

## MASTER

### Local search heuristics for pollution-routing problem with multiple vehicle types and deadlines

Saka, O.C.

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**Local Search Heuristics for  
Pollution-Routing Problem with  
Multiple Vehicle Types and  
Deadlines**

by

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## **ABSTRACT**

Vehicle Routing Problem (VRP) is one of the most widely studied problems in logistics literature. Up to now, many different types of exact solution methods and heuristics have been developed in order to deal with various variants of this computationally complex optimization problem. However, only a few researchers have included the concepts of speed control, fuel consumption and greenhouse gas (GHG) emissions in their studies.

The first part of this study is dedicated to a special variant of VRP called the Pollution-Routing Problem (PRP), which includes a comprehensive cost function that takes into account fuel consumption, GHG emissions and driver wages. An extension of PRP incorporating multiple vehicle types and deadlines is considered. Throughout this part, firstly two alternative exact solution methods are proposed: a Mixed Integer Programming model with a piecewise linear cost function and a Mixed Integer Second Order Cone Programming model, followed by local search heuristics with a special initialization algorithm and optimal travel time determination procedure. Results of experiments are interpreted in an extensive computational study.

In the second part (See Appendix E), the report of an applied project is represented. The project took place in a third-party logistics (3PL) company in the Netherlands with the aim of investigating the possible improvements that can be achieved via employing a multi-depot and automated planning approach.

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# CHAPTER 1

## INTRODUCTION

Logistics can be defined as the management of flow of resources between supply and demand points in order to meet certain requirements of customers. Physical item logistics can be considered as a combination of materials management and distribution [1]. Distribution costs make a major contribution to total costs in many types of organizations. Therefore, even the slightest reduction in the distribution costs draws the special attention of decision makers.

Vehicle Routing Problem (VRP) is a combinatorial optimization and integer programming problem seeking to find an optimal set of routes for a fleet of vehicles in order to satisfy the demand of a set of customers. VRP is considered to be one of the most critical elements in the physical distribution and logistics literature [2]. It has drawn the attention of researchers since early 1960s, and numerous kinds of methods have been presented for the sake of solving a wide range of VRP variants so far.

The aim of this study is to propose methodologies for finding optimal or near optimal solutions to a special variant of VRP, which takes into account fuel, greenhouse gas (GHG) emission and driver costs based on vehicle speeds and amount of load carried as well as the total distance traveled by the vehicles. This variant was named as the Pollution-Routing Problem (PRP) by Bektas and Laporte [3]. Even though the VRP literature is known to be quite rich, only a few authors have incorporated such a comprehensive cost function so far. In the majority of studies on VRP, researchers have tried to minimize the total travel distance over a variety of environmental settings. The consideration of the minimization of energy and GHG emissions is a quite recent subject. Global warming has become most serious environmental problem in today's world and GHG emitted by freight transportation is one of the significant contributors to this problem. Consequently, there is clearly room for improvement in VRPs in this respect. With this study, we intend to make a contribution to this field of research.

The work of Bektas and Laporte [3] serves as a basis for this study. Throughout this thesis, alternative approaches to PRP with multiple vehicle types and deadlines are addressed. Following this introductory chapter, a literature review on general VRPs and VRPs with fuel and energy considerations is represented. It is followed by a formal definition of the problem and description of two alternative mathematical programming formulations. The first model is an extension to the formulation presented by Bektas and Laporte [3] by means of the addition of multiple vehicle types. The second model we address is a Mixed Integer Second Order Cone Programming (MISOCP) formulation which provides complete control over the vehicle speeds and travel times. Afterwards, a sub-model for determining optimal vehicle speeds is introduced, and several properties in conjunction with this model are represented. These properties are used as a key in the development of an optimal speed determination procedure and computation of estimates to change in the total cost as a result of alterations in the structure of the vehicle routes.

Since VRP is a computationally complex problem, exact methods fail to generate optimal solutions even in small instances due to time and memory requirements. The addition of a complicated cost function renders it more difficult to solve. Therefore, employment of heuristic methods is a necessity for finding

good solutions to real-life instances. In Chapter 5, we present three basic local search heuristics for our specific problem. The first heuristic makes use of cost change estimates while searching for improving neighbor solutions whereas the second method selects exactly the best solution in the neighborhood. Hybrid heuristic is a mixture of these two approaches; it determines a certain number of promising alternatives based on the cost change estimates and select the best option among them by computing their exact solutions. Also, a fast solution heuristic is introduced at the end of Chapter 6, to be implemented to very large networks when the available solution time is quite limited. A comprehensive computational analysis is represented in Chapter 6 with comparisons between each solution approach and a further scenario analysis for changing environmental settings.

In this study, the problem addressed and approaches taken are different from the similar examples in the literature in several aspects. Firstly, the vehicle fleet is assumed to be heterogeneous, which allows vehicles of multiple types to be employed in the same routing strategy. Vehicle speeds and travel times are endogenously decided within the problem which creates the opportunity to explore more solution alternatives. In a large portion of heuristic approaches in the VRP literature, iterations are performed randomly whereas we make use of cost change estimations for determining the steps to be taken. We hope that this contribution will lead to further research initiatives and practical applications for a more efficient planning in distribution logistics.

In the next section we will give an overview of the literature on general VRPs and VRPs with fuel and energy considerations.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1. Literature on General VRPs

The vehicle routing problem (VRP) is a generalization of the *Traveling Salesman Problem* (TSP) and has the following differing characteristics: The problem allows a number of salesmen instead of one, which in the case of VRP are trucks, to depart from and return to the starting point (e.g. depot) after visiting a set of demand points (e.g. customers). The trucks have a fixed capacity and the total demand of the customers in a single trip cannot exceed the capacity of the vehicle attached to it. Given the distances between each location pair in the problem network, the problem becomes minimizing the total travel cost, which is generally expressed in terms of the total distance traveled, by finding one or more trips that start and end at the depot covering all the customers and satisfying the capacity constraints.

The first research in the field of VRP was conducted in late 1950s and early 1960s, and VRP practices has expanded to a broad range ever since. VRP was originally posed by Dantzig and Ramser [4] in 1959 under the name of *The Truck Dispatching Problem*. Clarke and Wright (1964) [5] carried this study one step further by incorporating the use of multiple vehicles. *Vehicle routing* term is firstly used by Golden et al. (1977) [6]. Since then, a large number of problem variants have emerged and the related literature has become quite disjoint and disparate.

In order to provide a guideline for keeping track of the developments in the VRP literature, Eksioglu et al. [7] presented a taxonomic framework for this problem. Figure 1 summarizes the classification framework that has been posed by Eksioglu et al. [7].

1. Type of Study	2.8. Backhauls	3.9. Vehicle homogeneity (Capacity)
1.1. Theory	2.8.1. Nodes request simultaneous pick ups and deliveries	3.9.1. Similar vehicles
1.2. Applied methods	2.8.2. Nodes request either linehaul or backhaul service, but not both	3.9.2. Load-specific vehicles
1.2.1. Exact methods	2.9. Node/Arc covering constraints	3.9.3. Heterogeneous vehicles
1.2.2. Heuristics	2.9.1. Precedence and coupling constraints	3.9.4. Customer-specific vehicles
1.2.3. Simulation	2.9.2. Subset covering constraints	3.10. Travel time
1.2.4. Real time solution methods	2.9.3. Re course allowed	3.10.1. Deterministic
1.3. Implementation documented	3. Problem Physical Characteristics	3.10.2. Function dependent (a function of current time)
1.4. Survey, review or meta-research	3.1. Transportation network design	3.10.3. Stochastic
2. Scenario Characteristics	3.1.1. Directed network	3.10.4. Unknown
2.1. Number of stops on route	3.1.2. Undirected network	3.11. Transportation cost
2.1.1. Known (deterministic)	3.2. Location of addresses (customers)	3.11.1. Travel time dependent
2.1.2. Partially known, partially probabilistic	3.2.1. Customers on nodes	3.11.2. Distance dependent
2.2. Load splitting constraint	3.2.2. Arc routing instances	3.11.3. Vehicle dependent
2.2.1. Splitting allowed	3.3. Geographical location of customers	3.11.4. Operation dependent
2.2.2. Splitting not allowed	3.3.1. Urban (scattered with a pattern)	3.11.5. Function of lateness
2.3. Customer service demand quantity	3.3.2. Rural (randomly scattered)	3.11.6. Implied hazard/risk related
2.3.1. Deterministic	3.3.3. Mixed	4. Information Characteristics
2.3.2. Stochastic	3.4. Number of points of origin	4.1. Evolution of information
2.3.3. Unknown	3.4.1. Single origin	4.1.1. Static
2.4. Request times of new customers	3.4.2. Multiple origins	4.1.2. Partially dynamic
2.4.1. Deterministic	3.5. Number of points of loading/unloading facilities (depot)	4.2. Quality of information
2.4.2. Stochastic	3.5.1. Single depot	4.2.1. Known (Deterministic)
2.4.3. Unknown	3.5.2. Multiple depots	4.2.2. Stochastic
2.5. On site service/waiting times	3.6. Time window type	4.2.3. Forecast
2.5.1. Deterministic	3.6.1. Restriction on customers	4.2.4. Unknown (Real-time)
2.5.2. Time dependent	3.6.2. Restriction on roads	4.3. Availability of information
2.5.3. Vehicle type dependent	3.6.3. Restriction on depot/hubs	4.3.1. Local
2.5.4. Stochastic	3.6.4. Restriction on drivers/vehicle	4.3.2. Global
2.5.5. Unknown	3.7. Number of vehicles	4.4. Processing of information
2.6. Time window structure	3.7.1. Exactly $n$ vehicles ( <i>TSP in this segment</i> )	4.4.1. Centralized
2.6.1. Soft time windows	3.7.2. Up to $n$ vehicles	4.4.2. Decentralized
2.6.2. Strict time windows	3.7.3. Unlimited number of vehicles	5. Data Characteristics
2.6.3. Mix of both	3.8. Capacity consideration	5.1. Data Used
2.7. Time horizon	3.8.1. Capacitated vehicles	5.1.1. Real world data
2.7.1. Single period	3.8.2. Uncapacitated vehicles	5.1.2. Synthetic data
2.7.2. Multi period		5.1.3. Both real and synthetic data
		5.2. No data used

Figure 1. Taxonomy of the VRP Literature [7]

Eksioglu et al. [7] suggest that the VRP literature can be classified into 5 main headings. The first point of separation is the type of study. We will generally focus on applied methods. Scenario characteristics and physical characteristics designate how the problem environment is set up. Due to the nature of the problem, it can be adapted to numerous scenarios, which is the main reason for the branching out of the relevant studies in the literature. Information and data characteristics determine the properties of the information and data used in these studies.

Let us summarize the VRP practices in terms of the applied solution methods, scenarios and physical characteristics.

### 2.1.1. Applied Solution Methods

In the VRP literature, different kinds of exact solution and heuristic methodologies have been developed so far. VRP is an NP-hard problem [8] and heuristics generate more time efficient solutions than exact solution methods, especially for large problem instances. Therefore more focus has been concentrated on heuristic approaches.

### Exact Solution Methods:

Laporte and Norbert [9] suggest that the exact solution methods in VRP literature can be classified into three categories:

- i) Direct tree search methods
- ii) Dynamic programming (DP)
- iii) Integer linear programming (ILP)

With direct tree search methods, it is possible to sequentially generate routes via branch and bound trees [9]. Laporte et al. [10] have posed an assignment based lower bound and a related branch and bound algorithm for asymmetric capacitated VRPs whereas Christofides et al. [11] have presented tree search algorithms incorporating lower bounds computed from shortest spanning k-degree centre tree and q-routes. Hadjiconstantinou et al. [12] have used these lower bounds and obtained improved solutions.

The use of dynamic programming (DP) for solving VRPs was first brought into question by Eilon et al. [13] in 1971. The method they have proposed required an excessive number of computations due to the growth of the state space. Christofides et al. [14] have provided a state-space relaxation procedure for reducing the number of states. DP applications were made use of by several other authors, usually in case of dynamic or stochastic demands, including [15], [16] and [17]. However, DP has not drawn as much attention as other solution methodologies in the VRP literature.

The last category, ILP formulations is a fairly broad subject and a large number of authors have presented different formulations that deal with the problem from distinct approaches. Magnanti [18] suggests that the ILP formulations for VRPs can be categorized into three subjects:

- Integer linear programming (ILP)
  - a. Set partitioning formulations
  - b. Vehicle flow formulations
  - c. Commodity flow formulations

The idea of formulating VRP as a set partitioning problem was initiated by Balinski and Quandt [19]. The formulation they propose is quite simple: Assume that  $j \in J$  represents a feasible route,  $c_j$  is the cost of performing route  $j$  and  $a_{ij}$  is a binary parameter that takes the value of 1 if node  $i \in N$  is included in route  $j$  and 0, otherwise. Note that the depot is denoted by 0 and binary decision variable  $x_j$  becomes 1 if route  $j$  is active and 0, otherwise. The resulting set partitioning formulation (SPF) is:

$$\text{Minimize } \sum_{j \in J} c_j x_j \quad (2.1)$$

subject to

$$\sum_{j \in J} a_{ij} x_j = 1 \quad \forall i \in N \setminus \{0\} \quad (2.2)$$

$$x_j \in \{0, 1\} \quad \forall j \in J \quad (2.3)$$



Expression (2.1) stands for the cost function to be minimized and constraint (2.2) assures that each customer is visited exactly once. Laporte [2] states that there are two main difficulties in solving such a formulation. Since the number of feasible routes gets significantly higher as the problem size grows, the number of binary variables also increases dramatically and it becomes more and more difficult to capture the total cost of a route. In order to overcome these difficulties, many authors have employed a column generation approach, in which a linear program is initialized with a small subset of variables one or more variables are added step by step if they appear to be improving [20]. Agarwal et al. [21] were the first ones to follow this approach. Other instances of similar practices can be found in [22], [23], [24], [25], [26] and [27].

Vehicle flow formulations are by far the most widely used models among the exact methods for VRPs. This type of models can be classified in general by the use of a two-index or three-index vehicle flow variable. In two-index formulations, binary flow variable  $x_{ij}$  represents whether a vehicle travels from node  $i$  to node  $j$  and in three-index formulations, a new dimension  $k$  is used for differentiating vehicles or vehicle types. The following simple two-index model, which was presented by Toth and Vigo [28] is a basic illustration of vehicle flow models:

$$\text{Minimize } \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} \quad (2.4)$$

subject to

$$\sum_{i \in N} x_{ij} = 1 \quad \forall j \in N \setminus \{0\} \quad (2.5)$$

$$\sum_{j \in N} x_{ij} = 1 \quad \forall i \in N \setminus \{0\} \quad (2.6)$$

$$\sum_{i \in N} x_{i0} = K \quad (2.7)$$

$$\sum_{j \in N} x_{0j} = K \quad (2.8)$$

$$\sum_{i \notin S} \sum_{j \in S} x_{ij} \geq r(S) \quad \forall S \subseteq N \setminus \{0\}, S \neq \emptyset \quad (2.9)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in N \quad (2.10)$$

$x_{ij}$  becomes 1 if arc  $(i, j)$  belongs to the optimal solution, and 0 otherwise.  $c_{ij}$  is the cost associated with traveling from node  $i$  to node  $j$ .  $N$  is the set of all vertices and  $K$  is the number of vehicles. (2.5) and (2.6) makes sure that each vertex representing a customer has exactly one entering and leaving arc. (2.7) and (2.8) impose the degree requirements for the depot vertex. Given that  $r(S)$  is the minimum number of vehicles to serve a customer set  $S$ , the capacity-cut constraints (2.9) satisfy both the connectivity of the solution and the vehicle capacity requirements.

Alternative two-index vehicle flow formulations have been proposed by Kulkarni and Bhave [29] and Laporte et al. [30]. Golden et al. [6] and Fisher and Jaikumar [31] have presented three-index models.

There are numerous other vehicle flow formulations in the literature, applied to various scenarios and physical settings.

Commodity flow formulations incorporate the effect of the amount of load carried on an arc in addition to other considerations. An early commodity flow formulation example is posed by Gavish and Graves [32] and a more recent study can be found in Baldacci et al. [33], which is based study of Finke et al. [34] on the two-commodity network flow approach to TSP.

### **Heuristics:**

Heuristics have been found to generate high quality solutions much faster than the exact methods; therefore the majority of the research interest has been concentrated on heuristic methods. In the literature, there are different classifications of heuristic approaches to VRPs. In a recent study, Laporte [35] has categorized the heuristics in the VRP literature as follows:

- i) Classical heuristics
  - a. The savings algorithm
  - b. Set partitioning heuristics
  - c. Cluster-first, route-second heuristics
  - d. Improvement heuristics
- ii) Metaheuristics
  - a. Local search
  - b. Population search
  - c. Learning mechanisms

The savings algorithm was introduced by Clarke and Wright [5] in 1964. It starts with the assumption that each node is visited by a separate vehicle i.e. the number of routes is equal to the number of customer nodes, initially. At each iteration, amount of saving that could be made by combining two routes is calculated for each route pair, and the most improving option is performed until no promising combinations can be found. Many extensions of the savings heuristics have been proposed up to now. Some of the important variants have been presented by Golden et al. [6], Solomon [36], Landeghem [37] and Paessens [38].

Set partitioning heuristics generate feasible routes (often called *petals*) and determine a best combination via solving a set partitioning problem [39]. Sweep-based algorithm [40] is an early example of this methodology. In this method, usually 1-petal routes (routes that can be performed by one vehicle) and 2-petal routes (two routes with two vehicles) are generated and an optimal combination is obtained by solution of a set partitioning problem. Foster and Ryan [41] and Renaud et al. [42] are among the authors who have employed this approach in their work.

Cluster-first, route-second type heuristics are also known as two-phase heuristics in the VRP literature. It was posed by Fisher and Jaikumar [31]. In the first phase customers are clustered by means of solution of a generalized assignment problem (GAP) and in the second phase routes are generated by solving a TSP.

Improvement heuristics are based on generating neighborhood structures. A neighborhood can be defined as the set of all possible solutions that can be obtained by a single modification on the current solution [20]. These modifications are usually in the form of inter-route or intra-route moves and a large number of

algorithms have been proposed within this broad context [35]. Laporte [43] has summarized intra-route heuristics for TSPs, whereas Thompson and Psaraftis [44] have put forward a good inter-route procedure in which routes are selected and moves are performed on the basis of a circular permutation. Figliozzi [45] has presented an iterative approach that contains an improvement procedure which aims to balance routes considering the slack capacity based on the number of customers, travel distance and time. Improvement methods are also used in many different types of complex VRP related algorithms and metaheuristics.

Metaheuristics have become quite popular recently in VRP studies. In general, a metaheuristic can be defined as a master guideline that modifies the operations of sub-heuristics by the combination of different concepts for identifying more and higher quality search opportunities [20]. In recent metaheuristics usually more than one method are employed and a hybrid approach is taken. The majority of metaheuristics contain a local search procedure, which is an improvement based method that aims to advance to a better solution in the neighborhood at each iteration. Tabu search is the most popular type of local search methods and numerous VRP studies have been conducted using this approach. In tabu search method, the heuristic is allowed to move to worse solutions within a limited step size, in order to get away from local optimal solutions and get closer to the global optimum as much as possible. Tabu search practices that have made a major impact in VRP literature can be found in [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57] and [58]. Another local search based method is the simulated annealing heuristic. In this approach, a random solution is drawn from the neighborhood; it is always accepted if it is better than the best known solution and it is still accepted with a certain probability even if it is found to be worse. The heuristic becomes more selective as it progresses. Example applications of this method can be found in [46], [59] and [60].

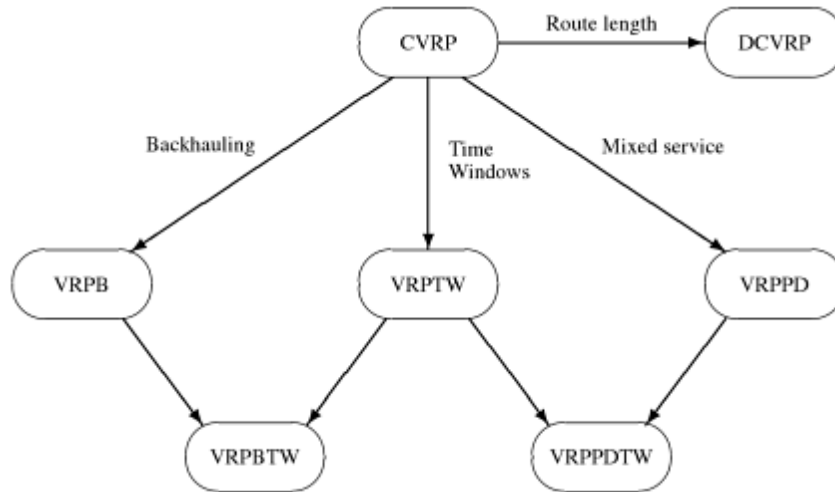
Genetic algorithms are the best-known examples of population search heuristics [35]. In this type of approaches sequences of customers are considered as chromosomes and improving offspring generated from parent solutions replace the worst elements of the population. Baker and Ayechev [61], Berger and Barkaoui [62], Prins [63], Mester and Bräysy [64] and Nagata [65] have presented an important piece of work in this area.

Learning mechanisms are among the interesting approaches to VRPs. The most common form of this type of practices is the ant colony optimization. This approach is based on the behavior of ants searching for food. Bell and McMullen [66], Mazzeo and Loiseau [67] and Xiao and Jiang-qing [68] have proposed ant colony optimization algorithms for VRPs.

### **2.1.2. Scenarios and Physical Characteristics**

VRP has been adapted to numerous real-life scenarios and it has become a quite attractive area of research for researchers especially for the past 20 years. A large number of extensions have been made to the classical VRP, also known as the *Capacitated Vehicle Routing Problem (CVRP)* and countless problem variants have become available in the literature. In this section, let us present a brief summary of the most commonly addressed VRP variants in the literature.

Toth and Vigo [28] have proposed the following classification to VRPs based on problem scenarios:



**Figure 2. Classification of VRP Problems and their Interconnections [28]**

This classification takes into account four basic extensions and their combinations. In CVRP, it is assumed that there exists a single depot from which all customer demands, which are deterministic and known in advance, are satisfied. Capacity limitations are imposed on the vehicles and the objective is to minimize the total cost, which can be a function of travel distance and/or total travel time. DCVRP relates to *Distance-Constrained VRP*, which is a special case of CVRP in which a restriction on the total distance of a route is applied.

VRPTW stands for *VRP with Time Windows*, which is by far the most commonly studied variant of VRP. In this kind of problems each customer has a specific time interval to be served. Single period time horizon assumption is valid in most of the studies, and the time windows are specified within this period. This extension makes it obligatory to include travel times in the problem formulations and solution approaches. As it is stated in [7], strict, soft or mixed type of time windows can be imposed by the customers. In the case of strict time windows, the solutions are feasible only if time window restrictions are fully satisfied. On the other hand, time windows can be extended by paying a penalty cost in scenarios with soft time windows.

Backhauling is another important dimension that is added to VRPs. In *VRPs with Backhauling* (VRPB), separate sets of *linehaul* and *backhaul* customers exist; to which goods are delivered and from which goods are picked up, respectively. Usually it is required that all linehaul customers are served before visiting any backhaul customers within a route.

*VRPs with Pickups and Deliveries* (VRPPD) is similar to VRPB; the differing point is that customers have delivery and pickup demands at the same time in VRPPD. It is assumed that the goods are delivered to a customer and returns are collected right after that. Vehicle capacity constraints are set up to such that this operation is allowed. Both VRPB and VRPPD can incorporate time windows and the resulting problem variants are named as *VRPs with Backhauling and Time Windows* (VRPBTW) and *VRPs with Pickup and Deliveries and Time Windows* (VRPPDTW).

Gendreau et al. [69] have presented additional extensions to VRP. These variants are named as follows:

- VRPs with Multiple Use of Vehicles
- Fleet Size and Mix VRP (FSMVRP)
- Multiple Depot VRP (MDVRP)
- Periodic VRP (PVRP)
- Dynamic VRP (DVRP)

In VRP with Multiple Use of Vehicles, it is assumed that a vehicle may be assigned to more than one route if necessary, as opposed to classical VRP. In this scenario, vehicles can be loaded and unloaded for several times. It might be useful to apply this setting to problem environments in which network size and vehicle capacities are relatively small.

FSMVRP assumes that the fleet sizes are changeable and multiple vehicle types with different capacities and routing costs are allowed. Its objective is to determine the number of vehicles of each type to be used to satisfy all the customer demand together with a coherent routing policy. It can be further categorized into sub-variants based on characteristics such that whether the fleet size is assumed to be limited or not, fixed cost of operating a vehicle is considered or not and routing costs depend or does not depend on the vehicle capacity.

In MDVRP there are more than one depot at different locations having their own vehicle fleets. Depending on the problem scenario, a customer may or may not be assigned to a specific depot and vehicles may or may not need to start and end their routes in the same depot.

PVRP takes into account a time horizon that consists of more than one period, as opposed to the classical VRP. DVRP assumes that not all customer data is known beforehand, and the plans can be revised during the execution. In this type of problems, a re-scheduling action should be taken in order to include new customers within the existing routes.

In addition to these basic VRP variants, there are other important applied scenarios in the VRP literature. Several authors have incorporated stochastic demands and time-dependent travel times in their work. Load splitting is another topic which has drawn the attention of VRP researchers. The computation of transportation costs is another important research question in this field and on the contrary to classical approaches which aimed to optimize the total travel distances, efforts have been put forward to estimate more realistic cost figures taking into account fuel consumption, driver wages and environmental issues such as greenhouse gas emissions.

Many of the problem scenarios presented in this section have been combined with each other in order to model more and more real-life problems in a more realistic way. This has caused the area of research to extend to a broad range and the problems in this field to become quite disjoint and disparate.

## **2.2. Literature on VRPs with Fuel and Energy Considerations**

Inclusion of fuel and energy costs and environmental matters is a quite recent trend in VRP literature. In majority of VRP practices, total travel distance is chosen as the key performance indicator (KPI), which is only a determinant of the actual total cost in reality. Within the last few years, researchers have started to develop solution strategies for VRP taking into account fuel and energy consumption together with an effort to minimize greenhouse gas emission.

Kara et al. [70] have presented an energy minimizing approach to VRP by means of an objective function based on both travel distances and gross weight of the vehicles. They used a basic physics equation: work equals friction force times distance. Assuming that the coefficient of friction is constant on roads, Kara et al. [70] have tried to minimize a cost function which consists of simply sums of weight times distance figures. They have modeled this problem as an ILP, and shown in a small instance that there can be a considerable difference in the total energy consumption between distance-minimizing and energy-minimizing formulations.

Xiao et al. [71] have considered the effect of vehicle load on the total fuel consumption. They assumed that there was a linear relationship between the gross weight of the vehicle and the amount of fuel consumption per kilometer and formulated the problem taking into account both a fixed vehicle usage cost and fuel cost. They have proposed a simulated annealing heuristic for solving the problem.

Kuo and Wang [72] have developed a tabu search heuristic in order to find feasible vehicle routes while minimizing the total fuel consumption. They have incorporated the effect of vehicle speed into the fuel cost. It is assumed that the vehicles may travel at low, medium and high speed levels and the unit fuel costs are given for each of these levels. Kuo and Wang [72] took the travel speed as a parameter and observed the effect of this by conducting experiments on four different data sets with differing travel speed patterns.

### 2.2.1. Consideration of Greenhouse Gas Emissions

As the problem of global warming have become serious in recent years, the reduction of greenhouse gas (GHG) emissions, which act as a main determinant of the atmospheric structure of the Earth, have gained more importance. Transportation is found to be the second largest contributor to GHGs by a portion of 28% [73].

An early study incorporating environmental matters in vehicle routing was conducted by Palmer [74]. Palmer [74] modeled the problem using a commercial vehicle routing and scheduling software with the aim of minimizing carbon dioxide emissions.

The reduction of GHGs in VRPs was first taken into consideration within an Integer Linear Programming (ILP) model by Bektas and Laporte [3] in 2011. They have introduced a new problem variant called *The Pollution-Routing Problem* (PRP) in which they have imposed an artificial cost on the amount of greenhouse gas emitted as a result of the fuel burn.

Bektas and Laporte [3] have defined four types of objective functions in their analysis: A distance-minimizing objective ( $P_D$ ), a weighted load-minimizing objective ( $P_L$ ), an energy-minimizing objective ( $P_E$ ) and a cost-minimizing objective ( $P_C$ ) that incorporates  $P_D$ ,  $P_L$  and  $P_E$ .  $P_D$  and  $P_L$  operates with a constant speed assumption and  $P_D$  stands for nothing but the classical VRP or VRPTW with the of addition of time windows.  $P_C$  is a comprehensive objective that takes into account travel distance, gross weight of the vehicles, travel speeds and driver wages. With the combination of these, a total cost function that composes of fuel, emission and driver costs has been developed.

Integer programming formulations has been proposed using these different objectives and the comparative results in terms of fuel, emission and driver costs have been analyzed under different parameter settings

such as inclusion or exclusion of time windows, changing customer demands, vehicle weights and capacities and alternative fleet sizes.

Two recent studies have already taken PRP to a higher level. Demir et al. [75] have proposed an adaptive large neighborhood search for PRP in which they have successfully implemented a complex heuristic algorithm with a speed optimization procedure and achieved time efficient and high quality solutions for relatively big networks. Pradenas et al. [76] have presented an extension of PRP with the inclusion of backhauling. They have developed a scatter search algorithm, which is a type of population search approach, and implemented their methodology to up to 100 customer-networks from the literature. A more detailed review on PRP can be found in the PhD dissertation of Demir [77].

The work of Bektas and Laporte [3] has given birth to a new area of research in VRP literature. It constitutes a good starting point for further research and it has also provided a basis for our study.

## CHAPTER 3

### PROBLEM DEFINITION & MATHEMATICAL PROGRAMMING FORMULATIONS

In this chapter we give a formal definition of our problem with the explanation of all the assumptions made in order to represent where our study stands in VRP literature. Afterwards we propose mathematical programming formulations for solving the problem to optimality.

#### 3.1. General Assumptions

As in classical VRPs, we assume that there is a single depot in our system. The distribution schedules are one-time plans and the time horizon is assumed to be a single period e.g. one day. We assume that all goods to be delivered are identical. The vehicle fleet consists of trucks of multiple types with differing curb weights and load carrying capacities. The fleet size is fixed and no additional trucks can be acquired. However it is not obligatory that all of the vehicles in the fleet will be used. At time zero all vehicles are assumed to be loaded and ready to depart from the depot.

All vehicles start their tours at time zero. Vehicles are assumed to travel with a constant speed along each road segment. The time required for reaching the projected travel speed and slowing down to stop is neglected. However, the speed of the vehicle can be different at each link. Each vehicle returns to the depot upon completing its respective tour. All waiting times are assumed to be equal to zero and all types of unexpected delays are disregarded, that is, a vehicle never becomes idle until it returns to the depot.

Customers have predefined deadlines. Vehicles must arrive at the customers before their deadlines. Once they arrive at the customer sites, vehicles wait there for a certain service time. It is considered as the unloading time of goods; therefore it is directly linked with the amount of goods to be unloaded at the customer sites. After the unloading process is completed, vehicles immediately start their next journey which can either be for delivery to a new customer or returning to the depot. Each vehicle is used only once; it is not used anymore after it returns to the depot. There is no restriction on the total tour times.

In order to show where our problem stands in VRP literature, in Table 1, we summarize these assumptions based on the applicable scenario and physical characteristics that are presented in the classification in Figure 1 [7].

**Table 1. Problem Characteristics and Our Assumptions**

Characteristic	Our Assumption
Load splitting	<i>Splitting not allowed</i>
Customer service demand quantity	<i>Deterministic</i>
On site service/waiting times	<i>Deterministic, based on the amount of goods to be unloaded</i>
Time window structure	<i>Strict deadlines, start time window is 0 for all customers</i>
Time horizon	<i>Single period</i>
Backhauls	<i>No backhauling, only linehaul customers</i>
Transportation network design	<i>Directed network</i>



Location of addresses (customers)	<i>Customers on nodes</i>
Number of depots	<i>Single depot</i>
Time window type	<i>Restriction on customers</i>
Number of vehicles	<i>Up to n vehicles</i>
Capacity consideration	<i>Capacitated vehicles</i>
Vehicle homogeneity	<i>Heterogeneous vehicles</i>
Travel time	<i>Endogenously determined</i>
Transportation cost	<i>A function of travel time, distance and gross weight of the vehicle</i>

### 3.2. Formal Definition of the Problem

Our problem is defined on complete graph  $G(N, A)$  where  $N = \{0, 1, \dots, n\}$  is the set of all nodes in the graph with the depot located at node 0 and  $A$  is the set of arcs between each pair of nodes. The set of customer nodes is represented by  $N_c = \{1, 2, \dots, n\}$ . For every arc  $(i, j)$  in  $A$ , the distance between nodes  $i$  and  $j$  is defined as  $d_{ij}$ . For all  $i \in N_c$ , there is a positive demand of  $q_i$  to be satisfied. For each customer, there is an associated unloading time which is directly proportional to the demand of that customer. The unloading time is computed by multiplying the demand with the unit unloading time:  $\tau$ . It is assumed that the vehicle fleet consists of vehicles of differing types and the set of vehicle types is denoted by  $V = \{1, 2, \dots, m\}$ . The capacity and the curb weight of the vehicles of type  $k$  are denoted by  $Q_k$  and  $w_k$ , respectively.

In the study of Bektas and Laporte [3] the theoretical energy consumption is calculated. On the other hand, Demir et al. [75] has presented a more practical calculation procedure which computes the fuel consumption taking into account the properties of the engine as well, resulting in a lower estimation, approximately by 25%. The objective function we consider is exactly the same as the cost minimizing objective ( $P_C$ ) defined by Bektas and Laporte [3]. It consists of fuel, emission and driver costs. In the basis of fuel and emission costs lies the amount of energy consumed by the vehicles. The energy consumption is calculated considering the road conditions such as the travel distance and the angle of the road, and vehicle specific conditions such as the gross weight of the vehicle (including the curb weight and the load carried), vehicle speed and the resulting air resistance. Driver cost is the wage paid to drivers which is calculated on hourly basis. The unit cost of fuel and greenhouse gas emission are denoted by  $c_f$  and  $c_e$ , respectively, whereas the hourly driver wage rate is  $p$ .

We assume that the size of the vehicle fleet is fixed and the number of vehicles of type  $k$  in the fleet is represented by  $h_k$ . Each customer  $i \in N_c$  has a strict deadline  $b_i$ , and the arrival time at the customers cannot exceed these specific deadlines. There are limitations on the vehicle speeds as well. As in [3], we consider only vehicle speeds above 40 km/h since the fuel consumption is expected to increase as the vehicle speed falls below approximately 40 km/h. Therefore, the lower bound for the vehicle speed ( $\underline{v}_{ij}$ ) is 40 km/h for all  $(i, j) \in A$ . Moreover, for each road link  $(i, j)$ , there exists a maximum speed limit of  $\bar{v}_{ij}$ . According to the European Commission report [78] the speed limit for heavy good vehicles varies between 80 km/h and 100 km/h on motorways in European Union countries. Based on this information, we assume that the upper speed limit ( $\bar{v}_{ij}$ ) is 90 km/h for all  $(i, j) \in A$ .

In the calculation of energy consumption we use two special parameters as they are defined in [3]. The first one is an arc-specific constant ( $\alpha_{ij}$ ) which depends on a number of factors. This constant is found by the following equation:

$$\alpha_{ij} = a + g \sin \theta_{ij} + C_r g \cos \theta_{ij} \quad (3.1)$$

Note that  $a$  is the acceleration of the vehicle,  $g$  is the gravitational acceleration,  $\theta_{ij}$  is the angle of the road segment ( $i, j$ ) and  $C_r$  is the rolling resistance coefficient. The vehicle specific constant ( $\beta_k$ ) on the other hand, is calculated as follows:

$$\beta_k = 0.5 C_d A_k \rho \quad (3.2)$$

where  $C_d$  is the drag (air resistance) coefficient,  $A_k$  is the frontal area of the vehicle and  $\rho$  is the air density. The fuel and emission costs of traversing arc ( $i, j$ ) with a vehicle of type  $k$  carrying a load of  $f_{ij}$  and traveling with a speed of  $v_{ij}$  are computed with the following formulas:

$$\text{Fuel Cost:} \quad c_f \alpha_{ij} d_{ij} (w_k + f_{ij}) + c_f \beta_k d_{ij} v_{ij}^2 \quad (3.3)$$

$$\text{Emission Cost:} \quad c_e \alpha_{ij} d_{ij} (w_k + f_{ij}) + c_e \beta_k d_{ij} v_{ij}^2 \quad (3.4)$$

The summation of fuel and emission costs can be denoted as:

$$\text{Fuel and Emission Cost:} \quad (c_e + c_f) [\alpha_{ij} d_{ij} (w_k + f_{ij}) + \beta_k d_{ij} v_{ij}^2] \quad (3.5)$$

The driver costs are simply calculated by multiplying the hourly wage rate with the total travel time. We propose the following mathematical programming model for this problem:

Decision Variables:

$$x_{ijk}: \begin{cases} 1, & \text{if arc } (i, j) \text{ is used by vehicle type } k \\ 0, & \text{otherwise} \end{cases}$$

$f_{ij}$ : Amount of load carried along arc ( $i, j$ )

$v_{ij}$ : Vehicle speed on arc ( $i, j$ )

$t_{ij}$ : Travel time on arc ( $i, j$ )

$y_j$ : Time at which the service at node  $j$  starts

$s_j$ : Total time spent on the route that has node  $j$  as the last visited node

Model 0 (M0)

$$\text{Minimize} \quad \sum_{k \in V} \sum_{(i, j) \in A} (c_f + c_e) \alpha_{ij} d_{ij} w_k x_{ijk} \quad (3.6)$$

$$+ \sum_{(i,j) \in A} (c_f + c_e) \alpha_{ij} d_{ij} f_{ij} \quad (3.7)$$

$$+ \sum_{k \in V} \sum_{(i,j) \in A} (c_f + c_e) d_{ij} \beta_k v_{ij}^2 x_{ijk} \quad (3.8)$$

$$+ \sum_{j \in N_c} p s_j \quad (3.9)$$

subject to

$$\sum_{j \in N} x_{0jk} \leq h_k \quad \forall k \in V \quad (3.10)$$

$$\sum_{j \in N} \sum_{k \in V} x_{ijk} = 1 \quad \forall i \in N_c \quad (3.11)$$

$$\sum_{i \in N} \sum_{k \in V} x_{ijk} = 1 \quad \forall j \in N_c \quad (3.12)$$

$$\sum_{i \in N} x_{ipk} - \sum_{j \in N} x_{pjk} = 0 \quad \forall k \in V, \forall p \in N_c \quad (3.13)$$

$$\sum_{k \in V} x_{ijk} + \sum_{k \in V} x_{jik} \leq 1 \quad \forall (i,j), (j,i) \in A \quad (3.14)$$

$$\sum_{j \in N} f_{ji} - \sum_{j \in N} f_{ij} = q_i \quad \forall i \in N_c \quad (3.15)$$

$$\sum_{k \in V} q_j x_{ijk} \leq f_{ij} \leq \sum_{k \in V} (Q_k - q_i) x_{ijk} \quad \forall (i,j) \in A \quad (3.16)$$

$$y_i - y_j + q_i \tau + t_{ij} \leq M \left( 1 - \sum_{k \in V} x_{ijk} \right) \quad i \in N, j \in N_c, i \neq j \quad (3.17)$$

$$y_j + q_i \tau - s_j + t_{j0} \leq M \left( 1 - \sum_{k \in V} x_{j0k} \right) \quad \forall j \in N_c \quad (3.18)$$

$$t_{ij} v_{ij} \geq d_{ij} \quad \forall (i,j) \in A \quad (3.19)$$

$$y_i \leq b_i \quad \forall i \in N_c \quad (3.20)$$

$$\underline{v}_{ij} \leq v_{ij} \leq \bar{v}_{ij} \quad \forall (i,j) \in A \quad (3.21)$$

$$f_{ij} \geq 0 \quad \forall (i,j) \in A \quad (3.22)$$

$$s_j, y_j \geq 0 \quad \forall j \in N_c \quad (3.23)$$

$$x_{ijk} \in \{0,1\} \quad \forall (i,j) \in A, \forall k \in V \quad (3.24)$$

In the objective function of the model, parts (3.6) and (3.7) point out the fuel and emission cost that incurred due to the curb weight of the vehicle and the load carried, respectively. Part (3.8) is directly linked with the speed of the vehicle, and (3.9) represents the driver cost.

Constraint (3.10) specifies the maximum number of vehicles to be used for each vehicle type. (3.11) and (3.12) guarantee that each customer node is visited exactly once and (3.13) ensures the consistency of vehicle types in these visits. (3.14) is a supplementary two-node subtour breaking constraint. Constraint (3.15) balances the flow and (3.16) guarantees that the demands are satisfied and the vehicle capacities are not exceeded. The arrival times at customer nodes and the route completion times are determined by constraints (3.17) and (3.18). The travel time – vehicle speed relationship is ensured by constraint (3.19). Deadlines and speed restrictions are imposed by constraints (3.20) and (3.21), respectively. Note that subtours are eliminated via constraints (3.15) through (3.17), and  $M$  is a sufficiently large number.

This initial formulation can be classified as a *Mixed-Integer Nonlinear Program* (MINLP) and it has nonlinear components both in the objective function (3.8) and in the constraints (3.19). We would expect this model to generate routes and determine speeds of the vehicles on each arc such that the total fuel, emission and driver costs are minimized. However, this is an NP-complete problem and in its current form, it is among the most difficult optimization problems [79].

We propose two alternative formulations to this problem. The first option we consider follows the same logic as in [3]: Using distinct speed levels and getting rid of the nonlinear terms by discretizing the vehicle speed; which results in a *Mixed-Integer Program* (MIP) with a piecewise-linear cost function.

The second way we address is getting rid of the non-linear terms in the objective function only and reformulating the model as a *Mixed-Integer Second Order Cone Problem* (MISOCP).

In the next section, these two formulations are represented.

### 3.3. Mathematical Programming Formulations

#### 3.3.1. MIP Formulation with a Piecewise Linear Cost Function

In this formulation, it is assumed that the vehicle speed on a road link equals to one of the predetermined

speed levels. Hence, the vehicle speed is not considered as a continuous decision variable anymore. The speed range is divided into intervals of equal length, of which endpoints represent distinct speed levels.

Assume that the set of speed levels is denoted by  $L$  and the number of elements in this set is  $|L|$ . Then the speed level values ( $v_l$ ) are defined as follows:

$$v_l = \underline{v}_{ij} + l \times \left( \frac{\bar{v}_{ij} - \underline{v}_{ij}}{|L| - 1} \right) \text{ for } l = 0, 1, \dots, (|L| - 1) \quad (3.25)$$

In order to make sure that a specific speed level is assigned to a road segment, a new binary variable is needed in the formulation. The new variable is defined as:

$$z_{ijkl}: \begin{cases} 1, & \text{if a vehicle of type } k \text{ travels with a speed of } v_l \text{ on arc } (i, j) \\ 0, & \text{otherwise} \end{cases}$$

The resulting formulation is represented below.

*Model 1 (M1)*

$$\begin{aligned} \text{Minimize} \quad & (3.6) + (3.7) + (3.9) \\ & + \sum_{k \in V} \sum_{(i,j) \in A} \sum_{l \in L} (c_f + c_e) d_{ij} \beta_k v_l^2 z_{ijkl} \quad (3.26) \end{aligned}$$

*subject to*

$$(3.10) - (3.16), (3.20), (3.22) - (3.24)$$

$$\sum_{v_l \in L} z_{ijkl} = x_{ijk} \quad \forall (i, j) \in A, \forall k \in V \quad (3.27)$$

$$y_i - y_j + q_i \tau + \left( \sum_{k \in V} \sum_{l \in L} \frac{d_{ij} z_{ijkl}}{v_l} \right) \leq M \left( 1 - \sum_{k \in V} x_{ijk} \right) \quad i \in N, j \in N_c, i \neq j \quad (3.28)$$

$$y_j + q_i \tau - s_j + \left( \sum_{k \in V} \sum_{l \in L} \frac{d_{j0} z_{j0kl}}{v_l} \right) \leq M \left( 1 - \sum_{k \in V} x_{j0k} \right) \quad \forall j \in N_c \quad (3.29)$$

$$z_{ijkl} \in \{0, 1\} \quad \forall (i, j) \in A, \forall k \in V, \forall l \in L \quad (3.30)$$

In this formulation,  $v_l$  is defined as a parameter. The variable denoting travel times ( $t_{ij}$ ) and constraint (3.19) are removed from the model. In the objective function, expression (3.26) replaces (3.8). In the new setting, only decision variable in expression (3.26) is  $z_{ijkl}$ ; therefore the quadratic term in the objective function is eliminated.

Constraints (3.28) and (3.29) are similar to (3.17) and (3.18); the only difference is the appearance of travel time. Now that speed is not considered a decision variable, travel time is calculated by distance over speed without violating the linearity of the constraint.

The main advantage of this model is that the non-linearity both in the objective function and in the constraints is completely removed. However, a binary variable with 4 indices is introduced in exchange for this improvement, which renders the problem more difficult to solve. Computational experiments will expose the efficiency of this formulation.

### 3.3.2. Mixed-Integer Second Order Cone Programming (MISOCP) Formulation

The MIP model (M1) has some drawbacks, especially in terms of violation of continuous vehicle speed assumption and excessive number of binary variables. In this section, we propose a new formulation which tries to overcome the handicaps of the previous model with an alternative approach.

In this formulation, we linearize the objective function and add quadratic constraints with new decision variables. We make necessary changes in the constraints to transform the model to a *Second-Order Cone Program* (SOCP). SOCP, in which a linear objective function is minimized over the intersection of an affine subspace with the Cartesian products of second-order (Lorentz) cones, is an important class of convex optimization problems [80]. There exist several efficient primal-dual interior point methods for solving problems of this type [81]. Let us first introduce the new variables included in the formulation and then describe how the MISOCP model is set up.

$v_{ijk}^+$ : Speed – up of vehicle type  $k$  on arc  $(i, j)$  above the minimum speed limit  $\underline{v}_{ij}$

$v_{ijk}$ : Actual speed of vehicle type  $k$  on arc  $(i, j)$

$\gamma_{ij}^-, \gamma_{ij}^+, \sigma_{ijk}^-, \sigma_{ijk}^+, \theta_{ijk}$ : Dummy variables used for linearizing the objective function and reorganizing the constraints

Recall expression (3.8) in the objective function of the original formulation (M0).

$$\sum_{k \in V} \sum_{(i,j) \in A} (c_f + c_e) d_{ij} \beta_k v_{ij}^2 x_{ijk} \quad (3.8)$$

#### Proposition 1

Expression (3.8) can be completely linearized and written as

$$\sum_{k \in V} \sum_{(i,j) \in A} (c_f + c_e) d_{ij} \beta_k (\underline{v}_{ij}^2 x_{ijk} + 2 \underline{v}_{ij} v_{ijk}^+ + \theta_{ijk}) \quad (3.31)$$

in the MISOCP formulation by means of introducing new variables and adding new constraints.

**Proof to Proposition 1**

In order to linearize this expression, we need to get rid of the non-linear term:  $v_{ij}^2 x_{ijk}$ . We define a new variable,  $v_{ijk}$  to replace  $v_{ij}$  and represent it as:

$$v_{ijk} = \underline{v}_{ij} + v_{ijk}^+ \quad (3.32)$$

With the new 3-index variable, assume that we have the term  $v_{ijk}^2 x_{ijk}$  in the objective function. Also taking into account equation (3.32), it can be rewritten it as:

$$v_{ijk}^2 x_{ijk} = (\underline{v}_{ij} x_{ijk} + v_{ijk}^+ x_{ijk})^2 = \underline{v}_{ij}^2 x_{ijk}^2 + 2\underline{v}_{ij} v_{ijk}^+ x_{ijk}^2 + (v_{ijk}^+)^2 x_{ijk}^2 \quad (3.33)$$

Since  $x_{ijk}$  is a binary variable and  $\underline{v}_{ij}$  is a constant parameter,  $\underline{v}_{ij}^2 x_{ijk}^2$  can be written as  $\underline{v}_{ij}^2 x_{ijk}$ . We change the speed limitation constraint (3.21) as  $\underline{v}_{ij} x_{ijk} \leq v_{ijk} \leq \bar{v}_{ij} x_{ijk}$  (3.34). Right hand side of this constraint satisfies that  $v_{ijk}$  can take a positive value only if  $x_{ijk}$  is 1, which forces us to write (3.32) as an inequality,  $v_{ijk} \leq \underline{v}_{ij} + v_{ijk}^+$  (3.35). By means of constraints (3.34) and (3.35), it is guaranteed that  $v_{ijk}^+$  will also take a positive value only if  $x_{ijk}$  is 1 since it is included as a positive term in the objective function. Therefore,  $2\underline{v}_{ij} v_{ijk}^+$  can be employed instead of  $2\underline{v}_{ij} v_{ijk}^+ x_{ijk}^2$  and  $(v_{ijk}^+)^2$  can be used instead of  $(v_{ijk}^+)^2 x_{ijk}^2$  within the objective function. In this way, expression (3.33) is reduced to the following:

$$\underline{v}_{ij}^2 x_{ijk} + 2\underline{v}_{ij} v_{ijk}^+ + (v_{ijk}^+)^2 \quad (3.36)$$

In this expression  $(v_{ijk}^+)^2$  is the only remaining quadratic term. We refer to [82] for reformulating this part. We define new variable  $\theta_{ijk}$  where

$$S_C = \{(x_{ijk}, v_{ijk}^+, \theta_{ijk}) \in \mathbb{R}^3: (v_{ijk}^+)^2 \leq \theta_{ijk} x_{ijk}, 0 \leq v_{ijk}^+ \leq (\bar{v}_{ij} - \underline{v}_{ij}) x_{ijk}, 0 \leq x_{ijk} \leq 1, \theta_{ijk} \geq 0\}$$

is a convex set for  $\forall (i, j) \in A, \forall k \in V$ .

We replace  $(v_{ijk}^+)^2$  by  $\theta_{ijk}$  and completely linearize the objective function. Next, we reformulate  $(v_{ijk}^+)^2 \leq \theta_{ijk} x_{ijk}$  as a second-order cone constraint. Let us define two additional decision variables  $\sigma_{ijk}^-$ , and  $\sigma_{ijk}^+$  such that  $\sigma_{ijk}^- = \theta_{ijk} - x_{ijk}$  (3.37) and  $\sigma_{ijk}^+ = \theta_{ijk} + x_{ijk}$  (3.38) for  $\forall (i, j) \in A, \forall k \in V$ . Using these expressions, we obtain the second order cone constraint as follows:

$$\begin{aligned} (v_{ijk}^+)^2 \leq \theta_{ijk} x_{ijk} &\rightarrow 4(v_{ijk}^+)^2 \leq 4\theta_{ijk} x_{ijk} \\ &\rightarrow (2v_{ijk}^+)^2 + \theta_{ijk}^2 + x_{ijk}^2 \leq \theta_{ijk}^2 + x_{ijk}^2 + 4\theta_{ijk} x_{ijk} \\ &\rightarrow (2v_{ijk}^+)^2 + \theta_{ijk}^2 - 2\theta_{ijk} x_{ijk} + x_{ijk}^2 \leq \theta_{ijk}^2 + x_{ijk}^2 + 2\theta_{ijk} x_{ijk} \\ &\rightarrow (2v_{ijk}^+)^2 + (\theta_{ijk} - x_{ijk})^2 - (\theta_{ijk} + x_{ijk})^2 \leq 0 \\ &\rightarrow (2v_{ijk}^+)^2 + (\sigma_{ijk}^-)^2 - (\sigma_{ijk}^+)^2 \leq 0 \end{aligned} \quad (3.39)$$

**Proposition 2**

Expression  $t_{ij}v_{ij} \geq d_{ij}$  (3.19) can be written as a second order cone constraint by introducing two new dummy variables.

**Proof to Proposition 2**

The same strategy as in Proposition 1 is applied for rearranging  $t_{ij}v_{ij} \geq d_{ij}$  (3.19). We can rewrite (3.19) with the 3-index speed variable as  $t_{ij}v_{ijk} \geq d_{ij}x_{ijk}^2$  (3.40). New dummy variables  $\gamma_{ij}^-$  and  $\gamma_{ij}^+$  are defined as  $\gamma_{ij}^- = \sum_{k \in V} v_{ijk} - t_{ij}$  (3.41) and  $\gamma_{ij}^+ = \sum_{k \in V} v_{ijk} + t_{ij}$  (3.42) for  $\forall (i, j) \in A$ . The second order cone constraint is found as:

$$\begin{aligned}
& \sum_{k \in V} d_{ij}x_{ijk}^2 \leq \sum_{k \in V} t_{ij}v_{ijk} \rightarrow \sum_{k \in V} 4d_{ij}x_{ijk}^2 \leq \sum_{k \in V} 4t_{ij}v_{ijk} \\
& \rightarrow \sum_{k \in V} 4d_{ij}x_{ijk}^2 + t_{ij}^2 + \sum_{k \in V} v_{ijk}^2 \leq t_{ij}^2 + \sum_{k \in V} v_{ijk}^2 + 4t_{ij} \sum_{k \in V} v_{ijk} \\
& \rightarrow \sum_{k \in V} 4d_{ij}x_{ijk}^2 + t_{ij}^2 - 2t_{ij} \sum_{k \in V} v_{ijk} + x_{ijk}^2 \leq t_{ij}^2 + 2t_{ij} \sum_{k \in V} v_{ijk} + \sum_{k \in V} v_{ijk}^2 \\
& \rightarrow \sum_{k \in V} 4d_{ij}x_{ijk}^2 + \left( t_{ij} - \sum_{k \in V} v_{ijk} \right)^2 - \left( t_{ij} + \sum_{k \in V} v_{ijk} \right)^2 \leq 0 \\
& \rightarrow \sum_{k \in V} 4d_{ij}x_{ijk}^2 + (\gamma_{ij}^-)^2 - (\gamma_{ij}^+)^2 \leq 0 \quad (3.43)
\end{aligned}$$

The resulting MISOCP formulation can be represented as follows:

*Model 2 (M2)*

$$\begin{aligned}
& \text{Minimize} \quad (3.6) + (3.7) + (3.9) \\
& \quad + \sum_{k \in V} \sum_{(i,j) \in A} (c_f + c_e) d_{ij} \beta_k (\underline{v}_{ij}^2 x_{ijk} + 2\underline{v}_{ij} v_{ijk}^+ + \theta_{ijk}) \quad (3.31)
\end{aligned}$$

subject to

$$(3.10) - (3.18), (3.20), (3.22) - (3.24)$$

$$\underline{v}_{ij} x_{ijk} \leq v_{ijk} \leq \bar{v}_{ij} x_{ijk} \quad \forall (i, j) \in A, \forall k \in V \quad (3.34)$$

$$v_{ijk} \leq \underline{v}_{ij} + v_{ijk}^+ \quad \forall (i, j) \in A, \forall k \in V \quad (3.35)$$



$$\sigma_{ijk}^- = \theta_{ijk} - x_{ijk} \quad \forall (i, j) \in A, \forall k \in V \quad (3.37)$$

$$\sigma_{ijk}^+ = \theta_{ijk} + x_{ijk} \quad \forall (i, j) \in A, \forall k \in V \quad (3.38)$$

$$(2v_{ijk}^+)^2 + (\sigma_{ij}^-)^2 - (\sigma_{ij}^+)^2 \leq 0 \quad \forall (i, j) \in A, \forall k \in V \quad (3.39)$$

$$\gamma_{ij}^- = \sum_{k \in V} v_{ijk} - t_{ij} \quad \forall (i, j) \in A \quad (3.41)$$

$$\gamma_{ij}^+ = \sum_{k \in V} v_{ijk} + t_{ij} \quad \forall (i, j) \in A \quad (3.42)$$

$$\sum_{k \in V} 4d_{ij}x_{ijk}^2 + (\gamma_{ij}^-)^2 - (\gamma_{ij}^+)^2 \leq 0 \quad \forall (i, j) \in A \quad (3.43)$$

$$\gamma_{ij}^-, \gamma_{ij}^+ \geq 0 \quad \forall (i, j) \in A \quad (3.44)$$

$$v_{ijk}^+, \theta_{ijk}, \sigma_{ijk}^-, \sigma_{ijk}^+ \geq 0 \quad \forall (i, j) \in A, \forall k \in V \quad (3.45)$$

We expect this model to generate more accurate solutions with slightly better objective function values since the vehicle speeds will be derived more precisely than the MIP formulation. In the extreme case where deadlines are far too restrictive, different optimal routes may be generated, which would result in a significantly better objective function value. However, the problem is still difficult to solve due to the newly added quadratic constraints and remaining binary variables. Implications of the changes in the formulation will be observed in the computational studies.

### 3.3.3. Preliminary Model Runs

After the mathematical models have been set up, it is necessary to determine the values of the problem parameters. Several preliminary runs were conducted to compare the models and test the models internally. The parameter values used in these runs can be found with the necessary explanations for their selection in Appendix A.1. We specify the number of vehicle types as 2 and run our experiments on a network containing 11 Dutch cities (1 Depot + 10 Customer locations). A map illustrating the customer and depot locations in the network is available in Appendix A.2. All computational experiments were run on a computer with a quad-core 2.00 GHz processor, 8 GB ram and 64-bit operating system. Mathematical models were solved by CPLEX 12.3.

First of all we would like to decide on the number of speed levels in the MIP formulation (M1). We consider three alternative configurations: 6, 11 and 26 speed levels with speed intervals of 10 km/h, 5 km/h and 2 km/h, respectively. 10 replications with differing demand and deadlines are run for each model with a time limit of 1000 seconds. Average results are tabulated below, detailed results can be found in Appendix B.1.

**Table 2. MIP Model - Differing Number of Speed Levels**

Number of Speed Levels	Final Solution (£)	LP Bound (£)	Gap (%)	Average Speed (km/h)		
				General	Type 1 Vehicle	Type 2 Vehicle
6	384.80	259.60	32.2%	61.49	64.10	59.00
11	376.46	280.12	25.4%	59.33	63.27	55.54
26	379.39	235.10	38.0%	57.91	60.15	55.65

The option with 6 speed levels yields the worst results in terms of the total cost (£384.80). This is an expected result due to the low precision of this model, which results in the highest average vehicle speeds. The third alternative on the other hand, gives the lowest LP bound and the largest solution gap. This makes sense since this model has the highest number of binary variables which makes it the most difficult to solve one among the options. Moreover, the total cost of this model is greater than that of the 11-speed levels case even though the average speed is higher for vehicles of both types. This is mainly because the best solution could not be improved further than the 11 speed levels option within the specified time limit due to the difficulty caused by the higher number of variables. Under these circumstances, the option with 11 speed levels seems to be the best alternative in terms of the solution quality. Thus, we decide to proceed with this setting in the following runs.

Due to the computational complexity of the problem, solving the models optimally requires too much CPU time. Therefore, there is need for putting a limitation on the total solution time. We conduct an experiment to see the effect of imposing a time limit of 1000 seconds on M1. Table 3 displays the average results of 10 replications with and without time limits. Detailed results can be found in Appendix B.2.

**Table 3. Effect of Time Limit**

Time Limit	Final Solution (£)	Total Solution Time (sec)	Solution Status	LP Bound (£)	Gap	Total Travel Distance (km)	Total Travel Time (h)
1000 seconds	376.46	1001	Feasible	280.12	25.4%	1057.19	17.86
None	369.01	52352	Optimal	368.98	0.0%	1049.07	18.30
% Difference	-1.98%	-	-	+31.72%	-	-0.77%	+2.48%

Looking at the average results, it can be derived that the difference between the final solutions of the time limited and no-time-limit cases is very small (1.98%), although the difference between total solution times is fairly high. Based on these results, we prefer to keep the time limit at 1000 seconds for 10-customer-networks, and increase it directly proportional to the number of customers in the network for the sake of time efficiency in our experiments.

After determining the appropriate limitation on the solution time, we proceed to the stage of comparing models M1 and M2. 8 test cases with differing parameter values were constructed accordingly. These cases are described as follows:

- C1: Base case
- C2: Fleet size is doubled
- C3: Fuel and emission costs are doubled
- C4: Driver wage is doubled
- C5: Deadlines are loosened
- C6: Deadlines are tightened

- C7: Low customer demand
- C8: High customer demand with a larger fleet size

10 replications were run for each case. The details of how parameters were changed and results of each replication can be found in Appendix B.3. Average results of 10 replications for each case are displayed in Table 4. Note that only results of replications in which a feasible solution could be found were taken into account while calculating the average figures.

**Table 4. Comparison of M1 and M2 under Different Test Cases**

	Changed Parameter	Final Solution (£)			LP Bound (£)			Gap	
		M1	M2	% Diff	M1	M2	% Diff	M1	M2
<b>C1</b>	None	376.46	368.56	-2.1%	280.12	219.93	-21.5%	25.4%	40.2%
<b>C2</b>	Fleet Size	334.36	332.44	-0.6%	249.33	165.95	-33.4%	25.3%	50.0%
<b>C3</b>	Fuel & Emission Costs	595.41	585.33	-1.7%	459.84	390.97	-15.0%	22.4%	32.8%
<b>C4</b>	Driver Wage	515.57	504.19	-2.2%	349.22	211.34	-39.5%	31.9%	58.0%
<b>C5</b>	Deadlines (+)	348.18	345.85	-0.7%	251.14	202.41	-19.4%	27.8%	41.5%
<b>C6</b>	Deadlines (-)	419.32	413.86	-1.3%	321.78	232.18	-27.8%	22.5%	43.6%
<b>C7</b>	Customer Demands (-)	351.87	364.48	+3.6%	220.64	125.18	-43.3%	37.3%	65.4%
<b>C8</b>	Customer Demands (+)	526.41	525.68	-0.1%	519.51	458.32	-11.8%	1.3%	12.8%
<b>Overall Average</b>		<b>433.45</b>	<b>430.05</b>	<b>-0.8%</b>	<b>331.45</b>	<b>250.78</b>	<b>-24.3%</b>	<b>24.2%</b>	<b>43.0%</b>

Average results obtained by solving the models with a time limit of 1000 seconds indicate that the MISOCP model (M2) performs slightly better than the MIP model (M1) in terms of the total cost. Only exception to this situation is observed in C7, where the customer demands were assumed to be fairly low. This can be interpreted as that the MIP model is more efficient in the case where the vehicle capacity constraint is loose. A similar observation can be made for other cases as well. In C2 and C5, where the constraints are less tight, the final solution value of M2 becomes closer to that of M1.

On the other hand, LP bound was improved significantly further in M1. Consequently, the percentage gap between the LP bound and best integer solution is much wider in M2 than in M1. However, since we work with time limits, we focus on good feasible solutions rather than guaranteeing the optimality. Since the difference between the final solution values of M1 and M2 is not significant in the test cases, we decide to proceed with both alternatives in the future experimentations.

VRP itself is a computationally difficult problem, the extensions we have made renders it even more difficult to solve. The results of the preliminary experiments support this fact. It turns out that it is not possible to obtain optimal solutions even for very small networks in a reasonable amount of time. Therefore, the need for an alternative, time efficient approach is obvious. In the following parts we explain the development of heuristic approaches for this purpose.

In the next chapter, we consider a sub-problem where the route for a vehicle is given and optimal speeds (or travel times) along the route is to be found.

## CHAPTER 4

### A SUB-MODEL TO DETERMINE THE OPTIMAL VEHICLE SPEEDS

#### 4.1. Speed Optimizing Sub-model

Preliminary experiments have shown that mathematical programming formulations fail to generate optimal solutions to our problem within a reasonable amount of time, even in small networks. Since the real-world data sets are much larger, the problem becomes more difficult to solve, which creates a need for an alternative approach.

In order to develop a new method to solve our problem, firstly we address the problem in two parts. The first part involves the construction of the routes, and the second part is about determining the optimal speeds along these routes. In this section we present a speed optimization model and two basic propositions which will form the basis of the route construction algorithm.

A speed optimization algorithm has been proposed by Demir et al. [75] which calculates the optimal speeds first, and changes the speed in case of violation of the time windows. In this study, we present a similar speed optimization procedure compatible with multiple vehicle types and deadlines.

Assume that feasible routes have been initially constructed and a vehicle has been assigned to a route. In this case, the minimum cost solution could be obtained by finding the optimal vehicle speed or travel time on each link in each route. In this chapter, we provide a new nonlinear programming formulation that aims to find optimal vehicle speeds on predetermined routes to minimize the total cost. We introduce the following additional notation for this new mathematical model. Let the set of routes be denoted by  $R$ .  $A_r$  represents the set of all arcs in route  $r \in R$ . Similarly,  $k_r$  shows the type of vehicle that is assigned to route  $r$ . Index  $i_r$  stands for the  $i^{\text{th}}$  node in route  $r$  given that the depot node is assumed to be the  $0^{\text{th}}$  node, and  $s(r)$  indicates the size of route  $r$  (number of customer nodes in  $r$ ).  $v_{ij}$  and  $t_{ij}$  denote vehicle speed and travel time on arc  $(i, j)$ , respectively, as presented in the earlier formulations.

*Model 3 (M3)*

$$\text{Minimize } \sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) \alpha_{ij} d_{ij} w_{k_r} \quad (4.1)$$

$$+ \sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) \alpha_{ij} d_{ij} f_{ij} \quad (4.2)$$

$$+ \sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) d_{ij} \beta_{k_r} v_{ij}^2 \quad (4.3)$$

$$+ \sum_{r \in R} \sum_{(i,j) \in A_r} p t_{ij} + \sum_{j \in N_c} q_j \tau \quad (4.4)$$

*subject to*

$$\sum_{i=1}^l (t_{(i-1)r, i_r} + q_{(i-1)r} \tau) \leq b_{l_r} \quad \forall r \in R, \forall l \in \{1, 2, \dots, s(r)\} \quad (4.5)$$

$$t_{ij} v_{ij} = d_{ij} \quad \forall r \in R, \forall (i, j) \in A_r \quad (4.6)$$

$$\underline{v}_{ij} \leq v_{ij} \leq \bar{v}_{ij} \quad \forall r \in R, \forall (i, j) \in A_r \quad (4.7)$$

$$t_{ij} \geq 0 \quad \forall r \in R, \forall (i, j) \in A_r \quad (4.8)$$

In this sub-model, (4.1) and (4.2) are constant terms since the routes are predetermined. We simplify the formulation by defining a new constant,

$$K_R = \sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) \alpha_{ij} d_{ij} w_{k_r} + \sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) \alpha_{ij} d_{ij} f_{ij} + \sum_{j \in N_c} q_j \tau$$

and replacing  $v_{ij}$ 's with  $d_{ij}/t_{ij}$  in order to make all further computations in terms of travel times. Moreover, we define new parameters  $\bar{t}_{ij} = d_{ij}/\underline{v}_{ij}$  and  $\underline{t}_{ij} = d_{ij}/\bar{v}_{ij}$ . Given that there are  $|R|$  routes in the initial solution, the problem can be split into  $|R|$  separate problems, each of which determines the optimal vehicle speeds throughout a single route. Objective function of the model is denoted by  $Z_{r^*}$ . The model for solving the speed problem for a single representative route ( $r^*$ ) is given below:

*Model 4 (M4)*

$$\text{Minimize} \quad Z_{r^*} = K_{r^*} + \sum_{(i,j) \in A_{r^*}} [(c_f + c_e) d_{ij}^3 \beta_{k_r} t_{ij}^{-2} + p t_{ij}] \quad (4.9)$$

subject to

$$\sum_{i=1}^l (t_{(i-1)r^*, i_{r^*}} + q_{(i-1)r^*} \tau) \leq b_{l_{r^*}} \quad \forall l \in \{1, 2, \dots, s(r^*)\} \quad (4.10)$$

$$\underline{t}_{ij} \leq t_{ij} \leq \bar{t}_{ij} \quad \forall (i, j) \in A_{r^*} \quad (4.11)$$

## 4.2. Properties of the Sub-model

The objective function of M4 (4.9) is convex on interval  $(0, +\infty)$ . Let us introduce function  $f(t_{ij}) = (c_f + c_e) d_{ij}^3 \beta_{k_r} t_{ij}^{-2}$ . We assume that there exist non-negative dual variables  $\mu_l$  associated with the constraint set (4.10),  $\underline{\lambda}_{ij}$  and  $\bar{\lambda}_{ij}$ , associated with the constraint set (4.11). Under these circumstances, the lagrangian function ( $L$ ) of M4 can be written as follows:

$$\begin{aligned}
L(t, \mu, \underline{\lambda}, \bar{\lambda}) = & K_{r^*} + \sum_{(i,j) \in A_{r^*}} (f(t_{ij}) + pt_{ij}) + \sum_{l=1}^{s(r^*)} \mu_l \left( \sum_{i=1}^l (t_{(i-1)r^*, i_{r^*}} + q_{(i-1)r^*} \tau) - b_{l_{r^*}} \right) \\
& + \sum_{(i,j) \in A_{r^*}} \underline{\lambda}_{ij} (\underline{t}_{ij} - t_{ij}) + \sum_{(i,j) \in A_{r^*}} \bar{\lambda}_{ij} (t_{ij} - \bar{t}_{ij})
\end{aligned} \quad (4.12)$$

According to *Karush-Kuhn-Tucker* (KKT) conditions,  $t^*$  is an optimal solution to M4 only if  $\partial L(t_{ij}^*)/\partial t_{ij} = 0$  for all  $(i, j) \in A_{r^*}$ . Given that  $i, j$  and  $k$  are successive nodes in route  $r^*$ , the following equations must hold in case of optimality:

$$\frac{\partial L}{\partial t_{ij}}(t^*, \mu^*, \underline{\lambda}^*, \bar{\lambda}^*) = f'(t_{ij}^*) + p + \sum_{l=j}^{s(r^*)} \mu_l^* - \underline{\lambda}_{ij}^* + \bar{\lambda}_{ij}^* = 0 \quad (4.13)$$

$$\frac{\partial L}{\partial t_{jk}}(t^*, \mu^*, \underline{\lambda}^*, \bar{\lambda}^*) = f'(t_{jk}^*) + p + \sum_{l=k}^{s(r^*)} \mu_l^* - \underline{\lambda}_{jk}^* + \bar{\lambda}_{jk}^* = 0 \quad (4.14)$$

Based on these two equations, we come up with three basic propositions regarding M4.

### **Proposition 3**

Consider two successive arcs  $(i, j)$  and  $(j, k)$  on a given route. In the optimal solution to M4, if deadline constraint for node  $j$  and speed limitation constraints for arcs  $(i, j)$  and  $(j, k)$  are not tight, then  $\partial Z_{r^*}(t_{ij}^*)/\partial t_{ij} = \partial Z_{r^*}(t_{jk}^*)/\partial t_{jk}$ . If the deadline constraint for node  $j$  is tight in contrast to the speed limitation constraints, then  $\partial Z_{r^*}(t_{ij}^*)/\partial t_{ij} \leq \partial Z_{r^*}(t_{jk}^*)/\partial t_{jk}$ .

### **Proof to Proposition 3**

Provided that the speed limitation constraints are loose, all dual variables associated with these constraints  $(\underline{\lambda}_{ij}, \underline{\lambda}_{jk}, \bar{\lambda}_{ij}, \bar{\lambda}_{jk})$  should take the value of zero due to the complementary slackness conditions. In this case, subtracting (4.14) from (4.13), we obtain:

$$f'(t_{ij}^*) - f'(t_{jk}^*) + \mu_j^* = 0 \quad (4.15)$$

If the deadline constraint for node  $j$  is not tight, then  $\mu_j^* = 0$  due to complementary slackness. In this case,  $f'(t_{ij}^*) = f'(t_{jk}^*)$ . Since  $\partial Z_{r^*}(t_{ij}^*)/\partial t_{ij} = f'(t_{ij}^*) + p$  and  $\partial Z_{r^*}(t_{jk}^*)/\partial t_{jk} = f'(t_{jk}^*) + p$ , this leads to:

$$\frac{\partial Z_{r^*}}{\partial t_{ij}}(t_{ij}^*) = \frac{\partial Z_{r^*}}{\partial t_{jk}}(t_{jk}^*) \quad (4.16)$$

In case the deadline constraint is loose, then  $\mu_j^*$  will take a non-negative value, which results in:

$$\frac{\partial Z_{r^*}}{\partial t_{ij}}(t_{ij}^*) \leq \frac{\partial Z_{r^*}}{\partial t_{jk}}(t_{jk}^*) \quad (4.17)$$

**Proposition 4**

In the optimal solution to M4, if deadline constraint for node  $j$  and travel time constraints for arcs  $(i, j)$  and  $(j, k)$  are not tight, then  $v_{ij}^* = v_{jk}^*$ . If the deadline constraint for node  $j$  is tight in contrast to the speed limitation constraints, then  $v_{ij}^* \geq v_{jk}^*$ .

**Proof to Proposition 4**

Firstly, let us write  $f'(t_{ij}^*)$  in open form and replace  $t_{ij}^*$  by  $d_{ij}^*/v_{ij}^*$ :

$$f'(t_{ij}^*) = -2(c_f + c_e)d_{ij}^3 \beta_{k_r}(t_{ij}^*)^{-3} = -2(c_f + c_e)d_{ij}^3 \beta_{k_r} \frac{(v_{ij}^*)^3}{d_{ij}^3} = -2(c_f + c_e)\beta_{k_r}(v_{ij}^*)^3 \quad (4.18)$$

Now, let us rewrite equation (4.15) using expression (4.18):

$$-2(c_f + c_e)\beta_{k_r}(v_{ij}^*)^3 + 2(c_f + c_e)\beta_{k_r}(v_{jk}^*)^3 + \mu_j^* = 0 \quad (4.19)$$

If the deadline constraint for node  $j$  is loose,  $\mu_j^* = 0$ , which leads to

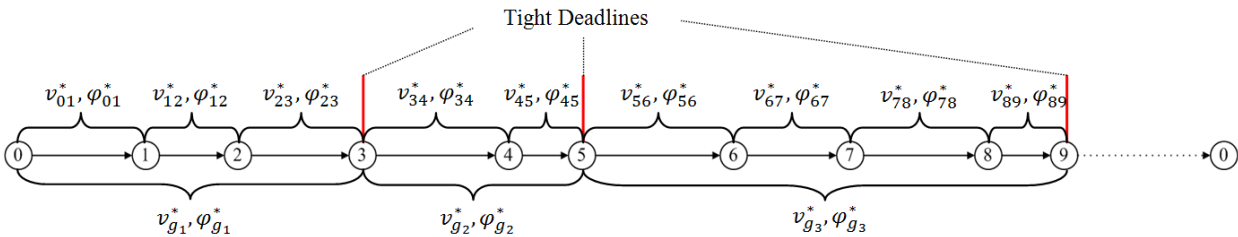
$$2(c_f + c_e)\beta_{k_r}(v_{ij}^*)^3 = 2(c_f + c_e)\beta_{k_r}(v_{jk}^*)^3 \rightarrow v_{ij}^* = v_{jk}^* \quad (4.20).$$

Otherwise, if the constraint is tight, then  $\mu_j^* \geq 0$  and

$$2(c_f + c_e)\beta_{k_r}(v_{ij}^*)^3 \geq 2(c_f + c_e)\beta_{k_r}(v_{jk}^*)^3 \rightarrow v_{ij}^* \geq v_{jk}^* \quad (4.21).$$

**Corollary 1**

Let  $\partial Z_{r^*}(t_{ij}^*)/\partial t_{ij}$  be denoted by  $\varphi_{ij}^*$ . In the light of propositions 1 and 2, an arbitrary route  $r^*$  is expected to have the following structure:



**Figure 3. Illustration of a Route**

As it can be observed in Figure 3, arcs between tight deadlines can be grouped together (as  $g_1$ ,  $g_2$  and  $g_3$ ) with common vehicle speeds ( $v_{g_1}^*$ ,  $v_{g_2}^*$  and  $v_{g_3}^*$ ) and partial derivative of objective function values ( $\varphi_{g_1}^*$ ,  $\varphi_{g_2}^*$  and  $\varphi_{g_3}^*$ ). For this specific case, the following conditions hold:

$$\varphi_{01}^* = \varphi_{12}^* = \varphi_{23}^* = \varphi_{g_1}^*, \quad \varphi_{34}^* = \varphi_{45}^* = \varphi_{g_2}^*, \quad \varphi_{56}^* = \varphi_{67}^* = \varphi_{78}^* = \varphi_{89}^* = \varphi_{g_3}^*$$

$$\text{and } \varphi_{g_1}^* \leq \varphi_{g_2}^* \leq \varphi_{g_3}^* \leq \dots,$$

$$v_{01}^* = v_{12}^* = v_{23}^* = v_{g_1}^*, \quad v_{34}^* = v_{45}^* = v_{g_2}^*, \quad v_{56}^* = v_{67}^* = v_{78}^* = v_{89}^* = v_{g_3}^*$$

$$\text{and } v_{g_1}^* \geq v_{g_2}^* \geq v_{g_3}^* \geq \dots.$$

### **Potential Use of the Properties**

The partial derivative of the objective function with respect to the travel time ( $\varphi_{ij}^*$ ) represents the marginal cost of changing the travel time. By using this figure, one can estimate the cost change that will occur if a new customer is added to a route, or an existing customer is removed.

Assume that a new customer is added into the  $l^{\text{th}}$  group ( $g_l$ ) of route  $r^*$ . If the vehicle continues to travel at its current speed along  $g_l$ , suppose that the total travel time on route  $r^*$  will increase by  $\Delta t$ . In this case, the total driver cost will also increase by  $p(\Delta t)$ . However, if there is a restrictive deadline for  $g_l$ , it will not be possible to let the travel time increase. Therefore, there is need for speeding up the vehicle in order to catch the deadline. We estimate the change in the total cost by speeding up the vehicle to reduce the travel time by  $\Delta t$  as  $-\varphi_{g_l}^*(\Delta t)$ . Assuming that  $(i, j)$  is any arc in group  $g_l$ , and  $t_{ij}^*$  is the optimal travel speed on this arc before the addition of the new customer, we estimate the change in the total cost as:

$$-\varphi_{g_l}^*(\Delta t) + p(\Delta t) = (-f'(t_{ij}^*) - p)(\Delta t) + p(\Delta t) = -f'(t_{ij}^*)(\Delta t) \quad (4.23)$$

A similar logic can be applied to the case where a customer is removed from the route. These type of expressions will be useful for estimating the change in the total cost when the structure of a route changes.

The properties we have defined also serve as a basis for developing methods to find optimal speeds without solving any mathematical programming models. The structure given in Corollary 1 can be directly applied to a route in which the sequence of customers is known, in order to determine vehicle speeds.

In the next chapter we present three heuristic algorithms for our problem, in which we make use of the properties described in this section.



## CHAPTER 5

### LOCAL SEARCH HEURISTICS

As we mentioned earlier, it is very difficult to find the optimal solution to our problem via mathematical programs in short CPU times. In the previous section, a mathematical model that determines optimal vehicle speeds has been presented. In this part, we introduce three local search heuristics to find fast and good solutions.

The general flow in our heuristic algorithms is similar to each other. All algorithms start with the generation of an initial solution. At each step, a neighborhood search procedure is executed. This cycle goes on until no better feasible solutions can be found. Below high level diagram summarizes the general structure of our heuristic algorithms.

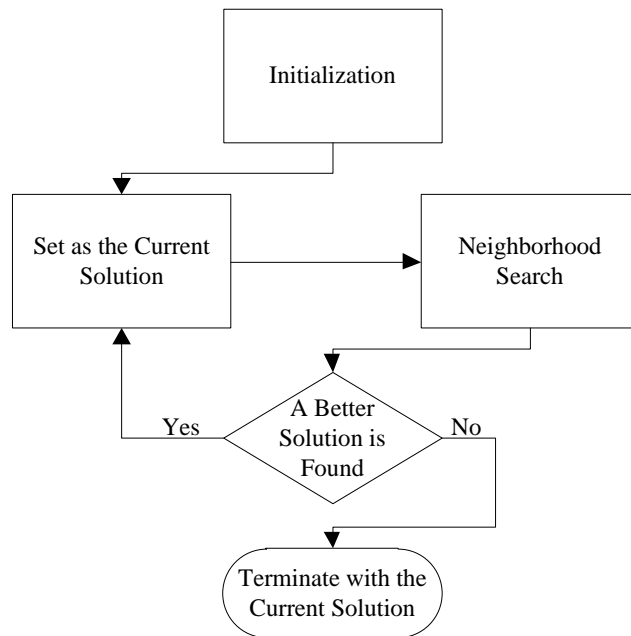


Figure 4. High-Level Diagram of Heuristic Algorithms

In this section, we will present the construction of the neighborhood structure and the evaluation of neighbor solutions followed by the initialization methodology. Finally the three heuristic algorithms will be explained in detail.

#### 5.1. Neighborhood Construction

In local search heuristics, neighborhood structure is the main determinant of solution speed and quality. Larger neighborhoods yield better quality solutions whereas smaller neighborhoods enable us to reach the final solutions much faster. As Gendrau and Tarantilis [83] state, in local search procedures neighborhoods are constructed via node or arc exchange operations. In our heuristics, the neighborhood is generated by simple node-exchange operations. We name these operations as *moves*. In the algorithms we

propose, all moves are examined for all possible combinations for each node in the network. Let us define the three moves that will be used in our solution methodology.

### 5.1.1. Inter-Route Relocate

Inter-route relocation move is defined as removing one of the nodes from its current position and inserting it into a different route. It is similar to a *shift process* defined by Osman [46]. This move is illustrated in the following figure.

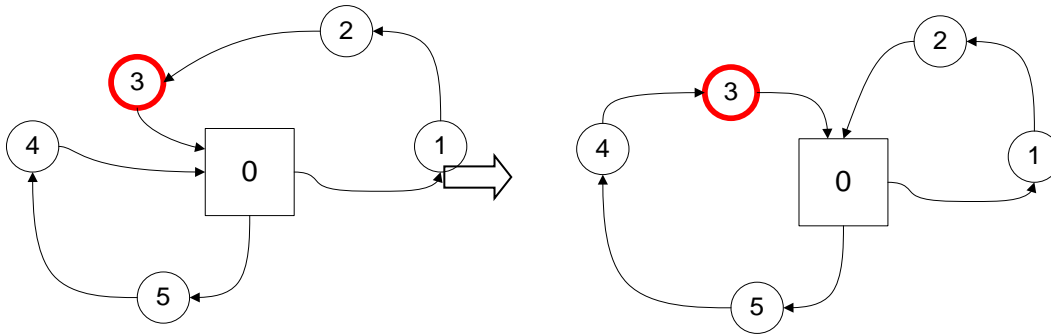


Figure 5. Inter-Route Relocation

In the illustration above, node 3 is moved from one route to another. This operation has several effects such as deletion of some existing arcs and addition of some new arcs to the solution. It is also possible that a new route is constructed, or an existing route is deleted as a result of relocation. Note that the formation of a new route is possible only if there are more vehicles in the fleet than the number of existing routes. The newly formed or removed route contains only one node. This situation is demonstrated in Figure 7.

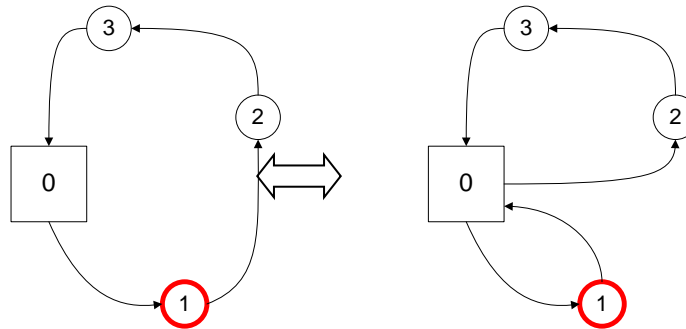


Figure 6. Inter-Route Relocation – 2

### 5.1.2. Exchange

In exchange move, a node is swapped with a second node from a different route. This move is firstly introduced by Osman [46] as the *interchange process*. Each node can be located at all possible positions in their new route. Route sizes remain the same as a result of this operation. The following figure shows the case where nodes 3 and 6 are exchanged.

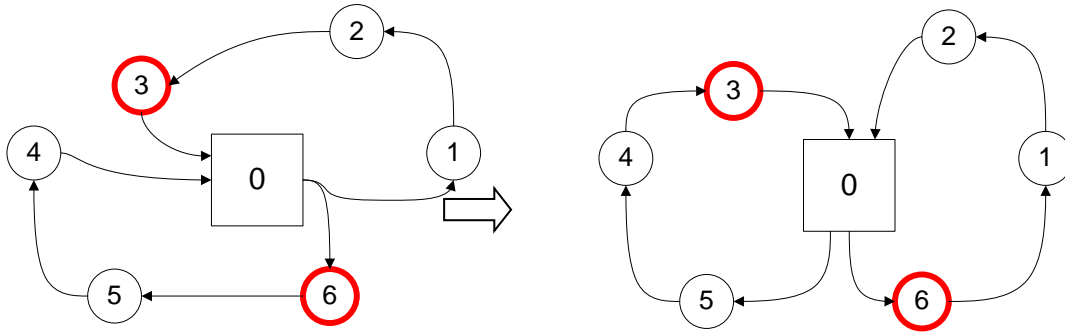


Figure 7. Exchange Move

### 5.1.3. Intra-Route Relocate

Intra-route relocate is similar to inter-route relocate except that it applies to a single route. In this operation, a node is selected and moved to a different position within its current route. If it is relocated in an adjacent position, then the resulting case will be nothing but the switch of two adjacent nodes. The following figure illustrates the intra-route relocation where node 1 is relocated to two possible positions in the route in two cases.

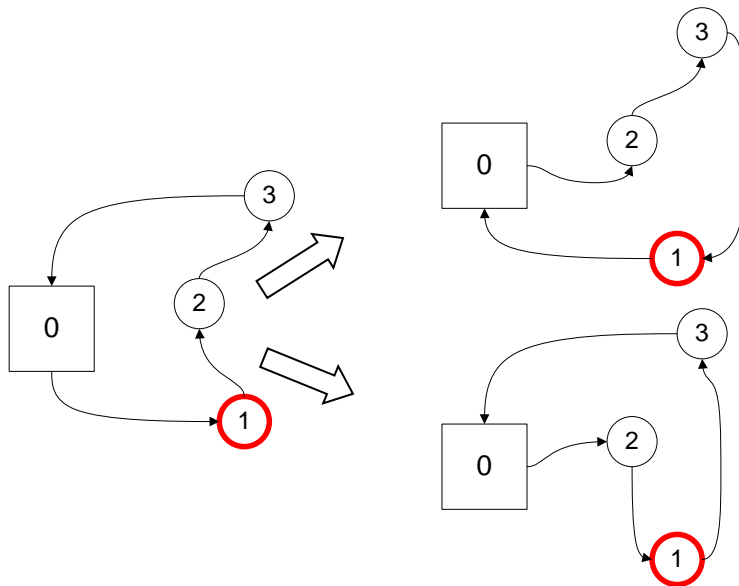


Figure 8. Intra-Route Relocation

In our heuristic algorithms, at each step, all possible combinations of these three moves will be evaluated and the best feasible solution among the alternatives will be decided. The best solution will be set as the new solution and the same procedure will be repeated.

After explaining how the neighborhoods are constructed at each step, let us move on to the evaluation of each solution in the neighborhood.

## 5.2. Evaluation of Neighbor Solutions

Once the current solution has been set, it is possible to generate neighbor solutions by performing the moves that have been introduced in the previous section. In order to determine which solutions are favorable, there is need for a mechanism to evaluate neighbor solutions. This is the step where our heuristic algorithms get separated from each other. We propose two alternative approaches in this regard. The first method is to estimate the change in the total cost by taking into account the deleted and newly formed arcs in the solution with weight, speed and driver time considerations. The second approach that we consider is finding the exact solutions resulting from the moves by means of an algorithm that calculates the optimal vehicle speeds. In the detailed analysis of our heuristics, we will introduce a third option: Employing a hybrid strategy which computes the exact solutions for a limited number of instances, which are determined using the estimates in the first method. For now, let us explain the two evaluation approaches in detail.

### 5.2.1. Estimation of Change in the Total Cost

In order to estimate the change in the total cost resulting from a move, firstly we address the composition of the total cost function. Let us recall the objective function of M3:

$$\sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) \alpha_{ij} d_{ij} w_{k_r} \quad (4.1)$$

$$+ \sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) \alpha_{ij} d_{ij} f_{ij} \quad (4.2)$$

$$+ \sum_{r \in R} \sum_{(i,j) \in A_r} (c_f + c_e) d_{ij} \beta_{k_r} v_{ij}^2 \quad (4.3)$$

$$+ \sum_{r \in R} \sum_{(i,j) \in A_r} p t_{ij} + \sum_{j \in N_c} q_j \tau \quad (4.4)$$

This cost function consists of four cost components. First three expressions represent the fuel and greenhouse emission costs. (4.1) emerges from the curb weight of the vehicle while (4.2) incorporates the fuel and emission cost based on the load carried by the vehicle. The third component (4.3) denotes the fuel and emission cost due to the speed of the vehicle and (4.4) stands for the total driver cost.

When a move is performed some arcs will be removed from the basis and replaced with new ones. Even this single alteration affects all cost components. Amount of load carried on a route will change since each customer node has different demands. Vehicle speed and driver times will also be subject to alterations due to differing deadline parameters and route distances.

In our cost change estimation procedure, we compute four different figures based on the cases mentioned above. These are:

$E_1$ : Cost change due to the curb weight of the vehicle

$E_2$ : Cost change due to the load carried by the vehicle

$E_3$ : Cost change due to the current speed of the vehicle

$E_4$ : Cost change due to speeding up or slowing down the vehicle (considering the driver costs)

Let us explain in detail how these figures are obtained.

### **Computation of $E_1$**

The cost related to the curb weight of the vehicle is directly linked with the term  $\alpha_{ij}d_{ij}$ . Since  $\alpha_{ij}$  is assumed to be constant for all  $(i, j) \in A$ , the parameter that matters is  $d_{ij}$ . Since the arcs in the basis will be replaced, the value of expression (4.1) in the total cost function is also subject to change. This change can be explicitly calculated. Detailed calculations are given in Appendix C.1 for each type of move.

### **Computation of $E_2$**

Computation of  $E_2$  is a bit more complicated than  $E_1$ . Assume that a new node is inserted in a route. In this case, the demand of the newly inserted node is carried along the road up to this node, which creates an extra cost. In the opposite case, the load carrying cost decreases for the related route. More complex situations may occur when two nodes are exchanged or a node is relocated within a route. Nevertheless, the exact cost change can be calculated by taking into account all possible cases resulting from the moves. Detailed calculations in this regard can be found in Appendix C.2.

### **Computation of $E_3$**

As a result of the moves, we expect the vehicle speeds to change due to the deadlines of the inserted or removed nodes. However at first, disregarding the new deadline limitations we assume that the current vehicle speeds does not change for the affected road segments, and compute how the cost would be different due to the changing arcs in the basis. Formulas for these calculations are given in Appendix C.3.

### **Computation of $E_4$**

The most difficult part of estimating the change in total cost is determining how the vehicle speeds and hence the total travel times would change as a result of a move. When a move is performed, an excessive number of cases may occur based on the locations of inserted and removed nodes, and the speed-change behavior could be different in each of these cases. On a single arc, for estimating the change in the cost by increasing the vehicle speed and decreasing the travel time, or vice versa, we make use of the previously defined derivative terms:  $f'(t)$ . In order to evaluate the alternative cases, we propose a complex procedure which tries to examine as much instances as possible. This procedure is explained in Appendix C.4 with all the details and relevant assumptions.

After all figures from  $E_1$  to  $E_4$  are computed, it is possible to find the estimation of the change in the total cost ( $E_T$ ) by summing up the values.

$$E_T = E_1 + E_2 + E_3 + E_4 \quad (5.1)$$

$E_T$  it is an indicator of how good the move would be and it is calculated for each possible move. In this way, a list of promising moves, a list of moves is generated, which can be used to decide which moves will be actualized.

After explaining the methodology of computing the cost change estimates, let us move on to our second neighborhood evaluation approach, finding the exact solution.

### 5.2.2. Calculation of the Exact Solution

Estimating the change in the total cost was one of the two ways of evaluating the neighbor solutions. The other approach we propose is calculating the exact solution for each possible move. We know that the optimal solution to the speed determination problem has unique characteristics. In this section, we introduce a simple algorithm that computes optimal vehicle speeds (or travel times) and the resulting total cost using these characteristics.

Recall Figure 3, which illustrates the structure of a route. A route consists of several arc groups that have common time derivative and vehicle speed values. The reason behind the formation of these groups is the restriction of customer deadlines. If no deadlines were imposed, then the optimal vehicle speeds could be easily calculated since the objective function of the problem is convex on interval  $(0, +\infty)$ . Assuming that the deadlines are not restrictive, we can rewrite the mathematical model M4 as:

*Model 5 (M5)*

$$\text{Minimize} \quad K_{r^*} + \sum_{(i,j) \in A_{r^*}} [(c_f + c_e)d_{ij}^3 \beta_{k_r} t_{ij}^{-2} + p t_{ij}] \quad (4.9)$$

subject to

$$\underline{t}_{ij} \leq t_{ij} \leq \bar{t}_{ij} \quad \forall (i,j) \in A_{r^*} \quad (4.11)$$

Where  $f(t_{ij}) = (c_f + c_e)d_{ij}^3 \beta_{k_r} t_{ij}^{-2}$ , the lagrangian function of this model can be written as:

$$L(t, \underline{\lambda}, \bar{\lambda}) = K_{r^*} + \sum_{(i,j) \in A_{r^*}} f(t_{ij}) + p t_{ij} + \sum_{(i,j) \in A_{r^*}} \underline{\lambda}_{ij} (\underline{t}_{ij} - t_{ij}) + \sum_{(i,j) \in A_{r^*}} \bar{\lambda}_{ij} (t_{ij} - \bar{t}_{ij}) \quad (5.2)$$

According to KKT conditions,  $t^*$  is an optimal solution to M5 only if  $\partial L(t^*, \underline{\lambda}^*, \bar{\lambda}^*) / \partial t_{ij} = 0$  for all  $(i,j) \in A_{r^*}$ . For an arbitrary arc  $(i,j)$ , the following equality must hold for optimality:

$$\frac{\partial L}{\partial t_{ij}}(t^*, \underline{\lambda}^*, \bar{\lambda}^*) = f'(t_{ij}^*) + p - \underline{\lambda}_{ij}^* + \bar{\lambda}_{ij}^* = 0 \quad (5.3)$$

Since  $\varphi_{ij}^* = \partial Z_{r^*}(t_{ij}^*) / \partial t_{ij} = f'(t_{ij}^*) + p$ , equation (5.3) is rewritten as:

$$\varphi_{ij}^* - \underline{\lambda}_{ij}^* + \bar{\lambda}_{ij}^* = 0 \quad (5.4)$$

When  $\underline{t}_{ij} < t_{ij}^* < \bar{t}_{ij}$ , both  $\underline{\lambda}_{ij}^*$  and  $\bar{\lambda}_{ij}^*$  are equal to zero, therefore:

$$\varphi_{ij}^* = 0 \quad (5.5)$$

Considering our problem parameters, preliminary experimentations have shown that the inequality  $\underline{t}_{ij} < t_{ij}^* < \bar{t}_{ij}$  is true in all cases for all types of vehicles. Therefore we conclude that equation (5.5) always holds for our specific problem.

At the beginning of our algorithm for determining the optimal travel times, for all routes, we set  $\varphi_{ij}^* = 0$  for all arcs in the route and calculate the corresponding  $t_{ij}^*$  values. Starting from the last customer node, we go backwards through the route and check whether deadlines are exceeded or not. In the case where the deadline is exceeded for a node, all arcs starting from the depot up to that node are grouped such that the vehicle speed each arc are equal to each other and the arrival time to the node equals its deadline. This process continues at each route until reaching the depot node. At the end, the structure demonstrated at Figure 3 is obtained and the optimal solution can be easily calculated using the optimal travel time values for each arc in the basis. This procedure is demonstrated in detail in Figure 9.

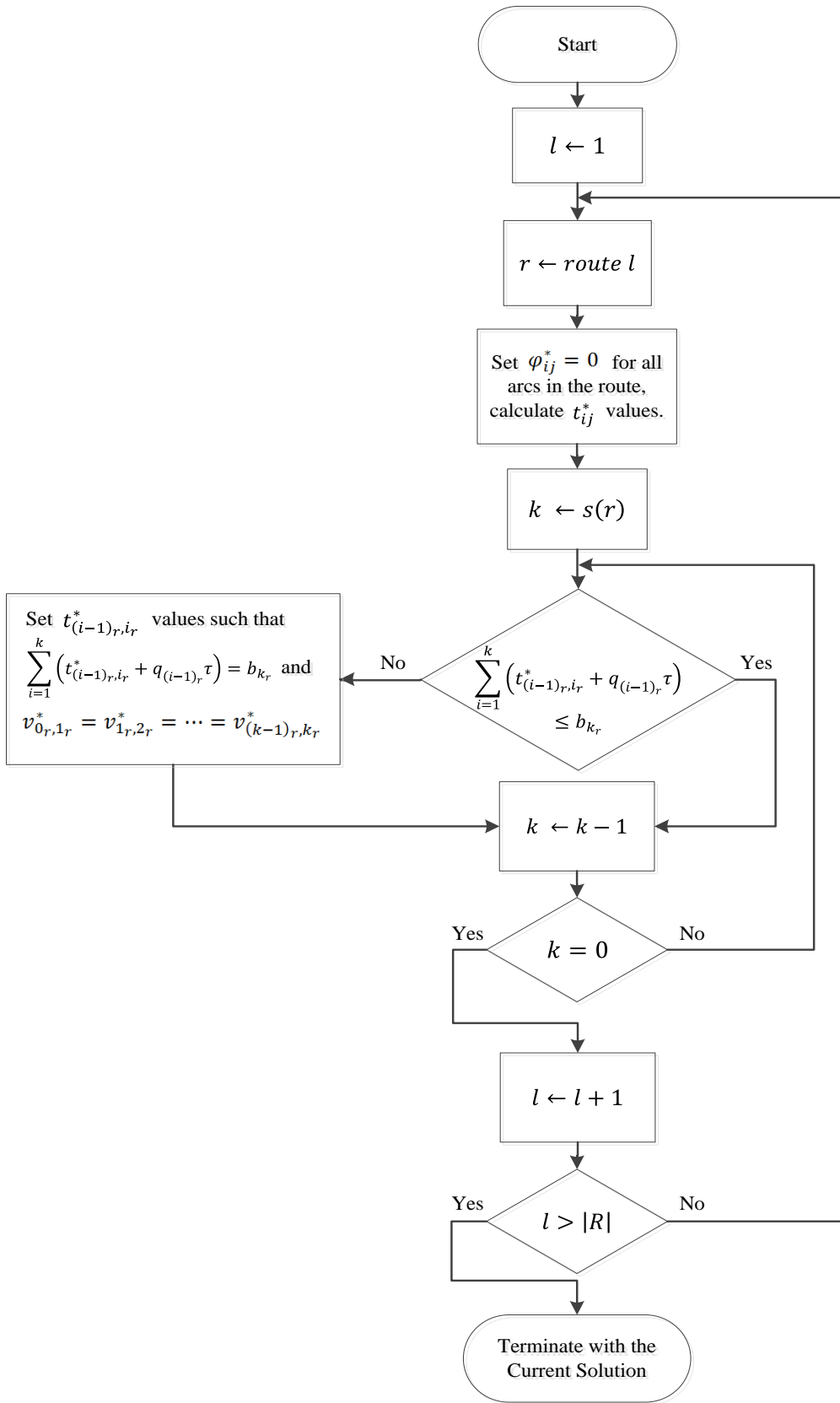


Figure 9. Travel Time Determination Procedure



After the travel time determination procedure is executed, the total cost is calculated using the  $t_{ij}^*$  values. By this way, the optimal solution is calculated without having to solve any mathematical programming models.

### 5.3. Generation of Initial Solution

We need to have a starting solution in order to be able to perform the moves that have been defined and make our heuristics work. The methodologies that generate a solution from the start are called *construction algorithms* in the VRP literature. Honsy [84] suggests that there are two main types of construction methods for VRPs, namely, *sequential* and *parallel construction* algorithms. The most famous type of sequential construction algorithm is the *insertion heuristics* described by Solomon [36]. In this approach, each customer node is inserted to a single route as long as there is enough capacity in the associated vehicle. A new route is generated only if the capacity of the vehicle would be exceeded otherwise. Our methodology is similar to the parallel construction algorithm introduced by Potvin and Rousseau [85], where it is allowed to add nodes to any of the possible routes.

The flow of our initialization procedure can be explained as follows:

- Step 1.* Generate empty tours for each vehicle in the fleet.
- Step 2.* Create a list of customer nodes where the nodes are arranged in ascending order based on  $b_j/d_{0j}$  values for all  $j \in N_c$ .
- Step 3.* Take the first node in the list. For each possible position that the node can be inserted, check if the insertion is feasible with regard to vehicle capacity and deadline restrictions. For the feasible insertions, execute the travel time determination procedure and calculate the resulting total costs.
- Step 4.* List all possible insertions in ascending order based on their total cost values.
- Step 5.* If the insertion list is empty, **STOP! No feasible solutions can be found.** Otherwise, apply the insertion at the top of the list, clear the insertion list and go to *Step 6*.
- Step 6.* Delete the entry at the top of the node list. If there are no items left in the node list, **STOP! The initial solution has been generated.** Otherwise go to *Step 3*.

In local search heuristics, the quality of initial solution is interrelated with the quality of the final solution most of the time. In order to generate a good initial solution, we have specifically chosen the  $b_j/d_{0j}$  parameter for providing a basis for listing the nodes. Three alternative options ( $b_j$ ,  $d_{0j}$ , and  $b_j/d_{0j}$ ) have been compared in the preliminary experiments, and selecting  $b_j/d_{0j}$  has yielded the best results.

### 5.4. Heuristic Algorithms

#### 5.4.1. Cost Change Estimates Heuristic (H1)

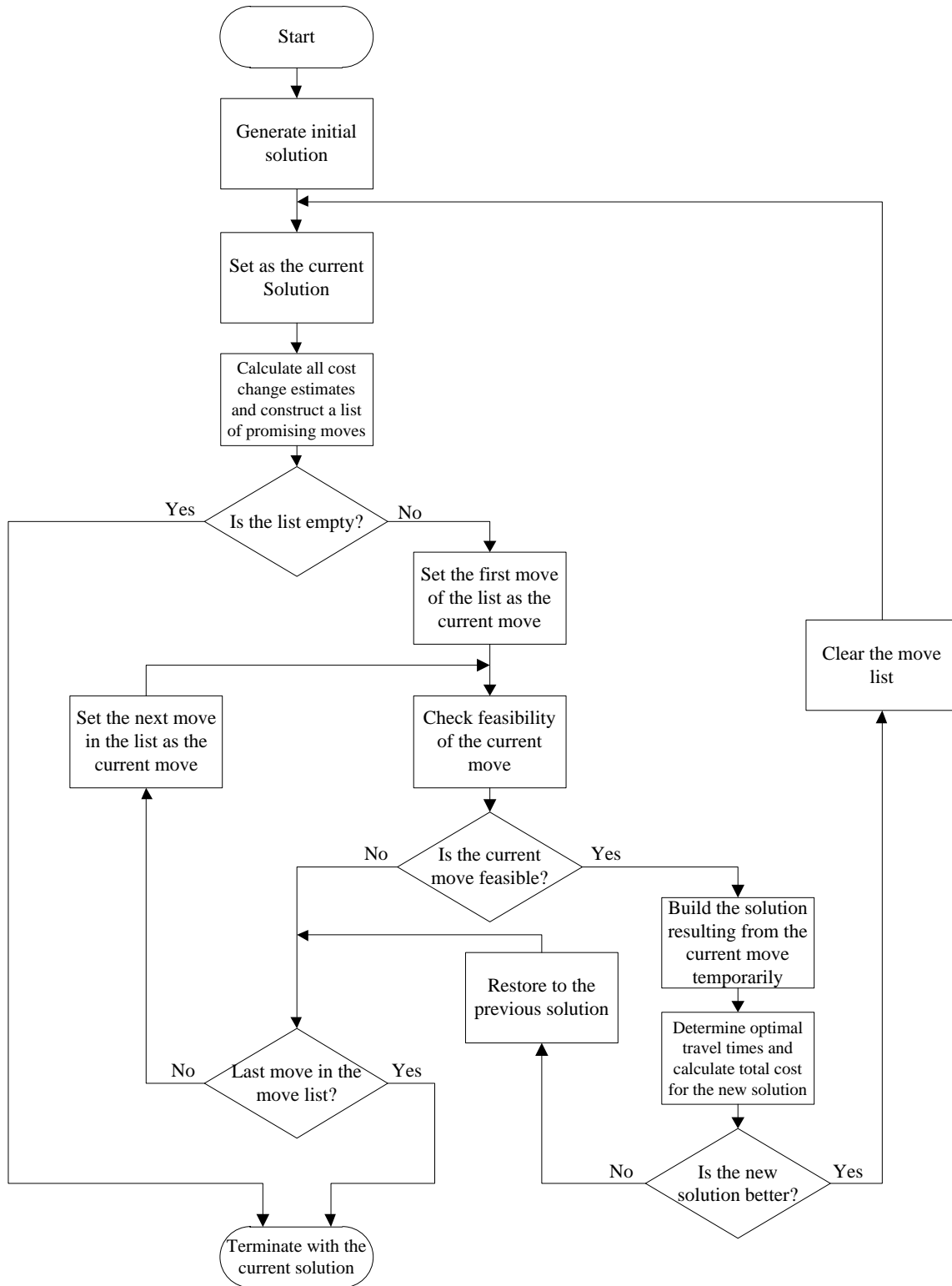
The first heuristic algorithm we propose aims to produce good solutions by constructing the neighborhood based on cost estimate figures specifically calculated for each move. The algorithm starts with the

generation of the initial solution by the method which has been explained in the previous section. After the solution is set as current, the cost change estimates are calculated for each move on the current routes and a list of promising moves is constructed in which moves are listed in ascending order based on their cost change estimates.

In order to determine whether a move is promising or not, we use a special parameter called *promising move indicator*. A move is considered to be promising if its cost change estimate is below the value of the promising move indicator, and left out of the promising moves list otherwise.

Once the promising moves list is completed, the algorithm checks the feasibility of the moves in the list beginning from the entry at the top. If a move is feasible in terms of both vehicle capacity and deadline constraints, the move is performed temporarily and the travel times are determined via the procedure explained in Section 5.2. The resulting total costs are calculated, and the move is rendered permanent if the calculated total cost value is lower than that of the best solution found so far.

The algorithm runs until the promising moves list becomes empty or no improving moves can be found. The following diagram illustrates the running principles of this algorithm in more detail.



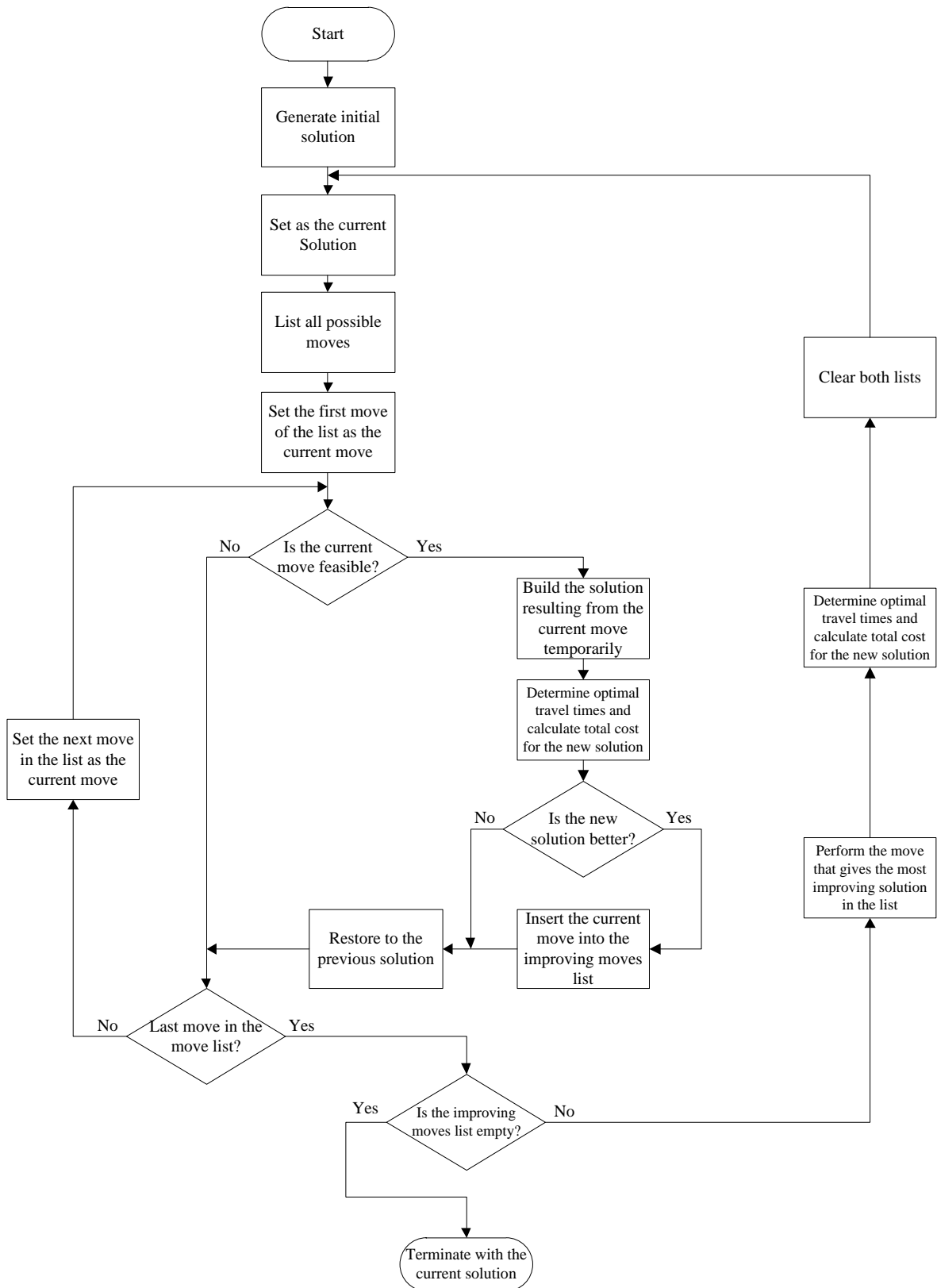
**Figure 10. Cost Change Estimates Heuristic**

#### 5.4.2. Exact Cost Calculation – Based Heuristic (H2)

The basic idea behind the second heuristic is to make sure that the best move is implemented at each step throughout the algorithm. Therefore, the neighborhood is constructed based on actual cost change figures rather than estimates as in H1.

The algorithm again starts with the generation of initial solution. Each possible move is checked for feasibility and the exact solution is calculated after the travel time determination procedure for the ones which pass the feasibility check. If the total cost is actually improved by a move, that move is inserted in the *improving moves list*. The most improving move in the improving moves list is performed and the resulting new solution is set as the current solution.

The algorithm goes until no improving solutions can be found. Calculating exact total costs for each feasible move may not seem a time efficient method, but it can be a good test case for evaluating the accuracy of the cost change estimates in H1. The flow of H2 is represented in Figure 12.



**Figure 11. Exact Solution Heuristic**

### 5.4.3. Hybrid Heuristic (H3)

Up to now, we have described two heuristic algorithms, one of which operates based on the cost change estimates and the other chooses the most improving move at each step by examining the actual cost figures. We expect H1 to produce solutions faster than H2. Since we employ a local search procedure, the quality of the final solution does not necessarily depend on the selection of the most improving moves throughout the algorithm. However executing H2 may cause a slight lack of quality in the solutions since it is not guaranteed that the best move is performed at each step.

The third heuristic we propose (H3) combines the powerful aspects of the first two methods. Firstly, the initial solution is generated as usual, followed by the construction of the promising moves list as in H1. We define a new move list called the *best moves list* which has a limited size. Beginning from the top of the promising moves list, we check the feasibility of the moves and calculate the exact solutions. If the solution found is better than the current best solution, the move is inserted in the best moves list.

Once the best moves list is filled in completely or the end of promising moves list is reached, move with the most improving solution in the best moves list is performed and the resulting solution is set as current. The algorithm continues until there are no improving moves left.

The key point of H3 is determining the size of the best moves list. If the size of the list is set as one, then the heuristic becomes exactly the same as H1. Alternatively, having a list size of  $+\infty$  renders the algorithm the same as H2. As the size of the list becomes larger, H3 becomes slower, but evaluates more options.

The flow of this heuristic is displayed in Figure 13 in more detail.

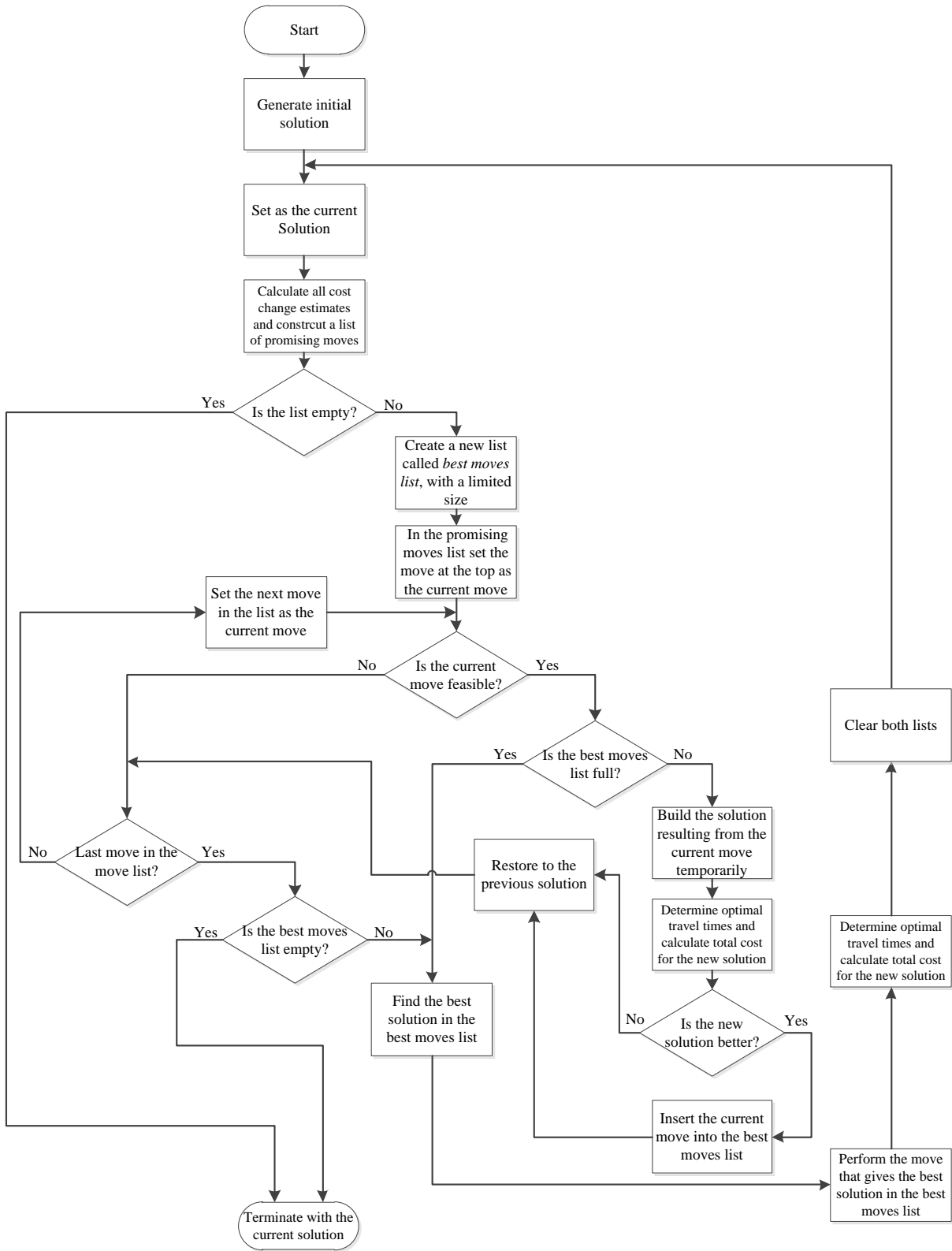


Figure 12. Hybrid Heuristic

## CHAPTER 6

### COMPUTATIONAL ANALYSIS

So far, two mathematical programming models and three heuristic algorithms have been proposed for the solution of our specific problem. We continue with an extensive computational study which evaluates each of these solution approaches under a variety of parameter settings to determine which of the methods are more appropriate to be used under what circumstances.

Throughout this chapter, we will firstly examine the base scenario with fixed solution parameters for alternative network sizes. Afterwards, we will share the results of our experiments with different parameter settings in order to observe the separate effects of different factors.

In all our experiments, we used distance matrices ( $d_{ij}$  values) provided by Demir et al. [75] which are composed of UK cities. Alternatives with 10, 15, 20, 25, 50, 75, 100, 150 and 200 customer nodes were examined. Heuristic algorithms were programmed in C++ and all computational experiments were run on a computer with a quad-core 2.00 GHz processor, 8 GB ram and 64-bit operating system.

#### 6.1. Analysis of the Base Scenario

The main purpose of the base scenario analysis is to evaluate the performance of each solution approach under a fixed parameter setting. We observe both solution quality and the efficiency of each method by changing problem size only.

The values of the parameters we use are the same as those specified in Appendix A.1 except for the problem network. Since the problem size is subject to change, we change the fleet sizes accordingly. The number of vehicles in the fleet for each network size alternative is given in Appendix D.1. Promising move indicator is set as £10 for H1 and H3, and the size of the best moves list is specified as 100 for H3. As it was mentioned in Section 3.3.3 regarding the preliminary model runs, it is necessary to impose a time limit for running the mathematical programming models. Since it would have been too time consuming to put a high limit on the solution time for all cases, we decided to use a time limit of  $100|N_c|$  seconds, where  $|N_c|$  denotes the number of customers in the network.

Under these settings, the mathematical programs were able to generate a feasible solution up to 50-customer networks; no feasible solutions were found for networks with 75 customer nodes and more. Therefore, we are able to compare the models and the heuristics only for 10, 15, 20, 25 and 50-customer cases.

We divide this section into three parts. In the first part, model and heuristic results are interpreted, in the second part, heuristics are compared with each other and in the last part, general deductions from the results of the base scenario are presented.

##### 6.1.1. Comparison of Model and Heuristic Results

The quality of solutions generated by heuristic algorithms can be evaluated by comparing them with those



generated by mathematical models. 10 replications with different distance matrices and random number seeds were run for each solution method (M1, M2, H1, H2 and H3) and 10, 15, 20, 25 and 50-customer instances. Detailed results of these experiments are given in Appendix D.2. Average values of 10 replications are represented in Table 5.

**Table 5. Comparison of Models and Heuristics – Average Results**

		Models					
		M1			M2		
Network Size	Time Limit	Total Cost (£)	Total CPU Time (sec)	GAP	Total Cost (£)	Total CPU Time (sec)	GAP
10	1000 sec	171.87	1003.94	26.67%	170.38	1001.10	37.38%
15	1500 sec	222.27	1503.04	63.34%	217.80	1503.14	66.69%
20	2000 sec	288.48	2011.02	67.17%	296.61	2037.36	70.76%
25	2500 sec	293.43	2518.61	65.91%	309.59	2569.24	70.94%
50 <sup>1</sup>	5000 sec	693.60	5317.15	70.60%	731.02	5804.75	75.14%
		Heuristics					
		H1		H2		H3	
Network Size	Time Limit	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
10	-	178.58	0.015	179.35	0.015	179.35	0.021
15	-	229.98	0.034	236.59	0.034	236.59	0.042
20	-	279.79	0.042	276.18	0.069	276.18	0.051
25	-	290.21	0.107	287.06	0.202	287.06	0.135
50	-	587.07	1.197	569.92	4.915	569.92	1.419

At first glance, it can be clearly observed that the total cost figures of models are better than those of the heuristics for network sizes below 20 customers. On the other hand, each heuristic approach performs better than each of the models for networks populated by 20 or more customers. In order to make a better comparison, the average model and heuristics results in terms of total cost are illustrated as follows:

**Table 6. Average Model and Heuristic Results**

Network Size	Average Model Result (£)	Average Heuristic Result (£)	% Difference
10	171.12	179.09	+4.66%
15	220.04	234.39	+6.52%
20	292.54	277.38	-5.18%
25	301.51	288.11	-4.44%
50	712.31	575.64	-19.19%

The results suggest that the absolute percentage differences are in the range of 4% - 7% up to the 25-customer variant. The difference gets significantly larger (19.19%) as the network size reaches 50 customers. The time required to obtain these results is dramatically higher for mathematical models compared to the heuristics. From a decision-making point of view, if there is enough time for running all the procedures, it would be nice to see both model and heuristic results for networks with 25 or less customers in order to select the best option. Otherwise, heuristic algorithms are still capable of generating good solutions even for small networks.

<sup>1</sup> In one of the replications, no feasible solution could be found by M2. Therefore the figures of the 50-customer case are calculated as the average of 9 replications.

Another observation that can be made regarding the models is that M2 performs better than M1 in 10 and 15-customer networks whereas the results of M1 get better as the network size grows. This result suggests that it is more appropriate to employ M2 in networks that have less than 20 customers, and employ M1 otherwise.

### 6.1.2. Comparison of Heuristics

After comparing heuristics to mathematical models, let us move on to the comparison of heuristics algorithms with each other. Obviously, the most important criteria for this comparison are total cost and solution time figures. The results of all 10 replications for each network size alternative are given in Appendix D.3. See Table 7 for the average figures.

**Table 7. Comparison of Heuristics – Average Results**

Network Size	H1		H2		H3	
	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
<b>10</b>	178.58	0.015	179.35	0.015	179.35	0.021
<b>15</b>	229.98	0.034	236.59	0.034	236.59	0.042
<b>20</b>	279.79	0.042	276.18	0.069	276.18	0.051
<b>25</b>	290.21	0.107	287.06	0.202	287.06	0.135
<b>50</b>	583.95	1.208	563.88	5.214	563.88	1.479
<b>75</b>	866.64	5.791	866.05	32.881	869.03	6.426
<b>100</b>	1062.04	22.511	1062.99	161.395	1062.14	32.971
<b>150</b>	1404.63	166.670	1450.06	1249.38	1464.80	215.154
<b>200</b>	1805.41	726.741	1808.83	6019.96	1791.46	983.031

Average results clearly show that the total cost figures are very close to each other in general. In spite of our expectations that H2 would generate better results than H1, there is no significant difference between the solution quality of these two approaches and in some cases H1 performs even better than H2. It is not possible to claim that an approach outweighs others in terms of total cost and there is no visible trend in the comparative performances of the methods depending on the network size. For instance, the average results of H3 seem to be the best among all heuristic results in 200-customer case and the worst in the 150-customer case. Similarly, the percentage difference between the results of H1 and H2 is very small (0.089%) in 100-customer case, it gets larger in 150-customer case (3.13%) and again decreases in the 200-customer case (0.19%).

H2 and H3 yields exactly the same results up to 50-customer variant. Their average results become different in networks that include 75 customers or more. This was an expected result since H3 is designed as nothing but the application of H2 on a limited number of promising moves. The results indicate that the calculated cost estimates are quite accurate. It is not difficult to predict that the results of H2 and H3 would have been the same for larger networks as well if the size of the best moves list was set to a larger value.

Total CPU time is another important aspect for evaluation of heuristic performances. Average CPU times are illustrated in the following chart for each network size alternative.

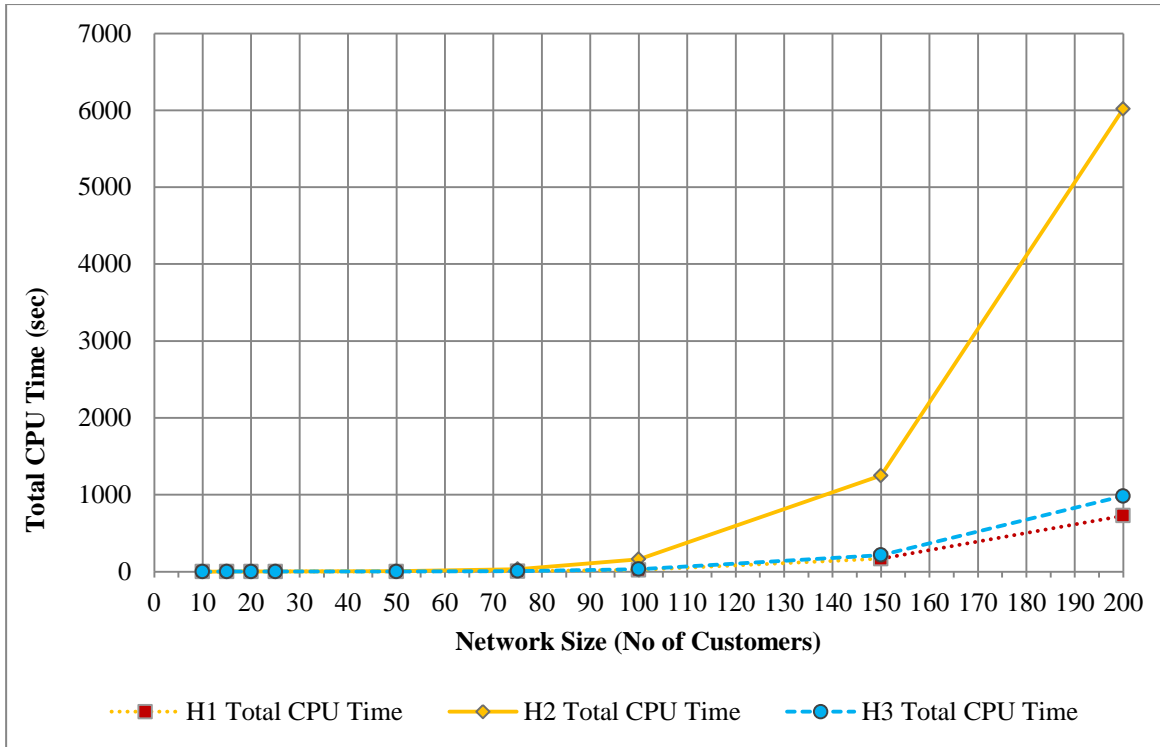


Figure 13. Average CPU Times

Figure 14 clearly shows that the solution time of H2 gets significantly higher than those of H1 and H3 as the network size becomes larger. Since there is no noteworthy difference between the total cost figures of H2 and H3, it can be concluded that H3 outweighs H2 when the solution time is taken into consideration together with the total cost.

### 6.1.3. General Interpretations

Up to now, performances of the solution methods have been compared to each other in terms of total cost and CPU time figures within this section. In this part, we interpret other results that can be derived from the solutions in order to create a better understanding of the methodologies introduced in this study.

#### Vehicle Usage

The following table shows the average number of vehicles used by each vehicle type and solution variant.

Table 8. Average Vehicle Usage by Vehicle Type

Network Size	Fleet Size (Vehicle Type 1)	Number of Vehicles Used (Vehicle Type 1)					Fleet Size (Vehicle Type 2)	Number of Vehicles Used (Vehicle Type 2)				
		M1	M2	H1	H2	H3		M1	M2	H1	H2	H3
10	1	0.9	0.9	1	1	1	1	1	1	1	1	1
15	2	1.7	1.8	2	2	2	1	1	1	1	1	1
20	2	2	2	2	2	2	2	1.4	1.6	1.2	1.2	1.2
25	2	1.8	2	2	2	2	2	2	1.9	2	2	2
50	4	3.8	3.9	4	4	4	4	3.8	3.7	2.9	2.9	2.9
75	5	-	-	5	5	5	5	-	-	4.2	4.2	4.2

<b>100</b>	<b>8</b>	-	-	8	8	8	<b>8</b>	-	-	4.9	5	5
<b>150</b>	<b>10</b>	-	-	10	10	10	<b>10</b>	-	-	7.9	7.9	7.9
<b>200</b>	<b>15</b>	-	-	15	15	15	<b>15</b>	-	-	9.7	9.7	9.7

By analyzing the fleet size and number of used vehicles figures, the fleet utilization is found as 98.3% and 83.7% for vehicle types 1 and 2, respectively. Breaking these figures down to solution types, we realize that 100% of type-1 vehicles are utilized in heuristic results whereas this rate stays at 93.7% for models. The utilization of type-2 vehicles is found as 93.2% in model and 86.5% in heuristic results for network sizes below 75 customers.

Type-1 vehicles are smaller in terms of weight and capacity. The results clearly show that the heuristics try to utilize as much small vehicles as possible whereas models behave in a more balanced manner. This can be seen as a consequence of the network structure and absence of vehicle acquisition cost. It can be claimed that the utilization of the bigger vehicles would have been larger if the customer locations were further from each other and a fixed cost was applied for employing an additional vehicle.

### **Distribution of Costs**

Total cost figure comprises of fuel and emission costs which are related to the weight of the vehicles and the load they carry and vehicle speed, and the wages of drivers which is calculated on time basis. The percentages of each of these costs within the total cost are demonstrated as follows:

**Table 9. Distribution of Costs**

Network Size	Cost Percentages				
	Fuel and Emission Cost			Driver Cost	Total
	Curb Weight	Load Carried	Vehicle Speed		
<b>10</b>	23.64%	6.31%	24.51%	45.54%	100%
<b>15</b>	20.60%	6.71%	25.03%	47.66%	100%
<b>20</b>	21.39%	8.05%	26.43%	44.14%	100%
<b>25</b>	22.45%	8.27%	23.86%	45.42%	100%
<b>50</b>	21.95%	8.70%	25.34%	44.01%	100%
<b>75</b>	22.14%	9.97%	24.68%	43.21%	100%
<b>100</b>	21.06%	9.38%	24.17%	45.39%	100%
<b>150</b>	21.77%	9.93%	23.27%	45.03%	100%
<b>200</b>	21.12%	9.53%	23.36%	45.99%	100%
<b>Average</b>	<b>21.79%</b>	<b>8.54%</b>	<b>24.52%</b>	<b>45.15%</b>	<b>100%</b>

Cost percentages do not show a significant trend based on the network growth. Driver cost is the primary determinant of total cost with a percentage of 45.15% on average. It is followed by the fuel and emission costs resulting from the vehicle speed (24.52%) and the cost related with the curb weight of the vehicle (21.79%). Cost associated with the amount of load carried constitutes the least important portion of total cost with an average percentage of 8.54%.

Percentage of total fuel and emission cost is 54.85%, which is slightly higher than that of the driver cost. However, fuel and emission cost is made up of distinct components which cannot be controlled together. Since the driver cost is the most important figure in the composition of the total cost, it can be stated that

minimizing total travel times is the primary interest of our solution methods while keeping an eye on other cost components.

### Vehicle Speeds

In the base scenario, average vehicle speeds are found to be as follows by vehicle type and solution method:

**Table 10. Average Vehicle Speeds**

	Average Speed (km/h)					Total Average (km/h)	Optimal Speed (km/h)	% Difference (Average Spd – Optimal Spd)
	M1	M2	H1	H2	H3			
<b>Vehicle Type 1</b>	53.54	52.85	53.48	53.46	53.48	<b>53.36</b>	<b>52.74</b>	<b>+1.18%</b>
<b>Vehicle Type 2</b>	46.82	49.57	49.00	48.56	48.53	<b>48.50</b>	<b>46.81</b>	<b>+3.60%</b>

The optimal speed is the speed that is calculated under the assumption that no deadline constraints are imposed by customers. Each vehicle type has a different optimal speed due to the difference in the vehicle specific parameters. Vehicles never travel at speeds below their respective optimal speed levels since in that case the driver cost would become more dominant and eventually total cost would be larger.

Results of the experiments suggest that average speed figures are slightly higher than optimal speed values; by 1.18% for vehicle type 1 and 3.6% for vehicle type 2. We would expect these differences to be higher if the deadline restrictions were more restrictive.

### Analysis of Heuristic Moves

We have previously defined three types of moves to be used in the heuristics. In this section we analyze the number of moves performed and the percentage of improvement provided by each move type on the initial solution.

The detailed results by heuristic types are given in Appendix D.4. The following table represents the average results by network size. Note that inter-route relocate, exchange and intra-route relocate moves are denoted by move type 1, 2 and 3, respectively.

**Table 11. Heuristic Moves – Average Results**

	Move Type	Network Size (Number of Customers)								
		10	15	20	25	50	75	100	150	200
<b>Number of Moves</b>	<b>1</b>	0.47	0.77	1.07	1.53	5.17	5.43	7.43	10.33	14.30
	<b>2</b>	1.10	2.60	3.63	5.33	16.70	26.70	48.10	83.20	131.33
	<b>3</b>	0.83	0.73	0.77	1.67	2.03	3.43	4.80	5.83	9.17
	<b>Total</b>	<b>2.40</b>	<b>4.10</b>	<b>5.47</b>	<b>8.53</b>	<b>23.90</b>	<b>35.57</b>	<b>60.33</b>	<b>99.37</b>	<b>154.8</b>
<b>Move Type Percentage</b>	<b>1</b>	19.4%	18.3%	19.4%	18%	21.7%	15.1%	12.3%	10.4%	9.3%
	<b>2</b>	45.8%	64.1%	66.6%	62.6%	69.8%	75%	79.7%	83.8%	84.8%
	<b>3</b>	34.7%	17.7%	14.1%	19.4%	8.4%	9.9%	8%	5.8%	5.8%
	<b>Total</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>
<b>% Improvement</b>	<b>1</b>	2.6%	2.0%	1.7%	2.8%	4.4%	2.3%	2.1%	2.1%	1.7%

	2	12.5%	13.3%	13.3%	12.8%	23.5%	25.8%	31.9%	35.2%	35.8%
	3	0.6%	0.5%	0.3%	0.9%	0.3%	0.8%	0.5%	0.3%	0.5%
	Total	15.7%	15.9%	15.4%	16.5%	28.1%	29%	34.5%	37.6%	37.9%

As it can be derived from the figures, the majority of the moves performed are of type 2. As the number of customers in the network increase, the number of exchange moves performed increases at a similar rate whereas the rate of increase of both types of relocation moves is much smaller compared to exchange move. This situation can be clearly observed in Figure 14.

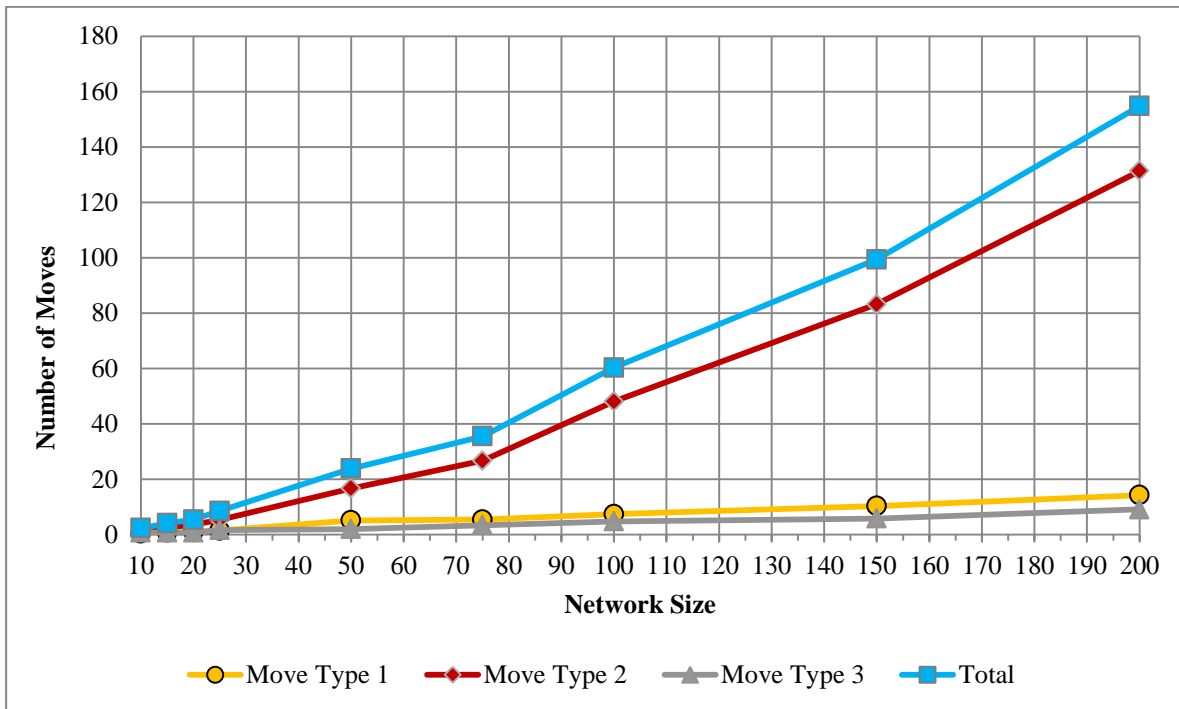


Figure 14. Number of Moves by Network Size

While the number of moves performed increases, the percentage of improvement on initial solution achieved by these movements also increases with the network size in general. The percentage improvement remains steady around 16% up to 25-customer network, increases in a steep manner as the number of customers in the network becomes 50, and continues to increase with a decreasing rate. Exchange type move is the primary determinant of the improvement percentage. These results are illustrated in Figure 16.

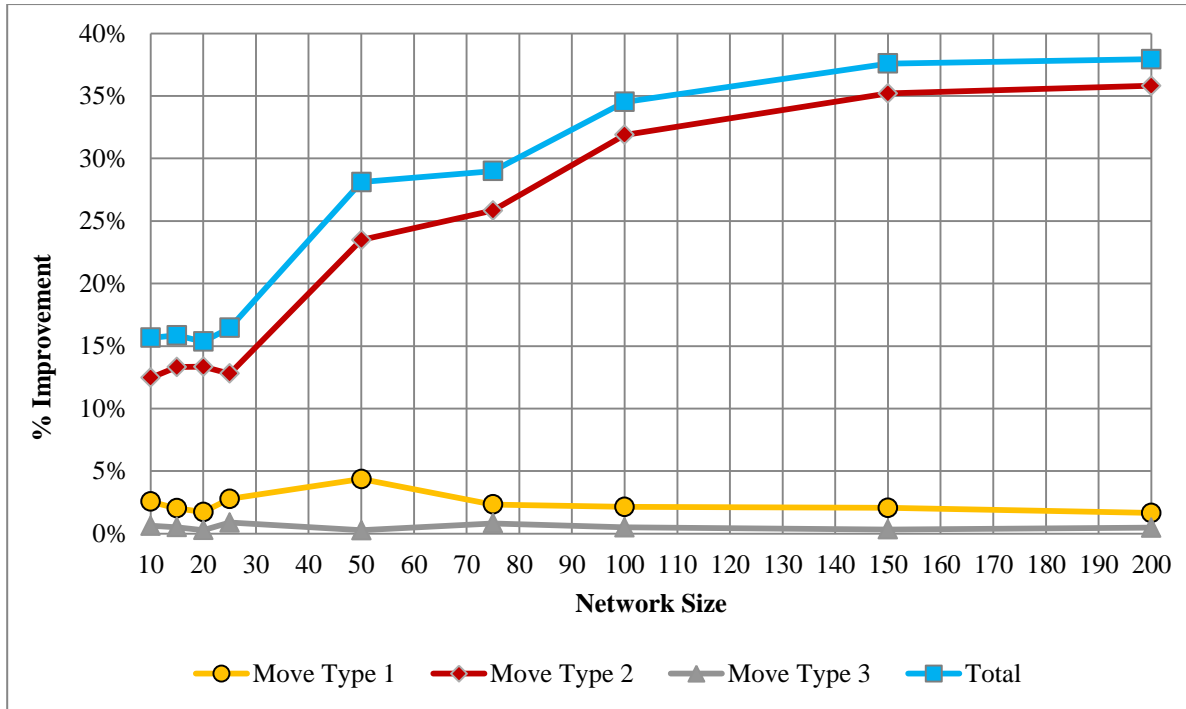


Figure 15. Percentage Improvement of Each Move Type by Network Size

## 6.2. Further Scenario Analysis

In this section we investigate how the results would change under different assumptions. We analyze the effect of changing the values of internal parameters of heuristics and external problem parameters such as fleet size, deadlines, unit costs and demands. Finally we present two alternative heuristics: A fast solution algorithm to obtain acceptable results in very short run times and a modification of H1 such that the total travel distance is to be minimized instead of the comprehensive cost function.

Throughout this section, the base scenario will be denoted by BS and the scenarios to be tested are defined as follows:

- S1: The value of Promising Move Indicator (PMI) is changed
- S2: The size of Best Moves List (BML) is lowered
- S3: The size of BML is raised
- S4: Deadlines are tightened
- S5: Deadlines are loosened
- S6: Demand per customer is increased
- S7: Demand per customer is decreased
- S8: Driver wages are doubled
- S9: Unit fuel and emission costs are doubled
- S10: Fleet size is reduced
- S11: Fleet size is expanded

- S12: Fast solution heuristic
- S13: Distance minimizing objective

### 6.2.1. Changing Internal Parameters

Internal parameters represent method specific values that we are actually capable of changing. We change the values of the promising move indicator (PMI) and the size of the best moves list (BML) and observe how the results are affected.

#### Changing the Value of PMI

PMI determines whether a move is to be treated as promising or not based on its cost change estimate. Recall that cost change estimates are calculated for each move in the neighborhood in H1 and H3. If the cost change estimate is below PMI, then the move is inserted in the promising moves list. In the base scenario (BS), this value was set to £10, that is, all moves with a cost change estimate below £10 were candidate moves to be implemented at each step.

In this part, we change the value of PMI to £0 and see how the results of H1 and H3 are affected. We obtain the results for 20, 50, 100 and 200-customer network variants. This setting is denoted by S1. The detailed results containing the outputs of all replications are given in Appendix D.5. The average results of the replications can be found in Table 12.

**Table 12. Changing the Value of PMI – Average Results**

Network Size	Total Cost (£)					
	H1			H3		
	BS	S1	% Diff	BS	S1	% Diff
20	279.79	277.47	-0.79%	276.18	276.37	+0.06%
50	583.95	586.88	+0.44%	563.88	565.58	+0.27%
100	1062.04	1062.18	+0.26%	1062.14	1054.52	-0.69%
200	1805.41	1852.88	+2.73%	1791.46	1806.95	+0.91%
Network Size	Total CPU Time (sec)					
	H1			H3		
	BS	S1	% Diff	BS	S1	% Diff
20	0.04	0.05	-	0.05	0.05	-
50	1.21	0.90	-22.71%	1.48	0.91	-35.39%
100	22.51	14.35	-30.24%	32.97	14.55	-50.45%
200	726.74	356.07	-49.45%	983.03	315.02	-67.42%

Average results show that in most of the cases the percentage difference between the total cost figures of BS and S1 is below 1% for both H1 and H3. The only exception is the 200-customer network variant of H1, where S1 yielded an average total cost value that is 2.73% worse than that of BS.

On the other hand, looking at the run times we can easily claim that the slight worsening in the total cost figures can be disregarded. A dramatic improvement in the total CPU times for both H1 and H3 is observed with the change of the PMI value. Moreover, the difference between the average total CPU times of BS and S1 increases as the network size grows. In the 200-customers case, the percentage improvement in terms of total CPU time reaches almost 50% for H1 and 67.5% for H3.



In the light of these findings, we can state that setting the PMI value to £0 is a better option than keeping it as £10, especially in large networks.

### Changing the Size of BML

Another important internal parameter is the size of the best moves list (BML), which is composed of the best feasible moves of the promising moves list in H3. As the size of BML gets larger, exact solutions of more promising moves are calculated and the chances that the best move is selected at each step increases.

In BS, the size of BML was set to 100. In this part we explore the results of alternative cases where this value is specified as 10 and 250. S2 and S3 denote the scenarios in which the size of BML is set to 10 and 250, respectively. The detailed results of BS, S2 and S3 for 20, 50, 100 and 200-customer networks can be found in Appendix D.6 and the average results are represented as follows:

**Table 13. Changing the Size of BML – Average Results**

Network Size	Total Cost (£)			Total CPU Time (sec)		
	S2	BS	S3	S2	BS	S3
20	284.39	276.18	284.39	0.047	0.051	0.051
50	563.80	563.88	563.88	1.399	1.479	1.515
100	1049.57	1062.14	1060.21	34.321	32.971	32.938
200	1815.34	1791.46	1808.83	926.38	983.03	979.49

As opposed to the case of changing PMI, setting the size of BML to different values has no significant effect on total cost and CPU time figures. The average results are close to each other and no trend can be observed in neither of the settings. Therefore, it is not possible to claim that any one of these options is superior to others.

### **6.2.2. Changing External Parameters**

In the base scenario, the same environmental parameters were employed in each replication. In this section we change deadlines, customer demands, driver, fuel and emission costs and fleet sizes in order to observe the effects of different external conditions.

Experiments are conducted for network sizes of 20, 100 and 200 customers. H2 is not executed due to solution time considerations. Models are run for 20-customer variants only with a time limit of 2000 seconds. Let us represent the outcomes of these experiments compared to those of the base scenario.

### Changing the Deadlines

As it is mentioned in Appendix A.1, in the base scenario (BS) deadlines are randomly generated by the formula  $b_j = B_1 + B_2$  for all  $j \in N_c$  where  $B_1$  and  $B_2$  are uniform random variables;  $B_1 \sim U(2,5)$  and  $B_2 \sim U(0,5)$ . In this part we analyze the cases where the deadlines become more restrictive and they are completely lifted.

S4 denotes the setting where the deadlines are tighter such that  $B_1 \sim U(2,3)$  and  $B_2 \sim U(0,3)$ . S5 is the scenario where there is no deadline restriction. The detailed results of these experiments are given in Appendix D.7. The average results are tabulated as follows:

**Table 14. Changing the Deadlines – Average Results**

Network Size	Total Cost (£)						GAP					
	M1			M2			M1			M2		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
20	305.5	288.5	266.7	315.8	296.6	282.5	68.8%	67.2%	65.0%	72.3%	70.8%	69.5%
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
20	289.9	279.8	271.8	287.2	276.2	271.8	0.04	0.04	0.04	0.04	0.05	0.05
100	1079.4	1062.0	1127.4	1095.7	1062.1	1094.0	21.59	22.51	15.51	26.99	32.97	23.74
200	1795.3	1805.4	1860.2	1820.7	1791.5	1872.4	626.3	726.7	468.2	740.2	983.0	663.1

Firstly, let us have a look at the results of 20-customer variant. As expected, the average total cost value is higher when deadlines are restrictive and lower when deadlines are removed. Accordingly, the gap between the LP bound and the best integer solution gets lower for M1 and M2, as the deadlines are loosened. Both H1 and H2 yield better results than M1 and M2 in terms of total cost in S4 and BS. However in S5, average total cost of M2 is lower than that of H1 and H3.

As the network size becomes larger, as opposed to our expectations, the worst total cost figures are obtained in S5. This indicates that our heuristic algorithms find better results when the deadlines are restrictive and they cannot utilize the opportunity created by lifting the deadlines.

Although total cost values found in S5 are slightly higher than those found in S4 and BS, the average total CPU time of S5 is significantly lower. This makes sense when we have a quick look at the average number of moves performed at each replication. For instance, regarding H1, 180.2 moves were performed per replication in BS, and 132 moves were performed per replication in S5. Similarly, regarding H3, 146.7 and 115.6 moves were performed on average in BS and S5, respectively. This means that our heuristics tend to find close solutions in BS and S5, but within a shorter time and with less number of moves in S5.

Let us also have a brief look at the effect of changing the deadlines on vehicle speeds. The average vehicle speeds were found to be 54.97 km/h, 52.03 km/h and 50.3 km/h in S4, BS and S5, respectively. It is reasonable that the highest average speed is observed when the deadlines are the tightest and it decreases as the deadline constraints are loosened. However the differences are not much significant.

**Changing Customer Demands**

In the base scenario (BS), the customer demands were generated randomly, where  $q_j$  comes from a uniform distribution between 0.1 and 1.0 tones, for all  $j \in N_c$ . In this part, we would like to see how our solution approaches will react to changes in the demand per customer.

We propose two new scenarios, S6 and S7. In S6, we assume that demand sizes are larger, and  $q_j \sim U(1, 3)$  for  $j \in N_c$ . In S7, the demand per customer is assumed to be lower and  $q_j \sim U(0.01, 0.1)$  for  $j \in N_c$ .

Since the number of vehicles in the fleet were determined taking into account the customer demands in BS, it would not be sufficient for the larger demands in S6. Therefore, we increase the number of vehicles

on hand accordingly in S6. The adjusted fleet sizes S6 can be found in Appendix D.8 together with the detailed results of experiments. Average results are given in Table 15.

**Table 15. Changing Customer Demands – Average Results<sup>2</sup>**

Network Size	Total Cost (£)						GAP					
	M1			M2			M1			M2		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
20	508.6	279.5	179.1	521.9	289.3	203.1	46.2%	67.0%	71.4%	56.3%	70.5%	75.0%
Network Size	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
20	502.8	270.9	180.8	502.8	267.9	184.4	0.06	0.05	0.05	0.08	0.06	0.05
100	2143.3	1046.8	563.0	2148.8	1047.1	569.8	15.69	23.52	70.30	21.61	34.97	69.57
200	3977.7	1805.4	856.8	3969.9	1791.5	867.7	460.2	726.7	3456.1	566.6	983.0	2765.7

The outcomes of the experiments show that effect of the demand growth on the results is significant. The average total cost values of S6 are nearly twice that of BS in all network sizes. Similarly, total cost is much lower in S7, compared to BS. On the other hand, as the total cost decreases when the demand growth become smaller, both the total CPU time for the heuristics and the gap between the LP bound and the best integer solution for the models increases. It can be said that the problem gets more difficult to solve as the demand per customer gets lower since the vehicle capacity limitation becomes non-restrictive and hence number of improvement opportunities gets larger for small demand growths.

Another observation that can be made is the change in the vehicle usage and the number of customers visited per tour with respect to the change in the demand growth. Table 16 illustrates these figures based on network size and vehicle type for each scenario.

**Table 16. Vehicle Usage and Number of Customers Visited per Tour**

Network Size	Vehicle Type	S6		BS		S7	
		No of Vehicles Used	No of Customers per Tour	No of Vehicles Used	No of Customers per Tour	No of Vehicles Used	No of Customers per Tour
20	1	3.4	1.7	2	5.3	1.8	11.8
	2	4.8	3	1.3	8	0	0
100	1	20.9	1.4	8	5.8	4.1	24.8
	2	21.3	3.3	5	10.9	0	0
200	1	41.3	1.4	15	5.8	6	33.5
	2	43.2	3.3	9.7	11.7	0	0

As expected, the number of vehicles used is high and the number of customers visited per tour is low when the demand per customer is large. In S7, no type-2 vehicles are used. It shows that it is more advantageous to use small vehicles when possible. In S6 and BS the number of vehicles used increases as the network size gets larger, and the number of customers per tour remains almost steady. However in S7, the number of customers visited at each tour increases significantly as the network size grows. This case indicates that the vehicle capacity is the main determinant of the number of customers in a single tour.

<sup>2</sup> In the 20-customer networks, 3 out of 10 and in 100 customer networks, 1 out of 10 replications turned out to be infeasible. These are the average results of 7 and 9 replications for 20 and 100-customer networks, respectively.

Let us also have a look at the composition of costs after the change in the customer demands. The following table summarizes how costs are distributed:

**Table 17. Distribution of Costs – Changing Customer Demands**

Scenario	Cost Percentages				
	Fuel and Emission Cost			Driver Cost	Total
	Curb Weight	Load Carried	Vehicle Speed		
S6	23.43%	10.26%	21.51%	44.81%	100.00%
BS	21.25%	8.74%	24.89%	45.12%	100.00%
S7	15.93%	2.94%	41.58%	39.55%	100.00%

The highest percentage of the fuel and emission costs due to the load carried is observed in S6 (10.26%), where the demand per customer assumed to be larger than the other scenarios. In S7 the customer demands are assumed to be quite low and the share of load carrying costs is very small (2.94%). An interesting finding is that the percentage of costs related to the vehicle speed is significantly higher in S7 compared to the other scenarios. The reason behind this is that the number of customers per tour is fairly higher in S7, which causes the vehicles to travel at higher speeds to catch the deadlines of all customers. Vehicle speed figures support our deduction. The average vehicle speeds are 48.98 km/h, 52.05 km/h and 66.31 km/h for scenarios S6, BS and S7, respectively. Since the average speed is significantly higher in S7 than S6 and BS, it is reasonable that the percentage of speed costs is higher as well.

**Changing Driver Wages, and Unit Fuel and Emission Costs**

For the base scenario, driver wages, unit fuel and emission costs are given in Appendix A.1. In this part, we propose two scenarios in which it is assumed that driver wages are doubled (S8) and unit fuel and emission costs are doubled (S9). Comparative results of the experiments conducted for these scenarios are represented in Appendix D.9 in detail and the average figures can be seen in Table 18.

**Table 18. Changing Driver Wages, Unit Fuel and Emission Costs – Average Results**

Network Size	Total Cost (£)						GAP					
	M1			M2			M1			M2		
	BS	S8	S9	BS	S8	S9	BS	S8	S9	BS	S8	S9
20	288.5	391.7	421.9	296.6	413.6	432.0	67.2%	74.9%	56.6%	70.8%	79.0%	59.8%
20	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S8	S9	BS	S8	S9	BS	S8	S9	BS	S8	S9
20	279.8	379.3	416.6	276.2	385.4	409.7	0.04	0.05	0.05	0.05	0.06	0.06
100	1062.0	1540.1	1585.3	1062.1	1513.1	1572.3	22.51	20.42	16.65	32.97	26.22	18.81
200	1805.4	2527.8	2672.6	1791.5	2569.8	2708.0	726.7	568.2	443.1	983.0	742.6	456.2

These results show that the total cost increases by approximately 40% and 48% on average, when the driver wages are doubled and the unit fuel and emission costs are doubled, respectively. Average total CPU times of the heuristics and the solution gaps of the models are lower in S9 compared to BS and S8.

The distribution of costs for these scenarios appears to be as follows:

**Table 19. Distribution of Costs – Changing Driver Wages and Unit Fuel and Emission Costs**

Scenario	Cost Percentages				
	Fuel and Emission Cost			Driver Cost	Total
	Curb Weight	Load Carried	Vehicle Speed		
<b>BS</b>	21.25%	8.74%	24.89%	45.12%	100.00%
<b>S8</b>	15.08%	6.74%	26.13%	52.05%	100.00%
<b>S9</b>	28.31%	12.20%	21.81%	37.68%	100.00%

The figures show that doubling the driver wages resulted in an increase from 45.12% to 52.05% in the percentage of driver cost and doubling unit fuel and emission costs raised the percentage of total fuel and emission costs from 54.88% to 62.32%. Average vehicle speed also increased from 52.05 km/h to 64.17 which also caused a slight rise in the percentage of speed cost. It is possible to claim that the reason behind the increase in the average vehicle speed in S8 is to reduce travel times, which are directly related with the driver costs.

**Changing the Fleet Size**

The number of vehicles in the fleet is a parameter which is always subject to change. In the base scenario (BS), the fleet size is chosen such that the solutions are ensured to be feasible without having excessive number of vehicles. In this section, we propose two new scenarios: In S10, we restrict the fleet size a little further and in S11, the fleet size is assumed to be unlimited for both vehicle types (such that each vehicle can visit a single customer).

The number of vehicles on hand, and the number of vehicles used are represented in the table below for S10, BS and S11:

**Table 20. Changing Fleet Sizes and Vehicle Usage**

Network Size	Vehicle Type	S10		BS		S11	
		Fleet Size	Avg No of Vehicles Used	Fleet Size	Avg No of Vehicles Used	Fleet Size	Avg No of Vehicles Used
<b>20</b>	<b>1</b>	1	1	2	2	20	4.15
	<b>2</b>	2	1.95	2	1.35	20	0.25
<b>100</b>	<b>1</b>	6	6	8	8	100	18.95
	<b>2</b>	6	5.6	8	4.95	100	0
<b>200</b>	<b>1</b>	12	12	15	15	200	37.8
	<b>2</b>	12	11	15	9.7	200	0

It is apparent that small type-1 vehicles are tried to be used as much as possible whereas the utilization of type-2 vehicles is lower. In S10 and BS, where the fleet size is restrictive, the utilization of type-1 vehicles is 100%, that is, all vehicles are used in all replications of the experiments for these scenarios. On the other hand, in S10, no type-2 vehicles were used at all experiments for 100 and 200-customer networks, whereas one type-2 vehicle is used only in a few replications when the network size is 20.

The average results of these experiments in terms of total cost, solution gap and total CPU time are given in Table 21, and the detailed results can be found in Appendix D.10.

**Table 21. Changing the Fleet Size – Average Results**

Network Size	Total Cost (£)						GAP					
	M1			M2			M1			M2		
	S10	BS	S11	S10	BS	S11	S10	BS	S11	S10	BS	S11
20	335.9	288.5	260.4	324.0	296.6	269.5	71.3%	67.2%	64.0%	72.5%	70.8%	68.0%
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S10	BS	S11	S10	BS	S11	S10	BS	S11	S10	BS	S11
20	285.4	279.8	274.7	284.4	276.2	271.4	0.05	0.04	0.06	0.05	0.05	0.08
100 <sup>3</sup>	1068.0	1046.8	1037.8	1057.2	1047.1	1057.8	26.29	23.52	64.79	31.58	34.97	94.08
200	1810.0	1805.4	1843.9	1811.4	1791.5	1848.3	727.4	726.7	1577.4	946.0	983.0	1947.8

In these scenarios, we observe a similar situation to the case where deadlines were changed. The results of experiments in 20-customer networks clearly show that the heuristics perform significantly better than the models when the fleet size constraint is restrictive, but the model results seem to be better under the unlimited fleet size assumption. In the case where the network size is 200, both average total cost and total CPU time figures of S11 are worse than those of S10 and BS. This clearly indicates that our heuristics work properly when the number of vehicles that can be used is limited.

### 6.2.3. Alternative Heuristics

#### Fast Solution Heuristic

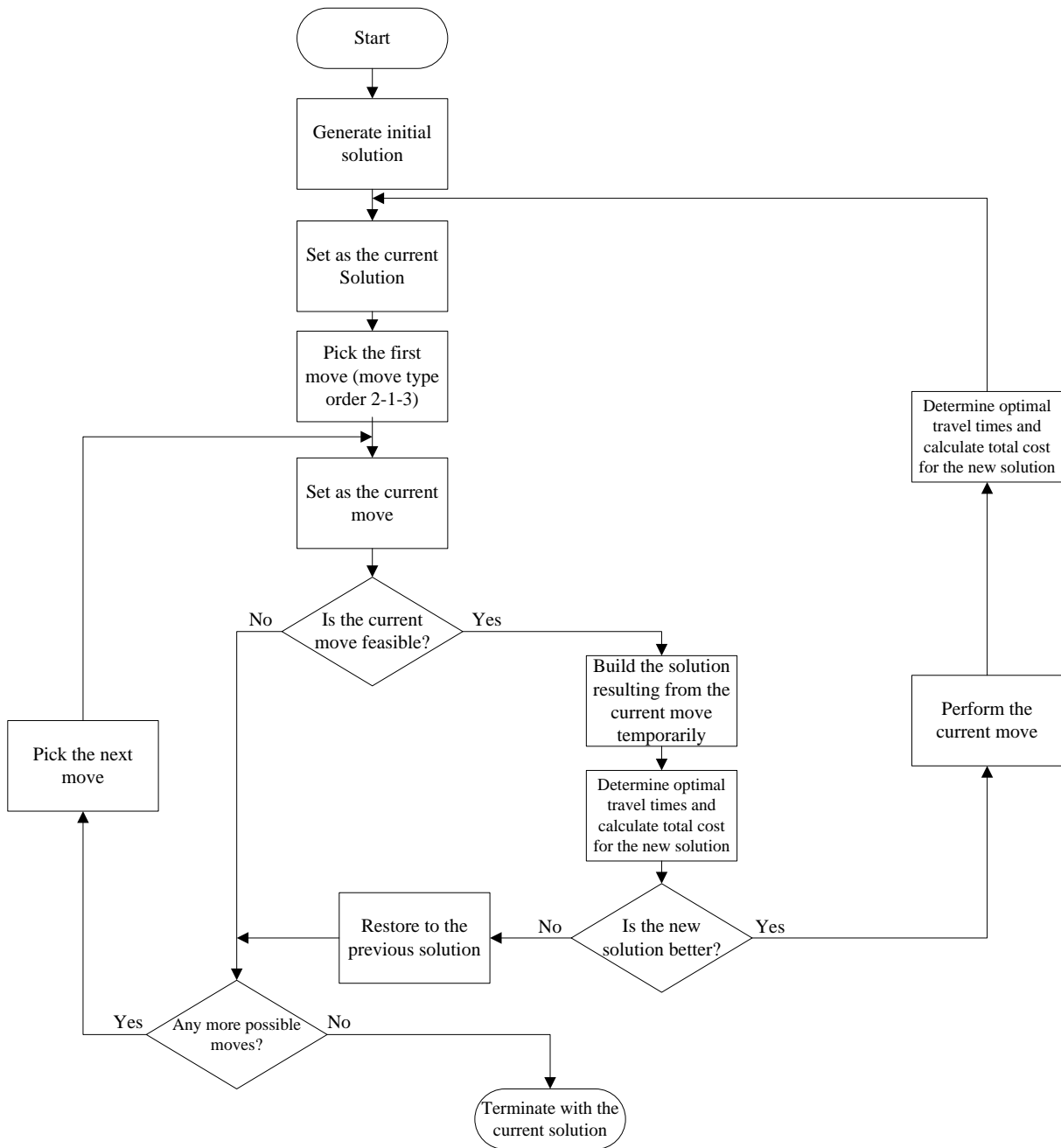
In the heuristic methods that have been proposed so far, final solutions are reached in several hundreds of seconds in 200-customer networks and it is possible to expect that the total CPU time of the heuristics will increase gradually if the network size gets larger.

In this section we introduce a new approach which aims to generate fast solutions using the moves that have been previously defined. The purpose of proposing this approach is providing an alternative solution method for relatively large networks when the available time for solution is quite low.

Fast solution heuristic (FSH) is quite simple compared to H1, H2 and H3. The idea is to select and perform the first improving feasible move regardless of the cost estimates and without constructing any lists. The results of the initial heuristics showed that type 2 moves were applied most frequently, followed by type 1 and type 3 moves. Therefore in FSH, exchange (type 2) moves are examined firstly, inter-route relocate (type 1) moves, secondly and intra-route relocate (type 3) moves, thirdly.

The flow of FSH is illustrated in Figure 16.

<sup>3</sup> One of the replications turned out to be infeasible in S10 where the network size is 100. Therefore the average results are calculated based on the results of 9 replications.



**Figure 16. Fast Solution Heuristic**

The detailed results of FSH runs can be found in Appendix D.11. The average comparative results are represented in Table 22.

**Table 22. Fast Solution Heuristic vs. H1 and H3 – Average Results**

Network Size	Total Cost (£)			Total CPU Time (sec)		
	Average of H1 and H3	FSH	% Difference	Average of H1 and H3	FSH	% Difference
10	178.97	182.16	+1.79%	0.02	0.03	-
15	233.29	246.28	+5.57%	0.04	0.04	-
20	277.99	291.88	+5.00%	0.05	0.06	-
25	288.64	311.20	+7.82%	0.12	0.08	-
50	573.92	665.37	+15.94%	1.34	0.34	-74.86%
75	867.84	1008.74	+16.24%	6.11	1.12	-81.60%
100	1062.09	1266.47	+19.24%	27.74	2.97	-89.29%
150	1434.72	1767.58	+23.20%	190.91	14.51	-92.40%
200	1798.44	2220.23	+23.45%	854.89	43.47	-94.92%

The results clearly show that there is a significant difference in the quality of solutions in terms of the total cost. In the 200-customer network variant, the average total cost obtained by FSH is 23.45% higher than the average result of H1 and H3. However, the total CPU time of FSH is dramatically low compared to H1 and H3 in all cases. These findings support our suggestion: FSH should only be used in large networks and when the time available for generating a solution is quite limited.

**Distance Minimizing Heuristic**

So far, in all heuristics presented, the main objective was to minimize a cost function that takes into account several factors including vehicle speed, carried load and travel distance. In this part, our aim is to find a solution with the minimum total distance traveled. We modify H1 such that the promising moves list is constructed by means of the changes in the travel distance resulting from the moves, rather than the cost change estimates. In this way, at each step the option which reduces the total travel distance most is selected and performed.

We name this modification as the *Distance Minimizing Heuristic* (DMH). We analyze both total cost and total distance figures of the existing heuristics (H1 and H3) and DMH. The average results are given in Table 23 and detailed outcomes can be found in Appendix D.12.

**Table 23. Distance Minimizing Heuristic vs. H1 and H3 – Average Results**

Network Size	Total Cost (£)			Total Travel Distance (km)		
	Average of H1 and H3	DMH	% Difference	Average of H1 and H3	DMH	% Difference
10	178.97	182.76	+2.12%	514.18	510.56	-0.70%
15	233.28	237.46	+1.79%	723.38	711.66	-1.62%
20	277.99	278.95	+0.35%	801.32	759.25	-5.25%
25	288.63	293.72	+1.76%	827.86	789.87	-4.59%
50	573.91	628.47	+9.51%	1628.90	1698.08	+4.25%
75	867.84	913.56	+5.27%	2377.59	2351.62	-1.09%
100	1062.09	1144.13	+7.72%	3039.18	3110.34	+2.34%
150	1434.71	1464.56	+2.08%	4000.12	3960.15	-1.00%
200	1798.44	1850.18	+2.88%	5132.17	5142.67	+0.20%

In all network sizes, DMH yields worse total cost values than the average of H1 and H3. Moreover, no significant advantage of DMH can be observed in terms of the travel distance either. In some cases, DMH



performs worse than H1 and H3 both in total cost and total distance. These outcomes show that employing a comprehensive cost function is always beneficial when the total cost is the main interest, and it does not have a significant disadvantage if the aim is to minimize the total travel distance.

## CHAPTER 7

### CONCLUSIONS & FUTURE RESEARCH DIRECTIONS

Throughout this study, two basic mathematical programming formulations (M1 and M2), a sub-model for speed optimization, three basic heuristic approaches (H1, H2 and H3) and a fast solution heuristic (FSH) have been presented for PRP with multiple vehicle types and deadlines. Using the properties of the sub-model, an optimal travel time determination procedure has been developed and a computation scheme for cost change estimates has been put forward, both of which have been employed within the heuristic approaches.

Computational results showed that the mathematical programming models were too slow for finding the optimal solution to our problem. Outcomes of preliminary experiments indicates that the average CPU time required for solving M1 to optimality was approximately 50,000 seconds in a network with only 10 customers. Therefore, models were run with a time limit, which was set directly proportional to the network size, and feasible solutions generated within this time limit were taken into consideration in the computational analysis. No feasible solutions could be found for networks with 75 or more customers.

When the results of models and heuristics were compared in the basic scenario, it was observed that models performed better than the heuristics in terms of total cost by approximately 5% in networks sizes of 10 and 15 customers. In bigger networks, heuristic results were better; in 50-customer network, difference was found to be 19%. The situation was similar in most of the scenarios that were examined in the further scenario analysis except for the cases where deadlines and vehicle capacity limitations are relaxed. In those cases, models yielded lower costs than heuristics in the 20-customer network. However, when these constraints are tightened, exactly opposite case was observed; the difference between the total cost figures of heuristics and models became even higher than usual. Based on these facts, it can be claimed that it would be more reasonable to make use of mathematical models in less restrictive instances and our heuristics in the cases where limitations are tighter.

No significant difference was observed between the total cost figures of the three heuristics. However, the total CPU time consumed by H2 was found to be significantly higher than H1 and H3. Consequently, H2 were not involved in further scenario analysis.

Promising move indicator (PMI) is a threshold value which is used for classifying a move as promising or not based on its cost change estimate. Changing the value of PMI resulted in dramatic changes in the total CPU times of H1 and H3. By decreasing its value from 10 (base scenario value) to 0, total solution time dropped from 750-1000 seconds to 300-350 seconds with a very little increase in the total cost value in 200-customer networks. Therefore it would be more reasonable to take PMI as 0 in real-life instances for the sake of time efficiency.

Fast solution algorithm (FSH) has been proposed for the cases where there is need for a very quick solution in large problem instances. Its performance was admirable in terms of CPU time, as it was able to generate its final solution in about 43 seconds for a 200-customer network. However the gap between total cost figures of FSH and other heuristics get larger as the network size grows and it is advised to be used only in very urgent situations.

In order to see the effect of employing a comprehensive objective function, a new heuristic with a distance-minimizing objective was executed and the outcomes have shown that the distance minimizing heuristic (DMH) performs worse than the existing heuristics in all network sizes based on the total cost, and it does not have a significant advantage over H1 and H3 in terms of the total distance traveled.

The methodologies proposed in this study have been compared internally under a variety of parameter settings so far. These analyses have been useful in terms of giving an idea about the working dynamics of each solution approach. For a better understanding of the quality of solutions generated by these methodologies, results can be compared with those of similar studies in the literature.

The heuristics we have presented can be easily adapted to different variants of VRP. Deadlines can be converted to classical time windows by an alteration in the travel time determination procedure. Incorporation of backhauling and pickup and delivery type customers would be quite simple with a small adjustment in the calculation of cost change estimates. The fixed fleet size assumption can be extended by introducing a vehicle acquisition cost and revising the cost change estimates accordingly.

Another research direction would be to change the solution method to solve the problem more efficiently. In our heuristics, only three simple move operations and straightforward local search methods were employed. However, there exist very powerful heuristic and metaheuristic algorithms in VRP literature which can be adapted to our case for a higher solution quality. The study of Demir et al. [75] is a nice example of incorporation of a more complicated neighborhood search procedure for PRP.

With this study, we have tried to make a contribution to a special variant of VRP which takes into account environmental matters. There is much room for improvement in this field of research and we hope that this contribution will play a role in the development of further research initiatives in the future.

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## APPENDIX A

### PARAMETER SELECTION

#### Appendix A.1. Parameter Selection

Parameter selections are made in conjunction with Bektas and Laporte [3]. Vehicle based data are obtained from Akcelik and Besley [86].

**Table 24. Parameter Selection**

$a$	$0 \text{ m/s}^2$
$g$	$9.81 \text{ m/s}^2$
$\theta_{ij}$	$0^\circ$
$C_r$	$0.01$
$\tau$	$0.025 \text{ h/t}$
$\rho$	$1.2041 \text{ kg/m}^3$
$c_f$	$\text{£ } 1/\text{L}$
$c_e$	$\text{£ } 27/\text{t}$
$p$	$\text{£ } 8/\text{h}$
$\underline{v}_{ij}$	$40 \text{ km/h}$
$\bar{v}_{ij}$	$90 \text{ km/h}$

	$C_d$	$h$	$Q$	$A$	$w$
<b>Vehicle Type 1</b>	0.7	1	3 t	$5 \text{ m}^2$	3 t
<b>Vehicle Type 2</b>	0.77	1	7 t	$6.5 \text{ m}^2$	7 t

Some unit conversions have been made to obtain the figures that can be directly used in the formulations. The actual fuel cost needs to be expressed in terms of  $\text{£}/\text{Joules}$ . As it is stated in [3], one liter of gasoline provides 8.8 kWh of energy. One kWh equals 3,600,000 J and with the assumption of an engine efficiency of 20%,  $c_f$  can be calculated as:

$$c_f = (\text{£}1/\text{L}) \times \left( \frac{1}{8.8 \times 3,600,000 \times 0.20} \text{L/J} \right) = \text{£}1.58 \times 10^{-7} / \text{J}$$

It is assumed that one liter of gasoline contains 2.32 kg of  $\text{CO}_2$  [3]. Considering the fact that  $c_e$  is given as  $\text{£}27/\text{t}$ , it can be calculated in terms of  $\text{£}/\text{J}$  as follows:

$$c_e = (\text{£}27/\text{t}) \times (0.001 \text{ t/kg}) \times (2.32 \text{ kg/L}) \times \left( \frac{1}{8.8 \times 3,600,000 \times 0.20} \text{L/J} \right) = \text{£}9.89 \times 10^{-9} / \text{J}$$

Note that these parameters are subject to change. Especially the cost of fuel and emissions are different today, due to the changes in the oil market and assessment of environmental effects.

Deadlines (in terms of hours) are estimated by the formula  $b_j = B_1 + B_2$  for all  $j \in N_c$  where  $B_1$  and  $B_2$  are uniform random variables:  $B_1 \sim U(2,5)$  and  $B_2 \sim U(0,5)$ .

Customer demands (in terms of tones) are also assumed to be uniform. We denote the demand per customer by  $q_j$  where  $q_j \sim U(0.01, 0.1)$  for  $j \in N_c$ .

## Appendix A.2. Depot and Customer Locations on NL Network



Figure 17. Depot and Customer Locations on NL Network

## APPENDIX B

### PRELIMINARY MODEL RUNS – DETAILED RESULTS

#### Appendix B.1. MIP Model - Differing Number of Speed Levels – Detailed Results

Table 25. MIP Model - Differing Number of Speed Levels – Detailed Results

Rep <sup>4</sup> No	Final Solution (£)	LP Bound (£)	Gap (%)	Average Speed (km/h)		
				General	Type 1 Vehicle	Type 2 Vehicle
<b>6 Speed Levels: 40 km/h - 50 km/h - 60 km/h ... 90 km/h</b>						
1	429.21	227.79	46.9%	70.90	74.14	67.41
2	380.10	233.58	38.5%	59.40	56.03	63.32
3	375.32	271.42	27.7%	52.93	56.15	50.00
4	408.44	269.40	34.0%	63.06	64.58	61.65
5	365.04	313.82	14.0%	57.18	63.44	50.00
6	377.66	213.49	43.5%	60.84	60.83	60.84
7	392.95	311.73	20.7%	67.59	63.34	74.91
8	369.03	338.26	8.3%	59.71	68.92	50.00
9	354.76	214.07	39.7%	54.44	54.97	53.67
10	395.46	202.43	48.8%	68.81	78.57	58.23
<b>Average</b>	<b>384.80</b>	<b>259.60</b>	<b>32.2%</b>	<b>61.49</b>	<b>64.10</b>	<b>59.00</b>
<b>11 Speed Levels: 40 km/h - 45 km/h - 50 km/h ... 90 km/h</b>						
1	352.02	295.96	15.9%	54.10	56.34	51.07
2	376.62	316.33	16.0%	60.14	58.89	61.49
3	404.64	215.76	46.7%	62.45	69.24	57.24
4	402.97	341.50	15.3%	63.52	66.92	60.54
5	361.46	304.09	15.9%	58.57	64.83	51.36
6	386.04	207.45	46.3%	61.32	62.59	59.73
7	388.03	320.66	17.4%	56.06	63.17	50.35
8	366.73	318.50	13.2%	58.05	70.59	46.13
9	359.34	195.95	45.5%	57.00	57.23	56.66
10	366.73	284.95	22.3%	62.05	62.93	60.81
<b>Average</b>	<b>376.46</b>	<b>280.12</b>	<b>25.4%</b>	<b>59.33</b>	<b>63.27</b>	<b>55.54</b>
<b>26 Speed Levels: 40 km/h - 42 km/h - 44 km/h ... 90 km/h</b>						
1	375.88	240.78	35.9%	54.22	57.16	52.42
2	376.24	237.16	37.0%	59.05	56.66	61.74
3	389.24	200.57	48.5%	53.95	52.00	56.03
4	406.22	262.76	35.3%	62.67	66.89	58.67
5	364.01	255.55	29.8%	57.98	63.28	51.65
6	367.51	233.77	36.4%	59.14	61.31	56.33
7	395.96	255.35	35.5%	54.53	53.09	56.02
8	365.90	243.08	33.6%	57.91	69.29	46.75
9	392.79	209.59	46.6%	60.11	59.94	60.34
10	360.15	212.38	41.0%	59.58	61.86	56.53
<b>Average</b>	<b>379.39</b>	<b>235.10</b>	<b>38.0%</b>	<b>57.91</b>	<b>60.15</b>	<b>55.65</b>

<sup>4</sup> Replication number

## Appendix B.2. Effect of Time Limit - Detailed Results

Table 26. Effect of Time Limit - Detailed Results

Rep No	Final Solution (£)	Total Solution Time (sec)	Solution Status	LP Bound (£)	Gap (%)	Total Travel Distance (km)	Total Travel Time (h)
	Time Limit: 1000 seconds						
1	352.02	1000	Feasible	295.96	15.9%	1042.5	19.27
2	376.62	1001	Feasible	316.33	16.0%	1047.8	17.42
3	404.64	1002	Feasible	215.76	46.7%	1089.4	17.45
4	402.97	1001	Feasible	341.50	15.3%	1070.8	16.86
5	361.46	1001	Feasible	304.09	15.9%	1025.6	17.51
6	386.04	1000	Feasible	207.45	46.3%	1096.2	17.88
7	388.03	1003	Feasible	320.66	17.4%	1101.9	19.65
8	366.73	1001	Feasible	318.50	13.2%	1025.6	17.67
9	359.34	1000	Feasible	195.95	45.5%	1042.5	18.29
10	366.73	1000	Feasible	284.95	22.3%	1029.6	16.59
<b>Avg</b>	<b>376.46</b>	<b>1001</b>	<b>-</b>	<b>280.12</b>	<b>25.4%</b>	<b>1057.19</b>	<b>17.86</b>
No Time Limit							
1	351.14	10075	Optimal	351.12	0.0%	1042.5	19.32
2	376.62	7118	Optimal	376.59	0.0%	1047.8	17.42
3	374.81	154898	Optimal	374.78	0.0%	1074.8	20.60
4	402.72	7096	Optimal	402.68	0.0%	1070.8	16.88
5	361.46	10360	Optimal	361.42	0.0%	1025.6	17.51
6	356.71	48860	Optimal	356.67	0.0%	1042.5	17.90
7	387.64	17072	Optimal	387.60	0.0%	1101.9	19.68
8	366.54	5787	Optimal	366.52	0.0%	1025.6	17.67
9	351.89	154970	Optimal	351.86	0.0%	1029.6	18.89
10	360.57	107280	Optimal	360.54	0.0%	1029.6	17.14
<b>Avg</b>	<b>369.01</b>	<b>52352</b>	<b>-</b>	<b>368.98</b>	<b>0.0%</b>	<b>1049.07</b>	<b>18.30</b>

## Appendix B.3. Comparison of M1 and M2 - Detailed Results

Table 27. Comparison of M1 and M2 - Detailed Results

Rep No.	C1: Base Case									
	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	352.02	350.46	-0.4%	295.96	216.78	-26.8%	15.9%	38.1%	Feasible	Feasible
2	376.62	376.00	-0.2%	316.33	203.31	-35.7%	16.0%	45.9%	Feasible	Feasible
3	404.64	374.28	-7.5%	215.76	214.58	-0.5%	46.7%	42.7%	Feasible	Feasible
4	402.97	401.89	-0.3%	341.50	229.98	-32.7%	15.3%	42.8%	Feasible	Feasible
5	361.46	360.95	-0.1%	304.09	231.63	-23.8%	15.9%	35.8%	Feasible	Feasible
6	386.04	355.64	-7.9%	207.45	197.34	-4.9%	46.3%	44.5%	Feasible	Feasible
7	388.03	389.32	+0.3%	320.66	215.81	-32.7%	17.4%	44.6%	Feasible	Feasible
8	366.73	365.69	-0.3%	318.50	223.59	-29.8%	13.2%	38.9%	Feasible	Feasible
9	359.34	351.40	-2.2%	195.95	221.94	+13.3%	45.5%	36.8%	Feasible	Feasible
10	366.73	359.97	-1.8%	284.95	244.36	-14.2%	22.3%	32.1%	Feasible	Feasible
<b>Avg</b>	<b>376.46</b>	<b>368.56</b>	<b>-2.1%</b>	<b>280.12</b>	<b>219.93</b>	<b>-21.5%</b>	<b>25.4%</b>	<b>40.2%</b>	<b>-</b>	<b>-</b>

C2: Fleet Extension – $h_1 = 2, h_2 = 2$										
Rep No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	310.19	309.75	-0.1%	219.09	163.71	-25.3%	29.4%	47.1%	Feasible	Feasible
2	342.62	346.90	1.3%	303.17	164.87	-45.6%	11.5%	52.5%	Feasible	Feasible
3	324.27	323.78	-0.2%	218.40	162.23	-25.7%	32.6%	49.9%	Feasible	Feasible
4	355.05	352.12	-0.8%	234.35	167.79	-28.4%	34.0%	52.3%	Feasible	Feasible
5	351.44	350.99	-0.1%	303.55	165.68	-45.4%	13.6%	52.8%	Feasible	Feasible
6	315.03	314.05	-0.3%	287.44	159.61	-44.5%	8.8%	49.2%	Feasible	Feasible
7	332.56	331.90	-0.2%	302.13	156.10	-48.3%	9.1%	53.0%	Feasible	Feasible
8	328.79	326.14	-0.8%	239.30	178.39	-25.5%	27.2%	45.3%	Feasible	Feasible
9	341.39	328.37	-3.8%	169.73	156.06	-8.1%	50.3%	52.5%	Feasible	Feasible
10	342.22	340.40	-0.5%	216.18	185.06	-14.4%	36.8%	45.6%	Feasible	Feasible
<b>Avg</b>	<b>334.36</b>	<b>332.44</b>	<b>-0.6%</b>	<b>249.33</b>	<b>165.95</b>	<b>-33.4%</b>	<b>25.3%</b>	<b>50.0%</b>	-	-
C3: Doubled Fuel and Emission Costs – $c_f = £2/L, c_e = £54/kg$										
Rep No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	563.31	538.49	-4.4%	399.64	392.10	-1.9%	29.1%	27.2%	Feasible	Feasible
2	642.15	635.03	-1.1%	426.14	366.58	-14.0%	33.6%	42.3%	Feasible	Feasible
3	597.01	574.69	-3.7%	392.61	388.74	-1.0%	34.2%	32.4%	Feasible	Feasible
4	684.15	662.04	-3.2%	443.69	363.56	-18.1%	35.1%	45.1%	Feasible	Feasible
5	573.35	570.77	-0.4%	527.63	426.50	-19.2%	8.0%	25.3%	Feasible	Feasible
6	561.52	558.18	-0.6%	498.26	383.04	-23.1%	11.3%	31.4%	Feasible	Feasible
7	614.59	605.96	-1.4%	517.70	392.77	-24.1%	15.8%	35.2%	Feasible	Feasible
8	587.64	584.95	-0.5%	477.87	360.10	-24.6%	18.7%	38.4%	Feasible	Feasible
9	551.18	545.94	-0.9%	403.35	418.46	+3.7%	26.8%	23.4%	Feasible	Feasible
10	579.25	577.27	-0.3%	511.48	417.84	-18.3%	11.7%	27.6%	Feasible	Feasible
<b>Avg</b>	<b>595.41</b>	<b>585.33</b>	<b>-1.7%</b>	<b>459.84</b>	<b>390.97</b>	<b>-15.0%</b>	<b>22.4%</b>	<b>32.8%</b>	-	-
C4: Doubled Driver Wage – $p = £16/h$										
Rep No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	496.45	498.58	0.4%	392.89	226.93	-42.2%	20.9%	54.5%	Feasible	Feasible
2	511.00	510.85	0.0%	408.97	189.86	-53.6%	20.0%	62.8%	Feasible	Feasible
3	520.69	519.43	-0.2%	324.07	201.51	-37.8%	37.8%	61.2%	Feasible	Feasible
4	530.06	529.07	-0.2%	397.09	211.35	-46.8%	25.1%	60.1%	Feasible	Feasible
5	495.77	495.20	-0.1%	422.28	215.41	-49.0%	14.8%	56.5%	Feasible	Feasible
6	556.34	494.24	-11.2%	230.44	193.47	-16.0%	58.6%	60.9%	Feasible	Feasible
7	510.82	510.25	-0.1%	428.76	217.00	-49.4%	16.1%	57.5%	Feasible	Feasible
8	543.13	496.76	-8.5%	262.16	214.06	-18.3%	51.7%	56.9%	Feasible	Feasible
9	492.82	492.54	-0.1%	389.91	211.95	-45.6%	20.9%	57.0%	Feasible	Feasible
10	498.67	494.98	-0.7%	235.63	231.86	-1.6%	52.7%	53.2%	Feasible	Feasible
<b>Avg</b>	<b>515.57</b>	<b>504.19</b>	<b>-2.2%</b>	<b>349.22</b>	<b>211.34</b>	<b>-39.5%</b>	<b>31.9%</b>	<b>58.0%</b>	-	-
C5: Loose Deadlines – Parameters: $B_1 \sim U(5,10)$ and $B_2 \sim U(0,5)$										
Rep No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	353.68	345.56	-2.3%	200.02	209.47	4.7%	43.4%	39.4%	Feasible	Feasible
2	334.74	334.15	-0.2%	257.63	200.72	-22.1%	23.0%	39.9%	Feasible	Feasible
3	361.72	361.19	-0.1%	257.90	199.35	-22.7%	28.7%	44.8%	Feasible	Feasible
4	347.17	343.13	-1.2%	236.14	194.56	-17.6%	32.0%	43.3%	Feasible	Feasible
5	347.81	347.40	-0.1%	284.08	215.87	-24.0%	18.3%	37.9%	Feasible	Feasible
6	348.79	345.37	-1.0%	250.36	190.52	-23.9%	28.2%	44.8%	Feasible	Feasible
7	344.60	344.14	-0.1%	279.77	196.12	-29.9%	18.8%	43.0%	Feasible	Feasible
8	343.26	342.64	-0.2%	259.72	190.70	-26.6%	24.3%	44.3%	Feasible	Feasible

9	347.52	347.10	-0.1%	293.48	199.04	-32.2%	15.5%	42.7%	Feasible	Feasible
10	352.54	347.83	-1.3%	192.29	227.76	18.4%	45.5%	34.5%	Feasible	Feasible
<b>Avg</b>	<b>348.18</b>	<b>345.85</b>	<b>-0.7%</b>	<b>251.14</b>	<b>202.41</b>	<b>-19.4%</b>	<b>27.8%</b>	<b>41.5%</b>	-	-

**C6: Tight Deadlines – Parameters:  $B_1 \sim U(2,4)$  and  $B_2 \sim U(0,3)$**

Rep No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	388.61	386.08	-0.7%	305.33	262.89	-13.9%	21.4%	31.9%	Feasible	Feasible
2	483.13	476.18	-1.4%	348.62	238.51	-31.6%	27.8%	49.9%	Feasible	Feasible
3	409.80	412.50	+0.7%	334.07	199.12	-40.4%	18.5%	51.7%	Feasible	Feasible
4	-	-	-	-	-	-	-	-	Unknown	Unknown
5	-	477.82	-	-	206.39	-	-	56.8%	Unknown	Feasible
6	389.52	387.60	-0.5%	359.92	228.53	-36.5%	7.6%	41.0%	Feasible	Feasible
7	451.36	422.44	-6.4%	232.86	244.22	4.9%	48.4%	42.2%	Feasible	Feasible
8	467.83	-	-	301.70	-	-	35.5%	-	Feasible	Unknown
9	393.47	398.38	+1.2%	349.85	219.79	-37.2%	11.1%	44.8%	Feasible	Feasible
10	-	431.97	-	-	248.09	-	-	42.6%	Unknown	Feasible
<b>Avg</b>	<b>419.32</b>	<b>413.86</b>	<b>-1.3%</b>	<b>321.78</b>	<b>232.18</b>	<b>-27.8%</b>	<b>22.5%</b>	<b>43.6%</b>	-	-

**C7: Low Customer Demand – Parameters:  $q_j \sim U(0.1, 0.2)$**

Rep No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	365.56	338.99	-7.3%	154.04	133.39	-13.4%	57.9%	60.7%	Feasible	Feasible
2	360.07	413.76	+14.9%	172.71	116.90	-32.3%	52.0%	71.7%	Feasible	Feasible
3	334.90	334.27	-0.2%	217.01	126.38	-41.8%	35.2%	62.2%	Feasible	Feasible
4	381.62	388.03	+1.7%	311.36	124.59	-60.0%	18.4%	67.9%	Feasible	Feasible
5	349.16	394.53	+13.0%	163.56	121.36	-25.8%	53.2%	69.2%	Feasible	Feasible
6	342.52	370.76	+8.2%	256.51	123.14	-52.0%	25.1%	66.8%	Feasible	Feasible
7	357.00	352.03	-1.4%	188.74	122.78	-34.9%	47.1%	65.1%	Feasible	Feasible
8	349.49	380.39	+8.8%	314.19	124.57	-60.4%	10.1%	67.3%	Feasible	Feasible
9	338.62	333.14	-1.6%	155.53	128.25	-17.5%	54.1%	61.5%	Feasible	Feasible
10	339.79	338.93	-0.3%	272.77	130.45	-52.2%	19.7%	61.5%	Feasible	Feasible
<b>Avg</b>	<b>351.87</b>	<b>364.48</b>	<b>3.6%</b>	<b>220.64</b>	<b>125.18</b>	<b>-43.3%</b>	<b>37.3%</b>	<b>65.4%</b>	-	-

**C8: High Customer Demand – Parameters:  $q_j \sim U(1, 3)$ , Fleet Extension –  $h_1 = 3, h_2 = 3$**

Rep No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
1	512.64	512.09	-0.1%	512.64	444.37	-13.3%	0.0%	13.2%	Optimal	Feasible
2	519.46	518.95	-0.1%	457.56	447.30	-2.2%	11.9%	13.8%	Feasible	Feasible
3	508.17	507.57	-0.1%	508.11	445.94	-12.2%	0.0%	12.1%	Optimal	Feasible
4	565.32	565.11	0.0%	565.27	481.09	-14.9%	0.0%	14.9%	Optimal	Feasible
5	555.29	554.01	-0.2%	555.29	475.64	-14.3%	0.0%	14.1%	Optimal	Feasible
6	476.28	475.88	-0.1%	476.24	409.74	-14.0%	0.0%	13.9%	Optimal	Feasible
7	518.23	517.30	-0.2%	518.19	421.95	-18.6%	0.0%	18.4%	Optimal	Feasible
8	543.50	542.91	-0.1%	543.49	461.51	-15.1%	0.0%	15.0%	Optimal	Feasible
9	-	-	-	-	-	-	-	-	Infeasible	Infeasible
10	538.82	537.32	-0.3%	538.82	537.32	-0.3%	0.0%	0.0%	Optimal	Optimal
<b>Avg</b>	<b>526.41</b>	<b>525.68</b>	<b>-0.1%</b>	<b>519.51</b>	<b>458.32</b>	<b>-11.8%</b>	<b>1.3%</b>	<b>12.8%</b>	-	-

**Overall**

Repl No.	Final Solution (£)			LP Bound (£)			GAP (%)		Solution Status	
	M1	M2	% Diff	M1	M2	% Diff	M1	M2	M1	M2
<b>Avg</b>	<b>433.45</b>	<b>430.05</b>	<b>-0.8%</b>	<b>331.45</b>	<b>250.78</b>	<b>-24.3%</b>	<b>24.2%</b>	<b>43.0%</b>	-	-



## APPENDIX C

### CALCULATION OF COST CHANGE ESTIMATES

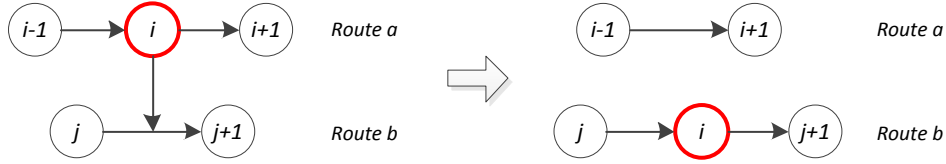
In this section, all formulas used in the calculation of the cost change estimates for each move are represented. Recall that the estimation of the change in the total cost ( $E_T$ ) is composed of four cost components  $E_1$ ,  $E_2$ ,  $E_3$  and  $E_4$  which are linked with the curb weight of the vehicle, load carried by the vehicle, current speed of the vehicle, and speeding up (or slowing down) the vehicle, respectively. In the following formulations, for the sake of simplicity we introduce two new parameters:  $c_p = c_e + c_f$  and  $\omega_{ij} = \alpha_{ij}d_{ij}$ .

#### Appendix C.1. Calculation of $E_1$

In this part, the formulas used in the calculation of  $E_1$  are given for each move. The basis for calculations is the removed and newly formed arcs as a result of the moves.

##### Inter-Route Relocate:

$i^{\text{th}}$  node of route  $a$  is relocated between  $j^{\text{th}}$  and  $(j + 1)^{\text{th}}$  nodes of route  $b$ .

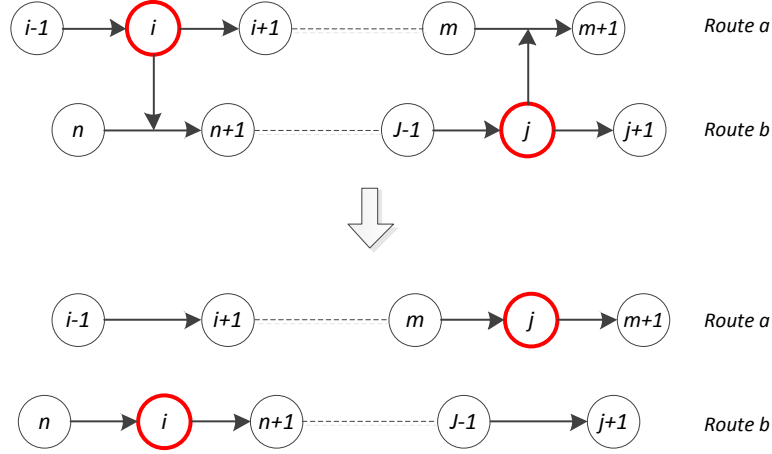


**Figure 18. Inter-Route Relocate**

$$E_1 = c_p w_{k_a} (\omega_{(i-1)_a(i+1)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a(i+1)_a}) + c_p w_{k_b} (\omega_{j_b i_a} + \omega_{i_a(j+1)_b} - \omega_{j_b(j+1)_b})$$

##### Exchange:

$i^{\text{th}}$  node of route  $a$  and  $j^{\text{th}}$  node of route  $b$  are exchanged.  $i^{\text{th}}$  node of route  $a$  is placed between  $n^{\text{th}}$  and  $(n + 1)^{\text{th}}$  nodes of route  $b$  whereas  $j^{\text{th}}$  node of route  $b$  is placed between  $m^{\text{th}}$  and  $(m + 1)^{\text{th}}$  nodes of route  $a$ .



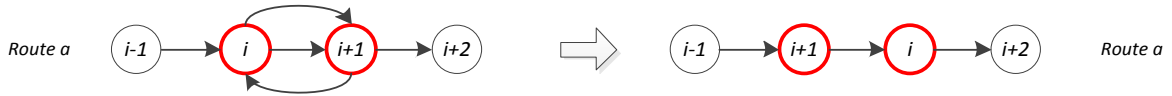
**Figure 19. Exchange**

$$E_1 = c_p W_{k_a} (\omega_{(i-1)_a(i+1)_a} + \omega_{m_a j_b} + \omega_{j_b(m+1)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a(i+1)_a} - \omega_{m_a(m+1)_a}) \\ + c_p W_{k_b} (\omega_{(j-1)_b(j+1)_b} + \omega_{n_b i_a} + \omega_{i_a(n+1)_b} - \omega_{(j-1)_b j_b} - \omega_{j_b(j+1)_b} - \omega_{n_b(n+1)_b})$$

**Intra-Route Relocate:**

Case 1: Two consecutive nodes are switched

$i^{th}$  and  $(i + 1)^{th}$  nodes of route  $a$  are switched.

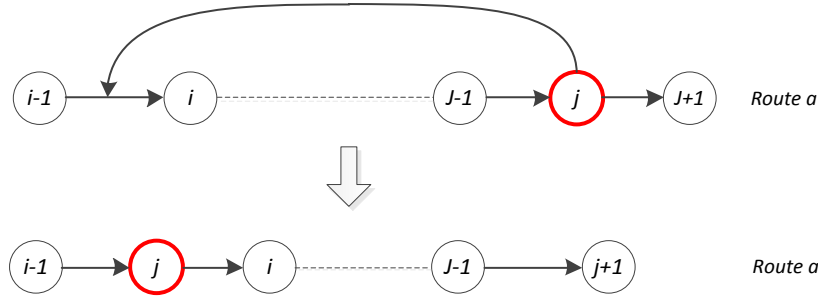


**Figure 20. Intra-Route Relocate - Case 1**

$$E_1 = c_p W_{k_a} (\omega_{(i-1)_a(i+1)_a} + \omega_{(i+1)_a i_a} + \omega_{i_a(i+2)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a(i+1)_a} - \omega_{(i+1)_a(i+2)_a})$$

Case 2: Relocating to a Further Location – Backward Move

$j^{th}$  node of route  $a$  is relocated between  $(i - 1)^{th}$  and  $i^{th}$  nodes of the same route where  $j > (i + 1)$ .

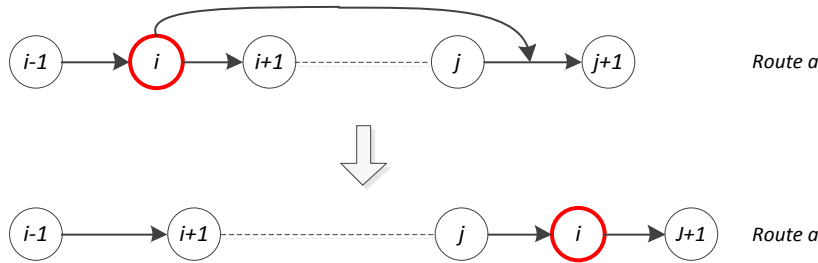


**Figure 21. Intra-Route Relocate - Case 2**

$$E_1 = c_p w_{k_a} (\omega_{(i-1)_a j_a} + \omega_{j_a i_a} + \omega_{(j-1)_a (j+1)_a} - \omega_{(i-1)_a i_a} - \omega_{(j-1)_a j_a} - \omega_{j_a (j+1)_a})$$

Case 3: Relocating to a Further Location – Forward Move

$i^{\text{th}}$  node of route  $a$  is relocated between  $j^{\text{th}}$  and  $(j + 1)^{\text{th}}$  nodes of the same route where  $j > (i + 1)$ .



**Figure 22. Intra-Route Relocate - Case 3**

$$E_1 = c_p w_{k_a} (\omega_{(i-1)_a (i+1)_a} + \omega_{j_a i_a} + \omega_{i_a (j+1)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a (i+1)_a} - \omega_{j_a (j+1)_a})$$

## Appendix C.2. Calculation of $E_2$

The cost related with the weight of the load carried by the vehicle changes when a move is performed. This change can be exactly calculated using the demands of the customer nodes and removed and newly added arcs in the basis. The basic idea behind the calculations is that, up to the  $i^{\text{th}}$  node of a sample route, the total demand of customers  $i, (i + 1) \dots$  up to the final customer in the route is carried. In the formulas given for each move, this structure has been preserved.

### Inter-Route Relocate:

$i^{\text{th}}$  node of route  $a$  is relocated between  $j^{\text{th}}$  and  $(j + 1)^{\text{th}}$  nodes of route  $b$ . (See Figure 19)

$$E_2 = \sum_{r=0}^{j-1} c_p \omega_{r_b (r+1)_b} q_{i_a} - \sum_{r=0}^{i-1} c_p \omega_{r_a (r+1)_a} q_{i_a} + c_p (\omega_{j_b i_a} + \omega_{i_a (j+1)_b} - \omega_{(j-1)_b (j+1)_b}) \left[ \sum_{r=j+1}^{s(b)} q_{r_b} \right]$$

$$+c_e\omega_{j_b i_a} q_{i_a} + c_p(\omega_{(i-1)_a(i+1)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a(i+1)_a}) \left[ \sum_{r=i+1}^{s(a)} q_{r_a} \right]$$

**Exchange:**

$i^{\text{th}}$  node of route  $a$  and  $j^{\text{th}}$  node of route  $b$  are exchanged.  $i^{\text{th}}$  node of route  $a$  is placed between  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  nodes of route  $b$  whereas  $j^{\text{th}}$  node of route  $b$  is placed between  $m^{\text{th}}$  and  $(m+1)^{\text{th}}$  nodes of route  $a$ . (See Figure 20)

$$\begin{aligned} E_2 = & \sum_{r=0}^{j-2} c_p \omega_{r_b(r+1)_b} q_{i_a} + \sum_{r=0}^{i-2} c_p \omega_{r_a(r+1)_a} q_{j_b} - \sum_{r=0}^{i-1} c_p \omega_{r_a(r+1)_a} q_{i_a} - \sum_{r=0}^{j-1} c_p \omega_{r_b(r+1)_b} q_{j_b} \\ & + c_p(\omega_{m_a j_b} + \omega_{j_b(m+1)_a} + \omega_{(i-1)_a(i+1)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a(i+1)_a} - \omega_{(m-1)_a(m+1)_a}) \left[ \sum_{r=i+1}^{s(a)} q_{r_a} \right] \\ & + c_p(\omega_{n_b i_a} + \omega_{i_a(n+1)_b} + \omega_{(j-1)_b(j+1)_b} - \omega_{(j-1)_b j_b} - \omega_{j_b(j+1)_b} - \omega_{(n-1)_b(n+1)_b}) \left[ \sum_{r=j+1}^{s(b)} q_{r_b} \right] \\ & + c_p \omega_{n_b i_a} q_{i_a} + c_p \omega_{j_b(m+1)_a} q_{j_b} \end{aligned}$$

**Intra-Route Relocate:**

Case 1: Two consecutive nodes are switched

$i^{\text{th}}$  and  $(i+1)^{\text{th}}$  nodes of route  $a$  are switched. (See Figure 21)

$$\begin{aligned} E_2 = & c_p(\omega_{(i-1)_a(i+1)_a} + \omega_{(i+1)_a i_a} - \omega_{(i-1)_a i_a}) q_{i_a} + c_p(\omega_{(i-1)_a(i+1)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a(i+1)_a}) q_{(i+1)_a} \\ & + c_p(\omega_{(i-1)_a(i+1)_a} + \omega_{(i+1)_a i_a} + \omega_{i_a(i+2)_a} - \omega_{(i-1)_a i_a} - \omega_{i_a(i+1)_a} - \omega_{(i+1)_a(i+2)_a}) \left[ \sum_{r=i+2}^{s(a)} q_{r_a} \right] \end{aligned}$$

Case 2: Relocating to a Further Location – Backward Move

$j^{\text{th}}$  node of route  $a$  is relocated between  $(i-1)^{\text{th}}$  and  $i^{\text{th}}$  nodes of the same route where  $j > (i+1)$ . (See Figure 22)

$$\begin{aligned} E_2 = & c_p \omega_{(i-1)_a j_a} \left[ \sum_{r=i}^{s(a)} q_{r_a} \right] + c_p \omega_{j_a i_a} \left[ \left( \sum_{r=i}^{s(a)} q_{r_a} \right) - q_{j_a} \right] - c_p \omega_{(j-1)_a(j+1)_a} \left[ \sum_{r=j+1}^{s(a)} q_{r_a} \right] \\ & - c_p \omega_{(i-1)_a i_a} \left[ \sum_{r=i}^{s(a)} q_{r_a} \right] - \sum_{r=i}^{j-2} c_p \omega_{r_a(r+1)_a} q_{j_a} - c_p \omega_{(j-1)_a j_a} \left[ \sum_{r=j}^{s(a)} q_{r_a} \right] - c_p \omega_{j_a(j+1)_a} \left[ \sum_{r=j+1}^{s(a)} q_{r_a} \right] \end{aligned}$$

Case 3: Relocating to a Further Location – Forward Move

$i^{\text{th}}$  node of route  $a$  is relocated between  $j^{\text{th}}$  and  $(j+1)^{\text{th}}$  nodes of the same route where  $j > (i+1)$ . (See Figure 23)

$$\begin{aligned}
E_2 = & c_p \omega_{(i-1)_a(i+1)_a} \left[ \sum_{r=i}^{s(a)} q_{r_a} \right] + c_p \omega_{j_a i_a} \left[ \left( \sum_{r=j+1}^{s(a)} q_{r_a} \right) + q_{i_a} \right] + c_p \omega_{i_a(j+1)_a} \left[ \sum_{r=j+1}^{s(a)} q_{r_a} \right] \\
& - c_p \omega_{(i-1)_a i_a} \left[ \sum_{r=i}^{s(a)} q_{r_a} \right] - c_p \omega_{i_a(i+1)_a} \left[ \sum_{r=i+1}^{s(a)} q_{r_a} \right] + \sum_{r=i+1}^{j-1} c_p \omega_{r_a(r+1)_a} q_{i_a} - c_p \omega_{j_a(j+1)_a} \left[ \sum_{r=j+1}^{s(a)} q_{r_a} \right]
\end{aligned}$$

### Appendix C.3. Calculation of $E_3$

In this part, we assume that the vehicles travel at the same speeds as they were used to in the affected road segments. For instance, if the  $i^{\text{th}}$  node is removed from a sample route  $a$ , and the vehicle speed on arc  $((i-1)_a, i_a)$  was  $v_{(i-1)_a i_a}$ , then it is assumed that  $v_{(i-1)_a(i+1)_a} = v_{(i-1)_a i_a}$ . Alternatively, if a new node  $j$  is inserted between the  $(i-1)^{\text{th}}$  and  $i^{\text{th}}$  nodes in route  $a$ , then it is assumed that  $v_{(i-1)_a j} = v_{j i_a} = v_{(i-1)_a i_a}$ . The resulting cost change estimates are calculated for each case as follows.

#### Inter-Route Relocate:

$i^{\text{th}}$  node of route  $a$  is relocated between  $j^{\text{th}}$  and  $(j+1)^{\text{th}}$  nodes of route  $b$ . (See Figure 19)

$$\begin{aligned}
E_3 = & c_p \beta_{k_a} \left( d_{(i-1)_a(i+1)_a} (v_{(i-1)_a(i+1)_a})^2 - d_{(i-1)_a i_a} (v_{i_a(i+1)_a})^2 - d_{i_a(i+1)_a} (v_{n_{(r_a,i),n_{(r_a,i+1)}}})^2 \right) \\
& + c_p \beta_{k_b} \left( d_{j_b i_a} (v_{j_b i_a})^{-2} + d_{i_a(j+1)_b} (v_{i_a(j+1)_b})^{-2} - d_{j_b(j+1)_b} (v_{j_b(j+1)_b})^{-2} \right)
\end{aligned}$$

#### Exchange:

$i^{\text{th}}$  node of route  $a$  and  $j^{\text{th}}$  node of route  $b$  are exchanged.  $i^{\text{th}}$  node of route  $a$  is placed between  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  nodes of route  $b$  whereas  $j^{\text{th}}$  node of route  $b$  is placed between  $m^{\text{th}}$  and  $(m+1)^{\text{th}}$  nodes of route  $a$ . (See Figure 20)

$$\begin{aligned}
E_3 = & c_p \beta_{k_a} \left( d_{(i-1)_a(i+1)_a} (v_{(i-1)_a(i+1)_a})^2 + d_{m_a j_b} (v_{m_a j_b})^2 + d_{j_b(m+1)_a} (v_{j_b(m+1)_a})^2 \right. \\
& \quad \left. - d_{(i-1)_a i_a} (v_{(i-1)_a i_a})^2 - d_{i_a(i+1)_a} (v_{i_a(i+1)_a})^2 - d_{m_a(m+1)_a} (v_{m_a(m+1)_a})^2 \right) \\
& + c_p \beta_{k_b} \left( d_{(j-1)_b(j+1)_b} (v_{(j-1)_b(j+1)_b})^2 + d_{n_b i_a} (v_{n_b i_a})^2 + d_{i_a(n+1)_b} (v_{i_a(n+1)_b})^2 \right. \\
& \quad \left. - d_{(j-1)_b j_b} (v_{(j-1)_b j_b})^2 - d_{j_b(j+1)_b} (v_{j_b(j+1)_b})^2 - d_{n_b(n+1)_b} (v_{n_b(n+1)_b})^2 \right)
\end{aligned}$$

#### Intra-Route Relocate:

Case 1: Two consecutive nodes are switched

$i^{\text{th}}$  and  $(i+1)^{\text{th}}$  nodes of route  $a$  are switched. (See Figure 21)

$$\begin{aligned}
E_3 = & c_p \beta_{k_a} \left( d_{(i-1)_a(i+1)_a} (v_{(i-1)_a(i+1)_a})^2 + d_{(i+1)_a i_a} (v_{(i+1)_a i_a})^2 + d_{i_a(i+2)_a} (v_{i_a(i+2)_a})^2 \right. \\
& \quad \left. - d_{(i-1)_a i_a} (v_{(i-1)_a i_a})^2 - d_{i_a(i+1)_a} (v_{i_a(i+1)_a})^2 - d_{(i+1)_a(i+2)_a} (v_{(i+1)_a(i+2)_a})^2 \right)
\end{aligned}$$

Case 2: Relocating to a Further Location – Backward Move

$j^{\text{th}}$  node of route  $a$  is relocated between  $(i - 1)^{\text{th}}$  and  $i^{\text{th}}$  nodes of the same route where  $j > (i + 1)$ . (See Figure 22)

$$E_3 = c_p \beta_{k_a} \left( d_{(i-1)_a j_a} (v_{(i-1)_a j_a})^2 + d_{j_a i_a} (v_{j_a i_a})^2 + d_{(j-1)_a (j+1)_a} (v_{(j-1)_a (j+1)_a})^2 \right. \\ \left. - d_{(i-1)_a i_a} (v_{(i-1)_a i_a})^2 - d_{(j-1)_a j_a} (v_{(j-1)_a j_a})^2 - d_{j_a (j+1)_a} (v_{j_a (j+1)_a})^2 \right)$$

Case 3: Relocating to a Further Location – Forward Move

$i^{\text{th}}$  node of route  $a$  is relocated between  $j^{\text{th}}$  and  $(j + 1)^{\text{th}}$  nodes of the same route where  $j > (i + 1)$ . (See Figure 23)

$$E_3 = c_p \beta_{k_a} \left( d_{(i-1)_a (i+1)_a} (v_{(i-1)_a (i+1)_a})^2 + d_{j_a i_a} (v_{j_a i_a})^2 + d_{i_a (j+1)_a} (v_{i_a (j+1)_a})^2 \right. \\ \left. - d_{(i-1)_a i_a} (v_{(i-1)_a i_a})^2 - d_{i_a (i+1)_a} (v_{i_a (i+1)_a})^2 - d_{j_a (j+1)_a} (v_{j_a (j+1)_a})^2 \right)$$

## Appendix C.4. Calculation of $E_4$

Each move results in a change in the total cost either due to the change in the vehicle speed or the total travel time which directly affects the driver costs. After performing a move, one of the following three main cases occurs in one or more locations in a route:

1. If the affected area is in the last arc group of the route, the change is directly reflected in the driver cost. Given that the change in the travel time is  $\Delta t$ , the change in the total cost will be  $p(\Delta t)$ .
2. If the affected area is not in the last arc group and  $\Delta t$  is positive, then the vehicle is speeded up to catch the deadline of the associated arc group. The change in the total cost is estimated as  $-\delta_l(\Delta t)$  where  $\delta_l$  is the derivative of function  $f(t)$  for all arcs in group  $l$ , that is:

$$\delta_l = f'(t_{ij}) = -2(c_f + c_e) d_{ij}^3 \beta_{k_r} (t_{ij})^{-3}$$

where  $(i, j)$  is any arc in the arc group  $l$  in route  $r$ .

3. If the affected area is not in the last arc group and  $\Delta t$  is negative, then two alternative cases may occur. In the first case, the vehicle is slowed down and the travel time remains the same. Alternatively, the vehicle speed is kept constant and travel time is reduced. The change in the total cost is estimated as  $\min\{p(\Delta t), -\delta_l(\Delta t)\}$ .

Keeping in mind these three rules, we estimate  $E_4$  for each type of move as follows:

### Inter-Route Relocate:

$i^{\text{th}}$  node of route  $a$  is relocated between  $j^{\text{th}}$  and  $(j + 1)^{\text{th}}$  nodes of route  $b$ . (See Figure 19) Assume that arc  $(j_b, (j + 1)_b)$  is in the  $l^{\text{th}}$  arc group, and arc  $(i_a, (i + 1)_a)$  is in the  $k^{\text{th}}$  arc group of their routes.

Let

$$\Delta t_1 = t_{j_b i_a} + t_{i_a(j+1)_b} - t_{j_b(j+1)_b} + q_{i_a} \tau$$

$$\Delta t_2 = t_{(i-1)_a(i+1)_a} - t_{(i-1)_a i_a} - t_{i_a(i+1)_a} - q_{i_a} \tau$$

If the  $k^{\text{th}}$  group is not the last arc group of route  $b$ ,

$$E_4 = -\delta_l(\Delta t_1) + \min\{p(\Delta t_2), -\delta_k(\Delta t_2)\}$$

Else,

$$E_4 = p(\Delta t_1) + \min\{p(\Delta t_2), -\delta_k(\Delta t_2)\}$$

**Exchange:**

$i^{\text{th}}$  node of route  $a$  and  $j^{\text{th}}$  node of route  $b$  are exchanged.  $i^{\text{th}}$  node of route  $a$  is placed between  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  nodes of route  $b$  whereas  $j^{\text{th}}$  node of route  $b$  is placed between  $m^{\text{th}}$  and  $(m+1)^{\text{th}}$  nodes of route  $a$ . (See Figure 20) Assume that arc  $(m_a, (m+1)_a)$  is in the  $l^{\text{th}}$  arc group, and arc  $(i_a, (i+1)_a)$  is in the  $k^{\text{th}}$  arc group of route  $a$ .

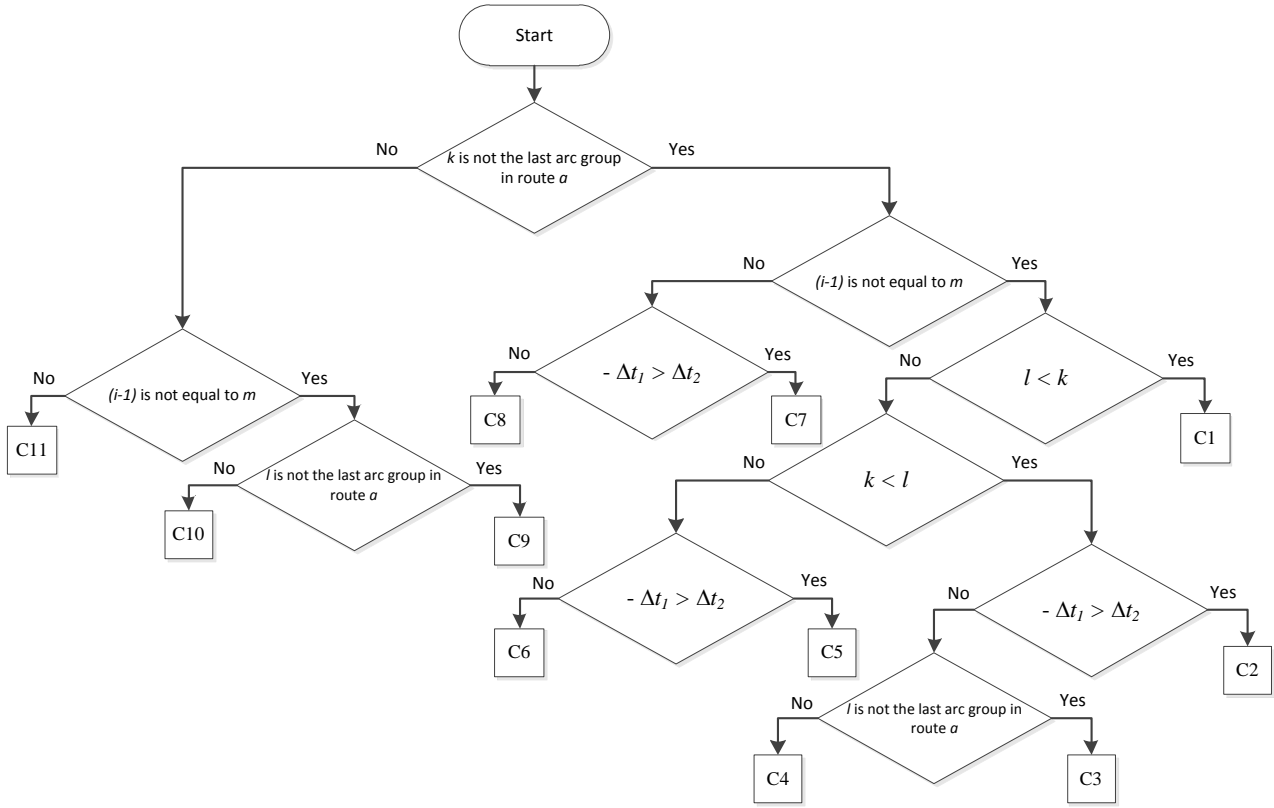
We set  $E_4 = E_{4a} + E_{4b}$  where  $E_{4a}$  and  $E_{4b}$  are estimate for routes  $a$  and  $b$ , respectively. Let us represent the calculation of  $E_{4a}$  in detail:

Let

$$\Delta t_1 = t_{(i-1)_a(i+1)_a} - t_{(i-1)_a i_a} - t_{i_a(i+1)_a} - q_{i_a} \tau$$

$$\Delta t_2 = t_{m_a j_b} + t_{j_b(m+1)_a} - t_{m_a(m+1)_a} + q_{j_b} \tau$$

Since many different states may occur as a result of the exchange move, we represent all possible cases within a decision flow diagram:



**Figure 23. Exchange – Decision Flow**

For each case illustrated above,  $E_{4a}$  can be calculated via the following formulas:

$$C1 \rightarrow E_{4a} = -\delta_l(\Delta t_2) + \min\{-\delta_k(\Delta t_1), p(\Delta t_1)\}$$

$$C2 \rightarrow E_{4a} = \min\{p(\Delta t_1 + \Delta t_2), -\delta_k(\Delta t_1) - \delta_l(\Delta t_2)\}$$

$$C3 \rightarrow E_{4a} = \min\{-\delta_k(\Delta t_1) - \delta_l(\Delta t_2), -\delta_l(\Delta t_1 + \Delta t_2)\}$$

$$C4 \rightarrow E_{4a} = \min\{p(\Delta t_1 + \Delta t_2), -\delta_k(\Delta t_1) + p(\Delta t_2)\}$$

$$C5 \rightarrow E_{4a} = \min\{p(\Delta t_1 + \Delta t_2), -\delta_l(\Delta t_1 + \Delta t_2)\}$$

$$C6 \rightarrow E_{4a} = -\delta_l(\Delta t_1 + \Delta t_2)$$

$$C7 \rightarrow E_{4a} = \min\{p(\Delta t_1 + \Delta t_2), -\delta_k(\Delta t_1 + \Delta t_2)\}$$

$$C8 \rightarrow E_{4a} = -\delta_k(\Delta t_1 + \Delta t_2)$$

$$C9 \rightarrow E_{4a} = p(\Delta t_1 + \Delta t_2)$$

$$C10 \rightarrow E_{4a} = p(\Delta t_1) - \delta_l(\Delta t_2)$$

$$C11 \rightarrow E_{4a} = p(\Delta t_1 + \Delta t_2)$$

Similar decision flow and formulas can be applied to route  $b$ . After  $E_{4a}$  and  $E_{4b}$  is calculated,  $E_4$  is easily found as:  $E_4 = E_{4a} + E_{4b}$ .



### **Intra-Route Relocate:**

#### Case 1: Two consecutive nodes are switched

$i^{\text{th}}$  and  $(i + 1)^{\text{th}}$  nodes of route  $a$  are switched. (See Figure 21) Assume that arc  $(i_a, (i + 1)_a)$  is in the  $k^{\text{th}}$  arc group of route  $a$ .

Let

$$\Delta t_1 = t_{(i-1)_a(i+1)_a} + t_{(i+1)_a i_a} + t_{i_a(i+2)_a} - t_{(i-1)_a i_a} - t_{i_a(i+1)_a} - t_{(i+1)_a(i+2)_a}$$

If the  $k^{\text{th}}$  arc group is not the last group in route  $a$  and  $\Delta t_1 > 0$ ,

$$E_4 = -\delta_k(\Delta t_1)$$

If the  $k^{\text{th}}$  arc group is not the last group in route  $a$  and  $\Delta t_1 \leq 0$ ,

$$E_4 = \min\{p(\Delta t_1), -\delta_k(\Delta t_1)\}$$

If the  $k^{\text{th}}$  arc group is the last group in route  $a$ ,

$$E_4 = p(\Delta t_1)$$

#### Case 2: Relocating to a Further Location – Backward Move

$j^{\text{th}}$  node of route  $a$  is relocated between  $(i - 1)^{\text{th}}$  and  $i^{\text{th}}$  nodes of the same route where  $j > (i + 1)$ . (See Figure 22) Assume that arc  $(j_a, (j + 1)_a)$  is in the  $l^{\text{th}}$  arc group, and arc  $((i - 1)_a, i_a)$  is in the  $k^{\text{th}}$  arc group of route  $a$ .

Let

$$\Delta t_1 = t_{(i-1)_a j_a} + t_{j_a(i+1)_a} - t_{(i-1)_a i_a} + q_{j_a} \tau$$

$$\Delta t_2 = t_{(j-1)_a(j+1)_a} - t_{(j-1)_a j_a} - t_{j_a(j+1)_a} - q_{j_a} \tau$$

Again, we illustrate the possible cases that may occur with a decision flow diagram:

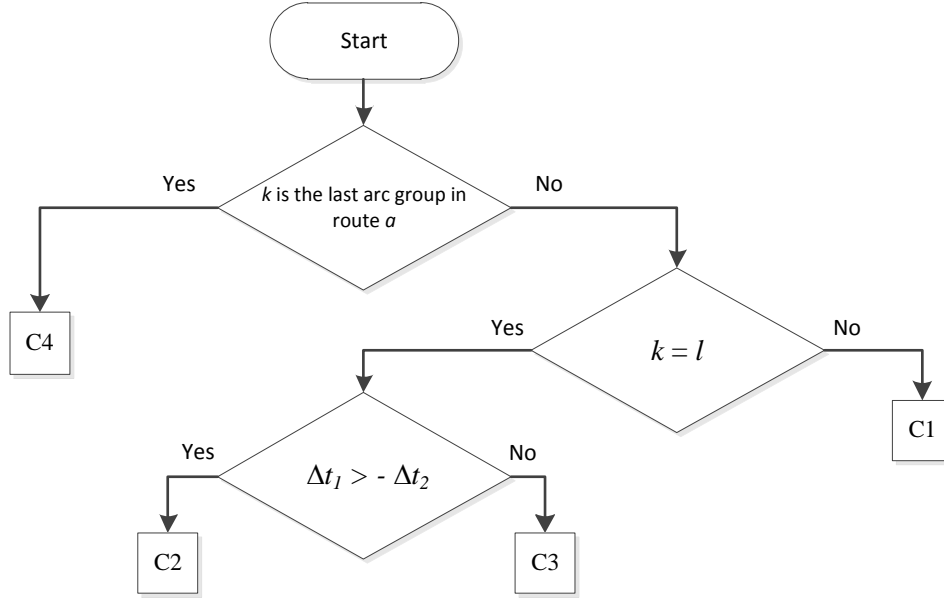


Figure 24. Intra-Route Relocate Case 2 – Decision Flow

And  $E_4$  can be calculated for each case as:

$$C1 \rightarrow E_4 = -\delta_k(\Delta t_1) + \min\{-\delta_l(\Delta t_2), p(\Delta t_2)\}$$

$$C2 \rightarrow E_4 = -\delta_k(\Delta t_1 + \Delta t_2)$$

$$C3 \rightarrow E_4 = \min\{-\delta_k(\Delta t_1 + \Delta t_2), p(\Delta t_1 + \Delta t_2)\}$$

$$C4 \rightarrow E_4 = p(\Delta t_1 + \Delta t_2)$$

Case 3: Relocating to a Further Location – Forward Move

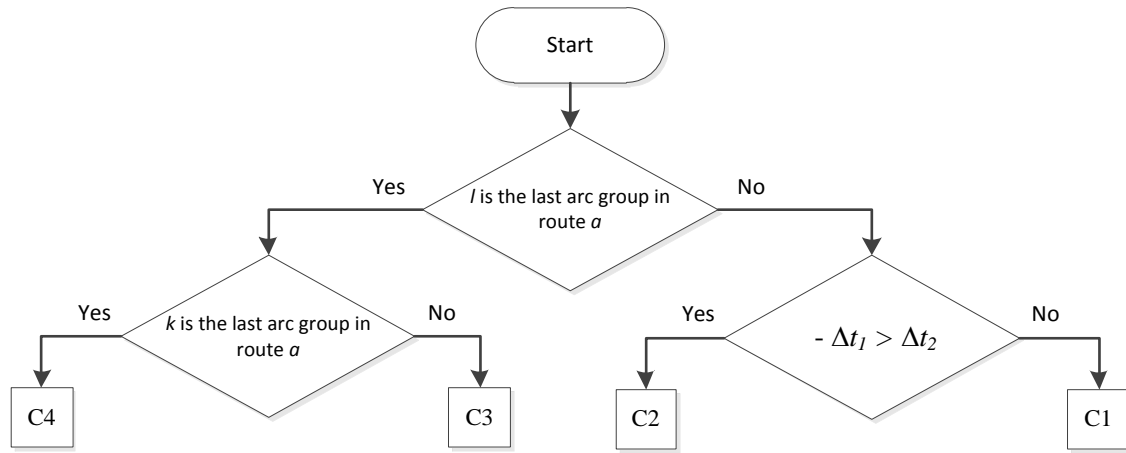
$i^{th}$  node of route  $a$  is relocated between  $j^{th}$  and  $(j + 1)^{th}$  nodes of the same route where  $j > (i + 1)$ . (See Figure 23) Assume that arc  $(j_a, (j + 1)_a)$  is in the  $l^{th}$  arc group, and arc  $(i_a, (i + 1)_a)$  is in the  $k^{th}$  arc group of route  $a$ .

Let

$$\Delta t_1 = t_{(i-1)_a(i+1)_a} - t_{(i-1)_a i_a} - t_{i_a(i+1)_a} - q_{i_a} \tau$$

$$\Delta t_2 = t_{j_a i_a} + t_{i_a(j+1)_a} - t_{(j-1)_a(j+1)_a} + q_{i_a} \tau$$

The decision flow diagram for this case:



**Figure 25. Intra-Route Relocate Case 3 – Decision Flow**

Calculation of  $E_4$ :

$$C1 \rightarrow E_4 = \min\{-\delta_k(\Delta t_1) - \delta_l(\Delta t_2), -\delta_l(\Delta t_1 + \Delta t_2)\}$$

$$C2 \rightarrow E_4 = \min\{(\min\{-\delta_l, p\})(\Delta t_1 + \Delta t_2), -\delta_k(\Delta t_1 + \Delta t_2)\}$$

$$C3 \rightarrow E_4 = \min\{-\delta_k(\Delta t_1) + p(\Delta t_2), p(\Delta t_1 + \Delta t_2)\}$$

$$C4 \rightarrow E_4 = p(\Delta t_1 + \Delta t_2)$$

## APPENDIX D

### COMPUTATIONAL ANALYSIS – DETAILED RESULTS

#### Appendix D.1. Number of Vehicles in the Fleet for Alternative Network Sizes

Table 28. Number of Vehicles in the Fleet for Alternative Network Sizes

# of Customers in the Network	Number of Vehicles in the Fleet	
	Vehicle Type 1 (Capacity: 3 tones)	Vehicle Type 2 (Capacity: 7 tones)
10	1	1
15	2	1
20	2	2
25	2	2
50	4	4
75	5	5
100	8	8
150	10	10
200	15	15

#### Appendix D.2. Models and Heuristics – Detailed Results

Table 29. Models and Heuristics – Detailed Results

Network Size: 10 Customers												
Rep No	M1			M2			H1		H2		H3	
	Total Cost (£)	Total CPU Time (sec)	GAP	Total Cost (£)	Total CPU Time (sec)	GAP	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
1	133.35	1000.17	17.86%	133.23	1000.82	35.81%	145.45	0.022	145.45	0.018	145.45	0.024
2	187.74	1016.69	22.55%	187.42	1001.32	41.42%	210.49	0.017	210.49	0.016	210.49	0.019
3	170.76	1001.88	42.80%	169.29	1001.73	38.73%	170.57	0.016	170.57	0.024	170.57	0.032
4	176.71	1001.63	42.13%	166.31	1000.41	41.48%	167.66	0.014	167.66	0.013	167.66	0.016
5	151.97	1000.16	32.85%	152.34	1000.36	43.37%	158.68	0.008	158.68	0.007	158.68	0.011
6	199.13	1000.16	26.47%	198.42	1000.97	45.56%	198.99	0.023	206.67	0.019	206.67	0.026
7	176.39	1000.55	14.55%	178.04	1000.86	39.04%	202.58	0.008	202.58	0.008	202.58	0.012
8	195.61	1015.25	14.51%	195.38	1001.83	36.33%	195.38	0.009	195.38	0.008	195.38	0.010
9	160.12	1001.32	28.85%	156.83	1002.50	23.31%	166.74	0.024	166.74	0.024	166.74	0.036
10	166.87	1001.61	24.08%	166.49	1000.15	28.74%	169.25	0.013	169.25	0.014	169.25	0.020
Avg	171.87	1003.94	26.67%	170.38	1001.1	37.38%	178.58	0.015	179.35	0.015	179.35	0.021
Network Size: 15 Customers												
Rep No	M1			M2			H1		H2		H3	
	Total Cost (£)	Total CPU Time (sec)	GAP	Total Cost (£)	Total CPU Time (sec)	GAP	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
1	231.54	1501.66	57.57%	247.06	1501.84	65.99%	249.93	0.046	249.93	0.058	249.93	0.070
2	193.41	1501.56	65.31%	186.08	1503.73	66.02%	212.59	0.020	212.59	0.027	212.59	0.050
3	248.79	1503.33	60.20%	249.74	1503.86	67.18%	241.81	0.069	306.59	0.036	306.59	0.051

4	255.59	1501.81	64.21%	225.59	1503.59	64.57%	250.98	0.023	250.98	0.018	250.98	0.030
5	296.22	1503.03	62.65%	293.72	1503.11	67.23%	313.85	0.031	313.85	0.033	313.85	0.050
6	206.40	1502.34	65.12%	195.22	1502.93	66.15%	202.57	0.021	202.57	0.025	202.57	0.031
7	207.30	1501.51	65.84%	219.80	1505.81	69.81%	220.96	0.028	220.96	0.036	220.96	0.036
8	150.52	1507.19	62.96%	140.00	1502.30	66.48%	157.68	0.022	157.68	0.026	157.68	0.019
9	226.43	1504.32	64.42%	214.09	1502.41	66.11%	238.14	0.057	238.48	0.066	238.48	0.064
10	206.48	1503.61	65.13%	206.69	1501.82	67.35%	211.25	0.020	212.21	0.019	212.21	0.018
<b>Avg</b>	<b>222.27</b>	<b>1503.03</b>	<b>63.34%</b>	<b>217.80</b>	<b>1503.14</b>	<b>66.69%</b>	<b>229.98</b>	<b>0.034</b>	<b>236.59</b>	<b>0.034</b>	<b>236.59</b>	<b>0.042</b>
<b>Network Size: 20 Customers</b>												
<b>Rep No</b>	<b>M1</b>			<b>M2</b>			<b>H1</b>		<b>H2</b>		<b>H3</b>	
	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>GAP</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>GAP</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>
1	352.96	2012.34	69.22%	372.39	2040.12	73.04%	336.59	0.039	321.51	0.097	321.51	0.043
2	370.31	2012.16	70.26%	372.17	2035.39	72.96%	314.43	0.036	314.43	0.054	314.43	0.059
3	195.60	2007.72	65.14%	217.45	2044.21	71.24%	201.42	0.043	202.02	0.067	202.02	0.05
4	288.35	2011.04	66.63%	307.08	2043.97	71.57%	294.88	0.077	273.69	0.103	273.69	0.079
5	256.62	2010.17	63.95%	273.98	2030.93	68.78%	299.93	0.041	299.93	0.074	299.93	0.045
6	307.30	2011.68	66.75%	326.78	2020.77	71.42%	309.72	0.058	309.72	0.091	309.72	0.081
7	204.31	2012.16	64.00%	225.99	2037.46	68.80%	211.52	0.056	211.52	0.086	211.52	0.07
8	259.55	2012.95	69.32%	241.09	2051.32	69.13%	217.48	0.022	217.48	0.039	217.48	0.027
9	379.70	2007.18	68.71%	351.04	2042.74	69.63%	363.68	0.013	363.23	0.02	363.23	0.016
10	270.01	2012.76	67.74%	278.04	2026.73	70.99%	248.21	0.034	248.21	0.058	248.21	0.042
<b>Avg</b>	<b>288.47</b>	<b>2011.02</b>	<b>67.17%</b>	<b>296.61</b>	<b>2037.36</b>	<b>70.76%</b>	<b>279.79</b>	<b>0.042</b>	<b>276.18</b>	<b>0.069</b>	<b>276.18</b>	<b>0.051</b>
<b>Network Size: 25 Customers</b>												
<b>Rep No</b>	<b>M1</b>			<b>M2</b>			<b>H1</b>		<b>H2</b>		<b>H3</b>	
	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>GAP</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>GAP</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>
1	280.76	2530.98	64.92%	293.14	2553.70	69.74%	307.53	0.074	276.39	0.175	276.39	0.137
2	311.88	2516.17	65.07%	322.49	2623.42	69.49%	313.20	0.114	314.92	0.200	314.92	0.169
3	205.19	2518.96	66.69%	204.02	2537.53	69.90%	178.07	0.041	178.07	0.072	178.07	0.052
4	266.86	2507.13	68.21%	277.14	2544.29	71.24%	282.54	0.105	282.54	0.231	282.54	0.148
5	320.68	2511.54	63.12%	351.94	2593.04	70.65%	278.59	0.171	285.12	0.219	285.12	0.160
6	269.89	2526.27	67.23%	324.93	2607.52	75.12%	327.63	0.057	327.63	0.064	327.63	0.056
7	344.18	2514.35	68.55%	337.27	2558.66	70.95%	306.83	0.086	305.18	0.225	305.18	0.136
8	371.59	2514.63	70.18%	339.40	2569.37	70.70%	334.29	0.138	333.68	0.282	333.68	0.168
9	255.79	2525.64	65.77%	278.89	2547.14	71.26%	253.62	0.107	257.01	0.211	257.01	0.116
10	307.41	2520.39	59.35%	366.63	2557.73	70.38%	319.73	0.172	310.02	0.340	310.02	0.205
<b>Avg</b>	<b>293.42</b>	<b>2518.61</b>	<b>65.91%</b>	<b>309.59</b>	<b>2569.24</b>	<b>70.94%</b>	<b>290.21</b>	<b>0.107</b>	<b>287.06</b>	<b>0.202</b>	<b>287.06</b>	<b>0.135</b>
<b>Network Size: 50 Customers</b>												
<b>Rep No</b>	<b>M1</b>			<b>M2</b>			<b>H1</b>		<b>H2</b>		<b>H3</b>	
	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>GAP</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>GAP</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>
1	600.23	5080.87	69.02%	686.26	5606.40	74.99%	512.97	1.458	517.44	8.067	517.44	2.007
2	626.58	5256.59	67.16%	720.64	5643.09	73.97%	670.37	0.767	600.47	3.225	600.47	0.978
3	1177.34	5363.70	82.69%	807.81	5873.50	76.55%	528.14	2.080	546.11	7.981	546.11	2.254
4	710.66	5474.51	69.76%	867.90	5572.68	77.56%	650.88	1.336	667.05	3.502	667.05	1.307
5	723.43	5205.81	72.72%	763.60	5644.34	75.49%	680.73	0.753	624.82	3.917	624.82	0.973
6	534.59	5426.90	69.17%	548.79	5697.03	72.06%	484.89	0.906	483.73	4.339	483.73	0.885
7	510.71	5660.02	68.61%	620.90	5778.10	75.67%	503.52	1.080	542.36	1.472	542.36	0.565
8	634.32	5379.83	72.23%	-	5656.23	-	555.84	1.314	509.49	7.908	509.49	2.016
9	752.42	5317.08	70.76%	796.70	5958.12	74.80%	617.75	1.089	600.34	5.151	600.34	1.194

<b>10</b>	606.42	5068.83	65.47%	766.50	6469.47	75.12%	634.37	1.301	546.93	6.578	546.93	2.612
<b>Avg</b>	<b>687.67</b>	<b>5323.41</b>	<b>70.76%</b>	<b>731.01</b>	<b>5789.89</b>	<b>75.14%</b>	<b>583.95</b>	<b>1.208</b>	<b>563.87</b>	<b>5.214</b>	<b>563.87</b>	<b>1.479</b>
<b>Avg<sup>5</sup></b>	<b>693.60</b>	<b>5317.14</b>	<b>70.60%</b>	<b>731.01</b>	<b>5804.74</b>	<b>75.14%</b>	<b>587.07</b>	<b>1.197</b>	<b>569.92</b>	<b>4.915</b>	<b>569.92</b>	<b>1.419</b>

## Appendix D.3. Heuristics – Detailed Results

Table 30. Heuristics – Detailed Results

Network Size: 10 Customers						
	H1		H2		H3	
Rep No	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
1	145.45	0.022	145.45	0.018	145.45	0.024
2	210.49	0.017	210.49	0.016	210.49	0.019
3	170.57	0.016	170.57	0.024	170.57	0.032
4	167.66	0.014	167.66	0.013	167.66	0.016
5	158.69	0.008	158.69	0.007	158.69	0.011
6	198.99	0.023	206.67	0.019	206.67	0.026
7	202.59	0.008	202.59	0.008	202.59	0.012
8	195.39	0.009	195.39	0.008	195.39	0.010
9	166.74	0.024	166.74	0.024	166.74	0.036
10	169.25	0.013	169.25	0.014	169.25	0.020
<b>Avg</b>	<b>178.58</b>	<b>0.015</b>	<b>179.35</b>	<b>0.015</b>	<b>179.35</b>	<b>0.021</b>
Network Size: 15 Customers						
	H1		H2		H3	
Rep No	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
1	249.93	0.046	249.93	0.058	249.93	0.070
2	212.60	0.020	212.60	0.027	212.60	0.050
3	241.81	0.069	306.60	0.036	306.60	0.051
4	250.98	0.023	250.98	0.018	250.98	0.030
5	313.86	0.031	313.86	0.033	313.86	0.050
6	202.58	0.021	202.58	0.025	202.58	0.031
7	220.96	0.028	220.96	0.036	220.96	0.036
8	157.69	0.022	157.69	0.026	157.69	0.019
9	238.14	0.057	238.48	0.066	238.48	0.064
10	211.25	0.020	212.21	0.019	212.21	0.018
<b>Avg</b>	<b>229.98</b>	<b>0.034</b>	<b>236.59</b>	<b>0.034</b>	<b>236.59</b>	<b>0.042</b>
Network Size: 20 Customers						
	H1		H2		H3	
Rep No	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
1	336.59	0.039	321.51	0.097	321.51	0.043
2	314.43	0.036	314.43	0.054	314.43	0.059
3	201.43	0.043	202.02	0.067	202.02	0.05
4	294.89	0.077	273.70	0.103	273.70	0.079
5	299.93	0.041	299.94	0.074	299.94	0.045
6	309.76	0.058	309.76	0.091	309.76	0.081
7	211.52	0.056	211.52	0.086	211.52	0.07
8	217.49	0.022	217.49	0.039	217.49	0.027
9	363.69	0.013	363.23	0.02	363.23	0.016

<sup>5</sup> The second line of average results in 50-customer case is based on 9 replications since in one of the replications no feasible solutions could be found by M2.

<b>10</b>	248.21	0.034	248.21	0.058	248.21	0.042
<b>Avg</b>	<b>279.79</b>	<b>0.042</b>	<b>276.18</b>	<b>0.069</b>	<b>276.18</b>	<b>0.051</b>
<b>Network Size: 25 Customers</b>						
<b>H1</b>		<b>H2</b>		<b>H3</b>		
<b>Rep No</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>
1	307.53	0.074	276.40	0.175	276.40	0.137
2	313.21	0.114	314.93	0.200	314.93	0.169
3	178.07	0.041	178.07	0.072	178.07	0.052
4	282.54	0.105	282.54	0.231	282.54	0.148
5	278.60	0.171	285.13	0.219	285.13	0.160
6	327.63	0.057	327.63	0.064	327.63	0.056
7	306.84	0.086	305.19	0.225	305.19	0.136
8	334.29	0.138	333.68	0.282	333.68	0.168
9	253.63	0.107	257.01	0.211	257.01	0.116
10	319.73	0.172	310.02	0.340	310.02	0.205
<b>Avg</b>	<b>290.21</b>	<b>0.107</b>	<b>287.06</b>	<b>0.202</b>	<b>287.06</b>	<b>0.135</b>
<b>Network Size: 50 Customers</b>						
<b>H1</b>		<b>H2</b>		<b>H3</b>		
<b>Rep No</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>
1	512.97	1.458	517.44	8.067	517.44	2.007
2	670.37	0.767	600.47	3.225	600.47	0.978
3	528.15	2.080	546.12	7.981	546.12	2.254
4	650.88	1.336	667.06	3.502	667.06	1.307
5	680.74	0.753	624.83	3.917	624.83	0.973
6	484.90	0.906	483.73	4.339	483.73	0.885
7	503.52	1.080	542.37	1.472	542.37	0.565
8	555.85	1.314	509.49	7.908	509.49	2.016
9	617.76	1.089	600.35	5.151	600.35	1.194
10	634.37	1.301	546.93	6.578	546.93	2.612
<b>Avg</b>	<b>583.95</b>	<b>1.208</b>	<b>563.88</b>	<b>5.214</b>	<b>563.88</b>	<b>1.479</b>
<b>Network Size: 75 Customers</b>						
<b>H1</b>		<b>H2</b>		<b>H3</b>		
<b>Rep No</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>
1	984.84	5.117	950.08	31.696	950.08	6.063
2	761.70	5.392	709.27	34.234	709.27	6.585
3	933.91	5.602	904.06	33.379	953.56	5.873
4	658.65	5.882	661.49	35.399	661.49	7.023
5	881.98	4.813	812.65	36.073	812.65	6.906
6	946.23	6.707	992.76	32.333	981.91	6.720
7	777.62	9.087	818.80	44.327	818.80	10.399
8	917.12	4.171	903.57	38.207	894.74	5.432
9	930.66	4.894	944.37	23.152	944.37	4.995
10	873.75	6.248	963.43	20.008	963.43	4.259
<b>Avg</b>	<b>866.64</b>	<b>5.791</b>	<b>866.05</b>	<b>32.881</b>	<b>869.03</b>	<b>6.426</b>
<b>Network Size: 100 Customers</b>						
<b>H1</b>		<b>H2</b>		<b>H3</b>		
<b>Rep No</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>
1	1125.81	20.39	1080.70	160.20	1080.70	25.86
2	1199.61	13.46	1197.93	99.47	1197.93	14.94
3	1008.77	47.73	1021.74	196.70	1041.04	74.58
4	911.73	22.11	968.45	154.28	940.67	37.35

5	934.26	31.03	958.69	149.51	958.69	41.75	
6	1220.08	16.19	1182.18	151.89	1182.18	25.06	
7	1156.16	18.16	1103.78	152.23	1103.78	24.78	
8	1015.58	20.08	1132.51	170.13	1132.51	25.00	
9	940.29	15.40	937.96	159.65	937.96	30.86	
10	1108.08	20.55	1045.98	219.89	1045.98	29.52	
<b>Avg</b>	<b>1062.04</b>	<b>22.511</b>	<b>1062.99</b>	<b>161.40</b>	<b>1062.14</b>	<b>32.97</b>	
<b>Network Size: 150 Customers</b>							
		<b>H1</b>		<b>H2</b>		<b>H3</b>	
<b>Rep No</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	
1	1214.38	157.55	1333.44	943.52	1287.75	212.88	
2	1544.46	133.71	1567.92	1065.68	1567.92	130.71	
3	1309.36	189.39	1318.16	1208.93	1318.16	327.55	
4	1501.93	149.51	1482.18	1349.27	1462.23	195.75	
5	1418.15	137.00	1395.95	1055.05	1395.95	175.59	
6	1223.98	199.68	1271.06	1644.75	1362.13	241.78	
7	1469.91	128.67	1602.67	978.63	1602.67	114.43	
8	1260.90	276.03	1370.49	1600.36	1370.49	461.92	
9	1656.11	142.06	1702.27	1297.76	1702.27	156.02	
10	1447.14	153.11	1456.44	1349.52	1578.39	134.90	
<b>Avg</b>	<b>1404.63</b>	<b>166.67</b>	<b>1450.06</b>	<b>1249.35</b>	<b>1464.80</b>	<b>215.15</b>	
<b>Network Size: 200 Customers</b>							
		<b>H1</b>		<b>H2</b>		<b>H3</b>	
<b>Rep No</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	<b>Total Cost (£)</b>	<b>Total CPU Time (sec)</b>	
1	1970.96	909.88	1810.27	6668.15	1810.27	1329.01	
2	1964.22	924.67	1933.37	6498.18	1933.37	1082.80	
3	1773.43	648.50	1719.08	5539.73	1719.08	978.37	
4	1630.35	765.64	1697.24	5836.97	1681.12	1041.05	
5	1993.11	564.47	1960.81	5542.82	1919.58	880.56	
6	1732.29	517.72	1669.11	5517.39	1658.51	745.64	
7	1750.43	622.15	1675.33	7012.85	1675.33	894.64	
8	1712.29	766.54	1855.73	5069.51	1774.56	1046.06	
9	1713.88	770.36	1783.96	4821.82	1759.45	902.42	
10	1813.15	777.48	1983.36	7692.14	1983.36	929.77	
<b>Avg</b>	<b>1805.41</b>	<b>726.74</b>	<b>1808.83</b>	<b>6019.96</b>	<b>1791.46</b>	<b>983.03</b>	

## Appendix D.4. Heuristic Moves – Detailed Results

Table 31. Heuristic Moves – Detailed Results

<b>Network Size: 10 Customers</b>												
<b>Move Type</b>	<b>Number of Moves</b>				<b>Move Type Percentage</b>				<b>% Improvement</b>			
	<b>H1</b>	<b>H2</b>	<b>H3</b>	<b>Avg</b>	<b>H1</b>	<b>H2</b>	<b>H3</b>	<b>Avg</b>	<b>H1</b>	<b>H2</b>	<b>H3</b>	<b>Avg</b>
1	0.6	0.4	0.4	0.47	25%	16.7%	16.7%	19.5%	3.3%	2.2%	2.2%	2.6%
2	1.1	1.1	1.1	1.10	45.8%	45.8%	45.8%	45.8%	12.2%	12.6%	12.6%	12.5%
3	0.7	0.9	0.9	0.83	29.2%	37.6%	37.5%	34.7%	0.5%	0.7%	0.7%	0.6%
<b>Total</b>	<b>2.4</b>	<b>2.4</b>	<b>2.4</b>	<b>2.40</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>15.9%</b>	<b>15.5%</b>	<b>15.5%</b>	<b>15.7%</b>
<b>Network Size: 15 Customers</b>												
<b>Move Type</b>	<b>Number of Moves</b>				<b>Move Type Percentage</b>				<b>% Improvement</b>			
	<b>H1</b>	<b>H2</b>	<b>H3</b>	<b>Avg</b>	<b>H1</b>	<b>H2</b>	<b>H3</b>	<b>Avg</b>	<b>H1</b>	<b>H2</b>	<b>H3</b>	<b>Avg</b>
1	1.1	0.6	0.6	0.77	21.6%	16.7%	16.7%	18.3%	3.1%	1.5%	1.5%	2%



<b>2</b>	3	2.4	2.4	2.60	58.8%	66.7%	66.7%	64.1%	13.8%	13.1%	13.1%	13.3%
<b>3</b>	1	0.6	0.6	0.73	19.6%	16.6%	16.6%	17.6%	0.5%	0.5%	0.5%	0.5%
<b>Total</b>	<b>5.1</b>	<b>3.6</b>	<b>3.6</b>	<b>4.10</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>17.4%</b>	<b>15.1%</b>	<b>15.1%</b>	<b>15.8%</b>
<b>Network Size: 20 Customers</b>												
Move Type	Number of Moves				Move Type Percentage				% Improvement			
	H1	H2	H3	Avg	H1	H2	H3	Avg	H1	H2	H3	Avg
<b>1</b>	1.4	0.9	0.9	1.07	24.1%	17%	17%	19.4%	1.5%	1.8%	1.8%	1.7%
<b>2</b>	3.7	3.6	3.6	3.63	63.8%	67.9%	67.9%	66.5%	12.7%	13.7%	13.7%	13.3%
<b>3</b>	0.7	0.8	0.8	0.77	121%	15.1%	15.1%	14.1%	0.4%	0.2%	0.2%	0.3%
<b>Total</b>	<b>5.8</b>	<b>5.3</b>	<b>5.3</b>	<b>5.47</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>14.6%</b>	<b>15.7%</b>	<b>15.7%</b>	<b>15.4%</b>
<b>Network Size: 25 Customers</b>												
Move Type	Number of Moves				Move Type Percentage				% Improvement			
	H1	H2	H3	Avg	H1	H2	H3	Avg	H1	H2	H3	Avg
<b>1</b>	1.6	1.5	1.5	1.53	17.4%	18.3%	18.3%	18%	3.3%	2.5%	2.5%	2.8%
<b>2</b>	5.6	5.2	5.2	5.33	60.9%	63.4%	63.4%	62.6%	12.1%	13.1%	13.1%	12.8%
<b>3</b>	2	1.5	1.5	1.67	21.7%	18.3%	18.3%	19.4%	0.5%	1.1%	1.1%	0.9%
<b>Total</b>	<b>9.2</b>	<b>8.2</b>	<b>8.2</b>	<b>8.53</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>15.9%</b>	<b>16.8%</b>	<b>16.8%</b>	<b>16.5%</b>
<b>Network Size: 50 Customers</b>												
Move Type	Number of Moves				Move Type Percentage				% Improvement			
	H1	H2	H3	Avg	H1	H2	H3	Avg	H1	H2	H3	Avg
<b>1</b>	4.9	5.3	5.3	5.17	18.9%	23.2%	23.2%	21.7%	4.5%	4.3%	4.3%	4.4%
<b>2</b>	18.3	15.9	15.9	16.70	70.7%	69.4%	69.4%	69.8%	21.6%	24.4%	24.4%	23.5%
<b>3</b>	2.7	1.7	1.7	2.03	10.4%	7.4%	7.4%	8.4%	0.3%	0.2%	0.2%	0.3%
<b>Total</b>	<b>25.9</b>	<b>22.9</b>	<b>22.9</b>	<b>23.90</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>26.4%</b>	<b>28.9%</b>	<b>29%</b>	<b>28.1%</b>
<b>Network Size: 75 Customers</b>												
Move Type	Number of Moves				Move Type Percentage				% Improvement			
	H1	H2	H3	Avg	H1	H2	H3	Avg	H1	H2	H3	Avg
<b>1</b>	7	4.7	4.6	5.43	17.2%	14%	14.2%	15.1%	3%	2%	2%	2.3%
<b>2</b>	30.7	25.1	24.3	26.70	75.4%	74.7%	75%	75.0%	25.3%	26.2%	26%	25.8%
<b>3</b>	3	3.8	3.5	3.43	7.4%	11.3%	10.8%	9.8%	0.8%	0.9%	0.8%	0.8%
<b>Total</b>	<b>40.7</b>	<b>33.6</b>	<b>32.4</b>	<b>35.57</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>29%</b>	<b>29.1%</b>	<b>28.8%</b>	<b>29%</b>
<b>Network Size: 100 Customers</b>												
Move Type	Number of Moves				Move Type Percentage				% Improvement			
	H1	H2	H3	Avg	H1	H2	H3	Avg	H1	H2	H3	Avg
<b>1</b>	6.3	7.8	8.2	7.43	10.6%	13%	13.4%	12.3%	2.5%	1.8%	2.1%	2.1%
<b>2</b>	48.2	47.9	48.2	48.10	80.9%	79.5%	78.7%	79.7%	31.6%	32.2%	31.9%	31.9%
<b>3</b>	5.1	4.5	4.8	4.80	8.57%	7.5%	7.9%	8%	0.4%	0.5%	0.6%	0.5%
<b>Total</b>	<b>59.6</b>	<b>60.2</b>	<b>61.2</b>	<b>60.33</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>34.6%</b>	<b>34.5%</b>	<b>34.5%</b>	<b>34.5%</b>
<b>Network Size: 150 Customers</b>												
Move Type	Number of Moves				Move Type Percentage				% Improvement			
	H1	H2	H3	Avg	H1	H2	H3	Avg	H1	H2	H3	Avg
<b>1</b>	13	8.9	9.1	10.33	10.7%	10%	10.4%	10.4%	3.2%	1.5%	1.5%	2.1%
<b>2</b>	101.3	74.9	73.4	83.20	83%	84.4%	84.1%	83.8%	35.4%	35.1%	35.1%	35.2%
<b>3</b>	7.8	4.9	4.8	5.83	6.3%	5.5%	5.5%	5.8%	0.3%	0.4%	0.4%	0.3%
<b>Total</b>	<b>122.1</b>	<b>88.7</b>	<b>87.3</b>	<b>99.37</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>38.9%</b>	<b>36.9%</b>	<b>36.9%</b>	<b>37.6%</b>
<b>Network Size: 200 Customers</b>												
Move Type	Number of Moves				Move Type Percentage				% Improvement			
	H1	H2	H3	Avg	H1	H2	H3	Avg	H1	H2	H3	Avg
<b>1</b>	14.8	12.7	15.4	14.30	8.2%	9.2%	10.5%	9.3%	1.8%	1.5%	1.7%	1.6%
<b>2</b>	153.2	117.6	123.2	131.3	85%	85.5%	84%	84.8%	35.5%	35.8%	36.2%	35.8%
<b>3</b>	12.2	7.2	8.1	9.17	6.8%	5.3%	5.5%	5.8%	0.5%	0.4%	0.5%	0.5%
<b>Total</b>	<b>180.2</b>	<b>137.5</b>	<b>146.7</b>	<b>154.8</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>37.8%</b>	<b>37.7%</b>	<b>38.3%</b>	<b>37.9%</b>

## Appendix D.5. Changing the Value of PMI – Detailed Results

Table 32. Changing the Value of PMI – Detailed Results

Rep No	Network Size: 10 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff
1	336.59	336.59	0.00%	321.51	321.51	0.00%	0.039	0.044	-	0.043	0.068	-
2	314.43	314.43	0.00%	314.43	314.43	0.00%	0.036	0.088	-	0.059	0.101	-
3	201.43	201.43	0.00%	202.02	202.02	0.00%	0.043	0.025	-	0.05	0.025	-
4	294.89	270.24	-8.36%	273.70	273.70	0.00%	0.077	0.095	-	0.079	0.065	-
5	299.94	299.94	0.00%	299.94	299.94	0.00%	0.041	0.037	-	0.045	0.042	-
6	309.73	311.18	0.47%	309.73	311.18	0.47%	0.058	0.006	-	0.081	0.005	-
7	211.52	211.52	0.00%	211.52	211.52	0.00%	0.056	0.038	-	0.07	0.061	-
8	217.49	217.49	0.00%	217.49	217.49	0.00%	0.022	0.025	-	0.027	0.032	-
9	363.69	363.69	0.00%	363.23	363.69	0.13%	0.013	0.026	-	0.016	0.035	-
10	248.21	248.21	0.00%	248.21	248.21	0.00%	0.034	0.072	-	0.042	0.097	-
Avg	279.79	277.47	-0.79%	276.18	276.37	0.06%	0.042	0.046	-	0.051	0.053	-
Rep No	Network Size: 50 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff
1	512.97	512.97	0.00%	517.44	517.44	0.00%	1.458	1.133	-22.29%	2.007	1.343	-33.08%
2	670.37	670.37	0.00%	600.47	600.47	0.00%	0.767	0.631	-17.73%	0.978	0.722	-26.18%
3	528.15	521.97	-1.17%	546.12	542.86	-0.60%	2.080	1.424	-31.54%	2.254	1.105	-50.98%
4	650.88	653.28	0.37%	667.06	667.06	0.00%	1.336	0.894	-33.08%	1.307	0.776	-40.63%
5	680.74	661.38	-2.84%	624.83	624.89	0.01%	0.753	0.875	16.20%	0.973	0.796	-18.19%
6	484.90	485.06	0.03%	483.73	483.73	0.00%	0.906	0.718	-20.75%	0.885	0.640	-27.68%
7	503.52	503.52	0.00%	542.37	542.37	0.00%	1.080	0.837	-22.50%	0.565	0.356	-36.99%
8	555.85	533.13	-4.09%	509.49	506.10	-0.66%	1.314	0.960	-26.94%	2.016	1.342	-33.43%
9	617.76	692.51	12.10%	600.35	621.16	3.47%	1.089	0.759	-30.30%	1.194	0.817	-31.57%
10	634.37	634.56	0.03%	546.93	549.68	0.50%	1.301	0.804	-38.20%	2.612	1.170	-55.21%
Avg	583.95	586.88	0.44%	563.88	565.58	0.27%	1.208	0.904	-22.71%	1.479	0.907	-35.39%
Rep No	Network Size: 100 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff
1	1125.81	1126.19	0.03%	1080.70	1080.70	0.00%	20.393	14.960	-26.64%	25.864	13.739	-46.88%
2	1199.61	1175.33	-2.02%	1197.93	1197.93	0.00%	13.457	10.923	-18.83%	14.942	9.642	-35.47%
3	1008.77	1008.77	0.00%	1041.04	1001.34	-3.81%	47.726	18.284	-61.69%	74.580	18.499	-75.20%
4	911.73	911.73	0.00%	940.67	959.07	1.96%	22.111	14.464	-34.58%	37.351	14.937	-60.01%
5	934.26	955.18	2.24%	958.69	958.89	0.02%	31.033	13.030	-58.01%	41.754	13.122	-68.57%
6	1220.08	1103.81	-9.53%	1182.18	1180.06	-0.18%	16.186	17.566	8.53%	25.062	14.401	-42.54%
7	1156.16	1156.16	0.00%	1103.78	1060.37	-3.93%	18.164	12.643	-30.40%	24.781	16.985	-31.46%
8	1015.58	1129.83	11.25%	1132.51	1129.75	-0.24%	20.084	15.145	-24.59%	24.995	12.443	-50.22%
9	940.29	940.29	0.00%	937.96	935.37	-0.28%	15.400	9.307	-39.56%	30.862	12.179	-60.54%
10	1108.08	1114.53	0.58%	1045.98	1041.76	-0.40%	20.553	17.135	-16.63%	29.515	19.597	-33.60%
Avg	1062.04	1062.18	0.26%	1062.14	1054.52	-0.69%	22.511	14.346	-30.24%	32.971	14.554	-50.45%
Rep No	Network Size: 200 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff	BS	S1	% Diff
1	1970.96	2051.04	4.06%	1810.27	1825.50	0.84%	909.88	316.81	-65.18%	1329.01	334.42	-74.84%
2	1964.22	2037.47	3.73%	1933.37	2030.14	5.01%	924.67	401.34	-56.60%	1082.80	298.74	-72.41%
3	1773.43	1781.84	0.47%	1719.08	1661.31	-3.36%	648.50	358.47	-44.72%	978.37	344.97	-64.74%

4	1630.35	1688.39	3.56%	1681.12	1735.92	3.26%	765.64	404.24	-47.20%	1041.05	293.53	-71.80%
5	1993.11	1927.41	-3.30%	1919.58	1916.74	-0.15%	564.47	341.58	-39.49%	880.56	330.48	-62.47%
6	1732.29	1623.91	-6.26%	1658.51	1664.41	0.36%	517.72	343.73	-33.61%	745.64	276.04	-62.98%
7	1750.43	1789.97	2.26%	1675.33	1721.66	2.77%	622.15	376.95	-39.41%	894.64	305.95	-65.80%
8	1712.29	1945.08	13.60%	1774.56	1844.62	3.95%	766.54	308.46	-59.76%	1046.06	320.54	-69.36%
9	1713.88	1870.82	9.16%	1759.45	1783.96	1.39%	770.36	215.94	-71.97%	902.42	242.19	-73.16%
10	1813.15	1812.85	-0.02%	1983.36	1885.19	-4.95%	777.48	493.18	-36.57%	929.77	403.28	-56.63%
<b>Avg</b>	<b>1805.41</b>	<b>1852.88</b>	<b>2.73%</b>	<b>1791.46</b>	<b>1806.95</b>	<b>0.91%</b>	<b>726.74</b>	<b>356.07</b>	<b>-49.45%</b>	<b>983.03</b>	<b>315.02</b>	<b>-67.42%</b>

## Appendix D.6. Changing the Size of BML - Detailed Results

Table 33. Changing the Size of BML - Detailed Results

Rep No	Network Size: 20 Customers					
	Total Cost (£)			Total CPU Time (sec)		
	S2	BS	S3	S2	BS	S3
1	373.37	321.513	373.37	0.041	0.043	0.043
2	354.09	314.433	354.09	0.05	0.059	0.052
3	220.18	202.021	220.18	0.048	0.05	0.051
4	266.04	273.699	266.04	0.077	0.079	0.079
5	280.21	299.937	280.21	0.042	0.045	0.049
6	322.29	309.725	322.29	0.074	0.081	0.079
7	198.25	211.524	198.25	0.064	0.07	0.073
8	221.93	217.488	221.93	0.024	0.027	0.035
9	359.57	363.232	359.57	0.014	0.016	0.018
10	247.94	248.213	247.94	0.035	0.042	0.034
<b>Avg</b>	<b>284.39</b>	<b>276.18</b>	<b>284.39</b>	<b>0.05</b>	<b>0.05</b>	<b>0.051</b>
Rep No	Network Size: 50 Customers					
	Total Cost (£)			Total CPU Time (sec)		
	S2	BS	S3	S2	BS	S3
1	517.44	517.44	517.44	1.885	2.007	2.038
2	600.47	600.474	600.47	0.992	0.978	1.000
3	555.79	546.118	546.12	1.652	2.254	2.336
4	618.25	667.058	667.06	1.774	1.307	1.339
5	624.83	624.827	624.83	0.953	0.973	0.998
6	483.73	483.731	483.73	0.82	0.885	0.902
7	542.37	542.365	542.37	0.542	0.565	0.582
8	534.16	509.491	509.49	1.695	2.016	2.080
9	600.35	600.345	600.35	1.099	1.194	1.192
10	560.60	546.933	546.93	2.574	2.612	2.678
<b>Avg</b>	<b>563.80</b>	<b>563.88</b>	<b>563.88</b>	<b>1.40</b>	<b>1.48</b>	<b>1.515</b>
Rep No	Network Size: 100 Customers					
	Total Cost (£)			Total CPU Time (sec)		
	S2	BS	S3	S2	BS	S3
1	1110.64	1080.7	1080.70	27.408	25.864	26.117
2	1211.57	1197.93	1197.93	15.697	14.942	15.352
3	1016.45	1041.04	1021.74	98.107	74.58	71.163
4	978.33	940.67	940.67	28.484	37.351	37.413
5	900.26	958.692	958.69	42.994	41.754	42.944
6	1056.54	1182.18	1182.18	32.557	25.062	25.373
7	1112.65	1103.78	1103.78	26.224	24.781	24.910
8	1094.49	1132.51	1132.51	25.046	24.995	25.296

9	935.17	937.961	937.96	26.809	30.862	31.106
10	1079.58	1045.98	1045.98	19.880	29.515	29.701
<b>Avg</b>	<b>1049.57</b>	<b>1062.14</b>	<b>1060.21</b>	<b>34.32</b>	<b>32.97</b>	<b>32.938</b>
<b>Rep No</b>	<b>Network Size: 200 Customers</b>					
	<b>Total Cost (£)</b>			<b>Total CPU Time (sec)</b>		
	<b>S2</b>	<b>BS</b>	<b>S3</b>	<b>S2</b>	<b>BS</b>	<b>S3</b>
1	1804.53	1810.27	1810.27	1065.60	1329.01	1339.72
2	1908.21	1933.37	1933.37	987.79	1082.80	1096.42
3	1734.84	1719.08	1719.08	1249.55	978.37	1002.29
4	1755.85	1681.12	1697.24	1034.59	1041.05	1126.02
5	1915.21	1919.58	1960.81	842.70	880.56	653.38
6	1719.42	1658.51	1669.11	679.46	745.64	967.26
7	1670.98	1675.33	1675.33	1040.23	894.64	926.34
8	1930.20	1774.56	1855.73	658.94	1046.06	798.02
9	1793.23	1759.45	1783.96	861.38	902.42	948.97
10	1920.93	1983.36	1983.36	843.55	929.77	936.46
<b>Avg</b>	<b>1815.34</b>	<b>1791.46</b>	<b>1808.83</b>	<b>926.38</b>	<b>983.03</b>	<b>979.49</b>

## Appendix D.7. Changing the Deadlines – Detailed Results

Table 34. Changing the Deadlines – Detailed Results

<b>Rep No</b>	<b>Network Size: 20 Customers</b>											
	<b>Total Cost (£)</b>						<b>GAP (%)</b>					
	<b>M1</b>			<b>M2</b>			<b>M1</b>			<b>M2</b>		
	<b>S4</b>	<b>BS</b>	<b>S5</b>	<b>S4</b>	<b>BS</b>	<b>S5</b>	<b>S4</b>	<b>BS</b>	<b>S5</b>	<b>S4</b>	<b>BS</b>	<b>S5</b>
1	417.8	353.0	303.5	429.1	372.4	339.5	74.1%	69.2%	64.6%	76.6%	73.0%	70.6%
2	364.1	370.3	310.3	407.3	372.2	330.7	69.4%	70.3%	64.9%	75.3%	73.0%	69.5%
3	209.7	195.6	195.0	202.0	217.5	203.2	67.4%	65.1%	65.2%	69.0%	71.2%	69.0%
4	381.5	288.4	273.2	313.7	307.1	300.1	74.6%	66.6%	64.4%	72.1%	71.6%	70.8%
5	269.7	256.6	259.4	298.9	274.0	282.1	65.7%	63.9%	63.9%	71.3%	68.8%	69.6%
6	309.0	307.3	295.0	360.9	326.8	313.5	66.8%	66.8%	65.5%	74.2%	71.4%	70.2%
7	217.7	204.3	202.4	224.5	226.0	228.5	66.1%	64.0%	63.7%	68.6%	68.8%	69.1%
8	262.1	259.6	258.5	277.9	241.1	244.3	69.5%	69.3%	69.4%	73.2%	69.1%	69.5%
9	331.8	379.7	309.8	359.7	351.0	306.8	64.2%	68.7%	62.0%	70.7%	69.6%	65.6%
10	292.0	270.0	260.4	284.0	278.0	276.8	70.2%	67.7%	66.6%	71.7%	71.0%	70.8%
<b>Avg</b>	<b>305.5</b>	<b>288.5</b>	<b>266.7</b>	<b>315.8</b>	<b>296.6</b>	<b>282.5</b>	<b>68.8%</b>	<b>67.2%</b>	<b>65.0%</b>	<b>72.3%</b>	<b>70.8%</b>	<b>69.5%</b>
<b>Rep No</b>	<b>Network Size: 50 Customers</b>											
	<b>Total Cost (£)</b>						<b>Total CPU Time (sec)</b>					
	<b>H1</b>			<b>H3</b>			<b>H1</b>			<b>H3</b>		
	<b>S4</b>	<b>BS</b>	<b>S5</b>	<b>S4</b>	<b>BS</b>	<b>S5</b>	<b>S4</b>	<b>BS</b>	<b>S5</b>	<b>S4</b>	<b>BS</b>	<b>S5</b>
1	347.7	336.6	303.7	347.7	321.5	303.7	0.03	0.04	0.06	0.03	0.04	0.08
2	356.9	314.4	290.7	333.7	314.4	290.7	0.04	0.04	0.04	0.04	0.06	0.05
3	207.4	201.4	197.4	207.4	202.0	197.4	0.01	0.04	0.02	0.01	0.05	0.02
4	297.0	294.9	286.8	297.0	273.7	286.8	0.05	0.08	0.05	0.04	0.08	0.06
5	308.0	299.9	276.6	284.7	299.9	276.6	0.06	0.04	0.03	0.06	0.05	0.04
6	309.6	309.7	302.6	309.6	309.7	302.6	0.03	0.06	0.08	0.03	0.08	0.09
7	210.3	211.5	220.3	212.8	211.5	220.3	0.05	0.06	0.02	0.07	0.07	0.02
8	245.9	217.5	217.5	220.3	217.5	217.5	0.02	0.02	0.03	0.07	0.03	0.04
9	337.6	363.7	361.6	380.3	363.2	361.6	0.07	0.01	0.02	0.06	0.02	0.03
10	278.6	248.2	261.0	278.6	248.2	261.0	0.04	0.03	0.03	0.05	0.04	0.05
<b>Avg</b>	<b>289.9</b>	<b>279.8</b>	<b>271.8</b>	<b>287.2</b>	<b>276.2</b>	<b>271.8</b>	<b>0.04</b>	<b>0.04</b>	<b>0.04</b>	<b>0.04</b>	<b>0.05</b>	<b>0.05</b>

Rep No	Network Size: 100 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S4	BS	S5	S4	BS	S5	S4	BS	S5	S4	BS	S5
1	1160.5	1125.8	1210.1	1217.1	1080.7	1207.3	19.13	20.39	16.41	17.34	25.86	25.33
2	1275.5	1199.6	1284.2	1261.2	1197.9	1220.1	19.06	13.46	12.98	15.71	14.94	17.03
3	1023.2	1008.8	974.8	1034.4	1041.0	1009.6	54.94	47.73	23.80	86.77	74.58	46.09
4	1041.5	911.7	1034.7	1063.8	940.7	970.2	24.45	22.11	19.20	29.66	37.35	24.74
5	972.7	934.3	971.9	925.5	958.7	959.4	15.67	31.03	18.74	24.89	41.75	29.51
6	1116.4	1220.1	1288.6	1089.6	1182.2	1237.2	15.32	16.19	12.50	18.08	25.06	22.21
7	1067.3	1156.2	1119.4	1050.3	1103.8	1118.3	13.78	18.16	8.13	18.90	24.78	10.45
8	1190.3	1015.6	1192.4	1204.9	1132.5	1067.7	13.88	20.08	18.12	17.78	25.00	25.15
9	940.7	940.3	939.9	1045.5	938.0	1002.0	15.34	15.40	15.55	18.89	30.86	16.25
10	1006.3	1108.1	1258.0	1064.5	1046.0	1148.2	24.37	20.55	9.71	21.90	29.52	20.64
Avg	<b>1079.4</b>	<b>1062.0</b>	<b>1127.4</b>	<b>1095.7</b>	<b>1062.1</b>	<b>1094.0</b>	<b>21.59</b>	<b>22.51</b>	<b>15.51</b>	<b>26.99</b>	<b>32.97</b>	<b>23.74</b>
Rep No	Network Size: 200 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S4	BS	S5	S4	BS	S5	S4	BS	S5	S4	BS	S5
1	1766.9	1971.0	1862.4	1899.7	1810.3	2028.8	776.2	909.9	511.3	916.7	1329.0	587.7
2	1952.8	1964.2	1942.2	2014.8	1933.4	1978.3	886.5	924.7	442.1	752.5	1082.8	600.7
3	1745.4	1773.4	1922.0	1710.4	1719.1	1751.2	744.2	648.5	379.4	865.6	978.4	606.8
4	1689.7	1630.4	1790.1	1628.0	1681.1	1770.7	535.8	765.6	420.5	781.3	1041.1	670.6
5	1907.1	1993.1	1934.6	1828.2	1919.6	1959.7	609.8	564.5	539.0	956.7	880.6	597.0
6	1714.5	1732.3	1841.6	1781.4	1658.5	1820.2	518.8	517.7	456.5	627.9	745.6	778.0
7	1693.3	1750.4	1741.1	1706.7	1675.3	1808.1	440.5	622.1	542.4	600.2	894.6	753.1
8	1807.9	1712.3	1774.8	1844.0	1774.6	1795.8	532.6	766.5	482.7	587.9	1046.1	607.1
9	1751.6	1713.9	1821.0	1735.5	1759.5	1789.9	526.5	770.4	441.5	649.5	902.4	568.1
10	1923.4	1813.2	1971.9	2058.1	1983.4	2021.0	692.3	777.5	466.7	664.1	929.8	861.6
Avg	<b>1795.3</b>	<b>1805.4</b>	<b>1860.2</b>	<b>1820.7</b>	<b>1791.5</b>	<b>1872.4</b>	<b>626.3</b>	<b>726.7</b>	<b>468.2</b>	<b>740.2</b>	<b>983.0</b>	<b>663.1</b>

## Appendix D.8. Number of Vehicles in the Fleet in BS and S6

Table 35. Number of Vehicles in the Fleet in BS and S6

Network Size	Vehicle Type	Number of Vehicles	
		BS	S6
20	1	2	5
	2	2	5
100	1	8	23
	2	8	23
200	1	15	45
	2	15	45

**Table 36. Changing Customer Demands – Detailed Results**

Rep No	Network Size: 20 Customers											
	Total Cost (£)						GAP (%)					
	M1			M2			M1			M2		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
1	-	353.0	219.9	-	372.4	245.2	-	69.2%	73.8%	-	73.0%	77.5%
2	597.3	370.3	219.6	615.6	372.2	245.1	46.2%	70.3%	72.1%	56.1%	73.0%	75.8%
3	-	195.6	122.9	-	217.5	136.2	-	65.1%	70.9%	-	71.2%	74.4%
4	487.6	288.4	214.6	505.1	307.1	239.2	45.2%	66.6%	73.2%	55.8%	71.6%	75.8%
5	524.9	256.6	189.3	524.3	274.0	198.2	48.5%	63.9%	72.5%	56.9%	68.8%	74.1%
6	517.8	307.3	211.4	523.6	326.8	259.1	46.1%	66.8%	69.7%	55.9%	71.4%	75.9%
7	460.2	204.3	101.5	453.8	226.0	103.8	44.9%	64.0%	68.6%	53.5%	68.8%	69.6%
8	439.6	259.6	163.1	482.5	241.1	169.7	46.2%	69.3%	71.3%	58.1%	69.1%	73.8%
9	-	379.7	224.3	-	351.0	330.5	-	68.7%	71.6%	-	69.6%	81.2%
10	532.8	270.0	154.4	548.2	278.0	206.3	46.3%	67.7%	72.2%	58.0%	71.0%	80.1%
<b>Avg</b>	<b>508.6</b>	<b>288.5</b>	<b>182.1</b>	<b>521.9</b>	<b>296.6</b>	<b>213.3</b>	<b>46.2%</b>	<b>67.2%</b>	<b>71.6%</b>	<b>56.3%</b>	<b>70.8%</b>	<b>75.8%</b>
Rep No	Network Size: 50 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
1	-	336.6	229.2	-	321.5	229.2	-	0.04	0.06	-	0.04	0.07
2	586.2	314.4	225.0	586.2	314.4	225.0	0.09	0.04	0.04	0.12	0.06	0.03
3	-	201.4	131.7	-	202.0	131.7	-	0.04	0.01	-	0.05	0.01
4	483.8	294.9	185.1	483.8	273.7	213.7	0.07	0.08	0.13	0.10	0.08	0.13
5	516.6	299.9	184.8	516.3	299.9	181.0	0.03	0.04	0.03	0.03	0.05	0.06
6	475.5	309.7	213.8	475.5	309.7	213.8	0.05	0.06	0.11	0.08	0.08	0.12
7	488.0	211.5	99.3	488.0	211.5	99.3	0.05	0.06	0.00	0.07	0.07	0.00
8	448.3	217.5	182.2	448.3	217.5	182.2	0.08	0.02	0.01	0.12	0.03	0.01
9	-	363.7	220.4	-	363.2	220.4	-	0.01	0.01	-	0.02	0.01
10	521.3	248.2	175.6	521.3	248.2	175.6	0.04	0.03	0.01	0.05	0.04	0.01
<b>Avg</b>	<b>502.8</b>	<b>279.8</b>	<b>184.7</b>	<b>502.8</b>	<b>276.2</b>	<b>187.2</b>	<b>0.06</b>	<b>0.04</b>	<b>0.04</b>	<b>0.08</b>	<b>0.05</b>	<b>0.04</b>
Rep No	Network Size: 100 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
1	2372.3	1125.8	612.5	2391.7	1080.7	623.1	17.1	20.4	54.5	25.5	25.9	60.7
2	-	1199.6	621.1	-	1197.9	621.1	-	13.5	38.9	-	14.9	36.2
3	2317.9	1008.8	532.7	2302.6	1041.0	571.2	23.1	47.7	120.7	34.4	74.6	140.6
4	1904.7	911.7	533.8	1942.5	940.7	533.8	18.8	22.1	55.8	24.5	37.4	45.0
5	1868.6	934.3	511.6	1851.0	958.7	531.3	20.6	31.0	78.8	27.3	41.8	63.4
6	2248.8	1220.1	565.6	2304.8	1182.2	564.1	12.0	16.2	107.6	16.4	25.1	93.9
7	2157.5	1156.2	552.5	2129.3	1103.8	536.4	11.6	18.2	58.5	17.8	24.8	68.3
8	2300.2	1015.6	564.6	2310.8	1132.5	567.0	12.8	20.1	67.4	17.9	25.0	76.8
9	1928.3	940.3	549.9	1959.8	938.0	549.9	14.4	15.4	14.9	16.5	30.9	15.8
10	2191.4	1108.1	643.9	2146.3	1046.0	651.3	10.8	20.6	74.5	14.1	29.5	61.6
<b>Avg</b>	<b>2143.3</b>	<b>1062.0</b>	<b>568.8</b>	<b>2148.8</b>	<b>1062.1</b>	<b>574.9</b>	<b>15.69</b>	<b>22.51</b>	<b>67.16</b>	<b>21.61</b>	<b>32.97</b>	<b>66.23</b>
Rep No	Network Size: 200 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S6	BS	S7	S6	BS	S7	S6	BS	S7	S6	BS	S7
1	4047.2	1971.0	796.5	3997.5	1810.3	768.0	508.9	909.9	2639.8	697.9	1329.0	2840.2
2	4215.1	1964.2	926.9	4191.5	1933.4	922.6	406.6	924.7	2746.2	591.2	1082.8	2060.3
3	3871.1	1773.4	1035.2	3839.6	1719.1	1013.9	466.8	648.5	4166.8	613.4	978.4	3027.8
4	3601.0	1630.4	762.4	3760.7	1681.1	765.9	551.5	765.6	4474.8	504.9	1041.1	2985.3

5	4398.9	1993.1	909.0	4335.2	1919.6	929.1	399.3	564.5	3478.2	504.1	880.6	2321.0
6	3605.2	1732.3	764.0	3587.8	1658.5	761.5	506.1	517.7	2743.3	647.1	745.6	2508.8
7	3810.3	1750.4	956.9	3815.3	1675.3	957.4	355.5	622.1	2196.1	427.9	894.6	1634.4
8	4077.2	1712.3	782.6	4093.9	1774.6	816.5	469.2	766.5	4084.4	547.8	1046.1	3013.2
9	3720.3	1713.9	755.2	3730.8	1759.5	778.2	573.7	770.4	3991.3	594.0	902.4	4967.3
10	4430.3	1813.2	879.5	4346.3	1983.4	963.9	364.1	777.5	4040.4	537.8	929.8	2299.0
Avg	<b>3977.7</b>	<b>1805.4</b>	<b>856.8</b>	<b>3969.9</b>	<b>1791.5</b>	<b>867.7</b>	<b>460.2</b>	<b>726.7</b>	<b>3456.1</b>	<b>566.6</b>	<b>983.0</b>	<b>2765.7</b>

## Appendix D.9. Changing Driver Wages, Unit Fuel and Emission Costs – Detailed Results

Table 37. Changing Driver Wages, Unit Fuel and Emission Costs – Detailed Results

Rep No	Network Size: 20 Customers											
	Total Cost (£)						GAP (%)					
	M1			M2			M1			M2		
	BS	S8	S9	BS	S8	S9	BS	S8	S9	BS	S8	S9
1	353.0	428.5	487.9	372.4	538.7	528.4	69.2%	73.8%	57.0%	73.0%	81.3%	62.7%
2	370.3	492.4	495.7	372.2	482.8	535.6	70.3%	76.5%	57.1%	73.0%	79.2%	62.3%
3	195.6	284.1	297.9	217.5	275.2	286.8	65.1%	74.8%	55.6%	71.2%	76.9%	56.5%
4	288.4	395.6	444.1	307.1	421.4	465.2	66.6%	74.2%	57.8%	71.6%	79.2%	62.5%
5	256.6	382.8	374.0	274.0	400.1	427.6	63.9%	74.9%	51.0%	68.8%	78.6%	60.0%
6	307.3	440.1	501.0	326.8	446.0	465.5	66.8%	75.8%	60.2%	71.4%	79.0%	59.9%
7	204.3	305.5	312.2	226.0	329.3	323.6	64.0%	75.3%	53.4%	68.8%	78.6%	56.5%
8	259.6	324.8	410.1	241.1	365.6	339.7	69.3%	74.5%	62.0%	69.1%	79.7%	56.1%
9	379.7	489.0	494.8	351.0	487.3	526.7	68.7%	74.4%	54.2%	69.6%	78.1%	59.6%
10	270.0	373.7	401.8	278.0	390.0	420.3	67.7%	75.3%	57.7%	71.0%	79.3%	61.6%
Avg	<b>288.5</b>	<b>391.7</b>	<b>421.9</b>	<b>296.6</b>	<b>413.6</b>	<b>432.0</b>	<b>67.2%</b>	<b>74.9%</b>	<b>56.6%</b>	<b>70.8%</b>	<b>79.0%</b>	<b>59.8%</b>
Rep No	Network Size: 50 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S8	S9	BS	S8	S9	BS	S8	S9	BS	S8	S9
1	336.6	416.1	480.4	321.5	420.3	481.6	0.04	0.08	0.06	0.04	0.08	0.07
2	314.4	440.4	411.8	314.4	486.8	411.8	0.04	0.09	0.03	0.06	0.06	0.03
3	201.4	280.0	313.1	202.0	280.9	313.1	0.04	0.02	0.01	0.05	0.04	0.01
4	294.9	368.9	399.9	273.7	378.0	420.9	0.08	0.07	0.10	0.08	0.07	0.09
5	299.9	408.5	508.4	299.9	408.5	417.4	0.04	0.03	0.05	0.05	0.05	0.13
6	309.7	424.6	473.5	309.7	424.6	473.5	0.06	0.03	0.01	0.08	0.05	0.00
7	211.5	302.3	309.6	211.5	302.3	309.6	0.06	0.05	0.05	0.07	0.09	0.08
8	217.5	310.9	356.4	217.5	310.9	356.4	0.02	0.04	0.02	0.03	0.06	0.03
9	363.7	489.4	543.8	363.2	489.4	543.8	0.01	0.03	0.06	0.02	0.04	0.04
10	248.2	352.2	368.9	248.2	352.2	368.9	0.03	0.09	0.10	0.04	0.11	0.11
Avg	<b>279.8</b>	<b>379.3</b>	<b>416.6</b>	<b>276.2</b>	<b>385.4</b>	<b>409.7</b>	<b>0.04</b>	<b>0.05</b>	<b>0.05</b>	<b>0.05</b>	<b>0.06</b>	<b>0.06</b>
Rep No	Network Size: 100 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S8	S9	BS	S8	S9	BS	S8	S9	BS	S8	S9
1	1125.8	1729.8	1755.8	1080.7	1710.6	1591.3	20.39	16.10	16.80	25.86	20.30	21.40
2	1199.6	1732.1	1876.6	1197.9	1722.1	1824.7	13.46	23.79	12.38	14.94	23.31	12.85
3	1008.8	1406.9	1571.4	1041.0	1467.0	1391.1	47.73	32.61	23.61	74.58	47.94	38.83
4	911.7	1430.0	1456.2	940.7	1473.8	1380.7	22.11	17.77	10.50	37.35	20.77	23.18
5	934.3	1304.5	1333.4	958.7	1329.5	1365.0	31.03	27.18	16.88	41.75	33.84	18.67

6	1220.1	1635.5	1596.0	1182.2	1587.5	1717.8	16.19	25.13	23.20	25.06	27.34	18.10
7	1156.2	1567.7	1459.8	1103.8	1463.0	1473.6	18.16	15.83	17.18	24.78	26.00	15.72
8	1015.6	1637.6	1616.6	1132.5	1603.2	1547.8	20.08	14.08	18.40	25.00	16.49	15.15
9	940.3	1429.6	1386.1	938.0	1287.1	1416.9	15.40	13.42	15.67	30.86	24.00	15.94
10	1108.1	1526.8	1801.0	1046.0	1487.3	2014.4	20.55	18.26	11.88	29.52	22.19	8.24
Avg	<b>1062.0</b>	<b>1540.1</b>	<b>1585.3</b>	<b>1062.1</b>	<b>1513.1</b>	<b>1572.3</b>	<b>22.51</b>	<b>20.42</b>	<b>16.65</b>	<b>32.97</b>	<b>26.22</b>	<b>18.81</b>
Rep No	Network Size: 200 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	BS	S8	S9	BS	S8	S9	BS	S8	S9	BS	S8	S9
1	1971.0	2594.0	2759.3	1810.3	2625.8	2717.2	909.9	603.0	534.9	1329.0	827.7	535.6
2	1964.2	2682.4	2818.9	1933.4	2499.2	2863.7	924.7	727.5	470.3	1082.8	1104.8	505.2
3	1773.4	2506.0	2772.2	1719.1	2392.2	2722.1	648.5	555.8	504.2	978.4	824.5	575.5
4	1630.4	2312.0	2433.1	1681.1	2461.7	2643.7	765.6	580.0	541.3	1041.1	654.5	427.7
5	1993.1	2643.0	2855.6	1919.6	2762.8	2910.3	564.5	543.7	418.8	880.6	622.1	434.6
6	1732.3	2421.6	2550.6	1658.5	2461.4	2566.0	517.7	485.4	281.3	745.6	717.5	370.4
7	1750.4	2434.3	2493.0	1675.3	2488.1	2490.8	622.1	537.8	355.5	894.6	625.9	396.2
8	1712.3	2590.0	2673.3	1774.6	2789.4	2723.9	766.5	521.9	405.2	1046.1	550.4	428.3
9	1713.9	2491.9	2601.7	1759.5	2570.2	2585.3	770.4	425.8	462.8	902.4	561.5	426.2
10	1813.2	2603.1	2767.9	1983.4	2647.5	2856.6	777.5	701.1	456.5	929.8	936.6	462.2
Avg	<b>1805.4</b>	<b>2527.8</b>	<b>2672.6</b>	<b>1791.5</b>	<b>2569.8</b>	<b>2708.0</b>	<b>726.7</b>	<b>568.2</b>	<b>443.1</b>	<b>983.0</b>	<b>742.6</b>	<b>456.2</b>

## Appendix D.10. Changing the Fleet Size – Detailed Results

Table 38. Changing the Fleet Size – Detailed Results

Rep No	Network Size: 20 Customers											
	Total Cost (£)						GAP (%)					
	M1			M2			M1			M2		
	S10	BS	S11	S10	BS	S11	S10	BS	S11	S10	BS	S11
1	511.9	353.0	293.1	453.0	372.4	316.0	78.5%	69.2%	62.5%	77.6%	73.0%	68.8%
2	400.2	370.3	295.5	359.8	372.2	322.8	72.7%	70.3%	62.4%	71.9%	73.0%	68.8%
3	217.0	195.6	184.8	192.5	217.5	181.1	68.6%	65.1%	63.2%	67.1%	71.2%	65.4%
4	326.9	288.4	275.8	333.0	307.1	289.0	70.4%	66.6%	65.0%	73.2%	71.6%	69.9%
5	314.5	256.6	267.5	287.4	274.0	265.0	70.3%	63.9%	65.0%	69.6%	68.8%	67.8%
6	333.5	307.3	284.7	405.6	326.8	305.3	69.3%	66.8%	64.5%	76.6%	71.4%	69.5%
7	215.6	204.3	218.7	236.5	226.0	214.0	65.7%	64.0%	66.3%	70.1%	68.8%	66.9%
8	299.5	259.6	222.6	299.0	241.1	230.7	73.5%	69.3%	64.2%	75.0%	69.1%	67.7%
9	416.5	379.7	305.3	349.8	351.0	306.1	71.4%	68.7%	61.3%	69.2%	69.6%	65.8%
10	323.3	270.0	255.6	323.8	278.0	265.2	72.9%	67.7%	65.8%	75.1%	71.0%	69.6%
Avg	<b>335.9</b>	<b>288.5</b>	<b>260.4</b>	<b>324.0</b>	<b>296.6</b>	<b>269.5</b>	<b>71.3%</b>	<b>67.2%</b>	<b>64.0%</b>	<b>72.5%</b>	<b>70.8%</b>	<b>68.0%</b>
Rep No	Network Size: 50 Customers											
	Total Cost (£)						Total CPU Time (sec)					
	H1			H3			H1			H3		
	S10	BS	S11	S10	BS	S11	S10	BS	S11	S10	BS	S11
1	373.4	336.6	315.4	373.4	321.5	315.4	0.07	0.04	0.05	0.05	0.04	0.08
2	354.1	314.4	328.4	354.1	314.4	300.8	0.05	0.04	0.06	0.05	0.06	0.08
3	220.2	201.4	188.6	220.2	202.0	188.6	0.05	0.04	0.04	0.05	0.05	0.05
4	266.0	294.9	261.0	266.0	273.7	287.0	0.08	0.08	0.13	0.08	0.08	0.11
5	286.1	299.9	306.1	280.2	299.9	274.6	0.04	0.04	0.05	0.05	0.05	0.11
6	326.4	309.7	299.1	322.3	309.7	299.1	0.05	0.06	0.04	0.08	0.08	0.05
7	198.2	211.5	222.9	198.2	211.5	222.9	0.05	0.06	0.04	0.07	0.07	0.05



<b>8</b>	221.9	217.5	211.1	221.9	217.5	211.1	0.02	0.02	0.08	0.03	0.03	0.11
<b>9</b>	359.6	363.7	351.2	359.6	363.2	351.3	0.01	0.01	0.07	0.02	0.02	0.07
<b>10</b>	247.9	248.2	263.6	247.9	248.2	263.6	0.03	0.03	0.07	0.04	0.04	0.09
<b>Avg</b>	<b>285.4</b>	<b>279.8</b>	<b>274.7</b>	<b>284.4</b>	<b>276.2</b>	<b>271.4</b>	<b>0.05</b>	<b>0.04</b>	<b>0.06</b>	<b>0.05</b>	<b>0.05</b>	<b>0.08</b>
<b>Rep No</b>	<b>Network Size: 100 Customers</b>											
	<b>Total Cost (£)</b>						<b>Total CPU Time (sec)</b>					
	<b>H1</b>			<b>H3</b>			<b>H1</b>			<b>H3</b>		
	<b>S10</b>	<b>BS</b>	<b>S11</b>	<b>S10</b>	<b>BS</b>	<b>S11</b>	<b>S10</b>	<b>BS</b>	<b>S11</b>	<b>S10</b>	<b>BS</b>	<b>S11</b>
<b>1</b>	1227.3	1125.8	1227.9	1211.4	1080.7	1199.5	26.30	20.39	68.68	35.90	25.86	102.33
<b>2</b>	-	-	-	-	-	-	-	-	-	-	-	-
<b>3</b>	966.1	1008.8	1040.7	1100.7	1041.0	1080.9	60.65	47.73	90.07	50.64	74.58	135.15
<b>4</b>	1004.8	911.7	943.6	868.1	940.7	938.2	20.07	22.11	79.83	40.97	37.35	130.09
<b>5</b>	879.8	934.3	898.6	908.3	958.7	861.0	42.74	31.03	92.00	47.67	41.75	152.86
<b>6</b>	1241.1	1220.1	1075.8	1242.3	1182.2	1119.3	19.28	16.19	52.09	19.33	25.06	60.67
<b>7</b>	1116.2	1156.2	1078.7	955.8	1103.8	1109.9	16.69	18.16	42.35	31.03	24.78	68.78
<b>8</b>	1037.6	1015.6	1083.1	1164.1	1132.5	1125.3	21.49	20.08	50.24	11.25	25.00	65.14
<b>9</b>	942.4	940.3	921.5	949.3	938.0	910.1	15.64	15.40	63.95	30.45	30.86	84.87
<b>10</b>	1196.1	1108.1	1070.0	1114.5	1046.0	1175.9	13.74	20.55	43.93	16.95	29.52	46.81
<b>Avg</b>	<b>1068.0</b>	<b>1046.8</b>	<b>1037.8</b>	<b>1057.2</b>	<b>1047.1</b>	<b>1057.8</b>	<b>26.29</b>	<b>23.52</b>	<b>64.79</b>	<b>31.58</b>	<b>34.97</b>	<b>94.08</b>
<b>Rep No</b>	<b>Network Size: 200 Customers</b>											
	<b>Total Cost (£)</b>						<b>Total CPU Time (sec)</b>					
	<b>H1</b>			<b>H3</b>			<b>H1</b>			<b>H3</b>		
	<b>S10</b>	<b>BS</b>	<b>S11</b>	<b>S10</b>	<b>BS</b>	<b>S11</b>	<b>S10</b>	<b>BS</b>	<b>S11</b>	<b>S10</b>	<b>BS</b>	<b>S11</b>
<b>1</b>	1898.6	1971.0	1879.7	1835.7	1810.3	1850.9	607.0	909.9	1782.8	1094.5	1329.0	2652.2
<b>2</b>	1906.5	1964.2	2042.5	1866.7	1933.4	1945.3	963.9	924.7	1575.2	1228.8	1082.8	2576.7
<b>3</b>	1745.1	1773.4	1825.0	1688.0	1719.1	1876.0	845.2	648.5	1214.5	1350.7	978.4	1518.4
<b>4</b>	1612.5	1630.4	1684.4	1675.2	1681.1	1649.7	839.4	765.6	2094.6	731.6	1041.1	2171.5
<b>5</b>	1936.5	1993.1	1912.2	1989.2	1919.6	1980.2	843.4	564.5	1040.0	806.5	880.6	1246.0
<b>6</b>	1705.4	1732.3	1793.0	1761.8	1658.5	1819.1	612.3	517.7	1736.2	744.3	745.6	2134.3
<b>7</b>	1674.8	1750.4	1662.8	1780.6	1675.3	1756.1	591.3	622.1	2214.7	707.3	894.6	2033.7
<b>8</b>	1963.5	1712.3	1855.2	1972.4	1774.6	1837.1	591.6	766.5	1224.1	855.6	1046.1	1460.6
<b>9</b>	1735.8	1713.9	1785.6	1642.7	1759.5	1790.5	645.4	770.4	1457.8	1091.3	902.4	2057.5
<b>10</b>	1921.2	1813.2	1999.0	1901.5	1983.4	1977.8	734.1	777.5	1433.9	849.0	929.8	1626.7
<b>Avg</b>	<b>1810.0</b>	<b>1805.4</b>	<b>1843.9</b>	<b>1811.4</b>	<b>1791.5</b>	<b>1848.3</b>	<b>727.4</b>	<b>726.7</b>	<b>1577.4</b>	<b>946.0</b>	<b>983.0</b>	<b>1947.8</b>

## Appendix D.11. Fast Solution Heuristic – Detailed Results

Table 39. Fast Solution Heuristic – Detailed Results

Rep No	10 Customers		15 Customers		20 Customers	
	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
<b>1</b>	145.92	0.03	277.24	0.05	326.80	0.08
<b>2</b>	210.49	0.02	213.07	0.02	354.53	0.08
<b>3</b>	174.76	0.04	296.69	0.07	207.29	0.05
<b>4</b>	167.66	0.02	250.98	0.02	299.88	0.07
<b>5</b>	158.69	0.01	338.17	0.05	311.94	0.08
<b>6</b>	206.67	0.03	219.20	0.04	310.16	0.02
<b>7</b>	211.95	0.02	224.66	0.03	221.63	0.06
<b>8</b>	219.20	0.02	164.21	0.04	248.76	0.05
<b>9</b>	156.83	0.06	245.27	0.06	371.17	0.06
<b>10</b>	169.44	0.02	233.33	0.01	266.66	0.09
<b>Avg</b>	<b>182.16</b>	<b>0.03</b>	<b>246.28</b>	<b>0.04</b>	<b>291.88</b>	<b>0.06</b>

	25 Customers		50 Customers		75 Customers	
Rep No	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
1	335.51	0.09	594.76	0.38	1117.31	1.23
2	323.37	0.11	707.01	0.28	882.97	0.95
3	201.72	0.08	678.83	0.47	1052.16	1.18
4	287.08	0.10	748.03	0.31	807.37	1.07
5	317.51	0.07	736.22	0.26	984.47	1.00
6	327.63	0.04	518.79	0.38	1195.19	1.26
7	306.00	0.11	586.44	0.26	1032.21	1.05
8	411.32	0.03	644.02	0.43	985.98	1.26
9	268.43	0.08	796.39	0.25	978.28	1.22
10	333.40	0.13	643.25	0.36	1051.43	1.03
<b>Avg</b>	<b>311.20</b>	<b>0.08</b>	<b>665.37</b>	<b>0.34</b>	<b>1008.74</b>	<b>1.12</b>
	100 Customers		150 Customers		200 Customers	
Rep No	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)	Total Cost (£)	Total CPU Time (sec)
1	1365.31	2.99	1549.26	14.13	2194.88	41.98
2	1397.87	2.37	1896.12	12.75	2355.01	43.43
3	1185.21	3.02	1646.12	13.10	2112.69	43.02
4	1085.26	2.87	1834.91	12.52	2129.87	45.29
5	1101.75	3.43	1598.91	15.20	2399.03	41.81
6	1383.01	2.72	1613.32	15.11	2075.43	40.82
7	1312.23	3.76	1838.77	16.09	2194.58	44.97
8	1307.33	3.04	1741.60	16.17	2184.88	44.38
9	1108.35	2.55	2155.21	13.81	2260.89	42.07
10	1418.42	2.98	1801.58	16.19	2295.08	46.92
<b>Avg</b>	<b>1266.47</b>	<b>2.97</b>	<b>1767.58</b>	<b>14.51</b>	<b>2220.23</b>	<b>43.47</b>

## Appendix D.12. Distance Minimizing Heuristic - Detailed Results

Table 40. Distance Minimizing Heuristic - Detailed Results

Network Size - 10 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	145.45	148.73	430.56	440.54
2	210.49	210.49	603.62	603.62
3	170.57	170.57	478.02	478.02
4	167.66	190.60	480.09	479.96
5	158.69	151.80	483.18	442.07
6	202.83	203.91	577.64	572.91
7	202.59	217.94	570.68	570.70
8	195.39	196.59	567.80	567.85
9	166.74	167.77	467.05	466.72
10	169.25	169.25	483.18	483.18
<b>Avg</b>	<b>178.97</b>	<b>182.76</b>	<b>514.18</b>	<b>510.56</b>

Network Size - 15 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	249.93	254.08	771.00	788.93
2	212.60	251.66	665.04	771.89
3	274.21	250.26	837.99	744.25
4	250.98	278.98	785.54	824.50
5	313.86	305.96	910.03	798.02
6	202.58	207.64	651.01	650.91
7	220.96	224.59	722.13	697.24
8	157.69	143.44	483.54	450.63
9	238.31	240.42	746.04	711.16
10	211.73	217.58	661.53	679.03
<b>Avg</b>	<b>233.28</b>	<b>237.46</b>	<b>723.38</b>	<b>711.66</b>

Network Size - 20 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	329.05	312.97	938.74	866.75
2	314.43	375.84	904.19	932.14
3	201.73	198.32	570.98	567.55
4	284.29	271.34	848.43	747.28
5	299.94	276.35	808.18	727.95
6	309.73	309.16	922.96	876.77
7	211.52	213.39	643.21	646.47
8	217.49	219.81	657.62	654.88
9	363.46	345.57	989.65	847.99
10	248.21	266.74	729.29	724.68
<b>Avg</b>	<b>277.99</b>	<b>278.95</b>	<b>801.32</b>	<b>759.25</b>

Network Size - 25 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	291.97	361.12	825.90	840.60
2	314.07	317.22	885.13	844.72
3	178.07	180.11	514.64	513.92
4	282.54	261.03	848.35	733.58
5	281.86	292.34	796.89	811.98
6	327.63	248.58	950.80	721.59
7	306.01	322.30	872.69	853.08
8	333.99	360.08	974.28	980.30
9	255.32	264.37	720.40	724.67
10	314.88	330.08	889.51	874.26
<b>Avg</b>	<b>288.63</b>	<b>293.72</b>	<b>827.86</b>	<b>789.87</b>

Network Size - 50 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	515.21	659.96	1464.51	1698.49
2	635.42	684.45	1784.03	1778.84
3	537.13	544.20	1534.23	1509.21
4	658.97	646.56	1826.21	1739.28
5	652.78	713.45	1820.03	1941.76
6	484.31	488.57	1419.84	1336.16
7	522.94	622.38	1517.17	1787.44
8	532.67	607.73	1510.62	1637.90
9	609.05	702.74	1703.09	1833.87
10	590.65	614.65	1709.25	1717.88
<b>Avg</b>	<b>573.91</b>	<b>628.47</b>	<b>1628.90</b>	<b>1698.08</b>

Network Size - 75 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	967.46	895.79	2607.10	2355.21
2	735.49	860.93	2041.03	2238.74
3	943.74	1034.11	2479.15	2577.37
4	660.07	732.23	1870.30	2032.54
5	847.31	865.26	2407.98	2363.47
6	964.07	880.07	2571.16	2237.06
7	798.21	842.31	2238.10	2163.93
8	905.93	999.42	2473.51	2382.68
9	937.52	1017.16	2585.86	2653.50
10	918.59	1008.32	2501.69	2511.67
<b>Avg</b>	<b>867.84</b>	<b>913.56</b>	<b>2377.59</b>	<b>2351.62</b>

Network Size - 100 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	1103.26	1210.98	3137.63	3313.76
2	1198.77	1532.71	3361.57	4033.83
3	1024.91	978.93	2890.73	2750.05
4	926.20	1142.25	2700.89	3032.89
5	946.48	974.57	2769.04	2601.19
6	1201.13	1131.01	3390.35	3066.21
7	1129.97	1081.46	3231.36	3027.73
8	1074.05	1240.27	3077.45	3297.61
9	939.12	868.19	2736.97	2459.65
10	1077.03	1280.91	3095.81	3520.45
<b>Avg</b>	<b>1062.09</b>	<b>1144.13</b>	<b>3039.18</b>	<b>3110.34</b>

Network Size - 150 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	1251.07	1408.77	3574.80	3831.44
2	1556.19	1618.88	4201.75	4296.89
3	1313.76	1394.42	3695.45	3767.18
4	1482.08	1440.15	4116.74	3855.54
5	1407.05	1391.65	4018.87	3915.88
6	1293.06	1301.14	3614.02	3621.12
7	1536.29	1459.00	4270.16	3933.34
8	1315.70	1412.23	3691.41	3846.52
9	1679.19	1612.76	4589.26	4312.48
10	1512.77	1606.57	4228.80	4221.12
<b>Avg</b>	<b>1434.71</b>	<b>1464.56</b>	<b>4000.12</b>	<b>3960.15</b>

Network Size - 200 Cities				
Rep No	Total Cost (£)		Total Distance (km)	
	Average of H1 and H3	DMH	Average of H1 and H3	DMH
1	1890.62	1924.57	5383.91	5312.93
2	1948.80	1793.03	5416.95	4926.25
3	1746.26	1728.50	4998.89	4832.58
4	1655.74	1829.50	4849.15	5056.41
5	1956.35	1905.82	5568.50	5309.33
6	1695.40	1773.25	4852.56	5057.58
7	1712.88	1896.65	4970.22	5220.98
8	1743.43	1896.08	4982.20	5289.92
9	1736.67	1873.90	4996.03	5190.68
10	1898.26	1880.48	5303.32	5230.08
<b>Avg</b>	<b>1798.44</b>	<b>1850.18</b>	<b>5132.17</b>	<b>5142.67</b>

## **APPENDIX E**

### **(RE)DESIGN OF A MULTI-DEPOT DISTRIBUTION SYSTEM**

#### **Abstract**

In this study, we will investigate the improvements that can be achieved via multi-depot (re)design of the Dutch distribution system of Kuehne + Nagel (K+N), a 3PL company who provides integrated services including all aspects of logistics planning, control and execution. Currently, the planning of both forward and reverse logistics activities is done manually by planners of K+N on site. K+N Netherlands would like to employ a centralized and automatic system to generate distribution plans. It is predicted that, this will result in higher efficiency, especially in terms of the total kilometers traveled, total number of vehicles used, utilization of vehicles and the percentage of loaded kilometers traveled by each vehicle. These improvements may lead to significant amount of cost savings for the company, which renders the problem worth investigating.

## **Appendix E.1. Third-Party Logistics**

Accounting for 10-15% of the final cost of a finished product, logistics costs constitute a significant portion of total costs in many industries. Therefore, it is possible to make major cost reductions by reducing logistics-related costs. Firms have several options to perform their logistics functions in an efficient manner. One way is to outsource these functions, which is referred to as “Third-party Logistics” (3PL) or “Contract Logistics”. Lieb et al. [87] define 3PL as “*the use of external companies to perform logistics functions that have traditionally been performed within an organization*”. There are several reasons why utilization of a 3PL provider would be beneficial for a firm. Nemoto & Tezuka [88] suggest that using 3PL is advantageous in terms of benefiting from economies of scale and economies of scope, which would result in an increase in the net value of the firm via cost reduction, and a decrease in their financial risks by saving on capital investments. Skjoett-Larsen [89] claims that 3PL arrangements also increase market coverage, improve service levels and provide greater flexibility. Some 3PL companies offer complete supply chain solutions on warehousing, order fulfillment, and especially value-added services such as repackaging, re-labeling, assembly, light manufacturing and repair. Moreover, it creates an opportunity for the firms to get into reverse logistics programs without interrupting forward flows [90].

Kuehne + Nagel Netherlands has a contract logistics branch which performs 3PL logistics activities for a variety of business sectors. The project environment basically consists of this branch, and all the analyses and efforts for improvement are carried out in this area.

The rest of this report consists of a company description, current system analysis, the formal definition of the research problem and evaluation of extension possibilities.

## **Appendix E.2. Kuehne + Nagel (K+N)**

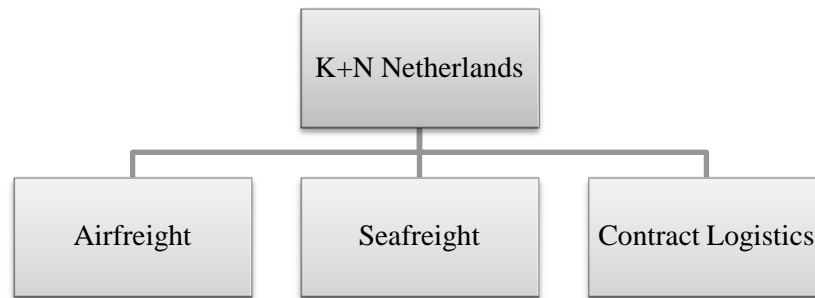
Kuehne + Nagel, one of the world’s leading logistics providers, was founded by August Kuehne and Friedrich Nagel in Bremen, Germany, in 1890. Today, K+N Group has more than 900 offices in over 100 countries, with over 60,000 employees. K+N offices are mainly located in Africa, Asia Pacific, Europe, Middle East, and North America. The capabilities of the company are:

- Seafreight
- Airfreight
- Contract Logistics & Lead Logistics
- Road & Rail Logistics

K+N is the number one seafreight forwarder in the world. It is among the global top 3 companies in air cargo forwarding and contract logistics, and European top 6 companies in road and rail logistics. The company provides logistics services to virtually all key industry sectors including aerospace, automotive, FMCG, high technology, industrials, oil and gas logistics, pharma and healthcare and retail [91].

### **Appendix E.2.1. K+N Netherlands**

Kuehne + Nagel has been present in the Netherlands since 1955 and currently employs 2500 staff at 27 locations. Key business activities of K+N within the Netherlands are as follows:



**Figure 26. Key Business Activities of K+N in the Netherlands**

Both in sea- and airfreight, Kuehne + Nagel ranks among the country's top three providers, and its more than 500,000 m<sup>2</sup> of warehousing space rank it among the top five players in contract logistics. [92]

*K+N Netherlands Contract Logistics Branch:*

The contract logistics (CL) branch of K+N Netherlands consists of 4 business units, namely:

- Fast Moving Consumer Goods (FMCG)
- Technology Solutions
- Returns
- Transport Nederland

**FMCG:** FMCG is the biggest division of contract logistics within the country and K+N Netherlands plays an important role in this market. K+N is responsible from both storage of retail goods and distribution of them to retail distribution centers. The following facilities are allocated to this business unit:

- Ede
- Nieuwegein
- Oud-Beijerland
- Raamsdonksveer
- Utrecht
- Veghel

**Technology Solutions:** Having a total warehousing surface of more than 100,000 m<sup>2</sup> and over 300 employees, Technology Solutions Business Unit specializes in after sales market in terms of both 'reverse logistics' and 'service logistics'. Products such as engines, small spare parts, complete communication systems, printers, are stored within the warehouses in Born, Helmond, Moerdijk, Tiel, Wijchen and Zoetermeer.

**Returns:** At the moment, this Business Unit consists of six return centers. These centers are responsible for the assimilation of all return goods, especially returning from the 800 Albert Heijn stores in the Netherlands. The most important return goods are boxes, packaging and charge carriers. About 1,000,000 items per week are being returned. The return centers are located in:

- Pijnacker

- 's-Hertogenbosch
- Tilburg
- Vaassen
- Zaandam
- Zwolle

**Transport Nederland:** This is the newest Business unit in the contract logistics division of K+N Netherlands. It is one of the few asset-based transport organizations in K+N Global and has the responsibility over 160 vehicles, both owned and subcontracted.

In total, there are 18 warehouses and return centers utilized by K+N Netherlands CL. These facilities and customers served from each facility can be seen on the map in Figure 27.

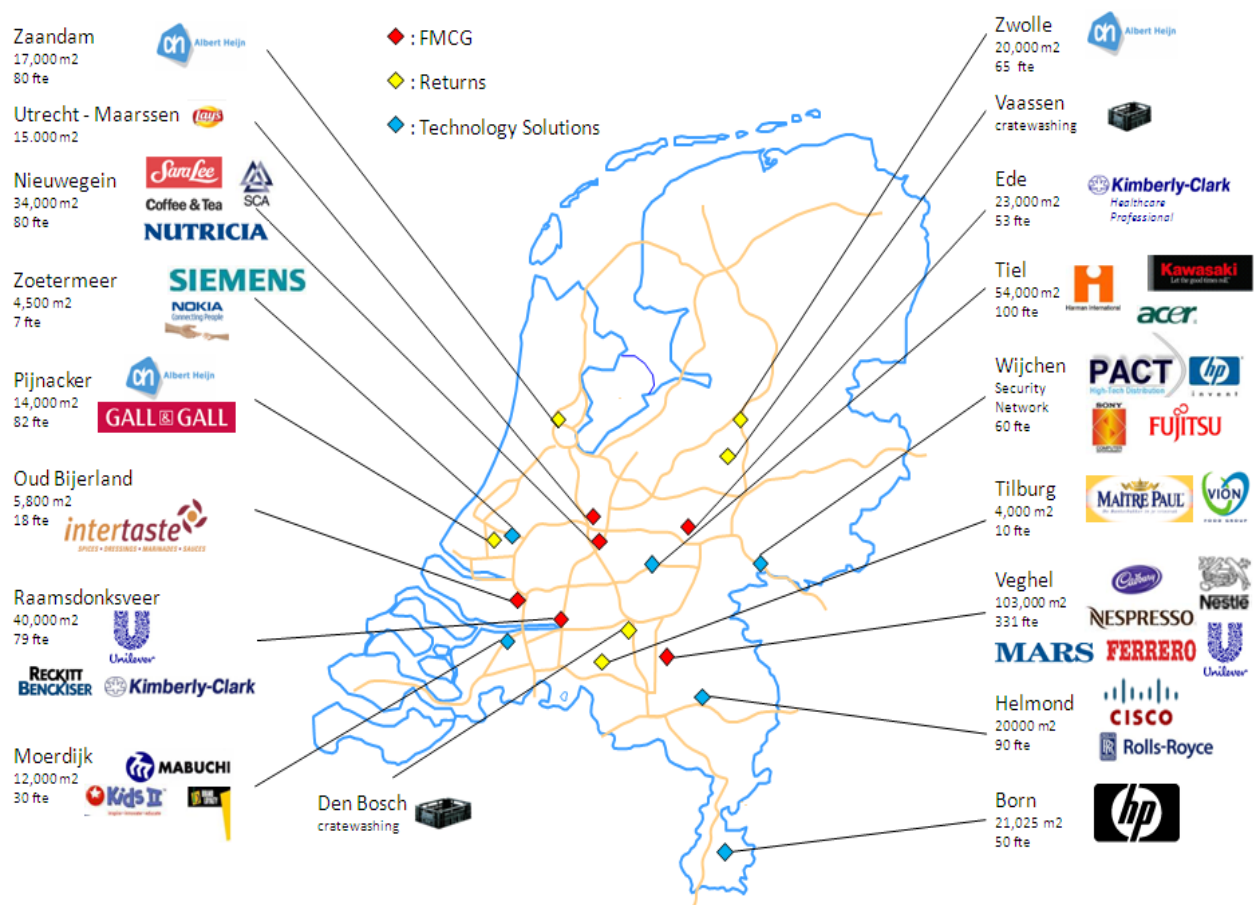


Figure 27. K+N Netherlands - Contract Logistics

### Appendix E.3. Analysis of the Current System

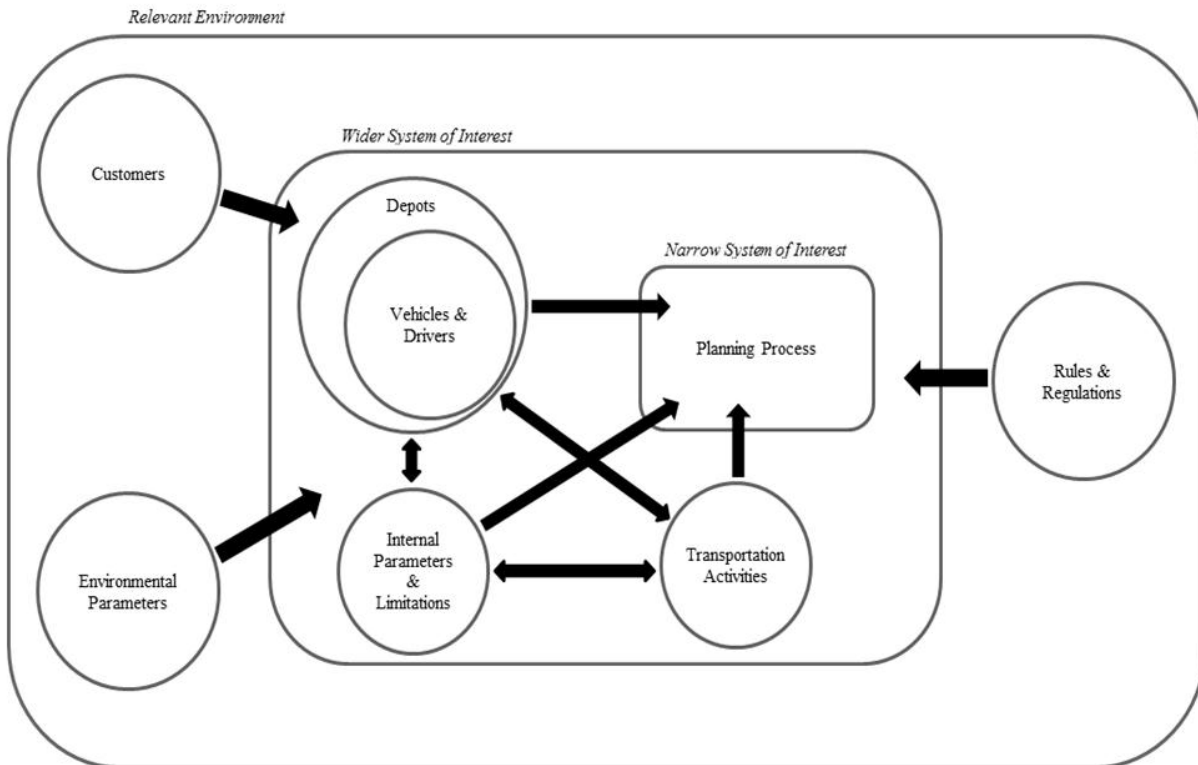
The first step of designing a new system is the analysis of the current one. Once the elements and processes within a system are analyzed, its characteristics are identified and the focus area is explicitly specified, then an appropriate problem definition can be made and possible alterations in the system can be discussed. Daellenbach [93] have proposed the following approach for describing a system:



Specify;

- The transformation processes or activities of the system
- System boundaries (narrow system of interest, wider system and relevant environment)
- Components and subsystems of the narrow system
- Inputs from the environment and the wider system and the outputs from the narrow system

Let us represent the system from our point of view with a 3-layer, high-level system boundary diagram in which borders are set such that the narrow system, wider system and environment are separated from each other:



**Figure 28. System Boundary Diagram**

The diagram above represents the elements of the system with the appropriate boundary setting. Now, let us discuss each element one by one and identify the relationships between them, shown by the arrows.

### **Appendix E.3.1. Relevant Environment**

The system environment consists of the uncontrollable external elements which do affect the system, but in an indirect manner and not to a significant extent [94]. These elements provide inputs to the wider system of interest.

Customers: Customers constitute the most important element of the system environment. We cannot control our customers directly; on the contrary, they provide inputs to our system via transport orders with

particular specifications such as origin and destination points, type and amount of products to be carried, delivery dates and time windows.

Environmental Parameters: There are some environmental parameters influencing the nature of the system. The most important ones are related to the road network which K+N utilizes. These are the distances and between pick-up and delivery points, existing route alternatives, the travel times and the expected vehicle speeds on specific road segments depending on the time of the day.

Rules & Regulations: This element includes all kinds of restrictions imposed by the law, such as carbon dioxide emission limits, maximum driving speed on different types of roads and any other similar standards specified.

### **Appendix E.3.2. Wider System of Interest**

Wider system of interest is known to be the containing and controlling system [93]. It receives inputs from the environment and it is always in interaction with the narrow system. Depots, vehicles assigned to each depot, drivers, internal limitations and transportation activities constitute the wider system of interest.

Depots: As specified by K+N, in this project, our focus will be on *FMCG* and *Returns* business units, and therefore the depots we are interested in are the ones that are used by these purposes. The characteristics, key capabilities and main customers of FMCG facilities are given in Appendix E.11.

There are six return centers of K+N in the Netherlands, located in Pijnacker, 's-Hertogenbosch Tilburg, Vaassen, Zaandam and Zwolle. Most of the facilities are taken over from Albert Heijn and they serve to this main customer of the *Returns* unit. Characteristics and key capabilities of the facilities in Pijnacker, Tilburg and Vaassen are represented in Appendix E.12.

Vehicles & Drivers: Vehicles are important components of the system in terms of carrying out the transportation activities and fulfilling the customer orders. The fleet size and mix of K+N Netherlands is dynamic; additional charter trucks can be utilized in case of excessive demands. There are three basic types of trucks, namely:

- Normal trucks
- Combination trucks
- LZVs (extra-long trucks)

These trucks are either owned by the company, subcontracted or chartered. According to the current fleet data of K+N Netherlands, 93 vehicles are owned by the CL division, and the fleet consists of 70 normal trucks, 20 combination trailers, 3 LZVs. Each vehicle has a specific base depot, at which it starts and ends its trips. These depots are Veghel, Vaassen, Nieuwegein and Raamsdonksveer, respectively.

Different types of trucks have different capacities in terms of weight and volume, and consume different amounts of fuel per kilometer; hence they have different fuel costs. These facts should be taken into consideration while preparing the transportation plans.

An integral part of this system is the drivers. In order to fulfill an order, both a vehicle and a driver should be assigned to the corresponding shipment. Having analyzed the 2-week trip data of K+N Netherlands, we have observed that vehicle-driver pairs are subject to change; a driver can be assigned to different vehicles

as well as different drivers can be assigned to a single vehicle for each shipment. Within the context of this project, the most important information that should be kept in mind regarding the drivers is the cost of overnight stays.

*Internal Parameters & Limitations:* In addition to the parameters provided and restrictions imposed and by the elements in the system environment, there are also some internal parameters and limitations that have a direct impact on the planning process. The following factors should be considered in preparing the transportation plans:

- ***Cooling:*** Some types of products are supposed to be kept cold during the shipment. These products should be carried by vehicles having cooling equipment.
- ***Contamination Effects:*** Some products could be hazardous to others. In these cases, these products should be transported in separate vehicles. For instance, foods can be contaminated if they are carried together with detergents. This type of interactions between products should be avoided.
- ***Crate Wash:*** Crates should be cleaned in a regular scheme or after carrying some particular products. For instance carrying pet food causes its smell to permeate the crate and other products that are going to be carried in the same crate afterwards might get affected. A need for crate wash arises in such a case.
- ***Overnight Stays:*** The maximum number of days of overnight stay for a driver is an important limitation on deciding the length of a trip, and it should be taken into account in the planning phase.
- ***Loading & Unloading Times:*** In addition to the time spent on the road, loading and unloading times of the vehicles are also important parameters and they should be included in the planning.
- ***Working Hours at Each Depot:*** As well as time windows specified by the customers, opening and closing hours of the depots are also important parameters for determining pickup or delivery times for distributions or returns.
- ***Vehicle Holding Capacity:*** Each depot has a vehicle holding capacity. Both the maximum number of vehicles that can be present at the parking area at the same time and the maximum number of vehicles that can be loaded/unloaded simultaneously are specific parameters for each depot and should be considered.

As it is seen in the above definitions, all these parameters and limitations are interrelated with depots, vehicles, drivers and all types of shipments, and together they form an important element of the controlling wider system.

*Transportation Activities:* There are several different types of transportation within the logistics network of K+N Netherlands *FMCG* and *Returns* units. These modes of transportation are listed as follows:

- ***Primary Transport:*** Collection of products from sourcing units and carrying them to K+N depots
- ***Secondary Transport:*** Delivery of products from K+N depots to retail distribution centers
- ***Returns:***
  - Shipment of wastes, boxes, recycle bottles and empty crates from Albert Heijn distribution centers to K+N return centers
  - Scheduled returns for collecting goods from retail distribution centers and delivering them to K+N return centers
  - Return of wrong deliveries from retail distribution centers to K+N return centers
- ***Inter K+N:*** Delivery of all types of products between K+N depots

- **Direct Shipments:** Pick-up and delivery of an order from the customer site to a specific delivery address

Each customer order arrive in the form of one of the shipments types above. Afterwards, these orders are planned such that each one is assigned to a trip carried out by a specific vehicle and a driver. Therefore, the relationship between the transportation activities and the other elements in the wider system is pretty strong.

### Appendix E.3.3. Narrow System of Interest

Narrow system of interest represents the focus area of this project. The elements in this area are changeable and we have the full control over them. In our case, the narrow system consists of the planning process of the orders in K+N Netherlands CL, *FMCG* and *Returns* business units. Our purpose is to change the planning process taking into consideration all the interacting components in the outer systems and improve the performance of the whole system. Let us first explain how the planning process is carried out currently.

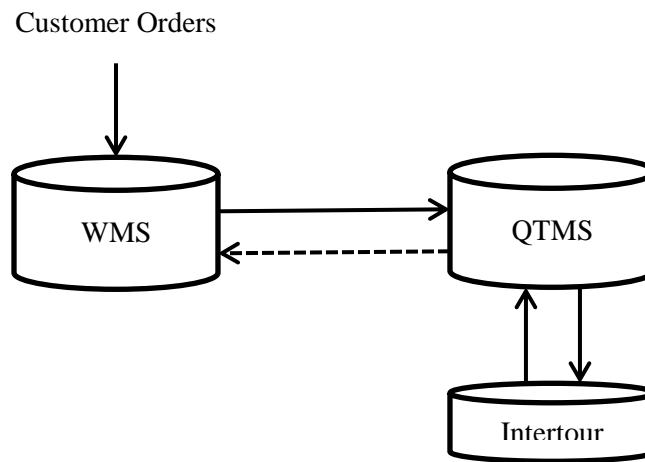


Figure 29. Transportation Planning Process - Current Setup

In the current setup, customer orders arrive at Warehouse Management System (WMS). After being processed there, they are transferred to the Qarius Transport Management System (QTMS) for planning. The routes are planned using the transport optimization tool ‘Intertour’ and a signal is sent back to WMS when the transportation plans are completed.

One customer order, in our case a transport order, consists of one or more transport order lines. Each transport order has a unique order number and each transport order line is specified by a line number. Transport order lines correspond to shipments and contain specific information such as origin and destination addresses, amount of goods to be transported, planned delivery dates and available time windows for the delivery.

The purpose of planning is basically to assign several shipments to a trip, which is to be realized by a single vehicle and driver. Currently, the planning over the network is done manually by the planners on site, each of whom is responsible for his/her own trucks. The planners are supposed to take into account

both inbound and outbound logistics and optimize the flows. However, this integration cannot be fully achieved because of the complexity of the network.

*Intertour* is an optimization tool that can generate minimum cost routes for the planned trips. However, planners use it just to visualize the network and they plan the routes manually. Therefore, the optimization part of the software is not utilized.

There are three types of orders according to their lead times. The planning of the first type of orders is made 48 hours prior to the shipment. The second type orders and the rush orders, which are of the third type, are planned 24 hours and 8-12 hours beforehand, respectively.

As we have mentioned before, currently the planning is performed regionally and manually. The company anticipates that the performance of the current system can be improved by automating the planning process by making use of planning software and implementing a central planning approach in which the orders for multiple depots are planned simultaneously. This perspective constitutes the starting point of this project. Having the necessary knowledge of the system and the anticipated direction of the project, we can now move on to the formal definition of the research problem to be studied.

#### Appendix E.4. Identification of the Research Problem

The research problem can be defined in the light of the insights given by the current system analysis. It is anticipated that the current way of planning is open to improvements; hence it must be shown that the system behaves inefficiently with respect to the current standards. One of the most important indicators of efficiency in logistics is the share of empty distance in total distance traveled. Having analyzed the past data, this ratio is found to be around 42% in K+N Netherlands (See also Table 42). In The Netherlands, this percentage is 23% for national, and 18% for international transport on average [95]. This is an indication of inefficiency. The possible causes to this problem are represented in the following cause-and-effect diagram:

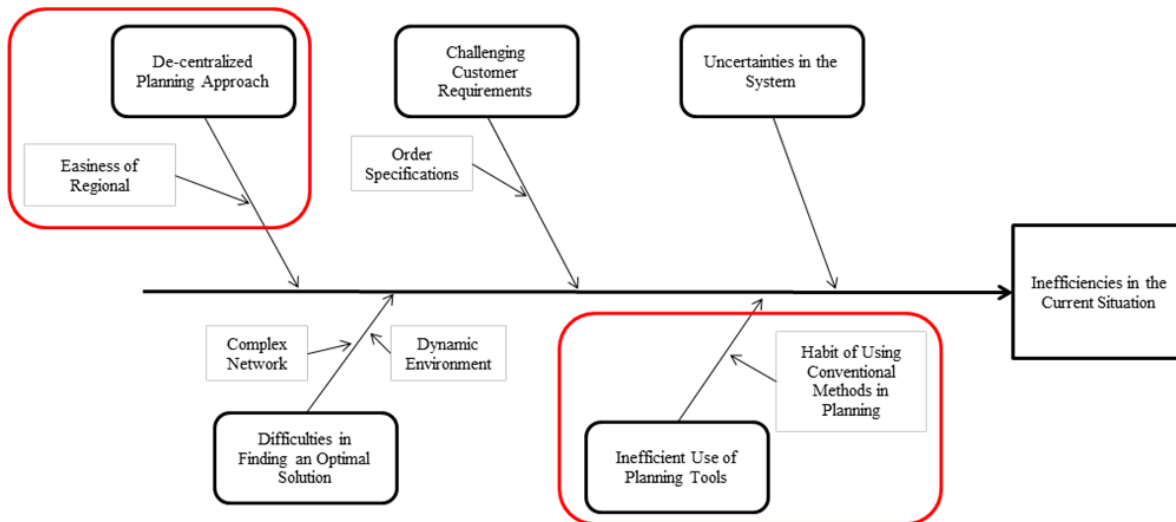


Figure 30. Cause-And-Effect Diagram

As it has been stated before, currently, a de-centralized planning approach is employed which is basically because of the simplicity this type of planning. Challenging customer requirements appearing in the form of order details such as strict time windows or similar restrictive specifications are other important factors rendering the planning more difficult to make, which results in inefficient results. The amount of uncertainty in the system is high; many parameters such as travel times, loading and unloading times, vehicle conditions are subject to changes depending on many different environmental factors. Even if the uncertainties in the system are not taken into account, and everything is assumed to be deterministic, it is still very difficult to determine an optimal plan because of the complexity of the network, and dynamic system environment. The complexity stems from the high number of customers, different types of shipments and the amount of resources to be planned, and the system is dynamic because continuously arriving customer orders. The final possible cause of the inefficiency is the inefficient use of automated planning tools. Currently, all the planning is done manually since the planners are used to prepare the plans in this way; therefore the possible benefits of making use of an automated tool are not fully utilized.

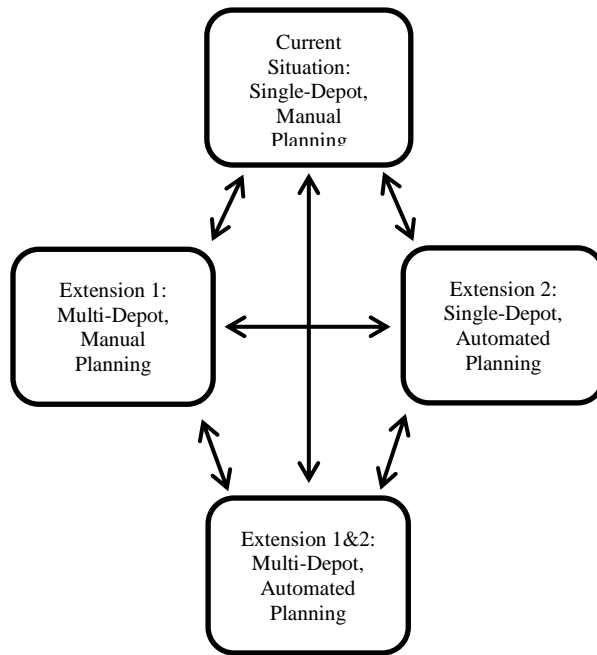
In this project we will work on the extensions in two highlighted causes: de-centralized planning and inefficient use of planning tools. Accordingly, our focus will be concentrated on investigating the benefits of two possible changes in the planning process:

- Multi-depot planning
- Automated planning (using a planning software)

According to the interviews held in the company, the current way of planning can be classified as “single-depot, manual planning”. Each vehicle owned by the company belongs to a specific depot at which it starts and ends its trips. Since the trips are planned separately for each depot, it becomes a standard single-depot case. Nevertheless, it has been mentioned that planners of different depots are usually in touch during the planning process, and shipments belonging to different depots can be combined or exchanged in some cases. Since everything is done manually, it is not known to what extent this integration is achieved. Therefore, the current setting will be assumed to be “single-depot” for now, and the level of inclusion of multi-depot aspects in the current setting will be investigated later on.

#### **Appendix E.4.1. Basic Scenarios to Be Compared**

In order to see the effects of implementing multi-depot and automated planning, three possible extensions are taken into consideration. The changes in the system are going to be observed by comparing the performances of the system in the following cases:



**Figure 31. Scenarios to Be Constructed**

#### **Appendix E.4.2. Key Performance Indicators**

In order to measure the system performance in each case, we need to define some key performance indicators (KPIs) and interpret the changes in the levels of these indicators under the changing conditions. The KPIs should be set in such a way that all possible changes that could occur by performing a multi-depot, automated planning with respect to the current way of planning can be clearly observed. For this purpose, the KPIs are defined as follows:

- Total distance traveled
  - Full kilometers traveled
  - Empty kilometers traveled
  - Percentage of empty kilometers
- The number of vehicles used per day and vehicle utilization
- Net working hours
- Average length of a trip
- The number of day and night shifts

Total traveled distance is an important and the most common indicator of performance for transportation systems. We expect that more efficient transport plans can be generated in terms of total distance by employing an automated planning system. Once the total distance is reduced, it is reasonable to anticipate a reduction in the net working time as well.

Multi-depot planning is expected to reduce empty distance traveled, hence the percentage of empty kilometers. More shipments are expected to be assigned to less number of vehicles, which would lead to a decrease in the number of vehicles used per day and an increase in average length of a trip.

The number of day and night shifts depends on both the time windows specified by the customers, and the length of the trips. If no time windows were set, we would expect the number of night shifts to increase because of having longer trips. The actual situation will be better understood after analyzing the alternatives quantitatively.

Having defined the appropriate KPIs, we can start analyzing each case in detail.

### Appendix E.5. Current Situation (Single-Depot, Manual Planning)

All the analyses on the current situation are performed using the figures obtained from the 2-week representative QTMS data, which were collected from 11.09.2011 (Sunday) to 24.09.2011 (Saturday). The data are analyzed on the order, shipment and trip basis.

#### Appendix 5.1. Order Based Analysis

In the representative 2-week period, a total of 1532 customer orders were planned. Each customer order consists of one or more deliveries. The total number of deliveries observed in this period is 15820; therefore on average, each customer order contains 10.33 deliveries. The daily analysis of orders and deliveries can be seen in the following figure:

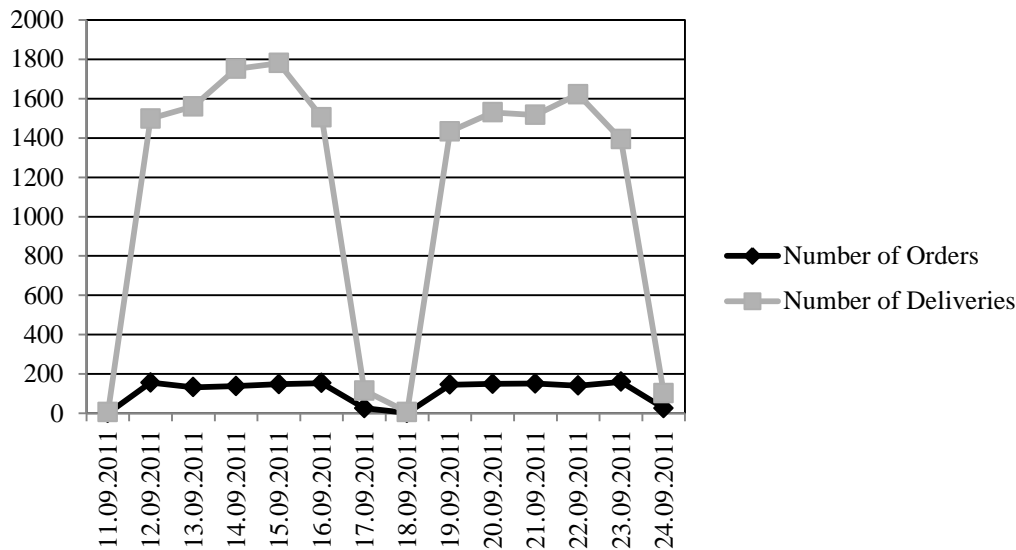


Figure 32. Daily Analysis of Orders & Deliveries

As it can be derived from the figure, the daily number of orders and deliveries differ significantly between weekdays and weekends. Therefore, the average number of orders and deliveries per day are calculated for weekdays, Saturdays and Sundays, separately:

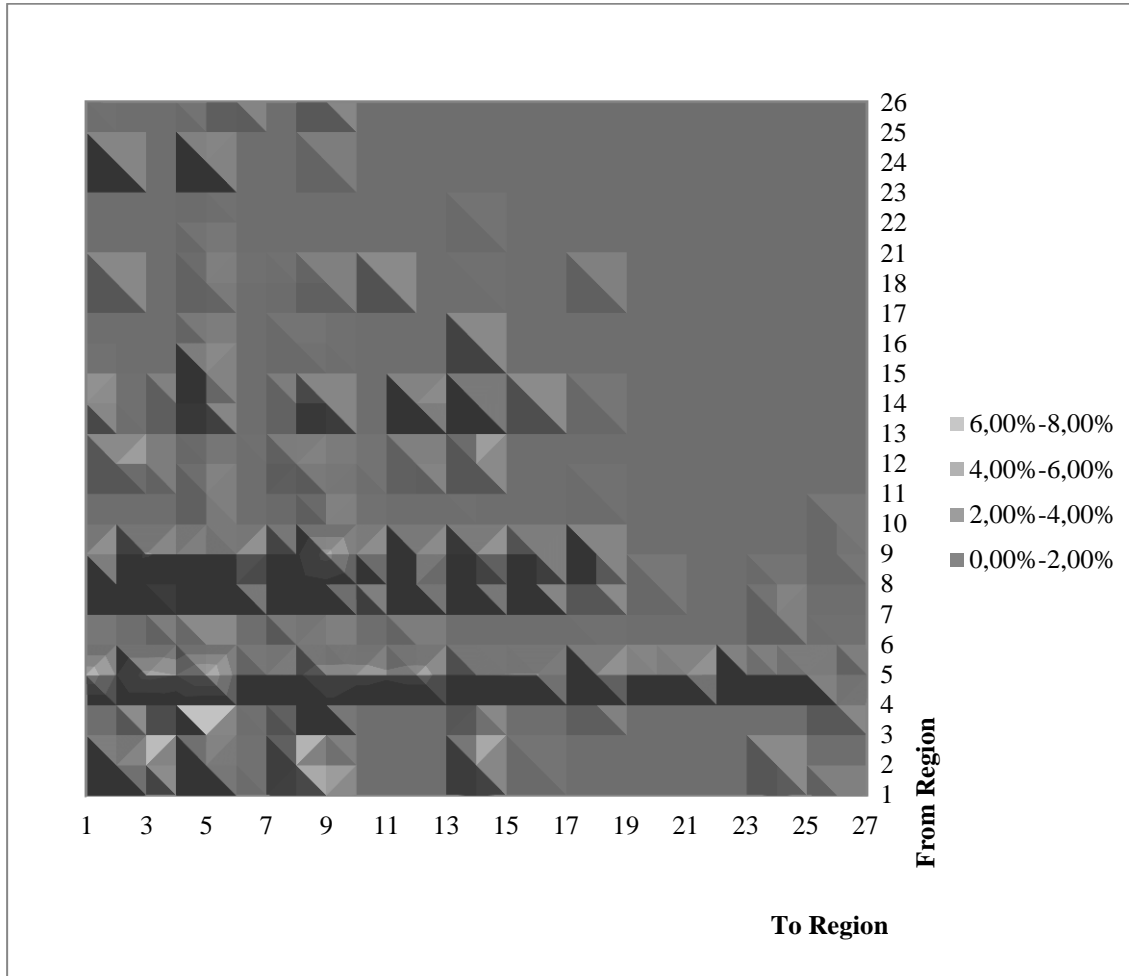
Table 41. Orders & Deliveries per Day

	Orders per Day	Deliveries per Day
Weekdays	147.7	1558.9
Saturdays	26	109
Sundays	1.5	6.5



As the figures show, weekdays much busier than weekends in terms of planned orders. Especially, on Sundays there are either one or two orders, which become almost negligible in the planning.

Orders are also analyzed based on their origin-destination pairs. In order to make such a comparison, the geographical region on which transportation activities are carried out is divided into 27 sub-regions based on the post codes of the locations. Maps showing these sub-regions can be found in Appendix E.13. The following surface chart represents the usage frequencies of each origin-destination pair in the form of from-to regions:



**Figure 33. Usage Frequencies of Origin-Destination Pairs**

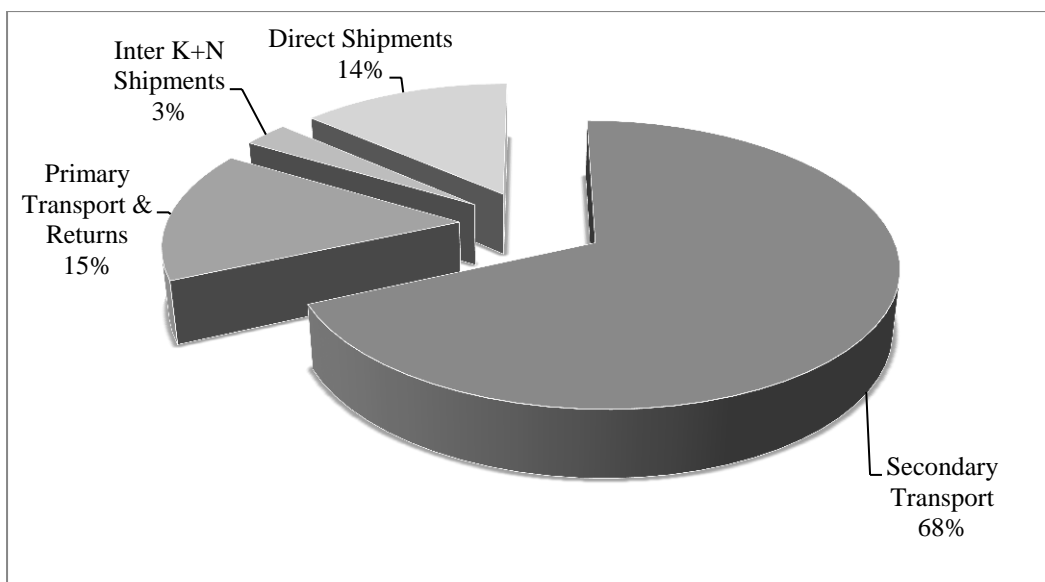
This figure shows that regions 5, 8 and 9 are the most active regions in terms of sending orders. Similarly, regions 1, 3, 5, 9 and 12 contain the most frequently used destination points.

After analyzing orders based on their dates and origin-destination regions, let us analyze the system based on shipments.

### **Appendix E.5.2. Shipment Based Analysis**

In this part, the shipments are going to be classified according to the definitions given in Section 3.2 (Transportation Activities Segment), and their respective depots. We assume that each drop, consisting of one or more line items which have the same origin and destination pairs, represents a shipment. In the selected 2-week period, there are 12286 shipments, and 18605 deliveries in total. This allows us to say that each shipment is represented by 1.51 line items on average.

In Section 3.2, we have defined the primary transport; as the shipment of goods from sourcing units to K+N depots, secondary transport; as the distribution of goods from depots to customer locations, returns; as shipment of goods from customer sites to K+N depots, inter K+N shipments; as shipments between K+N facilities and direct shipments; as shipments in which goods are picked up from and delivered to customer sites. Let us combine primary transport and returns, and classify them as shipments from outside locations to K+N depots. The percentages of each type of shipments are found to be as follows:

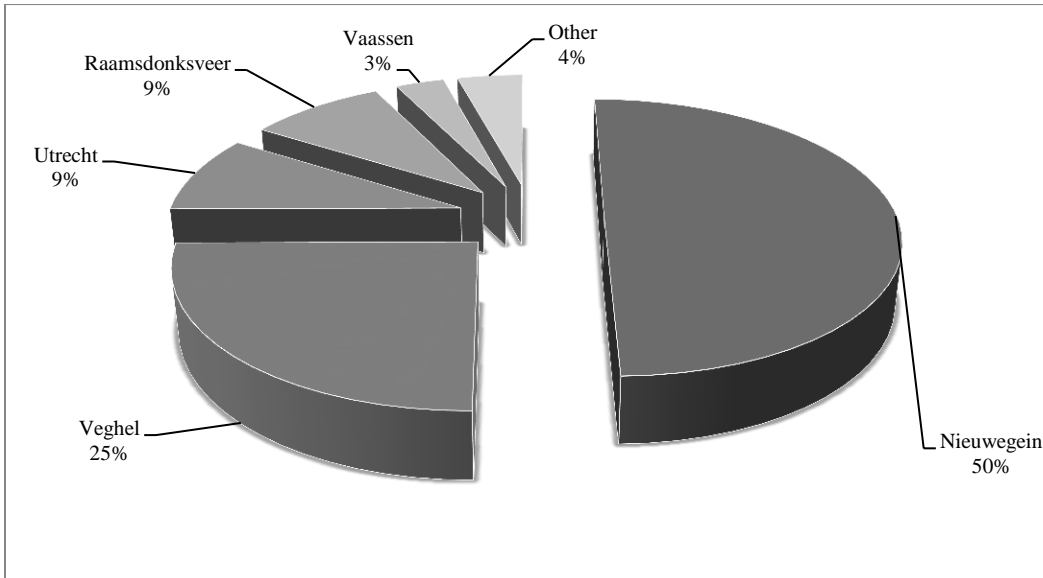


**Figure 34. Classification of Shipments**

Majority of the shipments (68.3%) are of the secondary transport type, which is the distribution from depots to customers. 15.25% of them are shipments into K+N depots from outside. Quantity of direct shipments from one customer site to another is slightly lower than that. Inter K+N shipments, on the other hand, constitute only a very small proportion of transportation activities.

Now, let us analyze which facilities are used in what frequency for the first three categories of shipments (secondary transport, primary transport & returns, inter K+N shipments).

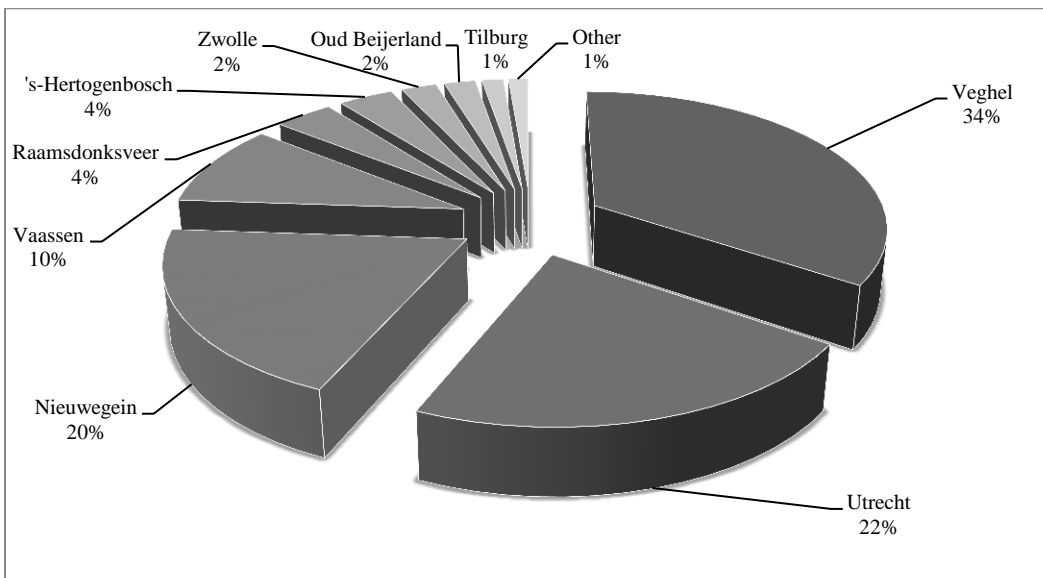
Secondary Transport: Shipments of this type are made from the following K+N facilities:



**Figure 35. Secondary Transport - Usage Frequencies of K+N Facilities**

Most of the distribution type shipments are from K+N facilities in Nieuwegein and Veghel. Utrecht, Raamsdonksveer and Vaassen are other notable depots used in secondary transport. The remaining small proportion (4%) of shipments have the base facilities Zwolle, Tilburg, Oud Beijerland, 's-Hertogenbosch, Delfgauw, Wijchen, Zaandam, Ede, Eindhout-Laakdal, Moerdijk and Soest.

Primary Transport & Returns: Shipments from customer sites or sourcing units arrive at the following K+N facilities with the given frequencies:

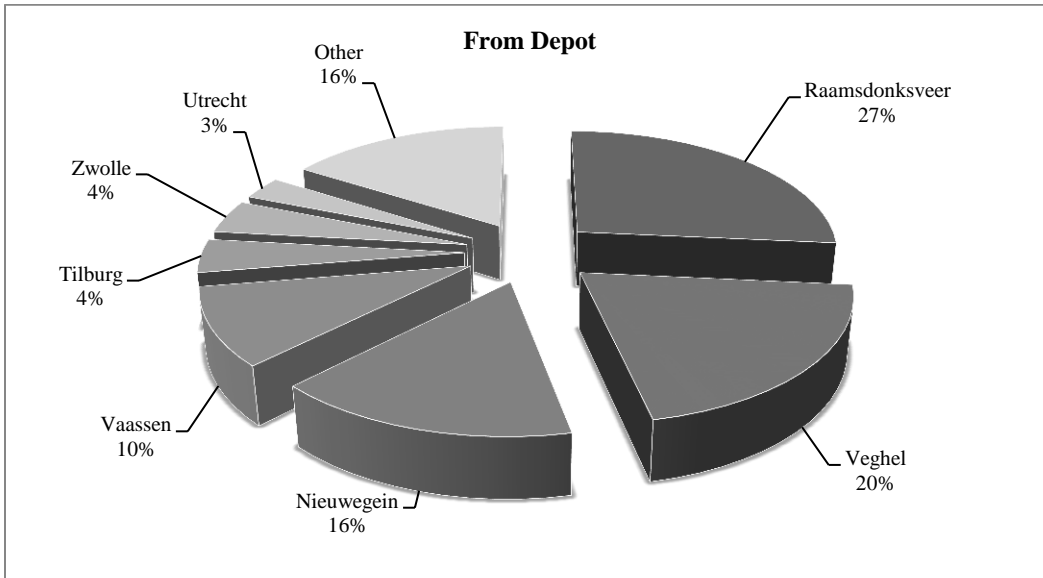


**Figure 36. Primary Transport & Returns - Usage Frequencies of K+N Facilities**

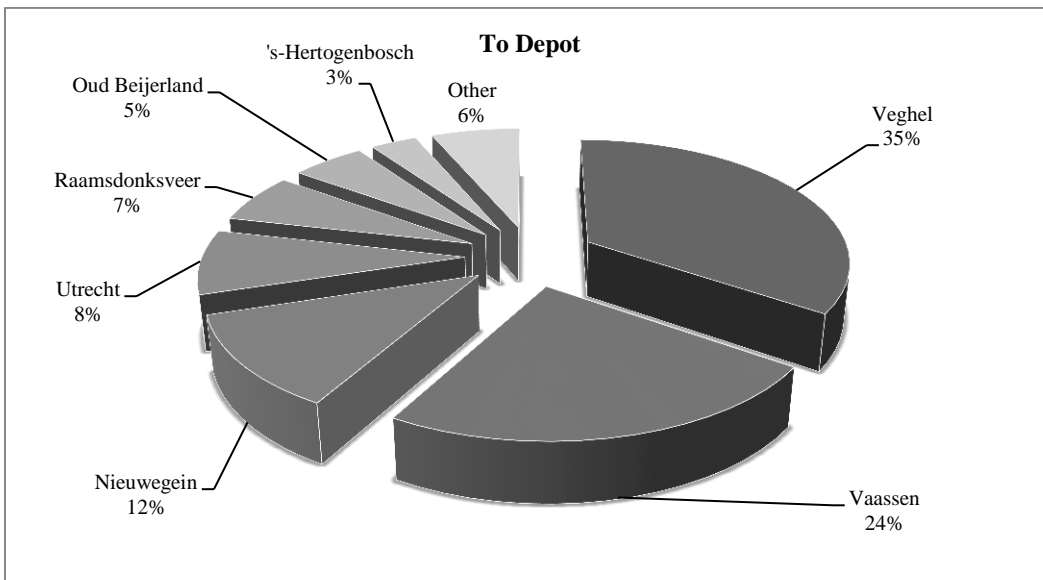
Veghel, Utrecht and Nieuwegein are again, actively used facilities for shipments of this type. Vaassen, which is an important return center, receives 9.6% of these shipments. Other depots used for this purpose

are Raamsdonksveer, 's-Hertogenbosch, Zwolle, Oud Beijerland, Tilburg, Wijchen, Eindhoven-Laakdal and Ede.

Inter K+N Shipments: The shipments between K+N depots are represented as follows:



**Figure 37. Inter K+N Shipments (From Depot) - Usage Frequencies of K+N Facilities**



**Figure 38. Inter K+N Shipments (To Depot) - Usage Frequencies of K+N Facilities**

Regarding inter K+N shipments; Raamsdonksveer, Veghel and Nieuwegein are the most frequently used pick up locations and the most of the shipments are delivered to Veghel, Vaassen and Nieuwegein. Facilities in Veghel and Nieuwegein are utilized in both sides of shipments, whereas Raamsdonksveer is actively used in sending, and Vaassen, in receiving.

### Appendix E.5.3. Trip Based Analysis

A trip is the combination of one or more shipments coming from different or the same customer orders with an assigned vehicle. A total of 3482 trips were planned in the representative 2-week period. Recalling that the number of shipments in this period was 12286, we can conclude that one trip contains 3.53 shipments on average.

In Section 3.2 it is mentioned that K+N utilizes its own, subcontracted, or chartered trucks in its operations. The following table represents the quantity and percentage of trips with respect to the type of performing vehicle.

**Table 42. Trips per Vehicle Types**

	Quantity	Percentage
Owned by the Company	1633	46.90%
Subcontract	1120	32.17%
Charter	729	20.94%
<b>Total</b>	<b>3482</b>	<b>100.00%</b>

In nearly half of the trips, company owned trucks are used. However, it can be seen that subcontracted and chartered vehicles are also utilized with a significant proportion. In the 2-week data, many of the statistical figures for trips performed by chartered trucks are missing. Therefore, the rest of the analyses on trips will be based on trips utilizing company owned and subcontracted trucks.

The number of vehicles used and the trips performed per vehicle for these two types of trucks are shown below:

**Table 43. Truck Usage per Type**

	Number of Trucks Used	Trips per Truck
Company Owned	93	17.56
Subcontracted	82	13.66

In the 2-week period, all 93 vehicles in the fleet of K+N are used. Each company owned truck performed 17.56 trips on average whereas this figure is 13.66 for subcontracted trucks, which is 22.2% less than those performed by the trucks owned by K+N.

After representing these descriptive quantitative figures regarding the base scenario, let us now introduce the current values of KPIs defined in Section 4.2. Figures related to trips carried out by chartered vehicles are disregarded due to lack of data.

**Table 44.KPIs for Current Situation**

	Per Trip	Total
Full Kilometers	153.18	421709.6
Empty Kilometers	109.67	301926.1
Total Distance Traveled	262.85	723635.7
Percentage of Empty Kilometers	-	41.77%
Trip Duration (hrs)	8.05	22159.1
Number of Day Shifts	-	2140
Number of Night Shifts	-	613
Percentage of Night Shifts	-	22.27%

Having calculated the KPIs for the base scenario, we can move on to investigation of the benefits extensions proposed.

### **Appendix E.6. Extension 1: Multi-Depot, Manual Planning**

The first extension that will be considered is utilizing multi-depot planning manually. Since the number of orders and corresponding shipments are too large for an initial quantitative analysis, we decide to take some samples from the 2-week data and look for ways to improve the system with appropriate adjustments towards the aim of a multi-depot planning approach.

We take and analyze two sets of samples; one sample on the trip base from a single day, and another on the vehicle base for several days. Let us continue with the trip based analysis.

#### **Appendix E.6.1. Sample 1: Trip Based Analysis**

The first sample we take consists of 50 trips performed on 23.09.2011. In order to see the ways of improvement more easily, we visualize these trips on a map. The following figure shows the routes of these trips. Note that each trip is represented with a different colour, dashed lines show the trips including pickups or deliveries abroad, and repeated routes are shown only once.



**Figure 39. Route Display of the Sample Trips**

The improvements on these existing trips are tried to be achieved intuitively by combining trips or switching shipments between trips. After spending some time on this search, we come up with the possible changes in the following trips:



**Figure 40. Trips to Be Modified**

We expect the system performance to increase if we combine trips 1 and 2, 3 and 4 and 5 and 6 in the above figure. All steps of the original and modified trips are represented in Appendix E.14. The values of the KPIs for the initial conditions, situation after the alterations and the corresponding percentage differences are as follows:



**Table 45. Results of Alterations within the First Sample**

	<b>Original</b>	<b>After Alterations</b>	<b>% Difference</b>
<b>Number of Trips</b>	50	47	-6.00%
<b>Full Kilometers</b>	14054	14054	-
<b>Empty Kilometers</b>	6547.4	5765.2	-11.95%
<b>Total Distance Traveled</b>	20601	19819.2	-3.79%
<b>Percentage of Full Kilometers</b>	64.82%	70.91%	+9.40%
<b>Total Travel Time</b>	696.66	688.71	-1.14%
<b>Average Trip Duration (hrs)</b>	13.93	14.65	+5.19%
<b>Average Trip Distance (km)</b>	412.02	421.69	+2.35%
<b># of Trucks Used</b>	47	44	-6.38%

As it can be derived from the table above, empty kilometers traveled was reduced significantly. Full kilometers traveled remained the same and therefore total distance traveled decreased whereas percentage of full kilometers increased. Average trip duration and distance increased as expected, and the number of vehicles used dropped by 3.

These figures show that the system performance can be improved by this kind of manual adjustments, but it is realized that the manual planning aspects cannot be fully implemented on a sample of trips taken from a single day. Therefore, in order to observe the benefits of multi-depot planning exclusively, we decide to take another sample, this time from a 3-day period and based on routes of the vehicles on these days rather than trips.

### **Appendix E.6.2. Sample 2: Vehicle Based Analysis**

The vehicle based analysis is done by taking a sample of 10 vehicles, 4 of which have Veghel, 3 of which have Vaassen, 2 of which have Nieuwegein and 1, has Raamsdonksveer as base depot. All the shipments performed by these trucks in 12-14 September, 2011 are examined carefully. Even for this small set of shipments we observe that:

- A truck based in Vaassen (T307), is also used for shipments from Raamsdonksveer
- A truck based in Vaassen (T480), is also used for shipments from Veghel
- A truck based in Nieuwegein (T326), is also used for shipments from Veghel
- A truck based in Raamsdonksveer (T335), is also used for shipments from Vaassen
- A truck based in Veghel (T457), is also used for shipments from Raamsdonksveer

This analysis shows that half of the vehicles are utilized by facilities apart from their base depots in this 3-day period. This proves that the “single-depot” assumption we have made at the beginning is not true for the current situation. It has now become apparent that planners actually put some effort to use multi-depot planning approach in order to increase efficiency. Therefore, it is not possible to classify the current situation as a pure single-depot plan, and the comparisons will be done between the following scenarios after this point.

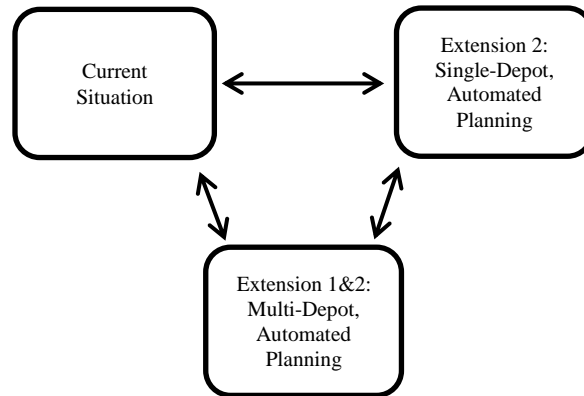


Figure 41. Scenarios to Be Constructed - Revised

## Appendix E.7. Automated Planning Tool – Shortrec

As stated in Chapter 4, one of the objectives of this project is to identify possible benefits of employing an automated planning approach instead of current manual planning philosophy. K+N Netherlands currently works in collaboration with ORTEC, a company that provides transportation planning and routing software solutions. Engineers in K+N are using a product of ORTEC, called Shortrec, an automated trip routing and scheduling system that optimizes transport and distribution planning in tactical and operational levels, for their simulations.

At the beginning of this project, the purpose was to use a new planning tool, ORTEC Transport and Distribution (OTD), a more advanced program that focuses on real-time route planning and dispatching. By utilizing OTD, the company would be able to completely automate the planning process and integrate it with the execution of all transportation activities. However, implementation of the software could not be completed within the time frame of this project; hence all the simulations were conducted using the existing planning tool: Shortrec.

Let us explain the types of inputs and outputs of the software, solution methodology and program settings used for simulations.

### Appendix E.7.1. Inputs & Outputs

For this project, version 7.2 of Shortrec was used. Environmental parameters (see Section 3.1) such as the road network in the Benelux region, distances and driving speeds between each location represented by postal codes are predefined in the software. Initially 9 depots were present, located in Oud Beijerland, Nieuwegein, Utrecht, Raamsdonksveer, Veghel, Helmond, Vaassen, Grimbergen (Belgium) and Mechelen (Belgium). Two main types of inputs are supposed to be entered by the user:

1. Orders
2. Combinations

Orders: Customer orders can be imported to Shortrec via spreadsheets in *csv* format. Many parameters can be specified in these files. The most important ones are: order number, full address and name of the client, delivery date, type of shipment (delivery or return), amount of load in terms of weight and volume,

fixed and variable loading and unloading times, time windows, admitted vehicle types and admitted depots.

Combinations: With the word “combination”, we refer to a vehicle-driver pair. Combinations can be imported to Shortrec in the same way as orders. In the associated spreadsheet, vehicle number, normal working and overtime hours of the driver, starting and ending locations, vehicle capacity, costs, admitted depots, allowed overload and many other parameters can be specified. Therefore the fleet of the company is defined in this part.

After the orders and combinations are imported, planning can be done manually using the planning board, or automatically. When the planning is finished, several types of reports can be generated.

SHV report: It is an excel file and the most comprehensive type of report. It contains a summary report representing total costs, kilometers, working time, amount of load lifted and delivered, number of trips, orders and average speed. Additionally, use of trucks and depots, customer visits, and trips are shown in detail in separate sheets. It is possible to conduct different kinds of performance analyses based on the data represented in SHV report.

Other types of output reports are:

Violations of Constraints: Shows which constraints e.g. vehicle capacity, are violated.

Performance of Vehicles: Shows kilometers traveled, worktime, duration, amount picked up and delivered for each trip, on a vehicle by vehicle basis.

Order Overview: Shows which orders are assigned to which combinations.

Compact Report: It is a more detailed version of the performance of vehicles report. It represents each order within each trip with the corresponding customer address, duration, loaded and unloaded amounts on vehicle by vehicle basis.

### **Appendix E.7.2. Solution Methodology**

Simply, Shortrec aims at assigning orders and combinations to trips at the minimum cost while satisfying vehicle capacity and timing constraints. It uses a two-phase strategy with construction and improvement algorithms. Construction algorithms are employed for generating basic solutions. A basic solution consists of the preliminary allocation of orders to the schedule. Construction algorithms are usually in the form of insertion heuristics. Insertion heuristics are fast algorithms that construct feasible solutions and have proven to be popular methods for solving many different kinds of vehicle routing and scheduling problems [96].

Once the feasible solution has been constructed, Shortrec tries to improve this solution by means of iterative improvement algorithms. It uses a series of optimization methods to define a total solution. The following operations are conducted within the improvement procedure:

- Optimization within trip: Sequence of orders within a trip is changed.
- Relocation of orders: Options are assessed by moving one or more orders from one trip to another.

- Equalization of working hours: Working hours are tried to be distributed evenly across the fleet.
- Selection of cheapest vehicles: Trips are tried to be assigned to vehicles that can operate at the lowest cost.
- Trip swapping: Each pair of trips is assessed to see whether a cost reduction can be achieved via swapping them over.
- Stop swapping: In a trip pairing, some of the orders are swapped from one to another and vice versa.
- Optimization between trips: A combination of trip and stop swapping options.

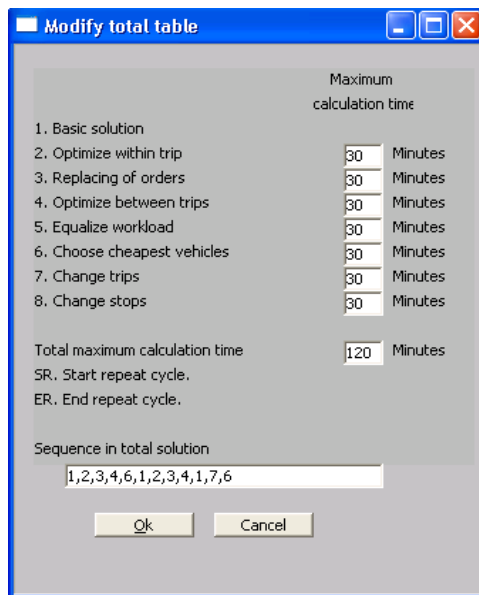
Each method is optional and the program user can specify which options to be operated in which order. The appropriate sequence of options changes depending on the distribution profile of the company.

### Appendix E.7.3. Program Settings and General Assumptions

Shortrec Settings: Shortrec has many different kinds of options and parameter settings for generating plans. The general settings should be maintained at each simulation for comparing different scenarios.

One of the options that could be specified in Shortrec is that it either works with flexible restrictions on time windows, vehicle capacity and maximum worktime or not. In all the simulation runs, restrictions are assumed to be non-flexible unless stated otherwise.

Lunch break is not considered, overtime is allowed. Orders having excessive amount of load are split so that vehicle capacities are not exceeded. Correction for distance within postal code is taken as 3 kilometers and the associated time is assumed to be 3 minutes. The sequence of methods used in total solution is represented in the following figure:



**Figure 42. Sequence of Options in Total Solution**

This sequence has been applied to all scenarios in order to maintain consistency. After these specifications are done, general assumptions should be made regarding depots, combinations and orders.

Assumptions on Depots: As stated in Section 7.1, 9 depots were predefined in Shortrec, namely Oud Beijerland, Nieuwegein, Utrecht, Raamsdonksveer, Veghel, Helmond, Vaassen, Grimbergen (Belgium) and Mechelen (Belgium). The first simulations were carried out using the initial depot settings. However due to the fact that direct shipments (from customer to customer) could not be planned in the current version of Shortrec, we were not able to include a significant portion of orders in our simulation. Moreover, we observed that the depots in Helmond and Mechelen were not used by any of the orders in our 2-week order set, so they could have been disregarded from the planning. Therefore we decided that new depots should be defined and the 2 redundant depots can be deleted in order to enhance the effectiveness of our plan.

In order to determine which additional depots are to be defined in Shortrec, the order set is examined and the cities that are most frequently used (in terms of both sending and receiving) are identified. Accordingly, in addition to the current 7 depots, 5 K+N facilities and 3 customer locations are defined as new depots. The customers that are defined as depots are Smiths Food Group BV (Broek Op Langedijk), Simon Loos (Wognum) and Plukon Poultry BV (Wezep). The final list of the depots used in the simulations is as follows:

**Table 46. List of Depots Defined in Shortrec**

	<b>Depot Name</b>	<b>Town</b>	<b>Post Code</b>
<b>Existing Depots</b>	GRI	Grimbergen	BE-1850
	NIE	Nieuwegein	3439NA
	ODU	Oud-Beijerland	3261MA
	RAA	Raamsdonksveer	4941VX
	UTC	Utrecht	3542AB
	VAA	Vaassen	8171MC
	VEG	Veghel	5466AA
<b>New K+N Facilities</b>	DEL	Delfgauw	2645EE
	HER	's-Hertogenbosch	5215ME
	TIL	Tilburg	5015BT
	ZAA	Zaandam	1507CJ
	ZWO	Zwolle	8028PS
<b>Customer Locations</b>	BRO	Broek Op Langedijk	1721PP
	WEZ	Wezep	8091AZ
	WOG	Wognum	1687JB

At each depot, there is a fixed loading time of 25 minutes and variable loading time of 1 minute per palette as the default setting. All depots are assumed to be open from 00:01 until 23:59 every day. Parking space and other limitations related to availability of depots are not taken into consideration. Moreover, note that for the depots where there are actually more than one K+N facilities (e.g. Veghel) only one post code is taken into consideration and orders belonging to nearby facilities are assigned to one single post code.

Assumptions on Combinations: In Section 3.2, 93 vehicles owned by the company are briefly explained. Also in Section 5.3, it is shown that 82 subcontract trucks used in the 2-week representative period. Each company owned truck has a base depot where it starts and ends its trips. The distribution of the 70 company owned normal trucks is known and represented in the following table:

**Table 47. Distribution of Normal Trucks**

Depot	Number of Trucks
Veghel	29
Vaassen	19
Nieuwegein	13
Raamsdonksveer	9

This scheme is preserved in Shortrec. 3 LZV's are assigned to Utrecht. It is not known which combination trucks belong to which depots. Therefore 20 combination trucks are evenly distributed to FMCG depots in Veghel, Nieuwegein, Raamsdonksveer, Utrecht and Our Beijerland. Similarly, 82 subcontract trucks are assigned to Veghel, Vaassen, Nieuwegein, Raamsdonksveer, Utrecht and Our Beijerland. All subcontract trucks are assumed to be normal trucks. Charter trucks are not taken into consideration. These depot admissions are reflected in the associated input file.

Each combination has a specific capacity, daily fixed cost and variable cost per kilometer. Those parameters are obtained from the previous simulations run in the company. However, after several trials, it has been realized that it is more appropriate to raise the fixed cost of utilizing a combination truck from 400 to 450. Capacities are in the form of number of CHEP pallets. The parameters of each truck type are given as follows:

**Table 48. Capacities and Costs of Combinations**

Vehicle Type	Capacity (CHEP)	Fixed Cost (per day)	Variable Cost (per km)
Normal Truck	26	400	0.36
Combination Truck	30	450	0.36
LZV	42	450	0.36
Subcontract Truck	26	500	0.36

Several simulations were performed using the fleet consisting of 175 trucks. However, the software was able to plan only 75% of the orders with this vehicle fleet. Therefore, the number of subcontract trucks in the fleet was increased up a point that Shortrec becomes able to plan at least 90% of the orders. These additional subcontract trucks were evenly distributed to the same depots mentioned above.

Other assumptions related to combinations:

- Working hours of combinations is set to be 30 hours in order to allow trucks turn back to their depots after midnight
- LZV's are only allowed to have trips between Utrecht and Broek Op Langedijk (Smiths Food Group BV)
- All other vehicles can be utilized by any of the depots
- Each vehicle starts and ends its trip at its own depot. (For the sake of ideal multi-depot planning, it would be better if this assumption could have been eliminated, but doing so causes all trips to start from a single, randomly selected depot.)
- Maximum allowed overload is 4 pallets for each vehicle.

Assumptions on Orders: The orders to be imported to Shortrec come from the same 2-week data set that has been used for previous analyses. Each transport order line can be converted to an order for Shortrec. There are 15820 transport order lines in the original data file. However, in order that KPIs can be comparable, the orders assigned to charter trucks are disregarded. That gives a total of 9451 transport order lines, handled by either K+N or subcontract vehicles.

As it is mentioned before, initially 7 depots were present on Shortrec. Since direct shipments cannot be planned by the software, at first the orders from or to the existing depots are considered. These order lines are assumed to have the shipment types 10 for outbound (secondary transport), and 20 for inbound (primary transport & returns) deliveries. At the next step, orders connected to other K+N facilities that are located in the same cities as existing depots are taken into consideration. These are entitled as type 11 for outbound, and 21 for inbound shipments. In order to increase the number of orders to be planned, orders related to non-K+N facilities in the cities with depots are also assigned to the respective depots in their cities. These are classified as type 12 and type 22 shipments for outbound and inbound logistics, respectively.

Once the new depots are defined, associated orders can be added to the order set. Orders connected to the cities with the additional depots are given type 14 for outbound, type 24 for inbound deliveries, and all are assigned to the corresponding depots in their cities. The resulting numbers of orders for each shipment type are represented as follows:

**Table 49. Classification of Orders to Be Imported to Shortrec**

	Shipment Type	Number of Orders
Outbound Shipments	10	5745
	11	912
	12	86
	14	612
	<b>Outbound Total</b>	<b>7355</b>
Inbound Shipments	20	1223
	21	301
	22	167
	24	192
	<b>Inbound Total</b>	<b>1883</b>
	<b>Final Total</b>	<b>9238</b>

9238 out of 9451 orders (97.7%) can be imported to Shortrec. These order lines are converted into a Shortrec input file in csv format. Excel VBA programming is used for extracting the appropriate fields from the original data file and arranging them in order that they become readable by Shortrec. The assumptions made while preparing the input file are as follows:

- Loading and unloading times at non-depot locations in K+N have already been calculated by Kokten [97] by means of a regression analysis. The same values are used for Shortrec simulations. Fixed loading and unloading times are assumed to be 24 and 30 minutes, respectively. Variable time for loading and unloading is taken as 1 minute per pallet.
- Decimal pallet loads are rounded up to integers

- Unloading time windows are taken into account for outbound deliveries; loading time windows are used for inbound deliveries.
- Normal trucks and combination trucks can be utilized by all orders; LZVs can only be used in trips between Utrecht and Broek Op Langedijk.
- For the orders without timing restrictions, time windows are set from 00:00 to 23:59.

Other physical restrictions such as cooling, contamination effects, crate wash and carbon dioxide emission limits are not taken into account in Shortrec simulations. Having defined the assumptions of the simulations models, we can move on to the simulation results for various cases.

## **Appendix E.8. Automated Plans – Simulation Results**

### **Appendix E.8.1. Extension 2: Single-Depot, Automated Planning**

In this setting, orders of each depot are planned separately. Therefore, orders from different depots cannot be combined in the same trip. The same vehicle set is employed for each simulation; so it is possible that a vehicle is used by different depots at the same time. As a result, vehicle utilization is not an appropriate KPI for this case.

Since 15 depots are defined, 15 separate simulations were performed in Shortrec for each depot. Some of the orders were eliminated in order to maintain consistency with other cases. The summary reports of the simulation results of each depot can be found in Appendix E.15. The resulting values of KPIs considering the total of these 15 simulations are found to be as follows:

**Table 50. Single-Depot, Automated Planning Simulation Results**

Number of Planned Orders	8924
Number of Trips	4021
Total Km	750252
Empty Km	288053
Empty Km Percentage	38.39%
Net Working Hours	20682.3
Net Trip Time (hrs)	5.14
Kms per Trip	186.58
Orders per Trip	2.22
Number of Day Shifts	2149
Number of Night Shifts	1872
Percentage of Night Shifts	46.56%

### **Appendix E.8.2. Extensions 1&2: Multi-Depot, Automated Planning**

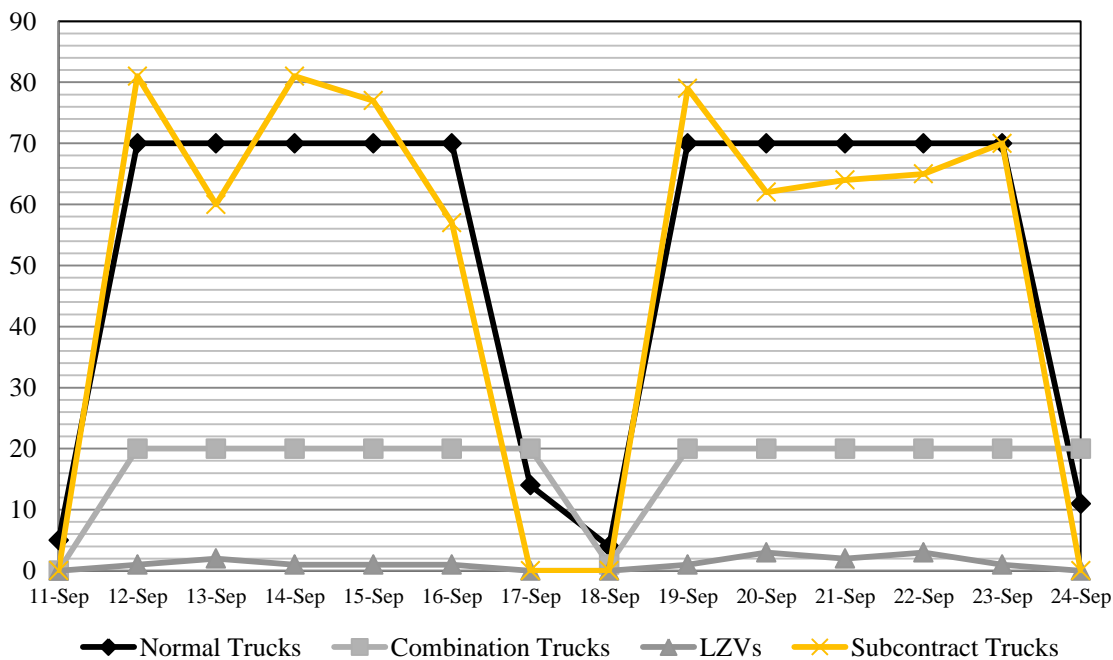
In this setting, all orders are planned in a single simulation and all the vehicles (except for LZVs) can be utilized by any of the depots. This yields a centralized approach which is trying to be achieved via multi-depot planning. The summary report of the simulation is given in Appendix E.16. KPIs for this case are represented as follows:



**Table 51. Multi-Depot Automated Planning Simulation Results**

Number of Planned Orders	8965
Number of Trips	3959
Total Km	692344
Empty Km	226747
Empty Km Percentage	32.75%
Net Working Hours	19809
Net Trip Time (hrs)	5.00
Kms per Trip	174.88
Orders per Trip	2.26
Number of Day Shifts	2095
Number of Night Shifts	1864
Percentage of Night Shifts	47.08%

Another important performance measure for the system is vehicle usage. Both the daily number of trucks used and the percentage of time that a truck is utilized are calculated for each vehicle type. The following figure shows the daily vehicle usage per type for the 2-week planning period.



**Figure 43. Extensions 1&2: Number of Vehicles Used Per Day**

The figure above demonstrates that all the normal and subcontract trucks in the vehicle fleet are used on weekdays. The utilization of subcontract trucks on the other hand, changes with the order volume from day to day on weekdays, and become zero in the weekends. On Saturdays and Sundays, the number of orders to be planned is significantly lower and on these days, trips are performed by company owned trucks instead of subcontract ones. This implies that K+N trucks have a priority over subcontracts, which is a reasonable consequence.

In addition to daily truck usage, the percentage of time each vehicle is utilized can also be derived from the simulation results. The following table represents the exact vehicle utilizations for each truck type on weekdays, Saturdays and Sundays:

**Table 52. Extensions 1&2: Vehicle Utilization on Hour Basis**

	Weekdays	Saturdays	Sundays
Normal Trucks	61.82%	8.73%	1.97%
Combination Trucks	90.52%	65.14%	1.63%
LZVs	42.43%	0.00%	0.00%
Subcontract Trucks	25.56%	0.00%	0.00%

Combination trucks are the most commonly used vehicles in both weekdays and Saturdays. This is mainly because the software tries to take advantage of the high capacity of these vehicles. The reason for low utilization of LZVs is that these vehicles are admitted to only one depot (UTC).

In order to compare the results of these scenarios, we need to make sure that exactly the same set of orders is taken into consideration in all cases. For this purpose, the results of the analysis of base scenario are overhauled. Let us explain what kinds of changes are done on these results.

### **Appendix E.8.3. Base Scenario - Adjustments**

We mentioned before that not all non-charter orders were imported to Shortrec, and neither all the imported orders could be planned. Moreover, some of the orders that were successfully imported and planned were missing performance records e.g. kilometers traveled and working hours in the data file.

In order to be able to give comparable results for the base scenario, the following adjustments are made:

- Orders that were imported and planned in Shortrec are determined using the “Order Overview” report.
- These orders are marked in the QTMS file using macros in Excel.
- The orders which were used in automated plans but missed performance records are identified (which corresponds to only 0.75% of the total).
- This small set of orders is imported to Shortrec and planned with multi-depot approach. Summary report of this simulation can be found in Appendix E.17.
- Results of the simulation are added to the figures obtained from the original data file.

After these alterations, the following results are obtained:

**Table 53. Current Situation - Adjusted Results**

Number of Orders	8542
Number of Trips	2732
Total Km	722485
Empty Km	302593.9
Empty Km Percentage	41.88%
Net Working Hours	22007.4
Net Trip Time (hrs)	8.06
Kms per Trip	264.45

Orders per Trip	3.13
Number of Day Shifts	2138
Number of Night Shifts	594
Percentage of Night Shifts	21.74%

The daily numbers of trucks used are illustrated in the following graph:

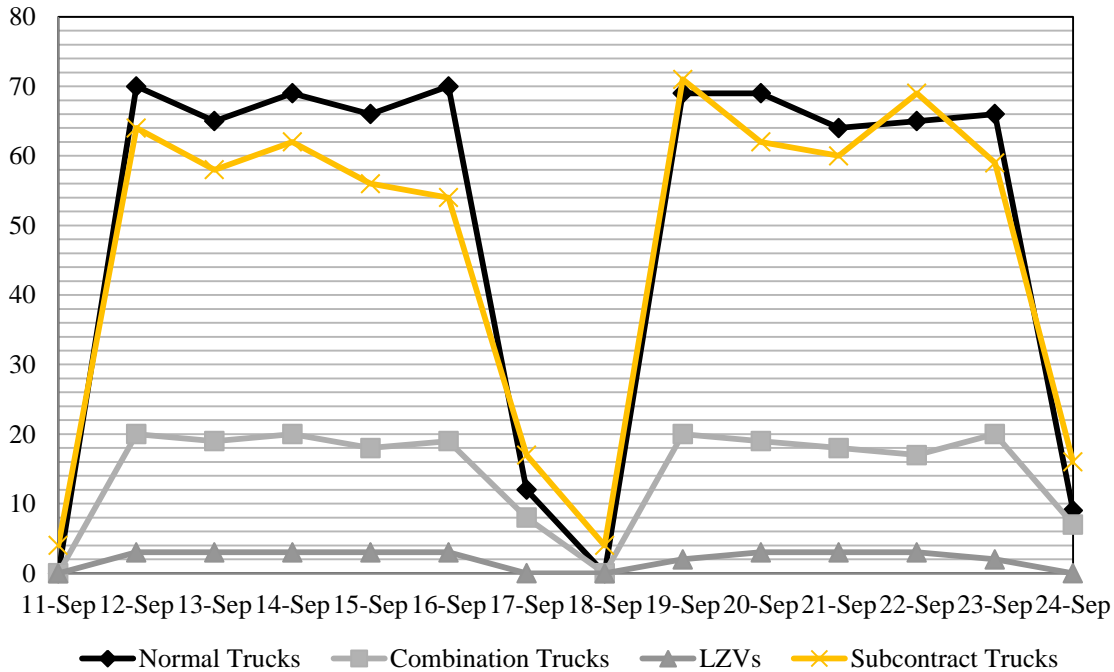


Figure 44. Base Scenario: Number of Vehicles Used Per Day

The daily truck usage figure is very similar to that of the multi-depot, automated planning case, except that the number of subcontract trucks used is somewhat lower in some days. The percentages of time that the vehicles are utilized are tabulated as follows:

Table 54. Base Scenario: Vehicle Utilization on Hour Basis

	Weekdays	Saturdays	Sundays
Normal Trucks	55.22%	5.55%	0.00%
Combination Trucks	72.58%	11.02%	0.00%
LZVs	66.91%	0.00%	0.00%
Subcontract Trucks	41.35%	7.38%	1.53%

All KPIs for the base scenario and automated planning extensions are represented. From now on, these results can be compared and interpreted.

#### Appendix E.8.4. Comparison of Scenarios

So far, 3 basic scenarios have been constructed by means of KPIs. These scenarios are: current situation (CS), single-depot automated planning (SD) and multi-depot automated planning (MD). In order that the results can be comparable, the same order set is maintained in each case. The numbers of orders planned seem to be different under each scenario, but this is mainly because Shortrec splits these orders in different ways. There were small deviations between each setting, e.g. a few number of orders were planned in one scenario, but couldn't be planned in the others, but these are eliminated by scaling the results based on the total quantity of pallets delivered ( $\approx 122400$  pallets for each case).

In the following table, results of each pair of cases are represented with the percentage differences in their values:

**Table 55.Scenario Results & Differences**

	CS	SD	% Difference (SD-CS)	CS	MD	% Difference (MD-CS)	SD	MD	% Difference (MD-SD)
<b>Number of Orders</b>	8542	8924	+4.47%	8542	8965	+4.95%	8924	8965	+0.46%
<b>Number of Trips</b>	2732	4021	+47.18%	2732	3959	+44.91%	4021	3959	-1.54%
<b>Total Km</b>	722485	750252	+3.84%	722485	692344	-4.17%	750252	692344	-7.72%
<b>Empty Km</b>	302593.9	288053	-4.81%	302593.9	226747	-25.07%	288053	226747	-21.28%
<b>Empty Km Percentage</b>	41.88%	38.39%	-8.33%	41.88%	32.75%	-21.80%	38.39%	32.75%	-14.70%
<b>Net Working Hours</b>	22007.4	20682.3	-6.02%	22007.4	19809	-9.99%	20682.3	19809	-4.22%
<b>Net Trip Time (hrs)</b>	8.06	5.14	-36.15%	8.06	5.00	-37.89%	5.14	5.00	-2.72%
<b>Kms per Trip</b>	264.45	186.58	-29.45%	264.45	174.88	-33.87%	186.58	174.88	-6.27%
<b>Orders per Trip</b>	3.13	2.22	-29.02%	3.13	2.26	-27.58%	2.22	2.26	+2.03%
<b>Number of Day Shifts</b>	2138	2149	+0.51%	2138	2095	-2.01%	2149	2095	-2.51%
<b>Number of Night Shifts</b>	594	1872	+215.15%	594	1864	+213.80%	1872	1864	-0.43%
<b>Percentage of Night Shifts</b>	21.74%	46.56%	+114.12%	21.74%	47.08%	+116.55%	46.56%	47.08%	+1.13%

*SD-CS Comparison:* In Section 6.2, it was mentioned that it is not possible to categorize the current way of planning as “single-depot” or “multi-depot”. Therefore we expect the SD setting to perform better depending on the automation and worse in terms of pure single-depot planning. Looking at the results, it can be seen that the total distance traveled is 3.84% higher in the SD case. However, it acts better in terms of empty kilometers, empty kilometer percentage and net working hours. It is not possible to state which setting is favorable in terms of these figures; a detailed cost analysis is required to arrive at this kind of a conclusion.

The reason why the number of trips is significantly higher in SD is that Shortrec counts each departure from-arrival at a depot pair as one trip. In the QTMS file on the other hand, trips including several departures and arrivals can be given a single trip number. Hence, this difference is caused by the diverse definition of trips and it explains all the significant changes in the figures related to trips.

The percentage of night shifts is significantly higher in SD case. So, it is possible to claim that Shortrec tries to distribute the deliveries evenly throughout the day rather than compressing them in the day shift.

MD-CS Comparison: A quick browse through the figures shows that MD setting performs significantly better than CS in terms of all critical KPIs. The most notable improvement (25.07%) is in the empty distance traveled, which was one of the most desirable results regarding the MD setting. The large differences in the figures related to trips are caused by the definition of trips, just like in the previous comparison. These two cases can also be compared in terms of vehicle utilizations:

**Table 56.MD-CS, Differences Between Vehicle Utilizations**

	Weekdays			Saturdays			Sundays		
	CS	MD	% Difference	CS	MD	Difference	CS	MD	% Difference
<b>Normal Trucks</b>	55.22%	61.82%	+11.96%	5.55%	8.73%	+57.22%	0.00%	1.97%	-
<b>Combination Trucks</b>	72.58%	90.52%	+24.71%	11.02%	65.14%	+491.13%	0.00%	1.63%	-
<b>LZVs</b>	66.91%	42.43%	-36.59%	0.00%	0.00%	-	0.00%	0.00%	-
<b>Subcontract Trucks</b>	41.35%	25.56%	-38.18%	7.38%	0.00%	-100.00%	1.53%	0.00%	-100.00%

The utilization of normal and combination trucks is higher, and that of LZVs and subcontract trucks is lower in the MD setting in all cases. As we stated before, the low utilization of LZVs in the MD case is a result of the single depot assignment to these trucks. Apart from this, the high utilization company owned trucks and low utilization of subcontract trucks is beneficial, in terms of efficiently using the current fleet and getting rid of the extra costs created by subcontracting.

MD-SD Comparison: It is apparent that MD is better than SD in terms of total kilometers, empty kilometers, empty kilometer percentage and net working hours. Moreover, more orders are carried in a single trip and trip length is shorter in terms of both duration and distance in the MD case. These are all implications of a more efficient planning approach.

It has been made clear that the MD setting (multi-depot, automated planning) performs significantly better than the two other basic scenarios. Now, some additional scenarios can be tested on the favorable setting, and it can be seen whether it is possible to improve the system any further.

## **Appendix E.9. Further Scenario Analysis**

5 alternative scenarios are developed in order to see the effects of some further alterations in the system. The following changes are going to be assessed in this chapter:

- S1 & S2: A hybrid planning approach is employed, in which some depots are planned together and the others are done separately.
- S3: LZVs are assigned to all depots.
- S4: Vehicles with altering origin-destination depots are utilized.
- S5: Time windows are relaxed.

### Appendix E.9.1. Scenarios S1 & S2 – Employment of a Hybrid Planning Approach

As a step in transition to multi-depot planning, firstly orders of 2 or 3 depots can be planned together and others can be added to the centralized planning system one by one later on. In S1, it is going to be assumed that the orders of Nieuwegein and Utrecht are planned together and others are done separately as in the SD setting. The reason why these two depots are chosen is that they are geographically close to each other, and the orders that belong to them constitute an important portion of the total order set. In S2, Veghel is added to the centralized planning, and the orders of these 3 depots are planned together. In the following tables results of these scenarios are compared to each other and to SD and MD settings:

Table 57. Comparison of Scenarios, S1-S2-SD-MD

	SD	S1	% Difference	S1	MD	% Difference
Number of Trips	4021	4011	-0.25%	4011	3959	-1.30%
Total Km	750252	753970	+0.50%	753970	692344	-8.17%
Empty Km	288053	305805	+6.16%	305805	226747	-25.85%
Empty Km Percentage	38.39%	40.56%	+5.64%	40.56%	32.75%	-19.25%
Net Working Hours	20682.3	20761	+0.38%	20761	19809	-4.59%
Net Trip Time (hrs)	5.14	5.18	+0.63%	5.18	5.00	-3.33%
Kms per Trip	186.58	187.98	+0.75%	187.98	174.88	-6.97%

	SD	S2	% Difference	S2	MD	% Difference
Number of Trips	4021	4004	-0.42%	4004	3959	-1.12%
Total Km	750252	739777	-1.40%	739777	692344	-6.41%
Empty Km	288053	290954	+1.01%	290954	226747	-22.07%
Empty Km Percentage	38.39%	39.33%	+2.44%	39.33%	32.75%	-16.73%
Net Working Hours	20682.3	20480	-0.98%	20480	19809	-3.28%
Net Trip Time (hrs)	5.14	5.11	-0.56%	5.11	5.00	-2.18%
Kms per Trip	186.58	184.76	-0.98%	184.76	174.88	-5.35%

	S1	S2	% Difference
Number of Trips	4011	4004	-0.17%
Total Km	753970	739777	-1.88%
Empty Km	305805	290954	-4.86%
Empty Km Percentage	40.56%	39.33%	-3.03%
Net Working Hours	20761	20480	-1.35%
Net Trip Time (hrs)	5.18	5.11	-1.18%
Kms per Trip	187.98	184.76	-1.71%

Having a quick look at the results, it can be observed that S1 gives slightly worse results than SD in terms of all KPIs. This implies that combining the planning processes in two geographically close depots is not

beneficial. Unsurprisingly, the difference between the results of S1 and MD are even higher. S2 performs better than SD in terms of total kilometers and net working hours. However the empty kilometer percentage is higher than that of SD, and there is a significant gap between MD and S2. As expected, S2 outweighs S1 in all figures.

The outcome of this analysis is that a hybrid approach is not much beneficial in terms of single and multi-depot planning. In order to take advantage of the multi-depot planning approach, the whole system should be centralized.

### Appendix E.9.2. Scenario S3: General Use of LZVs

In the basic scenarios (SD and MD), LZVs are assumed to operate only between Utrecht and Broek Op Langedijk. We would like to know what would be the effect of using these high-capacity trucks throughout the whole network. In order to simulate this scenario, input files including orders and combinations are altered accordingly, and the results are compared to those of the MD setting as follows:

**Table 58. Comparison of Results, S3 - MD**

	MD	S3	% Difference
Number of Orders	8965	8958	-0.08%
Number of Trips	3959	3890	-1.74%
Total Km	692344	674954	-2.51%
Empty Km	226747	213961	-5.64%
Empty Km Percentage	32.75%	31.70%	-3.21%
Net Working Hours	19809	19567	-1.22%
Net Trip Time (hrs)	5.00	5.03	+0.53%
Kms per Trip	174.88	173.51	-0.78%
Orders per Trip	2.26	2.30	+1.69%
Number of Day Shifts	2095	2199	+4.96%
Number of Night Shifts	1864	1691	-9.28%
Percentage of Night Shifts	47.08%	43.47%	-7.67%

Since the general use of LZV adds more capacity to the system without generating any extra costs, the system performance is improved in terms of all KPIs. This result implies that it is beneficial for the company to employ LZVs as much as possible. It is expected that modifying the usage conditions of LZVs also changes the vehicle utilizations. The comparison of vehicle utilizations between S3 and MD is represented below:

**Table 59. S3-MD, Differences Between Vehicle Utilizations**

	Weekdays			Saturdays			Sundays		
	MD	S3	% Diff	MD	S3	% Diff	MD	S3	% Diff
Normal Trucks	61.82%	59.34%	-4.02%	8.73%	10.75%	+23.20%	1.97%	2.53%	+28.22%
Combination Trucks	90.52%	84.06%	-7.13%	65.14%	11.93%	-81.69%	1.63%	0.00%	-100.00%
LZVs	42.43%	102.64%	+141.91%	0.00%	97.92%	-	0.00%	10.79%	-
Subcontract Trucks	25.56%	13.32%	-47.90%	0.00%	0.00%	-	0.00%	0.00%	-

These figures show that the utilization of LZVs significantly increases whereas that of the others decreases. In weekdays, utilization of LZVs becomes more than 100%; which is a result of the assumption that the working hours are 30 hours per day, in order to allow trucks turn back to their depots after midnight.

Utilization of combination trucks dramatically drops in the weekends. Instead of these, normal trucks and LZVs carry out the deliveries on these days. Subcontract trucks become even less effective, which gives way to getting rid of this type of vehicles and saving costs.

### Appendix E.9.3. Scenario S4: Altering Start and End Depots

In the base simulations, each vehicle is assumed to start from its origin depot, regardless of which depot's orders it is going to deliver, and end its trip at the same home depot. In an ideal multi-depot planning system we would not wish such a restriction so that the each vehicle in fleet becomes usable by the entire network.

In order to see what would be the effect of having such a system, we create combinations for each origin-destination pair in sufficient quantities. Since 15 depots are present, we have  $15 \times 15 = 225$  different types of origin-destination pairs. Using these 225 combination types, a total of 1512 trucks are generated. The same set of trucks is also used for a controlling simulation (CS'), where the starting and ending depots are kept the same.

All trucks generated are of the normal type, having a capacity of 26 pallets. The simulation is performed for a 3-day period between 14.09.2011 and 16.09.2011. The results of the simulation of S4 and the controlling simulation (CS') are tabulated as follows:

**Table 60. Comparison of Results, S4-CS'**

	CS'	S4	% Difference
Number of Orders	3200	3204	+0.13%
Number of Trips	1499	1453	-3.07%
Total Km	266794	235867	-11.59%
Empty Km	90875	63017	-30.66%
Empty Km Percentage	34.06%	26.72%	-21.56%
Net Working Hours	7318	6841	-6.52%
Net Trip Time (hrs)	4.88	4.71	-3.56%
Kms per Trip	177.98	162.33	-8.79%
Orders per Trip	2.13	2.21	+3.29%

It is apparent that the system performs much better in S4. Both the empty kilometer percentage and the total kilometers traveled drops significantly under this scenario. The main reason behind this is that empty vehicles which complete their deliveries will return to the nearest depot instead of their home depot. This brings a huge save on empty kilometers. This extension should be certainly considered in detail and adaptation of this approach to the planning system can generate great benefits for the company

### Appendix E.9.4. Scenario S5: Relaxation of Time Windows



In order to see how restrictive the timing limitations are, time windows are relaxed for 2 hours for each order, and the simulation is performed keeping all other settings the same as the MD case. It is observed that 9646 orders are planned which is 7.6% higher than the number of orders that were be planned in the MD setting.

In order to compare the two scenarios, the results of the simulation of S5 are scaled with respect to the total quantity of pallets delivered in MD. The following outcomes are obtained after the adjustments:

**Table 61. Comparison of Results, S5-MD**

	MD	S5	% Difference
<b>Total Km</b>	692344	641798	-7.30%
<b>Empty Km</b>	226747	217570	-4.05%
<b>Empty Km Percentage</b>	32.75%	33.90%	+3.51%
<b>Net Working Hours</b>	19809	18750	-5.35%

These figures demonstrate that relaxed time windows enable the software to plan the orders more efficiently in terms of total kilometers and net working hours. However, percentage of empty kilometers gets worse under this assumption.

More scenarios were tried to be tested in order to further improve the system, however due to the limited capabilities of the simulation software, attempts to test additional scenarios did not end up positively. It would be interesting to see the effects of multi-day planning with overnight stays or allowing trailers to be transferred on barges, but it was not possible to evaluate the advantages of these options with the current software.

## **Appendix E.10. Conclusions & Further Research Directions**

Throughout this project, potential benefits of employing multiple-depot and automated planning approaches in the contract logistics branch of a 3PL company, K+N Netherlands have been investigated. Four basic scenarios were constructed at the beginning, in order to test the effects of multi-depot and automated procedures both separately and jointly. After the description of the problem environment with a systems perspective, the current situation in the company has been analyzed, followed by an effort to examine the impact of implementing multi-depot planning, manually.

There were indications of both single-depot and multi-depot procedures in the current system, therefore the number basic scenarios to be compared has been reduced to three, consisting of the base (current) scenario and automated planning cases for both single-depot (SD) and multi-depot (MD) approaches.

Transport planning tool Shortrec has been utilized for simulating automated plans. After identifying the inputs, outputs, settings and assumptions of the software, basic scenarios were simulated and their results were interpreted. It has been realized that MD setting outweighs the other cases in terms of all the KPIs. However, there has not been found any obvious advantages of the SD case with regard to the current situation.

Having tested some additional scenarios, it has been noticed that using a hybrid approach between single and multi-depot planning is not beneficial for the company; but significant improvements can be achieved

via opening the LZVs for general use, having vehicles with altering origin and destination depots and relaxing the time windows.

Automated planning is a good way of generating good solutions in a significantly shorter time period than manual planning. However due to the limited capabilities of the planning software on hand, it was not possible to simulate more practical scenarios on the system. With a more advanced planning tool, we believe that more accurate results can be obtained with more detailed specifications.

Shortrec is mostly used for tactical planning purposes. It requires that all the orders are known and stationary. However, we mentioned before that as well as orders given 48h and 24h beforehand, there are also rush orders that should be attached to existing trips, adding dynamism to the actual system. Therefore it is not possible to automate the whole planning system with Shortrec; but an alternative tool that allows real-time execution is required for this purpose. OTD has these capabilities and the transition to automated planning can be completed only after the implementation of this software is finished.

The current version of the Shortrec only allows single-day planning; all the orders that are planned together have the same delivery day. However we expect that if multi-day planning was allowed, more efficient plans could have been generated. This extension appears to be a good case to be examined in the future.

Empty kilometer percentage is a good indicator of multi-depot planning. The lower this measure gets, it is understood that the more effective the multi-depot planning has been applied. In this project, the most significant improvement in terms of empty kilometer percentage has been achieved by defining numerous vehicles with every possible start-end depot combination. It would be better to see this impact by using the original vehicle fleet and freeing the start and end locations; but Shortrec is not capable of doing it in this way. The evaluation of this option with a more advanced planning tool is another good research direction.

Since the statistics such as kilometers traveled or working hours for trips performed by charter trucks are not kept in QTMS data pack, it was not possible to include an important portion of the orders in the comparisons. More realistic evaluations can be done having this information on hand.

Throughout the project, all the evaluations have been made in terms of the defined KPIs. A more detailed study taking into consideration the actual cost terms can be carried out and more precise interpretations can be made about the costs and benefits of each option to the company.

In summary, this study has shown that it is possible to improve the current system by implementing an automated multi-depot planning approach and this transition can be best performed with a more advanced planning tool. There exists other improvement opportunities which would constitute interesting research topics for the future.

## Appendix E.11. K+N Netherlands FMCG Facilities

**Table 62. K+N Netherlands FMCG Facilities**

### *Facility Ede*

Characteristics	Key Capabilities	Main Customers
<ul style="list-style-type: none"> <li>▪ 23,000 m<sup>2</sup></li> <li>▪ 10.0 m free height</li> <li>▪ Fully sprinkled</li> <li>▪ Centrally located</li> </ul>	<ul style="list-style-type: none"> <li>▪ Warehousing and Co-packing</li> <li>▪ RF based/paperless operation</li> <li>▪ Focused on non-food customers</li> <li>▪ Multilingual</li> </ul>	<ul style="list-style-type: none"> <li>▪ Kimberly-Clark Professional</li> <li>▪ Kimberly-Clark Healthcare</li> </ul>

### *Facility Nieuwegein*

Characteristics	Key Capabilities	Main Customers
<ul style="list-style-type: none"> <li>▪ 34,000 m<sup>2</sup></li> <li>▪ Automatic Layer Picker (ALP)</li> <li>▪ Centrally located</li> </ul>	<ul style="list-style-type: none"> <li>▪ FMCG Food oriented</li> <li>▪ Multilingual</li> <li>▪ Complete service offering</li> <li>▪ National transport</li> </ul>	<ul style="list-style-type: none"> <li>▪ Sara Lee</li> <li>▪ Nutricia</li> <li>▪ SCA</li> <li>▪ Britvic</li> </ul>

### *Facility Oud-Beijerland*

Characteristics	Key Capabilities	Main Customers
<ul style="list-style-type: none"> <li>▪ 5,600 m<sup>2</sup> ambient storage</li> <li>▪ Temperature controlled area – Dry storage                             <ul style="list-style-type: none"> <li>– 5 degrees</li> <li>– 200 m<sup>2</sup></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>▪ Warehousing and Co-packing</li> <li>▪ Transport management</li> <li>▪ Custom solutions</li> <li>▪ Multilingual</li> <li>▪ National distribution</li> </ul>	<ul style="list-style-type: none"> <li>▪ Intertaste</li> </ul>

### *Facility Raamsdonksveer*

Characteristics	Key Capabilities	Main Customers
<ul style="list-style-type: none"> <li>▪ 40,000 m<sup>2</sup></li> <li>▪ 36 loading docks</li> <li>▪ Automatic Layer Picker (ALP)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Manufacturing Consolidation Center</li> <li>▪ National distribution</li> <li>▪ Co-packing</li> <li>▪ Very high security level</li> <li>▪ RF based/paperless operation</li> <li>▪ Multilingual</li> </ul>	<ul style="list-style-type: none"> <li>▪ Kimberly-Clark Consumer</li> <li>▪ Reckitt Benckiser</li> <li>▪ Unilever Home and Personal Care</li> </ul>

### *Facility Utrecht*

Characteristics	Key Capabilities	Main Customers
<ul style="list-style-type: none"> <li>▪ 15,000 m<sup>2</sup></li> <li>▪ 12,500 full pallets storage</li> <li>▪ 20 loading docks</li> <li>▪ Drive-in-shelving system</li> </ul>	<ul style="list-style-type: none"> <li>▪ FMCG Food oriented</li> <li>▪ Manufacturing consolidation center</li> <li>▪ National distribution</li> </ul>	<ul style="list-style-type: none"> <li>▪ Duyvis</li> <li>▪ Quaker</li> <li>▪ Smiths / Lay's</li> </ul>

### *Facility Veghel*

Characteristics	Key Capabilities	Main Customers
<ul style="list-style-type: none"> <li>▪ 103,000 m<sup>2</sup> (3 buildings)</li> <li>▪ High bay warehouse</li> </ul>	<ul style="list-style-type: none"> <li>▪ Warehousing (ALP) / Factory warehousing</li> <li>▪ Raw materials and packaging</li> </ul>	<ul style="list-style-type: none"> <li>▪ Unilever Foods</li> <li>▪ Mars</li> </ul>

	<ul style="list-style-type: none"> <li>▪ National distribution / Primary transport</li> <li>▪ Transport co-ordination</li> <li>▪ Co-packing</li> <li>▪ RF based/paperless operation</li> <li>▪ Complete service offering</li> </ul>	
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## Appendix E.12. K+N Netherlands Return Centers

Table 63.K+N Netherlands Return Centers

### *Facility Pijnacker*

Characteristics	Key Capabilities
<ul style="list-style-type: none"> <li>▪ 14,000 m<sup>2</sup></li> <li>▪ 700 – 1,000 trucks a day</li> </ul>	<ul style="list-style-type: none"> <li>▪ In-house Return Center <ul style="list-style-type: none"> <li>–Processing of returns and waste</li> <li>–Processing of re-usable packaging</li> <li>–Crate and pallet rental / washing</li> <li>–RF based/paperless operation</li> <li>–Multilingual</li> </ul> </li> </ul>

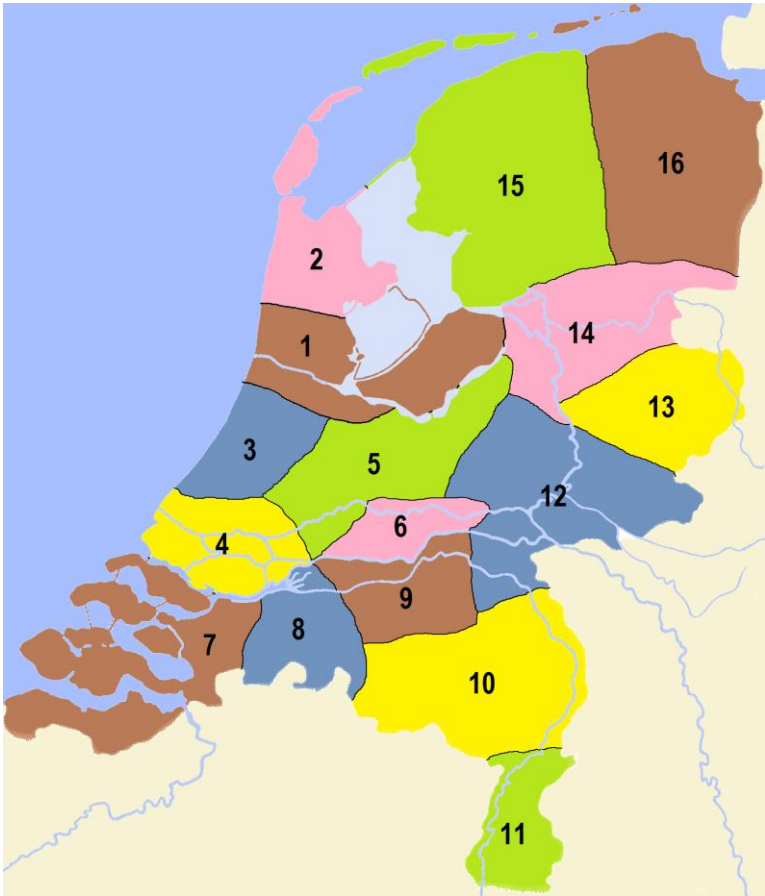
### *Facility Tilburg*

Characteristics	Key Capabilities
<ul style="list-style-type: none"> <li>▪ 1,000 m<sup>2</sup> crate washing machine</li> <li>▪ 14,000 m<sup>2</sup> sorting area</li> <li>▪ 1,750 m<sup>2</sup> docks in-out</li> </ul>	<ul style="list-style-type: none"> <li>▪ Multi-user crate and washing center</li> <li>▪ Crates rental / pallet rental</li> <li>▪ FTL transport</li> <li>▪ Pool management</li> <li>▪ Receiving, sorting and sending returns</li> <li>▪ Control returning goods</li> </ul>

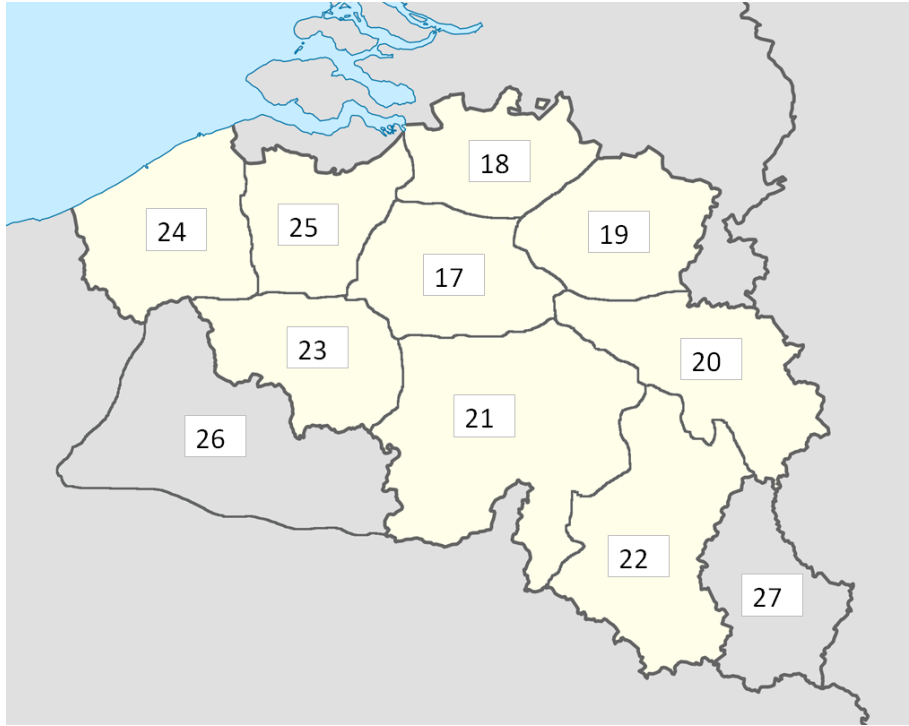
### *Facility Vaassen*

Characteristics	Key Capabilities
<ul style="list-style-type: none"> <li>▪ 16,000 m<sup>2</sup></li> <li>▪ Crate washing machines</li> </ul>	<ul style="list-style-type: none"> <li>▪ National platform chilled distribution</li> <li>▪ Cross docking activities</li> <li>▪ Conditioned warehousing</li> <li>▪ Services (crate and pallet rental / washing)</li> <li>▪ Complete chilled service offerings</li> </ul>

**Appendix E.13. Subregion Maps**



**Figure 45. Subregions on the Map – 1**



**Figure 46. Subregions on the Map – 2**

## Appendix E.14. Steps of the Original and Modified Trips

Table 64.Extension 1: Steps of the Original and Modified Trips

Original Case						
Trip No	From City	To City	Kilometers	Driving Time	Full?	Note
1	Nieuwegein	Groningen	201	125	1	
1	Groningen	Sneek	80.6	56	1	
1	Sneek	Nieuwegein	152	94	0	
2	Nieuwegein	Bolsward	160	98	0	
2	Bolsward	Nieuwegein	160	98	1	
3	Raamsdonksveer	Utrecht	168.6	120	0	x3
3	Utrecht	Raamsdonksveer	168.6	120	1	x3
4	Nieuwegein	Raamsdonksveer	312.9	238	0	x7
4	Raamsdonksveer	Nieuwegein	312.9	238	1	x7
5	Nieuwegein	Olen	122	102	0	
5	Olen	Willebroek	59.7	44	1	
5	Willebroek	Schoten	28.9	26	0	
5	Schoten	Beersel Lot	66.6	50	1	
5	Beersel Lot	Nieuwegein	184	108	0	
6	Veghel	Schoten	113	76	0	
6	Schoten	Zellik	52.7	52	1	
6	Zellik	Veghel	160	97	0	

Total Km	2503.5
Empty Km	1401.4
Total Driving Time (hrs)	29.03

After Modifications						
Trip No	From City	To City	Kilometers	Driving Time	Full?	Note
1'	Nieuwegein	Groningen	201	125	1	
1'	Groningen	Sneek	80.6	56	1	
1'	Sneek	Bolsward	13.9	13	0	
1'	Bolsward	Nieuwegein	160	98	1	
2'	Nieuwegein	Utrecht	36	57	0	x3
2'	Utrecht	Raamsdonksveer	168.6	120	1	x3
2'	Raamsdonksveer	Nieuwegein	312.9	238	1	x7
2'	Nieuwegein	Raamsdonksveer	178.8	136	0	x4
3'	Veghel	Schoten	113	76	0	
3'	Schoten	Zellik	52.7	52	1	
3'	Zellik	Olen	74.6	70	0	
3'	Olen	Willebroek	59.7	44	1	
3'	Willebroek	Schoten	28.9	26	0	
3'	Schoten	Beersel Lot	66.6	50	1	
3'	Beersel Lot	Veghel	174	104	0	

Total Km	1721.3
Empty Km	619.2
Total Driving Time (hrs)	21.08

## Appendix E.15. Simulation Results of Single-Depot, Automated Planning

Table 65. Simulation Results of Single-Depot, Automated Planning

BRO			
Total costs:	22786	Normal workingtime:	561:10:00
Nof trucks:	7	Overtime:	00:00
Nof orders:	112	Driving time:	416:49:00
Nof unscheduled orders	2	Unloading time:	92:33:00
Nof trips:	90	Loading time	47:15:00
Nof km:	26535	Waiting time (pause):	04:33
Tot. quantity () delivered:	148517	orders/trip:	1.24
Tot. quantity () lifted:	122506	Nof km/:	0.18
Tot. quantity (unscheduled):	6000	Nof km/trip:	294.83
		/trip:	1650.19
		Cost/:	0.15
		Average speed (km/hr):	63.66

DEL			
Total costs:	7453	Normal workingtime:	152:27:00
Nof trucks:	4	Overtime:	00:00
Nof orders:	43	Driving time:	86:44:00
Nof unscheduled orders	0	Unloading time:	34:19:00
Nof trips:	37	Loading time	30:42:00
Nof km:	4339	Waiting time (pause):	00:42
Tot. quantity () delivered:	91609	orders/trip:	1.16
Tot. quantity () lifted:	0	Nof km/:	0.05
Tot. quantity (unscheduled):	0	Nof km/trip:	117.27
		/trip:	2475.92
		Cost/:	0.08
		Average speed (km/hr):	50.03

GRI			
Total costs:	1446	Normal workingtime:	12:51
Nof trucks:	1	Overtime:	00:00
Nof orders:	3	Driving time:	09:54
Nof unscheduled orders	0	Unloading time:	01:36
Nof trips:	3	Loading time	01:21
Nof km:	684	Waiting time (pause):	00:00
Tot. quantity () delivered:	640	orders/trip:	1
Tot. quantity () lifted:	0	Nof km/:	1.07
Tot. quantity (unscheduled):	0	Nof km/trip:	228
		/trip:	213.33
		Cost/:	2.26
		Average speed (km/hr):	69.09

HER			
Total costs:	11355	Normal workingtime:	330:15:00
Nof trucks:	8	Overtime:	00:00
Nof orders:	89	Driving time:	92:43:00
Nof unscheduled orders	0	Unloading time:	77:21:00



Nof trips:	71	Loading time	37:24:00
Nof km:	4901	Waiting time (pause):	122:47:00
Tot. quantity () delivered:	114400	orders/trip:	1.25
Tot. quantity () lifted:	116904	Nof km/:	0.04
Tot. quantity (unscheduled):	0	Nof km/trip:	69.03
		/trip:	1611.27
		Cost/:	0.1
		Average speed (km/hr):	52.86

NIE			
Total costs:	146035	Normal workingtime:	3689:11:00
Nof trucks:	41	Overtime:	00:00
Nof orders:	1546	Driving time:	1503:43:00
Nof unscheduled orders	0	Unloading time:	899:53:00
Nof trips:	506	Loading time	359:14:00
Nof km:	91280	Waiting time (pause):	926:21:00
Tot. quantity () delivered:	1010584	orders/trip:	3.06
Tot. quantity () lifted:	643475	Nof km/:	0.09
Tot. quantity (unscheduled):	0	Nof km/trip:	180.4
		/trip:	1997.2
		Cost/:	0.14
		Average speed (km/hr):	60.7

OUD			
Total costs:	19393	Normal workingtime:	426:13:00
Nof trucks:	9	Overtime:	00:00
Nof orders:	95	Driving time:	232:00:00
Nof unscheduled orders	0	Unloading time:	80:23:00
Nof trips:	72	Loading time	32:38:00
Nof km:	12125	Waiting time (pause):	81:12:00
Tot. quantity () delivered:	98221	orders/trip:	1.32
Tot. quantity () lifted:	139588	Nof km/:	0.12
Tot. quantity (unscheduled):	0	Nof km/trip:	168.4
		/trip:	1364.18
		Cost/:	0.2
		Average speed (km/hr):	52.26

RAA			
Total costs:	171036	Normal workingtime:	3910:05:00
Nof trucks:	62	Overtime:	00:00
Nof orders:	1304	Driving time:	1735:37:00
Nof unscheduled orders	36	Unloading time:	706:26:00
Nof trips:	502	Loading time	376:17:00
Nof km:	107722	Waiting time (pause):	1091:45:00
Tot. quantity () delivered:	1035367	orders/trip:	2.6
Tot. quantity () lifted:	113298	Nof km/:	0.1
Tot. quantity (unscheduled):	142500	Nof km/trip:	214.59
		/trip:	2062.48
		Cost/:	0.17
		Average speed (km/hr):	62.07

TIL			
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Total costs:	23146	Normal workingtime:	514:07:00
Nof trucks:	14	Overtime:	00:00
Nof orders:	164	Driving time:	219:09:00
Nof unscheduled orders	0	Unloading time:	132:19:00
Nof trips:	126	Loading time	81:05:00
Nof km:	12847	Waiting time (pause):	81:34:00
Tot. quantity () delivered:	249091	orders/trip:	1.3
Tot. quantity () lifted:	126484	Nof km/:	0.05
Tot. quantity (unscheduled):	0	Nof km/trip:	101.96
		/trip:	1976.91
		Cost/:	0.09
		Average speed (km/hr):	58.62

UTC			
Total costs:	185515	Normal workingtime:	4538:52:00
Nof trucks:	43	Overtime:	00:00
Nof orders:	1239	Driving time:	2414:58:00
Nof unscheduled orders	59	Unloading time:	953:32:00
Nof trips:	813	Loading time	513:50:00
Nof km:	152234	Waiting time (pause):	656:32:00
Tot. quantity () delivered:	1553429	orders/trip:	1.52
Tot. quantity () lifted:	990857	Nof km/:	0.1
Tot. quantity (unscheduled):	134540	Nof km/trip:	187.25
		/trip:	1910.74
		Cost/:	0.12
		Average speed (km/hr):	63.04

VAA			
Total costs:	49866	Normal workingtime:	1176:32:00
Nof trucks:	12	Overtime:	00:00
Nof orders:	364	Driving time:	600:51:00
Nof unscheduled orders	1	Unloading time:	288:48:00
Nof trips:	243	Loading time	154:35:00
Nof km:	34379	Waiting time (pause):	132:18:00
Tot. quantity () delivered:	452746	orders/trip:	1.5
Tot. quantity () lifted:	406667	Nof km/:	0.08
Tot. quantity (unscheduled):	0	Nof km/trip:	141.48
		/trip:	1863.15
		Cost/:	0.11
		Average speed (km/hr):	57.22

VEG			
Total costs:	339723	Normal workingtime:	8794:02:00
Nof trucks:	88	Overtime:	00:00
Nof orders:	3624	Driving time:	3984:15:00
Nof unscheduled orders	84	Unloading time:	1992:34:00
Nof trips:	1241	Loading time	816:39:00
Nof km:	231620	Waiting time (pause):	2000:34:00
Tot. quantity () delivered:	2353962	orders/trip:	2.92
Tot. quantity () lifted:	1609004	Nof km/:	0.1
Tot. quantity (unscheduled):	178662	Nof km/trip:	186.64
		/trip:	1896.83

Cost/:	0.14
Average speed (km/hr):	58.13

WEZ			
Total costs:	6619	Normal workingtime:	115:00:00
Nof trucks:	2	Overtime:	00:00
Nof orders:	31	Driving time:	58:56:00
Nof unscheduled orders	0	Unloading time:	24:36:00
Nof trips:	24	Loading time	19:06
Nof km:	3810	Waiting time (pause):	12:22
Tot. quantity () delivered:	54690	orders/trip:	1.29
Tot. quantity () lifted:	0	Nof km/:	0.07
Tot. quantity (unscheduled):	0	Nof km/trip:	158.75
		/trip:	2278.75
		Cost/:	0.12
		Average speed (km/hr):	64.65

WOG			
Total costs:	43751	Normal workingtime:	847:31:00
Nof trucks:	15	Overtime:	00:00
Nof orders:	123	Driving time:	578:07:00
Nof unscheduled orders	59	Unloading time:	112:20:00
Nof trips:	116	Loading time	85:38:00
Nof km:	39558	Waiting time (pause):	71:26:00
Tot. quantity () delivered:	276300	orders/trip:	1.06
Tot. quantity () lifted:	59900	Nof km/:	0.14
Tot. quantity (unscheduled):	166000	Nof km/trip:	341.02
		/trip:	2381.9
		Cost/:	0.16
		Average speed (km/hr):	68.43

ZAA			
Total costs:	11371	Normal workingtime:	237:12:00
Nof trucks:	6	Overtime:	00:00
Nof orders:	48	Driving time:	147:37:00
Nof unscheduled orders	0	Unloading time:	41:25:00
Nof trips:	46	Loading time	35:27:00
Nof km:	9947	Waiting time (pause):	12:43
Tot. quantity () delivered:	110183	orders/trip:	1.04
Tot. quantity () lifted:	1063	Nof km/:	0.09
Tot. quantity (unscheduled):	0	Nof km/trip:	216.24
		/trip:	2395.28
		Cost/:	0.1
		Average speed (km/hr):	67.38

ZWO			
Total costs:	29794	Normal workingtime:	637:05:00
Nof trucks:	10	Overtime:	00:00
Nof orders:	139	Driving time:	347:34:00
Nof unscheduled orders	0	Unloading time:	126:49:00
Nof trips:	131	Loading time	97:16:00
Nof km:	18271	Waiting time (pause):	65:26:00

Tot. quantity () delivered:	296150	orders/trip:	1.06
Tot. quantity () lifted:	62200	Nof km/:	0.06
Tot. quantity (unscheduled):	0	Nof km/trip:	139.47
		/trip:	2260.69
		Cost/:	0.1
		Average speed (km/hr):	52.57

## Appendix E.16. Simulation Results of Multi-Depot, Automated Planning

**Table 66. Simulation Results of Multi-Depot, Automated Planning**

Total costs:	1016757	Normal workingtime:	24503:51
Nof trucks:	231	Overtime:	00:00
Nof orders:	8965	Driving time:	11525:37
Nof unscheduled orders	889	Unloading time:	5569:53:00
Nof trips:	3959	Loading time	2713:33:00
Nof km:	692344	Waiting time (pause):	4694:48:00
Tot. quantity () delivered:	7843089	orders/trip:	2.26
Tot. quantity () lifted:	4397016	Nof km/:	0.09
Tot. quantity (unscheduled):	2466660	Nof km/trip:	174.88
		/trip:	1981.08
		Cost/:	0.13
		Average speed (km/hr):	60.07

## Appendix E.17. Simulation Results of Base Case, Remaining Orders

**Table 67. Simulation Results of Base Case, Remaining Orders**

Total costs:	14375	Normal workingtime:	266:08:00
Nof trucks:	14	Overtime:	433:12:00
Nof orders:	64	Driving time:	183:44:00
Nof unscheduled orders	1	Unloading time:	45:09:00
Nof trips:	41	Loading time	19:53
Nof km:	12192	Waiting time (pause):	450:34:00
Tot. quantity () delivered:	56763	orders/trip:	1.56
Tot. quantity () lifted:	45016	Nof km/:	0.21
Tot. quantity (unscheduled):	3000	Nof km/trip:	297.37
		/trip:	1384.46
		Cost/:	0.25
		Average speed (km/hr):	66.36