MASTER

Multi-component maintenance planning of ABM- and CBM- components in a maritime setting

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Multi-component maintenance planning of ABM- and CBM-components in a maritime setting

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Abstract

In this master thesis, the maintenance modelling for the maritime sector is addressed. The maritime sector is characterized by the cost difference between operational states. E.g., when the ship leaves the harbour, performing maintenance becomes more expensive and time consuming. With the existence of operational state cost differences, the cost factor becomes time dependent. The traditional maintenance models do not support this. This project proposes a model that incorporates the operational state cost differences, dependencies between components and the integration of an age-based maintenance model with a condition-based maintenance model. A heuristic to perform the clustering of the age-based maintenance model is constructed. For the condition-based maintenance model, a dynamic programming model is constructed. The age-based maintenance and condition-based maintenance models both have a single-component and a multi-component variant. The clustering benefit of the multi-component model is evaluated in a case study; the maintenance planning of the Marlin weapon system owned by the Royal Netherlands Navy. A sensitivity analysis is performed to get insights in the influence of the input variables on the maintenance planning.

Executive summary

This project is performed as part of the MaSeLMa project. It serves as the final requirement to fulfil the master course Operations Management and Logistics. The project is conducted at the Royal Netherlands Navy.

This project focusses on the maintenance planning in a maritime setting. The maritime setting is characterized by having different operational states. The RNLN distinguishes four operational states: harbour, transit, mission, and in-dock overhaul. These operational states differ from a maintenance perspective, because the maintenance costs differ among them. Typically, the ships leave the harbour frequently during the between in dock overhauls. The difference in maintenance cost can be caused by the geographical distance between the ships and the harbour, by the mission abandonment that might be caused by component failure, or by the unavailability of equipment and tools at some of the operational states. The large cost difference between the operational states asks for a sophisticated maintenance planning. The classical maintenance models assume that the maintenance cost are stationary and therefore time-independent (Nahmias, 2009). This project proposes a model that incorporates the operational states in the maintenance planning.

The maritime sector typically uses sophisticated systems that are very costly. The spare parts, maintenance tasks and labour can be very expensive. In addition, the system architecture is often complex. To be able to perform maintenance tasks, the system needs to be dismantled and the tasks need to be prepared first. The time and costs that are incurred to do this can be referred to as a structural dependency. To prevent superfluous effort and costs on the maintenance tasks, it might be interesting to cluster multiple maintenance tasks. When the planning of multiple maintenance tasks is done at once, the structural dependencies between components can be included in the decision-making. This project proposes multi-component models to incorporate the structural dependencies between components.

An upcoming trend in the maritime sector is condition measurement and condition-based maintenance. More and more sensors are installed in the equipment and research is done into physics of failure. This way, the maintenance planning can be based on the actual condition or degradation of the system. However, condition-measurement can be too costly or might be unreliable for some components. It is unreasonable to assume that condition-based maintenance will be applied on all components within the foreseeable future. Hence, age-based maintenance models and condition-based maintenance models are expected to be used alongside. This project will propose a way to integrate the maintenance planning of components that are managed according to an age-based policy and components that are managed according to a condition-based policy. The proposed condition-based model can also be used for usage-based maintained components.

The research question of this project is formulated as:

"How can a multi-component policy be used to integrate a condition-based maintenance policy with a static maintenance planning in a setting with operational states?"

To answer this question, multiple maintenance models are constructed. First, the single-component age-based maintenance problem is modelled such that it incorporates the operational states. This model uses a heuristic to find cost efficient maintenance dates. Then, a multi-component age-based maintenance model is constructed. This model quantifies the cost that is incurred when a maintenance task is advanced or postponed with respect to the single-component optimal date. When maintenance is advanced, the expectation is that more preventive maintenance will be performed. When maintenance is postponed, the expectation is that more corrective maintenance is performed. If the cost for rescheduling the maintenance task is lower than the set-up cost that can be saved by clustering, it is beneficial to cluster the maintenance tasks. A heuristic is proposed to perform the clustering of the maintenance tasks in a relatively fast and close to optimal manner. The optimality of the heuristic is 97.2% on average.

A condition-based maintenance model is constructed. A dynamic programming approach is used to do this. The model decides whether it is beneficial to perform maintenance now or to postpone the maintenance. The expected cost of both options is expressed to quantify the consequences of the decision. A single-component and a multi-component variant are built. The multi-component variant incorporates the structural dependencies between the condition-based maintained components and the age-based maintained components. This way the planning of both types of components is integrated.

The performance of the proposed models is evaluated in the case study. The Marlin weapon system of the RNLN is used. The performance of the proposed models is compared with a classical age-based maintenance model. In addition, the multi-component models are compared to the single-component models to evaluate the clustering benefit. The proposed age-based maintenance models perform much better than the classical maintenance model. A 76% lower maintenance cost is obtained by the proposed models for the reference parameter settings. The classical maintenance model does not avoid maintenance during the high cost operational states and is thereby an inappropriate model for the maritime setting. The clustering benefit of the proposed models depends on the parameter settings. In the parameter setting that is used as a reference, the clustering benefit of the multi-component models is low. This is caused by the highly clustered nature of the parameter setting. The high cost difference between operational states in combination with a low corrective maintenance penalty and a mission planning with a low amount of state changes will cause the single-component planning to be highly clustered even without including the dependencies between components in the decision-making.

In the sensitivity analysis, a strong interrelation between the effects of the variables on the clustering benefit is found. For example, when the corrective penalty (cost difference between preventive and corrective maintenance) becomes higher, the single-component maintenance planning becomes less clustered. In that case, small differences in the maintenance cost between components are enlarged and will spread out the single-component optimal maintenance dates. This increases the potential clustering benefit. The operational state cost difference makes the maintenance planning focus on the end of the harbour periods and has the opposite effect as the corrective penalty. Due to the operational states and their cost difference, most of the maintenance decisions are mostly about maintaining the component just before one mission or just before the other. When parameters change, one might not observe big differences in the maintenance planning until the change in parameter values becomes high enough to shift the maintenance date to the harbour state before a preceding or succeeding mission. This can be a large difference in timing, hence a sudden change in the maintenance timing and resulting maintenance cost and availability might occur.

In a nutshell, the clustering benefit is higher (compared to the reference setting) for higher set-up cost levels, when more components can be clustered, when the operational state cost difference is lower, when the corrective penalty is higher, when the failure parameters of the components are less similar and when the mission schedule has shorter operational states and more state changes. In these cases, the clustering benefit ranges from to 0-20% of the total expected maintenance cost that is obtained by the single-component model. This is a noteworthy cost-saving.

Besides the potential cost savings, the model should be used for the insights it gives into the penalty cost for rescheduling and the influence of the dependencies between components on the maintenance planning. The sensitivity analysis can be consulted to get insight in the clustering benefit in different parameter settings. In this project, a tool is created that can be used to do numerical tests and evaluations and to optimize the maintenance planning.

Acknowledgement

Writing this chapter will conclude my work on this thesis. This will be the end of a six year long journey as a student in Eindhoven. It has been a period involving personal growth, learning and joy. I want to thank several people that helped me along the way.

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Next, I want to thank Arnold Boomstra for supervising me from the side of the RNLN. I want to thank you for your willingness to support this project when we struggled to find an application for the RNLN and for your help on finding an appropriate application when the direction of this project seemed unclear.

I want to thank my parents for giving me the opportunity to go to college and for supporting me along the way. Sometimes, it might have been unclear for you what I was so busy with here in Brabant, but you kept encouraging me nonetheless. At last, I want to thank my wife, Lisanne, for her ongoing support and motivation to continue at times of disappointments and frustration. I know you have been looking forward to the time that is to come. Finally, here we are.

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List of abbreviations

Abbreviation	**Meaning**
ABM	*Age-based maintenance*
ABM-components	*Components of which the maintenance is managed according to an age-based maintenance policy*
CBM	*Condition-based maintenance*
CBM-components	*Components of which the maintenance is managed according to a condition-based maintenance policy*
DLM	*Depot level maintenance*
DMI	*Directorate Material Sustainment (Directie Materiele Instandhouding)*
ILM	*Intermediate level maintenance*
ILS	*Integrated logistics support*
MaSeLMa	*Integrated Maintenance and Service Logistics Concepts for Maritime Assets*
MTTF	*Mean time to failure or mean time between failures*
OLM	*Operational level maintenance*
RNLN	*Royal Netherlands Navy*

1 Introduction

In this chapter, the research problem is introduced. The approach and the scope of this project are described and the relevance of the project is explained.

1.1 Research environment

This research is conducted as part of the MaSelMa project. MaSelMa is an abbreviation of Integrated Maintenance and Service Logistics Concepts for Maritime Assets. The project aims to reduce the total maintenance cost and to increase the availability of the assets in the maritime sector. In the maritime sector, maintenance cost is a significant part of the exploitation cost of the assets. The assets that are used in the maritime sector are typically complex and expensive. In addition, the sector has to deal with high availability targets. Therefore, a proper maintenance organization is crucial.

This project is conducted at the RNLN. The Royal Netherland Navy (RNLN) is the oldest service of the Dutch armed force. It provides safety at sea, operating single-handed or together with other armed forces or with its allies. It performs crisis management, humanitarian assistance and disaster relief for the Netherlands and abroad. It has a wide variation of ships, containing state of the art equipment. The Directorate Material Sustainment (DMI) is in charge of maintaining the ships and the equipment and is ensuring a high availability of the fleet. The DMI owns a repair-shop with advanced equipment in which the components are repaired. It also owns a warehouse containing the spare parts inventory.

1.2 Research problem

The assets in the maritime sector face a wide variety of environmental conditions as they travel around the world. The use of the assets does vary over time, as the ships are used in different operational states. In these operational states the degradation behaviour of the assets might be different, as the intensity of the use of the asset differs among them. Also the cost and effort for performing maintenance is likely to be different among the operational state.

Due to the complexity of the assets, dependencies between the components with respect to the maintenance activities are likely to be present. Under the existence of dependencies, cost savings can be made by combining or clustering maintenance activities.

A simple example of a dependency is set-up cost that is incurred when the system is maintained. Cost can be saved by combining maintenance activities so the set-up costs are only incurred once. When a part needs maintenance action, the entire system must be shut down in most situations. As availability is crucial for the RNLN, downtime can be saved when maintenance tasks are combined.

This project addresses the maintenance planning for setting in which the asset owner must deal with operational states and dependencies between the components.

1.3 Background of the problem

The problem is based on the maintenance organization at the RNLN. In this section, the current maintenance organization is explained. Preceding research is performed in the MaSelMa project; this project is the third project in line and should complement the preceding research, which is also explained in this section.

1.3.1 Maintenance organization at the RNLN

At the RNLN, an approach called Integrated Logistic Support (ILS) is used. This approach is highly structured; having distinct phases, predefined documentation, standards and tests. The DMI (Directorate Material Sustainment) is responsible for most of the process. The lifetime of the system is depicted in Figure 1.

Three phases can be distinguished: purchase, exploitation, decommissioning. The focus of this project will be on the exploitation phase.

In the first phase, purchase, the possibility to buy the new system is investigated. The system that fulfils the requirements best is selected. Because the RNLN deals with complex systems, the engineers from the DMI are trained and educated about the specifics of the system. Then, the engineers from the DMI construct the maintenance program and negotiate about the documentation and the standards concerning the use and maintenance of the ships, because it should fit the way the DMI works. The operational practices are highly captured by standards and protocols.

During the exploitation phase, the system is in operation. Regarding the maintenance during this phase, three levels can be distinguished: operational level maintenance (OLM), intermediate level maintenance (ILM), and depot level maintenance (DLM). The OLM concerns the regular tasks the ship operators have to do to maintain the ship. Again, this is captured in the documentation and protocols to a large extent, this can be preventive maintenance and corrective maintenance (if not too complicated). Often, there is no time for extensive investigation of the problems, as the mission plans cannot be delayed too much.

The ILM is performed by engineers of the DMI. ILM typically concerns tasks or tests that require complex skills or equipment. This can be preventive and corrective. The ILM can be done a couple times a year when the ship is in the harbour. These periods are scheduled for maintenance and cannot be changed, because the mission plans are binding.

The DLM concerns a major overhaul while the ship is docked for a long period. This happens once per 4 years. This large overhaul contains a standard maintenance list of tasks that are always performed (SOL) and tasks that are identified in the inspection before maintenance (ASOL). The systems are dismantled from the ship. When the system is in the repair-shop, it will be disassembled into subcomponents.

The decommissioning phase is the last phase of the system's lifetime at the RNLN. First, the system undergoes maintenance, to restore it to the state as prescribed by the standards. Then the system is sold or the usable parts are disassembled and stored. When the system is sold, the buyer will be educated about the system and its documentation.

The responsibilities of the maintenance management are allocated per system. The decision-makers in the maintenance management are different per system. There is of course some feedback between the actors of different systems, but this is not done in a structured manner. Therefore, cross-system maintenance planning is hard in the current organizational structure.

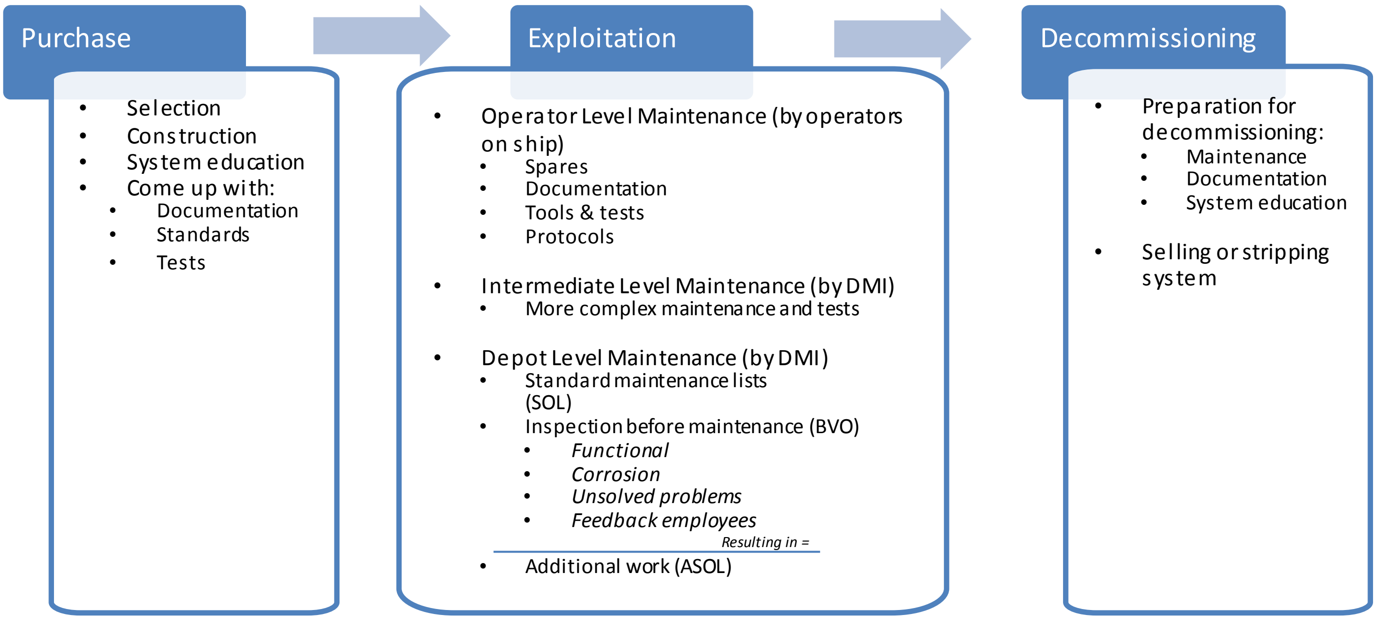

Figure 1 – Lifetime of a system from the point of view of the DMI

1.3.2 Preceding research

In the MaSeLMa project, the possibilities to use condition-based maintenance are evaluated. The first step to do this is getting insight into the part degradation. Politis (2015) has performed a project that aims to predict the part degradation of radar systems, based on the physics of failure. His model will predict the remaining useful life estimation of the part, and probability that it fails earlier. The research of Eruguz (2015) uses the degradation predictions to optimize a condition based maintenance strategy in a single component setting. It takes into account: different operational states, different mission severities, different component criticality per mission, and different repair quality for the operational states. Four thresholds that trigger maintenance action are introduced and optimized:

- Preventive replacement threshold; *when degradation reaches threshold while the ship is in the harbour, preventive replacement is triggered.*
- Dock replacement threshold; *when degradation reaches threshold while the ship is in major overhaul, preventive replacement is triggered.*
- Spare part ordering threshold; *when degradation reaches threshold, a spare part is ordered at the central warehouse.*
- Spare part allocation threshold; *before the ship leaves the harbour, put a spare part on board if the degradation is above a certain level and the component is critical for the next mission.*

A proper implementation of a new model or strategy is very important for practitioners, otherwise the advantage of the proposed strategy will not be obtained. The method that is currently used by the RNLN is highly structured, using predefined documentation, standards and tests. This makes the maintenance planning to a large extent static. The maintenance management would profit from a solid analytic model to base the decision-making on. As explained above, the possibility to use dynamic approaches such as condition based maintenance is investigated. However, it is unrealistic to expect the proposed condition-based maintenance (CBM) to be used for all components. It is interesting to investigate the interaction between the static ABM and the dynamic CBM approach, as the two are expected to be used alongside.

1.4 Research questions

The currently used and previously investigated maintenance strategies consider a single component at the time. Currently, there is no structural way to incorporate the planning of multiple components at once. In contrast, this project investigates the dependency between components, as they are expected to have a significant influence on the optimality of the strategy. Up-time and availability are very important for the RNLN. This makes clustering maintenance activities attractive. In this project a multi-component model and a heuristic solutions method are proposed. The research question of this project is:

How can a multi-component policy be used to integrate a condition-based maintenance policy with a static maintenance planning in a setting with operational states?

This research question requires multiple issues to be investigated. To give insight in the steps to be taken in this project, the research question is divided into sub questions:

1. *How can the two policies (ABM and CBM) be modelled such that they incorporate the operational states?*
2. *How can the interaction between the two policies (ABM and CBM), in terms of structural dependencies between components, be modelled?*
3. *How can the two policies be integrated, to optimize the maintenance planning in terms of costs?*

Figure 2 shows the contribution of this project (in italics) with respect to the previous research in the MaSelMa project. The research questions are formulated such that they will cover the highlighted area in the maintenance problem.

Chapter 3.1 presents the ABM-model incorporating the operational states, and chapter 3.2 contains the CBM-model incorporation the operational states. In these chapters the first research is answered. The dependencies between components are elaborated in chapter 2.1, answering the second research question. The third research question is answered in chapter 3.3; here the integration of the ABM and the CBM models is discussed.

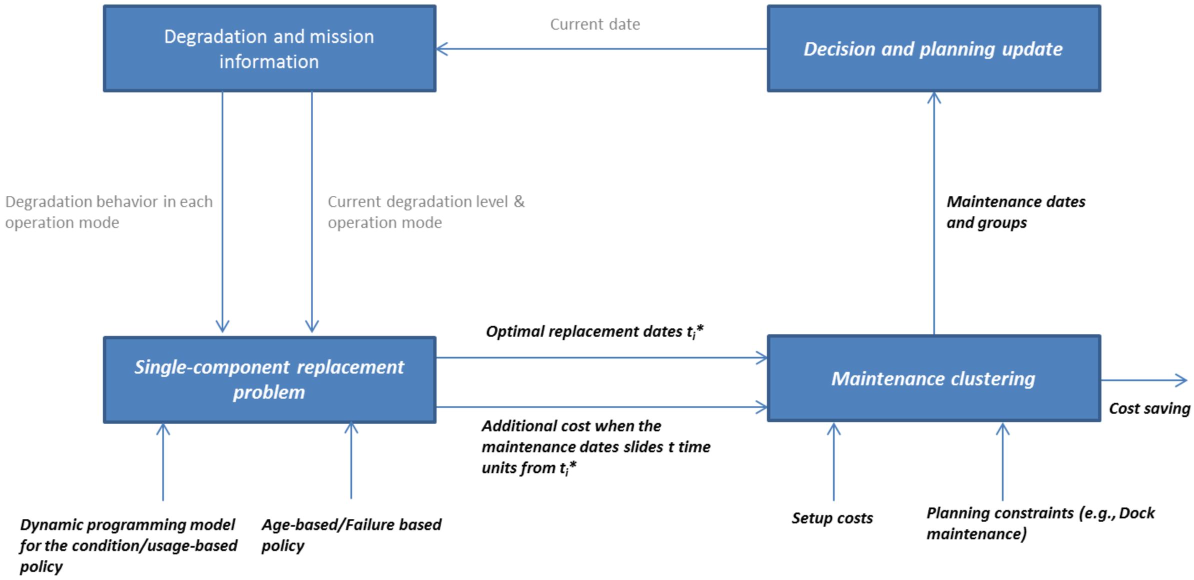

Figure 2 - Representation of the maintenance problem and the contribution of this project

1.5 Scope and boundary

This project focusses on the planning aspect of the maintenance management. The problem is investigated in a multi-component setting, because dependencies between components are expected to have a significant influence on the total maintenance cost. The model will incorporate the structural dependencies between components. In chapter 2.1 is elaborated on the dependencies between components.

The multi-component maintenance planning considering the operational states is complicated, so the other aspects of the maintenance management, such as spare parts management and the goods flow in the repair shop, is not in the scope of this project.

This project proposes a strategy that incorporates the planning of age-based components with the planning of condition-based components. The model can also be applied to usage-based components instead of condition-based components. In the case-study, the usage-based variant is used.

In this thesis, a case study is performed. The case that is investigated is the Marlin weapon system that is used on the fleet of the RNLN. This system is rather new to the RNLN, and the maintenance organization is more flexible than other systems that are in use. There are relatively many tasks categorized as ILM. Dependencies between components are expected to have a significant impact on the total maintenance cost. Therefore this system is a suitable candidate for this research.

1.6 Methodology

This section elaborates on the methodology of this project. First, the structure of the research is explained and the different stages are shown. Then, the approach of this project is explained. In this project different models are built. The relation between these models is explained here.

1.6.1 Research structure

To structure this project the research framework of Mitroff, Betz, Pondy, and Sagasti (1974) is used. This framework accommodates for problem solving in which the system under investigation is regarded as a whole, in contrast to only considering a subsection. This framework distinguishes four parts in the methodology:

1. Conceptualization
2. Modelling
3. Model solving
4. Implementation

This is visualized in Figure 3. The conceptualization phase focusses on conceptualizing the problem. In this phase the system characteristics are evaluated. The variables that influence the problem are explained and decisions are made on which variables will be included in the model. The scope and the boundaries of the project are discussed. In the modelling phase, the problem and its dynamics are captured in a quantitative model. Here, the mathematical model is built and explained. In the model solving phase, the model is used to solve the actual problem. Also a sensitivity analysis is performed in this stage. The last stage covers the implementation of the proposed model. Here is discussed how the new strategy can be implemented and what issues can be encountered. During the project, a lot of feedback loops are in place. Issues that are encountered in one phase can change the decisions made in previous phases. During the process, one should make sure that the decisions made in all phases are aligned.

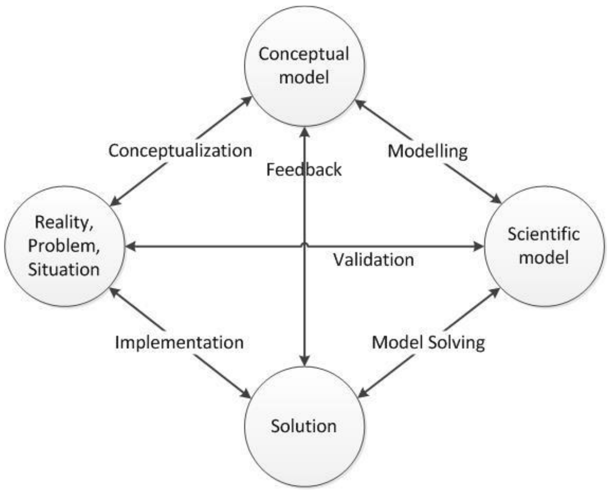

Figure 3 - Research framework introduced by Mitroff, et al. (1974)

In chapter 1.9 is explained in which section of this report the different stages are covered.

1.6.2 Approach

To solve the research problem and evaluate its dynamics, four mathematical models are created. The goal of the project is to incorporate multi-component effects in the maintenance management, as well as to integrate an age-based maintenance approach with a condition- or usage-based maintenance approach. Due to the difference in operational states that occurs in the maritime sector, the traditional models do not apply.

The single-component age-based maintenance model is created first. The resulting maintenance cost from this model is used in the next model; the multi-component age-based maintenance model. Solving this model is complicated, hence a heuristic is used to optimize the maintenance planning.

The resulting maintenance schedule of the age-based maintenance model is used as input for the condition- or usage-based maintenance model. For the condition-based model, both a single- and a multi-component model are created.

This project investigates the impact of structural dependencies between components and evaluates the benefit of a multi-component approach over a single-component approach. To evaluate the advantage of the multi-component approach in terms of cost, the single-component models are created.

1.7 Deliverables

This project investigates and proposes new way of optimizing the maintenance management. First, insight is given into the dependencies between components and their impact on the maintenance planning. Currently at the DMI, there is no structural way to incorporate these dependencies in the decision-making.

Second, a method is constructed to incorporate the operational states in the maintenance planning and their impact on the maintenance management. To the best of our knowledge, such a model is not present in the maintenance literature, so the model itself is an important deliverable of this project.

Third, a clustering algorithm is proposed that can be used to optimize the maintenance planning of the ABM- components in terms of cost.

Fourth, a condition-based model is created that incorporates the operational states. This model has a single-component and a multi-component variant that allows for maintenance clustering. This model can also be applied in a usage-based setting. The multi-component model does also consider clustering opportunities with ABM-components.

Fifth, the models are translated into a planning tool in VBA/Excel that can be used by the maintenance engineers to evaluate and optimize the maintenance planning. This can be used to compare the performance of the different models.

At last, a case study is performed on the Marlin weapon system. The maintenance planning of this system is evaluated and optimized in terms of cost. The case study incorporates a sensitivity analysis that gives insight in the influence of the input parameters on the performance of the models.

1.8 Relevance

In the literature, multiple multi-component maintenance models are presented. In these models, the components are assigned to a group of components that are maintained at a fixed interval. These types of policies can be referred to as block-policies. (Wildeman, Dekker, & Smit, 1997) (van Dijkhuizen & van Harten, 1997). Using grouping algorithms is an alternative approach. Here, the single-component optimal maintenance dates are determined first. Then, cost of grouping is expressed using penalty functions. After that, problem reduction principles are used to simplify the optimization (Vu, Do Van, Barros, & Berenguer, 2012) (Wildeman, Dekker, & Smit, 1997). This approach is more similar to the approach used in this project. However, the above described approaches rely heavily on the assumption that the cost structure of the problem is stationary. An optimal maintenance interval can be used permanently, because the maintenance cost are not time-dependent. In the maritime setting, this is not the case. This research is a relevant addition to the current literature on multi-component maintenance modelling as it introduces incorporating the operational states into the maintenance planning. In addition, the ability to incorporate the structural dependencies between components is granted. To the best of our knowledge, such a model is not available in the current literature. A new dynamic programming model is created to optimize the maintenance planning for condition-based or usage-based components. This model also incorporates the operational state cost difference. The traditional CBM-models use a preventive replacement threshold to trigger preventive maintenance. (Wang, Chu, & Mao, 2008) This threshold is optimized, such that the sum of the expected preventive and corrective maintenance cost is minimal. When the maintenance costs are time-dependent, a static maintenance threshold will not be appropriate. The decision-making should not only be made based on the current degradation threshold, but should also incorporate the asset’s operational schedule. Therefore, a dynamic programming model is constructed to optimize the maintenance planning. In addition, this project proposes a way to integrate the maintenance planning of ABM-components and CBM-components

The RNLN currently does not use such a detailed quantitative method for their maintenance management. Clustering maintenance tasks is done based on the insight in the cost structure and experience of the installation managers rather than using a structured analytical method. This project proposes a new policy that can be used for the maintenance planning of the weapon system on which the model is tested, but also for other systems. The model in itself is general, so it can be used for other systems where an operational state cost difference exists. The maritime sector is not the only sector that would profit from a model that incorporates operational states. An organisation that has to deal with deviating corrective and preventive cost due to changing operational states can use this model for their maintenance management.

1.9 Report outline

This section gives an outline of the contents of each chapter. The chapters are linked to a phase of the research framework of Mitroff, et al. (1974). This is shown in Table 1.

Table 1 – The outline of the report and the research phases of Mitroff, et al. (1974)

Chapter	Contents	Research phase
1	Introduction	*Conceptualization*
2.1	Dependencies between components	*Conceptualization & modelling*
2.2	Operational states	*Conceptualization*
2.3	Component criticality	*Conceptualization*
3.1	Age-based maintenance	*Modelling*
3.1.1	Single-component model	*Modelling*
3.1.2	Multi-component model	*Modelling*
3.1.3	ABM clustering algorithm	*Model solving*
3.2	Condition-based maintenance	*Modelling & model solving*
3.3	Integration ABM and CBM	*Modelling & model solving*
4	Case study	*Model solving*
4.2	Reference results	*Model solving*
4.3	Sensitivity analysis	*Model solving*
5	Conclusions	*Implementation*

The next chapters contain the conceptualization phase. The dependencies between components and how they can be modelled is discussed in chapter 2.1. The operational states are conceptualized in chapter 2.2. The conceptualization of the component criticality is discussed in chapter 2.3. In the next part, the modelling is discussed. First, the age-based models are presented. Chapter 3.1.1 contains the single-component age-based maintenance model. Then, the multi-component model is presented. Chapter 3.1.2.1 contains the penalty functions that are used in the model. The clustering algorithms are discussed in chapter 3.1.3. Then the clustering algorithms are evaluated on their performance, this is done in chapter 3.1.4. Thereafter, the condition-based model is presented in chapter 3.2. The integration of the ABM and CBM models is discussed in chapter 3.3. After that, the case study is introduced. The assumptions and parameter settings are explained in chapters 4.1 and chapter 4.2 contains the results of the models with the reference parameter settings as input. Chapter 4.3 presents a sensitivity analysis. At last, the conclusions that can be drawn from this project are presented. Chapter 5.1 explains how the research question is answered. The limitations of this project are presented in chapter 5.2. The implementation is discussed in chapter 5.3 and recommendations are given in this chapter. Then, future research directions are suggested in chapter 5.4.

2 Conceptualization

In this chapter, the factors that influence the decision-making are explained and conceptualized.

2.1 Dependencies between components

To optimize the multi-component maintenance problem, the dependencies between components must be investigated. The literature distinguishes three types of dependencies: economic, structural and stochastic dependencies.

2.1.1 Economic dependency

Economic dependency refers to the costs caused by unavailability of the system. In a production line setting, the entire production line is stopped during maintenance of a component. This makes maintaining multiple systems attractive, because the tasks can be done in parallel or time-savings can be made. In the case of the RNLN, there is no such thing as production loss. However, for many tasks, the ship need to be at the home harbour, during the harbour periods the system is unavailable for use. This can be seen as some sort of economical dependency. However, it is hard to quantify the cost for system unavailability. In addition, when the amount of maintenance instances can be reduced, the planning will most likely not have less home harbour instances, as the ship also visits the harbour for other purposes than maintenance. This does not mean that economic dependencies do not occur in the case of the RNLN, but they are rather ambiguous and hard to quantify. Therefore they will not be included in the case study. The models could include economical dependency as long as it can be quantified.

2.1.2 Stochastic dependency

Another type of dependency is stochastic dependency. This can be seen as the dependence between the failure behaviour of the components in a system. Failure or high degradation of a component can cause a higher failure or degradation rate among the other components. This type of dependency is hard to model. Most maintenance models assume that failure behaviour or degradation of the components is independent of the failure behaviour or degradation of the other components. Incorporating this kind of dependency would create a very complex model. For the RNLN, there might be some stochastic dependency between components. However, the size of this effect is unknown. In this project is assumed that there are no stochastic dependencies between components because of the reasons mentioned above.

2.1.3 Structural dependency

A structural dependency refers to the dependencies that arise due to the physical lay-out of the system. In complicated systems, maintaining a part often requires other components to be dismantled as well. The time and effort this takes can be seen as set-up cost that can be shared. For the RNLN the main part of the structural dependencies between components are time related. The set-up time that can be avoided can be translated to cost using the wage of the engineers.

The model will consider structural dependencies. The structural dependencies between components are modelled as a set-up cost. The set-up cost can be modelled in different ways.

A simple form of a set-up configuration is a common set-up cost for all components. In that case, the set-up cost that is avoided is equal to $(m-1) \cdot S$, where m is the amount of components that are clustered on a specific date. This is depicted in Figure 4a. Often, there are different groups of components that share a common set-up. These groups can be optimized separately, as the dependencies only exist within the groups.

Some companies have to deal with a more complicated set-up configuration. To determine which setups can be avoided, one must first divide the setups into parts that can be shared and component specific parts. Van Dijkhuizen & Van Harten (1997) use such a set-up configuration. They transform a production system into a maintenance tree. This is illustrated by Figure 4b. Here, setup i is related to component/task i. The cost of the setup part is notated on the arcs. The total setup cost of a maintenance task is found when going back into the tree. Hence, the components with setups at the end of the tree have larger setups than the ones at the beginning of the tree.

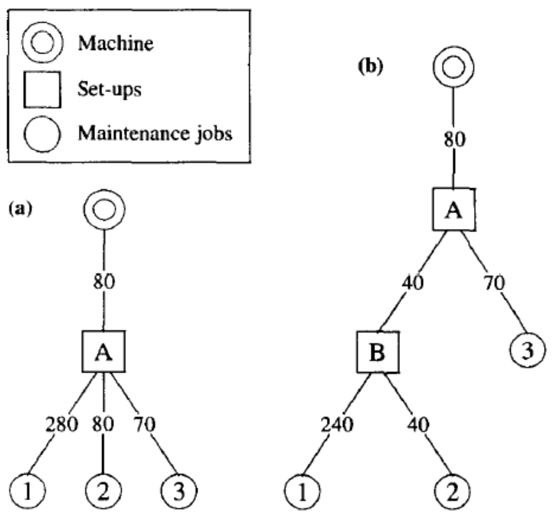

Figure 4 - Example of Van Dijkhuizen & Van Harten (1997), (a) set-up configuration with common set-ups. (b) Maintenance tree set-up configuration.

To determine the setup costs that are saved by rescheduling the maintenance on component i to a certain period, one can check whether the setups that are needed for component i are already performed that period. The saved set-up cost for a rescheduling option is referred to as $SS_i^{\tau_i^*+k}$. Note that the saved set-up cost can change each time a component is rescheduled.

The model can incorporate all these types of set-up configurations. It only takes some extra modelling effort to determine which set-up costs are saved with a certain clustering option, in case of the maintenance tree configuration. The tool developed in this project includes the possibility to use each of these set-up configurations.

2.2 Operational states

In the maritime sector, the assets are typically used in different operational states. The existence of operational states asks for a different kind of maintenance model. In contrast with a production line setting, it raises some specific dynamics to the maintenance planning. The opportunity to perform maintenance can be absent in some of the operational states, or it exists but with a high cost attached to it. Another issue with respect to the existence of operational states is that the failure or the degradation behaviour can be different per operational state. The assets are usually used more intensely in some of the operational states and less in others. This makes the decision making in the maintenance management more complicated and requires a more sophisticated mathematical model.

For the RNLN, four states can be distinguished:

1. Transit
2. Mission
3. Home harbour
4. In-dock overhaul

These states are different from a maintenance perspective for the following reasons. When the ship is in state 3 and 4 are it is in the home harbour, which includes an advanced repair shop. This means that the advanced equipment and tools can be used in these states. In addition, specialized engineers will perform the maintenance tasks. This is not the case when a ship is in operational state 1 and 2. Military engineers are on board when a ship leaves the harbour, but these are generalists with a broad set of skills, unable to perform the complex tasks that can arise for the sophisticated systems. Also, only basic equipment can be carried on a mission. For this reason, the complex maintenance tasks can only be performed when the ship is in states 3 and 4. In state 4 the ship undergoes major overhaul, this happens once in four years. This corresponds to the depot level maintenance described in chapter 1.3.1. Operational state 1 and 2 are expected to differ from a maintenance perspective, as for some systems the systems are used more intensely during missions. This results in a different failure behaviour among states 1 and 2. Also, the consequences of system failure during mission are more severe as the mission might be abandoned when the system breaks down. The mission schedule is often known for the upcoming years.

2.3 Component criticality

It can be very costly to perform corrective maintenance in some operational states. However, not every component is critical all the time. For some components, it is allowed to postpone corrective maintenance until the asset enters an operational state with low maintenance cost. This influences the decision-making for non-critical components and hence it should be included in the model. This is also the case for the RNLN. Some missions do not require all systems to be functioning. In that case, the maintenance on the component will be postponed until the ship visits the harbour and the component can be repaired at low cost. When the criticality of the components during the upcoming missions is known, it can be incorporated in the maintenance planning. This can be done for the condition-based maintenance model, see chapter 3.2. However, for the age-based maintenance model this is much harder, because a static model is used, see chapter 3.1. In a static model, it is much harder to model the behaviour of the system in case of failure of a non-critical component. Due to the time-limit of this project, the criticality of the components is only incorporated in the CBM-model.

3 Mathematical model

The mathematical modelling consists of two main parts; the age-based maintenance part and the condition-based maintenance part. For the ABM part, the model first addresses the single-component optimization problem as visualized in Figure 2. Based on the optimal replacement date of each component, the penalty cost for deviating from this date are determined. This is used as input for the clustering algorithm. Two methods to solve the clustering problem are presented, the performance of these methods is evaluated with a scenario analysis. For the condition-based/usage-based components a dynamic programming model is used.

3.1 Age-based maintenance

3.1.1 Single component model

The first step in the optimization procedure is to find the optimal maintenance date in a single-component setting. Here the dependencies between components are ignored.

One can argue that it is reasonable to assume that the degradation rate is dependent on the use of the system. In the maritime sector, the assets are used in different operational states. Therefore, distinguishing between degradation behaviours in the operational states of the asset is interesting. When the degradation rate is dependent on the operational state, the failure probabilities become more difficult. This is shown in Appendix B. The probability functions contain many convolution terms. This is hard to model and will result in an high computation time. In addition, the degradation rate for the different operational state is unknown.

Assuming an equal degradation rate for the operational states might be reasonable for many systems in the maritime sector. At the RNLN, the different degradation rates for the operational states are unknown, and in the MaSeLMa project this seems to be the same for other companies in the maritime sector. For this reason and for the ease of modelling the failure rate is assumed to be equal across the operational states.

In the classical renewal theory, the average maintenance cost is expressed as the following function (Nahmias, 2009):

$$C_i(\tau_i) = \frac{ECC(\tau_i)}{ECL(\tau_i)}$$

$with;$

$ECC(\cdot) \ = \ expected\ cycle\ cost;$

$ECL(\cdot) = \ expected\ cycle\ length;$

$\tau_i = the\ maintenance\ date\ of\ component\ i;$

Here, the maintenance costs are time-independent. Hence, the optimal maintenance date is time-independent and remains optimal permanently. In this project the maintenance costs are time-dependent, so an adjustment is needed. The average maintenance cost of component i, $C_i(\tau_i)$, can be modelled as a variation on the classical function. The numerator contains the expected maintenance cost per time unit of the upcoming life of the component.

The corrective maintenance cost differs per operational state, so the expected corrective cost is summated over the lifetime of the component until $\tau - 1$. Here, $c_{i,t}^{\text{corr}}$ and $c_{i,t}^{\text{prev}}$ represent the cost of performing corrective and, respectively, preventive maintenance for component *i* during the operational state at time *t*. S_i represents the set-up cost of component i. $F_i(t)$ represents the cumulative probability of failure unto time *t*. The average maintenance cost is expressed by the following function:

$$C_i(\tau_i) = \frac{\sum_{t=1}^{\tau_i - 1} (c_{i,t}^{\text{corr}} + S_i) \cdot \big(F_i(t) - F_i(t-1)\big) + (c_{i,\tau_i}^{\text{prev}} + S_i) \cdot \big(1 - F_i(\tau_i - 1)\big)}{\int_{t=1}^{\tau_i - 1} t \cdot f_i(t) dt + \tau_i \cdot \big(1 - F_i(\tau_i - 1)\big)}$$

When a failure occurs at the period for which preventive maintenance is planned, it is assumed that the system can still be maintained at the preventive maintenance cost, as the tasks are already planned for that period. When the function is used to find all maintenance dates within the horizon, the time should not be set to zero, but the current time should be subtracted in the failure probability terms, this is shown in the algorithm in the clustering algorithm in chapter 3.1.3. For simplicity, this is left out in the function above.

The maintenance planning and the ship schedule are done on a discrete scale, therefore a summation is used to calculate the expected corrective cost. The lifetime of the components is expected to be continuously distributed, because the age of a component is measured on a continuous scale. For more information on the lifetime distributions, see Appendix A.

When the function presented above is minimized, the optimal maintenance date of that life of the component is found. The optimal value is not found by setting the derivative to zero, because the cost function might not be unimodal. The optimal value can be found by calculating the cost resulting from all maintenance times until the horizon and selecting the optimal maintenance time. In software packages as Excel/VBA this can be done quickly.

This is a myopic policy, because only the upcoming maintenance date is optimized, instead of the all maintenance occurrences until the scheduling horizon. This policy is an approximate approach and it is rather easy to model. To be able to optimize all the maintenance decisions until the horizon a dynamic programming model would be needed, which is more complicated. The myopic issue is discussed in more detail in chapter 3.1.3.3.

Due to the state dependent corrective cost, the function is not necessarily unimodal as said before. For an Erlang distributed time to failure, and some arbitrary failure and cost parameters, the average maintenance cost per time unit is shown in Figure 5. The operational schedule of the asset is shown in

Table 2, with: 3=harbour state (low preventive and corrective maintenance costs), 1=transit state (intermediate preventive and corrective maintenance costs), 2= mission state (high preventive and corrective cost).

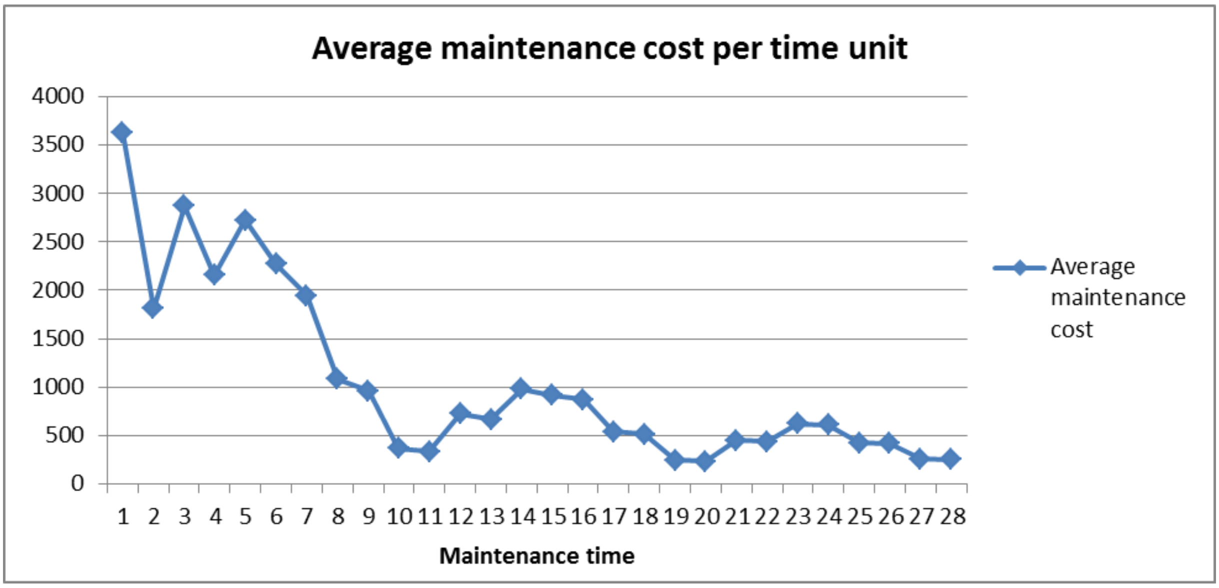

Figure 5 - Example of the average cost function depending on the maintenance timing

Table 2 - Operational schedule of the asset of Figure 5

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Oper. state	3	3	1	1	2	2	2	1	1	3	3	1	1	2	2	2	1	1	3	3	1	1	2	2	1	1	3	3

3.1.2 Multi-component model

The multi-component optimization part constitutes from two parts. First, the cost of rescheduling is modelled by means of a penalty function. Second, the clustering algorithm explains how the cost of rescheduling and the set-up cost can be used to optimize the maintenance planning.

3.1.2.1 *Penalty functions*

Due to the time-dependent maintenance cost and the non-stationary asset schedule, using maintenance policies that involve fixed intervals such as block policies or grouping algorithms, are not appropriate for this setting. Each maintenance occurrence must be optimized to evaluate the clustering possibilities. To do this, penalty functions can be used. The penalty function expresses the cost of deviating from the single-component optimal maintenance date. This cost can be compared with the cost saving that can be attained by clustering to evaluate the clustering decision.

Most of the clustering models for age-based maintained components presented in the literature, use penalty functions such as: $h_i(\tau_i^* + k) = C_i(\tau_i^* + k) - C_i(\tau_i^*) - kC_i(\tau_i^*)$ ((Wildeman, Dekker, & Smit, 1997); (Vu, Do Van, Barros, & Berenguer, 2012)). Here, $h_i(\tau_i^* + k)$ represents the penalty cost with τ_i^* as the single-component optimal maintenance date, and k as the size of the deviation from the single-component optimal date. $C_i(x)$ are the long term costs of performing maintenance at time x.

Such a function gives no insight in the underlying costs that constitute the penalty for rescheduling and hence it is hard to adjust such a function to incorporate different maintenance cost per operational state. Therefore, a more explicit penalty function is used to incorporate the operational state cost difference. Because the mission plans are known for the upcoming period, it is possible to incorporate this in the model.

The penalty function is based on the difference in the expected corrective cost between the optional date and the single-component optimal date. Additionally, the expected difference in the preventive cost between the optional date and the single-component optimal date must be included. The expected corrective and preventive cost are modelled the same as in the average maintenance cost function.

The penalty that is incurred for advancing or postponing maintenance contains another factor. A cost factor is included that represents the expected penalty that is incurred by advancing or the expected saving that is acquired by postponing the maintenance date. In the function from the literature, this is included by the factor $-kC_i(\tau_i^*)$. However, this is a simplistic representation. For large values of k, the potential postponement gain is high, but the probability that the component survives until the maintenance date is low, so the potential postponement gain is multiplied by the probability of survival. When the component fails before the maintenance date a partial postponement gain is acquired, again this is multiplied by the probability of failure at that time.
The penalty function for a postponement can be expressed as:

$$For\ k > 0:$$

$$h_i(\tau_i^* + k) = \sum_{t=1}^{\tau_i^*+k-1} c_{i,t}^{\text{corr}} \cdot \big(F_i(t) - F_i(t-1)\big) + c_{i,\tau_i^*+k}^{\text{prev}} \cdot \big(1 - F_i(\tau_i^* + k)\big)$$

$$- \left(\sum_{t=1}^{\tau_i^*} c_{i,t}^{\text{corr}} \cdot \big(F_i(t) - F_i(t-1)\big) + c_{i,\tau_i^*}^{\text{prev}} \cdot \big(1 - F_i(\tau_i^*)\big) \right)$$

$$- k \cdot C_i(\tau_i^*)\big(1 - F_i(\tau_i^* + k)\big) - \sum_{l=1}^{k-1} l \cdot C_i(\tau_i^*)\big(F_i(\tau_i^* + l) - F_i(\tau_i^* + l - 1)\big)$$

The two expected corrective cost summations can be subtracted from each other. Then, the corrective cost summations can be notated in a similar way as the postponement gain summation. The penalty function can be simplified to:

$$For\ k > 0:$$

$$h_i(\tau_i^* + k) = \sum_{l=0}^{k-1} c_{i,\tau_i^*+l}^{\text{corr}} \cdot \big(F_i(\tau_i^* + l) - F_i(\tau_i^* + l - 1)\big) + c_{i,\tau_i^*+k}^{\text{prev}} \cdot \big(1 - F_i(\tau_i^* + k - 1)\big)$$
$$- c_{i,\tau_i^*}^{\text{prev}} \cdot \big(1 - F_i(\tau_i^* - 1)\big) - k \cdot C_i(\tau_i^*)\big(1 - F_i(\tau_i^* + k - 1)\big)$$
$$- \sum_{l=1}^{k-1} l \cdot C_i(\tau_i^*)\big(F_i(\tau_i^* + l) - F_i(\tau_i^* + l - 1)\big)$$

For a maintenance advancement, the penalty function can be expressed as:

$For\, k < 0$:

$$h_i\big(\tau_i^{n^*} + k\big) = -\sum_{l=k}^{-1} c_{i,l}^{\text{corr}} \cdot \big(F_i(\tau_i^* + l) - F_i(\tau_i^* + l - 1)\big) + c_{i,\tau_i^*+k}^{\text{prev}} \cdot \big(1 - F_i(\tau_i^* + k - 1)\big)$$
$$- c_{i,\tau_i^*}^{\text{prev}} \cdot \big(1 - F_i(\tau_i^* - 1)\big) - k \cdot C_i(\tau_i^*)\big(1 - F_i(\tau_i^* - 1)\big)$$
$$- \sum_{l=k+1}^{-1} (k - l) \cdot C_i(\tau_i^*)\big(F_i(\tau_i^* + l) - F_i(\tau_i^* + l - 1)\big)$$

Due to the cost difference between the operational states, the penalty functions are not unimodal. When there is no cost difference between the operational states, the function simplifies to a unimodal function.

Figure 6 shows the penalty cost function depending on the maintenance timing. In this example the same data is used as in the average maintenance cost graph in Figure 5. The penalty cost is zero at the optimal maintenance time in terms of average maintenance cost. The order of magnitude is different because the average maintenance cost is per time unit whereas the penalty cost is not. In the penalty function, the postponement gain is incorporated explicitly. A similar pattern is notable in the penalty cost figure and in the average maintenance cost figure, but in the penalty cost, the operational state cost difference is more pronounced. From this picture, the relevance of incorporating the operational states is clear. If one would use averages for the maintenance cost during the horizon, a totally different cost figure would be created. A deviation from the single component optimal date will result in a very high penalty cost if the maintenance is planned in the high cost operational state, but the cost for rescheduling to the preceding and subsequent low cost operational state are relatively low.

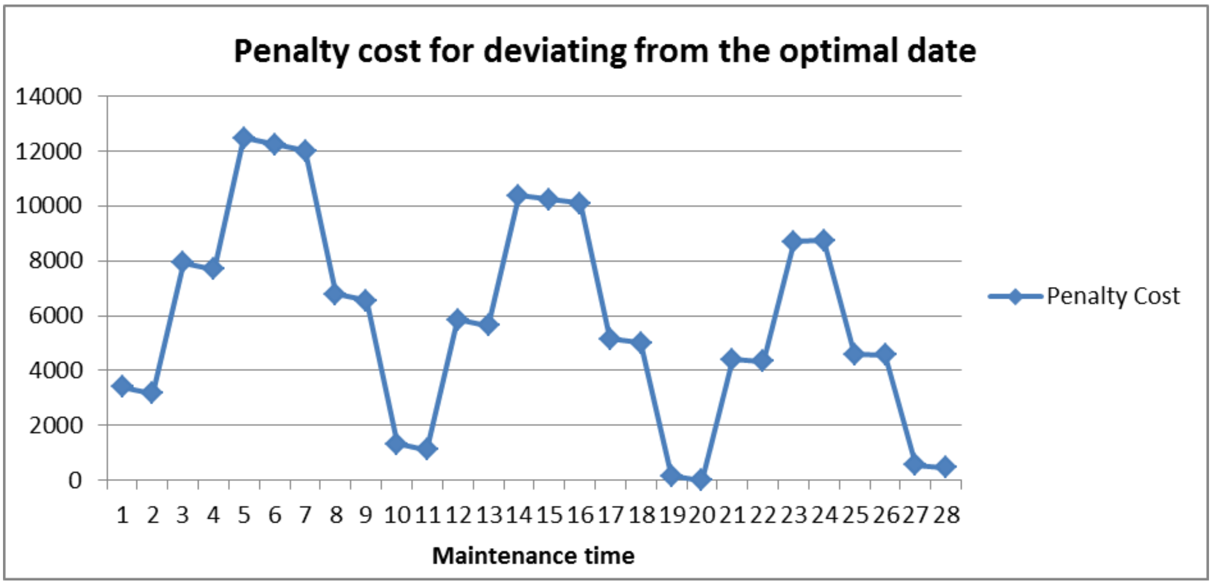

Figure 6 - Example of the penalty cost function depending on the maintenance timing

3.1.3 Clustering algorithm

Solving the clustering problem can be done in multiple ways. A basic way is to perform a full enumeration of the possibilities and select the option with the lowest cost. However, this is time consuming and intractable for long horizons and many components. Wildeman, Dekker, & Smit (1997) propose a method to reduce the solution space of the clustering problem. They introduce rules that must be satisfied by the optimal solution. However, their problem reduction approach assumes that the penalty function is convex. This is not true for this model, so the problem reduction method does not apply. However, the problem is reduced to some extend by only considering the dates of which the penalty cost are lower than the set-up cost of the component. For long horizons, this prevents the algorithm to evaluate dates in the future that are clearly not beneficial.

In the literature, clustering algorithms that are proposed often assume fixed maintenance frequencies or frequency constraints, instead of including the probability of failure in the model (Budai, Huisman, & Dekker, 2006) (van Dijkhuizen & van Harten, 1997). This allows for a strong problem reduction and thereby a less complex solution space. However, the introduction of operational states does not allow for such a model, as the maintenance frequency constraints are based on a stationary system. The maintenance costs are non-stationary over time in this model. Because of the complexity of the problem and the flexibility of this approach, a greedy type model is built. To the best of our knowledge, greedy heuristics are not often used for maintenance clustering, but they have been shown to perform well. (van Dijkhuizen & van Harten, 1997)

3.1.3.1 *Greedy heuristic*

The greedy algorithm finds a local optimum very fast, but it does not guaranty a globally optimal solution. The optimality gap of the greedy heuristic will be evaluated on a small instance, to give insights in the strength of the heuristic and the influence of the input parameters on the solution quality. The algorithm includes the iteration through time to plan all maintenance task occurrences within the horizon. A rolling horizon approach is used. The implications of this are explained in chapter 3.1.3.3. A variable for the CurrentTime is introduced in the calculations to be able to iterate through time.

1. Greedy heuristic:

1.1. $CurrentTime = 1, Age(i) = 0\ for\ \forall i.$

1.2. Fill set N with all components: $N = \{1, \ldots, n\}$.

1.3. Optimize the single-component model for each component in the set N.

$For\ i = 1\ to\ |N|$

$If\ Age(i) = 0, Then;$

Calculate $C_i(\tau_i)$:

$For\ \tau_i = CurrentTime\ to\ horizon$

$$C_i(\tau_i) = \frac{\sum_{t=CurrenTime}^{\tau_i-1}\left(c_{i,t}^{\text{corr}} + S_i\right)\cdot\left(F_i(t-CurrenTime+1)-F_i(t-CurrenTime)\right)+\left(c_{i,\tau_i}^{\text{prev}}+S_i\right)\cdot\left(1-F_i(\tau_i-CurrenTime)\right)}{\int_{t=CurrenTime}^{\tau_i-1} t\cdot f_i(t)\,dt+\tau_i\cdot\left(1-F_i(\tau_i-CurrenTime)\right)}$$

$Next\ \tau_i$

Find τ_i^* for which $C_i(\tau_i^*)$ is minimal.

Determine the penalty cost $h_i(\tau_i^* + k)$:

$For\ k = CurrentTime - \tau_i^*\ to\ horizon - \tau_i^*$

$If\ k > 0, Then;$

$$h_i(\tau_i^*+k) = \sum_{l=0}^{k-1} c_{i,\tau_i^*+l}^{\text{corr}}\cdot\left(F_i(\tau_i^*+l-CurrenTime+1)-F_i(\tau_i^*+l-CurrenTime)\right)$$
$$+c_{i,\tau_i^*+k}^{\text{prev}}\cdot\left(1-F_i(\tau_i^*+k-CurrenTime)\right)-c_{i,\tau_i^*}^{\text{prev}}\cdot\left(1-F_i(\tau_i^*-CurrenTime)\right)$$
$$-k\cdot C_i(\tau_i^*)\left(1-F_i(\tau_i^*+k-CurrenTime)\right)-\sum_{l=1}^{k-1} l\cdot C_i(\tau_i^*)\left(F_i(\tau_i^*+l-CurrenTime+1)-F_i(\tau_i^*+l-CurrenTime)\right)$$

$If\ k < 0, Then;$

$$h_i(\tau_i^{n^*}+k) = -\sum_{l=k}^{-1} c_{i,l}^{\text{corr}}\cdot\left(F_i(\tau_i^*+l-CurrenTime+1)-F_i(\tau_i^*+l-CurrenTime)\right)$$
$$+c_{i,\tau_i^*+k}^{\text{prev}}\cdot\left(1-F_i(\tau_i^*+k-CurrenTime)\right)-c_{i,\tau_i^*}^{\text{prev}}\cdot\left(1-F_i(\tau_i^*-CurrenTime)\right)$$
$$-k\cdot C_i(\tau_i^*)\left(1-F_i(\tau_i^*-CurrenTime)\right)-\sum_{l=k+1}^{-1}(k-l)\cdot C_i(\tau_i^*)\left(F_i(\tau_i^*+l-CurrenTime+1)-F_i(\tau_i^*+l-CurrenTime)\right)$$

$End\ if$

$Next\ k$

$If\ Age(i) > 0, Then;$

Use $C_i(\tau_i^*)$ and $h_i(\tau_i^*+k), for\ k = \{CurrentTime - \tau_i^*, \ldots, horizon - \tau_i^*\}$ as calculated in the previous time iteration.

$End\ if$

$Next\ i$

1.4. Based on the asset schedule, calculate the clustering benefit for each period until the horizon, using the penalty cost and the saved setup cost. The saved setup cost $SS_i^{\tau_i^*+k}$ can change each time a component is scheduled and should be updated each iteration.

$For\ i = 1\ to\ n$

$For\ k = CurrentTime - \tau_i^* to\ \ horizon - \tau_i^*$

$Benefit(i,k) = SS_i^{\tau_i^*+k} - h_i(\tau_i^*+k)$

$Next\ k$

$Next\ i$

1.5. Solve the clustering problem using the greedy heuristic logic:

1.5.1. Start with a blank schedule.

1.5.2. Select the largest $Benefit(i,k)$ of all values of k and of all components in the set N and schedule this component on date $\tau_i^* + k$, by adding the new maintenance occurrence to the planning matrix: $CurrentPlanning(i,m) = \tau_i^* + k$. Where m is the amount of maintenance occurrences, including the newly planned occurrence.

1.5.3. Remove component i from the set N. If $N \neq \{\emptyset\}$; go back to step 1.4. Else; continue with 2.1.

2. Improvement heuristic:

2.1. Set the improvement indicator x to $x = 0$. Start with the first component, $i = 1$.

2.2. Store the latest planning in a vector: $Latestplanning(i) = CurrentPlanning(i, latest\ )$. Remove the maintenance date of the last maintenance occurrence of component i from the schedule: set $CurrentPlanning(i,m) = \emptyset$.

2.3. Reconsider the latest planning of component i, knowing the maintenance planning of the other components. Update the possible set-up saving, $SS_i^{\tau_i^*+k}$. Schedule the maintenance of component i to the date, $\tau_i^* + k$, with the largest clustering benefit: $Benefit(i,k) = SS_i^{\tau_i^*+k} - h_i(\tau_i^* + k)\ for\ k = \{CurrentTime, \ldots, horizon\}$.

2.4. $If\ \tau_i^* + k \neq LatestPlanning(i)$, set the improvement indicator to $x = x + 1$.

2.5. Next component, $i = i + 1$.

$If\ i \leq n, Then;\ go\ to\ step\ 2.2, Else; continue\ at\ step\ 6.2.$

2.6. When all components are reconsidered, $i = n + 1$, check if any improvements were made.

$If\ x \geq 1, Then; go\ to\ step\ 2.1, Else; continue\ at\ step\ 3.1.$

3. Iteration through time:

3.1. Determine the step in time that can be made. First, retrieve the latest maintenance occurrence of each component: $LatestPlanning(i) = CurrentPlanning(i, latest)\ for\ \forall i.$

3.2. Update the age of the components.

$For\ \forall i,\ If\ LatestPlanning(i) = Min(NewPlanning), Then;$

$Age(i) = 0$

$Else;\ Age(i) = Age(i) +\ Min(LatestPlanning) - CurrentTime.$

3.3. Make the step in time. $CurrentTime = Min(LatestPlanning) + 1.$

3.4. $If\ CurrentTime \leq horizon, Then; go\ to\ step\ 1.2,$

$Else; finish.$

The first heuristic considers the planning of each task occurrence once, starting with a blank schedule. It is possible that the heuristic selects a date for the first component that has a high benefit or no penalty cost for that component only. When the schedule of the other components is known, it might be beneficial to reschedule that component to another date with a slightly higher penalty cost, but with a higher set-up saving. To adjust for this effect, an improvement heuristic is proposed. This heuristic re-evaluates the maintenance schedule of each component, knowing the schedule of the other components. The improvement heuristic can does only reconsider the maintenance date of one component at the time. When the planning of a component of a group will cause the remainder of the group to be clustered at its maintenance date, the improvement heuristic will not correct this.

3.1.3.2 *Enumeration*

The enumeration approach finds the optimal solution guaranteed. However, it is a very non-sophisticated method. The enumeration algorithm is used to evaluate the performance of the Greedy heuristic. The algorithm only includes the optimization of the decision-making at one certain point in time, instead of iterating throughout the horizon. The enumeration method is presented below:

1. **Enumeration algorithm**
   1.1. Determine the optimal maintenance date for each component $i \in N, with\, N = \{1, \dots, n\}$, as explained in chapter 3.1.1.
   1.2. Based on the asset schedule, calculate the penalty cost for deviating from the optimal maintenance date over the scheduling horizon for each component, as explained in chapter 3.1.2.1.
   1.3. Determine which clustering options are available for the components $i \in N$:
   Only consider a rescheduling date for component i if the penalty cost at that date are lower than its setup cost: $S_i - h_i(\tau_i^* + k_i) > 0$
   1.4. Complete enumeration:
      1.4.1. Determine for each combination of clustering options of all components the total clustering benefit: $\sum_{i=1}^{n} SS_i^{\tau_i^*+k_i} - h_i(\tau_i^* + k_i)\ for\ k_1, \dots, k_n \in \{1, \dots, horizon\}$.
      1.4.2. Select the scheduling combination with the highest total benefit.

3.1.3.3 *Myopic decisions and rolling horizon*

A system can often have a wide variety of maintenance tasks. These maintenance tasks can have dissimilar time intervals; some tasks are performed multiple times a year, while others are only performed once in a couple years. Therefore there are reciprocal effects between the occurrences of the maintenance tasks. Clustering the two tasks with very dissimilar time intervals becomes viable when the task with the short interval is already performed a couple times. The planning of the short interval component at this moment is influenced by the timing of the preceding times the task is performed. However, the decision-making is myopic; the timing of the succeeding maintenance occurrences is not incorporated in the decision-making. This way, the model might plan the first couple maintenance occurrences of the short interval component such that when clustering with long interval tasks becomes viable, the short interval component has a very inconvenient timing for clustering.

Another effect of the myopic decision-making is caused by the non-stationary asset schedule. The asset schedule at the time of the succeeding task occurrences can be different from the asset schedule at the time of the current task occurrence. The timing of a certain task occurrence influences the timing of the succeeding task occurrences; when a task is postponed at its first occurrence, the succeeding occurrences logically shift along. A postponement of the maintenance early in the horizon might cause the timing of the maintenance occurrences later in the horizon to be unfortunate. This effect is apparent in both the single-component as the multi-component ABM-model.

These myopic decision effects are not included in this model, as it would make the model much more complex. Future research can be done into reciprocal effects and in how to optimize all maintenance occurrences within a model.

To incorporate multiple occurrences of a maintenance task within a horizon, a rolling horizon approach can be used. This is shown in the greedy algorithm. The planning of reciprocal tasks is done by optimizing the maintenance planning each time a maintenance task is performed preventively or correctively. To iterate through time, the model will make a step in time towards the first upcoming preventive maintenance task that is planned. The penalty costs of the components that have no planned or unplanned maintenance do not have to be recalculated as the cost terms are unchanged. However, maintenance planning of these components can be changed, because new opportunities to save set-up cost arise each time another component is (re)scheduled.

3.1.4 Evaluation of the optimization methods

In this section the optimality of the greedy heuristic is evaluated.

3.1.4.1 *Scenario analysis configuration*

The results of the greedy heuristic are compared with the results of the enumeration algorithm in a setting with a few components. In this evaluation 6 components are used. A horizon of 20 time units is used. The asset schedule that is used for this evaluation is shown in Table 3. In this case, 3 is the low cost operational state and 1 and 2 are the medium and high cost operational states respectively. The set-up configuration that is used is one general set-up for all components. The set-up costs are used as a variable in the scenario analysis, as explained later on.

Table 3 – Asset schedule used for the evaluation

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Operational state	3	3	1	1	2	2	2	1	1	3	3	1	1	2	2	2	1	1	3	3

A scenario analysis is performed, so the performance of the optimization methods in different settings can be compared. The scenarios are composed with four variables with each three or four levels. The failure behaviour is an important factor in the maintenance planning. Therefore, the level of the failure distribution parameters is one of the variables. The failure distribution parameters are chosen such that the three levels represent the equality of the failure behaviour of the components. One level consists of similar failure parameters, another level consist of failure parameters with an intermediate deviation, and the third level consist of strongly deviating failure parameters.

The Erlang distribution is used for this analysis. The variable representing the failure behaviour is the mean time to failure (MTTF). When the scale parameter μ_i is kept fixed at 8, the rate parameter can be determined by:
$\lambda_i^l = \mu_i / MTTF_i^l$. The three levels ($l$) are:

$$MTTF_i^l = \begin{cases} U(8;12) & for\ l = 1 \\ U(5;15) & for\ l = 2 \\ U(0;20) & for\ l = 3 \end{cases}$$

Here, $U(a;b)$ represents the uniform distribution with lower a and upper bound b.

The second variable involves the set-up cost. When there are high set-up costs, clustering maintenance activities is clearly more attractive. More clustering options are available when the set-up costs are high, hence the clustering is more complex. Four levels of set-up cost are considered. The variable representing the set-up costs is modelled such that it is dependent on the preventive maintenance cost of the first component in the low cost operational state. This done because the ratio between the set-up cost and the other maintenance costs is what determines the attractiveness of clustering. Here, $C_{1,prev}(3)$ represents the preventive maintenance cost of the first component in the low cost operational state.

$$S_i^l = \begin{cases} {}^1/_2 \cdot C_{1,prev}(3) & for\ l = 1 \\ 1 \cdot C_{1,prev}(3) & for\ l = 2 \\ 2 \cdot C_{1,prev}(3) & for\ l = 3 \\ 10 \cdot C_{1,prev}(3) & for\ l = 4 \end{cases}$$

The third variable is the difference in cost between the operational states. When the difference in cost is large, the maintenance planning will be more focused on the low cost operational state. When the difference in operational states is less, more clustering options are available. Again, three levels of cost difference are considered. The operational state cost difference is modelled as a state penalty. The preventive maintenance cost at the low cost state is multiplied with the state penalty to get the preventive maintenance cost of the other operational states.

$$State\ penalty_{Op.St.}^l = \begin{cases} Op.St. & 1 & 2 \\ for\ l = 1 & U(1; 1.5) & U(1.5; 2) \\ for\ l = 2 & U(1.5; 3) & U(3; 6) \\ for\ l = 3 & U(3; 6) & U(6; 12) \end{cases}$$

The fourth variable is the difference in cost between corrective and preventive maintenance. This determines the attractiveness of preventive maintenance. This also influences whether the model tends to risk failure of components in order to cluster maintenance.

$$Corrective\ penalty^l = \begin{cases} 1.25 & for\ l = 1 \\ 2 & for\ l = 2 \\ 3 & for\ l = 3 \\ 10 & for\ l = 4 \end{cases}$$

The preventive and corrective maintenance cost will be based on the value of the preventive maintenance cost in the low cost operational state (state 3 in this case) and on the state penalty and the corrective penalty:

Operational state	**1**	**2**	**3**
Preventive maintenance	$C_{i,prev}(3) * State\ penalty_1^l$	$C_{i,prev}(3) * State\ penalty_2^l$	$C_{i,prev}(3) = U(50;300)$
Corrective maintenance	$C_{i,prev}(3) * State\ penalty_1^l * Corrective\ penalty^l$	$C_{i,prev}(3) * State\ penalty_2^l * Corrective\ penalty^l$	$C_{i,prev}(3) * Corrective\ penalty^l$

3.1.4.2 ***Scenario analysis results***

The heuristic is evaluated on its optimality. The optimality modelled as the profit that is obtained by the heuristic compared to the profit obtained by the enumeration procedure: $\frac{profit_{heuristic}}{profit_{optimal}}$.

The results are shown in Appendix C. The greedy heuristic that is proposed performs well. The mean optimality is 97.2%. It finds the optimal solution in more than 70% of the experiments. When the heuristic does not find the optimal solution, the optimality is still above 70% for all experiments. When the results are evaluated in more detail, the variables that influence the optimality of the greedy heuristic can be distinguished. Each variable is evaluated on its influence on the amount of non-optimal solutions found and on the optimality directly. It is important to test whether the Greedy heuristic finds the optimal solution, but it is also interesting to evaluate whether the solutions found are close to optimal. When the heuristic finds a solution that is non-optimal, but the difference in profit is close to zero, the use of the Greedy heuristic can be justified. Therefore both indicators are used to evaluate the performance of the optimization method.

The equality of the MTTF's across the components has a positive effect on the optimality of the proposed Greedy heuristic. Table 4 shows the percentage of times the heuristic did not find the optimal solution and the average optimality per scenario level. The MTTF and the maintenance cost values are randomly generated, hence it must be tested if the effect is significant or caused by randomness. This is done by an ANOVA analysis. The Greedy heuristic finds the optimal solution less often when the failure behaviour becomes more unequal. Also the optimality decreases with an increasing MTTF inequality. ANOVA shows that this effect is significant, see Appendix C - Table 13. When the failure behaviour of the components is unequal, there are more local optima. Therefore the Greedy heuristic is more likely to find non-optimal solutions. This explains the decreasing optimality when the failure behaviour becomes less equal.

Table 4 – Scenario analysis of the effect of the similarity of the failure behaviour

MTTF level l	**1**	**2**	**3**
% Non-optimal	18,750	31,250	39,583
Average optimality	0,9900	0,9673	0,9600

The set-up cost does not seem to affect the optimality of the Greedy heuristic, see Table 5. Only the lowest value for the set-up cost seems to have a slightly higher optimality and lower amount of non-optimal solutions found. However, the effect of the set-up cost on the optimality is not significant, see Appendix C - Table 14. The optimality of the Greedy heuristic is not affected because the size of the set-up cost only influences the attractiveness of clustering, but it does not change the structure of the solution. The heuristic is not expected to have a higher probability to end up in a local optima when the set-up cost are higher, because all (instead of some) clustering options become more attractive.

Table 5 - Scenario analysis of the effect of the set-up cost size

Set-up cost	0,5	1	2	10
% Non-optimal	16,667	36,111	33,333	33,333
Average optimality	0,9815	0,9663	0,9727	0,9692

The variable representing the difference in maintenance cost between the operational states shows a remarkable influence on the optimality, see Table 6. It seems that for the intermediate operational state difference the Greedy heuristic performs worse than for the low and high differences. ANOVA shows that the difference in optimality between the state penalty levels is significant, see Appendix C - Table 15. When the operational states have a low cost difference, the penalty cost function will be (close to) unimodal. This way, the Greedy heuristic is less likely to come up with a local optimum. For large cost differences among the operational states the penalty cost will be multimodal, but the amount of local optima will be small, as the maintenance possibilities will be limited to the low cost operational state. Therefore, the Greedy is more likely to result in the optimal solution. However, for intermediate cost difference among the operational states, the penalty cost will be multimodal, but there will be relatively many maintenance probabilities.

Table 6 - Scenario analysis of the effect of operational state cost difference

State penalty level l	1	2	3
% Non-optimal	20,833	50,000	18,750
Average optimality	0,9869	0,9467	0,9836

The effect of the difference in corrective and preventive maintenance cost shows a similar pattern as the effect of the state penalty, in terms of the percentage of non-optimal solutions. However this pattern is not noticeable in the optimality of the solution, see Table 7. The optimality seems positively influenced by the corrective penalty. However, ANOVA shows that this effect is not significant, see Appendix C - Table 16. The optimality of the heuristic at level 10 is significantly higher than the other levels together, see Appendix C - Table 17. The difference between corrective and preventive maintenance influences the benefit of preventive maintenance rather than that it influences clustering benefit. This explains the insensitivity of the optimality to this variable at the first three levels. The positive effect on the optimality for the extreme value is caused by the tendency to plan very conservative to avoid corrective maintenance. The penalty cost will be high. For such high values for the corrective penalty, there are very few clustering options available, which makes the clustering problem much less complex and hence the Greedy heuristic finds the optimal solution with a high probability.

Table 7 - Scenario analysis of the effect of cost difference in corrective and preventive maintenance

Corrective penalty	1,25	2	3	10
% Non-optimal	33,333	38,889	33,333	13,889
Average optimality	0,9617	0,9662	0,9713	0,9904

As said before, the greedy heuristic is much faster than the enumeration procedure. For an instance of 10 components and a horizon of 38 time units, the greedy heuristic finds its solution in around 2.5

seconds, whereas the enumeration procedure takes 27 seconds to find a solution. This difference will become polynomially larger when the instance size increases.

3.2 Condition-based maintenance

In this chapter, the condition-based maintenance model is presented. First, the general model is explained. Thereafter, the criticality aspect is introduced in the model. At last, insight is given into the structure of the solution.

3.2.1 Dynamic programming model

When there is information about the condition of the components available, the maintenance planning can be done dynamically. This information can be in the form of condition measurements or in component usage. The model that will be proposed can be used for condition-based maintenance components and for usage-based maintenance components. From now on, the model is regarded to be condition-based in its notations. The model can easily be translated to a usage-based problem by assuming a degradation probability density function that represents the usage behaviour of the asset.

The condition-based maintenance problem will be solved in a different manner than the age-based maintenance planning. For the condition-based problem real-time information is available. A dynamic programming problem is used. This means that the decision-making at a certain time period depends on the decision-making at the later time periods. E.g., at each state is decided whether the component is maintained during that time period, or if the maintenance is postponed. It is assumed that the preventive maintenance tasks are performed at the end of a time period. When preventive maintenance is performed, the component starts with zero degradation at the next time period. When preventive maintenance is planned for a certain time period and the component fails during that time period, preventive instead of corrective maintenance cost are incurred because the maintenance is already planned.

When the component is maintained at a certain state, the cost for preventively maintaining the component at that date and, depending on the planning of the other (ABM and CBM) components, set-up cost are incurred. In addition, the future expected maintenance cost starting at zero degradation for the next period is incurred. The cost for the postponement option depends on the failure probability and the decision at the succeeding states. The value function of component i at time t is represented by $V_{i,t}(u)$. $V_{i,t}(u)$ is the maintenance cost depending on the maintenance decision a and the current degradation u. The discrete value function is expressed as:

$$V_{i,t}(u) = Min_{a\in\{0,1\}}\left\{a\cdot\left[S_{i,t} + C_{i,t}^{\text{prev}} + V_{i,t+1}(0)\right] + (1-a)\right.$$
$$\left.\cdot\left[\sum_{x=0}^{F-1-u} P_{i,t}(x) * V_{i,t+1}(u+x) + \sum_{x=F-u}^{\infty} P_{i,t}(x)\cdot\left(S_{i,t} + C_{i,t}^{\text{corr}} + V_{i,t+1}(0)\right)\right]\right\}$$

Here, a represents the decision whether to replace the component at the current state. $a = 1$ means that the component is maintained at this period, $a = 0$ means that the maintenance is postponed. F is the degradation threshold. $P_{i,t}(x)$ is the probability of incurring a degradation of size x at period t for component i. $S_{i,t}$ represents the possible set-up cost that is incurred. The set-up cost is dependent on the planning of the other components; when another component with a shared set-up is planned on that period, no set-up cost is incurred. $C_{i,t}^{\text{prev}}$ and $C_{i,t}^{\text{corr}}$ represent the preventive, and respectively, the corrective maintenance cost of component i at the operational state during period t.

The degradation distribution is assumed to be discrete in the function above. It is possible to use a continuous degradation distribution mathematically, but the model will become intractable in software packages as Excel/VBA.

To make a decision at a certain point in time, the value function $V_{i,t}(u)$ will incorporate all decisions at the succeeding time point until the horizon. Hence, the computational size becomes large for long horizons. However, the value function at a certain time with a certain degradation level can be reused after they are calculated for the first decision. The first decision incorporates $V_{i,t+1}(0)$ and $\sum_{x=u}^{F-1} V_{i,t+1}(u + x)$. These terms cover all terms that are needed for the decisions at the upcoming times.

To optimize this problem, it is convenient to build a value matrix containing all the future value's that are needed for the decision-making. This matrix will be two dimensional and ranging from $u = 0 \; to \; F - 1$, $t = Current \; time + 1 \; to \; horizon$. First the values at the last column are determined: $t = horizon$ for $u = 0 \; to \; F - 1$. At $t = horizon$ the value function simplifies to: $V_{i,horizon}(u) = Min_{a\in\{0,1\}}\{a \cdot [S_{i,horizon} + C_{i,horizon}^{\text{prev}}] + (1 - a) \cdot [\sum_{x=F-u}^{\infty} P_{i,horizon}(x) \cdot (S_{i,t} + C_{i,horizon}^{\text{corr}})]\}$. The value to insert into the matrix will already incorporate the decision on a, given that the accumulated degradation at period t is u. The values at the last column can be used to determine the values at the second last column and so on. When all values in the value matrix are determined, the actual decision-making can be done. This is done by calculating the value function term at the current time and the current degradation level and calculating what decision results in the minimal expected maintenance cost. If the value function is minimal for $a = 1$, the maintenance will be performed this period. Otherwise the maintenance is postponed. During the next period, the decision is made again. As said before, the value matrix can be reused to make this decision.

3.2.2 Criticality

In chapter 2.3 component criticality is introduced. This can be included in the dynamic programming problem. When a non-critical component fails, the maintenance of this component is postponed to the first occasion that the asset enters the low-cost operational state. This can be modelled using a binary variable representing the criticality of a component during the states in the planning: $cr_{i,t}$. It makes sense to have a criticality variable for the whole system, instead for the components separately, because the system will not work (properly) when a component has failed, but the criticality can be modelled either way. The value function is extended with a term representing what happens in case of the failure of a non-critical component. In that case, no direct cost factor is incurred. The usage in the next period will remain at the same level or increase.

$$V_{i,t}(u) = Min_{a\in\{0,1\}}\Bigg\{a \cdot \left[S_{i,t} + C_{i,t}^{\text{prev}} + V_{i,t+1}(0)\right] + (1-a) \cdot cr_{i,t} \cdot \left[\sum_{x=0}^{F-1-u} P_{i,t}(x) * V_{i,t+1}(u+x) + \sum_{x=F-u}^{\infty} P_{i,t}(x) \cdot \left(S_{i,t} + C_{i,t}^{\text{corr}} + V_{i,t+1}(0)\right)\right] + (1-a) \cdot (1-cr_{i,t}) \cdot \left[\sum_{x=0}^{\infty} P_{i,t}(x) \cdot V_{i,t+1}(u+x)\right]\Bigg\}$$

To model the behaviour of the system in case the maintenance on a non-critical failed component is postponed, an additional row to the value function matrix is added for $V_{i,t}(F)$:

$$V_{i,t}(F) = cr_{i,t} \cdot \left[S_{i,t} + C_{i,t}^{\text{corr}} + V_{i,t+1}(0)\right] + \left(1 - cr_{i,t}\right) \cdot \left[V_{i,t+1}(u+x)\right]$$

There is no decision-making involved in this function, because the system is maintained the first time the system becomes critical. The criticality of the components can be added to the asset schedule. When a system is non-critical, the criticality variable $cr_{i,t}$ will be 0 until the asset enters the low cost operational state. Setting the criticality variable to 1 for all low cost operational states will force the model to repair the component the first time the asset enters the low cost operational state.

3.2.3 Value function solution structure

To illustrate the structure of the value function, the expected maintenance cost resulting from the decision making at a certain moment in time are plotted for different degradation levels. The multi-component model is used. Here, the data from the case study is used. For simplicity, the system is assumed to be critical all the time. Note that the graphs do not represent the decision-making for a component during the entire horizon. They represent the decision-making at a point in time, if the degradation is the specified amount at that time. In reality, the degradation will increase over time depending on the mission schedule and the decision-making; this is not incorporated in the graphs. In Figure 7, the value function structure for a degradation of 0 is shown. The expected cost for postponing the maintenance are always less than the expected cost for maintaining, because maintaining the component makes no sense in these cases. The four high cost operational states are clearly visible in the cost resulting from maintaining; see Table 9 for the asset schedule that is used. The three local minima in the graph for maintaining at time 67 and 124 are caused by clustering possibilities.

Figure 8 shows the value function structure for a degradation of 8, the degradation threshold is 17 for this component. Note that the cost resulting from maintaining is equal for the three graphs. The cost resulting from the decision to maintain is not dependent on the current degradation level, as can be seen in the value function expression. The expected cost for postponing is higher than in the graph where the degradation is zero, because the failure risk for postponing is higher. Figure 8 shows that the model would decide to maintain, when the component has a degradation of 8, at times 14, 67 and 124.

Figure 9 shows the value function structure for a degradation level just below the failure threshold. In this case, postponing the maintenance is risky. The peaks in the postponement cost are during the moments the degradation is expected (for simplicity, the expected degradation is set to be incurred at the end of each mission, this will result in the same decision-making as when the degradation is incurred throughout the mission state). At these moments, it is better to maintain the component preventively during missions, than to let the component fail. However, in the graph is also visible that when the degradation is just below the failure threshold during the low cost operational state before the missions, the model will decide to maintain the component. Note that the model will decide to postpone at the start of the low cost operational states, because it knows that the component will not fail until the end of this state and that will be deciding to maintain the component then.

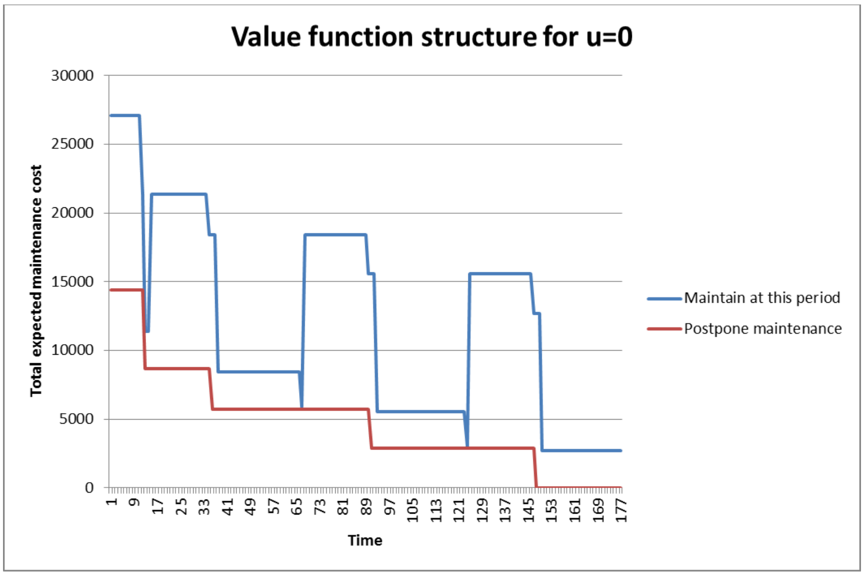

Figure 7 – The structure of the value function throughout the horizon, for a degradation level of 0

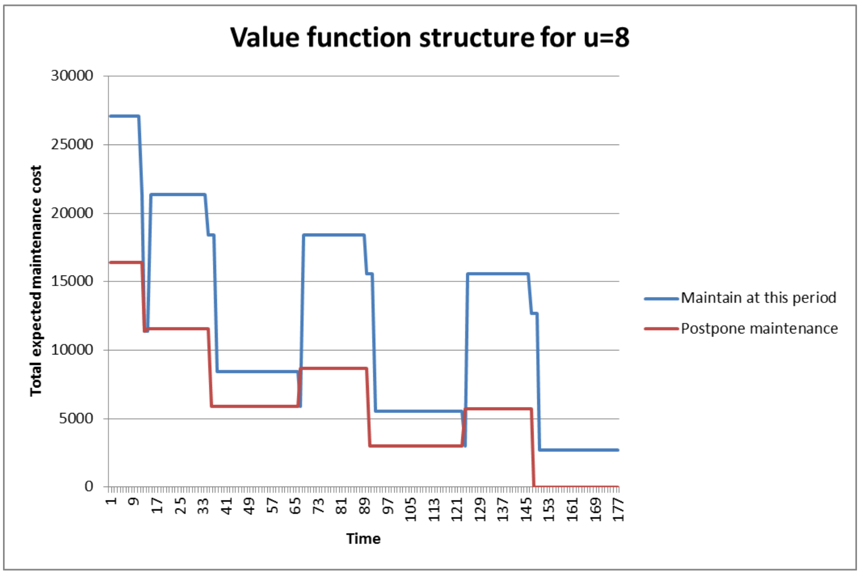

Figure 8 - The structure of the value function throughout the horizon, for a degradation level of 8

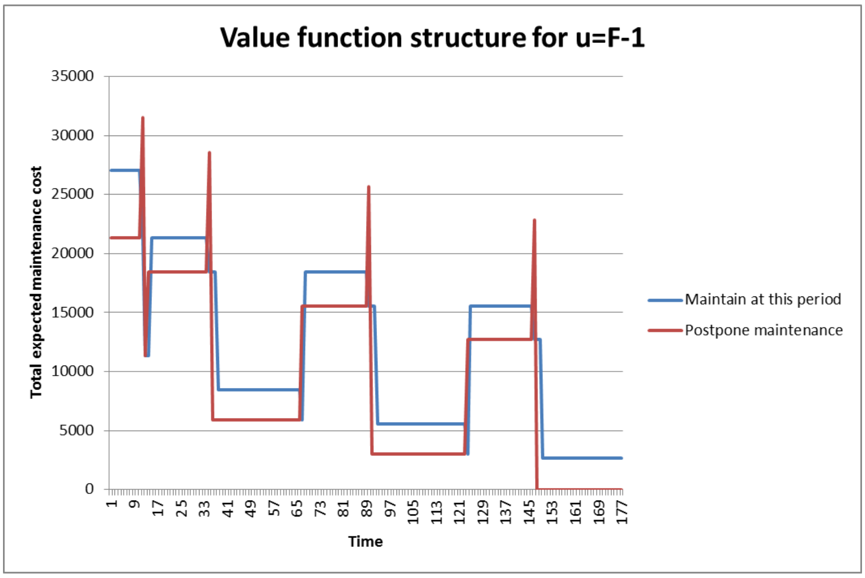

Figure 9 - The structure of the value function throughout the horizon, for a degradation level of F-1

3.3 Integration ABM and CBM

The multi-component condition-based model incorporates the structural dependencies between components. When the maintenance planning of the age-based components is optimized, the decision-making for the CBM-components can be done including clustering possibilities with age-based components. This way, the planning of age-based and condition-based components can be integrated. The planning of the condition-based components is not used in the decision-making in the age-based components planning, because the condition-based problem is dynamic. The decision-making considers whether to perform maintenance now or to postpone, instead of deciding upon the maintenance date directly. Of course, the model can be run iteratively to retrieve the maintenance date, but the decision-making is reconsidered each time a condition-update is received. This makes the maintenance date too volatile to base the maintenance planning of the ABM-components on.

Optimizing the ABM-model by clustering among ABM-components only and thereafter optimizing the CBM-model, makes more sense.

When there are many ABM-components and few CBM-components, the order of scheduling the CBM-components is not expected to have a significant influence on the outcomes. However, when there are relatively many CBM-components, the dependencies between these components will influence the decision-making. It will be intractable to optimize the scheduling of all CBM-components at once. Therefore, the order of the optimization of the components should be chosen wisely. An easy way to do this is to plan the components in order of their expected maintenance frequency. The components with the lowest maintenance frequencies are planned first, because these components are expected to have the lowest clustering flexibility for the following reasons. These components typically have low condition thresholds or fast degradation. Postponing these components will cause a relatively high expected corrective cost. Because these components are maintained frequently, the planning of the other components can benefit from a lot of possible set-up savings when the high frequency components are planned first.

4 Case study

The model is applied and tested on the maintenance management of the Marlin weapon system. This Marlin weapon system is an anti-material gun with a 30 mm calibre. The system is manufactured by OTO Melara. The system is installed on Joint Support Ships (also known as the 'Karel Doorman class') and on Oceangoing Patrol Vessels (also known as the 'Holland class'). In total, the RNLN has six Marlin systems in use. The Marlin gun is a highly accurate system that is mainly used to target smaller fast moving vessels. The system contains its own electro-optical aiming devices and can remotely be controlled via the ship's command management system.

Figure 10 - Picture of the Marlin weapon system

The Marlin weapon system is relatively new to the DMI and hence its maintenance organization is not as settled as it is for the older systems. Whereas for most systems the maintenance is organized such that most tasks are clustered in the major overhaul, the Marlin weapon system still has a lot of ILM tasks. Applying the model proposed in this thesis on these ILM tasks might be interesting.

Unfortunately, not all the inputs for the model are available for the case study. In addition, the available data has proven to be unreliable in reality. Therefore, the results of this case study will be presented in the form of a sensibility analysis. This way, the influence of each input parameter is evaluated. When more accurate data becomes available, the output of the sensitivity analysis can be consulted for the expected benefit of the proposed model.

4.1 Data and assumptions

The dataset contains 16 tasks that can be clustered. Three of these components have usage-based lifetimes. The degradation of these components is assumed to be determined by the amount of rounds fired.

The dependency between components is a structural dependency; the set-up time that is incurred for dismantling components and preparing the tasks. This is expressed in costs by multiplying the time with the hourly wage of the maintenance engineers. An hourly cost of €64,- can be used. The tasks for which the set-up can be shared are shown in the 'shared set-up combination nr.' column in Table 8.

The lifetime of the components of the Marlin is highly dependent on the way the system is used and maintained. After firing the gun, the barrel must be cleaned properly, otherwise failure is most likely to occur. There are multiple cleaning and inspection tasks that must be performed properly by the crew on board, otherwise the time to failure will decrease drastically. Human errors are not incorporated in this model, they are neglected.

Unfortunately, there is a lot of data missing for the case study and some of the available data is unreliable. Therefore, a reference parameter setting is created. Based on this parameter setting, the sensitivity analysis can be performed. The reference parameter settings are explained below. Table 8 shows the theoretic and the reference parameter settings.

Table 8 – Components used in the case study and their parameters

Task	PROCEDURE	Observed maintenance frequency (per year)	Theoretic ABM MTTF (weeks)	Theoretic UBM threshold (rounds)	Reference MTTF (weeks)	Reference failure threshold (rounds)	Reference coefficient of variance	Shared set-up combination nr.	Set-up time saved (hours)	Preventive maintenance costs for the harbour state
1	Feeder assembly inspection and maintenance	1	2799,39		90		0.408	6	4	3367,3
2	Receiver assembly inspection and maintenance	1	2799,39		90		0.408	6	4	2781,89
3	Lubrication roller path bearing (T-bearing Maint)	1	5042,17		120		0.408	4	1	530,5
4	Training toothed sector lubr. check (T-bearing Maint)	1	5042,17		120		0.408	4	1	1035,92
5	Replace Elevation shock absorbers bellows	0,2	492,63		492,63		0.408	5	0,5	502,47
6	TN reduction gearbox oil level check	2	133,65		70		0.408	2	1	81,49
7	Train reduction gearbox maintenance	0,5	534,58		230		0.408	3	1	375,9
8	EL reduction gearbox oil level check	2	133,65		80		0.408	2	1	151,9
9	Elevation reduction gearbox maintenance	0,5	534,58		230		0.408	3	1	375,9
10	Inside washing IR camera	1	125,75		125,75		0.408	1	0.5	906,55

11	Inside Washing of the Daylight Camera Optics	1	125,75		125,75		0.408	1	0,5	546,41
12	Inside Wash laser Rangefinder	1	125,75		125,75		0.408	1	0,5	546,41
13	Replace chute protection sleeves	0,2	262,20		350		0.408	5	0,5	740,1
14	Firing pin protrusion inspection	0,70		1000		1700		6	4	134,01
15	Barrel assembly inspection	0,35		2000		2400		6	4	805,52
16	Receiver assy: breech assembly inspection	1		25000		25000		6	4	327,6

It is assumed that after each maintenance task the component is restored to an as good as new state. In the data, there are multiple tasks on one component in some cases. These tasks concern parts within the component. There is only a MTTF measure available for the entire component. The MTTF measure of the parts will be determined by allocating the MTTF of the component using the weight of the frequency of the part in the current planning, such that the MTTF measures follow the equation $\frac{1}{MTTF_{system}} = \frac{1}{MTTF_1} + \frac{1}{MTTF_2} + \cdots$. Here, the $MTTF_i's$ are the MTTF's of the components to be determined, and $MTTF_{system}$ is the known MTTF of the system. When the relations between the maintenance frequencies are known, the equation can be solved.

As shown in Table 8, the theoretical MTTF measures contradict the maintenance frequencies that are observed in practice. The DMI identifies various reasons that might explain this difference. Therefore it is reasonable to adjust the MTTF measures to create a maintenance planning that reflects reality. This is done by iteratively changing the MTTF values and determining the single-components optimal maintenance date. The single-component maintenance date is determined at different moments in the mission schedule to decrease the influence of the mission schedule. Also, the relation between the lifetime distribution parameters and the maintenance date, as described in the next paragraph, is evaluated. The MTTF measures are adjusted until the resulting single-component optimal maintenance dates match the observed maintenance frequency.

From this MTTF value, the distribution parameters are derived. An Erlang distribution is used, hence the shape and the scale parameter have to be estimated. Only the ratio between the two parameters is known as only the first moment is available. The relation between the two parameters and the MTTF is: $MTTF(X) = \frac{\mu}{\lambda}$. The variation equals: $Var(X) = \frac{\mu}{\lambda^2}$. λ influences the variance in a quadratic way and the MTTF in a linear way. Therefore, choosing high values for μ and λ while keeping the MTTF as fixed, results in a low variation. When low values for both parameters are chosen while keeping the MTTF as fixed, the variation is relatively high. The variance in the lifetime distribution is expected to be large, hence relatively low values for the parameters can be chosen. The coefficient of variance

(cv_i) is only dependent on μ_i in case of the Erlang distribution: $cv_i = \frac{\sqrt{Var(X)}}{\mu_i} = \frac{1}{\sqrt{\mu_i}}$. To have an equal coefficient of variance across all components, μ_i is chosen to be equal for all (ABM-) components. However, guessing the right parameter values remains doubtful. In the sensitivity analysis, the influence of both parameters is investigated. The parameter values chosen for the reference parameter setting are shown in Table 8.

The usage-based components can be incorporated using the expected operating hours or the expected rounds fired per mission. Currently, only general expectations are available to base the planning on. The rounds fired during an upcoming mission are assumed to be random and uniformly distributed. The usage-based components have a usage threshold. Currently the components are preventively maintained when this threshold is reached. The same threshold is used for the usage-based model. Here is assumed that failure occurs when the threshold is used, while in reality the thresholds are just the recommendations of the usage before maintenance. A failure threshold is expected to be higher than a preventive maintenance threshold. Therefore, a range of threshold values is taken in the sensitivity analysis.

Performing maintenance while in transit or mission is costly. Failure of a critical component during a mission might cause the abortion of the mission. In addition, it might be necessary to fly in an engineer and spare parts. Unfortunately, the exact maintenance cost per component in the transit and mission state are not at hand. Therefore, the maintenance cost of the transit and mission state will be based on the maintenance cost of the harbour state and raised with a certain increment. As the value of this increment is unknown, various values will be included in the sensitivity analysis. Based on expert opinion from the MaSeLMa participants the following estimates are available: a cost increment of €10000 for both the mission and the transit state. This is used as reference value.

The engineers at the DMI claim that there is no cost difference in cost between corrective and preventive maintenance other than the operational state cost difference. However, while there may not be a difference in the cost of the tangible maintenance tasks whether it is performed preventively or correctively, there will be a disadvantage in unexpected maintenance. As corrective maintenance cannot be planned, unwanted downtime is likely to occur in case of failure. Due to the high operational availability needs of the RNLN, failure must be penalized within the system. Also, preventive maintenance is much easier to manage and prepare. This makes it reasonable to include a corrective penalty. For the reference parameter setting a value of 2 is chosen as corrective penalty. This means that corrective maintenance is 2 times as expensive as preventive maintenance.

A realistic mission schedule is created for the case study, see Table 9. This schedule is based on the mission schedules that are observed at the Navy. The real mission schedules are confidential, but the following mission schedule provides a realistic example. The mission schedule covers the maintenance cycle of 4 years. Each four years the system enters major overhaul (DLM). This project focusses on ILM so the horizon of the planning will be the cycle of the system between two major overhauls, which is 178 weeks. The lengths of the operational states are presented in chronological order. Here, operational state 1 represents the transit state, operational state 2 represents the mission state and operational state 3 represents the home harbour state. The first state in the schedule is a transit state. In reality, just after the ship leaves major overhaul, it enters a phase where the ship is built up and restored to be ready for missions. In this phase the systems are tested and the crew is trained. The ship is relatively close to the harbour. The transit periods after missions are regarded as non-critical as it is reasonable to assume that the weapon system is not used on the transit back to the home harbour.

The criticality of the system is also included in the schedule. The criticality is incorporated in the UBM model. Based on expert opinion is assumed that the system is non-critical in 10% of the missions. During the transit states after missions, it is reasonable to assume that the system is non-critical. When the system fails on the transit back to the harbour, the maintenance can be postponed until the harbour is reached. The reference parameter setting is shown in Table 9.

For the UBM-components, the failure threshold is unknown. As explained before, only the usage replacement threshold that is proposed by the OEM is available. The failure threshold is expected to be higher, as the usage replacement threshold is a preventive replacement trigger. To determine a realistic failure threshold value for the reference parameter setting, the system usage during the missions should be determined first. For the UBM-components of the Marlin weapon system, the amount of rounds fired during a mission is used as the degradation mode. On average, the amount of rounds fired per mission is 700. The exact usage is not known on beforehand. To simulate the uncertainty in the usage, the amount of rounds fired on a mission is assumed to follow a uniform distribution. The range of the uniform distribution is assumed to be $\{0.5 \cdot E[u_t]; 1.5 \cdot E[u_t]\}$, where $E[u_t]$ represents the expected usage at time t. The expected usage during the missions is shown in Table 9. During the first transit period, in which the ship is restored to an operational condition, only a small amount of rounds will be fired.

Table 9 – Mission schedule for the reference parameter setting

Operational state	1	3	1	2	1	3	1	2	1	3	1	2	1	3
Length	12	2	2	19	2	30	2	21	2	32	2	21	2	29
Criticality	1	1	1	1	0	1	0	0	0	1	1	1	0	1
Expected usage $E[u_t]$	300			700				700				700		

Based on the mission schedule, the usage failure thresholds for the reference parameter setting are determined. This is done by changing the failure threshold until the single-component UBM optimization results in a maintenance frequency that matches the maintenance frequency that is observed in practice. The reference parameter setting for the failure threshold is shown in Table 8.

To evaluate the benefit of the proposed model, the resulting maintenance cost is compared with the maintenance cost that would be incurred without clustering. The single-component model can 'accidently' result in shared set-ups. Note that, in Table 8, the components with shareable set-ups also have comparable failure parameters. Therefore, the actual benefit of the proposed ABM-model must be compensated for the set-ups that are shared within the results of the single-component model. To determine this, the single-component optimal maintenance date is chosen as the maintenance date. Then, the total expected maintenance cost is calculated while compensating for the set-ups that are saved in case tasks are maintained at the same date.

The single-component CBM-model maintenance cost can be determined by using the model without dependencies between components. The decision-making in case the dependencies between components are neglected is used to determine the (actual) expected maintenance cost with 'accidental' set-up cost savings.

This project focusses on the planning of the ILM-level tasks. Hence, a planning horizon will be the period between two major overhauls or DLM periods. Because major overhaul is performed after this horizon, it makes little sense to maintain the component just before the end of the horizon. Therefore, at time $horizon + 1$ no set-up cost will be incurred. This will cause the model to postpone the maintenance towards the DLM period when the expected corrective cost do not exceed the cost of performing maintenance before the major overhaul.

The suggested ABM-models incorporate the operational state cost difference and are expected to perform well. Therefore, the results will be compared with a less sophisticated model in addition. This model uses the classical renewal theory function $C_i(\tau_i) = \frac{ECC(\tau_i)}{ECL(\tau_i)}$. This model will not distinguish between operational states and use a weighted average of the operational state cost to determine the preventive and corrective maintenance cost. Here, the operational state costs are weighted by the amount weeks a certain state occurs in the mission schedule.

Two types of availability are considered; overall availability and mission availability. Overall availability includes all downtime as a result of performing maintenance. Mission availability only considers downtime as a result of maintenance during missions. This distinction is made because the RNLN might value availability during missions in particular and does not mind if increasing mission availability leads to additional downtime during the harbour periods. The expected duration of a task is known and used for the downtime. Additionally, the set-up time for each task is known, this is incurred as downtime when the maintenance task could not be clustered. When the ship is in the transit and mission state, an additional downtime of one week is assumed to be incurred. This can be seen as the time that is needed to ship the spare part and the engineer and to prepare the tasks. The availability measure will be based on the expected downtime of all components combined. Here is not compensated for the possibility that two components are down at the same time. In that case, the components are down in parallel and in reality the system would only incur the downtime once. However, for simplicity, this is not incorporated in the availability calculations. The results are primarily used to compare the performance of the models. Whether or not this aspect is included is not expected to give one model a significant advantage over the others, so the comparison is not expected to be biased by this assumption.

For the ABM-models, the downtime is calculated by multiplying the downtime for preventive maintenance by the probability of survival until the maintenance date + the probability of failure at a certain period multiplied by downtime for corrective maintenance during this period depending on the operational state. The total expected downtime of all components is added up. Then, the total expected downtime of all components together is divided by the horizon length to obtain the overall availability. The mission availability is obtained by only including the downtime during missions.

For the UBM-models, the availability can be determined using a similar dynamic programming model as for the maintenance planning. Instead of including the consequences of a decision in terms of cost, this function includes the consequences in terms of downtime. The following function shows this availability function. Here, $D_{i,t}$ and $S_{i,t}$ represent the task duration and setup time of component i during period t respectively. The task duration is dependent on the operational state as explained earlier. The setup time is only incurred when the maintenance of this task is not clustered.

$$Av_{i,t}(u) = \Bigg\{ a \cdot \left[S_{i,t} + D_{i,t} + Av_{i,t+1}(0)\right] + (1-a) \cdot \left[\sum_{x=0}^{F-1-u} P_{i,t}(x) * Av_{i,t+1}(u+x) + \sum_{x=F-u}^{\infty} P_{i,t}(x) \cdot \left(S_{i,t} + D_{i,t} + Av_{i,t+1}(0)\right) \right] \Bigg\}$$

To determine the mission availability, the same formula is used, but the downtime is only incurred when the ship is in a mission. The decision-making of the regular UBM-models used as input in the availability model to determine the resulting availability.

4.2 Reference results

This chapter discusses the results of the models when the reference parameter settings are used as input. As mentioned before, the reference parameter setting serves primarily as a basis for the sensitivity analysis. Although the reference parameter setting is ought to be as realistic as possible, guessing the right values was doubtful in some cases. Therefore, these results should not be viewed as directly applicable to the case of the Navy, or other cases. The sensitivity analysis can be consulted to get insight in the performance of the model for a specific parameter setting. The model itself can be used to find the planning that is proposed with this model. The reference results are presented in Table 10 and the resulting planning from the single-component models is shown in Table 11. The costs shown in Table 10 are the expected cost for the entire horizon. The set-up combinations that can be made are shown in Table 8.

The models that incorporate the operational state cost difference perform much better than the basic policy. The proposed ABM models result in a 76,6% cost saving compared with the basic policy. In addition, the proposed models obtain an 8% mission availability improvement. This shows the importance of incorporating the operational state difference in the maintenance modelling. The maritime environment asks for a tailored maintenance model, which makes the traditional maintenance models very unsuitable and suboptimal. The basic policy does not pay attention to the mission schedule in any way. Hence, it will schedule preventive maintenance at very unfortunate times.

For the ABM-components the multi-component approach does not show an advantage over the single-component approach and results in the same planning. All the set-ups that are saved in the multi-component solution where already saved in the single-component solution. The reason behind this is the equality of the failure parameters of the components that have dependencies between them. This is reinforced by the high cost difference between operational states. It is suboptimal to schedule maintenance somewhere halfway a harbour operational state, as waiting to the end of this state does not increase the expected corrective cost that much. Therefore, the maintenance planning will already be grouped on the end of the operational states. Additionally, the mission planning contains only a few quite long operational states. There are only four harbour periods, and hence there are only four practical maintenance dates. This increases the probability of 'accidentally' saved set-ups with the single-component model, which leaves little room for additional clustering. In the sensitivity analysis various levels for the corrective penalty, the cost difference between operational states and different mission schedules are chosen to further investigate this effect.

For the UBM-components, the multi-component approach has a slight advantage over the single-component approach. The multi-component approach yields a cost saving of 0.8%. Note that the single-component UBM-model does not preventively maintain component 14 before the second mission. This mission is non-critical in the reference planning, hence the model chooses to risk a probable failure. The corrective maintenance can be postponed until the ship arrives in the harbour at week 93. The multi-component chooses to preventively maintain the component, because a set-up can be saved at week 67. This causes the slight advantage of the multi-component model. The overall clustering benefit is low because of the highly clustered nature of the ABM single-component solution and its focus on the end of the operational states. For the UBM-components, there is no reason to schedule the maintenance halfway the harbour period, because the system is not used during harbour periods. As both the solutions of the ABM- and the UBM-components are already clustered on the end of the operational states for this parameter setting, a multi-component approach does not have a big advantage over a single-component approach.

Table 10 – The results from the different models using the reference parameter setting

	Single-component ABM model	Multi-component ABM model	Basic policy ABM model	Single-component UBM model	Multi-component UBM model
Total expected maintenance cost	46046,86	46046,86	196367,36	32693,25	32423,25
Overall availability	0,9882	0,9882	0,9643	0,9887	0,9879
Mission availability	0,9959	0,9959	0,9223	0,9933	0,9926

Table 11 – Maintenance planning resulting from the single-component models

Task/ Comp.	Planned maintenance times (weeks)			
1		67	124	179
2		67	124	179
3		67	124	179
4		67	124	179
5			124	179
6	14	67	124	179
7		67	124	179
8	14	67	124	179
9		67	124	179
10		67	124	179
11		67	124	179
12		67	124	179
13			124	179
14		93		179
15			124	179
16				179

4.3 Sensitivity analysis

To be able to draw conclusions that are not strictly depending on the reliability of the data, a sensitivity analysis is performed. The influence of several input parameters on the behaviour of the model is evaluated. The results are evaluated on the clustering benefit. The clustering benefit is calculated as the cost difference between the multi-component and the single-component model, as a percentage of the single-component expected maintenance cost.

First, the performance of the models in terms of availability is evaluated. The proposed models focus on cost primarily. While availability is an important factor in the maintenance management, it can only be incorporated indirectly in the models. Chapter 4.3.1 gives insights in the relation between total maintenance cost and availability and the performance of the models on these factors.

Then, the benefit of maintenance clustering is evaluated by comparing the results of single-component models with the multi-component models. Seven parameters are investigated in this sensitivity analysis; the failure behaviour, the difference in maintenance cost between the operational states, the difference in maintenance cost between corrective maintenance and preventive maintenance, the size of the structural dependencies between components, the failure threshold for the UBM-components, the variance in the usage during missions, and the criticality of the system for the UBM-components. The variables are evaluated on their primary effects on the behaviour of the model. However, from the reference results is seen that the multi-component benefit is hampered due to multiple effects at once. Therefore, a scenario analysis of the combinations of the variable levels that seem to be interrelated is performed. The MTTF inequality, corrective penalty, operational state cost difference, the set-up cost and the mission planning are incorporated in this scenario analysis. These variables determine the 'accidentally' clustered nature of the single-component solution. Chapter 4.3.2 explores scenarios that have less of a clustered nature in itself and thereby a higher clustering potential, and how these scenarios are formed. To limit the size of the experiment, the other variables are investigated separately.

4.3.1 Availability versus cost

The models predominantly focus on saving cost. However, optimizing system availability can be an important goal in the maintenance planning. The model incorporates the availability indirectly via the corrective penalty and the operational state cost difference. One can penalize downtime or unavailability during missions by choosing high values for the operational state cost difference. This section evaluates the performance of the models in terms of availability. An 'efficient frontier' can be drawn to compare the trade-off between availability and cost of the difference models. The focus on availability is increased by increasing the operational state cost difference. The following operational state cost difference values are used: $\{5000, 7500, 10000, 12500, 15000, 17500\}$.

The efficient frontiers between availability and maintenance cost of the proposed ABM models are presented in Figure 11. The efficient frontiers show a stepwise shape. When the operational state cost difference increases, the model decides is it should keep maintaining a component after a certain mission or to advance the maintenance to before this mission. The mission states are long, so this decision involves big difference in timing. Hence, the maintenance planning is expected to remain the same for small deviations of the operational state cost difference. When the operational state cost difference becomes high enough, the maintenance planning suddenly shifts towards a more conservative mission and results in a higher availability. Note that the efficient frontier of the multi-component model obtains a small decrease in maintenance cost at the fourth operational state cost difference level. The solution at the third level is suboptimal due to the heuristic optimization approach. The increase in operational state cost difference changes the solution structure in such a way that the model obtains the additional clustering benefit.

The multi-component models obtain at least equal availability levels and maintenance cost. For most of the operational state cost difference levels, the multi-component model obtains an availability increase and a cost decrease. In these cases, maintenance clustering is done by advancing maintenance tasks, resulting in saved set-up cost and lower expected downtime. The multi-component model could decide to postpone maintenance and result in a cost saving at a slightly lower availability level, but due to the high operational state difference, the penalty cost for postponing are high. This way, the availability is indirectly assured and will not be suffered much to obtain clustering benefits in terms of cost.

The efficient frontiers between availability and maintenance cost of the proposed UBM models are presented in Figure 12. The availability of the multi-component solution is not increased by the increasing operational state cost difference. This can be explained by the low amount missions in the mission planning. The multi-component model obtains a low risk and high availability solution at the lower operational state differences. The multi-component model has already advanced the maintenance timing to cluster the tasks, so the higher operational state cost difference will not make the maintenance planning more conservative.

Figure 13 shows the efficient frontier of the multi-component model compared to the basic policy. Clearly, the proposed multi-component model is superior to the basic policy on both availability and cost. The proposed single-component model is left out for the sake of visibility, but as shown in the other graphs, it will have a similar advantage over the basic policy. This comparison shows the importance of incorporating the operational state cost difference in the maintenance modelling. The basic policy does not avoid maintenance during missions in any way. Depending on the operational state cost difference level, the maintenance will be accidentally planned during harbour periods or during transit and mission periods. This explains the S-shape in the basic policy frontier.

The benefit of the multi-component model over the single-component model is rather small. This is caused by the highly clustered nature of the single-component solution, as said before. The next sections will focus on the clustering benefit and the parameter settings that influence this.

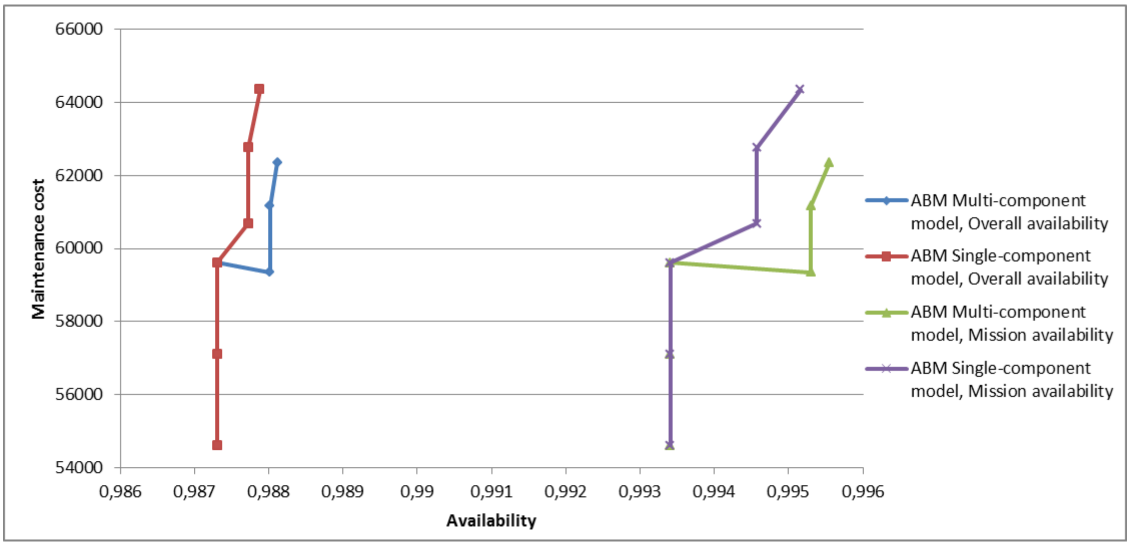

Figure 11 – The efficient frontier between both the availability measures and maintenance cost for the ABM models

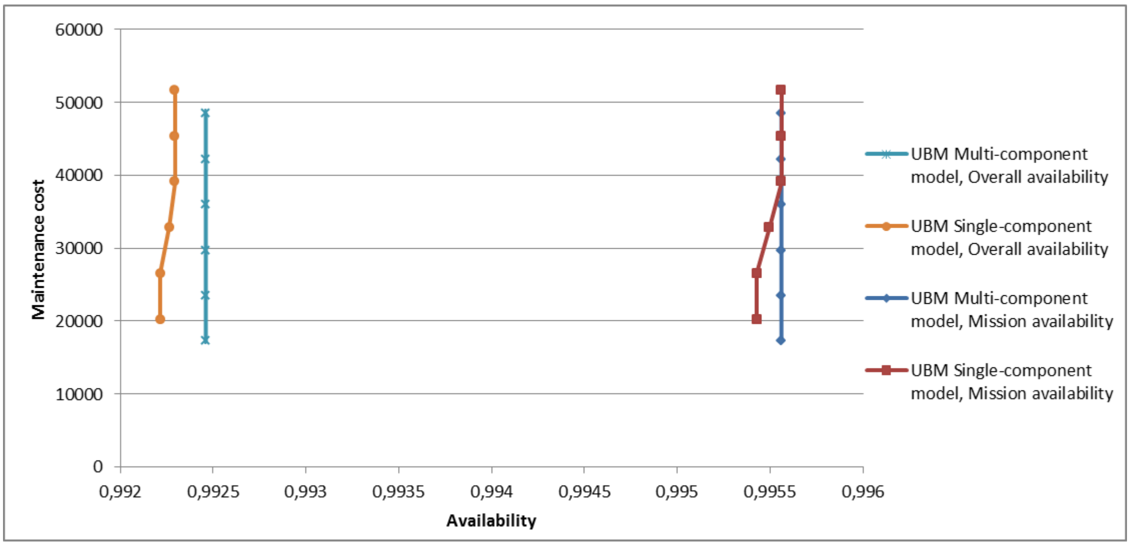

Figure 12 – The efficient frontier both the availability measures and maintenance cost for the UBM models

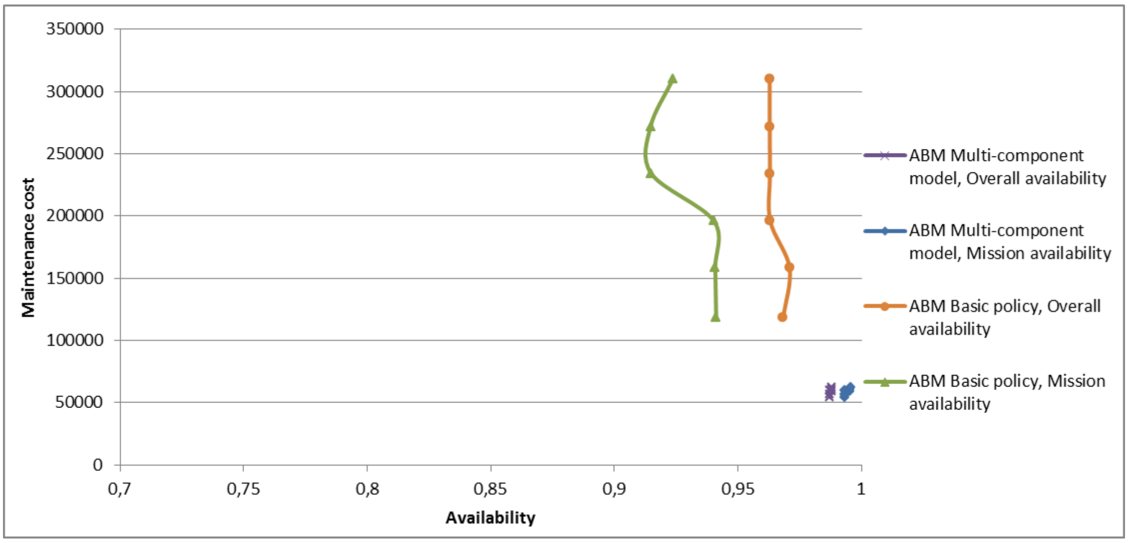

Figure 13 – The efficient frontier between the two availability measures and maintenance cost for the proposed multi-component model and the basic policy

4.3.2 Parameter settings with high clustering potential

The scenario analysis is constructed as follows. For the difference in maintenance cost between the operational states, three different levels are used: $\{0, 5000, 10000\}$. These are the cost increments for both the mission and the transit state. The value of 0 is included to evaluate the behaviour of the system around the boundaries of the parameters. This setting removes the operational state aspect of the problem.

For the corrective penalty, three levels are used: $\{2, 5, 10\}$. As the corrective penalty used for the reference setting is relatively low, more extreme values are evaluated. Other companies might observe much higher corrective penalties, so it is interesting to evaluate the effects of higher values on the performance of the models.

The failure behaviour in the reference parameter setting is very similar among components with dependencies between them. To evaluate the performance of a multi-component model, a less similar scenario is evaluated. To do this, the MTTF values of the components that have dependencies between them are pulled apart. When two components share set-ups, the MTTF of the first component is lowered by 25%, and the other component is increased with 25%. When three components share set-ups, the MTTF's of the first and the third component are decreased and increased with 25% respectively, while the MTTF of the second component is left as it is.

The set-up costs in the reference parameter setting are low compared to the other maintenance cost. The model is expected to perform better at higher levels of set-up cost. Therefore, a high set-up cost scenario is created by multiplying the set-up cost by 10.

The mission schedule in the reference parameter setting has a large influence on the results. The maintenance planning is highly focussed on the end of the harbour periods. There are a few long harbour periods in the mission schedule, this leads to a few ‘viable’ maintenance possibilities. To increase the amount of maintenance possibilities and to make the maintenance planning more interesting, an alternative mission schedule is made. In this alternative schedule, the harbour periods in the reference mission schedule are split up into three parts; two harbour periods with a transit period in between. This might also be a realistic scenario for the RNLN, as the harbour periods can also be used for training and testing purposes.

To limit the runtime of the scenario analysis, the last UBM-component is left out. The high replacement threshold makes the solution space of the dynamic problem very large, and the planning of this component is not interesting because it will survive during the entire horizon, despite the changes in the variables.

4.3.2.1 *Operational state cost difference*

The results of the entire scenario analysis are shown in Appendix E. For this analysis the advantage of the multi-component ABM-model over the basic policy is added. As expected, the proposed ABM-models do not have an advantage when there is no operational state difference. There is only a clustering benefit of 1.087% with respect to the single-component ABM-models. The benefit of the proposed ABM-models with respect to the basic policy quickly increases when the operational state difference increases. The advantage increases up to 70% at the reference operational state cost difference level. This advantage is mainly dependent on the operational state cost difference. The next parts of the sensitivity analysis focus on the clustering benefit of the multi-component models with respect to the single-component models.

For the usage-based components the clustering benefit goes up to 21%. The clustering benefit is higher for the lower operational state cost differences, as shown in Figure 14. Here the reference corrective penalty, the reference mission schedule and the high set-up cost are used. When the operational state cost difference is lower, the penalty cost is lower, which makes clustering less costly.

The ABM-model seems unaffected by the operational state cost difference for these parameter settings. The effect of the operational state cost difference is moderated by the other variables as well. This is shown in the next sections.

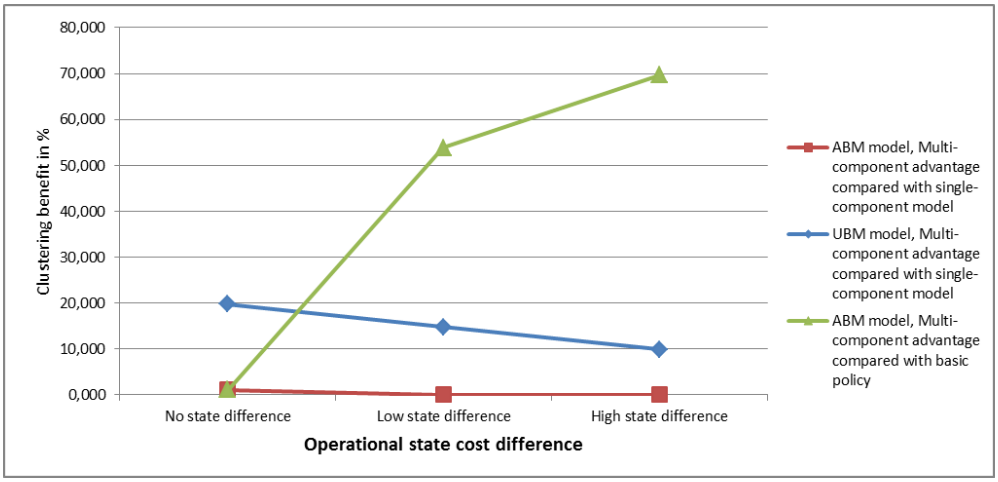

Figure 14 – The ABM and UBM clustering benefit and the advantage of the multi-component ABM-model over the basic policy as a function of the operational state cost difference

4.3.2.2 *MTTF equality*

The single-component replacement times can become less equal by changing various variables. One way to do this is changing the MTTF values. This makes the single-component planning less clustered and increases the clustering potential of the multi-component model. However, this effect is mediated by the operational state cost difference. This is shown in Figure 15, here the set-up cost is high and the other parameters are as in the reference setting. A high operational state cost difference will increase the penalty cost for rescheduling maintenance tasks, so clustering will be more expensive. That explains the lower clustering benefit at higher operational state cost difference.

The scenario without operational state cost differences shows a strange result. This happens because lack of operational state cost difference and the low corrective maintenance cost will weaken the need for preventive maintenance and lower the penalty cost. This makes the likelihood for the myopic algorithm to end up in a suboptimal solution high. However, when a longer horizon is chosen this effect diminishes. When the horizon is four times as long with the same parameter setting, the clustering disadvantage decreases to 1.1%

The relation between the MTTF inequality and the operational state cost difference is moderated by the corrective penalty. This is shown in Figure 16. In this figure, the high set-up costs are used, and the other parameters are as in the reference setting. Clearly, the MTTF inequality only has a positive effect at the low corrective penalty level. At the higher corrective penalty level, unequal MTTF settings will result in a lower clustering benefit. For a high corrective penalty, the penalty costs will be higher. This makes the unequal MTTF measures disadvantageous.

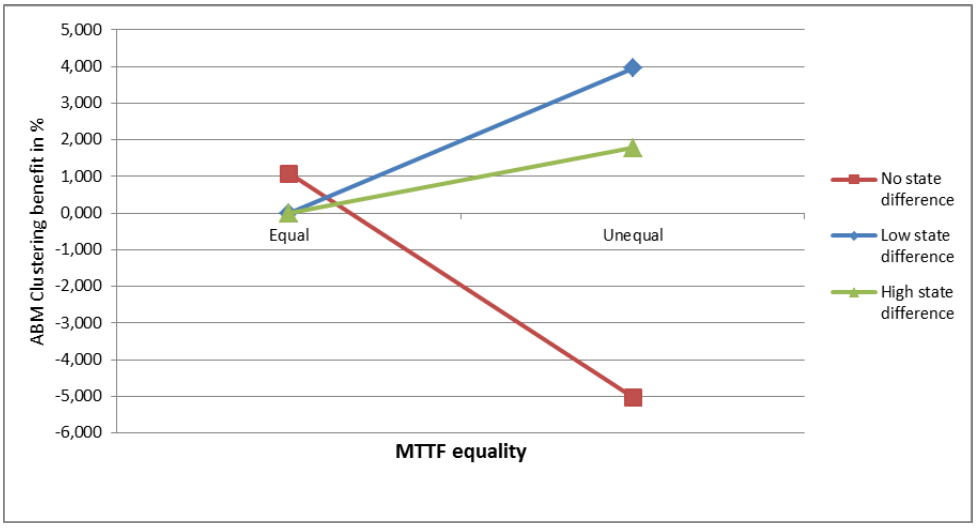

Figure 15 – The relation between the MTTF inequality and the operational state cost difference in terms of clustering benefit

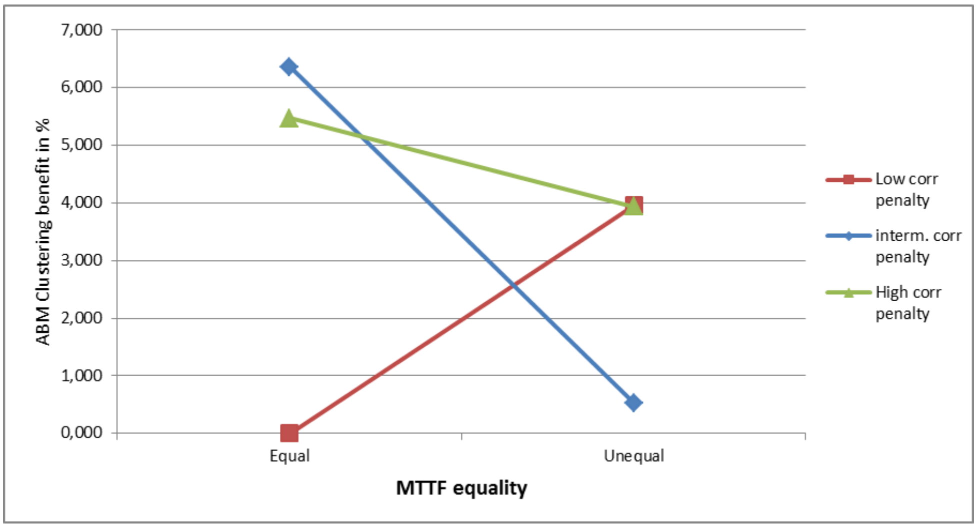

Figure 16 – The relation between the MTTF equality and the corrective penalty in terms of clustering benefit

4.3.2.3 *Corrective penalty*

The clustered nature of the single-component solution is dependent on the corrective penalty. A high corrective penalty makes the difference in maintenance cost between components extra influential in terms of the maintenance planning. A slight difference in preventive maintenance cost between components is enforced strongly by a high corrective penalty, which will spread out the (single-component) maintenance planning of the components. Figure 17 shows the clustering benefit for the different corrective penalty levels, here the set-up cost are high and the other parameters are as in the reference setting. The clustering benefit graphs of the low and high operational state cost difference show a stepwise behaviour when the corrective penalty increases. This has a similar reason as what caused the stepwise shape in the efficient frontier between availability and cost in Figure 11. A small increase in corrective penalty is not expected to change the maintenance planning much because the maintenance planning is strongly dependent on the mission schedule. When the corrective penalty increases enough to spread out the single-component solution such that it enables new clustering benefits, a clustering benefit is obtained. When the corrective penalty does not increase enough to spread out the single-component solution such that it enables new clustering possibilities, it will only increase the penalty cost for rescheduling and decrease the clustering benefit. This causes an oscillation effect. This effect is mediated by the operational state cost difference; the high operational state cost difference requires larger corrective penalty increases to show the same behaviour.

The graphs show an increasing tendency in the clustering benefit, partly because the corrective penalty spreads out the single-component solution, but it also increases the maintenance frequency to cope with the increased maintenance cost in case of failure. The latter will increase the amount of clustering possibilities. The increasing tendency will diminish when the corrective penalty increases as can be seen in the graph of no operational state cost difference.

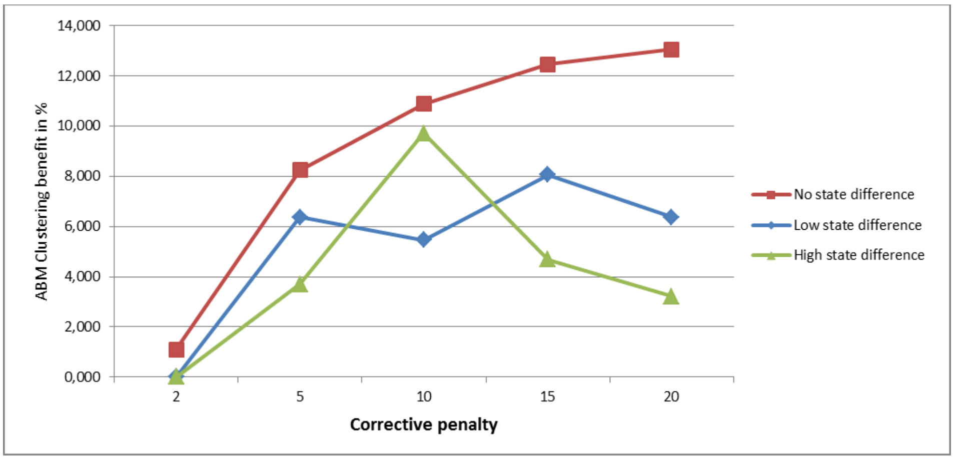

Figure 17 – The effect of the corrective penalty on the clustering benefit, at no operational state cost difference

4.3.2.4 *Mission schedule*

As said before, the mission schedule enlarges the highly clustered nature of the single-component solution. In the alternative schedule, the harbour periods are split up to increase the diversity in the planning. This increases the clustering benefit, as shown in Figure 18. Here, the high set-up cost and the reference settings for the other parameters are used. For the scenario without operational state cost differences the mission planning is irrelevant. AS expected, the clustering benefit is higher for the alternative planning. For the alternative planning, some of the components are maintained before the additional transit periods and others are maintained after the additional transit period in the single-component solution. For most of the components, the multi-component model introduces new maintenance occurrences such that the component is maintained both before and after the additional transit state. This will decrease the expected corrective cost and this can be done without incurring additional set-up cost, because the additional tasks can be clustered.

The clustering benefit is higher for the high operational state difference than for the lower operational state difference, because the expected corrective cost that are decreased by the additional maintenance occurrences is higher when the operational state differences are high.

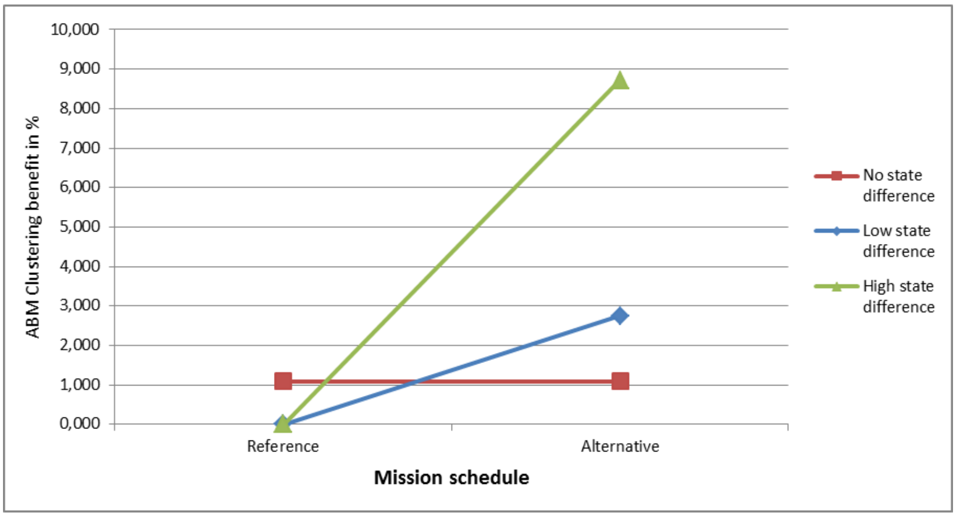

Figure 18 – The difference in clustering benefit as a function of the mission schedule, for the low and high operational state cost difference scenario

4.3.2.5 *Dependencies between components*

The dependencies between components are an important part of this project. Clearly, the clustering benefit is larger for higher set-up cost values. The reference set-up costs for the case study are quite low. In addition, the components have dependencies with at most 2 other components. When all maintenance tasks can be combined to save set-ups, clustering becomes more attractive. When the operational state cost difference is 5000, the set-up cost are high, the alternative mission schedule is used, and the other inputs are set to the reference setting, a clustering benefit of 19.8% is obtained for the ABM components and a clustering benefit of 14.8% for the UBM components. This corresponds with a total expected cost saving of more than €21000 for the 4 years maintenance cycle.

4.3.3 Lifetime variance

The available data about the failure behaviour of the components is very poor. As said before, the lack of knowledge about the lifetime variance makes it hard to draw solid conclusions about the failure behaviour. Therefore, the influence of the lifetime variance is evaluated.

This is done by changing the variance, while keeping the MTTF equal. The following coefficients of variation are investigated: {0.408, 0.500, 0.707}. Lower variance levels do not change the results for this parameter setting. Figure 19 shows the clustering benefit of the ABM-model for different operational state cost differences, here the high set-up cost are used and the other parameters are as in the reference setting. For the high operational state cost difference, a higher variance leads to a higher maintenance cost in general, due to the increased uncertainty. More interestingly, at the higher variance levels, the multi-component ABM-model obtains a small clustering benefit, which is not obtained at the lower coefficient of variation levels. This is caused by the additional clustering opportunities that occur due to the more conservative planning. This effect is not apparent at the lower operational state differences. At the lower operational state cost differences the need for preventive maintenance is lower. When the variance increases, scheduling maintenance earlier does not decrease the risk of failure enough to make additional maintenance beneficial, hence the maintenance planning becomes less conservative. This decreases the amount of clustering opportunities and thereby the clustering benefit. The influence of the variance on the clustering benefit is small, choosing different levels for the coefficient of variation among components does not make a substantial difference.

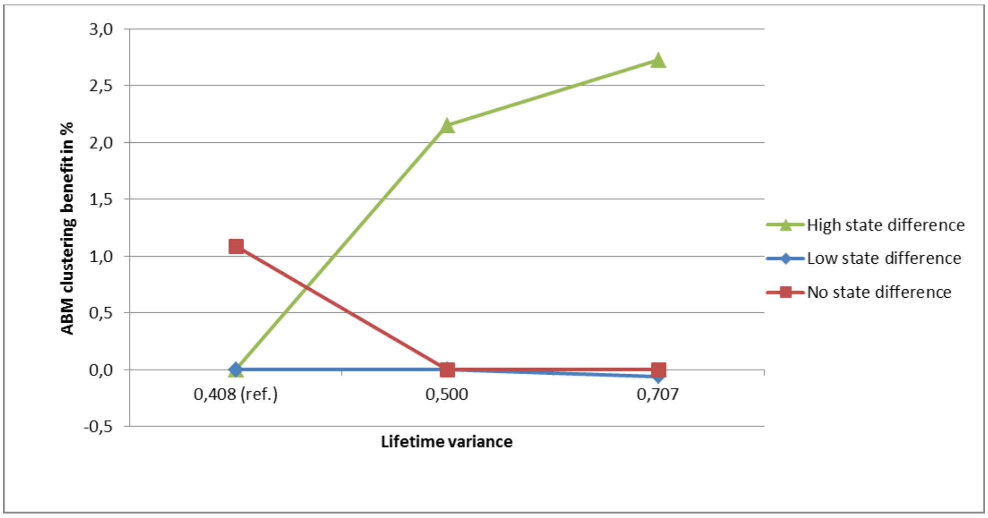

Figure 19 - The expected UBM clustering benefit at different lifetime variance levels

4.3.4 Usage degradation thresholds

The failure thresholds are varied as well, because they are not available for this project. Only the current usage replacement thresholds are available. As said before, these might not be equal to the underlying failure thresholds. Therefore, a range of values is used to multiply the reference failure threshold with; $\{0.5, 1, 1.5\}$. Higher values are not relevant, as no maintenance for the UBM-components will be planned during the horizon. The resulting clustering benefits are shown in Figure 20, here the high set-up costs are used, and for the other parameters the reference parameter settings are used. As expected, the expected maintenance cost for the UBM-components decreases when the failure threshold increases. The clustering benefit of the multi-component UBM-model decreases when the failure thresholds decrease, because the additional maintenance could not be clustered. For the high failure threshold scenario, the amount of maintenance occurrences needed decreases, hence the clustering benefit diminishes.

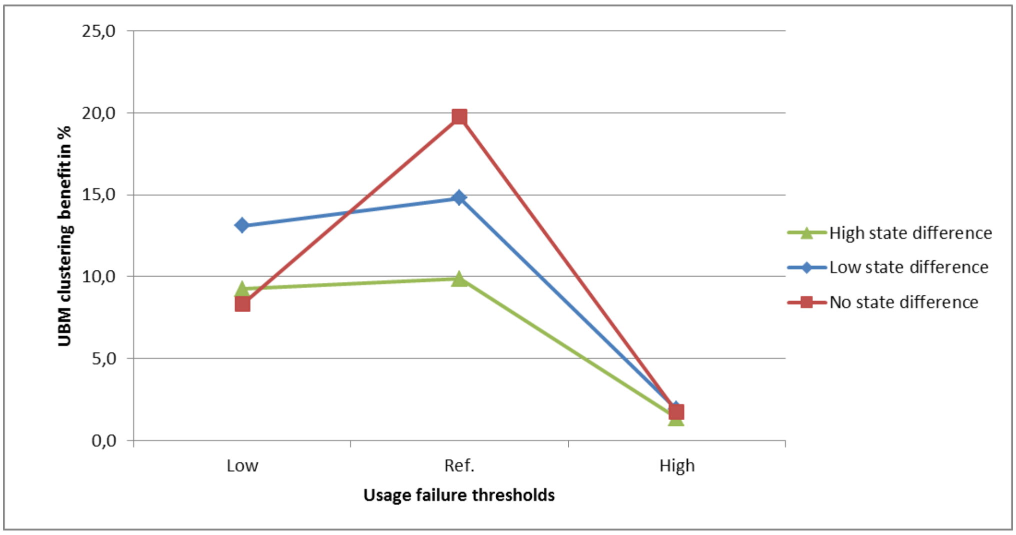

Figure 20 - The expected UBM clustering benefit at different failure threshold levels for different operational state differences

4.3.5 Usage variance

When the maintenance of the components is planned, expectations of the usage during the upcoming missions are known. In the reference setting, the actual usage during the missions is assumed to be uniformly distributed within $\{0.5 - 1.5\}$ times the expected usage. This is a rough assumption. To give insight in the outcomes of the model at other variation levels, three variation scenarios are compared: $\{0.75 - 1.25, 0.5 - 1.5, 0.25 - 1.75\}$. These are referred to as low, reference, high usage variance respectively. The resulting clustering benefit for the UBM-model is shown in Figure 21, here the high set-up cost are used and the reference parameter setting for the other parameters. The effect of changing the usage variance is surprising. The total expected maintenance cost for the UBM-models seems to decrease when the variance increases, while one would expect the opposite. This is caused by the low amount of missions and the criticality of the system. In the reference setting, the system is non-critical during the second mission. This makes the model inclined to postpone the maintenance on beforehand of this mission. When this is done, and the usage variance is large, the probability that the usage does not exceed the threshold will be larger than when the usage variance is low. When the usage level just before the second mission is such that incurring the expected usage of the second mission will make the usage level to exceed the failure threshold, having a high usage variance increases the probability of survival. The clustering benefit is increasing when the usage variance level increases. At low variance levels, advancing or postponing maintenance tasks is very expensive as the usage during the upcoming missions is certain. It will be easy to see whether the system will fail during the postponed or advanced periods. When the system is certainly going to fail in case of a postponement or certainly not going to fail in case of an advancement, the penalty cost will be high. This explains the increasing clustering benefit for higher levels of variance.

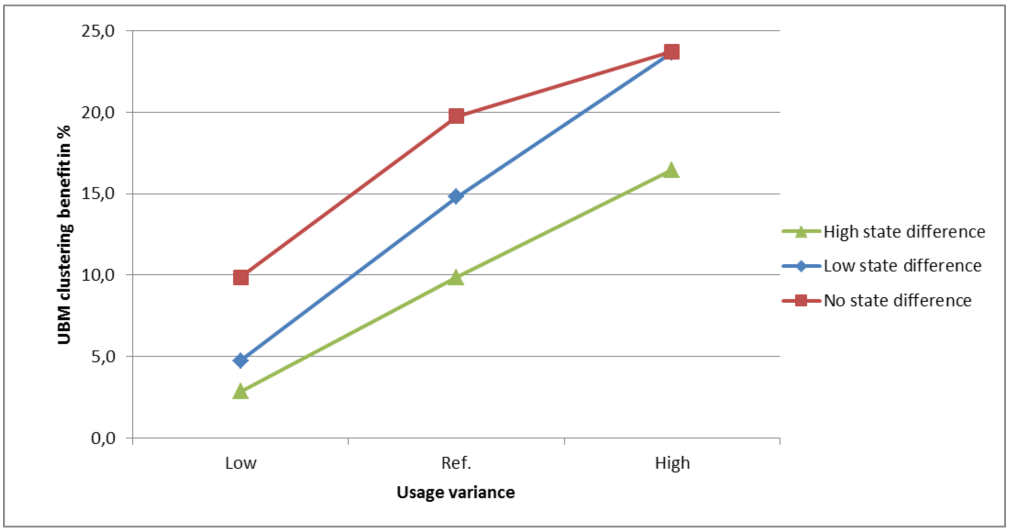

Figure 21 – The expected UBM clustering benefit at different usage variance levels for different operational state differences

4.3.6 Criticality

The proposed UBM-model can incorporate criticality, as shown in chapter 3.2. Three scenarios are created to evaluate the influence of the criticality of the system: the system is critical during all of the missions, the system is critical most of the missions like in the reference setting, the system is non-critical during all of the missions. These are referred to as low, intermediate and high criticality. The resulting clustering benefit at different operational state cost differences is shown in Figure 22, here the high set-up cost are used, the other parameters are as in the reference setting. As expected, the system criticality has a strong influence on the expected maintenance cost of the UBM-components. When the system is non-critical during the missions, the consequences of failure during missions are much less severe. Hence, the penalty cost for rescheduling are much lower, which partially explains the higher clustering benefit at the low criticality levels. However, the high clustering benefit is also caused by the much lower total maintenance costs. As the clustering benefit is relative to the single-component maintenance cost, a small cluster saving will result in high clustering benefit. For the low operational state cost difference, the effect is the same. However, the clustering benefit remains at a higher level, because of the lower penalty cost for rescheduling. When there are no operational state cost differences, the criticality issue is not relevant. The clustering benefit is unaffected in that case.

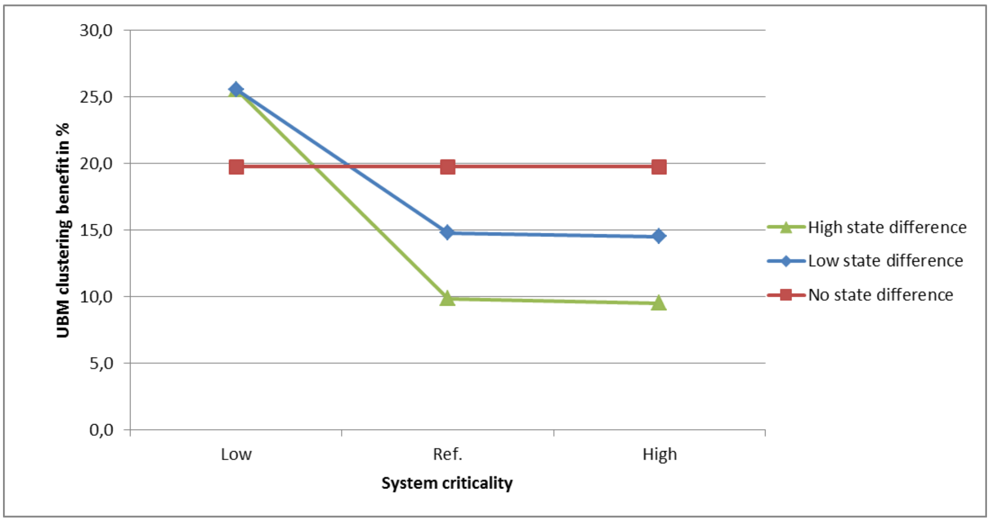

Figure 22 - The expected UBM clustering benefit depending on the system criticality level for different operational state cost difference levels

5 Conclusions

In this section, the conclusions that can be drawn from this project are elaborated. First, the main research question is addressed. Next, the limitations of this project and the proposed models are evaluated. Thereafter, the implementation is addressed and recommendations to the RNLN are given. At last, suggestions for future research are given to complement the research in this field.

5.1 Research question

Throughout this report, the research question and its sub-questions are answered. To give a clear answer to the research question, this section will summarize how the research question is answered. The research question is formulated as:

How can a multi-component approach be used to integrate a condition-based maintenance policy with a static maintenance planning in a setting with operational states?

This research question contains three important elements, which might be worth distinguishing: 1. *the use of a multi-component approach to cluster the maintenance tasks*, 2. *the integration of a condition-based policy and a static maintenance planning*, and 3. *the incorporation of the operational states in the model*. To be able to answer this research question, four models are built. The second element requires modelling both a condition-based maintenance model and an age-based maintenance model (which is the static maintenance model). These models incorporate the operational states that characterize the maritime setting, to satisfy the third element. To satisfy the first element, both a single-component variant and a multi-component variant of the models are built. The single-component models serve as a basis for the multi-component models and are used to compare the performance of the multi-components with. The condition-based maintenance model can be integrated with the age-based maintenance model by considering the dependencies between components of both approaches. First, the (static) multi-component age-based maintenance model is solved to find a (close to) optimal maintenance planning for the ABM-components, and then the (dynamic) condition-based maintenance planning is scheduled based on the ABM-schedule. In the case study, the model is tested on the maintenance management of the Marlin weapon system. The proposed models perform much better than a basic ABM model that does not incorporate the operational state cost difference. A 76% cost reduction and an 8% mission availability increase is obtained by the proposed ABM-models in the reference parameter setting. From the case study and the sensitivity analysis can be concluded that the benefit of a multi-component model with respect to a single-component model strongly depends on the parameter settings. The clustering benefit ranges from 0%-10% of the total expected maintenance cost with peaks of 20%. Certainly, one cannot conclude that the multi-component model is superior to the single-component model invariably. The single-component models that are proposed in this project incorporate the operational states, so the performance of these models can be good for the maintenance planning in a maritime setting. However, the clustering benefits that are obtained by the model can be noteworthy. The sensitivity analysis can be consulted to get insight in the clustering benefit at certain settings. Independent of the parameter setting, the model should be used for the insights it gives into the penalty costs and the effects of the dependencies between components.

5.2 Limitations

One of the limitations of this project is the lack of reliable data to test the proposed models on. Unfortunately, the available data in this project is very minor. This makes it hard to draw strong and reliable conclusions about the performance of the model. Knowledge about the failure behaviour of the components must be gained and the lifetime distribution that is used must be validated. Also, more detailed cost factors for preventive and corrective maintenance during the different operational states should be gathered, to perform a proper planning optimization.

The results of the multi-component model are compared to the results of the single-component model and to a basic policy, because no current practice data is available for this project. The advantage over the basic policy is big as it is an inappropriate model for the maritime sector. However, it would be interesting to compare the performance of the proposed models with more pragmatic approaches. The comparisons in the sensitivity analysis give insight into the benefit of clustering predominantly, leaving the performance of the model compared with the current practice unevaluated.

The proposed age-based models use a myopic approach; they only consider the optimization of one maintenance occurrence at the time. However, most tasks are maintained multiple times during the horizon. The model does not take the future decision-making into account, like the dynamic programming approach that is used for the CBM-model does. This can sometimes lead to a suboptimal maintenance planning. This is also seen in the results of the sensitivity analysis. Sometimes the multi-component model performs worse than the single-component model.

The multi-component age-based model is quite complex and is therefore solved using a greedy heuristic. The performance of this heuristic is quite high, as described in chapter 3.1.4. However, due to the time-limits of this project, no other optimization methods are evaluated. Other heuristics might result in a higher optimality score.

This project considers the structural dependencies between components. Economic dependencies and stochastic dependencies are not incorporated in the model. To be able to capture all multi-component dynamics in the model, the economic and stochastic dependencies should be researched and incorporated in the model. Due to time limits, this is outside the scope of this project.

5.3 Implementation and recommendations

A successful implementation of a new strategy or approach requires proper attention for the implementation phase. This section will elaborate on the issues that should be dealt with to implement the results of this project properly.

A first implementation issue is the lack of reliable data. As said before, data availability is a problem in this project. The results of the model are quite sensible towards changes of the input variables as seen in the sensitivity analysis. When the data remains missing or unreliable, the model should be used with cause. The tool can be used to perform analyses and get insight the consequences of changes in the inputs on the maintenance management. The data that should be collected is: reliable failure parameters and fitting a failure distribution for the ABM-components, the preventive and corrective maintenance cost for all operational states per component and per mission, the set-up cost and the set-up configuration, the system criticality during the missions, the failure thresholds of the UBM-components and the expected usage during missions and a fitting probability distribution.

When more accurate data is acquired, one should evaluate if the model represents the cost factors in an accurate way. One should think about the differences between the operational states and if the division into three operational states is sufficient. This might differ per system or component. The model can be extended for a larger amount of components, a longer horizon and more operational states. In addition, one should evaluate if the Erlang distribution that is used in this project is representative for the failure behaviour of the components. Other distributions can be used, as long as one can model its probability density function as described in Appendix A. The same holds if the model is to be applied on other systems. The model is formulated in a general manner, so it can be applied to other systems. However, one should check if the assumptions that are made in the modelling, are reasonable for that particular system.

To be able to acquire reliable data, a closer cooperation with the OEM might be needed. Effort to acquire reliable data and better insights in the failure behaviour might enable a big improvement for the maintenance and reliability management within the RNLN. Condition measurement technology becomes more accurate and applicable in a lot of fields these days. The newest trends like 'the internet of things' might offer big opportunities for the maintenance management of the RNLN. Even if these developments might not be rewarding on the short run, it might be very valuable to invest in these technologies.

The tool that is constructed in this project can be used to find a (close to) optimal maintenance planning that incorporates the multi-component effects and the operational states. But when one is not certain about the reliability of the input parameters, or one wants to evaluate different planning options for reasons outside the scope of the model; the penalty functions can be used to evaluate the effects of deviating from the single-component optimal date for the ABM-components in terms of cost.

This model only incorporates the maintenance planning on system level. Dependencies between different systems on the ships or between ships are not incorporated, so if they are expected to have a significant influence on the maintenance planning, one should interpret the results of this model with the knowledge about these dependencies. The dependencies between systems and ships can occur because the capacity of the workshop might be shared for the maintenance of other systems and ships.

The proposed model is not a substitute for the ILS program that is used. The model can be used as part or analytical tool within the ILS program, as the ILS program is more like a comprehensive maintenance management technique. The model can be used to do evaluations and tests of different scenarios and solutions and to come up with detailed analytics. When the model is used to base the maintenance planning on, the other aspects within the ILS program must be aligned to have a comprehensive maintenance strategy. For example, the inventory management must be aligned to fit the maintenance planning. The maintenance managers should evaluate the influence of the new planning on the capacity of the repair shop as well as on the workflow within the repair shop. The model does not take these aspects into account, hence these should be managed such that the maintenance management as a whole is efficient. When the new planning approach changes the goods- and workflow within the repair shop drastically, additional research might be needed to streamline the entire maintenance operation.

The tool is modelled in VBA, so an Excel package is needed to use it. This software package is already available at the DMI. The tool is designed to be used by people that do not have coding experience. Currently, the tool supports the planning of 16 ABM-tasks and 8 CBM/UBM-tasks. This can be extended, but small changes are needed in the code and in the lay-out of the sheet. This is also the case when the amount of operational states changes. A tutorial will be given at the DMI to explain how the tool works and how these aspects can be adjusted.

5.4 Future research

This section will suggest future research directions that complement the research in this field. As said before, this project optimizes the maintenance planning on a system level. Hence, dependencies with other systems on the ships or with other ships are not incorporated. Future research must be done to evaluate if these types of dependencies are interesting to incorporate.

The model can easily be extended to incorporate more structural dependencies. However, incorporating economic and stochastic dependencies requires more effort. Hence, an interesting research direction is the research on other types of dependencies and how to incorporate them. To incorporate economic dependencies, one could add a term that represents the cost in case of downtime to the cost functions. This term should be dependent on the planning of the other components. When one component is already maintained, the tasks can be done in parallel and the downtime cost is incurred only once.

Incorporating stochastic dependencies is complicated because the proposed models assume that the failure behaviour of a component is independent of the failure behaviour of the other components. In the traditional maintenance literature this assumption is also made. More theoretical research into mathematics and statistics is needed to incorporate this type of dependency.

In this project a greedy type heuristic is used to solve the ABM multi-component problem. This approach is quick and performs rather well, but it might be interesting to investigate other optimization methods. Other methods might perform better or more consistent. In addition, in the limitations section is discussed that the ABM-models are myopic in nature. They only consider one maintenance occurrence of a task at once, instead of optimizing all maintenance occurrences within the horizon. This can lead to a suboptimal maintenance planning. Future research can be done into approaches that optimize all maintenance decisions within the horizon. To do this, a dynamic programming model can be used. An expression must be found for the remaining lifetime distributions, which will appear in the value function.

This project incorporates the differences between operational states only in terms of maintenance cost. However, it might be reasonable to assume that the degradation rate of a component is also state-dependent. Due to the mathematical complexity of such a model and the time limits of this project, the state-dependent degradation behaviour is not incorporated. Incorporating this would make an interesting future research project.

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7 Appendix A

An appropriate probability distribution has to be chosen to describe the time to failure. When failure data is available, failure probabilities can be fitted on it to find an appropriate one. For this model, a continuous-time lifetime distribution is assumed. This seems the most reasonable, as there is no reason to assume the components degrade only on a discrete basis.

For this model, different distribution functions can be used. When the data shows that the failure rate is increasing over time (IFR), preventive maintenance is interesting. Probability distributions that allow parameter settings such that the failure rate is increasing can be used for this model.

For complex probability distributions it can be hard to derive the expected cycle length of the maintenance cycle (the denominator of the average maintenance cost). For example, for the Weibull distribution $\int_{t=1}^{\tau_i-1} t \cdot f_i(t)dt$ cannot be solved analytically. The function must be solved numerically, which is intractable in the accessible software packages as Excel.

Less complicated distributions as the Erlang distribution are more suitable for this model, because the expected cycle length function can be solved analytically, this is shown below.

Derivation of the failure part of the expected cycle length for an Erlang distribution:

$$\int_{t=0}^{\tau} t \cdot f(t)dt = \int_{t=0}^{\tau} \frac{(\lambda t)^{\mu}}{(\mu-1)!} e^{-\lambda t}\ dt$$

$$= \left[-\frac{(\lambda t)^{\mu}}{(\mu-1)!}\frac{1}{\lambda}e^{-\lambda t}\right]_{t=0}^{\tau} - \int_{t=0}^{\tau} -\frac{\mu \cdot \lambda^{\mu} t^{\mu-1}}{(\mu-1)!}\frac{1}{\lambda}e^{-\lambda t}dt$$

$$= -\frac{(\lambda \tau)^{\mu}}{(\mu-1)!}\frac{1}{\lambda}e^{-\lambda \tau} + \frac{\mu}{\lambda}\cdot F(\tau)$$

8 Appendix B

The failure probabilities can be modelled as follows. The probability of failure at a certain time t can be written as the probability that the degradation in the previous states is less then the failure threshold, multiplied by the probability that the degradation exceeds the failure threshold at time t. Convolution terms are needed to determine the survival probability at the previous states. Let L denote the failure threshold for the degradation. Let $f_{n_t}(l_t)$ denote the probability density function of incurring l_t degradation in state n_t.

The probability of failure at time t equals:

$$P(failure\ at\ t) = \int_{l_1=0}^{L} f_{n_1}(l_1) \cdot \int_{l_2=0}^{L-l_1} f_{n_2}(l_2) \cdot \dots \dots \cdot \int_{l_{t-1}=0}^{L-\sum_{u=1}^{t-2} l_u} f_{n_{t-1}}(l_{t-1}) \cdot \int_{l_t=L-\sum_{u=1}^{t-1} l_u}^{\infty} f_{n_t}(l_t) dl_t\, dl_{t-1} \dots dl_2 dl_1$$

This can be generalized to:

$$P(failure\ at\ t) = \prod_{z=1}^{t-1} \left[\int_{l_z=0}^{L-\sum_{u=1}^{z-1} l_u} f_{n_z}(l_z)\, dl_z \right] \cdot \int_{l_t=L-\sum_{u=1}^{t-1} l_u}^{\infty} f_{n_t}(l_t) dl_t$$

9 Appendix C

Table 12 - Results of the optimization method evaluation

Experiment	MTTF level	Corrective penalty	State penalty level	Set-up size scenario	Enumeration	Greedy	Greedy + improvements	Improvement iterations	Greedy optimality
1	1	1,25	1	0,5	407,50	407,50	407,50	0	1
2	1	1,25	1	1	815,00	815,00	815,00	0	1
3	1	1,25	1	2	1630,00	1630,00	1630,00	0	1
4	1	1,25	1	10	8150,00	8150,00	8150,00	0	1
5	2	1,25	1	0,5	407,50	407,50	407,50	0	1
6	2	1,25	1	1	815,00	815,00	815,00	0	1
7	2	1,25	1	2	1630,00	1630,00	1630,00	0	1
8	2	1,25	1	10	8150,00	8150,00	8150,00	0	1
9	3	1,25	1	0,5	407,50	407,50	407,50	0	1
10	3	1,25	1	1	815,00	815,00	815,00	0	1
11	3	1,25	1	2	1630,00	1630,00	1630,00	0	1
12	3	1,25	1	10	8150,00	8150,00	8150,00	0	1
13	1	2	1	0,5	448,44	448,44	448,44	0	1
14	1	2	1	1	846,66	846,66	846,66	0	1
15	1	2	1	2	1674,93	1674,93	1674,93	0	1
16	1	2	1	10	8151,54	8151,54	8151,54	0	1
17	2	2	1	0,5	471,66	471,66	471,66	0	1
18	2	2	1	1	871,31	871,31	871,31	0	1
19	2	2	1	2	1632,26	1413,43	1413,43	0	0,8659
20	2	2	1	10	8151,03	8151,03	8151,03	0	1
21	3	2	1	0,5	410,03	410,03	410,03	0	1
22	3	2	1	1	822,91	822,91	822,91	0	1
23	3	2	1	2	1636,36	1576,28	1576,28	0	0,9633
24	3	2	1	10	8153,64	8111,01	8111,01	0	0,9948
25	1	3	1	0,5	458,32	458,32	458,32	0	1
26	1	3	1	1	869,35	869,35	869,35	0	1
27	1	3	1	2	1737,96	1737,96	1737,96	0	1
28	1	3	1	10	8152,38	8103,19	8103,19	0	0,994
29	2	3	1	0,5	446,71	446,71	446,71	0	1
30	2	3	1	1	874,32	874,32	874,32	0	1
31	2	3	1	2	1362,85	1198,57	1198,57	0	0,8795
32	2	3	1	10	7647,37	7477,36	7477,36	0	0,9778
33	3	3	1	0,5	306,15	306,15	306,15	0	1
34	3	3	1	1	611,82	534,35	534,35	0	0,8734
35	3	3	1	2	1642,19	1642,19	1642,19	0	1
36	3	3	1	10	7353,50	7345,04	7345,04	0	0,9988
37	1	10	1	0,5	407,50	407,50	407,50	0	1
38	1	10	1	1	815,00	815,00	815,00	0	1
39	1	10	1	2	1630,00	1630,00	1630,00	0	1
40	1	10	1	10	8150,00	8150,00	8150,00	0	1
41	2	10	1	0,5	354,75	354,75	354,75	0	1
42	2	10	1	1	656,57	656,57	656,57	0	1
43	2	10	1	2	1561,51	1561,51	1561,51	0	1
44	2	10	1	10	8061,19	8061,19	8061,19	0	1
45	3	10	1	0,5	235,22	207,06	207,06	0	0,8803
46	3	10	1	1	535,02	432,54	505,63	1	0,9451
47	3	10	1	2	1407,97	1407,97	1407,97	0	1
48	3	10	1	10	8116,79	8116,79	8116,79	0	1
49	1	1,25	2	0,5	415,48	415,48	415,48	0	1
50	1	1,25	2	1	815,00	655,67	655,67	0	0,8045
51	1	1,25	2	2	1650,59	1650,59	1650,59	0	1
52	1	1,25	2	10	8150,00	8150,00	8150,00	0	1
53	2	1,25	2	0,5	396,09	168,23	305,98	1	0,7725
54	2	1,25	2	1	815,00	695,07	695,07	0	0,8528
55	2	1,25	2	2	1630,00	1495,49	1495,49	0	0,9175
56	2	1,25	2	10	8176,43	8176,43	8176,43	0	1
57	3	1,25	2	0,5	407,50	288,52	288,52	0	0,708
58	3	1,25	2	1	815,00	782,91	782,91	0	0,9606
59	3	1,25	2	2	1630,00	1575,22	1575,22	0	0,9664
60	3	1,25	2	10	7541,36	7441,59	7441,59	0	0,9868
61	1	2	2	0,5	601,22	601,22	601,22	0	1
62	1	2	2	1	850,53	727,67	727,67	0	0,8555
63	1	2	2	2	1630,00	1544,77	1544,77	0	0,9477
64	1	2	2	10	8339,04	8339,04	8339,04	0	1
65	2	2	2	0,5	587,95	525,44	587,95	1	1
66	2	2	2	1	508,12	489,00	489,00	0	0,9624
67	2	2	2	2	1478,96	1164,49	1478,96	1	1
68	2	2	2	10	7304,87	6756,00	6756,00	0	0,9249
69	3	2	2	0,5	244,50	163,00	244,50	1	1
70	3	2	2	1	594,62	594,62	594,62	0	1
71	3	2	2	2	1291,58	976,99	1278,04	1	0,9895
72	3	2	2	10	7409,35	5344,78	5344,78	0	0,7214
73	1	3	2	0,5	471,22	468,34	468,34	0	0,9939

74	1	3	2	1	691,91	691,91	691,91	0	1
75	1	3	2	2	1390,77	1390,77	1390,77	0	1
76	1	3	2	10	7610,57	7547,30	7547,30	0	0,9917
77	2	3	2	0,5	280,32	280,32	280,32	0	1
78	2	3	2	1	642,06	622,78	622,78	0	0,97
79	2	3	2	2	1382,34	812,46	1077,56	1	0,7795
80	2	3	2	10	8044,28	3009,55	8044,28	2	1
81	3	3	2	0,5	279,88	275,60	275,60	0	0,9847
82	3	3	2	1	652,00	528,34	528,34	0	0,8103
83	3	3	2	2	1304,00	1304,00	1304,00	0	1
84	3	3	2	10	7918,83	5650,11	5650,11	0	0,7135
85	1	10	2	0,5	407,50	407,50	407,50	0	1
86	1	10	2	1	815,00	815,00	815,00	0	1
87	1	10	2	2	1630,00	1630,00	1630,00	0	1
88	1	10	2	10	8150,00	8150,00	8150,00	0	1
89	2	10	2	0,5	407,50	407,50	407,50	0	1
90	2	10	2	1	696,09	580,91	652,00	1	0,9367
91	2	10	2	2	1630,00	1630,00	1630,00	0	1
92	2	10	2	10	8150,00	8150,00	8150,00	0	1
93	3	10	2	0,5	326,00	198,81	326,00	1	1
94	3	10	2	1	695,60	554,27	652,00	1	0,9373
95	3	10	2	2	1304,00	1245,28	1245,28	0	0,955
96	3	10	2	10	8078,96	8078,96	8078,96	0	1
97	1	1,25	3	0,5	422,29	419,87	419,87	0	0,9943
98	1	1,25	3	1	715,50	715,50	715,50	0	1
99	1	1,25	3	2	1844,82	1743,96	1743,96	0	0,9453
100	1	1,25	3	10	7372,61	7372,61	7372,61	0	1
101	2	1,25	3	0,5	336,39	336,39	336,39	0	1
102	2	1,25	3	1	576,80	547,23	547,23	0	0,9487
103	2	1,25	3	2	1640,18	1640,18	1640,18	0	1
104	2	1,25	3	10	7753,02	5929,81	5929,81	0	0,7648
105	3	1,25	3	0,5	320,59	320,59	320,59	0	1
106	3	1,25	3	1	815,00	815,00	815,00	0	1
107	3	1,25	3	2	1304,00	1304,00	1304,00	0	1
108	3	1,25	3	10	6753,81	6753,81	6753,81	0	1
109	1	2	3	0,5	326,00	326,00	326,00	0	1
110	1	2	3	1	652,00	652,00	652,00	0	1
111	1	2	3	2	1311,73	1191,46	1304,00	1	0,9941
112	1	2	3	10	7609,14	7609,14	7609,14	0	1
113	2	2	3	0,5	326,00	326,00	326,00	0	1
114	2	2	3	1	652,00	604,98	604,98	0	0,9279
115	2	2	3	2	1408,57	1408,57	1408,57	0	1
116	2	2	3	10	7044,97	6686,59	6686,59	0	0,9491
117	3	2	3	0,5	326,00	326,00	326,00	0	1
118	3	2	3	1	670,18	670,18	670,18	0	1
119	3	2	3	2	1304,00	1063,10	1063,10	0	0,8153
120	3	2	3	10	7598,96	6635,51	6635,51	0	0,8732
121	1	3	3	0,5	407,50	407,50	407,50	0	1
122	1	3	3	1	815,00	815,00	815,00	0	1
123	1	3	3	2	1630,00	1630,00	1630,00	0	1
124	1	3	3	10	8150,00	8150,00	8150,00	0	1
125	2	3	3	0,5	326,00	326,00	326,00	0	1
126	2	3	3	1	748,18	748,18	748,18	0	1
127	2	3	3	2	1515,26	1515,26	1515,26	0	1
128	2	3	3	10	8150,00	8150,00	8150,00	0	1
129	3	3	3	0,5	303,04	110,41	303,04	2	1
130	3	3	3	1	706,70	652,00	706,70	2	1
131	3	3	3	2	1499,26	1499,26	1499,26	0	1
132	3	3	3	10	7896,48	7896,48	7896,48	0	1
133	1	10	3	0,5	407,50	407,50	407,50	0	1
134	1	10	3	1	815,00	815,00	815,00	0	1
135	1	10	3	2	1630,00	1630,00	1630,00	0	1
136	1	10	3	10	8150,00	8150,00	8150,00	0	1
137	2	10	3	0,5	407,50	407,50	407,50	0	1
138	2	10	3	1	815,00	815,00	815,00	0	1
139	2	10	3	2	1630,00	1630,00	1630,00	0	1
140	2	10	3	10	8150,00	8150,00	8150,00	0	1
141	3	10	3	0,5	326,00	326,00	326,00	0	1
142	3	10	3	1	769,96	493,31	769,96	1	1
143	3	10	3	2	1368,76	1368,76	1368,76	0	1
144	3	10	3	10	8150,00	8150,00	8150,00	0	1

10 Appendix D

Table 13 – ANOVA of the effect of dissimilar failure behaviour on the optimality of the Greedy heuristic

ANOVA

Greedy optimality

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	,024	2	,012	3,044	,051
Within Groups	,546	141	,004		
Total	,570	143			

Table 14 – ANOVA of the effect of the set-up size on the optimality of the Greedy heuristic

ANOVA

Greedy optimality

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	,005	3	,002	,389	,761
Within Groups	,565	140	,004		
Total	,570	143			

Table 15 - ANOVA of the effect of the cost difference between operational states on the optimality of the Greedy heuristic

ANOVA

Greedy optimality

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	,048	2	,024	6,450	,002
Within Groups	,522	141	,004		
Total	,570	143			

Table 16 - ANOVA of the effect of the cost difference between operational states on the optimality of the Greedy heuristic

ANOVA

Greedy optimality

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	,017	3	,006	1,448	,231
Within Groups	,553	140	,004		
Total	,570	143			

Table 17 - ANOVA of the effect of the cost difference between operational states on the optimality of the Greedy heuristic comparing variable level 10 with levels {1,25; 2; 3} together

ANOVA

Greedy optimality

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	,016	1	,016	3,971	,048
Within Groups	,555	142	,004		
Total	,570	143			

11 Appendix E

Table 18 – The results of the scenario analysis of the sensitivity analysis

Exp.	Operational state cost difference	MTTF inequality	Corrective penalty	Setup size	Asset schedule	ABM - Single component	ABM - Multi-component	UBM - Single-component	UBM - Multi-component	ABM clustering benefit in %	UBM clustering benefit in %
1	0	ref.	2	low	ref.	32436,39	32216,71	1887,50	1790,97	0,677	5,114
2	5000	ref.	2	low	ref.	42992,44	42570,14	12123,75	11853,75	0,982	2,227
3	10000	ref.	2	low	ref.	46046,86	46046,86	22123,75	21853,75	0,000	1,220
4	0	unequal	2	low	ref.	36201,96	35756,26	1887,50	1768,44	1,231	6,308
5	5000	unequal	2	low	ref.	47257,24	47257,24	12123,75	11853,75	0,000	2,227
6	10000	unequal	2	low	ref.	52949,49	52624,23	22123,75	21853,75	0,614	1,220
7	0	ref.	5	low	ref.	52618,15	52344,48	3549,58	3453,82	0,520	2,698
8	5000	ref.	5	low	ref.	70158,37	69588,77	26388,65	25888,08	0,812	1,897
9	10000	ref.	5	low	ref.	80312,38	80360,67	48888,65	48388,08	-0,060	1,024
10	0	unequal	5	low	ref.	58342,10	57899,69	3549,58	3346,32	0,758	5,726
11	5000	unequal	5	low	ref.	76172,08	73942,08	26388,65	25888,08	2,928	1,897
12	10000	unequal	5	low	ref.	84823,10	86635,25	48888,65	48388,08	-2,136	1,024
13	0	ref.	10	low	ref.	65628,35	65310,73	5319,13	5027,37	0,484	5,485
14	5000	ref.	10	low	ref.	90864,62	90141,22	35593,00	34825,00	0,796	2,158
15	10000	ref.	10	low	ref.	104586,96	103146,57	65593,00	64825,00	1,377	1,171
16	0	unequal	10	low	ref.	72530,56	74405,72	5319,13	5027,37	-2,585	5,485
17	5000	unequal	10	low	ref.	100574,23	98194,87	35593,00	34825,00	2,366	2,158
18	10000	unequal	10	low	ref.	121959,26	116351,44	65593,00	64825,00	4,598	1,171
19	0	ref.	2	high	ref.	38141,05	37726,53	6286,06	5045,33	1,087	19,738
20	5000	ref.	2	high	ref.	54613,22	54613,22	17290,16	14733,94	0,000	14,784
21	10000	ref.	2	high	ref.	59610,10	59610,10	27440,18	24733,94	0,000	9,862
22	0	unequal	2	high	ref.	38632,56	40570,98	6286,06	4938,20	-5,018	21,442
23	5000	unequal	2	high	ref.	60791,23	58383,81	17290,16	14733,94	3,960	14,784
24	10000	unequal	2	high	ref.	65773,13	64597,78	27440,18	24733,94	1,787	9,862
25	0	ref.	5	high	ref.	70784,61	64945,42	8258,69	6495,56	8,249	21,349
26	5000	ref.	5	high	ref.	80722,20	75584,51	34284,77	31495,56	6,365	8,135
27	10000	ref.	5	high	ref.	86593,32	83391,26	57312,65	52420,08	3,698	8,537
28	0	unequal	5	high	ref.	76231,43	70905,58	8258,69	6495,56	6,986	21,349
29	5000	unequal	5	high	ref.	95304,08	94800,30	34284,77	31495,56	0,529	8,135
30	10000	unequal	5	high	ref.	115980,55	111634,70	57312,65	52420,08	3,747	8,537
31	0	ref.	10	high	ref.	90301,10	80464,28	11243,46	9431,59	10,893	16,115
32	5000	ref.	10	high	ref.	114798,31	108520,49	47481,34	44179,06	5,469	6,955
33	10000	ref.	10	high	ref.	141642,94	127877,99	79993,00	72313,00	9,718	9,601
34	0	unequal	10	high	ref.	99462,66	92519,09	11243,46	9431,59	6,981	16,115
35	5000	unequal	10	high	ref.	132369,97	127157,59	47481,34	44179,06	3,938	6,955
36	10000	unequal	10	high	ref.	148724,72	145219,64	79993,00	72313,00	2,357	9,601
37	0	ref.	2	low	alt.	32436,39	32216,71	1887,50	1790,97	0,677	5,114
38	5000	ref.	2	low	alt.	55403,05	54105,27	12123,75	11853,75	2,342	2,227
39	10000	ref.	2	low	alt.	66613,47	66613,47	22123,75	21853,75	0,000	1,220
40	0	unequal	2	low	alt.	36201,96	35756,26	1887,50	1768,44	1,231	6,308
41	5000	unequal	2	low	alt.	61547,10	61305,23	12123,75	11853,75	0,393	2,227
42	10000	unequal	2	low	alt.	72197,65	73897,48	22123,75	21853,75	-2,354	1,220
43	0	ref.	5	low	alt.	52618,15	52344,48	3549,58	3453,82	0,520	2,698
44	5000	ref.	5	low	alt.	81050,92	80685,47	26388,65	25888,08	0,451	1,897
45	10000	ref.	5	low	alt.	87986,68	87434,07	48888,65	48388,08	0,628	1,024
46	0	unequal	5	low	alt.	58342,10	57899,69	3549,58	3346,32	0,758	5,726
47	5000	unequal	5	low	alt.	85224,94	84940,91	26388,65	25888,08	0,333	1,897
48	10000	unequal	5	low	alt.	96536,65	96536,65	48888,65	48388,08	0,000	1,024
49	0	ref.	10	low	alt.	65628,35	65310,73	5319,13	5027,37	0,484	5,485
50	5000	ref.	10	low	alt.	92326,49	91288,74	35593,00	34825,00	1,124	2,158
51	10000	ref.	10	low	alt.	107746,25	107358,62	65593,00	64825,00	0,360	1,171
52	0	unequal	10	low	alt.	72530,56	74405,72	5319,13	5027,37	-2,585	5,485
53	5000	unequal	10	low	alt.	102812,72	102096,34	35593,00	34825,00	0,697	2,158
54	10000	unequal	10	low	alt.	124008,43	122752,71	65593,00	64825,00	1,013	1,171
55	0	ref.	2	high	alt.	38141,05	37726,53	6286,06	5045,33	1,087	19,738
56	5000	ref.	2	high	alt.	66368,98	64545,48	17290,16	14733,94	2,748	14,784
57	10000	ref.	2	high	alt.	84192,61	76851,78	27440,18	24733,94	8,719	9,862
58	0	unequal	2	high	alt.	38632,56	40570,98	6286,06	4938,20	-5,018	21,442
59	5000	unequal	2	high	alt.	81992,54	76094,58	17290,16	14733,94	7,193	14,784
60	10000	unequal	2	high	alt.	98346,99	93485,45	27440,18	24733,94	4,943	9,862
61	0	ref.	5	high	alt.	70784,61	64945,42	8258,69	6495,56	8,249	21,349
62	5000	ref.	5	high	alt.	97971,60	96858,74	34284,77	31495,56	1,136	8,135

63	10000	ref.	5	high	alt.	116995,57	116876,62	57312,65	52420,08	0,102	8,537
64	0	unequal	5	high	alt.	76231,43	70905,58	8258,69	6495,56	6,986	21,349
65	5000	unequal	5	high	alt.	106686,36	109538,94	34284,77	31495,56	-2,674	8,135
66	10000	unequal	5	high	alt.	125696,94	124828,05	57312,65	52420,08	0,691	8,537
67	0	ref.	10	high	alt.	90301,10	80464,28	11243,46	9431,59	10,893	16,115
68	5000	ref.	10	high	alt.	124601,67	122976,71	47481,34	44179,06	1,304	6,955
69	10000	ref.	10	high	alt.	128363,30	128536,30	79993,00	72313,00	-0,135	9,601
70	0	unequal	10	high	alt.	99462,66	92519,09	11243,46	9431,59	6,981	16,115
71	5000	unequal	10	high	alt.	143344,90	131761,96	47481,34	44179,06	8,080	6,955
72	10000	unequal	10	high	alt.	156174,32	148812,73	79993,00	72313,00	4,714	9,601