $c_\beta = 0,2$

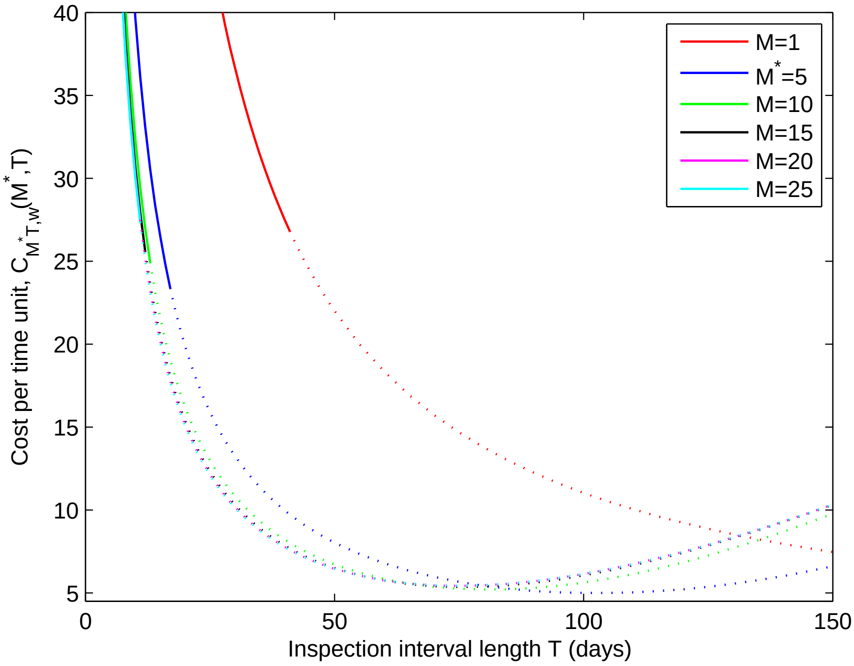


(a) Cost evaluation A3 model with $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$ and $\beta_0 = 0,2$ under various M and T values

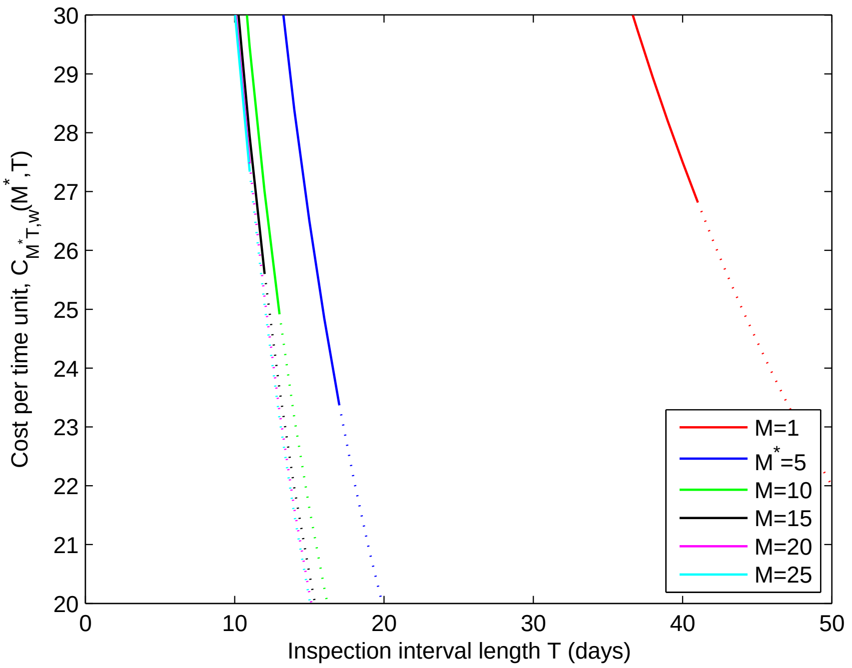


(b) Cost evaluation A3 model with $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$ and $\beta = 0,2$ under various M and T values (zoomed)

Figure F.14: Cost evaluation for the constrained A3 model, $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$ and $\beta_0 = 0,2$

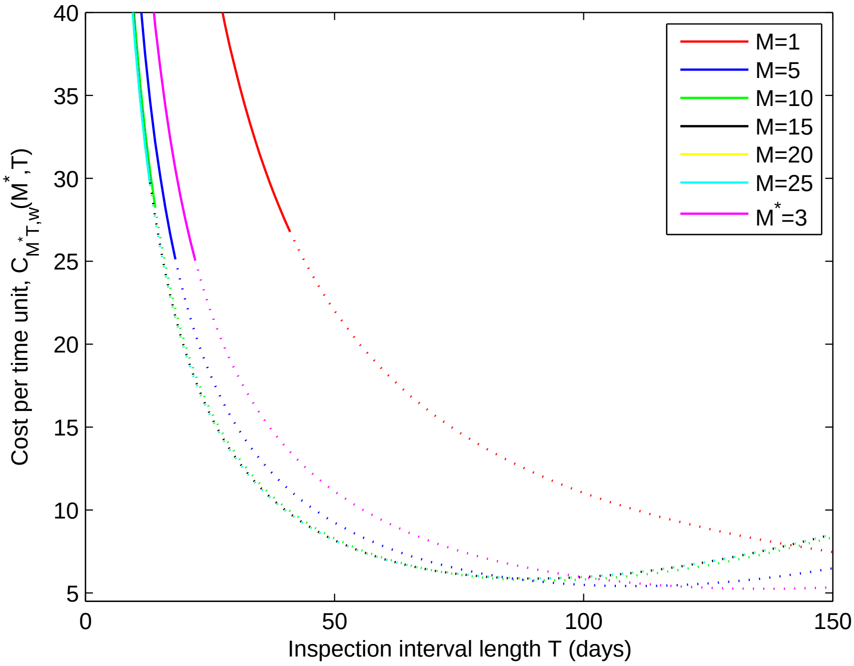


(a) Cost evaluation A3 model with $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$ under various M and T values

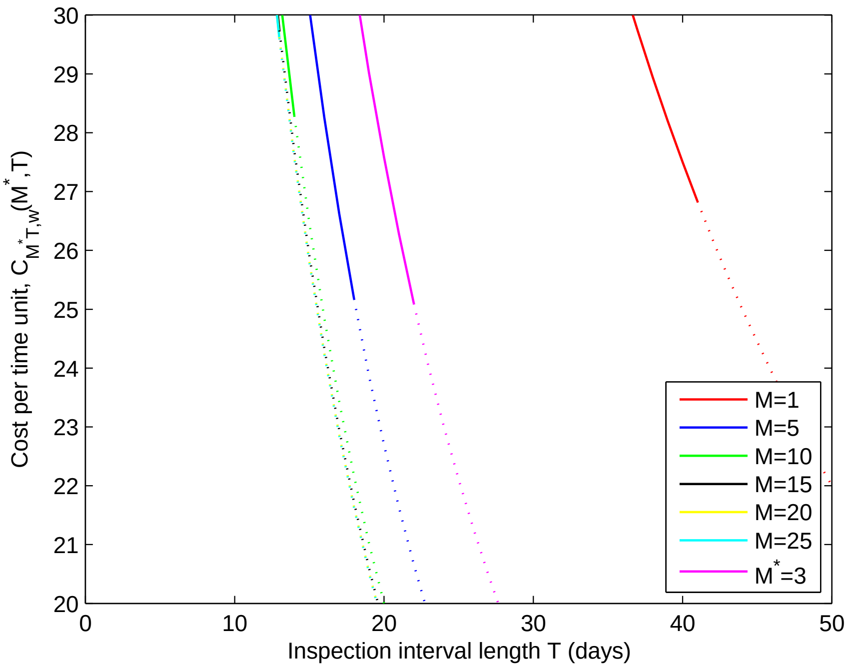


(b) Cost evaluation A3 model with $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$ under various M and T values (zoomed)

Figure F.15: Cost evaluation for the constrained A3 model, $\alpha_0 = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$



(a) Cost evaluation A3 model with $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$ under various M and T values

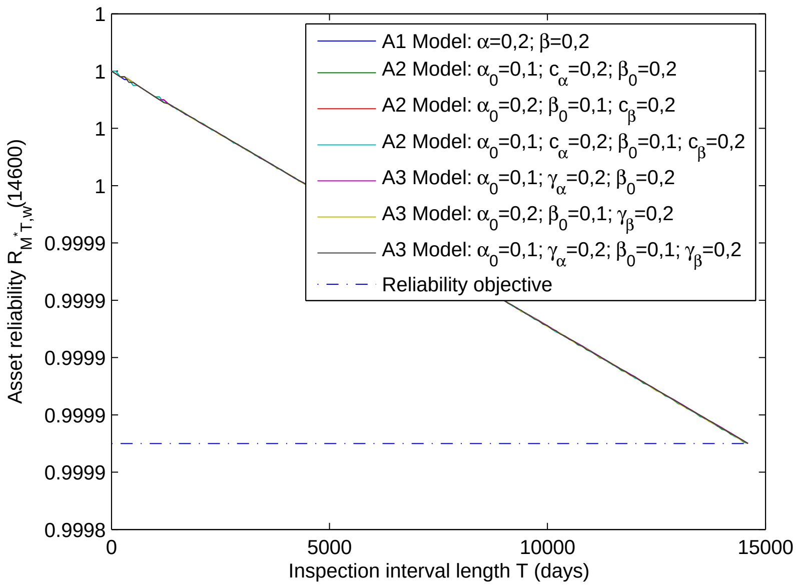


(b) Cost evaluation A3 model with $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$ under various M and T values (zoomed)

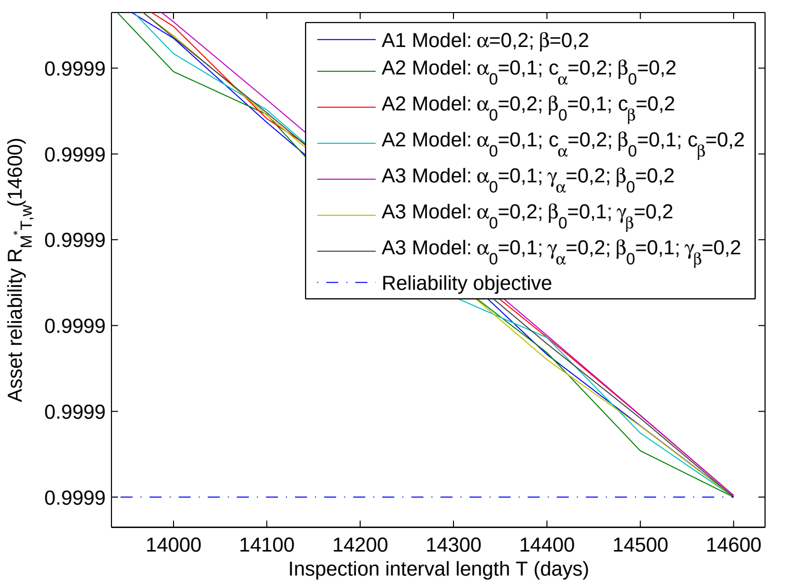
Figure F.16: Cost evaluation for the constrained A3 model, $\alpha_0 = 0,1$, $\gamma_\alpha = 0,2$, $\beta_0 = 0,1$ and $\gamma_\beta = 0,2$

F.4 Reliability evaluation figures

For the reliability evaluation figure, the reliability of the cost optimal maintenance schedules under the reliability constraint are assessed. This implies that the figure presented offers a check whether the cost optimal maintenance schedule derived under the reliability constraint actually meets the reliability constraint set. The reliability evaluation figure is presented in Figure F.17. Note that Subfigure F.17a denotes the unzoomed variant and Subfigure F.17b presents the check that the reliability constraint is not violated at the time over which the reliability objective is set, i.e. $R_{obj}(14.600)$.



(a) Reliability evaluation for cases 8-14



(b) Reliability evaluation for cases 8-14 (zoomed)

Figure F.17: Asset reliability evaluation for constrained cases 8-14

G

Reliability Evaluation Table

In this Appendix, Table G.1 presents a tabular overview on the reliability evaluation under various values for the reliability objective parameter. Table G.1 follows the same structure as the table presenting the numerical results in Chapter 5, Table 5.1. Note that Figure 5.3 is based on the results from the table presented here.

Table G.1: Optimal maintenance schedules under various reliability objectives

Case	**Model**	**Distribution parameters**				**Reliability parameter**	**Cost parameters**			**Probability parameters**								**Optimal Maintenance schedule**		
		δ_X	θ_X	δ_H	θ_H	R_{obj}	c_0	c_p	c_f	α	β	α_0	β_0	c_α	c_β	γ_α	γ_β	M^*	T^*	$C(M^*, T^*)$
1	A1	2,5	1234	2,5	203	**0,80000**	100	1.000	100.000	0,2	0,2							2	162,18	5,21
2	A2	2,5	1234	2,5	203	**0,80000**	100	1.000	100.000			0,1	0,2	0,2				5	96,37	4,82
3	A2	2,5	1234	2,5	203	**0,80000**	100	1.000	100.000			0,2	0,1		0,2			1	271,71	5,24
4	A2	2,5	1234	2,5	203	**0,80000**	100	1.000	100.000			0,1	0,1	0,2	0,2			6	88,37	4,84
5	A3	2,5	1234	2,5	203	**0,80000**	100	1.000	100.000			0,1	0,2			0,2		1	271,71	5,24
6	A3	2,5	1234	2,5	203	**0,80000**	100	1.000	100.000			0,2	0,1				0,2	4	113,42	4,97
7	A3	2,5	1234	2,5	203	**0,80000**	100	1.000	100.000			0,1	0,1			0,2	0,2	2	169,32	5,23
8	A1	2,5	1234	2,5	203	**0,82000**	100	1.000	100.000	0,2	0,2							2	162,18	5,21
9	A2	2,5	1234	2,5	203	**0,82000**	100	1.000	100.000			0,1	0,2	0,2				5	96,37	4,82
10	A2	2,5	1234	2,5	203	**0,82000**	100	1.000	100.000			0,2	0,1		0,2			1	271,71	5,24
11	A2	2,5	1234	2,5	203	**0,82000**	100	1.000	100.000			0,1	0,1	0,2	0,2			6	88,37	4,84
12	A3	2,5	1234	2,5	203	**0,82000**	100	1.000	100.000			0,1	0,2			0,2		1	271,71	5,24
13	A3	2,5	1234	2,5	203	**0,82000**	100	1.000	100.000			0,2	0,1				0,2	4	113,42	4,97
14	A3	2,5	1234	2,5	203	**0,82000**	100	1.000	100.000			0,1	0,1			0,2	0,2	2	169,32	5,23
15	A1	2,5	1234	2,5	203	**0,84000**	100	1.000	100.000	0,2	0,2							2	162,18	5,21
16	A2	2,5	1234	2,5	203	**0,84000**	100	1.000	100.000			0,1	0,2	0,2				5	96,37	4,82
17	A2	2,5	1234	2,5	203	**0,84000**	100	1.000	100.000			0,2	0,1		0,2			1	271,56	5,24
18	A2	2,5	1234	2,5	203	**0,84000**	100	1.000	100.000			0,1	0,1	0,2	0,2			6	88,37	4,84
19	A3	2,5	1234	2,5	203	**0,84000**	100	1.000	100.000			0,1	0,2			0,2		1	271,56	5,24
20	A3	2,5	1234	2,5	203	**0,84000**	100	1.000	100.000			0,2	0,1				0,2	4	113,42	4,97
21	A3	2,5	1234	2,5	203	**0,84000**	100	1.000	100.000			0,1	0,1			0,2	0,2	2	169,32	5,23
22	A1	2,5	1234	2,5	203	**0,86000**	100	1.000	100.000	0,2	0,2							2	160,66	5,22
23	A2	2,5	1234	2,5	203	**0,86000**	100	1.000	100.000			0,1	0,2	0,2				5	96,37	4,82
24	A2	2,5	1234	2,5	203	**0,86000**	100	1.000	100.000			0,2	0,1		0,2			2	159,37	5,25
25	A2	2,5	1234	2,5	203	**0,86000**	100	1.000	100.000			0,1	0,1	0,2	0,2			6	88,37	4,84
26	A3	2,5	1234	2,5	203	**0,86000**	100	1.000	100.000			0,1	0,2			0,2		1	259,86	5,26
27	A3	2,5	1234	2,5	203	**0,86000**	100	1.000	100.000			0,2	0,1				0,2	4	113,42	4,97
28	A3	2,5	1234	2,5	203	**0,86000**	100	1.000	100.000			0,1	0,1			0,2	0,2	2	169,32	5,23
29	A1	2,5	1234	2,5	203	**0,88000**	100	1.000	100.000	0,2	0,2							2	153,30	5,24
30	A2	2,5	1234	2,5	203	**0,88000**	100	1.000	100.000			0,1	0,2	0,2				5	96,37	4,82
31	A2	2,5	1234	2,5	203	**0,88000**	100	1.000	100.000			0,2	0,1		0,2			2	151,65	5,28
32	A2	2,5	1234	2,5	203	**0,88000**	100	1.000	100.000			0,1	0,1	0,2	0,2			6	88,37	4,84
33	A3	2,5	1234	2,5	203	**0,88000**	100	1.000	100.000			0,1	0,2			0,2		1	247,83	5,31

Case	Model	Distribution parameters				Reliability parameter	Cost parameters			Probability parameters								Optimal Maintenance schedule		
		δ_X	θ_X	δ_H	θ_H	R_{obj}	c_0	c_p	c_f	α	β	α_0	β_0	c_α	c_β	γ_α	γ_β	M^*	T^*	$C(M^*,T^*)$
34	A3	2,5	1234	2,5	203	**0,88000**	100	1.000	100.000			0,2	0,1				0,2	4	112,73	4,97
35	A3	2,5	1234	2,5	203	**0,88000**	100	1.000	100.000			0,1	0,1			0,2	0,2	2	161,94	5,25
36	A1	2,5	1234	2,5	203	**0,90000**	100	1.000	100.000	0,2	0,2							3	114,28	5,31
37	A2	2,5	1234	2,5	203	**0,90000**	100	1.000	100.000			0,1	0,2	0,2				6	85,56	4,84
38	A2	2,5	1234	2,5	203	**0,90000**	100	1.000	100.000			0,2	0,1		0,2			2	143,86	5,26
39	A2	2,5	1234	2,5	203	**0,90000**	100	1.000	100.000			0,1	0,1	0,2	0,2			6	85,25	4,85
40	A3	2,5	1234	2,5	203	**0,90000**	100	1.000	100.000			0,1	0,2			0,2		1	234,55	5,41
41	A3	2,5	1234	2,5	203	**0,90000**	100	1.000	100.000			0,2	0,1				0,2	4	106,68	5,00
42	A3	2,5	1234	2,5	203	**0,90000**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	126,47	5,30
43	A1	2,5	1234	2,5	203	**0,92000**	100	1.000	100.000	0,2	0,2							3	106,98	5,44
44	A2	2,5	1234	2,5	203	**0,92000**	100	1.000	100.000			0,1	0,2	0,2				7	75,88	4,90
45	A2	2,5	1234	2,5	203	**0,92000**	100	1.000	100.000			0,2	0,1		0,2			3	106,13	5,48
46	A2	2,5	1234	2,5	203	**0,92000**	100	1.000	100.000			0,1	0,1	0,2	0,2			7	75,69	4,91
47	A3	2,5	1234	2,5	203	**0,92000**	100	1.000	100.000			0,1	0,2			0,2		1	219,81	5,57
48	A3	2,5	1234	2,5	203	**0,92000**	100	1.000	100.000			0,2	0,1				0,2	4	100,13	5,09
49	A3	2,5	1234	2,5	203	**0,92000**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	118,26	5,41
50	A1	2,5	1234	2,5	203	**0,94000**	100	1.000	100.000	0,2	0,2							3	98,46	5,68
51	A2	2,5	1234	2,5	203	**0,94000**	100	1.000	100.000			0,1	0,2	0,2				8	66,92	5,05
52	A2	2,5	1234	2,5	203	**0,94000**	100	1.000	100.000			0,2	0,1		0,2			3	97,73	5,72
53	A2	2,5	1234	2,5	203	**0,94000**	100	1.000	100.000			0,1	0,1	0,2	0,2			8	66,84	5,05
54	A3	2,5	1234	2,5	203	**0,94000**	100	1.000	100.000			0,1	0,2			0,2		1	202,28	5,86
55	A3	2,5	1234	2,5	203	**0,94000**	100	1.000	100.000			0,2	0,1				0,2	5	84,30	5,27
56	A3	2,5	1234	2,5	203	**0,94000**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	109,14	5,63
57	A1	2,5	1234	2,5	203	**0,96000**	100	1.000	100.000	0,2	0,2							3	87,91	6,13
58	A2	2,5	1234	2,5	203	**0,96000**	100	1.000	100.000			0,1	0,2	0,2				8	59,77	5,35
59	A2	2,5	1234	2,5	203	**0,96000**	100	1.000	100.000			0,2	0,1		0,2			3	87,28	6,17
60	A2	2,5	1234	2,5	203	**0,96000**	100	1.000	100.000			0,1	0,1	0,2	0,2			8	59,69	5,36
61	A3	2,5	1234	2,5	203	**0,96000**	100	1.000	100.000			0,1	0,2			0,2		2	113,97	6,36
62	A3	2,5	1234	2,5	203	**0,96000**	100	1.000	100.000			0,2	0,1				0,2	5	75,37	5,65
63	A3	2,5	1234	2,5	203	**0,96000**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	97,29	6,06
64	A1	2,5	1234	2,5	203	**0,98000**	100	1.000	100.000	0,2	0,2							3	72,92	7,16
65	A2	2,5	1234	2,5	203	**0,98000**	100	1.000	100.000			0,1	0,2	0,2				10	46,67	6,10
66	A2	2,5	1234	2,5	203	**0,98000**	100	1.000	100.000			0,2	0,1		0,2			3	72,34	7,21
67	A2	2,5	1234	2,5	203	**0,98000**	100	1.000	100.000			0,1	0,1	0,2	0,2			10	46,67	6,10
68	A3	2,5	1234	2,5	203	**0,98000**	100	1.000	100.000			0,1	0,2			0,2		2	94,59	7,43
69	A3	2,5	1234	2,5	203	**0,98000**	100	1.000	100.000			0,2	0,1				0,2	6	58,54	6,54
70	A3	2,5	1234	2,5	203	**0,98000**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	80,82	7,05
71	A1	2,5	1234	2,5	203	**0,99000**	100	1.000	100.000	0,2	0,2							3	60,74	8,48
72	A2	2,5	1234	2,5	203	**0,99000**	100	1.000	100.000			0,1	0,2	0,2				12	37,40	7,08
73	A2	2,5	1234	2,5	203	**0,99000**	100	1.000	100.000			0,2	0,1		0,2			3	60,35	8,53
74	A2	2,5	1234	2,5	203	**0,99000**	100	1.000	100.000			0,1	0,1	0,2	0,2			13	36,89	7,07
75	A3	2,5	1234	2,5	203	**0,99000**	100	1.000	100.000			0,1	0,2			0,2		2	78,93	8,79
76	A3	2,5	1234	2,5	203	**0,99000**	100	1.000	100.000			0,2	0,1				0,2	6	48,75	7,72
77	A3	2,5	1234	2,5	203	**0,99000**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	67,38	8,33
78	A1	2,5	1234	2,5	203	**0,99500**	100	1.000	100.000	0,2	0,2							3	50,8395	10,072
79	A2	2,5	1234	2,5	203	**0,99500**	100	1.000	100.000			0,1	0,2	0,2				13	30,84	8,28
80	A2	2,5	1234	2,5	203	**0,99500**	100	1.000	100.000			0,2	0,1		0,2			3	50,46	10,15
81	A2	2,5	1234	2,5	203	**0,99500**	100	1.000	100.000			0,1	0,1	0,2	0,2			13	30,88	8,27
82	A3	2,5	1234	2,5	203	**0,99500**	100	1.000	100.000			0,1	0,2			0,2		2	66,01	10,45
83	A3	2,5	1234	2,5	203	**0,99500**	100	1.000	100.000			0,2	0,1				0,2	6	40,80	9,15
84	A3	2,5	1234	2,5	203	**0,99500**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	56,35	9,89
85	A1	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000	0,2	0,2							3	24,30	20,98
86	A2	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,2	0,2				20	14,00	16,27
87	A2	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,2	0,1		0,2			4	20,76	21,14
88	A2	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,1	0,2	0,2			22	13,91	16,24
89	A3	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,2			0,2		2	31,54	21,75
90	A3	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,2	0,1				0,2	6	19,38	19,15
91	A3	2,5	1234	2,5	203	**0,99973**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	26,83	20,61
92	A1	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000	0,2	0,2							3	20,23	25,20
93	A2	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,2	0,2				22	11,58	19,30
94	A2	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,2	0,1		0,2			4	17,28	25,39
95	A2	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,1	0,2	0,2			22	11,61	19,25

Case	**Model**	**Distribution parameters**				**Reliability parameter**	**Cost parameters**			**Probability parameters**								**Optimal Maintenance schedule**		
		δ_X	θ_X	δ_H	θ_H	R_{obj}	c_0	c_p	c_f	α	β	α_0	β_0	c_α	c_β	γ_α	γ_β	M^*	T^*	$C(M^*,T^*)$
96	A3	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,2			0,2		2	26,25	26,12
97	A3	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,2	0,1				0,2	5	17,24	23,07
98	A3	2,5	1234	2,5	203	**0,99987**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	22,29	24,77
99	A1	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000	0,2	0,2							3	18,45	27,63
100	A2	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,2	0,2				23	10,54	21,04
101	A2	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,2	0,1		0,2			4	15,76	27,83
102	A2	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,1	0,2	0,2			25	10,50	20,99
103	A3	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,2			0,2		2	25,26	27,14
104	A3	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,2	0,1				0,2	5	15,69	25,33
105	A3	2,5	1234	2,5	203	**0,99991**	100	1.000	100.000			0,1	0,1			0,2	0,2	3	20,31	27,17

H

Calculating a two phase MTTF

This chapter of the Appendix elaborates on the derivation of a two phase MTTF. The asset's mean time to failure $MTTF$ consists of two parts: time to defect x and the delay time h. The mean value for the time to failure can, by definition, be mathematically expressed by the mean values of the time to defect $E[X]$ and the delay time $E[H]$. To formally present the relation:

$$MTTF = E[X] + E[H]$$

Based on the relation above the $MTTF$ is decomposed into the expected value or the mean value of the time to defect $E[X]$ and the mean value of the delay time $E[H]$. On a continuous scale both mean values can be derived from their own probability distributions. From basic probability theory one then obtains the expression for the $MTTF$ in terms of integrals of the time to defect x and delay time h.

$$MTTF = E[X] + E[H] = \int_0^\infty x f_X(x) dx + \int_0^\infty h f_H(h) dh$$

I

MATLAB Scripts

I.1 Graphical User Interface

Listing I.1: Graphical User Interface

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%   Master Thesis                              %%%%%
%%%%%   Eindhoven University of Technology         %%%%%
%%%%%   NedTrain                                   %%%%%
%%%%%                                              %%%%%
%%%%%   J.P.C. Driessen (0633109)                  %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Init
%Clear the dash all open variables and figures
close all
clear all
clc

%General parameters
M_con = 25;          % Last inspection instance M in the enumeration
stepsize = 1e-10;    % Stepsize of the reliability analysis
alpha_0 = 0.10;      % Inevitable probability of false positive
beta_0 = 0.10;       % Inevitable probability of false negative
c_alpha = 0.20;      % Scaling variable of dependent false positive probability
c_beta = 0.20;       % Scaling variable of dependent false positive probability
c_0 = 1e2;           % Inspection costs
c_p = 1e3;           % Maintenance action costs
c_f = 1e5;           % Failure costs
A = 365*40;          % Number of days for the reliability requirement
Robj = 0.99987;      % Objective reliability from (0,A)
approach = 2;        % Number of the approach under consideration

%Time to defect and delay time parameters
theta_X = 1234;      % Weibull scale parameter of time to defect
delta_X = 2.5;       % Weibull shape paramter of time to defect (if 1 then EXP)
theta_D = 203;       % Weibull scale parameter of delay time
delta_D = 2.5;       % Weibull shape paramter of delay time (if 1 then EXP)

MTTF = wblstat(theta_X,delta_X)+wblstat(theta_D,delta_D);
int_UB = wblinv(1-1e-15,theta_D,delta_D);

%Note that using an integral limit which yields probability values near 0
%improves code speed tremendously compared to the infinity case.

opts2 = optimset('fmincon');
opts2 = optimset(opts2, 'Algorithm', 'interior-point');
```

```
%% Cost evaluation
%Note that low values of the objective reliability function result in
%long evaluation times because the T_UB --> infinity. To remedy, author
%suggests to artificially alter the boundaries for T considered to
%reasonable values.

tic;
Tmax=0;
for M =1:M_con

    nonlcon = @(T)fn_reliability_optimisation(M,T,c_0,c_p,c_f, Robj, ...
        stepsize,approach, c_alpha, c_beta, alpha_0, beta_0, theta_X,...
        delta_X,theta_D, delta_D,MTTF,int_UB,A);

    T_UB = fmincon(@(T) -T,5,-1,-1,[],[],[],[],nonlcon,opts2)

    if T_UB>Tmax
       Tmax = T_UB;
    end


    for T=1:floor(T_UB)
        cost(T,M) = fn_cost_evaluation(M,T,c_0,c_p,c_f,...
            approach, c_alpha, c_beta, alpha_0, beta_0, theta_X, ...
            delta_X,theta_D, delta_D,int_UB);
    end
    cost(T+1:Tmax,M) = NaN;

end
toc
figure(2)
plot(cost);

Legend=cell(M_con,1);
for iter=1:M_con
  Legend{iter}=strcat('M=', num2str(iter));
end
legend(Legend);

%% Cost function optimisation
% Enable the matlabpool function by
% matlabpool
opts = optimset('fmincon');
opts = optimset(opts, 'Algorithm', 'interior-point','UseParallel',...
    'always');

Aineq = [-1];
bineq = [-1];
Aeq = [];
beq = [];
lb = [];
ub = [];
nonlcon = [];
Interval = zeros(1,M_con);
fval = zeros(1,M_con);
LB = zeros(1,M_con);
UB = zeros(1,M_con);
timer = cputime;

for M=1:M_con
    tic
    M
    nonlcon = @(T)fn_reliability_optimisation(M,T,c_0,c_p,c_f, Robj,...
        stepsize,approach, c_alpha, c_beta, alpha_0, beta_0, theta_X, ...
        delta_X,theta_D, delta_D,MTTF,int_UB,A);

    %first row denotes the case of Robj = 0 in case nothing is entered.
    %When it is entered the first corresponds to the optimisation given the
    %Robj
    if M==1

        [Interval(1,M),fval(1,M)] = fmincon(@(T)fn_cost_evaluation...
            (M,T,c_0,c_p,c_f, approach, c_alpha, c_beta, alpha_0,...
```

```
            beta_0, theta_X, delta_X,theta_D,...
            delta_D,int_UB),MTTF,Aineq,bineq,Aeq,beq,lb,ub,nonlcon,opts)
    else
        [Interval(1,M),fval(1,M)] = fmincon(@(T)fn_cost_evaluation...
            (M,T,c_0,c_p,c_f, approach, c_alpha, c_beta, alpha_0,...
            beta_0, theta_X, delta_X,theta_D,...
            delta_D,int_UB),Interval(1,M-1)*0.9,Aineq,bineq,Aeq,...
            beq,lb,ub,nonlcon,opts)

    end

    %Determining the upper and lower bounds of the solution
    [lowval(1,M),lowval(2,M),lowval(3,M),lowval(4,M)] = ...
        fn_cost_evaluation(M,Interval(1,M),c_0,c_p,c_f, ...
        approach, 0, 0, alpha_0, beta_0, theta_X, delta_X,theta_D,...
        delta_D,int_UB);

    [lowval2(1,M),lowval2(2,M),lowval2(3,M),lowval2(4,M)] = ...
        fn_cost_evaluation(M,Interval(1,M),c_0,c_p,c_f,...
        approach, 0, 0, alpha_0+c_alpha, beta_0, theta_X, delta_X,...
        theta_D, delta_D,int_UB);

    [highval(1,M),highval(2,M),highval(3,M),highval(4,M)] = ...
        fn_cost_evaluation(M,Interval(1,M),c_0,c_p,c_f, ...
        approach, 0, 0 , alpha_0, beta_0+c_beta, theta_X, delta_X,...
        theta_D, delta_D,int_UB);

    [highval2(1,M),highval2(2,M),highval2(3,M),highval2(4,M)] = ...
        fn_cost_evaluation(M,Interval(1,M),c_0,c_p,c_f, ...
        approach, 0, 0, alpha_0+c_alpha, beta_0+c_beta, theta_X,...
        delta_X,theta_D, delta_D,int_UB);

    EK_vec = [lowval(2,M) lowval2(2,M) highval(2,M) highval2(2,M)];
    Pcp_vec = [lowval(3,M) lowval2(3,M) highval(3,M) highval2(3,M)];
    ECL_vec = [lowval(4,M) lowval2(4,M) highval(4,M) highval2(4,M)];

    LB(M) = (min(EK_vec)*c_0+c_f+(c_p-c_f)* ...
        max(Pcp_vec))/(max(ECL_vec));

    UB(M) = (max(EK_vec)*c_0+c_f+(c_p-c_f)* ...
        min(Pcp_vec))/(min(ECL_vec));
    toc
end

Interval2 = Interval.';
fval2 = fval.';
LB2 = LB.';
UB2 = UB.';
```

I.2 Cost function

Listing I.2: Cost evaluation function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%   Master Thesis                       %%%%%
%%%%%   Eindhoven University of Technology  %%%%%
%%%%%   NedTrain                            %%%%%
%%%%%                                       %%%%%
%%%%%   J.P.C. Driessen (0633109)           %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Cost evaluation %%

function [y, EK, P_cp, ECL] = fn_cost_evaluation...
    (M,T,c_0,c_p,c_f, approach, c_alpha, c_beta, alpha_0, beta_0,...
    theta_X, delta_X,theta_D, delta_D,int_UB)

EK = 0;
P_cp = 0;
```

```
ECL = 0;

Term1 = zeros(M-1,M-1);
Term2 = Term1;
Term3 = zeros(1,M-1);
Term4 = Term3;

%% Calculations
for j=1:M-1
    hmin = @(x) j*T-x;
    hmax = @(x) (j+1)*T-x;
    for i=1:j
        %Term 1 E(K) and Term 1 P_cp and Term 3 ECL
        Term1(i,j) = integral2(@(x,h) ...
            fn_one_minus_alpha(approach, alpha_0, c_alpha,x,i-1, T).* ...
            fn_beta(approach, beta_0, c_beta, x, h, i, j-1, T).*...
            fn_one_minus_beta(approach, beta_0,c_beta, j, x, h, T).*...
            delta_X./theta_X.*(x./theta_X).^(delta_X-1).*exp(-(x./ ...
            theta_X).^delta_X).*delta_D./theta_D.*(h./theta_D).^ ...
            (delta_D-1).*exp(-(h./theta_D).^delta_D),(i-1)*T,i*T, ...
            hmin,int_UB);

        %Term 2
        Term2(i,j) = integral2(@(x,h) ...
            fn_one_minus_alpha(approach, alpha_0, c_alpha,x,i-1,T) .* ...
            fn_beta(approach, beta_0, c_beta, x, h, i, j, T).*...
            delta_X./theta_X.*(x./theta_X).^(delta_X-1).*exp(-(x./ ...
            theta_X).^delta_X).*delta_D./theta_D.*(h./theta_D).^ ...
            (delta_D-1).*exp(-(h./theta_D).^delta_D), (i-1)*T, i*T, ...
            hmin, hmax);
    end

    % Term 3
    Term3(j) = integral(@(x) wblcdf((j+1)*T-x,theta_D ...
        , delta_D).*fn_one_minus_alpha(approach, alpha_0, c_alpha, ...
        x,j,T).*delta_X./theta_X.*(x./theta_X).^(delta_X-1).*...
        exp(-(x./theta_X).^delta_X), j*T, (j+1)*T);

    % Term 4 P_EK and Term 2 P_cp and Term 6 ECL

    Term4(j) = integral(@(x) delta_X./theta_X.*(x./theta_X).^(delta_X-1)...
        .*exp(-(x./theta_X).^delta_X) .* ...
        fn_alpha(approach, alpha_0, c_alpha,x,j,T)...
        .*fn_one_minus_alpha(approach, alpha_0, c_alpha,x ,j-1, T),...
        j*T,inf);

end

if M>1
    EK = sum([1:M-1].*(sum(Term1,1)+sum(Term2,1)+Term3+Term4));
    P_cp = sum((sum(Term1,1)+Term4));
    ECL = ECL + sum([1:M-1].*T.*(sum(Term1,1)+Term4));
end

hmin = @(x) M*T-x;
for i=1:M
    % Term 5 EK and term 3 P_cp and Term 4 ECL
    Term5(i) = integral2(@(x,h)...
        fn_one_minus_alpha(approach, alpha_0, c_alpha,x,i-1, T) .* ...
               fn_beta(approach, beta_0, c_beta, x, h, i, M-1, T).*...
               delta_X./theta_X.*(x./theta_X).^(delta_X-1).*exp(-(x./ ...
               theta_X).^delta_X).*delta_D./theta_D.*(h./theta_D).^ ...
               (delta_D-1).*exp(-(h./theta_D).^delta_D),(i-1)*T,i*T,...
               hmin,int_UB);

    %Term 1 ECL
    hmax = @(x) i*T-x;
    Term1_ECL (i) = integral2(@(x,h) (x+h).* ...
        fn_one_minus_alpha(approach, alpha_0, c_alpha,x,i-1, T) .* ...
        delta_X./theta_X.*(x./theta_X).^(delta_X-1).*exp(-(x./theta_X)...
        .^delta_X).*delta_D./theta_D.*(h./theta_D).^(delta_D-1)...
        .*exp(-(h./theta_D).^delta_D), (i-1)*T, i*T, 0, hmax);
end
```

```
%Term 6 P_EK and Term 4 P_cp and Term 5 ECL
Term6 = integral(@(x) fn_one_minus_alpha(approach, alpha_0, c_alpha,x,...
    M-1, T).*delta_X./theta_X.*(x./theta_X).^(delta_X-1)...
    .*exp(-(x./theta_X).^delta_X),M*T,inf);

EK = EK + M*(sum(Term5) + Term6);

% Term 4 P_cp
P_cp = P_cp + sum(Term5) + Term6;

ECL = ECL + sum(Term1_ECL) + M*T*(sum(Term5) + Term6);

%% Calculating expected cycle length

for i=1:M-1
    for j=i:M-1
        % Term 2
        hmin = @(x) j*T-x;
        hmax = @(x) (j+1)*T-x;
        ECL = ECL + integral2(@(x,h)...
            fn_one_minus_alpha(approach, alpha_0, c_alpha,x,i-1, T) .* ...
            fn_beta(approach, beta_0, c_beta, x, h, i, j, T).*...
            (x+h).*delta_X./theta_X.*(x./theta_X).^(delta_X-1)...
            .*exp(-(x./theta_X).^delta_X).*delta_D./theta_D.*(h./ ...
            theta_D).^(delta_D-1).*exp(-(h./theta_D).^delta_D),...
            (i-1)*T,i*T,hmin,hmax);
    end
end

%% Calculating average costs
y = (c_0*EK+P_cp*c_p+(1-P_cp)*c_f)/ECL;

end
```

I.3 Reliability function

Listing I.3: Reliability evaluation function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%   Master Thesis                      %%%%%
%%%%%   Eindhoven University of Technology %%%%%
%%%%%   NedTrain                           %%%%%
%%%%%                                      %%%%%
%%%%%   J.P.C. Driessen (0633109)          %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Reliability optimisation %%

function [c ceq] = fn_reliability_optimisation(M,T,c_0,c_p,c_f, Robj, ...
    stepsize, approach, c_alpha, c_beta, alpha_0, beta_0, theta_X, ...
    delta_X,theta_D,delta_D, MTTF,int_UB,A)

R=zeros(round(M),1);

Cycle_max = floor(A/(M*T));
s_value_max = ceil((A-Cycle_max*M*T)/T);

if Cycle_max >= 1
    for s=1:M
        for j=1:s-1
            for i=j:s-1
               hmin = @(x) i*T-x;
               Term1(j,i) = integral2(@(x,h) fn_one_minus_beta(approach, beta_0, ...
                   c_beta, i, x, h, T) ...
                   .*fn_one_minus_alpha(approach, alpha_0, c_alpha,x,j-1, T) ...
                   .*fn_beta(approach, beta_0, c_beta, x, h, j, i-1, T).*...
                    delta_X./theta_X.*(x./theta_X).^(delta_X-1).*exp(-(x./...
                    theta_X).^delta_X).*delta_D./theta_D.*(h./theta_D).^...
                    (delta_D-1).*exp(-(h./theta_D).^delta_D),(j-1)*T, j*T,...
```

```
                    hmin,int_UB);
            end
        end

        t=s*T-stepsize;
        if s>1
            Term1_sum = sum(Term1,1);

            for i=1:length(Term1_sum)
              Rel(:,i) = R(s-i);
            end
            R(s)=R(s)+sum(Rel.*Term1_sum);
        end

        for j=1:s-1
           %Term 2
           hmin = @(x) t-x;
           R(s)=R(s)+integral2(@(x,h) fn_beta(approach, beta_0, c_beta, x,...
               h, j, s-1, T).*fn_one_minus_alpha(approach, alpha_0, ...
               c_alpha,x,j-1, T).*delta_X./theta_X.*(x./theta_X).^...
               (delta_X-1).*exp(-(x./theta_X).^delta_X).*delta_D./theta_D.*...
               (h./theta_D).^(delta_D-1).*exp(-(h./theta_D).^delta_D),...
               (j-1)*T,j*T,hmin,int_UB);
           %Term 3
           R(s)=R(s)+ integral(@(x) fn_one_minus_alpha(approach, alpha_0,...
               c_alpha,x,j-1, T).*fn_alpha(approach, alpha_0, c_alpha,...
               x, j,T).*delta_X./theta_X.*(x./theta_X).^(delta_X-1)...
               .*exp(-(x./theta_X).^delta_X),j*T,inf)*R(s-j);
        end

        %Term 4
        R(s)=R(s)+integral(@(x) fn_one_minus_alpha(approach, alpha_0,...
            c_alpha,x,s-1, T).*delta_X./theta_X.*(x./theta_X).^...
            (delta_X-1).*exp(-(x./theta_X).^delta_X)...
            .*exp(-((t-x)./theta_D).^delta_D), (s-1)*T, t);
        %Term 5
        R(s)=R(s)+integral(@(x) fn_one_minus_alpha(approach, alpha_0,...
            c_alpha,x,s-1, T).*delta_X./theta_X.*(x./theta_X).^...
            (delta_X-1).*exp(-(x./theta_X).^delta_X),t,inf);
    end
    R2 = R(M);
else
    R2 = 1;
end

% HERE COMES THE OTHER LOOP TO CALCULATE THE LAST INSTANCE!!

clear R
R=zeros(round(s_value_max),1);
clear Term1
clear Rel
clear Term1_sum

for s=1:s_value_max
    for j=1:s-1
        for i=j:s-1
           hmin = @(x) i*T-x;
           Term1(j,i) = integral2(@(x,h) fn_one_minus_beta(approach, beta_0, ...
               c_beta, i, x, h, T) ...
               .*fn_one_minus_alpha(approach, alpha_0, c_alpha,x,j-1, T) ...
               .*fn_beta(approach, beta_0, c_beta, x, h, j, i-1, T).*...
                delta_X./theta_X.*(x./theta_X).^(delta_X-1).*exp(-(x./...
                theta_X).^delta_X).*delta_D./theta_D.*(h./theta_D).^...
                (delta_D-1).*exp(-(h./theta_D).^delta_D),(j-1)*T, j*T,...
                hmin,int_UB);
        end
    end
    if s>1
        Term1_sum = sum(Term1,1);

        for i=1:length(Term1_sum)
          Rel(:,i) = R(s-i);
        end
```

```
        R(s)=R(s)+sum(Rel.*Term1_sum);
    end


    if s<s_value_max
        t=s*T-stepsize;
    else
        t = A - Cycle_max*M*T-stepsize;
    end

    for j=1:s-1
       %Term 2
       hmin = @(x) t-x;
       R(s)=R(s)+integral2(@(x,h) fn_beta(approach, beta_0, c_beta, x,...
           h, j, s-1, T).*fn_one_minus_alpha(approach, alpha_0, ...
           c_alpha,x,j-1, T).*delta_X./theta_X.*(x./theta_X).^...
           (delta_X-1).*exp(-(x./theta_X).^delta_X).*delta_D./theta_D.*...
           (h./theta_D).^(delta_D-1).*exp(-(h./theta_D).^delta_D),...
           (j-1)*T,j*T,hmin,int_UB);
       %Term 3
       R(s)=R(s)+ integral(@(x) fn_one_minus_alpha(approach, alpha_0,...
           c_alpha,x,j-1, T).*fn_alpha(approach, alpha_0, c_alpha,...
           x, j,T).*delta_X./theta_X.*(x./theta_X).^(delta_X-1)...
           .*exp(-(x./theta_X).^delta_X),j*T,inf)*R(s-j);
    end

    %Term 4
    R(s)=R(s)+integral(@(x) fn_one_minus_alpha(approach, alpha_0,...
        c_alpha,x,s-1, T).*delta_X./theta_X.*(x./theta_X).^...
        (delta_X-1).*exp(-(x./theta_X).^delta_X)...
        .*exp(-((t-x)./theta_D).^delta_D), (s-1)*T, t);
    %Term 5
    R(s)=R(s)+integral(@(x) fn_one_minus_alpha(approach, alpha_0,...
        c_alpha,x,s-1, T).*delta_X./theta_X.*(x./theta_X).^...
        (delta_X-1).*exp(-(x./theta_X).^delta_X),t,inf);
end
if s_value_max ==0
    R3 = R2^(Cycle_max);
else
    R3 = R(s_value_max)*R2^(Cycle_max);
end

c=Robj-R3;
ceq=[];
```

I.4 α function

Listing I.4: α function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%   Master Thesis                     %%%%%
%%%%%   Eindhoven University of Technology %%%%%
%%%%%   NedTrain                          %%%%%
%%%%%                                     %%%%%
%%%%%   J.P.C. Driessen (0633109)         %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Alpha %%

function y = fn_alpha(approach, alpha_0, c_alpha, x, q, T)

    if approach == 1
        y = alpha_0+c_alpha;

    elseif approach == 2
       y = alpha_0+c_alpha*q.*T./x;
    elseif approach == 3
       y = alpha_0+c_alpha*(1-1/T);
    end
```

```
end
```

I.5 $1-\alpha$ function

Listing I.5: 1-α function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%   Master Thesis                        %%%%%
%%%%%   Eindhoven University of Technology   %%%%%
%%%%%   NedTrain                             %%%%%
%%%%%                                        %%%%%
%%%%%   J.P.C. Driessen (0633109)            %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% 1-Alpha %%

function y = fn_one_minus_alpha(approach, alpha_0, c_alpha, x...
    ,prod_UB, T)

    if approach == 1
        y = (1-(alpha_0+c_alpha))^(prod_UB);

    elseif approach == 2

       y=1;
       for n=1:prod_UB
           alpha_H = (c_alpha*n.*T./x);
           y = y.*(1-(alpha_0+alpha_H));
       end
    elseif approach == 3
        y = (1-(alpha_0+c_alpha*(1-1/T)))^(prod_UB);

    end
end
```

I.6 β function

Listing I.6: β function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%   Master Thesis                        %%%%%
%%%%%   Eindhoven University of Technology   %%%%%
%%%%%   NedTrain                             %%%%%
%%%%%                                        %%%%%
%%%%%   J.P.C. Driessen (0633109)            %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% Beta %%

function y = fn_beta(approach, beta_0, c_beta, x, h, prod_LB, prod_UB, T)

    if approach == 1
        y = (beta_0+c_beta)^(prod_UB-prod_LB+1);

    elseif approach == 2

       y=1;
       for k=prod_LB:prod_UB
           beta_H = c_beta*(1-(k*T-x)./h);
           y = y.*(beta_0+beta_H);
       end

    elseif approach == 3
       y = (beta_0+c_beta*1/T)^(prod_UB-prod_LB+1);
```

```
    end
end
```

I.7 $1 - \beta$ function

Listing I.7: β function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%   Master Thesis                         %%%%%
%%%%%   Eindhoven University of Technology    %%%%%
%%%%%   NedTrain                              %%%%%
%%%%%                                         %%%%%
%%%%%   J.P.C. Driessen (0633109)             %%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% 1-Beta %%

function y = fn_one_minus_beta(approach, beta_0, c_beta, q, x, h, T)

    if approach == 1
        y = 1-(beta_0+c_beta);

    elseif approach == 2

           beta_H = c_beta*(1-(q*T-x)./h);
           y = 1-(beta_0+beta_H);

    elseif approach == 3
        y = (1-(beta_0+c_beta*1/T));
    end
end
```